Solutions Manual

to accompany

STRUCTURAL DYNAMICS
Theory and Applications

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STRUCTURAL DYNAMICS
Theory and Applications

by

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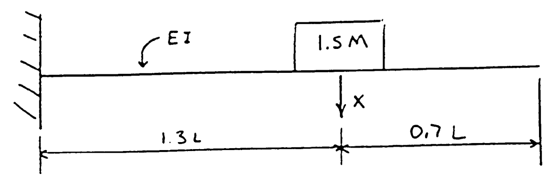
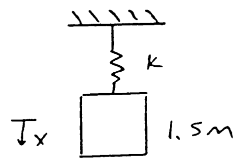
2.1



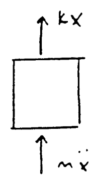
SOLUTION

d'ALEMBERT'S PRINCIPLE

$$\Sigma(\text{FORCES})_x - m\ddot{x} = 0$$



FBD



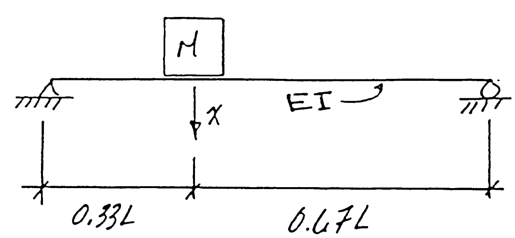
$$kx + m\ddot{x} = 0$$

$$\ddot{x} + \frac{k}{m}x = 0 \qquad \text{EQUATION OF MOTION}$$

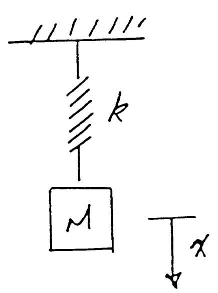
$$k = \frac{3EI}{(1.3L)^3}$$

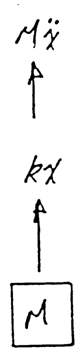
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{3EI}{1.5M(1.3L)^3}} = 0.954\sqrt{\frac{EI}{ML^3}}$$

2.2



Solution:





2.2 Cont.

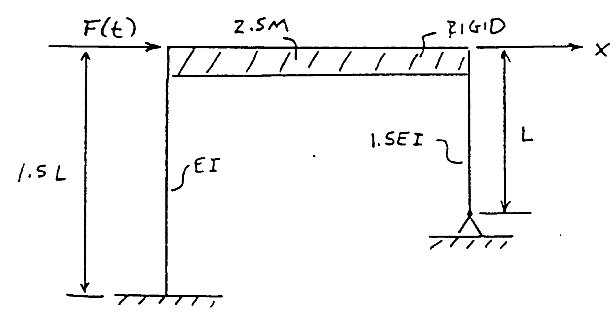
Equation of motion: $M\ddot{x} + kx = 0$ or $\ddot{x} + \frac{k}{M}x = 0$

$$k = \frac{6EIL}{(0.33L)(L-0.33L)[2L(0.33L)-(0.33L)^2-(0.33L)^2}$$

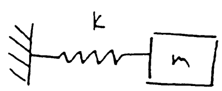
$$k = \frac{61.37EI}{L^3}$$

Natural Frequency: $\omega = \sqrt{\frac{k}{M}} = 7.834\sqrt{\frac{EI}{ML^3}}$

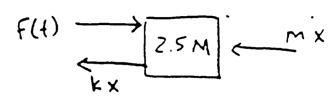
2.3



SOLUTION



FBD



$m = 2.5M$

$\Sigma(\text{FORCES})_x - m\ddot{x} = 0$

$F(t) - kx - m\ddot{x} = 0$

$\ddot{x} + \frac{k}{m}x = \frac{F(t)}{m}$ EQUATION OF MOTION

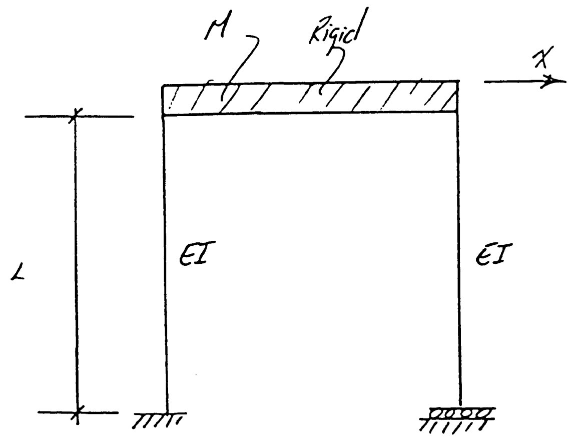
$$k = \frac{12EI}{(1.5L)^3} + \frac{3(1.5EI)}{L^3}$$

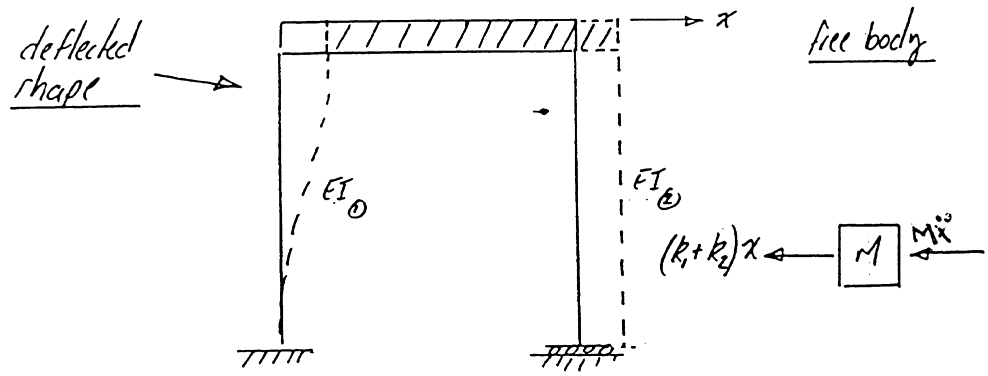
$$= \frac{12(30\times10^6)(150)}{(1.5\times12\times12.0)^3} + \frac{3(1.5)(30\times10^6)(150)}{(12.0\times12)^3}$$

$$= 12{,}140 \text{ LB/IN}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{12{,}140 \text{ LB/IN}}{2.5(1.0 \text{ LB}\cdot\text{sec}^2/\text{IN})}} = 69.7 \text{ rad/sec}$$

2.4





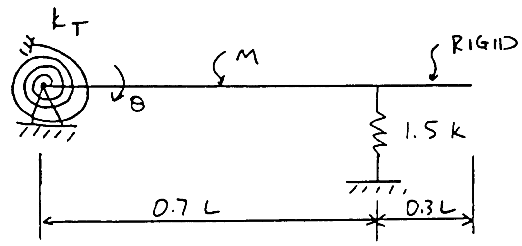
equation of motion: $\boxed{M\ddot{x} + kx = 0 \quad \text{or} \quad \ddot{x} + \frac{k}{M}x = 0}$ ANS

$k_{①} = \frac{12EI}{L^3}$

$k_{②} = 0$

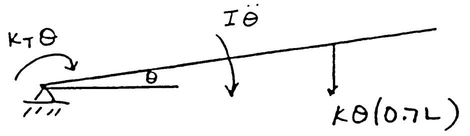
natural freq. : $\boxed{\omega = \sqrt{\frac{k}{M}} = \left(\frac{12EI}{ML^3}\right)^{1/2}}$ ANS

2.5



2.5 Cont.

SOLUTION



$\Delta = 0.7L\sin\theta \approx 0.7L\theta$ FOR SMALL θ

$I = \frac{mL^2}{3}$ (ABOUT PIVOT PT)

$$\Sigma M - I\ddot{\theta} = 0$$

$$K_T\theta + k(0.7L)\Delta + I\ddot{\theta} = 0$$

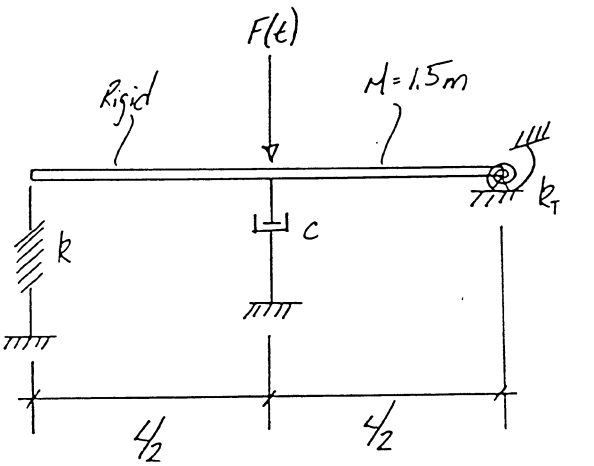
$$k_T\theta + k(0.7L)^2\theta + I\ddot{\theta} = 0$$

$$\frac{mL^2}{3}\ddot{\theta} + k(0.7L)^2\theta + k_T\theta = 0$$

$$\ddot{\theta} + \frac{3\left[k(0.7L)^2 + K_T\right]}{mL^2}\theta = 0 \quad \text{EQUATION OF MOTION}$$

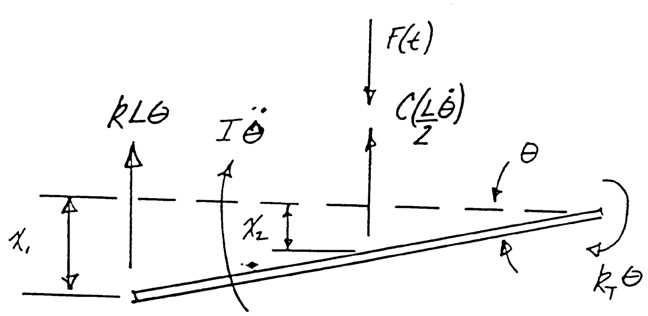
$$\omega = \sqrt{\frac{3k(0.7L)^2 + 3k_T}{mL^2}} \quad \text{NATURAL FREQUENCY}$$

2.6



Solution:

free body



$x_1 = L\sin\theta$ small disp $= L\theta$

$x_2 = \frac{L}{2}\sin\theta$ small disp $= \frac{L}{2}\theta$

$\dot{x}_2 = \frac{L}{2}\dot{\theta}$

2.6 Cont.

$$I = \frac{ML^2}{3} = \frac{1.5mL^2}{3} = \frac{mL^2}{2}$$

equation of motion:

$$k_T\theta + kL\theta(L) + I\ddot{\theta} + c\left(\frac{L}{2}\dot{\theta}\right)\left(\frac{L}{2}\right) = F(t)\left(\frac{L}{2}\right)$$

$$k_T\theta + kL^2\theta + I\ddot{\theta} + \frac{cL^2}{4}\dot{\theta} = F(t)\left(\frac{L}{2}\right)$$

$$I\ddot{\theta} + \frac{cL^2}{4}\dot{\theta} + (kL^2 + k_T)\theta = F(t)\left(\frac{L}{2}\right)$$

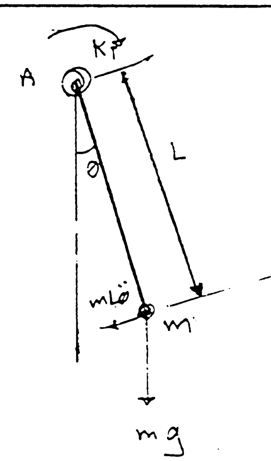
$$\left(\frac{mL^2}{2}\right)\ddot{\theta} + \frac{cL^2}{4}\dot{\theta} + (kL^2 + k_T)\theta = F(t)\left(\frac{L}{2}\right)$$

$$\boxed{\ddot{\theta} + \frac{c}{2m}\dot{\theta} + \frac{2(kL^2 + k_T)}{mL^2}\theta = F(t)\left(\frac{1}{mL}\right)}$$ ANS

natural frequency:

$$\boxed{\omega = \sqrt{\frac{2(kL^2 + k_T)}{mL^2}}}$$ ANS

2.7



$$\Sigma M_A = I_A \alpha$$

$$mL\ddot{\theta}(L) + mgL\sin\theta + k_t\theta = 0$$

$$mL^2\ddot{\theta} + (mgL\sin\theta + k_t\theta) = 0$$

2.7 Cont.

for small values of θ

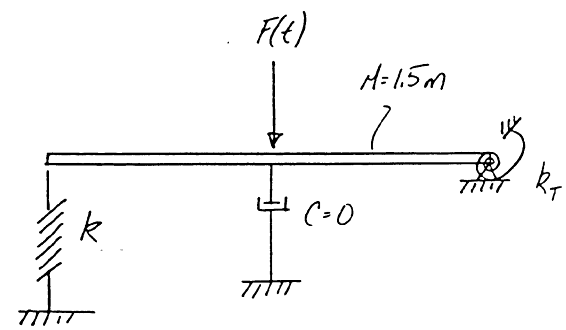
$\sin\theta = \theta$

$$mL^2\ddot{\theta} + (mgL + k_t)\,\theta = 0$$

$$\ddot{\theta} + \left(\frac{g}{L} + \frac{k_t}{mL^2}\right)\theta = 0$$

$$\omega = \sqrt{\frac{g}{L} + \frac{k_t}{mL^2}}$$

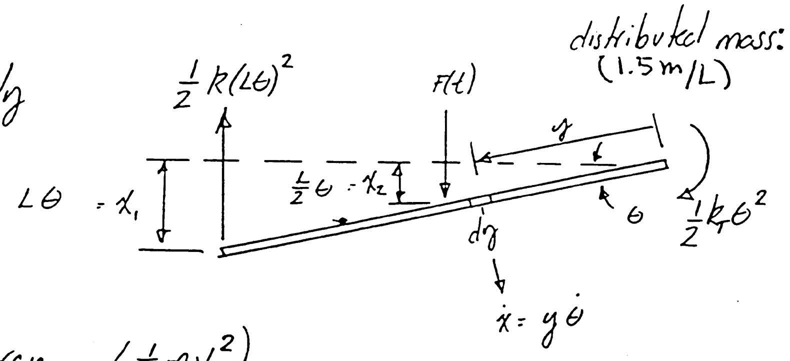
2.8



Assume a conservative system (i.e. no damping)

Solution:

free body



Kinetic Energy $\left(\frac{1}{2}mv^2\right)$

$$\frac{1}{2}\int_0^L \frac{1.5m}{L}(y\dot{\theta})^2\,dy = \frac{3}{4}\int_0^L \frac{m}{L}\dot{\theta}^2 y^2\,dy = \frac{3}{4}\left(\frac{m}{3L}\dot{\theta}^2 y^3\right)\Big|_0^L$$

$$\boxed{T = \frac{mL^2\dot{\theta}^2}{4}}$$

2.8 Cont.

Potential Energy

$$V = \frac{1}{2}k(L\theta)^2 + \frac{1}{2}k_T\theta^2 - F(t)\left(\frac{L}{2}\right)\theta$$

TOTAL WORK $(T+V) =$ constant

$$\frac{mL^2\dot{\theta}^2}{4} + \frac{1}{2}k(L\theta)^2 + \frac{1}{2}k_T\theta^2 - F(t)\left(\frac{L}{2}\right)\theta = \text{constant}$$

$$\frac{d(T+V)}{d\theta} = 0 = \frac{mL^2}{2}\ddot{\theta}\cancel{\dot{\theta}} + kL^2\cancel{\dot{\theta}}\theta + k_T\cancel{\dot{\theta}}\theta - F(t)\left(\frac{L}{2}\right)\cancel{\dot{\theta}}$$

equation of motion:

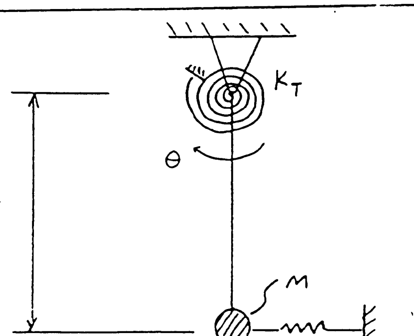
$$\frac{mL^2}{2}\ddot{\theta} + (kL^2 + k_T)\theta = F(t)\left(\frac{L}{2}\right)$$

$$\boxed{\ddot{\theta} + \frac{2(kL^2+k_T)}{mL^2}\theta = F(t)\left(\frac{1}{mL}\right)}$$ ANS

natural frequency:

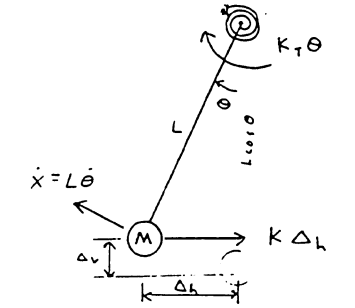
$$\boxed{\omega = \sqrt{\frac{2(kL^2+k_T)}{mL^2}}}$$ ANS

2.9



2.9 Cont.

SOLUTION



$\Delta_h = L\sin\theta \doteq L\theta$

$\Delta_v = L - L\cos\theta = L(1-\cos\theta)$

KINETIC ENERGY

$$T = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}m(L\dot{\theta})^2$$

POTENTIAL ENERGY

$$V = mg\Delta_v + \frac{1}{2}K\Delta_h^2 + \frac{1}{2}k_T\theta^2$$
$$= mgL(1-\cos\theta) + \frac{1}{2}K(L\theta)^2 + \frac{1}{2}k_T\theta^2$$

ENERGY METHOD

$$T + V = \text{CONSTANT}$$

$$\frac{d}{dt}(T+V) = 0$$

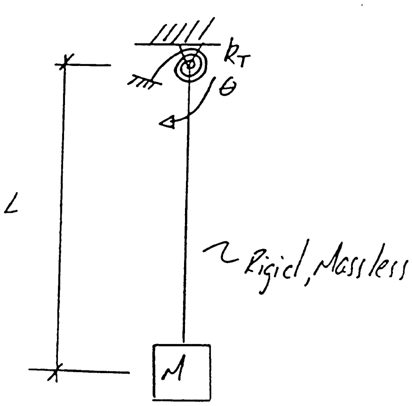
$$mL^2\dot{\theta}\ddot{\theta} + mgL(\sin\theta)\dot{\theta} + KL^2\theta\dot{\theta} + k_T\theta\dot{\theta} = 0$$

$$mL^2\ddot{\theta} + mgL\sin\theta + KL^2\theta + k_T\theta = 0$$

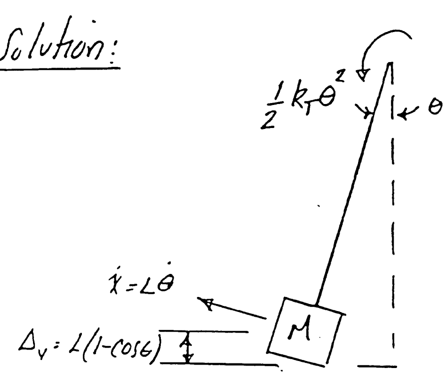
$$\ddot{\theta} + \left(\frac{MgL + KL^2 + k_T}{ML^2}\right)\theta = 0 \qquad \text{EQUATION OF MOTION}$$

$$\omega = \sqrt{\frac{MgL + KL^2 + k_T}{ML^2}} \qquad \text{NATURAL FREQUENCY}$$

2.10



Solution:



2.10 cont.

Kinetic Energy (T) $\frac{1}{2}mv^2$

$$\frac{1}{2}M(L\dot{\theta})^2$$

Potential energy: (V)

$$Mg\,L(1-\cos\theta) + \frac{1}{2}k_T\theta^2$$

TOTAL WORK: (T+V)

$$\frac{1}{2}ML^2\dot{\theta}^2 + \frac{1}{2}k_T\theta^2 + MgL(1-\cos\theta) = \text{constant}$$

$$\frac{d(T+V)}{d\theta} = 0 = ML^2\ddot{\theta}\cancel{\dot{\theta}} + k_T\cancel{\dot{\theta}}\theta + MgL\,\sin\theta\,\cancel{\dot{\theta}}$$

$$ML^2\ddot{\theta} + k_T\theta + MgL\theta = 0$$

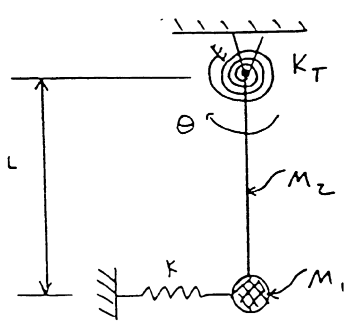
$$ML^2\ddot{\theta} + (k_T + MgL)\theta = 0$$

$$\boxed{\ddot{\theta} + \left(\frac{k_T + MgL}{ML^2}\right)\theta = 0}$$ ANS

natural frequency:

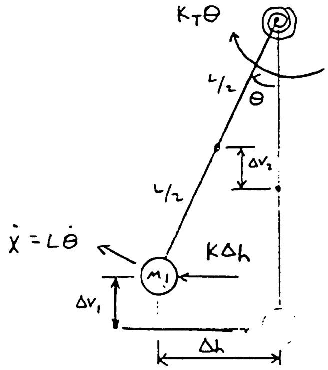
$$\boxed{\omega = \sqrt{\frac{k_T + MgL}{ML^2}}}$$ ANS

2.11



2.11 Cont.

SOLUTION



$$\Delta h = L\sin\theta \approx L\theta$$
$$\Delta V_1 = L(1-\cos\theta)$$
$$\Delta V_2 = \tfrac{L}{2}(1-\cos\theta)$$

$$T = \tfrac{1}{2} m_1 \dot{x}^2 + \tfrac{1}{2} I_0 \dot{\theta}^2$$
$$= \tfrac{1}{2} m_1 (L\dot{\theta})^2 + \tfrac{1}{2}\left(\tfrac{1}{3} m_2 L^2\right)\dot{\theta}^2$$
$$= \tfrac{1}{2} m_1 L^2 \dot{\theta}^2 + \tfrac{1}{6} m_2 L^2 \dot{\theta}^2$$
$$V = m_1 g \Delta V_1 + m_2 g \Delta V_2 + \tfrac{1}{2} k \Delta h^2 + \tfrac{1}{2} k_T \theta^2$$
$$= m_1 g L(1-\cos\theta) + m_2 g \tfrac{L}{2}(1-\cos\theta) + \tfrac{1}{2} k (L\theta)^2 + \tfrac{1}{2} k_T \theta^2$$

ENERGY METHOD

$$T + V = \text{CONSTANT}$$
$$\frac{d}{dt}(T+V) = 0$$

$$m_1 L^2 \dot{\theta}\ddot{\theta} + \tfrac{1}{3} m_2 L^2 \dot{\theta}\ddot{\theta} + m_1 g L(\sin\theta)\dot{\theta} + m_2 g \tfrac{L}{2}(\sin\theta)\dot{\theta} + k L^2 \theta\dot{\theta} + k_T \theta\dot{\theta} = 0$$

$$m_1 L^2 \ddot{\theta} + \tfrac{1}{3} m_2 L^2 \ddot{\theta} + m_1 g L\theta + m_2 g \tfrac{L}{2}\theta + kL^2\theta + k_T\theta = 0$$

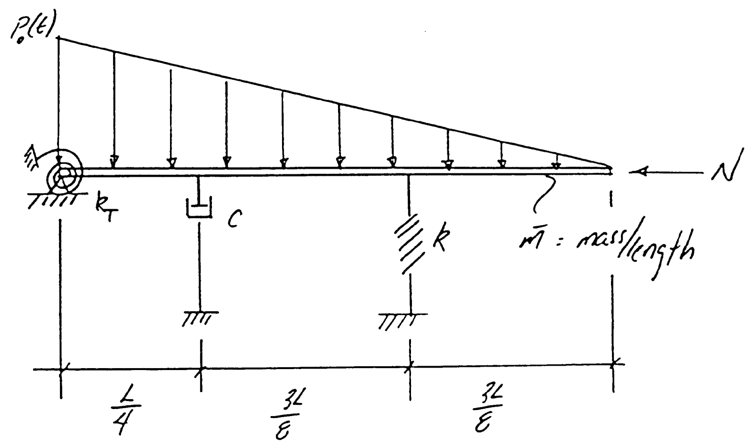
$$\ddot{\theta} + \frac{m_1 g L + \frac{1}{2} m_2 g L + kL^2 + k_T}{m_1 L^2 + \frac{1}{3} m_2 L^2}\,\theta = 0$$

EQUATION OF MOTION

$$\omega = \sqrt{\frac{m_1 g L + \frac{1}{2} m_2 g L + kL^2 + k_T}{m_1 L^2 + \frac{1}{3} m_2 L^2}}$$

NATURAL FREQUENCY

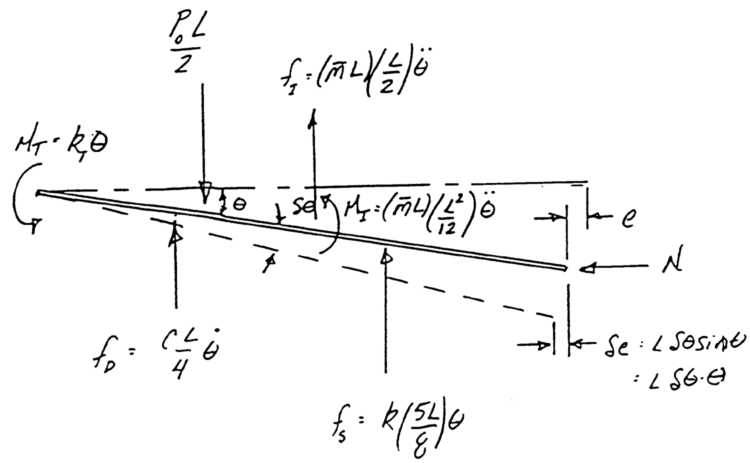
2.12



2.12 Cont.

Solution:

free body



equation of motion:

$$-f_s\left(\frac{5L}{8}\delta\theta\right) - f_D\left(\frac{L}{4}\delta\theta\right) - M_T\,\delta\theta + N\,\delta e + \frac{P_oL}{2}\left(\frac{L}{3}\delta\theta\right)$$

$$\Longrightarrow \quad - f_I\left(\frac{L}{2}\delta\theta\right) - M_I\,\delta\theta = 0$$

$$-\frac{25}{64}L^2k\theta\cancel{\delta\theta} - c\frac{L^2}{16}\dot{\theta}\cancel{\delta\theta} - k_T\theta\cancel{\delta\theta} + NL\theta\cancel{\delta\theta} + \frac{P_oL^2}{6}\cancel{\delta\theta}$$

$$\Longrightarrow \quad -\frac{\bar{m}L^3}{4}\ddot{\theta}\cancel{\delta\theta} - \frac{\bar{m}L^3}{12}\ddot{\theta}\cancel{\delta\theta} = 0$$

$$\Longrightarrow -\frac{\bar{m}L^3}{3}\ddot{\theta} - \frac{cL^2}{16}\dot{\theta} - \left(\frac{25}{64}kL^2 + k_T - NL\right)\theta + \frac{P_oL^2}{6} = 0$$

$$\boxed{\frac{\bar{m}L^3}{3}\ddot{\theta} + \frac{cL^2}{16}\dot{\theta} + \left(\frac{25}{64}kL^2 + k_T - NL\right)\theta = \frac{P_oL^2}{6}} \quad \text{ANS}$$

Natural frequency: $\omega = \sqrt{\frac{k}{m}}$

$$\boxed{\omega = \sqrt{\frac{\frac{25}{64}kL^2 + k_T - NL}{\frac{\bar{m}L^3}{3}}}} \quad \text{ANS}$$