

## UNIT 2A

### TIME OUT TO THINK

Pg. 74. Answers will vary. This question should help the student think about the units associated with everyday numbers. Even if they pick a page number from the newspaper or magazine, note that this has units of “pages.” This would be a good topic for a discussion either during or outside of class.

Pg. 76. Using  $\frac{60 \text{ s}}{1 \text{ min}}$ , the solution would be

$$3000 \text{ s} \times \frac{60 \text{ s}}{1 \text{ min}} = 18,000 \text{ s}^2 / \text{min} . \quad \text{Since the}$$

question asked for minutes, you would know there was an error since the units of  $\text{s}^2/\text{min}$  are incorrect.

Pg. 78. Answers will vary, but this question should help the student make the connection between abstract ideas of measurement and the ancient origin of units as body measures. This would be a good topic for a discussion either during or outside of class.

Pg. 80. Literally, a megabuck is a million dollars. But it is used colloquially to mean “a lot of money.” Another example of the use of metric prefixes is “nanotechnology” for very small machines. This would be a good topic for a discussion either during or outside of class.

Pg. 84(1<sup>st</sup>). Even without carrying out the conversions, it should be obvious that this is a Fahrenheit temperature;  $59^\circ\text{C}$  is more than halfway between the freezing and boiling points of water, which means it is well over  $100^\circ\text{F}$  (a precise conversion shows it is  $138.2^\circ\text{F}$ ). No populated place on Earth gets this hot, so the forecast of  $59^\circ$  must refer to a Fahrenheit temperature.

Pg. 84(2<sup>nd</sup>).  $\frac{1}{1.624} = 0.6158$ ,  $\frac{1}{1.005} = 0.9950$ ,

$$\frac{1}{1.320} = 0.7576, \quad \frac{1}{0.0120} = 83.33, \quad \text{and}$$

$$\frac{1}{0.07855} = 12.73. \quad \text{The values in the two columns}$$

are reciprocals of one another (at least approximately). For example, the conversion factor used to convert from euros to U.S. dollars is  $\frac{\$1.320}{1 \text{ euro}}$  and the conversion factor to change from

U.S. dollars to euros is  $\frac{1 \text{ euro}}{\$1.320}$ . These numbers

are reciprocals.

### QUICK QUIZ

1. **a.** Think of the unit *miles per hour*; the unit of *mile* is divided by the unit of *hour*.
2. **b.** The area of a square is its length multiplied by its width (these are, of course, equal for squares), and thus a square of side length 2 mi has area of  $2 \text{ mi} \times 2 \text{ mi} = 4 \text{ mi}^2$ .
3. **c.** When multiplying quantities that have units, the units are also multiplied, so  $\text{ft}^2 \times \text{ft} = \text{ft}^3$ .
4. **b.**  $1 \text{ mi}^3 = (1760 \text{ yd})^3 = 1760^3 \text{ yd}^3$ .
5. **c.**  $1 \text{ ft}^2 = 12 \text{ in} \times 12 \text{ in} = 144 \text{ in}^2$ .
6. **a.** Divide both sides of  $1 \text{ L} = 1.057 \text{ qt}$  by 1 L.
7. **c.** The metric prefix *kilo* means 1000, so a kilometer is 1000 meters.
8. **c.** Water boils at  $100^\circ\text{C}$  (at sea level), so  $110^\circ\text{C}$  is boiling hot.
9. **a.** Apples are most likely to be sold by units of weight (or more accurately, mass), and thus euros per kilogram is the best answer.
10. **b.**  $\$1.32$  per euro means  $1 \text{ euro} = \$1.32$ , which is more than  $\$1$ .

### DOES IT MAKE SENSE?

7. Does not make sense. 35 miles is a distance, not a speed.
8. Does not make sense. Two  $\text{ft}^2$  describes an area, not a volume.
9. Makes sense. Liquids are measured in liters, and since one liter is about a quart, drinking two liters is a reasonable thing to do.
10. Does not make sense. First of all, we use the unit of kilogram to measure mass, not weight, though mass and weight are often used interchangeably in everyday conversation. Even so, a bicyclist with a mass of 300 kg would weigh more than 650 pounds (on the surface of the earth), which is an unheard of weight for a professional cyclist.
11. Makes sense. 10,000 meters is 10 kilometers, which is about 6.2 miles, a common length for foot races. Anyone who can run six back-to-back 9-minute miles has no trouble running 10,000 meters in less than an hour.
12. Does not make sense. The unit of meter measures length, not volume.

## BASIC SKILLS AND CONCEPTS

13. a.  $\frac{3}{4} \times \frac{1}{2} = \frac{3 \cdot 1}{4 \cdot 2} = \frac{3}{8}$   
 b.  $\frac{2}{3} \times \frac{3}{5} = \frac{2 \cdot 3}{3 \cdot 5} = \frac{2}{5}$   
 c.  $\frac{1}{2} + \frac{3}{2} = \frac{1+3}{2} = \frac{4}{2} = 2$   
 d.  $\frac{2}{3} + \frac{1}{6} = \frac{4}{6} + \frac{1}{6} = \frac{4+1}{6} = \frac{5}{6}$   
 e.  $\frac{2}{3} \times \frac{1}{4} = \frac{2 \cdot 1}{3 \cdot 4} = \frac{2}{12} = \frac{1}{6}$   
 f.  $\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{2+3}{8} = \frac{5}{8}$   
 g.  $\frac{5}{8} - \frac{1}{4} = \frac{5}{8} - \frac{2}{8} = \frac{5-2}{8} = \frac{3}{8}$   
 h.  $\frac{3}{2} \times \frac{2}{3} = \frac{3 \cdot 2}{2 \cdot 3} = 1$
14. a.  $\frac{1}{3} + \frac{1}{5} = \frac{5}{15} + \frac{3}{15} = \frac{8}{15}$   
 b.  $\frac{10}{3} \times \frac{3}{7} = \frac{10 \cdot 3}{3 \cdot 7} = \frac{10}{7} = 1\frac{3}{7}$   
 c.  $\frac{3}{4} - \frac{1}{8} = \frac{6}{8} - \frac{1}{8} = \frac{5}{8}$   
 d.  $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} = \frac{6}{12} + \frac{8}{12} + \frac{9}{12} = \frac{23}{12} = 1\frac{11}{12}$   
 e.  $\frac{6}{5} + \frac{4}{15} = \frac{18}{15} + \frac{4}{15} = \frac{22}{15} = 1\frac{7}{15}$   
 f.  $\frac{3}{5} \times \frac{2}{7} = \frac{3 \cdot 2}{5 \cdot 7} = \frac{6}{35}$   
 g.  $\frac{1}{3} + \frac{13}{6} = \frac{2}{6} + \frac{13}{6} = \frac{15}{6} = \frac{5}{2} = 2\frac{1}{2}$   
 h.  $\frac{3}{5} \times \frac{10}{3} \times \frac{3}{2} = \frac{3 \cdot 10 \cdot 3}{5 \cdot 3 \cdot 2} = \frac{3}{1} = 3$
15. Answers may vary depending on whether fractions are reduced.
- a.  $3.5 = \frac{35}{10} = \frac{7}{2}$       b.  $0.3 = \frac{3}{10}$   
 c.  $0.05 = \frac{5}{100} = \frac{1}{20}$       d.  $4.1 = \frac{41}{10}$   
 e.  $2.15 = \frac{215}{100} = \frac{43}{20}$       f.  $0.35 = \frac{35}{100} = \frac{7}{20}$   
 g.  $0.98 = \frac{98}{100} = \frac{49}{50}$       h.  $4.01 = \frac{401}{100}$

16. Answers may vary depending on whether fractions are reduced.

- a.  $2.75 = \frac{275}{100} = \frac{11}{4}$       b.  $0.45 = \frac{45}{100} = \frac{9}{20}$   
 c.  $0.005 = \frac{5}{1000} = \frac{1}{200}$       d.  $1.16 = \frac{116}{100} = \frac{29}{25}$   
 e.  $6.5 = \frac{65}{10} = \frac{13}{2}$       f.  $4.123 = \frac{4123}{1000}$   
 g.  $0.0003 = \frac{3}{10,000}$       h.  $0.034 = \frac{34}{1000} = \frac{17}{500}$
17. a.  $\frac{1}{4} = 0.25$       b.  $\frac{3}{8} = 0.375$   
 c.  $\frac{2}{3} \approx 0.667$       d.  $\frac{3}{5} = 0.6$   
 e.  $\frac{13}{2} = 6.5$       f.  $\frac{23}{6} \approx 3.833$   
 g.  $\frac{103}{50} = 2.06$       h.  $\frac{42}{26} \approx 1.615$
18. a.  $\frac{1}{5} = 0.2$       b.  $\frac{4}{9} \approx 0.444$   
 c.  $\frac{4}{11} \approx 0.364$       d.  $\frac{12}{7} \approx 1.714$   
 e.  $\frac{28}{9} \approx 3.111$       f.  $\frac{56}{11} \approx 5.091$   
 g.  $\frac{102}{49} \approx 2.082$       h.  $\frac{15}{4} = 3.75$
19. a.  $10^4 \times 10^7 = 10^{4+7} = 10^{11}$   
 b.  $10^5 \times 10^{-3} = 10^{5-3} = 10^2$   
 c.  $10^6 \div 10^2 = 10^{6-2} = 10^4$   
 d.  $\frac{10^8}{10^{-4}} = 10^{8-(-4)} = 10^{12}$   
 e.  $\frac{10^{12}}{10^{-4}} = 10^{12-(-4)} = 10^{16}$   
 f.  $10^{23} \times 10^{-23} = 10^{23-23} = 10^0 = 1$   
 g.  $10^4 + 10^2 = 10,000 + 100 = 10,100$   
 h.  $10^{15} \div 10^{-5} = 10^{15-(-5)} = 10^{20}$
20. a.  $10^{-2} \times 10^{-6} = 10^{-2+(-6)} = 10^{-8}$   
 b.  $\frac{10^{-6}}{10^{-8}} = 10^{-6-(-8)} = 10^2$   
 c.  $10^{12} \times 10^{23} = 10^{12+23} = 10^{35}$   
 d.  $\frac{10^{-4}}{10^5} = 10^{-4-5} = 10^{-9}$   
 e.  $\frac{10^{25}}{10^{15}} = 10^{25-15} = 10^{10}$

20. (continued)
- f.  $10^1 + 10^0 = 10 + 1 = 11$
- g.  $10^2 + 10^{-1} = 100 + 0.1 = 100.1$
- h.  $10^2 - 10^1 = 100 - 10 = 90$
21.  $3.5 \text{ lb} \times \frac{\$0.90}{1 \text{ lb}} = \$3.15$
22.  $23 \text{ baseballs} \times \frac{5.25 \text{ oz}}{1 \text{ baseball}} = 120.75 \text{ oz}$
23.  $6 \text{ months} \times \frac{\$3200}{1 \text{ month}} = \$19,200$
24.  $3000 \text{ people} \times \frac{1 \text{ building}}{150 \text{ people}} = 20 \text{ buildings}$
25. a. The area of the arena's floor is  $200 \text{ ft} \times 150 \text{ ft} = 30,000 \text{ ft}^2$ , and the volume of the arena is  $200 \text{ ft} \times 150 \text{ ft} \times 35 \text{ ft} = 1,050,000 \text{ ft}^3$ .
- b. The surface area of the pool is  $30 \text{ yd} \times 10 \text{ yd} = 300 \text{ yd}^2$ , and the volume of water it holds is  $30 \text{ yd} \times 10 \text{ yd} \times 0.3 \text{ yd} = 90 \text{ yd}^3$ .
- c. The area of the bed is  $25 \text{ ft} \times 8 \text{ ft} = 200 \text{ ft}^2$ , and the volume of soil it holds is  $25 \text{ ft} \times 8 \text{ ft} \times 1.5 \text{ ft} = 300 \text{ ft}^3$ .
26. a. The area of the warehouse floor is  $40 \text{ yd} \times 25 \text{ yd} = 1000 \text{ yd}^2$ , and the volume of the cartons is  $40 \text{ yd} \times 25 \text{ yd} \times 3 \text{ yd} = 3000 \text{ yd}^3$ .
- b. The area of the bed's floor is  $12 \text{ ft} \times 5 \text{ ft} = 60 \text{ ft}^2$ , and the volume of the bed is  $12 \text{ ft} \times 5 \text{ ft} \times 3.5 \text{ ft} = 210 \text{ ft}^3$ .
- c. The volume of the can is the area of its base multiplied by its height, which is  $6 \text{ in}^2 \times 4 \text{ in} = 24 \text{ in}^3$ .
27. Speed has units of miles per hour, or mi/hr.
28. The price of oranges has units of dollars per pound, or \$/lb.
29. The cost of carpet has units of dollars per square yard, or \$/yd<sup>2</sup>.
30. The flow rate has units of cubic feet per second, or cfs (or ft<sup>3</sup>/s).
31. The price of rice has units of yen per kilogram, or yen/kg.
32. The production rate has units of bagels per hour, or bagels/hr.
33. The daily consumption has units of gallons per person, or gal/person.
34. The density of rock has units of grams per cubic centimeter, or g/cm<sup>3</sup>.
35.  $24 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} = 288 \text{ in}$
36.  $24 \text{ ft} \times \frac{1 \text{ yd}}{3 \text{ ft}} = 8 \text{ yd}$
37.  $25 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}} = 1500 \text{ s}$
38.  $32 \text{ year} \times \frac{365 \text{ day}}{1 \text{ year}} = 11,680 \text{ days}$
39.  $2.5 \text{ hr} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{60 \text{ s}}{1 \text{ min}} = 9000 \text{ s}$
40.  $17,200 \frac{\text{mi}}{\text{hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} \approx 4.78 \frac{\text{mi}}{\text{s}}$
41.  $3 \text{ tr} \times \frac{365 \text{ day}}{1 \text{ tr}} \times \frac{24 \text{ hr}}{1 \text{ day}} = 26,280 \text{ hr}$
42.  $26,500 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ yd}}{3 \text{ ft}} \times \frac{1 \text{ mi}}{1760 \text{ yd}} \approx 0.42 \text{ mi}$
43. Note that  $1 \text{ ft} = 12 \text{ in}$ , and thus  $(1 \text{ ft})^2 = (12 \text{ in})^2$ , which means  $1 \text{ ft}^2 = 144 \text{ in}^2$ . This can also be written as  $\frac{1 \text{ ft}^2}{144 \text{ in}^2} = 1$ , or  $\frac{144 \text{ in}^2}{1 \text{ ft}^2} = 1$ .
44. Since  $1 \text{ m} = 100 \text{ cm}$ , we have  $(1 \text{ m})^3 = (100 \text{ cm})^3$ , which means  $1 \text{ m}^3 = 1,000,000 \text{ cm}^3$ . This can also be written as  $\frac{1 \text{ m}^3}{1,000,000 \text{ cm}^3} = 1$ , or  $\frac{1,000,000 \text{ cm}^3}{1 \text{ m}^3} = 1$ .
45. The volume of the sidewalk is  $4 \text{ ft} \times 200 \text{ ft} \times 0.5 \text{ ft} = 400 \text{ ft}^3$ . Since  $1 \text{ yd} = 3 \text{ ft}$ , we know  $1 \text{ yd}^3 = 27 \text{ ft}^3$ , and this can be used to convert to cubic yards.
- $$400 \text{ ft}^3 \times \frac{1 \text{ yd}^3}{27 \text{ ft}^3} = 14.8 \text{ yd}^3$$

46. The area of the yard is
- $20 \text{ yd} \times 12 \text{ yd} = 240 \text{ yd}^2$
- .

Since  $1 \text{ yd} = 3 \text{ ft}$ , we know  $1 \text{ yd}^2 = 9 \text{ ft}^2$ , and this can be used to convert to square feet.

$$240 \text{ yd}^2 \times \frac{9 \text{ ft}^2}{1 \text{ yd}^2} = 2160 \text{ ft}^2$$

47. Use the fact that
- $1 \text{ yd}^3 = 27 \text{ ft}^3$
- (see Exercise 45).

$$320 \text{ ft}^3 \times \frac{1 \text{ yd}^3}{27 \text{ ft}^3} = 11.9 \text{ yd}^3$$

48. Since
- $1 \text{ ft} = 12 \text{ in}$
- ,
- $(1 \text{ ft})^3 = (12 \text{ in})^3$
- , which means

$1 \text{ ft}^3 = 1728 \text{ in}^3$ . Therefore,

$$4 \text{ ft}^3 \times \frac{1728 \text{ in}^3}{1 \text{ ft}^3} = 6912 \text{ in}^3$$

49. a.
- $10 \text{ furlongs} \times \frac{1 \text{ mi}}{8 \text{ furlongs}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ yd}}{3 \text{ ft}}$
- 
- $\times \frac{1 \text{ rod}}{5.5 \text{ yd}} = 400 \text{ rods}$

$$\text{b. } 10 \text{ furlongs} \times \frac{1 \text{ mi}}{8 \text{ furlongs}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ fathom}}{6 \text{ ft}} = 1100 \text{ fathoms}$$

50. a.
- $36,198 \text{ ft} \times \frac{1 \text{ fathom}}{6 \text{ ft}} = 6033 \text{ fathoms}$

$$\text{b. } 36,198 \text{ ft} \times \frac{1 \text{ naut. mi}}{6076.1 \text{ ft}} \times \frac{1 \text{ marine league}}{3 \text{ naut. mi}} \approx 1.99 \text{ marine leagues}$$

51. Convert a cubic foot of water into pounds.

$$1 \text{ ft}^3 (\text{of water}) \times \frac{7.48 \text{ gal}}{1 \text{ ft}^3} \times \frac{8.33 \text{ lb}}{1 \text{ gal}} = 62.3 \text{ lb}$$

Now convert that answer into ounces.

$$62.3 \text{ lb} \times \frac{16 \text{ oz}}{1 \text{ lb}} = 997 \text{ oz (av)}$$

52. Convert six-packs to gallons.

$$10 \text{ six-packs} \times \frac{6 \text{ cans}}{1 \text{ six-pack}} \times \frac{12 \text{ oz}}{1 \text{ can}} \times \frac{1 \text{ pint}}{16 \text{ oz}} \times \frac{1 \text{ qt}}{2 \text{ pint}} \times \frac{1 \text{ gal}}{4 \text{ qt}} = 5.625 \text{ gal}$$

53. Use the fact that 1 nautical mile = 6076.1 feet to convert knots into mph.

$$46 \text{ knots} = \frac{46 \text{ naut. mi}}{\text{hr}} \times \frac{6076.1 \text{ ft}}{1 \text{ naut. mi}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} = 52.94 \frac{\text{mi}}{\text{hr}}$$

54. The volume of the room is
- $4 \text{ yd} \times 4 \text{ yd} \times 2 \text{ yd} = 32 \text{ yd}^3$
- .

Convert cubic yards into cords.

$$32 \text{ yd}^3 \times \left( \frac{3 \text{ ft}}{1 \text{ yd}} \right)^3 \times \frac{1 \text{ cord}}{128 \text{ ft}^3} = 6.75 \text{ cords}$$

55. You can tell the factor by which the first unit is larger than the second by dividing the second into the first. Rewrite metric prefixes as powers of 10, and then simplify, as shown below.

$$\frac{1 \text{ m}}{1 \text{ mm}} = \frac{10^0 \text{ m}}{10^{-3} \text{ m}} = \frac{10^0}{10^{-3}} = 10^{0-(-3)} = 10^3. \text{ This means a meter is 1000 times as large as a millimeter.}$$

56. See Exercise 55.

$$\frac{1 \text{ kg}}{1 \text{ mg}} = \frac{10^3 \text{ g}}{10^{-3} \text{ g}} = 10^{3-(-3)} = 10^6, \text{ so a kilogram is 1,000,000 times as large as a milligram.}$$

57. See Exercise 55.

$$\frac{1 \text{ L}}{1 \text{ mL}} = \frac{1 \text{ L}}{10^{-3} \text{ L}} = \frac{1}{10^{-3}} = 10^3 = 1000, \text{ so a liter is 1000 times larger than a milliliter.}$$

58. See Exercise 55.

$$\frac{1 \text{ km}}{1 \mu\text{m}} = \frac{10^3 \text{ m}}{10^{-6} \text{ m}} = 10^{3-(-6)} = 10^9, \text{ so a kilometer is 1,000,000,000 times as large as a micrometer.}$$

59. See Exercise 55.

$$\frac{1 \text{ m}^2}{1 \text{ cm}^2} = \frac{1 \text{ m}^2}{(10^{-2} \text{ m})^2} = \frac{1 \text{ m}^2}{10^{-4} \text{ m}^2} = \frac{1}{10^{-4}} = 10^4, \text{ so a square meter is 10,000 times as large as a square centimeter.}$$

60. See Exercise 55.

$$\frac{1 \text{ m}^3}{1 \text{ mm}^3} = \frac{1 \text{ m}^3}{(10^{-3} \text{ m})^3} = \frac{1 \text{ m}^3}{10^{-9} \text{ m}^3} = 10^9, \text{ so a cubic meter is 1,000,000,000 times as large as a cubic millimeter.}$$

- 61.
- $22 \text{ lb} \times \frac{2.205 \text{ lb}}{1 \text{ kg}} = 48.51 \text{ lb}$
- . (Note: you'll get an answer of 48.50 lb if you use the conversion
- $0.4536 \text{ kg} = 1 \text{ lb}$
- ).

$$62. 160 \text{ cm} \times \frac{1 \text{ in}}{2.54 \text{ cm}} = 62.99 \text{ in}$$

$$63. 16 \text{ qt} \times \frac{1 \text{ L}}{1.057 \text{ qt}} = 15.14 \text{ L}$$

64. Square both sides of the conversion  $1 \text{ km} = 0.6214 \text{ mi}$  to find the conversion between square miles and square kilometers.

$$(1 \text{ km})^2 = (0.6214 \text{ mi})^2 \Rightarrow 1 \text{ km}^2 = 0.38614 \text{ mi}^2.$$

Now use this to complete the problem.

$$2 \text{ km}^2 \times \frac{0.38614 \text{ mi}^2}{1 \text{ km}^2} = 0.77 \text{ mi}^2.$$

65.  $\frac{55 \text{ mi}}{\text{hr}} \times \frac{1.6093 \text{ km}}{1 \text{ mi}} = 88.51 \frac{\text{km}}{\text{hr}}$

66.  $23 \frac{\text{m}}{\text{s}} \times \frac{1 \text{ km}}{1000 \text{ m}} \times \frac{1 \text{ mi}}{1.6093 \text{ km}} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \approx 51.45 \frac{\text{mi}}{\text{hr}}$

67. Cube both sides of the conversion  $2.54 \text{ cm} = 1 \text{ in}$  to find the conversion between cubic centimeters and cubic inches.

$$(2.54 \text{ cm})^3 = (1 \text{ in})^3 \Rightarrow 16.387 \text{ cm}^3 = 1 \text{ in}^3$$

Now use this to complete the problem.

$$300 \text{ in}^3 \times \frac{16.387 \text{ cm}^3}{1 \text{ in}^3} = 4916.12 \text{ cm}^3$$

68.  $\frac{18 \text{ g}}{1 \text{ cm}^3} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \frac{2.205 \text{ lb}}{1 \text{ kg}} \times \left( \frac{2.54 \text{ cm}}{1 \text{ in}} \right)^3 = 0.650 \frac{\text{lb}}{\text{in}^3}$

69. a. Use  $C = \frac{F - 32}{1.8}$ .  $C = \frac{45 - 32}{1.8} = 7.2^\circ \text{C}$

b. Use  $F = 1.8C + 32$ .  $F = 1.8(20) + 32 = 68^\circ \text{F}$ .

c.  $F = 1.8(-15) + 32 = 5^\circ \text{F}$

d.  $F = 1.8(-30) + 32 = -22^\circ \text{F}$

e.  $C = \frac{70 - 32}{1.8} = 21.1^\circ \text{C}$

70. a. Use  $F = 1.8C + 32$ .  $F = 1.8(-8) + 32 = 17.6^\circ \text{F}$

b. Use  $C = \frac{F - 32}{1.8}$ .  $C = \frac{15 - 32}{1.8} = -9.4^\circ \text{C}$

c.  $F = 1.8(15) + 32 = 59^\circ \text{F}$

d.  $C = \frac{75 - 32}{1.8} = 23.9^\circ \text{C}$

e.  $C = \frac{20 - 32}{1.8} = -6.7^\circ \text{C}$

71. a. Use  $C = K - 273.15$ .  $C = 50 - 273.15 = -223.15^\circ \text{C}$

b.  $C = 240 - 273.15 = -33.15^\circ \text{C}$ .

c. Use  $K = C + 273.15$ .  $K = 10 + 273.15 = 283.15 \text{ K}$

72. a. Use  $K = C + 273.15$ .  $K = -40 + 273.15 = 233.15 \text{ K}$

b. Use  $C = K - 273.15$ .  $C = 400 - 273.15 = 126.85^\circ \text{C}$

c.  $K = 125 + 273.15 = 398.15 \text{ K}$

73.  $60 \text{ pounds} \times \frac{\$1.624}{1 \text{ pound}} = \$97.44$  or

$$60 \text{ Pounds} \times \frac{\$1}{0.6158 \text{ Pound}} = \$97.43$$

74.  $31,000 \text{ Yen} \times \frac{\$0.012}{1 \text{ Yen}} = \$372.00$  or

$$31,000 \text{ Yen} \times \frac{\$1}{83.33 \text{ Yen}} = \$372.01$$

75.  $450 \text{ euros} \times \frac{\$1.320}{1 \text{ euro}} = \$594.00$  or

$$450 \text{ euros} \times \frac{\$1}{0.7576 \text{ euro}} = \$593.98$$

76.  $3000 \text{ pesos} \times \frac{\$0.07855}{1 \text{ peso}} \approx \$235.65$  or

$$3000 \text{ pesos} \times \frac{\$1}{12.73 \text{ peso}} = \$235.66$$

77.  $\frac{1.5 \text{ euro}}{1 \text{ L}} \times \frac{3.785 \text{ L}}{1 \text{ gal}} \times \frac{\$1.320}{1 \text{ euro}} = \frac{\$7.49}{\text{gal}}$

78.  $\frac{28 \text{ pesos}}{1 \text{ kg}} \times \frac{\$1}{12.73 \text{ pesos}} \times \frac{1 \text{ kg}}{2.205 \text{ lb}} = \frac{\$1.00}{\text{lb}}$

## FURTHER APPLICATIONS

79. a. Convert \$27,800,000 per 80 games into dollars per game.

$$\frac{\$27,800,000}{80 \text{ games}} = \$347,500/\text{game}$$

- b. Convert \$347,500 per game into dollars per minute.

$$\frac{\$347,500}{1 \text{ game}} \times \frac{1 \text{ game}}{48 \text{ min}} \approx \$7240/\text{min}$$

79. (continued)

c. His total hours spent training were  $\frac{40 \text{ hrs}}{1 \text{ game}} \times 80 \text{ games} = 3200 \text{ hours}$ . His total time

playing was  $\frac{48 \text{ min}}{1 \text{ game}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times 80 \text{ games} = 64 \text{ hr}$ .

The total time related to basketball was  $3200 \text{ hr} + 64 \text{ hr} = 3264 \text{ hr}$ . Convert \$27,800,000 per 3264 hours into dollars per hour.

$$\frac{\$27,800,000}{3264 \text{ hr}} = \$8517/\text{hr}$$

80. Convert 6 breaths per minute into liters of warmed air per day using 1 minute = 6 breaths, and 1 breath = 0.5 L.

$$\frac{6 \text{ breaths}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{0.5 \text{ L}}{1 \text{ breath}} \times \frac{24 \text{ hr}}{1 \text{ day}} = 4320 \frac{\text{L}}{\text{day}}$$

81. a. Convert 16 gigabytes into pages, using the fact that 2000 bytes are need to store 1 page of text.

$$16 \text{ Gbyte} \times \frac{1 \text{ billion bytes}}{1 \text{ Gbyte}} \times \frac{1 \text{ page}}{2000 \text{ bytes}} = 8,000,000 \text{ pages}$$

$$\text{b. } 8,000,000 \text{ pages} \times \frac{1 \text{ book}}{500 \text{ pages}} = 16,000 \text{ books}$$

82. Answers will vary depending on prices. The area of the yard is  $60 \text{ ft} \times 35 \text{ ft} = 2100 \text{ ft}^2$ .

a. The cost to plant the region with grass seed would be  $\frac{\text{Cost of grass seed}}{1 \text{ ft}^2} \times 2100 \text{ ft}^2$ .

b. The cost to cover the region with sod would be  $\frac{\text{Cost of sod}}{1 \text{ ft}^2} \times 2100 \text{ ft}^2$ .

c. The cost to cover the region with topsoil and plant bulbs would be  $\frac{\text{Cost of topsoil}}{\text{ft}^2} \times 2100 \text{ ft}^2 +$

$$\frac{\text{Cost}}{1 \text{ bulb}} \times \frac{2 \text{ bulbs}}{1 \text{ ft}^2} \times 2100 \text{ ft}^2.$$

$$83. 300 \text{ km} \times \frac{1 \text{ L}}{26 \text{ km}} \times \frac{1.50 \text{ euro}}{1 \text{ L}} \times \frac{\$1.320}{1 \text{ euro}} = \$22.85$$

$$84. \frac{1}{2} \text{ gal} \times \frac{3.785 \text{ L}}{1 \text{ gal}} \times \frac{100 \text{ pesos}}{0.8 \text{ L}} \times \frac{\$1}{12.73 \text{ pesos}} = \$18.58$$

$$85. \frac{16 \text{ pounds}}{1 \text{ m}^2} \times \left( \frac{1 \text{ m}}{1.094 \text{ yd}} \right)^2 \times \frac{\$1}{0.6158 \text{ pound}} = \frac{\$21.71}{\text{yd}^2}$$

86. Monte Carlo:

$$\frac{1150 \text{ euro}}{80 \text{ m}^2} \times \left( \frac{1 \text{ m}}{3.28 \text{ ft}} \right)^2 \times \frac{\$1.320}{1 \text{ euro}} = \$1.76/\text{ft}^2$$

Santa Fe:

$$\frac{\$800}{500 \text{ ft}^2} = \$1.60/\text{ft}^2$$

So Santa Fe is less expensive.

87. The Cullinan diamond weighs 3106 carats, and:

$$3106 \text{ carats} \times \frac{0.2 \text{ g}}{1 \text{ carat}} \times \frac{1000 \text{ mg}}{1 \text{ g}} = 621,200 \text{ mg}$$

$$3106 \text{ carats} \times \frac{0.2 \text{ g}}{1 \text{ carat}} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \frac{2.205 \text{ lb}}{1 \text{ kg}} = 1.37 \text{ lb}$$

88. The Star of Africa weighs 530.2 carats, which is 106,040 mg, and 0.23 lb (the conversions being identical to those shown in Exercise 87).

89. The Hope diamond weighs 45.52 carats, so:

$$45.52 \text{ carats} \times \frac{0.2 \text{ g}}{1 \text{ carat}} = 9.1 \text{ g and}$$

$$9.1 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \frac{2.205 \text{ lb}}{1 \text{ kg}} \times \frac{16 \text{ oz}}{1 \text{ lb}} = 0.32 \text{ oz.}$$

90. 14-karat gold is 58% of pure gold since  $\frac{14}{24} = 0.58$ .

91. 2.2 ounces of 16-karat gold is  $2.2 \text{ oz} \times \frac{16}{24} = 1.47 \text{ oz}$  of pure gold.

92. A 0.15 ounce diamond will weigh  $0.15 \text{ oz} \times \frac{437.5 \text{ grain}}{1 \text{ oz}} \times \frac{0.0648 \text{ g}}{1 \text{ grain}} \times \frac{1 \text{ carat}}{0.2 \text{ g}} = 21.26 \text{ carat}$ .

$$93. \text{ a. } 1 \text{ centiare} \times \frac{1 \text{ are}}{100 \text{ centiare}} \times \frac{100 \text{ m}^2}{1 \text{ are}} = 1 \text{ m}^2.$$

$$\text{b. } 1 \text{ km}^2 \times \left( \frac{1000 \text{ m}}{1 \text{ km}} \right)^2 \times \frac{1 \text{ are}}{100 \text{ m}^2} \times \frac{1 \text{ ha}}{100 \text{ are}} = 100 \text{ ha.}$$

93. (continued)

c. From the “By the Way” (Page 74), 1 acre = 43,560 ft<sup>2</sup>, so

$$1 \text{ ha} \times \frac{100 \text{ are}}{1 \text{ ha}} \times \frac{100 \text{ m}^2}{1 \text{ are}} \times \left( \frac{3.28 \text{ ft}}{100 \text{ m}} \right)^2 \\ \times \frac{1 \text{ acre}}{43,560 \text{ ft}^2} \approx 2.47 \text{ acres.}$$

d.  $\frac{10,000 \text{ euro}}{1 \text{ ha}} \times \frac{\$1.320}{1 \text{ euro}} \times \frac{1 \text{ ha}}{2.47 \text{ acre}}$   
 $\approx \$5344/\text{acre}$ , so \$5000/acre is the better option.

94. a.  $0.5 \text{ L} \times \frac{20 \text{ gtt}}{1 \text{ mL}} \times \frac{1000 \text{ mL}}{1 \text{ L}} = 10,000 \text{ gtt}$

b.  $1 \text{ L} \times \frac{60 \text{ gtt}}{1 \text{ mL}} \times \frac{1000 \text{ mL}}{1 \text{ L}} = 60,000 \text{ gtt}$

c.  $1 \text{ L} \times \frac{15 \text{ gtt}}{1 \text{ mL}} \times \frac{1000 \text{ mL}}{1 \text{ L}} = 15,000 \text{ gtt}$ ; The rate  
of infusion is  $\frac{15,000 \text{ gtt}}{5 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} = 50 \text{ gtt/min.}$

95.  $40 \text{ lb} \times \frac{1 \text{ kg}}{2.205 \text{ lb}} \approx 18 \text{ kg}$ , so the child should drink

$$1000 \text{ mL} + (18 - 10) \text{ kg} \times \frac{50 \text{ mL}}{1 \text{ kg}} = 1400 \text{ mL of}$$

fluid, which is  $1400 \text{ mL} \times \frac{1 \text{ oz}}{29.547 \text{ mL}} \times \frac{1 \text{ glass}}{8 \text{ oz}}$   
 $\approx 5.9 \text{ glasses or } 6 \text{ glasses per day.}$

96. The child is receiving  $\frac{200 \text{ mg}}{8 \text{ hr}} \times \frac{24 \text{ hr}}{1 \text{ day}}$   
 $= 600 \text{ mg/day}$ , which is  $\frac{600 \text{ mg/day}}{20 \text{ lb}} \times \frac{2.205 \text{ lb}}{1 \text{ kg}}$   
 $\approx 66 \text{ mg/kg/day}$ . This is not within the guidelines.

97. Answers will vary.

**TECHNOLOGY EXERCISES**

103. Answers will vary.

104. a. 100 furlongs = 20.1168 kilometers

b.  $23 \text{ (inches per day)} = 1.51252104 \times 10^{-5} \text{ miles per hour}$

c.  $100 \text{ drams} = 6.25 \text{ ounces}$

d.  $1 \text{ hectare} = 2.47105 \text{ acres}$

e.  $100 \text{ pascals} = 0.145037738 \text{ pounds per square inch}$

f.  $1 \text{ hectoliter} = 26.4172052 \text{ US gallons}$

**UNIT 2B****TIME OUT TO THINK**

Pg. 93. Answers will vary. Possible topics include problems involving density, rates of change, or area and volume of objects with differing units of measure. This would be a good topic for a discussion either during or outside of class.

Pg. 95(1<sup>st</sup>). Answers will vary. Note: many utility bills do not give the price per kilowatt-hour explicitly, but it can be calculated by dividing the total price charged for electricity by the metered usage in kilowatt-hours, both of which are almost always shown clearly.

Pg. 95(2<sup>nd</sup>). You have a greater total volume but essentially the same mass (except for the weight of air) when your lungs are filled with air, so your density is lower and you float better.

Pg. 98. Two beers contain enough alcohol to put you at 12 times the legal limit if all of it were in your bloodstream at once. Given the metabolic absorption rate, you should wait at least 3 hours before driving.

**QUICK QUIZ**

1. **b.** Speed is described by distance per unit of time, so dividing a distance by a time is the correct choice.

2. **b.** Intuitively, it makes sense to divide a volume (with units of, say, ft<sup>3</sup>) by a depth (with units of ft) to arrive at surface area (which has units of ft<sup>2</sup>).

3. **a.** Take dollars per gallon and divide it by miles per gallon, and you get

$$\frac{\$}{\text{gal}} \div \frac{\text{mi}}{\text{gal}} = \frac{\$}{\text{gal}} \times \frac{\text{gal}}{\text{mi}} = \frac{\$}{\text{mi}}.$$

4. **b.** A watt is defined to be one joule per second, which is a unit of power, not energy.
5. **c.** Energy used by an appliance is computed by multiplying the power rating by the amount of time the appliance is used, so you need to know how long the light bulb is on in order to compute the energy it uses in that time span.
6. **a.** Multiplying the population density, which has units of  $\frac{\text{people}}{\text{mi}^2}$ , by the area, which has units of  $\text{mi}^2$ , will result in an answer with units of people.
7. **b.** Concentrations of gases are often stated in parts per million (and the other two answers make no sense).
8. **c.** Multiplying by 30 kg will give the daily dose, which should be divided by three to determine the dose for one eight hour period.
9. **b.**  $4 \text{ L} \times \frac{0.08 \text{ gm}}{100 \text{ mL}} \times \frac{1000 \text{ mL}}{1 \text{ L}} = 0.08 \text{ gm} \times 40, = 3.2 \text{ gm}$
10. **c.**  $100 \text{ mi}^2 \times \frac{25 \text{ people}}{1 \text{ mi}^2} = 2500 \text{ people}$  and  $100 \text{ km}^2 \times \frac{25 \text{ people}}{1 \text{ km}^2} = 2500 \text{ people}$

### DOES IT MAKE SENSE?

5. Does not make sense. Using the familiar formula  $\text{distance} = \text{rate} \times \text{time}$ , one can see that it would be necessary to divide the distance by the rate (or speed) in order to compute the time, not the other way around.
6. Makes sense. Using the familiar formula  $\text{distance} = \text{rate} \times \text{time}$ , one can see that dividing the distance by the rate (or speed) will result in time.
7. Makes sense. There are 4184 joules in a Calorie, and when 10,000,000 joules is converted into Calories, the result is about 2400 Calories, which is a typical caloric intake for an active person.
8. Does not make sense. Utility companies bill you for energy use (typically in units of kW-hr), not power ratings (recall that the watt is a unit of power).
9. Does not make sense. The volume of a sphere is  $V = \frac{4}{3}\pi r^3$ , so the volume of a beach ball with a radius of 20 cm (about 8 inches) would be more than  $32,000 \text{ cm}^3$ . This translates into a mass of 320,000 grams if the density is 10 grams per  $\text{cm}^3$ , which is 320 kg. The mass of a beach ball isn't anywhere near that large.
10. Does not make sense.  $\frac{15 \text{ people}}{1 \text{ km}^2} \times \left(\frac{1 \text{ km}}{1000 \text{ m}}\right)^2 = 0.000015 \text{ people per square meter}$ , which is very small.
11. Makes sense. If the dose is based on weight, doubling the dose makes sense if you double the weight of the subject.
12. Makes sense. It is very likely that a person would be above the legal limit after two glasses of wine.

### BASIC SKILLS AND CONCEPTS

13. The car is traveling with speed  $\frac{45 \text{ mi}}{5 \text{ min}}$ , and this needs to be converted to mi/hr.  

$$\frac{45 \text{ mi}}{5 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}} = 540 \frac{\text{mi}}{\text{hr}}$$
14. Convert 1.2 cubic yards to dollars by using the conversion 1 cubic yard = \$24.  

$$1.2 \text{ yd}^3 \times \frac{\$24}{\text{yd}^3} = \$28.80$$
15. Convert 300 gallons to hours by using the conversion 3.2 gallons = 1 minute.  

$$300 \text{ gal} \times \frac{1 \text{ min}}{3.2 \text{ gal}} \times \frac{1 \text{ hr}}{60 \text{ min}} = 1.56 \text{ hr}$$
16. The skydiver is traveling with speed 614 mi/hr, and this needs to be converted to ft/s.  

$$\frac{614 \text{ mi}}{1 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} = 901 \frac{\text{ft}}{\text{s}}$$
17. Convert 2.5 ounces to dollars by using the conversion 1 oz = \$920.  

$$2.5 \text{ oz} \times \frac{\$920}{\text{oz}} = \$2300$$
18. You would earn 24 days  $\times \frac{8 \text{ hr}}{1 \text{ day}} \times \frac{\$8.50}{\text{hr}} = \$1632$  in that month.
19. First, note that 305 million people is 3050 groups of 100,000 people each. The mortality rate is  $\frac{565,650 \text{ deaths}}{305,000,000 \text{ people}} \times \frac{305,000,000 \text{ people}}{3050 \text{ groups of } 100,000} = 185 \text{ deaths/100,000 people}$ .



20. First, note that 305 million people is 3050 groups of 100,000 people each. The mortality rate is

$$\frac{310,000 \text{ deaths}}{305,000,000 \text{ people}} \times \frac{305,000,000 \text{ people}}{3050 \text{ groups of } 100,000} = 102 \text{ deaths/100,000 people.}$$

21. Convert 3 million births per year into births per minute.

$$\frac{3,000,000 \text{ births}}{\text{yr}} \times \frac{1 \text{ yr}}{365 \text{ d}} \times \frac{1 \text{ d}}{24 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} = 5.7 \frac{\text{births}}{\text{min}}$$

22. Divide the total miles travelled by the number of gallons to find the miles per gallon.

$$\frac{420 \text{ mi}}{12 \text{ gal}} = 35 \frac{\text{mi}}{\text{gal}}$$

23. The cost to drive 250 miles is

$$250 \text{ mi} \times \frac{1 \text{ gal}}{28 \text{ mi}} \times \frac{\$2.90}{1 \text{ gal}} = \$25.89$$

24. The salary in dollars per game is:

$$\frac{\$1,875,000}{160 \text{ games}} = \$11,719/\text{game}$$

25. Convert one year into hours of sleep.

$$1 \text{ yr} \times \frac{365 \text{ d}}{\text{yr}} \times \frac{8 \text{ hr (of sleep)}}{\text{d}} = 2920 \text{ hr}$$

26. Convert one lifetime into heart beats.

$$1 \text{ lifetime} \times \frac{80 \text{ yr}}{\text{lifetime}} \times \frac{365 \text{ d}}{\text{yr}} \times \frac{24 \text{ hr}}{1 \text{ d}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{70 \text{ beats}}{1 \text{ min}} = 2,943,360,000 \text{ beats}$$

27. The solution is wrong. It's helpful to include units on your numerical values. This would have signaled your error, as witness:

$$(\text{incorrect}) 0.11 \text{ lb} \div \frac{\$7.70}{\text{lb}} = \frac{0.11 \text{ lb}^2}{\$7.70} = 0.014 \frac{\text{lb}^2}{\$}$$

Notice that your solution, when the correct units are added, produces units of square pounds per dollar, which isn't very helpful, and doesn't answer the question. The correct solution is

$$0.11 \text{ lb} \times \frac{\$7.70}{\text{lb}} = \$0.85$$

28. The solution is wrong. Attach units to your numerical quantities so that you can determine whether the answer has correct units. When it

doesn't, this is an indication there's an error. Here's your incorrect solution with units attached:

$$\frac{5 \text{ mi}}{\text{hr}} \div 3 \text{ hr} = \frac{5 \text{ mi}}{\text{hr}} \times \frac{1}{3 \text{ hr}} = 1.7 \frac{\text{mi}}{\text{hr}^2}.$$

It's clear this is incorrect as the question asked how far did you go, and thus you should expect an answer with units of miles. The correct solution is

$$3 \text{ hr} \times \frac{5 \text{ mi}}{\text{hr}} = 15 \text{ mi}.$$

29. The solution is wrong. Units should be included with all quantities. When dividing by \$11, the unit of dollars goes with the 11 into the denominator (as shown below). Also, while it's reasonable to round an answer to the nearest tenth, it's more useful to round to the nearest hundredth in a problem like this, as you'll be comparing the price per pound of the large bag to the price per pound of the small bag, which is 39¢ per pound. Here's your solution with all units attached, division treated as it should be, and rounded to the hundredth-place:

$$50 \text{ lb} \div \$11 = \frac{50 \text{ lb}}{\$11} = 4.55 \frac{\text{lb}}{\$}.$$

This actually produces some useful information, as you can see a dollar buys 4.55 pounds of potatoes. If you're good with numbers, you can already see this is a better buy than 39¢ per pound, which is roughly 3 pounds for a dollar. But it's better to find the price per pound for the large bag:

$$\$11 \div 50 \text{ lb} = \frac{\$11}{50 \text{ lb}} = 0.22 \frac{\$}{\text{lb}} = 22¢ \text{ per pound}.$$

Now you can compare it to the price per pound of the small bag (39¢/lb) and tell which is the better buy.

30. The solution is wrong. Always include units with numerical quantities; this helps in deciding whether the correct procedure was followed. Here's your solution with units attached:

$$\frac{1500 \text{ Cal}}{\text{d}} \times \frac{140 \text{ Cal}}{\text{Coke}} = 210,000 \frac{\text{Cal}^2}{\text{d-Coke}}.$$

It's clear this calculation won't answer the question (note that as shown, there's nothing mathematically incorrect about the above calculation – it just doesn't provide useful information). The correct solution looks like this:

$$\frac{1500 \text{ Cal}}{\text{d}} \times \frac{1 \text{ Coke}}{140 \text{ Cal}} = 10.7 \frac{\text{Cokes}}{\text{d}}.$$

Thus you would need about 11 Cokes per day to meet your caloric needs.

$$\frac{\$3.99}{6 \text{ oz}} = \frac{\$0.665}{1 \text{ oz}} \quad \frac{\$9.49}{14 \text{ oz}} \approx \frac{\$0.678}{1 \text{ oz}}$$

The 6-ounce bottle is the better deal.

$$\frac{\$2.30}{12 \text{ eggs}} \approx \frac{\$0.19}{1 \text{ egg}} \quad \frac{\$5.50}{30 \text{ eggs}} \approx \frac{\$0.18}{1 \text{ egg}}$$

30 eggs is the better deal.

$$33. \text{ The 15-gallon tank will cost } \frac{\$55.20}{15 \text{ gal}} = \frac{\$3.68}{1 \text{ gal}} \text{ so}$$

the \$3.60/gal option is the better deal.

34. \$32/yd<sup>2</sup> per month:

$$\frac{\$32/\text{yd}^2 \times \left(\frac{1 \text{ yd}}{3 \text{ ft}}\right)^2}{30 \text{ days}} \approx \$0.12/\text{ft}^2/\text{day}$$

\$2/ft<sup>2</sup> per week:

$$\frac{\$2/\text{ft}^2}{7 \text{ days}} \approx \$0.29/\text{ft}^2/\text{day}$$

The \$32/yd<sup>2</sup> per month option is the better deal.

35. Convert 2000 miles into gallons by using the conversion 32 miles = 1 gallon.

$$2000 \text{ mi} \times \frac{1 \text{ gal}}{32 \text{ mi}} = 62.5 \text{ gal}$$

Yes; a car that has half the gas mileage would need twice as much gas. Halving the value of the denominator has the same effect as doubling the value of the fraction.

36. a. One full tank of gas for Car A costs  $12 \text{ gal} \times \frac{\$3.90}{1 \text{ gal}} = \$46.80$  and one full tank of

$$\text{gas for Car B costs } 20 \text{ gal} \times \frac{\$3.90}{1 \text{ gal}} = \$78.00.$$

$$\text{b. Car A will use } 3000 \text{ mi} \times \frac{1 \text{ gal}}{40 \text{ mi}} \times \frac{1 \text{ tank}}{12 \text{ gal}} = 6.25$$

tanks of gas and Car B will use

$$3000 \text{ mi} \times \frac{1 \text{ gal}}{30 \text{ mi}} \times \frac{1 \text{ tank}}{20 \text{ gal}} = 5 \text{ tanks of gas.}$$

$$\text{c. The driver of Car A will spend } 6.25 \text{ tanks} \times \frac{\$46.80}{1 \text{ tank}} = \$292.50 \text{ and the driver of}$$

$$\text{Car B will spend } 5 \text{ tanks} \times \frac{\$78.00}{1 \text{ tank}} = \$390.00.$$

$$37. \text{ a. The driving time when traveling at 55 miles per}$$

hour is  $2000 \text{ mi} \times \frac{1 \text{ hr}}{55 \text{ mi}} = 36.36 \text{ hr}$ , while the

time at 70 miles per hour is

$$2000 \text{ mi} \times \frac{1 \text{ hr}}{70 \text{ mi}} = 28.57 \text{ hr}.$$

b. Your car gets 38 miles to the gallon when driving at 55 mph, so the cost is

$$2000 \text{ mi} \times \frac{1 \text{ gal}}{38 \text{ mi}} \times \frac{\$3.90}{\text{gal}} = \$205.26.$$

Your car gets 32 miles to the gallon when driving at 70 mph, so the cost is

$$2000 \text{ mi} \times \frac{1 \text{ gal}}{32 \text{ mi}} \times \frac{\$3.90}{\text{gal}} = \$243.75.$$

$$38. \text{ a. The driving time when traveling at 60 miles per}$$

hour is  $1500 \text{ mi} \times \frac{1 \text{ hr}}{60 \text{ mi}} = 25 \text{ hr}$ , while the time at

$$75 \text{ miles per hour is } 1500 \text{ mi} \times \frac{1 \text{ hr}}{75 \text{ mi}} = 20 \text{ hr}.$$

b. Your car gets 32 miles to the gallon when driving at 60 mph, so the cost is

$$1500 \text{ mi} \times \frac{1 \text{ gal}}{32 \text{ mi}} \times \frac{\$3.90}{\text{gal}} = \$182.81.$$

Your car gets 25 miles to the gallon when driving at 75 mph, so the cost is

$$1500 \text{ mi} \times \frac{1 \text{ gal}}{25 \text{ mi}} \times \frac{\$3.90}{\text{gal}} = \$234.00.$$

$$39. \text{ a. The rise in sea level is found by dividing the}$$

volume of water by the Earth's surface area, so

$$\text{sea level rise} = \frac{2.5 \times 10^6 \text{ km}^3}{340 \times 10^6 \text{ km}^2} = 0.007 \text{ km} \quad \text{or}$$

about 7 meters.

40. If the volume of ash is divided by the area it covers, you'll get the average depth of the ash (see Exercise 39):

$$\text{average depth} = \frac{\text{volume}}{\text{area}} = \frac{100 \text{ km}^3}{600 \text{ km}^2}.$$

$$= 0.167 \text{ km} = 167 \text{ m}$$

41. Power is the rate at which energy is used, so the power is 100 Calories per mile. Convert this to joules per second, or watts.

$$\frac{100 \text{ Cal}}{1 \text{ mi}} \times \frac{1 \text{ mi}}{10 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{4184 \text{ J}}{1 \text{ Cal}} =$$

$$\frac{418,400 \text{ J}}{600 \text{ s}} = 697 \text{ W}$$

42. Power is the rate at which energy is used, so the power is 50 Calories per mile. Convert this to joules per second, or watts.

$$\frac{50 \text{ Cal}}{1 \text{ mi}} \times \frac{15 \text{ mi}}{1 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{4184 \text{ J}}{1 \text{ Cal}} = 871.7 \text{ W}$$

43. In order to compute the cost, we need to find the energy used by the bulbs in kW-hr, and then convert to cents. This can be done in a single chain of unit conversions, beginning with the idea that energy = power  $\times$  time.

75 watt bulb: One day will cost

$$75 \text{ W} \times \frac{12 \text{ hr}}{1 \text{ day}} \times \frac{1 \text{ kW}}{1000 \text{ W}} \times \frac{\$0.13}{1 \text{ kW-hr}} = \$0.117.$$

15 watt bulb: One day will cost

$$15 \text{ W} \times \frac{12 \text{ hr}}{1 \text{ day}} \times \frac{1 \text{ kW}}{1000 \text{ W}} \times \frac{\$0.13}{1 \text{ kW-hr}} = \$0.0234$$

You will save  $(\$0.117 - \$0.0234) \times 365 \text{ days} = \$34.16$  in one year.

44. In order to compute the cost, we need to find the energy used by the clothes dryer in kW-hr, and then convert to cents. This can be done in a single chain of unit conversions, beginning with the idea that energy = power  $\times$  time.

4000 watt clothes dryer: Average daily cost is

$$4000 \text{ W} \times 1 \text{ hr} \times \frac{1 \text{ kW}}{1000 \text{ W}} \times \frac{\$0.14}{1 \text{ kW-hr}} = \$0.56.$$

2000 watt clothes dryer: Average daily cost is

$$2000 \text{ W} \times 1 \text{ hr} \times \frac{1 \text{ kW}}{1000 \text{ W}} \times \frac{\$0.14}{1 \text{ kW-hr}} = \$0.28.$$

You will save  $(\$0.56 - \$0.28) \times 365 = \$102.20$  in one year.

45. The volume of the block is  $(3 \text{ cm})^3 = 27 \text{ cm}^3$ . Density is mass per unit volume, so the density of the block is  $\frac{20 \text{ g}}{27 \text{ cm}^3} = 0.74 \frac{\text{g}}{\text{cm}^3}$ . It will float in water because the density of water is  $1 \frac{\text{g}}{\text{cm}^3}$ .

46. Density is mass per unit volume, so the density of the sample of plutonium is  $\frac{1.98 \text{ g}}{0.1 \text{ cm}^3} = 19.8 \frac{\text{g}}{\text{cm}^3}$ . It will sink in water because the density of water is  $1 \frac{\text{g}}{\text{cm}^3}$ .

47. Population density is people per unit area, so the average population density of the U.S. is

$$\frac{306,000,000 \text{ people}}{3,500,000 \text{ mi}^2} = 87 \frac{\text{people}}{\text{mi}^2}.$$

48. The population density for Monaco is about

$$\frac{32,500 \text{ people}}{1.95 \text{ km}^2} = 16,700 \frac{\text{people}}{\text{km}^2},$$

$$\text{which is about } \frac{16,700 \frac{\text{people}}{\text{km}^2}}{31 \frac{\text{people}}{1 \text{ km}^2}} = 538, \text{ or more}$$

than 500 times greater than that of the United States.

49. The population density for New Jersey is

$$\frac{8,700,000 \text{ people}}{7417 \text{ mi}^2} = 1173 \frac{\text{people}}{\text{mi}^2}.$$

The population density for Alaska is

$$\frac{680,000 \text{ people}}{571,951 \text{ mi}^2} = 1.2 \frac{\text{people}}{\text{mi}^2},$$

which is smaller than New Jersey's population density.

50. Information density is bytes per unit area, so, depending on the formatting, the DVD's information density is either

$$\frac{4.7 \text{ Gbyte}}{134 \text{ cm}^2} \times \frac{10^9 \text{ byte}}{1 \text{ Gbyte}} = 35,000,000 \frac{\text{byte}}{\text{cm}^2} \text{ or}$$

$$\frac{8.5 \text{ Gbyte}}{134 \text{ cm}^2} \times \frac{10^9 \text{ byte}}{1 \text{ Gbyte}} = 63,000,000 \frac{\text{byte}}{\text{cm}^2}.$$

51. a. In one week, a 100-pound person would take  $1 \text{ week} \times \frac{7 \text{ days}}{1 \text{ week}} \times \frac{25 \text{ mg}}{6 \text{ hr}} \times \frac{24 \text{ hr}}{1 \text{ day}} = 700 \text{ mg}$ , so

$$\text{should take } 700 \text{ mg} \times \frac{1 \text{ tablet}}{12.5 \text{ mg}} = 56 \text{ tablets.}$$

b. A 100-pound person should take  $700 \text{ mg} \times \frac{5 \text{ mL}}{12.5 \text{ mg}} = 280 \text{ mL}$  of liquid Benadryl.

52. a. The dose is  $\frac{9000 \text{ units}}{1 \text{ kg}} \times \frac{250 \text{ mg}}{400,000 \text{ units}} = 5.625 \text{ mg/kg}$ .

b. A child would take 4 doses in a day, so a 20-kg child would receive

$$4 \times 20 \text{ kg} \times \frac{5.625 \text{ mg}}{1 \text{ kg}} = 450 \text{ mg}.$$

53. a. BAC is usually measured in units of grams of alcohol per 100 milliliters of blood. A woman who drinks two glasses of wine, each with 20 grams of alcohol, has consumed 40 grams of alcohol. If she has 4000 milliliters of blood, her BAC is

$$\frac{40 \text{ g}}{4000 \text{ mL}} = \frac{0.01 \text{ g}}{\text{mL}} \times \frac{100}{100} = \frac{1 \text{ g}}{100 \text{ mL}}.$$

It is fortunate that alcohol is not absorbed immediately, because if it were, the woman would most likely die – a BAC above 0.4 g/mL is typically enough to induce coma or death.

b. If alcohol is eliminated from the body at a rate of 10 grams per hour, then after 3 hours, 30 grams would have been eliminated. This leaves 10 grams in the woman's system, which means her

$$\text{BAC is } \frac{10 \text{ g}}{4000 \text{ mL}} = \frac{0.0025 \text{ g}}{\text{mL}} \times \frac{100}{100} = \frac{0.25 \text{ g}}{100 \text{ mL}}.$$

This is well above the legal limit for driving, so it is not safe to drive. Of course this solution assumes the woman survives 3 hours of lethal levels of alcohol in her body, because we have assumed all the alcohol is absorbed immediately. In reality, the situation is somewhat more complicated.

54. a. BAC is usually measured in units of grams of alcohol per 100 milliliters of blood. A man who drinks 8 ounces of hard liquor has consumed 70 grams of alcohol, and with 6000 milliliters of blood, his BAC is

$$\frac{70 \text{ g}}{6000 \text{ mL}} = \frac{0.0117 \text{ g}}{\text{mL}} \times \frac{100}{100} = \frac{1.17 \text{ g}}{100 \text{ mL}}.$$

It is fortunate that alcohol is not absorbed immediately, because if it were, the man would be dead – a BAC above 0.4g/mL is typically enough to induce coma or death.

b. If alcohol is eliminated from the body at a rate of 15 grams per hour, then after 4 hours, 60 grams would have been eliminated. This leaves 10 grams of alcohol in the man's system, which means his

$$\text{BAC is } \frac{10 \text{ g}}{6000 \text{ mL}} = \frac{0.0017 \text{ g}}{\text{mL}} \times \frac{100}{100} = \frac{0.17 \text{ g}}{100 \text{ mL}}.$$

This is well above the legal limit for driving, so it is not safe to drive. Of course this solution assumes the man survives 4 hours of lethal levels of alcohol in his body, because we have assumed all the alcohol is absorbed immediately. In reality, the situation is somewhat more complicated.

## FURTHER APPLICATIONS

55. a. A metric mile is  $1500 \text{ m} \times \frac{3.28 \text{ ft}}{1 \text{ m}} = 4920 \text{ ft}$ . A

USCS mile is 5280 ft. Since  $4920/5280 = 0.932$ , the metric mile is 93.2% of the USCS mile.

b. Men's mile: Note:  $(3:43:13 = 223.13 \text{ seconds})$

$$\frac{1 \text{ mi}}{223.13 \text{ s}} \times \frac{3600 \text{ s}}{1 \text{ hr}} = 16.13 \text{ mi/hr}$$

Men's metric mile: Note:  $(3:26:00 = 206 \text{ seconds})$

$$\frac{4920 \text{ ft}}{206 \text{ s}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \times \frac{3600 \text{ s}}{1 \text{ hr}} = 16.28 \text{ mi/hr}$$

The record holder in the metric mile ran at a faster pace.

c. Women's mile: Note:  $(4:12:56 = 252.56 \text{ seconds})$

$$\frac{1 \text{ mi}}{252.56 \text{ s}} \times \frac{3600 \text{ s}}{1 \text{ hr}} = 14.25 \text{ mi/hr}$$

Women's metric mile: Note:  $(3:50:46 = 230.46 \text{ seconds})$

$$\frac{4920 \text{ ft}}{230.46 \text{ s}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \times \frac{3600 \text{ s}}{1 \text{ hr}} = 14.56 \text{ mi/hr}$$

The record holder in the metric mile ran at a faster pace.

d. This would be true in both cases since, for both men and women, the record holder in the metric mile ran at a faster pace.

56. Buy 36 feet of the 12-ft wide roll, and cut off an 18-ft long piece to be laid along the edge with that dimension. Take the remaining piece (also 18 feet long) and cut off a 2-ft wide strip (which will be waste) – this will cover the remaining portion of the room. The cost is

$$36 \text{ ft} \times 12 \text{ ft} \times \frac{1 \text{ yd}^2}{9 \text{ ft}^2} \times \frac{\$28.50}{\text{yd}^2} = \$1368.$$

A cheaper method (\$1254) can be found with two seams.

57. a. The volume of the bath is  $6 \text{ ft} \times 3 \text{ ft} \times 2.5 \text{ ft} = 45 \text{ ft}^3$ . Fill it to the halfway point, and you'll use  $22.5 \text{ ft}^3$  of water (half of 45 is 22.5). (Interesting side note: it doesn't matter which of the bathtub's three dimensions you regard as the height – fill it halfway, and it's always  $22.5 \text{ ft}^3$  of water). When you take a shower, you use

57. (continued)

$$1 \text{ shower} \times \frac{10 \text{ min}}{\text{shower}} \times \frac{1.75 \text{ gal}}{\text{min}} \times \frac{1 \text{ ft}^3}{7.5 \text{ gal}} = 2.33 \text{ ft}^3$$

of water, and thus you use considerably more water when taking a bath.

b. Convert  $22.5 \text{ ft}^3$  of water (the water used in a bath) into minutes.

$$22.5 \text{ ft}^3 \times \frac{7.5 \text{ gal}}{1 \text{ ft}^3} \times \frac{1 \text{ min}}{1.75 \text{ gal}} = 96 \text{ min}.$$

c. Plug the drain in the bathtub, and mark the depth to which you would normally fill the tub when taking a bath. Take a shower, and note how long your shower lasts. Step out and towel off, but keep the shower running. When the water reaches your mark (you used a crayon, and not a pencil, right?), note the time it took to get there. You now have a sense of how many showers it takes to use the same amount of water as a bath. For example, suppose your shower took 12 minutes, and it takes a full hour (60 minutes) for the water to reach your mark. That would mean every bath uses as much water as five showers.

58. a. Convert 300,000 long tons into kilograms.

$$300,000 \text{ long ton} \times \frac{2240 \text{ lb}}{1 \text{ long ton}} \times \frac{1 \text{ kg}}{2.205 \text{ lb}} = 305,000,000 \text{ kg}$$

b. Convert 305,000,000 kilograms to cubic meters.

$$305,000,000 \text{ kg} \times \frac{1 \text{ m}^3}{850 \text{ kg}} = 359,000 \text{ m}^3$$

c. Convert 359,000 cubic meters to barrels of oil.

$$359,000 \text{ m}^3 \times \frac{1000 \text{ L}}{1 \text{ m}^3} \times \frac{1 \text{ gal}}{3.785 \text{ L}} \times \frac{1 \text{ barrel}}{42 \text{ gal}} = 2,260,000 \text{ barrels}.$$

c. Answers will vary. Assuming \$100/barrel, the value of oil in a full supertanker would be

$$\frac{\$100}{1 \text{ barrel}} \times 2,260,000 \text{ barrels} = \$226,000,000$$

or about \$226 million.

59. a. The depth of Lake Victoria is found by dividing the its volume by its surface area, so

$$\text{depth} = \frac{2750 \text{ km}^3}{68700 \text{ km}^2} = 0.03997 \text{ km} \text{ or about } 40 \text{ meters}.$$

b. The lost volume is

$$68700 \text{ km}^2 \times 10 \text{ ft} \times \frac{0.3048 \text{ m}}{1 \text{ ft}} \times \frac{1 \text{ km}}{1000 \text{ m}} = 209.3976 \text{ km}^3 \text{ or about } 210 \text{ cubic kilometers}.$$

c. The lost volume as a percentage is  $\frac{210 \text{ km}^3}{2750 \text{ km}^3} = 0.076$  or 7.6%.

60. a. Convert 9 billion gallons per day into cfs.

$$\frac{9,000,000,000 \text{ gal}}{\text{d}} \times \frac{1 \text{ ft}^3}{7.5 \text{ gal}} \times \frac{1 \text{ d}}{24 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 13,889 \frac{\text{ft}^3}{\text{s}}, \text{ or } 13,889 \text{ cfs},$$

which is about 46% of the flow rate of the Colorado River.

b. The volume of water entering the city over the course of a day, in cubic feet, was

$$9,000,000,000 \text{ gal} \times \frac{1 \text{ ft}^3}{7.5 \text{ gal}} = 1,200,000,000 \text{ ft}^3.$$

The area of the flooded portion of the city, in square feet, was

$$6 \text{ mi}^2 \times \left( \frac{5280 \text{ ft}}{1 \text{ mi}} \right)^2 = 167,270,400 \text{ ft}^2.$$

If the volume of water is divided by the area it covers, you'll get the average depth of the water (see Exercise 59):

$$\text{average depth} = \frac{\text{volume}}{\text{area}} = \frac{1,200,000,000 \text{ ft}^3}{167,270,400 \text{ ft}^2} = 7.2 \text{ ft}$$

Thus the water level rose about 7 feet in one day, assuming the entire 6 square miles was covered in that day.

61. Convert 1 week into cubic feet of water.

$$1 \text{ wk} \times \frac{7 \text{ d}}{1 \text{ wk}} \times \frac{24 \text{ hr}}{1 \text{ d}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{25,800 \text{ ft}^3}{\text{s}} = 15,603,840,000 \text{ ft}^3,$$

which is about 15.6 billion cubic feet. Since

$$\frac{15,603,840,000}{1,200,000,000,000} = 0.013, \text{ about } 1.3\% \text{ of the water in the reservoir was released}.$$

62. a. There are  $750 \text{ mL} \times \frac{5 \text{ mg}}{100 \text{ mL}} = 37.5 \text{ mg}$  of dextrose in 750 mL of D5W. The patient should be given  $50 \text{ mg} \times \frac{750 \text{ mL}}{37.5 \text{ mg}} = 1000 \text{ mL}$  of D5W.
- b. There are  $1.2 \text{ L} \times \frac{1000 \text{ mL}}{1 \text{ L}} \times \frac{0.9 \text{ mg}}{100 \text{ mL}} = 10.8 \text{ mg}$  of dextrose in 1.2 L of NS. The patient should be given  $15 \text{ mg} \times \frac{1.2 \text{ L}}{10.8 \text{ mg}} \times \frac{1000 \text{ mL}}{1 \text{ L}} = 1667 \text{ mL}$  of NS.
63. a. The flow rates are  $\frac{1.5 \text{ L}}{12 \text{ hr}} \times \frac{1000 \text{ mL}}{1 \text{ L}} = 125 \frac{\text{mL}}{\text{hr}}$  and  $125 \frac{\text{mL}}{\text{hr}} \times \frac{5 \text{ mg}}{100 \text{ L}} = 6.25 \frac{\text{mg}}{\text{hr}}$ .
- b. The rate is  $125 \frac{\text{mL}}{\text{hr}} \times \frac{15 \text{ gtt}}{1 \text{ mL}} = 1875 \frac{\text{gtt}}{\text{hr}}$ .
- c.  $12 \text{ hr} \times 6.25 \frac{\text{mg}}{\text{hr}} = 75 \text{ mg}$  of dextrose is delivered.
64. a. The flow rates are  $\frac{0.5 \text{ L}}{4 \text{ hr}} \times \frac{1000 \text{ mL}}{1 \text{ L}} = 125 \frac{\text{mL}}{\text{hr}}$  and  $125 \frac{\text{mL}}{\text{hr}} \times \frac{0.9 \text{ mg}}{100 \text{ L}} = 1.125 \frac{\text{mg}}{\text{hr}}$ .
- b. The rate is  $125 \frac{\text{mL}}{\text{hr}} \times \frac{20 \text{ gtt}}{1 \text{ mL}} = 2500 \frac{\text{gtt}}{\text{hr}}$ .
- c.  $4 \text{ hr} \times 1.125 \frac{\text{mg}}{\text{hr}} = 4.5 \text{ mg}$  of sodium chloride is delivered.
65. a. The flow rate is  $\frac{10 \text{ mL}}{1 \text{ hr}} \times \frac{300 \text{ mg}}{200 \text{ mL}} = 15 \frac{\text{mg}}{\text{hr}}$ .
- b. The infusion should last  $60 \text{ mg} \times \frac{1 \text{ hr}}{15 \text{ mg}} = 4$  hours.
66. a.  $40 \text{ kg} \times \frac{25 \text{ mg/kg}}{1 \text{ day}} = 1000 \text{ mg/day}$  and  $\frac{1000 \text{ mg}}{1 \text{ day}} \times \frac{1 \text{ capsule}}{250 \text{ mg}} = 4 \text{ capsules/day}$ , so the patient should take 1 capsule every six hours.
- b. From part a, you would need 250 mg in 6 hours, which requires 5 mL of solution in 6 hours, so the rate would be  $\frac{5 \text{ mL}}{6 \text{ hr}} \times \frac{60 \text{ gtt}}{1 \text{ mL}} = 50 \text{ gtt/hr}$ .
67. a.  $36 \text{ kg} \times \frac{50 \text{ mg/kg}}{1 \text{ day}} = 1800 \text{ mg/day}$  and  $\frac{1800 \text{ mg}}{1 \text{ day}} \times \frac{1 \text{ tablet}}{300 \text{ mg}} = 6 \text{ tablets/day}$ , so the patient should take 1 tablet every four hours.
- b. From part a, you would need 2 tablets or 600 mg in 8 hours, so  $600 \text{ mg} \times \frac{5 \text{ mL}}{200 \text{ mg}} = 15 \text{ mL}$  of solution in 8 hours, so the rate would be  $\frac{15 \text{ mL}}{8 \text{ hr}} \times \frac{15 \text{ gtt}}{1 \text{ mL}} = 28.13 \text{ gtt/hr}$ .
68. a. Your power is the rate at which you use energy, and thus your average power is 2500 Calories per day. Convert this to watts.
- $$\frac{2500 \text{ Cal}}{1 \text{ d}} \times \frac{1 \text{ d}}{24 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{4184 \text{ J}}{1 \text{ Cal}} = 121 \frac{\text{J}}{\text{s}}$$
- Since  $1 \text{ W} = 1 \text{ J/s}$ , this is 121 watts.
- b.  $\frac{2500 \text{ Cal}}{1 \text{ d}} \times \frac{365 \text{ d}}{1 \text{ yr}} \times \frac{4184 \text{ J}}{1 \text{ Cal}} = 3,817,900,000 \frac{\text{J}}{\text{yr}}$ , so you need about 3.8 billion joules each year from food, which is very close to 1% of your total energy consumption (3.8 billion/400 billion = 0.0095).
69. a. Convert kilowatt-hours into joules.
- $$900 \text{ kW-hr} \times \frac{3,600,000 \text{ J}}{1 \text{ kW-hr}} = 3,240,000,000 \text{ J}$$
- b. May has 31 days, so the average power is  $\frac{3,240,000,000 \text{ J}}{31 \text{ d}} \times \frac{1 \text{ d}}{24 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = \frac{1210 \text{ J}}{\text{s}} = 1210 \text{ W}$ .
- One could also begin with 900 kW-hr per 31 days, and convert that into watts, using  $1 \text{ watt} = 1 \text{ joule per second}$ .
- c. First, convert joules into liters.
- $$3,240,000,000 \text{ J} \times \frac{1 \text{ L}}{12,000,000 \text{ J}} = 270 \text{ L}$$
- Now convert liters into gallons.
- $$270 \text{ L} \times \frac{1 \text{ gal}}{3.785 \text{ L}} = 71.33 \text{ gal}$$

69. (continued)

Thus it would take 270 liters = 71.33 gallons of oil to provide the energy shown on the bill (assuming all the energy released by the burning oil could be captured and delivered to your home with no loss).

70. a. Convert kilowatt-hours into joules.

$$1050 \text{ kW-hr} \times \frac{3,600,000 \text{ J}}{1 \text{ kW-hr}} = 3,780,000,000 \text{ J}.$$

b. October has 31 days, so the average power is

$$\frac{3,780,000,000 \text{ J}}{31 \text{ d}} \times \frac{1 \text{ d}}{24 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = \frac{1411 \text{ J}}{\text{s}} = 1411 \text{ W}.$$

One could also begin with 1050 kW-hr per 31 days, and convert that into watts, using 1 watt = 1 joule per second.

c. First, convert joules into liters.

$$3,780,000,000 \text{ J} \times \frac{1 \text{ L}}{12,000,000 \text{ J}} = 315 \text{ L}.$$

Now convert liters into gallons.

$$315 \text{ L} \times \frac{1 \text{ gal}}{3.785 \text{ L}} = 83.22 \text{ gal}.$$

Thus it would take 315 liters = 83.22 gallons of oil to provide the energy shown on the bill (assuming all the energy released by the burning oil could be captured and delivered to your home with no loss).

71. In order to compute the cost, we need to find the energy used by the outdoor spa in kW-hr, and then convert to dollars. This can be done in a single chain of unit conversions, beginning with the idea that energy = power  $\times$  time (this is true because power is the rate at which energy is consumed; that is, power = energy/time) and assuming a 30-day month.

$$1500 \text{ W} \times 4 \text{ months} \times \frac{1 \text{ kW}}{1000 \text{ W}} \times \frac{30 \text{ d}}{1 \text{ month}} \times \frac{24 \text{ hr}}{1 \text{ d}} \times \frac{\$0.10}{1 \text{ kW-hr}} = \$432$$

72. Since energy = power  $\times$  time, the power plant can produce (assuming a 30-day month)

$$1190 \text{ MW} \times 1 \text{ month} \times \frac{30 \text{ d}}{1 \text{ month}} \times \frac{24 \text{ hr}}{1 \text{ d}} \times \frac{1000 \text{ kW}}{1 \text{ MW}} \approx 8.57 \times 10^8 \text{ kW-hr of energy each month. The amount of uranium used each month is given by}$$

$$8.57 \times 10^8 \frac{\text{kW-hr}}{\text{month}} \times \frac{1 \text{ kg}}{16 \times 10^6 \text{ kW-hr}} \approx$$

$$53.6 \frac{\text{kg}}{\text{month}}.$$

The plant can serve about 857,000 homes,

$$\text{because } 8.57 \times 10^8 \text{ kW-hr} \times \frac{1 \text{ home}}{1000 \text{ kW-hr}} =$$

857,000 homes.

73. Since energy = power  $\times$  time, the power plant can produce (assuming a 30-day month)

$$1.5 \times 10^9 \text{ W} \times 1 \text{ month} \times \frac{30 \text{ d}}{\text{month}} \times \frac{24 \text{ hr}}{\text{d}} \times \frac{1 \text{ kW}}{1000 \text{ W}}$$

$$= 1.08 \times 10^9 \text{ kW-hr} = 1.08 \text{ billion kW-hr}$$

of energy every month. The plant needs

$$1.08 \times 10^9 \frac{\text{kW-hr}}{1 \text{ month}} \times \frac{1 \text{ kg}}{450 \text{ kW-hr}} =$$

$$2.4 \times 10^6 \frac{\text{kg}}{\text{month}}, \text{ or } 2.4 \text{ million kg of coal every}$$

month. Because  $1.08 \times 10^9 \text{ kW-hr} \times \frac{1 \text{ home}}{1000 \text{ kW-hr}} = 1.08 \times 10^6 \text{ homes}$ , the plant can serve 1.08 million homes.

74. At 20% efficiency, this solar panel can generate 200 watts of power when exposed to direct sunlight. Since energy = power  $\times$  time, and because the panel receives the equivalent of 6 hours of direct sunlight, the panel can produce

$$200 \text{ W} \times 6 \text{ hr} \times \frac{1 \text{ kW}}{1000 \text{ W}} \times \frac{3,600,000 \text{ J}}{\text{kW-hr}} = 4,320,000 \text{ J}.$$

This occurs over the span of one day, so the panel produces an average power of  $\frac{4,320,000 \text{ J}}{\text{d}} \times \frac{1 \text{ d}}{24 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 50 \frac{\text{J}}{\text{s}} = 50 \text{ W}.$

75. In Exercise 74, each solar panel covers one square meter. You would need

$$1 \text{ kW} \times \frac{1000 \text{ W}}{1 \text{ kW}} \times \frac{1 \text{ panel}}{50 \text{ W}} = 20 \text{ panels},$$

which is 20 square meters of solar panels.

76. Energy = power  $\times$  time, so the energy produced by a wind turbine over the course of a year is

$$200 \text{ kW} \times 1 \text{ yr} \times \frac{365 \text{ d}}{1 \text{ yr}} \times \frac{24 \text{ hr}}{1 \text{ d}} = 1,752,000 \text{ kW-hr}.$$

76. (continued)

This is enough energy to serve

$$1,752,000 \text{ kW-hr} \times \frac{1 \text{ home}}{10,000 \text{ kW-hr}} = 175 \text{ homes}.$$

77. a. Energy = power  $\times$  time, so the energy capacity of the wind farms over the span of a year is

$$2.5 \times 10^9 \text{ W} \times 1 \text{ yr} \times \frac{365 \text{ d}}{1 \text{ yr}} \times \frac{24 \text{ hr}}{1 \text{ d}} \times \frac{1 \text{ kW}}{1000 \text{ W}} \times 0.30 \\ = 2.19 \times 10^{10} \text{ kW-hr}$$

Since the wind farms produce 30% of their capacity, on average, the energy produced is

$$= 2.19 \times 10^{10} \text{ kW-hr} \times 0.30 = 6.57 \times 10^9 \text{ kW-hr} \text{ or } 6,570,000,000 \text{ kW-hr}.$$

This is enough energy to serve

$$6.57 \times 10^9 \text{ kW-hr} \times \frac{1 \text{ home}}{10,000 \text{ kW-hr}} \\ = 657,000 \text{ homes}.$$

b. If the  $2.19 \times 10^{10}$  kW-hr of energy produced by wind farms were instead produced from fossil fuels, there would be

$$6.57 \times 10^9 \text{ kW-hr} \times \frac{1.5 \text{ lb}}{1 \text{ kW-hr}} = 9.855 \times 10^9 \text{ lb},$$

or about 9,855,000,000 pounds of carbon dioxide entering the atmosphere each year.

78. a. The volume of one hemlock tree would be about

$$\pi (15 \text{ in})^2 (120 \text{ ft}) = 84,823 \text{ in} \times \text{in} \times \text{ft} \quad \text{and}$$

$$84,823 \text{ in} \times \text{in} \times \text{ft} \times \frac{1 \text{ ft}}{12 \text{ in}} = 7069 \text{ ft} \times \text{ft} \times \text{in} \quad \text{or} \\ 7069 \text{ fbm}.$$

b. One 8 ft 2-by-4 contains about

$$8 \text{ ft} \times 1.5 \text{ in} \times 3.5 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} = 3.5 \text{ ft} \times \text{ft} \times \text{in} = 3.5$$

$$\text{fbm, so you could cut } \frac{150 \text{ fbm}}{3.5 \text{ fbm}} = 42.9 \text{ or } 43$$

whole boards. (Round up to next whole board.)

c. One 12 ft 2-by-6 is  $12 \text{ ft} \times 1.5 \text{ in} \times 5.5 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} = 8.25 \text{ ft} \times \text{ft} \times \text{in} = 8.25 \text{ fbm}$ , so the project would require  $75 \times 8.25 \text{ fbm} \approx 619 \text{ fbm}$ 

79. The stand would contain 24 hectares

$$\times \frac{10,000 \text{ m}^2}{1 \text{ hectare}} \times \frac{1 \text{ tree}}{20 \text{ m}^2} \times \frac{400 \text{ fbm}}{1 \text{ tree}} = 4,800,000 \text{ fbm}.$$

So there would be  $0.10 \times 4,800,000 \text{ fbm} = 480,000 \text{ fbm}$  if one-tenth of the trees were cut.Note: Exercise 93 in Section 2A shows the calculation of  $\frac{10,000 \text{ m}^2}{1 \text{ hectare}} = 1$ .

80. Case A:

$$\text{Income: } 50 \text{ acres} \times \frac{60 \text{ bushels}}{1 \text{ acre}} \times \frac{\$3.50}{1 \text{ bushel}} \\ = \$10,500.$$

$$\text{Cost: } 50 \text{ acres} \times \frac{100 \text{ lb}}{1 \text{ acre}} \times \frac{\$0.25}{1 \text{ lb}} = \$1250.$$

$$\text{Revenue: } \$10,500 - \$1250 = \$9250.$$

Case B:

$$\text{Income: } 50 \text{ acres} \times \frac{50 \text{ bushels}}{1 \text{ acre}} \times \frac{\$4.50}{1 \text{ bushel}} \\ = \$11,250.$$

$$\text{Cost: } 50 \text{ acres} \times \frac{70 \text{ lb}}{1 \text{ acre}} \times \frac{\$0.50}{1 \text{ lb}} = \$1750.$$

$$\text{Revenue: } \$11,250 - \$1750 = \$9500.$$

Case B has the higher revenue.

## UNIT 2C

### TIME OUT TO THINK

Pg. 104. Answers will vary. Recall that the four-step process is: Step 1: Understand the problem; Step 2: Devise a strategy for solving the problem; Step 3: Carry out your strategy, and revise it if necessary; Step 4: Look back to check, interpret, and explain your result.

Pg. 106. The combination of 5 child tickets and 4 adult tickets totals to \$130 ( $\$10 \times 5 + \$20 \times 4 = \$130$ ). Similar calculations will verify the remained child/ticket combinations. There cannot

be an even number of child tickets since the amount of money left for adult tickets would not be evenly divisible by \$20. For example, if there were 4 child tickets, the adult tickets would have to be worth  $\$130 - 4 \times \$10 = \$90$ , which is not divisible by \$20, so there could not be a whole number of tickets.

Pg. 110. Answers will vary. This would be a good topic for a discussion either during or outside of class.



Pg. 112. This question simply checks that students followed Example 6. This is a subjective question designed to spur debate.

Pg. 113. Answers will vary. This would be a good topic for a discussion either during or outside of class.

## QUICK QUIZ

1. **c.** Look at example 1 (*Box Office Receipts*) in this unit.
2. **a.** You must have done something wrong as gas mileage is reported in units of miles per gallon.
3. **b.** Common experience tells us that batteries can power a flashlight for many hours, not just a few minutes nor several years.
4. **b.** An elevator that carried only 10 kg couldn't accommodate even one person (a 150 lb person weighs about 75 kg), and hotel elevators aren't designed to carry hundreds of people (you'd need at least 100 people to reach 10,000 kg).
5. **b.** Refer to the discussion of Zeno's paradox in the text. There, it was shown that the sum of an infinite number of ever-smaller fraction is equal to 2, and thus we can eliminate answers **a** and **c**. This leaves **b**.
6. **a.** If you cut the cylinder along its length, and lay it flat, it will form a rectangle with width equal to the circumference of the cylinder (i.e. 10 in), and with length equal to the length of the cylinder.
7. **c.** A widow is a woman who survives the death of her husband, and thus the man is dead, making it impossible for him to marry anyone.
8. **c.** It could happen that the first 20 balls selected are odd, in which case the next two would have to be even, and this is the first time one can be certain of selecting two even balls.
9. **b.** The most likely explanation is that the A train always arrives 10 minutes after the B train. Suppose the A train arrives on the hour (12:00, 1:00, 2:00,...), while the B train arrives ten minutes before the hour (11:50, 12:50, 1:50,...). If Karen gets to the station in the first 50 minutes of the hour, she'll take the B train; otherwise she'll take the A train. Since an hour is 60 minutes long,  $\frac{5}{6}$  of the time, Karen will take the B train to the beach. Note that  $\frac{5}{6}$  of 30 days is 25 days, which is the number of times Karen went to the beach. The other scenarios *could* happen, but they aren't nearly as likely as the scenario in answer **b**.
10. **b.** Label the hamburgers as A, B, and C. Put burgers A and B on the grill. After 5 minutes, turn burger A, take burger B and put it on a plate, and

put burger C on the grill. After 10 minutes, burger A is cooked, while burgers B and C are half-cooked. Finish off burgers B and C in the final 5 minutes, and you've cooked all three in 15 minutes.

## DOES IT MAKE SENSE?

3. Does not make sense. There is no problem-solving recipe that can be applied to all problems.
4. Makes sense. It's generally a waste of time to blindly dive into the solution of a problem if you don't take the time to understand its nature.
5. Does not make sense. The four-step method is applicable to a wide variety of problems, and is not limited to mathematical problems. (It's not a cure-all, either, but it's quite useful, and worth learning).
6. Does not make sense. Pictures can be extremely beneficial in the problem solving process as they allow young and old alike to visualize the problem.

## BASIC SKILLS AND CONCEPTS

7. You won't be able to determine the exact number of cars and buses that passed through the toll booth, but the method of trial-and-error leads to the following possible solutions: (16 cars, 0 buses), (13 cars, 2 buses), (10 cars, 4 buses), (7 cars, 6 buses), (4 cars, 8 buses), (1 car, 10 buses). Along the way, you may have noticed that the number of buses must be an even number, because when it is odd, the remaining money cannot be divided evenly into \$2 (car) tolls.
8. The method of trial-and-error produces the following possible recipes, the ordered pairs are (Level 1, Level 2): (14, 0), (12, 1), (10, 2), (8, 3), (6, 4), (4, 5), (2, 6), (0, 7).
9. **a.** Based on the first race data, Jordan runs 200 meters in the time it takes Amari to run 190 meters. In the second race, Jordan will catch up to Amari 10 meters from the finish line (because Jordan has covered 200 meters at that point, and Amari has covered 190 meters). Jordan will win the race in the last ten meters, because Jordan runs faster than Amari.  
**b.** Jordan will be 5 meters from the finish line when Amari is 10 meters from the finish line (because Jordan has covered 200 meters at that point, and Amari has covered 190 meters). Jordan will win the race in the because Jordan runs faster than Amari and is in the lead.

9. (continued)

c. Jordan will be 15 meters from the finish line when Amari is 10 meters from the finish line (because Jordan has covered 200 meters at that point, and Amari has covered 190 meters). In the time Amari runs 10 meters, Jordan will only run

$$10 \text{ m}_{\text{Amari}} \times \frac{200 \text{ m}_{\text{Jordan}}}{190 \text{ m}_{\text{Amari}}} = 10.53 \text{ m}_{\text{Jordan}}, \quad \text{so}$$

Amari wins the race.

d. From part c, Jordan must start 10.53 meters behind the starting line, since that is how far she would run in the same time Amari finishes the last 10 meters of the race.

10. Note that after two hours, the cars collide, because the first travels at 80 km/hr, and thus covers 160 km, while the second travels at 100 km/hr, and covers 200 km. The canary is flying at 120 km/hr for the entire 2 hours, so it has flown 240 km.

11. On the second transfer, there are four possibilities to consider. A) All three marbles are black. B) Two are black, one is white. C) One is black, two are white. D) All three are white. In case A), after the transfer, there are no black marbles in the white pile, and no white marbles in the black pile. In case B), one white marble is transferred to the black pile, and one black marble is left in the white pile. In case C), two white marbles are transferred to the black pile, and two black marbles are left in the white pile. In case D), three white marbles are transferred to the black pile, and three black marbles are left in the white pile. In all four cases, there are as many white marbles in the black pile as there are black marbles in the white pile after the second transfer.

12. Cut the pipe along its length, and press it flat – this results in a rectangle with height = 20 cm, and width = circumference of pipe = 6 cm. As shown in example 5, a wrap of wire becomes the hypotenuse of a right triangle, whose base is 6 cm, and whose height, because there are 8 turns of wire, is  $1/8$  of 20 cm = 2.5 cm. By the Pythagorean theorem, the length of one turn of the wire is  $h = \sqrt{6^2 + 2.5^2} = 6.5$  cm, and thus the total length of wire required is 52 cm (8 turns of wire, each 6.5 cm long).

13. Proceeding as in example 6, the circular arc formed by the bowed track can be approximated by two congruent right triangles. Since 1 km is 100,000 cm, the base of one of the triangles is 50,000 cm, and its hypotenuse is 50,005 cm. The height of the triangle can be computed as

$$h = \sqrt{50,005^2 - 50,000^2} = 707 \text{ cm},$$

which is about 7.1 meters.

14. Answers will vary.

15. Yes, imagine that at the same time the monk leaves the monastery to walk up the mountain, his twin brother leaves the temple and walks down the mountain. Clearly, the two must pass each other somewhere along the path.

## FURTHER APPLICATIONS

16. a. Yes;  $P = 2(7) + 2(3) = 20$ ;  $A = 7 \times 3 = 21$

b. Yes;  $P = 2(8) + 2(2) = 20$ ;  $A = 8 \times 2 = 16$

c. By trial and error, possible dimensions ( $l, w$ ) are:

(9,1);  $P = 2(9) + 2(1) = 20$ ;  $A = 9 \times 1 = 9$

(6,4);  $P = 2(6) + 2(4) = 20$ ;  $A = 6 \times 4 = 24$

(5,5);  $P = 2(5) + 2(5) = 20$ ;  $A = 5 \times 5 = 25$

Note that reversing the length and width will result in the same areas, so the maximum area is when the yard is a square with sides of length 5 meters.

17. Note that at least one truck must have passed over the counter as there are an odd number of counts, and we must have an odd number of trucks for the same reason. Beginning with one truck, followed by 3, 5, 7, etc. trucks, and computing the number of cars that go along with these possible truck solutions, we arrive at the following answers: (1 truck, 16 cars), (3 t, 13 c), (5 t, 10 c), (7 t, 7 c), (9 t, 4 c), and (11 t, 1 c).

18. Following the hint given in the problem, the best place to start is with a drawing that shows the room cut along its vertical corners and laid flat. Figure 18a shows the room and the configuration of the amplifier and speaker before the cuts are made, and Figure 18b shows the room in “flat mode.” Note that there are at least two solutions to consider (a third will be presented in Figure 18c).

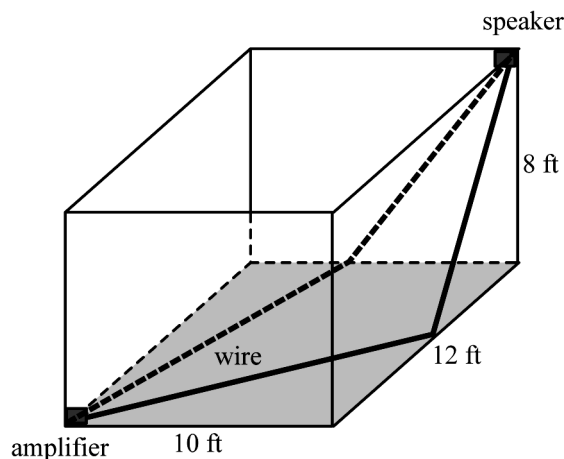


FIGURE 18a

18. (continued)

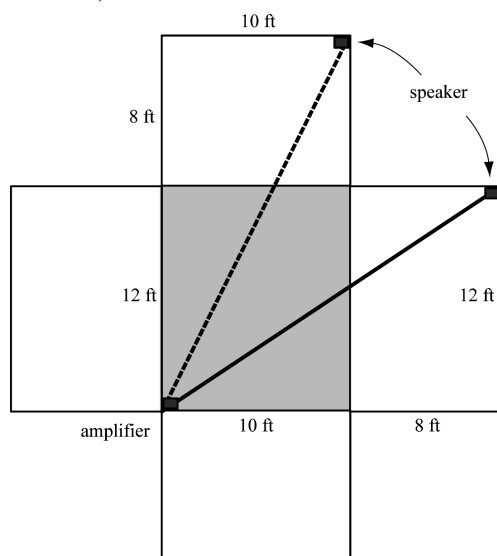


FIGURE 18b

In flat mode, it is clear that the shortest distance between the amplifier and the speaker is a straight line, though there are two possible paths (the upper right hand corner of the room, where the speaker is situated, must be represented twice, as shown in Figure 18b). In both cases, the distance from the amplifier to the speaker can be computed using the Pythagorean theorem.

Case 1 (lower triangle): the length of the wire is

$$\sqrt{18^2 + 12^2} = \sqrt{468} = 21.6 \text{ ft.}$$

Case 2 (upper triangle): the length of the wire is

$$\sqrt{20^2 + 10^2} = \sqrt{500} = 22.4 \text{ ft.}$$

Based on these calculations, the shortest length of wire needed to connect the amplifier to the speaker is 21.6 ft. However, there's a third possibility that is not evident when looking at Figure 18b. In order to see it, the room must be cut and laid flat in a different configuration, shown here.

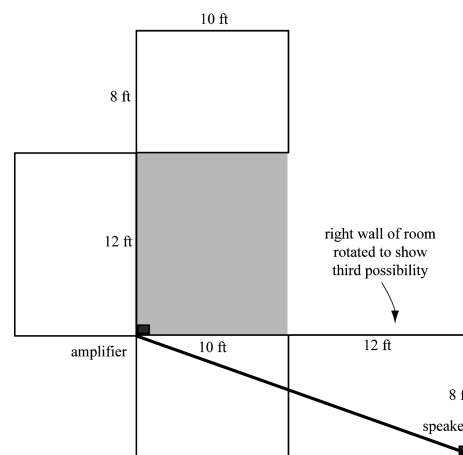


FIGURE 18c

This solution has the wire traveling along the walls of the room for its entire length, which is

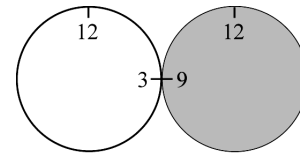
$$\sqrt{22^2 + 8^2} = \sqrt{548} = 23.4 \text{ ft.}$$

Since this does not produce a shorter length of wire than the previous solution, the answer remains 21.6 ft (different room dimensions might reveal that Figure 18c is the best choice). There are other ways to run the wire not shown here, such as rotating the back wall in a manner similar to Figure 18c (this gives an equivalent solution of 23.4 ft), or running the wire along the ceiling. However, the problem states the wire is to run along the floor and walls, so the ceiling solutions need not be considered. Try some solutions along the ceiling if you're curious – you'll find they are equivalent to running the wire along the floor.

19. a. The time spent running is  $2 \text{ mi} \div 4 \frac{\text{mi}}{\text{hr}}$   
 $= 2 \text{ mi} \times \frac{1 \text{ hr}}{4 \text{ mi}} = \frac{1}{2} \text{ hr.}$
- b. The time spent walking is  $2 \text{ mi} \div 2 \frac{\text{mi}}{\text{hr}}$   
 $= 2 \text{ mi} \times \frac{1 \text{ hr}}{2 \text{ mi}} = 1 \text{ hr.}$
- c. Not true; more time is spent walking than running.
- d. The average speed for the trip is  $4 \text{ mi} \div \frac{3}{2} \text{ hr}$   
 $4 \text{ mi} \times \frac{2}{3 \text{ hr}} = \frac{8}{3} \text{ mi/hr or about } 2.7 \text{ mi/hr.}$

20. In going from the first floor to the third floor, you have gone up two floors. Since it takes 30 seconds to do that, your pace is 15 seconds per floor. Walking from the first floor to the sixth floor means walking up five floors, and at 15 seconds per floor, this will take 75 seconds.
21. Reuben's birthday is December 31, and he's talking to you on January 1. Thus two days ago, he was 20 years old on December 30. He turned 21 the next day. "Later *next* year," refers to almost two years later, when he will turn 23 (*this* year on December 31, he will turn 22).
22. You might pick one of each kind of apple on the first three draws. On the fourth draw, you are guaranteed to pick an apple that matches one of the first three, so four draws are required.
23. That man is the son of the person speaking (who, in turn, is the son of his father, and an only child).
24. The blind fiddler is a woman.
25. The woman gained \$100 in each transaction, so she gained \$200 overall.
26. Select a fruit from the box labeled *Apples and Oranges*. If it's an apple, that box must be the *Apple* box (because its original label is incorrect, leaving only *Apples* or *Oranges* as its correct label). The correct label for the box labeled *Oranges* is either *Apples* or *Apples and Oranges*, but we just determined the box containing apples, so the correct label for this box is *Apples and Oranges*. There's only one choice left for the box labeled *Apples*, and that's *Oranges*. A similar argument allows one to determine the correct labeling for each box if an orange is selected first.
27. Select one ball from the first barrel, two balls from the second barrel, three from the third, and so on, ending with ten balls chosen from the tenth barrel. Find the weight of all 55 balls thus chosen. If all the balls weighed one ounce, the weight would be 55 ounces. But we know one of the barrels contains two-ounce balls. Suppose the first barrel contained the two-ounce balls – then the weighing would reveal a result of 56 ounces, and we'd know the first barrel was the one that contains two-ounce balls. If, in fact, the second barrel contained the two-ounce balls, the combined weight would be 57 ounces. A combined weight of 58 ounces means the third barrel contained the heavy balls. Continuing in this fashion, we see that a combined weight of  $n$  ounces more than 55 ounces corresponds to the  $n^{\text{th}}$  barrel containing the two-ounce balls.
28. Seven crossings are required. Trip 1: the woman crosses with the goose. Trip 2: the woman returns to the other shore by herself. Trip 3: the woman takes the mouse across. Trip 4: the woman returns with the goose. Trip 5: the woman brings the wolf across. Trip 6: the woman returns to the other shore by herself. Trip 7: the woman brings the goose over. In this way, the goose is never left alone with the wolf, nor the mouse with the goose. The foursome could get to the other side of the river in five crossings if you give rowing abilities to the animals, but the statement of the problem (and common sense) probably forbids it (the boat "...will hold only herself and one other animal"). Try it anyway for an interesting twist to the problem.
29. Turn both the 7-minute and 4-minute hourglasses over to begin timing. At 4 minutes, turn the 4-minute hourglass over again. At 7 minutes, turn the 7-minute hourglass over; there will be one minute of sand left in the 4-minute hourglass. At 8 minutes, the upper chamber of the 4-minute hourglass will be empty, and one minute of sand will have drained into the lower chamber of the 7-minute hourglass. Turn the 7-minute hourglass over to time the last minute. This is only one solution of many. Another: Turn both hourglasses over whenever their upper chambers are empty. After 12 minutes (three cycles of the 4-minute hourglass), there will be 5 minutes of sand in the bottom chamber of the 7-minute hourglass. Turn that hourglass on its side. Now you are prepared to measure a 9-minute interval whenever you please (use the 5 minutes of sand in the 7-minute hourglass, followed by one cycle of the 4-minute hourglass).
30. Ten rungs will be showing – the boat rises with the tide.
31. A balance scale is one that displays a needle in the middle when both sides of the scale are loaded with equal weights. Put six coins on each side of the scale. The side with the heavy coin will be lower, so discard the other six. Now put three coins of the remaining six on each side of the scale, and as before, discard the three that are in the light group. Finally, select two of the remaining three coins, and put one on each side of the scale. The scale will be even if the heavy coin is not on the scale. The scale will tip to one side if the heavy coin was in the final selection. This is just one solution – another begins by dividing the 12 coins into three groups of four coins. See if you can supply the logic necessary to find the heavy coin in that scenario.

32. It's possible you'll pick a sock of each color on the first two draws, but on the third draw, you are assured of making a match.
33. Since the gray, orange, and pink books are consecutive and the pink book cannot be rightmost, the only possible orders are *gray, orange, pink, other, other* or *other, gray, orange, pink, other*. The only way to add the brown and gold books so that the gold book is not leftmost and there are two books between the brown and gold books is the order *brown, gray, orange, pink, gold*.
34. The first prisoner would know he had a white hat if he saw red hats on prisoners 2 and 3. Since he doesn't know the color of his hat, prisoners 2 and 3 are both wearing white hats, or one has a white hat while the other has a red hat. If prisoner 2 saw a red hat on prisoner 3, he would know that he must have a white hat (he can't have a red hat while seeing red on prisoner 3, because then prisoner 1 would know his color). But he doesn't know, which means prisoner 3 must have a white hat. Prisoner 3 doesn't need to see the hats of the other prisoners; he can deduce that his hat is white simply by hearing the other two prisoners confess they don't know what hats they are wearing.
35. The visitor should patronize the barber with unkempt hair, because he's the barber that cuts the hair of the other barber (who has a splendid hair cut). Of course we are assuming that these barbers don't travel out of town to get their hair done.
36. The coin has revolved twice ( $720^\circ$  of revolution). The best way to see this is to duplicate the experiment. Another way to think about it: consider two circular clocks (instead of coins), nestled side by side, both in the same position, with 12-o'clock pointing straight ahead (see Figure 36). Roll the right clock around the left clock until the 12's are touching. The right clock has rotated  $180^\circ$ , but it's made only  $1/4$  of its journey around the left clock. By the time it makes the full journey around, it will have rotated  $720^\circ$ .



initial position of clocks

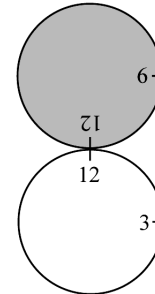
right clock has rotated  $180^\circ$   
(it's upside down), yet it has  
made only  $1/4$  of its journey  
around the left clock

FIGURE 36

37. When a clock chimes five times, there are four pauses between the chimes. Since it takes five seconds to chime five times, each pause lasts 1.25 seconds. There are nine pauses that need to be accounted for when the clock chimes 10 times, and thus it takes  $9 \times 1.25 = 11.25$  seconds to chime 10 times. This solution assumes it takes no time for the clock to chime once (we'd need to know the duration of the chime to solve it otherwise).
38. a. Assume the babies are named A, B, C, and D. If two babies are correctly tagged, the other two labels must be switched for their labels to be incorrect. Thus the problem boils down to counting the number of ways two of the babies can be correctly tagged. Listing all the options is the easiest way to count them. Babies AB could be correctly labeled (leaving babies C and D labeled incorrectly), and the other options for correct labels are AC, AD, BC, BD, and CD. This makes six different ways.
- b. If three of the babies have correct labels, the other baby must also have a correct label, which means there is no way to label three of the babies correctly while the fourth is labeled incorrectly.

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**44** CHAPTER 2: APPROACHES TO PROBLEM SOLVING

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39. Each guest has, in fact, spent \$9. But not all of the \$27 spent is resting in the cash register at the front desk. The front desk has \$25, the bellhop has \$2 (this adds up to the \$27 spent), and each guest has \$1, for a total of \$30. The reason the problem is perplexing is that we think of the \$2 held by the bellhop as a *positive* \$2, and so it should be added to the \$27 spent by the guests to somehow produce the original \$30. But from the perspective of the desk clerk running the cash register, the \$2 is a *negative* value – it represents money out of the till. \$30 (*positive*) came into the till, \$2 (*negative*) went to the bellhop, and \$1 (*negative*) went to each of the guests. Thus the equation we should be looking at is  $30 - 2 - 1 - 1 - 1 = \$25$  in the cash register (which is correct), or  $\$27 - \$2 = \$25$  (also correct), but *not*  $\$27 + \$2$  (which is just silly).