

Solution to Problems in Chapter 17, Section 17.10

17.1. In words, the conservation relation is:

$$\left[\begin{array}{l} \text{Rate of Energy} \\ \text{Accumulation} \\ \text{within the system} \end{array} \right] = \left[\begin{array}{l} \text{Net Rate of Energy} \\ \text{Transfer Across} \\ \text{System Surfaces} \end{array} \right] + \left[\begin{array}{l} \text{Rate of Work} \\ \text{Done on the} \\ \text{System} \end{array} \right] + \left[\begin{array}{l} \text{Rate of Energy} \\ \text{Production} \\ \text{within the system} \end{array} \right]$$

Using a rectangular control volume and the definition of the system energy per unit mass (Equation (17.2.3)) and energy flux (Equation (17.2.4))

$$\Delta x \Delta y \Delta z \rho \frac{\partial \hat{E}}{\partial t} = (e_x|_x - e_x|_{x+\Delta x}) \Delta y \Delta z + (e_y|_y - e_y|_{y+\Delta y}) \Delta x \Delta z + (e_z|_z - e_z|_{z+\Delta z}) \Delta x \Delta y + (\dot{W}_t + \dot{Q}_p^*) \Delta x \Delta y \Delta z$$

Dividing by the volume element $\Delta x \Delta y \Delta z$, taking the limit as the volume goes to zero and using the definition of the derivative yields:

$$\rho \frac{\partial \hat{E}}{\partial t} = -\frac{\partial e_x}{\partial x} - \frac{\partial e_y}{\partial y} - \frac{\partial e_z}{\partial z} + \dot{W}_t + \dot{Q}_p^* \quad (\text{S17.1.1})$$

Using the definition of the divergence of a vector (Equation (A.3.10)), Equation (S17.1.1) becomes

$$\rho \frac{\partial \hat{E}}{\partial t} = -\nabla \cdot \mathbf{e} + \dot{W}_t + \dot{Q}_p^* \quad (\text{S17.1.2})$$

Using the definition of \mathbf{e} , Equation (17.2.4), the divergence of \mathbf{e} is:

$$\nabla \cdot \mathbf{e} = \nabla \cdot (\rho \hat{E} \mathbf{v} + \mathbf{q}) = \rho \hat{E} \nabla \cdot \mathbf{v} + \mathbf{v} \cdot \nabla \rho \hat{E} + \nabla \cdot \mathbf{q} \quad (\text{S17.1.3})$$

For an incompressible fluid, $\nabla \cdot \mathbf{v} = 0$ and ρ is a constant. As a result, Equation (S17.1.3) reduces to:

$$\nabla \cdot \mathbf{e} = \nabla \cdot (\rho \hat{E} \mathbf{v} + \mathbf{q}) = \rho \mathbf{v} \cdot \nabla \hat{E} + \nabla \cdot \mathbf{q} \quad (\text{S17.1.4})$$

Inserting Equation (S17.1.4) into Equation (S17.1.2)

$$\rho \frac{\partial \hat{E}}{\partial t} = -\rho \mathbf{v} \cdot \nabla \hat{E} - \nabla \cdot \mathbf{q} + \dot{W}_t + \dot{Q}_p^* \quad (\text{S17.1.5})$$

Moving both terms with the system energy to the left hand side of Equation (S17.1.5) yields:

$$\rho \left(\frac{\partial \hat{E}}{\partial t} + \mathbf{v} \cdot \nabla \hat{E} \right) = -\nabla \cdot \mathbf{q} + \dot{W}_t + \dot{Q}_p^* \quad (\text{S17.1.5})$$

Lastly, the total rate of work represents work done by fluid stresses ($\nabla \cdot (\boldsymbol{\sigma} \cdot \mathbf{v}) = -\nabla \cdot (p\mathbf{v}) + \nabla \cdot (\boldsymbol{\tau} \cdot \mathbf{v})$), body forces ($\mathbf{F} \cdot \mathbf{v}$) and other types of mechanical work by the body (\dot{W}). Inserting these terms into Equation (S17.1.5) yields Equation (17.2.6)

$$\rho \left(\frac{\partial \hat{E}}{\partial t} + \mathbf{v} \cdot \nabla \hat{E} \right) = -\nabla \cdot \mathbf{q} - \nabla \cdot (p\mathbf{v}) + \nabla \cdot (\boldsymbol{\tau} \cdot \mathbf{v}) + \mathbf{F} \cdot \mathbf{v} + \dot{W} + \dot{Q}_p^* \quad (\text{S17.1.6})$$

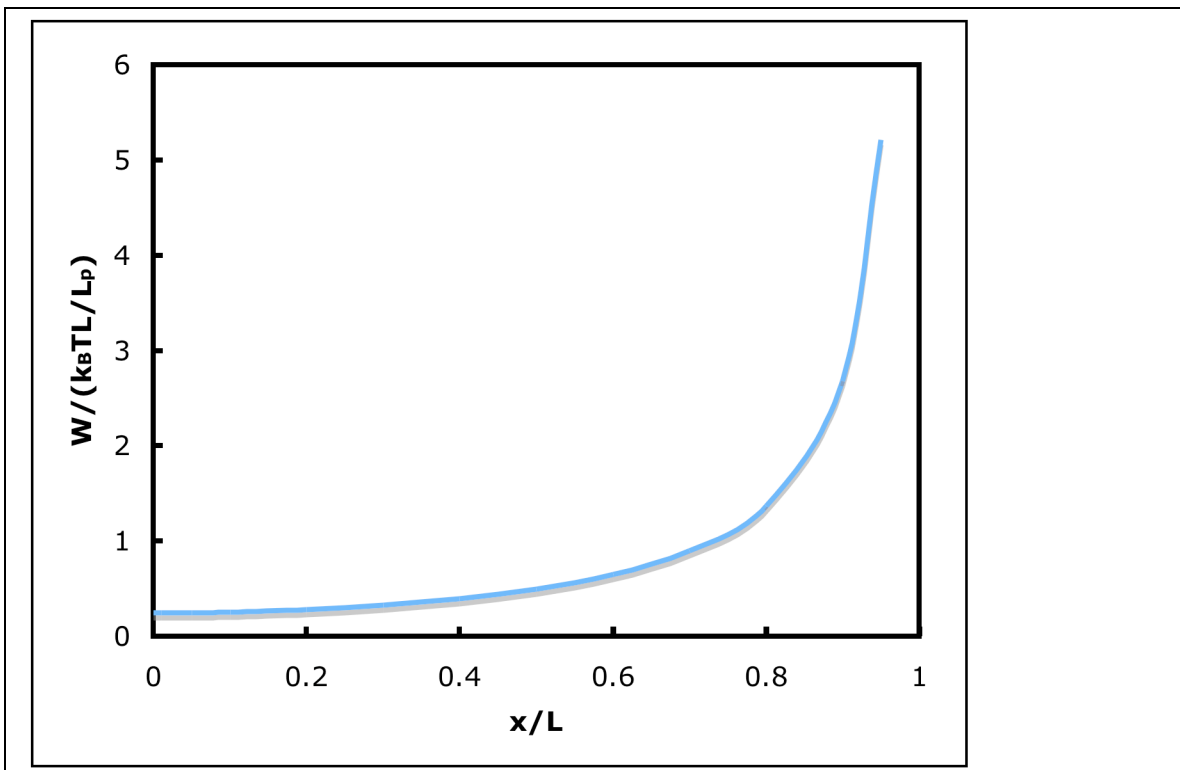
17.2. The work is: $W = \int \mathbf{F} \cdot \mathbf{n} dx = \int F dx$ since the force and unit outward normal are both positive. Normally, a protein is present in a specific conformation which is much less than the maximum length, known as the contour length, L . The contour length is the length of the polymer if each chain element were aligned along a line.

Substituting for the wormlike chain model:

$$W = \left(\frac{k_B T}{L_p} \right) \int_0^x \left(0.25 \left(1 - \frac{x}{L} \right)^{-2} - 0.25 + \frac{x}{L} \right) dx = \left(\frac{k_B T}{L_p} \right) \left(0.25 L \left(1 - \frac{x}{L} \right)^{-1} - 0.25 x + \frac{x^2}{2L} \right)$$

$$W = \left(\frac{k_B T L}{L_p} \right) \left(0.25 \left(1 - \frac{x}{L} \right)^{-1} - 0.25 \frac{x}{L} + \frac{x^2}{2L^2} \right)$$

This result is shown in the graph below. At low extensions, the relation is linear. However, as x approaches L , the work increases dramatically. The flexibility of the polymer arises from the arrangement of the chains. As the polymer elongates, more work must be done to extend the polymer to overcome the tendency for the chains to move freely and to extend each element.



17.3. Note: The equation listed in the problem statement should be:

$$\dot{\Phi}_v = \boldsymbol{\tau} : \nabla \mathbf{v} = \mu \left(\frac{dv_z}{dr} \right)^2$$

The shear stress tensor for a Newtonian fluid is:

$$\boldsymbol{\tau} = \mu (\nabla \mathbf{v} + (\nabla \mathbf{v})^T) \quad (\text{S17.3.1})$$

$$\dot{\Phi}_v = \boldsymbol{\tau} : \nabla \mathbf{v} = \mu (\nabla \mathbf{v} + (\nabla \mathbf{v})^T) : \nabla \mathbf{v} \quad (\text{S17.3.2})$$

Using the summation convention for vectors and tensors

$$\mu (\nabla \mathbf{v} + (\nabla \mathbf{v})^T) : \nabla \mathbf{v} = \mu \left(\sum_{i=1}^3 \sum_{j=1}^3 \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \mathbf{e}_i \mathbf{e}_j \right) : \left(\sum_{k=1}^3 \sum_{l=1}^3 \frac{\partial v_k}{\partial x_l} \mathbf{e}_k \mathbf{e}_l \right) \quad (\text{S17.3.3})$$

Since $\mathbf{e}_i \mathbf{e}_j : \mathbf{e}_k \mathbf{e}_l = (\mathbf{e}_j \cdot \mathbf{e}_k)(\mathbf{e}_i \cdot \mathbf{e}_l) = \delta_{jk} \delta_{il}$, Equation (S17.3.3) becomes:

$$\mu (\nabla \mathbf{v} + (\nabla \mathbf{v})^T) : \nabla \mathbf{v} = \mu \sum_{i=1}^3 \sum_{j=1}^3 \left(\left(\frac{\partial v_i}{\partial x_j} \right) \left(\frac{\partial v_j}{\partial x_i} \right) + \left(\frac{\partial v_j}{\partial x_i} \right)^2 \right) \quad (\text{S17.3.4})$$

For fully developed steady, laminar flow in a cylindrical tube of radius R ,

$$\boldsymbol{\tau} = \mu \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right) \mathbf{e}_r \mathbf{e}_z$$

Using the symmetry property of the shear stress, $\tau_{ij} = \tau_{ji}$:

$$\begin{aligned} \dot{\Phi}_v &= \mu \left(\frac{\partial v_z}{\partial r} \mathbf{e}_r \mathbf{e}_z + \frac{\partial v_r}{\partial z} \mathbf{e}_z \mathbf{e}_r \right) : \frac{\partial v_z}{\partial r} \mathbf{e}_r \mathbf{e}_z \\ &= \mu \left(\frac{\partial v_z}{\partial r} \mathbf{e}_r \mathbf{e}_z + \frac{\partial v_z}{\partial r} \mathbf{e}_z \mathbf{e}_r \right) : \frac{\partial v_z}{\partial r} \mathbf{e}_r \mathbf{e}_z = \mu \left(\frac{\partial v_z}{\partial r} \frac{\partial v_z}{\partial r} \right) = \mu \left(\frac{\partial v_z}{\partial r} \right)^2 \end{aligned} \quad (\text{S17.3.5})$$

For laminar flow in a tube

$$v_z = 2 \langle v_z \rangle \left(1 - \frac{r^2}{R^2} \right) \quad (\text{S17.3.6a})$$

$$\frac{\partial v_z}{\partial r} = \frac{2 \langle v_z \rangle r}{R^2} \quad (\text{S17.3.6b})$$

The velocity gradient is maximum at $r = R$. Thus, the maximum value of viscous dissipation is:

$$\dot{\Phi}_v = \mu \left(\frac{4 \langle v_z \rangle}{R} \right)^2 = \frac{16 \mu \langle v_z \rangle^2}{R^2} \quad (\text{S17.3.7})$$

In terms of flow rate

$$\dot{\Phi}_v \Big|_{\max} = \frac{16 \mu Q^2}{\pi^2 R^6} \quad (\text{S17.3.8})$$

For the data given:

$$\dot{\Phi}_v \Big|_{\max} = \frac{16(0.01 \text{ g cm}^{-1} \text{ s}^{-1})(83.3 \text{ cm}^3 \text{ s}^{-1})^2}{\pi^2 (1.5 \text{ cm})^6} = 22.22 \text{ g cm}^{-1} \text{ s}^{-3} = 2.22 \text{ kg m}^{-1} \text{ s}^{-3}$$

$$\dot{\Phi}_v \Big|_{\max} = 2.22 \text{ Pa s}^{-1} = 2.22 \text{ J m}^{-3} \text{ s}^{-1} = 2.22 \text{ W m}^{-3}$$

To compute the maximum heating in blood arising from viscous dissipation, assume steady radial conduction with viscous dissipation. From Equations (17.2.8), (17.2.9), (17.2.12) and (S17.3.6b), the following result is obtained.

$$\frac{k}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) = -\dot{\Phi}_v = -\frac{16\mu Q^2 r^2}{\pi^2 R^8} \quad (\text{S17.3.9})$$

The boundary conditions are that for $r = 0$, the flux is zero and at $r = R$, $T = T_0$. Integrating Equation (S17.3.9) once yields:

$$\frac{dT}{dr} = -\frac{4\mu Q^2 r^3}{k\pi^2 R^8} + \frac{C}{r} \quad (\text{S17.3.10})$$

From the boundary condition at $r = 0$, $C = 0$. Integrating Equation (S17.3.10) yields:

$$T = -\frac{\mu Q^2 r^4}{k\pi^2 R^8} + C_2 = -\frac{\dot{\Phi}_v \Big|_{\max} r^4}{kR^2} + C_2 \quad (\text{S17.3.11})$$

From the boundary condition at $r = R$, C_2 is

$$C_2 = T_0 + \frac{\dot{\Phi}_v \Big|_{\max} R^2}{k} \quad (\text{S17.3.12})$$

The temperature profile is:

$$T = T_0 + \frac{\dot{\Phi}_v \Big|_{\max} R^2}{k} \left(1 - \frac{r^4}{R^4} \right) \quad (\text{S17.3.13})$$

The maximum temperature difference occurs between $r = 0$ and $r = R$:

$$\Delta T_{\max} = T(r=0) - T_0 = \frac{\dot{\Phi}_v \Big|_{\max} R^2}{k} \quad (\text{S17.3.14})$$

For the value of the viscous dissipation obtained above and the thermal conductivity of blood (Table 17.2):

$$\Delta T_{\max} = \frac{(2.22 \text{ W m}^{-3})(0.015 \text{ m})^2}{0.642 \text{ W m}^{-1} \text{K}^{-1}} = 0.00078 \text{ K} \quad (\text{S17.3.15})$$

Thus, viscous dissipation has a very minor effect on the temperature of blood and can be neglected.

17.4. For steady conduction for a spherical surface of radius R, Equation (17.2.14c) simplifies to:

$$\frac{k}{r^2} \frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) = 0 \quad (\text{S17.4.1})$$

The boundary conditions are at $r = R$, $T = T_0$ and as $r \rightarrow \infty$, $T = T_\infty$. Integrating equation (S17.4.1) twice yields:

$$T = \frac{C_1}{r} + C_2 \quad (\text{S17.4.2})$$

From the boundary condition as $r \rightarrow \infty$, $C_2 = T_\infty$. At $r = R$

$$C_1 = (T_0 - T_\infty)R \quad (\text{S17.4.3})$$

The temperature profile is:

$$T = (T_0 - T_\infty) \frac{R}{r} + T_\infty \quad (\text{S17.4.4})$$

To obtain the Nusselt number, compute the flux at $r = R$ and apply the definition of the heat transfer coefficient:

$$-k \left. \frac{dT}{dr} \right|_{r=R} = k \frac{(T_0 - T_\infty)}{R} = h_m (T_0 - T_\infty) \quad (\text{S17.4.5})$$

The heat transfer coefficient for conduction is:

$$h_m = \frac{k}{R} = 2 \frac{k}{D} \quad (\text{S17.4.6})$$

Using this result in the definition of the Nusselt number:

$$Nu_m = \frac{h_m D}{k} = 2 \frac{kD}{kD} = 2 \quad (\text{S17.4.7})$$

17.5. The definition of β is given by Equation (17.4.7)

$$\beta = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p \quad (\text{S17.5.1})$$

From the ideal gas relationship, $PV = nRT$. For a fixed number of moles, $V = nRT/P$ and the derivative in Equation (S17.5.1) is:

$$\left(\frac{\partial V}{\partial T} \right)_p = \frac{nR}{P} \quad (\text{S17.5.2})$$

Thus,

$$\beta = \frac{nR}{VP} = \frac{1}{T} \quad (\text{S17.5.3})$$

since $T = PV/nR$ for an ideal gas.

17.6. For this problem, assume unsteady conduction in a tissue of thickness $2L$. Based upon analogy with unsteady diffusion in a region of half thickness of L , the time to reach steady state is $2L^2/\alpha$. While specific thermal diffusivities for tissue are not provided in Table 17.2, a reasonable value, between water and fat, is $1.1 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$. For the half-thickness of $125 \mu\text{m} = 1.25 \times 10^{-4} \text{ m}$, the time to reach steady state is 0.284 s . So, one would expect uniform temperatures in well perfused tissues.

17.7. Note: The phase change during freezing is discussed in Section 17.3.4, not Section 17.3.3.

The rate of growth of the ice front is $\frac{dX}{dt}$. X is given by Equation (17.3.26b). Thus,

$$\frac{dX}{dt} = C \sqrt{\frac{\alpha_s}{t}} \quad (\text{S17.7.1})$$

C is dimensionless and is provided by solving Equation (17.3.31) or Equation (17.3.33). Values of C are tabulated in Table 17.3 for several different values of $T_m - T_0$ and α_s is given in Table 17.2 as $1.06 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$. For a value of $T_m - T_0 = 10 \text{ C}$, $C = 0.183$ and the derivative in Equation (S17.7.1) is $(1.8448 \times 10^{-4})t^{-1/2} \text{ m s}^{-1}$.

17.8. This problem is a modification of the problem presented in Example 6.6. Thus, Equation (6.7.25) applies for the distribution of vapor concentration in a column of height δ .

$$\frac{d}{dy} \left(\frac{1}{1-x} \frac{dx}{dy} \right) = 0 \quad (\text{S17.8.1})$$

The boundary conditions are that, at $y = h$, $x = x_a$ which is the vapor pressure at the given temperature and pressure. At $y = h + \delta$, $x = x_s$, the relative humidity in the air. Integrating Equation (S17.8.1) once yields:

$$\frac{dx}{dy} = C_1 (1-x) \quad (\text{S17.8.2})$$

Integrating again,

$$\ln(1-x) = -C_1 y + C_2 \quad (\text{S17.8.3})$$

Applying the boundary conditions:

$$\ln(1-x_s) = -C_1 h + C_2 \quad (\text{S17.8.4a})$$

$$\ln(1-x_a) = -C_1 (h + \delta) + C_2 \quad (\text{S17.8.4b})$$

Subtracting (S17.8.4b) from Equation (S17.8.4a)

$$C_1 = \frac{1}{\delta} \ln \left(\frac{1-x_a}{1-x_s} \right) \quad (\text{S17.8.4c})$$

Inserting Equation (S17.8.4c) in Equation (S17.8.4b) and solving for C_2 yields;

$$\ln(1-x_a) = -\ln \left(\frac{1-x_s}{1-x_s} \right) \left(\frac{h+\delta}{\delta} \right) + C_2 \quad (\text{S17.8.4d})$$

$$C_2 = \ln(1 - x_a) + \ln\left(\frac{1 - x_s}{1 - x_a}\right)\left(\frac{h + \delta}{\delta}\right) \quad (\text{S17.8.4d})$$

The solution is:

$$\ln\left(\frac{1 - x}{1 - x_a}\right) = -\frac{y}{\delta} \ln\left(\frac{1 - x_s}{1 - x_a}\right) + \left(\frac{h + \delta}{\delta}\right) \ln\left(\frac{1 - x_s}{1 - x_a}\right) \quad (\text{S17.8.4d})$$

Add the term $\ln((1-x_a)/(1-x_s))$ to each side:

$$\ln\left(\frac{1 - x}{1 - x_a}\right) + \ln\left(\frac{1 - x_a}{1 - x_s}\right) = -\frac{y}{\delta} \ln\left(\frac{1 - x_s}{1 - x_a}\right) + \left(\frac{h}{\delta} + 1\right) \ln\left(\frac{1 - x_s}{1 - x_a}\right) + \ln\left(\frac{1 - x_a}{1 - x_s}\right) \quad (\text{S17.8.5})$$

Collect terms

$$\ln\left(\frac{1 - x}{1 - x_s}\right) = \frac{y}{\delta} \ln\left(\frac{1 - x_a}{1 - x_s}\right) - \frac{h}{\delta} \ln\left(\frac{1 - x_a}{1 - x_s}\right) = \frac{y - h}{\delta} \ln\left(\frac{1 - x_a}{1 - x_s}\right) \quad (\text{S17.8.6})$$

Raising each side to the power e:

$$\left(\frac{1 - x}{1 - x_s}\right) = \left(\frac{1 - x_a}{1 - x_s}\right)^{\frac{y-h}{\delta}} \quad (\text{S17.8.7})$$

17.9. The vapor flux is given by Equation (17.5.11)

$$N_{y=h} = \frac{cD_{w,air}}{\delta} \ln\left(\frac{1 - x_a}{1 - x_s}\right)$$

where x_s is the partial pressure of water in air at saturation (vapor pressure/total air pressure) and x_a is the partial pressure of water/total air pressure. The quantity x_a can be expressed as $x_H x_s$, where x_H is the relative humidity. Using the data for Problem 17.10 and a total air pressure of 101,325 Pa. The quantity $c = p_{tot}/RT = 101,325 \text{ Pa}/(8.314 \text{ N m K}^{-1} \text{ mol}^{-1})(298 \text{ K}) = 40.90 \text{ mole m}^{-3}$. The diffusivity of water in air is provided in the text, p. 797, as $2.6 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$. Thus, $x_s = 0.0310$ at 25 C and 0.0728 at 40 C. For 20% relative humidity at 25 C.

$$N_{y=h} = \frac{(40.90 \text{ mol m}^{-3})(2.6 \times 10^{-5} \text{ m}^2 \text{ s}^{-1})}{0.0136 \text{ m}} \ln\left(\frac{1 - 0.20(0.031)}{1 - 0.031}\right) = 0.0020 \text{ mol m}^{-2} \text{ s}^{-1}$$

For 80% relative humidity, the flux is $0.00050 \text{ mol m}^{-2} \text{ s}^{-1}$.

17.10. The error can be computed from the ratio of Equations (17.5.12) to Equation (17.5.13):

$$\text{error} = x_s - x_a \ln\left(\frac{1 - x_s}{1 - x_a}\right) = x_s (1 - x_H) \ln\left(\frac{1 - x_s}{1 - x_s x_H}\right)$$

At 25 C the error is -0.000226 and at 40 C the error rises to -0.00128. Thus, Equation (17.5.13) is a good approximation.

17.11. Since the enthalpy of vaporization is a function of temperature, application of Equation (17.5.25) or Equation (17.5.26) is done iteratively. That is, the enthalpy of vaporization is updated, once the temperature at the air-sweat interface is calculated. The flux for the

evaporating liquid is temperature independent and was found to be $0.001 \text{ mol m}^{-2} \text{ s}^{-1}$ for 60% relative humidity. For the calculation reported in the text, Equation (17.5.5a) was used and $\Delta\bar{H}_{vap}$ was determined for a temperature of 25 C. Using T equal to 37 C, $\Delta\bar{H}_{vap} = 54047.6 \text{ J mol}^{-1}$. The temperature drop is 0.444 C and the energy flux is $54.05 \text{ J m}^{-2} \text{ s}^{-1}$. Updating the values at T = 309.7 K, the temperature drop is 0.444 C and the energy flux is $54.15 \text{ J m}^{-2} \text{ s}^{-1}$. These values are within 1% of the values obtained for T = 310.00 K.

17.12. Use Equation (17.4.3) to calculate the Nusselt number. The Prandtl number does not vary significantly with temperature and a value of 0.72 is commonly used for air. The kinemtic viscosity of air $0.1327 \text{ cm}^2 \text{ s}^{-1} = 1.327 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$. As noted on page 797, a characteristic diameter for a typical female is 0.304 m. The following table lists values of Re, Nu, h and q for various wind speeds. The energy flux can be quite substantial and is reduced significantly by clothing.

v, miles/h	v, m/s	Re	Nu	h, $\text{W m}^{-2} \text{ K}^{-1}$	q, W m^{-2}
1	0.447	10241	54.64	4.49	143.80
2	0.894	20482	80.86	6.65	212.79
5	2.235	51206	140.33	11.54	369.30
10	4.470	102412	220.05	18.10	579.08
25	11.176	256029	420.46	34.58	1106.48

17.13. Start with the definition of the Grashof number, Equation (17.4.22)

$$Gr = \frac{\rho^2 g \beta \Delta T L^3}{\mu^2}$$

The definition of β in terms of the density is given by Equation (17.4.6)

$$\rho \approx \rho_0 - \rho_0 \beta \Delta T$$

Let $\Delta\rho = \rho_0 - \rho$. Thus, $\Delta\rho \approx \rho_0 \beta \Delta T$. Assuming that density in the definition of the Grashof number is the value at the reference temperature, ρ_0 , the Grashof number becomes:

$$Gr = \frac{\rho_0^2 g \beta \Delta T L^3}{\mu^2} = \frac{\rho_0^2 g \Delta\rho L^3}{\mu^2 \rho_0} = \frac{\rho_0 g \Delta\rho L^3}{\mu^2}$$

17.14. For free convection, Equation (17.4.5) is used for flow over a sphere. The viscosity ratio is 0.900 and Pr = 0.72.

	v, miles/h	v, m/s	Diameter, m	Re	Nu
adult	10	4.47	0.178	60050	164.29
child	10	4.47	0.124	41820	133.60

For free convection, the Grashof number is calculated using Equation (17.4.22) with L equal to the diameter and $\beta = 1/T$ where T is the air temperature (273.15 K). Equation (17.4.24) is used to determine the Nusselt number for a flat plate. The correlation for spheres is found in reference [18], page 301.

$$Nu = 2.0 + 0.43(\text{Pr} Gr)^{1/4}$$

	Diameter, m	Gr	Nu, flat plate	Nu, sphere
adult	0.178	42697290	38.57	34.02
child	0.124	14422151	29.40	26.41

For radiation, the energy flux is given by Equation (17.2.19c). Treating the absorptivity and emissivity as the same, the flux equals $q = \sigma e(T_b^4 - T_{air}^4)$. A heat transfer coefficient can be defined as $h = q/\Delta T$ and a Nussel number determined. Results are:

q_{rad}	h	Nu adult	Nu child
193.44	5.23	37.22	25.93

Comparing results, the free convection and radiation terms are comparable and are about 20% of the value for forced convection.

17.15. Note, there is a typographical error in the text and Equation (17.5.25) should be:

$$T_a = T_{air} + \left(\frac{\Delta \bar{H}^{vap} N_{y=h} h}{k_l} - \Delta T \right) \frac{\left[\exp\left(\frac{\rho \hat{C}_p}{C_{vap} k_a} N_{y=h} (y-h) \right) - \exp\left(\frac{\rho \hat{C}_p}{C_{vap} k_a} N_{y=h} \delta \right) \right]}{\frac{k_a}{k_l} \left(\frac{h N_{y=h} \rho \hat{C}_p}{C_{vap} k_a} \right) - \left[1 - \exp\left(\frac{\rho \hat{C}_p}{C_{vap} k_a} N_{y=h} \delta \right) \right]} \quad (17.5.25)$$

Begin with Equation (17.5.21) for air and Equation (17.5.24) for the liquid.

$$T_a = \frac{a_1 C_{vap} k_a}{N_{y=h} \rho \hat{C}_p} \exp\left(\frac{\rho \hat{C}_p}{C_{vap} k_a} N_{y=h} y \right) + a_2 \quad (17.5.21)$$

$$T_l = a_3 y + a_4 \quad (17.5.24)$$

The boundary conditions are:

$$y = 0 \quad T_l = T_b \quad (S17.15.1a)$$

$$y = h \quad T_l = T_a \quad (S17.15.1b)$$

$$k_a \left. \frac{dT_a}{dy} \right|_{y=h} - k_l \left. \frac{dT_l}{dy} \right|_{y=h} = \Delta \bar{H}^{vap} N_{y=h} \quad (S17.15.1c)$$

$$y = h + \delta \quad T_a = T_{air} \quad (S17.15.1a)$$

From the boundary condition at $y = 0$

$$a_4 = T_b \quad (S17.15.2a)$$

and

$$T_l = a_3 y + T_b \quad (S17.15.2b)$$

From the boundary condition at $y = h + \delta$,

$$a_2 = T_{\text{air}} - \frac{a_1 C_{\text{vap}} k_a}{N_{y=h} \rho \hat{C}_p} \exp\left(\frac{\rho \hat{C}_p}{C_{\text{vap}} k_a} N_{y=h} (h + \delta)\right) \quad (\text{S17.15.3a})$$

$$T_a = T_{\text{air}} + \frac{a_1 C_{\text{vap}} k_a}{N_{y=h} \rho \hat{C}_p} \left[\exp\left(\frac{\rho \hat{C}_p}{C_{\text{vap}} k_a} N_{y=h} y\right) - \exp\left(\frac{\rho \hat{C}_p}{C_{\text{vap}} k_a} N_{y=h} (h + \delta)\right) \right] \quad (\text{S17.15.3b})$$

Equating Equations (S17.15.2b) and (S17.15.3b) at $y = h$,

$$a_3 h + T_b = T_{\text{air}} + \frac{a_1 C_{\text{vap}} k_a}{N_{y=h} \rho \hat{C}_p} \left[\exp\left(\frac{\rho \hat{C}_p}{C_{\text{vap}} k_a} N_{y=h} h\right) - \exp\left(\frac{\rho \hat{C}_p}{C_{\text{vap}} k_a} N_{y=h} (h + \delta)\right) \right] \quad (\text{S17.15.4a})$$

$$a_3 = -\frac{\Delta T}{h} + \frac{a_1 C_{\text{vap}} k_a}{h N_{y=h} \rho \hat{C}_p} \exp\left(\frac{\rho \hat{C}_p}{C_{\text{vap}} k_a} N_{y=h} h\right) \left[1 - \exp\left(\frac{\rho \hat{C}_p}{C_{\text{vap}} k_a} N_{y=h} \delta\right) \right] \quad (\text{S17.15.4b})$$

where $\Delta T = T_b - T_{\text{air}}$.

The liquid temperature is

$$T_l = -\Delta T \frac{y}{h} + \frac{y a_1 C_{\text{vap}} k_a}{h N_{y=h} \rho \hat{C}_p} \exp\left(\frac{\rho \hat{C}_p}{C_{\text{vap}} k_a} N_{y=h} h\right) \left[1 - \exp\left(\frac{\rho \hat{C}_p}{C_{\text{vap}} k_a} N_{y=h} \delta\right) \right] + T_b \quad (\text{S17.15.5})$$

Use Equations (S17.15.3b) and (S17.15.5) to compute the derivatives of the temperature. The boundary condition, Equation (S17.15.1c), becomes:

$$a_1 k_a \exp\left(\frac{\rho \hat{C}_p}{C_{\text{vap}} k_a} N_{y=h} h\right) + \frac{k_l \Delta T}{h} - \frac{k_l a_1 C_{\text{vap}} k_a}{h N_{y=h} \rho \hat{C}_p} \exp\left(\frac{\rho \hat{C}_p}{C_{\text{vap}} k_a} N_{y=h} h\right) \left[1 - \exp\left(\frac{\rho \hat{C}_p}{C_{\text{vap}} k_a} N_{y=h} \delta\right) \right] = \Delta \bar{H}^{\text{vap}} N_{y=h} \quad (\text{S17.15.6})$$

Solving for a_1 :

$$a_1 = \frac{\exp\left(-\frac{\rho \hat{C}_p}{C_{\text{vap}} k_a} N_{y=h} h\right) \left(\Delta \bar{H}^{\text{vap}} N_{y=h} - \frac{k_l \Delta T}{h} \right)}{k_a - \frac{k_l C_{\text{vap}} k_a}{h N_{y=h} \rho \hat{C}_p} \left[1 - \exp\left(\frac{\rho \hat{C}_p}{C_{\text{vap}} k_a} N_{y=h} \delta\right) \right]} \quad (\text{S17.15.7})$$

Inserting this expression for a_1 into Equation (S17.15.3b) yields the final result for the air temperature.

$$T_a = T_{air} + \left(\frac{\Delta \bar{H}^{vap} N_{y=h} - k_l \Delta T}{h} \right) \frac{\frac{C_{vap} k_a}{N_{y=h} \rho \hat{C}_p} \left[\exp \left(\frac{\rho \hat{C}_p}{C_{vap} k_a} N_{y=h} (y-h) \right) - \exp \left(\frac{\rho \hat{C}_p}{C_{vap} k_a} N_{y=h} \delta \right) \right]}{k_a - \frac{k_l C_{vap} k_a}{h N_{y=h} \rho \hat{C}_p} \left[1 - \exp \left(\frac{\rho \hat{C}_p}{C_{vap} k_a} N_{y=h} \delta \right) \right]} \quad (S17.15.8a)$$

Rearrange to yield the correct form of Equation (17.5.25)

$$T_a = T_{air} + \left(\frac{\Delta \bar{H}^{vap} N_{y=h} h - \Delta T}{k_l} \right) \frac{\left[\exp \left(\frac{\rho \hat{C}_p}{C_{vap} k_a} N_{y=h} (y-h) \right) - \exp \left(\frac{\rho \hat{C}_p}{C_{vap} k_a} N_{y=h} \delta \right) \right]}{\frac{k_a}{k_l} \left(\frac{h N_{y=h} \rho \hat{C}_p}{C_{vap} k_a} \right) - \left[1 - \exp \left(\frac{\rho \hat{C}_p}{C_{vap} k_a} N_{y=h} \delta \right) \right]} \quad (17.5.25)$$

At $y = h$, Equation (17.5.25) is:

$$T_a = T_{air} + \left(\frac{\Delta \bar{H}^{vap} N_{y=h} h - \Delta T}{k_l} \right) \frac{\left[1 - \exp \left(\frac{\rho \hat{C}_p}{C_{vap} k_a} N_{y=h} \delta \right) \right]}{\frac{k_a}{k_l} \left(\frac{h N_{y=h} \rho \hat{C}_p}{C_{vap} k_a} \right) - \left[1 - \exp \left(\frac{\rho \hat{C}_p}{C_{vap} k_a} N_{y=h} \delta \right) \right]} \quad (S17.15.8b)$$

The group $\frac{h N_{y=h} \rho \hat{C}_p}{C_{vap} k_a}$ can be rewritten as the following by using Equation (17.5.17):

$$\frac{h N_{y=h} \rho \hat{C}_p}{C_{vap} k_a} = \frac{h C_{vap} v_y \rho \hat{C}_p}{C_{vap} k_a} = \frac{h v_y \rho \hat{C}_p}{k_a} = Pe_{air}$$

$$T_a = T_{air} + \left(\frac{\Delta \bar{H}^{vap} N_{y=h} h - \Delta T}{k_l} \right) \frac{\left[1 - \exp(Pe_{air} \delta / h) \right]}{\frac{k_a}{k_l} (Pe_{air}) - \left[1 - \exp(Pe_{air} \delta / h) \right]} \quad (S17.15.8c)$$

The thermal Peclet number for air is 0.20, which is larger than the value for sweat, but still much less than 1. For the case of conduction only, energy transport through the liquid is unchanged. Equation (17.5.17) for the air simplifies to:

$$\frac{d^2 T_a}{dy^2} = 0 \quad (S17.15.9)$$

After integration we obtain:

$$T_a = a_1 y + a_2$$

$$\text{At } y = h + \delta,$$

$$a_2 = T_{air} - a_1 (h + \delta)$$

$$T_a = a_1(y - (h + \delta)) + T_{air}$$

$$T_l = a_3 y + T_b$$

Equating the air and sweat temperatures at $y = h$:

$$a_3 h + T_b = -a_1 \delta + T_{air}$$

$$a_3 = -a_1 \frac{\delta}{h} + \frac{T_{air} - T_b}{h} = -a_1 \frac{\delta}{h} - \frac{\Delta T}{h}$$

$$T_l = \left[-a_1 \frac{\delta}{h} - \frac{\Delta T}{h} \right] y + T_b$$

Use these results for T_a and T_l to compute the derivatives in Equation (S17.15.1c)

$$k_a a_1 + k_l \left(a_1 \frac{\delta}{h} + \frac{\Delta T}{h} \right) = \Delta \bar{H}^{vap} N_{y=h} \quad (\text{S17.15.10a})$$

Solving for a_1 :

$$a_1 \left(k_a + k_l \frac{\delta}{h} \right) = \Delta \bar{H}^{vap} N_{y=h} - k_l \frac{\Delta T}{h} \quad (\text{S17.15.10b})$$

$$a_1 = \frac{\Delta \bar{H}^{vap} N_{y=h} - k_l \frac{\Delta T}{h}}{\left(k_a + k_l \frac{\delta}{h} \right)} \quad (\text{S17.15.10c})$$

The resulting expression for the air temperature is:

$$T_a = T_{air} + \left(\frac{k_l \frac{\Delta T}{h} - \Delta \bar{H}^{vap} N_{y=h}}{\left(k_a + k_l \frac{\delta}{h} \right)} \right) ((h + \delta) - y) = T_{air} + \left(\frac{\Delta T - \frac{h \Delta \bar{H}^{vap} N_{y=h}}{k_l}}{\left(\frac{k_a}{k_l} \right) h + \delta} \right) ((h + \delta) - y)$$

for $y = h$

$$T_a(y = h) = T_{air} + \left(\frac{\Delta T - \frac{h \Delta \bar{H}^{vap} N_{y=h}}{k_l}}{\left(\frac{k_a}{k_l} \right) h + \delta} \right) \delta \quad (\text{S17.15.11})$$

For values of h (0.005 m) and δ (0.0136 m) provided in Section 17.5 and thermal conductivities of air and water in Table 17.2, $\frac{\delta}{\left(\frac{k_a}{k_l} \right) h + \delta} = 0.985$. Thus, the approximation presented in

Equation (17.5.26) is reasonable. Further, Equation (17.5.26) arises as a limiting value of Equation (17.5.25) when $k_a/k_l Pe \ll 1$.

If vaporization does not occur, then $\Delta\bar{H}^{vap} = 0$ and Equation (17.5.27) results.

17.16. From Table 2.4, the blood vessel diameters range from 6×10^{-6} m to 5×10^{-5} m. Corresponding mean velocities range from 2×10^{-4} to 0.001 m s⁻¹. The Pe ranges from 0.0068 to 0.284. Blood vessel densities range from 2.0×10^8 vessels m⁻² to 2.22×10^9 vessels m⁻². The ratio of thermal conductivities between blood and tissue range from 1.5 to 3. From Equation (17.7.4), k_{eff}/k_{tissue} ranges from 1.00 to 5.21.