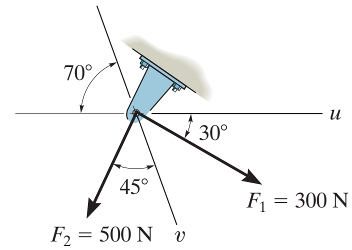
2–1. Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive u axis.



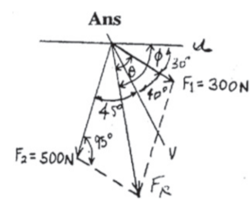
SOLUTION

$$F_R = \sqrt{(300)^2 + (500)^2 - 2(300)(500)\cos 95^\circ} = 605.1 = 605 \text{ N}$$ **Ans.**

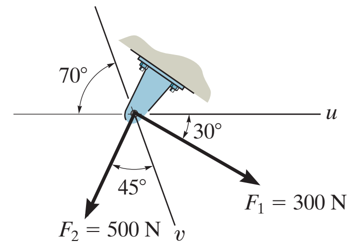
$$\frac{605.1}{\sin 95^\circ} = \frac{500}{\sin \theta}$$

$$\theta = 55.40^\circ$$

$$\phi = 55.40^\circ + 30^\circ = 85.4^\circ$$ **Ans.**



2–2. Resolve the force $\mathbf{F}_1$ into components acting along the u and v axes and determine the magnitudes of the components.



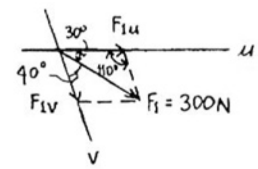
SOLUTION

$$\frac{F_{1u}}{\sin 40^\circ} = \frac{300}{\sin 110^\circ}$$

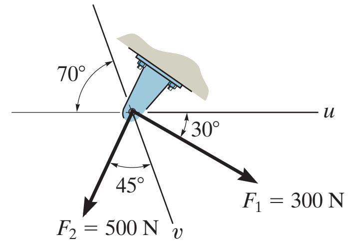
$$F_{1u} = 205 \text{ N}$$ **Ans.**

$$\frac{F_{1v}}{\sin 30^\circ} = \frac{300}{\sin 110^\circ}$$

$$F_{1v} = 160 \text{ N}$$ **Ans.**



2–3. Resolve the force $\mathbf{F}_2$ into components acting along the u and v axes and determine the magnitudes of the components.



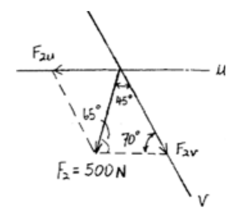
SOLUTION

$$\frac{F_{2u}}{\sin 45^\circ} = \frac{500}{\sin 70^\circ}$$

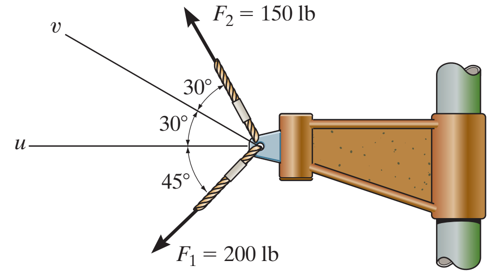
$$F_{2u} = 376 \text{ N} \qquad \textbf{Ans.}$$

$$\frac{F_{2v}}{\sin 65^\circ} = \frac{500}{\sin 70^\circ}$$

$$F_{2v} = 482 \text{ N} \qquad \textbf{Ans.}$$



***2–4.** Determine the magnitude of the resultant force acting on the bracket and its direction measured counterclockwise from the positive u axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

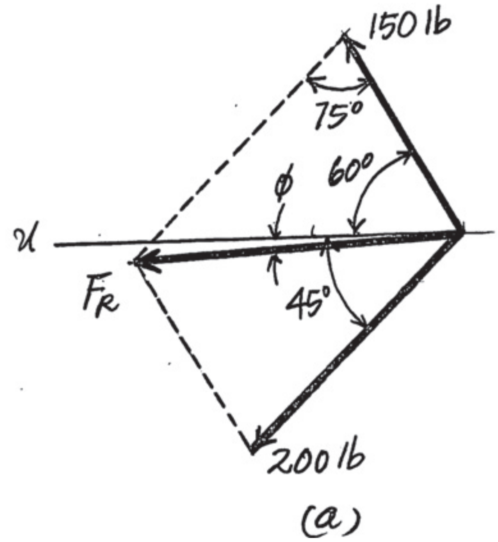
$$F_R = \sqrt{200^2 + 150^2 - 2(200)(150)\cos 75°}$$
$$= 216.72 \text{ lb} = 217 \text{ lb} \qquad \textbf{Ans.}$$

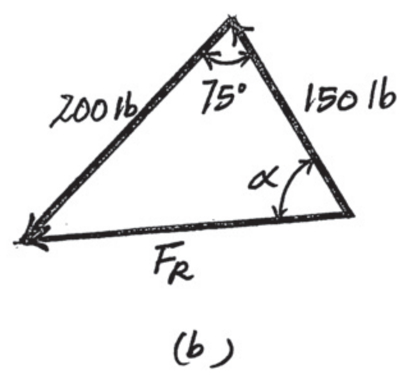
Applying the law of sines to Fig. *b* and using this result yields

$$\frac{\sin\alpha}{200} = \frac{\sin 75°}{216.72} \qquad \alpha = 63.05°$$

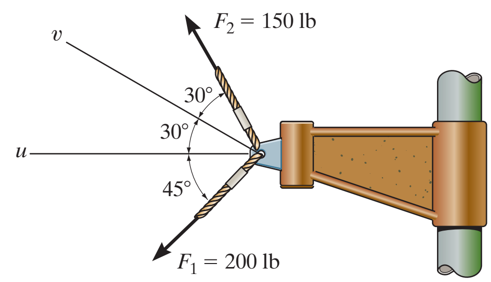
Thus, the direction angle ϕ of $\mathbf{F}_R$, measured counterclockwise from the positive u axis, is

$$\phi = \alpha - 60° = 63.05° - 60° = 3.05° \qquad \textbf{Ans.}$$





2–5. Resolve $\mathbf{F}_1$ into components along the u and v axes, and determine the magnitudes of these components.



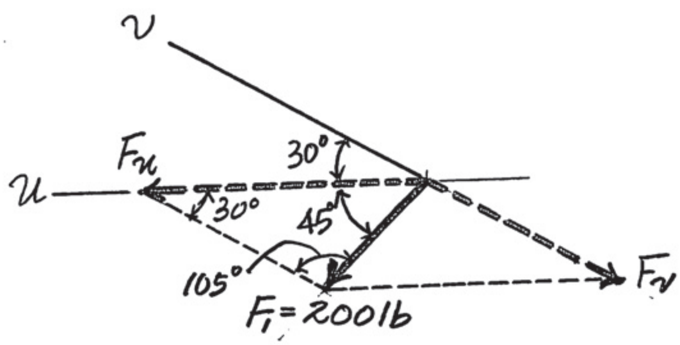
SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. a and b, respectively.

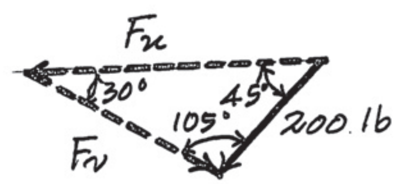
Applying the law of sines to Fig. b, yields

$$\frac{F_u}{\sin 105°} = \frac{200}{\sin 30°} \qquad F_u = 386 \text{ lb}$$ **Ans.**

$$\frac{F_v}{\sin 45°} = \frac{200}{\sin 30°} \qquad F_v = 283 \text{ lb}$$ **Ans.**

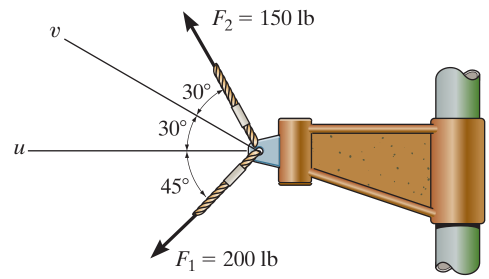


(a)



(b)

2–6. Resolve $\mathbf{F}_2$ into components along the u and v axes, and determine the magnitudes of these components.



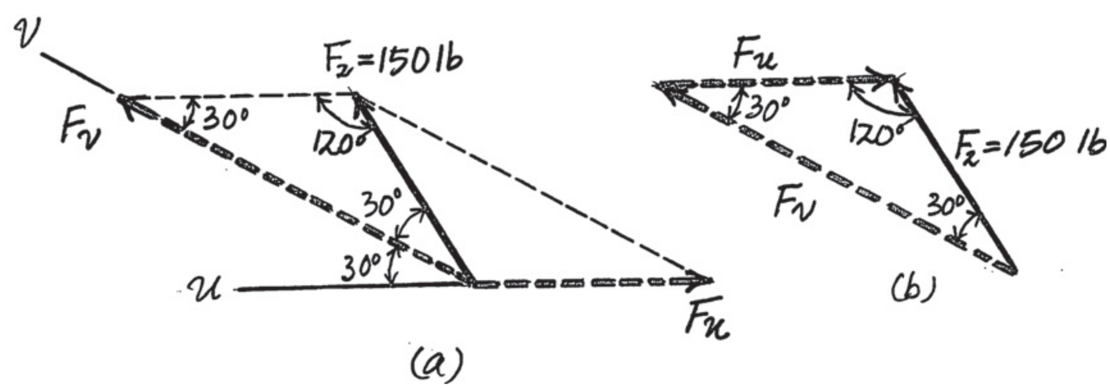
SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

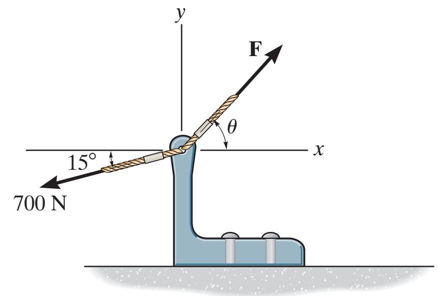
Applying the law of sines to Fig. *b*,

$$\frac{F_u}{\sin 30°} = \frac{150}{\sin 30°} \qquad F_u = 150 \text{ lb} \qquad \textbf{Ans.}$$

$$\frac{F_v}{\sin 120°} = \frac{150}{\sin 30°} \qquad F_v = 260 \text{ lb} \qquad \textbf{Ans.}$$



2–7. If $\theta = 60^\circ$ and $F = 450$ N, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of consines to Fig. *b*,

$$F_R = \sqrt{700^2 + 450^2 - 2(700)(450)\cos 45^\circ}$$

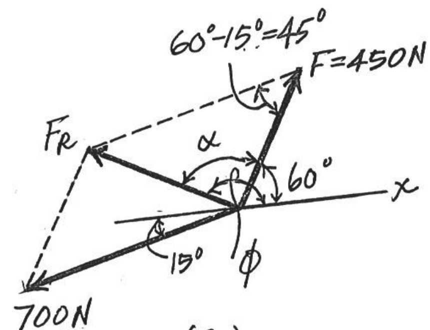
$$= 497.01 \text{ N} = 497 \text{ N} \qquad \textbf{Ans.}$$

This yields

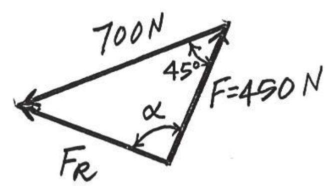
$$\frac{\sin \alpha}{700} = \frac{\sin 45^\circ}{497.01} \qquad \alpha = 95.19^\circ$$

Thus, the direction of angle ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis, is

$$\phi = \alpha + 60^\circ = 95.19^\circ + 60^\circ = 155^\circ \qquad \textbf{Ans.}$$

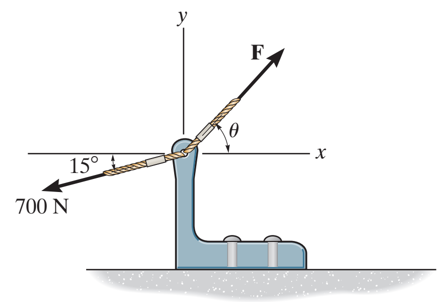


(a)



(b)

***2–8.** If the magnitude of the resultant force is to be 500 N, directed along the positive y axis, determine the magnitude of force **F** and its direction θ.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

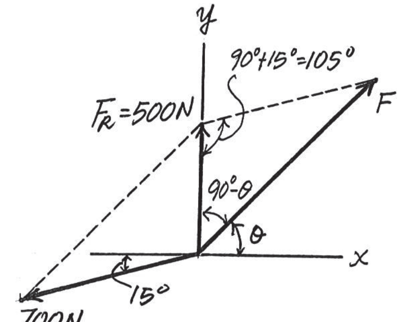
$$F = \sqrt{500^2 + 700^2 - 2(500)(700)\cos 105^\circ}$$

$$= 959.78\text{ N} = 960\text{ N} \qquad \textbf{Ans.}$$

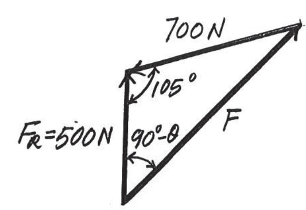
Applying the law of sines to Fig. *b*, and using this result, yields

$$\frac{\sin(90^\circ + \theta)}{700} = \frac{\sin 105^\circ}{959.78}$$

$$\theta = 45.2^\circ \qquad \textbf{Ans.}$$

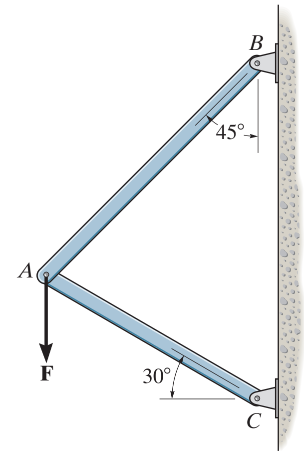


(a)



(b)

2–9. The vertical force **F** acts downward at A on the two-membered frame. Determine the magnitudes of the two components of **F** directed along the axes of AB and AC. Set $F = 500$ N.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

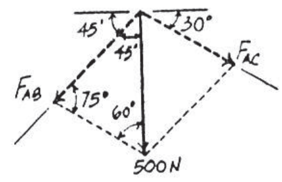
Trigonometry: Using the law of sines (Fig. *b*), we have

$$\frac{F_{AB}}{\sin 60°} = \frac{500}{\sin 75°}$$

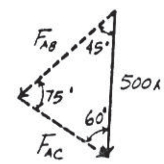
$$F_{AB} = 448 \text{ N}$$ **Ans.**

$$\frac{F_{AC}}{\sin 45°} = \frac{500}{\sin 75°}$$

$$F_{AC} = 366 \text{ N}$$ **Ans.**

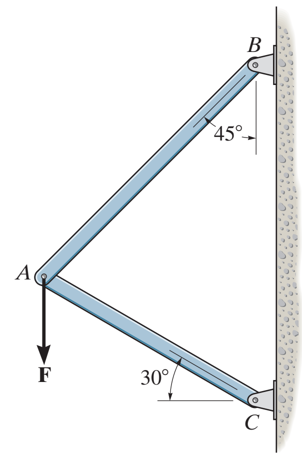


(a)



(b)

2–10. Solve Prob. 2–9 with $F = 350$ lb.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

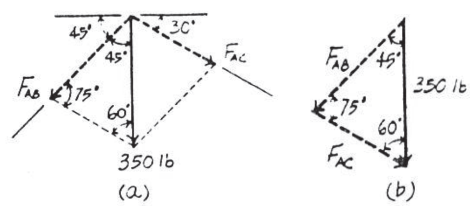
Trigonometry: Using the law of sines (Fig. *b*), we have

$$\frac{F_{AB}}{\sin 60^\circ} = \frac{350}{\sin 75^\circ}$$

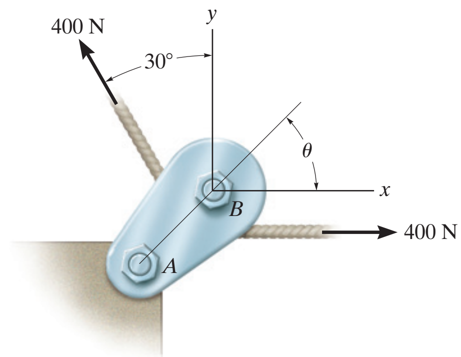
$$F_{AB} = 314 \text{ lb}$$ **Ans.**

$$\frac{F_{AC}}{\sin 45^\circ} = \frac{350}{\sin 75^\circ}$$

$$F_{AC} = 256 \text{ lb}$$ **Ans.**



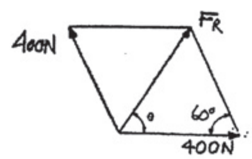
2–11. If the tension in the cable is 400 N, determine the magnitude and direction of the resultant force acting on the pulley. This angle is the same angle θ of line AB on the tailboard block.

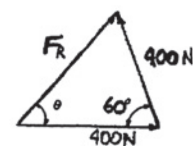


SOLUTION

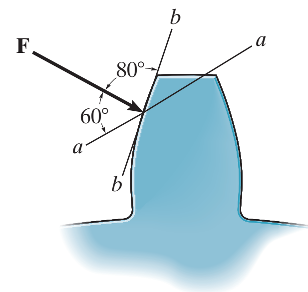
$F_R = \sqrt{(400)^2 + (400)^2 - 2(400)(400)\cos 60°} = 400\text{ N}$ **Ans**

$\dfrac{\sin\theta}{400} = \dfrac{\sin 60°}{400}$; $\theta = 60°$ **Ans**





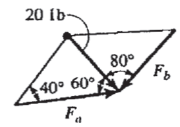
***2–12.** The force acting on the gear tooth is $F = 20$ lb. Resolve this force into two components acting along the lines *aa* and *bb*.



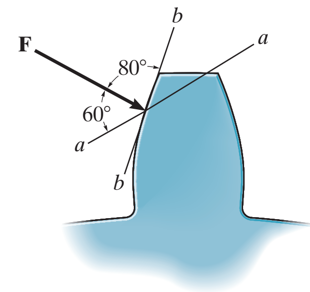
SOLUTION

$$\frac{20}{\sin 40^\circ} = \frac{F_a}{\sin 80^\circ}; \qquad F_a = 30.6 \text{ lb}$$ **Ans.**

$$\frac{20}{\sin 40^\circ} = \frac{F_b}{\sin 60^\circ}; \qquad F_b = 26.9 \text{ lb}$$ **Ans.**



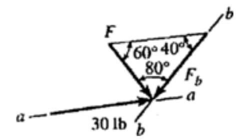
2–13. The component of force **F** acting along line *aa* is required to be 30 lb. Determine the magnitude of **F** and its component along line *bb*.



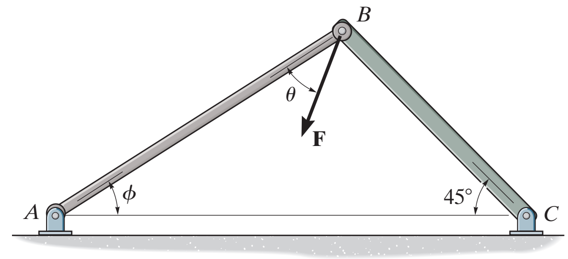
SOLUTION

$$\frac{30}{\sin 80^\circ} = \frac{F}{\sin 40^\circ}; \qquad F = 19.6 \text{ lb} \qquad \textbf{Ans.}$$

$$\frac{30}{\sin 80^\circ} = \frac{F_b}{\sin 60^\circ}; \qquad F_b = 26.4 \text{ lb} \qquad \textbf{Ans.}$$

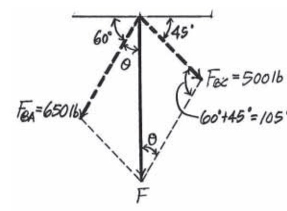


2–14. Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A, and the component acting along member BC is 500 lb, directed from B towards C. Determine the magnitude of **F** and its direction θ. Set $\phi = 60°$.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. a and b, respectively.



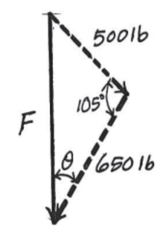
(a)

Applying the law of cosines to Fig. b,

$$F = \sqrt{500^2 + 650^2 - 2(500)(650)\cos 105°}$$

$$= 916.91 \text{ lb} = 917 \text{ lb}$$ **Ans.**

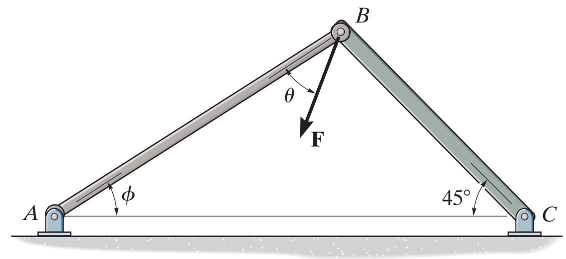
Using this result and applying the law of sines to Fig. b, yields



(b)

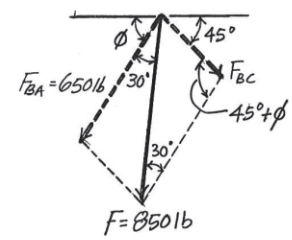
$$\frac{\sin\theta}{500} = \frac{\sin 105°}{916.91} \qquad \theta = 31.8°$$ **Ans.**

2–15. Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A. Determine the required angle ϕ ($0° \leq \phi \leq 90°$) and the component acting along member BC. Set $F = 850$ lb and $\theta = 30°$.

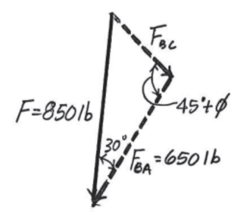


SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



(a)



(b)

Applying the law of cosines to Fig. *b*,

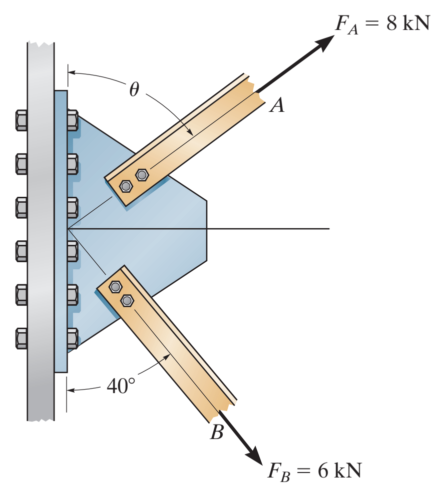
$$F_{BC} = \sqrt{850^2 + 650^2 - 2(850)(650)\cos 30°}$$

$$= 433.64 \text{ lb} = 434 \text{ lb} \qquad \textbf{Ans.}$$

Using this result and applying the sine law to Fig. *b*, yields

$$\frac{\sin(45° + \phi)}{850} = \frac{\sin 30°}{433.64} \qquad \phi = 56.5° \qquad \textbf{Ans.}$$

***2–16.** The plate is subjected to the two forces at A and B as shown. If $\theta = 60°$, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of cosines (Fig. *b*), we have

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100°}$$

$$= 10.80 \text{ kN} = 10.8 \text{ kN}$$

The angle θ can be determined using law of sines (Fig. *b*).

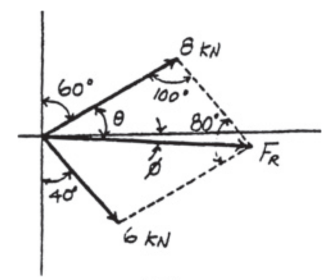
$$\frac{\sin\theta}{6} = \frac{\sin 100°}{10.80}$$

$$\sin\theta = 0.5470$$

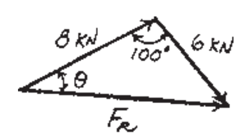
$$\theta = 33.16°$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured from the x axis is

$$\phi = 33.16° - 30° = 3.16°$$

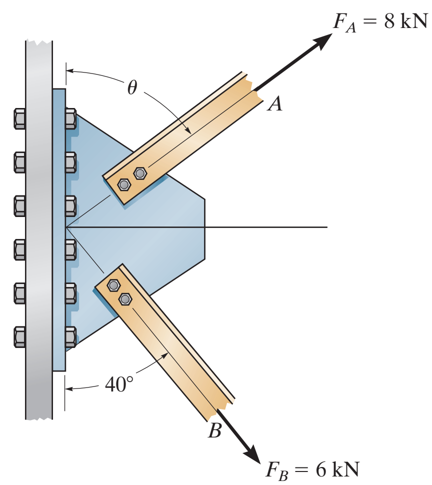


(a)



(b)

2–17. Determine the angle θ for connecting member A to the plate so that the resultant force of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed horizontally to the right. Also, what is the magnitude of the resultant force?



SOLUTION

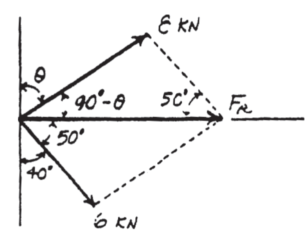
Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of sines (Fig .*b*), we have

$$\frac{\sin\,(90° - \theta)}{6} = \frac{\sin 50°}{8}$$

$$\sin\,(90° - \theta) = 0.5745$$

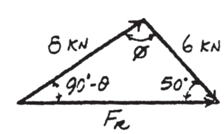
$$\theta = 54.93° = 54.9° \qquad \textbf{Ans.}$$



From the triangle, $\phi = 180° - (90° - 54.93°) - 50° = 94.93°$. Thus, using law of cosines, the magnitude of $\mathbf{F}_R$ is

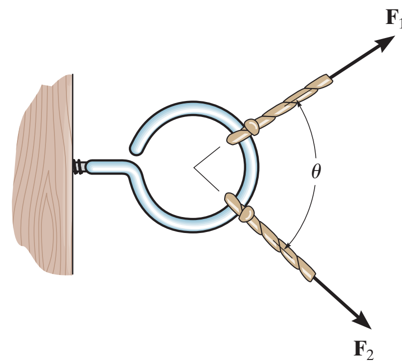
$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 94.93°}$$

$$= 10.4 \text{ kN} \qquad \textbf{Ans.}$$



(b)

2–18. Two forces act on the screw eye. If $F_1 = 400$ N and $F_2 = 600$ N, determine the angle θ ($0° \leq \theta \leq 180°$) between them, so that the resultant force has a magnitude of $F_R = 800$ N.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively. Applying law of cosines to Fig. *b*,

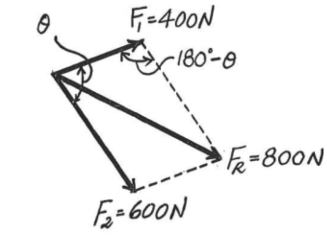
$$800 = \sqrt{400^2 + 600^2 - 2(400)(600)\cos(180° - \theta°)}$$

$$800^2 = 400^2 + 600^2 - 480000\cos(180° - \theta)$$

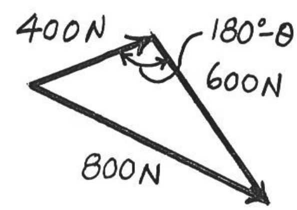
$$\cos(180° - \theta) = -0.25$$

$$180° - \theta = 104.48$$

$$\theta = 75.52° = 75.5° \qquad \textbf{Ans.}$$

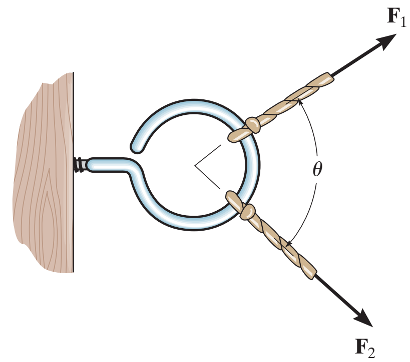


(a)



(b)

2–19. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the screw eye. If their lines of action are at an angle θ apart and the magnitude of each force is $F_1 = F_2 = F$, determine the magnitude of the resultant force $\mathbf{F}_R$ and the angle between $\mathbf{F}_R$ and $\mathbf{F}_1$.



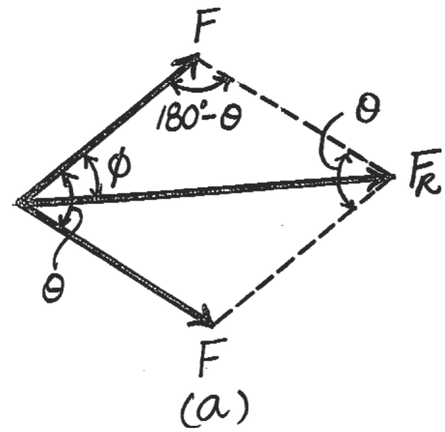
SOLUTION

$$\frac{F}{\sin \phi} = \frac{F}{\sin (\theta - \phi)}$$

$$\sin (\theta - \phi) = \sin \phi$$

$$\theta - \phi = \phi$$

$$\phi = \frac{\theta}{2} \qquad \textbf{Ans.}$$



$$F_R = \sqrt{(F)^2 + (F)^2 - 2(F)(F)\cos(180^\circ - \theta)}$$

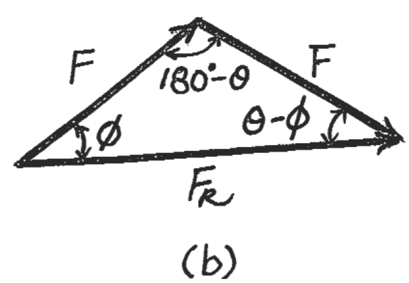
Since $\cos(180^\circ - \theta) = -\cos\theta$

$$F_R = F\left(\sqrt{2}\right)\sqrt{1 + \cos\theta}$$

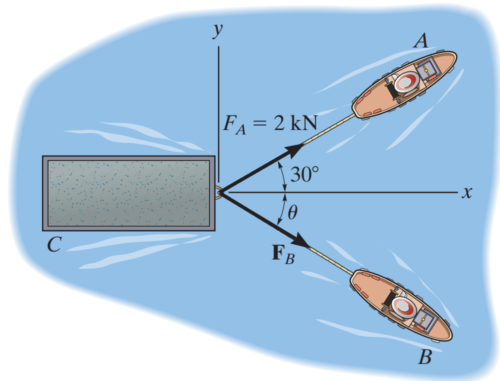
Since $\cos\left(\dfrac{\theta}{2}\right) = \sqrt{\dfrac{1 + \cos\theta}{2}}$

Then

$$F_R = 2F\cos\left(\frac{\theta}{2}\right) \qquad \textbf{Ans.}$$

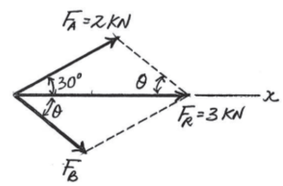


***2–20.** If the resultant force of the two tugboats is 3 kN, directed along the positive x axis, determine the required magnitude of force $\mathbf{F}_B$ and its direction θ.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



(a)

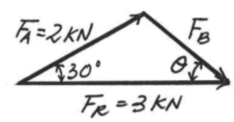
Applying the law of cosines to Fig. *b*,

$$F_B = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 30^\circ}$$

$$= 1.615\text{kN} = 1.61 \text{ kN} \qquad \textbf{Ans.}$$

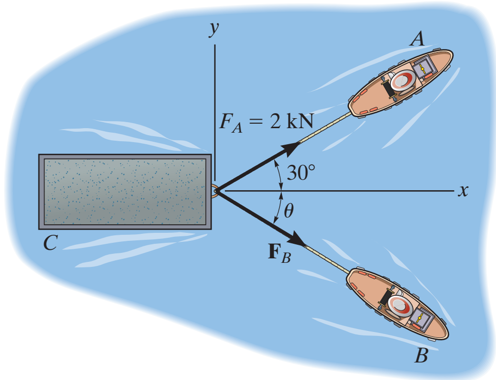
Using this result and applying the law of sines to Fig. *b*, yields

$$\frac{\sin\theta}{2} = \frac{\sin 30^\circ}{1.615} \qquad \theta = 38.3^\circ \qquad \textbf{Ans.}$$



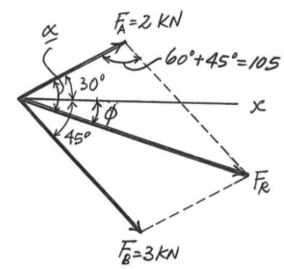
(b)

2–21. If $F_B = 3$ kN and $\theta = 45°$, determine the magnitude of the resultant force of the two tugboats and its direction measured clockwise from the positive x axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



(a)

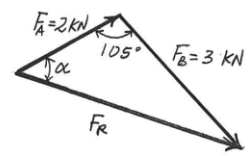
Applying the law of cosines to Fig. *b*,

$$F_R = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 105°}$$

$$= 4.013 \text{ kN} = 4.01 \text{ kN} \qquad \textbf{Ans.}$$

Using this result and applying the law of sines to Fig. *b*, yields

$$\frac{\sin\alpha}{3} = \frac{\sin 105°}{4.013} \qquad \alpha = 46.22°$$

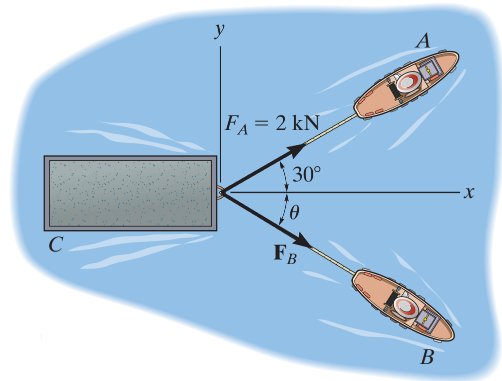


(b)

Thus, the direction angle ϕ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\phi = \alpha - 30° = 46.22° - 30° = 16.2° \qquad \textbf{Ans.}$$

2–22. If the resultant force of the two tugboats is required to be directed towards the positive x axis, and $\mathbf{F}_B$ is to be a minimum, determine the magnitude of $\mathbf{F}_R$ and $\mathbf{F}_B$ and the angle θ.

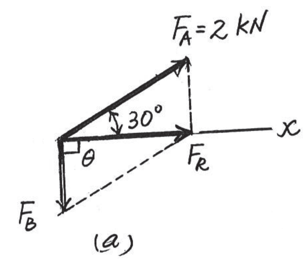


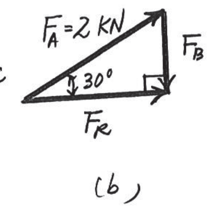
SOLUTION

For $\mathbf{F}_B$ to be minimum, it has to be directed perpendicular to $\mathbf{F}_R$. Thus,

$$\theta = 90^\circ \qquad \textbf{Ans.}$$

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.



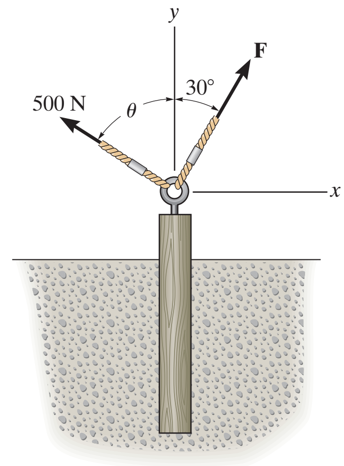


By applying simple trigonometry to Fig. *b*,

$$F_B = 2 \sin 30^\circ = 1 \text{ kN} \qquad \textbf{Ans.}$$

$$F_R = 2 \cos 30^\circ = 1.73 \text{ kN} \qquad \textbf{Ans.}$$

2–23. Two forces act on the screw eye. If $F = 600$ N, determine the magnitude of the resultant force and the angle θ if the resultant force is directed vertically upward.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b* respectively. Applying law of sines to Fig. *b*,

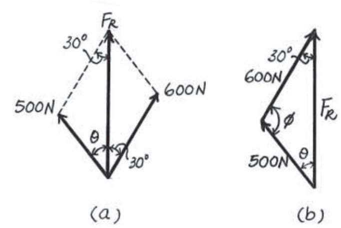
$$\frac{\sin\theta}{600} = \frac{\sin 30^\circ}{500}; \quad \sin\theta = 0.6 \quad \theta = 36.87^\circ = 36.9^\circ \qquad \textbf{Ans.}$$

Using the result of θ,

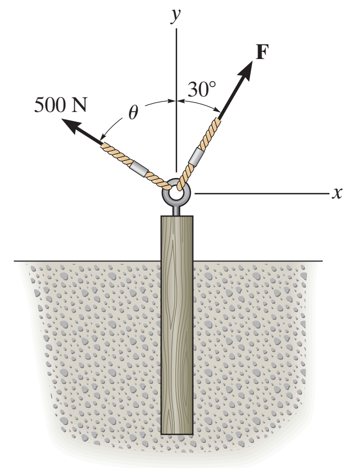
$$\phi = 180^\circ - 30^\circ - 36.87^\circ = 113.13^\circ$$

Again, applying law of sines using the result of ϕ,

$$\frac{F_R}{\sin 113.13^\circ} = \frac{500}{\sin 30^\circ}; \quad F_R = 919.61 \text{ N} = 920 \text{ N} \qquad \textbf{Ans.}$$



***2–24.** Two forces are applied at the end of a screw eye in order to remove the post. Determine the angle θ ($0° \le \theta \le 90°$) and the magnitude of force **F** so that the resultant force acting on the post is directed vertically upward and has a magnitude of 750 N.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of sines (Fig. *b*), we have

$$\frac{\sin\phi}{750} = \frac{\sin 30°}{500}$$

$$\sin\phi = 0.750$$

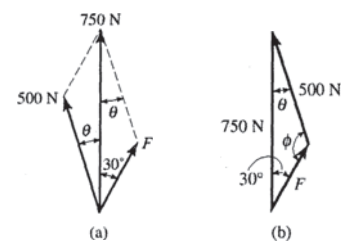
$$\phi = 131.41° \text{ (By observation, } \phi > 90°\text{)}$$

Thus,

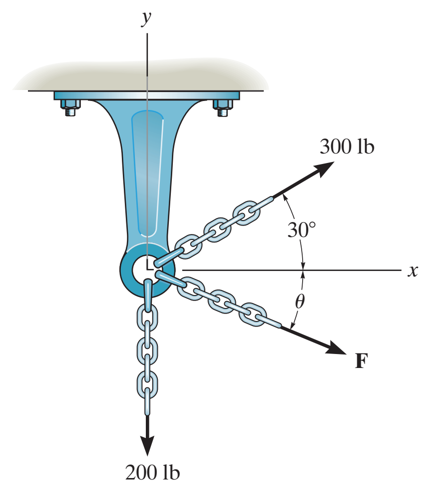
$$\theta = 180° - 30° - 131.41° = 18.59° = 18.6° \qquad \textbf{Ans.}$$

$$\frac{F}{\sin 18.59°} = \frac{500}{\sin 30°}$$

$$F = 319 \text{ N} \qquad \textbf{Ans.}$$



2–25. Three chains act on the bracket such that they create a resultant force having a magnitude of 500 lb. If two of the chains are subjected to known forces, as shown, determine the angle θ of the third chain measured clockwise from the positive x axis, so that the magnitude of force **F** in this chain is a *minimum*. All forces lie in the x–y plane. What is the magnitude of **F**? *Hint*: First find the resultant of the two known forces. Force **F** acts in this direction.



SOLUTION

Cosine law:

$$F_{R1} = \sqrt{300^2 + 200^2 - 2(300)(200)\cos 60^\circ} = 264.6 \text{ lb}$$

Sine law:

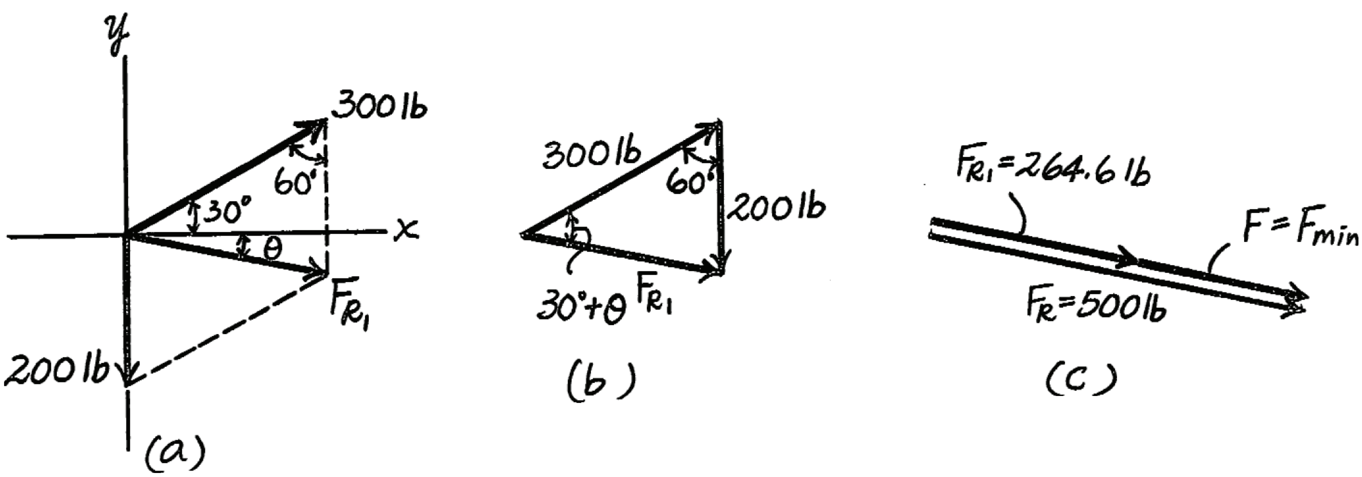
$$\frac{\sin(30^\circ + \theta)}{200} = \frac{\sin 60^\circ}{264.6} \qquad \theta = 10.9^\circ \qquad \textbf{Ans.}$$

When **F** is directed along $\mathbf{F}_{R1}$, F will be minimum to create the resultant force.

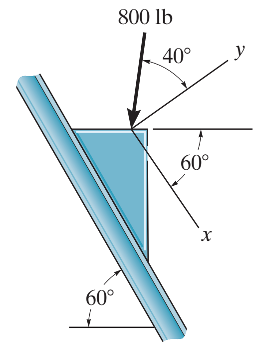
$$F_R = F_{R1} + F$$

$$500 = 264.6 + F_{\min}$$

$$F_{\min} = 235 \text{ lb} \qquad \textbf{Ans.}$$



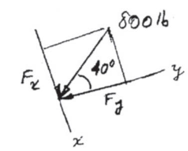
2–26. Determine the x and y components of the 800-lb force.



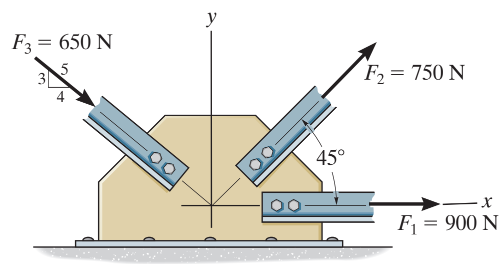
SOLUTION

$F_x = 800 \sin 40° = 514 \text{ lb}$ **Ans.**

$F_y = -800 \cos 40° = -613 \text{ lb}$ **Ans.**



2–27. Resolve each force acting on the gusset plate into its x and y components, and express each force as a Cartesian vector.



SOLUTION

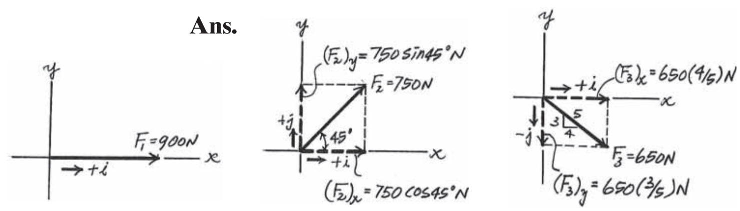
$\mathbf{F}_1 = \{900(+\mathbf{i})\} = \{900\mathbf{i}\}$ N **Ans.**

$\mathbf{F}_2 = \{750 \cos 45°(+\mathbf{i}) + 750 \sin 45°(+\mathbf{j})\}$ N

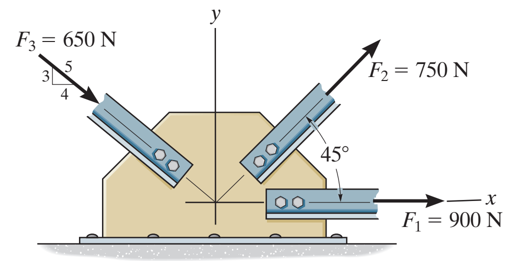
$= \{530\mathbf{i} + 530\mathbf{j}\}$ N **Ans.**

$$\mathbf{F}_3 = \left\{650\left(\frac{4}{5}\right)(+\mathbf{i}) + 650\left(\frac{3}{5}\right)(-\mathbf{j})\right\} \text{ N}$$

$= \{520\,\mathbf{i} - 390\mathbf{j})\}$ N **Ans.**



***2–28.** Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 900\text{ N} \qquad (F_1)_y = 0$$

$$(F_2)_x = 750\cos 45^\circ = 530.33\text{ N} \qquad (F_2)_y = 750\sin 45^\circ = 530.33\text{ N}$$

$$(F_3)_x = 650\left(\frac{4}{5}\right) = 520\text{ N} \qquad (F_3)_y = 650\left(\frac{3}{5}\right) = 390\text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 900 + 530.33 + 520 = 1950.33\text{ N}\rightarrow$$

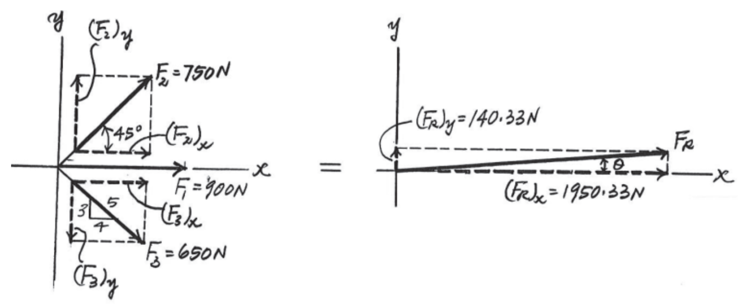
$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 530.33 - 390 = 140.33\text{ N}\uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{1950.33^2 + 140.33^2} = 1955\text{ N} = 1.96\text{ kN} \quad \textbf{Ans.}$$

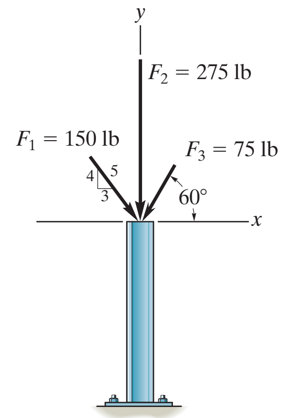
The direction angle θ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{140.33}{1950.33}\right) = 4.12^\circ \quad \textbf{Ans.}$$



(a)

2–29. Express each of the three forces acting on the column in Cartesian vector form and compute the magnitude of the resultant force.



SOLUTION

$$\mathbf{F}_1 = 150\left(\frac{3}{5}\right)\mathbf{i} - 150\left(\frac{4}{5}\right)\mathbf{j}$$

$$\mathbf{F}_1 = \{90\mathbf{i} - 120\mathbf{j}\} \text{ lb}$$ **Ans.**

$$\mathbf{F}_2 = \{-275\mathbf{j}\} \text{ lb}$$ **Ans.**

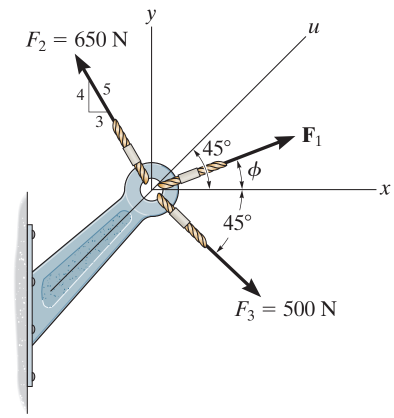
$$\mathbf{F}_3 = -75 \cos 60°\mathbf{i} - 75 \sin 60°\mathbf{j}$$

$$\mathbf{F}_3 = \{-37.5\mathbf{i} - 65.0\mathbf{j}\} \text{ lb}$$ **Ans.**

$$\mathbf{F}_R = \Sigma\mathbf{F} = \{52.5\mathbf{i} - 460\mathbf{j}\} \text{ lb}$$

$$F_R = \sqrt{(52.5)^2 + (-460)^2} = 463 \text{ lb}$$ **Ans.**

2–30. The magnitude of the resultant force acting on the bracket is to be 400 N. Determine the magnitude of $\mathbf{F}_1$ if $\phi = 30°$.



SOLUTION

Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = F_1 \cos 30° = 0.8660F_1 \qquad (F_1)_y = F_1 \sin 30° = 0.5F_1$$

$$(F_2)_x = 650\left(\frac{3}{5}\right) = 390 \text{ N} \qquad (F_2)_y = 650\left(\frac{4}{5}\right) = 520 \text{ N}$$

$$(F_3)_x = 500 \cos 45° = 353.55 \text{ N} \qquad (F_3)_y = 500 \sin 45° = 353.55 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 0.8660F_1 - 390 + 353.55 = 0.8660F_1 - 36.45$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 0.5F_1 + 520 - 353.55 = 0.5F_1 + 166.45$$

Since the magnitude of the resultant force is $\mathbf{F}_R = 400$ N, we can write

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}}$$

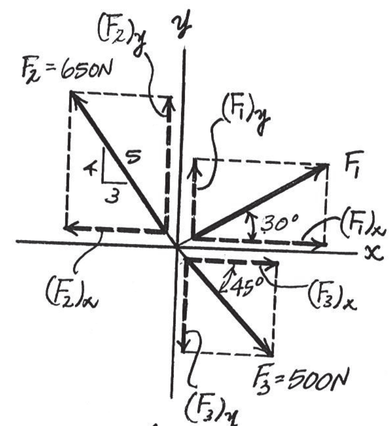
$$400 = \sqrt{(0.8660F_1 - 36.45)^2 + (0.5F_1 + 166.45)^2}$$

$$F_1^{\,2} + 103.32F_1 - 130967.17 = 0 \qquad \textbf{Ans.}$$

Solving,

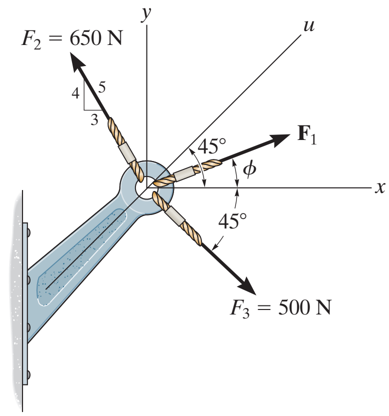
$$F_1 = 314 \text{ N} \qquad \text{or} \qquad F_1 = -417 \text{ N} \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{F}_1 = 417$ N must act in the opposite sense to that shown in the figure.



(a)

2–31. If the resultant force acting on the bracket is to be directed along the positive u axis, and the magnitude of $\mathbf{F}_1$ is required to be *minimum*, determine the magnitudes of the resultant force and $\mathbf{F}_1$.



SOLUTION

Rectangular Components: By referring to Figs. a and b, the x and y components of $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$(F_1)_x = F_1 \cos\phi$ $\qquad$ $(F_1)_y = F_1 \sin\phi$

$(F_2)_x = 650\left(\frac{3}{5}\right) = 390\text{ N}$ $\qquad$ $(F_2)_y = 650\left(\frac{4}{5}\right) = 520\text{ N}$

$(F_3)_x = 500\cos 45^\circ = 353.55\text{ N}$ $\qquad$ $(F_3)_y = 500\sin 45^\circ = 353.55\text{ N}$

$(F_R)_x = F_R\cos 45^\circ = 0.7071F_R$ $\qquad$ $(F_R)_y = F_R\sin 45^\circ = 0.7071F_R$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \qquad 0.7071F_R = F_1\cos\phi - 390 + 353.55 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \qquad 0.7071F_R = F_1\sin\phi + 520 - 353.55 \qquad \textbf{(2)}$$

Eliminating F_R from Eqs. (1) and (2), yields

$$F_1 = \frac{202.89}{\cos\phi - \sin\phi} \qquad \textbf{(3)}$$

The first derivative of Eq. (3) is

$$\frac{dF_1}{d\phi} = \frac{\sin\phi + \cos\phi}{(\cos\phi - \sin\phi)^2} \qquad \textbf{(4)}$$

The second derivative of Eq. (3) is

$$\frac{d^2F_1}{d\phi^2} = \frac{2(\sin\phi + \cos\phi)^2}{(\cos\phi - \sin\phi)^3} + \frac{1}{\cos\phi - \sin\phi} \qquad \textbf{(5)}$$

For $\mathbf{F}_1$ to be minimum, $\frac{dF_1}{d\phi} = 0$. Thus, from Eq. (4)

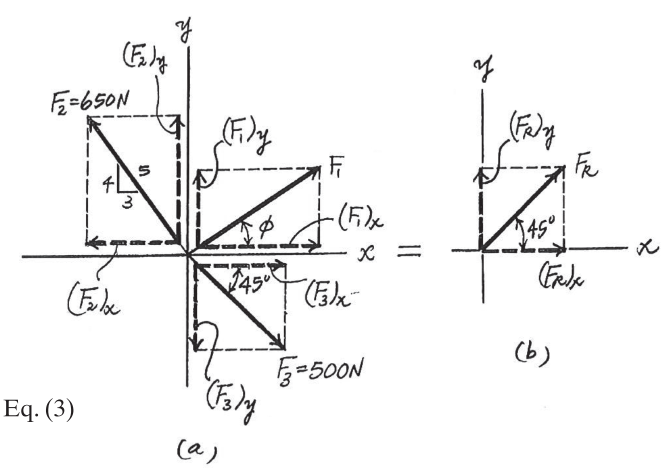
$$\sin\phi + \cos\phi = 0$$

$$\tan\phi = -1$$

$$\phi = -45^\circ$$

Substituting $\phi = -45^\circ$ into Eq. (5), yields

$$\frac{d^2F_1}{d\phi^2} = 0.7071 > 0$$



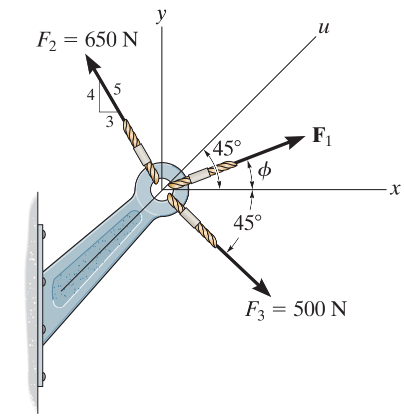
This shows that $\phi = -45^\circ$ indeed produces minimum F_1. Thus, from Eq. (3)

$$F_1 = \frac{202.89}{\cos(-45^\circ) - \sin(-45^\circ)} = 143.47\text{ N} = 143\text{ N} \qquad \textbf{Ans.}$$

Substituting $\phi = -45^\circ$ and $F_1 = 143.47$ N into either Eq. (1) or Eq. (2), yields

$$F_R = 91.9\text{ N} \qquad \textbf{Ans.}$$

***2–32.** If the magnitude of the resultant force acting on the bracket is 600 N, directed along the positive u axis, determine the magnitude of **F** and its direction ϕ.



SOLUTION

Rectangular Components: By referring to Figs. a and b, the x and y components of $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$(F_1)_x = F_1 \cos\phi$ $\qquad$ $(F_1)_y = F_1 \sin\phi$

$(F_2)_x = 650\left(\frac{3}{5}\right) = 390 \text{ N}$ $\qquad$ $(F_2)_y = 650\left(\frac{4}{5}\right) = 520 \text{ N}$

$(F_3)_x = 500 \cos 45° = 353.55 \text{ N}$ $\qquad$ $(F_3)_y = 500 \cos 45° = 353.55 \text{ N}$

$(F_R)_x = 600 \cos 45° = 424.26 \text{ N}$ $\qquad$ $(F_R)_y = 600 \sin 45° = 424.26 \text{ N}$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$\xrightarrow{+} \Sigma(F_R)_x = \Sigma F_x;$ $\qquad$ $424.26 = F_1 \cos\phi - 390 + 353.55$ $\qquad$ **(1)**

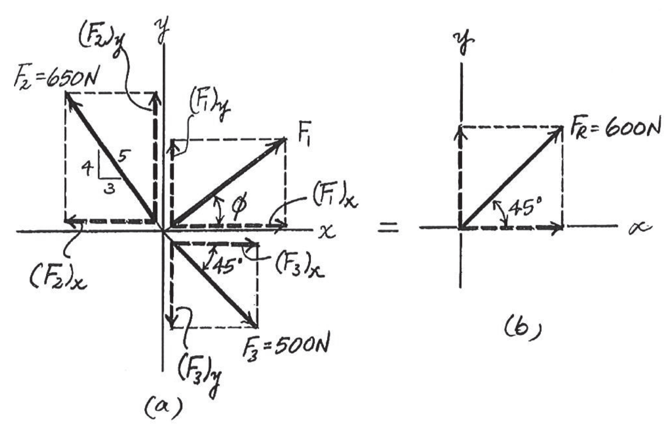
$$F_1 \cos\phi = 460.71$$

$+\uparrow \Sigma(F_R)_y = \Sigma F_y;$ $\qquad$ $424.26 = F_1 \sin\phi + 520 - 353.55$ $\qquad$ **(2)**

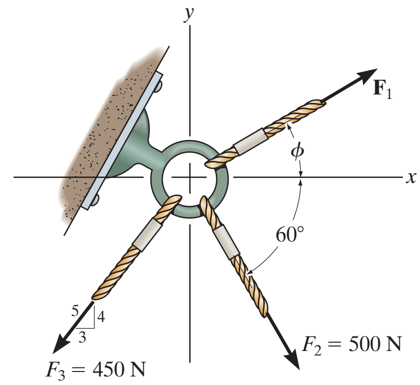
$$F_1 \sin\phi = 257.82$$

Solving Eqs. (1) and (2), yields

$\phi = 29.2°$ $\qquad$ $F_1 = 528 \text{ N}$ $\qquad$ **Ans.**



2–33. If $F_1 = 600$ N and $\phi = 30°$, determine the magnitude of the resultant force acting on the eyebolt and its direction, measured clockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of each force can be written as

$$(F_1)_x = 600 \cos 30° = 519.62 \text{ N} \quad (F_1)_y = 600 \sin 30° = 300 \text{ N}$$

$$(F_2)_x = 500 \cos 60° = 250 \text{ N} \quad (F_2)_y = 500 \sin 60° = 433.01 \text{ N}$$

$$(F_3)_x = 450\left(\frac{3}{5}\right) = 270 \text{ N} \qquad (F_3)_y = 450\left(\frac{4}{5}\right) = 360 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 519.62 + 250 - 270 = 499.62 \text{ N} \quad \rightarrow$$

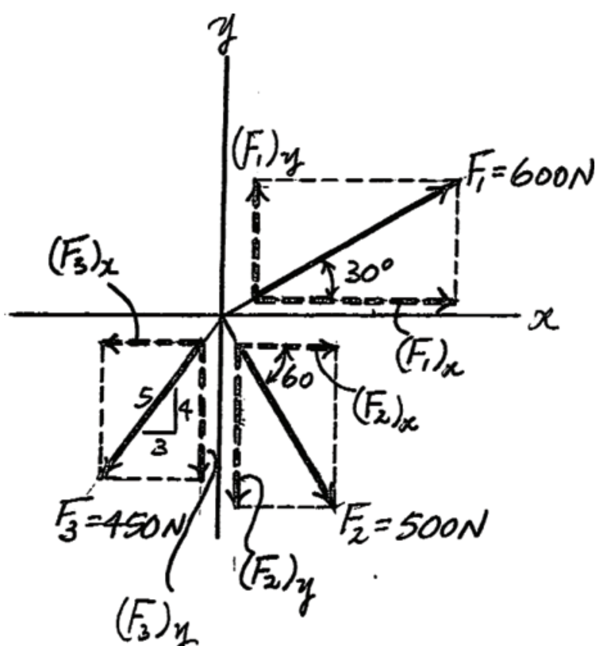
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 300 - 433.01 - 360 = -493.01 \text{ N} = 493.01 \text{ N} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

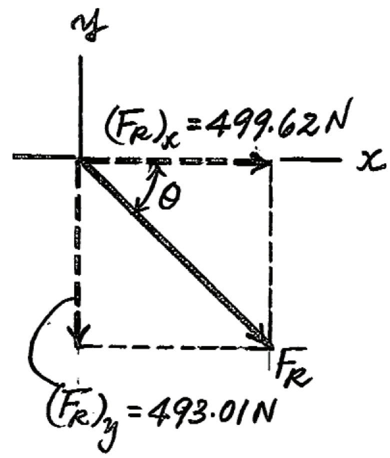
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{499.62^2 + 493.01^2} = 701.91 \text{ N} = 702 \text{ N} \qquad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, Fig. *b*, measured clockwise from the x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{493.01}{499.62}\right) = 44.6° \qquad \textbf{Ans.}$$

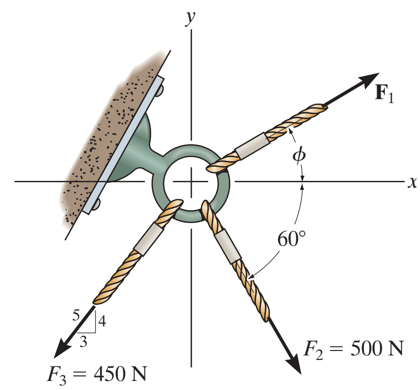


(a)



(b)

2–34. If the magnitude of the resultant force acting on the eyebolt is 600 N and its direction measured clockwise from the positive x axis is $\theta = 30°$, determine the magnitude of $\mathbf{F}_1$ and the angle ϕ.



SOLUTION

Rectangular Components: By referring to Figs. a and b, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1 \cos\phi \qquad (F_1)_y = F_1 \sin\phi$$

$$(F_2)_x = 500\cos 60° = 250\text{ N} \qquad (F_2)_y = 500\sin 60° = 433.01\text{ N}$$

$$(F_3)_x = 450\left(\frac{3}{5}\right) = 270\text{ N} \qquad (F_3)_y = 450\left(\frac{4}{5}\right) = 360\text{ N}$$

$$(F_R)_x = 600\cos 30° = 519.62\text{ N} \qquad (F_R)_y = 600\sin 30° = 300\text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \qquad 519.62 = F_1\cos\phi + 250 - 270$$

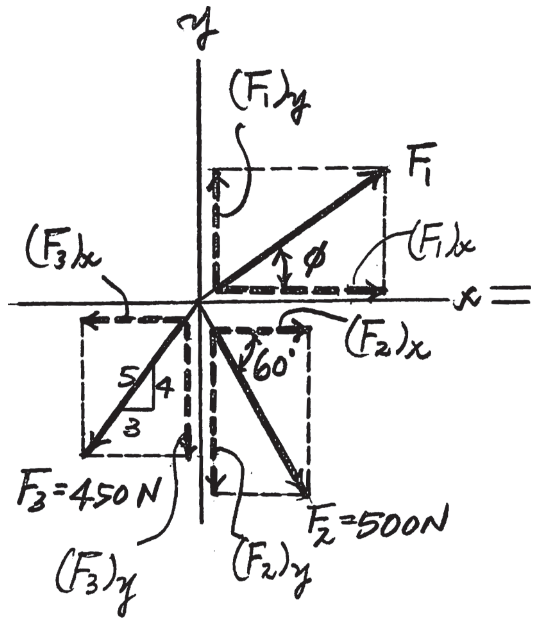
$$F_1\cos\phi = 539.62 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad -300 = F_1\sin\phi - 433.01 - 360$$

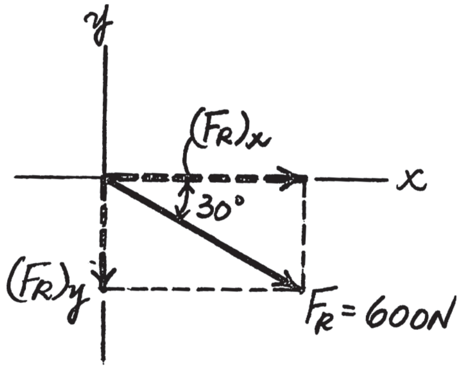
$$F_1\sin\phi = 493.01 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$$\phi = 42.4° \qquad F_1 = 731\text{ N} \qquad \textbf{Ans.}$$

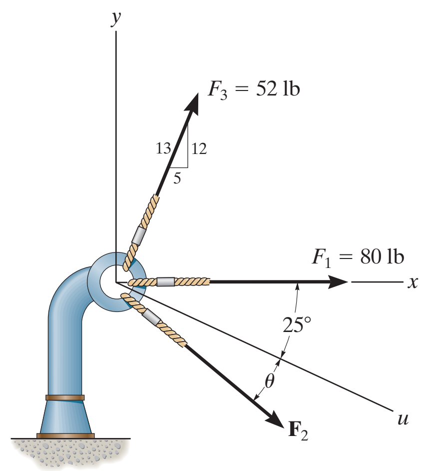


(a)



(b)

2–35. Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F}_2$ so that the resultant force is directed along the positive u axis and has a magnitude of 50 lb.



SOLUTION

Scalar Notation: Summing the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad 50 \cos 25° = 80 + 52\left(\frac{5}{13}\right) + F_2 \cos (25° + \theta)$$

$$F_2 \cos (25° + \theta) = -54.684 \tag{1}$$

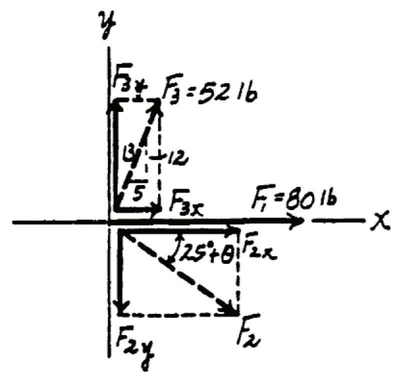
$$+\uparrow F_{R_y} = \Sigma F_y; \qquad -50 \sin 25° = 52\left(\frac{12}{13}\right) - F_2 \sin (25° + \theta)$$

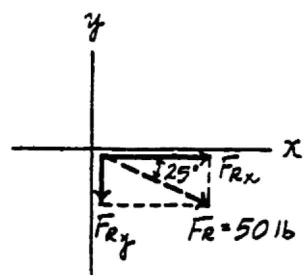
$$F_2 \sin (25° + \theta) = 69.131 \tag{2}$$

Solving Eqs. (1) and (2) yields

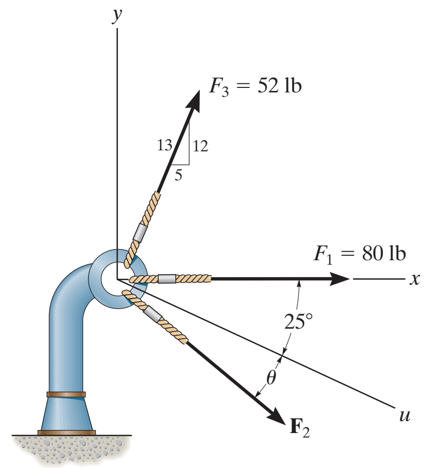
$$25° + \theta = 128.35° \qquad \theta = 103° \qquad \textbf{Ans.}$$

$$F_2 = 88.1 \text{ lb} \qquad \textbf{Ans.}$$





***2–36.** If $F_2 = 150$ lb and $\theta = 55°$, determine the magnitude and direction measured clockwise from the positive x axis, of the resultant force of the three forces acting on the bracket.



SOLUTION

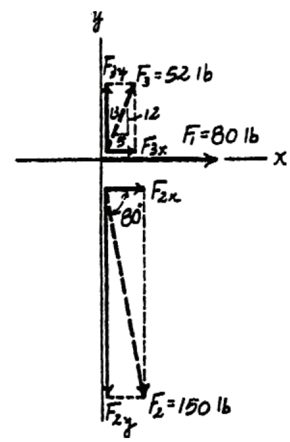
Scalar Notation: Summing the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 80 + 52\left(\frac{5}{13}\right) + 150 \cos 80°$$

$$= 126.05 \text{ lb} \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 52\left(\frac{12}{13}\right) - 150 \sin 80°$$

$$= -99.72 \text{ lb} = 99.72 \text{ lb} \downarrow$$

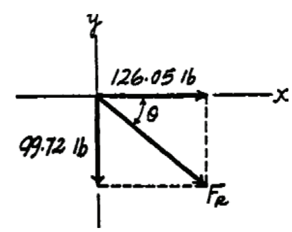


The magnitude of the resultant force $\mathbf{F}_R$ is

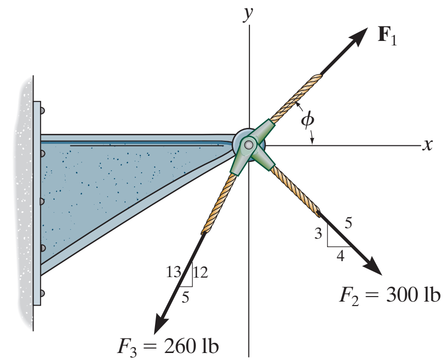
$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{126.05^2 + 99.72^2} = 161 \text{ lb} \qquad \textbf{Ans.}$$

The direction angle θ measured clockwise from positive x axis is

$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{99.72}{126.05}\right) = 38.3° \qquad \textbf{Ans.}$$



2–37. If $\phi = 30°$ and $F_1 = 250$ lb, determine the magnitude of the resultant force acting on the bracket and its direction measured clockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 250\cos 30° = 216.51 \text{ lb} \qquad (F_1)_y = 250\sin 30° = 125 \text{ lb}$$

$$(F_2)_x = 300\left(\frac{4}{5}\right) = 240 \text{ lb} \qquad (F_2)_y = 300\left(\frac{3}{5}\right) = 180 \text{ lb}$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100 \text{ lb} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240 \text{ lb}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 216.51 + 240 - 100 = 356.51 \text{ lb} \rightarrow$$

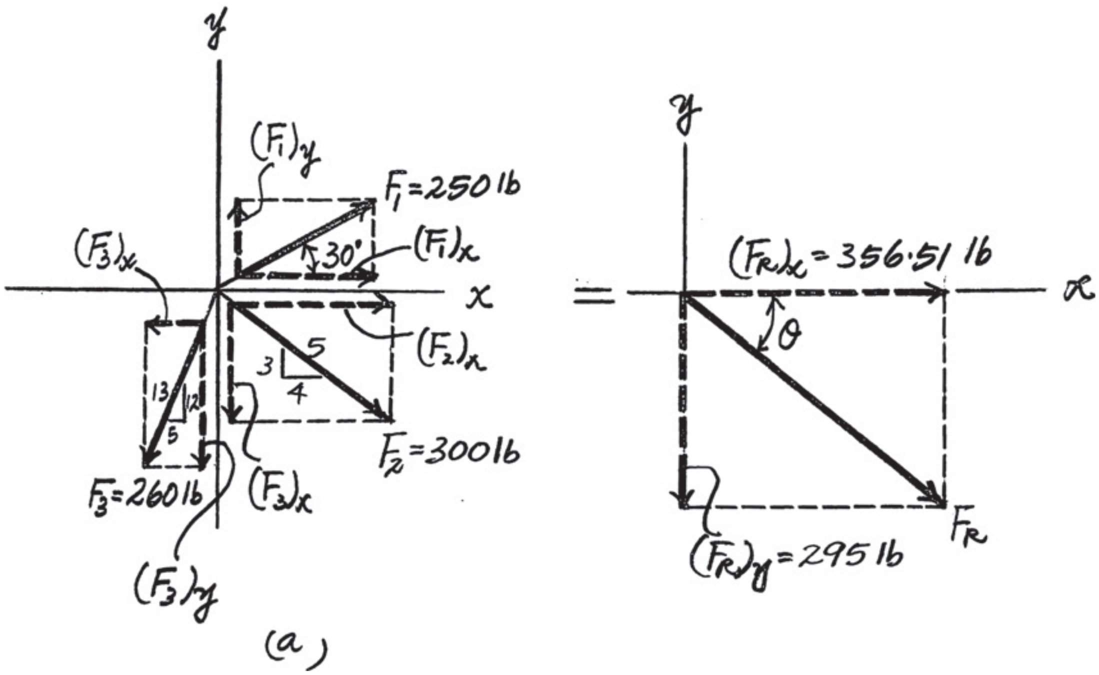
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 125 - 180 - 240 = -295 \text{ lb} = 295 \text{ lb} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{356.51^2 + 295^2} = 463 \text{ lb}$$ **Ans.**

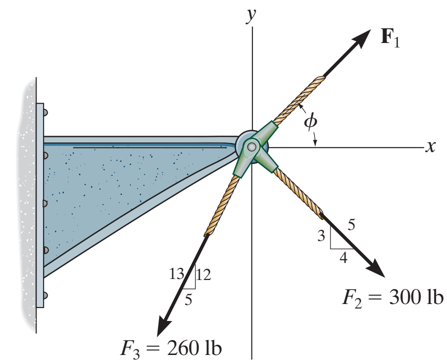
The direction angle θ of $\mathbf{F}_R$, Fig. *b*, measured clockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{295}{356.51}\right) = 39.6°$$ **Ans.**



(a)

2–38. If the magnitude of the resultant force acting on the bracket is 400 lb directed along the positive x axis, determine the magnitude of $\mathbf{F}_1$ and its direction ϕ.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1 \cos\phi \qquad (F_1)_y = F_1 \sin\phi$$

$$(F_2)_x = 300\left(\frac{4}{5}\right) = 240 \text{ lb} \qquad (F_2)_y = 300\left(\frac{3}{5}\right) = 180 \text{ lb}$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100 \text{ lb} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240 \text{ lb}$$

$$(F_R)_x = 400 \text{ lb} \qquad (F_R)_y = 0$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad 400 = F_1 \cos\phi + 240 - 100$$

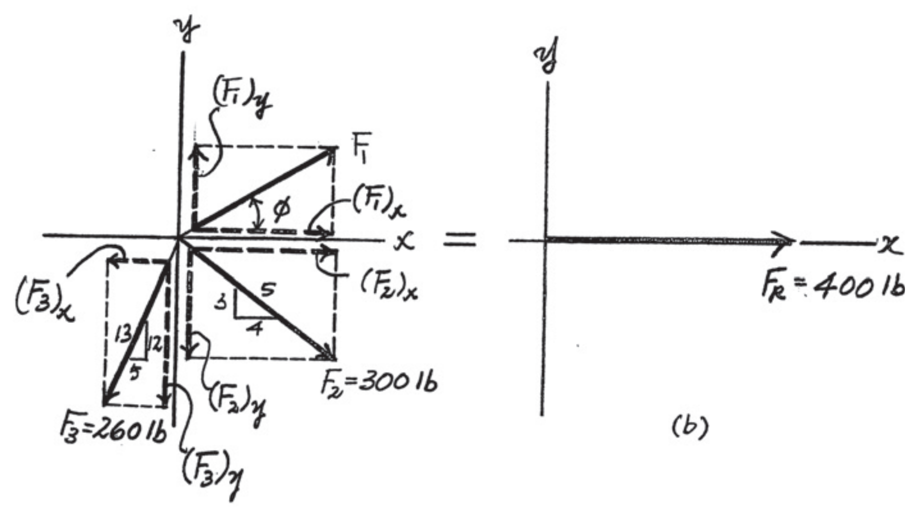
$$F_1 \cos\phi = 260 \qquad (1)$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad 0 = F_1 \sin\phi - 180 - 240$$

$$F_1 \sin\phi = 420 \qquad (2)$$

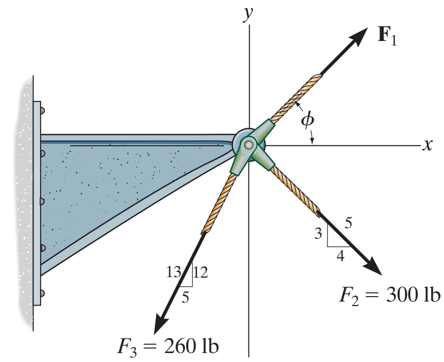
Solving Eqs. (1) and (2), yields

$$\phi = 58.2^\circ \qquad F_1 = 494 \text{ lb}$$ **Ans.**



(a) (b)

2–39. If the resultant force acting on the bracket is to be directed along the positive x axis and the magnitude of $\mathbf{F}_1$ is required to be a minimum, determine the magnitudes of the resultant force and $\mathbf{F}_1$.



SOLUTION

Rectangular Components: By referring to Figs. a and b, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1 \cos\phi \qquad (F_1)_y = F_1 \sin\phi$$

$$(F_2)_x = 300\left(\frac{4}{5}\right) = 240 \text{ lb} \qquad (F_2)_y = 300\left(\frac{3}{5}\right) = 180 \text{ lb}$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100 \text{ lb} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240 \text{ lb}$$

$$(F_R)_x = F_R \qquad (F_R)_y = 0$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad 0 = F_1 \sin\phi - 180 - 240$$

$$F_1 = \frac{420}{\sin\phi} \qquad (1)$$

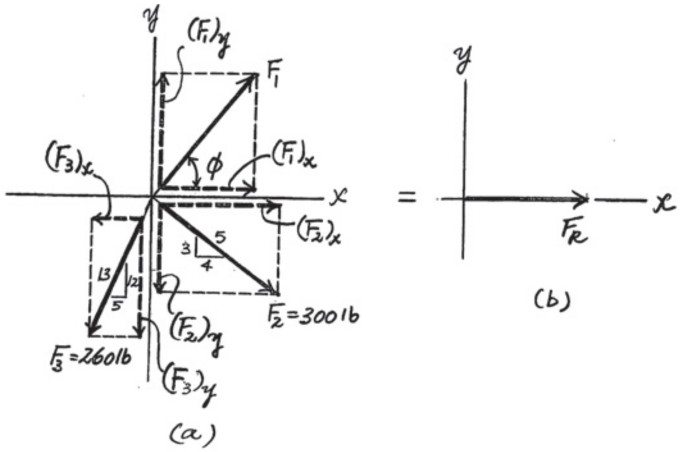
$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \quad F_R = F_1 \cos\phi + 240 - 100 \qquad (2)$$

By inspecting Eq. (1), we realize that F_1 is minimum when $\sin\phi = 1$ or $\phi = 90°$. Thus,

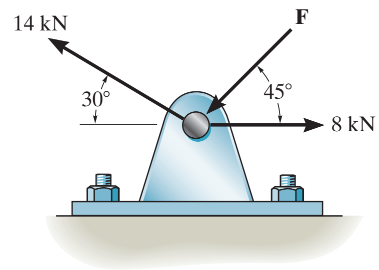
$F_1 = 420$ lb **Ans.**

Substituting these results into Eq. (2), yields

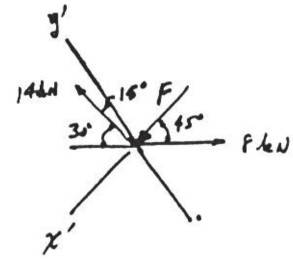
$F_R = 140$ lb **Ans.**



***2–40.** Determine the magnitude of force **F** so that the resultant force of the three forces is as small as possible. What is the magnitude of the resultant force?



SOLUTION



$\xrightarrow{+} F_{Rx} = \Sigma F_x;$ $\qquad F_{Rz} = 8 - F\cos 45° - 14\cos 30°$

$\qquad = -4.1244 - F\cos 45°$

$+\uparrow F_{Ry} = \Sigma F_y;$ $\qquad F_{Ry} = -F\sin 45° + 14\sin 30°$

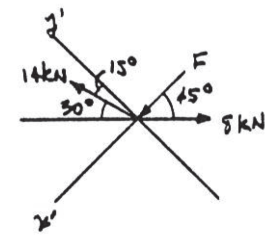
$\qquad = 7 - F\sin 45°$

$$F_R^2 = (-4.1244 - F\cos 45°)^2 + (7 - F\sin 45°)^2 \qquad \textbf{(1)}$$

$$2F_R\frac{dF_R}{dF} = 2(-4.1244 - F\cos 45°)(-\cos 45°) + 2(7 - F\sin 45°)(-\sin 45°) = 0$$

$F = 2.03\text{ kN}$ **Ans.**

From Eq. (1); $\qquad F_R = 7.87\text{ kN}$ **Ans.**



Also, from the figure require

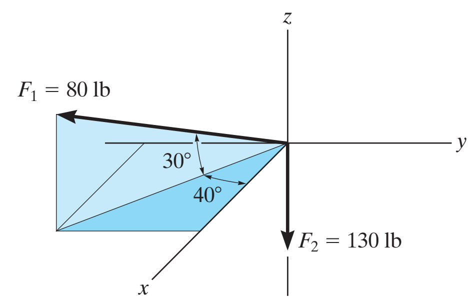
$(F_R)_{x'} = 0 = \Sigma F_{x'};$ $\qquad F + 14\sin 15° - 8\cos 45° = 0$

$F = 2.03\text{ kN}$ **Ans.**

$(F_R)_{y'} = \Sigma F_{y'};$ $\qquad F_R = 14\cos 15° - 8\sin 45°$

$F_R = 7.87\text{ kN}$ **Ans.**

2–41. Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the coordinate system.



SOLUTION

$\mathbf{F}_1 = \{80 \cos 30° \cos 40° \mathbf{i} - 80 \cos 30° \sin 40° \mathbf{j} + 80 \sin 30° \mathbf{k}\}$ lb

$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb

$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

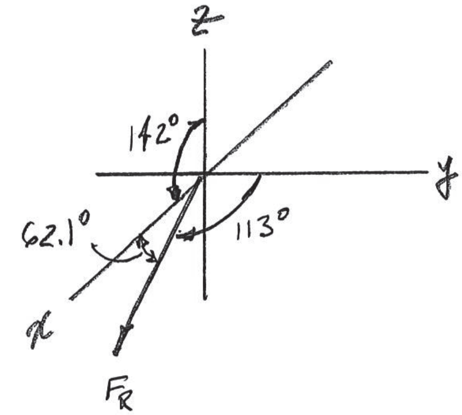
$\mathbf{F}_R = \{53.1\mathbf{i} - 44.5\mathbf{j} - 90.0\mathbf{k}\}$ lb

$F_R = \sqrt{(53.1)^2 + (-44.5)^2 + (-90.0)^2} = 114$ lb **Ans.**

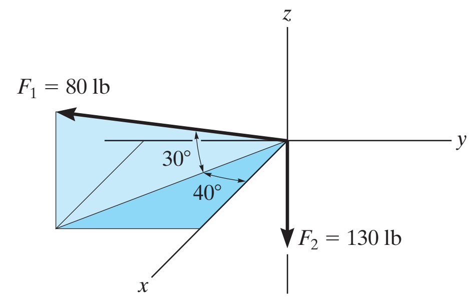
$\alpha = \cos^{-1}\left(\frac{53.1}{113.6}\right) = 62.1°$ **Ans.**

$\beta = \cos^{-1}\left(\frac{-44.5}{113.6}\right) = 113°$ **Ans.**

$\gamma = \cos^{-1}\left(\frac{-90.0}{113.6}\right) = 142°$ **Ans.**



2–42. Specify the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_2$ and express each force as a Cartesian vector.



SOLUTION

$$\mathbf{F}_1 = \{80 \cos 30° \cos 40°\mathbf{i} - 80 \cos 30° \sin 40°\mathbf{j} + 80 \sin 30°\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\} \text{ lb}$$ **Ans.**

$$\alpha_1 = \cos^{-1}\left(\frac{53.1}{80}\right) = 48.4°$$ **Ans.**

$$\beta_1 = \cos^{-1}\left(\frac{-44.5}{80}\right) = 124°$$ **Ans.**

$$\gamma_1 = \cos^{-1}\left(\frac{40}{80}\right) = 60°$$ **Ans.**

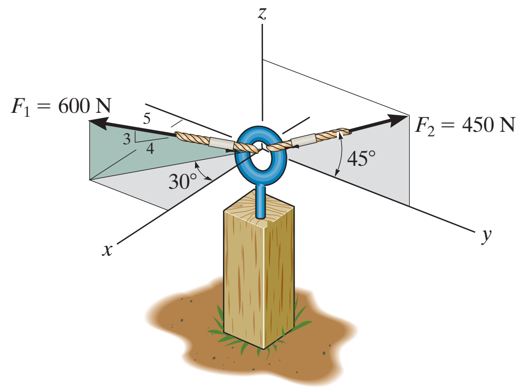
$$\mathbf{F}_2 = \{-130\mathbf{k}\} \text{ lb}$$ **Ans.**

$$\alpha_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90°$$ **Ans.**

$$\beta_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90°$$ **Ans.**

$$\gamma_2 = \cos^{-1}\left(\frac{-130}{130}\right) = 180°$$ **Ans.**

2–43. Determine the coordinate direction angles of force $\mathbf{F}_1$.



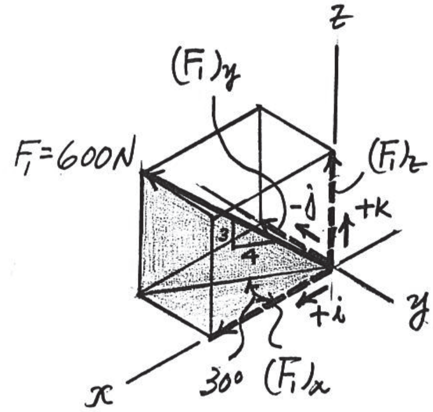
SOLUTION

Rectangular Components: By referring to Figs. *a*, the *x*, *y*, and *z* components of $\mathbf{F}_1$ can be written as

$$(F_1)_x = 600\left(\frac{4}{5}\right)\cos 30°\text{ N} \qquad (F_1)_y = 600\left(\frac{4}{5}\right)\sin 30°\text{ N} \qquad (F_1)_z = 600\left(\frac{3}{5}\right)\text{ N}$$

Thus, $\mathbf{F}_1$ expressed in Cartesian vector form can be written as

$$\mathbf{F}_1 = 600\left\{\frac{4}{5}\cos 30°(+\mathbf{i}) + \frac{4}{5}\sin 30°(-\mathbf{j}) + \frac{3}{5}(+\mathbf{k})\right\}\text{ N}$$
$$= 600[0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k}]\text{ N}$$



(a)

Therefore, the unit vector for $\mathbf{F}_1$ is given by

$$\mathbf{u}_{F_1} = \frac{\mathbf{F}_1}{F_1} = \frac{600(0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k}}{600} = 0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k}$$

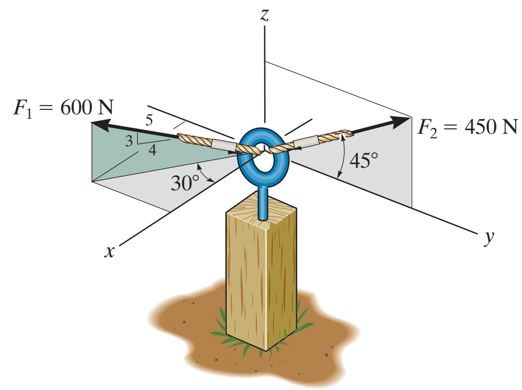
The coordinate direction angles of $\mathbf{F}_1$ are

$$\alpha = \cos^{-1}(u_{F_1})_x = \cos^{-1}(0.6928) = 46.1° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}(u_{F_1})_y = \cos^{-1}(-0.4) = 114° \qquad \textbf{Ans.}$$

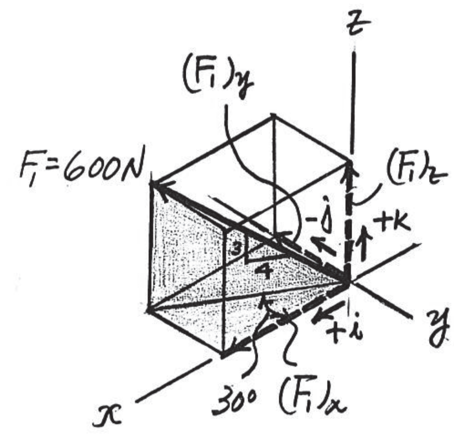
$$\gamma = \cos^{-1}(u_{F_1})_z = \cos^{-1}(0.6) = 53.1° \qquad \textbf{Ans.}$$

***2–44.** Determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.



SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. a and b, respectively, they are expressed in Cartesian vector form as



(a)

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)\cos 30°(+\mathbf{i}) + 600\left(\frac{4}{5}\right)\sin 30°(-\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

$$= \{415.69\mathbf{i} - 240\mathbf{j} + 360\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_2 = 0\mathbf{i} + 450 \cos 45°(+\mathbf{j}) + 450 \sin 45°(+\mathbf{k})$$

$$= \{318.20\mathbf{j} + 318.20\mathbf{k}\} \text{ N}$$

Resultant Force: The resultant force acting on the eyebolt can be obtained by vectorally adding $\mathbf{F}_1$ and $\mathbf{F}_2$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

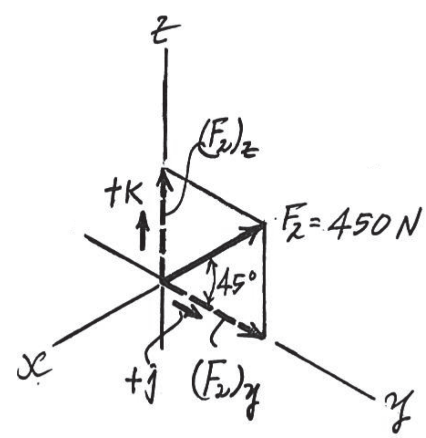
$$= (415.69\mathbf{i} - 240\mathbf{j} + 360\mathbf{k}) + (318.20\mathbf{j} + 318.20\mathbf{k})$$

$$= \{415.69\mathbf{i} + 78.20\mathbf{j} + 678.20\mathbf{k}\} \text{ N}$$

The magnitude of $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2} + (F_R)_z^{\,2}}$$

$$= \sqrt{(415.69)^2 + (78.20)^2 + (678.20)^2} = 799.29 \text{ N} = 799 \text{ N} \qquad \textbf{Ans.}$$



(b)

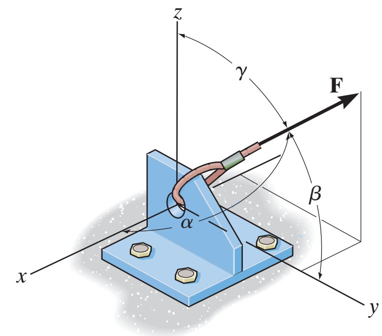
The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{415.69}{799.29}\right) = 58.7° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{78.20}{799.29}\right) = 84.4° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{678.20}{799.29}\right) = 32.0° \qquad \textbf{Ans.}$$

2–45. The force **F** acts on the bracket within the octant shown. If $F = 400$ N, $\beta = 60°$, and $\gamma = 45°$, determine the x, y, z components of **F**.



SOLUTION

Coordinate Direction Angles: Since β and γ are known, the third angle α can be determined from

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$\cos^2\alpha + \cos^2 60° + \cos^2 45° = 1$$
$$\cos\alpha = \pm 0.5$$

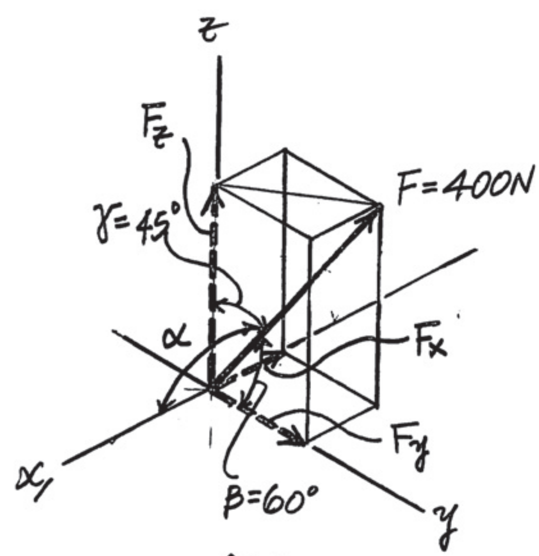
Since **F** is in the octant shown in Fig. *a*, θ_x must be greater than 90°. Thus,

$\alpha = \cos^{-1}(-0.5) = 120°$.

Rectangular Components: By referring to Fig. *a*, the x, y, and z components of **F** can be written as

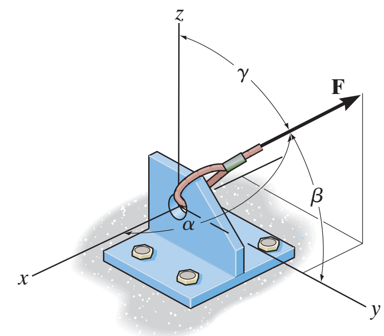
$$F_x = F\cos\alpha = 400\cos 120° = -200\text{ N} \qquad \textbf{Ans.}$$
$$F_y = F\cos\beta = 400\cos 60° = 200\text{ N} \qquad \textbf{Ans.}$$
$$F_z = F\cos\gamma = 400\cos 45° = 283\text{ N} \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{F}_x$ is directed towards the negative x axis.



(a)

2–46. The force **F** acts on the bracket within the octant shown. If the magnitudes of the x and z components of **F** are $F_x = 300$ N and $F_z = 600$ N, respectively, and $\beta = 60°$, determine the magnitude of **F** and its y component. Also, find the coordinate direction angles α and γ.



SOLUTION

Rectangular Components: The magnitude of **F** is given by

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F = \sqrt{300^2 + F_y^2 + 600^2}$$

$$F^2 = F_y^2 + 450\ 000 \qquad (1)$$

The magnitude of $\mathbf{F}_y$ is given by

$$F_y = F \cos 60° = 0.5F \qquad (2)$$

Solving Eqs. (1) and (2) yields

$$F = 774.60\text{ N} = 775\text{ N} \qquad \textbf{Ans.}$$

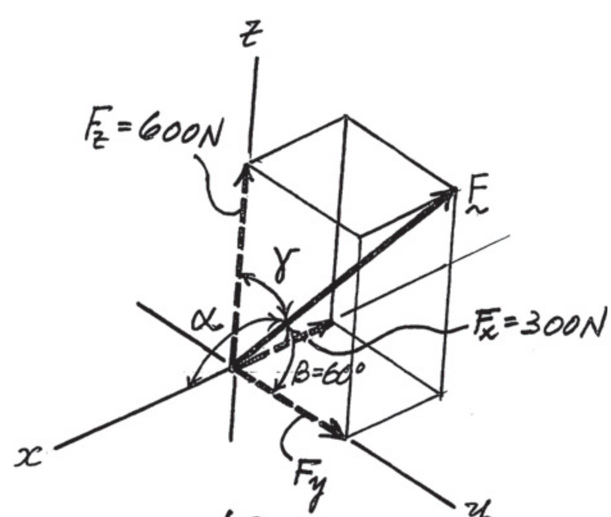
$$F_y = 387\text{ N} \qquad \textbf{Ans.}$$

Coordinate Direction Angles: Since **F** is contained in the octant so that $\mathbf{F}_x$ is directed towards the negative x axis, the coordinate direction angle θ_x is given by

$$\alpha = \cos^{-1}\left(\frac{-F_x}{F}\right) = \cos^{-1}\left(\frac{-300}{774.60}\right) = 113° \qquad \textbf{Ans.}$$

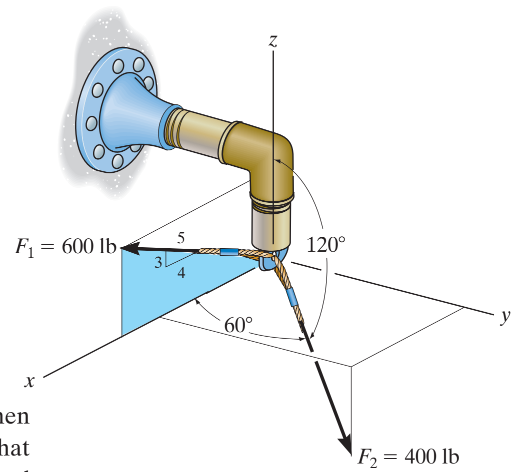
The third coordinate direction angle is

$$\gamma = \cos^{-1}\left(\frac{-F_z}{F}\right) = \cos^{-1}\left(\frac{600}{774.60}\right) = 39.2° \qquad \textbf{Ans.}$$



(a)

2–47. Express each force acting on the pipe assembly in Cartesian vector form.



SOLUTION

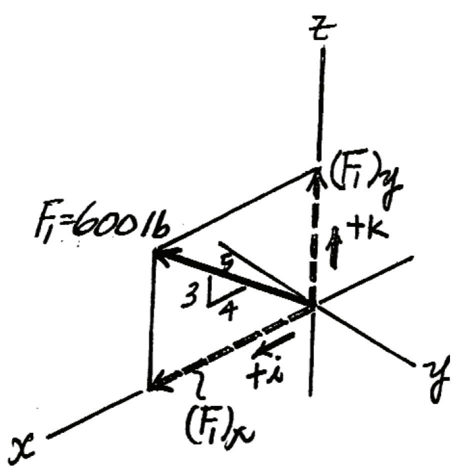
Rectangular Components: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\beta_2 = \pm\sqrt{1 - \cos^2 60° - \cos^2 120°} = \pm 0.7071$. However, it is required that $\beta_2 < 90°$, thus, $\beta_2 = \cos^{-1}(0.7071) = 45°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)(+\mathbf{i}) + 0\mathbf{j} + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

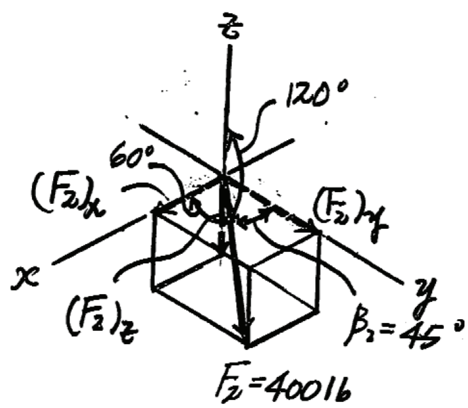
$$= [480\mathbf{i} + 360\mathbf{k}]\ \text{lb} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = 400\cos 60°\mathbf{i} + 400\cos 45°\mathbf{j} + 400\cos 120°\mathbf{k}$$

$$= [200\mathbf{i} + 283\mathbf{j} - 200\mathbf{k}]\ \text{lb} \qquad \textbf{Ans.}$$

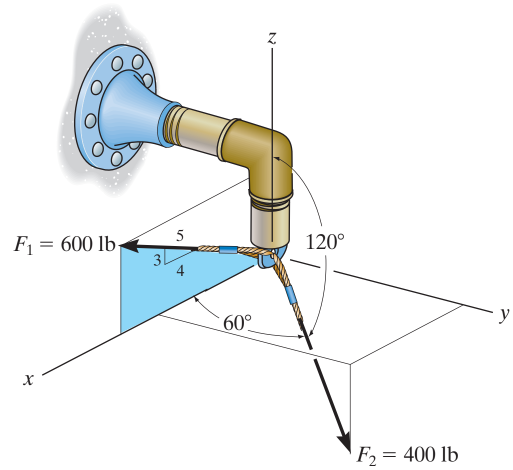


(a)



(b)

***2–48.** Determine the magnitude and the direction of the resultant force acting on the pipe assembly.



SOLUTION

Force Vectors: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\gamma_2 = \pm\sqrt{1 - \cos^2 60° - \cos^2 120°} = \pm\ 0.7071$. However, it is required that $\beta_2 < 90°$, thus, $\beta_2 = \cos^{-1}(0.7071) = 45°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)(+\mathbf{i}) + 0\mathbf{j} + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

$$= \{480\mathbf{i} + 360\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_2 = 400\cos 60°\mathbf{i} + 400\cos 45°\mathbf{j} + 400\cos 120°\mathbf{k}$$

$$= \{200\mathbf{i} + 282.84\mathbf{j} - 200\mathbf{k}\}\text{ lb}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}_2$ vectorally, we obtain $\mathbf{F}_R$.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (480\mathbf{i} + 360\mathbf{k}) + (200\mathbf{i} + 282.84\mathbf{j} - 200\mathbf{k})$$

$$= \{680\mathbf{i} + 282.84\mathbf{j} + 160\mathbf{k}\}\text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$

$$= \sqrt{680^2 + 282.84^2 + 160^2} = 753.66\text{ lb} = 754\text{ lb} \qquad \textbf{Ans.}$$

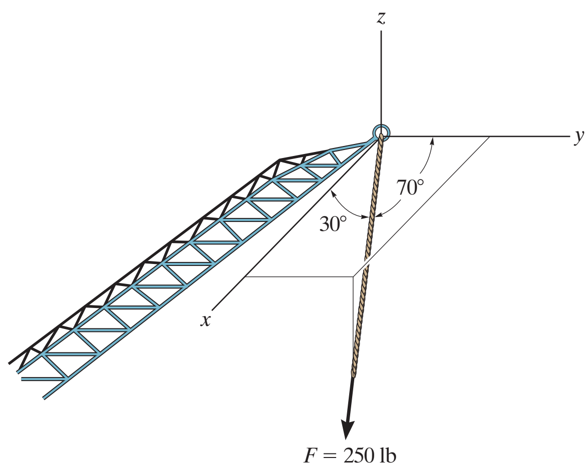
The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{680}{753.66}\right) = 25.5° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{282.84}{753.66}\right) = 68.0° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{160}{753.66}\right) = 77.7° \qquad \textbf{Ans.}$$

2–49. Determine the magnitude and coordinate direction angles of $\mathbf{F}_1 = \{60\mathbf{i} - 50\mathbf{j} + 40\mathbf{k}\}$ N and $\mathbf{F}_2 = \{-40\mathbf{i} - 85\mathbf{j} + 30\mathbf{k}\}$ N. Sketch each force on an x, y, z reference frame.



SOLUTION

$\mathbf{F}_1 = 60\,\mathbf{i} - 50\,\mathbf{j} + 40\,\mathbf{k}$

$F_1 = \sqrt{(60)^2 + (-50)^2 + (40)^2} = 87.7496 = 87.7 \text{ N}$ **Ans.**

$\alpha_1 = \cos^{-1}\left(\dfrac{60}{87.7496}\right) = 46.9°$ **Ans.**

$\beta_1 = \cos^{-1}\left(\dfrac{-50}{87.7496}\right) = 125°$ **Ans.**

$\gamma_1 = \cos^{-1}\left(\dfrac{40}{87.7496}\right) = 62.9°$ **Ans.**

$\mathbf{F}_2 = -40\,\mathbf{i} - 85\,\mathbf{j} + 30\,\mathbf{k}$

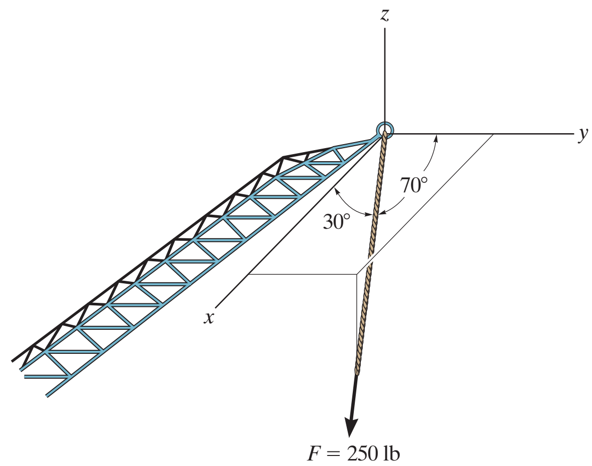
$F_2 = \sqrt{(-40)^2 + (-85)^2 + (30)^2} = 98.615 = 98.6 \text{ N}$ **Ans.**

$\alpha_2 = \cos^{-1}\left(\dfrac{-40}{98.615}\right) = 114°$ **Ans.**

$\beta_2 = \cos^{-1}\left(\dfrac{-85}{98.615}\right) = 150°$ **Ans.**

$\gamma_2 = \cos^{-1}\left(\dfrac{30}{98.615}\right) = 72.3°$ **Ans.**

2–50. The cable at the end of the crane boom exerts a force of 250 lb on the boom as shown. Express **F** as a Cartesian vector.



SOLUTION

Cartesian Vector Notation: With $\alpha = 30°$ and $\beta = 70°$, the third coordinate direction angle γ can be determined using Eq. 2–8.

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\cos^2 30° + \cos^2 70° + \cos^2 \gamma = 1$$

$$\cos \gamma = \pm 0.3647$$

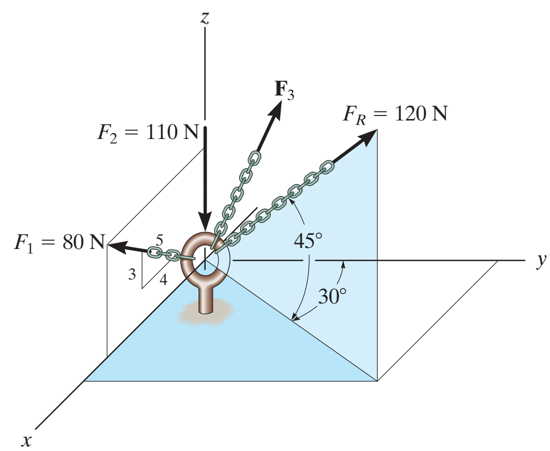
$$\gamma = 68.61° \text{ or } 111.39°$$

By inspection, $\gamma = 111.39°$ since the force **F** is directed in negative octant.

$$\mathbf{F} = 250\{\cos 30°\mathbf{i} + \cos 70°\mathbf{j} + \cos 111.39°\}\text{ lb}$$

$$= \{217\mathbf{i} + 85.5\mathbf{j} - 91.2\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

2–51. Three forces act on the ring. If the resultant force $\mathbf{F}_R$ has a magnitude and direction as shown, determine the magnitude and the coordinate direction angles of force $\mathbf{F}_3$.



SOLUTION

Cartesian Vector Notation:

$$\mathbf{F}_R = 120\{\cos 45°\sin 30°\mathbf{i} + \cos 45°\cos 30°\mathbf{j} + \sin 45°\mathbf{k}\}\text{ N}$$

$$= \{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_1 = 80\left\{\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right\}\text{ N} = \{64.0\mathbf{i} + 48.0\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = \{-110\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_3 = \{F_{3_x}\mathbf{i} + F_{3_y}\mathbf{j} + F_{3_z}\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$\{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\} = \left\{\left(64.0 + F_{3_x}\right)\mathbf{i} + F_{3_y}\mathbf{j} + \left(48.0 - 110 + F_{3_z}\right)\mathbf{k}\right\}$$

Equating **i**, **j** and **k** components, we have

$$64.0 + F_{3_x} = 42.43 \qquad F_{3_x} = -21.57\text{ N}$$

$$F_{3_y} = 73.48\text{ N}$$

$$48.0 - 110 + F_{3_z} = 84.85 \qquad F_{3_z} = 146.85\text{ N}$$

The magnitude of force $\mathbf{F}_3$ is

$$F_3 = \sqrt{F_{3_x}^2 + F_{3_y}^2 + F_{3_z}^2}$$

$$= \sqrt{(-21.57)^2 + 73.48^2 + 146.85^2}$$

$$= 165.62\text{ N} = 166\text{ N} \qquad \textbf{Ans.}$$

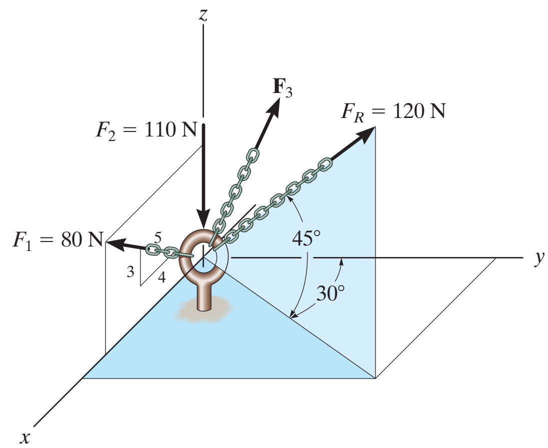
The coordinate direction angles for $\mathbf{F}_3$ are

$$\cos\alpha = \frac{F_{3_x}}{F_3} = \frac{-21.57}{165.62} \qquad \alpha = 97.5° \qquad \textbf{Ans.}$$

$$\cos\beta = \frac{F_{3_y}}{F_3} = \frac{73.48}{165.62} \qquad \beta = 63.7° \qquad \textbf{Ans.}$$

$$\cos\gamma = \ = \frac{F_{3_z}}{F_3} = \frac{146.85}{165.62} \qquad \gamma = 27.5° \qquad \textbf{Ans.}$$

***2–52.** Determine the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_R$.



SOLUTION

Unit Vector of $\mathbf{F}_1$ and $\mathbf{F}_R$:

$$\mathbf{u}_{F_1} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k} = 0.8\mathbf{i} + 0.6\mathbf{k}$$

$$\mathbf{u}_R = \cos 45° \sin 30°\mathbf{i} + \cos 45° \cos 30°\mathbf{j} + \sin 45°\mathbf{k}$$

$$= 0.3536\mathbf{i} + 0.6124\mathbf{j} + 0.7071\mathbf{k}$$

Thus, the coordinate direction angles $\mathbf{F}_1$ and $\mathbf{F}_R$ are

$\cos \alpha_{F_1} = 0.8 \qquad \alpha_{F_1} = 36.9°$ **Ans.**

$\cos \beta_{F_1} = 0 \qquad \beta_{F_1} = 90.0°$ **Ans.**

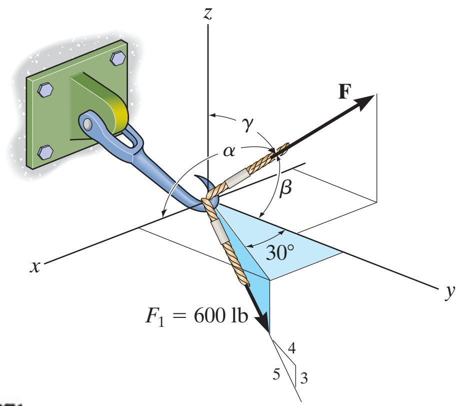
$\cos \gamma_{F_1} = 0.6 \qquad \gamma_{F_1} = 53.1°$ **Ans.**

$\cos \alpha_R = 0.3536 \qquad \alpha_R = 69.3°$ **Ans.**

$\cos \beta_R = 0.6124 \qquad \beta_R = 52.2°$ **Ans.**

$\cos \gamma_R = 0.7071 \qquad \gamma_R = 45.0°$ **Ans.**

2–53. If $\alpha = 120°$, $\beta < 90°$, $\gamma = 60°$, and $F = 400$ lb, determine the magnitude and coordinate direction angles of the resultant force acting on the hook.



SOLUTION

Force Vectors: Since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, then $\cos\beta = \pm\sqrt{1 - \cos^2 120° - \cos^2 60°} = \pm 0.7071$. However, it is required that $\beta < 90°$, thus, $\beta = \cos^{-1}(0.7071) = 45°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$, can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)\sin 30°(+\mathbf{i}) + 600\left(\frac{4}{5}\right)\cos 30°(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(-\mathbf{k})$$
$$= \{240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}\} \text{ lb}$$
$$\mathbf{F} = 400\cos 120°\mathbf{i} + 400\cos 45°\mathbf{j} + 400\cos 60°\mathbf{k}$$
$$= \{-200\mathbf{i} + 282.84\mathbf{j} + 200\mathbf{k}\} \text{ lb}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}$ vectorally, we obtain $\mathbf{F}_R$.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$
$$= (240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}) + (-200\mathbf{i} + 282.84\mathbf{j} + 200\mathbf{k})$$
$$= \{40\mathbf{i} + 698.53\mathbf{j} - 160\mathbf{k}\} \text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

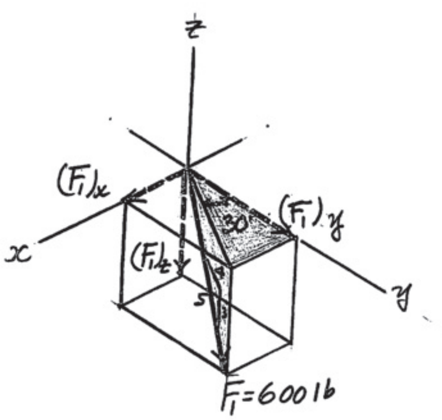
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$
$$= \sqrt{(40)^2 + (698.53)^2 + (-160)^2} = 717.74 \text{ lb} = 718 \text{ lb} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

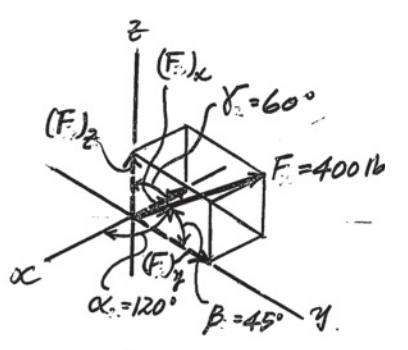
$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{40}{717.74}\right) = 86.8° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{698.53}{717.74}\right) = 13.3° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-160}{717.74}\right) = 103° \qquad \textbf{Ans.}$$

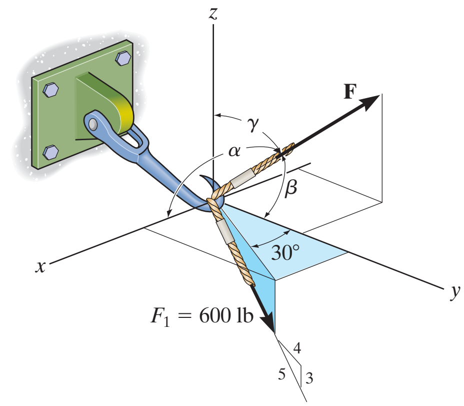


(a)



(b)

2–54. If the resultant force acting on the hook is $\mathbf{F}_R = \{-200\mathbf{i} + 800\mathbf{j} + 150\mathbf{k}\}$ lb, determine the magnitude and coordinate direction angles of $\mathbf{F}$.



SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}$ into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)\sin 30°(+\mathbf{i}) + 600\left(\frac{4}{5}\right)\cos 30°(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(-\mathbf{k})$$
$$= \{240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}\}\text{ lb}$$
$$\mathbf{F} = F\cos\alpha\mathbf{i} + F\cos\beta\mathbf{j} + F\cos\gamma\mathbf{k}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}_2$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$
$$-200\mathbf{i} + 800\mathbf{j} + 150\mathbf{k} = (240\mathbf{i} + 415.69\mathbf{j} - 360\mathbf{k}) + (F\cos\theta_x\mathbf{i} + F\cos\theta_y\mathbf{j} + F\cos\theta_z\mathbf{k})$$
$$-200\mathbf{i} + 800\mathbf{j} + 150\mathbf{k} = (240 + F\cos\alpha)\mathbf{i} + (415.69 + F\cos\beta)\mathbf{j} + (F\cos\gamma - 360)\mathbf{k}$$

Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components, we have

$$-200 = 240 + F\cos\theta_x$$
$$F\cos\alpha = -440 \qquad (1)$$

$$800 = 415.69 + F\cos\beta$$
$$F\cos\beta = 384.31 \qquad (2)$$

$$150 = F\cos\gamma - 360$$
$$F\cos\gamma = 510 \qquad (3)$$

Squaring and then adding Eqs. (1), (2), and (3), yields

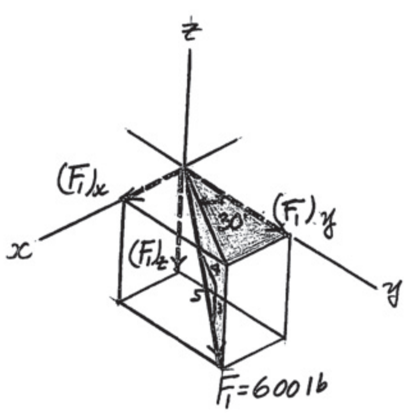
$$F^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 601392.49 \qquad (4)$$

However, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Thus, from Eq. (4)

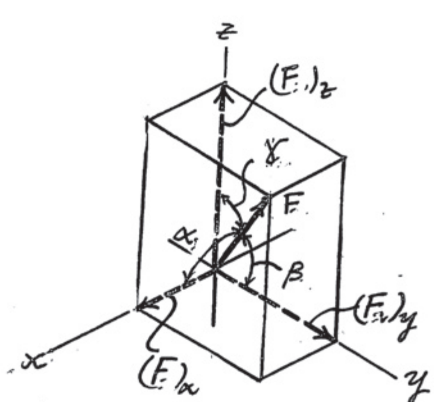
$$F = 775.49\text{ N} = 775\text{ N} \qquad \textbf{Ans.}$$

Substituting $F = 775.49$ N into Eqs. (1), (2), and (3), yields

$$\alpha = 125° \qquad \beta = 60.3° \qquad \gamma = 48.9° \qquad \textbf{Ans.}$$

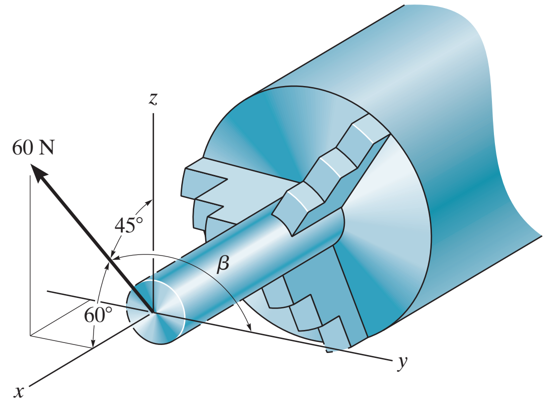


(a)



(b)

2–55. The stock mounted on the lathe is subjected to a force of 60 N. Determine the coordinate direction angle β and express the force as a Cartesian vector.



SOLUTION

$$1 = \sqrt{\cos^2\alpha + \cos^2\beta + \cos^2\gamma}$$

$$1 = \cos^2 60° + \cos^2\beta + \cos^2 45°$$

$$\cos\beta = \pm 0.5$$

$$\beta = 60°, 120°$$

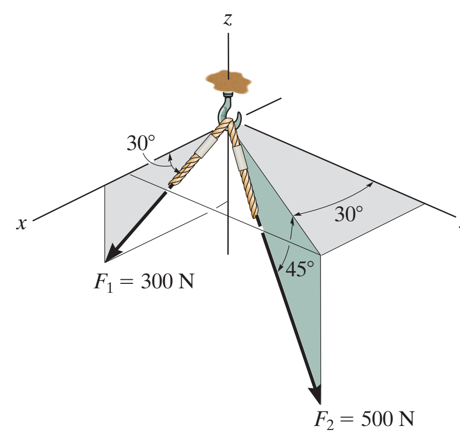
Use

$$\beta = 120° \qquad \textbf{Ans.}$$

$$F = 60\text{ N}(\cos 60°\mathbf{i} + \cos 120°\mathbf{j} + \cos 45°\mathbf{k})$$

$$= \{30\mathbf{i} - 30\mathbf{j} + 42.4\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

***2–56.** Express each force as a Cartesian vector.



SOLUTION

Rectangular Components: By referring to Figs. *a* and *b*, the *x*, *y*, and *z* components of $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written as

$(F_1)_x = 300 \cos 30° = 259.8$ N $\qquad (F_2)_x = 500 \cos 45° \sin 30° = 176.78$ N

$(F_1)_y = 0 \qquad (F_2)_y = 500 \cos 45° \cos 30° = 306.19$ N

$(F_1)_t = 300 \sin 30° = 150$ N $\qquad (F_2)_z = 500 \sin 45° = 353.55$ N

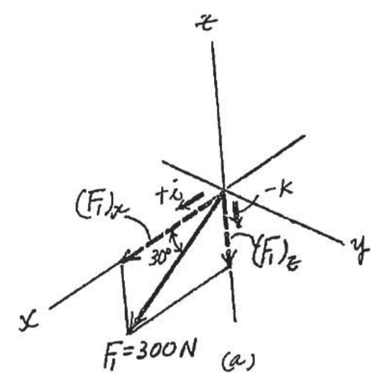
Thus, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written in Cartesian vector form as

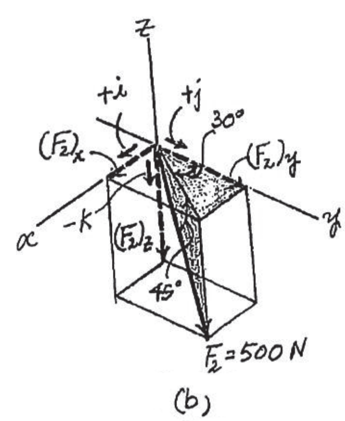
$$\mathbf{F}_1 = 259.81(+\mathbf{i}) + 0\mathbf{j} + 150(-\mathbf{k})$$

$$= \{260\mathbf{i} - 150\mathbf{k}\} \text{ N} \qquad \textbf{Ans.}$$

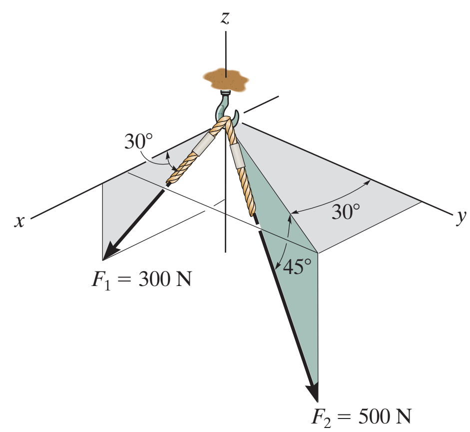
$$\mathbf{F}_2 = 176.78(+\mathbf{i}) + 306.19(+\mathbf{j}) + 353.55(-\mathbf{k})$$

$$= 2\{177\mathbf{i} + 306\mathbf{j} - 354\mathbf{k}\} \text{ N} \qquad \textbf{Ans.}$$





2–57. Determine the magnitude and coordinate direction angles of the resultant force acting on the hook.



SOLUTION

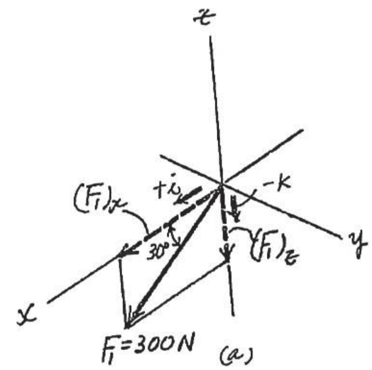
Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expessed in Cartesian vector form as

$$\begin{aligned}\mathbf{F}_1 &= 300\cos 30°(+\mathbf{i}) + 0\mathbf{j} + 300\sin 30°(-\mathbf{k})\\ &= \{259.81\mathbf{i} - 150\mathbf{k}\}\ \text{N}\end{aligned}$$

$$\begin{aligned}\mathbf{F}_2 &= 500\cos 45°\sin 30°(+\mathbf{i}) + 500\cos 45°\cos 30°(+\mathbf{j}) + 500\sin 45°(-\mathbf{k})\\ &= \{176.78\mathbf{i} - 306.19\mathbf{j} - 353.55\mathbf{k}\}\ \text{N}\end{aligned}$$

Resultant Force: The resultant force acting on the hook can be obtained by vectorally adding $\mathbf{F}_1$ and $\mathbf{F}_2$. Thus,

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2\\ &= (259.81\mathbf{i} - 150\mathbf{k}) + (176.78\mathbf{i} + 306.19\mathbf{j} - 353.55\mathbf{k})\\ &= \{436.58\mathbf{i}) + 306.19\mathbf{j} - 503.55\mathbf{k}\}\ \text{N}\end{aligned}$$



The magnitude of $\mathbf{F}_R$ is

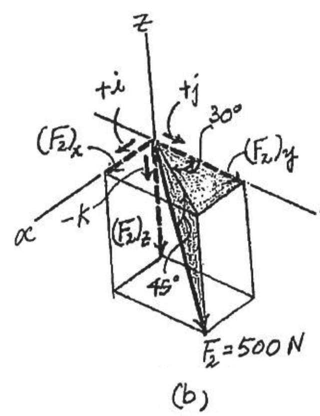
$$\begin{aligned}F_R &= \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2(F_R)_z{}^2}\\ &= \sqrt{(436.58)^2 + (306.19)^2 + (-503.55)^2} = 733.43\ \text{N} = 733\ \text{N} \qquad \textbf{Ans.}\end{aligned}$$

The coordinate direction angles of $\mathbf{F}_R$ are

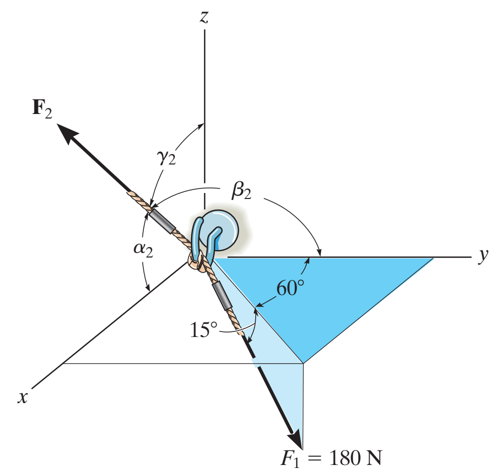
$$\theta_x = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{436.58}{733.43}\right) = 53.5° \qquad \textbf{Ans.}$$

$$\theta_y = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{306.19}{733.43}\right) = 65.3° \qquad \textbf{Ans.}$$

$$\theta_z = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-503.55}{733.43}\right) = 133° \qquad \textbf{Ans.}$$



2–58. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$ so that the resultant of the two forces acts along the positive x axis and has a magnitude of 500 N.



SOLUTION

$\mathbf{F}_1 = (180 \cos 15°) \sin 60° \,\mathbf{i} + (180 \cos 15°) \cos 60° \,\mathbf{j} - 180 \sin 15° \,\mathbf{k}$

$= 150.57\,\mathbf{i} + 86.93\,\mathbf{j} - 46.59\,\mathbf{k}$

$\mathbf{F}_2 = F_2 \cos \alpha_2 \,\mathbf{i} + F_2 \cos \beta_2 \,\mathbf{j} + F_2 \cos \gamma_2 \,\mathbf{k}$

$\mathbf{F}_R = \{500\,\mathbf{i}\}$ N

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

i components :

$500 = 150.57 + F_2 \cos \alpha_2$

$F_{2x} = F_2 \cos \alpha_2 = 349.43$

j components :

$0 = 86.93 + F_2 \cos \beta_2$

$F_{2y} = F_2 \cos \beta_2 = -86.93$

k components :

$0 = -46.59 + F_2 \cos \gamma_2$

$F_{2z} = F_2 \cos \gamma_2 = 46.59$

Thus,

$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2 + F_{2z}^2} = \sqrt{(349.43)^2 + (-86.93)^2 + (46.59)^2}$

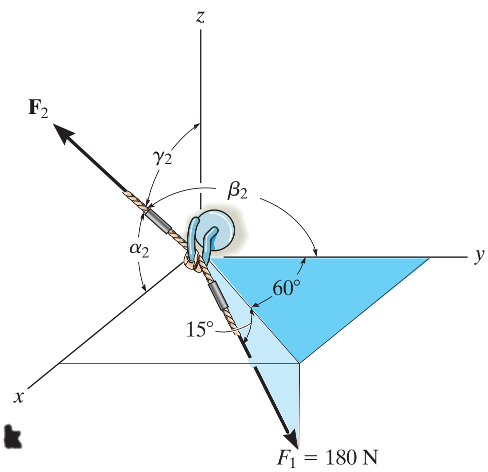
$F_2 = 363$ N **Ans**

$\alpha_2 = 15.8°$ **Ans**

$\beta_2 = 104°$ **Ans**

$\gamma_2 = 82.6°$ **Ans**

2–59. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$ so that the resultant of the two forces is zero.



SOLUTION

$$\mathbf{F}_1 = (180 \cos 15^\circ) \sin 60^\circ \, \mathbf{i} + (180 \cos 15^\circ) \cos 60^\circ \, \mathbf{j} - 180 \sin 15^\circ \, \mathbf{k}$$

$$= 150.57 \, \mathbf{i} + 86.93 \, \mathbf{j} - 46.59 \, \mathbf{k}$$

$$\mathbf{F}_2 = F_2 \cos \alpha_2 \, \mathbf{i} + F_2 \cos \beta_2 \, \mathbf{j} + F_2 \cos \gamma_2 \, \mathbf{k}$$

$$\mathbf{F}_R = \mathbf{0}$$

i components :

$$0 = 150.57 + F_2 \cos \alpha_2$$

$$F_2 \cos \alpha_2 = -150.57$$

j components :

$$0 = 86.93 + F_2 \cos \beta_2$$

$$F_2 \cos \beta_2 = -86.93$$

k components :

$$0 = -46.59 + F_2 \cos \gamma_2$$

$$F_2 \cos \gamma_2 = 46.59$$

$$F_2 = \sqrt{(-150.57)^2 + (-86.93)^2 + (46.59)^2}$$

Solving,

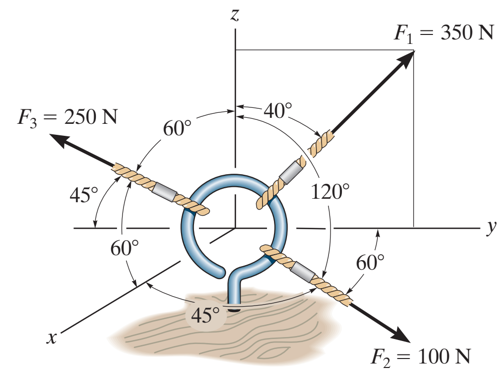
$F_2 = 180$ N **Ans**

$\alpha_2 = 147^\circ$ **Ans**

$\beta_2 = 119^\circ$ **Ans**

$\gamma_2 = 75.0^\circ$ **Ans**

***2–60.** The cables attached to the screw eye are subjected to the three forces shown. Express each force in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

Cartesian Vector Notation:

$$\mathbf{F}_1 = 350\{\sin 40°\mathbf{j} + \cos 40°\mathbf{k}\}\text{ N}$$
$$= \{224.98\mathbf{j} + 268.12\mathbf{k}\}\text{ N}$$
$$= \{225\mathbf{j} + 268\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = 100\{\cos 45°\mathbf{i} + \cos 60°\mathbf{j} + \cos 120°\mathbf{k}\}\text{ N}$$
$$= \{70.71\mathbf{i} + 50.0\mathbf{j} - 50.0\mathbf{k}\}\text{ N}$$
$$= \{70.7\mathbf{i} + 50.0\mathbf{j} - 50.0\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_3 = 250\{\cos 60°\mathbf{i} + \cos 135°\mathbf{j} + \cos 60°\mathbf{k}\}\text{ N}$$
$$= \{125.0\mathbf{i} - 176.78\mathbf{j} + 125.0\mathbf{k}\}\text{ N}$$
$$= \{125\mathbf{i} - 177\mathbf{j} + 125\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$= \{(70.71 + 125.0)\mathbf{i} + (224.98 + 50.0 - 176.78)\mathbf{j} + (268.12 - 50.0 + 125.0)\mathbf{k}\}\text{ N}$$
$$= \{195.71\mathbf{i} + 98.20\mathbf{j} + 343.12\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2}$$
$$= \sqrt{195.71^2 + 98.20^2 + 343.12^2}$$
$$= 407.03\text{ N} = 407\text{ N} \qquad \textbf{Ans.}$$

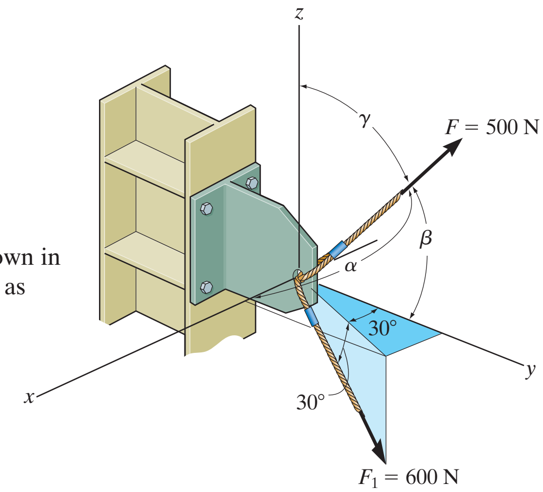
The coordinate direction angles are

$$\cos\alpha = \frac{F_{R_x}}{F_R} = \frac{195.71}{407.03} \qquad \alpha = 61.3° \qquad \textbf{Ans.}$$

$$\cos\beta = \frac{F_{R_y}}{F_R} = \frac{98.20}{407.03} \qquad \beta = 76.0° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{F_{R_z}}{F_R} = \frac{343.12}{407.03} \qquad \gamma = 32.5° \qquad \textbf{Ans.}$$

2–61. If the resultant force acting on the bracket is directed along the positive y axis, determine the magnitude of the resultant force and the coordinate direction angles of **F** so that $\beta < 90°$.



SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}$ into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and $\mathbf{F}$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 600\cos 30°\sin 30°(+\mathbf{i}) + 600\cos 30°\cos 30°(+\mathbf{j}) + 600\sin 30°(-\mathbf{k})$$

$$= \{259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}\}\ \text{N}$$

$$\mathbf{F} = 500\cos\alpha\mathbf{i} + 500\cos\beta\mathbf{j} + 500\cos\gamma\mathbf{k}$$

Since the resultant force $\mathbf{F}_R$ is directed towards the positive y axis, then

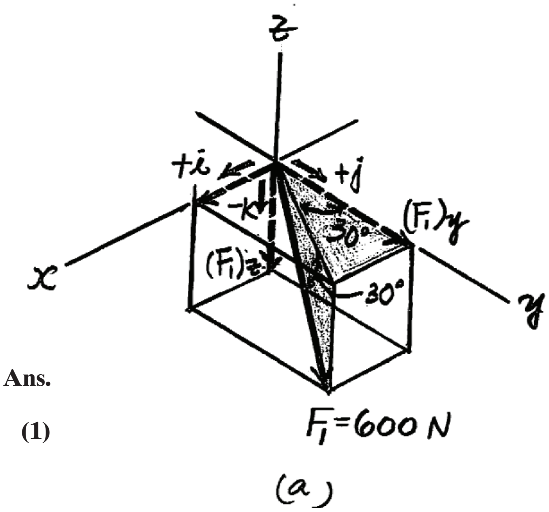
$$\mathbf{F}_R = F_R\,\mathbf{j}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$

$$F_R\,\mathbf{j} = (259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}) + (500\cos\alpha\mathbf{i} + 500\cos\beta\mathbf{j} + 500\cos\gamma\mathbf{k})$$

$$F_R\,\mathbf{j} = (259.81 + 500\cos\alpha)\mathbf{i} + (450 + 500\cos\beta)\mathbf{j} + (500\cos\gamma - 300)\mathbf{k}$$



Equating the **i**, **j**, and **k** components,

$$0 = 259.81 + 500\cos\alpha$$

$$\alpha = 121.31° = 121° \qquad \textbf{Ans.}$$

$$F_R = 450 + 500\cos\beta \qquad \textbf{(1)}$$

$$0 = 500\cos\gamma - 300$$

$$\gamma = 53.13° = 53.1° \qquad \textbf{Ans.}$$

However, since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, $\alpha = 121.31°$, and $\gamma = 53.13°$,

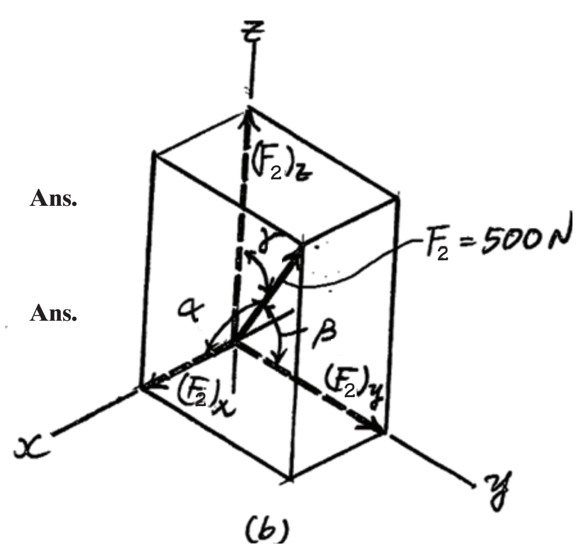
$$\cos\beta = \pm\sqrt{1 - \cos^2 121.31° - \cos^2 53.13°} = \pm 0.6083$$

If we substitute $\cos\beta = 0.6083$ into Eq. (1),

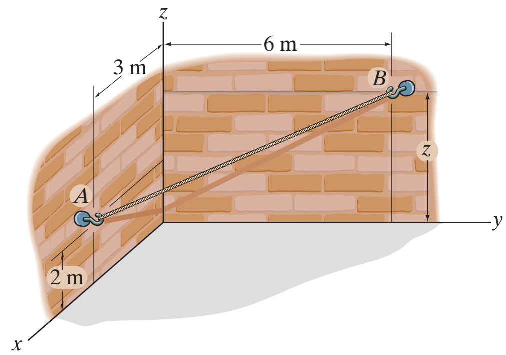
$$F_R = 450 + 500(0.6083) = 754\ \text{N} \qquad \textbf{Ans.}$$

and

$$\beta = \cos^{-1}(0.6083) = 52.5° \qquad \textbf{Ans.}$$



2–62. Determine the position vector **r** directed from point A to point B and the length of cord AB. Take $z = 4$ m.



SOLUTION

Position Vector: The coordinates for points A and B are $A(3, 0, 2)$ m and $B(0, 6, 4)$ m, respectively. Thus,

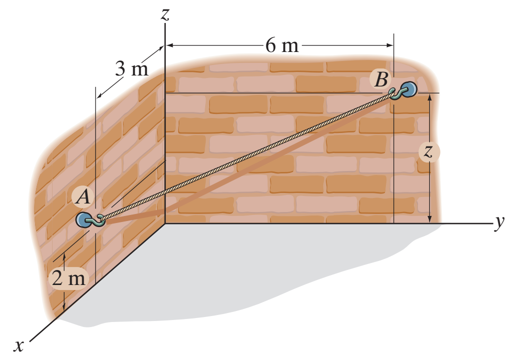
$$\mathbf{r}_{AB} = (0-3)\mathbf{i} + (6-0)\mathbf{j} + (4-2)\mathbf{k}$$

$$= \{-3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}\} \text{ m} \qquad \textbf{Ans.}$$

The length of cord AB is

$$r_{AB} = \sqrt{(-3)^2 + 6^2 + 2^2} = 7 \text{ m} \qquad \textbf{Ans.}$$

2–63. If the cord AB is 7.5 m long, determine the coordinate position $+z$ of point B.



SOLUTION

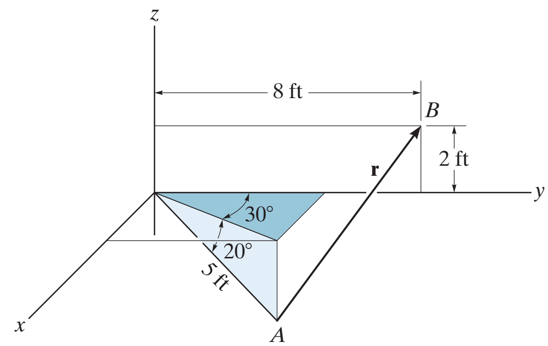
Position Vector: The coordinates for points A and B are $A(3, 0, 2)$ m and $B(0, 6, z)$ m, respectively. Thus,

$$\mathbf{r}_{AB} = (0-3)\mathbf{i} + (6-0)\mathbf{j} + (z-2)\mathbf{k}$$
$$= \{-3\mathbf{i} + 6\mathbf{j} + (z-2)\mathbf{k}\}\text{ m}$$

Since the length of cord is equal to the magnitude of $\mathbf{r}_{AB}$, then

$$r_{AB} = 7.5 = \sqrt{(-3)^2 + 6^2 + (z-2)^2}$$
$$56.25 = 45 + (z-2)^2$$
$$z - 2 = \pm 3.354$$
$$z = 5.35\text{ m} \qquad \textbf{Ans.}$$

***2–64.** Express the position vector **r** in Cartesian vector form; then determine its magnitude and coordinate direction angles.



SOLUTION

$$\mathbf{r} = (-5\cos 20^\circ \sin 30^\circ)\mathbf{i} + (8 - 5\cos 20^\circ \cos 30^\circ)\mathbf{j} + (2 + 5\sin 20^\circ)\mathbf{k}$$

$$\mathbf{r} = \{-2.35\mathbf{i} + 3.93\mathbf{j} + 3.71\mathbf{k}\}\ \text{ft}$$ **Ans.**

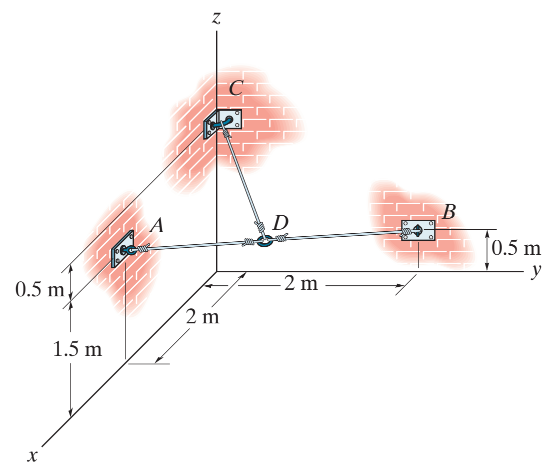
$$r = \sqrt{(-2.35)^2 + (3.93)^2 + (3.71)^2} = 5.89\ \text{ft}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{-2.35}{5.89}\right) = 113^\circ$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{3.93}{5.89}\right) = 48.2^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{3.71}{5.89}\right) = 51.0^\circ$$ **Ans.**

2–65. Determine the lengths of wires *AD*, *BD*, and *CD*. The ring at *D* is midway between *A* and *B*.



SOLUTION

$$D\left(\frac{2+0}{2}, \frac{0+2}{2}, \frac{1.5+0.5}{2}\right) \text{m} = D(1, 1, 1) \text{ m}$$

$$\mathbf{r}_{AD} = (1-2)\mathbf{i} + (1-0)\mathbf{j} + (1-1.5)\mathbf{k}$$

$$= -1\mathbf{i} + 1\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{r}_{BD} = (1-0)\mathbf{i} + (1-2)\mathbf{j} + (1-0.5)\mathbf{k}$$

$$= 1\mathbf{i} - 1\mathbf{j} + 0.5\mathbf{k}$$

$$\mathbf{r}_{CD} = (1-0)\mathbf{i} + (1-0)\mathbf{j} + (1-2)\mathbf{k}$$

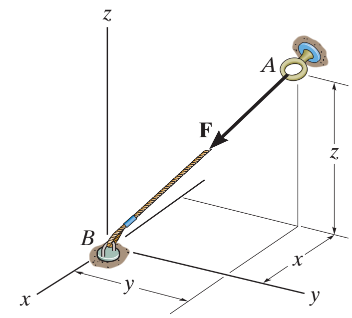
$$= 1\mathbf{i} + 1\mathbf{j} - 1\mathbf{k}$$

$r_{AD} = \sqrt{(-1)^2 + 1^2 + (-0.5)^2} = 1.50 \text{ m}$ **Ans.**

$r_{BD} = \sqrt{1^2 + (-1)^2 + 0.5^2} = 1.50 \text{ m}$ **Ans.**

$r_{CD} = \sqrt{1^2 + 1^2 + (-1)^2} = 1.73 \text{ m}$ **Ans.**

2–66. If $\mathbf{F} = \{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}\}$ N and cable AB is 9 m long, determine the x, y, z coordinates of point A.



SOLUTION

Position Vector: The position vector $\mathbf{r}_{AB}$, directed from point A to point B, is given by

$$\mathbf{r}_{AB} = [0 - (-x)]\mathbf{i} + (0 - y)\mathbf{j} + (0 - z)\mathbf{k}$$

$$= x\mathbf{i} - y\mathbf{j} - z\mathbf{k}$$

Unit Vector: Knowing the magnitude of $\mathbf{r}_{AB}$ is 9 m, the unit vector for $\mathbf{r}_{AB}$ is given by

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9}$$

The unit vector for force $\mathbf{F}$ is

$$\mathbf{u}_F = \frac{\mathbf{F}}{F} = \frac{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}}{\sqrt{350^2 + (-250)^2 + (-450)^2}} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Since force $\mathbf{F}$ is also directed from point A to point B, then

$$\mathbf{u}_{AB} = \mathbf{u}_F$$

$$\frac{x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

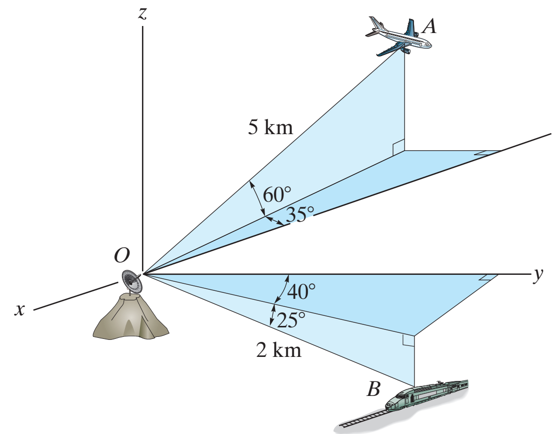
Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components,

$$\frac{x}{9} = 0.5623 \qquad x = 5.06 \text{ m} \qquad \textbf{Ans.}$$

$$\frac{-y}{9} = -0.4016 \qquad y = 3.61 \text{ m} \qquad \textbf{Ans.}$$

$$\frac{-z}{9} = 0.7229 \qquad z = 6.51 \text{ m} \qquad \textbf{Ans.}$$

2–67. At a given instant, the position of a plane at A and a train at B are measured relative to a radar antenna at O. Determine the distance d between A and B at this instant. To solve the problem, formulate a position vector, directed from A to B, and then determine its magnitude.



SOLUTION

Position Vector: The coordinates of points A and B are

$$A(-5\cos 60°\cos 35°, -5\cos 60°\sin 35°, 5\sin 60°)\text{ km}$$

$$= A(-2.048, -1.434, 4.330)\text{ km}$$

$$B(2\cos 25°\sin 40°, 2\cos 25°\cos 40°, -2\sin 25°)\text{ km}$$

$$= B(1.165, 1.389, -0.845)\text{ km}$$

The position vector $\mathbf{r}_{AB}$ can be established from the coordinates of points A and B.

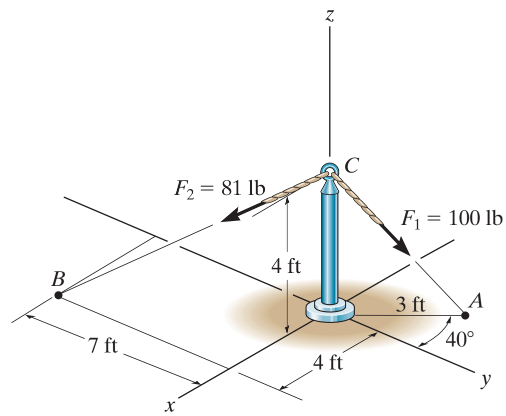
$$\mathbf{r}_{AB} = \{[1.165 - (-2.048)]\mathbf{i} + [1.389 - (-1.434)]\mathbf{j} + (-0.845 - 4.330)\mathbf{k}\}\text{ km}$$

$$= \{3.213\mathbf{i} + 2.822\mathbf{j} - 5.175)\mathbf{k}\}\text{ km}$$

The distance between points A and B is

$$d = r_{AB} = \sqrt{3.213^2 + 2.822^2 + (-5.175)^2} = 6.71\text{ km}$$ **Ans.**

***2–68.** Determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$$\mathbf{F}_1 = -100\left(\frac{3}{5}\right)\sin 40°\,\mathbf{i} + 100\left(\frac{3}{5}\right)\cos 40°\,\mathbf{j} - 100\left(\frac{4}{5}\right)\mathbf{k}$$

$$= \{-38.567\,\mathbf{i} + 45.963\,\mathbf{j} - 80\,\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_2 = 81\text{ lb}\left(\frac{4}{9}\,\mathbf{i} - \frac{7}{9}\,\mathbf{j} - \frac{4}{9}\,\mathbf{k}\right)$$

$$= \{36\,\mathbf{i} - 63\,\mathbf{j} - 36\,\mathbf{k}\}\text{lb}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = \{-2.567\,\mathbf{i} - 17.04\,\mathbf{j} - 116.0\,\mathbf{k}\}\text{ lb}$$

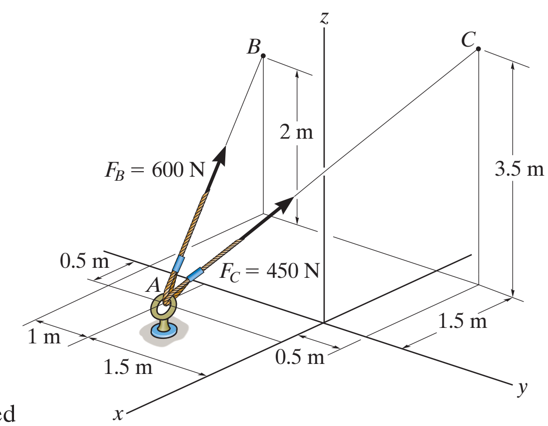
$$F_R = \sqrt{(-2.567)^2 + (-17.04)^2 + (-116.0)^2} = 117.27\text{ lb} = 117\text{ lb} \quad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{-2.567}{117.27}\right) = 91.3° \quad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{-17.04}{117.27}\right) = 98.4° \quad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-116.0}{117.27}\right) = 172° \quad \textbf{Ans}$$

2–69. Express $\mathbf{F}_B$ and $\mathbf{F}_C$ in Cartesian vector form.



SOLUTION

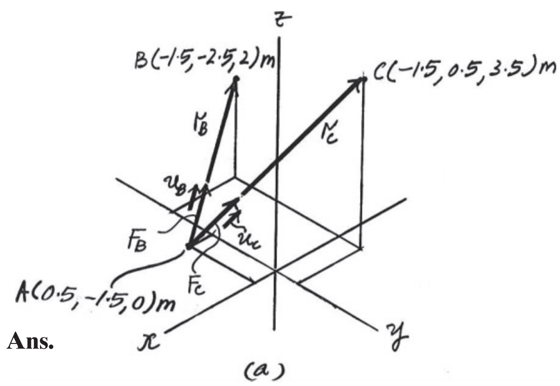
Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-1.5 - 0.5)\mathbf{i} + [-2.5 - (-1.5)]\mathbf{j} + (2 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [-2.5 - (-1.5)]^2 + (2 - 0)^2}}$$

$$= -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-1.5 - 0.5)\mathbf{i} + [0.5 - (-1.5)]\mathbf{j} + (3.5 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [0.5 - (-1.5)]^2 + (3.5 - 0)^2}}$$

$$= -\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}$$



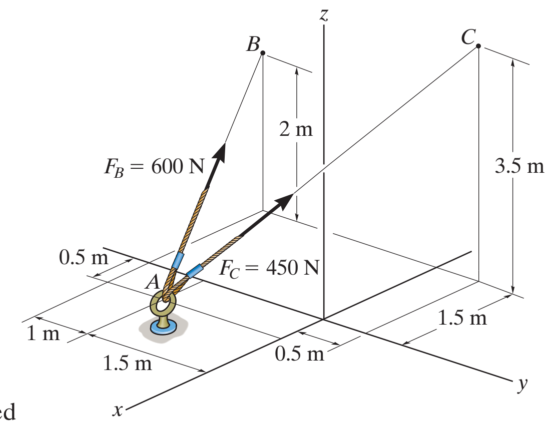
(a)

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 600\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_C = F_C\mathbf{u}_C = 450\left(-\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}\right) = \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\}\text{ N}$$ **Ans.**

2–70. Determine the magnitude and coordinate direction angles of the resultant force acting at A.

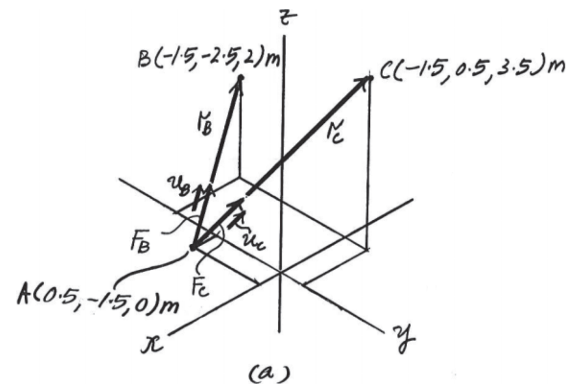


SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-1.5-0.5)\mathbf{i} + [-2.5-(-1.5)]\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(-1.5-0.5)^2 + [-2.5-(-1.5)]^2 + (2-0)^2}}$$

$$= -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$



$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-1.5-0.5)\mathbf{i} + [0.5-(-1.5)]\mathbf{j} + (3.5-0)\mathbf{k}}{\sqrt{(-1.5-0.5)^2 + [0.5-(-1.5)]^2 + (3.5-0)^2}}$$

$$= -\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 600\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 450\left(-\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}\right) = \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}) + (-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k})$$

$$= \{-600\mathbf{i} + 750\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(-600)^2 + 0^2 + 750^2} = 960.47\text{ N} = 960\text{ N}$$

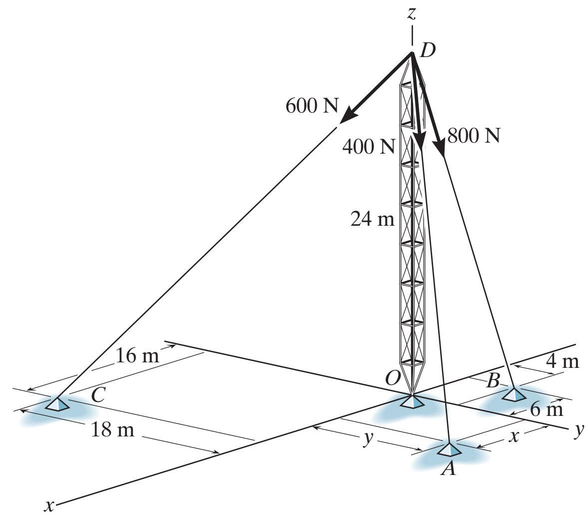
The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{-600}{960.47}\right) = 129^\circ$$ **Ans.**

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{0}{960.47}\right) = 90^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{760}{960.47}\right) = 38.7^\circ$$ **Ans.**

2–71. The tower is held in place by three cables. If the force of each cable acting on the tower is shown, determine the magnitude and coordinate direction angles α, β, γ of the resultant force. Take $x = 20$ m, $y = 15$ m.



SOLUTION

$$\mathbf{F}_{DA} = 400\left(\frac{20}{34.66}\mathbf{i} + \frac{15}{34.66}\mathbf{j} - \frac{24}{34.66}\mathbf{k}\right) \text{N}$$

$$\mathbf{F}_{DB} = 800\left(\frac{-6}{25.06}\mathbf{i} + \frac{4}{25.06}\mathbf{j} - \frac{24}{25.06}\mathbf{k}\right) \text{N}$$

$$\mathbf{F}_{DC} = 600\left(\frac{16}{34}\mathbf{i} - \frac{18}{34}\mathbf{j} - \frac{24}{34}\mathbf{k}\right) \text{N}$$

$$\mathbf{F}_R = \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC}$$

$$= \{321.66\mathbf{i} - 16.82\mathbf{j} - 1466.71\mathbf{k}\} \text{ N}$$

$$F_R = \sqrt{(321.66)^2 + (-16.82)^2 + (-1466.71)^2}$$

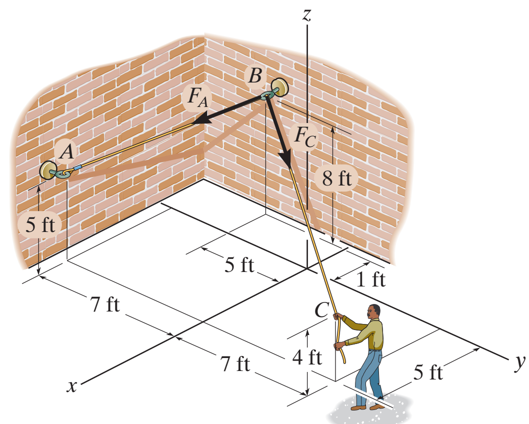
$$= 1501.66 \text{ N} = 1.50 \text{ kN}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{321.66}{1501.66}\right) = 77.6^\circ$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{-16.82}{1501.66}\right) = 90.6^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-1466.71}{1501.66}\right) = 168^\circ$$ **Ans.**

***2–72.** The man pulls on the rope at C with a force of 70 lb which causes the forces $\mathbf{F}_A$ and $\mathbf{F}_C$ at B to have this same magnitude. Express each of these two forces as Cartesian vectors.



SOLUTION

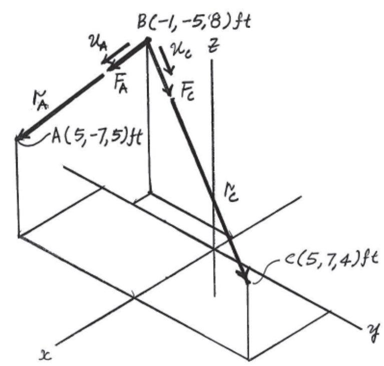
Unit Vectors: The coordinate points A, B, and C are shown in Fig. a. Thus,

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{[5-(-1)]\mathbf{i} + [-7-(-5)]\mathbf{j} + (5-8)\mathbf{k}}{\sqrt{[5-(-1)]^2 + [-7-(-5)]^2 + (5-8)^2}}$$

$$= \frac{6}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{[5-(-1)]\mathbf{i} + [-7(-5)]\mathbf{j} + (4-8)\mathbf{k}}{\sqrt{[5-(-1)]^2 + [-7(-5)]^2 + (4-8)^2}}$$

$$= \frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$



(a)

Force Vectors: Multiplying the magnitude of the force with its unit vector,

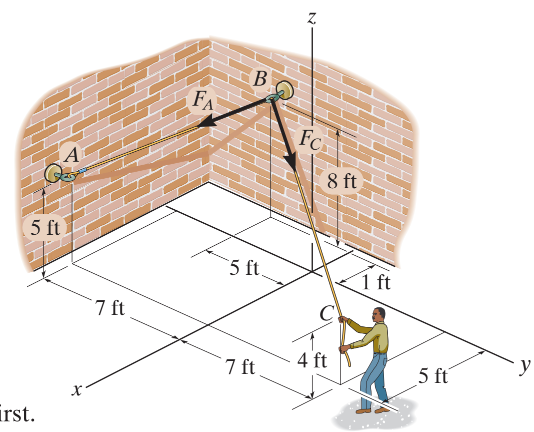
$$\mathbf{F}_A = F_A\mathbf{u}_A = 70\left(\frac{6}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}\right)$$

$$= \{60\mathbf{i} - 20\mathbf{j} + 30\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_C = F_C\mathbf{u}_C = 70\left(\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)$$

$$= \{30\mathbf{i} + 60\mathbf{j} + 20\mathbf{k}\}\text{ lb}$$ **Ans.**

2–73. The man pulls on the rope at C with a force of 70 lb which causes the forces $\mathbf{F}_A$ and $\mathbf{F}_C$ at B to have this same magnitude. Determine the magnitude and coordinate direction angles of the resultant force acting at B.



SOLUTION

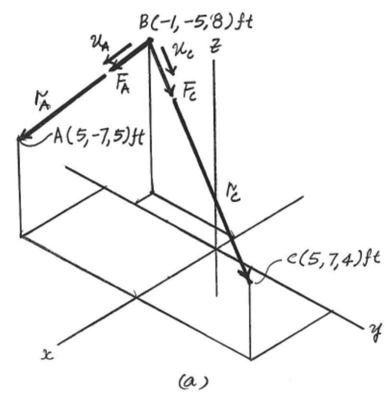
Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{[5-(-1)]\mathbf{i} + [-7(-5)]\mathbf{j} + (5-8)\mathbf{k}}{\sqrt{[5-(-1)]^2 + [-7(-5)]^2 + (5-8)^2}}$$

$$= \frac{6}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{[5-(-1)]\mathbf{i} + [-7(-5)]\mathbf{j} + (4-8)\mathbf{k}}{\sqrt{[5-(-1)]^2 + [-7(-5)]^2 + (4-8)^2}}$$

$$= \frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$



Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_A = F_A\mathbf{u}_A = 70\left(\frac{6}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}\right) = \{60\mathbf{i} - 20\mathbf{j} + 30\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 70\left(\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right) = \{30\mathbf{i} + 60\mathbf{j} + 20\mathbf{k}\}\text{ lb}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_C = (60\mathbf{i} - 20\mathbf{j} - 30\mathbf{k}) + (30\mathbf{i} + 60\mathbf{j} - 20\mathbf{k})$$

$$= \{90\mathbf{i} + 40\mathbf{j} - 50\mathbf{k}\}\text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(90)^2 + (40)^2 + (-50)^2} = 110.45\text{ lb} = 110\text{ lb} \qquad \textbf{Ans.}$$

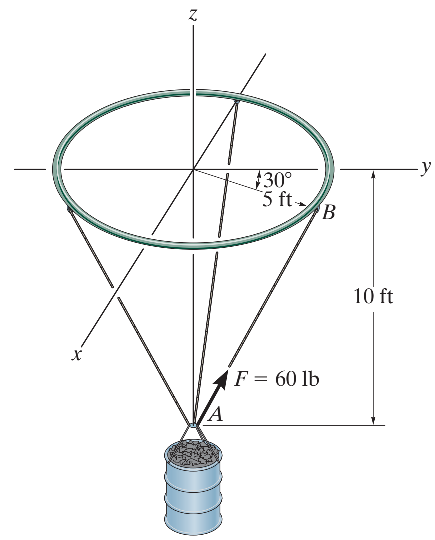
The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{90}{110.45}\right) = 35.4° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{40}{110.45}\right) = 68.8° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-50}{110.45}\right) = 117° \qquad \textbf{Ans.}$$

2–74. The load at A creates a force of 60 lb in wire AB. Express this force as a Cartesian vector acting on A and directed toward B as shown.



SOLUTION

Unit Vector: First determine the position vector $\mathbf{r}_{AB}$. The coordinates of point B are

$B\,(5\sin 30°, 5\cos 30°, 0)\text{ ft} = B\,(2.50, 4.330, 0)\text{ ft}$

Then

$$\mathbf{r}_{AB} = \{(2.50 - 0)\mathbf{i} + (4.330 - 0)\mathbf{j} + [0 - (-10)]\mathbf{k}\}\text{ ft}$$

$$= \{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{2.50^2 + 4.330^2 + 10.0^2} = 11.180\text{ ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}}{11.180}$$

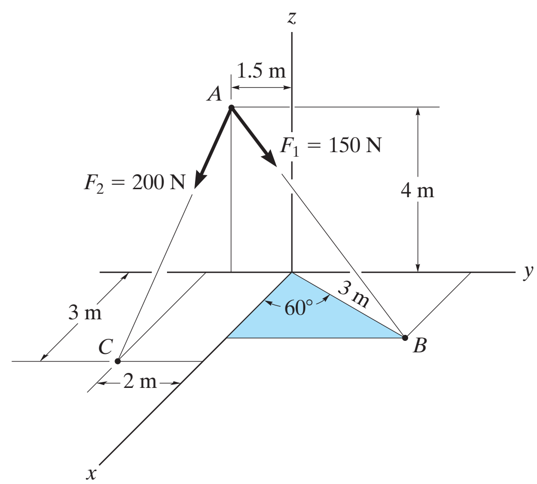
$$= 0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}$$

Force Vector:

$$\mathbf{F} = F\mathbf{u}_{AB} = 60\,\{0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}\}\text{ lb}$$

$$= \{13.4\mathbf{i} + 23.2\mathbf{j} + 53.7\mathbf{k}\}\text{ lb}$$ **Ans.**

2–75. Determine the magnitude and coordinate direction angles of the resultant force acting at point A.



SOLUTION

$$\mathbf{r}_{AC} = \{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}\}\text{ m}$$

$$|\mathbf{r}_{AC}| = \sqrt{3^2 + (-0.5)^2 + (-4)^2} = \sqrt{25.25} = 5.02494$$

$$\mathbf{F}_2 = 200\left(\frac{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}}{5.02494}\right) = (119.4044\mathbf{i} - 19.9007\mathbf{j} - 159.2059\mathbf{k})$$

$$\mathbf{r}_{AB} = (3\cos 60°\mathbf{i} + (1.5 + 3\sin 60°)\mathbf{j} - 4\mathbf{k})$$

$$\mathbf{r}_{AB} = (1.5\mathbf{i} + 4.0981\mathbf{j} + 4\mathbf{k})$$

$$|\mathbf{r}_{AB}| = \sqrt{(1.5)^2 + (4.0981)^2 + (-4)^2} = 5.9198$$

$$\mathbf{F}_1 = 150\left(\frac{1.5\mathbf{i} + 4.0981\mathbf{j} - 4\mathbf{k}}{5.9198}\right) = (38.0079\mathbf{i} + 103.8396\mathbf{j} - 101.3545\mathbf{k})$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = (157.4124\mathbf{i} + 83.9389\mathbf{j} - 260.5607\mathbf{k})$$

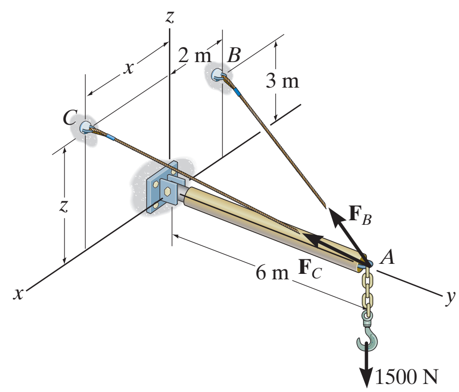
$$F_R = \sqrt{(157.4124)^2 + (83.9389)^2 + (-260.5604)^2} = 315.7786 = 316\text{ N} \qquad \textbf{Ans.}$$

$$\alpha = \cos^{-1}\left(\frac{157.4124}{315.7786}\right) = 60.100° = 60.1° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{83.9389}{315.7786}\right) = 74.585° = 74.6° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{-260.5607}{315.7786}\right) = 145.60° = 146° \qquad \textbf{Ans.}$$

***2–76.** Two cables are used to secure the overhang boom in position and support the 1500-N load. If the resultant force is directed along the boom from point A towards O, determine the magnitudes of the resultant force and forces $\mathbf{F}_B$ and $\mathbf{F}_C$. Set $x = 3$ m and $z = 2$ m.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ must be determined first. From Fig. a,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-2-0)\mathbf{i}+(0-6)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(-2-0)^2+(0-6)^2+(3-0)^2}} = -\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i}+(0-6)\mathbf{j}+(2-0)\mathbf{k}}{\sqrt{(3-0)^2+(0-6)^2+(2-0)^2}} = \frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = -\frac{2}{7}F_B\mathbf{i}-\frac{6}{7}F_B\mathbf{j}+\frac{3}{7}F_B\mathbf{k}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = \frac{3}{7}F_C\mathbf{i}-\frac{6}{7}F_C\mathbf{j}+\frac{2}{7}F_C\mathbf{k}$$

Since the resultant force $\mathbf{F}_R$ is directed along the negative y axis, and the load $\mathbf{W}$ is directed along the z axis, these two forces can be written as

$$\mathbf{F}_R = -F_R\mathbf{j} \qquad \text{and} \qquad \mathbf{W} = [-1500\mathbf{k}]\ \text{N}$$

Resultant Force: The vector addition of $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{W}$ is equal to $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{W}$$

$$-F_R\mathbf{j} = \left(-\frac{2}{7}F_B\mathbf{i}-\frac{6}{7}F_B\mathbf{j}+\frac{3}{7}F_B\mathbf{k}\right)+\left(\frac{3}{7}F_C\mathbf{i}-\frac{6}{7}F_C\mathbf{j}+\frac{2}{7}F_C\mathbf{k}\right)+(-1500\mathbf{k})$$

$$-F_R\mathbf{j} = \left(-\frac{2}{7}F_B+\frac{3}{7}F_C\right)\mathbf{i}+\left(-\frac{6}{7}F_B-\frac{6}{7}F_C\right)\mathbf{j}+\left(\frac{3}{7}F_B+\frac{2}{7}F_C-1500\right)\mathbf{k}$$

Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components,

$$0 = -\frac{2}{7}F_B + \frac{3}{7}F_C \qquad (1)$$

$$-F_R = -\frac{6}{7}F_B - \frac{6}{7}F_C \qquad (2)$$

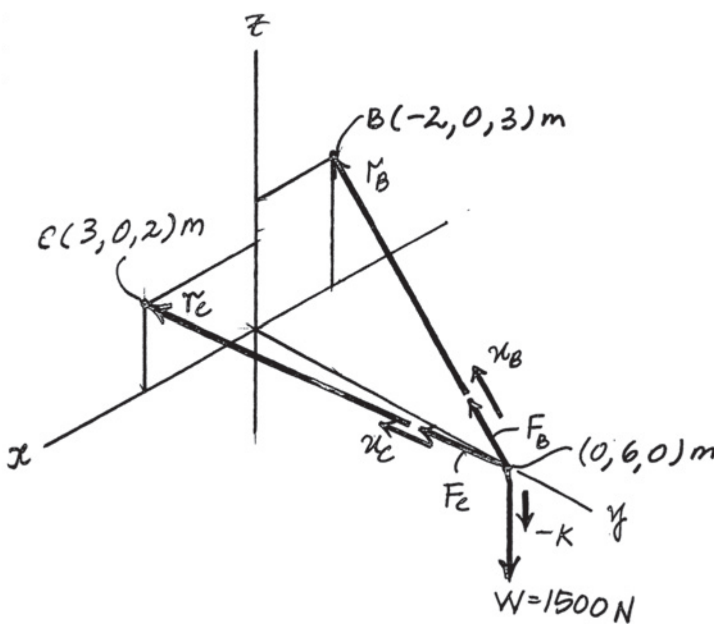
$$0 = \frac{3}{7}F_B + \frac{2}{7}F_C - 1500 \qquad (3)$$

Solving Eqs. (1), (2), and (3) yields

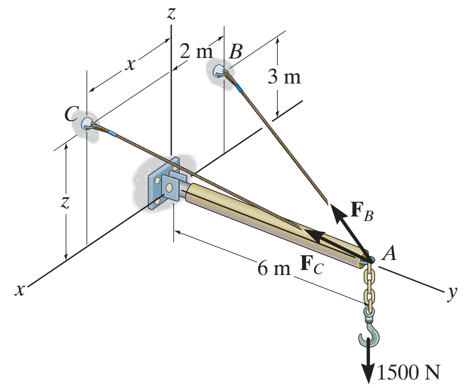
$F_C = 1615.38\text{ N} = 1.62\text{ kN}$ **Ans.**

$F_B = 2423.08\text{ N} = 2.42\text{ kN}$ **Ans.**

$F_R = 3461.53\text{ N} = 3.46\text{ kN}$ **Ans.**



2–77. Two cables are used to secure the overhang boom in position and support the 1500-N load. If the resultant force is directed along the boom from point *A* towards *O*, determine the values of *x* and *z* for the coordinates of point *C* and the magnitude of the resultant force. Set $F_B = 1610$ N and $F_C = 2400$ N.



SOLUTION

Force Vectors: From Fig. *a*,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-2-0)\mathbf{i}+(0-6)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(-2-0)^2+(0-6)^2+(3-0)^2}} = -\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}$$

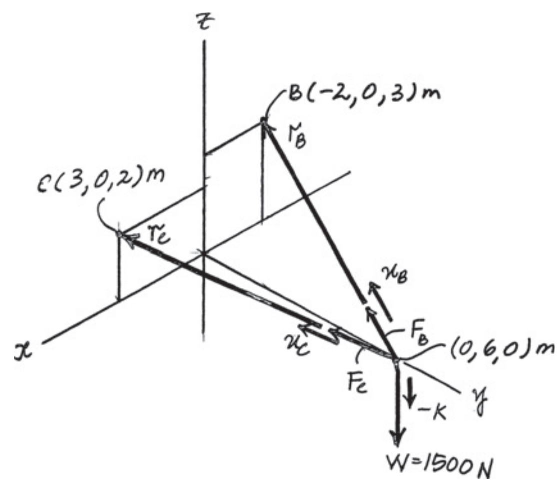
$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(x-0)\mathbf{i}+(0-6)\mathbf{j}+(z-0)\mathbf{k}}{\sqrt{(x-0)^2+(0-6)^2+(z-0)^2}} = \frac{x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{6}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{z}{\sqrt{x^2+z^2+36}}\mathbf{k}$$

Thus,

$$\mathbf{F}_B = F_B\mathbf{u}_B = 1610\left(-\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}\right) = [-460\mathbf{i}-1380\mathbf{j}+690\mathbf{k}]\text{N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 2400\left(\frac{x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{6}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{z}{\sqrt{x^2+z^2+36}}\mathbf{k}\right)$$

$$= \frac{2400x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{14\,400}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{2400z}{\sqrt{x^2+z^2+36}}\mathbf{k}$$



Since the resultant force $\mathbf{F}_R$ is directed along the negative *y* axis, and the load is directed along the *z* axis, these two forces can be written as

$$\mathbf{F}_R = -F_R\mathbf{j} \qquad \text{and} \qquad \mathbf{W} = [-1500\mathbf{k}]\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{W}$$

$$-F_R\mathbf{j} = (-460\mathbf{i}-1380\mathbf{j}+690\mathbf{k})+\left(\frac{2400x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{14\,400}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{2400z}{\sqrt{x^2+z^2+36}}\mathbf{k}\right)+(-1500\mathbf{k})$$

$$-F_R\mathbf{j} = \left(\frac{2400x}{\sqrt{x^2+z^2+36}}-460\right)\mathbf{i}-\left(\frac{14\,400}{\sqrt{x^2+z^2+36}}+1380\right)\mathbf{j}+\left(690+\frac{2400z}{\sqrt{x^2+z^2+36}}-1500\right)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = \frac{2400x}{\sqrt{x^2+z^2+36}}-460 \qquad \frac{2400x}{\sqrt{x^2+z^2+36}} = 460 \qquad (1)$$

$$-F_R = -\left(\frac{14400}{\sqrt{x^2+z^2+36}}+1380\right) \qquad F_R = \frac{14400}{\sqrt{x^2+z^2+36}}+1380 \qquad (2)$$

$$0 = 690+\frac{2400z}{\sqrt{x^2+z^2+36}}-1500 \qquad \frac{2400z}{\sqrt{x^2+z^2+36}} = 810 \qquad (3)$$

Dividing Eq. (1) by Eq. (3), yields

$$x = 0.5679z \qquad (4)$$

Substituting Eq. (4) into Eq. (1), and solving

$$z = 2.197\text{ m} = 2.20\text{ m} \qquad \textbf{Ans.}$$

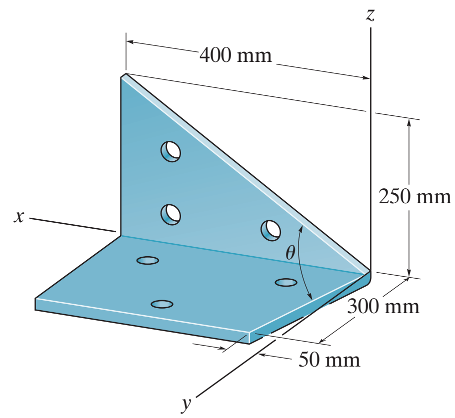
Substituting $z = 2.197$ m into Eq. (4), yields

$$x = 1.248\text{ m} = 1.25\text{ m} \qquad \textbf{Ans.}$$

Substituting $x = 1.248$ m and $z = 2.197$ m into Eq. (2), yields

$$F_R = 3591.85\text{ N} = 3.59\text{ kN} \qquad \textbf{Ans.}$$

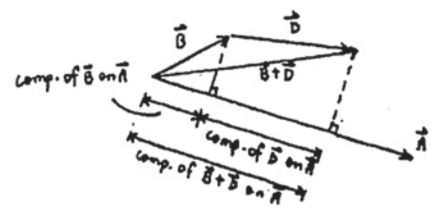
2–78. Given the three vectors **A, B,** and **D,** show that $\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D})$.



SOLUTION

Since the component of $(\mathbf{B}+\mathbf{D})$ is equal to the sum of the components of **B** and **D**, then

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{D} \qquad \text{(QED)}$$



Also,

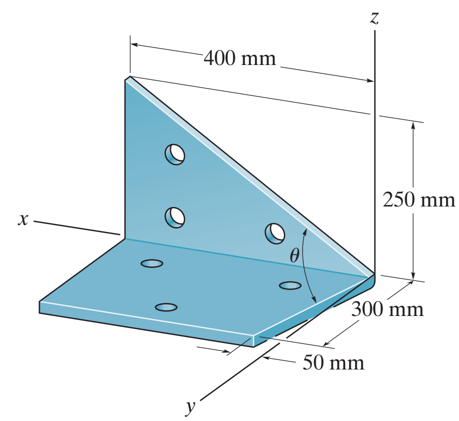
$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\cdot[(B_x + D_x)\mathbf{i} + (B_y + D_y)\mathbf{j} + (B_z + D_z)\mathbf{k}]$$

$$= A_x(B_x + D_x) + A_y(B_y + D_y) + A_z(B_z + D_z)$$

$$= (A_xB_x + A_yB_y + A_zB_z) + (A_xD_x + A_yD_y + A_zD_z)$$

$$= (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D}) \qquad \text{(QED)}$$

2–79. Determine the angle θ between the edges of the sheet-metal bracket.



SOLUTION

$\mathbf{r}_1 = \{400\mathbf{i} + 250\mathbf{k}\}$ mm ; $\quad r_1 = 471.70$ mm

$\mathbf{r}_2 = \{50\mathbf{i} + 300\mathbf{j}\}$ mm ; $\quad r_2 = 304.14$ mm

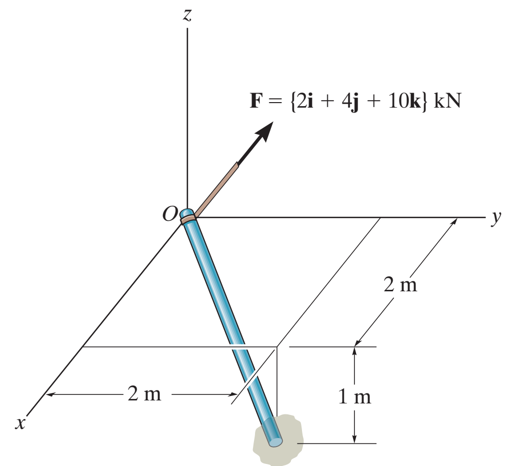
$$\mathbf{r}_1 \cdot \mathbf{r}_2 = (400)\,(50) + 0(300) + 250(0) = 20\,000$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{r_1\, r_2}\right)$$

$$= \cos^{-1}\left(\frac{20\,000}{(471.70)\,(304.14)}\right) = 82.0°$$

Ans.

***2–80.** Determine the projection of the force **F** along the pole.

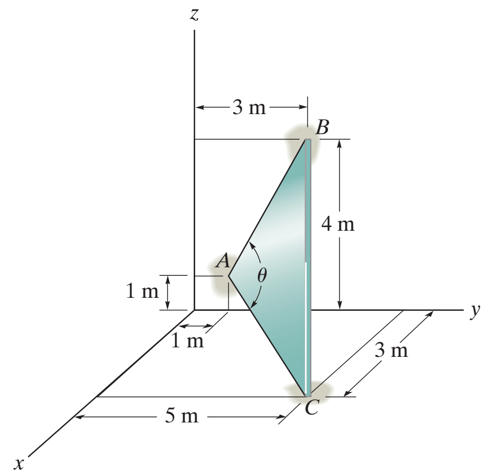


SOLUTION

$$\text{Proj } F = \mathbf{F}\cdot\mathbf{u}_a = (2\,\mathbf{i} + 4\,\mathbf{j} + 10\,\mathbf{k})\cdot\left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}\right)$$

$$\text{Proj } F = 0.667\text{ kN}$$

2–81. Determine the length of side BC of the triangular plate. Solve the problem by finding the magnitude of $\mathbf{r}_{BC}$; then check the result by first finding θ, r_{AB}, and r_{AC} and then using the cosine law.



SOLUTION

$$\mathbf{r}_{BC} = \{3\,\mathbf{i} + 2\,\mathbf{j} - 4\,\mathbf{k}\}\text{ m}$$

$$r_{BC} = \sqrt{(3)^2 + (2)^2 + (-4)^2} = 5.39\text{ m} \qquad \textbf{Ans}$$

Also,

$$\mathbf{r}_{AC} = \{3\,\mathbf{i} + 4\,\mathbf{j} - 1\,\mathbf{k}\}\text{ m}$$

$$r_{AC} = \sqrt{(3)^2 + (4)^2 + (-1)^2} = 5.0990\text{ m}$$

$$\mathbf{r}_{AB} = \{2\,\mathbf{j} + 3\,\mathbf{k}\}\text{ m}$$

$$r_{AB} = \sqrt{(2)^2 + (3)^2} = 3.6056\text{ m}$$

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = 0 + 4(2) + (-1)(3) = 5$$

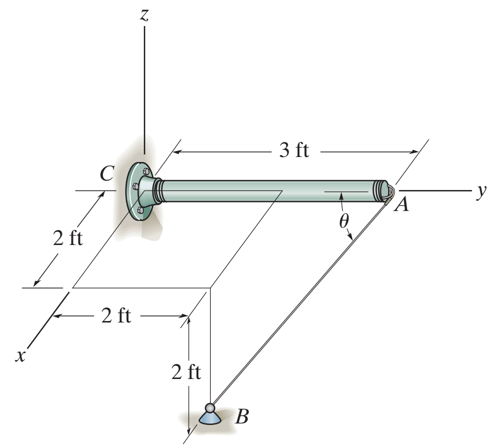
$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{r_{AC}\,r_{AB}}\right) = \cos^{-1}\frac{5}{(5.0990)(3.6056)}$$

$$\theta = 74.219^\circ$$

$$r_{BC} = \sqrt{(5.0990)^2 + (3.6056)^2 - 2(5.0990)(3.6056)\cos 74.219^\circ}$$

$$r_{BC} = 5.39\text{ m} \qquad \textbf{Ans}$$

2–82. Determine the angle θ between the y axis of the pole and the wire AB.



SOLUTION

Position Vector:

$$\mathbf{r}_{AC} = \{-3\mathbf{j}\}\text{ ft}$$

$$\mathbf{r}_{AB} = \{(2-0)\mathbf{i} + (2-3)\mathbf{j} + (-2-0)\mathbf{k}\}\text{ ft}$$

$$= \{2\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$

The magnitudes of the position vectors are

$$r_{AC} = 3.00\text{ ft} \qquad r_{AB} = \sqrt{2^2 + (-1)^2 + (-2)^2} = 3.00\text{ ft}$$

The Angles Between Two Vectors θ: The dot product of two vectors must be determined first.

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = (-3\mathbf{j}) \cdot (2\mathbf{i} - 1\mathbf{j} - 2\mathbf{k})$$

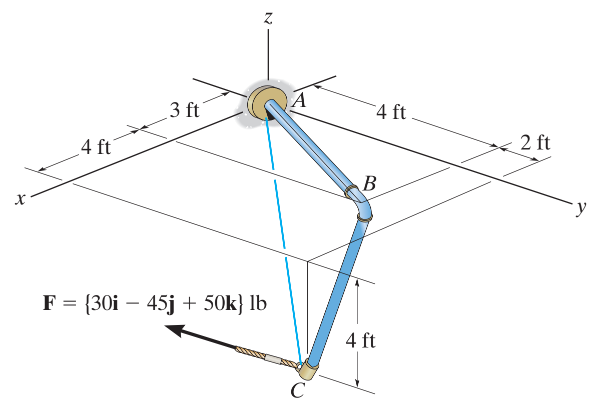
$$= 0(2) + (-3)(-1) + 0(-2)$$

$$= 3$$

Then,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AO} \cdot \mathbf{r}_{AB}}{r_{AO} r_{AB}}\right) = \cos^{-1}\left[\frac{3}{3.00(3.00)}\right] = 70.5^\circ \qquad \textbf{Ans.}$$

2–83. Determine the magnitudes of the components of **F** acting along and perpendicular to segment *BC* of the pipe assembly.



SOLUTION

Unit Vector: The unit vector $\mathbf{u}_{CB}$ must be determined first. From Fig. *a*

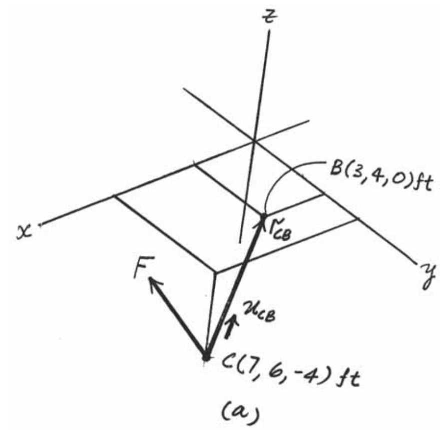
$$\mathbf{u}_{CB} = \frac{\mathbf{r}_{CB}}{r_{CB}} = \frac{(3-7)\mathbf{i} + (4-6)\mathbf{j} + [0-(-4)]\mathbf{k}}{\sqrt{(3-7)^2 + (4-6)^2 + [0-(-4)]^2}} = -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** parallel to segment *BC* of the pipe assembly is

$$(F_{BC})_{\text{pa}} = \mathbf{F}\cdot\mathbf{u}_{CB} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k})\cdot\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$

$$= (30)\left(-\frac{2}{3}\right) + (-45)\left(-\frac{1}{3}\right) + 50\left(\frac{2}{3}\right)$$

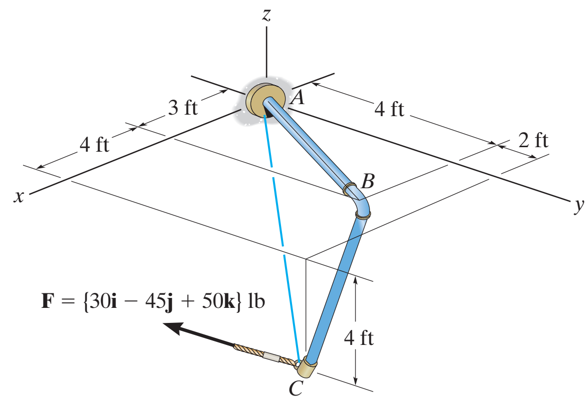
$$= 28.33 \text{ lb} = 28.3 \text{ lb}$$ **Ans.**



The magnitude of **F** is $F = \sqrt{30^2 + (-45)^2 + 50^2} = \sqrt{5425}$ lb. Thus, the magnitude of the component of **F** perpendicular to segment *BC* of the pipe assembly can be determined from

$$(F_{BC})_{\text{pr}} = \sqrt{F^2 - (F_{BC})_{\text{pa}}^{\,2}} = \sqrt{5425 - 28.33^2} = 68.0 \text{ lb}$$ **Ans.**

***2–84.** Determine the magnitude of the projected component of **F** along *AC*. Express this component as a Cartesian vector.



SOLUTION

Unit Vector: The unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*

$$\mathbf{u}_{AC} = \frac{(7-0)\mathbf{i} + (6-0)\mathbf{j} + (-4-0)\mathbf{k}}{\sqrt{(7-0)^2 + (6-0)^2 + (-4-0)^2}} = 0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** along line *AC* is

$$F_{AC} = \mathbf{F}\cdot\mathbf{u}_{AC} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k})\cdot(0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k})$$

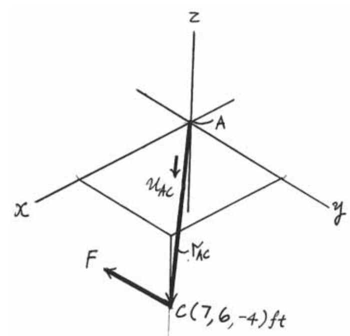
$$= (30)(0.6965) + (-45)(0.5970) + 50(-0.3980)$$

$$= 25.87 \text{ lb}$$ **Ans.**

Thus, $\mathbf{F}_{AC}$ expressed in Cartesian vector form is

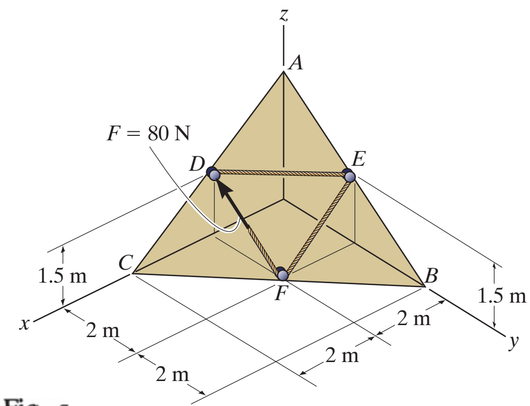
$$\mathrm{F}_{AC} = F_{AC}\,\mathbf{u}_{AC} = -25.87(0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k})$$

$$= \{-18.0\mathbf{i} - 15.4\mathbf{j} + 10.3\mathbf{k}\} \text{ lb}$$ **Ans.**



(a)

2–85. Determine the projection of force $F = 80$ N along line BC. Express the result as a Cartesian vector.



SOLUTION

Unit Vectors: The unit vectors $\mathbf{u}_{FD}$ and $\mathbf{u}_{FC}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{FD} = \frac{\mathbf{r}_{FD}}{r_{FD}} = \frac{(2-2)\mathbf{i}+(0-2)\mathbf{j}+(1.5-0)\mathbf{k}}{\sqrt{(2-2)^2+(0-2)^2+(1.5-0)^2}} = -\frac{4}{5}\mathbf{j}+\frac{3}{5}\mathbf{k}$$

$$\mathbf{u}_{FC} = \frac{\mathbf{r}_{FC}}{r_{FC}} = \frac{(4-2)\mathbf{i}+(0-2)\mathbf{j}+(0-0)\mathbf{k}}{\sqrt{(4-2)^2+(0-2)^2+(0-0)^2}} = 0.7071\mathbf{i}-0.7071\mathbf{j}$$

Thus, the force vector **F** is given by

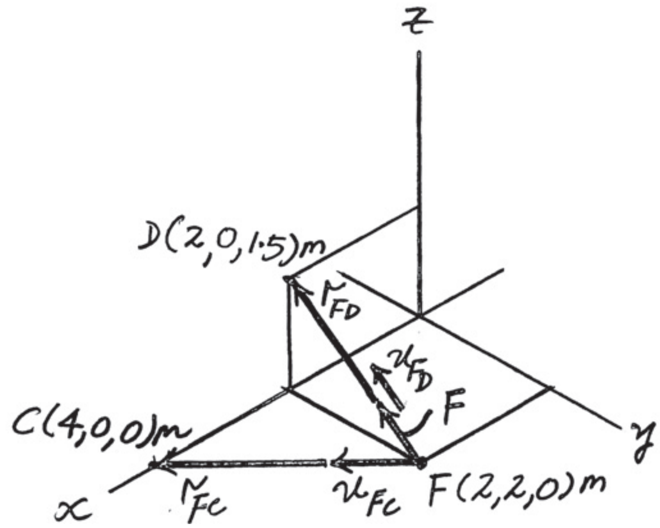
$$\mathbf{F} = F\mathbf{u}_{FD} = 80\left(-\frac{4}{5}\mathbf{j}+\frac{3}{5}\mathbf{k}\right) = [-64\mathbf{j}+48\mathbf{k}]\text{N}$$

Vector Dot Product: The magnitude of the projected component of **F** along line BC is

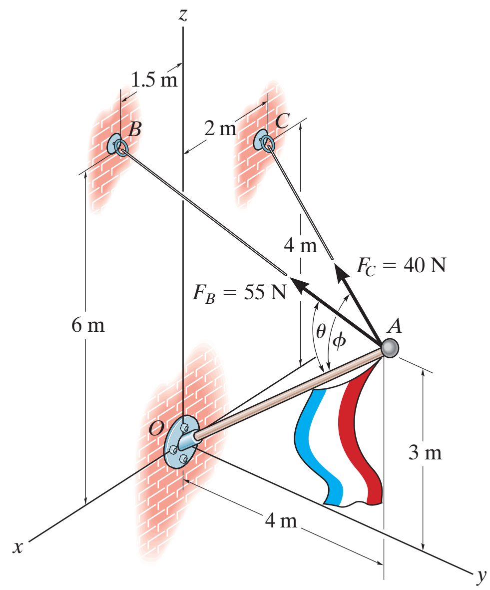
$$\begin{aligned} F_{BC} = \mathbf{F}\cdot\mathbf{u}_{FC} &= (-64\mathbf{j}+48\mathbf{k})\cdot(0.7071\mathbf{i}-0.7071\mathbf{j}) \\ &= (0)(0.7071)+(-64)(-0.7071)+48(0) \\ &= 45.25 = 45.2\text{ N} \end{aligned}$$ **Ans.**

The component of $\mathbf{F}_{BC}$ can be expressed in Cartesian vector form as

$$\begin{aligned} \mathbf{F}_{BC} = F_{BC}(\mathbf{u}_{FC}) &= 45.25(0.7071\mathbf{i}-0.7071\mathbf{j}) \\ &= \{32\mathbf{i}-32\mathbf{j}\}\text{ N} \end{aligned}$$ **Ans.**



2–86. Determine the angles θ and ϕ made between the axes OA of the flag pole and AB and AC, respectively, of each cable.



SOLUTION

$\mathbf{r}_{AC} = \{-2\mathbf{i} - 4\mathbf{j} + 1\mathbf{k}\}$ m ; $\quad r_{AC} = 4.58$ m

$\mathbf{r}_{AB} = \{1.5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}\}$ m; $\quad r_{AB} = 5.22$ m

$\mathbf{r}_{AO} = \{-4\mathbf{j} - 3\mathbf{k}\}$ m; $\quad r_{AO} = 5.00$ m

$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AO} = (1.5)(0) + (-4)(-4) + (3)(-3) = 7$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AO}}{r_{AB}\, r_{AO}}\right)$$

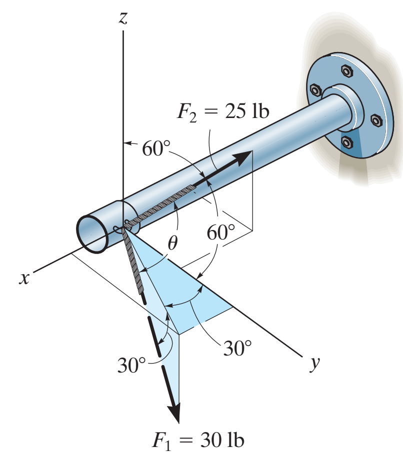
$$= \cos^{-1}\left(\frac{7}{5.22(5.00)}\right) = 74.4^\circ$$ **Ans.**

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AO} = (-2)(0) + (-4)(-4) + (1)(-3) = 13$$

$$\phi = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AO}}{r_{AC}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{13}{4.58(5.00)}\right) = 55.4^\circ$$ **Ans.**

2–87. Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.



SOLUTION

Force Vector:

$$\mathbf{u}_{F_1} = \cos 30° \sin 30°\mathbf{i} + \cos 30° \cos 30°\mathbf{j} - \sin 30°\mathbf{k}$$

$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{F}_1 = F_R\mathbf{u}_{F_1} = 30(0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k})\text{ lb}$$

$$= \{12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}\}\text{ lb}$$

Unit Vector: One can obtain the angle $\alpha = 135°$ for $\mathbf{F}_2$ using Eq. 2–8. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, with $\beta = 60°$ and $\gamma = 60°$. The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 135°\mathbf{i} + \cos 60°\mathbf{j} + \cos 60°\mathbf{k} = -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

Projected Component of $\mathbf{F}_1$ Along the Line of Action of $\mathbf{F}_2$:

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$

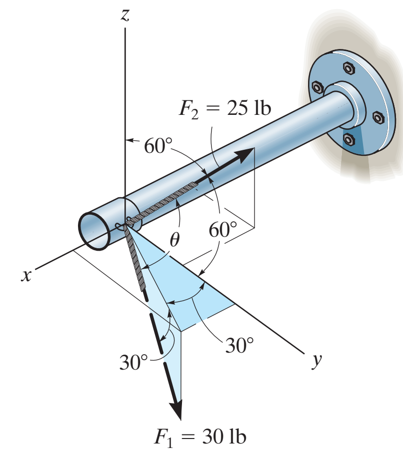
$$= (12.990)(-0.7071) + (22.5)(0.5) + (-15.0)(0.5)$$

$$= -5.44\text{ lb}$$

Negative sign indicates that the projected component of $(F_1)_{F_2}$ acts in the opposite sense of direction to that of $\mathbf{u}_{F_2}$.

The magnitude is $(F_1)_{F_2} = 5.44$ lb **Ans.**

***2–88.** Determine the angle θ between the two cables attached to the pipe.



SOLUTION

Unit Vectors:

$$\mathbf{u}_{F_1} = \cos 30° \sin 30°\mathbf{i} + \cos 30° \cos 30°\mathbf{j} - \sin 30°\mathbf{k}$$

$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 135°\mathbf{i} + \cos 60°\mathbf{j} + \cos 60°\mathbf{k}$$

$$= -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

The Angles Between Two Vectors θ:

$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$

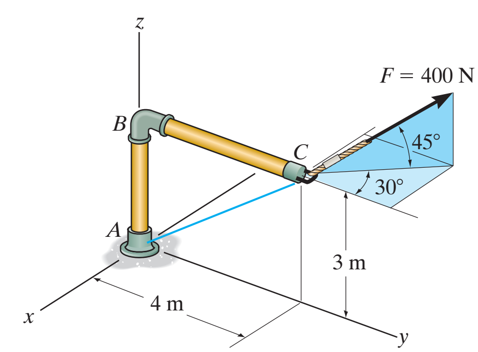
$$= 0.4330(-0.7071) + 0.75(0.5) + (-0.5)(0.5)$$

$$= -0.1812$$

Then,

$$\theta = \cos^{-1}\left(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}\right) = \cos^{-1}(-0.1812) = 100° \qquad \textbf{Ans.}$$

2–89. Determine the projection of force $F = 400$ N acting along line AC of the pipe assembly. Express the result as a Cartesian vector.



SOLUTION

Force and unit Vector: The force vector **F** and unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. (*a*)

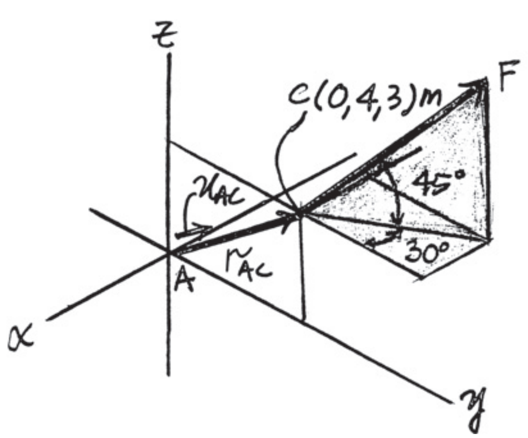
$$\mathbf{F} = 400(-\cos 45^\circ \sin 30^\circ \mathbf{i} + \cos 45^\circ \cos 30^\circ \mathbf{j} + \sin 45^\circ \mathbf{k})$$
$$= \{-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}\}$$
$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(0-0)\mathbf{i} + (4-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-0)^2 + (4-0)^2 + (3-0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** along line AC is

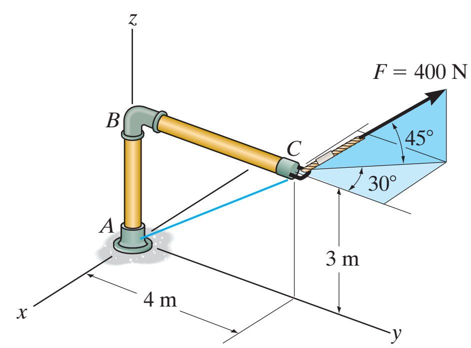
$$F_{AC} = \mathbf{F} \cdot \mathbf{u}_{AC} = (-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}) \cdot \left(\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right)$$
$$= (-141.42)(0) + 244.95\left(\frac{4}{5}\right) + 282.84\left(\frac{3}{5}\right)$$
$$= 365.66 \text{ lb}$$ **Ans.**

Thus, $\mathbf{F}_{AC}$ written in Cartesian vector form is

$$\mathbf{F}_{AC} = F_{AC}\mathbf{u}_{AC} = 365.66\left(\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right) = \{293\mathbf{j} + 219\mathbf{k}\} \text{ lb}$$ **Ans.**



2–90. Determine the magnitudes of the components of force $F = 400$ N acting parallel and perpendicular to segment BC of the pipe assembly.



SOLUTION

Force Vector: The force vector **F** must be determined first. From Fig. *a*,

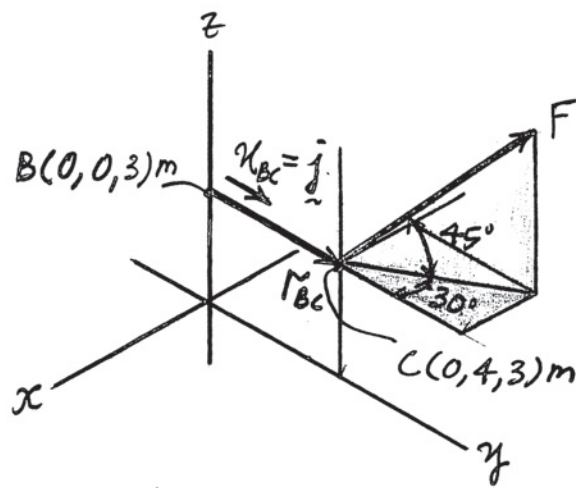
$$\mathbf{F} = 400(-\cos 45° \sin 30° \mathbf{i} + \cos 45° \cos 30° \mathbf{j} + \sin 45° \mathbf{k})$$
$$= \{-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}\} \text{ N}$$

Vector Dot Product: By inspecting Fig. (a) we notice that $\mathbf{u}_{BC} = \mathbf{j}$. Thus, the magnitude of the component of **F** parallel to segment BC of the pipe assembly is

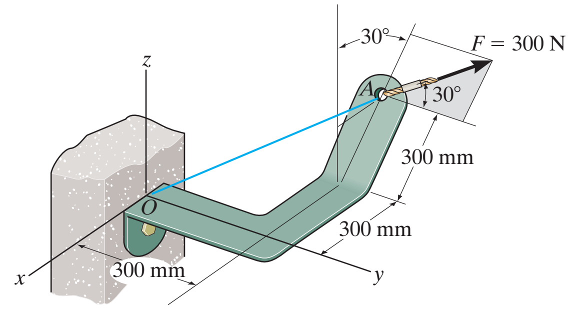
$$(F_{BC})_{\text{paral}} = \mathbf{F} \cdot \mathbf{j} = (-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}) \cdot \mathbf{j}$$
$$= -141.42(0) + 244.95(1) + 282.84(0)$$
$$= 244.95 \text{ lb} = 245 \text{ N} \qquad \textbf{Ans.}$$

The magnitude of the component of **F** perpendicular to segment BC of the pipe assembly can be determined from

$$(F_{BC})_{\text{per}} = \sqrt{F^2 - (F_{BC})_{\text{paral}}} = \sqrt{400^2 - 244.95^2} = 316 \text{ N} \qquad \textbf{Ans.}$$



2–91. Determine the magnitudes of the projected components of the force $F = 300$ N acting along the x and y axes.



SOLUTION

Force Vector: The force vector **F** must be determined first. From Fig. a,

$$\mathbf{F} = -300 \sin 30° \sin 30° \mathbf{i} + 300 \cos 30° \mathbf{j} + 300 \sin 30° \cos 30° \mathbf{k}$$

$$= [-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}] \text{ N}$$

Vector Dot Product: The magnitudes of the projected component of **F** along the x and y axes are

$$F_x = \mathbf{F} \cdot \mathbf{i} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \mathbf{i}$$

$$= -75(1) + 259.81(0) + 129.90(0)$$

$$= -75 \text{ N}$$

$$F_y = \mathbf{F} \cdot \mathbf{j} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \mathbf{j}$$

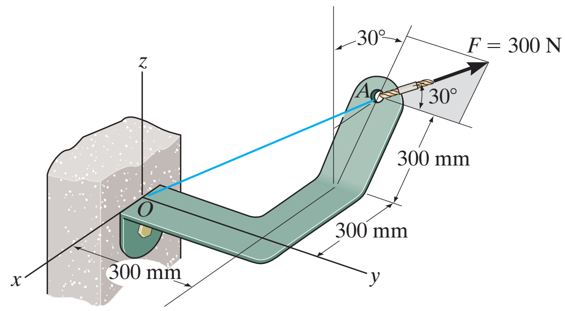
$$= -75(0) + 259.81(1) + 129.90(0)$$

$$= 260 \text{ N}$$

The negative sign indicates that $\mathbf{F}_x$ is directed towards the negative x axis. Thus

$$F_x = 75 \text{ N}, \qquad F_y = 260 \text{ N}$$ **Ans.**

***2–92.** Determine the magnitude of the projected component of the force $F = 300$ N acting along line OA.



SOLUTION

Force and Unit Vector: The force vector $\mathbf{F}$ and unit vector $\mathbf{u}_{OA}$ must be determined first. From Fig. a

$$\mathbf{F} = (-300 \sin 30° \sin 30° \mathbf{i} + 300 \cos 30° \mathbf{j} + 300 \sin 30° \cos 30° \mathbf{k})$$

$$= \{-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\} \text{ N}$$

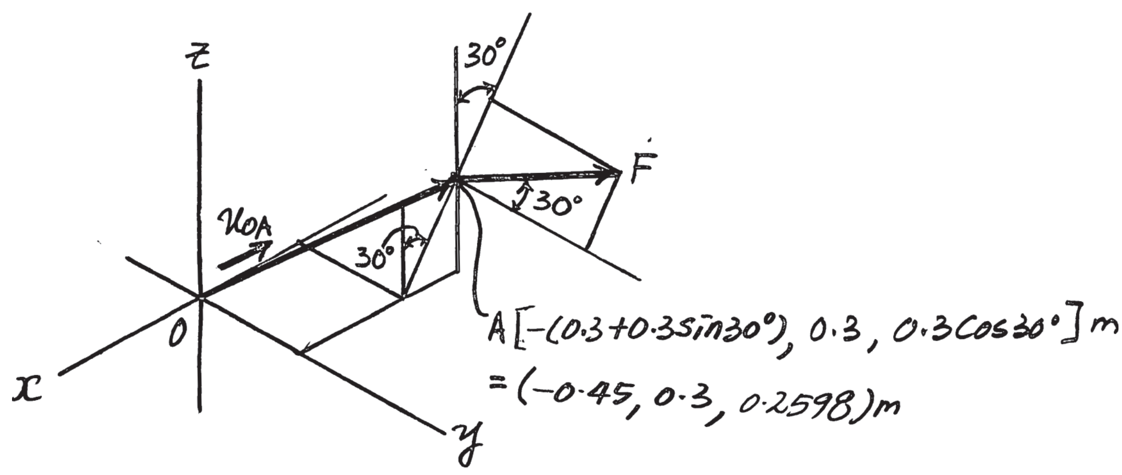
$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-0.45 - 0)\mathbf{i} + (0.3 - 0)\mathbf{j} + (0.2598 - 0)\mathbf{k}}{\sqrt{(-0.45 - 0)^2 + (0.3 - 0)^2 + (0.2598 - 0)^2}} = -0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of $\mathbf{F}$ along line OA is

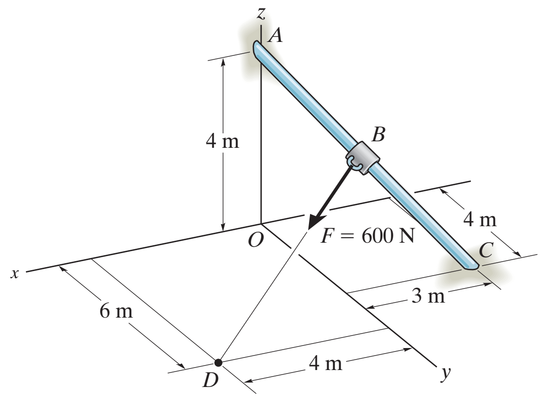
$$F_{OA} = \mathbf{F} \cdot \mathbf{u}_{OA} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \left(-0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}\right)$$

$$= (-75)(-0.75) + 259.81(0.5) + 129.90(0.4330)$$

$$= 242 \text{ N} \qquad \textbf{Ans.}$$



2–93. Determine the components of **F** that act along rod AC and perpendicular to it. Point B is located at the midpoint of the rod.



SOLUTION

$$\mathbf{r}_{AC} = (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}), \qquad r_{AC} = \sqrt{(-3)^2 + 4^2 + (-4)^2} = \sqrt{41}\text{ m}$$

$$\mathbf{r}_{AB} = \frac{\mathbf{r}_{AC}}{2} = \frac{-3\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}}{2} = -1.5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$$

$$\mathbf{r}_{AD} = \mathbf{r}_{AB} + \mathbf{r}_{BD}$$

$$\mathbf{r}_{BD} = \mathbf{r}_{AD} - \mathbf{r}_{AB}$$

$$= (4\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}) - (-1.5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$$

$$= \{5.5\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}\}\text{ m}$$

$$r_{BD} = \sqrt{(5.5)^2 + (4)^2 + (-2)^2} = 7.0887\text{ m}$$

$$\mathbf{F} = 600\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) = 465.528\mathbf{i} + 338.5659\mathbf{j} - 169.2829\mathbf{k}$$

Component of **F** along $\mathbf{r}_{AC}$ is $\mathbf{F}_{||}$

$$F_{||} = \frac{\mathbf{F}\cdot\mathbf{r}_{AC}}{r_{AC}} = \frac{(465.528\mathbf{i} + 338.5659\mathbf{j} - 169.2829\mathbf{k})\cdot(-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})}{\sqrt{41}}$$

$$F_{||} = 99.1408 = 99.1\text{ N}$$ **Ans.**

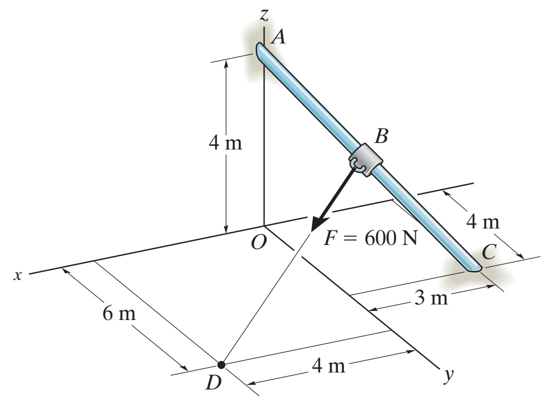
Component of F perpendicular to $\mathbf{r}_{AC}$ is $F_{\perp}$

$$F_{\perp}^2 + F_{||}^2 = F^2 = 600^2$$

$$F_{\perp}^2 = 600^2 - 99.1408^2$$

$$F_{\perp} = 591.75 = 592\text{ N}$$ **Ans.**

2–94. Determine the components of **F** that act along rod AC and perpendicular to it. Point B is located 3 m along the rod from end C.



SOLUTION

$\mathbf{r}_{CA} = 3\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$

$r_{CA} = 6.403124$

$\mathbf{r}_{CB} = \dfrac{3}{6.403124}(\mathbf{r}_{CA}) = 1.40556\mathbf{i} - 1.874085\mathbf{j} + 1.874085\mathbf{k}$

$\mathbf{r}_{OB} = \mathbf{r}_{OC} + \mathbf{r}_{CB}$

$= -3\mathbf{i} + 4\mathbf{j} + \mathbf{r}_{CB}$

$= -1.59444\mathbf{i} + 2.1259\mathbf{j} + 1.874085\mathbf{k}$

$\mathbf{r}_{OD} = \mathbf{r}_{OB} + \mathbf{r}_{BD}$

$\mathbf{r}_{BD} = \mathbf{r}_{OD} - \mathbf{r}_{OB} = (4\mathbf{i} + 6\mathbf{j}) - \mathbf{r}_{OB}$

$= 5.5944\mathbf{i} + 3.8741\mathbf{j} - 1.874085\mathbf{k}$

$r_{BD} = \sqrt{(5.5944)^2 + (3.8741)^2 + (-1.874085)^2} = 7.0582$

$\mathbf{F} = 600(\dfrac{\mathbf{r}_{BD}}{r_{BD}}) = 475.568\mathbf{i} + 329.326\mathbf{j} - 159.311\mathbf{k}$

$\mathbf{r}_{AC} = (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}), \qquad r_{AC} = \sqrt{41}$

Component of **F** along $\mathbf{r}_{AC}$ is $\mathbf{F}_{||}$

$$F_{||} = \frac{\mathbf{F}\cdot\mathbf{r}_{AC}}{r_{AC}} = \frac{(475.568\mathbf{i} + 329.326\mathbf{j} - 159.311\mathbf{k})\cdot(-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})}{\sqrt{41}}$$

$F_{||} = 82.4351 = 82.4\text{ N}$ **Ans.**

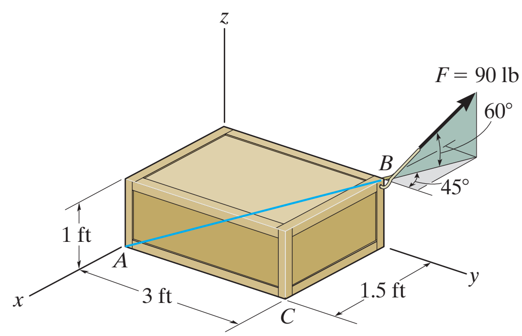
Component of **F** perpendicular to $\mathbf{r}_{AC}$ is $\mathbf{F}_{\perp}$

$F_{\perp}^2 + F_{||}^2 = F^2 = 600^2$

$F_{\perp}^2 = 600^2 - 82.4351^2$

$F_{\perp} = 594\text{ N}$ **Ans.**

2–95. Determine the magnitudes of the components of force $F = 90$ lb acting parallel and perpendicular to diagonal AB of the crate.



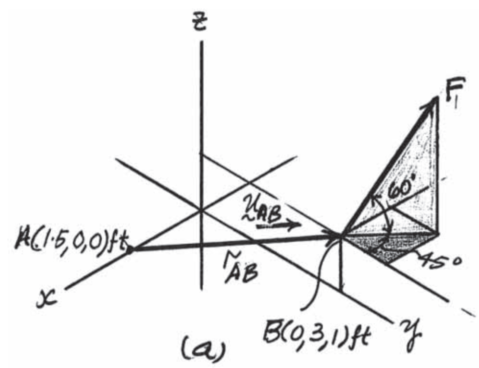
SOLUTION

Force and Unit Vector: The force vector $\mathbf{F}$ and unit vector $\mathbf{u}_{AB}$ must be determined first. From Fig. *a*

$$\mathbf{F} = 90(-\cos 60° \sin 45°\mathbf{i} + \cos 60° \cos 45°\mathbf{j} + \sin 60°\mathbf{k})$$

$$= \{-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}\} \text{ lb}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(0 - 1.5)\mathbf{i} + (3 - 0)\mathbf{j} + (1 - 0)\mathbf{k}}{\sqrt{(0 - 1.5)^2 + (3 - 0)^2 + (1 - 0)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$



Vector Dot Product: The magnitude of the projected component of $\mathbf{F}$ parallel to the diagonal AB is

$$[(F)_{AB}]_{\text{pa}} = \mathbf{F} \cdot \mathbf{u}_{AB} = (-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}) \cdot \left(-\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)$$

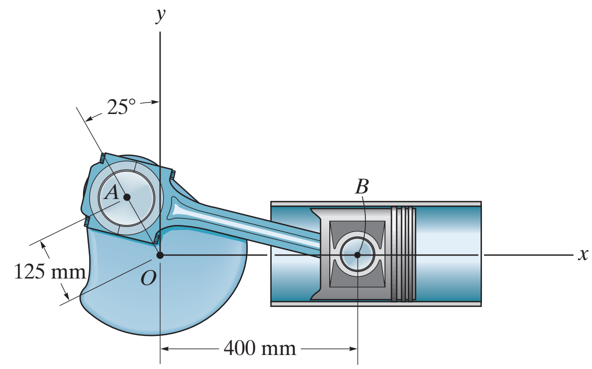
$$= (-31.82)\left(-\frac{3}{7}\right) + 31.82\left(\frac{6}{7}\right) + 77.94\left(\frac{2}{7}\right)$$

$$= 63.18 \text{ lb} = 63.2 \text{ lb}$$ **Ans.**

The magnitude of the component $\mathbf{F}$ perpendicular to the diagonal AB is

$$[(F)_{AB}]_{\text{pr}} = \sqrt{F^2 - [(F)_{AB}]_{\text{pa}}{}^2} = \sqrt{90^2 - 63.18^2} = 64.1 \text{ lb}$$ **Ans.**

***2–96.** Determine the length of the connecting rod AB by first formulating a Cartesian position vector from A to B and then determining its magnitude.



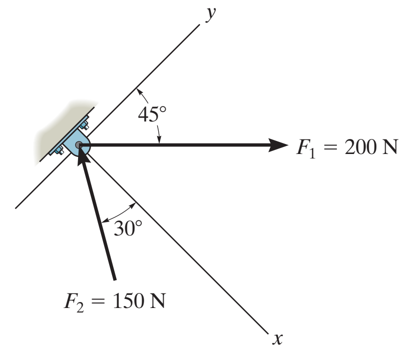
SOLUTION

$$\mathbf{r}_{AB} = [16 - (-5 \sin 30°)]\mathbf{i} + (0 - 5 \cos 30°)\,\mathbf{j}$$

$$= \{18.5\,\mathbf{i} - 4.330\,\mathbf{j}\} \text{ in.}$$

$$r_{AB} = \sqrt{(18.5)^2 + (4.330)^2} = 19.0 \text{ in.}$$ **Ans.**

2–97. Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$.



SOLUTION

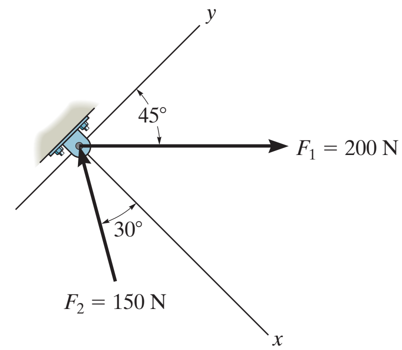
$F_{1x} = 200 \sin 45° = 141 \text{ N}$ **Ans.**

$F_{1y} = 200 \cos 45° = 141 \text{ N}$ **Ans.**

$F_{2x} = -150 \cos 30° = -130 \text{ N}$ **Ans.**

$F_{2y} = 150 \sin 30° = 75 \text{ N}$ **Ans.**

2–98. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

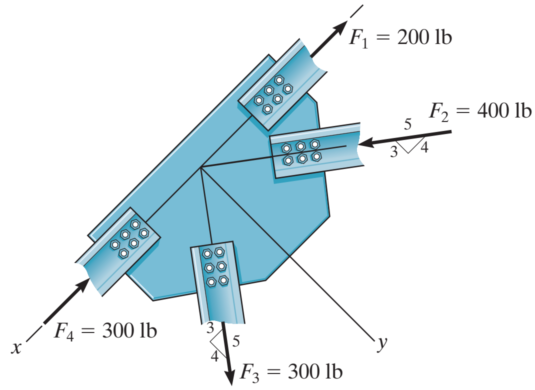
$+\searrow F_{Rx} = \Sigma F_x; \qquad F_{Rx} = -150 \cos 30° + 200 \sin 45° = 11.518 \text{ N}$

$\nearrow + F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 150 \sin 30° + 200 \cos 45° = 216.421 \text{ N}$

$F_R = \sqrt{(11.518)^2 + (216.421)^2} = 217 \text{ N}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{216.421}{11.518}\right) = 87.0°$ **Ans.**

2–99. Determine the x and y components of each force acting on the *gusset plate* of the bridge truss. Show that the resultant force is zero.



SOLUTION

$F_{1x} = -200$ lb **Ans.**

$F_{1y} = 0$ **Ans.**

$$F_{2x} = 400\left(\frac{4}{5}\right) = 320 \text{ lb}$$ **Ans.**

$$F_{2y} = -400\left(\frac{3}{5}\right) = -240 \text{ lb}$$ **Ans.**

$$F_{3x} = 300\left(\frac{3}{5}\right) = 180 \text{ lb}$$ **Ans.**

$$F_{3y} = 300\left(\frac{4}{5}\right) = 240 \text{ lb}$$ **Ans.**

$F_{4x} = -300$ lb **Ans.**

$F_{4y} = 0$ **Ans.**

$F_{Rx} = F_{1x} + F_{2x} + F_{3x} + F_{4x}$

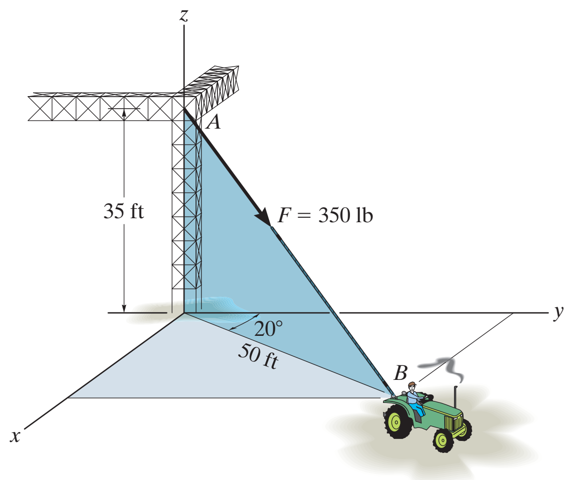
$F_{Rx} = -200 + 320 + 180 - 300 = 0$

$F_{Ry} = F_{1y} + F_{2y} + F_{3y} + F_{4y}$

$F_{Ry} = 0 - 240 + 240 + 0 = 0$

Thus, $F_R = 0$

***2–100.** The cable attached to the tractor at B exerts a force of 350 lb on the framework. Express this force as a Cartesian vector.



SOLUTION

$$\mathbf{r} = 50 \sin 20°\mathbf{i} + 50 \cos 20°\mathbf{j} - 35\mathbf{k}$$

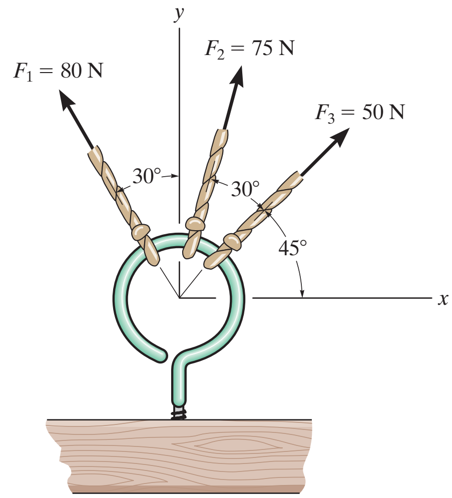
$$\mathbf{r} = \{17.10\mathbf{i} + 46.98\mathbf{j} - 35\mathbf{k}\}\ \text{ft}$$

$$r = \sqrt{(17.10)^2 + (46.98)^2 + (-35)^2} = 61.03\ \text{ft}$$

$$\mathbf{u} = \frac{\mathbf{r}}{r} = (0.280\mathbf{i} + 0.770\mathbf{j} - 0.573\mathbf{k})$$

$$\mathbf{F} = F\mathbf{u} = \{98.1\mathbf{i} + 269\mathbf{j} - 201\mathbf{k}\}\ \text{lb}$$ **Ans.**

2–101. Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_2$. Specify its direction measured counterclockwise from the positive x axis.



SOLUTION

$$F' = \sqrt{(80)^2 + (50)^2 - 2(80)(50)\cos 105^\circ} = 104.7 \text{ N}$$

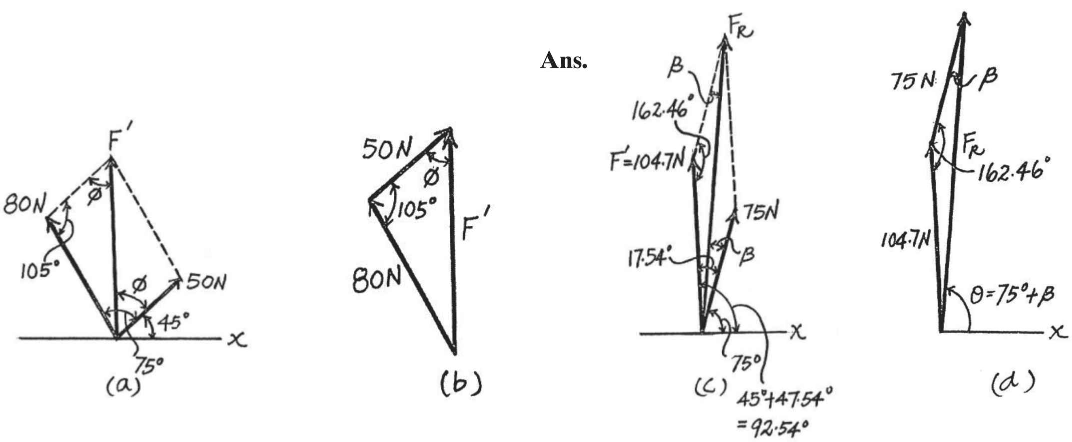
$$\frac{\sin\phi}{80} = \frac{\sin 105^\circ}{104.7}; \qquad \phi = 47.54^\circ$$

$$F_R = \sqrt{(104.7)^2 + (75)^2 - 2(104.7)(75)\cos 162.46^\circ}$$

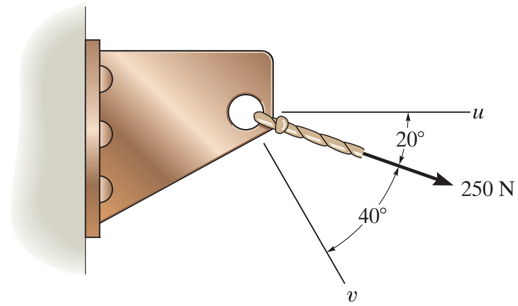
$$F_R = 177.7 = 178 \text{ N}$$ **Ans.**

$$\frac{\sin\beta}{104.7} = \frac{\sin 162.46^\circ}{177.7}; \qquad \beta = 10.23^\circ$$

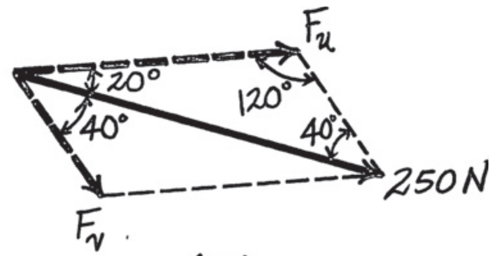
$$\theta = 75^\circ + 10.23^\circ = 85.2^\circ$$ **Ans.**



2–102. Resolve the 250-N force into components acting along the u and v axes and determine the magnitudes of these components.



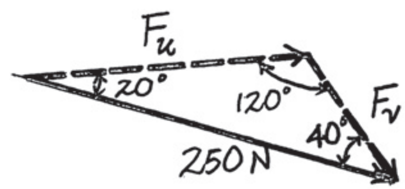
SOLUTION



(a)

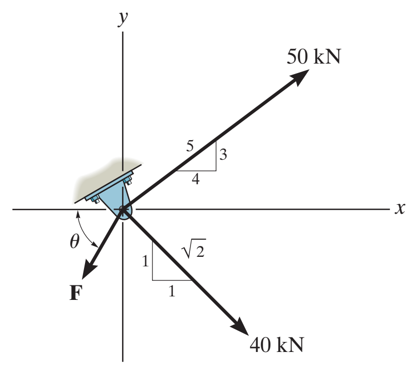
$$\frac{250}{\sin 120^\circ} = \frac{F_u}{\sin 40^\circ}; \qquad F_u = 186 \text{ N} \qquad \text{Ans}$$

$$\frac{250}{\sin 120^\circ} = \frac{F_v}{\sin 20^\circ}; \qquad F_v = 98.7 \text{ N} \qquad \text{Ans}$$



(b)

2–103. If $\theta = 60°$ and $F = 20$ kN, determine the magnitude of the resultant force and its direction measured clockwise from the positive x axis.



SOLUTION

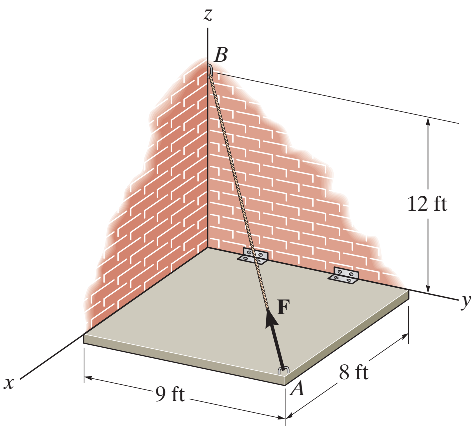
$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;$ $\quad F_{Rx} = 50\left(\frac{4}{5}\right) + \frac{1}{\sqrt{2}}(40) - 20\cos 60° = 58.28\text{ kN}$

$+\uparrow F_{Ry} = \Sigma F_y;$ $\quad F_{Ry} = 50\left(\frac{3}{5}\right) - \frac{1}{\sqrt{2}}(40) - 20\sin 60° = -15.60\text{ kN}$

$$F_R = \sqrt{(58.28)^2 + (-15.60)^2} = 60.3\text{ kN}$$ **Ans.**

$$\phi = \tan^{-1}\left[\frac{15.60}{58.28}\right] = 15.0°$$ **Ans.**

***2–104.** The hinged plate is supported by the cord AB. If the force in the cord is $F = 340$ lb, express this force, directed from A toward B, as a Cartesian vector. What is the length of the cord?



SOLUTION

Unit Vector:

$$\mathbf{r}_{AB} = \{(0 - 8)\mathbf{i} + (0 - 9)\mathbf{j} + (12 - 0)\mathbf{k}\} \text{ ft}$$

$$= \{-8\mathbf{i} - 9\mathbf{j} + 12\mathbf{k}\} \text{ ft}$$

$$r_{AB} = \sqrt{(-8)^2 + (-9)^2 + 12^2} = 17.0 \text{ ft} \qquad \textbf{Ans.}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-8\mathbf{i} - 9\mathbf{j} + 12\mathbf{k}}{17} = -\frac{8}{17}\mathbf{i} - \frac{9}{17}\mathbf{j} + \frac{12}{17}\mathbf{k}$$

Force Vector:

$$\mathbf{F} = F\mathbf{u}_{AB} = 340\left\{-\frac{8}{17}\mathbf{i} - \frac{9}{17}\mathbf{j} + \frac{12}{17}\mathbf{k}\right\} \text{ lb}$$

$$= \{-160\mathbf{i} - 180\mathbf{j} + 240\mathbf{k}\} \text{ lb} \qquad \textbf{Ans.}$$