

2-1.

If $\theta = 60^\circ$ and $F = 450$ N, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.

SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

$$F_R = \sqrt{700^2 + 450^2 - 2(700)(450) \cos 45^\circ}$$

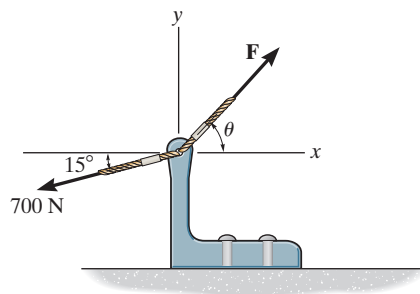
$$= 497.01 \text{ N} = 497 \text{ N}$$

This yields

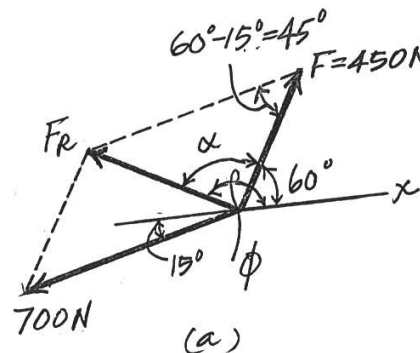
$$\frac{\sin \alpha}{700} = \frac{\sin 45^\circ}{497.01} \quad \alpha = 95.19^\circ$$

Thus, the direction of angle ϕ of \mathbf{F}_R measured counterclockwise from the positive x axis, is

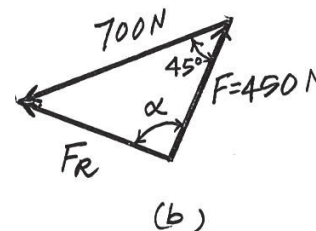
$$\phi = \alpha + 60^\circ = 95.19^\circ + 60^\circ = 155^\circ$$



Ans.



Ans.



Ans:

$$F_R = 497 \text{ N}$$

$$\phi = 155^\circ$$

2-2.

If the magnitude of the resultant force is to be 500 N, directed along the positive y axis, determine the magnitude of force \mathbf{F} and its direction θ .

SOLUTION

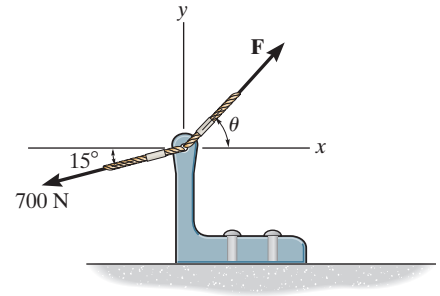
The parallelogram law of addition and the triangular rule are shown in Figs. a and b , respectively.

Applying the law of cosines to Fig. b ,

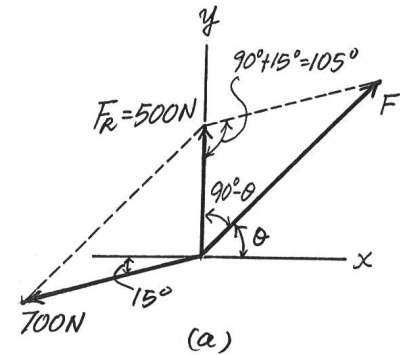
$$\begin{aligned} F &= \sqrt{500^2 + 700^2 - 2(500)(700) \cos 105^\circ} \\ &= 959.78 \text{ N} = 960 \text{ N} \end{aligned}$$

Applying the law of sines to Fig. b , and using this result, yields

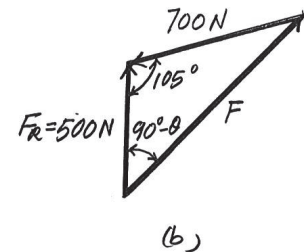
$$\begin{aligned} \frac{\sin (90^\circ + \theta)}{700} &= \frac{\sin 105^\circ}{959.78} \\ \theta &= 45.2^\circ \end{aligned}$$



Ans.



Ans.



Ans:

$$\begin{aligned} F &= 960 \text{ N} \\ \theta &= 45.2^\circ \end{aligned}$$

2-3.

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured counterclockwise from the positive x axis.

SOLUTION

$$F_R = \sqrt{(250)^2 + (375)^2 - 2(250)(375) \cos 75^\circ} = 393.2 = 393 \text{ lb}$$

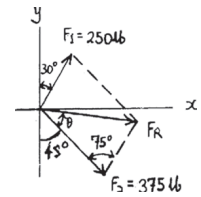
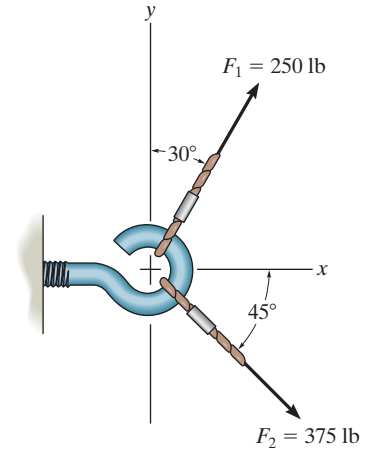
$$\frac{393.2}{\sin 75^\circ} = \frac{250}{\sin \theta}$$

$$\theta = 37.89^\circ$$

$$\phi = 360^\circ - 45^\circ + 37.89^\circ = 353^\circ$$

Ans.

Ans.



Ans:

$$F_R = 393 \text{ lb}$$

$$\phi = 353^\circ$$

*2-4.

Determine the magnitudes of the two components of **F** directed along members *AB* and *AC*. Set $F = 500$ N.

SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using the law of sines (Fig. *b*), we have

$$\frac{F_{AB}}{\sin 60^\circ} = \frac{500}{\sin 75^\circ}$$

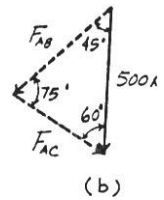
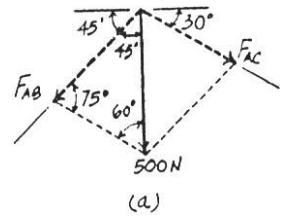
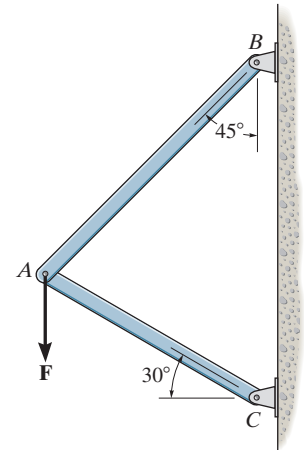
$$F_{AB} = 448 \text{ N}$$

$$\frac{F_{AC}}{\sin 45^\circ} = \frac{500}{\sin 75^\circ}$$

$$F_{AC} = 366 \text{ N}$$

Ans.

Ans.



Ans:

$$F_{AB} = 448 \text{ N}$$

$$F_{AC} = 366 \text{ N}$$

2-5.

Solve Prob. 2-4 with $F = 350$ lb.

SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

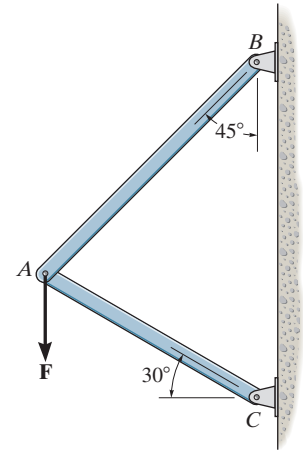
Trigonometry: Using the law of sines (Fig. *b*), we have

$$\frac{F_{AB}}{\sin 60^\circ} = \frac{350}{\sin 75^\circ}$$

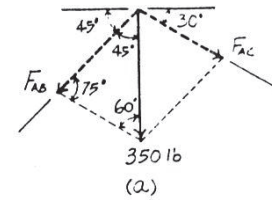
$$F_{AB} = 314 \text{ lb}$$

$$\frac{F_{AC}}{\sin 45^\circ} = \frac{350}{\sin 75^\circ}$$

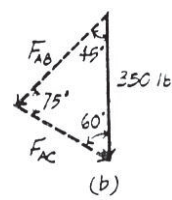
$$F_{AC} = 256 \text{ lb}$$



Ans.



Ans.



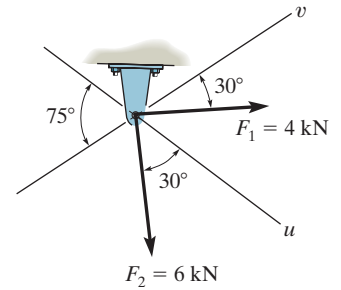
Ans:

$$F_{AB} = 314 \text{ lb}$$

$$F_{AC} = 256 \text{ lb}$$

2-6.

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive u axis.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.

Trigonometry. Applying Law of cosines by referring to Fig. *b*,

$$F_R = \sqrt{4^2 + 6^2 - 2(4)(6) \cos 105^\circ} = 8.026 \text{ kN} = 8.03 \text{ kN}$$

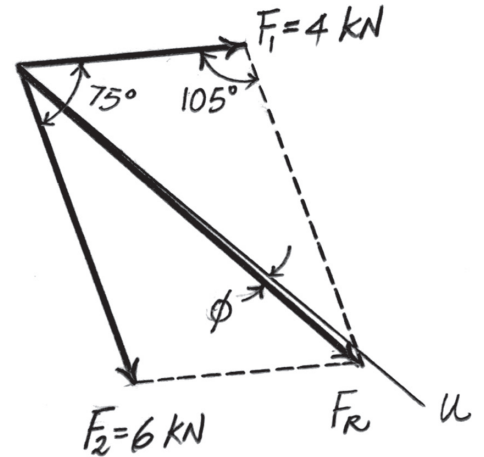
Using this result to apply Law of sines, Fig. *b*,

$$\frac{\sin \theta}{6} = \frac{\sin 105^\circ}{8.026}; \quad \theta = 46.22^\circ$$

Thus, the direction ϕ of \mathbf{F}_R measured clockwise from the positive u axis is

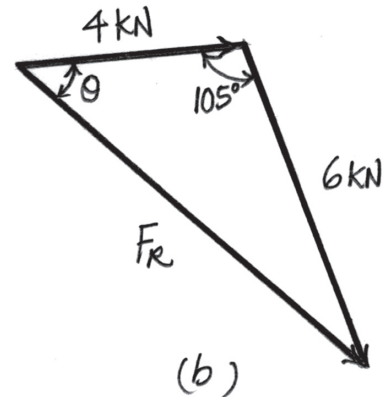
$$\phi = 46.22^\circ - 45^\circ = 1.22^\circ$$

Ans.



Ans.

(a)



(b)

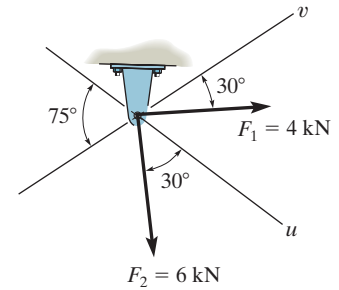
Ans:

$$F_R = 8.03 \text{ kN}$$

$$\phi = 1.22^\circ$$

2-7.

Resolve the force F_1 into components acting along the u and v axes and determine the magnitudes of the components.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.

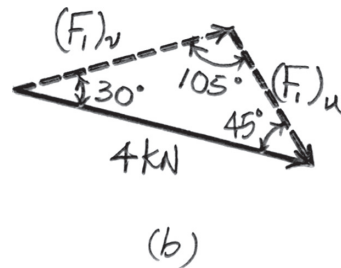
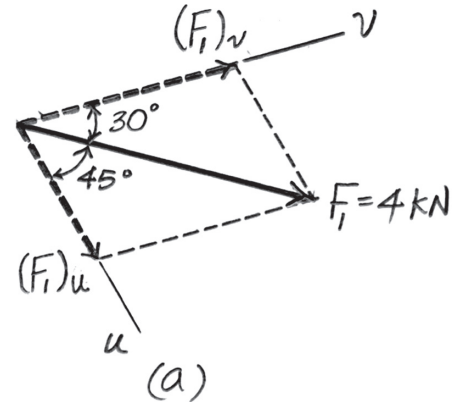
Trigonometry. Applying the sines law by referring to Fig. *b*.

$$\frac{(F_1)_v}{\sin 45^\circ} = \frac{4}{\sin 105^\circ}; \quad (F_1)_v = 2.928 \text{ kN} = 2.93 \text{ kN}$$

$$\frac{(F_1)_u}{\sin 30^\circ} = \frac{4}{\sin 105^\circ}; \quad (F_1)_u = 2.071 \text{ kN} = 2.07 \text{ kN}$$

Ans.

Ans.



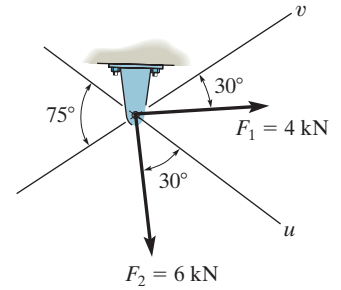
Ans:

$$(F_1)_v = 2.93 \text{ kN}$$

$$(F_1)_u = 2.07 \text{ kN}$$

*2-8.

Resolve the force \mathbf{F}_2 into components acting along the u and v axes and determine the magnitudes of the components.



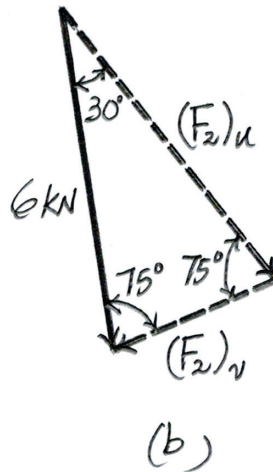
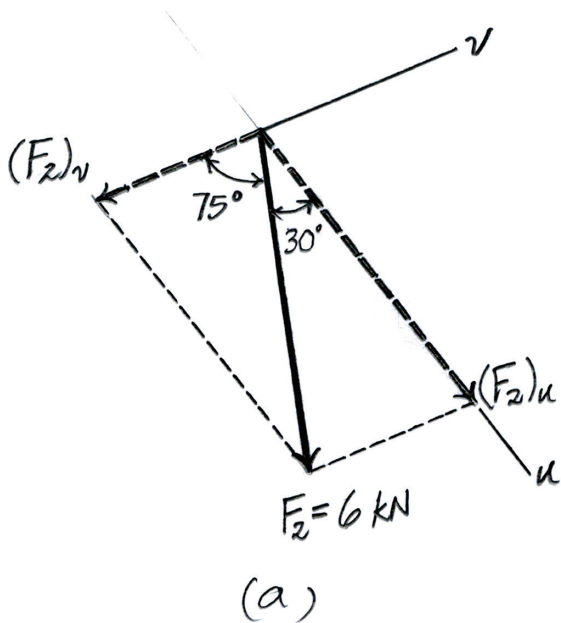
SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.

Trigonometry. Applying the sines law of referring to Fig. *b*,

$$\frac{(F_2)_u}{\sin 75^\circ} = \frac{6}{\sin 75^\circ}; \quad (F_2)_u = 6.00 \text{ kN} \quad \text{Ans.}$$

$$\frac{(F_2)_v}{\sin 30^\circ} = \frac{6}{\sin 75^\circ}; \quad (F_2)_v = 3.106 \text{ kN} = 3.11 \text{ kN} \quad \text{Ans.}$$



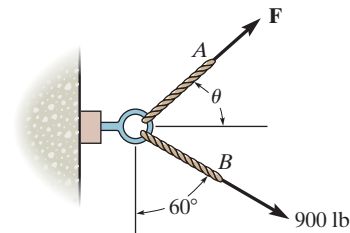
Ans:

$$(F_2)_u = 6.00 \text{ kN}$$

$$(F_2)_v = 3.11 \text{ kN}$$

2-9.

If the resultant force acting on the support is to be 1200 lb, directed horizontally to the right, determine the force **F** in rope *A* and the corresponding angle θ .



SOLUTION

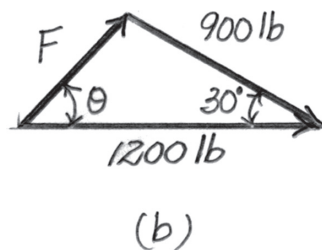
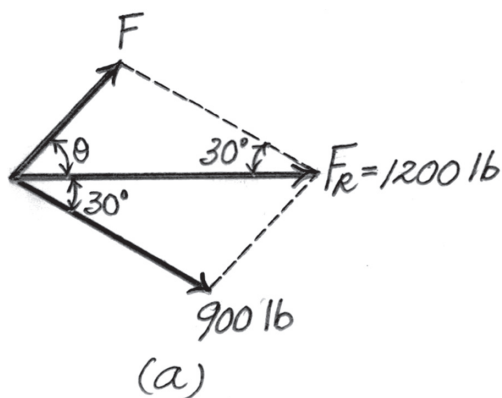
Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.

Trigonometry. Applying the law of cosines by referring to Fig. *b*,

$$F = \sqrt{900^2 + 1200^2 - 2(900)(1200) \cos 30^\circ} = 615.94 \text{ lb} = 616 \text{ lb} \quad \text{Ans.}$$

Using this result to apply the sines law, Fig. *b*,

$$\frac{\sin \theta}{900} = \frac{\sin 30^\circ}{615.94}; \quad \theta = 46.94^\circ = 46.9^\circ \quad \text{Ans.}$$



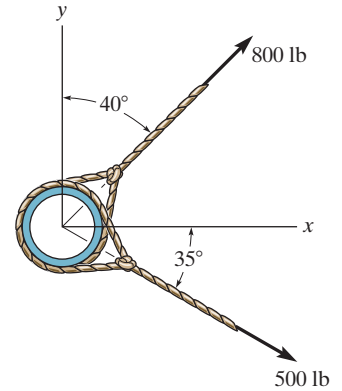
Ans:

$$F = 616 \text{ lb}$$

$$\theta = 46.9^\circ$$

2-10.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.

Trigonometry. Applying the law of cosines by referring to Fig. *b*,

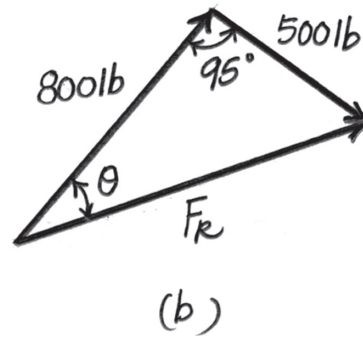
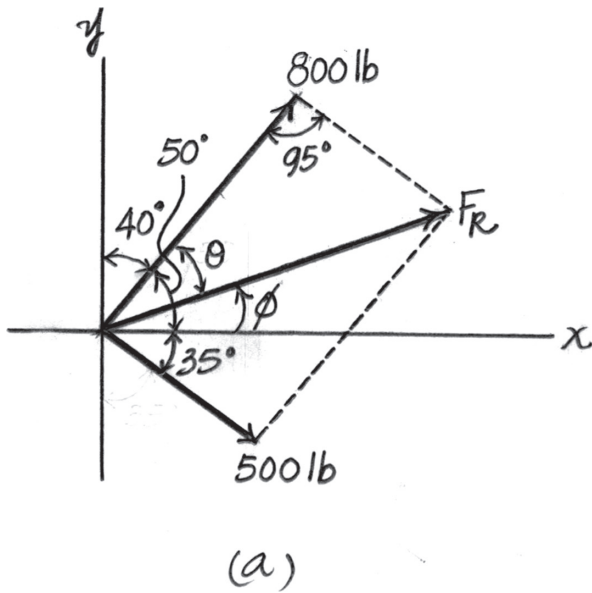
$$F_R = \sqrt{800^2 + 500^2 - 2(800)(500) \cos 95^\circ} = 979.66 \text{ lb} = 980 \text{ lb} \quad \text{Ans.}$$

Using this result to apply the sines law, Fig. *b*,

$$\frac{\sin \theta}{500} = \frac{\sin 95^\circ}{979.66}; \quad \theta = 30.56^\circ$$

Thus, the direction ϕ of \mathbf{F}_R measured counterclockwise from the positive x axis is

$$\phi = 50^\circ - 30.56^\circ = 19.44^\circ = 19.4^\circ \quad \text{Ans.}$$



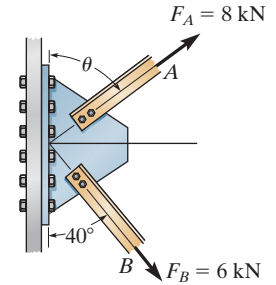
Ans:

$$F_R = 980 \text{ lb}$$

$$\phi = 19.4^\circ$$

2-11.

If $\theta = 60^\circ$, determine the magnitude of the resultant and its direction measured clockwise from the horizontal.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of cosines (Fig. *b*), we have

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6) \cos 100^\circ}$$

$$= 10.80 \text{ kN} = 10.8 \text{ kN}$$

The angle θ can be determined using law of sines (Fig. *b*).

$$\frac{\sin \theta}{6} = \frac{\sin 100^\circ}{10.80}$$

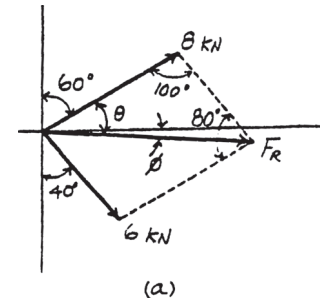
$$\sin \theta = 0.5470$$

$$\theta = 33.16^\circ$$

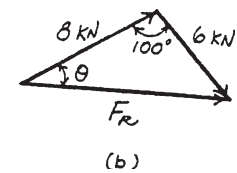
Thus, the direction ϕ of \mathbf{F}_R measured from the x axis is

$$\phi = 33.16^\circ - 30^\circ = 3.16^\circ$$

Ans.



Ans.



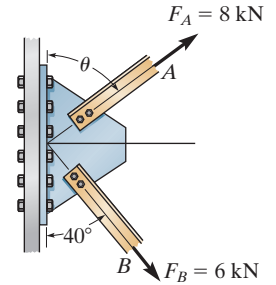
Ans:

$$F_R = 10.8 \text{ kN}$$

$$\phi = 3.16^\circ$$

***2-12.**

Determine the angle θ for connecting member A to the plate so that the resultant force of \mathbf{F}_A and \mathbf{F}_B is directed horizontally to the right. Also, what is the magnitude of the resultant force?



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of sines (Fig. *b*), we have

$$\frac{\sin(90^\circ - \theta)}{6} = \frac{\sin 50^\circ}{8}$$

$$\sin(90^\circ - \theta) = 0.5745$$

$$\theta = 54.93^\circ = 54.9^\circ$$

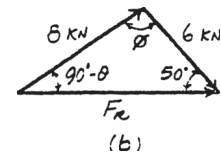
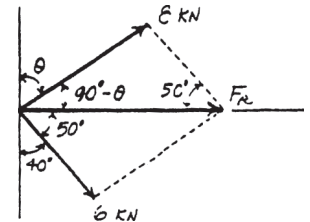
Ans.

From the triangle, $\phi = 180^\circ - (90^\circ - 54.93^\circ) - 50^\circ = 94.93^\circ$. Thus, using law of cosines, the magnitude of \mathbf{F}_R is

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6) \cos 94.93^\circ}$$

$$= 10.4 \text{ kN}$$

Ans.



Ans:

$$\theta = 54.9^\circ$$

$$F_R = 10.4 \text{ kN}$$

2-13.

The force acting on the gear tooth is $F = 20$ lb. Resolve this force into two components acting along the lines aa and bb .

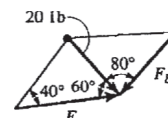
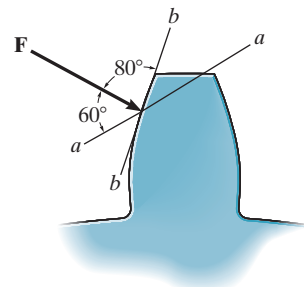
SOLUTION

$$\frac{20}{\sin 40^\circ} = \frac{F_a}{\sin 80^\circ}; \quad F_a = 30.6 \text{ lb}$$

Ans.

$$\frac{20}{\sin 40^\circ} = \frac{F_b}{\sin 60^\circ}; \quad F_b = 26.9 \text{ lb}$$

Ans.



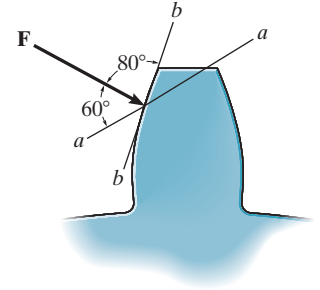
Ans:

$$F_a = 30.6 \text{ lb}$$

$$F_b = 26.9 \text{ lb}$$

2-14.

The component of force **F** acting along line *aa* is required to be 30 lb. Determine the magnitude of **F** and its component along line *bb*.



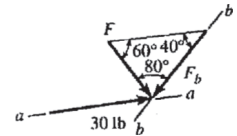
SOLUTION

$$\frac{30}{\sin 80^\circ} = \frac{F}{\sin 40^\circ}; \quad F = 19.6 \text{ lb}$$

Ans.

$$\frac{30}{\sin 80^\circ} = \frac{F_b}{\sin 60^\circ}; \quad F_b = 26.4 \text{ lb}$$

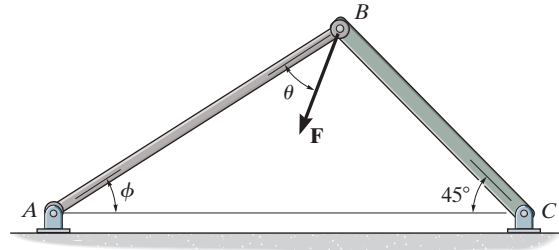
Ans.



Ans:
 $F = 19.6 \text{ lb}$
 $F_b = 26.4 \text{ lb}$

2-15.

Force **F** acts on the frame such that its component acting along member *AB* is 650 lb, directed from *B* towards *A*, and the component acting along member *BC* is 500 lb, directed from *B* towards *C*. Determine the magnitude of **F** and its direction θ . Set $\phi = 60^\circ$.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

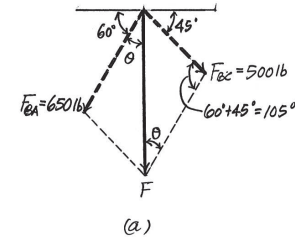
$$F = \sqrt{500^2 + 650^2 - 2(500)(650) \cos 105^\circ}$$

$$= 916.91 \text{ lb} = 917 \text{ lb}$$

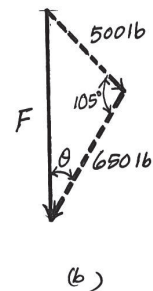
Using this result and applying the law of sines to Fig. *b* yields

$$\frac{\sin \theta}{500} = \frac{\sin 105^\circ}{916.91} \quad \theta = 31.8^\circ$$

Ans.



Ans.



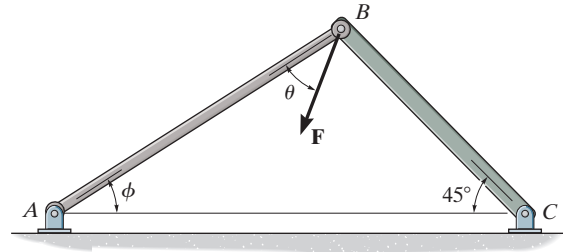
Ans:

$$F = 917 \text{ lb}$$

$$\theta = 31.8^\circ$$

*2-16.

Force \mathbf{F} acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A . Determine the required angle ϕ ($0^\circ \leq \phi \leq 45^\circ$) and the component acting along member BC . Set $F = 850$ lb and $\theta = 30^\circ$.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

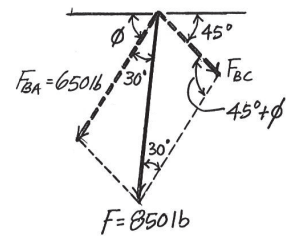
$$\begin{aligned} F_{BC} &= \sqrt{850^2 + 650^2 - 2(850)(650) \cos 30^\circ} \\ &= 433.64 \text{ lb} = 434 \text{ lb} \end{aligned}$$

Using this result and applying the sine law to Fig. *b* yields

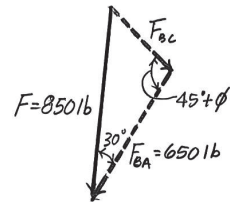
$$\frac{\sin (45^\circ + \phi)}{850} = \frac{\sin 30^\circ}{433.64} \quad \phi = 33.5^\circ$$

Ans.

Ans.



(a)



(b)

Ans:

$$\begin{aligned} F_{BC} &= 434 \text{ lb} \\ \phi &= 33.5^\circ \end{aligned}$$

2-17.

If $F_1 = 30$ lb and $F_2 = 40$ lb, determine the angles θ and ϕ so that the resultant force is directed along the positive x axis and has a magnitude of $F_R = 60$ lb.

SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*.

Trigonometry. Applying the law of cosine by referring to Fig. *b*,

$$40^2 = 30^2 + 60^2 - 2(30)(60) \cos \theta$$

$$\theta = 36.34^\circ = 36.3^\circ$$

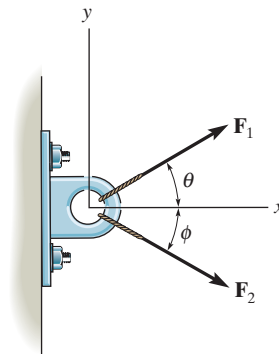
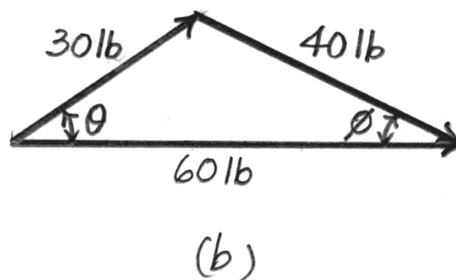
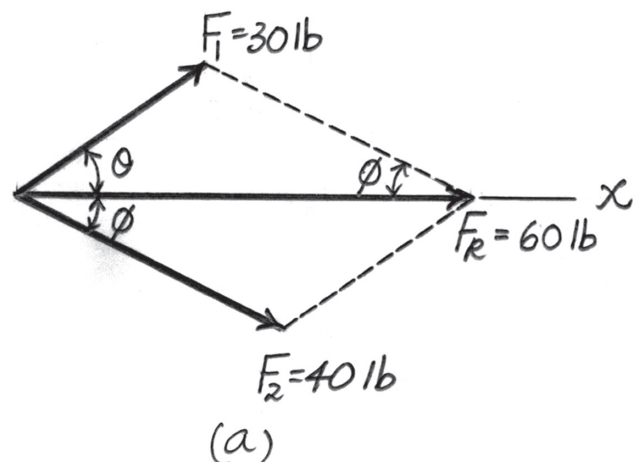
Ans.

And

$$30^2 = 40^2 + 60^2 - 2(40)(60) \cos \phi$$

$$\phi = 26.38^\circ = 26.4^\circ$$

Ans.



Ans:

$$\theta = 36.3^\circ$$

$$\phi = 26.4^\circ$$

2–18.

Determine the magnitude and direction θ of \mathbf{F}_A so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.

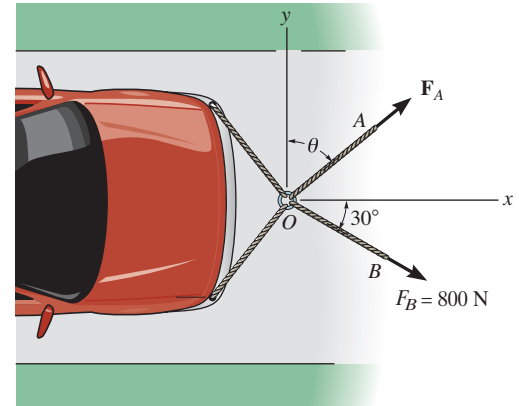
SOLUTION

$$\rightarrow F_{R_x} = \Sigma F_x; \quad F_{R_x} = F_A \sin \theta + 800 \cos 30^\circ = 1250$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = F_A \cos \theta - 800 \sin 30^\circ = 0$$

$$\theta = 54.3^\circ$$

$$F_A = 686 \text{ N}$$



Ans.

Ans.

Ans:
 $\theta = 54.3^\circ$
 $F_A = 686 \text{ N}$

2-19.

Determine the magnitude of the resultant force acting on the ring at O if $F_A = 750$ N and $\theta = 45^\circ$. What is its direction, measured counterclockwise from the positive x axis?

SOLUTION

Scalar Notation: Summing the force components algebraically, we have

$$\begin{aligned}\rightarrow F_{R_x} &= \Sigma F_x; & F_{R_x} &= 750 \sin 45^\circ + 800 \cos 30^\circ \\ & & &= 1223.15 \text{ N} \rightarrow\end{aligned}$$

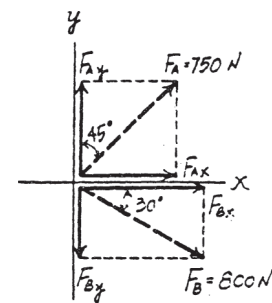
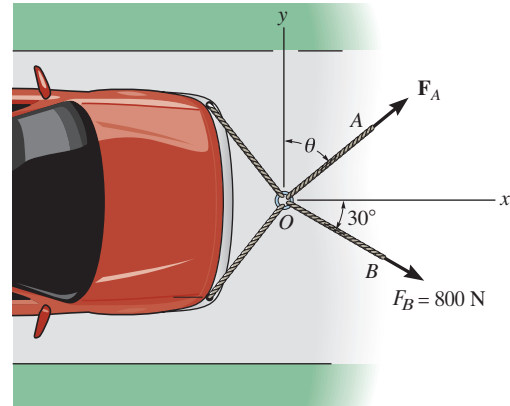
$$\begin{aligned}+\uparrow F_{R_y} &= \Sigma F_y; & F_{R_y} &= 750 \cos 45^\circ - 800 \sin 30^\circ \\ & & &= 130.33 \text{ N} \uparrow\end{aligned}$$

The magnitude of the resultant force \mathbf{F}_R is

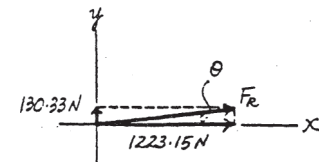
$$\begin{aligned}F_R &= \sqrt{F_{R_x}^2 + F_{R_y}^2} \\ &= \sqrt{1223.15^2 + 130.33^2} = 1230 \text{ N} = 1.23 \text{ kN}\end{aligned}$$

The directional angle θ measured counterclockwise from positive x axis is

$$\theta = \tan^{-1} \frac{F_{R_y}}{F_{R_x}} = \tan^{-1} \left(\frac{130.33}{1223.15} \right) = 6.08^\circ$$



Ans.



Ans.

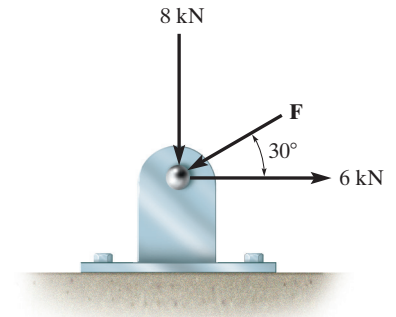
Ans:

$$F_R = 1.23 \text{ kN}$$

$$\theta = 6.08^\circ$$

*2-20.

Determine the magnitude of force \mathbf{F} so that the resultant \mathbf{F}_R of the three forces is as small as possible. What is the minimum magnitude of \mathbf{F}_R ?



SOLUTION

Parallelogram Law. The parallelogram laws of addition for 6 kN and 8 kN and then their resultant F' and F are shown in Figs. *a* and *b*, respectively. In order for F_R to be minimum, it must act perpendicular to \mathbf{F} .

Trigonometry. Referring to Fig. *b*,

$$F' = \sqrt{6^2 + 8^2} = 10.0 \text{ kN} \quad \theta = \tan^{-1}\left(\frac{8}{6}\right) = 53.13^\circ.$$

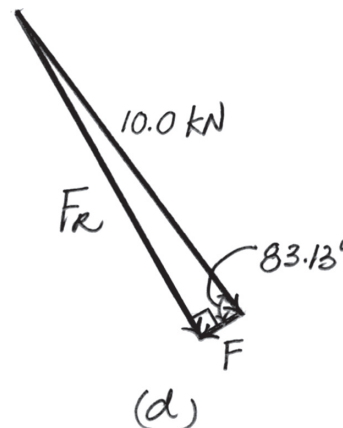
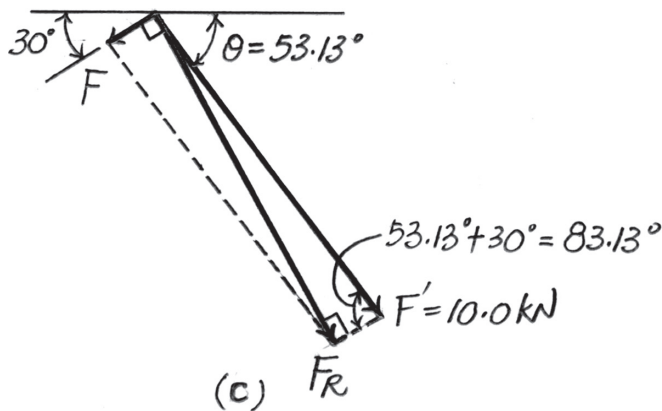
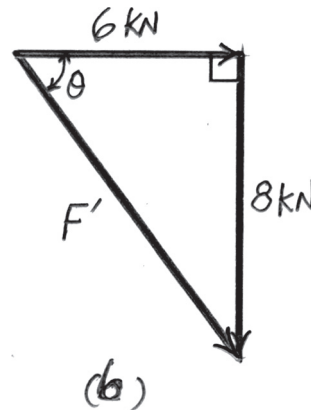
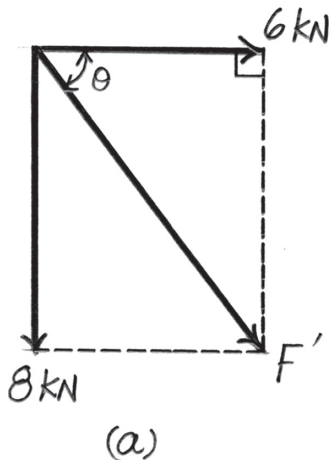
Referring to Figs. *c* and *d*,

$$F_R = 10.0 \sin 83.13^\circ = 9.928 \text{ kN} = 9.93 \text{ kN}$$

Ans.

$$F = 10.0 \cos 83.13^\circ = 1.196 \text{ kN} = 1.20 \text{ kN}$$

Ans.



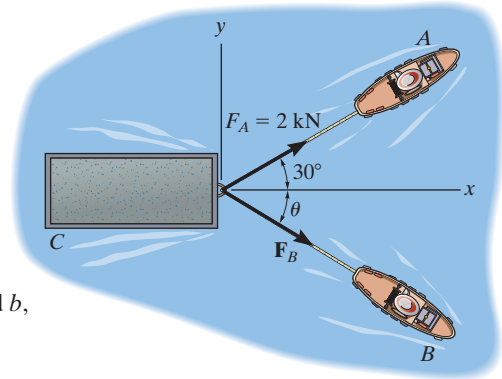
Ans:

$$F_R = 9.93 \text{ kN}$$

$$F = 1.20 \text{ kN}$$

2-21.

If the resultant force of the two tugboats is 3 kN, directed along the positive x axis, determine the required magnitude of force \mathbf{F}_B and its direction θ .



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. a and b , respectively.

Applying the law of cosines to Fig. b ,

$$F_B = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 30^\circ}$$

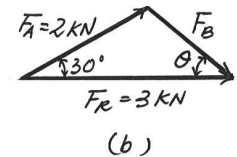
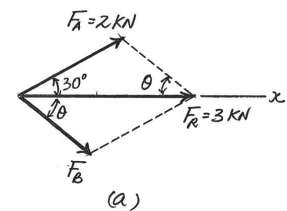
$$= 1.615 \text{ kN} = 1.61 \text{ kN}$$

Using this result and applying the law of sines to Fig. b yields

$$\frac{\sin \theta}{2} = \frac{\sin 30^\circ}{1.615} \quad \theta = 38.3^\circ$$

Ans.

Ans.



Ans:

$$F_B = 1.61 \text{ kN}$$

$$\theta = 38.3^\circ$$

2-22.

If $F_B = 3 \text{ kN}$ and $\theta = 45^\circ$, determine the magnitude of the resultant force and its direction measured clockwise from the positive x axis.

SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

$$\begin{aligned} F_R &= \sqrt{2^2 + 3^2 - 2(2)(3) \cos 105^\circ} \\ &= 4.013 \text{ kN} = 4.01 \text{ kN} \end{aligned}$$

Ans.

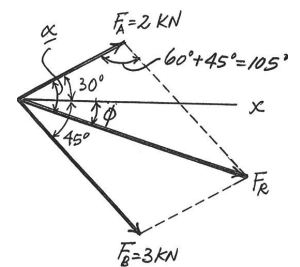
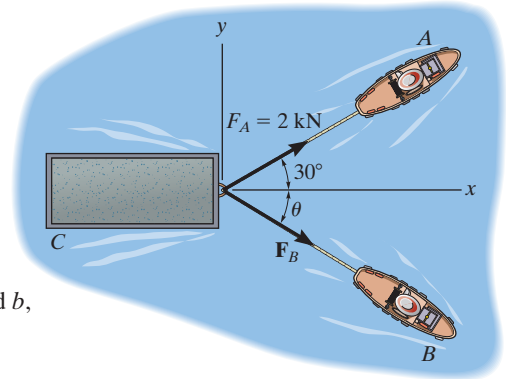
Using this result and applying the law of sines to Fig. *b* yields

$$\frac{\sin \alpha}{3} = \frac{\sin 105^\circ}{4.013} \quad \alpha = 46.22^\circ$$

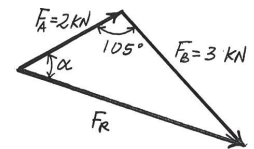
Thus, the direction angle ϕ of \mathbf{F}_R , measured clockwise from the positive x axis, is

$$\phi = \alpha - 30^\circ = 46.22^\circ - 30^\circ = 16.2^\circ$$

Ans.



(a)



(b)

Ans:

$$F_R = 4.01 \text{ kN}$$

$$\phi = 16.2^\circ$$

2-23.

If the resultant force of the two tugboats is required to be directed towards the positive x axis, and \mathbf{F}_B is to be a minimum, determine the magnitude of \mathbf{F}_R and \mathbf{F}_B and the angle θ .

SOLUTION

For \mathbf{F}_B to be minimum, it has to be directed perpendicular to \mathbf{F}_R . Thus,

$$\theta = 90^\circ$$

Ans.

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

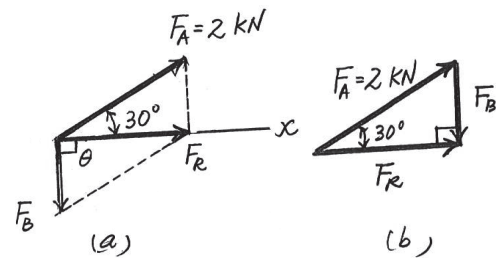
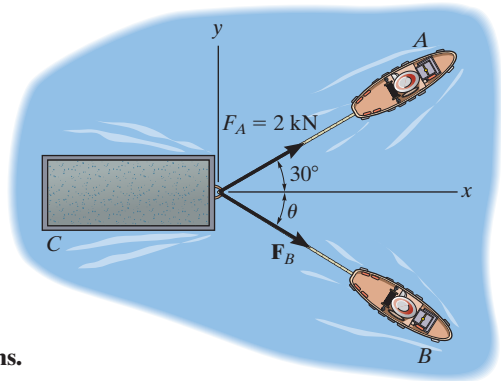
By applying simple trigonometry to Fig. *b*,

$$F_B = 2 \sin 30^\circ = 1 \text{ kN}$$

Ans.

$$F_R = 2 \cos 30^\circ = 1.73 \text{ kN}$$

Ans.



Ans:

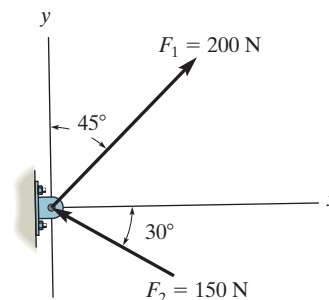
$$\theta = 90^\circ$$

$$F_B = 1 \text{ kN}$$

$$F_R = 1.73 \text{ kN}$$

***2-24.**

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a ,

$$\rightarrow (F_R)_x = \Sigma F_x; \quad (F_R)_x = 200 \sin 45^\circ - 150 \cos 30^\circ = 11.518 \text{ N} \rightarrow$$

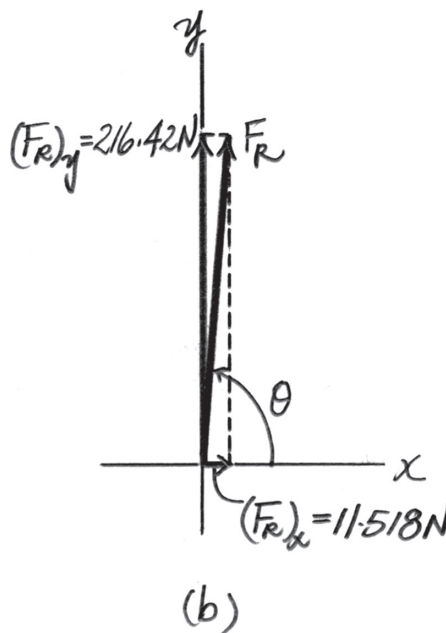
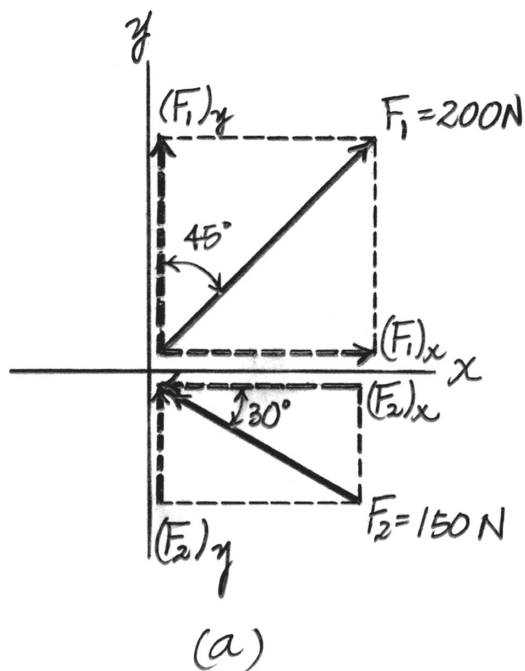
$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 200 \cos 45^\circ + 150 \sin 30^\circ = 216.42 \text{ N} \uparrow$$

Referring to Fig. b , the magnitude of the resultant force F_R is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{11.518^2 + 216.42^2} = 216.73 \text{ N} = 217 \text{ N} \quad \text{Ans.}$$

And the directional angle θ of \mathbf{F}_R measured counterclockwise from the positive x axis is

$$\theta = \tan^{-1} \left[\frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left(\frac{216.42}{11.518} \right) = 86.95^\circ = 87.0^\circ \quad \text{Ans.}$$



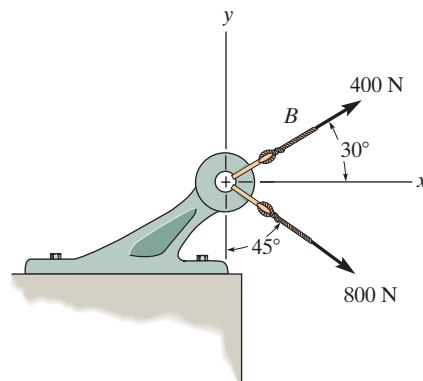
Ans:

$$F_R = 217 \text{ N}$$

$$\theta = 87.0^\circ$$

2-25.

Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes by referring to Fig. *a*,

$$\rightarrow (F_R)_x = \Sigma F_x; \quad (F_R)_x = 400 \cos 30^\circ + 800 \sin 45^\circ = 912.10 \text{ N} \rightarrow$$

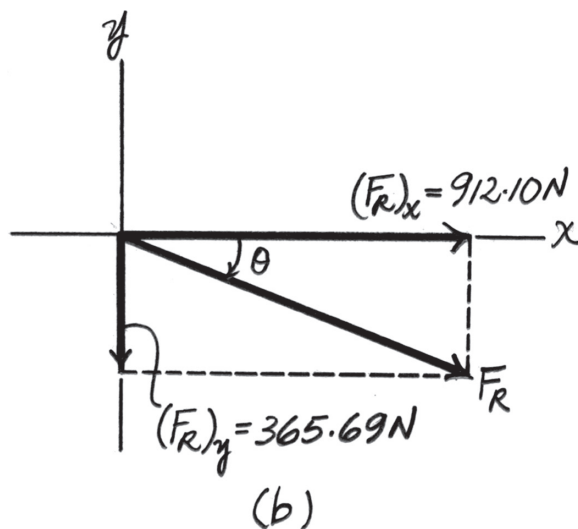
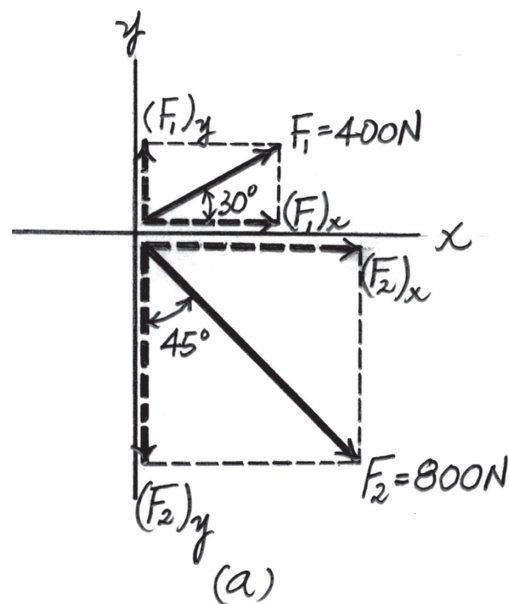
$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 400 \sin 30^\circ - 800 \cos 45^\circ = -365.69 \text{ N} = 365.69 \text{ N} \downarrow$$

Referring to Fig. *b*, the magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{912.10^2 + 365.69^2} = 982.67 \text{ N} = 983 \text{ N} \quad \text{Ans.}$$

And its directional angle θ measured clockwise from the positive x axis is

$$\theta = \tan^{-1} \left[\frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left(\frac{365.69}{912.10} \right) = 21.84^\circ = 21.8^\circ \quad \text{Ans.}$$



Ans:

$$F_R = 983 \text{ N}$$

$$\theta = 21.8^\circ$$

2-26.

Resolve \mathbf{F}_1 and \mathbf{F}_2 into their x and y components.

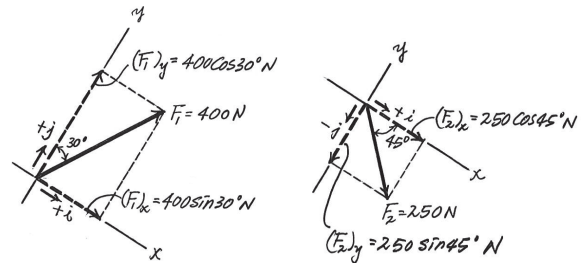
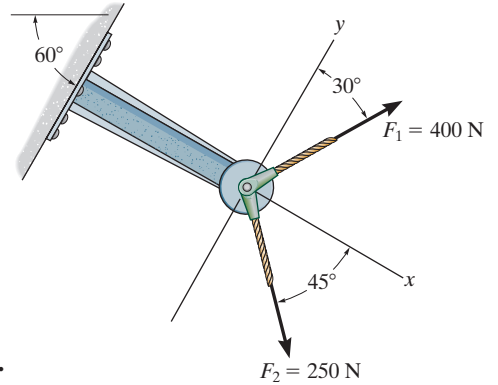
SOLUTION

$$\begin{aligned}\mathbf{F}_1 &= \{400 \sin 30^\circ(+\mathbf{i}) + 400 \cos 30^\circ(+\mathbf{j})\} \text{ N} \\ &= \{200\mathbf{i} + 346\mathbf{j}\} \text{ N}\end{aligned}$$

$$\begin{aligned}\mathbf{F}_2 &= \{250 \cos 45^\circ(+\mathbf{i}) + 250 \sin 45^\circ(-\mathbf{j})\} \text{ N} \\ &= \{177\mathbf{i} - 177\mathbf{j}\} \text{ N}\end{aligned}$$

Ans.

Ans.



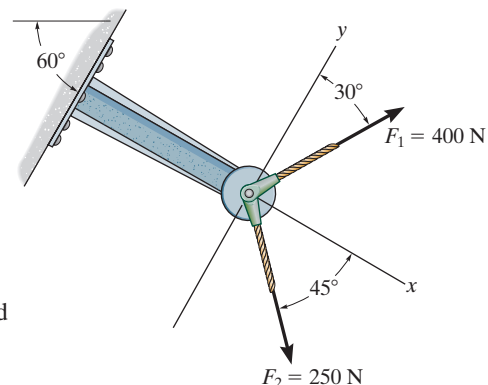
Ans:

$$\mathbf{F}_1 = \{200\mathbf{i} + 346\mathbf{j}\} \text{ N}$$

$$\mathbf{F}_2 = \{177\mathbf{i} - 177\mathbf{j}\} \text{ N}$$

2-27.

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of \mathbf{F}_1 and \mathbf{F}_2 can be written as

$$(F_1)_x = 400 \sin 30^\circ = 200 \text{ N} \quad (F_1)_y = 400 \cos 30^\circ = 346.41 \text{ N}$$

$$(F_2)_x = 250 \cos 45^\circ = 176.78 \text{ N} \quad (F_2)_y = 250 \sin 45^\circ = 176.78 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$+\rightarrow \Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 200 + 176.78 = 376.78 \text{ N}$$

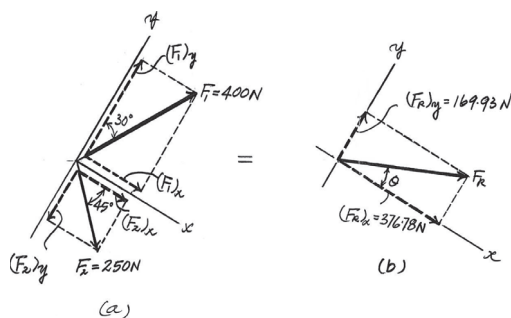
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 346.41 - 176.78 = 169.63 \text{ N} \uparrow$$

The magnitude of the resultant force \mathbf{F}_R is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{376.78^2 + 169.63^2} = 413 \text{ N} \quad \text{Ans.}$$

The direction angle θ of \mathbf{F}_R , Fig. *b*, measured counterclockwise from the positive axis, is

$$\theta = \tan^{-1} \left[\frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left(\frac{169.63}{376.78} \right) = 24.2^\circ \quad \text{Ans.}$$



Ans:

$$F_R = 413 \text{ N}$$

$$\theta = 24.2^\circ$$

***2-28.**

Resolve each force acting on the gusset plate into its x and y components, and express each force as a Cartesian vector.

SOLUTION

$$\mathbf{F}_1 = \{900(+\mathbf{i})\} = \{900\mathbf{i}\} \text{ N}$$

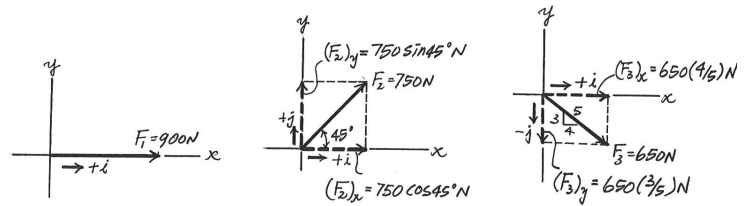
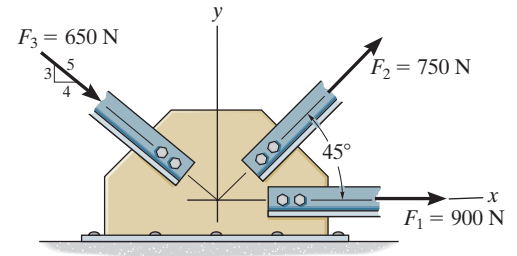
Ans.

$$\begin{aligned} \mathbf{F}_2 &= \{750 \cos 45^\circ(+\mathbf{i}) + 750 \sin 45^\circ(+\mathbf{j})\} \text{ N} \\ &= \{530\mathbf{i} + 530\mathbf{j}\} \text{ N} \end{aligned}$$

Ans.

$$\begin{aligned} \mathbf{F}_3 &= \left\{ 650 \left(\frac{4}{5} \right) (+\mathbf{i}) + 650 \left(\frac{3}{5} \right) (-\mathbf{j}) \right\} \text{ N} \\ &= \{520\mathbf{i} - 390\mathbf{j}\} \text{ N} \end{aligned}$$

Ans.



Ans:

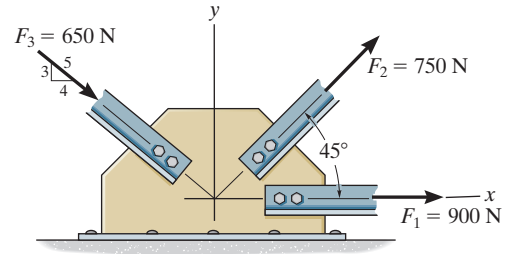
$$\mathbf{F}_1 = \{900\mathbf{i}\} \text{ N}$$

$$\mathbf{F}_2 = \{530\mathbf{i} + 530\mathbf{j}\} \text{ N}$$

$$\mathbf{F}_3 = \{520\mathbf{i} - 390\mathbf{j}\} \text{ N}$$

2-29.

Determine the magnitude of the resultant force acting on the gusset plate and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of \mathbf{F}_1 , \mathbf{F}_2 , and \mathbf{F}_3 can be written as

$$\begin{aligned} (F_1)_x &= 900 \text{ N} & (F_1)_y &= 0 \\ (F_2)_x &= 750 \cos 45^\circ = 530.33 \text{ N} & (F_2)_y &= 750 \sin 45^\circ = 530.33 \text{ N} \\ (F_3)_x &= 650 \left(\frac{4}{5}\right) = 520 \text{ N} & (F_3)_y &= 650 \left(\frac{3}{5}\right) = 390 \text{ N} \end{aligned}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

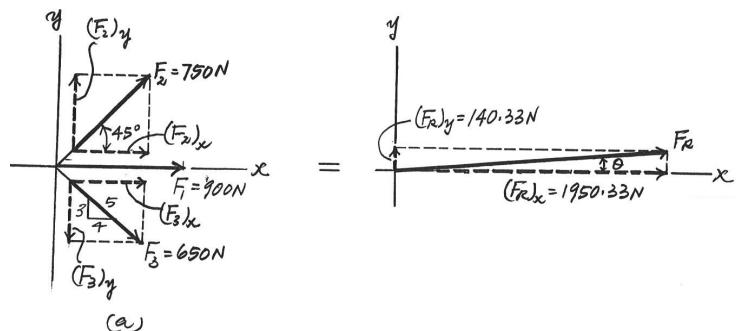
$$\begin{aligned} \rightarrow \Sigma(F_R)_x &= \Sigma F_x; & (F_R)_x &= 900 + 530.33 + 520 = 1950.33 \text{ N} \rightarrow \\ + \uparrow \Sigma(F_R)_y &= \Sigma F_y; & (F_R)_y &= 530.33 - 390 = 140.33 \text{ N} \uparrow \end{aligned}$$

The magnitude of the resultant force \mathbf{F}_R is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{1950.33^2 + 140.33^2} = 1955 \text{ N} = 1.96 \text{ kN} \quad \text{Ans.}$$

The direction angle θ of \mathbf{F}_R , measured clockwise from the positive x axis, is

$$\theta = \tan^{-1} \left[\frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left(\frac{140.33}{1950.33} \right) = 4.12^\circ \quad \text{Ans.}$$



Ans:

$$\begin{aligned} F_R &= 1.96 \text{ kN} \\ \theta &= 4.12^\circ \end{aligned}$$

2–30.

Express each of the three forces acting on the support in Cartesian vector form and determine the magnitude of the resultant force and its direction, measured clockwise from positive x axis.

SOLUTION

Cartesian Notation. Referring to Fig. *a*,

$$\mathbf{F}_1 = (F_1)_x \mathbf{i} + (F_1)_y \mathbf{j} = 50 \left(\frac{3}{5} \right) \mathbf{i} + 50 \left(\frac{4}{5} \right) \mathbf{j} = \{30 \mathbf{i} + 40 \mathbf{j}\} \text{ N} \quad \text{Ans.}$$

$$\begin{aligned} \mathbf{F}_2 &= -(F_2)_x \mathbf{i} - (F_2)_y \mathbf{j} = -80 \sin 15^\circ \mathbf{i} - 80 \cos 15^\circ \mathbf{j} \\ &= \{-20.71 \mathbf{i} - 77.27 \mathbf{j}\} \text{ N} \\ &= \{-20.7 \mathbf{i} - 77.3 \mathbf{j}\} \text{ N} \end{aligned}$$

$$\mathbf{F}_3 = (F_3)_x \mathbf{i} = \{30 \mathbf{i}\}$$

Thus, the resultant force is

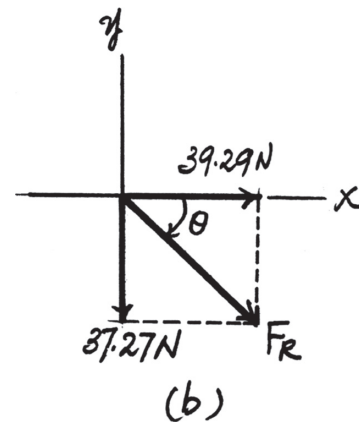
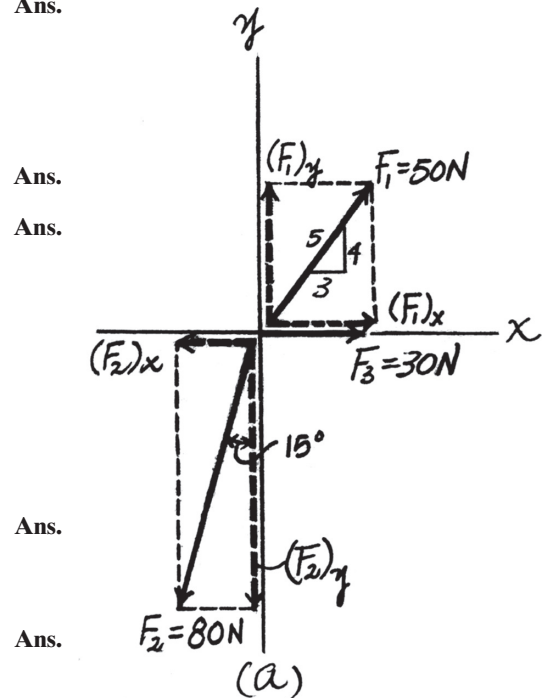
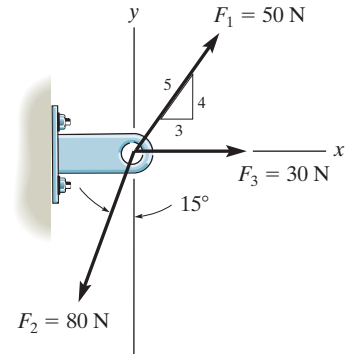
$$\begin{aligned} \mathbf{F}_R &= \Sigma \mathbf{F}; \quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= (30 \mathbf{i} + 40 \mathbf{j}) + (-20.71 \mathbf{i} - 77.27 \mathbf{j}) + 30 \mathbf{i} \\ &= \{39.29 \mathbf{i} - 37.27 \mathbf{j}\} \text{ N} \end{aligned}$$

Referring to Fig. *b*, the magnitude of \mathbf{F}_R is

$$F_R = \sqrt{39.29^2 + 37.27^2} = 54.16 \text{ N} = 54.2 \text{ N}$$

And its directional angle θ measured clockwise from the positive x axis is

$$\theta = \tan^{-1} \left(\frac{37.27}{39.29} \right) = 43.49^\circ = 43.5^\circ$$



Ans:

$$\mathbf{F}_1 = \{30 \mathbf{i} + 40 \mathbf{j}\} \text{ N}$$

$$\mathbf{F}_2 = \{-20.7 \mathbf{i} - 77.3 \mathbf{j}\} \text{ N}$$

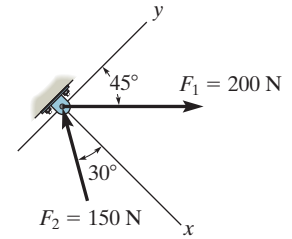
$$\mathbf{F}_3 = \{30 \mathbf{i}\}$$

$$F_R = 54.2 \text{ N}$$

$$\theta = 43.5^\circ$$

2–31.

Determine the x and y components of \mathbf{F}_1 and \mathbf{F}_2 .



SOLUTION

$$F_{1x} = 200 \sin 45^\circ = 141 \text{ N}$$

Ans.

$$F_{1y} = 200 \cos 45^\circ = 141 \text{ N}$$

Ans.

$$F_{2x} = -150 \cos 30^\circ = -130 \text{ N}$$

Ans.

$$F_{2y} = 150 \sin 30^\circ = 75 \text{ N}$$

Ans.

Ans:

$$F_{1x} = 141 \text{ N}$$

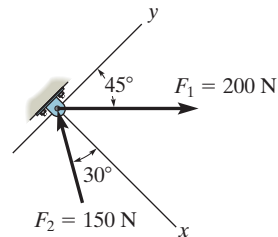
$$F_{1y} = 141 \text{ N}$$

$$F_{2x} = -130 \text{ N}$$

$$F_{2y} = 75 \text{ N}$$

***2-32.**

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

$$+\searrow F_{Rx} = \Sigma F_x; \quad F_{Rx} = -150 \cos 30^\circ + 200 \sin 45^\circ = 11.518\text{ N}$$

$$\nearrow +F_{Ry} = \Sigma F_y; \quad F_{Ry} = 150 \sin 30^\circ + 200 \cos 45^\circ = 216.421\text{ N}$$

$$F_R = \sqrt{(11.518)^2 + (216.421)^2} = 217\text{ N}$$

Ans.

$$\theta = \tan^{-1}\left(\frac{216.421}{11.518}\right) = 87.0^\circ$$

Ans.

Ans:

$$F_R = 217\text{ N}$$

$$\theta = 87.0^\circ$$

2-33.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.

SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a ,

$$\rightarrow (F_R)_x = \Sigma F_x; \quad (F_R)_x = 4 + 5 \cos 45^\circ - 8 \sin 15^\circ = 5.465 \text{ kN} \rightarrow$$

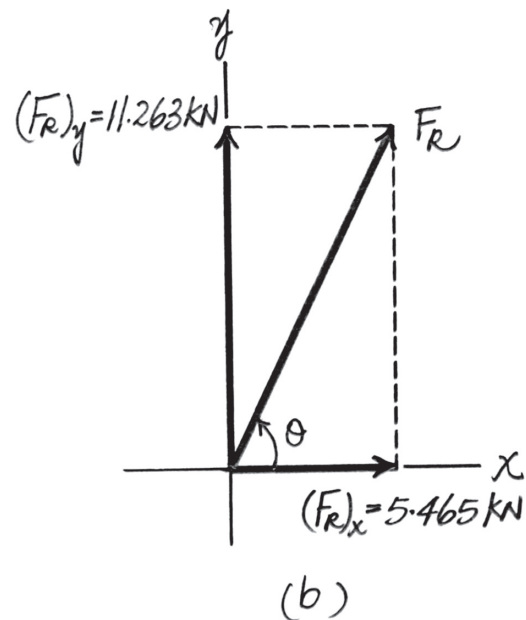
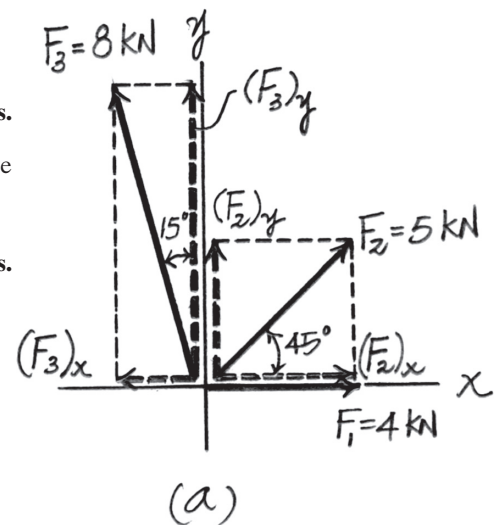
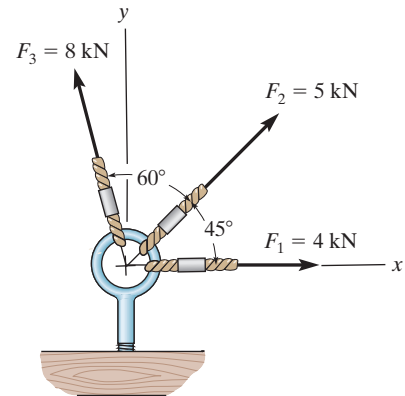
$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 5 \sin 45^\circ + 8 \cos 15^\circ = 11.263 \text{ kN} \uparrow$$

By referring to Fig. b , the magnitude of the resultant force F_R is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{5.465^2 + 11.263^2} = 12.52 \text{ kN} = 12.5 \text{ kN} \quad \text{Ans.}$$

And the directional angle θ of F_R measured counterclockwise from the positive x axis is

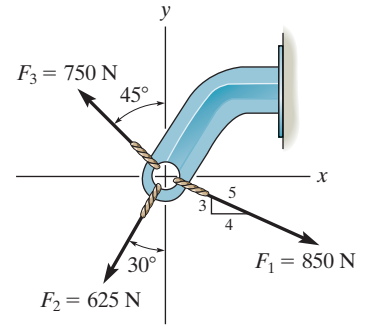
$$\theta = \tan^{-1} \left[\frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left(\frac{11.263}{5.465} \right) = 64.12^\circ = 64.1^\circ \quad \text{Ans.}$$



Ans:
 $F_R = 12.5 \text{ kN}$
 $\theta = 64.1^\circ$

2–34.

Express \mathbf{F}_1 , \mathbf{F}_2 , and \mathbf{F}_3 as Cartesian vectors.



SOLUTION

$$\begin{aligned}\mathbf{F}_1 &= \frac{4}{5}(850)\mathbf{i} - \frac{3}{5}(850)\mathbf{j} \\ &= \{680\mathbf{i} - 510\mathbf{j}\} \text{ N}\end{aligned}$$

Ans.

$$\begin{aligned}\mathbf{F}_2 &= -625 \sin 30^\circ \mathbf{i} - 625 \cos 30^\circ \mathbf{j} \\ &= \{-312\mathbf{i} - 541\mathbf{j}\} \text{ N}\end{aligned}$$

Ans.

$$\begin{aligned}\mathbf{F}_3 &= -750 \sin 45^\circ \mathbf{i} + 750 \cos 45^\circ \mathbf{j} \\ &= \{-530\mathbf{i} + 530\mathbf{j}\} \text{ N}\end{aligned}$$

Ans.

Ans:

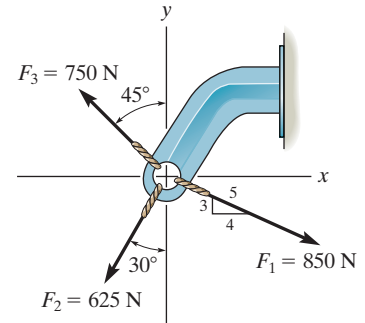
$$\mathbf{F}_1 = \{680\mathbf{i} - 510\mathbf{j}\} \text{ N}$$

$$\mathbf{F}_2 = \{-312\mathbf{i} - 541\mathbf{j}\} \text{ N}$$

$$\mathbf{F}_3 = \{-530\mathbf{i} + 530\mathbf{j}\} \text{ N}$$

2-35.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

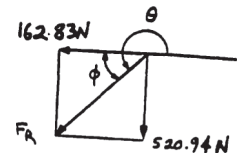
$$\rightarrow F_{Rx} = \Sigma F_x; \quad F_{Rx} = \frac{4}{5}(850) - 625 \sin 30^\circ - 750 \sin 45^\circ = -162.83 \text{ N}$$

$$+\uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = -\frac{3}{5}(850) - 625 \cos 30^\circ + 750 \cos 45^\circ = -520.94 \text{ N}$$

$$F_R = \sqrt{(-162.83)^2 + (-520.94)^2} = 546 \text{ N} \quad \text{Ans.}$$

$$\phi = \tan^{-1}\left(\frac{520.94}{162.83}\right) = 72.64^\circ$$

$$\theta = 180^\circ + 72.64^\circ = 253^\circ \quad \text{Ans.}$$



Ans:

$$F_R = 546 \text{ N}$$

$$\theta = 253^\circ$$

***2-36.**

Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.

SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a ,

$$\rightarrow (F_R)_x = \Sigma F_x; \quad (F_R)_x = 40\left(\frac{3}{5}\right) + 91\left(\frac{5}{13}\right) + 30 = 89 \text{ lb} \rightarrow$$

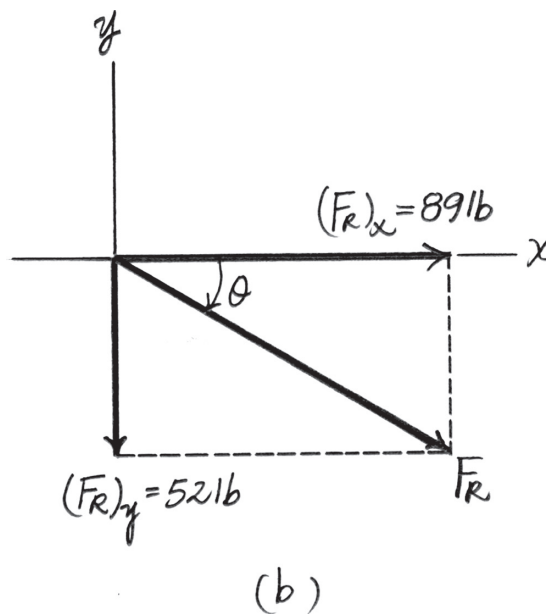
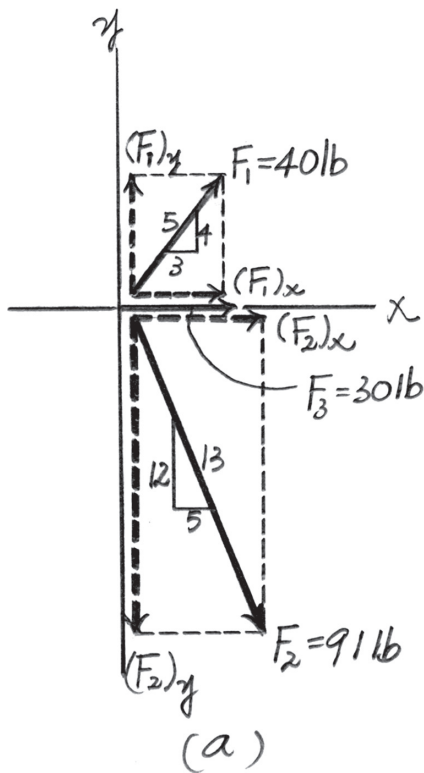
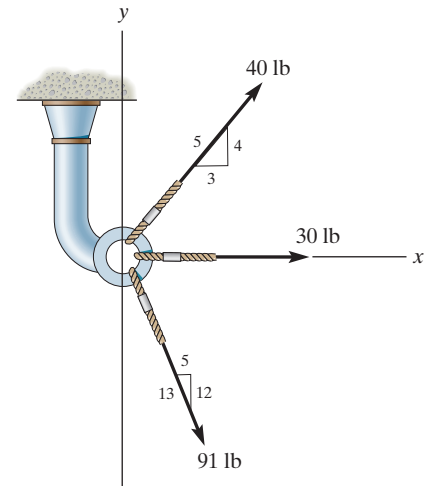
$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 40\left(\frac{4}{5}\right) - 91\left(\frac{12}{13}\right) = -52 \text{ lb} = 52 \text{ lb} \downarrow$$

By referring to Fig. b , the magnitude of resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{89^2 + 52^2} = 103.08 \text{ lb} = 103 \text{ lb} \quad \text{Ans.}$$

And its directional angle θ measured clockwise from the positive x axis is

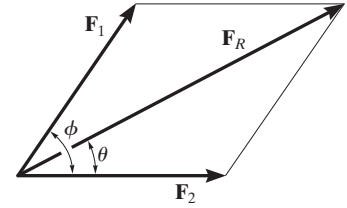
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{52}{89}\right) = 30.30^\circ = 30.3^\circ \quad \text{Ans.}$$



Ans:
 $F_R = 103 \text{ lb}$
 $\theta = 30.3^\circ$

2-37.

Determine the magnitude and direction θ of the resultant force \mathbf{F}_R . Express the result in terms of the magnitudes of the components \mathbf{F}_1 and \mathbf{F}_2 and the angle ϕ .



SOLUTION

$$F_R^2 = F_1^2 + F_2^2 - 2F_1F_2 \cos (180^\circ - \phi)$$

Since $\cos (180^\circ - \phi) = -\cos \phi$,

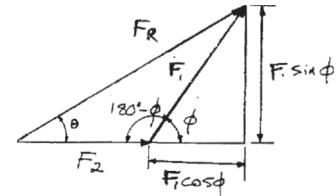
$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \phi}$$

From the figure,

$$\tan \theta = \frac{F_1 \sin \phi}{F_2 + F_1 \cos \phi}$$

$$\theta = \tan^{-1} \left(\frac{F_1 \sin \phi}{F_2 + F_1 \cos \phi} \right)$$

Ans.



Ans.

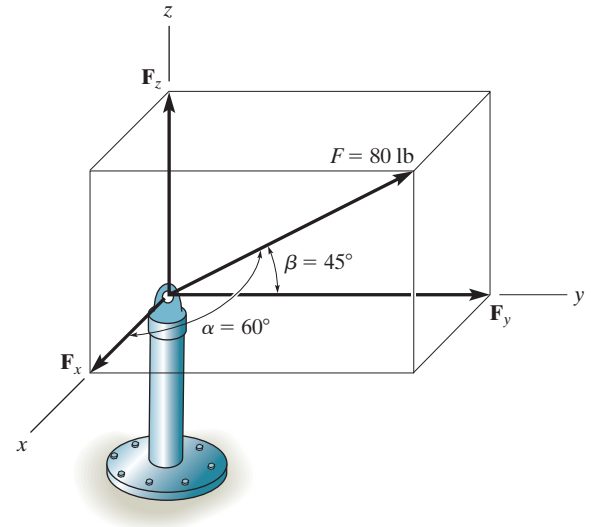
Ans:

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \phi}$$

$$\theta = \tan^{-1} \left(\frac{F_1 \sin \phi}{F_2 + F_1 \cos \phi} \right)$$

2–38.

The force **F** has a magnitude of 80 lb. Determine the magnitudes of the *x*, *y*, *z* components of **F**.



SOLUTION

$$1 = \cos^2 60^\circ + \cos^2 45^\circ + \cos^2 \gamma$$

Solving for the positive root, $\gamma = 60^\circ$

$$F_x = 80 \cos 60^\circ = 40.0 \text{ lb}$$

$$F_y = 80 \cos 45^\circ = 56.6 \text{ lb}$$

$$F_z = 80 \cos 60^\circ = 40.0 \text{ lb}$$

Ans.

Ans.

Ans.

Ans:

$$F_x = 40.0 \text{ lb}$$

$$F_y = 56.6 \text{ lb}$$

$$F_z = 40.0 \text{ lb}$$

2–39.

The bolt is subjected to the force \mathbf{F} , which has components acting along the x , y , z axes as shown. If the magnitude of \mathbf{F} is 80 N, and $\alpha = 60^\circ$ and $\gamma = 45^\circ$, determine the magnitudes of its components.

SOLUTION

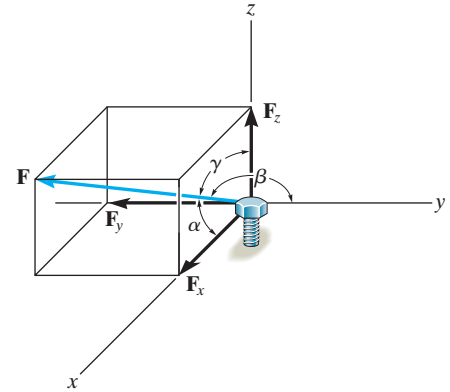
$$\begin{aligned}\cos\beta &= \sqrt{1 - \cos^2\alpha - \cos^2\gamma} \\ &= \sqrt{1 - \cos^2 60^\circ - \cos^2 45^\circ}\end{aligned}$$

$$\beta = 120^\circ$$

$$F_x = |80 \cos 60^\circ| = 40 \text{ N}$$

$$F_y = |80 \cos 120^\circ| = 40 \text{ N}$$

$$F_z = |80 \cos 45^\circ| = 56.6 \text{ N}$$



Ans.

Ans.

Ans.

Ans:

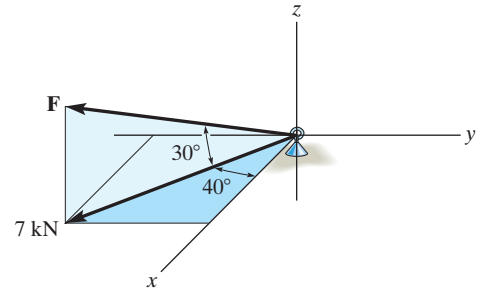
$$F_x = 40 \text{ N}$$

$$F_y = 40 \text{ N}$$

$$F_z = 56.6 \text{ N}$$

***2–40.**

Determine the magnitude and coordinate direction angles of the force \mathbf{F} acting on the support. The component of \mathbf{F} in the x – y plane is 7 kN.



SOLUTION

Coordinate Direction Angles. The unit vector of \mathbf{F} is

$$\begin{aligned}\mathbf{u}_F &= \cos 30^\circ \cos 40^\circ \mathbf{i} - \cos 30^\circ \sin 40^\circ \mathbf{j} + \sin 30^\circ \mathbf{k} \\ &= \{0.6634\mathbf{i} - 0.5567\mathbf{j} + 0.5\mathbf{k}\}\end{aligned}$$

Thus,

$$\cos \alpha = 0.6634; \quad \alpha = 48.44^\circ = 48.4^\circ \quad \text{Ans.}$$

$$\cos \beta = -0.5567; \quad \beta = 123.83^\circ = 124^\circ \quad \text{Ans.}$$

$$\cos \gamma = 0.5; \quad \gamma = 60^\circ \quad \text{Ans.}$$

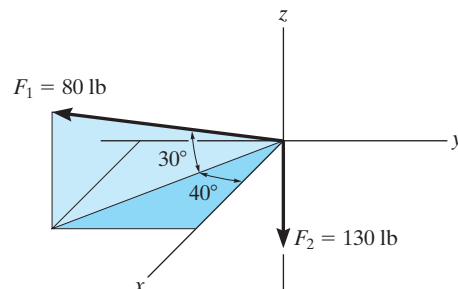
The magnitude of \mathbf{F} can be determined from

$$F \cos 30^\circ = 7; \quad F = 8.083 \text{ kN} = 8.08 \text{ kN} \quad \text{Ans.}$$

$$\begin{aligned}\text{Ans:} \\ \alpha &= 48.4^\circ \\ \beta &= 124^\circ \\ \gamma &= 60^\circ \\ F &= 8.08 \text{ kN}\end{aligned}$$

2-41.

Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the coordinate system.



SOLUTION

$$\mathbf{F}_1 = \{80 \cos 30^\circ \cos 40^\circ \mathbf{i} - 80 \cos 30^\circ \sin 40^\circ \mathbf{j} + 80 \sin 30^\circ \mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_2 = \{-130\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\mathbf{F}_R = \{53.1\mathbf{i} - 44.5\mathbf{j} - 90.0\mathbf{k}\} \text{ lb}$$

$$F_R = \sqrt{(53.1)^2 + (-44.5)^2 + (-90.0)^2} = 114 \text{ lb}$$

$$\alpha = \cos^{-1}\left(\frac{53.1}{113.6}\right) = 62.1^\circ$$

$$\beta = \cos^{-1}\left(\frac{-44.5}{113.6}\right) = 113^\circ$$

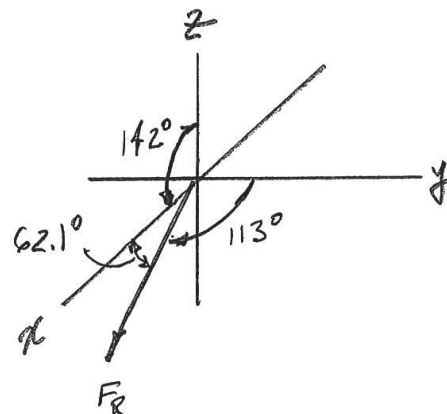
$$\gamma = \cos^{-1}\left(\frac{-90.0}{113.6}\right) = 142^\circ$$

Ans.

Ans.

Ans.

Ans.



Ans:

$$F_R = 114 \text{ lb}$$

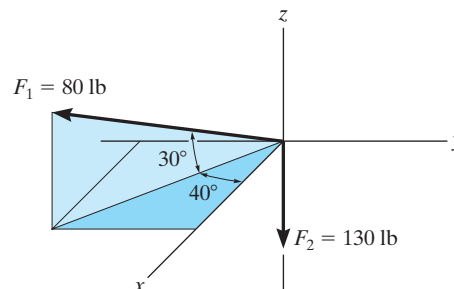
$$\alpha = 62.1^\circ$$

$$\beta = 113^\circ$$

$$\gamma = 142^\circ$$

2-42.

Specify the coordinate direction angles of \mathbf{F}_1 and \mathbf{F}_2 and express each force as a Cartesian vector.



SOLUTION

$$\mathbf{F}_1 = \{80 \cos 30^\circ \cos 40^\circ \mathbf{i} - 80 \cos 30^\circ \sin 40^\circ \mathbf{j} + 80 \sin 30^\circ \mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\} \text{ lb}$$

$$\alpha_1 = \cos^{-1}\left(\frac{53.1}{80}\right) = 48.4^\circ$$

$$\beta_1 = \cos^{-1}\left(\frac{-44.5}{80}\right) = 124^\circ$$

$$\gamma_1 = \cos^{-1}\left(\frac{40}{80}\right) = 60^\circ$$

$$\mathbf{F}_2 = \{-130\mathbf{k}\} \text{ lb}$$

$$\alpha_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90^\circ$$

$$\beta_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90^\circ$$

$$\gamma_2 = \cos^{-1}\left(\frac{-130}{130}\right) = 180^\circ$$

Ans.

Ans.

Ans.

Ans.

Ans.

Ans.

Ans.

Ans.

Ans:

$$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\} \text{ lb}$$

$$\alpha_1 = 48.4^\circ$$

$$\beta_1 = 124^\circ$$

$$\gamma_1 = 60^\circ$$

$$\mathbf{F}_2 = \{-130\mathbf{k}\} \text{ lb}$$

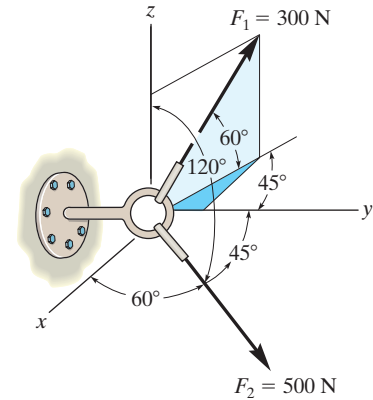
$$\alpha_2 = 90^\circ$$

$$\beta_2 = 90^\circ$$

$$\gamma_2 = 180^\circ$$

2-43.

Express each force in Cartesian vector form and then determine the resultant force. Find the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$$\begin{aligned}\mathbf{F}_1 &= 300(-\cos 60^\circ \sin 45^\circ \mathbf{i} + \cos 60^\circ \cos 45^\circ \mathbf{j} + \sin 60^\circ \mathbf{k}) \\ &= \{-106.07 \mathbf{i} + 106.07 \mathbf{j} + 259.81 \mathbf{k}\} \text{ N} \\ &= \{-106 \mathbf{i} + 106 \mathbf{j} + 260 \mathbf{k}\} \text{ N}\end{aligned}$$

Ans.

$$\begin{aligned}\mathbf{F}_2 &= 500(\cos 60^\circ \mathbf{i} + \cos 45^\circ \mathbf{j} + \cos 120^\circ \mathbf{k}) \\ &= \{250.0 \mathbf{i} + 353.55 \mathbf{j} - 250.0 \mathbf{k}\} \text{ N} \\ &= \{250 \mathbf{i} + 354 \mathbf{j} - 250 \mathbf{k}\} \text{ N}\end{aligned}$$

Ans.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\ &= -106.07 \mathbf{i} + 106.07 \mathbf{j} + 259.81 \mathbf{k} + 250.0 \mathbf{i} + 353.55 \mathbf{j} - 250.0 \mathbf{k} \\ &= 143.93 \mathbf{i} + 459.62 \mathbf{j} + 9.81 \mathbf{k} \\ &= \{144 \mathbf{i} + 460 \mathbf{j} + 9.81 \mathbf{k}\} \text{ N}\end{aligned}$$

Ans.

$$F_R = \sqrt{143.93^2 + 459.62^2 + 9.81^2} = 481.73 \text{ N} = 482 \text{ N}$$

Ans.

$$\mathbf{u}_{F_R} = \frac{\mathbf{F}_R}{F_R} = \frac{143.93 \mathbf{i} + 459.62 \mathbf{j} + 9.81 \mathbf{k}}{481.73} = 0.2988 \mathbf{i} + 0.9541 \mathbf{j} + 0.02036 \mathbf{k}$$

$$\cos \alpha = 0.2988 \quad \alpha = 72.6^\circ$$

Ans.

$$\cos \beta = 0.9541 \quad \beta = 17.4^\circ$$

Ans.

$$\cos \gamma = 0.02036 \quad \gamma = 88.8^\circ$$

Ans.

Ans:

$$\mathbf{F}_1 = \{-106 \mathbf{i} + 106 \mathbf{j} + 260 \mathbf{k}\} \text{ N}$$

$$\mathbf{F}_2 = \{250 \mathbf{i} + 354 \mathbf{j} - 250 \mathbf{k}\} \text{ N}$$

$$\mathbf{F}_R = \{144 \mathbf{i} + 460 \mathbf{j} + 9.81 \mathbf{k}\} \text{ N}$$

$$F_R = 482 \text{ N}$$

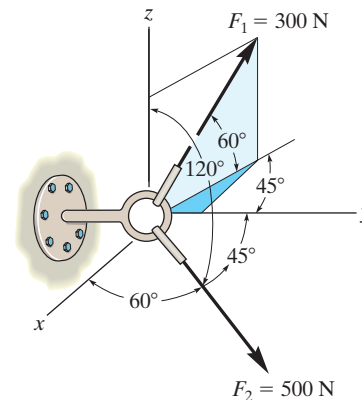
$$\alpha = 72.6^\circ$$

$$\beta = 17.4^\circ$$

$$\gamma = 88.8^\circ$$

***2-44.**

Determine the coordinate direction angles of \mathbf{F}_1 .



SOLUTION

$$\mathbf{F}_1 = 300(-\cos 60^\circ \sin 45^\circ \mathbf{i} + \cos 60^\circ \cos 45^\circ \mathbf{j} + \sin 60^\circ \mathbf{k})$$

$$= \{-106.07\mathbf{i} + 106.07\mathbf{j} + 259.81\mathbf{k}\} \text{ N}$$

$$= \{-106\mathbf{i} + 106\mathbf{j} + 260\mathbf{k}\} \text{ N}$$

$$\mathbf{u}_1 = \frac{\mathbf{F}_1}{300} = -0.3536\mathbf{i} + 0.3536\mathbf{j} + 0.8660\mathbf{k}$$

$$\alpha_1 = \cos^{-1}(-0.3536) = 111^\circ$$

Ans.

$$\beta_1 = \cos^{-1}(0.3536) = 69.3^\circ$$

Ans.

$$\gamma_1 = \cos^{-1}(0.8660) = 30.0^\circ$$

Ans.

Ans:

$$\alpha_1 = 111^\circ$$

$$\beta_1 = 69.3^\circ$$

$$\gamma_1 = 30.0^\circ$$

2-45.

Determine the magnitude and coordinate direction angles of \mathbf{F}_3 so that the resultant of the three forces acts along the positive y axis and has a magnitude of 600 lb.

SOLUTION

$$F_{Rx} = \Sigma F_x ; \quad 0 = -180 + 300 \cos 30^\circ \sin 40^\circ + F_3 \cos \alpha$$

$$F_{Ry} = \Sigma F_y ; \quad 600 = 300 \cos 30^\circ \cos 40^\circ + F_3 \cos \beta$$

$$F_{Rz} = \Sigma F_z ; \quad 0 = -300 \sin 30^\circ + F_3 \cos \gamma$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

Solving:

$$F_3 = 428 \text{ lb}$$

$$\alpha = 88.3^\circ$$

$$\beta = 20.6^\circ$$

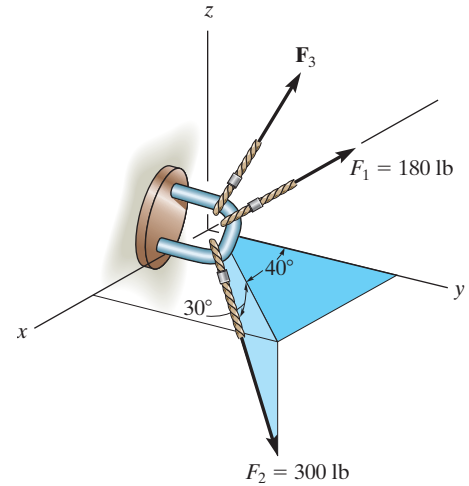
$$\gamma = 69.5^\circ$$

Ans.

Ans.

Ans.

Ans.



Ans:

$$F_3 = 428 \text{ lb}$$

$$\alpha = 88.3^\circ$$

$$\beta = 20.6^\circ$$

$$\gamma = 69.5^\circ$$

2-46.

Determine the magnitude and coordinate direction angles of \mathbf{F}_3 so that the resultant of the three forces is zero.

SOLUTION

$$F_{Rx} = \Sigma F_x; \quad 0 = -180 + 300 \cos 30^\circ \sin 40^\circ + F_3 \cos \alpha$$

$$F_{Ry} = \Sigma F_y; \quad 0 = 300 \cos 30^\circ \cos 40^\circ + F_3 \cos \beta$$

$$F_{Rz} = \Sigma F_z; \quad 0 = -300 \sin 30^\circ + F_3 \cos \gamma$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

Solving:

$$F_3 = 250 \text{ lb}$$

$$\alpha = 87.0^\circ$$

$$\beta = 143^\circ$$

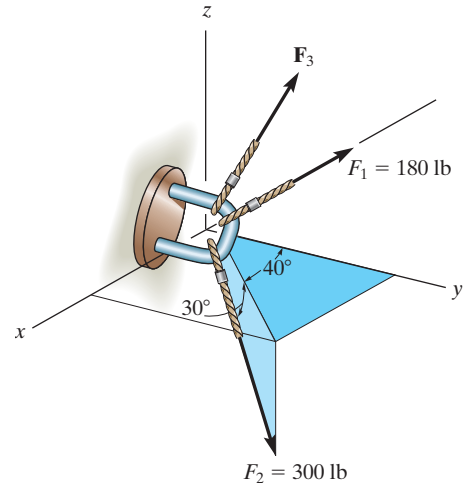
$$\gamma = 53.1^\circ$$

Ans.

Ans.

Ans.

Ans.



Ans:

$$F_3 = 250 \text{ lb}$$

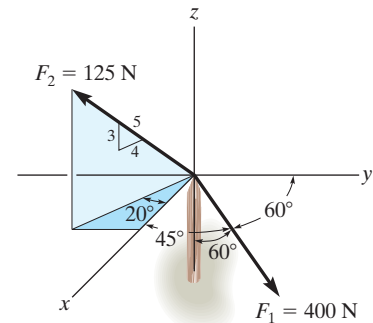
$$\alpha = 87.0^\circ$$

$$\beta = 143^\circ$$

$$\gamma = 53.1^\circ$$

2-47.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

Cartesian Vector Notation. For \mathbf{F}_1 and \mathbf{F}_2 ,

$$\mathbf{F}_1 = 400 (\cos 45^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} - \cos 60^\circ \mathbf{k}) = \{282.84\mathbf{i} + 200\mathbf{j} - 200\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_2 = 125 \left[\frac{4}{5} (\cos 20^\circ) \mathbf{i} - \frac{4}{5} (\sin 20^\circ) \mathbf{j} + \frac{3}{5} \mathbf{k} \right] = \{93.97\mathbf{i} - 34.20\mathbf{j} + 75.0\mathbf{k}\}$$

Resultant Force.

$$\begin{aligned} \mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\ &= \{282.84\mathbf{i} + 200\mathbf{j} - 200\mathbf{k}\} + \{93.97\mathbf{i} - 34.20\mathbf{j} + 75.0\mathbf{k}\} \\ &= \{376.81\mathbf{i} + 165.80\mathbf{j} - 125.00\mathbf{k}\} \text{ N} \end{aligned}$$

The magnitude of the resultant force is

$$\begin{aligned} F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{376.81^2 + 165.80^2 + (-125.00)^2} \\ &= 430.23 \text{ N} = 430 \text{ N} \end{aligned}$$

Ans.

The coordinate direction angles are

$$\cos \alpha = \frac{(F_R)_x}{F_R} = \frac{376.81}{430.23}; \quad \alpha = 28.86^\circ = 28.9^\circ$$

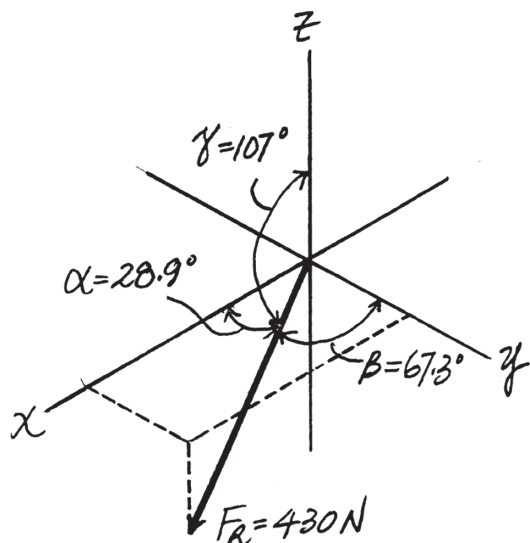
Ans.

$$\cos \beta = \frac{(F_R)_y}{F_R} = \frac{165.80}{430.23}; \quad \beta = 67.33^\circ = 67.3^\circ$$

Ans.

$$\cos \gamma = \frac{(F_R)_z}{F_R} = \frac{-125.00}{430.23}; \quad \gamma = 106.89^\circ = 107^\circ$$

Ans.



Ans:

$$F_R = 430 \text{ N}$$

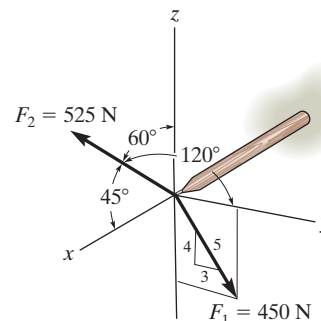
$$\alpha = 28.9^\circ$$

$$\beta = 67.3^\circ$$

$$\gamma = 107^\circ$$

*2-48.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

Cartesian Vector Notation. For \mathbf{F}_1 and \mathbf{F}_2 ,

$$\mathbf{F}_1 = 450 \left(\frac{3}{5} \mathbf{j} - \frac{4}{5} \mathbf{k} \right) = \{270\mathbf{j} - 360\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_2 = 525 (\cos 45^\circ \mathbf{i} + \cos 120^\circ \mathbf{j} + \cos 60^\circ \mathbf{k}) = \{371.23\mathbf{i} - 262.5\mathbf{j} + 262.5\mathbf{k}\} \text{ N}$$

Resultant Force.

$$\begin{aligned} \mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\ &= \{270\mathbf{j} - 360\mathbf{k}\} + \{371.23\mathbf{i} - 262.5\mathbf{j} + 262.5\mathbf{k}\} \\ &= \{371.23\mathbf{i} + 7.50\mathbf{j} - 97.5\mathbf{k}\} \text{ N} \end{aligned}$$

The magnitude of the resultant force is

$$\begin{aligned} F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{371.23^2 + 7.50^2 + (-97.5)^2} \\ &= 383.89 \text{ N} = 384 \text{ N} \end{aligned}$$

Ans.

The coordinate direction angles are

$$\cos \alpha = \frac{(F_R)_x}{F_R} = \frac{371.23}{383.89}; \quad \alpha = 14.76^\circ = 14.8^\circ$$

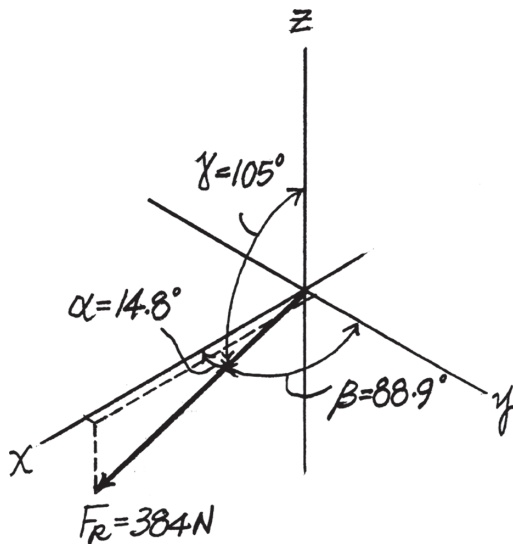
Ans.

$$\cos \beta = \frac{(F_R)_y}{F_R} = \frac{7.50}{383.89}; \quad \beta = 88.88^\circ = 88.9^\circ$$

Ans.

$$\cos \gamma = \frac{(F_R)_z}{F_R} = \frac{-97.5}{383.89}; \quad \gamma = 104.71^\circ = 105^\circ$$

Ans.

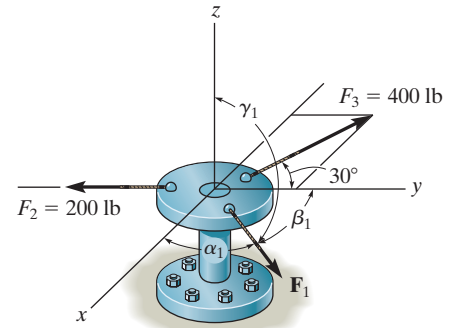


Ans:

$$\begin{aligned} F_R &= 384 \text{ N} \\ \alpha &= 14.8^\circ \\ \beta &= 88.9^\circ \\ \gamma &= 105^\circ \end{aligned}$$

2–49.

Determine the magnitude and coordinate direction angles α_1 , β_1 , γ_1 of \mathbf{F}_1 so that the resultant of the three forces acting on the bracket is $\mathbf{F}_R = \{-350\mathbf{k}\}$ lb.



SOLUTION

$$\mathbf{F}_1 = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k}$$

$$\mathbf{F}_2 = -200 \mathbf{j}$$

$$\begin{aligned} \mathbf{F}_3 &= -400 \sin 30^\circ \mathbf{i} + 400 \cos 30^\circ \mathbf{j} \\ &= -200 \mathbf{i} + 346.4 \mathbf{j} \end{aligned}$$

$$\mathbf{F}_R = \Sigma \mathbf{F}$$

$$-350 \mathbf{k} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k} - 200 \mathbf{j} - 200 \mathbf{i} + 346.4 \mathbf{j}$$

$$0 = F_x - 200; \quad F_x = 200 \text{ lb}$$

$$0 = F_y - 200 + 346.4; \quad F_y = -146.4 \text{ lb}$$

$$F_z = -350 \text{ lb}$$

$$F_1 = \sqrt{(200)^2 + (-146.4)^2 + (-350)^2}$$

$$F_1 = 425.9 \text{ lb} = 429 \text{ lb}$$

Ans.

$$\alpha_1 = \cos^{-1} \left(\frac{200}{428.9} \right) = 62.2^\circ$$

Ans.

$$\beta_1 = \cos^{-1} \left(\frac{-146.4}{428.9} \right) = 110^\circ$$

Ans.

$$\gamma_1 = \cos^{-1} \left(\frac{-350}{428.9} \right) = 145^\circ$$

Ans.

Ans:

$$F_1 = 429 \text{ lb}$$

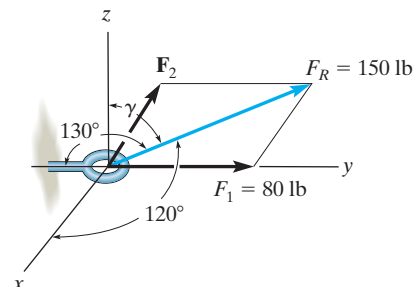
$$\alpha_1 = 62.2^\circ$$

$$\beta_1 = 110^\circ$$

$$\gamma_1 = 145^\circ$$

2-50.

If the resultant force \mathbf{F}_R has a magnitude of 150 lb and the coordinate direction angles shown, determine the magnitude of \mathbf{F}_2 and its coordinate direction angles.



SOLUTION

Cartesian Vector Notation. For \mathbf{F}_R , γ can be determined from

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\cos^2 120^\circ + \cos^2 50^\circ + \cos^2 \gamma = 1$$

$$\cos \gamma = \pm 0.5804$$

Here $\gamma < 90^\circ$, then

$$\gamma = 54.52^\circ$$

Thus

$$\begin{aligned} \mathbf{F}_R &= 150(\cos 120^\circ \mathbf{i} + \cos 50^\circ \mathbf{j} + \cos 54.52^\circ \mathbf{k}) \\ &= \{-75.0\mathbf{i} + 96.42\mathbf{j} + 87.05\mathbf{k}\} \text{ lb} \end{aligned}$$

Also

$$\mathbf{F}_1 = \{80\mathbf{j}\} \text{ lb}$$

Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\{-75.0\mathbf{i} + 96.42\mathbf{j} + 87.05\mathbf{k}\} = \{80\mathbf{j}\} + \mathbf{F}_2$$

$$\mathbf{F}_2 = \{-75.0\mathbf{i} + 16.42\mathbf{j} + 87.05\mathbf{k}\} \text{ lb}$$

Thus, the magnitude of \mathbf{F}_2 is

$$\begin{aligned} F_2 &= \sqrt{(F_2)_x^2 + (F_2)_y^2 + (F_2)_z^2} = \sqrt{(-75.0)^2 + 16.42^2 + 87.05^2} \\ &= 116.07 \text{ lb} = 116 \text{ lb} \end{aligned}$$

Ans.

And its coordinate direction angles are

$$\cos \alpha_2 = \frac{(F_2)_x}{F_2} = \frac{-75.0}{116.07}; \quad \alpha_2 = 130.25^\circ = 130^\circ \quad \text{Ans.}$$

$$\cos \beta_2 = \frac{(F_2)_y}{F_2} = \frac{16.42}{116.07}; \quad \beta_2 = 81.87^\circ = 81.9^\circ \quad \text{Ans.}$$

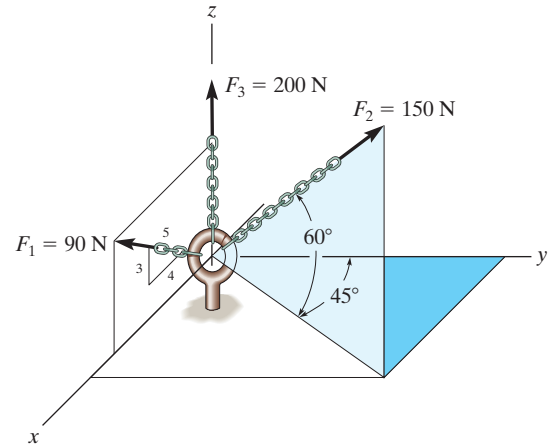
$$\cos \gamma_2 = \frac{(F_2)_z}{F_2} = \frac{87.05}{116.07}; \quad \gamma_2 = 41.41^\circ = 41.4^\circ \quad \text{Ans.}$$

Ans:

$$\begin{aligned} F_2 &= 116 \text{ lb} \\ \alpha_2 &= 130^\circ \\ \beta_2 &= 81.9^\circ \\ \gamma_2 &= 41.4^\circ \end{aligned}$$

2-51.

Express each force as a Cartesian vector.



SOLUTION

Cartesian Vector Notation. For \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 ,

$$\mathbf{F}_1 = 90 \left(-\frac{4}{5} \mathbf{i} + \frac{3}{5} \mathbf{j} \right) = \{-72.0\mathbf{i} + 54.0\mathbf{j}\} \text{ N}$$

$$\mathbf{F}_2 = 150 (\cos 60^\circ \sin 45^\circ \mathbf{i} + \cos 60^\circ \cos 45^\circ \mathbf{j} + \sin 60^\circ \mathbf{k})$$

$$= \{53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}\} \text{ N}$$

$$= \{53.0\mathbf{i} + 53.0\mathbf{j} + 130\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_3 = \{200 \mathbf{k}\}$$

Ans.

Ans.

Ans.

Ans:

$$\mathbf{F}_1 = \{72.0\mathbf{i} + 54.0\mathbf{j}\} \text{ N}$$

$$\mathbf{F}_2 = \{53.0\mathbf{i} + 53.0\mathbf{j} + 130\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_3 = \{200 \mathbf{k}\}$$

***2-52.**

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.

SOLUTION

Cartesian Vector Notation. For \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 ,

$$\mathbf{F}_1 = 90 \left(\frac{4}{5} \mathbf{i} + \frac{3}{5} \mathbf{k} \right) = \{72.0\mathbf{i} + 54.0\mathbf{k}\} \text{ N}$$

$$\begin{aligned} \mathbf{F}_2 &= 150 (\cos 60^\circ \sin 45^\circ \mathbf{i} + \cos 60^\circ \cos 45^\circ \mathbf{j} + \sin 60^\circ \mathbf{k}) \\ &= \{53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}\} \text{ N} \end{aligned}$$

$$\mathbf{F}_3 = \{200 \mathbf{k}\} \text{ N}$$

Resultant Force.

$$\begin{aligned} \mathbf{F} &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= (72.0\mathbf{i} + 54.0\mathbf{k}) + (53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}) + (200\mathbf{k}) \\ &= \{125.03\mathbf{i} + 53.03\mathbf{j} + 383.90\mathbf{k}\} \text{ N} \end{aligned}$$

The magnitude of the resultant force is

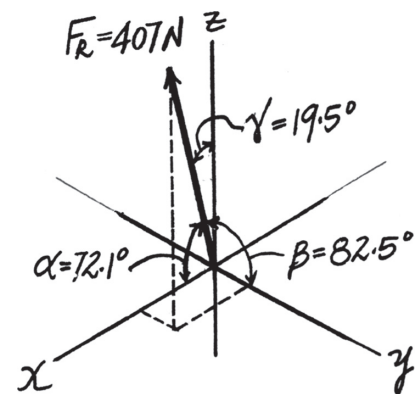
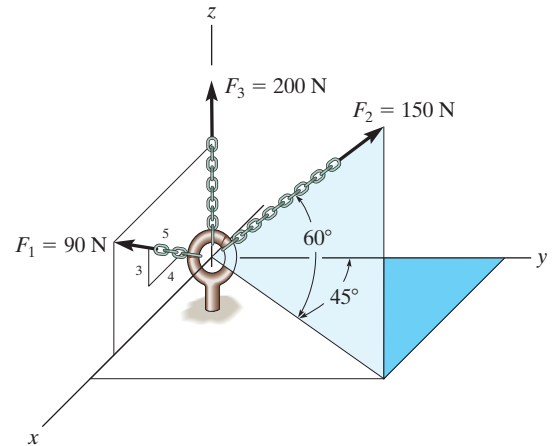
$$\begin{aligned} F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{125.03^2 + 53.03^2 + 383.90^2} \\ &= 407.22 \text{ N} = 407 \text{ N} \end{aligned}$$

And the coordinate direction angles are

$$\cos \alpha = \frac{(F_R)_x}{F_R} = \frac{125.03}{407.22}; \quad \alpha = 72.12^\circ = 72.1^\circ$$

$$\cos \beta = \frac{(F_R)_y}{F_R} = \frac{53.03}{407.22}; \quad \beta = 82.52^\circ = 82.5^\circ$$

$$\cos \gamma = \frac{(F_R)_z}{F_R} = \frac{383.90}{407.22}; \quad \gamma = 19.48^\circ = 19.5^\circ$$



Ans.

Ans.

Ans.

Ans.

Ans:

$$\begin{aligned} F_R &= 407 \text{ N} \\ \alpha &= 72.1^\circ \\ \beta &= 82.5^\circ \\ \gamma &= 19.5^\circ \end{aligned}$$

2-53.

The spur gear is subjected to the two forces. Express each force as a Cartesian vector.

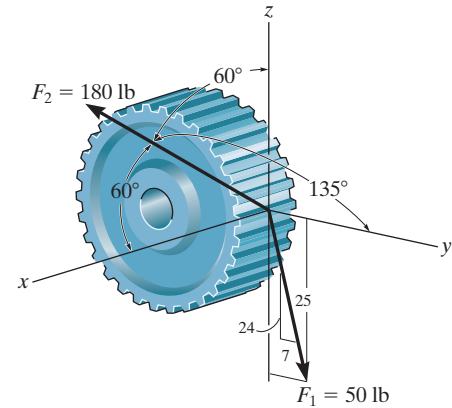
SOLUTION

$$\mathbf{F}_1 = \frac{7}{25}(50)\mathbf{j} - \frac{24}{25}(50)\mathbf{k} = \{14.0\mathbf{j} - 48.0\mathbf{k}\} \text{ lb}$$

$$\begin{aligned}\mathbf{F}_2 &= 180 \cos 60^\circ \mathbf{i} + 180 \cos 135^\circ \mathbf{j} + 180 \cos 60^\circ \mathbf{k} \\ &= \{90\mathbf{i} - 127\mathbf{j} + 90\mathbf{k}\} \text{ lb}\end{aligned}$$

Ans.

Ans.



Ans:

$$\mathbf{F}_1 = \{14.0\mathbf{j} - 48.0\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_2 = \{90\mathbf{i} - 127\mathbf{j} + 90\mathbf{k}\} \text{ lb}$$

2-54.

The spur gear is subjected to the two forces. Determine the resultant of the two forces and express the result as a Cartesian vector.

SOLUTION

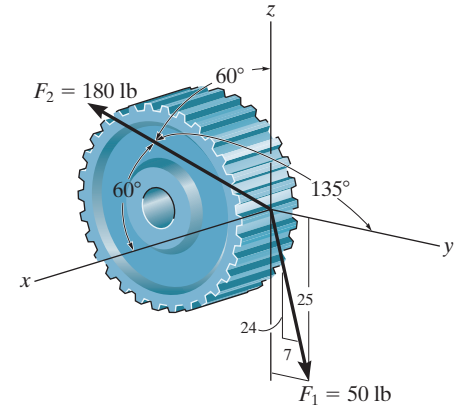
$$F_{Rx} = 180 \cos 60^\circ = 90$$

$$F_{Ry} = \frac{7}{25}(50) + 180 \cos 135^\circ = -113$$

$$F_{Rz} = -\frac{24}{25}(50) + 180 \cos 60^\circ = 42$$

$$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\} \text{ lb}$$

Ans.



Ans:

$$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\} \text{ lb}$$

2-55.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.

SOLUTION

Cartesian Vector Notation. For \mathbf{F}_1 and \mathbf{F}_2 ,

$$\begin{aligned}\mathbf{F}_1 &= 400 (\sin 60^\circ \cos 20^\circ \mathbf{i} - \sin 60^\circ \sin 20^\circ \mathbf{j} + \cos 60^\circ \mathbf{k}) \\ &= \{325.52\mathbf{i} - 118.48\mathbf{j} + 200\mathbf{k}\} \text{ N}\end{aligned}$$

$$\begin{aligned}\mathbf{F}_2 &= 500 (\cos 60^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 135^\circ \mathbf{k}) \\ &= \{250\mathbf{i} + 250\mathbf{j} - 353.55\mathbf{k}\} \text{ N}\end{aligned}$$

Resultant Force.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\ &= (325.52\mathbf{i} - 118.48\mathbf{j} + 200\mathbf{k}) + (250\mathbf{i} + 250\mathbf{j} - 353.55\mathbf{k}) \\ &= \{575.52\mathbf{i} + 131.52\mathbf{j} - 153.55\mathbf{k}\} \text{ N}\end{aligned}$$

The magnitude of the resultant force is

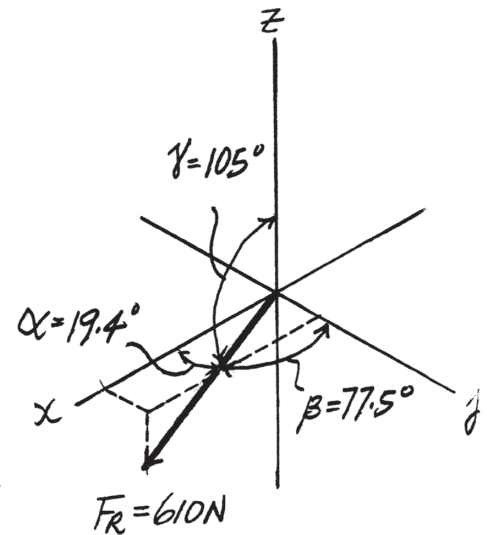
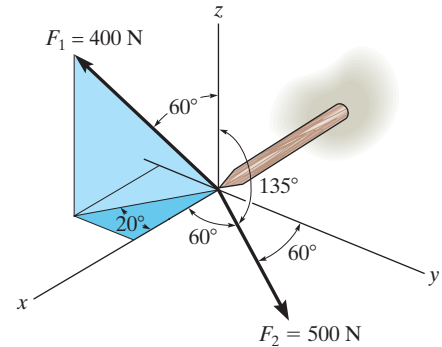
$$\begin{aligned}F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{575.52^2 + 131.52^2 + (-153.55)^2} \\ &= 610.00 \text{ N} = 610 \text{ N}\end{aligned}$$

The coordinate direction angles are

$$\cos \alpha = \frac{(F_R)_x}{F_R} = \frac{575.52}{610.00} \quad \alpha = 19.36^\circ = 19.4^\circ$$

$$\cos \beta = \frac{(F_R)_y}{F_R} = \frac{131.52}{610.00} \quad \beta = 77.549^\circ = 77.5^\circ$$

$$\cos \gamma = \frac{(F_R)_z}{F_R} = \frac{-153.55}{610.00} \quad \gamma = 104.58^\circ = 105^\circ$$



Ans.

Ans.

Ans.

Ans.

Ans:

$$\begin{aligned}F_R &= 610 \text{ N} \\ \alpha &= 19.4^\circ \\ \beta &= 77.5^\circ \\ \gamma &= 105^\circ\end{aligned}$$

***2-56.**

Determine the length of the connecting rod AB by first formulating a position vector from A to B and then determining its magnitude.

SOLUTION

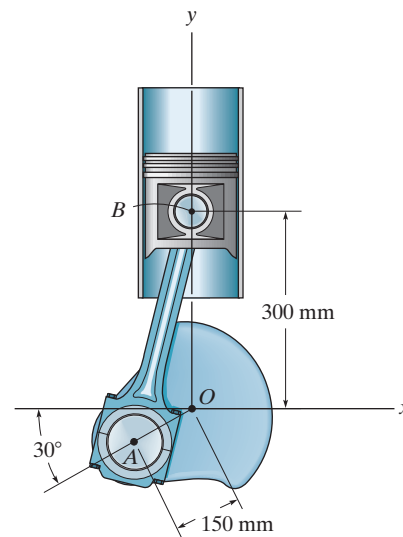
Position Vector. The coordinates of points A and B are $A(-150 \cos 30^\circ, -150 \sin 30^\circ)$ mm and $B(0, 300)$ mm respectively. Then

$$\begin{aligned}\mathbf{r}_{AB} &= [0 - (-150 \cos 30^\circ)]\mathbf{i} + [300 - (-150 \sin 30^\circ)]\mathbf{j} \\ &= \{129.90\mathbf{i} + 375\mathbf{j}\} \text{ mm}\end{aligned}$$

Thus, the magnitude of \mathbf{r}_{AB} is

$$r_{AB} = \sqrt{129.90^2 + 375^2} = 396.86 \text{ mm} = 397 \text{ mm}$$

Ans.



Ans:
 $r_{AB} = 397 \text{ mm}$

2-57.

Express force **F** as a Cartesian vector; then determine its coordinate direction angles.

SOLUTION

$$\mathbf{r}_{AB} = (5 + 10 \cos 70^\circ \sin 30^\circ)\mathbf{i} \\ + (-7 - 10 \cos 70^\circ \cos 30^\circ)\mathbf{j} - 10 \sin 70^\circ\mathbf{k}$$

$$\mathbf{r}_{AB} = \{6.710\mathbf{i} - 9.962\mathbf{j} - 9.397\mathbf{k}\} \text{ ft}$$

$$r_{AB} = \sqrt{(6.710)^2 + (-9.962)^2 + (-9.397)^2} = 15.25$$

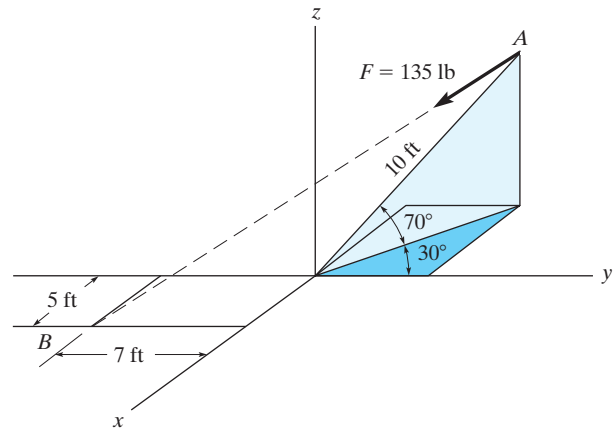
$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = (0.4400\mathbf{i} - 0.6532\mathbf{j} - 0.6162\mathbf{k})$$

$$\mathbf{F} = 135\mathbf{u}_{AB} = (59.40\mathbf{i} - 88.18\mathbf{j} - 83.18\mathbf{k}) \\ = \{59.4\mathbf{i} - 88.2\mathbf{j} - 83.2\mathbf{k}\} \text{ lb}$$

$$\alpha = \cos^{-1}\left(\frac{59.40}{135}\right) = 63.9^\circ$$

$$\beta = \cos^{-1}\left(\frac{-88.18}{135}\right) = 131^\circ$$

$$\gamma = \cos^{-1}\left(\frac{-83.18}{135}\right) = 128^\circ$$



Ans.

Ans.

Ans.

Ans.

Ans:

$$\mathbf{F} = \{59.4\mathbf{i} - 88.2\mathbf{j} - 83.2\mathbf{k}\} \text{ lb}$$

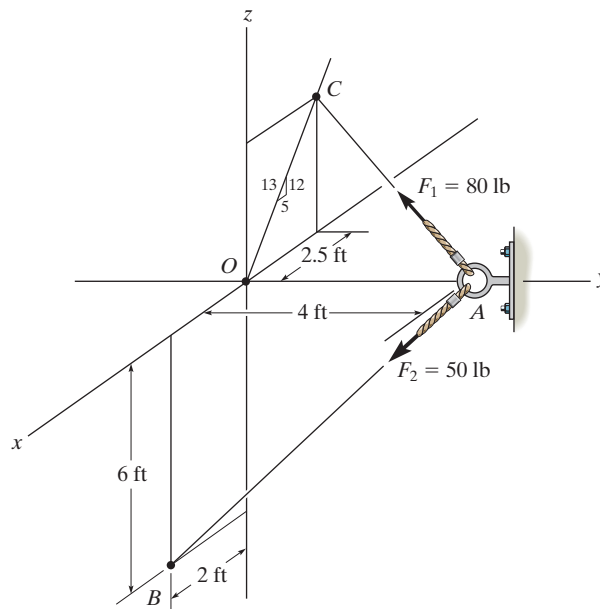
$$\alpha = 63.9^\circ$$

$$\beta = 131^\circ$$

$$\gamma = 128^\circ$$

2-58.

Express each force as a Cartesian vector, and then determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$$\mathbf{r}_{AC} = \left\{ -2.5 \mathbf{i} - 4 \mathbf{j} + \frac{12}{5} (2.5) \mathbf{k} \right\} \text{ ft}$$

$$\begin{aligned} \mathbf{F}_1 &= 80 \text{ lb} \left(\frac{\mathbf{r}_{AC}}{r_{AC}} \right) = -26.20 \mathbf{i} - 41.93 \mathbf{j} + 62.89 \mathbf{k} \\ &= \{-26.2 \mathbf{i} - 41.9 \mathbf{j} + 62.9 \mathbf{k}\} \text{ lb} \end{aligned}$$

$$\mathbf{r}_{AB} = \{2 \mathbf{i} - 4 \mathbf{j} - 6 \mathbf{k}\} \text{ ft}$$

$$\begin{aligned} \mathbf{F}_2 &= 50 \text{ lb} \left(\frac{\mathbf{r}_{AB}}{r_{AB}} \right) = 13.36 \mathbf{i} - 26.73 \mathbf{j} - 40.09 \mathbf{k} \\ &= \{13.4 \mathbf{i} - 26.7 \mathbf{j} - 40.1 \mathbf{k}\} \text{ lb} \end{aligned}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= -12.84 \mathbf{i} - 68.65 \mathbf{j} + 22.80 \mathbf{k}$$

$$= \{-12.8 \mathbf{i} - 68.7 \mathbf{j} + 22.8 \mathbf{k}\} \text{ lb}$$

$$F_R = \sqrt{(-12.84)^2 + (-68.65)^2 + (22.80)^2} = 73.47 = 73.5 \text{ lb}$$

$$\alpha = \cos^{-1} \left(\frac{-12.84}{73.47} \right) = 100^\circ$$

$$\beta = \cos^{-1} \left(\frac{-68.65}{73.47} \right) = 159^\circ$$

$$\gamma = \cos^{-1} \left(\frac{22.80}{73.47} \right) = 71.9^\circ$$

Ans.

Ans.

Ans.

Ans.

Ans.

Ans.

Ans:

$$\mathbf{F}_1 = \{-26.2 \mathbf{i} - 41.9 \mathbf{j} + 62.9 \mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_2 = \{13.4 \mathbf{i} - 26.7 \mathbf{j} - 40.1 \mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_R = 73.5 \text{ lb}$$

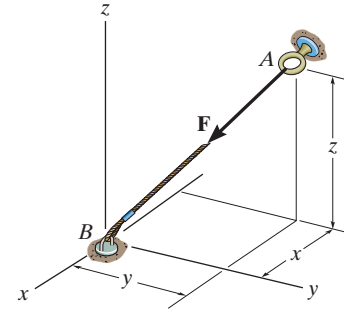
$$\alpha = 100^\circ$$

$$\beta = 159^\circ$$

$$\gamma = 71.9^\circ$$

2-59.

If $\mathbf{F} = \{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}\}$ N and cable AB is 9 m long, determine the x, y, z coordinates of point A .



SOLUTION

Position Vector: The position vector \mathbf{r}_{AB} , directed from point A to point B , is given by

$$\begin{aligned}\mathbf{r}_{AB} &= [0 - x]\mathbf{i} + (0 - y)\mathbf{j} + (0 - z)\mathbf{k} \\ &= -x\mathbf{i} - y\mathbf{j} - z\mathbf{k}\end{aligned}$$

Unit Vector: Knowing the magnitude of \mathbf{r}_{AB} is 9 m, the unit vector for \mathbf{r}_{AB} is given by

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9}$$

The unit vector for force \mathbf{F} is

$$\mathbf{u}_F = \frac{\mathbf{F}}{F} = \frac{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}}{\sqrt{350^2 + (-250)^2 + (-450)^2}} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Since force \mathbf{F} is also directed from point A to point B , then

$$\mathbf{u}_{AB} = \mathbf{u}_F$$

$$\frac{-x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9} = -0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Equating the \mathbf{i}, \mathbf{j} , and \mathbf{k} components,

$$\frac{x}{9} = -0.5623 \quad x = -5.06 \text{ m} \quad \text{Ans.}$$

$$\frac{-y}{9} = -0.4016 \quad y = 3.61 \text{ m} \quad \text{Ans.}$$

$$\frac{-z}{9} = 0.7229 \quad z = 6.51 \text{ m} \quad \text{Ans.}$$

Ans:

$$x = -5.06 \text{ m}$$

$$y = 3.61 \text{ m}$$

$$z = 6.51 \text{ m}$$

***2–60.**

The 8-m-long cable is anchored to the ground at A . If $x = 4$ m and $y = 2$ m, determine the coordinate z to the highest point of attachment along the column.

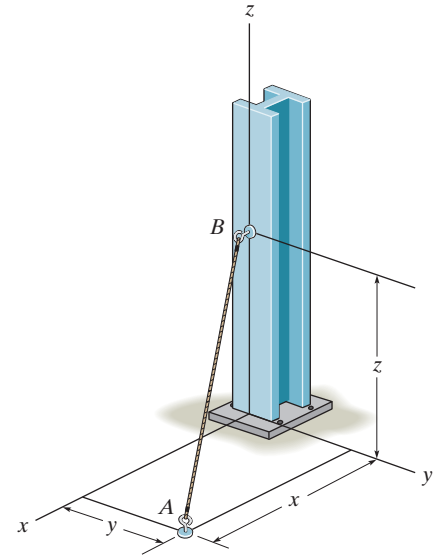
SOLUTION

$$\mathbf{r} = \{4\mathbf{i} + 2\mathbf{j} + z\mathbf{k}\} \text{ m}$$

$$r = \sqrt{(4)^2 + (2)^2 + (z)^2} = 8$$

$$z = 6.63 \text{ m}$$

Ans.



Ans:
6.63 m

2-61.

The 8-m-long cable is anchored to the ground at A . If $z = 5$ m, determine the location $+x, +y$ of the support at A . Choose a value such that $x = y$.

SOLUTION

$$\mathbf{r} = \{x\mathbf{i} + y\mathbf{j} + 5\mathbf{k}\} \text{ m}$$

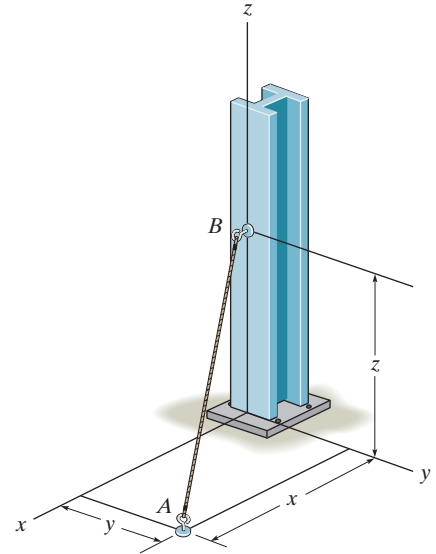
$$r = \sqrt{(x)^2 + (y)^2 + (5)^2} = 8$$

$$x = y, \text{ thus}$$

$$2x^2 = 8^2 - 5^2$$

$$x = y = 4.42 \text{ m}$$

Ans.



Ans:
 $x = y = 4.42 \text{ m}$

2-62.

Express each of the forces in Cartesian vector form and then determine the magnitude and coordinate direction angles of the resultant force

SOLUTION

Unit Vectors. The coordinates for points A, B and C are $(0, -0.75, 3)$ m, $B(2 \cos 40^\circ, 2 \sin 40^\circ, 0)$ m and $C(2, -1, 0)$ m, respectively.

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(2 \cos 40^\circ - 0)\mathbf{i} + [2 \sin 40^\circ - (-0.75)]\mathbf{j} + (0 - 3)\mathbf{k}}{\sqrt{(2 \cos 40^\circ - 0)^2 + [2 \sin 40^\circ - (-0.75)]^2 + (0 - 3)^2}}$$

$$= 0.3893\mathbf{i} + 0.5172\mathbf{j} - 0.7622\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(2 - 0)\mathbf{i} + [-1 - (-0.75)]\mathbf{j} + (0 - 3)\mathbf{k}}{\sqrt{(2 - 0)^2 + [-1 - (-0.75)]^2 + (0 - 3)^2}}$$

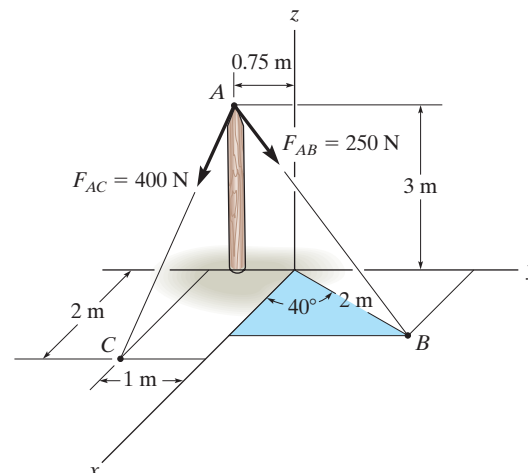
$$= 0.5534\mathbf{i} - 0.0692\mathbf{j} - 0.8301\mathbf{k}$$

Force Vectors

$$\mathbf{F}_{AB} = F_{AB} \mathbf{u}_{AB} = 250 (0.3893\mathbf{i} + 0.5172\mathbf{j} - 0.7622\mathbf{k})$$

$$= \{97.32\mathbf{i} + 129.30\mathbf{j} - 190.56\mathbf{k}\} \text{ N}$$

$$= \{97.3\mathbf{i} + 129\mathbf{j} - 191\mathbf{k}\} \text{ N}$$



Ans.

$$\mathbf{F}_{AC} = F_{AC} \mathbf{u}_{AC} = 400 (0.5534\mathbf{i} - 0.06917\mathbf{j} - 0.8301\mathbf{k})$$

$$= \{221.35\mathbf{i} - 27.67\mathbf{j} - 332.02\mathbf{k}\} \text{ N}$$

$$= \{221\mathbf{i} - 27.7\mathbf{j} - 332\mathbf{k}\} \text{ N}$$

Ans.

Resultant Force

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$$

$$= \{97.32\mathbf{i} + 129.30\mathbf{j} - 190.56\mathbf{k}\} + \{221.35\mathbf{i} - 27.67\mathbf{j} - 332.02\mathbf{k}\}$$

$$= \{318.67\mathbf{i} + 101.63\mathbf{j} - 522.58\mathbf{k}\} \text{ N}$$

The magnitude of \mathbf{F}_R is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{318.67^2 + 101.63^2 + (-522.58)^2}$$

$$= 620.46 \text{ N} = 620 \text{ N}$$

And its coordinate direction angles are

$$\cos \alpha = \frac{(F_R)_x}{F_R} = \frac{318.67}{620.46}; \quad \alpha = 59.10^\circ = 59.1^\circ$$

Ans.

$$\cos \beta = \frac{(F_R)_y}{F_R} = \frac{101.63}{620.46}; \quad \beta = 80.57^\circ = 80.6^\circ$$

Ans.

$$\cos \gamma = \frac{(F_R)_z}{F_R} = \frac{-522.58}{620.46}; \quad \gamma = 147.38^\circ = 147^\circ$$

Ans.

Ans:

$$\mathbf{F}_{AB} = \{97.3\mathbf{i} - 129\mathbf{j} - 191\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_{AC} = \{221\mathbf{i} - 27.7\mathbf{j} - 332\mathbf{k}\} \text{ N}$$

$$F_R = 620 \text{ N}$$

$$\alpha = 59.1^\circ$$

$$\beta = 80.6^\circ$$

$$\gamma = 147^\circ$$

2-63.

If $F_B = 560$ N and $F_C = 700$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.

SOLUTION

Force Vectors: The unit vectors \mathbf{u}_B and \mathbf{u}_C of \mathbf{F}_B and \mathbf{F}_C must be determined first. From Fig. a,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors \mathbf{F}_B and \mathbf{F}_C are given by

$$\mathbf{F}_B = F_B \mathbf{u}_B = 560 \left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right) = \{160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_C = F_C \mathbf{u}_C = 700 \left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right) = \{300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k}\} \text{ N}$$

Resultant Force:

$$\begin{aligned} \mathbf{F}_R &= \mathbf{F}_B + \mathbf{F}_C = (160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}) + (300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k}) \\ &= \{460\mathbf{i} - 40\mathbf{j} + 1080\mathbf{k}\} \text{ N} \end{aligned}$$

The magnitude of \mathbf{F}_R is

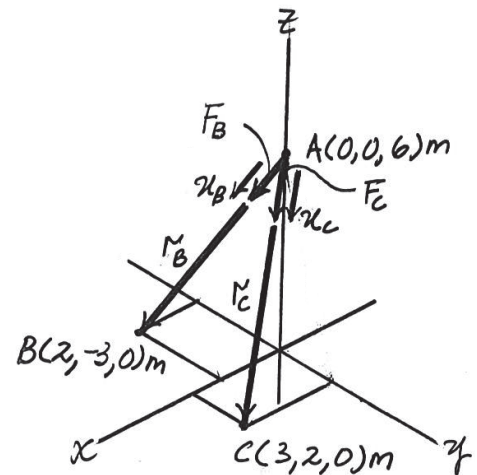
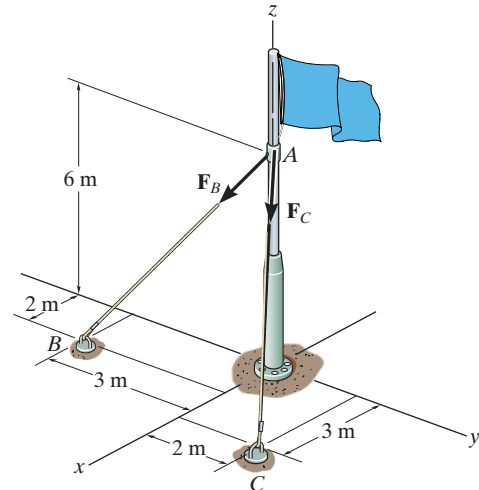
$$\begin{aligned} F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} \\ &= \sqrt{(460)^2 + (-40)^2 + (1080)^2} = 1174.56 \text{ N} = 1.17 \text{ kN} \end{aligned}$$

The coordinate direction angles of \mathbf{F}_R are

$$\alpha = \cos^{-1} \left[\frac{(F_R)_x}{F_R} \right] = \cos^{-1} \left(\frac{460}{1174.56} \right) = 66.9^\circ$$

$$\beta = \cos^{-1} \left[\frac{(F_R)_y}{F_R} \right] = \cos^{-1} \left(\frac{-40}{1174.56} \right) = 92.0^\circ$$

$$\gamma = \cos^{-1} \left[\frac{(F_R)_z}{F_R} \right] = \cos^{-1} \left(\frac{1080}{1174.56} \right) = 157^\circ$$



Ans.

Ans.

Ans.

Ans.

Ans:

$$F_R = 1.17 \text{ kN}$$

$$\alpha = 66.9^\circ$$

$$\beta = 92.0^\circ$$

$$\gamma = 157^\circ$$

***2-64.**

If $F_B = 700$ N, and $F_C = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.

SOLUTION

Force Vectors: The unit vectors \mathbf{u}_B and \mathbf{u}_C of \mathbf{F}_B and \mathbf{F}_C must be determined first. From Fig. *a*,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors \mathbf{F}_B and \mathbf{F}_C are given by

$$\mathbf{F}_B = F_B \mathbf{u}_B = 700 \left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right) = \{200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_C = F_C \mathbf{u}_C = 560 \left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right) = \{240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k}\} \text{ N}$$

Resultant Force:

$$\begin{aligned} \mathbf{F}_R &= \mathbf{F}_B + \mathbf{F}_C = (200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}) + (240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k}) \\ &= \{440\mathbf{i} - 140\mathbf{j} - 1080\mathbf{k}\} \text{ N} \end{aligned}$$

The magnitude of \mathbf{F}_R is

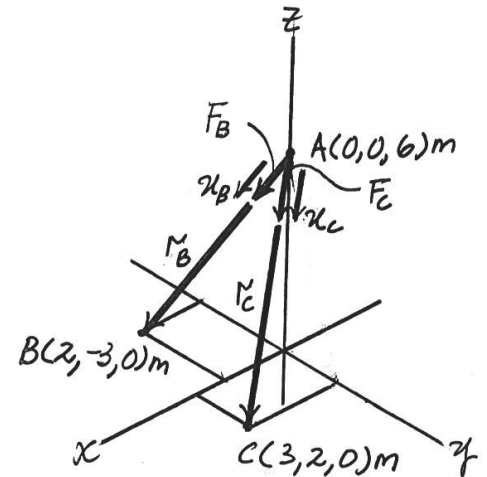
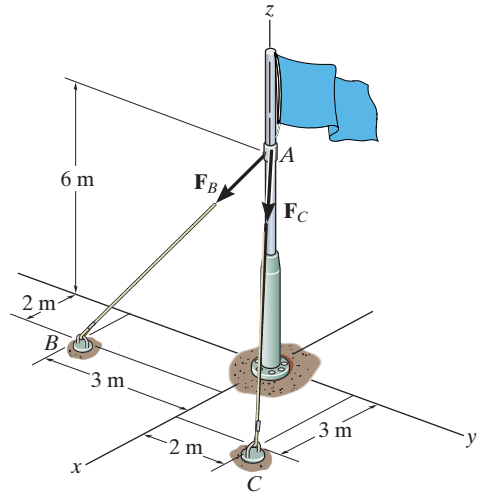
$$\begin{aligned} F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} \\ &= \sqrt{(440)^2 + (-140)^2 + (-1080)^2} = 1174.56 \text{ N} = 1.17 \text{ kN} \end{aligned}$$

The coordinate direction angles of \mathbf{F}_R are

$$\alpha = \cos^{-1} \left[\frac{(F_R)_x}{F_R} \right] = \cos^{-1} \left(\frac{440}{1174.56} \right) = 68.0^\circ$$

$$\beta = \cos^{-1} \left[\frac{(F_R)_y}{F_R} \right] = \cos^{-1} \left(\frac{-140}{1174.56} \right) = 96.8^\circ$$

$$\gamma = \cos^{-1} \left[\frac{(F_R)_z}{F_R} \right] = \cos^{-1} \left(\frac{-1080}{1174.56} \right) = 157^\circ$$



Ans.

Ans.

Ans.

Ans.

Ans:

$$F_R = 1.17 \text{ kN}$$

$$\alpha = 68.0^\circ$$

$$\beta = 96.8^\circ$$

$$\gamma = 157^\circ$$

2-65.

The plate is suspended using the three cables which exert the forces shown. Express each force as a Cartesian vector.

SOLUTION

$$\mathbf{F}_{BA} = 350 \left(\frac{\mathbf{r}_{BA}}{r_{BA}} \right) = 350 \left(-\frac{5}{16.031} \mathbf{i} + \frac{6}{16.031} \mathbf{j} + \frac{14}{16.031} \mathbf{k} \right)$$

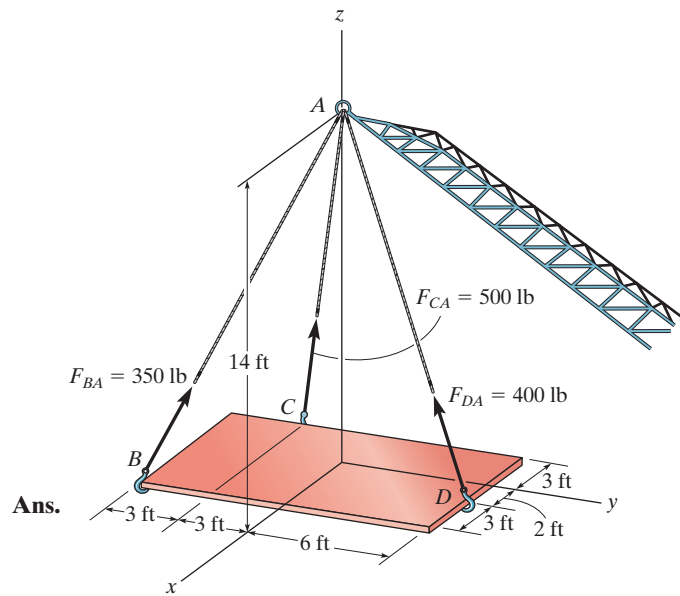
$$= \{-109 \mathbf{i} + 131 \mathbf{j} + 306 \mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_{CA} = 500 \left(\frac{\mathbf{r}_{CA}}{r_{CA}} \right) = 500 \left(\frac{3}{14.629} \mathbf{i} + \frac{3}{14.629} \mathbf{j} + \frac{14}{14.629} \mathbf{k} \right)$$

$$= \{103 \mathbf{i} + 103 \mathbf{j} + 479 \mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_{DA} = 400 \left(\frac{\mathbf{r}_{DA}}{r_{DA}} \right) = 400 \left(-\frac{2}{15.362} \mathbf{i} - \frac{6}{15.362} \mathbf{j} + \frac{14}{15.362} \mathbf{k} \right)$$

$$= \{-52.1 \mathbf{i} - 156 \mathbf{j} + 365 \mathbf{k}\} \text{ lb}$$



Ans.

Ans.

Ans.

Ans:

$$\mathbf{F}_{BA} = \{-109 \mathbf{i} + 131 \mathbf{j} + 306 \mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_{CA} = \{103 \mathbf{i} + 103 \mathbf{j} + 479 \mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_{DA} = \{-52.1 \mathbf{i} - 156 \mathbf{j} + 365 \mathbf{k}\} \text{ lb}$$

2-66.

Represent each cable force as a Cartesian vector.

SOLUTION

$$\mathbf{r}_C = (0 - 5)\mathbf{i} + (-2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\} \text{ m}$$

$$r_C = \sqrt{(-5)^2 + (-2)^2 + 3^2} = \sqrt{38} \text{ m}$$

$$\mathbf{r}_B = (0 - 5)\mathbf{i} + (2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}\} \text{ m}$$

$$r_B = \sqrt{(-5)^2 + 2^2 + 3^2} = \sqrt{38} \text{ m}$$

$$\mathbf{r}_E = (0 - 2)\mathbf{i} + (0 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-2\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}\} \text{ m}$$

$$r_E = \sqrt{(-2)^2 + 0^2 + 3^2} = \sqrt{13} \text{ m}$$

$$\mathbf{F} = F_u = F\left(\frac{\mathbf{r}}{r}\right)$$

$$\mathbf{F}_C = 400\left(\frac{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}}{\sqrt{38}}\right) = \{-324\mathbf{i} - 130\mathbf{j} + 195\mathbf{k}\} \text{ N}$$

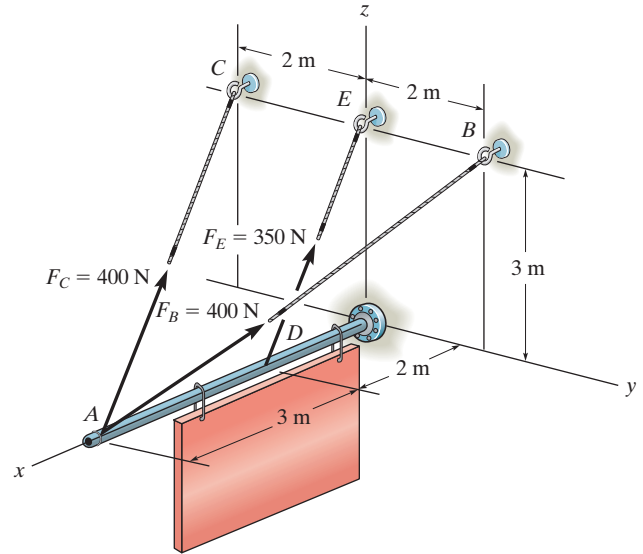
Ans.

$$\mathbf{F}_B = 400\left(\frac{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{\sqrt{38}}\right) = \{-324\mathbf{i} + 130\mathbf{j} + 195\mathbf{k}\} \text{ N}$$

Ans.

$$\mathbf{F}_E = 350\left(\frac{-2\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}}{\sqrt{13}}\right) = \{-194\mathbf{i} + 291\mathbf{k}\} \text{ N}$$

Ans.



Ans:

$$\mathbf{F}_C = \{-324\mathbf{i} - 130\mathbf{j} + 195\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_B = \{-324\mathbf{i} + 130\mathbf{j} + 195\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_E = \{-194\mathbf{i} + 291\mathbf{k}\} \text{ N}$$

2-67.

Determine the magnitude and coordinate direction angles of the resultant force of the two forces acting at point A.

SOLUTION

$$\mathbf{r}_C = (0 - 5)\mathbf{i} + (-2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\}$$

$$r_C = \sqrt{(-5)^2 + (-2)^2 + (3)^2} = \sqrt{38} \text{ m}$$

$$\mathbf{F}_C = 400 \left(\frac{\mathbf{r}_C}{r_C} \right) = 400 \left(\frac{(-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})}{\sqrt{38}} \right)$$

$$\mathbf{F}_C = (-324.4428\mathbf{i} - 129.777\mathbf{j} + 194.666\mathbf{k})$$

$$\mathbf{r}_B = (0 - 5)\mathbf{i} + (2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}\}$$

$$r_B = \sqrt{(-5)^2 + 2^2 + 3^2} = \sqrt{38} \text{ m}$$

$$\mathbf{F}_B = 400 \left(\frac{\mathbf{r}_B}{r_B} \right) = 400 \left(\frac{(-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})}{\sqrt{38}} \right)$$

$$\mathbf{F}_B = (-324.443\mathbf{i} + 129.777\mathbf{j} + 194.666\mathbf{k})$$

$$\mathbf{F}_R = \mathbf{F}_C + \mathbf{F}_B = (-648.89\mathbf{i} + 389.33\mathbf{k})$$

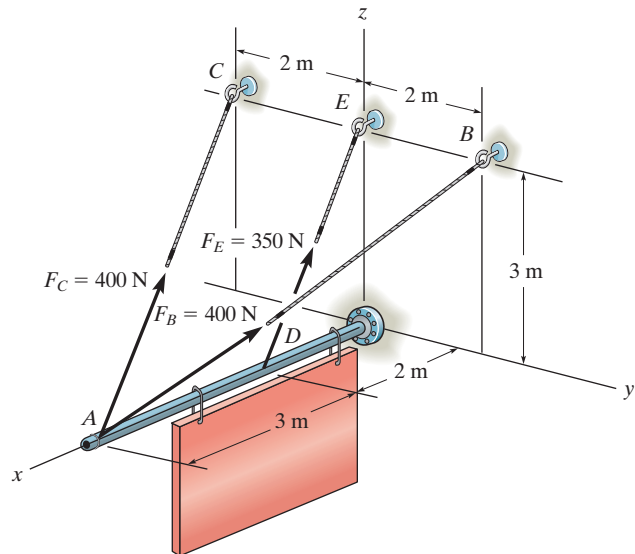
$$F_R = \sqrt{(-648.89)^2 + (389.33)^2 + 0^2} = 756.7242$$

$$F_R = 757 \text{ N}$$

$$\alpha = \cos^{-1} \left(\frac{-648.89}{756.7242} \right) = 149.03 = 149^\circ$$

$$\beta = \cos^{-1} \left(\frac{0}{756.7242} \right) = 90.0^\circ$$

$$\gamma = \cos^{-1} \left(\frac{389.33}{756.7242} \right) = 59.036 = 59.0^\circ$$



Ans.

Ans.

Ans.

Ans.

Ans:

$$F_R = 757 \text{ N}$$

$$\alpha = 149^\circ$$

$$\beta = 90.0^\circ$$

$$\gamma = 59.0^\circ$$

***2-68.**

The force \mathbf{F} has a magnitude of 80 lb and acts at the midpoint C of the rod. Express this force as a Cartesian vector.

SOLUTION

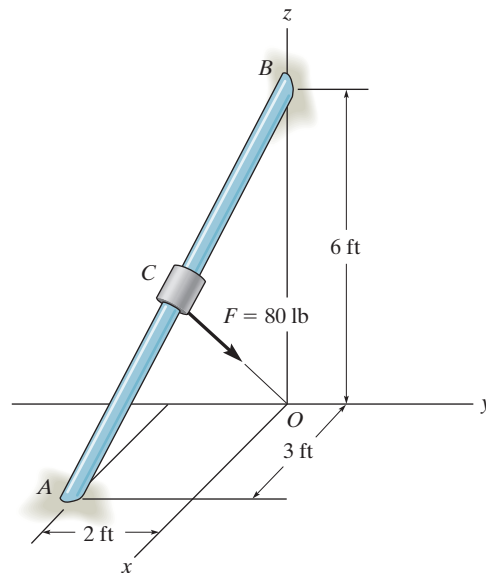
$$\mathbf{r}_{AB} = (-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k})$$

$$\mathbf{r}_{CB} = \frac{1}{2}\mathbf{r}_{AB} = (-1.5\mathbf{i} + 1\mathbf{j} + 3\mathbf{k})$$

$$\begin{aligned}\mathbf{r}_{CO} &= \mathbf{r}_{BO} + \mathbf{r}_{CB} \\ &= -6\mathbf{k} - 1.5\mathbf{i} + 1\mathbf{j} + 3\mathbf{k} \\ &= -1.5\mathbf{i} + 1\mathbf{j} - 3\mathbf{k}\end{aligned}$$

$$r_{CO} = 3.5$$

$$\mathbf{F} = 80 \left(\frac{\mathbf{r}_{CO}}{r_{CO}} \right) = \{-34.3\mathbf{i} + 22.9\mathbf{j} - 68.6\mathbf{k}\} \text{ lb}$$



Ans.

Ans:
 $\mathbf{F} = \{-34.3\mathbf{i} + 22.9\mathbf{j} - 68.6\mathbf{k}\} \text{ lb}$

2–69.

The load at A creates a force of 60 lb in wire AB . Express this force as a Cartesian vector.

SOLUTION

Unit Vector: First determine the position vector \mathbf{r}_{AB} . The coordinates of point B are

$$B (5 \sin 30^\circ, 5 \cos 30^\circ, 0) \text{ ft} = B (2.50, 4.330, 0) \text{ ft}$$

Then

$$\begin{aligned} \mathbf{r}_{AB} &= \{(2.50 - 0)\mathbf{i} + (4.330 - 0)\mathbf{j} + [0 - (-10)]\mathbf{k}\} \text{ ft} \\ &= \{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}\} \text{ ft} \end{aligned}$$

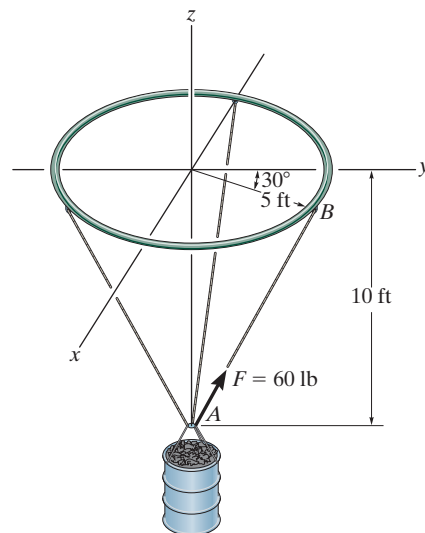
$$r_{AB} = \sqrt{2.50^2 + 4.330^2 + 10.0^2} = 11.180 \text{ ft}$$

$$\begin{aligned} \mathbf{u}_{AB} &= \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}}{11.180} \\ &= 0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k} \end{aligned}$$

Force Vector:

$$\begin{aligned} \mathbf{F} &= F\mathbf{u}_{AB} = 60 \{0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}\} \text{ lb} \\ &= \{13.4\mathbf{i} + 23.2\mathbf{j} + 53.7\mathbf{k}\} \text{ lb} \end{aligned}$$

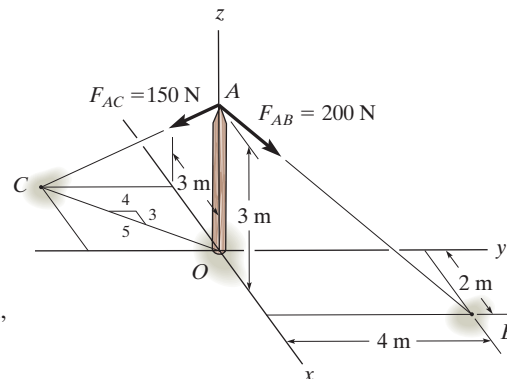
Ans.



Ans:
 $\mathbf{F} = \{13.4\mathbf{i} + 23.2\mathbf{j} + 53.7\mathbf{k}\} \text{ lb}$

2-70.

Determine the magnitude and coordinate direction angles of the resultant force acting at point A on the post.



SOLUTION

Unit Vector. The coordinates for points A , B and C are $A(0, 0, 3)$ m, $B(2, 4, 0)$ m, and $C(-3, -4, 0)$ m, respectively.

$$\mathbf{r}_{AB} = (2 - 0)\mathbf{i} + (4 - 0)\mathbf{j} + (0 - 3)\mathbf{k} = \{2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}\} \text{ m}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}}{\sqrt{2^2 + 4^2 + (-3)^2}} = \frac{2}{\sqrt{29}}\mathbf{i} + \frac{4}{\sqrt{29}}\mathbf{j} - \frac{3}{\sqrt{29}}\mathbf{k}$$

$$\mathbf{r}_{AC} = (-3 - 0)\mathbf{i} + (-4 - 0)\mathbf{j} + (0 - 3)\mathbf{k} = \{-3\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}\} \text{ m}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-3\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}}{\sqrt{(-3)^2 + (-4)^2 + (-3)^2}} = -\frac{3}{\sqrt{34}}\mathbf{i} - \frac{4}{\sqrt{34}}\mathbf{j} - \frac{3}{\sqrt{34}}\mathbf{k}$$

Force Vectors

$$\begin{aligned} \mathbf{F}_{AB} &= F_{AB} \mathbf{u}_{AB} = 200 \left(\frac{2}{\sqrt{29}}\mathbf{i} + \frac{4}{\sqrt{29}}\mathbf{j} - \frac{3}{\sqrt{29}}\mathbf{k} \right) \\ &= \{74.28\mathbf{i} + 148.56\mathbf{j} - 111.42\mathbf{k}\} \text{ N} \end{aligned}$$

$$\begin{aligned} \mathbf{F}_{AC} &= F_{AC} \mathbf{u}_{AC} = 150 \left(-\frac{3}{\sqrt{34}}\mathbf{i} - \frac{4}{\sqrt{34}}\mathbf{j} - \frac{3}{\sqrt{34}}\mathbf{k} \right) \\ &= \{-77.17\mathbf{i} - 102.90\mathbf{j} - 77.17\mathbf{k}\} \text{ N} \end{aligned}$$

Resultant Force

$$\begin{aligned} \mathbf{F}_R &= \mathbf{F}_{AB} + \mathbf{F}_{AC} \\ &= \{74.28\mathbf{i} + 148.56\mathbf{j} - 111.42\mathbf{k}\} + \{-77.17\mathbf{i} - 102.90\mathbf{j} - 77.17\mathbf{k}\} \\ &= \{-2.896\mathbf{i} + 45.66\mathbf{j} - 188.59\mathbf{k}\} \text{ N} \end{aligned}$$

The magnitude of the resultant force is

$$\begin{aligned} F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{(-2.896)^2 + 45.66^2 + (-188.59)^2} \\ &= 194.06 \text{ N} = 194 \text{ N} \end{aligned}$$

Ans.

And its coordinate direction angles are

$$\cos \alpha = \frac{(F_R)_x}{F_R} = \frac{-2.896}{194.06}; \quad \alpha = 90.86^\circ = 90.9^\circ \quad \text{Ans.}$$

$$\cos \beta = \frac{(F_R)_y}{F_R} = \frac{45.66}{194.06}; \quad \beta = 76.39^\circ = 76.4^\circ \quad \text{Ans.}$$

$$\cos \gamma = \frac{(F_R)_z}{F_R} = \frac{-188.59}{194.06}; \quad \gamma = 166.36^\circ = 166^\circ \quad \text{Ans.}$$

Ans:

$$\begin{aligned} F_R &= 194 \text{ N} \\ \alpha &= 90.9^\circ \\ \beta &= 76.4^\circ \\ \gamma &= 166^\circ \end{aligned}$$

2-71.

Given the three vectors **A**, **B**, and **D**, show that
 $\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D})$.

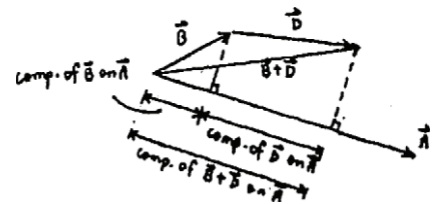
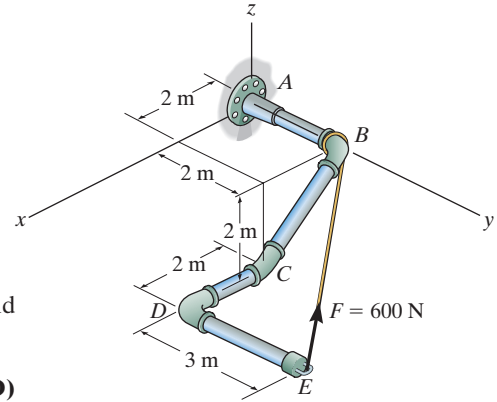
SOLUTION

Since the component of $(\mathbf{B} + \mathbf{D})$ is equal to the sum of the components of **B** and **D**, then

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{D} \quad (\text{QED})$$

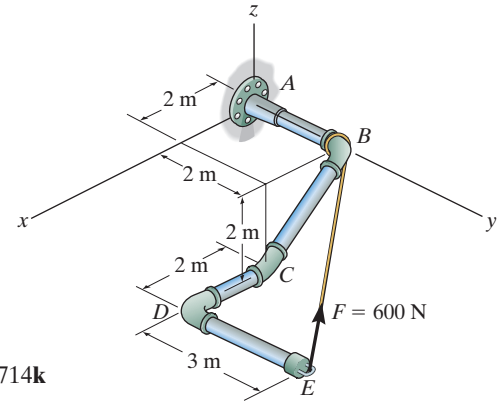
Also,

$$\begin{aligned} \mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) &= (A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}) \cdot [(B_x + D_x) \mathbf{i} + (B_y + D_y) \mathbf{j} + (B_z + D_z) \mathbf{k}] \\ &= A_x(B_x + D_x) + A_y(B_y + D_y) + A_z(B_z + D_z) \\ &= (A_x B_x + A_y B_y + A_z B_z) + (A_x D_x + A_y D_y + A_z D_z) \\ &= (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D}) \quad (\text{QED}) \end{aligned}$$



***2-72.**

Determine the magnitudes of the components of $F = 600 \text{ N}$ acting along and perpendicular to segment DE of the pipe assembly.



SOLUTION

Unit Vectors: The unit vectors \mathbf{u}_{EB} and \mathbf{u}_{ED} must be determined first. From Fig. *a*,

$$\mathbf{u}_{EB} = \frac{\mathbf{r}_{EB}}{r_{EB}} = \frac{(0 - 4)\mathbf{i} + (2 - 5)\mathbf{j} + [0 - (-2)]\mathbf{k}}{\sqrt{(0 - 4)^2 + (2 - 5)^2 + [0 - (-2)]^2}} = -0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}$$

$$\mathbf{u}_{ED} = -\mathbf{j}$$

Thus, the force vector \mathbf{F} is given by

$$\mathbf{F} = F\mathbf{u}_{EB} = 600(-0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}) = [-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}] \text{ N}$$

Vector Dot Product: The magnitude of the component of \mathbf{F} parallel to segment DE of the pipe assembly is

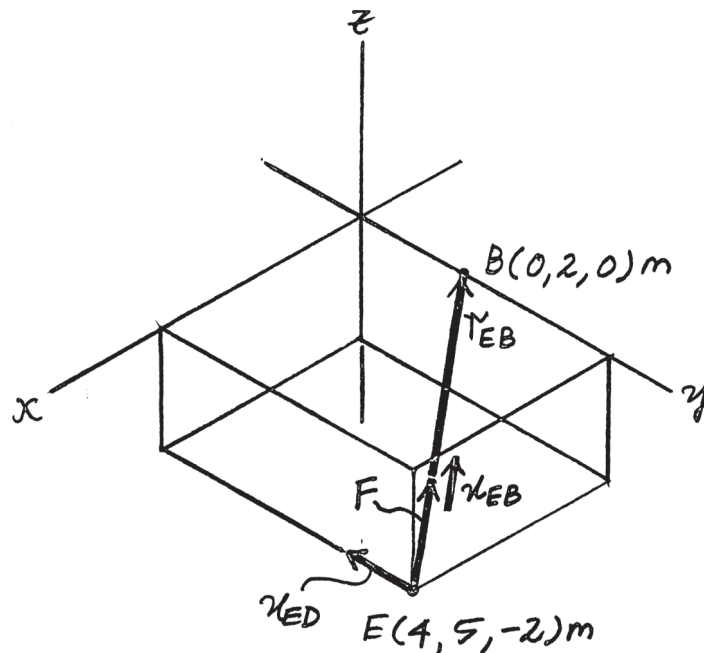
$$\begin{aligned} (F_{ED})_{\text{para}} &= \mathbf{F} \cdot \mathbf{u}_{ED} = (-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}) \cdot (-\mathbf{j}) \\ &= (-445.66)(0) + (-334.25)(-1) + (222.83)(0) \\ &= 334.25 = 334 \text{ N} \end{aligned}$$

Ans.

The component of \mathbf{F} perpendicular to segment DE of the pipe assembly is

$$(F_{ED})_{\text{per}} = \sqrt{F^2 - (F_{ED})_{\text{para}}^2} = \sqrt{600^2 - 334.25^2} = 498 \text{ N}$$

Ans.



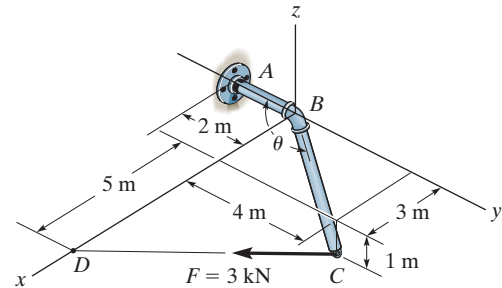
Ans:

$$(F_{ED})_{\parallel} = 334 \text{ N}$$

$$(F_{ED})_{\perp} = 498 \text{ N}$$

2-73.

Determine the angle θ between BA and BC .



SOLUTION

Unit Vectors. Here, the coordinates of points A , B and C are $A(0, -2, 0)$ m, $B(0, 0, 0)$ m and $C(3, 4, -1)$ m respectively. Thus, the unit vectors along BA and BC are

$$\mathbf{u}_{BA} = -\mathbf{j} \quad \mathbf{u}_{BC} = \frac{(3-0)\mathbf{i} + (4-0)\mathbf{j} + (-1-0)\mathbf{k}}{\sqrt{(3-0)^2 + (4-0)^2 + (-1-0)^2}} = \frac{3}{\sqrt{26}}\mathbf{i} + \frac{4}{\sqrt{26}}\mathbf{j} - \frac{1}{\sqrt{26}}\mathbf{k}$$

The Angle θ Between BA and BC .

$$\begin{aligned} \mathbf{u}_{BA} \cdot \mathbf{u}_{BC} &= (-\mathbf{j}) \cdot \left(\frac{3}{\sqrt{26}}\mathbf{i} + \frac{4}{\sqrt{26}}\mathbf{j} - \frac{1}{\sqrt{26}}\mathbf{k} \right) \\ &= (-1) \left(\frac{4}{\sqrt{26}} \right) = -\frac{4}{\sqrt{26}} \end{aligned}$$

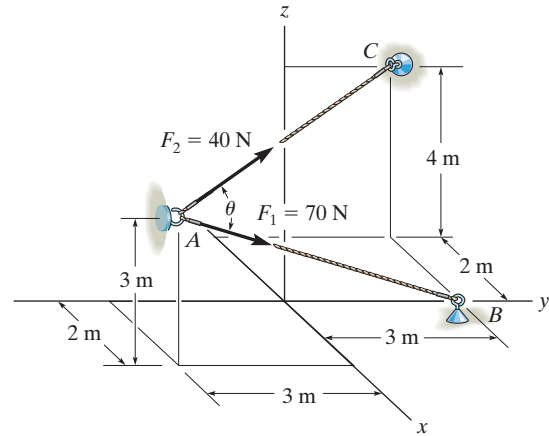
Then

$$\theta = \cos^{-1}(\mathbf{u}_{BA} \cdot \mathbf{u}_{BC}) = \cos^{-1}\left(-\frac{4}{\sqrt{26}}\right) = 141.67^\circ = 142^\circ \quad \text{Ans.}$$

Ans:
 $\theta = 142^\circ$

2-75.

Determine the angle θ between the two cables.



SOLUTION

Unit Vectors. Here, the coordinates of points A , B and C are $A(2, -3, 3)$ m, $B(0, 3, 0)$ and $C(-2, 3, 4)$ m, respectively. Thus, the unit vectors along AB and AC are

$$\mathbf{u}_{AB} = \frac{(0 - 2)\mathbf{i} + [3 - (-3)]\mathbf{j} + (0 - 3)\mathbf{k}}{\sqrt{(0 - 2)^2 + [3 - (-3)]^2 + (0 - 3)^2}} = -\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{(-2 - 2)\mathbf{i} + [3 - (-3)]\mathbf{j} + (4 - 3)\mathbf{k}}{\sqrt{(-2 - 2)^2 + [3 - (-3)]^2 + (4 - 3)^2}} = -\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}$$

The Angle θ Between AB and AC .

$$\begin{aligned}\mathbf{u}_{AB} \cdot \mathbf{u}_{AC} &= \left(-\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}\right) \cdot \left(-\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}\right) \\ &= \left(-\frac{2}{7}\right)\left(-\frac{4}{\sqrt{53}}\right) + \frac{6}{7}\left(\frac{6}{\sqrt{53}}\right) + \left(-\frac{3}{7}\right)\left(\frac{1}{\sqrt{53}}\right) \\ &= \frac{41}{7\sqrt{53}}\end{aligned}$$

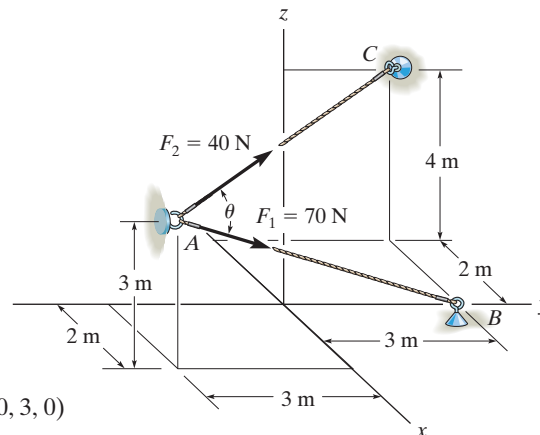
Then

$$\theta = \cos^{-1}(\mathbf{u}_{AB} \cdot \mathbf{u}_{AC}) = \cos^{-1}\left(\frac{41}{7\sqrt{53}}\right) = 36.43^\circ = 36.4^\circ \quad \text{Ans.}$$

Ans:
 $\theta = 36.4^\circ$

***2-76.**

Determine the magnitude of the projection of the force \mathbf{F}_1 along cable AC .



SOLUTION

Unit Vectors. Here, the coordinates of points A , B and C are $A(2, -3, 3)\text{ m}$, $B(0, 3, 0)$ and $C(-2, 3, 4)\text{ m}$, respectively. Thus, the unit vectors along AB and AC are

$$\mathbf{u}_{AB} = \frac{(0 - 2)\mathbf{i} + [3 - (-3)]\mathbf{j} + (0 - 3)\mathbf{k}}{\sqrt{(0 - 2)^2 + [3 - (-3)]^2 + (0 - 3)^2}} = -\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{(-2 - 2)\mathbf{i} + [3 - (-3)]\mathbf{j} + (4 - 3)\mathbf{k}}{\sqrt{(-2 - 2)^2 + [3 - (-3)]^2 + (4 - 3)^2}} = -\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}$$

Force Vector, For \mathbf{F}_1 ,

$$\mathbf{F}_1 = F_1 \mathbf{u}_{AB} = 70 \left(-\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k} \right) = \{-20\mathbf{i} + 60\mathbf{j} - 30\mathbf{k}\} \text{ N}$$

Projected Component of \mathbf{F}_1 . Along AC , it is

$$\begin{aligned} (F_1)_{AC} &= \mathbf{F}_1 \cdot \mathbf{u}_{AC} = (-20\mathbf{i} + 60\mathbf{j} - 30\mathbf{k}) \cdot \left(-\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k} \right) \\ &= (-20) \left(-\frac{4}{\sqrt{53}} \right) + 60 \left(\frac{6}{\sqrt{53}} \right) + (-30) \left(\frac{1}{\sqrt{53}} \right) \\ &= 56.32 \text{ N} = 56.3 \text{ N} \end{aligned}$$

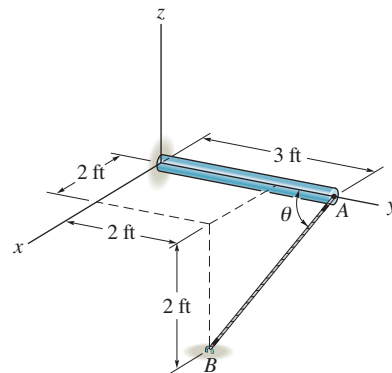
Ans.

The positive sign indicates that this component points in the same direction as \mathbf{u}_{AC} .

Ans:
 $(F_1)_{AC} = 56.3 \text{ N}$

2-77.

Determine the angle θ between the pole and the wire AB .



SOLUTION

Position Vector:

$$\mathbf{r}_{AC} = \{-3\mathbf{j}\} \text{ ft}$$

$$\begin{aligned}\mathbf{r}_{AB} &= \{(2 - 0)\mathbf{i} + (2 - 3)\mathbf{j} + (-2 - 0)\mathbf{k}\} \text{ ft} \\ &= \{2\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}\} \text{ ft}\end{aligned}$$

The magnitudes of the position vectors are

$$r_{AC} = 3.00 \text{ ft} \quad r_{AB} = \sqrt{2^2 + (-1)^2 + (-2)^2} = 3.00 \text{ ft}$$

The Angles Between Two Vectors θ : The dot product of two vectors must be determined first.

$$\begin{aligned}\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} &= (-3\mathbf{j}) \cdot (2\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}) \\ &= 0(2) + (-3)(-1) + 0(-2) \\ &= 3\end{aligned}$$

Then,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AO} \cdot \mathbf{r}_{AB}}{r_{AO}r_{AB}}\right) = \cos^{-1}\left[\frac{3}{3.00(3.00)}\right] = 70.5^\circ$$

Ans.

Ans:
 $\theta = 70.5^\circ$

2-78.

Determine the magnitude of the projection of the force along the u axis.

SOLUTION

Unit Vectors: The unit vectors \mathbf{u}_{OA} and \mathbf{u}_u must be determined first. From Fig. a ,

$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-2 - 0)\mathbf{i} + (4 - 0)\mathbf{j} + (4 - 0)\mathbf{k}}{\sqrt{(-2 - 0)^2 + (4 - 0)^2 + (4 - 0)^2}} = -\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

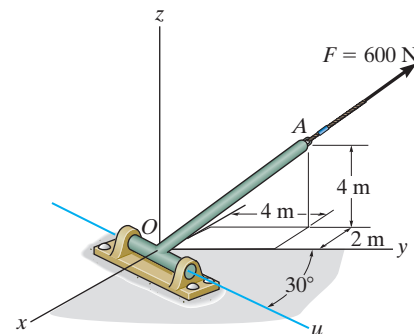
$$\mathbf{u}_u = \sin 30^\circ \mathbf{i} + \cos 30^\circ \mathbf{j}$$

Thus, the force vectors \mathbf{F} is given by

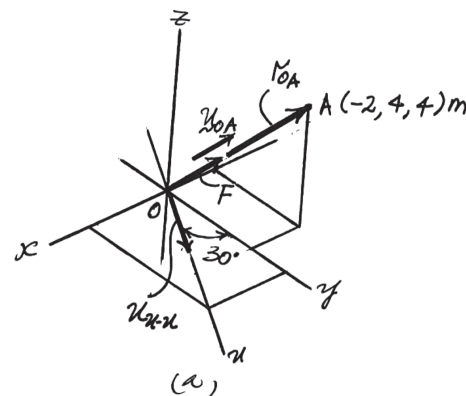
$$\mathbf{F} = F \mathbf{u}_{OA} = 600 \left(-\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k} \right) = \{-200\mathbf{i} + 400\mathbf{j} + 400\mathbf{k}\} \text{ N}$$

Vector Dot Product: The magnitude of the projected component of \mathbf{F} along the u axis is

$$\begin{aligned} F_u &= \mathbf{F} \cdot \mathbf{u}_u = (-200\mathbf{i} + 400\mathbf{j} + 400\mathbf{k}) \cdot (\sin 30^\circ \mathbf{i} + \cos 30^\circ \mathbf{j}) \\ &= (-200)(\sin 30^\circ) + 400(\cos 30^\circ) + 400(0) \\ &= 246 \text{ N} \end{aligned}$$



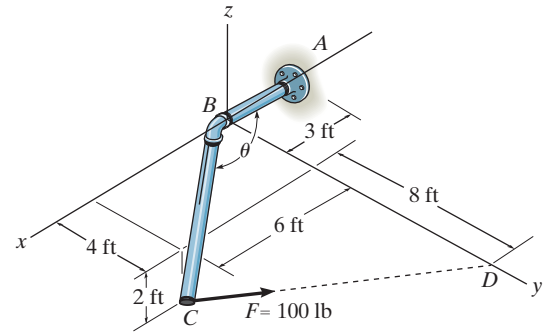
Ans.



Ans:
 $F_u = 246 \text{ N}$

2-79.

Determine the magnitude of the projected component of the 100-lb force acting along the axis BC of the pipe.



SOLUTION

$$\vec{\gamma}_{BC} = \{6\hat{i} + 4\hat{j} - 2\hat{k}\} \text{ ft}$$

$$\vec{F} = 100 \frac{\{-6\hat{i} + 8\hat{j} + 2\hat{k}\}}{\sqrt{(-6)^2 + 8^2 + 2^2}}$$

$$= \{-58.83\hat{i} + 78.45\hat{j} + 19.61\hat{k}\} \text{ lb}$$

$$F_p = \vec{F} \cdot \vec{\mu}_{BC} = \vec{F} \cdot \frac{\vec{\gamma}_{BC}}{|\vec{\gamma}_{BC}|} = \frac{-78.45}{7.483} = -10.48$$

$$F_p = 10.5 \text{ lb}$$

Ans.

Ans:
 $F_p = 10.5 \text{ lb}$

***2-80.**

Determine the angle θ between pipe segments BA and BC .

SOLUTION

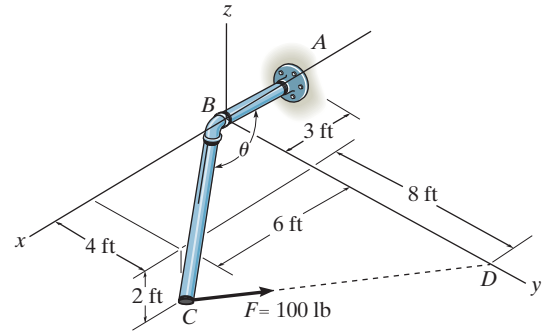
$$\vec{\gamma}_{BC} = \{6\hat{i} + 4\hat{j} - 2\hat{k}\} \text{ ft}$$

$$\vec{\gamma}_{BA} = \{-3\hat{i}\} \text{ ft}$$

$$\theta = \cos^{-1} \left(\frac{\vec{\gamma}_{BC} \cdot \vec{\gamma}_{BA}}{|\vec{\gamma}_{BC}| |\vec{\gamma}_{BA}|} \right) = \cos^{-1} \left(\frac{-18}{22.45} \right)$$

$$\theta = 143^\circ$$

Ans.



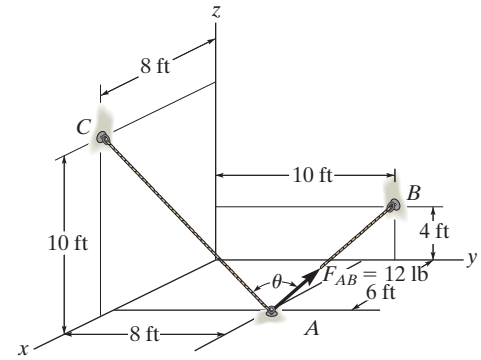
Ans:
 $\theta = 143^\circ$

2-81.

Determine the angle θ between the two cables.

SOLUTION

$$\begin{aligned}\theta &= \cos^{-1} \left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{r_{AC} r_{AB}} \right) \\ &= \cos^{-1} \left[\frac{(2\mathbf{i} - 8\mathbf{j} + 10\mathbf{k}) \cdot (-6\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})}{\sqrt{2^2 + (-8)^2 + 10^2} \sqrt{(-6)^2 + 2^2 + 4^2}} \right] \\ &= \cos^{-1} \left(\frac{12}{96.99} \right) \\ \theta &= 82.9^\circ\end{aligned}$$



Ans.

Ans:
 $\theta = 82.9^\circ$

2-82.

Determine the projected component of the force acting in the direction of cable AC . Express the result as a Cartesian vector.

SOLUTION

$$\mathbf{r}_{AC} = \{2\mathbf{i} - 8\mathbf{j} + 10\mathbf{k}\} \text{ ft}$$

$$\mathbf{r}_{AB} = \{-6\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}\} \text{ ft}$$

$$\mathbf{F}_{AB} = 12 \left(\frac{\mathbf{r}_{AB}}{r_{AB}} \right) = 12 \left(-\frac{6}{7.483}\mathbf{i} + \frac{2}{7.483}\mathbf{j} + \frac{4}{7.483}\mathbf{k} \right)$$

$$\mathbf{F}_{AB} = \{-9.621\mathbf{i} + 3.207\mathbf{j} + 6.414\mathbf{k}\} \text{ lb}$$

$$\mathbf{u}_{AC} = \frac{2}{12.961}\mathbf{i} - \frac{8}{12.961}\mathbf{j} + \frac{10}{12.961}\mathbf{k}$$

$$\text{Proj } \mathbf{F}_{AB} = \mathbf{F}_{AB} \cdot \mathbf{u}_{AC} = -9.621 \left(\frac{2}{12.961} \right) + 3.207 \left(-\frac{8}{12.961} \right) + 6.414 \left(\frac{10}{12.961} \right)$$

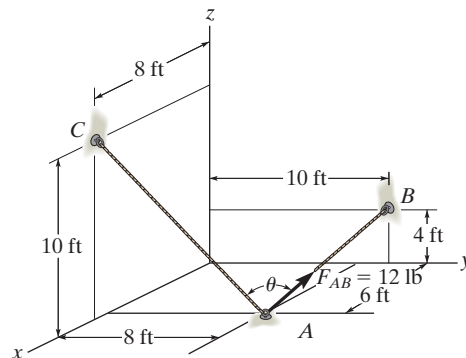
$$= 1.4846$$

$$\text{Proj } \mathbf{F}_{AB} = F_{AB} \mathbf{u}_{AC}$$

$$\text{Proj } \mathbf{F}_{AB} = (1.4846) \left[\frac{2}{12.962}\mathbf{i} - \frac{8}{12.962}\mathbf{j} + \frac{10}{12.962}\mathbf{k} \right]$$

$$\text{Proj } \mathbf{F}_{AB} = \{0.229\mathbf{i} - 0.916\mathbf{j} + 1.15\mathbf{k}\} \text{ lb}$$

Ans.



Ans:

$$\text{Proj } \mathbf{F}_{AB} = \{0.229\mathbf{i} - 0.916\mathbf{j} + 1.15\mathbf{k}\} \text{ lb}$$

2-83.

Determine the angles θ and ϕ between the flag pole and the cables AB and AC .

SOLUTION

$$\mathbf{r}_{AC} = \{-2\mathbf{i} - 4\mathbf{j} + 1\mathbf{k}\} \text{ m}; \quad r_{AC} = 4.58 \text{ m}$$

$$\mathbf{r}_{AB} = \{1.5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}\} \text{ m}; \quad r_{AB} = 5.22 \text{ m}$$

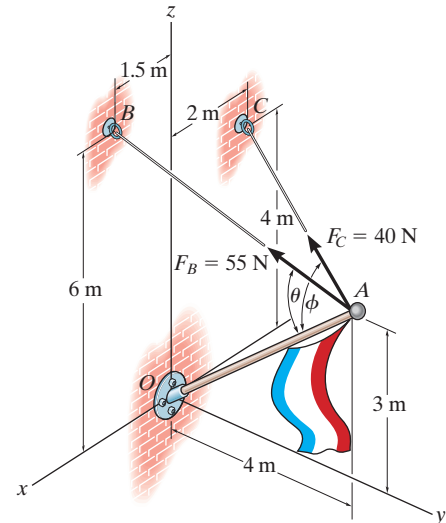
$$\mathbf{r}_{AO} = \{-4\mathbf{j} - 3\mathbf{k}\} \text{ m}; \quad r_{AO} = 5.00 \text{ m}$$

$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AO} = (1.5)(0) + (-4)(-4) + (3)(-3) = 7$$

$$\begin{aligned} \theta &= \cos^{-1} \left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AO}}{r_{AB} r_{AO}} \right) \\ &= \cos^{-1} \left(\frac{7}{5.22(5.00)} \right) = 74.4^\circ \end{aligned}$$

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AO} = (-2)(0) + (-4)(-4) + (1)(-3) = 13$$

$$\begin{aligned} \phi &= \cos^{-1} \left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AO}}{r_{AC} r_{AO}} \right) \\ &= \cos^{-1} \left(\frac{13}{4.58(5.00)} \right) = 55.4^\circ \end{aligned}$$



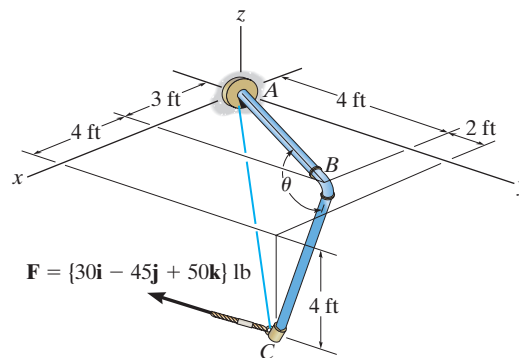
Ans.

Ans.

Ans:
 $\theta = 74.4^\circ$
 $\phi = 55.4^\circ$

***2-84.**

Determine the magnitudes of the components of \mathbf{F} acting along and perpendicular to segment BC of the pipe assembly.



SOLUTION

Unit Vector: The unit vector \mathbf{u}_{CB} must be determined first. From Fig. *a*,

$$\mathbf{u}_{CB} = \frac{\mathbf{r}_{CB}}{r_{CB}} = \frac{(3-7)\mathbf{i} + (4-6)\mathbf{j} + [0-(-4)]\mathbf{k}}{\sqrt{(3-7)^2 + (4-6)^2 + [0-(-4)]^2}} = -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of \mathbf{F} parallel to segment BC of the pipe assembly is

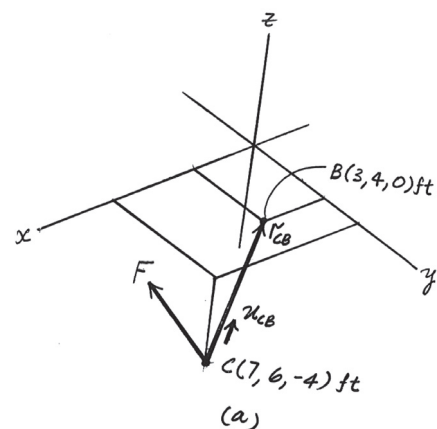
$$\begin{aligned} (F_{BC})_{pa} &= \mathbf{F} \cdot \mathbf{u}_{CB} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k}) \cdot \left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) \\ &= (30)\left(-\frac{2}{3}\right) + (-45)\left(-\frac{1}{3}\right) + 50\left(\frac{2}{3}\right) \\ &= 28.33 \text{ lb} = 28.3 \text{ lb} \end{aligned}$$

Ans.

The magnitude of \mathbf{F} is $F = \sqrt{30^2 + (-45)^2 + 50^2} = \sqrt{5425}$ lb. Thus, the magnitude of the component of \mathbf{F} perpendicular to segment BC of the pipe assembly can be determined from

$$(F_{BC})_{per} = \sqrt{F^2 - (F_{BC})_{pa}^2} = \sqrt{5425 - 28.33^2} = 68.0 \text{ lb}$$

Ans.

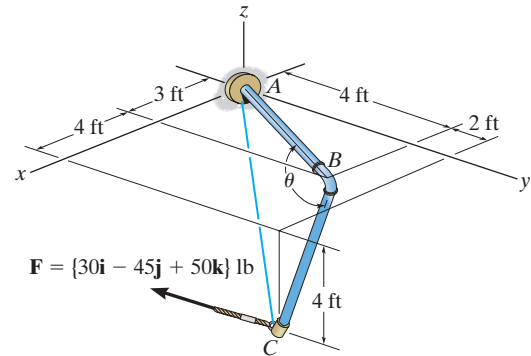


Ans:

$$\begin{aligned} (F_{BC})_{||} &= 28.3 \text{ lb} \\ (F_{BC})_{\perp} &= 68.0 \text{ lb} \end{aligned}$$

2-85.

Determine the magnitude of the projected component of \mathbf{F} along line AC . Express this component as a Cartesian vector.



SOLUTION

Unit Vector: The unit vector \mathbf{u}_{AC} must be determined first. From Fig. *a*,

$$\mathbf{u}_{AC} = \frac{(7-0)\mathbf{i} + (6-0)\mathbf{j} + (-4-0)\mathbf{k}}{\sqrt{(7-0)^2 + (6-0)^2 + (-4-0)^2}} = 0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of \mathbf{F} along line AC is

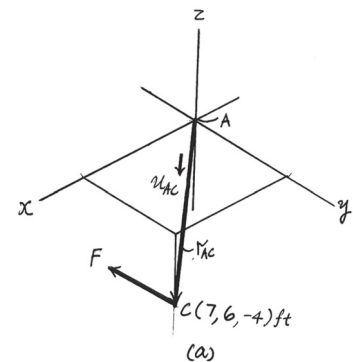
$$\begin{aligned} F_{AC} &= \mathbf{F} \cdot \mathbf{u}_{AC} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k}) \cdot (0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k}) \\ &= (30)(0.6965) + (-45)(0.5970) + 50(-0.3980) \\ &= 25.87 \text{ lb} \end{aligned}$$

Thus, \mathbf{F}_{AC} expressed in Cartesian vector form is

$$\begin{aligned} \mathbf{F}_{AC} &= F_{AC} \mathbf{u}_{AC} = 25.87(0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k}) \\ &= \{-18.0\mathbf{i} - 15.4\mathbf{j} + 10.3\mathbf{k}\} \text{ lb} \end{aligned}$$

Ans.

Ans.



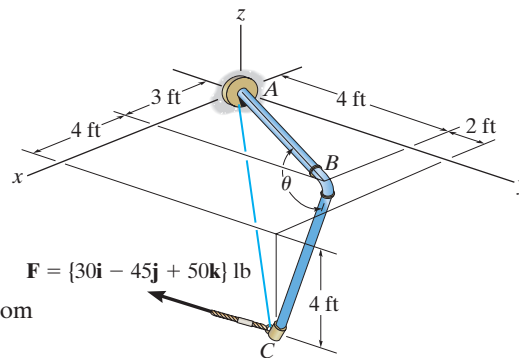
Ans:

$$F_{AC} = 25.87 \text{ lb}$$

$$\mathbf{F}_{AC} = \{-18.0\mathbf{i} - 15.4\mathbf{j} + 10.3\mathbf{k}\} \text{ lb}$$

2-86.

Determine the angle θ between the pipe segments BA and BC .



SOLUTION

Position Vectors: The position vectors \mathbf{r}_{BA} and \mathbf{r}_{BC} must be determined first. From Fig. a,

$$\mathbf{r}_{BA} = (0 - 3)\mathbf{i} + (0 - 4)\mathbf{j} + (0 - 0)\mathbf{k} = \{-3\mathbf{i} - 4\mathbf{j}\} \text{ ft}$$

$$\mathbf{r}_{BC} = (7 - 3)\mathbf{i} + (6 - 4)\mathbf{j} + (-4 - 0)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\} \text{ ft}$$

The magnitude of \mathbf{r}_{BA} and \mathbf{r}_{BC} are

$$r_{BA} = \sqrt{(-3)^2 + (-4)^2} = 5 \text{ ft}$$

$$r_{BC} = \sqrt{4^2 + 2^2 + (-4)^2} = 6 \text{ ft}$$

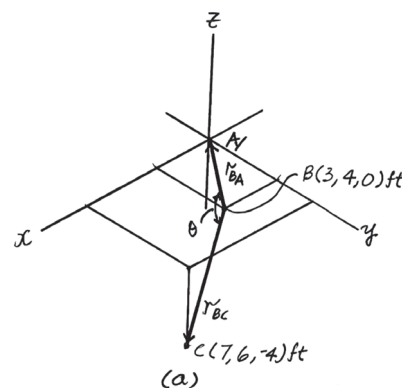
Vector Dot Product:

$$\begin{aligned} \mathbf{r}_{BA} \cdot \mathbf{r}_{BC} &= (-3\mathbf{i} - 4\mathbf{j}) \cdot (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}) \\ &= (-3)(4) + (-4)(2) + 0(-4) \\ &= -20 \text{ ft}^2 \end{aligned}$$

Thus,

$$\theta = \cos^{-1} \left(\frac{\mathbf{r}_{BA} \cdot \mathbf{r}_{BC}}{r_{BA} r_{BC}} \right) = \cos^{-1} \left[\frac{-20}{5(6)} \right] = 132^\circ$$

Ans.



Ans:
 $\theta = 132^\circ$

2-87.

If the force $F = 100$ N lies in the plane $DBEC$, which is parallel to the x - z plane, and makes an angle of 10° with the extended line DB as shown, determine the angle that \mathbf{F} makes with the diagonal AB of the crate.

SOLUTION

Use the x, y, z axes.

$$\mathbf{u}_{AB} = \left(\frac{-0.5\mathbf{i} + 0.2\mathbf{j} + 0.2\mathbf{k}}{0.57446} \right)$$

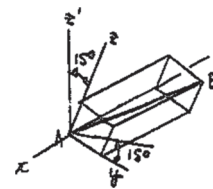
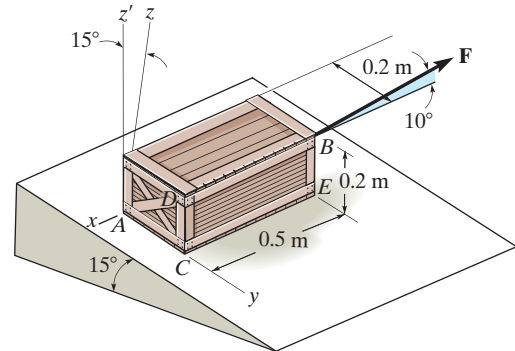
$$= -0.8704\mathbf{i} + 0.3482\mathbf{j} + 0.3482\mathbf{k}$$

$$\mathbf{F} = -100 \cos 10^\circ \mathbf{i} + 100 \sin 10^\circ \mathbf{k}$$

$$\theta = \cos^{-1} \left(\frac{\mathbf{F} \cdot \mathbf{u}_{AB}}{F u_{AB}} \right)$$

$$= \cos^{-1} \left(\frac{-100 (\cos 10^\circ)(-0.8704) + 0 + 100 \sin 10^\circ (0.3482)}{100(1)} \right)$$

$$= \cos^{-1} (0.9176) = 23.4^\circ$$

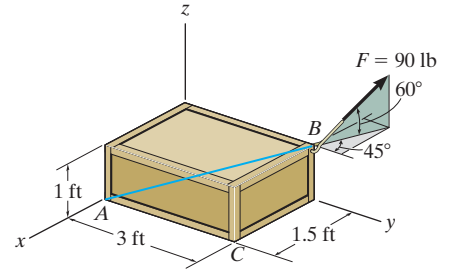


Ans.

Ans:
 $\theta = 23.4^\circ$

***2-88.**

Determine the magnitudes of the components of the force acting parallel and perpendicular to diagonal AB of the crate.



SOLUTION

Force and Unit Vector: The force vector \mathbf{F} and unit vector \mathbf{u}_{AB} must be determined first. From Fig. a ,

$$\mathbf{F} = 90(-\cos 60^\circ \sin 45^\circ \mathbf{i} + \cos 60^\circ \cos 45^\circ \mathbf{j} + \sin 60^\circ \mathbf{k})$$

$$= \{-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}\} \text{ lb}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(0 - 1.5)\mathbf{i} + (3 - 0)\mathbf{j} + (1 - 0)\mathbf{k}}{\sqrt{(0 - 1.5)^2 + (3 - 0)^2 + (1 - 0)^2}} = -\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of \mathbf{F} parallel to the diagonal AB is

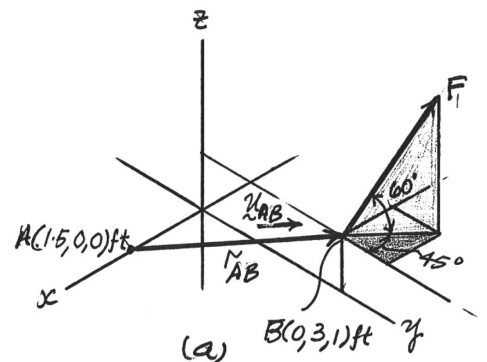
$$\begin{aligned} [(F)_{AB}]_{\text{pa}} &= \mathbf{F} \cdot \mathbf{u}_{AB} = (-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}) \cdot \left(-\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right) \\ &= (-31.82)\left(-\frac{3}{7}\right) + 31.82\left(\frac{6}{7}\right) + 77.94\left(\frac{2}{7}\right) \\ &= 63.18 \text{ lb} = 63.2 \text{ lb} \end{aligned}$$

Ans.

The magnitude of the component \mathbf{F} perpendicular to the diagonal AB is

$$[(F)_{AB}]_{\text{per}} = \sqrt{F^2 - [(F)_{AB}]_{\text{pa}}^2} = \sqrt{90^2 - 63.18^2} = 64.1 \text{ lb}$$

Ans.



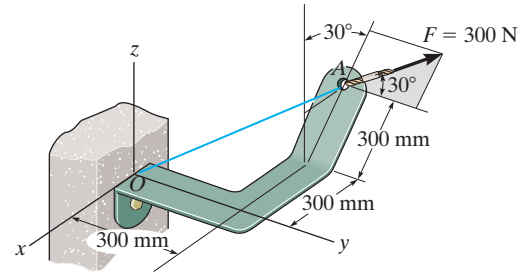
Ans:

$$[(F)_{AB}]_{\parallel} = 63.2 \text{ lb}$$

$$[(F)_{AB}]_{\perp} = 64.1 \text{ lb}$$

2-89.

Determine the magnitudes of the projected components of the force acting along the x and y axes.



SOLUTION

Force Vector: The force vector \mathbf{F} must be determined first. From Fig. a ,

$$\begin{aligned}\mathbf{F} &= -300 \sin 30^\circ \sin 30^\circ \mathbf{i} + 300 \cos 30^\circ \mathbf{j} + 300 \sin 30^\circ \cos 30^\circ \mathbf{k} \\ &= [-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}] \text{ N}\end{aligned}$$

Vector Dot Product: The magnitudes of the projected component of \mathbf{F} along the x and y axes are

$$\begin{aligned}F_x &= \mathbf{F} \cdot \mathbf{i} = (-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}) \cdot \mathbf{i} \\ &= -75(1) + 259.81(0) + 129.90(0) \\ &= -75 \text{ N}\end{aligned}$$

$$\begin{aligned}F_y &= \mathbf{F} \cdot \mathbf{j} = (-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}) \cdot \mathbf{j} \\ &= -75(0) + 259.81(1) + 129.90(0) \\ &= 260 \text{ N}\end{aligned}$$

The negative sign indicates that \mathbf{F}_x is directed towards the negative x axis. Thus

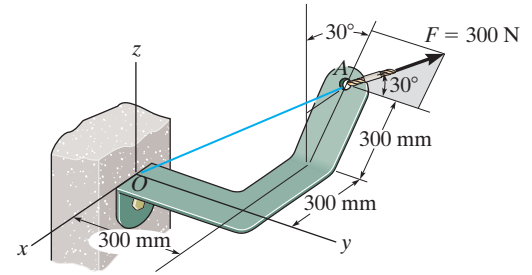
$$F_x = 75 \text{ N}, \quad F_y = 260 \text{ N} \quad \textbf{Ans.}$$

Ans:

$$\begin{aligned}F_x &= 75 \text{ N} \\ F_y &= 260 \text{ N}\end{aligned}$$

2-90.

Determine the magnitude of the projected component of the force acting along line OA .



SOLUTION

Force and Unit Vector: The force vector \mathbf{F} and unit vector \mathbf{u}_{OA} must be determined first. From Fig. a,

$$\mathbf{F} = (-300 \sin 30^\circ \sin 30^\circ \mathbf{i} + 300 \cos 30^\circ \mathbf{j} + 300 \sin 30^\circ \cos 30^\circ \mathbf{k})$$

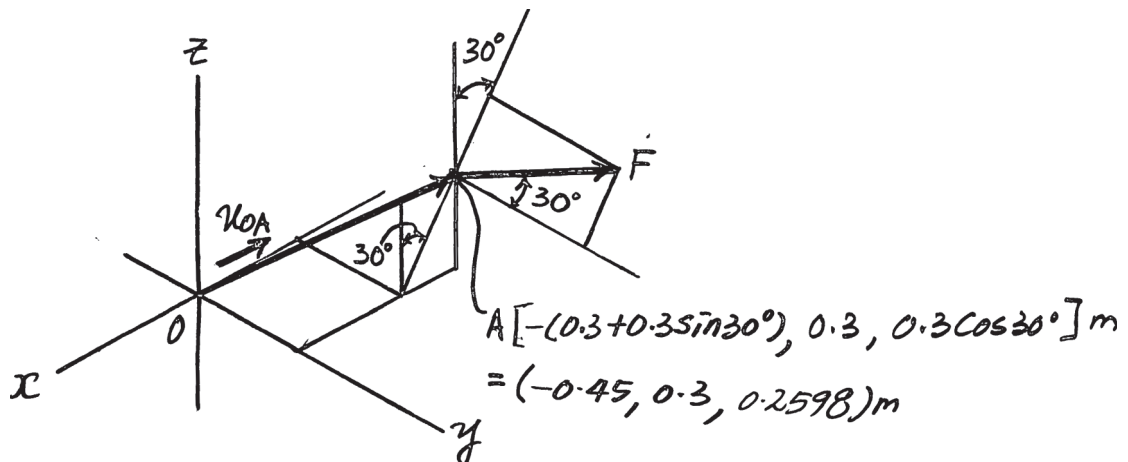
$$= \{-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\} \text{ N}$$

$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-0.45 - 0)\mathbf{i} + (0.3 - 0)\mathbf{j} + (0.2598 - 0)\mathbf{k}}{\sqrt{(-0.45 - 0)^2 + (0.3 - 0)^2 + (0.2598 - 0)^2}} = -0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of \mathbf{F} along line OA is

$$\begin{aligned} F_{OA} &= \mathbf{F} \cdot \mathbf{u}_{OA} = (-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}) \cdot (-0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}) \\ &= (-75)(-0.75) + 259.81(0.5) + 129.90(0.4330) \\ &= 242 \text{ N} \end{aligned}$$

Ans.



Ans:
 $F_{OA} = 242 \text{ N}$

2-91.

Two cables exert forces on the pipe. Determine the magnitude of the projected component of \mathbf{F}_1 along the line of action of \mathbf{F}_2 .

SOLUTION

Force Vector:

$$\begin{aligned}\mathbf{u}_{F_1} &= \cos 30^\circ \sin 30^\circ \mathbf{i} + \cos 30^\circ \cos 30^\circ \mathbf{j} - \sin 30^\circ \mathbf{k} \\ &= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{F}_1 &= F_R \mathbf{u}_{F_1} = 30(0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \text{ lb} \\ &= \{12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}\} \text{ lb}\end{aligned}$$

Unit Vector: One can obtain the angle $\alpha = 135^\circ$ for \mathbf{F}_2 using Eq. 2-8. $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$, with $\beta = 60^\circ$ and $\gamma = 60^\circ$. The unit vector along the line of action of \mathbf{F}_2 is

$$\mathbf{u}_{F_2} = \cos 135^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 60^\circ \mathbf{k} = -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

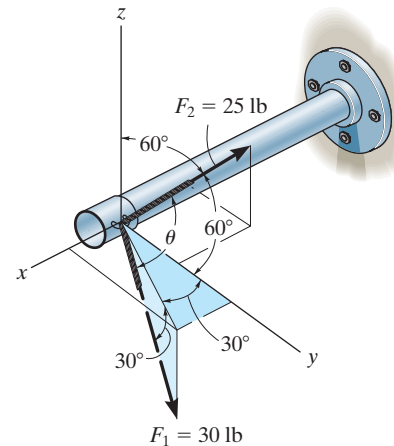
Projected Component of \mathbf{F}_1 Along the Line of Action of \mathbf{F}_2 :

$$\begin{aligned}(F_1)_{F_2} &= \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}) \\ &= (12.990)(-0.7071) + (22.5)(0.5) + (-15.0)(0.5) \\ &= -5.44 \text{ lb}\end{aligned}$$

Negative sign indicates that the projected component of $(F_1)_{F_2}$ acts in the opposite sense of direction to that of \mathbf{u}_{F_2} .

The magnitude is $(F_1)_{F_2} = 5.44 \text{ lb}$

Ans.



Ans:

The magnitude is $(F_1)_{F_2} = 5.44 \text{ lb}$

***2-92.**

Determine the angle θ between the two forces.

SOLUTION

Unit Vectors:

$$\begin{aligned}\mathbf{u}_{F_1} &= \cos 30^\circ \sin 30^\circ \mathbf{i} + \cos 30^\circ \cos 30^\circ \mathbf{j} - \sin 30^\circ \mathbf{k} \\ &= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{u}_{F_2} &= \cos 135^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 60^\circ \mathbf{k} \\ &= -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}\end{aligned}$$

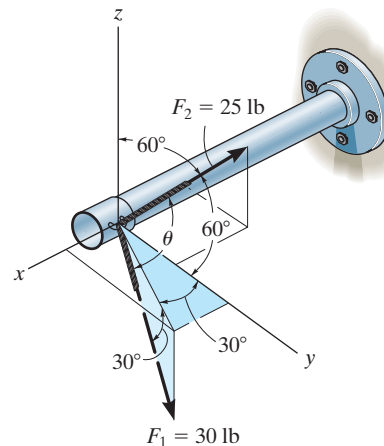
The Angles Between Two Vectors θ :

$$\begin{aligned}\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} &= (0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}) \\ &= 0.4330(-0.7071) + 0.75(0.5) + (-0.5)(0.5) \\ &= -0.1812\end{aligned}$$

Then,

$$\theta = \cos^{-1}(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}) = \cos^{-1}(-0.1812) = 100^\circ$$

Ans.



Ans:
 $\theta = 100^\circ$

***R2-4.**

The cable exerts a force of 250 lb on the crane boom as shown. Express this force as a Cartesian vector.

SOLUTION

Cartesian Vector Notation: With $\alpha = 30^\circ$ and $\beta = 70^\circ$, the third coordinate direction angle γ can be determined using Eq. 2-8.

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\cos^2 30^\circ + \cos^2 70^\circ + \cos^2 \gamma = 1$$

$$\cos \gamma = \pm 0.3647$$

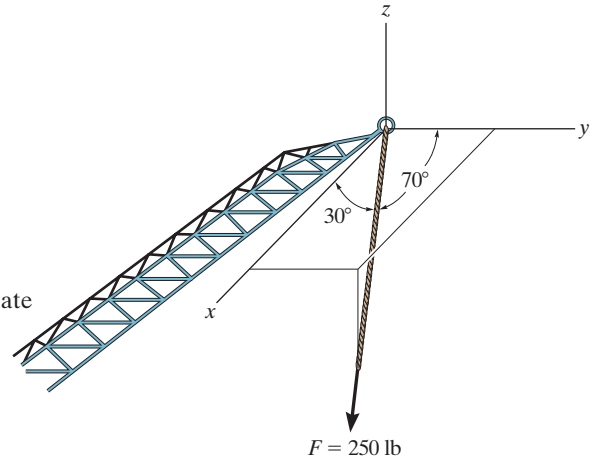
$$\gamma = 68.61^\circ \text{ or } 111.39^\circ$$

By inspection, $\gamma = 111.39^\circ$ since the force \mathbf{F} is directed in negative octant.

$$\mathbf{F} = 250\{\cos 30^\circ \mathbf{i} + \cos 70^\circ \mathbf{j} + \cos 111.39^\circ\} \text{ lb}$$

$$= \{217\mathbf{i} + 85.5\mathbf{j} - 91.2\mathbf{k}\} \text{ lb}$$

Ans.



Ans:
 $\mathbf{F} = \{217\mathbf{i} + 85.5\mathbf{j} - 91.2\mathbf{k}\} \text{ lb}$

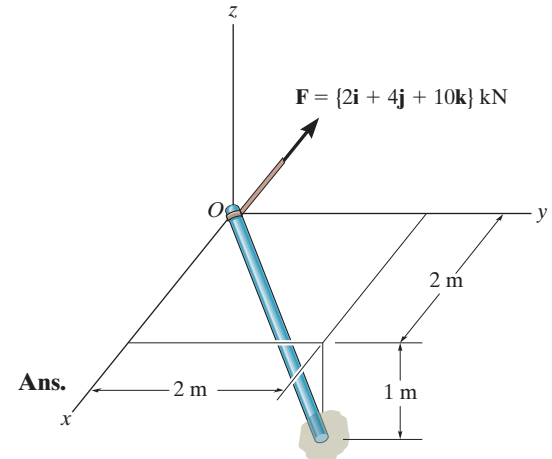
***R2-8.**

Determine the projection of the force \mathbf{F} along the pole.

SOLUTION

$$\text{Proj } F = \mathbf{F} \cdot \mathbf{u}_a = (2\mathbf{i} + 4\mathbf{j} + 10\mathbf{k}) \cdot \left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k} \right)$$

$$\text{Proj } F = 0.667 \text{ kN}$$



Ans:
 $F = 0.667 \text{ kN}$