Chapter 2
Set Theory

Check Points 2.1

1. Set L is the set of the first six lowercase letters in the English alphabet.

2. $M = \{\text{April, August}\}$

3. $O = \{1, 3, 5, 7, 9\}$

4. **a.** not the empty set; Many numbers meet the criteria to belong to this set.

   b. the empty set; No numbers meet the criteria, thus this set is empty

   c. not the empty set; "nothing" is not a set.

   d. not the empty set; This is a set that contains one element, that element is a set.

5. **a.** true; 8 is an element of the given set.

   b. true; r is not an element of the given set.

   c. false; {Monday} is a set and the set {Monday} is not an element of the given set.

6. **a.** $A = \{1, 2, 3\}$

   b. $B = \{15, 16, 17, \ldots\}$

   c. $O = \{1, 3, 5, \ldots\}$

7. **a.** {1, 2, 3, 4, ..., 199}

   b. {51, 52, 53, 54, ..., 200}

8. **a.** $n(A) = 5$; the set has 5 elements

   b. $n(B) = 1$; the set has only 1 element

   c. $n(C) = 8$; Though this set lists only five elements, the three dots indicate 12, 13, and 14 are also elements.

   d. $n(D) = 0$ because the set has no elements.

9. No, the sets are not equivalent. Set A has 5 elements yet set B has only 4 elements.

10. **a.** true; {O, L, D} = {D, O, L} because the sets contain exactly the same elements.

    b. false; The two sets do not contain exactly the same elements.

Concept and Vocabulary Check 2.1

1. roster; set builder

2. empty; $\varnothing$

3. is an element

4. natural numbers

5. cardinal; $n(A)$

6. equivalent

7. equal

Exercise Set 2.1

1. This is well defined and therefore it is a set.

2. This is well defined and therefore it is a set.

3. This is a matter of opinion and not well defined, thus it is not a set.

4. This is a matter of opinion and not well defined, thus it is not a set.

5. This is well defined and therefore it is a set.

6. This is well defined and therefore it is a set.

7. The set of known planets in our Solar System. Note to student: This exercise did not forget Pluto. In 2006, based on the requirement that a planet must dominate its own orbit, the International Astronomical Union removed Pluto from the list of planets.

8. The set of weekend days.

9. The set of months that begin with J.

10. The set of months that begin with A.

11. The set of natural numbers greater than 5.

12. The set of natural numbers greater than 8.

13. The set of natural numbers between 6 and 20, inclusive.

14. The set of natural numbers between 9 and 25, inclusive.

15. {winter, spring, summer, fall}

16. {April, June, September, November}

17. {September, October, November, December}

18. {e, f, g, h, i}

19. {1, 2, 3}

20. {1, 2, 3, 4, 5, 6}

21. {1, 3, 5, 7, 9, 11}

22. {2, 4, 6, 8}

23. {1, 2, 3, 4, 5}

24. {1, 2, 3, 4}

25. {6, 7, 8, 9, …}

26. {5, 6, 7, 8, …}

27. {7, 8, 9, 10}

28. {8, 9, 10, 11}

29. {10, 11, 12, 13, …, 79}

30. {15, 16, 17, 18, …, 59}}

31. {2}

32. {6}

33. not the empty set

34. not the empty set

35. empty set

36. empty set

37. not the empty set
Note that the number of women who served as U.S. president before 2016 is 0. Thus the number 0 is an element of the set.

38. not the empty set
Note that the number of living U.S. presidents born before 1200 is 0. Thus the number 0 is an element of the set.

39. empty set

40. empty set

41. empty set

42. empty set

43. not the empty set

44. not the empty set

45. not the empty set

46. not the empty set

47. true
3 is a member of the set.

48. true
6 is a member of the set.

49. true
12 is a member of the set.

50. true
10 is a member of the set.

51. false
5 is *not* a member of the set.

52. false
8 is *not* a member of the set.

53. true
11 is *not* a member of the set.

54. true.
17 is *not* a member of the set.

55. false
37 is a member of the set.

56. false
26 is a member of the set.

57. false
4 is a member of the set.

58. false
2 is *not* a member of the set.

59. true
13 is *not* a member of the set.

60. true
20 is *not* a member of the set.

61. false
16 is a member of the set.

62. false
19 is a member of the set.

63. false
The set {3} is *not* a member of the set.

64. false
The set {7} is *not* a member of the set.

65. true
–1 is *not* a natural number.

66. true
–2 is *not* a natural number.

67. $n(A) = 5$; There are 5 elements in the set.

68. $n(A) = 6$; There are 6 elements in the set.

69. $n(B) = 15$; There are 15 elements in the set.

70. $n(B) = 11$; There are 11 elements in the set.

71. $n(C) = 0$; There are *no* days of the week beginning with A.

72. $n(C) = 0$; There are no such months.

73. $n(D) = 1$; There is 1 element in the set.

74. $n(D) = 1$; There is 1 element in the set.

75. $n(A) = 4$; There is 4 elements in the set.

76. $n(A) = 3$; There is 3 elements in the set.

77. $n(B) = 5$; There is 5 elements in the set.

78. $n(B) = 7$; There is 7 elements in the set.

79. $n(C) = 0$; There are no elements in the set.

80. $n(C) = 0$; There are no elements in the set.

81. a. Not equivalent
The number of elements is not the same.

b. Not equal
The two sets contain different elements.

82. a. Equivalent
The number of elements is the same.

b. Not equal
The two sets contain different elements.

83. a. Equivalent
The number of elements is the same.

b. Not equal
The elements are not exactly the same.

84. a. Equivalent
The number of elements is the same.

b. Not equal
The elements are not exactly the same.

85. a. Equivalent
The number of elements is the same.

b. Equal
The elements are exactly the same.

86. a. Equivalent
Number of elements is the same.

b. Equal
The elements are exactly the same.

87. a. Equivalent
Number of elements is the same.

b. Not equal
The two sets contain different elements.

88. a. Equivalent
Number of elements is the same.

b. Not equal
The two sets contain different elements.

89. a. Equivalent
Number of elements is the same.

b. Equal
The elements are exactly the same.

90. a. Equivalent
Number of elements is the same.

b. Equal
The elements are exactly the same.

91. infinite

92. infinite

93. finite

94. finite

95. finite

96. finite

97. $\{x \mid x \in \mathbb{N} \text{ and } x \geq 61\}$

98. $\{x \mid x \in \mathbb{N} \text{ and } x \geq 36\}$

99. $\{x \mid x \in \mathbb{N} \text{ and } 61 \leq x \leq 89\}$

100. $\{x \mid x \in \mathbb{N} \text{ and } 36 \leq x \leq 59\}$

101. Answers will vary; an example is: $\{0, 1, 2, 3\}$ and $\{1, 2, 3, 4\}$.

102. Answers will vary; an example is: $\{x \mid x \in \mathbb{N} \text{ and } x < 5\}$ and $\{1, 2, 3, 4\}$.

103. Impossible. Equal sets have exactly the same elements. This would require that there also must be the same number of elements.

104. Answers will vary; an example is: $\{1\}$ and $\{1, 2\}$.

105. {New Zealand, Australia, United States}

106. {New Zealand, Australia, United States}

107. {Australia, United States, United Kingdom, Switzerland, Ireland}

108. {Australia, United States, United Kingdom, Switzerland}

109. {United Kingdom, Switzerland, Ireland}

110. {Switzerland, Ireland, Spain}

111. { }

112. {New Zealand}

113. {12, 19}

114. {10, 14, 16}

115. {20, 21}

116. {20, 21, 22}

117. There is not a one-to-one correspondence. These sets are not equivalent.

124. makes sense

125. does not make sense; Explanations will vary. Sample explanation: The natural numbers do not include negative numbers. Since the temperature will be below zero, a set that includes negative numbers would be necessary.

126. does not make sense; Explanations will vary. Sample explanation: There is not a one-to-one correspondence for men because two ages sleep for 8.3 hours.

127. makes sense

128. false; Changes to make the statement true will vary. A sample change is: If two sets are equal, they must be equivalent.

129. false; Changes to make the statement true will vary. A sample change is: If a roster set contains three dots, it is finite if there is an ending value after the three dots.

130. false; Changes to make the statement true will vary. A sample change is: The cardinality of the empty set is 0.

131. true

132. true

133. false; Changes to make the statement true will vary. A sample change is: Though that set has many values, it is still a finite set.

134. false; Changes to make the statement true will vary. A sample change is: Some finite sets could have so many elements that it would take longer than a trillion years to count them.

135. false; Changes to make the statement true will vary. A sample change is: If 0 is removed from a set, it will lower the cardinality of that set by one.

136. This question contains a paradox. Sweeney Todd cannot shave himself because he does not shave any men who shave themselves. That suggests that $s \notin A$ which implies $s \in B$. However, if Sweeney Todd does not shave himself, the question states he shaves all such men who do not shave themselves. That suggests that he does shave himself, giving $s \in A$ which implies $s \notin B$. Therefore, paradoxically, *s* belongs and does not belong in both sets.

a. no

b. no

Check Points 2.2

1. a. $\not\subseteq$; because 6, 9, and 11 are not in set *B*.

b. $\subseteq$; because all elements in set *A* are also in set *B*.

c. $\subseteq$; because all elements in set *A* are also in set *B*.

2. a. Both $\subseteq$ and $\subset$ are correct.

b. Both $\subseteq$ and $\subset$ are correct.

3. Yes, the empty set is a subset of any set.

4. a. 16 subsets, 15 proper subsets
There are 4 elements, which means there are 2^4 or 16 subsets. There are $2^4 - 1$ proper subsets or 15.

b. 64 subsets, 63 proper subsets
There are 6 elements, which means there are 2^6 or 64 subsets. There are $2^6 - 1$ proper subsets or 63.

Concept and Vocabulary Check 2.2

1. $A \subseteq B$; every element in set *A* is also an element in set *B*

2. $A \subset B$; sets *A* and *B* are not equal

3. the empty; subset

4. 2^n

5. $2^n - 1$

Exercise Set 2.2

1. $\subseteq$

2. $\subseteq$

3. $\not\subseteq$

4. $\not\subseteq$

5. $\not\subseteq$

6. $\not\subseteq$

7. $\not\subseteq$
Subset cannot be larger than the set.

8. $\not\subseteq$
Subset cannot be larger than the set.

9. $\subseteq$

10. $\subseteq$

11. $\not\subseteq$

12. $\not\subseteq$

13. $\subseteq$

14. $\subseteq$

15. $\not\subseteq$

16. $\not\subseteq$

17. $\subseteq$

18. $\subseteq$

19. $\subseteq$ or $\subset$

20. $\subseteq$ or $\subset$

21. $\subseteq$

22. $\subseteq$

23. neither

24. neither

25. both

26. both

27. $\subseteq$

28. $\subseteq$

29. $\subseteq$

30. $\subseteq$

31. both

32. both

33. both

34. neither

35. both

36. both

37. neither

38. neither

39. $\subseteq$

40. $\subseteq$

41. true

42. true

43. false
{Ralph} is a subset, not Ralph.

44. false
{Canada} is a subset, not Canada.

45. true

46. true

47. false
The symbol "$\varnothing$" is not a member of the set.

48. true

49. true

50. true

51. false
All elements of {1, 4} are members of {4, 1}

52. true

53. true

54. true

55. { } {Border Collie} {Poodle} {Border Collie, Poodle}

56. { } {Romeo} {Juliet} {Romeo, Juliet}

57. { } {t} {a} {b} {t, a} {t, b} {a, b} {t, a, b}

58. { } {I} {II} {III} {I, II} {I, III} {II, III} {I, II, III }

59. { } {0}

60. $\varnothing$

61. 16 subsets, 15 proper subsets
There are 4 elements, which means there are 2^4 or 16 subsets. There are $2^4 - 1$ proper subsets or 15.

62. 16 subsets, 15 proper subsets
There are 4 elements, which means there are 2^4 or 16 subsets. There are $2^4 - 1$ proper subsets or 15.

63. 64 subsets, 63 proper subsets
There are 6 elements, which means there are 2^6 or 64 subsets. There are $2^6 - 1$ proper subsets or 63.

64. 64 subsets, 63 proper subsets
There are 6 elements, which means there are 2^6 or 64 subsets. There are $2^6 - 1$ proper subsets or 63.

65. 128 subsets, 127 proper subsets
There are 7 elements, which means there are 2^7 or 128 subsets. There are $2^7 - 1$ proper subsets or 127.

66. 32 subsets, 31 proper subsets
There are 5 elements, which means there are 2^5 or 32 subsets. There are $2^5 - 1$ proper subsets or 31.

67. 8 subsets, 7 proper subsets
There are 3 elements, which means there are 2^3 or 8 subsets. There are $2^3 - 1$ proper subsets or 7.

68. 32 subsets, 31 proper subsets
There are 5 elements, which means there are 2^5 or 32 subsets. There are $2^5 - 1$ proper subsets or 31.

69. false; The set $\{1, 2, 3, ..., 1000\}$ has $2^{1000} - 1$ proper subsets.

70. false; The set $\{1, 2, 3, ..., 10{,}000\}$ has $2^{10{,}000} - 1$ proper subsets.

71. true

72. true

73. false; $\varnothing \subseteq \{\varnothing, \{\varnothing\}\}$

74. false; $\{\varnothing\} \subseteq \{\varnothing, \{\varnothing\}\}$

75. true

76. true

77. true

78. true

79. true

80. true

81. false; The set of subsets of {a, e, i, o, u} contains 2^5 or 32 elements.

82. false; The set of subsets of {a, b, c, d, e, f} contains 2^6 or 64 elements.

83. false; $D \subseteq T$

84. false; $R \subseteq T$

85. true

86. true

87. false; If $x \in W$, then $x \in D$.

88. false; If $x \in M$, then $x \in D$.

89. true

90. true

91. true

92. true

93. $2^5 =$ 32 option combinations

94. $2^9 = 512$ topping combinations

95. $2^6 =$ 64 viewing combinations

96. 2^4 = 16 response options

97. $2^8 =$ 256 city combinations

98. 2^7 = 128 viewing combinations

105. does not make sense; Explanations will vary. Sample explanation: The set's elements are not members of the other set.

106. makes sense

107. does not make sense; Explanations will vary. Sample explanation: The same formulas are used for each of the mentioned problems.

108. makes sense

109. false; Changes to make the statement true will vary. A sample change is: The set has one element and has $2^1 = 2$ subsets.

110. true

111. false; Changes to make the statement true will vary. A sample change is: The empty set does not have a proper subset.

112. false; Changes to make the statement true will vary. A sample change is: The set has two elements and has $2^2 = 4$ subsets.

113. 0, 5¢, 10¢, 25¢, 40¢, 15¢, 30¢, 35¢
Since there are 3 elements or coins, there are 2^3 or 8 different coin combinations.

114. Number of proper subsets is $2^n - 1$, which means there are 128 total subsets (127 + 1).
128 is 2^7, so there are 7 elements.

Check Points 2.3

1. **a.** {1, 5, 6, 7, 9}

b. {1, 5, 6}

c. {7, 9}

2. **a.** {a, b, c, d}

b. {e}

c. {e, f, g}

d. {f, g}

3. $A' = \{b, c, e\}$; those are the elements in U but not in A.

4. **a.** {1, 3, 5, $\underline{7}$, $\underline{10}$} ∩ {6, $\underline{7}$, $\underline{10}$, 11} = {7, 10}

b. {1, 2, 3} ∩ {4, 5, 6, 7} = ∅

c. {1, 2, 3} ∩ ∅ = ∅

5. **a.** {1, 3, 5, 7, 10} ∪ {6, 7, 10, 11}
= {1, 3, 5, 6, 7, 10, 11}

b. {1, 2, 3} ∪ {4, 5, 6, 7} = {1, 2, 3, 4, 5, 6, 7}

c. {1, 2, 3} ∪ ∅ = {1, 2, 3}

6. **a.** $A \cup B = \{b, c, e\}$
$(A \cup B)' = \{a, d\}$

b. $A' = \{a, d, e\}$
$B' = \{a, d\}$
$A' \cap B' = \{a, d\}$

7. **a.** {5}; region II

b. {2, 3, 7, 11, 13, 17, 19}; the complement of region II

c. {2, 3, 5, 7, 11, 13}; regions I, II, and III

d. {17, 19}; the complement of regions I, II, and III

e. {5, 7, 11, 13, 17, 19}; the complement of *A* united with *B*

f. {2, 3}; *A* intersected with the complement of *B*

8. $$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 26 + 11 - 9 \\ &= 28 \end{aligned}$$

Concept and Vocabulary Check 2.3

1. Venn diagrams
2. complement; A'
3. intersection; $A \cap B$
4. union; $A \cup B$
5. $n(A) + n(B) - n(A \cap B)$
6. true
7. false
8. true
9. false

Exercise Set 2.3

1. *U* is the set of all composers.
2. *U* is the set of all writers.
3. *U* is the set of all brands of soft drinks.
4. *U* is the set of all models of automobiles.
5. $A' = \{c, d, e\}$
6. $B' = \{a, b, f, g\}$
7. $C' = \{b, c, d, e, f\}$
8. $D' = \{g\}$
9. $A' = \{6, 7, 8, \ldots, 20\}$
10. $B' = \{1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$
11. $C' = \{2, 4, 6, 8, \ldots, 20\}$
12. $D' = \{1, 3, 5, 7, \ldots, 19\}$
13. $A' = \{21, 22, 23, 24, \ldots\}$
14. $B' = \{51, 52, 53, 54, \ldots\}$
15. $C' = \{1, 3, 5, 7, \ldots\}$
16. $D' = \{2, 4, 6, 8, \ldots\}$
17. $A = \{1, 3, 5, 7\}$
    $B = \{1, 2, 3\}$
    $A \cap B = \{1, 3\}$
18. $B = \{1, 2, 3\}$
    $C = \{2, 3, 4, 5, 6\}$
    $B \cap C = \{2, 3\}$
19. $A = \{1, 3, 5, 7\}$
    $B = \{1, 2, 3\}$
    $A \cup B = \{1, 2, 3, 5, 7\}$
20. $B = \{1, 2, 3\}$
    $C = \{2, 3, 4, 5, 6\}$
    $B \cup C = \{1, 2, 3, 4, 5, 6\}$
21. $A = \{1, 3, 5, 7\}$
    $U = \{1, 2, 3, 4, 5, 6, 7\}$
    $A' = \{2, 4, 6\}$
22. $B = \{1, 2, 3\}$
    $U = \{1, 2, 3, 4, 5, 6, 7\}$
    $B' = \{4, 5, 6, 7\}$

23. $A' = \{2, 4, 6\}$
$B' = \{4, 5, 6, 7\}$
$A' \cap B' = \{4, 6\}$

24. $B' = \{4, 5, 6, 7\}$
$C = \{2, 3, 4, 5, 6\}$
$B' \cap C = \{4, 5, 6\}$

25. $A = \{1, 3, 5, 7\}$
$C' = \{1, 7\}$
$A \cup C' = \{1, 3, 5, 7\}$

26. $B = \{1, 2, 3\}$
$C' = \{1, 7\}$
$B \cup C' = \{1, 2, 3, 7\}$

27. $A = \{1, 3, 5, 7\}$
$C = \{2, 3, 4, 5, 6\}$
$A \cap C = \{3, 5\}$
$(A \cap C)' = \{1, 2, 4, 6, 7\}$

28. $A = \{1, 3, 5, 7\}$
$B = \{1, 2, 3\}$
$A \cap B = \{1, 3\}$
$(A \cap B)' = \{2, 4, 5, 6, 7\}$

29. $A = \{1, 3, 5, 7\}$ $C = \{2, 3, 4, 5, 6\}$
$A' = \{2, 4, 6\}$ $C' = \{1, 7\}$
$A' \cup C' = \{1, 2, 4, 6, 7\}$

30. $A = \{1, 3, 5, 7\}$
$B = \{1, 2, 3\}$
$A' = \{2, 4, 6\}$
$B' = \{4, 5, 6, 7\}$
$A' \cup B' = \{2, 4, 5, 6, 7\}$

31. $A = \{1, 3, 5, 7\}$ $B = \{1, 2, 3\}$
$(A \cup B) = \{1, 2, 3, 5, 7\}$
$(A \cup B)' = \{4, 6\}$

32. $A = \{1, 3, 5, 7\}$
$C = \{2, 3, 4, 5, 6\}$
$A \cup C = \{1, 2, 3, 4, 5, 6, 7\}$
$(A \cup C)' = \varnothing$

33. $A = \{1, 3, 5,7\}$
$A \cup \varnothing = \{1, 3, 5, 7\}$

34. $C = \{2, 3, 4, 5, 6\}$
$C \cup \varnothing = \{2, 3, 4, 5, 6\}$

35. $A \cap \varnothing = \varnothing$

36. $C \cap \varnothing = \varnothing$

37. $A \cup U = U$
$U = \{1, 2, 3, 4, 5, 6, 7\}$

38. $B \cup U = U$
$U = \{1, 2, 3, 4, 5, 6, 7\}$

39. $A \cap U = A$
$A=\{1, 3, 5, 7\}$

40. $B \cap U = B$
$B = \{1, 2, 3\}$

41. $A = \{a, g, h\}$
$B = \{b, g, h\}$
$A \cap B = \{g, h\}$

42. $B = \{b, g, h\}$
$C = \{b, c, d, e, f\}$
$B \cap C = \{b\}$

43. $A = \{a, g, h\}$
$B = \{b, g, h\}$
$A \cup B = \{a, b, g, h\}$

44. $B = \{b, g, h\}$
$C = \{b, c, d, e, f\}$
$B \cup C = \{b, c, d, e, f, g, h\}$

45. $A = \{a, g, h\}$
$U = \{a, b, c, d, e, f, g, h\}$
$A' = \{b, c, d, e, f\}$

46. $B = \{b, g, h\}$
$U = \{a, b, c, d, e, f, g, h\}$
$B' = \{a, c, d, e, f\}$

47. $A' = \{b, c, d, e, f\}$
$B' = \{a, c, d, e, f\}$
$A' \cap B' = \{c, d, e, f\}$

48. $B' = \{a, c, d, e, f\}$
$C = \{b, c, d, e, f\}$
$B' \cap C = \{c, d, e, f\}$

49. $A = \{a, g, h\}$
$C' = \{a, g, h\}$
$A \cup C' = \{a, g, h\}$

50. $B = \{b, g, h\}$
$C = \{b, c, d, e, f\}$
$C' = \{a, g, h\}$
$B \cup C' = \{a, b, g, h\}$

51. $A = \{a, g, h\}$
$C = \{b, c, d, e, f\}$
$A \cap C = \varnothing$
$(A \cap C)' = \{a, b, c, d, e, f, g, h\}$

52. $A = \{a, g, h\}$
$B = \{b, g, h\}$
$A \cap B = \{g, h\}$
$(A \cap B)' = \{a, b, c, d, e, f\}$

53. $A' = \{b, c, d, e, f\}$
$C' = \{a, g, h\}$
$A' \cup C' = \{a, b, c, d, e, f, g, h\}$

54. $A' = \{b, c, d, e, f\}$
$B' = \{a, c, d, e, f\}$
$A' \cup B' = \{a, b, c, d, e, f\}$

55. $A = \{a, g, h\}$
$B = \{b, g, h\}$
$A \cup B = \{a, b, g, h\}$
$(A \cup B)' = \{c, d, e, f\}$

56. $A = \{a, g, h\}$
$C = \{b, c, d, e, f\}$
$A \cup C = \{a, b, c, d, e, f, g, h\}$
$(A \cup C)' = \varnothing$

57. $A \cup \varnothing = A$
$A = \{a, g, h\}$

58. $C \cup \varnothing = C$
$C = \{b, c, d, e, f\}$

59. $A \cap \varnothing = \varnothing$

60. $C \cap \varnothing = \varnothing$

61. $A = \{a, g, h\}$
$U = \{a, b, c, d, e, f, g, h\}$
$A \cup U = \{a, b, c, d, e, f, g, h\}$

62. $B = \{b, g, h\}$
$U = \{a, b, c, d, e, f, g, h\}$
$B \cup U = \{a, b, c, d, e, f, g, h\}$

63. $A = \{a, g, h\}$
$U = \{a, b, c, d, e, f, g, h\}$
$A \cap U = \{a, g, h\}$

64. $B = \{b, g, h\}$
$U = \{a, b, c, d, e, f, g, h\}$
$B \cap U = \{b, g, h\}$

65. $A = \{a, g, h\}$
$B = \{b, g, h\}$
$B' = \{a, c, d, e, f\}$
$A \cap B = \{g, h\}$
$(A \cap B) \cup B' = \{a, c, d, e, f, g, h\}$

66. $A = \{a, g, h\}$
$B = \{b, g, h\}$
$B' = \{a, c, d, e, f\}$
$A \cup B = \{a, b, g, h\}$
$(A \cup B) \cap B' = \{a\}$

67. $A = \{1, 3, 4, 7\}$

68. $B = \{2, 3, 5, 6, 7\}$

69. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

70. $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

71. $A \cap B = \{3, 7\}$

72. $A' = \{2, 5, 6, 8, 9\}$

73. $B' = \{1, 4, 8, 9\}$

74. $(A \cap B)' = \{1, 2, 4, 5, 6, 8, 9\}$

75. $(A \cup B)' = \{8, 9\}$

76. $A' = \{2, 5, 6, 8, 9\}$
$B = \{2, 3, 5, 6, 7\}$
$A' \cap B = \{2, 5, 6\}$

77. $A = \{1, 3, 4, 7\}$
$B' = \{1, 4, 8, 9\}$
$A \cap B' = \{1, 4\}$

78. $A = \{1, 3, 4, 7\}$
$B' = \{1, 4, 8, 9\}$
$A \cup B' = \{1, 3, 4, 7, 8, 9\}$

79. $B = \{\triangle$, two, four, six$\}$

80. $A = \{\triangle, \#, \$\}$

81. $A \cup B = \{\triangle, \#, \$, \text{ two, four, six}\}$

82. $A \cap B = \{\triangle\}$

83. $n(A \cup B) = n(\{\triangle, \#, \$, \text{ two, four, six}\}) = 6$

84. $n(A \cap B) = n(\{\triangle\}) = 1$

85. $n(A') = 5$

86. $n(B') = 4$

87. $(A \cap B)' = \{\#, \$, \text{ two, four, six, } 10, 01\}$

88. $(A \cup B)' = \{10, 01\}$

89. $A' \cap B = \{\text{two, four, six}\}$

90. $A \cap B' = \{\#, \$\}$

91. $n(U) - n(B) = 8 - 4 = 4$

92. $n(U) - n(A) = 8 - 3 = 5$

93.
$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 17 + 20 - 6 \\ &= 31 \end{aligned}$$

94.
$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 30 + 18 - 5 \\ &= 43 \end{aligned}$$

95.
$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 17 + 17 - 7 \\ &= 27 \end{aligned}$$

96.
$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 30 + 24 - 7 \\ &= 47 \end{aligned}$$

97. $A = \{1, 3, 5, 7\}$
$B = \{2, 4, 6, 8\}$
$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

98. $B = \{2, 4, 6, 8\}$
$C = \{2, 3, 4, 5\}$
$B \cup C = \{2, 3, 4, 5, 6, 8\}$

99. $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$A = \{1, 3, 5, 7\}$
$A \cap U = \{1, 3, 5, 7\}$

100. $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$A = \{1, 3, 5, 7\}$
$A \cup U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

101. $A = \{1, 3, 5, 7\}$
$C' = \{1, 6, 7, 8\}$
$A \cap C' = \{1, 7\}$

102. $A = \{1, 3, 5, 7\}$
$B' = \{1, 3, 5, 7\}$
$A \cap B' = \{1, 3, 5, 7\}$

103. $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$B = \{2, 4, 6, 8\}$
$C = \{2, 3, 4, 5\}$
$B \cap C = \{2, 4\}$
$(B \cap C)' = \{1, 3, 5, 6, 7, 8\}$

104. $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$A = \{1, 3, 5, 7\}$
$C = \{2, 3, 4, 5\}$
$A \cap C = \{3, 5\}$
$(A \cap C)' = \{1, 2, 4, 6, 7, 8\}$

105. $A \cup (A \cup B)'$
$= \{23, 29, 31, 37, 41, 43, 53, 59, 61, 67, 71\}$

106. $(A' \cap B) \cup (A \cap B) = \{41, 43, 47\}$

107.
$$\begin{aligned} n(U)\left[n(A \cup B) - n(A \cap B)\right] &= 12[7 - 2] \\ &= 12(5) = 60 \end{aligned}$$

108. $n(A \cap B)\left[n(A \cup B) - n(A')\right] = 2[7 - 6] = 2(1) = 2$

109. {Ashley, Mike, Josh}

110. {Mike, Josh, Emily, Hannah, Ethan}

111. {Ashley, Mike, Josh, Emily, Hannah, Ethan}

112. {Mike, Josh}

113. {Ashley}

114. {Emily, Hannah, Ethan}

115. {Jacob}

116. {Ashley, Mike, Josh, Emily, Hannah, Ethan, Jacob}

117. Region III, *elementary school teacher* is in set *B* but not set *A*.

118. Region I, *police officer* is in set *A* but not set *B*.

119. Region I, *surgeon* is in set *A* but not set *B*.

120. Region III, *banker* is in set *B* but not set *A*.

121. Region II, *family doctor* is in set *A* and set *B*.

122. Region II, *lawyer* is in set *A* and set *B*.

123. Region I, 11 is in set *A* but not set *B*.

124. Region II, 22 is in set *A* and set *B*.

125. Region IV, 15 is in neither set *A* nor set *B*.

126. Region IV, 17 is in neither set *A* nor set *B*.

127. Region II, 454 is in set *A* and set *B*.

128. Region I, 101 is in set *A* but not set *B*.

129. Region III, 9558 is in set *B* but not set *A*.

130. Region III, 9778 is in set *B* but not set *A*.

131. Region I, 9559 is in set *A* but not set *B*.

132. Region I, 9779 is in set *A* but not set *B*.

133. $\{\underline{1980}, \underline{1990}\} \cap \{\underline{1980}, \underline{1990}, 2000\}$
$= \{1980, 1990\}$

134. $\{1980, \underline{1990}, \underline{2000}\} \cap \{\underline{1990}, \underline{2000}\}$
$= \{1990, 2000\}$

135. $\{1980, 1990\} \cup \{1980, 1990, 2000\}$
$= \{1980, 1990, 2000\}$

136. $\{1980, 1990, 2000\} \cup \{1990, 2000\}$
$= \{1980, 1990, 2000\}$

137. $\{1990, 2000, 2010\} \cap \{1980\}$
$= \varnothing$

138. $\{1990, 2000, 2010\} \cup \{1980\}$
$= \{1980, 1990, 2000, 2010\}$

139. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 178 + 154 - 49$
$= 283$ people

140. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 96 + 97 - 29$
$= 164$ people

152. does not make sense; Explanations will vary. Sample explanation: Even with only one common element, the sets intersection will be shown by overlapping circles.

153. makes sense

154. does not make sense; Explanations will vary. Sample explanation: The given expression indicates that you should find the union of set *A* and set *B*, and then find the complement of the resulting set.

155. makes sense

156. false; Changes to make the statement true will vary. A sample change is:
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

157. true

158. false; Changes to make the statement true will vary. A sample change is: $A \subseteq (A \cup B)$

159. false; Changes to make the statement true will vary. A sample change is: If $A \subseteq B$, then $A \cup B = B$.

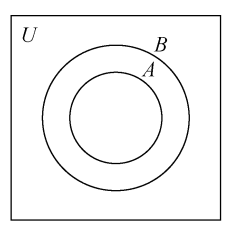
160. false; Changes to make the statement true will vary. A sample change is: $A \cup U = U$

161. false; Changes to make the statement true will vary. A sample change is: $A \cap \varnothing = \varnothing$

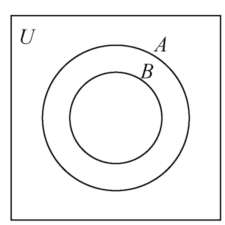
162. false; Changes to make the statement true will vary. A sample change is: If $A \subseteq B$, then $A \cap B = A$.

163. true

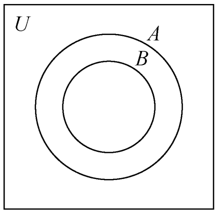
164.



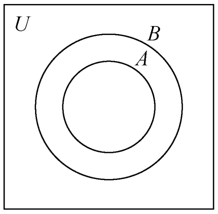
165.



166.



167.



Check Points 2.4

1. a. $A \cup (B \cap C) = \{a, b, c, d\} \cup \{b, f\}$
$= \{a, b, c, d, f\}$

b.

$(A \cup B) \cap (A \cup C) = \{a, b, c, d, f\} \cap \{a, b, c, d, f\}$
$= \{a, b, c, d, f\}$

c.

$A \cap (B \cup C') = \{a, b, c, d\} \cap (\{a, b, d, f\} \cup \{a, d, e\})$
$= \{a, b, c, d\} \cap \{a, b, d, e, f\}$
$= \{a, b, d\}$

2. a. C is represented by regions IV, V, VI, and VII.
Thus, $C = \{5, 6, 7, 8, 9\}$

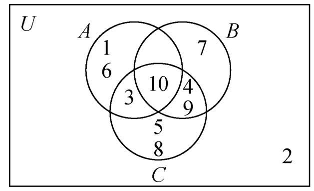
b. $B \cup C$ is represented by regions II, III, IV, V, VI, and VII.
Thus, $B \cup C = \{1, 2, 5, 6, 7, 8, 9, 10, 12\}$

c. $A \cap C$ is represented by regions IV and V.
Thus, $A \cap C = \{5, 6, 7\}$

d. B' is represented by regions I, IV, VII, and VIII.
Thus, $B' = \{3, 4, 6, 8, 11\}$

e. $A \cup B \cup C$ is represented by regions I, II, III, IV, V, VI, and VII.
Thus,
$A \cup B \cup C = \{1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12\}$

3.



4. a. $A \cup B$ is represented by regions I, II, and III. Therefore $(A \cup B)'$ is represented by region IV.

b. A' is represented by regions III and IV.
B' is represented by regions I and IV.
Therefore $A' \cap B'$ is represented by region IV.

c. $(A \cup B)' = A' \cap B'$ because they both represent region IV.

5. a. $B \cup C$ is represented by regions II, III, IV, V, VI, and VII.
Therefore $A \cap (B \cup C)$ is represented by regions II, IV, and V.

b. $A \cap B$ is represented by regions II and V.
$A \cap C$ is represented by regions IV and V.
Therefore $(A \cap B) \cup (A \cap C)$ is represented by regions II, IV, and V.

c. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ because they both represent region IV.

Concept and Vocabulary Check 2.4

1. inside parentheses

2. eight

3. false

4. true

Exercises 2.4

1. $B \cap C = \{2, 3\}$
$A \cup (B \cap C) = \{1, 2, 3, 5, 7\}$

2. $B \cup C = \{1, 2, 3, 4, 5, 6\}$
$A \cap (B \cup C) = \{1, 3, 5\}$

3. $A \cup B = \{1, 2, 3, 5, 7\}$
$A \cup C = \{1, 2, 3, 4, 5, 6, 7\}$
$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 5, 7\}$

4. $(A \cap B) = \{1, 3\}$
$(A \cap C) = \{3, 5\}$
$(A \cap B) \cup (A \cap C) = \{1, 3, 5\}$

5. $A' = \{2, 4, 6\}$ $C' = \{1, 7\}$
$B \cup C' = \{1, 2, 3, 7\}$
$A' \cap (B \cup C') = \{2\}$

6. $B' = \{4, 5, 6, 7\}$
$C' = \{1, 7\}$
$A \cup B' = \{1, 3, 4, 5, 6, 7\}$
$C' \cap (A \cup B') = \{1, 7\}$

7. $A' = \{2, 4, 6\}$ $C' = \{1, 7\}$
$A' \cap B = \{2\}$
$A' \cap C' = \varnothing$
$(A' \cap B) \cup (A' \cap C') = \{2\}$

8. $B' = \{4, 5, 6, 7\}$
$C' = \{1, 7\}$
$(C' \cap A) = \{1, 7\}$
$(C' \cap B') = \{7\}$
$(C' \cap A) \cup (C' \cap B') = \{1, 7\}$

9. $A = \{1, 3, 5, 7\}$
$B = \{1, 2, 3\}$
$C = \{2, 3, 4, 5, 6\}$
$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7\}$
$(A \cup B \cup C)' = \varnothing$

10. $A = \{1, 3, 5, 7\}$
$B = \{1, 2, 3\}$
$C = \{2, 3, 4, 5, 6\}$
$A \cap B \cap C = \{3\}$
$(A \cap B \cap C)' = \{1, 2, 4, 5, 6, 7\}$

11. $A = \{1, 3, 5, 7\}$
$B = \{1, 2, 3\}$
$A \cup B = \{1, 2, 3, 5, 7\}$
$(A \cup B)' = \{4, 6\}$
$C = \{2, 3, 4, 5, 6\}$
$(A \cup B)' \cap C = \{4, 6\}$

12. $B \cup C = \{1, 2, 3, 4, 5, 6\}$
$(B \cup C)' = \{7\}$
$(B \cup C)' \cap A = \{7\}$

13. $B \cap C = \{\text{b}\}$
$A \cup (B \cap C) = \{\text{a, b, g, h}\}$

14. $B \cup C = \{\text{b, c, d, e, f, g, h}\}$
$A \cap (B \cup C) = \{\text{g, h}\}$

15. $A \cup B = \{\text{a, b, g, h}\}$
$A \cup C = \{\text{a, b, c, d, e, f, g, h}\}$
$(A \cup B) \cap (A \cup C) = \{\text{a, b, g, h}\}$

16. $A \cap B = \{\text{g, h}\}$
$A \cap C = \varnothing$
$(A \cap B) \cup (A \cap C) = \{\text{g, h}\}$

17. $A' = \{\text{b,c, d, e, f}\}$
$C' = \{\text{a, g, h}\}$
$B \cup C' = \{\text{a, b, g, h}\}$
$A' \cap (B \cup C') = \{\text{b}\}$

18. $C' = \{\text{a, g, h}\}$
$B' = \{\text{a, c, d, e, f}\}$
$A \cup B' = \{\text{a, c, d, e, f, g, h}\}$
$C' \cap (A \cup B') = \{\text{a, g, h}\}$

19. $A' = \{\text{b, c, d, e, f}\}$
$A' \cap B = \{\text{b}\}$
$C' = \{\text{a, g, h}\}$
$A' \cap C' = \varnothing$
$(A' \cap B) \cup (A' \cap C') = \{\text{b}\}$

20. $C' = \{\text{a, g, h}\}$
$B' = \{\text{a, c, d, e, f}\}$
$C' \cap A = \{\text{a, g, h}\}$
$C' \cap B' = \{\text{a}\}$
$(C' \cap A) \cup (C' \cap B') = \{\text{a, g, h}\}$

21. $A \cup B \cup C = \{a, b, c, d, e, f, g, h\}$
$(A \cup B \cup C)' = \varnothing$

22. $A \cap B \cap C = \varnothing$
$(A \cap B \cap C)' = \{a, b, c, d, e, f, g, h\}$

23. $A \cup B = \{a, b, g, h\}$
$(A \cup B)' = \{c, d, e, f\}$
$(A \cup B)' \cap C = \{c, d, e, f\}$

24. $B \cup C = \{b, c, d, e, f, g, h\}$
$(B \cup C)' = \{a\}$
$(B \cup C)' \cap A = \{a\}$

25. II, III, V, VI

26. IV, V, VI, VII

27. I, II, IV, V, VI, VII

28. II, III, IV, V, VI, VII

29. II, V

30. IV, V

31. I, IV, VII, VIII

32. I, II, III, VIII

33. $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

34. $B = \{4, 5, 6, 9, 10, 11\}$

35. $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

36. $B = \{4, 5, 6, 9, 10, 11\}$
$C = \{6, 7, 8, 9, 12\}$
$B \cup C = \{4, 5, 6, 7, 8, 9, 10, 11, 12\}$

37. $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$B = \{4, 5, 6, 9, 10, 11\}$
$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$
$(A \cup B)' = \{12, 13\}$

38. $B = \{4, 5, 6, 9, 10, 11\}$
$C = \{6, 7, 8, 9, 12\}$
$B \cup C = \{4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$(B \cup C)' = \{1, 2, 3, 13\}$

39. The set contains the elements in the two regions where the circles representing sets *A* and *B* overlap.
$A \cap B = \{4, 5, 6\}$

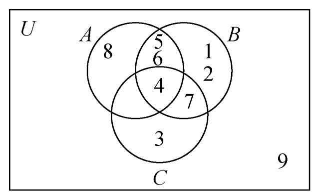
40. The set contains the elements in the two regions where the circles representing sets *A* and *C* overlap.
$A \cap C = \{6, 7, 8\}$

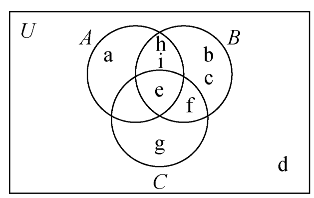
41. The set contains the element in the center region where the circles representing sets *A*, *B*, and *C* overlap.
$A \cap B \cap C = \{6\}$

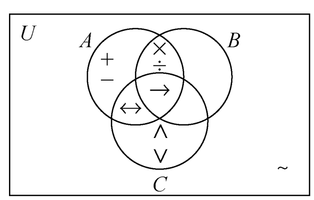
42. The set contains the elements in the seven regions of the circles representing sets *A*, *B*, and *C*. Only element 13 lies outside these regions.
$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

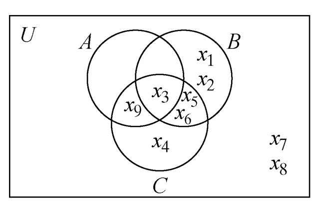
43. $A \cap B \cap C = \{6\}$
$(A \cap B \cap C)' = \{1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13\}$

44. $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$(A \cup B \cup C)' = \{13\}$

45.


46.


47.


48.


49. **a.** II

b. II

c. $A \cap B = B \cap A$

50. **a.** I, II, III

b. I, II, III

c. $A \cup B = B \cup A$

51. **a.** I, III, IV

b. IV

c. No, $(A \cap B)' \neq A' \cap B'$

52. **a.** IV

b. I, III, IV

c. No, $(A \cup B)' \neq A' \cup B'$

53. Set A is represented by regions I and II.
Set A' is represented by regions III and IV.
Set B is represented by regions II and III.
Set B' is represented by regions I and IV.
$A' \cup B$ is represented by regions II, III, and IV.
$A \cap B'$ is represented by region I.
Thus, $A' \cup B$ and $A \cap B'$ are not equal for all sets *A* and *B*.

54. Set A is represented by regions I and II.
Set A' is represented by regions III and IV.
Set B is represented by regions II and III.
Set B' is represented by regions I and IV.
$A' \cap B$ is represented by region III.
$A \cup B'$ is represented by regions I, II, and IV.
Thus, $A' \cap B$ and $A \cup B'$ are not equal for all sets *A* and *B*.

55. Set A is represented by regions I and II.
Set B is represented by regions II and III.
$(A \cup B)'$ is represented by region IV.
$(A \cap B)'$ is represented by regions I, III, and IV.
Thus, $(A \cup B)'$ and $(A \cap B)'$ are not equal for all sets *A* and *B*.

56. Set A is represented by regions I and II.
Set B is represented by regions II and III.
$(A \cup B)'$ is represented by region IV.
$A' \cap B$ is represented by region III.
Thus, $(A \cup B)'$ and $A' \cap B$ are not equal for all sets *A* and *B*.

57. Set A is represented by regions I and II.
Set A' is represented by regions III and IV.
Set B is represented by regions II and III.
Set B' is represented by regions I and IV.
$(A' \cap B)'$ is represented by regions I, II, and IV.
$A \cup B'$ is represented by regions I, II, and IV.
Thus, $A' \cap B$ and $A \cup B'$ are equal for all sets *A* and *B*.

58. Set A is represented by regions I and II.
Set A' is represented by regions III and IV.
Set B is represented by regions II and III.
Set B' is represented by regions I and IV.
$(A \cup B')'$ is represented by region III.
$A' \cap B$ is represented by region III.
Thus, $A' \cap B$ and $A \cup B'$ are equal for all sets *A* and *B*.

59. **a.** II, IV, V, VI, VII

b. II, IV, V, VI, VII

c. $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

60. **a.** IV, V, VI

b. IV, V, VI

c. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

61. **a.** II, IV, V

b. I, II, IV, V, VI

c. No
The results in **a** and **b** show $A \cap (B \cup C) \neq A \cup (B \cap C)$ because of the different regions represented.

62. **a.** II, IV, V, VI, VII

b. IV, V, VI

c. No
The results in **a** and **b** show $C \cup (B \cap A) \neq C \cap (B \cup A)$ because of the different regions represented.

63. The left expression is represented by regions II, IV, and V. The right expression is represented by regions II, IV, V, VI, and VII. Thus this statement is not true.

64. The left expression is represented by regions I, II, IV, V, and VI. The right expression is represented by regions IV, V, and VI. Thus this statement is not true.

65. Both expressions are represented by regions II, III, IV, V, and VI. Thus this statement is true and is a theorem.

66. Both expressions are represented by regions II, V, and VI. Thus this statement is true and is a theorem.

67. Both expressions are represented by region I. Thus this statement is true and is a theorem.

68. Both expressions are represented by regions I, II, III, IV, V, VII, and VIII. Thus this statement is true and is a theorem.

69. **a.** $A \cup (B' \cap C') = \{\text{c, e, f}\}$

$(A \cup B') \cap (A \cup C') = \{\text{c, e, f}\}$

b. $A \cup (B' \cap C') = \{1, 3, 5, 7, 8\}$

$(A \cup B') \cap (A \cup C') = \{1, 3, 5, 7, 8\}$

c. $A \cup (B' \cap C') = (A \cup B') \cap (A \cup C')$

d. $A \cup (B' \cap C')$ and $(A \cup B') \cap (A \cup C')$ are both represented by regions I, II, IV, V, and VIII. Thus, the conjecture in part c is a theorem.

70. **a.** $(A \cup B)' \cap C = \{4\}$

$A' \cap (B' \cap C) = \{4\}$

b. $(A \cup B)' \cap C = \{\text{e}\}$

$A' \cap (B' \cap C) = \{\text{e}\}$

c. $(A \cup B)' \cap C = A' \cap (B' \cap C)$

d. $(A \cup B)' \cap C$ and $A' \cap (B' \cap C)$ are both represented by regions IV, V, and VI. Thus, the conjecture in part c is a theorem.

71. $(A \cap B') \cap (A \cup B)$

72. $(A \cup B)'$

73. $A' \cup B$

74. $A' \cap B$

75. $(A \cap B) \cup C$

76. $A \cap (B \cup C)$

77. $A' \cap (B \cup C)$

78. $(A \cup B)' \cap C$

79. {Ann, Jose, Al, Gavin, Amy, Ron, Grace}

80. {Jose, Ron, Grace, Lee, Maria}

81. {Jose}

82. {Ann, Jose}

83. {Lily, Emma}

84. {Ron, Grace, Lee, Maria}

85. {Lily, Emma, Ann, Jose, Lee, Maria, Fred, Ben, Sheila, Ellen, Gary}

86. {Jose, Ron, Grace, Lee, Maria, Al, Gavin, Amy, Fred, Ben, Sheila, Ellen, Gary}

87. {Lily, Emma, Al, Gavin, Amy, Lee, Maria}

88. {Ann, Jose, Ron, Grace}

89. {Al, Gavin, Amy}

90. {Lily, Emma}

91. The set of students who scored 90% or above on exam 1 and exam 3 but not on exam 2 is the empty set.

92. The set of students who did not score 90% or higher on any of the exams.

93. Region I

94. Region III

95. Region V

96. Region V

97. Region VI

98. Region VIII

99. Region III

100. Region VI

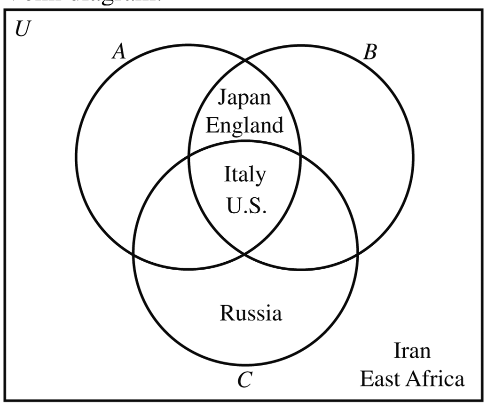
101. Region IV

102. Region VIII

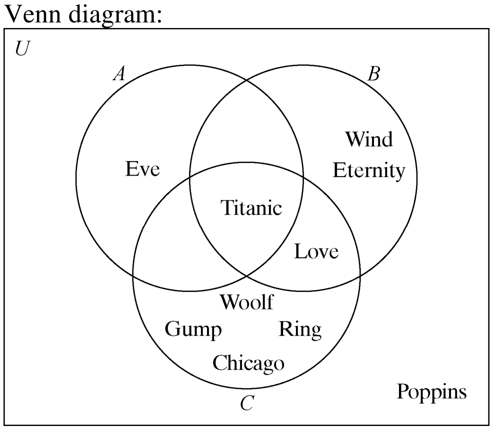
103. Region VI

104. Region I

105. Venn diagram:



106. Venn diagram:



109. does not make sense; Explanations will vary. Sample explanation: You should begin by placing elements in the innermost region.

110. makes sense

111. makes sense

112. does not make sense; Explanations will vary. Sample explanation: Finding examples, even many examples, is not enough to prove a conjecture.

113. AB^+

114. O^-

115. no

116. yes

Check Points 2.5

1. **a.** 55 + 20 = 75

b. 20 + 70 = 90

c. 20

d. 55 + 20 + 70 = 145

e. 55

f. 70

g. 30

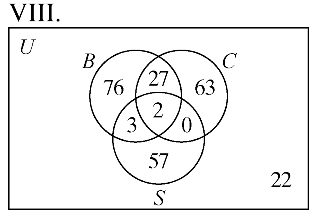
h. 55 + 20 + 70 + 30 =175

2. Start by placing 700 in region II.
Next place $1190-700$ or 490 in region III.
Since half of those surveyed were women, place $1000-700$ or 300 in region I.
Finally, place $2000-300-700-490$ or 510 in region IV.

a. 490 men agreed with the statement and are represented by region III.

b. 510 men disagreed with the statement and are represented by region IV.

3. Since 2 people collect all three items, begin by placing a 2 in region V.
Since 29 people collect baseball cards and comic books, 29 – 2 or 27 should be placed in region II.
Since 5 people collect baseball cards and stamps, 5 – 2 or 3 should be placed in region IV.
Since 2 people collect comic books and stamps, 2 – 2 or 0 should be placed in region VI.
Since 108 people collect baseball cards, 108 – 27 – 3 – 2 or 76 should be placed in region I.
Since 92 people collect comic books, 92 – 27 – 2 – 0 or 63 should be placed in region III.
Since 62 people collect stamps, 62 – 3 – 2 – 0 or 57 should be placed in region VII.
Since there were 250 people surveyed, place $250-76-27-63-3-2-0-57=22$ in region VIII.



4. **a.** 63 as represented by region III.

b. 3 as represented by region IV.

c. 136 as represented by regions I, IV, and VII.

d. 30 as represented by regions II, IV, and VI.

e. 228 as represented by regions I through VII.

f. 22 as represented by region VIII.

Concept and Vocabulary Check 2.5

1. and/but
2. or
3. not
4. innermost; subtraction
5. true
6. true
7. false
8. true

Exercise Set 2.5

1. 26
2. 20
3. 17
4. 11
5. 37
6. 9
7. 7
8. 44
9. Region I has 21 – 7 or 14 elements.
   Region III has 29 – 7 or 22 elements.
   Region IV has 48 – 14 – 7– 22 or 5 elements.
10. Region I has 23 – 7 or 16 elements.
    Region III has 27 – 7 or 20 elements.
    Region IV has 53 – 16 – 7– 20 or 10 elements.
11. 17 as represented by regions II, III, V, and VI.
12. 15 as represented by regions I, II, IV, and V.
13. 6 as represented by regions I and II.
14. 9 as represented by regions III and VI.
15. 28 as represented by regions I, II, IV, V, VI, and VII.
16. 24 as represented by regions I through VI.
17. 9 as represented by regions IV and V.
18. 8 as represented by regions II and V.
19. 3 as represented by region VI.
20. 2 as represented by region IV.
21. 19 as represented by regions III, VI, and VII.
22. 17 as represented by regions I, IV, and VII.
23. 21 as represented by regions I, III, and VII.
24. 6 as represented by regions II, IV, and VI.
25. 34 as represented by regions I through VII.
26. 13 as represented by regions II, IV, V, and VI.
27. Since $n(A\cap B)=3$, there is 1 element in region II.
    Since $n(A\cap C)=5$, there are 3 elements in region IV.
    Since $n(B\cap C)=3$, there is 1 element in region VI.
    Since $n(A)=11$, there are 5 elements in region I.
    Since $n(B)=8$, there are 4 elements in region III.
    Since $n(C)=14$, there are 8 elements in region VII.
    Since $n(U)=30$, there are 6 elements in region VIII.
28. Since $n(A\cap B)=6$, there are 4 elements in region II.
    Since $n(A\cap C)=7$, there are 5 elements in region IV.
    Since $n(B\cap C)=8$, there are 6 elements in region VI.
    Since $n(A)=21$, there are 10 elements in region I.
    Since $n(B)=15$, there are 3 elements in region III.
    Since $n(C)=14$, there is 1 element in region VII.
    Since $n(U)=32$, there is 1 element in region VIII.

29. Since $n(A \cap B \cap C) = 7$, there are 7 elements in region V.
Since $n(A \cap B) = 17$, there are 10 elements in region II.
Since $n(A \cap C) = 11$, there are 4 elements in region IV.
Since $n(B \cap C) = 8$, there is 1 element in region VI.
Since $n(A) = 26$, there are 5 elements in region I.
Since $n(B) = 21$, there are 3 elements in region III.
Since $n(C) = 18$, there are 6 elements in region VII.
Since $n(U) = 38$, there are 2 elements in region VIII.

30. Since $n(A \cap B \cap C) = 5$, there are 5 elements in region V.
Since $n(A \cap B) = 17$, there are 12 elements in region II.
Since $n(A \cap C) = 11$, there are 6 elements in region IV.
Since $n(B \cap C) = 9$, there are 4 elements in region VI.
Since $n(A) = 26$, there are 3 elements in region I.
Since $n(B) = 22$, there is 1 element in region III.
Since $n(C) = 25$, there are 10 elements in region VII.
Since $n(U) = 42$, there is 1 element in region VIII.

31. Since $n(A \cap B \cap C) = 2$, there are 2 elements in region V.
Since $n(A \cap B) = 6$, there are 4 elements in region II.
Since $n(A \cap C) = 9$, there are 7 elements in region IV.
Regions II, IV, and V contain a total of 13 elements, yet set A is stated to contain a total of only 10 elements. That is impossible.

32. Since $n(A \cap B \cap C) = 5$, there are 5 elements in region V.
Since $n(A \cap B) = 6$, there is 1 element in region II.
Since $n(A \cap C) = 9$, there are 4 elements in region IV.
Regions II, IV, and V contain a total of 10 elements, yet set A is stated to contain a total of only 8 elements. That is impossible.

33. $4+5+2+7=18$ respondents agreed with the statement.

34. $8+2+3+9=22$ respondents disagreed with the statement.

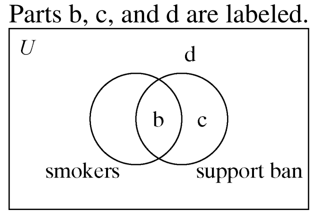
35. $2+7=9$ women agreed with the statement.

36. $4+2=6$ people who are not African American agreed with the statement.

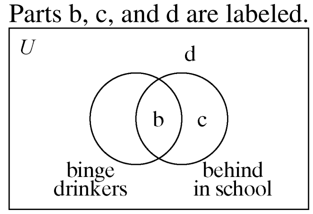
37. 9 women who are not African American disagreed with the statement.

38. 8 men who are not African American agreed with the statement.

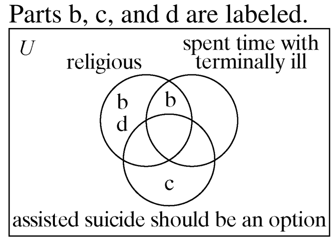
39. Parts b, c, and d are labeled.



40. Parts b, c, and d are labeled.

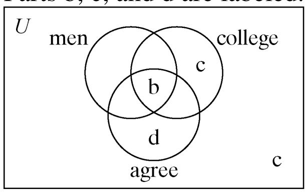


41. Parts b, c, and d are labeled.



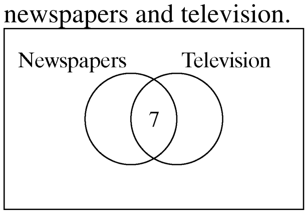
Answers for part e will vary.

42. Parts b, c, and d are labeled.



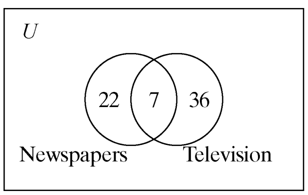
Answers for part e will vary.

43. Begin by placing 7 in the region that represents both newspapers and television.



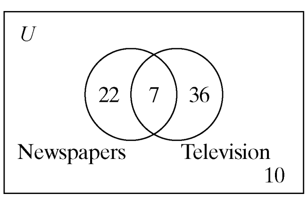
a. Since 29 students got news from newspapers, $29-7=22$ got news from only newspapers.

b. Since 43 students got news from television, $43-7=36$ got news from only television.

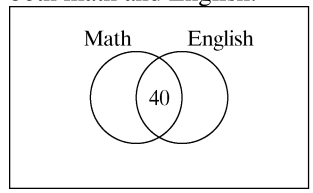


c. $22+7+36=65$ students who got news from newspapers or television.

d. Since 75 students were surveyed, $75-65=10$ students who did not get news from either.

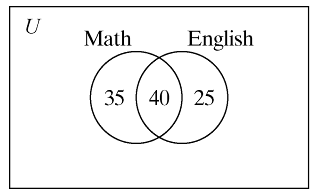


44. Begin by placing 40 in the region that represents both math and English.



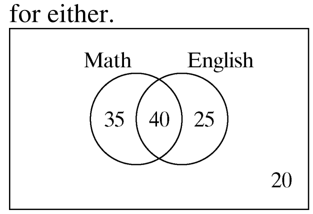
a. Since 75 students registered for math, $75-40=35$ registered for only math.

b. Since 65 students registered for English, $65-40=25$ registered for only English.

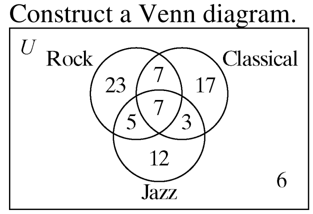


c. $35+40+25=100$ students who registered for math or English.

d. Since 120 students were surveyed, $120-100=20$ students who did not register for either.



45. Construct a Venn diagram.



a. 23

b. 3

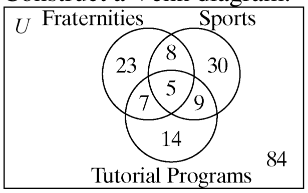
c. $17+3+12=32$

d. $23+17+12=52$

e. $7+3+5+7=22$

f. 6

46. Construct a Venn diagram.



a. 30

b. 8

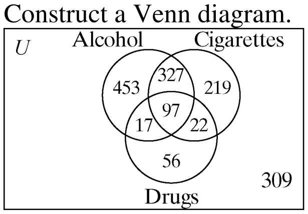
c. $23+8+30=61$

d. $23+30+14=67$

e. $8+7+9+5=29$

f. 84

47. Construct a Venn diagram.



a. 1500 (all eight regions)

b. 1135 (the six regions of sets *A* and *C*)

c. 56 (region VII)

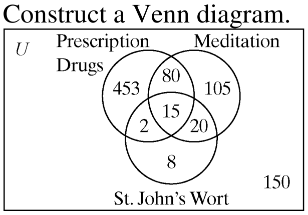
d. 327 (region II)

e. 526 (regions I, IV, and VII)

f. 366 (regions II, IV, and VI)

g. 1191 (regions I through VII)

48. Construct a Venn diagram.



a. 833 (all eight regions)

b. 675 (the six regions of sets *A* and *C*)

c. 8 (region VII)

d. 80 (region II)

e. 463 (regions I, IV, and VII)

f. 102 (regions II, IV, and VI)

g. 683 (regions I through VII)

51. does not make sense; Explanations will vary. Sample explanation: A survey problem could present the information in any order.

52. makes sense

53. does not make sense; Explanations will vary. Sample explanation: Since there is a circle to represent smokers, then nonsmokers are represented by being placed outside that circle, not in a separate circle.

54. does not make sense; Explanations will vary. Sample explanation: The bar graph does not indicate how the various ailments intersect.

55. false; Changes to make the statement true will vary. A sample change is: It is possible that some students are taking more than one of these courses. If so, then the number surveyed is less than 220.

56. true

57. false; Changes to make the statement true will vary. A sample change is: Then innermost region is the first region to be filled in.

58. true

59. **a.** 0; This would assume none of the psychology students were taking mathematics.

b. 30; This would assume all 30 students taking psychology were taking mathematics.

c. 60; $U = 150$ so with 90 taking mathematics, if we assume all the psychology students are taking mathematics courses, $U - 90 = 60$.

60. Under the conditions given concerning enrollment in math, chemistry, and psychology courses, the total number of students is 100, not 90

Chapter 2 Review Exercises

1. the set of days of the week beginning with the letter T

2. the set of natural numbers between 1 and 10, inclusive

3. {m, i, s}

4. {8, 9, 10, 11, 12}

5. {1, 2, 3, …, 30}

6. not empty

7. empty set

8. $\in$
93 is an element of the set.

9. $\notin$
{d} is a subset, not a member; "d" would be a member.

10. 12
12 months in the year.

11. 15

12. $\neq$
The two sets do not contain exactly the same elements.

13. $\neq$
One set is infinite. The other is finite.

14. Equivalent
Same number of elements, but different elements.

15. Equal and equivalent
The two sets have exactly the same elements.

16. finite

17. infinite

18. $\subseteq$

19. $\not\subseteq$

20. $\subseteq$

21. $\subseteq$

22. both

23. false; Texas is not a member of the set.

24. false; 4 is not a subset. {4} is a subset.

25. true

26. false; It is a subset but not a proper subset.

27. true

28. false; The set {six} has only one element so it has $2^1 = 2$ subsets.

29. true

30. $\varnothing$ {1} {5} {1, 5}
{1, 5} is not a proper subset.

31. There are 5 elements. This means there are $2^5 = 32$ subsets.
There are $2^5 - 1 = 31$ proper subsets.

32. {January, June, July}
There are 3 elements. This means there are $2^3 = 8$ subsets.
There are $2^3 - 1 = 7$ proper subsets.

33. $A \cap B = \{1, 2, 4\}$

34. $A \cup B' = \{1, 2, 3, 4, 6, 7, 8\}$

35. $A' \cap B = \{5\}$

36. $(A \cup B)' = \{6, 7, 8\}$

37. $A' \cap B' = \{6, 7, 8\}$

38. {4, 5, 6}

39. {2, 3, 6, 7}

40. {1, 4, 5, 6, 8, 9}

41. {4, 5}

42. {1, 2, 3, 6, 7, 8, 9}

43. {2, 3, 7}

44. {6}

45. {1, 2, 3, 4, 5, 6, 7, 8, 9}

46.
$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 25 + 17 - 9 \\ &= 33 \end{aligned}$$

47. $B \cap C = \{1, 5\}$
$A \cup (B \cap C) = \{1, 2, 3, 4, 5\}$

48. $A \cap C = \{1\}$
$(A \cap C)' = (2, 3, 4, 5, 6, 7, 8\}$
$(A \cap C)' \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

49. {c, d, e, f, k, p, r}

50. {f, p}

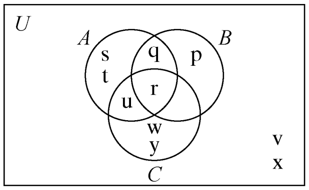
51. {c, d, f, k, p, r}

52. {c, d, e}

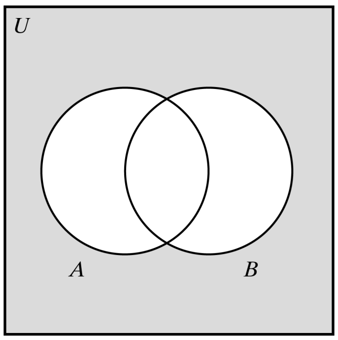
53. {a, b, c, d, e, g, h, p, r}

54. {f}

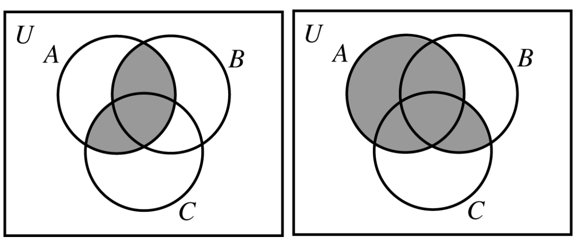
55.



56. The shaded regions are the same for $(A \cup B)'$ and $A' \cap B'$. Therefore $(A \cup B)' = A' \cap B'$

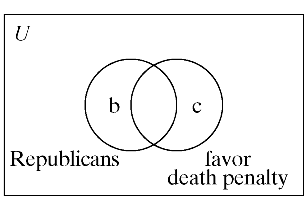


57. The statement is false because the shaded regions are different.

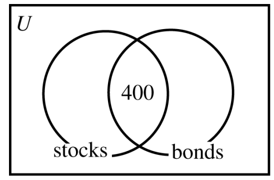


58. United States is in V; Italy is in IV; Turkey is in VIII; Norway is in V; Pakistan is in VIII; Iceland is in V; Mexico is in I

59. a. Parts b and c are labeled.

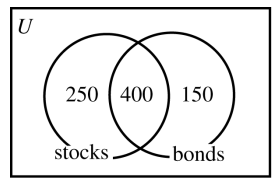


60. Begin by placing 400 in the region that represents both stocks and bonds.



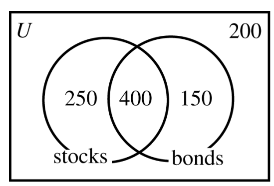
a. Since 650 respondents invested in stocks, $650 - 400 = 250$ invested in only stocks.

Furthermore, since 550 respondents invested in bonds, $550 - 400 = 150$ invested in only bonds. Place this data in the Venn diagram.

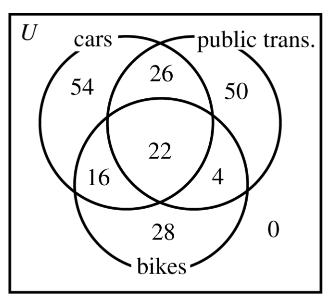


b. $250 + 400 + 150 = 800$ respondents invested in stocks or bonds.

c. Since 1000 people were surveyed, $1000 - 800 = 200$ respondents who did not invest in either.



61. Construct a Venn diagram.



a. 50

b. 26

c. $54 + 26 + 50 = 130$

d. $26 + 16 + 4 = 46$

e. 0

Chapter 2 Test

1. {18, 19, 20, 21, 22, 23, 24}

2. false, {6} is not an element of the set, but 6 is an element.

3. true, both sets have seven elements.

4. true

5. false, *g* is not an element in the larger set.

6. true

7. false, 14 is an element of the set.

8. false, Number of subsets: 2^N where *N* is the number of elements. There are 5 elements. $2^5 = 32$ subsets

9. false, ∅ is *not* a proper subset of itself.

10. ∅ {6} {9} {6, 9}
{6, 9} is not a proper subset.

11. {a, b, c, d, e, f}

12. $B \cap C = \{e\}$
$(B \cap C)' = \{a, b, c, d, f, g\}$

13. $C' = \{b, c, d, f\}$
$A \cap C' = \{b, c, d\}$

14. $A \cup B = \{a, b, c, d, e, f\}$
$(A \cup B) \cap C = \{a, e\}$

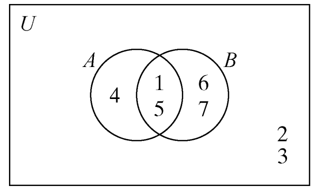
15. $B' = \{a, b, g\}$
$A \cup B' = \{a, b, c, d, g\}$
$n(A \cup B') = 5$

16. {b, c, d, i, j, k}

17. {a}

18. {a, f, h}

19.



20. Both expressions are represented by regions III, VI, and VII. Thus this statement is true and is a theorem.

21. **a.** region V

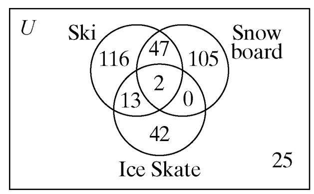
b. region VII

c. region IV

d. region I

e. region VI

22. **a.**



b. 263 (regions I, III, and VII)

c. 25 (region VIII)

d. 62 (regions II, IV, V, and VI)

e. 0 (region VI)

f. 147 (regions III, VI, and VII)

g. 116 (region I)