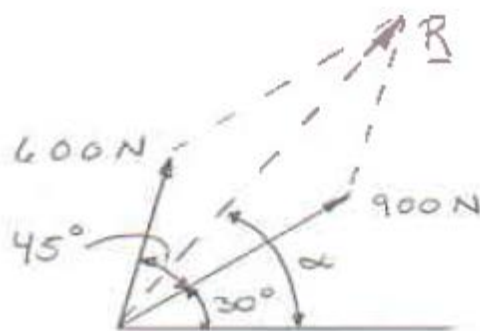


### PROBLEM 2.1

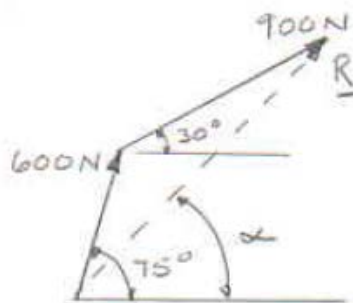
Two forces are applied as shown to a hook. Determine graphically the magnitude and direction of their resultant using (a) the parallelogram law, (b) the triangle rule.

### SOLUTION

(a) Parallelogram law:



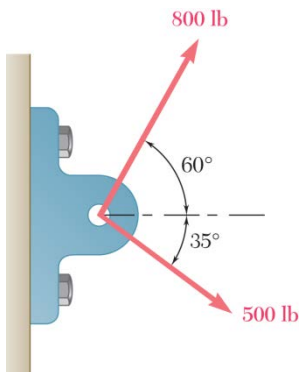
(b) Triangle rule:



We measure:

$$R = 1391 \text{ kN}, \quad \alpha = 47.8^\circ$$

$$\mathbf{R} = 1391 \text{ N} \nearrow 47.8^\circ \blacktriangleleft$$

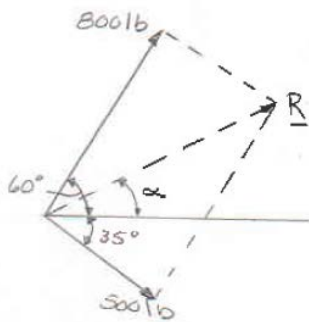


## PROBLEM 2.2

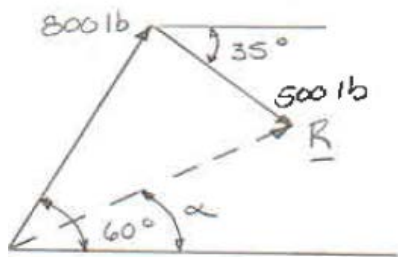
Two forces are applied as shown to a bracket support. Determine graphically the magnitude and direction of their resultant using (a) the parallelogram law, (b) the triangle rule.

## SOLUTION

(a) Parallelogram law:



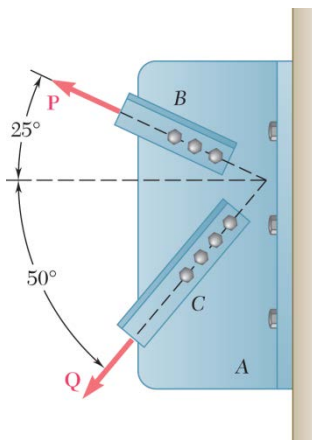
(b) Triangle rule:



We measure:

$$R = 906 \text{ lb}, \quad \alpha = 26.6^\circ$$

$$R = 906 \text{ lb} \nearrow 26.6^\circ \blacktriangleleft$$

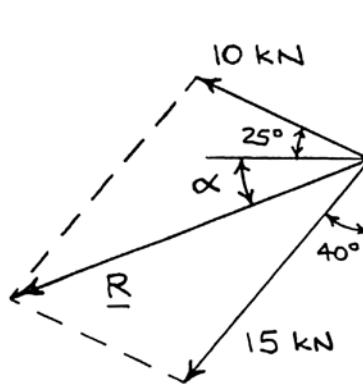


### PROBLEM 2.3

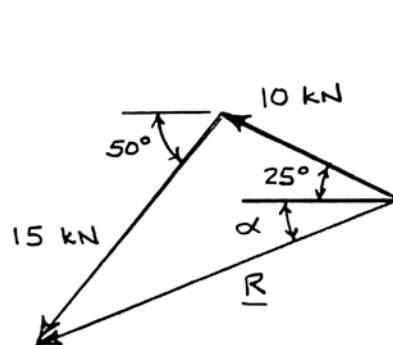
Two structural members  $B$  and  $C$  are bolted to bracket  $A$ . Knowing that both members are in tension and that  $P = 10$  kN and  $Q = 15$  kN, determine graphically the magnitude and direction of the resultant force exerted on the bracket using (a) the parallelogram law, (b) the triangle rule.

### SOLUTION

(a) Parallelogram law:



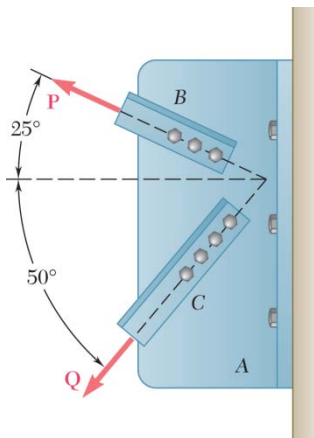
(b) Triangle rule:



We measure:

$$R = 20.1 \text{ kN}, \quad \alpha = 21.2^\circ$$

$$\mathbf{R} = 20.1 \text{ kN} \nearrow 21.2^\circ \nwarrow$$

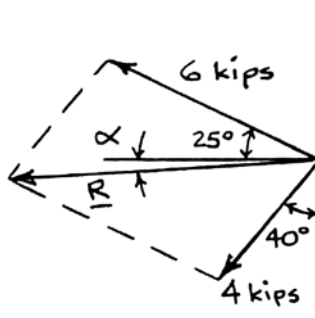


### PROBLEM 2.4

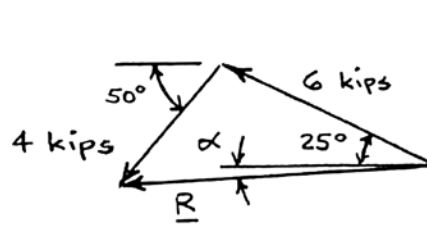
Two structural members  $B$  and  $C$  are bolted to bracket  $A$ . Knowing that both members are in tension and that  $P = 6$  kips and  $Q = 4$  kips, determine graphically the magnitude and direction of the resultant force exerted on the bracket using (a) the parallelogram law, (b) the triangle rule.

### SOLUTION

(a) Parallelogram law:



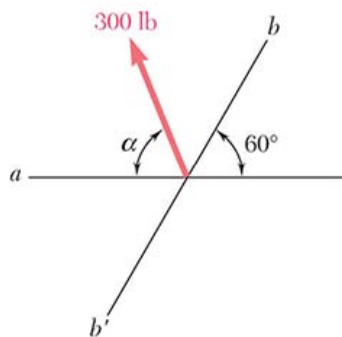
(b) Triangle rule:



We measure:

$$R = 8.03 \text{ kips}, \quad \alpha = 3.8^\circ$$

$$\mathbf{R} = 8.03 \text{ kips} \nearrow 3.8^\circ \blacktriangleleft$$



### PROBLEM 2.5

The 300-lb force is to be resolved into components along lines  $a-a'$  and  $b-b'$ . (a) Determine the angle  $\alpha$  by trigonometry knowing that the component along line  $a-a'$  is to be 240 lb. (b) What is the corresponding value of the component along  $b-b'$ ?

### SOLUTION

(a) Using the triangle rule and law of sines:

$$\frac{\sin \beta}{240 \text{ lb}} = \frac{\sin 60^\circ}{300 \text{ lb}}$$

$$\sin \beta = 0.69282$$

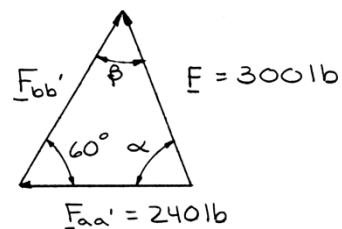
$$\beta = 43.854^\circ$$

$$\alpha + \beta + 60^\circ = 180^\circ$$

$$\alpha = 180^\circ - 60^\circ - 43.854^\circ$$

$$= 76.146^\circ$$

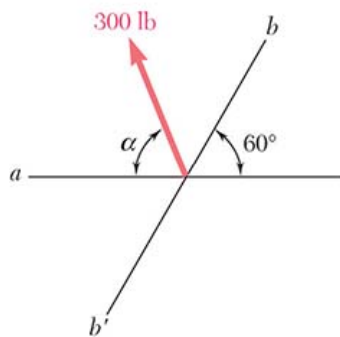
$$\alpha = 76.1^\circ \blacktriangleleft$$



(b) Law of sines:

$$\frac{F_{bb'}}{\sin 76.146^\circ} = \frac{300 \text{ lb}}{\sin 60^\circ}$$

$$F_{bb'} = 336 \text{ lb} \blacktriangleleft$$



### PROBLEM 2.6

The 300-lb force is to be resolved into components along lines  $a-a'$  and  $b-b'$ . (a) Determine the angle  $\alpha$  by trigonometry knowing that the component along line  $b-b'$  is to be 120 lb. (b) What is the corresponding value of the component along  $a-a'$ ?

### SOLUTION

Using the triangle rule and law of sines:

$$(a) \quad \frac{\sin \alpha}{120 \text{ lb}} = \frac{\sin 60^\circ}{300 \text{ lb}}$$

$$\sin \alpha = 0.34641$$

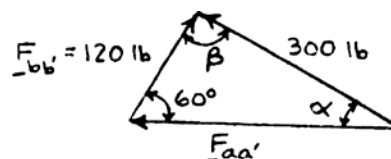
$$\alpha = 20.268^\circ$$

$$(b) \quad \alpha + \beta + 60^\circ = 180^\circ$$

$$\beta = 180^\circ - 60^\circ - 20.268^\circ$$

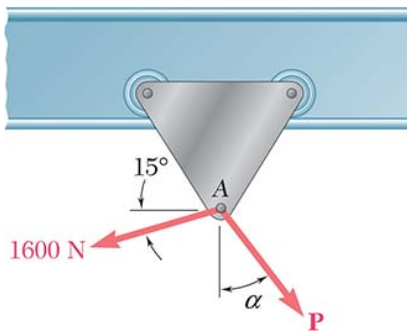
$$= 99.732^\circ$$

$$\frac{F_{aa'}}{\sin 99.732^\circ} = \frac{300 \text{ lb}}{\sin 60^\circ}$$



$$\alpha = 20.3^\circ \quad \blacktriangleleft$$

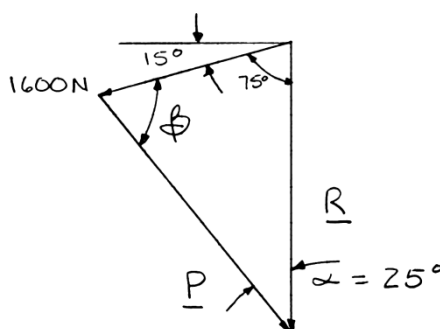
$$F_{aa'} = 341 \text{ lb} \quad \blacktriangleleft$$



### PROBLEM 2.7

A trolley that moves along a horizontal beam is acted upon by two forces as shown. (a) Knowing that  $\alpha = 25^\circ$ , determine by trigonometry the magnitude of the force  $\mathbf{P}$  so that the resultant force exerted on the trolley is vertical. (b) What is the corresponding magnitude of the resultant?

### SOLUTION



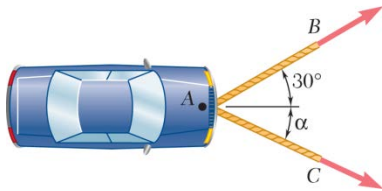
Using the triangle rule and the law of sines:

$$(a) \quad \frac{1600 \text{ N}}{\sin 25^\circ} = \frac{P}{\sin 75^\circ} \quad P = 3660 \text{ N} \quad \blacktriangleleft$$

$$(b) \quad \begin{aligned} 25^\circ + \beta + 75^\circ &= 180^\circ \\ \beta &= 180^\circ - 25^\circ - 75^\circ \\ &= 80^\circ \end{aligned}$$

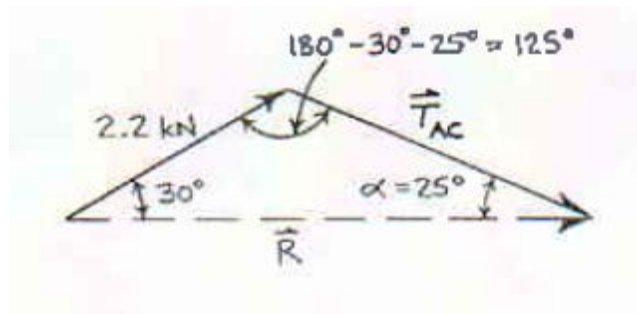
$$\frac{1600 \text{ N}}{\sin 25^\circ} = \frac{R}{\sin 80^\circ} \quad R = 3730 \text{ N} \quad \blacktriangleleft$$

### PROBLEM 2.8



A disabled automobile is pulled by means of two ropes as shown. The tension in rope  $AB$  is 2.2 kN, and the angle  $\alpha$  is  $25^\circ$ . Knowing that the resultant of the two forces applied at  $A$  is directed along the axis of the automobile, determine by trigonometry (a) the tension in rope  $AC$ , (b) the magnitude of the resultant of the two forces applied at  $A$ .

### SOLUTION



Using the law of sines:

$$\frac{T_{AC}}{\sin 30^\circ} = \frac{R}{\sin 125^\circ} = \frac{2.2 \text{ kN}}{\sin 25^\circ}$$

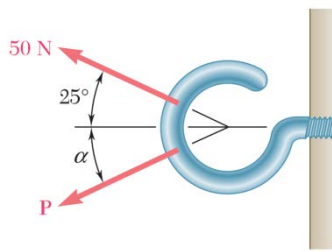
$$T_{AC} = 2.603 \text{ kN}$$

$$R = 4.264 \text{ kN}$$

$$(a) \quad T_{AC} = 2.60 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad R = 4.26 \text{ kN} \quad \blacktriangleleft$$





### PROBLEM 2.9

Two forces are applied as shown to a hook support. Knowing that the magnitude of  $\mathbf{P}$  is 35 N, determine by trigonometry (a) the required angle  $\alpha$  if the resultant  $\mathbf{R}$  of the two forces applied to the support is to be horizontal, (b) the corresponding magnitude of  $\mathbf{R}$ .

### SOLUTION

Using the triangle rule and law of sines:

$$(a) \quad \frac{\sin \alpha}{50 \text{ N}} = \frac{\sin 25^\circ}{35 \text{ N}}$$

$$\sin \alpha = 0.60374$$

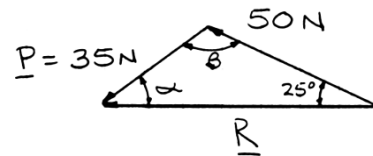
$$\alpha = 37.138^\circ$$

$$(b) \quad \alpha + \beta + 25^\circ = 180^\circ$$

$$\beta = 180^\circ - 25^\circ - 37.138^\circ$$

$$= 117.862^\circ$$

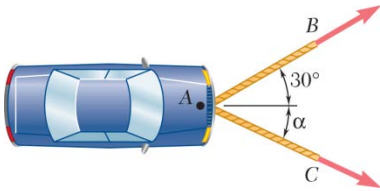
$$\frac{R}{\sin 117.862^\circ} = \frac{35 \text{ N}}{\sin 25^\circ}$$



$$\alpha = 37.1^\circ \quad \blacktriangleleft$$

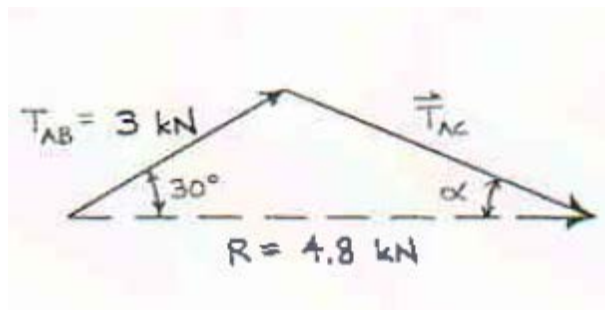
$$R = 73.2 \text{ N} \quad \blacktriangleleft$$

### PROBLEM 2.10



A disabled automobile is pulled by means of two ropes as shown. Knowing that the tension in rope  $AB$  is 3 kN, determine by trigonometry the tension in rope  $AC$  and the value of  $\alpha$  so that the resultant force exerted at  $A$  is a 4.8-kN force directed along the axis of the automobile.

### SOLUTION



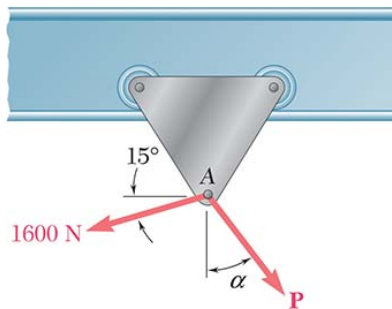
Using the law of cosines:

$$T_{AC}^2 = (3 \text{ kN})^2 + (4.8 \text{ kN})^2 - 2(3 \text{ kN})(4.8 \text{ kN}) \cos 30^\circ$$
$$T_{AC} = 2.6643 \text{ kN}$$

Using the law of sines:

$$\frac{\sin \alpha}{3 \text{ kN}} = \frac{\sin 30^\circ}{2.6643 \text{ kN}}$$
$$\alpha = 34.3^\circ$$

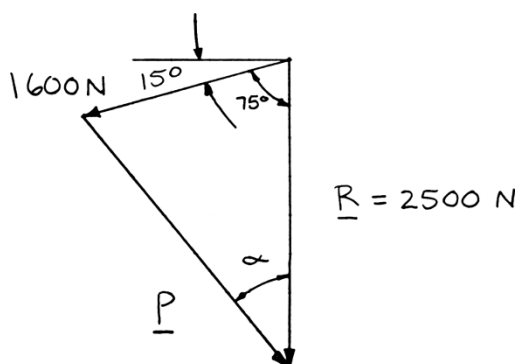
$$\mathbf{T_{AC} = 2.66 \text{ kN} \searrow 34.3^\circ \blacktriangleleft}$$



### PROBLEM 2.11

A trolley that moves along a horizontal beam is acted upon by two forces as shown. Determine by trigonometry the magnitude and direction of the force **P** so that the resultant is a vertical force of 2500 N.

### SOLUTION



Using the law of cosines:

$$P^2 = (1600 \text{ N})^2 + (2500 \text{ N})^2 - 2(1600 \text{ N})(2500 \text{ N})\cos 75^\circ$$

$$P = 2596 \text{ N}$$

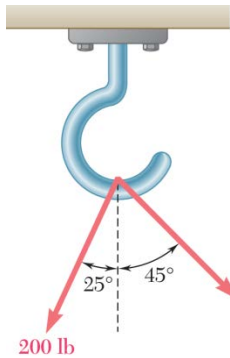
Using the law of sines:

$$\frac{\sin \alpha}{1600 \text{ N}} = \frac{\sin 75^\circ}{2596 \text{ N}}$$

$$\alpha = 36.5^\circ$$

**P** is directed  $90^\circ - 36.5^\circ$  or  $53.5^\circ$  below the horizontal.

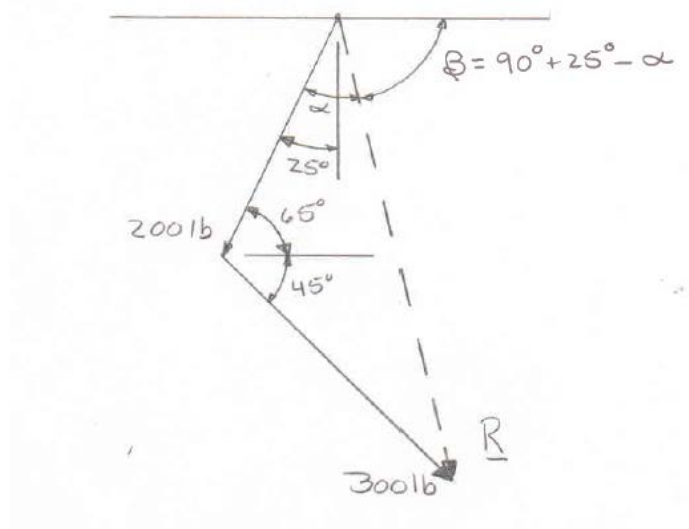
$$\mathbf{P} = 2600 \text{ N} \searrow 53.5^\circ \blacktriangleleft$$



### PROBLEM 2.12

For the hook support shown, determine by trigonometry the magnitude and direction of the resultant of the two forces applied to the support.

### SOLUTION



Using the law of cosines:

$$\begin{aligned}
 R^2 &= (200 \text{ lb})^2 + (300 \text{ lb})^2 \\
 &\quad - 2(200 \text{ lb})(300 \text{ lb}) \cos(45^\circ + 65^\circ) \\
 R &= 413.57 \text{ lb}
 \end{aligned}$$

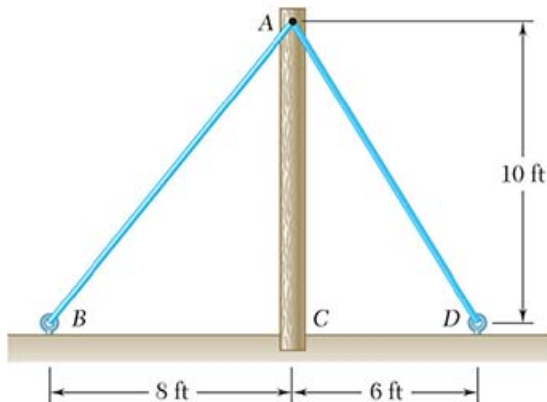
Using the law of sines:

$$\begin{aligned}
 \frac{\sin \alpha}{300 \text{ lb}} &= \frac{\sin(45^\circ + 65^\circ)}{413.57 \text{ lb}} \\
 \alpha &= 42.972^\circ
 \end{aligned}$$

$$\beta = 90^\circ + 25^\circ - 42.972^\circ$$

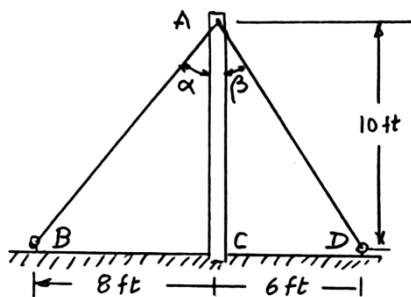
$$\mathbf{R} = 414 \text{ lb} \searrow 72.0^\circ \blacktriangleleft$$

### PROBLEM 2.13



The cable stays  $AB$  and  $AD$  help support pole  $AC$ . Knowing that the tension is 120 lb in  $AB$  and 40 lb in  $AD$ , determine graphically the magnitude and direction of the resultant of the forces exerted by the stays at  $A$  using (a) the parallelogram law, (b) the triangle rule.

### SOLUTION

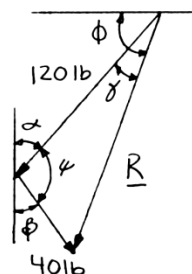


$$\tan \alpha = \frac{8}{10}$$

$$\alpha = 38.66^\circ$$

$$\tan \beta = \frac{6}{10}$$

$$\beta = 30.96^\circ$$



Using the triangle rule:

$$\alpha + \beta + \psi = 180^\circ$$

$$38.66^\circ + 30.96^\circ + \psi = 180^\circ$$

$$\psi = 110.38^\circ$$

Using the law of cosines:

$$R^2 = (120 \text{ lb})^2 + (40 \text{ lb})^2 - 2(120 \text{ lb})(40 \text{ lb})\cos 110.38^\circ$$

$$R = 139.08 \text{ lb}$$

Using the law of sines:

$$\frac{\sin \gamma}{40 \text{ lb}} = \frac{\sin 110.38^\circ}{139.08 \text{ lb}}$$

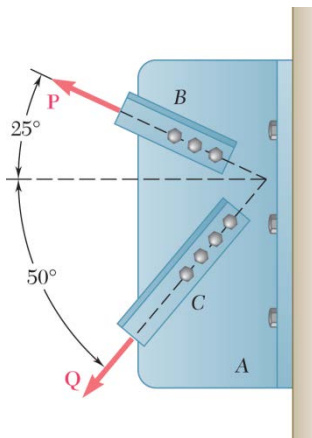
$$\gamma = 15.64^\circ$$

$$\phi = (90^\circ - \alpha) + \gamma$$

$$\phi = (90^\circ - 38.66^\circ) + 15.64^\circ$$

$$\phi = 66.98^\circ$$

$$\mathbf{R} = 139.1 \text{ lb} \nearrow 67.0^\circ \blacktriangleleft$$



### PROBLEM 2.14

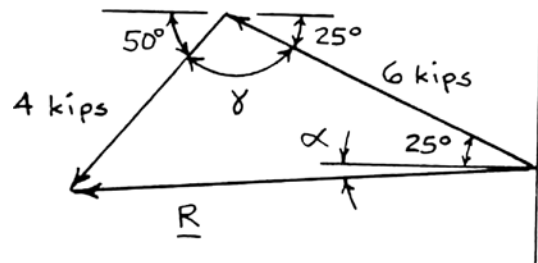
Solve Problem 2.4 by trigonometry.

**PROBLEM 2.4:** Two structural members *B* and *C* are bolted to bracket *A*. Knowing that both members are in tension and that  $P = 6$  kips and  $Q = 4$  kips, determine graphically the magnitude and direction of the resultant force exerted on the bracket using (a) the parallelogram law, (b) the triangle rule.

### SOLUTION

Using the force triangle and the laws of cosines and sines:

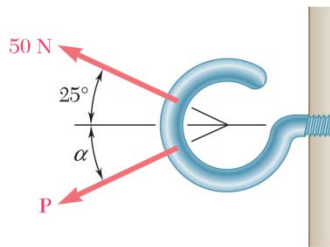
We have: 
$$\gamma = 180^\circ - (50^\circ + 25^\circ)$$
$$= 105^\circ$$



Then 
$$R^2 = (4 \text{ kips})^2 + (6 \text{ kips})^2 - 2(4 \text{ kips})(6 \text{ kips})\cos 105^\circ$$
$$= 64.423 \text{ kips}^2$$
$$R = 8.0264 \text{ kips}$$

And 
$$\frac{4 \text{ kips}}{\sin(25^\circ + \alpha)} = \frac{8.0264 \text{ kips}}{\sin 105^\circ}$$
$$\sin(25^\circ + \alpha) = 0.48137$$
$$25^\circ + \alpha = 28.775^\circ$$
$$\alpha = 3.775^\circ$$

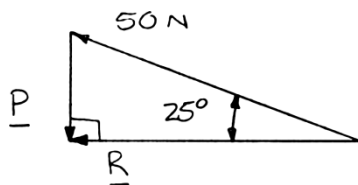
$$\mathbf{R} = 8.03 \text{ kips} \nearrow 3.8^\circ \blacktriangleleft$$



### PROBLEM 2.15

For the hook support of Prob. 2.9, determine by trigonometry (a) the magnitude and direction of the smallest force **P** for which the resultant **R** of the two forces applied to the support is horizontal, (b) the corresponding magnitude of **R**.

### SOLUTION



The smallest force  $P$  will be perpendicular to  $R$ .

(a)  $P = (50 \text{ N}) \sin 25^\circ$

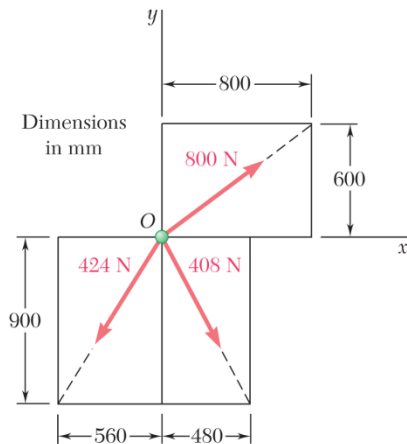
$P = 21.1 \text{ N} \downarrow \blacktriangleleft$

(b)  $R = (50 \text{ N}) \cos 25^\circ$

$R = 45.3 \text{ N} \blacktriangleleft$

## PROBLEM 2.16

Determine the  $x$  and  $y$  components of each of the forces shown.



## SOLUTION

Compute the following distances:

$$OA = \sqrt{(600)^2 + (800)^2} \\ = 1000 \text{ mm}$$

$$OB = \sqrt{(560)^2 + (900)^2} \\ = 1060 \text{ mm}$$

$$OC = \sqrt{(480)^2 + (900)^2} \\ = 1020 \text{ mm}$$

800-N Force:

$$F_x = +(800 \text{ N}) \frac{800}{1000}$$

$$F_x = +640 \text{ N} \blacktriangleleft$$

$$F_y = +(800 \text{ N}) \frac{600}{1000}$$

$$F_y = +480 \text{ N} \blacktriangleleft$$

424-N Force:

$$F_x = -(424 \text{ N}) \frac{560}{1060}$$

$$F_x = -224 \text{ N} \blacktriangleleft$$

$$F_y = -(424 \text{ N}) \frac{900}{1060}$$

$$F_y = -360 \text{ N} \blacktriangleleft$$

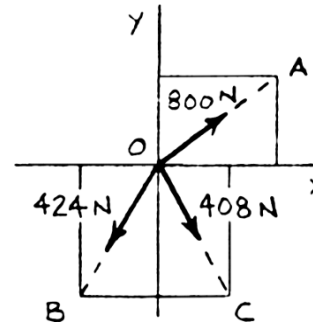
408-N Force:

$$F_x = +(408 \text{ N}) \frac{480}{1020}$$

$$F_x = +192.0 \text{ N} \blacktriangleleft$$

$$F_y = -(408 \text{ N}) \frac{900}{1020}$$

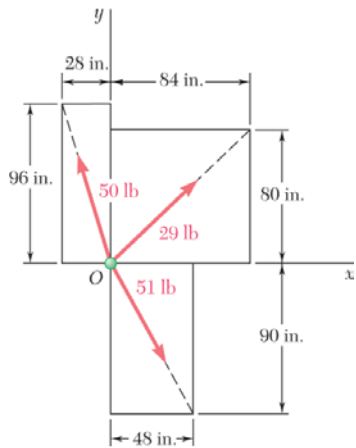
$$F_y = -360 \text{ N} \blacktriangleleft$$





## PROBLEM 2.17

Determine the  $x$  and  $y$  components of each of the forces shown.



## SOLUTION

Compute the following distances:

$$OA = \sqrt{(84)^2 + (80)^2} \\ = 116 \text{ in.}$$

$$OB = \sqrt{(28)^2 + (96)^2} \\ = 100 \text{ in.}$$

$$OC = \sqrt{(48)^2 + (90)^2} \\ = 102 \text{ in.}$$

29-lb Force:

$$F_x = +(29 \text{ lb}) \frac{84}{116}$$

$$F_x = +21.0 \text{ lb} \quad \blacktriangleleft$$

$$F_y = +(29 \text{ lb}) \frac{80}{116}$$

$$F_y = +20.0 \text{ lb} \quad \blacktriangleleft$$

50-lb Force:

$$F_x = -(50 \text{ lb}) \frac{28}{100}$$

$$F_x = -14.00 \text{ lb} \quad \blacktriangleleft$$

$$F_y = +(50 \text{ lb}) \frac{96}{100}$$

$$F_y = +48.0 \text{ lb} \quad \blacktriangleleft$$

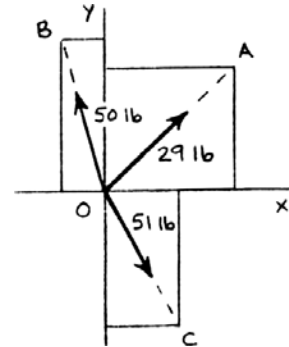
51-lb Force:

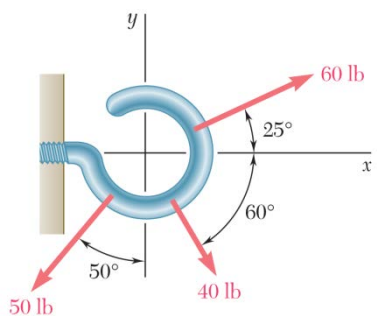
$$F_x = +(51 \text{ lb}) \frac{48}{102}$$

$$F_x = +24.0 \text{ lb} \quad \blacktriangleleft$$

$$F_y = -(51 \text{ lb}) \frac{90}{102}$$

$$F_y = -45.0 \text{ lb} \quad \blacktriangleleft$$





### PROBLEM 2.18

Determine the  $x$  and  $y$  components of each of the forces shown.

### SOLUTION

40-lb Force:

$$F_x = +(40 \text{ lb}) \cos 60^\circ$$

$$F_x = 20.0 \text{ lb} \quad \blacktriangleleft$$

$$F_y = -(40 \text{ lb}) \sin 60^\circ$$

$$F_y = -34.6 \text{ lb} \quad \blacktriangleleft$$

50-lb Force:

$$F_x = -(50 \text{ lb}) \sin 50^\circ$$

$$F_x = -38.3 \text{ lb} \quad \blacktriangleleft$$

$$F_y = -(50 \text{ lb}) \cos 50^\circ$$

$$F_y = -32.1 \text{ lb} \quad \blacktriangleleft$$

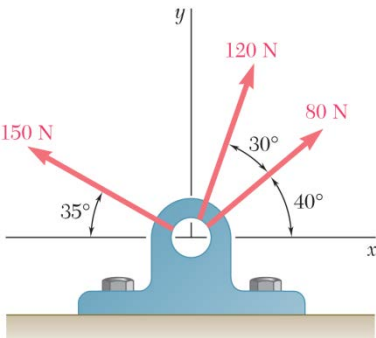
60-lb Force:

$$F_x = +(60 \text{ lb}) \cos 25^\circ$$

$$F_x = 54.4 \text{ lb} \quad \blacktriangleleft$$

$$F_y = +(60 \text{ lb}) \sin 25^\circ$$

$$F_y = 25.4 \text{ lb} \quad \blacktriangleleft$$



### PROBLEM 2.19

Determine the  $x$  and  $y$  components of each of the forces shown.

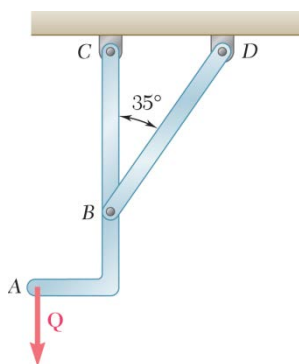
### SOLUTION

80-N Force:

120-N Force:

150-N Force:

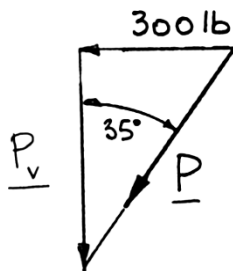
$F_x = +(80 \text{ N}) \cos 40^\circ$	$F_x = 61.3 \text{ N} \blacktriangleleft$
$F_y = +(80 \text{ N}) \sin 40^\circ$	$F_y = 51.4 \text{ N} \blacktriangleleft$
$F_x = +(120 \text{ N}) \cos 70^\circ$	$F_x = 41.0 \text{ N} \blacktriangleleft$
$F_y = +(120 \text{ N}) \sin 70^\circ$	$F_y = 112.8 \text{ N} \blacktriangleleft$
$F_x = -(150 \text{ N}) \cos 35^\circ$	$F_x = -122.9 \text{ N} \blacktriangleleft$
$F_y = +(150 \text{ N}) \sin 35^\circ$	$F_y = 86.0 \text{ N} \blacktriangleleft$



### PROBLEM 2.20

Member  $BD$  exerts on member  $ABC$  a force  $\mathbf{P}$  directed along line  $BD$ . Knowing that  $\mathbf{P}$  must have a 300-lb horizontal component, determine (a) the magnitude of the force  $\mathbf{P}$ , (b) its vertical component.

### SOLUTION



(a)

$$P \sin 35^\circ = 300 \text{ lb}$$

$$P = \frac{300 \text{ lb}}{\sin 35^\circ}$$

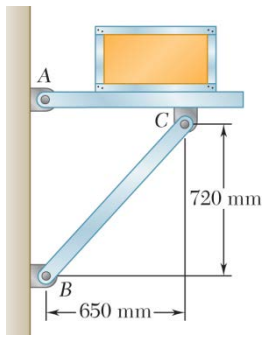
$$P = 523 \text{ lb} \quad \blacktriangleleft$$

(b) Vertical component

$$P_v = P \cos 35^\circ$$

$$= (523 \text{ lb}) \cos 35^\circ$$

$$P_v = 428 \text{ lb} \quad \blacktriangleleft$$



### PROBLEM 2.21

Member  $BC$  exerts on member  $AC$  a force  $\mathbf{P}$  directed along line  $BC$ . Knowing that  $\mathbf{P}$  must have a 325-N horizontal component, determine (a) the magnitude of the force  $\mathbf{P}$ , (b) its vertical component.

### SOLUTION

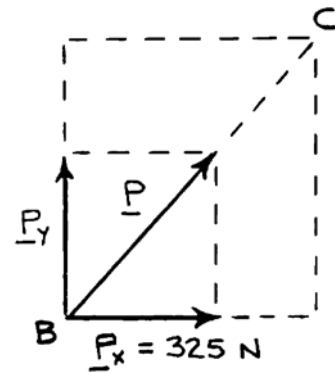
(a)

or

$$BC = \sqrt{(650 \text{ mm})^2 + (720 \text{ mm})^2} \\ = 970 \text{ mm}$$

$$P_x = P \left( \frac{650}{970} \right)$$

$$P = P_x \left( \frac{970}{650} \right) \\ = 325 \text{ N} \left( \frac{970}{650} \right) \\ = 485 \text{ N}$$

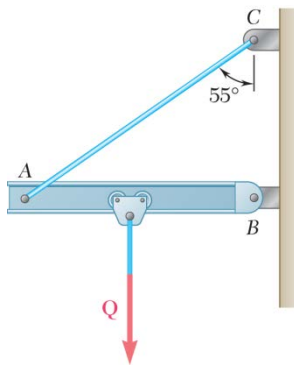


$$P = 485 \text{ N} \quad \blacktriangleleft$$

(b)

$$P_y = P \left( \frac{720}{970} \right) \\ = 485 \text{ N} \left( \frac{720}{970} \right) \\ = 360 \text{ N}$$

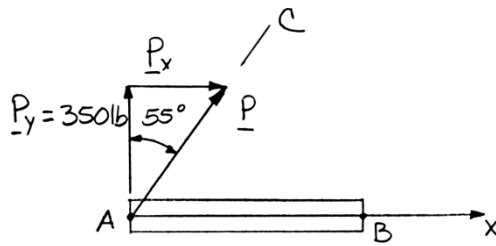
$$P_y = 360 \text{ N} \quad \blacktriangleleft$$



### PROBLEM 2.22

Cable  $AC$  exerts on beam  $AB$  a force  $\mathbf{P}$  directed along line  $AC$ . Knowing that  $\mathbf{P}$  must have a 350-lb vertical component, determine (a) the magnitude of the force  $\mathbf{P}$ , (b) its horizontal component.

### SOLUTION



(a)

$$P = \frac{P_y}{\cos 55^\circ}$$

$$= \frac{350 \text{ lb}}{\cos 55^\circ}$$

$$= 610.21 \text{ lb}$$

$$P = 610 \text{ lb} \quad \blacktriangleleft$$

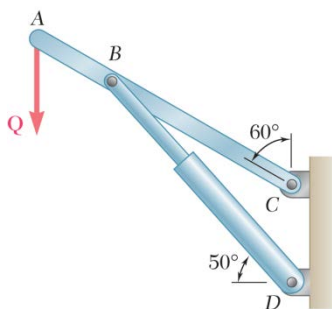
(b)

$$P_x = P \sin 55^\circ$$

$$= (610.21 \text{ lb}) \sin 55^\circ$$

$$= 499.85 \text{ lb}$$

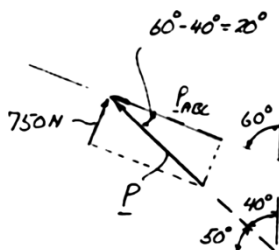
$$P_x = 500 \text{ lb} \quad \blacktriangleleft$$



### PROBLEM 2.23

The hydraulic cylinder  $BD$  exerts on member  $ABC$  a force  $\mathbf{P}$  directed along line  $BD$ . Knowing that  $\mathbf{P}$  must have a 750-N component perpendicular to member  $ABC$ , determine (a) the magnitude of the force  $\mathbf{P}$ , (b) its component parallel to  $ABC$ .

### SOLUTION



(a)

$$750 \text{ N} = P \sin 20^\circ$$

$$P = 2192.9 \text{ N}$$

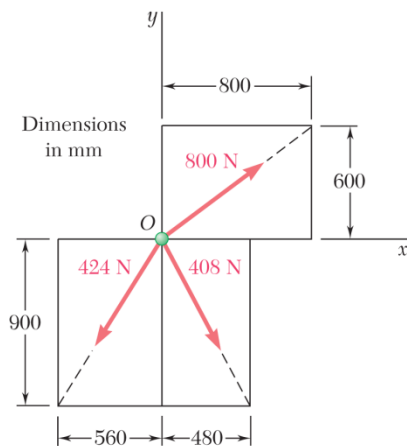
$$P = 2190 \text{ N} \quad \blacktriangleleft$$

(b)

$$P_{ABC} = P \cos 20^\circ$$

$$= (2192.9 \text{ N}) \cos 20^\circ$$

$$P_{ABC} = 2060 \text{ N} \quad \blacktriangleleft$$



### PROBLEM 2.24

Determine the resultant of the three forces of Problem 2.16.

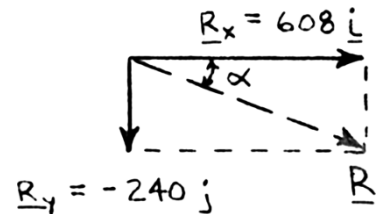
**PROBLEM 2.16** Determine the  $x$  and  $y$  components of each of the forces shown.

### SOLUTION

Components of the forces were determined in Problem 2.16:

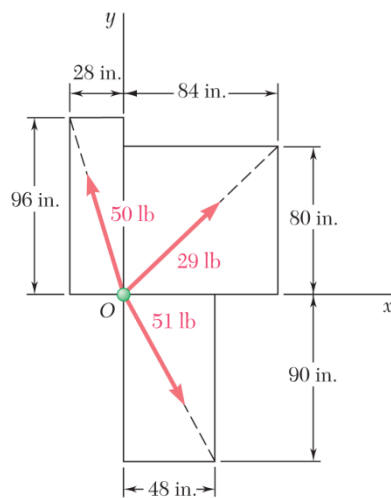
Force	$x$ Comp. (N)	$y$ Comp. (N)
800 lb	+640	+480
424 lb	-224	-360
408 lb	+192	-360
	$R_x = +608$	$R_y = -240$

$$\begin{aligned}
 \mathbf{R} &= R_x \mathbf{i} + R_y \mathbf{j} \\
 &= (608 \text{ lb})\mathbf{i} + (-240 \text{ lb})\mathbf{j} \\
 \tan \alpha &= \frac{R_y}{R_x} \\
 &= \frac{240}{608} \\
 \alpha &= 21.541^\circ \\
 R &= \frac{240 \text{ N}}{\sin(21.541^\circ)} \\
 &= 653.65 \text{ N}
 \end{aligned}$$



$$\mathbf{R} = 654 \text{ N} \searrow 21.5^\circ \blacktriangleleft$$





### PROBLEM 2.25

Determine the resultant of the three forces of Problem 2.17.

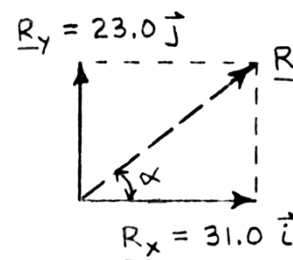
**PROBLEM 2.17** Determine the  $x$  and  $y$  components of each of the forces shown.

### SOLUTION

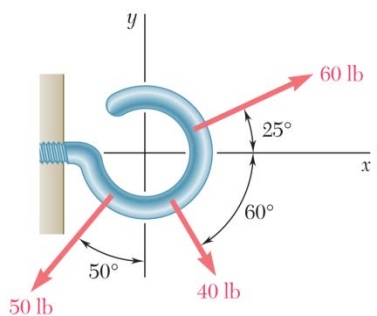
Components of the forces were determined in Problem 2.17:

Force	$x$ Comp. (lb)	$y$ Comp. (lb)
29 lb	+21.0	+20.0
50 lb	-14.00	+48.0
51 lb	+24.0	-45.0
	$R_x = +31.0$	$R_y = +23.0$

$$\begin{aligned}
 \mathbf{R} &= R_x \mathbf{i} + R_y \mathbf{j} \\
 &= (31.0 \text{ lb})\mathbf{i} + (23.0 \text{ lb})\mathbf{j} \\
 \tan \alpha &= \frac{R_y}{R_x} \\
 &= \frac{23.0}{31.0} \\
 \alpha &= 36.573^\circ \\
 R &= \frac{23.0 \text{ lb}}{\sin(36.573^\circ)} \\
 &= 38.601 \text{ lb}
 \end{aligned}$$



$$\mathbf{R} = 38.6 \text{ lb} \angle 36.6^\circ \blacktriangleleft$$



### PROBLEM 2.26

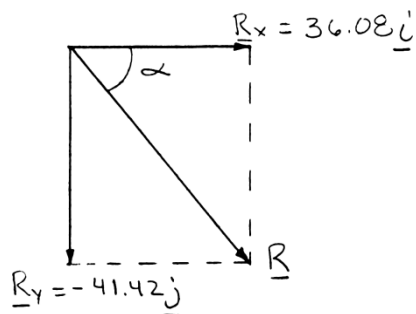
Determine the resultant of the three forces of Problem 2.18.

**PROBLEM 2.18** Determine the  $x$  and  $y$  components of each of the forces shown.

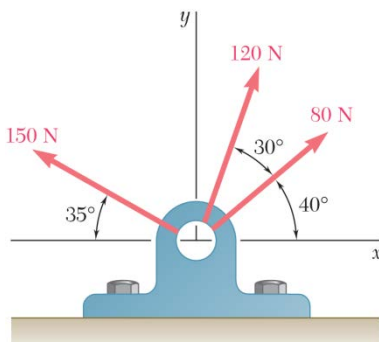
### SOLUTION

Force	$x$ Comp. (lb)	$y$ Comp. (lb)
40 lb	+20.00	-34.64
50 lb	-38.30	-32.14
60 lb	+54.38	+25.36
	$R_x = +36.08$	$R_y = -41.42$

$$\begin{aligned}
 \mathbf{R} &= R_x \mathbf{i} + R_y \mathbf{j} \\
 &= (+36.08 \text{ lb}) \mathbf{i} + (-41.42 \text{ lb}) \mathbf{j} \\
 \tan \alpha &= \frac{R_y}{R_x} \\
 \tan \alpha &= \frac{41.42 \text{ lb}}{36.08 \text{ lb}} \\
 \tan \alpha &= 1.14800 \\
 \alpha &= 48.942^\circ \\
 R &= \frac{41.42 \text{ lb}}{\sin 48.942^\circ}
 \end{aligned}$$



$$\mathbf{R} = 54.9 \text{ lb} \searrow 48.9^\circ \blacktriangleleft$$



### PROBLEM 2.27

Determine the resultant of the three forces of Problem 2.19.

**PROBLEM 2.19** Determine the  $x$  and  $y$  components of each of the forces shown.

### SOLUTION

Components of the forces were determined in Problem 2.19:

Force	$x$ Comp. (N)	$y$ Comp. (N)
80 N	+61.3	+51.4
120 N	+41.0	+112.8
150 N	-122.9	+86.0
	$R_x = -20.6$	$R_y = +250.2$

$$\mathbf{R} = R_x \mathbf{i} + R_y \mathbf{j}$$

$$= (-20.6 \text{ N})\mathbf{i} + (250.2 \text{ N})\mathbf{j}$$

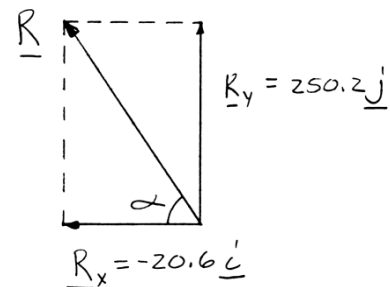
$$\tan \alpha = \frac{R_y}{R_x}$$

$$\tan \alpha = \frac{250.2 \text{ N}}{20.6 \text{ N}}$$

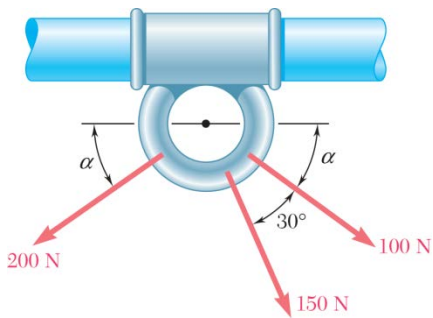
$$\tan \alpha = 12.1456$$

$$\alpha = 85.293^\circ$$

$$R = \frac{250.2 \text{ N}}{\sin 85.293^\circ}$$



$$\mathbf{R} = 251 \text{ N} \nearrow 85.3^\circ \blacktriangleleft$$



### PROBLEM 2.28

For the collar loaded as shown, determine (a) the required value of  $\alpha$  if the resultant of the three forces shown is to be vertical, (b) the corresponding magnitude of the resultant.

### SOLUTION

$$\begin{aligned}
 R_x &= \Sigma F_x \\
 &= (100 \text{ N}) \cos \alpha + (150 \text{ N}) \cos(\alpha + 30^\circ) - (200 \text{ N}) \cos \alpha \\
 R_x &= -(100 \text{ N}) \cos \alpha + (150 \text{ N}) \cos(\alpha + 30^\circ)
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 R_y &= \Sigma F_y \\
 &= -(100 \text{ N}) \sin \alpha - (150 \text{ N}) \sin(\alpha + 30^\circ) - (200 \text{ N}) \sin \alpha \\
 R_y &= -(300 \text{ N}) \sin \alpha - (150 \text{ N}) \sin(\alpha + 30^\circ)
 \end{aligned} \tag{2}$$

(a) For  $\mathbf{R}$  to be vertical, we must have  $R_x = 0$ . We make  $R_x = 0$  in Eq. (1):

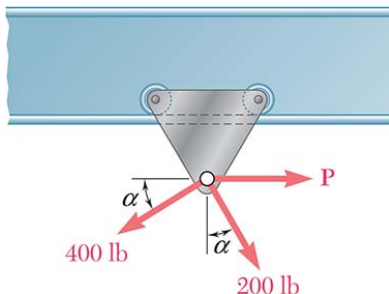
$$\begin{aligned}
 -100 \cos \alpha + 150 \cos(\alpha + 30^\circ) &= 0 \\
 -100 \cos \alpha + 150(\cos \alpha \cos 30^\circ - \sin \alpha \sin 30^\circ) &= 0 \\
 29.904 \cos \alpha &= 75 \sin \alpha \\
 \tan \alpha &= \frac{29.904}{75} \\
 &= 0.39872 \\
 \alpha &= 21.738^\circ
 \end{aligned} \tag{3}$$

$$\alpha = 21.7^\circ \quad \blacktriangleleft$$

(b) Substituting for  $\alpha$  in Eq. (2):

$$\begin{aligned}
 R_y &= -300 \sin 21.738^\circ - 150 \sin 51.738^\circ \\
 &= -228.89 \text{ N} \\
 R &= |R_y| = 228.89 \text{ N}
 \end{aligned} \tag{4}$$

$$R = 229 \text{ N} \quad \blacktriangleleft$$



### PROBLEM 2.29

A hoist trolley is subjected to the three forces shown. Knowing that  $\alpha = 40^\circ$ , determine (a) the required magnitude of the force **P** if the resultant of the three forces is to be vertical, (b) the corresponding magnitude of the resultant.

### SOLUTION

$$R_x = \rightarrow \Sigma F_x = P + (200 \text{ lb}) \sin 40^\circ - (400 \text{ lb}) \cos 40^\circ$$

$$R_x = P - 177.860 \text{ lb} \quad (1)$$

$$R_y = \downarrow \Sigma F_y = (200 \text{ lb}) \cos 40^\circ + (400 \text{ lb}) \sin 40^\circ$$

$$R_y = 410.32 \text{ lb} \quad (2)$$

(a) For **R** to be vertical, we must have  $R_x = 0$ .

Set

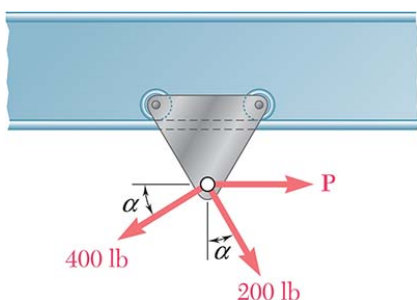
$$R_x = 0 \text{ in Eq. (1)}$$

$$0 = P - 177.860 \text{ lb}$$

$$P = 177.860 \text{ lb} \quad P = 177.9 \text{ lb} \blacktriangleleft$$

(b) Since **R** is to be vertical:

$$R = R_y = 410 \text{ lb} \quad R = 410 \text{ lb} \blacktriangleleft$$



### PROBLEM 2.30

A hoist trolley is subjected to the three forces shown. Knowing that  $P = 250$  lb, determine (a) the required value of  $\alpha$  if the resultant of the three forces is to be vertical, (b) the corresponding magnitude of the resultant.

### SOLUTION

$$R_x = \rightarrow \Sigma F_x = 250 \text{ lb} + (200 \text{ lb}) \sin \alpha - (400 \text{ lb}) \cos \alpha$$

$$R_x = 250 \text{ lb} + (200 \text{ lb}) \sin \alpha - (400 \text{ lb}) \cos \alpha \quad (1)$$

$$R_y = \downarrow \Sigma F_y = (200 \text{ lb}) \cos \alpha + (400 \text{ lb}) \sin \alpha$$

(a) For  $\mathbf{R}$  to be vertical, we must have  $R_x = 0$ .

Set

$$R_x = 0 \text{ in Eq. (1)}$$

$$0 = 250 \text{ lb} + (200 \text{ lb}) \sin \alpha - (400 \text{ lb}) \cos \alpha$$

$$(400 \text{ lb}) \cos \alpha = (200 \text{ lb}) \sin \alpha + 250 \text{ lb}$$

$$2 \cos \alpha = \sin \alpha + 1.25$$

$$4 \cos^2 \alpha = \sin^2 \alpha + 2.5 \sin \alpha + 1.5625$$

$$4(1 - \sin^2 \alpha) = \sin^2 \alpha + 2.5 \sin \alpha + 1.5625$$

$$0 = 5 \sin^2 \alpha + 2.5 \sin \alpha - 2.4375$$

Using the quadratic formula to solve for the roots gives

$$\sin \alpha = 0.49162$$

or

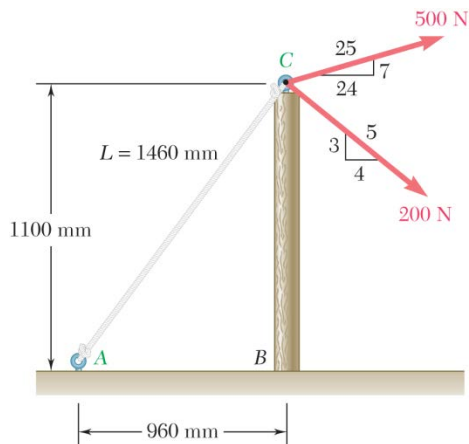
$$\alpha = 29.447^\circ$$

$$\alpha = 29.4^\circ \blacktriangleleft$$

(b) Since  $\mathbf{R}$  is to be vertical:

$$R = R_y = (200 \text{ lb}) \cos 29.447^\circ + (400 \text{ lb}) \sin 29.447^\circ$$

$$\mathbf{R} = 371 \text{ lb} \blacktriangleleft$$



### PROBLEM 2.31

For the post loaded as shown, determine (a) the required tension in rope AC if the resultant of the three forces exerted at point C is to be horizontal, (b) the corresponding magnitude of the resultant.

### SOLUTION

$$R_x = \Sigma F_x = -\frac{960}{1460}T_{AC} + \frac{24}{25}(500 \text{ N}) + \frac{4}{5}(200 \text{ N})$$

$$R_x = -\frac{48}{73}T_{AC} + 640 \text{ N} \quad (1)$$

$$R_y = \Sigma F_y = -\frac{1100}{1460}T_{AC} + \frac{7}{25}(500 \text{ N}) - \frac{3}{5}(200 \text{ N})$$

$$R_y = -\frac{55}{73}T_{AC} + 20 \text{ N} \quad (2)$$

(a) For  $\mathbf{R}$  to be horizontal, we must have  $R_y = 0$ .

Set  $R_y = 0$  in Eq. (2):  $-\frac{55}{73}T_{AC} + 20 \text{ N} = 0$

$$T_{AC} = 26.545 \text{ N}$$

$$T_{AC} = 26.5 \text{ N} \quad \blacktriangleleft$$

(b) Substituting for  $T_{AC}$  into Eq. (1) gives

$$R_x = -\frac{48}{73}(26.545 \text{ N}) + 640 \text{ N}$$

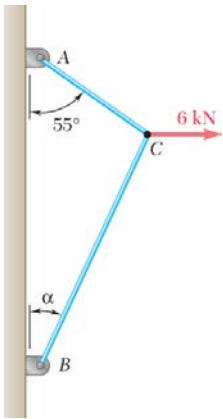
$$R_x = 622.55 \text{ N}$$

$$R = R_x = 623 \text{ N}$$

$$R = 623 \text{ N} \quad \blacktriangleleft$$

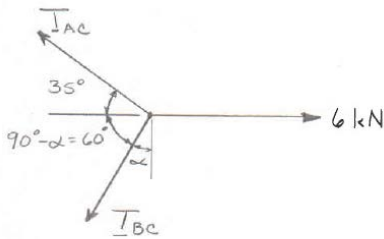
### PROBLEM 2.32

Two cables are tied together at  $C$  and are loaded as shown. Knowing that  $\alpha = 30^\circ$ , determine the tension ( $a$ ) in cable  $AC$ , ( $b$ ) in cable  $BC$ .

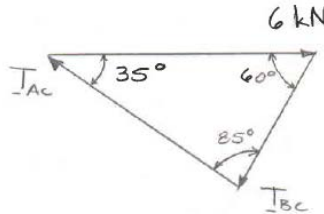


### SOLUTION

#### Free-Body Diagram



#### Force Triangle



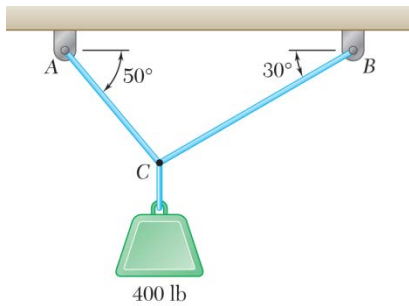
Law of sines:

$$\frac{T_{AC}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 35^\circ} = \frac{6 \text{ kN}}{\sin 85^\circ}$$

$$(a) \quad T_{AC} = \frac{6 \text{ kN}}{\sin 85^\circ} (\sin 60^\circ) \quad T_{AC} = 5.22 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad T_{BC} = \frac{6 \text{ kN}}{\sin 85^\circ} (\sin 35^\circ) \quad T_{BC} = 3.45 \text{ kN} \quad \blacktriangleleft$$



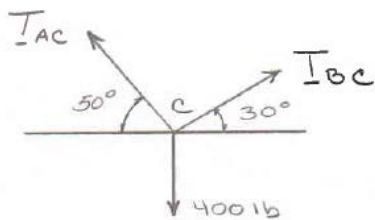


### PROBLEM 2.33

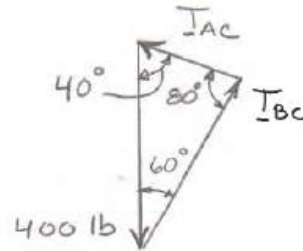
Two cables are tied together at  $C$  and are loaded as shown. Determine the tension (a) in cable  $AC$ , (b) in cable  $BC$ .

### SOLUTION

#### Free-Body Diagram



#### Force Triangle

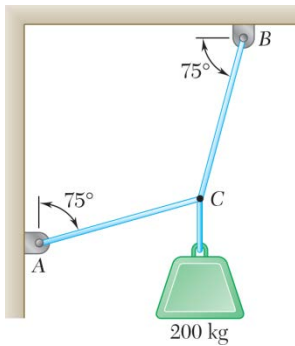


Law of sines:

$$\frac{T_{AC}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 40^\circ} = \frac{400 \text{ lb}}{\sin 80^\circ}$$

$$(a) \quad T_{AC} = \frac{400 \text{ lb}}{\sin 80^\circ} (\sin 60^\circ) \quad T_{AC} = 352 \text{ lb} \quad \blacktriangleleft$$

$$(b) \quad T_{BC} = \frac{400 \text{ lb}}{\sin 80^\circ} (\sin 40^\circ) \quad T_{BC} = 261 \text{ lb} \quad \blacktriangleleft$$

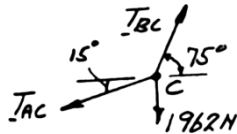


### PROBLEM 2.34

Two cables are tied together at  $C$  and are loaded as shown. Determine the tension (a) in cable  $AC$ , (b) in cable  $BC$ .

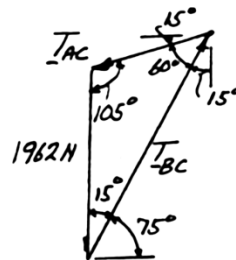
### SOLUTION

#### Free-Body Diagram



$$\begin{aligned} W &= mg \\ &= (200 \text{ kg})(9.81 \text{ m/s}^2) \\ &= 1962 \text{ N} \end{aligned}$$

#### Force Triangle

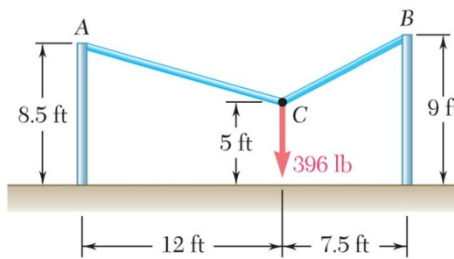


Law of sines:

$$\frac{T_{AC}}{\sin 15^\circ} = \frac{T_{BC}}{\sin 105^\circ} = \frac{1962 \text{ N}}{\sin 60^\circ}$$

$$(a) \quad T_{AC} = \frac{(1962 \text{ N}) \sin 15^\circ}{\sin 60^\circ} \quad T_{AC} = 586 \text{ N} \quad \blacktriangleleft$$

$$(b) \quad T_{BC} = \frac{(1962 \text{ N}) \sin 105^\circ}{\sin 60^\circ} \quad T_{BC} = 2190 \text{ N} \quad \blacktriangleleft$$



### PROBLEM 2.35

Two cables are tied together at  $C$  and loaded as shown. Determine the tension (a) in cable  $AC$ , (b) in cable  $BC$ .

### SOLUTION

$$\Sigma F_x = 0: -\frac{12 \text{ ft}}{12.5 \text{ ft}}T_{AC} + \frac{7.5 \text{ ft}}{8.5 \text{ ft}}T_{BC} = 0$$

$$T_{BC} = 1.08800T_{AC}$$

$$\Sigma F_y = 0: \frac{3.5 \text{ ft}}{12 \text{ ft}}T_{AC} + \frac{4 \text{ ft}}{8.5 \text{ ft}}T_{BC} - 396 \text{ lb} = 0$$

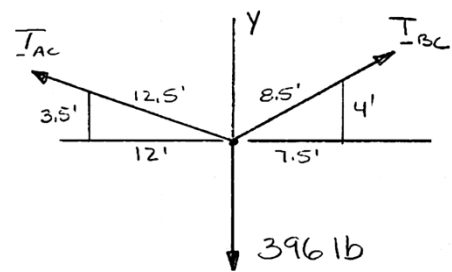
$$(a) \quad \frac{3.5 \text{ ft}}{12.5 \text{ ft}}T_{AC} + \frac{4 \text{ ft}}{8.5 \text{ ft}}(1.08800T_{AC}) - 396 \text{ lb} = 0$$

$$(0.28000 + 0.51200)T_{AC} = 396 \text{ lb}$$

$$T_{AC} = 500.0 \text{ lb}$$

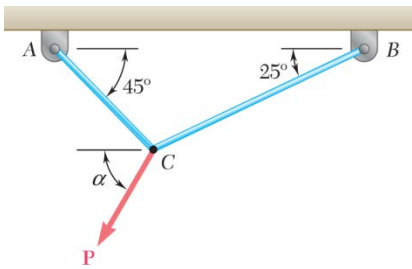
$$(b) \quad T_{BC} = (1.08800)(500.0 \text{ lb})$$

### Free Body Diagram at C:



$$T_{AC} = 500 \text{ lb} \quad \blacktriangleleft$$

$$T_{BC} = 544 \text{ lb} \quad \blacktriangleleft$$

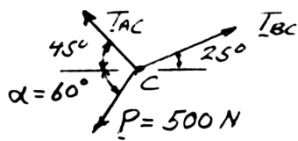


### PROBLEM 2.36

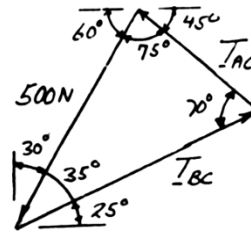
Two cables are tied together at  $C$  and are loaded as shown. Knowing that  $P = 500 \text{ N}$  and  $\alpha = 60^\circ$ , determine the tension in (a) in cable  $AC$ , (b) in cable  $BC$ .

### SOLUTION

#### Free-Body Diagram



#### Force Triangle

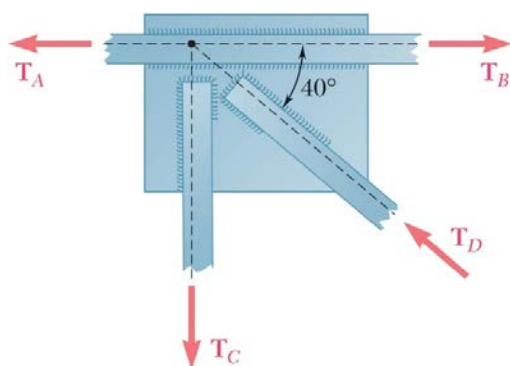


Law of sines:

$$\frac{T_{AC}}{\sin 35^\circ} = \frac{T_{BC}}{\sin 75^\circ} = \frac{500 \text{ N}}{\sin 70^\circ}$$

$$(a) \quad T_{AC} = \frac{500 \text{ N}}{\sin 70^\circ} \sin 35^\circ \quad T_{AC} = 305 \text{ N} \quad \blacktriangleleft$$

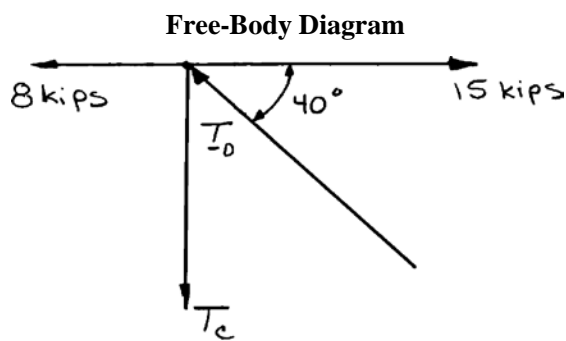
$$(b) \quad T_{BC} = \frac{500 \text{ N}}{\sin 70^\circ} \sin 75^\circ \quad T_{BC} = 514 \text{ N} \quad \blacktriangleleft$$



### PROBLEM 2.37

Two forces of magnitude  $T_A = 8$  kips and  $T_B = 15$  kips are applied as shown to a welded connection. Knowing that the connection is in equilibrium, determine the magnitudes of the forces  $T_C$  and  $T_D$ .

### SOLUTION



$$+\rightarrow \Sigma F_x = 0 \quad 15 \text{ kips} - 8 \text{ kips} - T_D \cos 40^\circ = 0$$

$$T_D = 9.1379 \text{ kips}$$

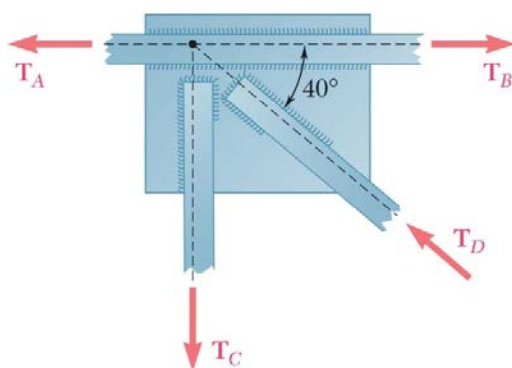
$$+\uparrow \Sigma F_y = 0 \quad T_D \sin 40^\circ - T_C = 0$$

$$(9.1379 \text{ kips}) \sin 40^\circ - T_C = 0$$

$$T_C = 5.8737 \text{ kips}$$

$$T_C = 5.87 \text{ kips} \quad \blacktriangleleft$$

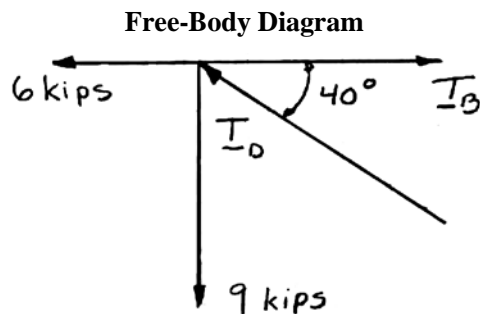
$$T_D = 9.14 \text{ kips} \quad \blacktriangleleft$$



### PROBLEM 2.38

Two forces of magnitude  $T_A = 6$  kips and  $T_C = 9$  kips are applied as shown to a welded connection. Knowing that the connection is in equilibrium, determine the magnitudes of the forces  $T_B$  and  $T_D$ .

### SOLUTION



$$\rightarrow \Sigma F_x = 0$$

$$T_B - 6 \text{ kips} - T_D \cos 40^\circ = 0 \quad (1)$$

$$\uparrow \Sigma F_y = 0$$

$$T_D \sin 40^\circ - 9 \text{ kips} = 0$$

$$T_D = \frac{9 \text{ kips}}{\sin 40^\circ}$$

$$T_D = 14.0015 \text{ kips}$$

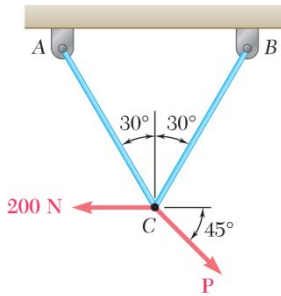
Substituting for  $T_D$  into Eq. (1) gives:

$$T_B - 6 \text{ kips} - (14.0015 \text{ kips}) \cos 40^\circ = 0$$

$$T_B = 16.7258 \text{ kips}$$

$$T_B = 16.73 \text{ kips} \quad \blacktriangleleft$$

$$T_D = 14.00 \text{ kips} \quad \blacktriangleleft$$

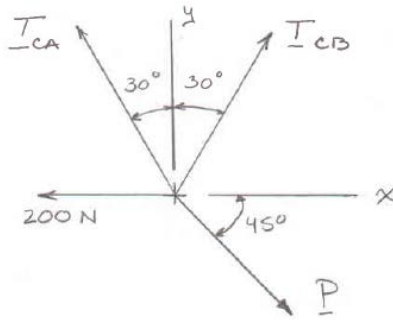


### PROBLEM 2.39

Two cables are tied together at  $C$  and are loaded as shown. Knowing that  $P = 300 \text{ N}$ , determine the tension in cables  $AC$  and  $BC$ .

### SOLUTION

#### Free-Body Diagram



$$+\rightarrow \Sigma F_x = 0 \quad -T_{CA} \sin 30^\circ + T_{CB} \sin 30^\circ - P \cos 45^\circ - 200 \text{ N} = 0$$

For  $P = 200 \text{ N}$  we have,

$$-0.5T_{CA} + 0.5T_{CB} + 212.13 - 200 = 0 \quad (1)$$

$$+\uparrow \Sigma F_y = 0 \quad T_{CA} \cos 30^\circ - T_{CB} \cos 30^\circ - P \sin 45^\circ = 0$$

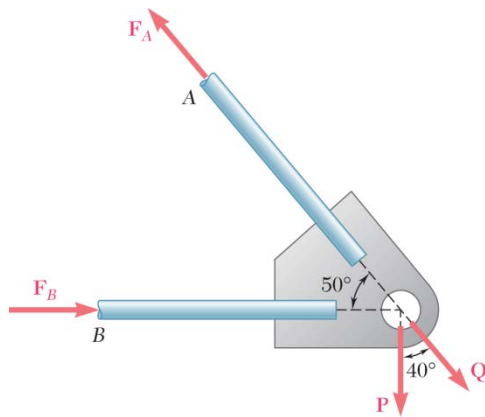
$$0.86603T_{CA} + 0.86603T_{CB} - 212.13 = 0 \quad (2)$$

Solving equations (1) and (2) simultaneously gives,

$$T_{CA} = 134.6 \text{ N} \quad \blacktriangleleft$$

$$T_{CB} = 110.4 \text{ N} \quad \blacktriangleleft$$

### PROBLEM 2.40



Two forces **P** and **Q** are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that  $P = 500 \text{ lb}$  and  $Q = 650 \text{ lb}$ , determine the magnitudes of the forces exerted on the rods **A** and **B**.

### SOLUTION

Resolving the forces into  $x$ - and  $y$ -directions:

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{F}_A + \mathbf{F}_B = 0$$

Substituting components:

$$\begin{aligned} \mathbf{R} = & -(500 \text{ lb})\mathbf{j} + [(650 \text{ lb}) \cos 50^\circ]\mathbf{i} \\ & - [(650 \text{ lb}) \sin 50^\circ]\mathbf{j} \\ & + F_B\mathbf{i} - (F_A \cos 50^\circ)\mathbf{i} + (F_A \sin 50^\circ)\mathbf{j} = 0 \end{aligned}$$

In the  $y$ -direction (one unknown force):

$$-500 \text{ lb} - (650 \text{ lb}) \sin 50^\circ + F_A \sin 50^\circ = 0$$

Thus,

$$\begin{aligned} F_A &= \frac{500 \text{ lb} + (650 \text{ lb}) \sin 50^\circ}{\sin 50^\circ} \\ &= 1302.70 \text{ lb} \end{aligned}$$

$$F_A = 1303 \text{ lb} \quad \blacktriangleleft$$

In the  $x$ -direction:

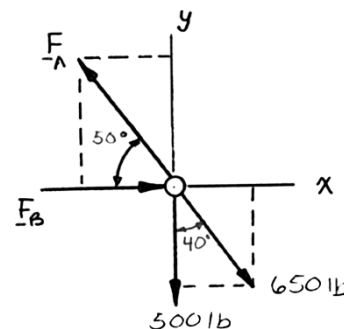
$$(650 \text{ lb}) \cos 50^\circ + F_B - F_A \cos 50^\circ = 0$$

Thus,

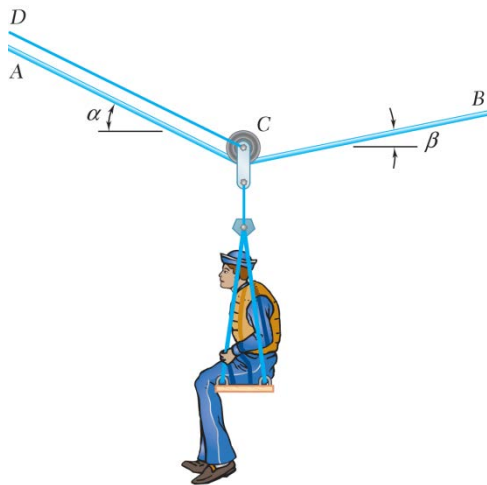
$$\begin{aligned} F_B &= F_A \cos 50^\circ - (650 \text{ lb}) \cos 50^\circ \\ &= (1302.70 \text{ lb}) \cos 50^\circ - (650 \text{ lb}) \cos 50^\circ \\ &= 419.55 \text{ lb} \end{aligned}$$

$$F_B = 420 \text{ lb} \quad \blacktriangleleft$$

Free-Body Diagram





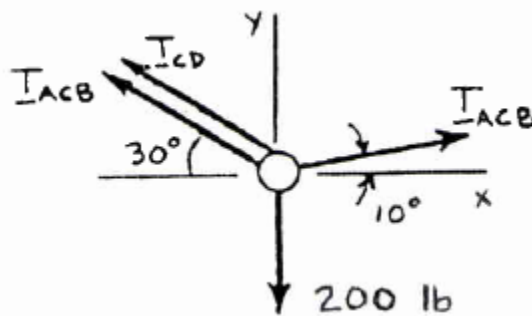


### PROBLEM 2.41

A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable  $ACB$  and is pulled at a constant speed by cable  $CD$ . Knowing that  $\alpha = 30^\circ$  and  $\beta = 10^\circ$  and that the combined weight of the boatswain's chair and the sailor is 200 lb, determine the tension (a) in the support cable  $ACB$ , (b) in the traction cable  $CD$ .

### SOLUTION

#### Free-Body Diagram



$$+\rightarrow \Sigma F_x = 0: T_{ACB} \cos 10^\circ - T_{ACB} \cos 30^\circ - T_{CD} \cos 30^\circ = 0$$

$$T_{CD} = 0.137158 T_{ACB} \quad (1)$$

$$+\uparrow \Sigma F_y = 0: T_{ACB} \sin 10^\circ + T_{ACB} \sin 30^\circ + T_{CD} \sin 30^\circ - 200 = 0$$

$$0.67365 T_{ACB} + 0.5 T_{CD} = 200 \quad (2)$$

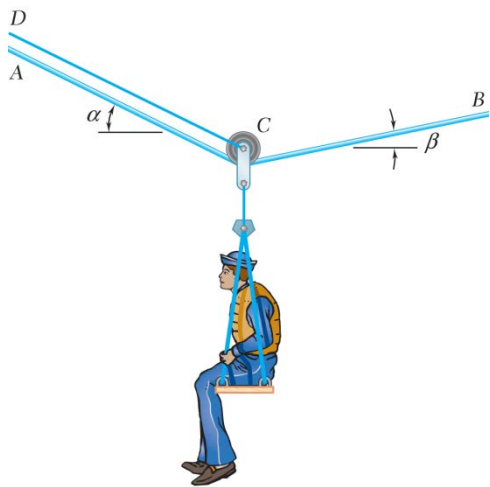
(a) Substitute (1) into (2):  $0.67365 T_{ACB} + 0.5(0.137158 T_{ACB}) = 200$

$$T_{ACB} = 269.46 \text{ lb}$$

$$T_{ACB} = 269 \text{ lb} \quad \blacktriangleleft$$

(b) From (1):  $T_{CD} = 0.137158(269.46 \text{ lb})$

$$T_{CD} = 37.0 \text{ lb} \quad \blacktriangleleft$$

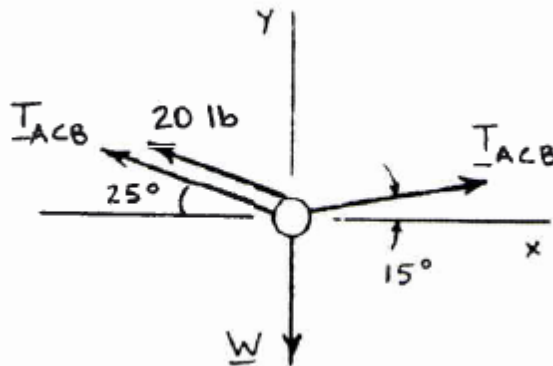


### PROBLEM 2.42

A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable  $ACB$  and is pulled at a constant speed by cable  $CD$ . Knowing that  $\alpha = 25^\circ$  and  $\beta = 15^\circ$  and that the tension in cable  $CD$  is 20 lb, determine (a) the combined weight of the boatswain's chair and the sailor, (b) the tension in the support cable  $ACB$ .

### SOLUTION

#### Free-Body Diagram



$$+\rightarrow \Sigma F_x = 0: T_{ACB} \cos 15^\circ - T_{ACB} \cos 25^\circ - (20 \text{ lb}) \cos 25^\circ = 0$$

$$T_{ACB} = 304.04 \text{ lb}$$

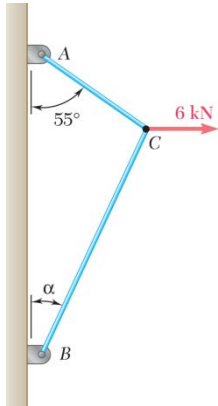
$$+\uparrow \Sigma F_y = 0: (304.04 \text{ lb}) \sin 15^\circ + (304.04 \text{ lb}) \sin 25^\circ$$

$$+ (20 \text{ lb}) \sin 25^\circ - W = 0$$

$$W = 215.64 \text{ lb}$$

$$(a) \quad W = 216 \text{ lb} \quad \blacktriangleleft$$

$$(b) \quad T_{ACB} = 304 \text{ lb} \quad \blacktriangleleft$$

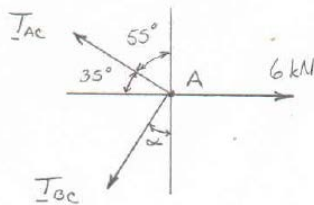


### PROBLEM 2.43

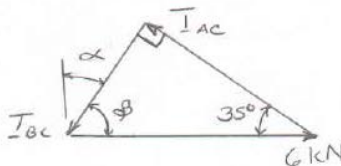
For the cables of prob. 2.32, find the value of  $\alpha$  for which the tension is as small as possible (a) in cable  $bc$ , (b) in both cables simultaneously. In each case determine the tension in each cable.

### SOLUTION

#### Free-Body Diagram



#### Force Triangle



(a) For a minimum tension in cable  $BC$ , set angle between cables to 90 degrees.

By inspection,

$$\alpha = 35.0^\circ \quad \blacktriangleleft$$

$$T_{AC} = (6 \text{ kN}) \cos 35^\circ$$

$$T_{AC} = 4.91 \text{ kN} \quad \blacktriangleleft$$

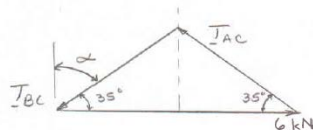
$$T_{BC} = (6 \text{ kN}) \sin 35^\circ$$

$$T_{BC} = 3.44 \text{ kN} \quad \blacktriangleleft$$

(b) For equal tension in both cables, the force triangle will be an isosceles.

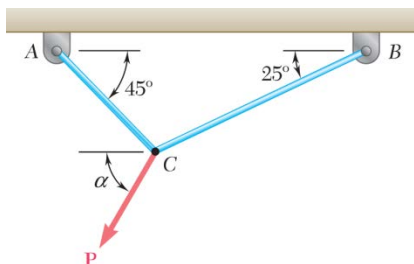
Therefore, by inspection,

$$\alpha = 55.0^\circ \quad \blacktriangleleft$$



$$T_{AC} = T_{BC} = (1/2) \frac{6 \text{ kN}}{\cos 35^\circ}$$

$$T_{AC} = T_{BC} = 3.66 \text{ kN} \quad \blacktriangleleft$$

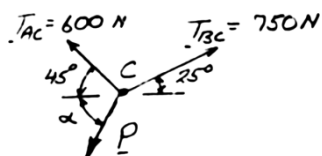


### PROBLEM 2.44

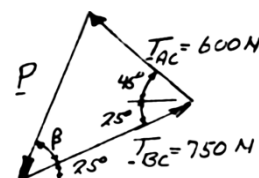
For the cables of Problem 2.36, it is known that the maximum allowable tension is 600 N in cable AC and 750 N in cable BC. Determine (a) the maximum force **P** that can be applied at C, (b) the corresponding value of  $\alpha$ .

### SOLUTION

#### Free-Body Diagram



#### Force Triangle



(a) Law of cosines

$$P^2 = (600)^2 + (750)^2 - 2(600)(750)\cos(25^\circ + 45^\circ)$$

$$P = 784.02 \text{ N}$$

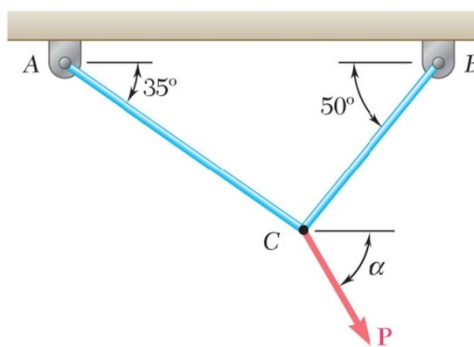
$$P = 784 \text{ N} \quad \blacktriangleleft$$

(b) Law of sines

$$\frac{\sin \beta}{600 \text{ N}} = \frac{\sin(25^\circ + 45^\circ)}{784.02 \text{ N}}$$

$$\beta = 46.0^\circ \quad \therefore \alpha = 46.0^\circ + 25^\circ$$

$$\alpha = 71.0^\circ \quad \blacktriangleleft$$

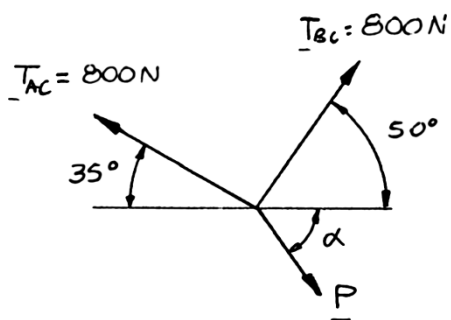


### PROBLEM 2.45

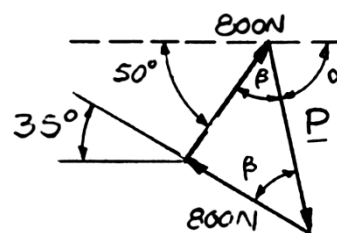
Two cables tied together at  $C$  are loaded as shown. Knowing that the maximum allowable tension in each cable is 800 N, determine  
 (a) the magnitude of the largest force  $P$  that can be applied at  $C$ ,  
 (b) the corresponding value of  $\alpha$ .

### SOLUTION

#### Free-Body Diagram: $C$



#### Force Triangle



Force triangle is isosceles with

$$2\beta = 180^\circ - 85^\circ$$

$$\beta = 47.5^\circ$$

(a)

$$P = 2(800 \text{ N})\cos 47.5^\circ = 1081 \text{ N}$$

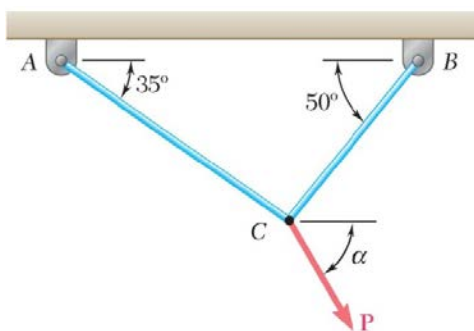
Since  $P > 0$ , the solution is correct.

$$P = 1081 \text{ N} \quad \blacktriangleleft$$

(b)

$$\alpha = 180^\circ - 50^\circ - 47.5^\circ = 82.5^\circ$$

$$\alpha = 82.5^\circ \quad \blacktriangleleft$$

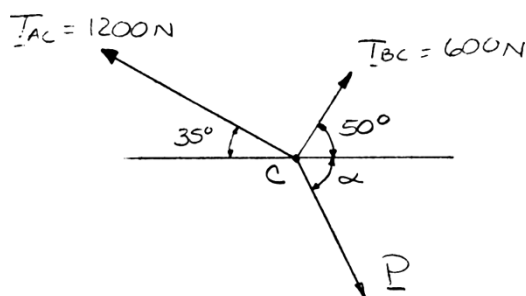


### PROBLEM 2.46

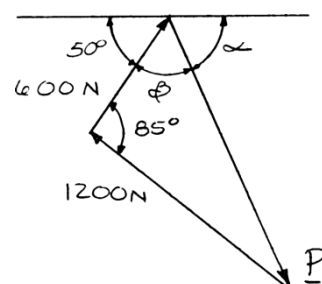
Two cables tied together at  $C$  are loaded as shown. Knowing that the maximum allowable tension is 1200 N in cable  $AC$  and 600 N in cable  $BC$ , determine (a) the magnitude of the largest force  $\mathbf{P}$  that can be applied at  $C$ , (b) the corresponding value of  $\alpha$ .

### SOLUTION

#### Free-Body Diagram



#### Force Triangle



(a) Law of cosines:

$$P^2 = (1200 \text{ N})^2 + (600 \text{ N})^2 - 2(1200 \text{ N})(600 \text{ N})\cos 85^\circ$$

$$P = 1294 \text{ N}$$

Since  $P < 1200 \text{ N}$ , the solution is correct.

$$P = 1294 \text{ N} \quad \blacktriangleleft$$

(b) Law of sines:

$$\frac{\sin \beta}{1200 \text{ N}} = \frac{\sin 85^\circ}{1294 \text{ N}}$$

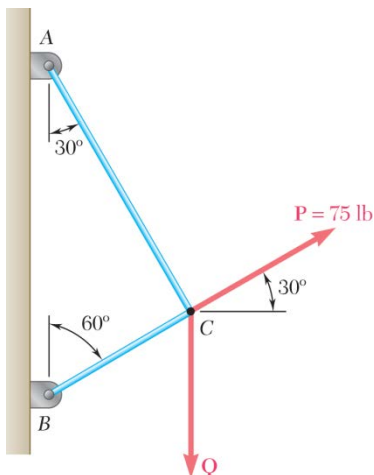
$$\beta = 67.5^\circ$$

$$\alpha = 180^\circ - 50^\circ - 67.5^\circ$$

$$\alpha = 62.5^\circ \quad \blacktriangleleft$$

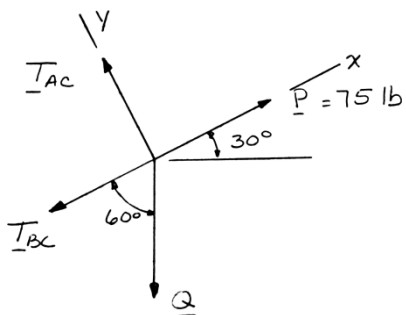
### PROBLEM 2.47

Two cables tied together at  $C$  are loaded as shown. Determine the range of values of  $Q$  for which the tension will not exceed 60 lb in either cable.



### SOLUTION

#### Free-Body Diagram



$$\Sigma F_x = 0: -T_{BC} - Q \cos 60^\circ + 75 \text{ lb} = 0$$

$$T_{BC} = 75 \text{ lb} - Q \cos 60^\circ \quad (1)$$

$$\Sigma F_y = 0: T_{AC} - Q \sin 60^\circ = 0$$

$$T_{AC} = Q \sin 60^\circ \quad (2)$$

Requirement:  $T_{AC} = 60 \text{ lb}:$

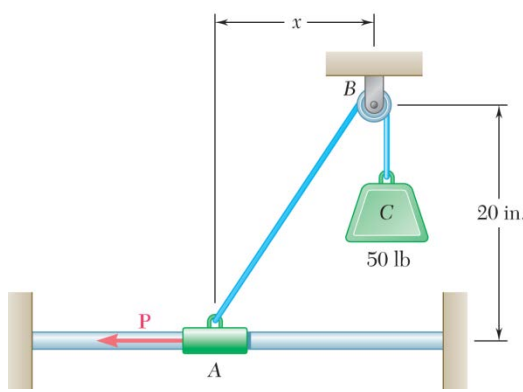
From Eq. (2):  $Q \sin 60^\circ = 60 \text{ lb}$

$$Q = 69.3 \text{ lb}$$

Requirement:  $T_{BC} = 60 \text{ lb}:$

From Eq. (1):  $75 \text{ lb} - Q \cos 60^\circ = 60 \text{ lb}$

$$Q = 30.0 \text{ lb} \quad 30.0 \text{ lb} \leq Q \leq 69.3 \text{ lb} \quad \blacktriangleleft$$

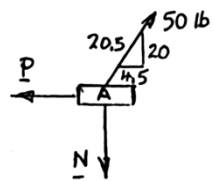
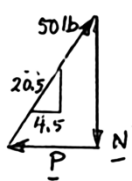


### PROBLEM 2.48

Collar A is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the magnitude of the force **P** required to maintain the equilibrium of the collar when (a)  $x = 4.5$  in., (b)  $x = 15$  in.

### SOLUTION

(a) **Free Body: Collar A**

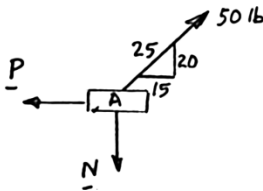
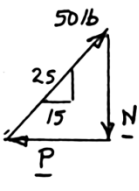



**Force Triangle**

$$\frac{P}{4.5} = \frac{50 \text{ lb}}{20.5}$$

$P = 10.98 \text{ lb} \quad \blacktriangleleft$

(b) **Free Body: Collar A**

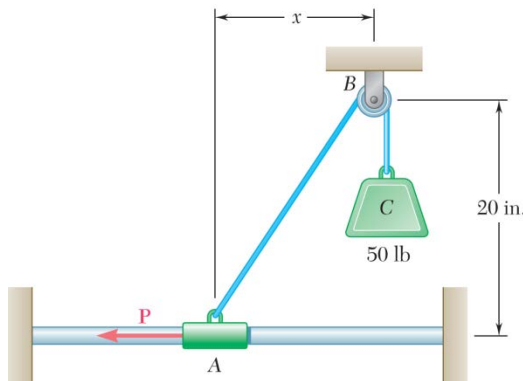



**Force Triangle**

$$\frac{P}{15} = \frac{50 \text{ lb}}{25}$$

$P = 30.0 \text{ lb} \quad \blacktriangleleft$



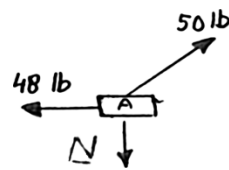


### PROBLEM 2.49


Collar A is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the distance  $x$  for which the collar is in equilibrium when  $P = 48$  lb.

### SOLUTION

**Free Body: Collar A**

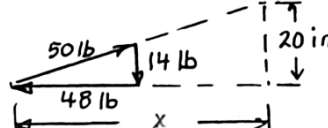


**Force Triangle**



$$N^2 = (50)^2 - (48)^2 = 196$$

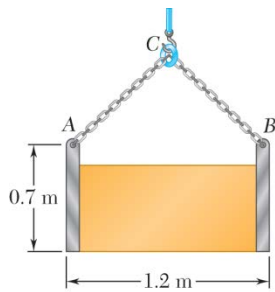
$$N = 14.00 \text{ lb}$$



$x = 68.6 \text{ in.} \quad \blacktriangleleft$

**Similar Triangles**

$$\frac{x}{20 \text{ in.}} = \frac{48 \text{ lb}}{14 \text{ lb}}$$

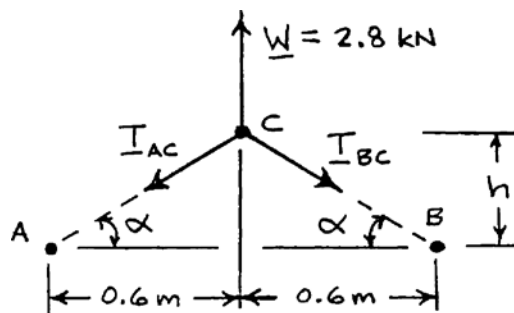


### PROBLEM 2.50

A movable bin and its contents have a combined weight of 2.8 kN. Determine the shortest chain sling  $ACB$  that can be used to lift the loaded bin if the tension in the chain is not to exceed 5 kN.

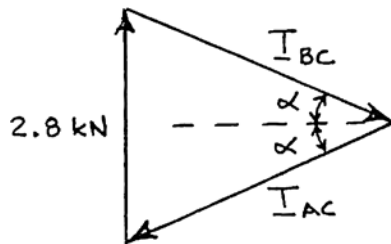
### SOLUTION

#### Free-Body Diagram



$$\tan \alpha = \frac{h}{0.6 \text{ m}} \quad (1)$$

#### Isosceles Force Triangle



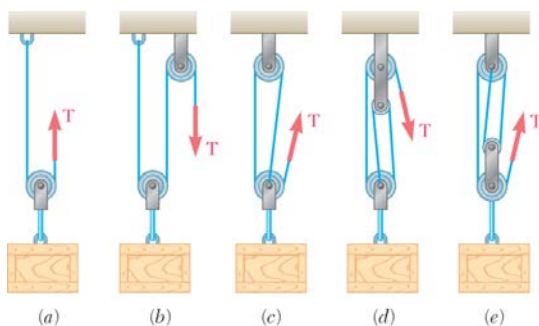
$$\begin{aligned} \text{Law of sines: } \sin \alpha &= \frac{\frac{1}{2}(2.8 \text{ kN})}{T_{AC}} \\ T_{AC} &= 5 \text{ kN} \\ \sin \alpha &= \frac{\frac{1}{2}(2.8 \text{ kN})}{5 \text{ kN}} \\ \alpha &= 16.2602^\circ \end{aligned}$$

$$\text{From Eq. (1): } \tan 16.2602^\circ = \frac{h}{0.6 \text{ m}} \quad \therefore h = 0.175000 \text{ m}$$

$$\begin{aligned} \text{Half-length of chain} = AC &= \sqrt{(0.6 \text{ m})^2 + (0.175 \text{ m})^2} \\ &= 0.625 \text{ m} \end{aligned}$$

$$\text{Total length: } = 2 \times 0.625 \text{ m}$$

$$1.250 \text{ m} \quad \blacktriangleleft$$

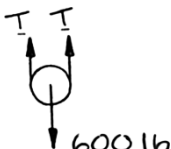


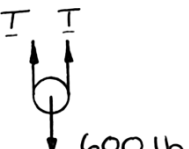
### PROBLEM 2.51

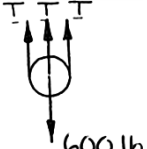
A 600-lb crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (*Hint:* The tension in the rope is the same on each side of a simple pulley. This can be proved by the methods of Ch. 4.)

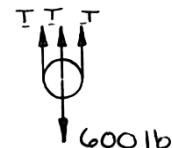
### SOLUTION


#### Free-Body Diagram of Pulley

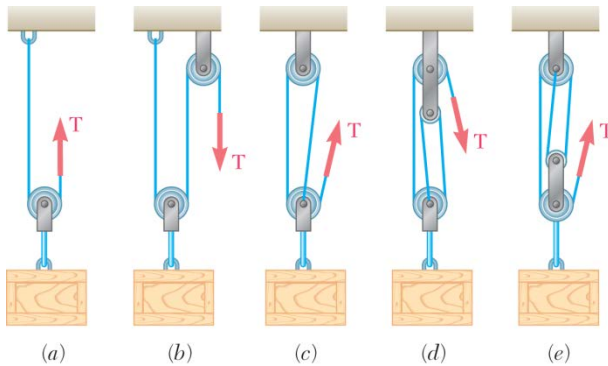
(a)  
$$+\uparrow \Sigma F_y = 0: 2T - (600 \text{ lb}) = 0$$
$$T = \frac{1}{2}(600 \text{ lb})$$
$$T = 300 \text{ lb} \quad \blacktriangleleft$$

(b)  
$$+\uparrow \Sigma F_y = 0: 2T - (600 \text{ lb}) = 0$$
$$T = \frac{1}{2}(600 \text{ lb})$$
$$T = 300 \text{ lb} \quad \blacktriangleleft$$

(c)  
$$+\uparrow \Sigma F_y = 0: 3T - (600 \text{ lb}) = 0$$
$$T = \frac{1}{3}(600 \text{ lb})$$
$$T = 200 \text{ lb} \quad \blacktriangleleft$$

(d)  
$$+\uparrow \Sigma F_y = 0: 3T - (600 \text{ lb}) = 0$$
$$T = \frac{1}{3}(600 \text{ lb})$$
$$T = 200 \text{ lb} \quad \blacktriangleleft$$

(e)  
$$+\uparrow \Sigma F_y = 0: 4T - (600 \text{ lb}) = 0$$
$$T = \frac{1}{4}(600 \text{ lb})$$
$$T = 150.0 \text{ lb} \quad \blacktriangleleft$$



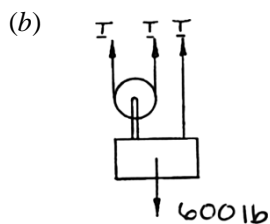
### PROBLEM 2.52

Solve Parts *b* and *d* of Problem 2.51, assuming that the free end of the rope is attached to the crate.

**PROBLEM 2.51** A 600-lb crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. . (*Hint:* The tension in the rope is the same on each side of a simple pulley. This can be proved by the methods of Ch. 4.)

### SOLUTION

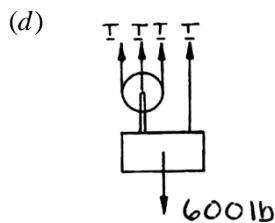
#### Free-Body Diagram of Pulley and Crate



$$+\uparrow \Sigma F_y = 0: 3T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{3}(600 \text{ lb})$$

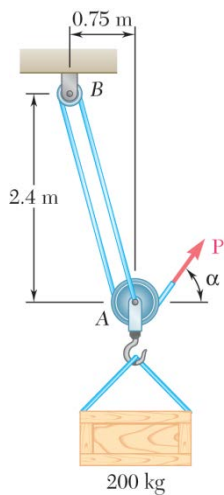
$$T = 200 \text{ lb} \quad \blacktriangleleft$$



$$+\uparrow \Sigma F_y = 0: 4T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{4}(600 \text{ lb})$$

$$T = 150.0 \text{ lb} \quad \blacktriangleleft$$

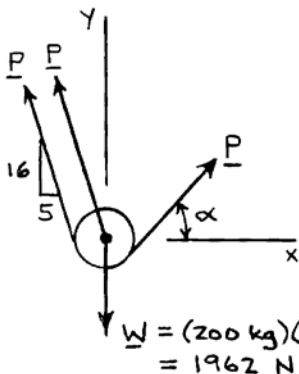


### PROBLEM 2.53

A 200-kg crate is to be supported by the rope-and-pulley arrangement shown. Determine the magnitude and direction of the force **P** that must be exerted on the free end of the rope to maintain equilibrium. (See the hint for Prob. 2.51.)

### SOLUTION

Free-Body Diagram: Pulley A



$$\rightarrow \Sigma F_x = 0: -2P\left(\frac{5}{\sqrt{281}}\right) + P \cos \alpha = 0$$

$$\cos \alpha = 0.59655$$

$$\alpha = \pm 53.377^\circ$$

For  $\alpha = +53.377^\circ$ :

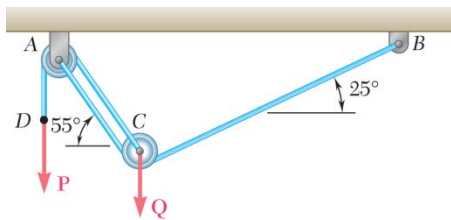
$$\uparrow \Sigma F_y = 0: 2P\left(\frac{16}{\sqrt{281}}\right) + P \sin 53.377^\circ - 1962 \text{ N} = 0$$

$$\mathbf{P} = 724 \text{ N } \nearrow 53.4^\circ \blacktriangleleft$$

For  $\alpha = -53.377^\circ$ :

$$\uparrow \Sigma F_y = 0: 2P\left(\frac{16}{\sqrt{281}}\right) + P \sin(-53.377^\circ) - 1962 \text{ N} = 0$$

$$\mathbf{P} = 1773 \text{ N } \searrow 53.4^\circ \blacktriangleleft$$

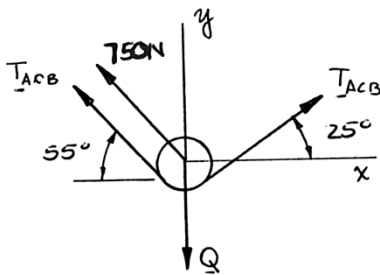


### PROBLEM 2.54

A load **Q** is applied to the pulley **C**, which can roll on the cable **ACB**. The pulley is held in the position shown by a second cable **CAD**, which passes over the pulley **A** and supports a load **P**. Knowing that  $P = 750 \text{ N}$ , determine (a) the tension in cable **ACB**, (b) the magnitude of load **Q**.

### SOLUTION

Free-Body Diagram: Pulley **C**



$$(a) \quad \pm \rightarrow \Sigma F_x = 0: \quad T_{ACB}(\cos 25^\circ - \cos 55^\circ) - (750 \text{ N})\cos 55^\circ = 0$$

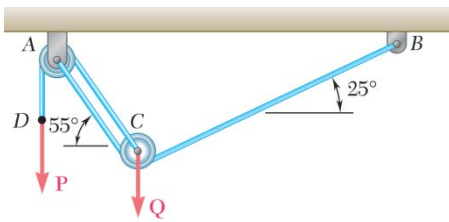
$$\text{Hence:} \quad T_{ACB} = 1292.88 \text{ N}$$

$$T_{ACB} = 1293 \text{ N} \quad \blacktriangleleft$$

$$(b) \quad +\uparrow \Sigma F_y = 0: \quad T_{ACB}(\sin 25^\circ + \sin 55^\circ) + (750 \text{ N})\sin 55^\circ - Q = 0$$

$$(1292.88 \text{ N})(\sin 25^\circ + \sin 55^\circ) + (750 \text{ N})\sin 55^\circ - Q = 0$$

$$\text{or} \quad Q = 2219.8 \text{ N} \quad Q = 2220 \text{ N} \quad \blacktriangleleft$$

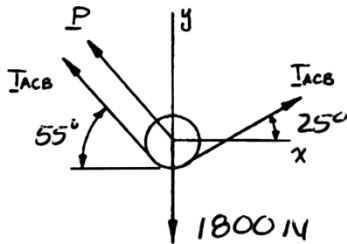


### PROBLEM 2.55

An 1800-N load **Q** is applied to the pulley **C**, which can roll on the cable **ACB**. The pulley is held in the position shown by a second cable **CAD**, which passes over the pulley **A** and supports a load **P**. Determine (a) the tension in cable **ACB**, (b) the magnitude of load **P**.

### SOLUTION

Free-Body Diagram: Pulley **C**



$$\rightarrow \Sigma F_x = 0: T_{ACB}(\cos 25^\circ - \cos 55^\circ) - P \cos 55^\circ = 0$$

or

$$P = 0.58010 T_{ACB} \quad (1)$$

$$+\uparrow \Sigma F_y = 0: T_{ACB}(\sin 25^\circ + \sin 55^\circ) + P \sin 55^\circ - 1800 \text{ N} = 0$$

or

$$1.24177 T_{ACB} + 0.81915 P = 1800 \text{ N} \quad (2)$$

(a) Substitute Equation (1) into Equation (2):

$$1.24177 T_{ACB} + 0.81915(0.58010 T_{ACB}) = 1800 \text{ N}$$

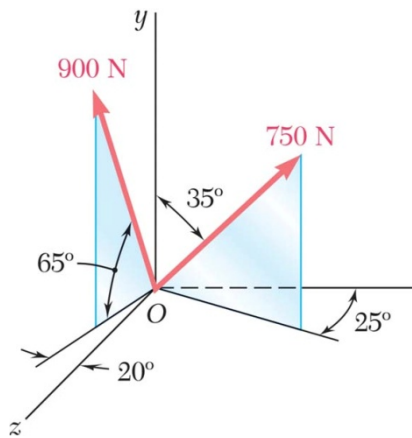
Hence:

$$T_{ACB} = 1048.37 \text{ N}$$

$$T_{ACB} = 1048 \text{ N} \quad \blacktriangleleft$$

(b) Using (1),  $P = 0.58010(1048.37 \text{ N}) = 608.16 \text{ N}$

$$P = 608 \text{ N} \quad \blacktriangleleft$$

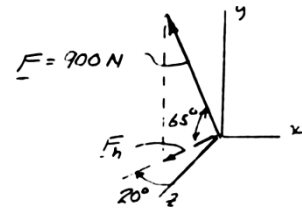


### PROBLEM 2.56

Determine (a) the  $x$ ,  $y$ , and  $z$  components of the 900-N force, (b) the angles  $\theta_x$ ,  $\theta_y$ , and  $\theta_z$  that the force forms with the coordinate axes.

### SOLUTION

$$\begin{aligned} F_h &= F \cos 65^\circ \\ &= (900 \text{ N}) \cos 65^\circ \\ F_h &= 380.36 \text{ N} \end{aligned}$$



(a) 
$$\begin{aligned} F_x &= F_h \sin 20^\circ \\ &= (380.36 \text{ N}) \sin 20^\circ \end{aligned}$$

$$F_x = -130.091 \text{ N},$$

$$F_x = -130.1 \text{ N} \quad \blacktriangleleft$$

$$\begin{aligned} F_y &= F \sin 65^\circ \\ &= (900 \text{ N}) \sin 65^\circ \end{aligned}$$

$$F_y = +815.68 \text{ N},$$

$$F_y = +816 \text{ N} \quad \blacktriangleleft$$

$$\begin{aligned} F_z &= F_h \cos 20^\circ \\ &= (380.36 \text{ N}) \cos 20^\circ \end{aligned}$$

$$F_z = +357.42 \text{ N}$$

$$F_z = +357 \text{ N} \quad \blacktriangleleft$$

(b) 
$$\cos \theta_x = \frac{F_x}{F} = \frac{-130.091 \text{ N}}{900 \text{ N}}$$

$$\theta_x = 98.3^\circ \quad \blacktriangleleft$$

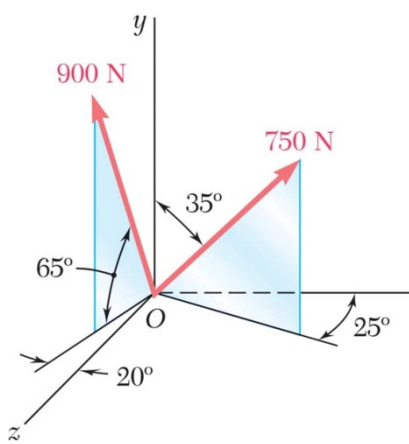
$$\cos \theta_y = \frac{F_y}{F} = \frac{+815.68 \text{ N}}{900 \text{ N}}$$

$$\theta_y = 25.0^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{+357.42 \text{ N}}{900 \text{ N}}$$

$$\theta_z = 66.6^\circ \quad \blacktriangleleft$$





### PROBLEM 2.57

Determine (a) the  $x$ ,  $y$ , and  $z$  components of the 750-N force, (b) the angles  $\theta_x$ ,  $\theta_y$ , and  $\theta_z$  that the force forms with the coordinate axes.

### SOLUTION

$$F_h = F \sin 35^\circ$$

$$= (750 \text{ N}) \sin 35^\circ$$

$$F_h = 430.18 \text{ N}$$
  

(a)

$$F_x = F_h \cos 25^\circ$$

$$= (430.18 \text{ N}) \cos 25^\circ$$

$$F_x = +389.88 \text{ N}, \quad F_x = +390 \text{ N} \blacktriangleleft$$
  

$$F_y = F \cos 35^\circ$$

$$= (750 \text{ N}) \cos 35^\circ$$

$$F_y = +614.36 \text{ N}, \quad F_y = +614 \text{ N} \blacktriangleleft$$
  

$$F_z = F_h \sin 25^\circ$$

$$= (430.18 \text{ N}) \sin 25^\circ$$

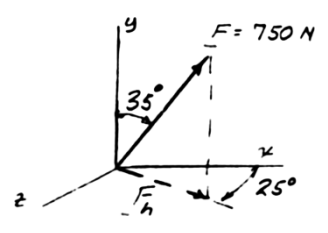
$$F_z = +181.802 \text{ N} \quad F_z = +181.8 \text{ N} \blacktriangleleft$$
  

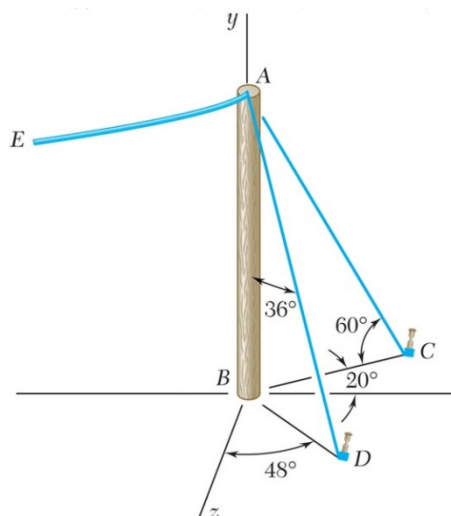
(b)

$$\cos \theta_x = \frac{F_x}{F} = \frac{+389.88 \text{ N}}{750 \text{ N}} \quad \theta_x = 58.7^\circ \blacktriangleleft$$
  

$$\cos \theta_y = \frac{F_y}{F} = \frac{+614.36 \text{ N}}{750 \text{ N}} \quad \theta_y = 35.0^\circ \blacktriangleleft$$
  

$$\cos \theta_z = \frac{F_z}{F} = \frac{+181.802 \text{ N}}{750 \text{ N}} \quad \theta_z = 76.0^\circ \blacktriangleleft$$





### PROBLEM 2.58

The end of the coaxial cable  $AE$  is attached to the pole  $AB$ , which is strengthened by the guy wires  $AC$  and  $AD$ . Knowing that the tension in wire  $AC$  is 120 lb, determine (a) the components of the force exerted by this wire on the pole, (b) the angles  $\theta_x$ ,  $\theta_y$ , and  $\theta_z$  that the force forms with the coordinate axes.

### SOLUTION

(a)

$$F_x = (120 \text{ lb}) \cos 60^\circ \cos 20^\circ$$

$$F_x = 56.382 \text{ lb}$$

$$F_x = +56.4 \text{ lb} \quad \blacktriangleleft$$

$$F_y = -(120 \text{ lb}) \sin 60^\circ$$

$$F_y = -103.923 \text{ lb}$$

$$F_y = -103.9 \text{ lb} \quad \blacktriangleleft$$

$$F_z = -(120 \text{ lb}) \cos 60^\circ \sin 20^\circ$$

$$F_z = -20.521 \text{ lb}$$

$$F_z = -20.5 \text{ lb} \quad \blacktriangleleft$$

(b)

$$\cos \theta_x = \frac{F_x}{F} = \frac{56.382 \text{ lb}}{120 \text{ lb}}$$

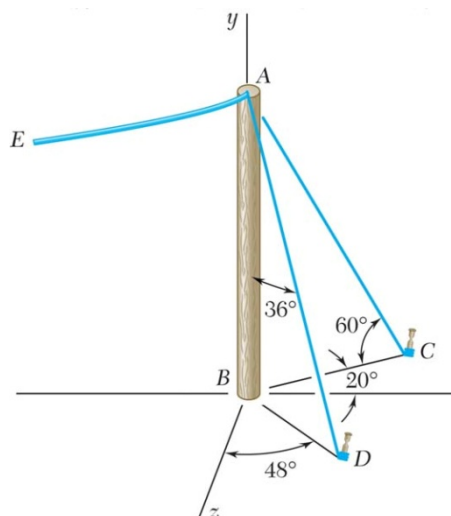
$$\theta_x = 62.0^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{-103.923 \text{ lb}}{120 \text{ lb}}$$

$$\theta_y = 150.0^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{-20.52 \text{ lb}}{120 \text{ lb}}$$

$$\theta_z = 99.8^\circ \quad \blacktriangleleft$$



### PROBLEM 2.59

The end of the coaxial cable  $AE$  is attached to the pole  $AB$ , which is strengthened by the guy wires  $AC$  and  $AD$ . Knowing that the tension in wire  $AD$  is 85 lb, determine (a) the components of the force exerted by this wire on the pole, (b) the angles  $\theta_x$ ,  $\theta_y$ , and  $\theta_z$  that the force forms with the coordinate axes.

### SOLUTION

(a)

$$F_x = (85 \text{ lb}) \sin 36^\circ \sin 48^\circ$$

$$= 37.129 \text{ lb}$$

$$F_x = 37.1 \text{ lb} \quad \blacktriangleleft$$

$$F_y = -(85 \text{ lb}) \cos 36^\circ$$

$$= -68.766 \text{ lb}$$

$$F_y = -68.8 \text{ lb} \quad \blacktriangleleft$$

$$F_z = (85 \text{ lb}) \sin 36^\circ \cos 48^\circ$$

$$= 33.431 \text{ lb}$$

$$F_z = 33.4 \text{ lb} \quad \blacktriangleleft$$

(b)

$$\cos \theta_x = \frac{F_x}{F} = \frac{37.129 \text{ lb}}{85 \text{ lb}}$$

$$\theta_x = 64.1^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{-68.766 \text{ lb}}{85 \text{ lb}}$$

$$\theta_y = 144.0^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{33.431 \text{ lb}}{85 \text{ lb}}$$

$$\theta_z = 66.8^\circ \quad \blacktriangleleft$$

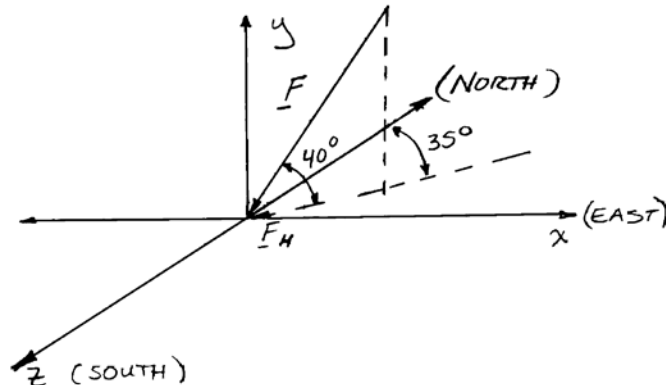
## PROBLEM 2.60

A gun is aimed at a point  $A$  located  $35^\circ$  east of north. Knowing that the barrel of the gun forms an angle of  $40^\circ$  with the horizontal and that the maximum recoil force is  $400\text{ N}$ , determine (a) the  $x$ ,  $y$ , and  $z$  components of that force, (b) the values of the angles  $\theta_x$ ,  $\theta_y$ , and  $\theta_z$  defining the direction of the recoil force. (Assume that the  $x$ ,  $y$ , and  $z$  axes are directed, respectively, east, up, and south.)

## SOLUTION

Recoil force

$$\begin{aligned} F &= 400\text{ N} \\ \therefore F_H &= (400\text{ N}) \cos 40^\circ \\ &= 306.42\text{ N} \end{aligned}$$



$$\begin{aligned} (a) \quad F_x &= -F_H \sin 35^\circ \\ &= -(306.42\text{ N}) \sin 35^\circ \\ &= -175.755\text{ N} \qquad F_x = -175.8\text{ N} \blacktriangleleft \end{aligned}$$

$$\begin{aligned} F_y &= -F \sin 40^\circ \\ &= -(400\text{ N}) \sin 40^\circ \\ &= -257.12\text{ N} \qquad F_y = -257\text{ N} \blacktriangleleft \end{aligned}$$

$$\begin{aligned} F_z &= +F_H \cos 35^\circ \\ &= +(306.42\text{ N}) \cos 35^\circ \\ &= +251.00\text{ N} \qquad F_z = +251\text{ N} \blacktriangleleft \end{aligned}$$

$$(b) \quad \cos \theta_x = \frac{F_x}{F} = \frac{-175.755\text{ N}}{400\text{ N}} \qquad \theta_x = 116.1^\circ \blacktriangleleft$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{-257.12\text{ N}}{400\text{ N}} \qquad \theta_y = 130.0^\circ \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{251.00\text{ N}}{400\text{ N}} \qquad \theta_z = 51.1^\circ \blacktriangleleft$$

## PROBLEM 2.61

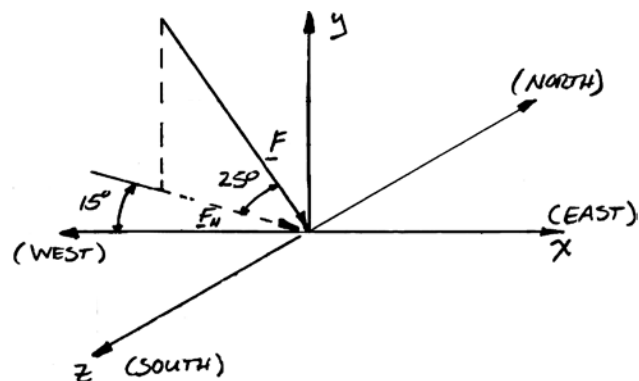
Solve Problem 2.60, assuming that point A is located  $15^\circ$  north of west and that the barrel of the gun forms an angle of  $25^\circ$  with the horizontal.

**PROBLEM 2.60** A gun is aimed at a point A located  $35^\circ$  east of north. Knowing that the barrel of the gun forms an angle of  $40^\circ$  with the horizontal and that the maximum recoil force is 400 N, determine (a) the  $x$ ,  $y$ , and  $z$  components of that force, (b) the values of the angles  $\theta_x$ ,  $\theta_y$ , and  $\theta_z$  defining the direction of the recoil force. (Assume that the  $x$ ,  $y$ , and  $z$  axes are directed, respectively, east, up, and south.)

## SOLUTION

Recoil force  $F = 400 \text{ N}$

$$\begin{aligned}\therefore F_H &= (400 \text{ N}) \cos 25^\circ \\ &= 362.52 \text{ N}\end{aligned}$$



$$\begin{aligned}(a) \quad F_x &= +F_H \cos 15^\circ \\ &= +(362.52 \text{ N}) \cos 15^\circ \\ &= +350.17 \text{ N} \qquad F_x = +350 \text{ N} \blacktriangleleft\end{aligned}$$

$$\begin{aligned}F_y &= -F \sin 25^\circ \\ &= -(400 \text{ N}) \sin 25^\circ \\ &= -169.047 \text{ N} \qquad F_y = -169.0 \text{ N} \blacktriangleleft\end{aligned}$$

$$\begin{aligned}F_z &= +F_H \sin 15^\circ \\ &= +(362.52 \text{ N}) \sin 15^\circ \\ &= +93.827 \text{ N} \qquad F_z = +93.8 \text{ N} \blacktriangleleft\end{aligned}$$

$$(b) \quad \cos \theta_x = \frac{F_x}{F} = \frac{+350.17 \text{ N}}{400 \text{ N}} \qquad \theta_x = 28.9^\circ \blacktriangleleft$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{-169.047 \text{ N}}{400 \text{ N}} \qquad \theta_y = 115.0^\circ \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{+93.827 \text{ N}}{400 \text{ N}} \qquad \theta_z = 76.4^\circ \blacktriangleleft$$

### PROBLEM 2.62

Determine the magnitude and direction of the force  $\mathbf{F} = (690 \text{ lb})\mathbf{i} + (300 \text{ lb})\mathbf{j} - (580 \text{ lb})\mathbf{k}$ .

### SOLUTION

$$\mathbf{F} = (690 \text{ lb})\mathbf{i} + (300 \text{ lb})\mathbf{j} - (580 \text{ lb})\mathbf{k}$$

$$\begin{aligned} F &= \sqrt{F_x^2 + F_y^2 + F_z^2} \\ &= \sqrt{(690 \text{ lb})^2 + (300 \text{ lb})^2 + (-580 \text{ lb})^2} \\ &= 950 \text{ lb} \end{aligned}$$

$$F = 950 \text{ lb} \quad \blacktriangleleft$$

$$\cos \theta_x = \frac{F_x}{F} = \frac{690 \text{ lb}}{950 \text{ lb}}$$

$$\theta_x = 43.4^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{300 \text{ lb}}{950 \text{ lb}}$$

$$\theta_y = 71.6^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{-580 \text{ lb}}{950 \text{ lb}}$$

$$\theta_z = 127.6^\circ \quad \blacktriangleleft$$

### PROBLEM 2.63

Determine the magnitude and direction of the force  $\mathbf{F} = (650 \text{ N})\mathbf{i} - (320 \text{ N})\mathbf{j} + (760 \text{ N})\mathbf{k}$ .

### SOLUTION

$$\mathbf{F} = (650 \text{ N})\mathbf{i} - (320 \text{ N})\mathbf{j} + (760 \text{ N})\mathbf{k}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$
$$= \sqrt{(650 \text{ N})^2 + (-320 \text{ N})^2 + (760 \text{ N})^2} \qquad F = 1050 \text{ N} \quad \blacktriangleleft$$

$$\cos \theta_x = \frac{F_x}{F} = \frac{650 \text{ N}}{1050 \text{ N}} \qquad \theta_x = 51.8^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{-320 \text{ N}}{1050 \text{ N}} \qquad \theta_y = 107.7^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{760 \text{ N}}{1050 \text{ N}} \qquad \theta_z = 43.6^\circ \quad \blacktriangleleft$$

### PROBLEM 2.64

A force acts at the origin of a coordinate system in a direction defined by the angles  $\theta_x = 69.3^\circ$  and  $\theta_z = 57.9^\circ$ . Knowing that the y component of the force is  $-174.0$  lb, determine (a) the angle  $\theta_y$ , (b) the other components and the magnitude of the force.

### SOLUTION

$$\begin{aligned}\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z &= 1 \\ \cos^2 (69.3^\circ) + \cos^2 \theta_y + \cos^2 (57.9^\circ) &= 1 \\ \cos \theta_y &= \pm 0.7699\end{aligned}$$

(a) Since  $F_y < 0$ , we choose  $\cos \theta_y = -0.7699$   $\therefore \theta_y = 140.3^\circ \blacktriangleleft$

(b)

$$\begin{aligned}F_y &= F \cos \theta_y \\ -174.0 \text{ lb} &= F(-0.7699) \\ F &= 226.0 \text{ lb} & F = 226 \text{ lb} \blacktriangleleft \\ F_x &= F \cos \theta_x = (226.0 \text{ lb}) \cos 69.3^\circ & F_x = 79.9 \text{ lb} \blacktriangleleft \\ F_z &= F \cos \theta_z = (226.0 \text{ lb}) \cos 57.9^\circ & F_z = 120.1 \text{ lb} \blacktriangleleft\end{aligned}$$



### PROBLEM 2.65

A force acts at the origin of a coordinate system in a direction defined by the angles  $\theta_x = 70.9^\circ$  and  $\theta_y = 144.9^\circ$ . Knowing that the  $z$  component of the force is  $-52.0$  lb, determine (a) the angle  $\theta_z$ , (b) the other components and the magnitude of the force.

### SOLUTION

$$\begin{aligned}\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z &= 1 \\ \cos^2 70.9^\circ + \cos^2 144.9^\circ + \cos^2 \theta_z &= 1 \\ \cos \theta_z &= \pm 0.47282\end{aligned}$$

(a) Since  $F_z < 0$ , we choose  $\cos \theta_z = -0.47282$   $\therefore \theta_z = 118.2^\circ \blacktriangleleft$

(b)

$$\begin{aligned}F_z &= F \cos \theta_z \\ -52.0 \text{ lb} &= F(-0.47282) \\ F &= 110.0 \text{ lb} & F = 110.0 \text{ lb} \blacktriangleleft \\ F_x &= F \cos \theta_x = (110.0 \text{ lb}) \cos 70.9^\circ & F_x = 36.0 \text{ lb} \blacktriangleleft \\ F_y &= F \cos \theta_y = (110.0 \text{ lb}) \cos 144.9^\circ & F_y = -90.0 \text{ lb} \blacktriangleleft\end{aligned}$$

### PROBLEM 2.66

A force acts at the origin of a coordinate system in a direction defined by the angles  $\theta_y = 55^\circ$  and  $\theta_z = 45^\circ$ . Knowing that the  $x$  component of the force is  $-500$  lb, determine (a) the angle  $\theta_x$ , (b) the other components and the magnitude of the force.

### SOLUTION

(a) We have

$$(\cos \theta_x)^2 + (\cos \theta_y)^2 + (\cos \theta_z)^2 = 1 \Rightarrow (\cos \theta_x)^2 = 1 - (\cos \theta_y)^2 - (\cos \theta_z)^2$$

Since  $F_x < 0$  we must have  $\cos \theta_x < 0$

Thus, taking the negative square root, from above, we have:

$$\cos \theta_x = -\sqrt{1 - (\cos 55^\circ)^2 - (\cos 45^\circ)^2} = -0.41353 \quad \theta_x = 114.4^\circ \quad \blacktriangleleft$$

(b) Then:

$$F = \frac{F_x}{\cos \theta_x} = \frac{-500 \text{ lb}}{-0.41353} = 1209.10 \text{ lb} \quad F = 1209 \text{ lb} \quad \blacktriangleleft$$

$$\text{and} \quad F_y = F \cos \theta_y = (1209.10 \text{ lb}) \cos 55^\circ \quad F_y = 694 \text{ lb} \quad \blacktriangleleft$$

$$F_z = F \cos \theta_z = (1209.10 \text{ lb}) \cos 45^\circ \quad F_z = 855 \text{ lb} \quad \blacktriangleleft$$

### PROBLEM 2.67

A force  $\mathbf{F}$  of magnitude 1200 N acts at the origin of a coordinate system. Knowing that  $\theta_x = 65^\circ$ ,  $\theta_y = 40^\circ$ , and  $F_z > 0$ , determine (a) the components of the force, (b) the angle  $\theta_z$ .

### SOLUTION

$$\begin{aligned}\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z &= 1 \\ \cos^2 65^\circ + \cos^2 40^\circ + \cos^2 \theta_z &= 1 \\ \cos \theta_z &= \pm 0.48432\end{aligned}$$

(b) Since  $F_z > 0$ , we choose  $\cos \theta_z = 0.48432$ , or  $\theta_z = 61.032^\circ$   $\therefore \theta_z = 61.0^\circ \blacktriangleleft$

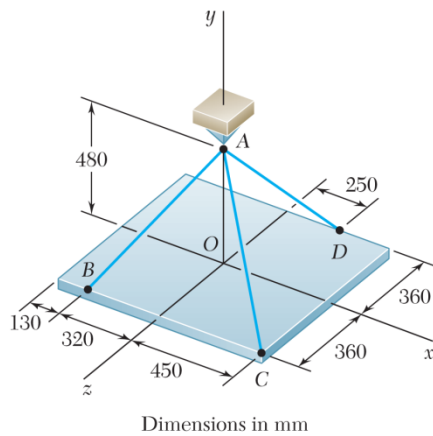
(a)  $F = 1200 \text{ N}$

$$F_x = F \cos \theta_x = (1200 \text{ N}) \cos 65^\circ \qquad F_x = 507 \text{ N} \blacktriangleleft$$

$$F_y = F \cos \theta_y = (1200 \text{ N}) \cos 40^\circ \qquad F_y = 919 \text{ N} \blacktriangleleft$$

$$F_z = F \cos \theta_z = (1200 \text{ N}) \cos 61.032^\circ \qquad F_z = 582 \text{ N} \blacktriangleleft$$

### PROBLEM 2.68



A rectangular plate is supported by three cables as shown. Knowing that the tension in cable  $AB$  is 408 N, determine the components of the force exerted on the plate at  $B$ .

### SOLUTION

We have:

$$\overline{BA} = +(320 \text{ mm})\mathbf{i} + (480 \text{ mm})\mathbf{j} - (360 \text{ mm})\mathbf{k} \quad BA = 680 \text{ mm}$$

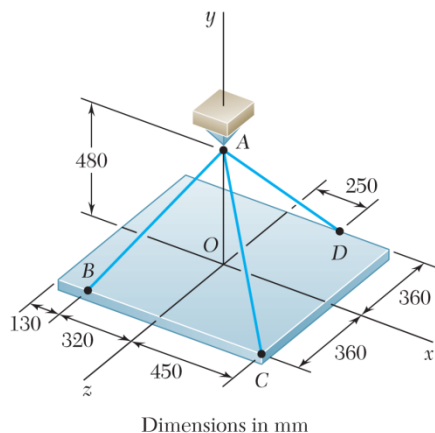
Thus:

$$\mathbf{F}_B = T_{BA} \lambda_{BA} = T_{BA} \frac{\overline{BA}}{BA} = T_{BA} \left( \frac{8}{17} \mathbf{i} + \frac{12}{17} \mathbf{j} - \frac{9}{17} \mathbf{k} \right)$$

$$\left( \frac{8}{17} T_{BA} \right) \mathbf{i} + \left( \frac{12}{17} T_{BA} \right) \mathbf{j} - \left( \frac{9}{17} T_{BA} \right) \mathbf{k} = 0$$

Setting  $T_{BA} = 408 \text{ N}$  yields,

$$F_x = +192.0 \text{ N}, \quad F_y = +288 \text{ N}, \quad F_z = -216 \text{ N} \quad \blacktriangleleft$$



### PROBLEM 2.69

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable  $AD$  is 429 N, determine the components of the force exerted on the plate at  $D$ .

### SOLUTION

We have:

$$\overrightarrow{DA} = -(250 \text{ mm})\mathbf{i} + (480 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \quad DA = 650 \text{ mm}$$

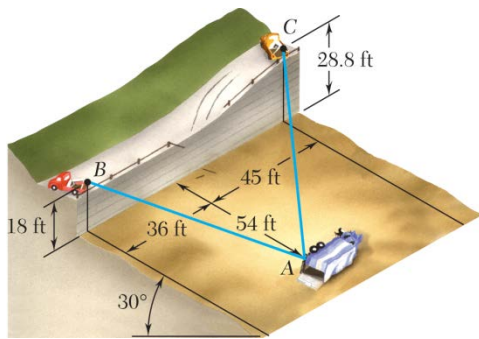
Thus:

$$\mathbf{F}_D = T_{DA} \lambda_{DA} = T_{DA} \frac{\overrightarrow{DA}}{DA} = T_{DA} \left( -\frac{5}{13} \mathbf{i} + \frac{48}{65} \mathbf{j} + \frac{36}{65} \mathbf{k} \right)$$

$$-\left( \frac{5}{13} T_{DA} \right) \mathbf{i} + \left( \frac{48}{65} T_{DA} \right) \mathbf{j} + \left( \frac{36}{65} T_{DA} \right) \mathbf{k} = 0$$

Setting  $T_{DA} = 429 \text{ N}$  yields,

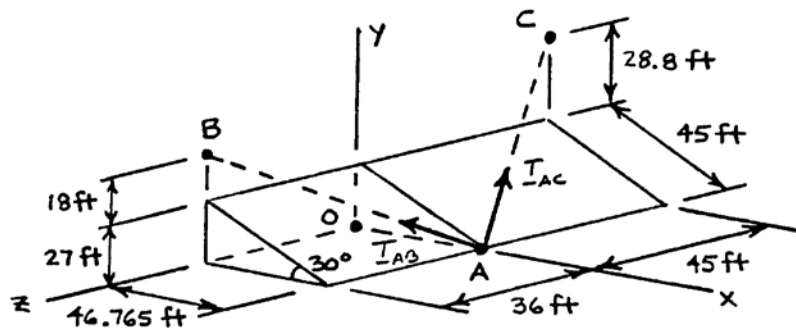
$$F_x = -165.0 \text{ N}, \quad F_y = +317 \text{ N}, \quad F_z = +238 \text{ N} \quad \blacktriangleleft$$



### PROBLEM 2.70

In order to move a wrecked truck, two cables are attached at  $A$  and pulled by winches  $B$  and  $C$  as shown. Knowing that the tension in cable  $AB$  is 2 kips, determine the components of the force exerted at  $A$  by the cable.

### SOLUTION



$$AB = 74.216 \text{ ft}$$

$$AC = 85.590 \text{ ft}$$

Cable  $AB$ :

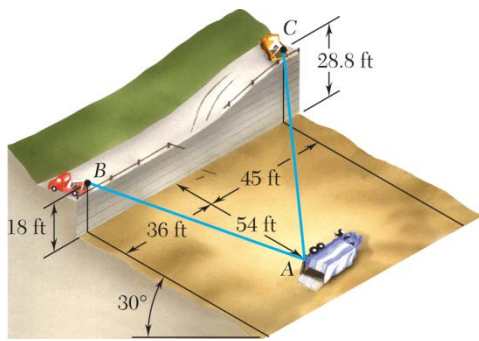
$$\lambda_{AB} = \frac{\overline{AB}}{AB} = \frac{(-46.765 \text{ ft})\mathbf{i} + (45 \text{ ft})\mathbf{j} + (36 \text{ ft})\mathbf{k}}{74.216 \text{ ft}}$$

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = \frac{-46.765\mathbf{i} + 45\mathbf{j} + 36\mathbf{k}}{74.216}$$

$$(T_{AB})_x = -1.260 \text{ kips} \quad \blacktriangleleft$$

$$(T_{AB})_y = +1.213 \text{ kips} \quad \blacktriangleleft$$

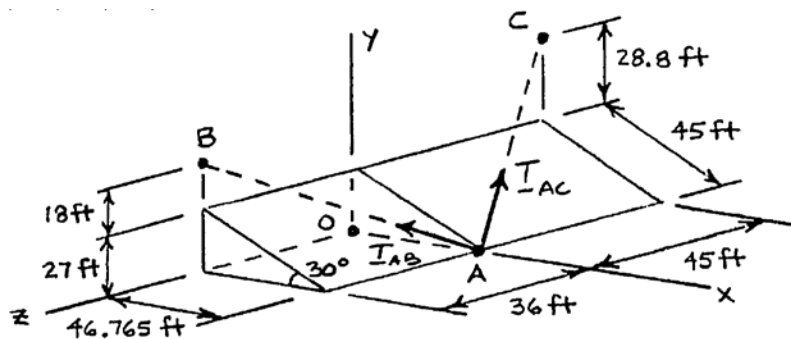
$$(T_{AB})_z = +0.970 \text{ kips} \quad \blacktriangleleft$$



### PROBLEM 2.71

In order to move a wrecked truck, two cables are attached at  $A$  and pulled by winches  $B$  and  $C$  as shown. Knowing that the tension in cable  $AC$  is 1.5 kips, determine the components of the force exerted at  $A$  by the cable.

### SOLUTION



$$AB = 74.216 \text{ ft}$$

$$AC = 85.590 \text{ ft}$$

Cable  $AB$ :

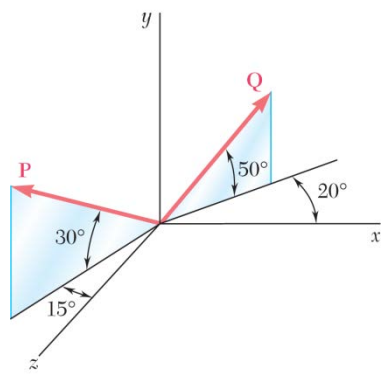
$$\lambda_{AC} = \frac{\overline{AC}}{AC} = \frac{(-46.765 \text{ ft})\mathbf{i} + (55.8 \text{ ft})\mathbf{j} + (-45 \text{ ft})\mathbf{k}}{85.590 \text{ ft}}$$

$$\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = (1.5 \text{ kips}) \frac{-46.765\mathbf{i} + 55.8\mathbf{j} - 45\mathbf{k}}{85.590}$$

$$(T_{AC})_x = -0.820 \text{ kips} \quad \blacktriangleleft$$

$$(T_{AC})_y = +0.978 \text{ kips} \quad \blacktriangleleft$$

$$(T_{AC})_z = -0.789 \text{ kips} \quad \blacktriangleleft$$



### PROBLEM 2.72

Find the magnitude and direction of the resultant of the two forces shown knowing that  $P = 300 \text{ N}$  and  $Q = 400 \text{ N}$ .

### SOLUTION

$$\mathbf{P} = (300 \text{ N})[-\cos 30^\circ \sin 15^\circ \mathbf{i} + \sin 30^\circ \mathbf{j} + \cos 30^\circ \cos 15^\circ \mathbf{k}]$$

$$= -(67.243 \text{ N})\mathbf{i} + (150 \text{ N})\mathbf{j} + (250.95 \text{ N})\mathbf{k}$$

$$\mathbf{Q} = (400 \text{ N})[\cos 50^\circ \cos 20^\circ \mathbf{i} + \sin 50^\circ \mathbf{j} - \cos 50^\circ \sin 20^\circ \mathbf{k}]$$

$$= (400 \text{ N})[0.60402\mathbf{i} + 0.76604\mathbf{j} - 0.21985\mathbf{k}]$$

$$= (241.61 \text{ N})\mathbf{i} + (306.42 \text{ N})\mathbf{j} - (87.939 \text{ N})\mathbf{k}$$

$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$

$$= (174.367 \text{ N})\mathbf{i} + (456.42 \text{ N})\mathbf{j} + (163.011 \text{ N})\mathbf{k}$$

$$R = \sqrt{(174.367 \text{ N})^2 + (456.42 \text{ N})^2 + (163.011 \text{ N})^2}$$

$$= 515.07 \text{ N}$$

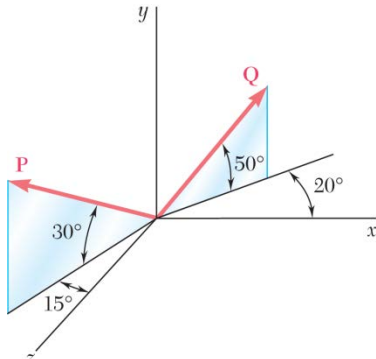
$$\cos \theta_x = \frac{R_x}{R} = \frac{174.367 \text{ N}}{515.07 \text{ N}} = 0.33853$$

$$\cos \theta_y = \frac{R_y}{R} = \frac{456.42 \text{ N}}{515.07 \text{ N}} = 0.88613$$

$$\cos \theta_z = \frac{R_z}{R} = \frac{163.011 \text{ N}}{515.07 \text{ N}} = 0.31648$$

$R = 515 \text{ N} \quad \blacktriangleleft$   
 $\theta_x = 70.2^\circ \quad \blacktriangleleft$   
 $\theta_y = 27.6^\circ \quad \blacktriangleleft$   
 $\theta_z = 71.5^\circ \quad \blacktriangleleft$





### PROBLEM 2.73

Find the magnitude and direction of the resultant of the two forces shown knowing that  $P = 400 \text{ N}$  and  $Q = 300 \text{ N}$ .

### SOLUTION

$$\begin{aligned} \mathbf{P} &= (400 \text{ N})[-\cos 30^\circ \sin 15^\circ \mathbf{i} + \sin 30^\circ \mathbf{j} + \cos 30^\circ \cos 15^\circ \mathbf{k}] \\ &= -(89.678 \text{ N})\mathbf{i} + (200 \text{ N})\mathbf{j} + (334.61 \text{ N})\mathbf{k} \\ \mathbf{Q} &= (300 \text{ N})[\cos 50^\circ \cos 20^\circ \mathbf{i} + \sin 50^\circ \mathbf{j} - \cos 50^\circ \sin 20^\circ \mathbf{k}] \\ &= (181.21 \text{ N})\mathbf{i} + (229.81 \text{ N})\mathbf{j} - (65.954 \text{ N})\mathbf{k} \\ \mathbf{R} &= \mathbf{P} + \mathbf{Q} \\ &= (91.532 \text{ N})\mathbf{i} + (429.81 \text{ N})\mathbf{j} + (268.66 \text{ N})\mathbf{k} \\ R &= \sqrt{(91.532 \text{ N})^2 + (429.81 \text{ N})^2 + (268.66 \text{ N})^2} \\ &= 515.07 \text{ N} \end{aligned}$$

$R = 515 \text{ N} \quad \blacktriangleleft$

$$\cos \theta_x = \frac{R_x}{R} = \frac{91.532 \text{ N}}{515.07 \text{ N}} = 0.177708$$

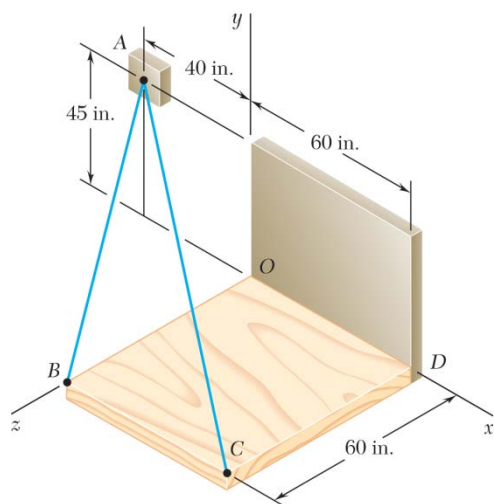
$\theta_x = 79.8^\circ \quad \blacktriangleleft$

$$\cos \theta_y = \frac{R_y}{R} = \frac{429.81 \text{ N}}{515.07 \text{ N}} = 0.83447$$

$\theta_y = 33.4^\circ \quad \blacktriangleleft$

$$\cos \theta_z = \frac{R_z}{R} = \frac{268.66 \text{ N}}{515.07 \text{ N}} = 0.52160$$

$\theta_z = 58.6^\circ \quad \blacktriangleleft$



### PROBLEM 2.74

Knowing that the tension is 425 lb in cable  $AB$  and 510 lb in cable  $AC$ , determine the magnitude and direction of the resultant of the forces exerted at  $A$  by the two cables.

### SOLUTION

$$\overline{AB} = (40 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}$$

$$AB = \sqrt{(40 \text{ in.})^2 + (45 \text{ in.})^2 + (60 \text{ in.})^2} = 85 \text{ in.}$$

$$\overline{AC} = (100 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}$$

$$AC = \sqrt{(100 \text{ in.})^2 + (45 \text{ in.})^2 + (60 \text{ in.})^2} = 125 \text{ in.}$$

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} = (425 \text{ lb}) \left[ \frac{(40 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}}{85 \text{ in.}} \right]$$

$$\mathbf{T}_{AB} = (200 \text{ lb})\mathbf{i} - (225 \text{ lb})\mathbf{j} + (300 \text{ lb})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = (510 \text{ lb}) \left[ \frac{(100 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}}{125 \text{ in.}} \right]$$

$$\mathbf{T}_{AC} = (408 \text{ lb})\mathbf{i} - (183.6 \text{ lb})\mathbf{j} + (244.8 \text{ lb})\mathbf{k}$$

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (608)\mathbf{i} - (408.6 \text{ lb})\mathbf{j} + (544.8 \text{ lb})\mathbf{k}$$

Then:

$$R = 912.92 \text{ lb}$$

$$R = 913 \text{ lb} \quad \blacktriangleleft$$

and

$$\cos \theta_x = \frac{608 \text{ lb}}{912.92 \text{ lb}} = 0.66599$$

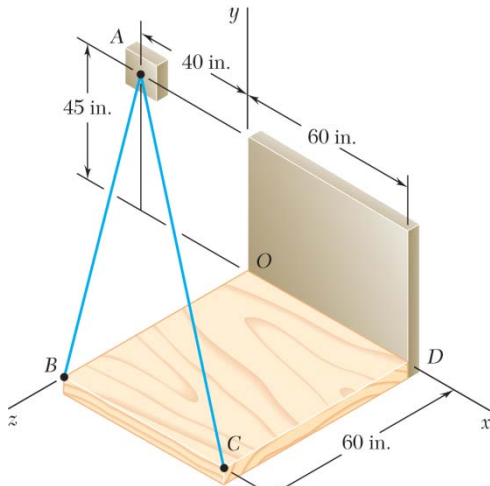
$$\theta_x = 48.2^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{408.6 \text{ lb}}{912.92 \text{ lb}} = -0.44757$$

$$\theta_y = 116.6^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{544.8 \text{ lb}}{912.92 \text{ lb}} = 0.59677$$

$$\theta_z = 53.4^\circ \quad \blacktriangleleft$$



### PROBLEM 2.75

Knowing that the tension is 510 lb in cable  $AB$  and 425 lb in cable  $AC$ , determine the magnitude and direction of the resultant of the forces exerted at  $A$  by the two cables.

### SOLUTION

$$\overrightarrow{AB} = (40 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}$$

$$AB = \sqrt{(40 \text{ in.})^2 + (45 \text{ in.})^2 + (60 \text{ in.})^2} = 85 \text{ in.}$$

$$\overrightarrow{AC} = (100 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}$$

$$AC = \sqrt{(100 \text{ in.})^2 + (45 \text{ in.})^2 + (60 \text{ in.})^2} = 125 \text{ in.}$$

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overrightarrow{AB}}{AB} = (510 \text{ lb}) \left[ \frac{(40 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}}{85 \text{ in.}} \right]$$

$$\mathbf{T}_{AB} = (240 \text{ lb})\mathbf{i} - (270 \text{ lb})\mathbf{j} + (360 \text{ lb})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overrightarrow{AC}}{AC} = (425 \text{ lb}) \left[ \frac{(100 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}}{125 \text{ in.}} \right]$$

$$\mathbf{T}_{AC} = (340 \text{ lb})\mathbf{i} - (153 \text{ lb})\mathbf{j} + (204 \text{ lb})\mathbf{k}$$

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (580 \text{ lb})\mathbf{i} - (423 \text{ lb})\mathbf{j} + (564 \text{ lb})\mathbf{k}$$

Then:

$$R = 912.92 \text{ lb}$$

$$R = 913 \text{ lb} \quad \blacktriangleleft$$

and

$$\cos \theta_x = \frac{580 \text{ lb}}{912.92 \text{ lb}} = 0.63532$$

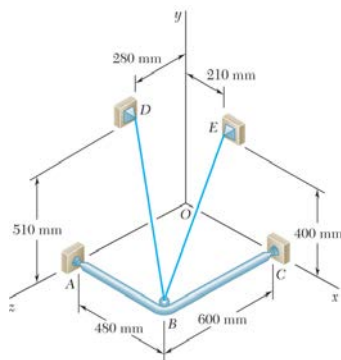
$$\theta_x = 50.6^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{-423 \text{ lb}}{912.92 \text{ lb}} = -0.46335$$

$$\theta_y = 117.6^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{564 \text{ lb}}{912.92 \text{ lb}} = 0.61780$$

$$\theta_z = 51.8^\circ \quad \blacktriangleleft$$



### PROBLEM 2.76

A frame  $ABC$  is supported in part by cable  $DBE$  that passes through a frictionless ring at  $B$ . Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at  $D$ .

### SOLUTION

$$\overrightarrow{BD} = -(480 \text{ mm})\mathbf{i} + (510 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k}$$

$$BD = \sqrt{(480 \text{ mm})^2 + (510 \text{ mm})^2 + (320 \text{ mm})^2} = 770 \text{ mm}$$

$$\begin{aligned}\mathbf{F}_{BD} &= T_{BD}\lambda_{BD} = T_{BD} \frac{\overrightarrow{BD}}{BD} \\ &= \frac{(385 \text{ N})}{(770 \text{ mm})} [-(480 \text{ mm})\mathbf{i} + (510 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k}] \\ &= -(240 \text{ N})\mathbf{i} + (255 \text{ N})\mathbf{j} - (160 \text{ N})\mathbf{k}\end{aligned}$$

$$\overrightarrow{BE} = -(270 \text{ mm})\mathbf{i} + (400 \text{ mm})\mathbf{j} - (600 \text{ mm})\mathbf{k}$$

$$BE = \sqrt{(270 \text{ mm})^2 + (400 \text{ mm})^2 + (600 \text{ mm})^2} = 770 \text{ mm}$$

$$\begin{aligned}\mathbf{F}_{BE} &= T_{BE}\lambda_{BE} = T_{BE} \frac{\overrightarrow{BE}}{BE} \\ &= \frac{(385 \text{ N})}{(770 \text{ mm})} [-(270 \text{ mm})\mathbf{i} + (400 \text{ mm})\mathbf{j} - (600 \text{ mm})\mathbf{k}] \\ &= -(135 \text{ N})\mathbf{i} + (200 \text{ N})\mathbf{j} - (300 \text{ N})\mathbf{k}\end{aligned}$$

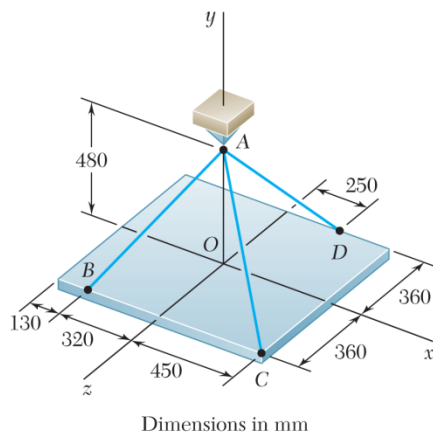
$$\mathbf{R} = \mathbf{F}_{BD} + \mathbf{F}_{BE} = -(375 \text{ N})\mathbf{i} + (455 \text{ N})\mathbf{j} - (460 \text{ N})\mathbf{k}$$

$$R = \sqrt{(375 \text{ N})^2 + (455 \text{ N})^2 + (460 \text{ N})^2} = 747.83 \text{ N} \quad R = 748 \text{ N} \quad \blacktriangleleft$$

$$\cos \theta_x = \frac{-375 \text{ N}}{747.83 \text{ N}} \quad \theta_x = 120.1^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{455 \text{ N}}{747.83 \text{ N}} \quad \theta_y = 52.5^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{-460 \text{ N}}{747.83 \text{ N}} \quad \theta_z = 128.0^\circ \quad \blacktriangleleft$$



### PROBLEM 2.77

For the plate of Prob. 2.68, determine the tensions in cables  $AB$  and  $AD$  knowing that the tension in cable  $AC$  is 54 N and that the resultant of the forces exerted by the three cables at  $A$  must be vertical.

### SOLUTION

We have:

$$\overrightarrow{AB} = -(320 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \quad AB = 680 \text{ mm}$$

$$\overrightarrow{AC} = (450 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \quad AC = 750 \text{ mm}$$

$$\overrightarrow{AD} = (250 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} - (360 \text{ mm})\mathbf{k} \quad AD = 650 \text{ mm}$$

Thus:

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overrightarrow{AB}}{AB} = \frac{T_{AB}}{680} (-320\mathbf{i} - 480\mathbf{j} + 360\mathbf{k})$$

$$\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overrightarrow{AC}}{AC} = \frac{54}{750} (450\mathbf{i} - 480\mathbf{j} + 360\mathbf{k})$$

$$\mathbf{T}_{AD} = T_{AD} \lambda_{AD} = T_{AD} \frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{650} (250\mathbf{i} - 480\mathbf{j} - 360\mathbf{k})$$

Substituting into the Eq.  $\mathbf{R} = \Sigma \mathbf{F}$  and factoring  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ :

$$\begin{aligned} \mathbf{R} = & \left( -\frac{320}{680}T_{AB} + 32.40 + \frac{250}{650}T_{AD} \right) \mathbf{i} \\ & + \left( -\frac{480}{680}T_{AB} - 34.560 - \frac{480}{650}T_{AD} \right) \mathbf{j} \\ & + \left( \frac{360}{680}T_{AB} + 25.920 - \frac{360}{650}T_{AD} \right) \mathbf{k} \end{aligned}$$

### SOLUTION (Continued)

Since **R** is vertical, the coefficients of **i** and **k** are zero:

$$\mathbf{i}: \quad -\frac{320}{680}T_{AB} + 32.40 + \frac{250}{650}T_{AD} = 0 \quad (1)$$

$$\mathbf{k}: \quad \frac{360}{680}T_{AB} + 25.920 - \frac{360}{650}T_{AD} = 0 \quad (2)$$

Multiply (1) by 3.6 and (2) by 2.5 then add:

$$-\frac{252}{680}T_{AB} + 181.440 = 0$$

$$T_{AB} = 489.60 \text{ N}$$

$$T_{AB} = 490 \text{ N} \quad \blacktriangleleft$$

Substitute into (2) and solve for  $T_{AD}$ :

$$\frac{360}{680}(489.60 \text{ N}) + 25.920 - \frac{360}{650}T_{AD} = 0$$

$$T_{AD} = 514.80 \text{ N}$$

$$T_{AD} = 515 \text{ N} \quad \blacktriangleleft$$

### PROBLEM 2.78

The boom  $OA$  carries a load  $\mathbf{P}$  and is supported by two cables as shown. Knowing that the tension in cable  $AB$  is 183 lb and that the resultant of the load  $\mathbf{P}$  and of the forces exerted at  $A$  by the two cables must be directed along  $OA$ , determine the tension in cable  $AC$ .

### SOLUTION

**Cable AB:**  $T_{AB} = 183 \text{ lb}$

$$\mathbf{T}_{AB} = T_{AB} \boldsymbol{\lambda}_{AB} = T_{AB} \frac{\overline{AB}}{AB} = (183 \text{ lb}) \frac{(-48 \text{ in.})\mathbf{i} + (29 \text{ in.})\mathbf{j} + (24 \text{ in.})\mathbf{k}}{61 \text{ in.}}$$

$$\mathbf{T}_{AB} = -(144 \text{ lb})\mathbf{i} + (87 \text{ lb})\mathbf{j} + (72 \text{ lb})\mathbf{k}$$

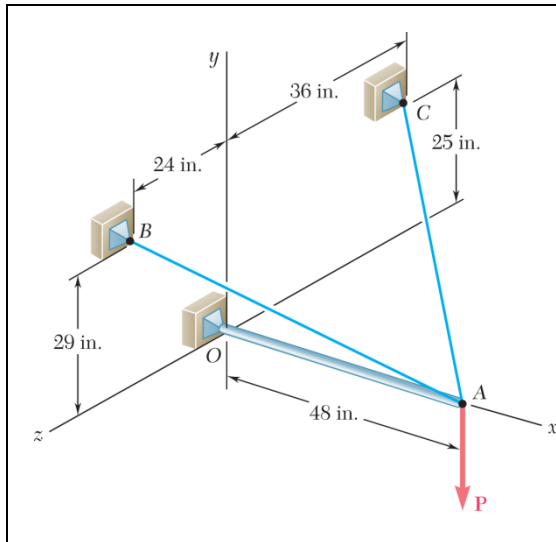
**Cable AC:**  $\mathbf{T}_{AC} = T_{AC} \boldsymbol{\lambda}_{AC} = T_{AC} \frac{\overline{AC}}{AC} = T_{AC} \frac{(-48 \text{ in.})\mathbf{i} + (25 \text{ in.})\mathbf{j} + (-36 \text{ in.})\mathbf{k}}{65 \text{ in.}}$

$$\mathbf{T}_{AC} = -\frac{48}{65}T_{AC}\mathbf{i} + \frac{25}{65}T_{AC}\mathbf{j} - \frac{36}{65}T_{AC}\mathbf{k}$$

**Load P:**  $\mathbf{P} = P\mathbf{j}$

For resultant to be directed along  $OA$ , i.e.,  $x$ -axis

$$R_z = 0: \quad \Sigma F_z = (72 \text{ lb}) - \frac{36}{65}T_{AC} = 0 \qquad T_{AC} = 130.0 \text{ lb} \quad \blacktriangleleft$$



### PROBLEM 2.79

For the boom and loading of Problem. 2.78, determine the magnitude of the load **P**.

**PROBLEM 2.78** The boom *OA* carries a load **P** and is supported by two cables as shown. Knowing that the tension in cable *AB* is 183 lb and that the resultant of the load **P** and of the forces exerted at *A* by the two cables must be directed along *OA*, determine the tension in cable *AC*.

### SOLUTION

See Problem 2.78. Since resultant must be directed along *OA*, i.e., the *x*-axis, we write

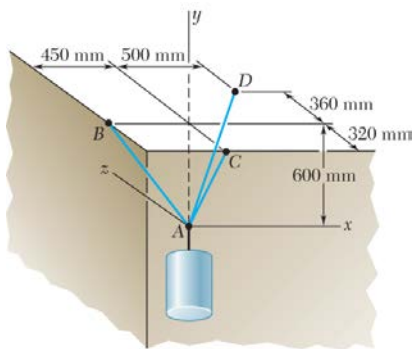
$$R_y = 0: \quad \Sigma F_y = (87 \text{ lb}) + \frac{25}{65} T_{AC} - P = 0$$

$T_{AC} = 130.0 \text{ lb}$  from Problem 2.97.

Then 
$$(87 \text{ lb}) + \frac{25}{65} (130.0 \text{ lb}) - P = 0$$

$$P = 137.0 \text{ lb} \quad \blacktriangleleft$$

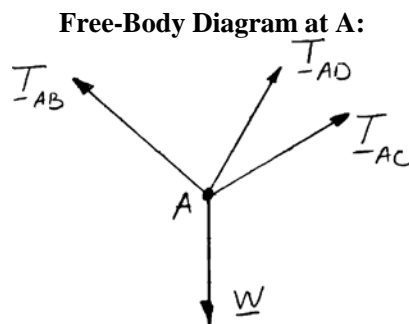




### PROBLEM 2.80

A container is supported by three cables that are attached to a ceiling as shown. Determine the weight  $W$  of the container, knowing that the tension in cable  $AB$  is 6 kN.

### SOLUTION



The forces applied at A are:

$\mathbf{T}_{AB}$ ,  $\mathbf{T}_{AC}$ ,  $\mathbf{T}_{AD}$ , and  $\mathbf{W}$

where  $\mathbf{W} = W\mathbf{j}$ . To express the other forces in terms of the unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ , we write

$$\overline{AB} = -(450 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} \quad AB = 750 \text{ mm}$$

$$\overline{AC} = +(600 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad AC = 680 \text{ mm}$$

$$\overline{AD} = +(500 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \quad AD = 860 \text{ mm}$$

and

$$\mathbf{T}_{AB} = \lambda_{AB}T_{AB} = T_{AB} \frac{\overline{AB}}{AB} = T_{AB} \frac{(-450 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j}}{750 \text{ mm}}$$

$$= \left( -\frac{45}{75}\mathbf{i} + \frac{60}{75}\mathbf{j} \right) T_{AB}$$

$$\mathbf{T}_{AC} = \lambda_{AC}T_{AC} = T_{AC} \frac{\overline{AC}}{AC} = T_{AC} \frac{(600 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k}}{680 \text{ mm}}$$

$$= \left( \frac{60}{68}\mathbf{j} - \frac{32}{68}\mathbf{k} \right) T_{AC}$$

$$\mathbf{T}_{AD} = \lambda_{AD}T_{AD} = T_{AD} \frac{\overline{AD}}{AD} = T_{AD} \frac{(500 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k}}{860 \text{ mm}}$$

$$= \left( \frac{50}{86}\mathbf{i} + \frac{60}{86}\mathbf{j} + \frac{36}{86}\mathbf{k} \right) T_{AD}$$

### SOLUTION (Continued)

*Equilibrium condition:*  $\Sigma F = 0: \therefore \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{W} = 0$

Substituting the expressions obtained for  $\mathbf{T}_{AB}$ ,  $\mathbf{T}_{AC}$ , and  $\mathbf{T}_{AD}$ ; factoring **i**, **j**, and **k**; and equating each of the coefficients to zero gives the following equations:

From **i**:  $-\frac{45}{75}T_{AB} + \frac{50}{86}T_{AD} = 0 \quad (1)$

From **j**:  $\frac{60}{75}T_{AB} + \frac{60}{68}T_{AC} + \frac{60}{86}T_{AD} - W = 0 \quad (2)$

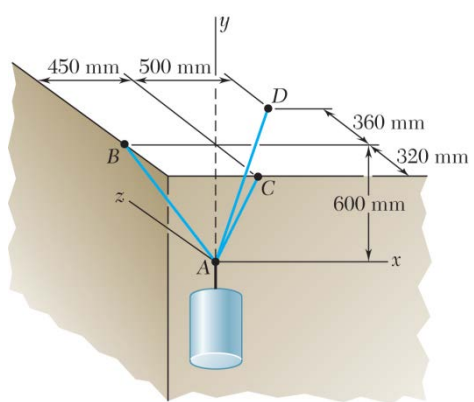
From **k**:  $-\frac{32}{68}T_{AC} + \frac{36}{86}T_{AD} = 0 \quad (3)$

Setting  $T_{AB} = 6 \text{ kN}$  in (1) and (2), and solving the resulting set of equations gives

$$T_{AC} = 6.1920 \text{ kN}$$

$$T_{AD} = 5.5080 \text{ kN}$$

$$W = 13.98 \text{ kN} \quad \blacktriangleleft$$

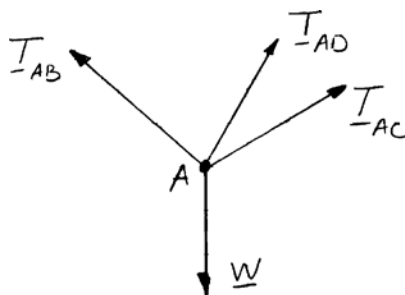


### PROBLEM 2.81

A container is supported by three cables that are attached to a ceiling as shown. Determine the weight  $W$  of the container, knowing that the tension in cable  $AD$  is 4.3 kN.

### SOLUTION

Free-Body Diagram at A:



The forces applied at A are:

$\mathbf{T}_{AB}$ ,  $\mathbf{T}_{AC}$ ,  $\mathbf{T}_{AD}$ , and  $\mathbf{W}$

where  $\mathbf{W} = W\mathbf{j}$ . To express the other forces in terms of the unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ , we write

$$\overline{AB} = -(450 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} \quad AB = 750 \text{ mm}$$

$$\overline{AC} = +(600 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad AC = 680 \text{ mm}$$

$$\overline{AD} = +(500 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \quad AD = 860 \text{ mm}$$

and

$$\begin{aligned} \mathbf{T}_{AB} &= \lambda_{AB} T_{AB} = T_{AB} \frac{\overline{AB}}{AB} = T_{AB} \frac{(-450 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j}}{750 \text{ mm}} \\ &= \left( -\frac{45}{75}\mathbf{i} + \frac{60}{75}\mathbf{j} \right) T_{AB} \end{aligned}$$

$$\begin{aligned} \mathbf{T}_{AC} &= \lambda_{AC} T_{AC} = T_{AC} \frac{\overline{AC}}{AC} = T_{AC} \frac{(600 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k}}{680 \text{ mm}} \\ &= \left( \frac{60}{68}\mathbf{j} - \frac{32}{68}\mathbf{k} \right) T_{AC} \end{aligned}$$

$$\begin{aligned} \mathbf{T}_{AD} &= \lambda_{AD} T_{AD} = T_{AD} \frac{\overline{AD}}{AD} = T_{AD} \frac{(500 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k}}{860 \text{ mm}} \\ &= \left( \frac{50}{86}\mathbf{i} + \frac{60}{86}\mathbf{j} + \frac{36}{86}\mathbf{k} \right) T_{AD} \end{aligned}$$

### PROBLEM 2.81 (Continued)

*Equilibrium condition:*  $\Sigma F = 0: \therefore \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{W} = 0$

Substituting the expressions obtained for  $\mathbf{T}_{AB}$ ,  $\mathbf{T}_{AC}$ , and  $\mathbf{T}_{AD}$ ; factoring **i**, **j**, and **k**; and equating each of the coefficients to zero gives the following equations:

From **i**: 
$$-\frac{45}{75}T_{AB} + \frac{50}{86}T_{AD} = 0$$

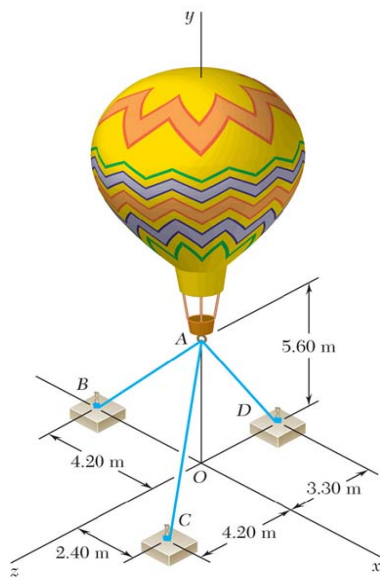
From **j**: 
$$\frac{60}{75}T_{AB} + \frac{60}{68}T_{AC} + \frac{60}{86}T_{AD} - W = 0$$

From **k**: 
$$-\frac{32}{68}T_{AC} + \frac{36}{86}T_{AD} = 0$$

Setting  $T_{AD} = 4.3 \text{ kN}$  into the above equations gives

$$T_{AB} = 4.1667 \text{ kN}$$

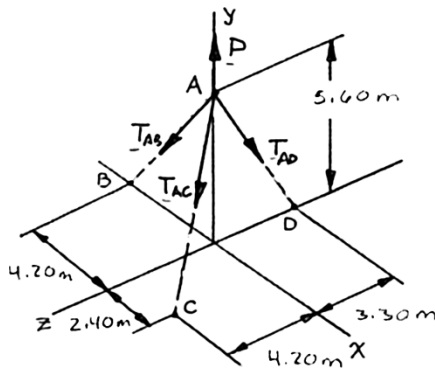
$$T_{AC} = 3.8250 \text{ kN} \quad W = 9.71 \text{ kN} \quad \blacktriangleleft$$



### PROBLEM 2.82

Three cables are used to tether a balloon as shown. Knowing that the balloon exerts an 800-N vertical force at A, determine the tension in each cable.

### SOLUTION



The forces applied at A are:

$\mathbf{T}_{AB}$ ,  $\mathbf{T}_{AC}$ ,  $\mathbf{T}_{AD}$ , and  $\mathbf{P}$

where  $\mathbf{P} = P\mathbf{j}$ . To express the other forces in terms of the unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ , we write

$$\overline{AB} = -(4.20 \text{ m})\mathbf{i} - (5.60 \text{ m})\mathbf{j} \quad AB = 7.00 \text{ m}$$

$$\overline{AC} = (2.40 \text{ m})\mathbf{i} - (5.60 \text{ m})\mathbf{j} + (4.20 \text{ m})\mathbf{k} \quad AC = 7.40 \text{ m}$$

$$\overline{AD} = -(5.60 \text{ m})\mathbf{j} - (3.30 \text{ m})\mathbf{k} \quad AD = 6.50 \text{ m}$$

and

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} = (-0.6\mathbf{i} - 0.8\mathbf{j})T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = (0.32432 - 0.75676\mathbf{j} + 0.56757\mathbf{k})T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\lambda_{AD} = T_{AD} \frac{\overline{AD}}{AD} = (-0.86154\mathbf{j} - 0.50769\mathbf{k})T_{AD}$$

### PROBLEM 2.82 (Continued)

Equilibrium condition  $\Sigma F = 0$ :  $\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$

Substituting the expressions obtained for  $\mathbf{T}_{AB}$ ,  $\mathbf{T}_{AC}$ , and  $\mathbf{T}_{AD}$  and factoring  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$ :

$$(-0.6T_{AB} + 0.32432T_{AC})\mathbf{i} + (-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P)\mathbf{j} \\ + (0.56757T_{AC} - 0.50769T_{AD})\mathbf{k} = 0$$

Equating to zero the coefficients of  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ :

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \quad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \quad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \quad (3)$$

From Eq. (1)  $T_{AB} = 0.54053T_{AC}$

From Eq. (3)  $T_{AD} = 1.11795T_{AC}$

Substituting for  $T_{AB}$  and  $T_{AD}$  in terms of  $T_{AC}$  into Eq. (2) gives:

$$-0.8(0.54053T_{AC}) - 0.75676T_{AC} - 0.86154(1.11795T_{AC}) + P = 0$$

$$2.1523T_{AC} = P; \quad P = 800 \text{ N}$$

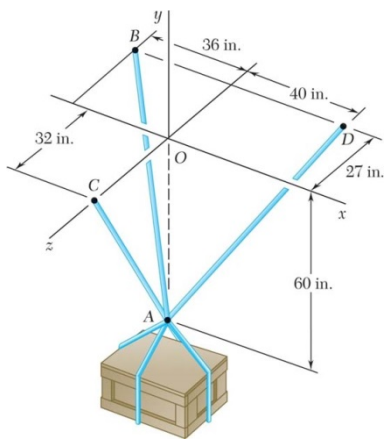
$$T_{AC} = \frac{800 \text{ N}}{2.1523} \\ = 371.69 \text{ N}$$

Substituting into expressions for  $T_{AB}$  and  $T_{AD}$  gives:

$$T_{AB} = 0.54053(371.69 \text{ N})$$

$$T_{AD} = 1.11795(371.69 \text{ N})$$

$$T_{AB} = 201 \text{ N}, \quad T_{AC} = 372 \text{ N}, \quad T_{AD} = 416 \text{ N} \quad \blacktriangleleft$$



### PROBLEM 2.83

A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable  $AD$  is 616 lb.

### SOLUTION

The forces applied at  $A$  are:

$$\mathbf{T}_{AB}, \mathbf{T}_{AC}, \mathbf{T}_{AD} \text{ and } \mathbf{W}$$

where  $\mathbf{P} = P\mathbf{j}$ . To express the other forces in terms of the unit vectors  $\mathbf{i}, \mathbf{j}, \mathbf{k}$ , we write

$$\overrightarrow{AB} = -(36 \text{ in.})\mathbf{i} + (60 \text{ in.})\mathbf{j} - (27 \text{ in.})\mathbf{k}$$

$$AB = 75 \text{ in.}$$

$$\overrightarrow{AC} = (60 \text{ in.})\mathbf{j} + (32 \text{ in.})\mathbf{k}$$

$$AC = 68 \text{ in.}$$

$$\overrightarrow{AD} = (40 \text{ in.})\mathbf{i} + (60 \text{ in.})\mathbf{j} - (27 \text{ in.})\mathbf{k}$$

$$AD = 77 \text{ in.}$$

and

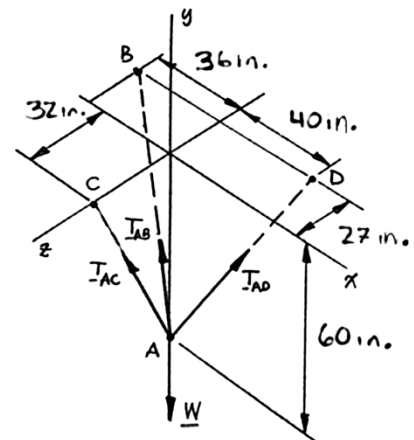
$$\begin{aligned} \mathbf{T}_{AB} &= T_{AB} \lambda_{AB} = T_{AB} \frac{\overrightarrow{AB}}{AB} \\ &= (-0.48\mathbf{i} + 0.8\mathbf{j} - 0.36\mathbf{k})T_{AB} \end{aligned}$$

$$\begin{aligned} \mathbf{T}_{AC} &= T_{AC} \lambda_{AC} = T_{AC} \frac{\overrightarrow{AC}}{AC} \\ &= (0.88235\mathbf{j} + 0.47059\mathbf{k})T_{AC} \end{aligned}$$

$$\begin{aligned} \mathbf{T}_{AD} &= T_{AD} \lambda_{AD} = T_{AD} \frac{\overrightarrow{AD}}{AD} \\ &= (0.51948\mathbf{i} + 0.77922\mathbf{j} - 0.35065\mathbf{k})T_{AD} \end{aligned}$$

Equilibrium Condition with  $\mathbf{W} = -W\mathbf{j}$

$$\Sigma \mathbf{F} = 0: \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} - W\mathbf{j} = 0$$



### PROBLEM 2.83 (Continued)

Substituting the expressions obtained for  $T_{AB}$ ,  $T_{AC}$ , and  $T_{AD}$  and factoring  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$ :

$$(-0.48T_{AB} + 0.51948T_{AD})\mathbf{i} + (0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W)\mathbf{j} \\ + (-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD})\mathbf{k} = 0$$

$$-0.48T_{AB} + 0.51948T_{AD} = 0$$

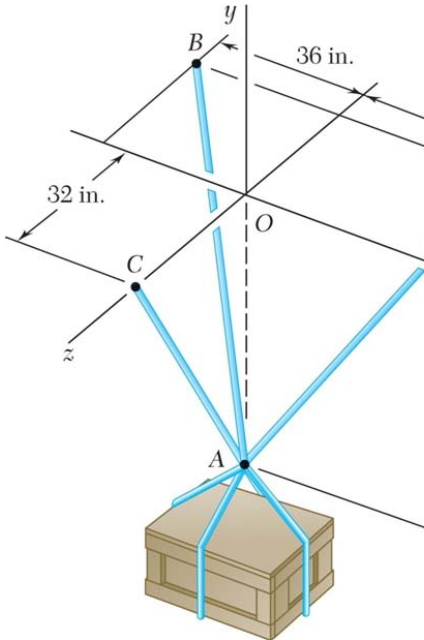
$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0$$

$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0$$

Substituting  $T_{AD} = 616 \text{ lb}$  in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives:

$$T_{AB} = 667.87 \text{ lb} \\ T_{AC} = 969.00 \text{ lb} \quad W = 1868 \text{ lb} \quad \blacktriangleleft$$





### PROBLEM 2.84

A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable AC is 544 lb.

### SOLUTION

See Problem 2.83 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB} + 0.51948T_{AD} = 0 \quad (1)$$

$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0 \quad (2)$$

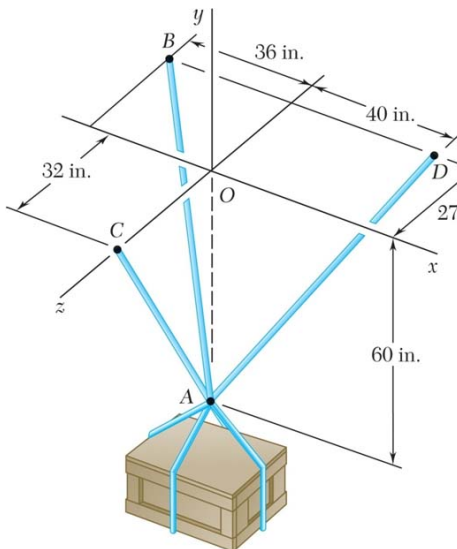
$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0 \quad (3)$$

Substituting  $T_{AC} = 544$  lb in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives:

$$T_{AB} = 374.27 \text{ lb}$$

$$T_{AD} = 345.82 \text{ lb}$$

$$W = 1049 \text{ lb} \quad \blacktriangleleft$$



### PROBLEM 2.85

A 1600-lb crate is supported by three cables as shown. Determine the tension in each cable.

### SOLUTION

See Problem 2.83 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB} + 0.51948T_{AD} = 0 \quad (1)$$

$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0 \quad (2)$$

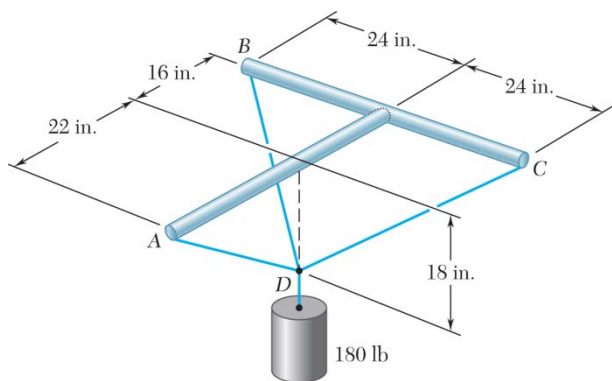
$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0 \quad (3)$$

Substituting  $W = 1600$  lb in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives

$$T_{AB} = 571 \text{ lb} \quad \blacktriangleleft$$

$$T_{AC} = 830 \text{ lb} \quad \blacktriangleleft$$

$$T_{AD} = 528 \text{ lb} \quad \blacktriangleleft$$

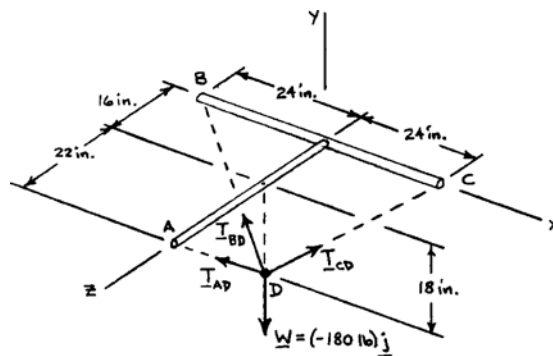


### PROBLEM 2.86

Three wires are connected at point  $D$ , which is located 18 in. below the T-shaped pipe support  $ABC$ . Determine the tension in each wire when a 180-lb cylinder is suspended from point  $D$  as shown.

### SOLUTION

#### Free-Body Diagram of Point $D$ :



The forces applied at  $D$  are:

$$\mathbf{T}_{DA}, \mathbf{T}_{DB}, \mathbf{T}_{DC} \text{ and } \mathbf{W}$$

where  $\mathbf{W} = -180.0 \text{ lb} \mathbf{j}$ . To express the other forces in terms of the unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ , we write

$$\overrightarrow{DA} = (18 \text{ in.})\mathbf{j} + (22 \text{ in.})\mathbf{k}$$

$$DA = 28.425 \text{ in.}$$

$$\overrightarrow{DB} = -(24 \text{ in.})\mathbf{i} + (18 \text{ in.})\mathbf{j} - (16 \text{ in.})\mathbf{k}$$

$$DB = 34.0 \text{ in.}$$

$$\overrightarrow{DC} = (24 \text{ in.})\mathbf{i} + (18 \text{ in.})\mathbf{j} - (16 \text{ in.})\mathbf{k}$$

$$DC = 34.0 \text{ in.}$$

### SOLUTION (Continued)

and

$$\begin{aligned}\mathbf{T}_{DA} &= T_{DA} \lambda_{DA} = T_{DA} \frac{\overline{DA}}{DA} \\ &= (0.63324\mathbf{j} + 0.77397\mathbf{k})T_{DA}\end{aligned}$$

$$\begin{aligned}\mathbf{T}_{DB} &= T_{DB} \lambda_{DB} = T_{DB} \frac{\overline{DB}}{DB} \\ &= (-0.70588\mathbf{i} + 0.52941\mathbf{j} - 0.47059\mathbf{k})T_{DB}\end{aligned}$$

$$\begin{aligned}\mathbf{T}_{DC} &= T_{DC} \lambda_{DC} = T_{DC} \frac{\overline{DC}}{DC} \\ &= (0.70588\mathbf{i} + 0.52941\mathbf{j} - 0.47059\mathbf{k})T_{DC}\end{aligned}$$

*Equilibrium Condition* with  $\mathbf{W} = -W\mathbf{j}$

$$\Sigma F = 0: \mathbf{T}_{DA} + \mathbf{T}_{DB} + \mathbf{T}_{DC} - W\mathbf{j} = 0$$

Substituting the expressions obtained for  $\mathbf{T}_{DA}$ ,  $\mathbf{T}_{DB}$ , and  $\mathbf{T}_{DC}$  and factoring  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$ :

$$\begin{aligned}&(-0.70588T_{DB} + 0.70588T_{DC})\mathbf{i} \\ &(0.63324T_{DA} + 0.52941T_{DB} + 0.52941T_{DC} - W)\mathbf{j} \\ &(0.77397T_{DA} - 0.47059T_{DB} - 0.47059T_{DC})\mathbf{k}\end{aligned}$$

Equating to zero the coefficients of  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ :

$$-0.70588T_{DB} + 0.70588T_{DC} = 0 \quad (1)$$

$$0.63324T_{DA} + 0.52941T_{DB} + 0.52941T_{DC} - W = 0 \quad (2)$$

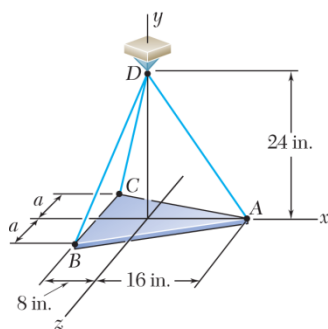
$$0.77397T_{DA} - 0.47059T_{DB} - 0.47059T_{DC} = 0 \quad (3)$$

Substituting  $W = 180$  lb in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms gives,

$$T_{DA} = 119.7 \text{ lb} \quad \blacktriangleleft$$

$$T_{DB} = 98.4 \text{ lb} \quad \blacktriangleleft$$

$$T_{DC} = 98.4 \text{ lb} \quad \blacktriangleleft$$



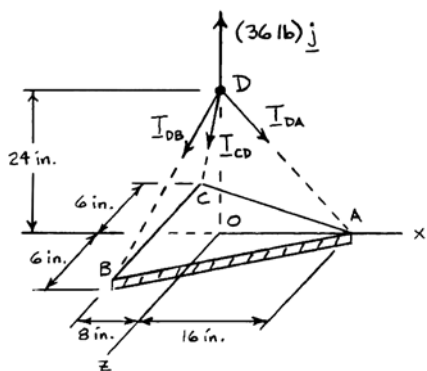
### PROBLEM 2.87

A 36-lb triangular plate is supported by three wires as shown. Determine the tension in each wire, knowing that  $a = 6$  in.

### SOLUTION

By Symmetry  $T_{DB} = T_{DC}$

Free-Body Diagram of Point  $D$ :



The forces applied at  $D$  are:

$\mathbf{T}_{DB}$ ,  $\mathbf{T}_{DC}$ ,  $\mathbf{T}_{DA}$ , and  $\mathbf{P}$

where  $\mathbf{P} = P\mathbf{j} = (36 \text{ lb})\mathbf{j}$ . To express the other forces in terms of the unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ , we write

$$\overline{DA} = (16 \text{ in.})\mathbf{i} - (24 \text{ in.})\mathbf{j} \quad DA = 28.844 \text{ in.}$$

$$\overline{DB} = -(8 \text{ in.})\mathbf{i} - (24 \text{ in.})\mathbf{j} + (6 \text{ in.})\mathbf{k} \quad DB = 26.0 \text{ in.}$$

$$\overline{DC} = -(8 \text{ in.})\mathbf{i} - (24 \text{ in.})\mathbf{j} - (6 \text{ in.})\mathbf{k} \quad DC = 26.0 \text{ in.}$$

and

$$\mathbf{T}_{DA} = T_{DA} \lambda_{DA} = T_{DA} \frac{\overline{DA}}{DA} = (0.5547\mathbf{i} - 0.83206\mathbf{j})T_{DA}$$

$$\mathbf{T}_{DB} = T_{DB} \lambda_{DB} = T_{DB} \frac{\overline{DB}}{DB} = (-0.30769\mathbf{i} - 0.92308\mathbf{j} + 0.23077\mathbf{k})T_{DB}$$

$$\mathbf{T}_{DC} = T_{DC} \lambda_{DC} = T_{DC} \frac{\overline{DC}}{DC} = (-0.30769\mathbf{i} - 0.92308\mathbf{j} - 0.23077\mathbf{k})T_{DC}$$

### SOLUTION (Continued)

*Equilibrium condition:*  $\Sigma F = 0: \mathbf{T}_{DA} + \mathbf{T}_{DB} + \mathbf{T}_{DC} + (36 \text{ lb})\mathbf{j} = 0$

Substituting the expressions obtained for  $\mathbf{T}_{DA}$ ,  $\mathbf{T}_{DB}$ , and  $\mathbf{T}_{DC}$  and factoring  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$ :

$$(0.55471T_{DA} - 0.30769T_{DB} - 0.30769T_{DC})\mathbf{i} + (-0.83206T_{DA} - 0.92308T_{DB} - 0.92308T_{DC} + 36 \text{ lb})\mathbf{j} \\ + (0.23077T_{DB} - 0.23077T_{DC})\mathbf{k} = 0$$

Equating to zero the coefficients of  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ :

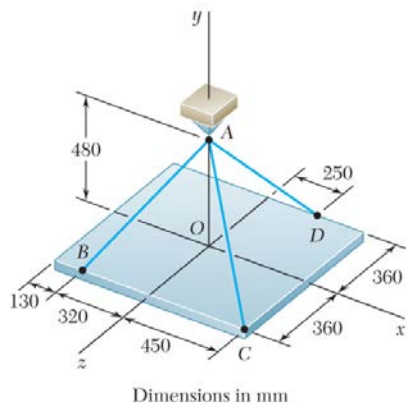
$$0.55471T_{DA} - 0.30769T_{DB} - 0.30769T_{DC} = 0 \quad (1)$$

$$-0.83206T_{DA} - 0.92308T_{DB} - 0.92308T_{DC} + 36 \text{ lb} = 0 \quad (2)$$

$$0.23077T_{DB} - 0.23077T_{DC} = 0 \quad (3)$$

Equation (3) confirms that  $T_{DB} = T_{DC}$ . Solving simultaneously gives,

$$T_{DA} = 14.42 \text{ lb}; \quad T_{DB} = T_{DC} = 13.00 \text{ lb} \quad \blacktriangleleft$$



### PROBLEM 2.88

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AC is 60 N, determine the weight of the plate.

### SOLUTION

We note that the weight of the plate is equal in magnitude to the force  $\mathbf{P}$  exerted by the support on Point A.

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

We have:

$$\overline{AB} = -(320 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \quad AB = 680 \text{ mm}$$

$$\overline{AC} = (450 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \quad AC = 750 \text{ mm}$$

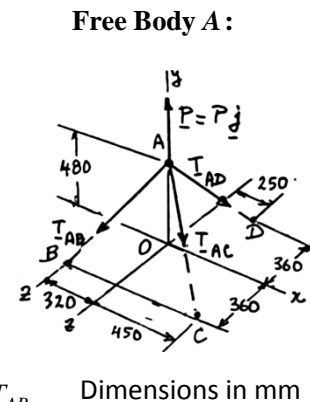
$$\overline{AD} = (250 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} - (360 \text{ mm})\mathbf{k} \quad AD = 650 \text{ mm}$$

Thus:

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} = \left( -\frac{8}{17}\mathbf{i} - \frac{12}{17}\mathbf{j} + \frac{9}{17}\mathbf{k} \right) T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = (0.6\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k}) T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD} \lambda_{AD} = T_{AD} \frac{\overline{AD}}{AD} = \left( \frac{5}{13}\mathbf{i} - \frac{9.6}{13}\mathbf{j} - \frac{7.2}{13}\mathbf{k} \right) T_{AD}$$



Substituting into the Eq.  $\Sigma \mathbf{F} = 0$  and factoring  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ :

$$\begin{aligned} & \left( -\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} \right) \mathbf{i} \\ & + \left( -\frac{12}{17}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P \right) \mathbf{j} \\ & + \left( \frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} \right) \mathbf{k} = 0 \end{aligned}$$

### SOLUTION (Continued)

Setting the coefficient of **i**, **j**, **k** equal to zero:

$$\mathbf{i}: \quad -\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \quad (1)$$

$$\mathbf{j}: \quad -\frac{12}{7}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P = 0 \quad (2)$$

$$\mathbf{k}: \quad \frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \quad (3)$$

Making  $T_{AC} = 60 \text{ N}$  in (1) and (3):

$$-\frac{8}{17}T_{AB} + 36 \text{ N} + \frac{5}{13}T_{AD} = 0 \quad (1')$$

$$\frac{9}{17}T_{AB} + 28.8 \text{ N} - \frac{7.2}{13}T_{AD} = 0 \quad (3')$$

Multiply (1') by 9, (3') by 8, and add:

$$554.4 \text{ N} - \frac{12.6}{13}T_{AD} = 0 \quad T_{AD} = 572.0 \text{ N}$$

Substitute into (1') and solve for  $T_{AB}$ :

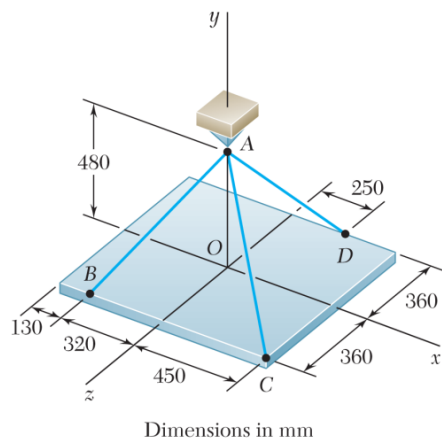
$$T_{AB} = \frac{17}{8} \left( 36 + \frac{5}{13} \times 572 \right) \quad T_{AB} = 544.0 \text{ N}$$

Substitute for the tensions in Eq. (2) and solve for  $P$ :

$$\begin{aligned} P &= \frac{12}{17}(544 \text{ N}) + 0.64(60 \text{ N}) + \frac{9.6}{13}(572 \text{ N}) \\ &= 844.8 \text{ N} \end{aligned}$$

Weight of plate =  $P = 845 \text{ N}$  ◀





### PROBLEM 2.89

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AD is 520 N, determine the weight of the plate.

### SOLUTION

See Problem 2.88 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \quad (1)$$

$$-\frac{12}{17}T_{AB} + 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P = 0 \quad (2)$$

$$\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \quad (3)$$

Making  $T_{AD} = 520 \text{ N}$  in Eqs. (1) and (3):

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + 200 \text{ N} = 0 \quad (1')$$

$$\frac{9}{17}T_{AB} + 0.48T_{AC} - 288 \text{ N} = 0 \quad (3')$$

Multiply (1') by 9, (3') by 8, and add:

$$9.24T_{AC} - 504 \text{ N} = 0 \quad T_{AC} = 54.5455 \text{ N}$$

Substitute into (1') and solve for  $T_{AB}$ :

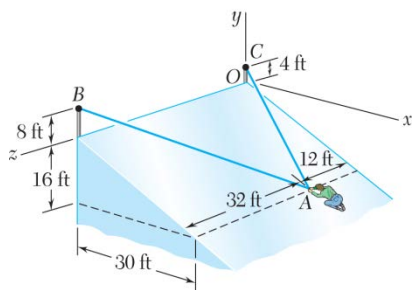
$$T_{AB} = \frac{17}{8}(0.6 \times 54.5455 + 200) \quad T_{AB} = 494.545 \text{ N}$$

Substitute for the tensions in Eq. (2) and solve for  $P$ :

$$P = \frac{12}{17}(494.545 \text{ N}) + 0.64(54.5455 \text{ N}) + \frac{9.6}{13}(520 \text{ N})$$

$$= 768.00 \text{ N}$$

Weight of plate =  $P = 768 \text{ N}$  ◀

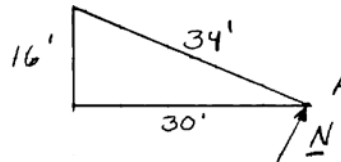
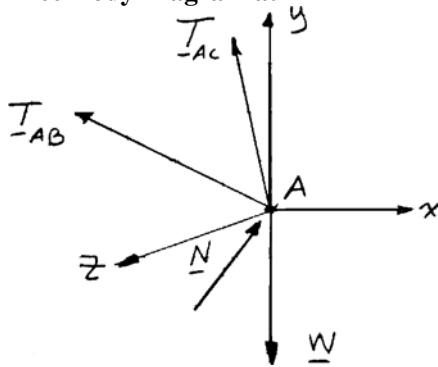


### PROBLEM 2.90

In trying to move across a slippery icy surface, a 175-lb man uses two ropes  $AB$  and  $AC$ . Knowing that the force exerted on the man by the icy surface is perpendicular to that surface, determine the tension in each rope.

### SOLUTION

Free-Body Diagram at  $A$



$$\mathbf{N} = N \left( \frac{16}{34} \mathbf{i} + \frac{30}{34} \mathbf{j} \right)$$

and  $\mathbf{W} = W \mathbf{j} = -(175 \text{ lb}) \mathbf{j}$

$$\begin{aligned} \mathbf{T}_{AC} &= T_{AC} \lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = T_{AC} \frac{(-30 \text{ ft}) \mathbf{i} + (20 \text{ ft}) \mathbf{j} - (12 \text{ ft}) \mathbf{k}}{38 \text{ ft}} \\ &= T_{AC} \left( -\frac{15}{19} \mathbf{i} + \frac{10}{19} \mathbf{j} - \frac{6}{19} \mathbf{k} \right) \\ \mathbf{T}_{AB} &= T_{AB} \lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} = T_{AB} \frac{(-30 \text{ ft}) \mathbf{i} + (24 \text{ ft}) \mathbf{j} + (32 \text{ ft}) \mathbf{k}}{50 \text{ ft}} \\ &= T_{AB} \left( -\frac{15}{25} \mathbf{i} + \frac{12}{25} \mathbf{j} + \frac{16}{25} \mathbf{k} \right) \end{aligned}$$

Equilibrium condition:  $\Sigma \mathbf{F} = 0$

$$\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{N} + \mathbf{W} = 0$$

### SOLUTION (Continued)

Substituting the expressions obtained for  $T_{AB}$ ,  $T_{AC}$ ,  $N$ , and  $W$ ; factoring **i**, **j**, and **k**; and equating each of the coefficients to zero gives the following equations:

From **i**: 
$$-\frac{15}{25}T_{AB} - \frac{15}{19}T_{AC} + \frac{16}{34}N = 0 \quad (1)$$

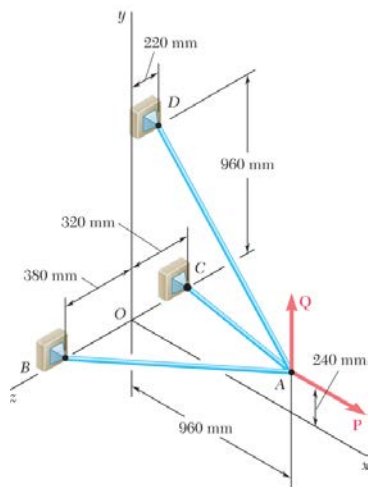
From **j**: 
$$\frac{12}{25}T_{AB} + \frac{10}{19}T_{AC} + \frac{30}{34}N - (175 \text{ lb}) = 0 \quad (2)$$

From **k**: 
$$\frac{16}{25}T_{AB} - \frac{6}{19}T_{AC} = 0 \quad (3)$$

Solving the resulting set of equations gives:

$$T_{AB} = 30.8 \text{ lb}; T_{AC} = 62.5 \text{ lb} \quad \blacktriangleleft$$





### PROBLEM 2.92

Three cables are connected at A, where the forces **P** and **Q** are applied as shown. Knowing that  $Q = 0$ , find the value of  $P$  for which the tension in cable AD is 305 N.

### SOLUTION

$$\Sigma \mathbf{F}_A = 0: \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} = 0 \quad \text{where} \quad \mathbf{P} = P\mathbf{i}$$

$$\overrightarrow{AB} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} + (380 \text{ mm})\mathbf{k} \quad AB = 1060 \text{ mm}$$

$$\overrightarrow{AC} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad AC = 1040 \text{ mm}$$

$$\overrightarrow{AD} = -(960 \text{ mm})\mathbf{i} + (720 \text{ mm})\mathbf{j} - (220 \text{ mm})\mathbf{k} \quad AD = 1220 \text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overrightarrow{AB}}{AB} = T_{AB} \left( -\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k} \right)$$

$$\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overrightarrow{AC}}{AC} = T_{AC} \left( -\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k} \right)$$

$$\begin{aligned} \mathbf{T}_{AD} &= T_{AD} \lambda_{AD} = \frac{305 \text{ N}}{1220 \text{ mm}} [(-960 \text{ mm})\mathbf{i} + (720 \text{ mm})\mathbf{j} - (220 \text{ mm})\mathbf{k}] \\ &= -(240 \text{ N})\mathbf{i} + (180 \text{ N})\mathbf{j} - (55 \text{ N})\mathbf{k} \end{aligned}$$

Substituting into  $\Sigma \mathbf{F}_A = 0$ , factoring **i**, **j**, **k**, and setting each coefficient equal to  $\phi$  gives:

$$\mathbf{i}: \quad P = \frac{48}{53}T_{AB} + \frac{12}{13}T_{AC} + 240 \text{ N} \quad (1)$$

$$\mathbf{j}: \quad \frac{12}{53}T_{AB} + \frac{3}{13}T_{AC} = 180 \text{ N} \quad (2)$$

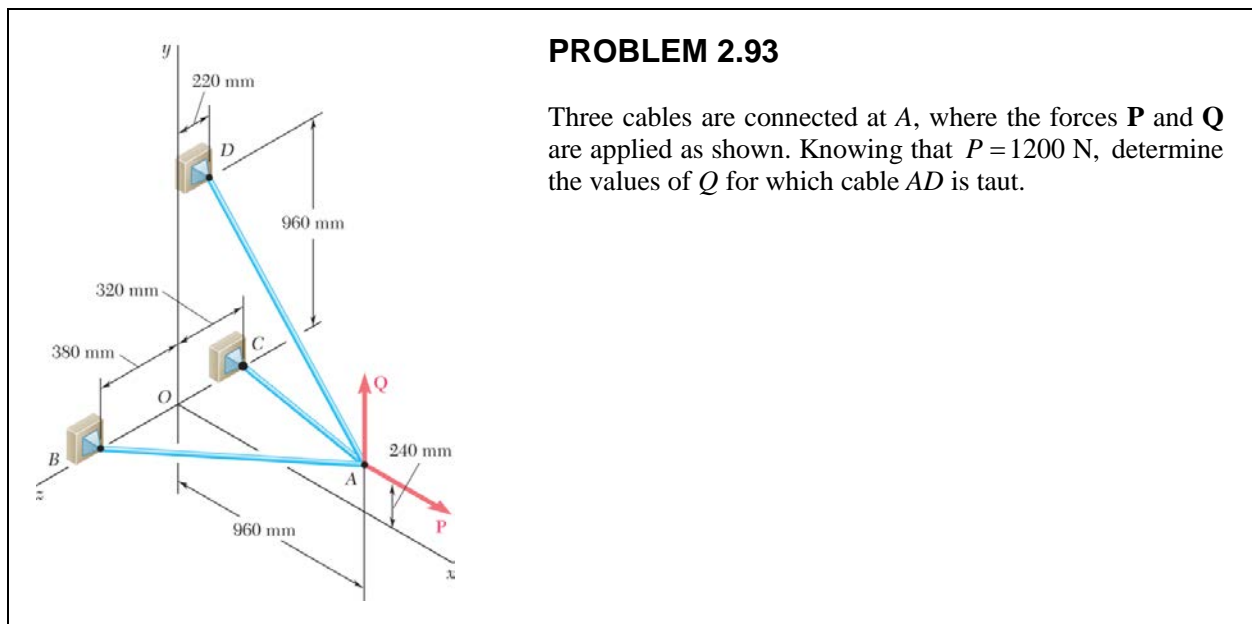
$$\mathbf{k}: \quad \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} = 55 \text{ N} \quad (3)$$

Solving the system of linear equations using conventional algorithms gives:

$$T_{AB} = 446.71 \text{ N}$$

$$T_{AC} = 341.71 \text{ N}$$

$$P = 960 \text{ N} \quad \blacktriangleleft$$



### SOLUTION

We assume that  $T_{AD} = 0$  and write  $\Sigma \mathbf{F}_A = 0$ :  $\mathbf{T}_{AB} + \mathbf{T}_{AC} + Q\mathbf{j} + (1200 \text{ N})\mathbf{i} = 0$

$$\overline{AB} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} + (380 \text{ mm})\mathbf{k} \quad AB = 1060 \text{ mm}$$

$$\overline{AC} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad AC = 1040 \text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} = \left( -\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k} \right) T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = \left( -\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k} \right) T_{AC}$$

Substituting into  $\Sigma \mathbf{F}_A = 0$ , factoring **i**, **j**, **k**, and setting each coefficient equal to  $\phi$  gives:

$$\mathbf{i}: -\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} + 1200 \text{ N} = 0 \quad (1)$$

$$\mathbf{j}: -\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + Q = 0 \quad (2)$$

$$\mathbf{k}: \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} = 0 \quad (3)$$

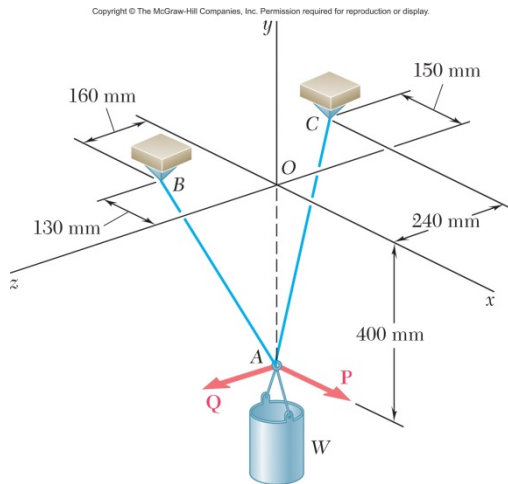
Solving the resulting system of linear equations using conventional algorithms gives:

$$T_{AB} = 605.71 \text{ N}$$

$$T_{AC} = 705.71 \text{ N}$$

$$Q = 300.00 \text{ N} \quad 0 \leq Q < 300 \text{ N} \blacktriangleleft$$

*Note:* This solution assumes that  $Q$  is directed upward as shown ( $Q \geq 0$ ), if negative values of  $Q$  are considered, cable  $AD$  remains taut, but  $AC$  becomes slack for  $Q = -460$  N.



### PROBLEM 2.94

A container of weight  $W$  is suspended from ring  $A$ . Cable  $BAC$  passes through the ring and is attached to fixed supports at  $B$  and  $C$ . Two forces  $\mathbf{P} = P\mathbf{i}$  and  $\mathbf{Q} = Q\mathbf{k}$  are applied to the ring to maintain the container in the position shown. Knowing that  $W = 376 \text{ N}$ , determine  $P$  and  $Q$ . (Hint: The tension is the same in both portions of cable  $BAC$ .)

### SOLUTION

$$\begin{aligned}\mathbf{T}_{AB} &= T\lambda_{AB} \\ &= T \frac{\overline{AB}}{AB} \\ &= T \frac{(-130 \text{ mm})\mathbf{i} + (400 \text{ mm})\mathbf{j} + (160 \text{ mm})\mathbf{k}}{450 \text{ mm}} \\ &= T \left( -\frac{13}{45}\mathbf{i} + \frac{40}{45}\mathbf{j} + \frac{16}{45}\mathbf{k} \right)\end{aligned}$$

$$\begin{aligned}\mathbf{T}_{AC} &= T\lambda_{AC} \\ &= T \frac{\overline{AC}}{AC} \\ &= T \frac{(-150 \text{ mm})\mathbf{i} + (400 \text{ mm})\mathbf{j} + (-240 \text{ mm})\mathbf{k}}{490 \text{ mm}} \\ &= T \left( -\frac{15}{49}\mathbf{i} + \frac{40}{49}\mathbf{j} - \frac{24}{49}\mathbf{k} \right)\end{aligned}$$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{Q} + \mathbf{P} + \mathbf{W} = 0$$

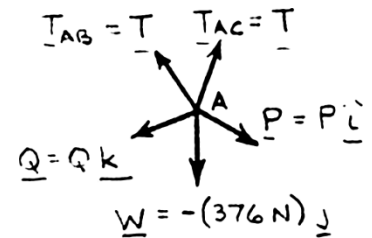
Setting coefficients of  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$  equal to zero:

$$\mathbf{i}: \quad -\frac{13}{45}T - \frac{15}{49}T + P = 0 \quad 0.59501T = P \quad (1)$$

$$\mathbf{j}: \quad +\frac{40}{45}T + \frac{40}{49}T - W = 0 \quad 1.70521T = W \quad (2)$$

$$\mathbf{k}: \quad +\frac{16}{45}T - \frac{24}{49}T + Q = 0 \quad 0.134240T = Q \quad (3)$$

Free-Body A:



**PROBLEM 2.94 (Continued)**

Data:

$$W = 376 \text{ N} \quad 1.70521T = 376 \text{ N} \quad T = 220.50 \text{ N}$$

$$0.59501(220.50 \text{ N}) = P$$

$$P = 131.2 \text{ N} \quad \blacktriangleleft$$

$$0.134240(220.50 \text{ N}) = Q$$

$$Q = 29.6 \text{ N} \quad \blacktriangleleft$$



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### PROBLEM 2.95

For the system of Problem 2.94, determine  $W$  and  $Q$  knowing that  $P = 164 \text{ N}$ .

**PROBLEM 2.94** A container of weight  $W$  is suspended from ring  $A$ . Cable  $BAC$  passes through the ring and is attached to fixed supports at  $B$  and  $C$ . Two forces  $\mathbf{P} = P\mathbf{i}$  and  $\mathbf{Q} = Q\mathbf{k}$  are applied to the ring to maintain the container in the position shown. Knowing that  $W = 376 \text{ N}$ , determine  $P$  and  $Q$ . (*Hint:* The tension is the same in both portions of cable  $BAC$ .)

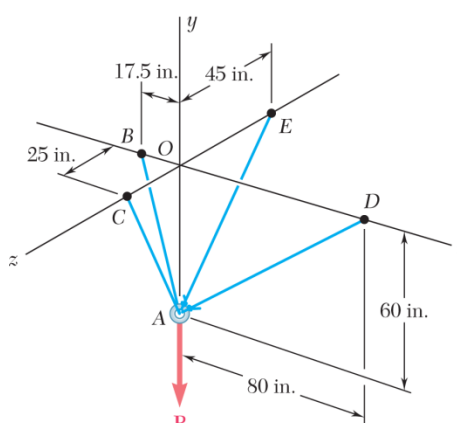
### SOLUTION

Refer to Problem 2.94 for the figure and analysis resulting in Equations (1), (2), and (3) for  $P$ ,  $W$ , and  $Q$  in terms of  $T$  below. Setting  $P = 164 \text{ N}$  we have:

Eq. (1):	$0.59501T = 164 \text{ N}$	$T = 275.63 \text{ N}$
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Eq. (2):	$1.70521(275.63 \text{ N}) = W$	$W = 470 \text{ N} \quad \blacktriangleleft$
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Eq. (3):	$0.134240(275.63 \text{ N}) = Q$	$Q = 37.0 \text{ N} \quad \blacktriangleleft$
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### PROBLEM 2.96

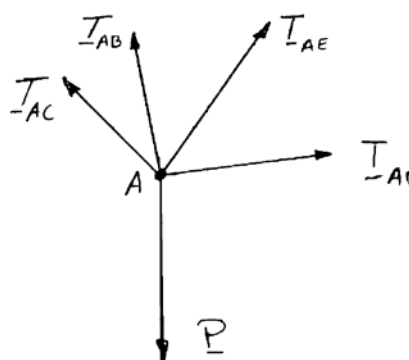
Cable  $BAC$  passes through a frictionless ring  $A$  and is attached to fixed supports at  $B$  and  $C$ , while cables  $AD$  and  $AE$  are both tied to the ring and are attached, respectively, to supports at  $D$  and  $E$ . Knowing that a 200-lb vertical load  $\mathbf{P}$  is applied to ring  $A$ , determine the tension in each of the three cables.

### SOLUTION

Since  $T_{BAC}$  = tension in cable  $BAC$ , it follows that

$$T_{AB} = T_{AC} = T_{BAC}$$

#### Free Body Diagram at A:



$$\mathbf{T}_{AB} = T_{BAC} \boldsymbol{\lambda}_{AB} = T_{BAC} \frac{(-17.5 \text{ in.})\mathbf{i} + (60 \text{ in.})\mathbf{j}}{62.5 \text{ in.}} = T_{BAC} \left( \frac{-17.5}{62.5} \mathbf{i} + \frac{60}{62.5} \mathbf{j} \right)$$

$$\mathbf{T}_{AC} = T_{BAC} \boldsymbol{\lambda}_{AC} = T_{BAC} \frac{(60 \text{ in.})\mathbf{i} + (25 \text{ in.})\mathbf{k}}{65 \text{ in.}} = T_{BAC} \left( \frac{60}{65} \mathbf{j} + \frac{25}{65} \mathbf{k} \right)$$

$$\mathbf{T}_{AD} = T_{AD} \boldsymbol{\lambda}_{AD} = T_{AD} \frac{(80 \text{ in.})\mathbf{i} + (60 \text{ in.})\mathbf{j}}{100 \text{ in.}} = T_{AD} \left( \frac{4}{5} \mathbf{i} + \frac{3}{5} \mathbf{j} \right)$$

$$\mathbf{T}_{AE} = T_{AE} \boldsymbol{\lambda}_{AE} = T_{AE} \frac{(60 \text{ in.})\mathbf{j} - (45 \text{ in.})\mathbf{k}}{75 \text{ in.}} = T_{AE} \left( \frac{4}{5} \mathbf{j} - \frac{3}{5} \mathbf{k} \right)$$

### SOLUTION Continued

Substituting into  $\Sigma \mathbf{F}_A = 0$ , setting  $\mathbf{P} = (-200 \text{ lb})\mathbf{j}$ , and setting the coefficients of  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$  equal to  $\phi$ , we obtain the following three equilibrium equations:

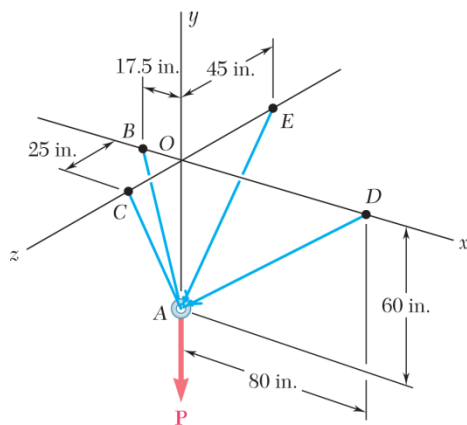
$$\text{From } \mathbf{i}: -\frac{17.5}{62.5}T_{BAC} + \frac{4}{5}T_{AD} = 0 \quad (1)$$

$$\text{From } \mathbf{j}: \left( \frac{60}{62.5} + \frac{60}{65} \right)T_{BAC} + \frac{3}{5}T_{AD} + \frac{4}{5}T_{AE} - 200 \text{ lb} = 0 \quad (2)$$

$$\text{From } \mathbf{k}: \frac{25}{65}T_{BAC} - \frac{3}{5}T_{AE} = 0 \quad (3)$$

Solving the system of linear equations using conventional algorithms gives:

$$T_{BAC} = 76.7 \text{ lb}; T_{AD} = 26.9 \text{ lb}; T_{AE} = 49.2 \text{ lb} \quad \blacktriangleleft$$



### PROBLEM 2.97

Knowing that the tension in cable  $AE$  of Prob. 2.96 is 75 lb, determine (a) the magnitude of the load  $\mathbf{P}$ , (b) the tension in cables  $BAC$  and  $AD$ .

**PROBLEM 2.96** Cable  $BAC$  passes through a frictionless ring  $A$  and is attached to fixed supports at  $B$  and  $C$ , while cables  $AD$  and  $AE$  are both tied to the ring and are attached, respectively, to supports at  $D$  and  $E$ . Knowing that a 200-lb vertical load  $\mathbf{P}$  is applied to ring  $A$ , determine the tension in each of the three cables.

### SOLUTION

Refer to the solution to Problem 2.96 for the figure and analysis leading to the following set of equilibrium equations, Equation (2) being modified to include  $P\mathbf{j}$  as an unknown quantity:

$$-\frac{17.5}{62.5}T_{BAC} + \frac{4}{5}T_{AD} = 0 \quad (1)$$

$$\left(\frac{60}{62.5} + \frac{60}{65}\right)T_{BAC} + \frac{3}{5}T_{AD} + \frac{4}{5}T_{AE} - P = 0 \quad (2)$$

$$\frac{25}{65}T_{BAC} - \frac{3}{5}T_{AE} = 0 \quad (3)$$

Substituting for  $T_{AE} = 75$  lb and solving simultaneously gives:

$$(a) \quad P = 305 \text{ lb} \quad \blacktriangleleft$$

$$(b) \quad T_{BAC} = 117.0 \text{ lb}; \quad T_{AD} = 40.9 \text{ lb} \quad \blacktriangleleft$$

### SOLUTION Continued

Then from the specifications of the problem,  $y = 155 \text{ mm} = 0.155 \text{ m}$

$$z^2 = 0.23563 \text{ m}^2 - (0.155 \text{ m})^2$$

$$z = 0.46 \text{ m}$$

and

$$\begin{aligned} (a) \quad T_{AB} &= \frac{341 \text{ N}}{0.155(1.90476)} \\ &= 1155.00 \text{ N} \end{aligned}$$

or

$$T_{AB} = 1155 \text{ N} \quad \blacktriangleleft$$

and

$$\begin{aligned} (b) \quad Q &= \frac{341 \text{ N}(0.46 \text{ m})(0.866)}{(0.155 \text{ m})} \\ &= (1012.00 \text{ N}) \end{aligned}$$

or

$$Q = 1012 \text{ N} \quad \blacktriangleleft$$

### PROBLEM 2.98

A container of weight  $W$  is suspended from ring  $A$ , to which cables  $AC$  and  $AE$  are attached. A force  $\mathbf{P}$  is applied to the end  $F$  of a third cable that passes over a pulley at  $B$  and through ring  $A$  and that is attached to a support at  $D$ . Knowing that  $W = 1000 \text{ N}$ , determine the magnitude of  $P$ . (Hint: The tension is the same in all portions of cable  $FBAD$ .)

### SOLUTION

The (vector) force in each cable can be written as the product of the (scalar) force and the unit vector along the cable. That is, with

$$\begin{aligned}\overline{AB} &= -(0.78 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (0 \text{ m})\mathbf{k} \\ AB &= \sqrt{(-0.78 \text{ m})^2 + (1.6 \text{ m})^2 + (0)^2} \\ &= 1.78 \text{ m}\end{aligned}$$

$$\begin{aligned}\mathbf{T}_{AB} &= T\lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} \\ &= \frac{T_{AB}}{1.78 \text{ m}} [-(0.78 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (0 \text{ m})\mathbf{k}] \\ \mathbf{T}_{AB} &= T_{AB} (-0.4382\mathbf{i} + 0.8989\mathbf{j} + 0\mathbf{k})\end{aligned}$$

and

$$\begin{aligned}\overline{AC} &= (0)\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (1.2 \text{ m})\mathbf{k} \\ AC &= \sqrt{(0)^2 + (1.6 \text{ m})^2 + (1.2 \text{ m})^2} = 2 \text{ m} \\ \mathbf{T}_{AC} &= T\lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = \frac{T_{AC}}{2 \text{ m}} [(0)\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (1.2 \text{ m})\mathbf{k}] \\ \mathbf{T}_{AC} &= T_{AC} (0.8\mathbf{j} + 0.6\mathbf{k})\end{aligned}$$

and

$$\begin{aligned}\overline{AD} &= (1.3 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (0.4 \text{ m})\mathbf{k} \\ AD &= \sqrt{(1.3 \text{ m})^2 + (1.6 \text{ m})^2 + (0.4 \text{ m})^2} = 2.1 \text{ m} \\ \mathbf{T}_{AD} &= T\lambda_{AD} = T_{AD} \frac{\overline{AD}}{AD} = \frac{T_{AD}}{2.1 \text{ m}} [(1.3 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (0.4 \text{ m})\mathbf{k}] \\ \mathbf{T}_{AD} &= T_{AD} (0.6190\mathbf{i} + 0.7619\mathbf{j} + 0.1905\mathbf{k})\end{aligned}$$

### SOLUTION Continued

Finally,

$$\begin{aligned}\overline{AE} &= -(0.4 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} - (0.86 \text{ m})\mathbf{k} \\ AE &= \sqrt{(-0.4 \text{ m})^2 + (1.6 \text{ m})^2 + (-0.86 \text{ m})^2} = 1.86 \text{ m} \\ \mathbf{T}_{AE} &= T_{AE} \frac{\overline{AE}}{AE} \\ &= \frac{T_{AE}}{1.86 \text{ m}} [-(0.4 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} - (0.86 \text{ m})\mathbf{k}] \\ \mathbf{T}_{AE} &= T_{AE} (-0.215\mathbf{i} + 0.8602\mathbf{j} - 0.4624\mathbf{k})\end{aligned}$$

With the weight of the container  $\mathbf{W} = -W\mathbf{j}$ , at A we have:

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} - W\mathbf{j} = 0$$

Equating the factors of  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$  to zero, we obtain the following linear algebraic equations:

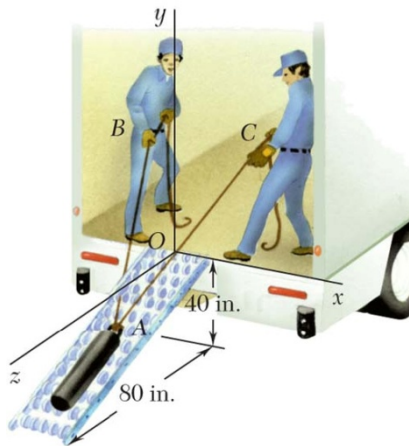
$$-0.4382T_{AB} + 0.6190T_{AD} - 0.2151T_{AE} = 0 \quad (1)$$

$$0.8989T_{AB} + 0.8T_{AC} + 0.7619T_{AD} + 0.8602T_{AE} - W = 0 \quad (2)$$

$$0.6T_{AC} + 0.1905T_{AD} - 0.4624T_{AE} = 0 \quad (3)$$

Knowing that  $W = 1000 \text{ N}$  and that because of the pulley system at B  $T_{AB} = T_{AD} = P$ , where  $P$  is the externally applied (unknown) force, we can solve the system of linear Equations (1), (2) and (3) uniquely for  $P$ .

$$P = 378 \text{ N} \quad \blacktriangleleft$$



### PROBLEM 2.99

Using two ropes and a roller chute, two workers are unloading a 200-lb cast-iron counterweight from a truck. Knowing that at the instant shown the counterweight is kept from moving and that the positions of Points A, B, and C are, respectively,  $A(0, -20 \text{ in.}, 40 \text{ in.})$ ,  $B(-40 \text{ in.}, 50 \text{ in.}, 0)$ , and  $C(45 \text{ in.}, 40 \text{ in.}, 0)$ , and assuming that no friction exists between the counterweight and the chute, determine the tension in each rope. (*Hint: Since there is no friction, the force exerted by the chute on the counterweight must be perpendicular to the chute.*)

### SOLUTION

From the geometry of the chute:

$$\begin{aligned} \mathbf{N} &= \frac{N}{\sqrt{5}}(2\mathbf{j} + \mathbf{k}) \\ &= N(0.8944\mathbf{j} + 0.4472\mathbf{k}) \end{aligned}$$

The force in each rope can be written as the product of the magnitude of the force and the unit vector along the cable. Thus, with

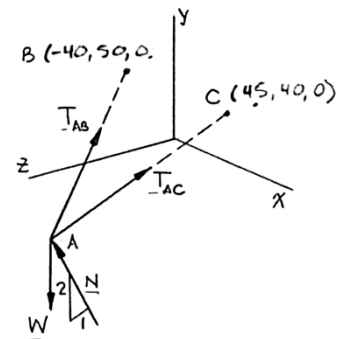
$$\begin{aligned} \overline{AB} &= (40 \text{ in.})\mathbf{i} + (70 \text{ in.})\mathbf{j} - (40 \text{ in.})\mathbf{k} \\ AB &= \sqrt{(40 \text{ in.})^2 + (70 \text{ in.})^2 + (40 \text{ in.})^2} \\ &= 90 \text{ in.} \\ \mathbf{T}_{AB} &= T_{AB} \frac{\overline{AB}}{AB} \\ &= \frac{T_{AB}}{90 \text{ in.}} [(-40 \text{ in.})\mathbf{i} + (70 \text{ in.})\mathbf{j} - (40 \text{ in.})\mathbf{k}] \\ \mathbf{T}_{AB} &= T_{AB} \left( -\frac{4}{9}\mathbf{i} + \frac{7}{9}\mathbf{j} - \frac{4}{9}\mathbf{k} \right) \end{aligned}$$

and

$$\begin{aligned} \overline{AC} &= (45 \text{ in.})\mathbf{i} + (60 \text{ in.})\mathbf{j} - (40 \text{ in.})\mathbf{k} \\ AC &= \sqrt{(45 \text{ in.})^2 + (60 \text{ in.})^2 + (40 \text{ in.})^2} = 85 \text{ in.} \\ \mathbf{T}_{AC} &= T_{AC} \frac{\overline{AC}}{AC} \\ &= \frac{T_{AC}}{85 \text{ in.}} [(45 \text{ in.})\mathbf{i} + (60 \text{ in.})\mathbf{j} - (40 \text{ in.})\mathbf{k}] \\ \mathbf{T}_{AC} &= T_{AC} \left( \frac{9}{17}\mathbf{i} + \frac{12}{17}\mathbf{j} - \frac{8}{17}\mathbf{k} \right) \end{aligned}$$

Then:

$$\Sigma \mathbf{F} = 0: \quad \mathbf{N} + \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{W} = 0$$





### SOLUTION Continued

With  $W = 200$  lb, and equating the factors of **i**, **j**, and **k** to zero, we obtain the linear algebraic equations:

$$\mathbf{i}: -\frac{4}{9}T_{AB} + \frac{9}{17}T_{AC} = 0 \quad (1)$$

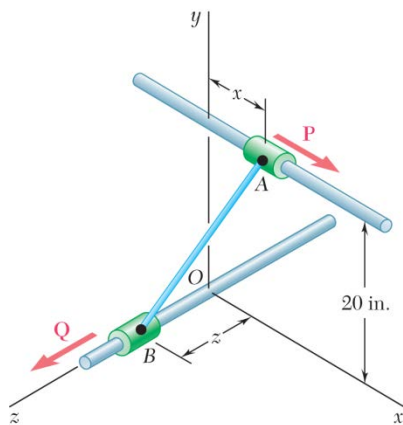
$$\mathbf{j}: \frac{7}{9}T_{AB} + \frac{12}{17}T_{AC} + \frac{2}{\sqrt{5}} - 200 \text{ lb} = 0 \quad (2)$$

$$\mathbf{k}: -\frac{4}{9}T_{AB} - \frac{8}{17}T_{AC} + \frac{1}{\sqrt{5}}N = 0 \quad (3)$$

Using conventional methods for solving linear algebraic equations we obtain:

$$T_{AB} = 65.6 \text{ lb} \quad \blacktriangleleft$$

$$T_{AC} = 55.1 \text{ lb} \quad \blacktriangleleft$$

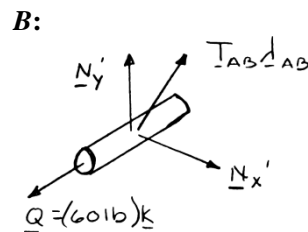
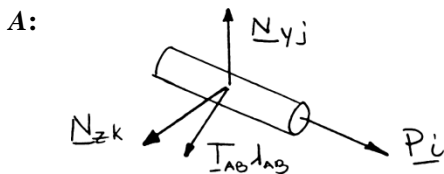


### PROBLEM 2.100

Collars *A* and *B* are connected by a 25-in.-long wire and can slide freely on frictionless rods. If a 60-lb force *Q* is applied to collar *B* as shown, determine (a) the tension in the wire when  $x = 9$  in., (b) the corresponding magnitude of the force *P* required to maintain the equilibrium of the system.

### SOLUTION

#### Free-Body Diagrams of Collars:



$$\lambda_{AB} = \frac{\overline{AB}}{AB} = \frac{-x\mathbf{i} - (20 \text{ in.})\mathbf{j} + z\mathbf{k}}{25 \text{ in.}}$$

Collar *A*:  $\Sigma \mathbf{F} = 0: P\mathbf{i} + N_y\mathbf{j} + N_z\mathbf{k} + T_{AB}\lambda_{AB} = 0$

Substitute for  $\lambda_{AB}$  and set coefficient of  $\mathbf{i}$  equal to zero:

$$P - \frac{T_{AB}x}{25 \text{ in.}} = 0 \quad (1)$$

Collar *B*:  $\Sigma \mathbf{F} = 0: (60 \text{ lb})\mathbf{k} + N'_x\mathbf{i} + N'_y\mathbf{j} - T_{AB}\lambda_{AB} = 0$

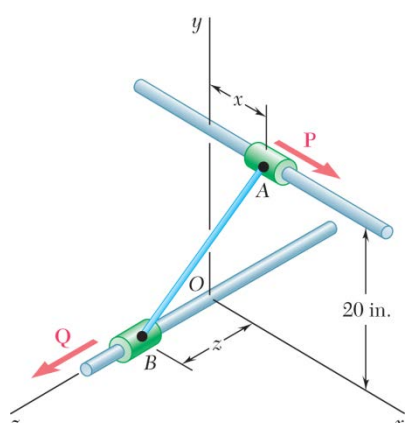
Substitute for  $\lambda_{AB}$  and set coefficient of  $\mathbf{k}$  equal to zero:

$$60 \text{ lb} - \frac{T_{AB}z}{25 \text{ in.}} = 0 \quad (2)$$

(a)  $x = 9 \text{ in.}$   $(9 \text{ in.})^2 + (20 \text{ in.})^2 + z^2 = (25 \text{ in.})^2$   
 $z = 12 \text{ in.}$

From Eq. (2):  $\frac{60 \text{ lb} - T_{AB}(12 \text{ in.})}{25 \text{ in.}} = 0$   $T_{AB} = 125.0 \text{ lb} \blacktriangleleft$

(b) From Eq. (1):  $P = \frac{(125.0 \text{ lb})(9 \text{ in.})}{25 \text{ in.}}$   $P = 45.0 \text{ lb} \blacktriangleleft$



### PROBLEM 2.101

Collars A and B are connected by a 25-in.-long wire and can slide freely on frictionless rods. Determine the distances  $x$  and  $z$  for which the equilibrium of the system is maintained when  $P = 120$  lb and  $Q = 60$  lb.

### SOLUTION

See Problem 2.100 for the diagrams and analysis leading to Equations (1) and (2) below:

$$P = \frac{T_{AB}x}{25 \text{ in.}} = 0 \quad (1)$$

$$60 \text{ lb} - \frac{T_{AB}z}{25 \text{ in.}} = 0 \quad (2)$$

For  $P = 120$  lb, Eq. (1) yields

$$T_{AB}x = (25 \text{ in.})(20 \text{ lb}) \quad (1')$$

From Eq. (2):

$$T_{AB}z = (25 \text{ in.})(60 \text{ lb}) \quad (2')$$

Dividing Eq. (1') by (2'),

$$\frac{x}{z} = 2 \quad (3)$$

Now write

$$x^2 + z^2 + (20 \text{ in.})^2 = (25 \text{ in.})^2 \quad (4)$$

Solving (3) and (4) simultaneously,

$$4z^2 + z^2 + 400 = 625$$

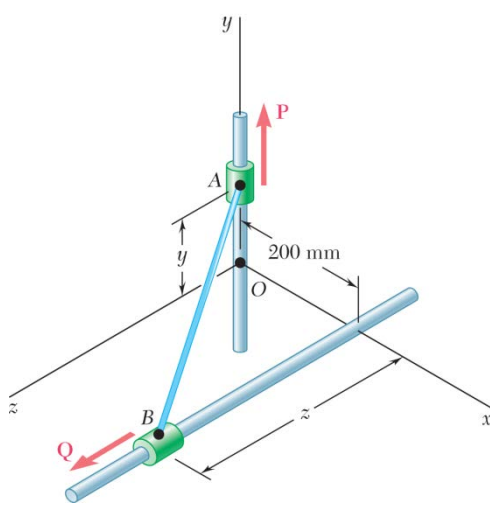
$$z^2 = 45$$

$$z = 6.7082 \text{ in.}$$

From Eq. (3):

$$x = 2z = 2(6.7082 \text{ in.}) = 13.4164 \text{ in.}$$

$x = 13.42 \text{ in.}, \quad z = 6.71 \text{ in.} \quad \blacktriangleleft$



### PROBLEM 2.102

Collars *A* and *B* are connected by a 525-mm-long wire and can slide freely on frictionless rods. If a force  $\mathbf{P} = (341 \text{ N})\mathbf{j}$  is applied to collar *A*, determine (a) the tension in the wire when  $y = 155 \text{ mm}$ , (b) the magnitude of the force  $\mathbf{Q}$  required to maintain the equilibrium of the system.

### SOLUTION

For both Problems 2.102 and 2.103:

$$(AB)^2 = x^2 + y^2 + z^2$$

Here  $(0.525 \text{ m})^2 = (0.20 \text{ m})^2 + y^2 + z^2$

or  $y^2 + z^2 = 0.23563 \text{ m}^2$

Thus, when  $y$  given,  $z$  is determined,

Now 
$$\begin{aligned} \lambda_{AB} &= \frac{\overline{AB}}{AB} \\ &= \frac{1}{0.525 \text{ m}} (0.20\mathbf{i} - y\mathbf{j} + z\mathbf{k})\text{m} \\ &= 0.38095\mathbf{i} - 1.90476y\mathbf{j} + 1.90476z\mathbf{k} \end{aligned}$$

Where  $y$  and  $z$  are in units of meters, m.

From the F.B. Diagram of collar *A*:  $\Sigma \mathbf{F} = 0: N_x\mathbf{i} + N_z\mathbf{k} + P\mathbf{j} + T_{AB}\lambda_{AB} = 0$

Setting the  $\mathbf{j}$  coefficient to zero gives  $P - (1.90476y)T_{AB} = 0$

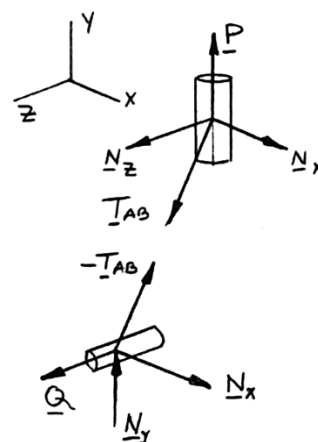
With 
$$\begin{aligned} P &= 341 \text{ N} \\ T_{AB} &= \frac{341 \text{ N}}{1.90476y} \end{aligned}$$

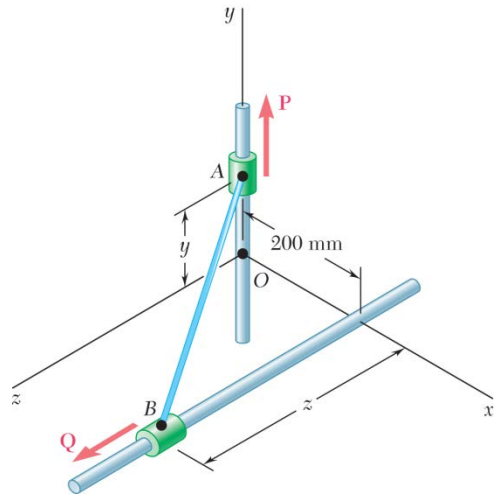
Now, from the free body diagram of collar *B*:  $\Sigma \mathbf{F} = 0: N_x\mathbf{i} + N_y\mathbf{j} + Q\mathbf{k} - T_{AB}\lambda_{AB} = 0$

Setting the  $\mathbf{k}$  coefficient to zero gives  $Q - T_{AB}(1.90476z) = 0$

And using the above result for  $T_{AB}$ , we have 
$$Q = T_{AB}z = \frac{341 \text{ N}}{(1.90476)y} (1.90476z) = \frac{(341 \text{ N})(z)}{y}$$

Free-Body Diagrams of Collars:





### PROBLEM 2.103

Solve Problem 2.102 assuming that  $y = 275$  mm.

**PROBLEM 2.102** Collars  $A$  and  $B$  are connected by a 525-mm-long wire and can slide freely on frictionless rods. If a force  $\mathbf{P} = (341 \text{ N})\mathbf{j}$  is applied to collar  $A$ , determine (a) the tension in the wire when  $y = 155$  mm, (b) the magnitude of the force  $\mathbf{Q}$  required to maintain the equilibrium of the system.

### SOLUTION

From the analysis of Problem 2.102, particularly the results:

$$y^2 + z^2 = 0.23563 \text{ m}^2$$

$$T_{AB} = \frac{341 \text{ N}}{1.90476y}$$

$$Q = \frac{341 \text{ N}}{y}z$$

With  $y = 275 \text{ mm} = 0.275 \text{ m}$ , we obtain:

$$z^2 = 0.23563 \text{ m}^2 - (0.275 \text{ m})^2$$

$$z = 0.40 \text{ m}$$

and

$$(a) \quad T_{AB} = \frac{341 \text{ N}}{(1.90476)(0.275 \text{ m})} = 651.00$$

or

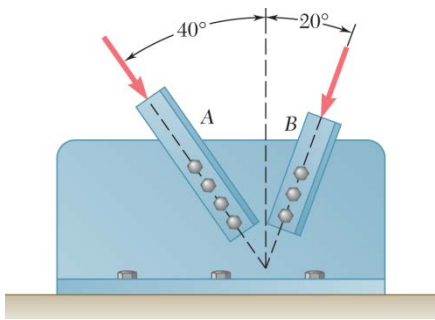
$$T_{AB} = 651 \text{ N} \quad \blacktriangleleft$$

and

$$(b) \quad Q = \frac{341 \text{ N}(0.40 \text{ m})}{(0.275 \text{ m})}$$

or

$$Q = 496 \text{ N} \quad \blacktriangleleft$$



### PROBLEM 2.104

Two structural members *A* and *B* are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 15 kN in member *A* and 10 kN in member *B*, determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members *A* and *B*.

### SOLUTION

Using the force triangle and the laws of cosines and sines, we have

$$\begin{aligned}\gamma &= 180^\circ - (40^\circ + 20^\circ) \\ &= 120^\circ\end{aligned}$$

Then

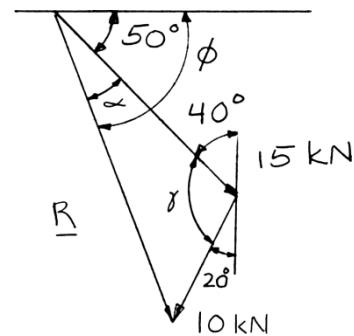
$$\begin{aligned}R^2 &= (15 \text{ kN})^2 + (10 \text{ kN})^2 \\ &\quad - 2(15 \text{ kN})(10 \text{ kN})\cos 120^\circ \\ &= 475 \text{ kN}^2 \\ R &= 21.794 \text{ kN}\end{aligned}$$

and

$$\begin{aligned}\frac{10 \text{ kN}}{\sin \alpha} &= \frac{21.794 \text{ kN}}{\sin 120^\circ} \\ \sin \alpha &= \left( \frac{10 \text{ kN}}{21.794 \text{ kN}} \right) \sin 120^\circ \\ &= 0.39737 \\ \alpha &= 23.414\end{aligned}$$

Hence:

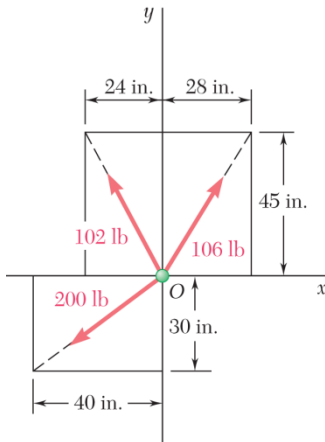
$$\phi = \alpha + 50^\circ = 73.414$$



$$\mathbf{R} = 21.8 \text{ kN} \searrow 73.4^\circ \blacktriangleleft$$

## PROBLEM 2.105

Determine the  $x$  and  $y$  components of each of the forces shown.



## SOLUTION

Compute the following distances:

$$OA = \sqrt{(24 \text{ in.})^2 + (45 \text{ in.})^2} = 51.0 \text{ in.}$$

$$OB = \sqrt{(28 \text{ in.})^2 + (45 \text{ in.})^2} = 53.0 \text{ in.}$$

$$OC = \sqrt{(40 \text{ in.})^2 + (30 \text{ in.})^2} = 50.0 \text{ in.}$$

102-lb Force:

$$F_x = -102 \text{ lb} \frac{24 \text{ in.}}{51.0 \text{ in.}}$$

$$F_x = -48.0 \text{ lb} \quad \blacktriangleleft$$

$$F_y = +102 \text{ lb} \frac{45 \text{ in.}}{51.0 \text{ in.}}$$

$$F_y = +90.0 \text{ lb} \quad \blacktriangleleft$$

106-lb Force:

$$F_x = +106 \text{ lb} \frac{28 \text{ in.}}{53.0 \text{ in.}}$$

$$F_x = +56.0 \text{ lb} \quad \blacktriangleleft$$

$$F_y = +106 \text{ lb} \frac{45 \text{ in.}}{53.0 \text{ in.}}$$

$$F_y = +90.0 \text{ lb} \quad \blacktriangleleft$$

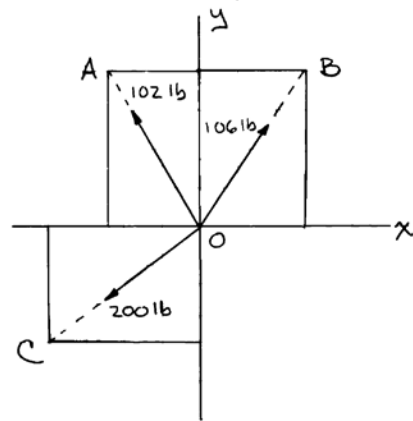
200-lb Force:

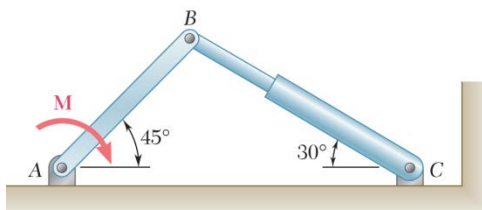
$$F_x = -200 \text{ lb} \frac{40 \text{ in.}}{50.0 \text{ in.}}$$

$$F_x = -160.0 \text{ lb} \quad \blacktriangleleft$$

$$F_y = -200 \text{ lb} \frac{30 \text{ in.}}{50.0 \text{ in.}}$$

$$F_y = -120.0 \text{ lb} \quad \blacktriangleleft$$





### PROBLEM 2.106

The hydraulic cylinder  $BC$  exerts on member  $AB$  a force  $\mathbf{P}$  directed along line  $BC$ . Knowing that  $\mathbf{P}$  must have a 600-N component perpendicular to member  $AB$ , determine (a) the magnitude of the force  $\mathbf{P}$ , (b) its component along line  $AB$ .

### SOLUTION

$$180^\circ = 45^\circ + \alpha + 90^\circ + 30^\circ$$

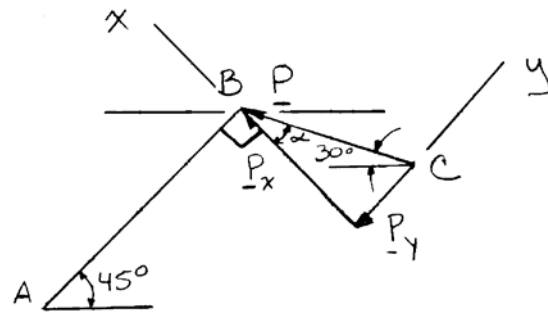
$$\alpha = 180^\circ - 45^\circ - 90^\circ - 30^\circ$$

$$= 15^\circ$$

(a)

$$\cos \alpha = \frac{P_x}{P}$$

$$\begin{aligned} P &= \frac{P_x}{\cos \alpha} \\ &= \frac{600 \text{ N}}{\cos 15^\circ} \\ &= 621.17 \text{ N} \end{aligned}$$



$$P = 621 \text{ N} \quad \blacktriangleleft$$

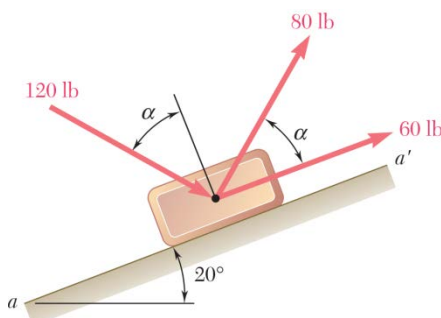
(b)

$$\tan \alpha = \frac{P_y}{P_x}$$

$$\begin{aligned} P_y &= P_x \tan \alpha \\ &= (600 \text{ N}) \tan 15^\circ \\ &= 160.770 \text{ N} \end{aligned}$$

$$P_y = 160.8 \text{ N} \quad \blacktriangleleft$$





### PROBLEM 2.107

Knowing that  $\alpha = 40^\circ$ , determine the resultant of the three forces shown.

### SOLUTION

60-lb Force:  $F_x = (60 \text{ lb}) \cos 20^\circ = 56.382 \text{ lb}$   
 $F_y = (60 \text{ lb}) \sin 20^\circ = 20.521 \text{ lb}$

80-lb Force:  $F_x = (80 \text{ lb}) \cos 60^\circ = 40.000 \text{ lb}$   
 $F_y = (80 \text{ lb}) \sin 60^\circ = 69.282 \text{ lb}$

120-lb Force:  $F_x = (120 \text{ lb}) \cos 30^\circ = 103.923 \text{ lb}$   
 $F_y = -(120 \text{ lb}) \sin 30^\circ = -60.000 \text{ lb}$

and  $R_x = \Sigma F_x = 200.305 \text{ lb}$   
 $R_y = \Sigma F_y = 29.803 \text{ lb}$

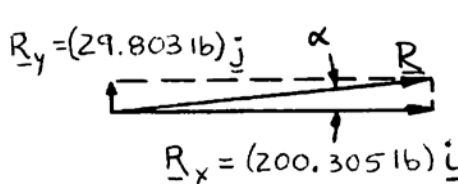
$$R = \sqrt{(200.305 \text{ lb})^2 + (29.803 \text{ lb})^2}$$

$$= 202.510 \text{ lb}$$

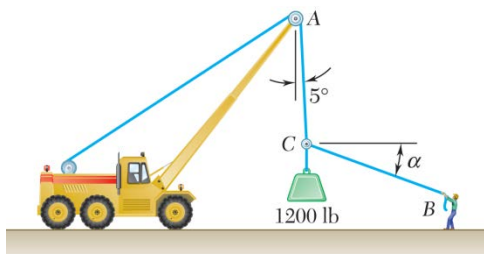
Further:  $\tan \alpha = \frac{29.803}{200.305}$

$$\alpha = \tan^{-1} \frac{29.803}{200.305}$$

$$= 8.46^\circ$$



**R = 203 lb  $\nearrow$  8.46° ◀**

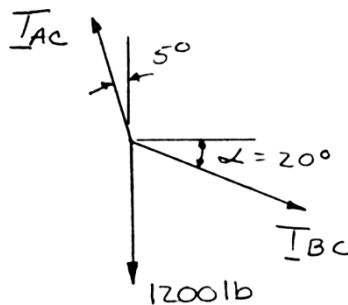


### PROBLEM 2.108

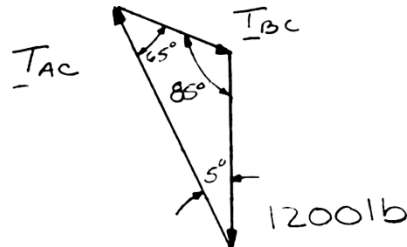
Knowing that  $\alpha = 20^\circ$ , determine the tension (a) in cable AC, (b) in rope BC.

### SOLUTION

Free-Body Diagram



Force Triangle

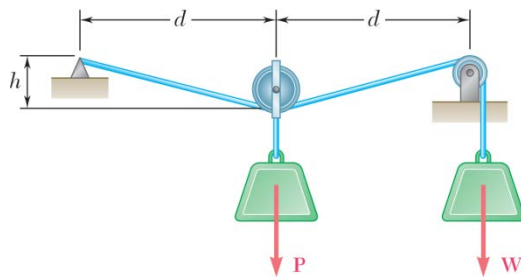


Law of sines:

$$\frac{T_{AC}}{\sin 110^\circ} = \frac{T_{BC}}{\sin 5^\circ} = \frac{1200 \text{ lb}}{\sin 65^\circ}$$

(a)  $T_{AC} = \frac{1200 \text{ lb}}{\sin 65^\circ} \sin 110^\circ$   $T_{AC} = 1244 \text{ lb} \blacktriangleleft$

(b)  $T_{BC} = \frac{1200 \text{ lb}}{\sin 65^\circ} \sin 5^\circ$   $T_{BC} = 115.4 \text{ lb} \blacktriangleleft$

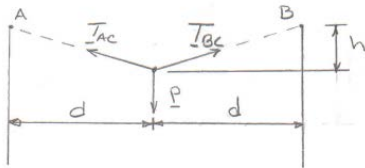


### PROBLEM 2.109

For  $W = 800 \text{ N}$ ,  $P = 200 \text{ N}$ , and  $d = 600 \text{ mm}$ , determine the value of  $h$  consistent with equilibrium.

### SOLUTION

#### Free-Body Diagram



$$T_{AC} = T_{BC} = 800 \text{ N}$$

$$AC = BC = \sqrt{(h^2 + d^2)}$$

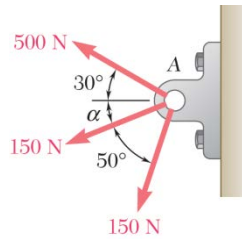
$$\Sigma F_y = 0: 2(800 \text{ N}) \frac{h}{\sqrt{h^2 + d^2}} - P = 0$$

$$800 = \frac{P}{2} \sqrt{1 + \left(\frac{d}{h}\right)^2}$$

Data:  $P = 200 \text{ N}$ ,  $d = 600 \text{ mm}$  and solving for  $h$

$$800 \text{ N} = \frac{200 \text{ N}}{2} \sqrt{1 + \left(\frac{600 \text{ mm}}{h}\right)^2}$$

$$h = 75.6 \text{ mm} \quad \blacktriangleleft$$

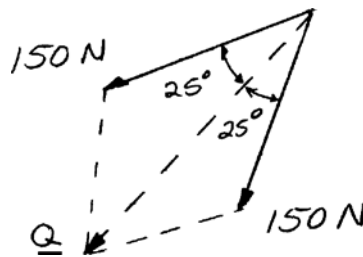


### PROBLEM 2.110

Three forces are applied to a bracket as shown. The directions of the two 150-N forces may vary, but the angle between these forces is always  $50^\circ$ . Determine the range of values of  $\alpha$  for which the magnitude of the resultant of the forces acting at A is less than 600 N.

### SOLUTION

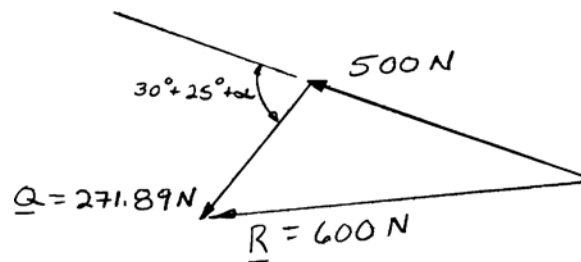
Combine the two 150-N forces into a resultant force  $Q$ :



$$Q = 2(150 \text{ N}) \cos 25^\circ$$

$$= 271.89 \text{ N}$$

Equivalent loading at A:



Using the law of cosines:

$$(600 \text{ N})^2 = (500 \text{ N})^2 + (271.89 \text{ N})^2 + 2(500 \text{ N})(271.89 \text{ N}) \cos(55^\circ + \alpha)$$

$$\cos(55^\circ + \alpha) = 0.132685$$

Two values for  $\alpha$ :

$$55^\circ + \alpha = 82.375^\circ$$

$$\alpha = 27.4^\circ$$

or

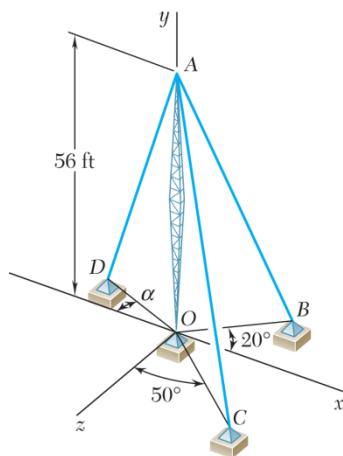
$$55^\circ + \alpha = -82.375^\circ$$

$$55^\circ + \alpha = 360^\circ - 82.375^\circ$$

$$\alpha = 222.6^\circ$$

For  $R < 600 \text{ lb}$ :

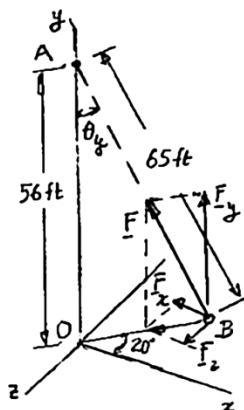
$$27.4^\circ < \alpha < 222.6^\circ \quad \blacktriangleleft$$



### PROBLEM 2.111

Cable  $AB$  is 65 ft long, and the tension in that cable is 3900 lb. Determine (a) the  $x$ ,  $y$ , and  $z$  components of the force exerted by the cable on the anchor  $B$ , (b) the angles  $\theta_x$ ,  $\theta_y$ , and  $\theta_z$  defining the direction of that force.

### SOLUTION



From triangle  $AOB$ :

$$\begin{aligned}\cos \theta_y &= \frac{56 \text{ ft}}{65 \text{ ft}} \\ &= 0.86154 \\ \theta_y &= 30.51^\circ\end{aligned}$$

(a)

$$\begin{aligned}F_x &= -F \sin \theta_y \cos 20^\circ \\ &= -(3900 \text{ lb}) \sin 30.51^\circ \cos 20^\circ\end{aligned}$$

$$F_x = -1861 \text{ lb} \quad \blacktriangleleft$$

$$F_y = +F \cos \theta_y = (3900 \text{ lb})(0.86154) \quad F_y = +3360 \text{ lb} \quad \blacktriangleleft$$

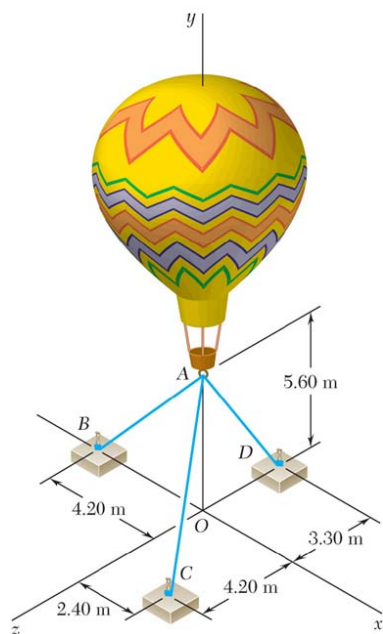
$$F_z = +(3900 \text{ lb}) \sin 30.51^\circ \sin 20^\circ \quad F_z = +677 \text{ lb} \quad \blacktriangleleft$$

(b)

$$\cos \theta_x = \frac{F_x}{F} = -\frac{1861 \text{ lb}}{3900 \text{ lb}} = -0.4771 \quad \theta_x = 118.5^\circ \quad \blacktriangleleft$$

$$\text{From above:} \quad \theta_y = 30.51^\circ \quad \theta_y = 30.5^\circ \quad \blacktriangleleft$$

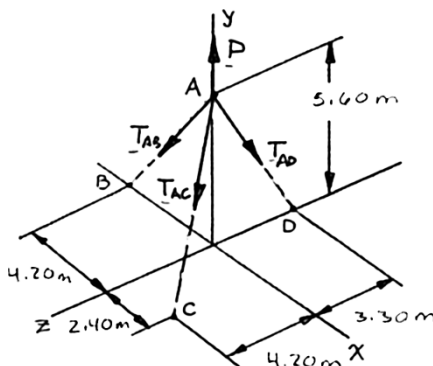
$$\cos \theta_z = \frac{F_z}{F} = +\frac{677 \text{ lb}}{3900 \text{ lb}} = +0.1736 \quad \theta_z = 80.0^\circ \quad \blacktriangleleft$$



### PROBLEM 2.112

Three cables are used to tether a balloon as shown. Determine the vertical force  $\mathbf{P}$  exerted by the balloon at A knowing that the tension in cable AB is 259 N.

### SOLUTION



The forces applied at A are:

$\mathbf{T}_{AB}$ ,  $\mathbf{T}_{AC}$ ,  $\mathbf{T}_{AD}$ , and  $\mathbf{P}$

where  $\mathbf{P} = P\mathbf{j}$ . To express the other forces in terms of the unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ , we write

$$\overline{AB} = -(4.20 \text{ m})\mathbf{i} - (5.60 \text{ m})\mathbf{j} \quad AB = 7.00 \text{ m}$$

$$\overline{AC} = (2.40 \text{ m})\mathbf{i} - (5.60 \text{ m})\mathbf{j} + (4.20 \text{ m})\mathbf{k} \quad AC = 7.40 \text{ m}$$

$$\overline{AD} = -(5.60 \text{ m})\mathbf{j} - (3.30 \text{ m})\mathbf{k} \quad AD = 6.50 \text{ m}$$

and

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} = (-0.6\mathbf{i} - 0.8\mathbf{j})T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = (0.32432 - 0.75676\mathbf{j} + 0.56757\mathbf{k})T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\lambda_{AD} = T_{AD} \frac{\overline{AD}}{AD} = (-0.86154\mathbf{j} - 0.50769\mathbf{k})T_{AD}$$

### SOLUTION Continued

*Equilibrium condition*  $\Sigma F = 0: \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$

Substituting the expressions obtained for  $\mathbf{T}_{AB}$ ,  $\mathbf{T}_{AC}$ , and  $\mathbf{T}_{AD}$  and factoring  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$ :

$$(-0.6T_{AB} + 0.32432T_{AC})\mathbf{i} + (-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P)\mathbf{j} \\ + (0.56757T_{AC} - 0.50769T_{AD})\mathbf{k} = 0$$

Equating to zero the coefficients of  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ :

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \quad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \quad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \quad (3)$$

Setting  $T_{AB} = 259 \text{ N}$  in (1) and (2), and solving the resulting set of equations gives

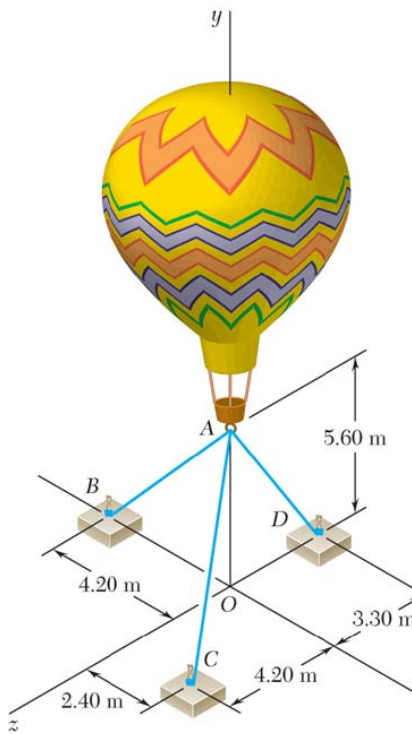
$$T_{AC} = 479.15 \text{ N}$$

$$T_{AD} = 535.66 \text{ N}$$

$$\mathbf{P} = 1031 \text{ N } \uparrow \blacktriangleleft$$

### PROBLEM 2.113

Three cables are used to tether a balloon as shown. Determine the vertical force **P** exerted by the balloon at A knowing that the tension in cable AC is 444 N.



### SOLUTION

See Problem 2.112 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \quad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \quad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \quad (3)$$

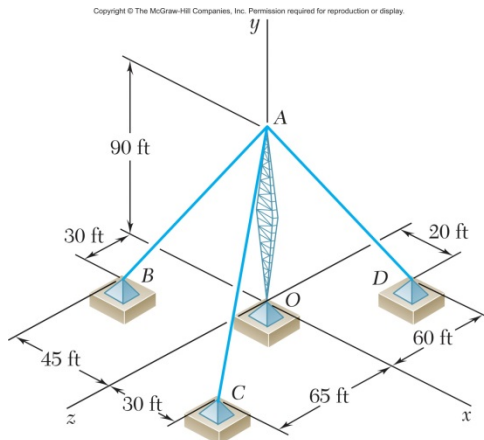
Substituting  $T_{AC} = 444 \text{ N}$  in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms gives

$$T_{AB} = 240 \text{ N}$$

$$T_{AD} = 496.36 \text{ N}$$

$$\mathbf{P} = 956 \text{ N} \uparrow \blacktriangleleft$$





### PROBLEM 2.114

A transmission tower is held by three guy wires attached to a pin at  $A$  and anchored by bolts at  $B$ ,  $C$ , and  $D$ . If the tension in wire  $AB$  is 630 lb, determine the vertical force  $\mathbf{P}$  exerted by the tower on the pin at  $A$ .

### SOLUTION

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

$$\overline{AB} = -45\mathbf{i} - 90\mathbf{j} + 30\mathbf{k} \quad AB = 105 \text{ ft}$$

$$\overline{AC} = 30\mathbf{i} - 90\mathbf{j} + 65\mathbf{k} \quad AC = 115 \text{ ft}$$

$$\overline{AD} = 20\mathbf{i} - 90\mathbf{j} - 60\mathbf{k} \quad AD = 110 \text{ ft}$$

We write

$$\begin{aligned} \mathbf{T}_{AB} &= T_{AB} \lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} \\ &= \left( -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k} \right) T_{AB} \end{aligned}$$

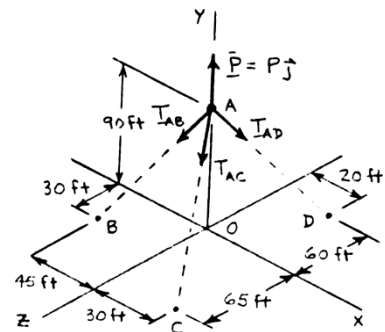
$$\begin{aligned} \mathbf{T}_{AC} &= T_{AC} \lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} \\ &= \left( \frac{6}{23}\mathbf{i} - \frac{18}{23}\mathbf{j} + \frac{13}{23}\mathbf{k} \right) T_{AC} \end{aligned}$$

$$\begin{aligned} \mathbf{T}_{AD} &= T_{AD} \lambda_{AD} = T_{AD} \frac{\overline{AD}}{AD} \\ &= \left( \frac{2}{11}\mathbf{i} - \frac{9}{11}\mathbf{j} - \frac{6}{11}\mathbf{k} \right) T_{AD} \end{aligned}$$

Substituting into the Eq.  $\Sigma \mathbf{F} = 0$  and factoring  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ :

$$\begin{aligned} &\left( -\frac{3}{7}T_{AB} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} \right) \mathbf{i} \\ &+ \left( -\frac{6}{7}T_{AB} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P \right) \mathbf{j} \\ &+ \left( \frac{2}{7}T_{AB} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} \right) \mathbf{k} = 0 \end{aligned}$$

Free Body A :



### PROBLEM 2.114 (Continued)

Setting the coefficients of **i**, **j**, **k**, equal to zero:

$$\mathbf{i}: \quad -\frac{3}{7}T_{AB} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} = 0 \quad (1)$$

$$\mathbf{j}: \quad -\frac{6}{7}T_{AB} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P = 0 \quad (2)$$

$$\mathbf{k}: \quad \frac{2}{7}T_{AB} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} = 0 \quad (3)$$

Set  $T_{AB} = 630$  lb in Eqs. (1) – (3):

$$-270 \text{ lb} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} = 0 \quad (1')$$

$$-540 \text{ lb} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P = 0 \quad (2')$$

$$180 \text{ lb} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} = 0 \quad (3')$$

Solving,  $T_{AC} = 467.42$  lb  $T_{AD} = 814.35$  lb  $P = 1572.10$  lb

$P = 1572$  lb ◀

### PROBLEM 2.115

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AD is 520 N, determine the weight of the plate.

Dimensions in mm

### SOLUTION

See Problem 2.114 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \quad (1)$$

$$-\frac{12}{17}T_{AB} + 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P = 0 \quad (2)$$

$$\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \quad (3)$$

Making  $T_{AD} = 520$  N in Eqs. (1) and (3):

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + 200 \text{ N} = 0 \quad (1')$$

$$\frac{9}{17}T_{AB} + 0.48T_{AC} - 288 \text{ N} = 0 \quad (3')$$

Multiply (1') by 9, (3') by 8, and add:

$$9.24T_{AC} - 504 \text{ N} = 0 \quad T_{AC} = 54.5455 \text{ N}$$

Substitute into (1') and solve for  $T_{AB}$ :

$$T_{AB} = \frac{17}{8}(0.6 \times 54.5455 + 200) \quad T_{AB} = 494.545 \text{ N}$$

Substitute for the tensions in Eq. (2) and solve for  $P$ :

$$\begin{aligned} P &= \frac{12}{17}(494.545 \text{ N}) + 0.64(54.5455 \text{ N}) + \frac{9.6}{13}(520 \text{ N}) \\ &= 768.00 \text{ N} \end{aligned}$$

Weight of plate =  $P = 768$  N ◀