Chapter 2 - Mathematical Description of Continuous-Time Signals

Solutions

Exercises With Answers in Text

Signal Functions

1. If $g(t) = 7e^{-2t-3}$ write out and simplify

(a) $g(3) = 7e^{-9} = 8.6387 \times 10^{-4}$

(b) $g(2-t) = 7e^{-2(2-t)-3} = 7e^{-7+2t}$

(c) $g(t/10+4) = 7e^{-t/5-11}$

(d) $g(jt) = 7e^{-j2t-3}$

(e) $$\frac{g(jt)+g(-jt)}{2} = 7e^{-3}\frac{e^{-j2t}+e^{j2t}}{2} = 7e^{-3}\cos(2t) = 0.3485\cos(2t)$$

(f) $$\frac{g\left(\frac{jt-3}{2}\right)+g\left(\frac{-jt-3}{2}\right)}{2} = 7\frac{e^{-jt}+e^{jt}}{2} = 7\cos(t)$$

2. If $g(x) = x^2 - 4x + 4$ write out and simplify

(a) $g(z) = z^2 - 4z + 4$

(b) $g(u+v) = (u+v)^2 - 4(u+v) + 4 = u^2 + v^2 + 2uv - 4u - 4v + 4$

(c) $g(e^{jt}) = (e^{jt})^2 - 4e^{jt} + 4 = e^{j2t} - 4e^{jt} + 4 = (e^{jt} - 2)^2$

(d) $g(g(t)) = g(t^2 - 4t + 4) = (t^2 - 4t + 4)^2 - 4(t^2 - 4t + 4) + 4$

$g(g(t)) = t^4 - 8t^3 + 20t^2 - 16t + 4$

(e) $g(2) = 4 - 8 + 4 = 0$

3. Find the magnitudes and phases of these complex quantities.

(a) $e^{-(3+j2.3)}$

$$\left|e^{-(3+j2.3)}\right| = \left|e^{-3}\right|\left|e^{-j2.3}\right| = e^{-3}\left|\cos(2.3) - j\sin(2.3)\right|$$

$$\left|e^{-(3+j2.3)}\right| = e^{-3}\sqrt{\cos^2(2.3)+\sin^2(2.3)} = e^{-3} = 0.0498$$

$$\measuredangle e^{-(3+j2.3)} = \underbrace{\measuredangle e^{-3}}_{=0} + \measuredangle e^{-j2.3} = \tan^{-1}\left(-\frac{\sin(2.3)}{\cos(2.3)}\right) = -2.3 \text{ radians}$$

(b) e^{2-j6}

$$\left|e^{2-j6}\right| = \left|e^{2}\right|\left|e^{-j6}\right| = e^{2} = 7.3891$$

$$\measuredangle e^{2-j6} = \underbrace{\measuredangle e^{2}}_{=0} + \measuredangle e^{-j6} = \tan^{-1}\left(-\frac{\sin(6)}{\cos(6)}\right) = -6 \text{ radians}$$

(c) $\dfrac{100}{8+j13}$

$$\left|\frac{100}{8+j13}\right| = \frac{|100|}{|8+j13|} = \frac{100}{\sqrt{8^2+13^2}} = 6.5512$$

$$\measuredangle \frac{100}{8+j13} = \underbrace{\measuredangle 100}_{=0} - \measuredangle(8+j13) = -\tan\left(\frac{13}{8}\right) = -1.0191 \text{ radians}$$

4. Let $G(f) = \dfrac{j4f}{2+j7f/11}$.

(a) What value does the magnitude of this function approach as *f* approaches positive infinity?

$$\lim_{f\to\infty}\left|\frac{j4f}{2+j7f/11}\right| = \frac{|j4f|}{|j7f/11|} = \frac{4}{7/11} = \frac{44}{7} \cong 6.285$$

(b) What value (in radians) does the phase of this function approach as f approaches zero from the positive side?

$$\lim_{f\to 0^+}\left[\measuredangle j4f - \measuredangle\left(2+j7f/11\right)\right] = \lim_{f\to 0^+}\left[\measuredangle j4f - \measuredangle 2\right] = \pi/2 - 0 = \pi/2$$

5. Let $\mathrm{X}(f) = \dfrac{jf}{jf+10}$

(a) Find the magnitude $\left|\mathrm{X}(4)\right|$ and the angle $\measuredangle \mathrm{X}(4)$ in radians

$$\mathrm{X}(f) = \frac{jf}{jf+10} \Rightarrow \mathrm{X}(4) = \frac{j4}{j4+10} = \frac{4e^{j\pi/2}}{10.77e^{j0.3805}} = 0.3714e^{j1.19}$$

$$\left|\mathrm{X}(4)\right| = 0.3714 \qquad \measuredangle \mathrm{X}(4) = 1.19$$

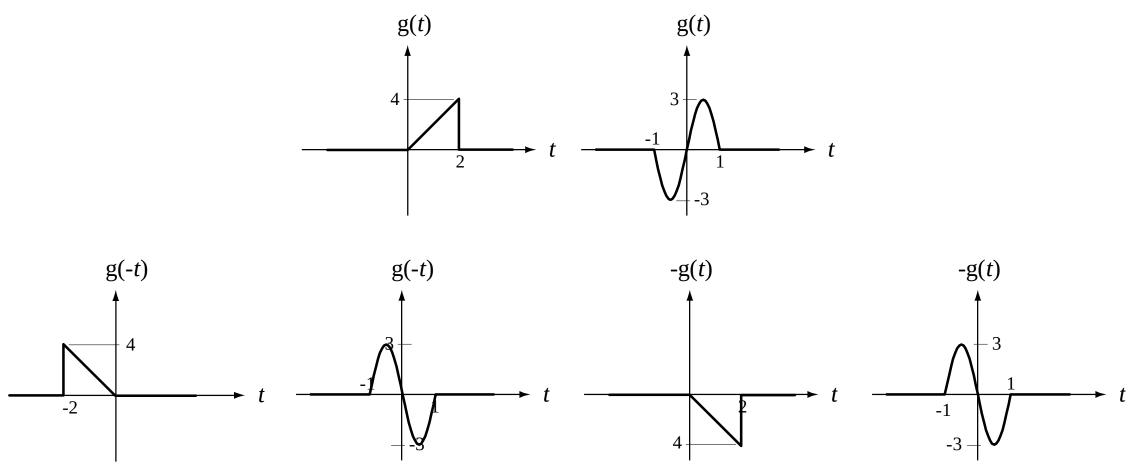
(b) What value (in radians) does $\measuredangle \mathrm{X}(f)$ approach as f approaches zero from the positive side?

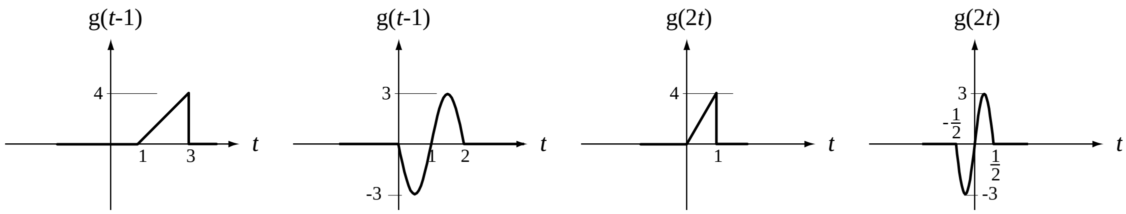
$$\measuredangle \mathrm{X}(f) = \lim_{f\to 0^+}\left[\measuredangle jf - \measuredangle\left(jf+10\right)\right] = \pi/2 - 0 = \pi/2$$

Shifting and Scaling

6. For each function $\mathrm{g}(t)$ graph $\mathrm{g}(-t)$, $-\mathrm{g}(t)$, $\mathrm{g}(t-1)$, and $\mathrm{g}(2t)$.

(a) (b)





7. Find the values of the following signals at the indicated times.

(a) $x(t) = 2\,\text{rect}(t/4)$, $x(-1) = 2\,\text{rect}(-1/4) = 2$

(b) $x(t) = 5\text{rect}(t/2)\text{sgn}(2t)$, $x(0.5) = 5\text{rect}(1/4)\text{sgn}(1) = 5$

(c) $x(t) = 9\,\text{rect}(t/10)\text{sgn}(3(t-2))$, $x(1) = 9\,\text{rect}(1/10)\text{sgn}(-3) = -9$

8. For each pair of functions in Figure E-8 provide the values of the constants A, t_0 and w in the functional transformation $g_2(t) = A g_1((t - t_0)/w)$.

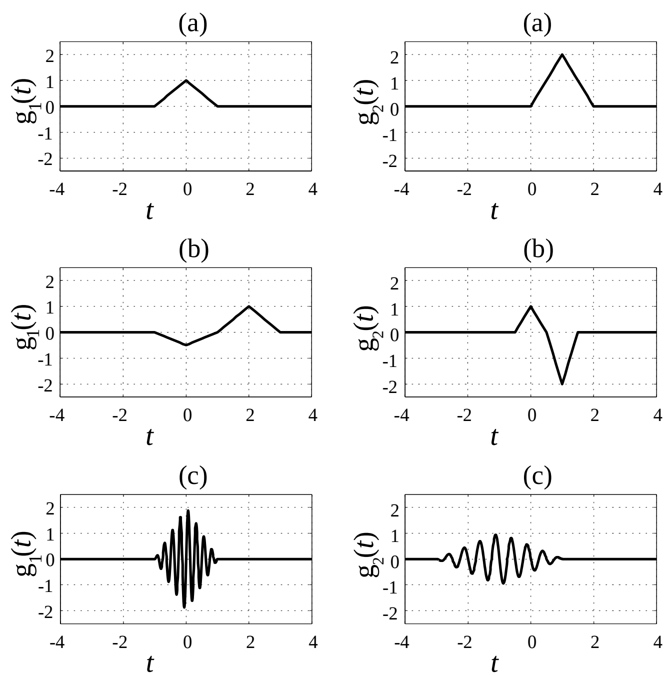


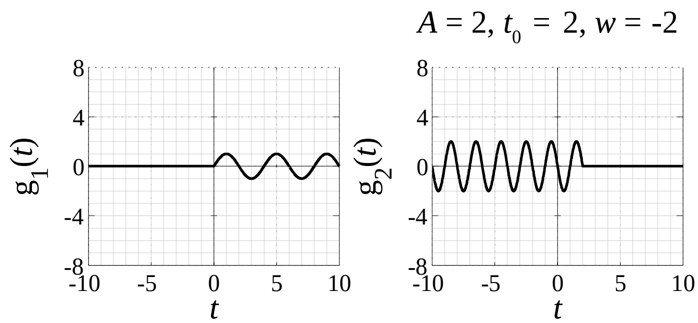
Figure E-8

Answers: (a) $A = 2, t_0 = 1, w = 1$, (b) $A = -2, t_0 = 0, w = 1/2$, (c) $A = -1/2, t_0 = -1, w = 2$

9. For each pair of functions in Figure E-9 provide the values of the constants A,

t_0 and a in the functional transformation $\mathrm{g}_2(t) = A\,\mathrm{g}_1\left(w\left(t - t_0\right)\right)$.

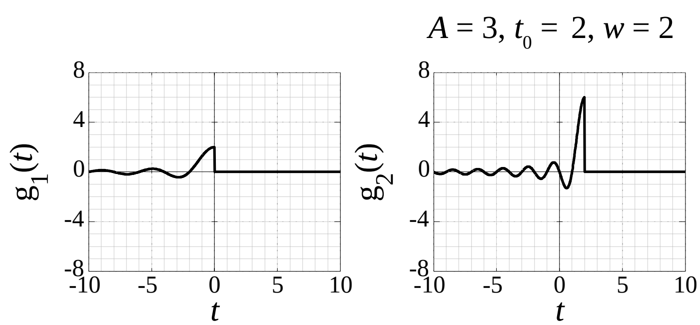
(a)



Amplitude comparison yields $A = 2$. Time scale comparison yields $w = -2$.

$$\mathrm{g}_2(2) = 2\,\mathrm{g}_1\left(-2\left(2 - t_0\right)\right) = 2\,\mathrm{g}_1(0) \Rightarrow -4 + 2t_0 = 0 \Rightarrow t_0 = 2$$

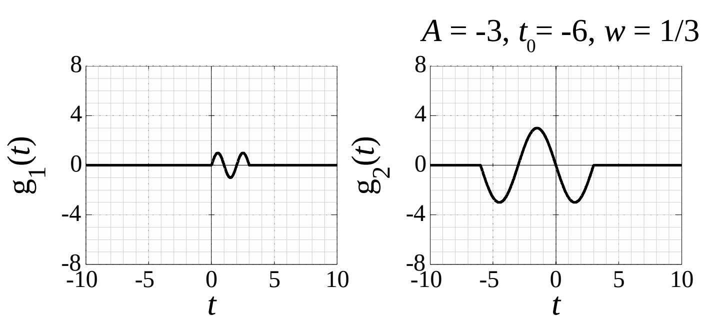
(b)



Amplitude comparison yields $A = 3$. Time scale comparison yields $w = 2$.

$$\mathrm{g}_2(2) = 3\mathrm{g}_1\left(2\left(2 - t_0\right)\right) = 3\mathrm{g}_1(0) \Rightarrow 4 - 2t_0 = 0 \Rightarrow t_0 = 2$$

(c)



Amplitude comparison yields $A = -3$. Time scale comparison yields $w = 1/3$.

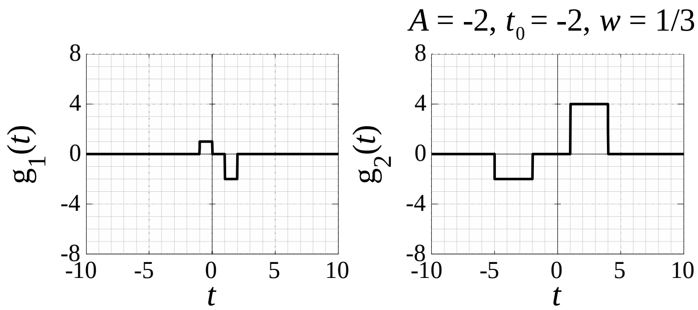
$$\mathrm{g}_2(0) = -3\mathrm{g}_1\left((1/3)\left(0 - t_0\right)\right) = -3\mathrm{g}_1(2) \Rightarrow -t_0/3 = 2 \Rightarrow t_0 = -6$$

OR

Amplitude comparison yields $A = -3$. Time scale comparison yields $w = -1/3$.

$$\mathrm{g}_2(3) = -3\mathrm{g}_1\left((-1/3)\left(3 - t_0\right)\right) = -3\mathrm{g}_1(0) \Rightarrow t_0/3 - 1 = 0 \Rightarrow t_0 = 3$$

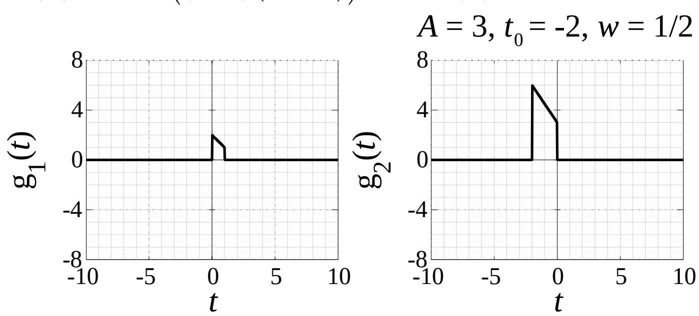
(d)



Amplitude comparison yields $A=-2$. Time scale comparison yields $w=1/3$.

$$g_2(4)=-2\,g_1\left((1/3)(4-t_0)\right)=-3g_1(2)\Rightarrow -t_0/3+4/3=2\Rightarrow t_0=-2$$

(e)



Amplitude comparison yields $A=3$. Time scale comparison yields $w=1/2$.

$$g_2(0)=3g_1\left((1/2)(0-t_0)\right)=3g_1(1)\Rightarrow -t_0/2=1\Rightarrow t_0=-2$$

Figure E-9

10. In Figure E-10 is plotted a function $g_1(t)$ which is zero for all time outside the range plotted. Let some other functions be defined by

$$g_2(t)=3g_1(2-t) \quad , \quad g_3(t)=-2\,g_1(t/4) \quad , \quad g_4(t)=g_1\left(\frac{t-3}{2}\right)$$

Find these values.

(a) $g_2(1)=-3$

(b) $g_3(-1)=-3.5$

(c) $\left[g_4(t)g_3(t)\right]_{t=2}=\frac{3}{2}\times(-1)=-\frac{3}{2}$

(d) $\int_{-3}^{-1} g_4(t)\,dt$

The function $g_4(t)$ is linear between the integration limits and the area under it is a triangle. The base width is 2 and the height is -2. Therefore the area is -2.

$$\int_{-3}^{-1} g_4(t)\,dt=-2$$

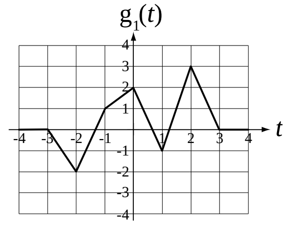


Figure E-10

11. A function $\mathrm{G}(f)$ is defined by

$$\mathrm{G}(f) = e^{-j2\pi f}\,\mathrm{rect}(f/2) \;.$$

Graph the magnitude and phase of $\mathrm{G}(f-10)+\mathrm{G}(f+10)$ over the range, $-20 < f < 20$.

First imagine what $\mathrm{G}(f)$ looks like. It consists of a rectangle centered at $f = 0$ of width, 2, multiplied by a complex exponential. Therefore for frequencies greater than one in magnitude it is zero. Its magnitude is simply the magnitude of the rectangle function because the magnitude of the complex exponential is one for any *f*.

$$\left|e^{-j2\pi f}\right| = \left|\cos(-2\pi f) + j\sin(-2\pi f)\right| = \left|\cos(2\pi f) - j\sin(2\pi f)\right|$$

$$\left|e^{-j2\pi f}\right| = \sqrt{\cos^2(2\pi f) + \sin^2(2\pi f)} = 1$$

The phase (angle) of $\mathrm{G}(f)$ is simply the phase of the complex exponential between $f = -1$ and $f = 1$ and undefined outside that range because the phase of the rectangle function is zero between $f = -1$ and $f = 1$and undefined outside that range and the phase of a product is the sum of the phases. The phase of the complex exponential is

$$\measuredangle e^{-j2\pi f} = \measuredangle\left(\cos(2\pi f) - j\sin(2\pi f)\right) = \tan^{-1}\left(-\frac{\sin(2\pi f)}{\cos(2\pi f)}\right) = -\tan^{-1}\left(\frac{\sin(2\pi f)}{\cos(2\pi f)}\right)$$

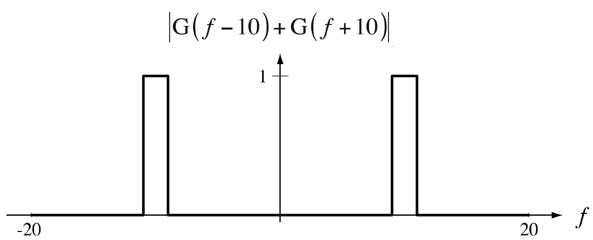
$$\measuredangle e^{-j2\pi f} = -\tan^{-1}\left(\tan(2\pi f)\right)$$

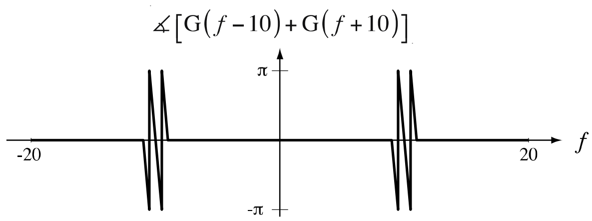
The inverse tangent function is multiple-valued. Therefore there are multiple correct answers for this phase. The simplest of them is found by choosing

$$\measuredangle e^{-j2\pi f} = -2\pi f$$

which is simply the coefficient of *j* in the original complex exponential expression. A more general solution would be $\measuredangle e^{-j2\pi f} = -2\pi f + 2n\pi$, n an integer. The solution of the original problem is simply this solution except shifted up and down by 10 in *f* and added.

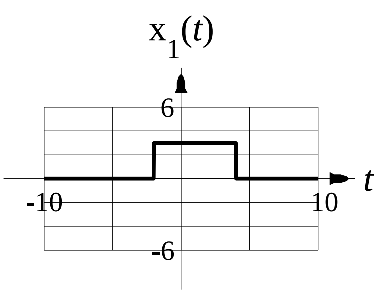
$$\mathrm{G}(f-10)+\mathrm{G}(f+10)=e^{-j2\pi(f-10)}\,\mathrm{rect}\left(\frac{f-10}{2}\right)+e^{-j2\pi(f+10)}\,\mathrm{rect}\left(\frac{f+10}{2}\right)$$

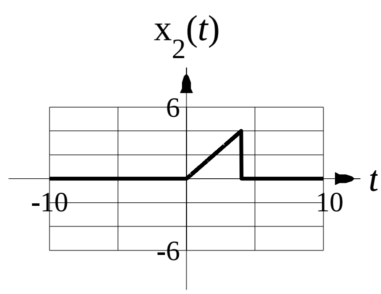




12. Let $\mathrm{x}_1(t) = 3\mathrm{rect}((t-1)/6)$ and $\mathrm{x}_2(t) = \mathrm{ramp}(t)[\mathrm{u}(t)-\mathrm{u}(t-4)]$.

(a) Graph them in the time range $-10 < t < 10$. Put scale numbers on the vertical axis so that actual numerical values could be read from the graph.

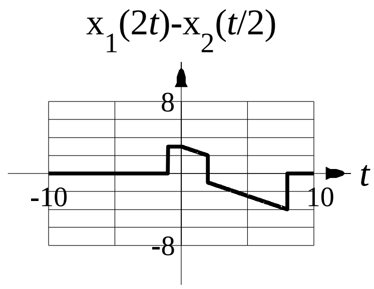




(b) Graph $\mathrm{x}(t) = \mathrm{x}_1(2t) - \mathrm{x}_2(t/2)$ in the time range $-10 < t < 10$. Put scale numbers on the vertical axis so that actual numerical values could be read from the graph.

$$x_1(2t) = 3\text{rect}((2t-1)/6) \text{ and } x_2(t/2) = \text{ramp}(t/2)[\text{u}(t/2) - \text{u}(t/2-4)]$$

$$x_1(2t) = 3\text{rect}((t-1/2)/3) \text{ and } x_2(t/2) = (1/2)\text{ramp}(t)[\text{u}(t) - \text{u}(t-8)]$$



13. Write an expression consisting of a summation of unit step functions to represent a signal which consists of rectangular pulses of width 6 ms and height 3 which occur at a uniform rate of 100 pulses per second with the leading edge of the first pulse occurring at time $t = 0$.

$$x(t) = 3\sum_{n=0}^{\infty}[\text{u}(t-0.01n) - \text{u}(t-0.01n-0.006)]$$

14. Find the strengths of the following impulses.

(a) $-3\delta(-4t)$

$$-3\delta(-4t) = -3 \times \frac{1}{|-4|}\delta(t) \Rightarrow \text{Strength is } -3/4$$

(b) $5\delta(3(t-1))$

$$5\delta(3(t-1)) = \frac{5}{3}\delta(t-1) \Rightarrow \text{Strength is } \frac{5}{3}$$

15. Find the strengths and spacing between the impulses in the periodic impulse $-9\delta_{11}(5t)$.

$$-9\delta_{11}(5t) = -9\sum_{k=-\infty}^{\infty}\delta(5t-11k) = -\frac{9}{5}\sum_{k=-\infty}^{\infty}\delta(t-11k/5)$$

The strengths are all -9/5 and the spacing between them is 11/5.

Derivatives and Integrals of Functions

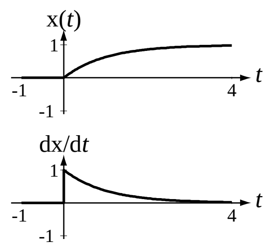
16. Graph the derivative of $\mathrm{x}(t)=\left(1-e^{-t}\right)\mathrm{u}(t)$.

This function is constant zero for all time before time, $t=0$, therefore its derivative during that time is zero. This function is a constant minus a decaying exponential after time, $t=0$, and its derivative in that time is therefore a positive decaying exponential.

$$\mathrm{x}'(t)=\begin{cases} e^{-t} & , \ t>0 \\ 0 & , \ t<0 \end{cases}$$

Strictly speaking, its derivative is not defined at exactly $t=0$. Since the value of a physical signal at a single point has no impact on any physical system (as long as it is finite) we can choose any finite value at time, $t=0$, without changing the effect of this signal on any physical system. If we choose 1/2, then we can write the derivative as

$$\mathrm{x}'(t)=e^{-t}\,\mathrm{u}(t) \ .$$



Alternate Solution using the chain rule of differentiation and the fact that the impulse occurs at time $t=0$.

$$\frac{d}{dt}\left(\mathrm{x}(t)\right)=\underbrace{\left(1-e^{-t}\right)}_{=0 \text{ for } t=0}\delta(t)+e^{-t}\,\mathrm{u}(t)=e^{-t}\,\mathrm{u}(t)$$

17. Find the numerical value of each integral.

(a) $\int_{-2}^{11}\mathrm{u}(4-t)\,dt=\int_{-2}^{4}dt=6$

(b) $\int_{-1}^{8}\left[\delta(t+3)-2\delta(4t)\right]dt$

$$\int_{-1}^{8}[\delta(t+3)-2\delta(4t)]dt=\int_{-1}^{8}\delta(t+3)dt-2\int_{-1}^{8}\delta(4t)dt$$

$$\int_{-1}^{8}[\delta(t+3)-2\delta(4t)]dt=0-2\times 1/4\int_{-1}^{8}\delta(t)dt=-1/2$$

(c) $\int_{1/2}^{5/2}\delta_2(3t)dt$

$$\int_{1/2}^{5/2}\delta_2(3t)dt=\int_{1/2}^{5/2}\sum_{n=-\infty}^{\infty}\delta(3t-2n)dt=\frac{1}{3}\int_{1/2}^{5/2}\sum_{n=-\infty}^{\infty}\delta(t-2n/3)dt$$

$$\int_{1/2}^{5/2}\delta_2(3t)dt=\frac{1}{3}[1+1+1]=1$$

(d) $\int_{-\infty}^{\infty}\delta(t+4)\mathrm{ramp}(-2t)dt=\mathrm{ramp}(-2(-4))=\mathrm{ramp}(8)=8$

(e) $\int_{-3}^{10}\mathrm{ramp}(2t-4)dt=\int_{2}^{10}(2t-4)dt=\left[t^2-4t\right]_2^{10}=100-40-4+8=64$

(f) $\int_{11}^{82}3\sin(200t)\delta(t-7)dt=0$ (Impulse does not occur between 11 and 82.)

(g) $\int_{-5}^{5}\sin(\pi t/20)dt=\underline{0}$ Odd function integrated over symmetrical limits.

(h) $\int_{-2}^{10}39t^2\delta_4(t-1)dt=39\int_{-2}^{10}t^2\sum_{k=-\infty}^{\infty}\delta(t-1-4k)dt$

$$\int_{-2}^{10}39t^2\delta_4(t-1)dt=39\int_{-2}^{10}t^2[\delta(t-1)+\delta(t-5)+\delta(t-9)]dt$$

$$\int_{-2}^{10}39t^2\delta_4(t-1)dt=39(1^2+5^2+9^2)=4173$$

(i) $\int_{-\infty}^{\infty}e^{-18t}\,\mathrm{u}(t)\delta(10t-2)dt$.

$$\int_{-\infty}^{\infty} e^{-18t}\,\mathrm{u}(t)\,\delta(10t-2)\,dt = \int_{-\infty}^{\infty} e^{-18t}\,\mathrm{u}(t)\,\delta(10(t-1/5))\,dt$$

Using the scaling property of the impulse,

$$\int_{-\infty}^{\infty} e^{-18t}\,\mathrm{u}(t)\,\delta(10t-2)\,dt = \frac{1}{10}\int_{-\infty}^{\infty} e^{-18t}\,\mathrm{u}(t)\,\delta(t-1/5)\,dt$$

Using the sampling property of the impulse,

$$\int_{-\infty}^{\infty} e^{-18t}\,\mathrm{u}(t)\,\delta(10t-2)\,dt = \frac{1}{10}e^{-18/5} = 0.002732$$

(j) $\int_{2}^{9} 9\delta((t-4)/5)\,dt = 45\int_{2}^{9}\delta(t-4)\,dt = 45$

(k) $\int_{-6}^{3} 5\delta(3(t-4))\,dt = \frac{5}{3}\int_{-6}^{3}\delta(t-4)\,dt = 0$

(l) $\int_{-\infty}^{\infty} \mathrm{ramp}(3t)\,\delta(t-4)\,dt = \mathrm{ramp}(3\times 4) = 12$

(m) $\int_{1}^{17} \delta_3(t)\cos(2\pi t/3)\,dt$

The impulses in the periodic impulse occur at ...-9,-6,-3,0,3,6,9,12,15,18,... At each of these points the cosine value is the same, one, because its period is 3 seconds also. So the integral value is simple the sum of the strengths of the impulse that occur in the time range, 1 to 17 seconds

$$\int_{1}^{17} \delta_3(t)\cos(2\pi t/3)\,dt = 5$$

18. Graph the integral from negative infinity to time *t* of the functions in Figure E-18 which are zero for all time $t < 0$.

This is the integral $\int_{-\infty}^{t} \mathrm{g}(\tau)\,d\tau$ which, in geometrical terms, is the accumulated area under the function $\mathrm{g}(t)$ from time $-\infty$ to time *t*. For the case of the two back-to-back rectangular pulses, there is no accumulated area until after time $t = 0$ and then in the time interval $0 < t < 1$ the area accumulates linearly with time up to a maximum area of one at time $t = 1$. In the second time interval $1 < t < 2$ the area is linearly declining at half the rate at which it increased in the first time interval $0 < t < 1$ down to a value of 1/2 where it stays because there is no accumulation of area for $t > 2$.

In the second case of the triangular-shaped function, the area does not accumulate linearly, but rather non-linearly because the integral of a linear function is a second-degree polynomial. The rate of accumulation of area is increasing up to time $t = 1$ and then decreasing (but still positive) until time $t = 2$ at which time it stops completely. The final value of the accumulated area must be the total area of the triangle, which, in this case, is one.

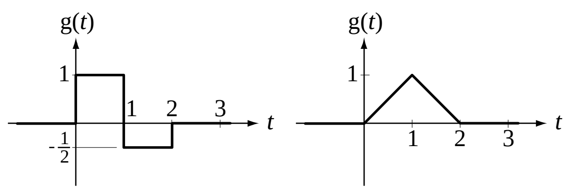
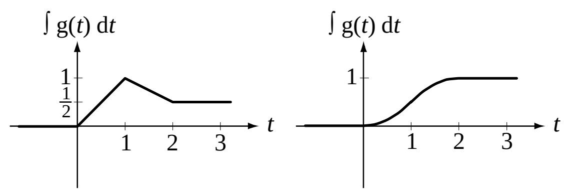


Figure E-18



19. If $4\,\mathrm{u}(t-5) = \frac{d}{dt}(\mathrm{x}(t))$, what is the function $\mathrm{x}(t)$? $\mathrm{x}(t) = 4\,\mathrm{ramp}(t-5)$

Generalized Derivative

20. The generalized derivative of $18\,\mathrm{rect}\left(\frac{t-2}{3}\right)$ consists of two impulses.

Find their numerical locations and strengths.

Impulse #1: Location is t = 0.5 and strength is 18.

Impulse #2: Location is t = 3.5 and strength is -18.

Even and Odd Functions

21. Classify the following functions as even, odd or neither .

(a) $\cos(2\pi t)\,\mathrm{tri}(t-1)$ Neither

Cosine is even but the shifted triangle is neither even nor odd which means it has a non-zero even part and a non-zero odd part. So the product also has a non-zero even part and a non-zero odd part.

(b) $\sin(2\pi t)\text{rect}(t/5)$ Odd

Sine is odd and rectangle is even. Therefore the product is odd.

22. An even function $\text{g}(t)$ is described over the time range $0 < t < 10$ by

$$\text{g}(t) = \begin{cases} 2t & , \ 0 < t < 3 \\ 15 - 3t & , \ 3 < t < 7 \\ -2 & , \ 7 < t < 10 \end{cases} .$$

(a) What is the value of $\text{g}(t)$ at time $t = -5$?

Since $\text{g}(t)$ is even, $\text{g}(t) = g(-t) \Rightarrow \text{g}(-5) = \text{g}(5) = 15 - 3 \times 5 = 0$.

(b) What is the value of the first derivative of g(t) at time $t = -6$?

Since $\text{g}(t)$ is even,

$$\frac{d}{dt}\text{g}(t) = -\frac{d}{dt}\text{g}(-t) \Rightarrow \left[\frac{d}{dt}\text{g}(t)\right]_{t=-6} = -\left[\frac{d}{dt}\text{g}(t)\right]_{t=6} = -(-3) = 3.$$

23. Find the even and odd parts of these functions.

(a) $\text{g}(t) = 2t^2 - 3t + 6$

$$\text{g}_e(t) = \frac{2t^2 - 3t + 6 + 2(-t)^2 - 3(-t) + 6}{2} = \frac{4t^2 + 12}{2} = 2t^2 + 6$$

$$\text{g}_o(t) = \frac{2t^2 - 3t + 6 - 2(-t)^2 + 3(-t) - 6}{2} = \frac{-6t}{2} = -3t$$

(b) $\text{g}(t) = 20\cos(40\pi t - \pi/4)$

$$\text{g}_e(t) = \frac{20\cos(40\pi t - \pi/4) + 20\cos(-40\pi t - \pi/4)}{2}$$

Using $\cos(z_1 + z_2) = \cos(z_1)\cos(z_2) - \sin(z_1)\sin(z_2)$,

$$g_e(t) = \frac{\left\{ \begin{array}{l} 20\left[\cos(40\pi t)\cos(-\pi/4) - \sin(40\pi t)\sin(-\pi/4)\right] \\ +20\left[\cos(-40\pi t)\cos(-\pi/4) - \sin(-40\pi t)\sin(-/4)\right] \end{array} \right\}}{2}$$

$$g_e(t) = \frac{\left\{ \begin{array}{l} 20\left[\cos(40\pi t)\cos(\pi/4) + \sin(40\pi t)\sin(\pi/4)\right] \\ +20\left[\cos(40\pi t)\cos(\pi/4) - \sin(40\pi t)\sin(\pi/4)\right] \end{array} \right\}}{2}$$

$$g_e(t) = 20\cos(\pi/4)\cos(40\pi t) = \left(20/\sqrt{2}\right)\cos(40\pi t)$$

$$g_o(t) = \frac{20\cos(40\pi t - \pi/4) - 20\cos(-40\pi t - \pi/4)}{2}$$

Using $\cos(z_1 + z_2) = \cos(z_1)\cos(z_2) - \sin(z_1)\sin(z_2)$,

$$g_o(t) = \frac{\left\{ \begin{array}{l} 20\left[\cos(40\pi t)\cos(-\pi/4) - \sin(40\pi t)\sin(-\pi/4)\right] \\ -20\left[\cos(-40\pi t)\cos(-\pi/4) - \sin(-40\pi t)\sin(-\pi/4)\right] \end{array} \right\}}{2}$$

$$g_o(t) = \frac{\left\{ \begin{array}{l} 20\left[\cos(40\pi t)\cos(\pi/4) + \sin(40\pi t)\sin(\pi/4)\right] \\ -20\left[\cos(40\pi t)\cos(\pi/4) - \sin(40\pi t)\sin(\pi/4)\right] \end{array} \right\}}{2}$$

$$g_o(t) = 20\sin(\pi/4)\sin(40\pi t) = \left(20/\sqrt{2}\right)\sin(40\pi t)$$

(c) $$g(t) = \frac{2t^2 - 3t + 6}{1+t}$$

$$g_e(t) = \frac{\dfrac{2t^2 - 3t + 6}{1+t} + \dfrac{2t^2 + 3t + 6}{1-t}}{2}$$

$$g_e(t) = \frac{\dfrac{\left(2t^2 - 3t + 6\right)(1-t) + \left(2t^2 + 3t + 6\right)(1+t)}{(1+t)(1-t)}}{2}$$

$$g_e(t) = \frac{4t^2 + 12 + 6t^2}{2(1-t^2)} = \frac{6+5t^2}{1-t^2}$$

$$g_o(t) = \frac{\dfrac{2t^2-3t+6}{1+t} - \dfrac{2t^2+3t+6}{1-t}}{2}$$

$$g_o(t) = \frac{\dfrac{(2t^2-3t+6)(1-t)-(2t^2+3t+6)(1+t)}{(1+t)(1-t)}}{2}$$

$$g_o(t) = \frac{-6t-4t^3-12t}{2(1-t^2)} = -t\frac{2t^2+9}{1-t^2}$$

(d) $g(t) = t(2-t^2)(1+4t^2)$

$$g(t) = \underbrace{t}_{\text{odd}}\underbrace{(2-t^2)}_{\text{even}}\underbrace{(1+4t^2)}_{\text{even}}$$

Therefore $g(t)$ is odd, $g_e(t) = 0$ and $g_o(t) = t(2-t^2)(1+4t^2)$

(e) $g(t) = t(2-t)(1+4t)$

$$g_e(t) = \frac{t(2-t)(1+4t)+(-t)(2+t)(1-4t)}{2} \qquad g_e(t) = 7t^2$$

$$g_o(t) = \frac{t(2-t)(1+4t)-(-t)(2+t)(1-4t)}{2} \qquad g_o(t) = t(2-4t^2)$$

(f) $g(t) = \dfrac{20-4t^2+7t}{1+|t|}$

$$g_e(t) = \frac{\dfrac{20-4t^2+7t}{1+|t|} + \dfrac{20-4t^2-7t}{1+|-t|}}{2} = \frac{\dfrac{40-8t^2}{1+|t|}}{2} = \frac{20-4t^2}{1+|t|}$$

$$g_o(t) = \frac{\dfrac{20-4t^2+7t}{1+|t|} - \dfrac{20-4t^2-7t}{1+|-t|}}{2} = \frac{\dfrac{14t}{1+|t|}}{2} = \frac{7t}{1+|t|}$$

24. Graph the even and odd parts of the functions in Figure E-24.

To graph the even part of graphically-defined functions like these, first graph $g(-t)$. Then add it (graphically, point by point) to $g(t)$ and (graphically) divide the sum by two. Then, to graph the odd part, subtract $g(-t)$ from $g(t)$ (graphically) and divide the difference by two.

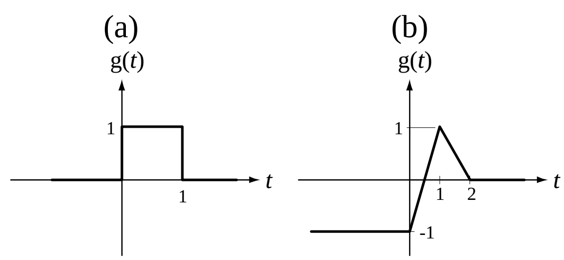
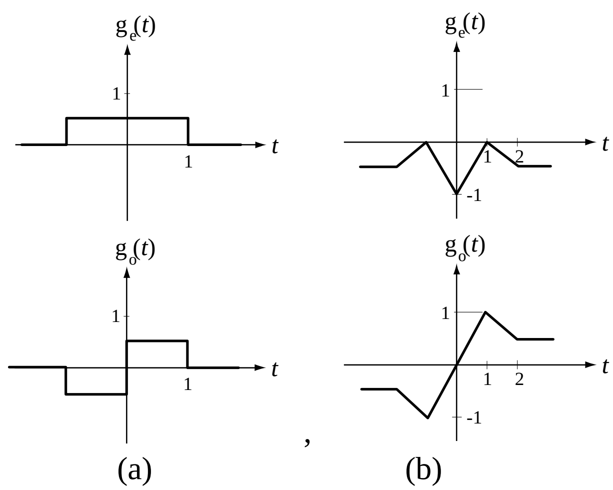
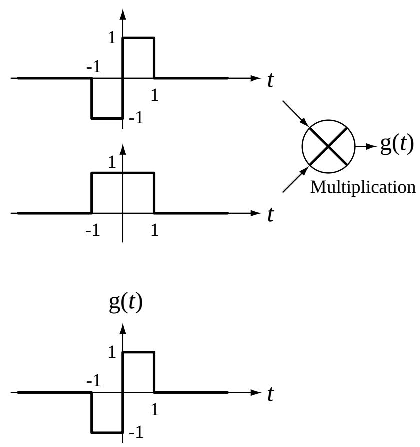


Figure E-24



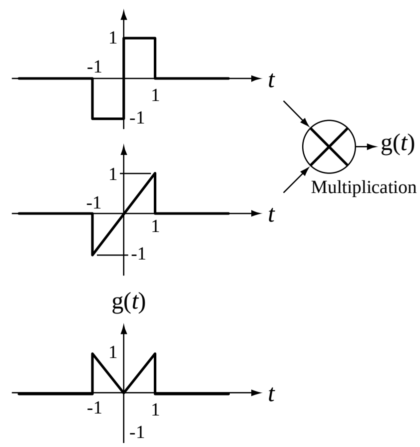
25. Graph the indicated product or quotient $g(t)$ of the functions in Figure E-25.

(a)



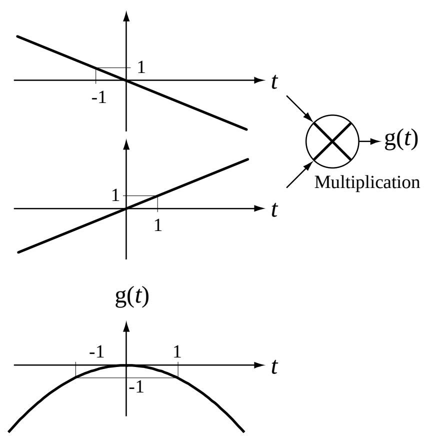
g(t)
Multiplication
g(t)

(b)



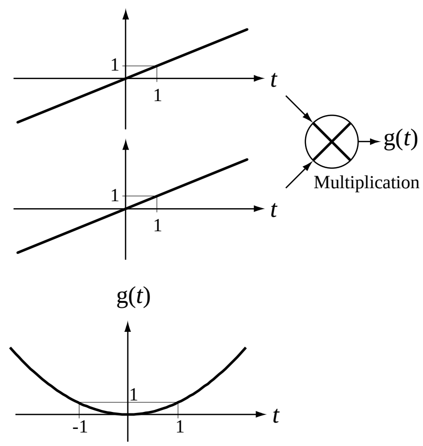
g(t)
Multiplication
g(t)

(c)



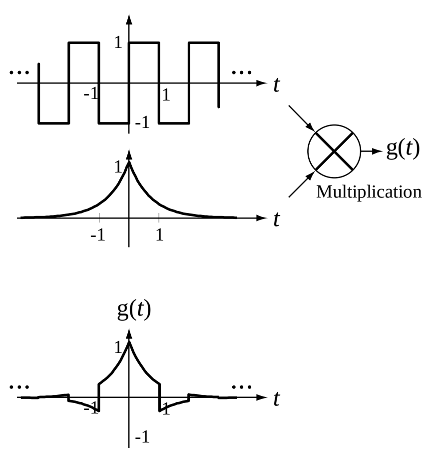
g(t)
Multiplication
g(t)

(d)



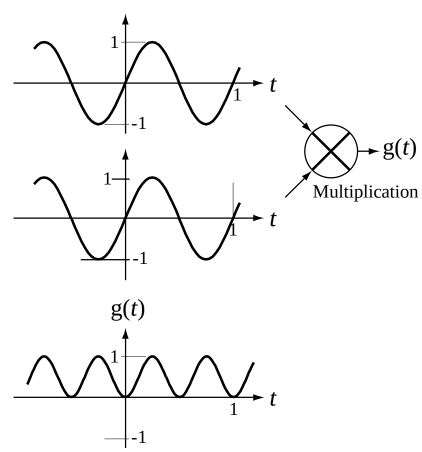
g(t)
Multiplication
g(t)

(e)



g(t)
Multiplication
g(t)

(f)



g(t)
Multiplication
g(t)

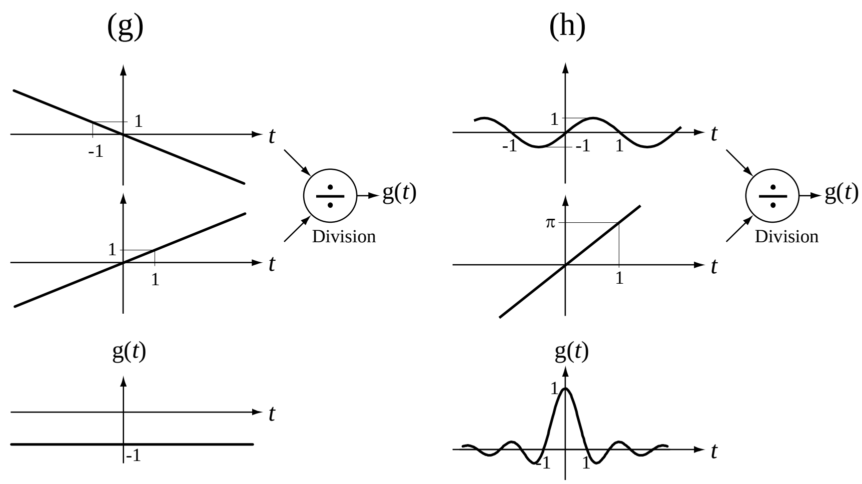


Figure E-25

26. Use the properties of integrals of even and odd functions to evaluate these integrals in the quickest way.

(a) $$\int_{-1}^{1}(2+t)\,dt = \int_{-1}^{1}\underbrace{2}_{\text{even}}\,dt + \int_{-1}^{1}\underbrace{t}_{\text{odd}}\,dt = 2\int_{0}^{1}2\,dt = 4$$

(b) $$\int_{-1/20}^{1/20}\left[4\cos(10\pi t)+8\sin(5\pi t)\right]dt = \int_{-1/20}^{1/20}\underbrace{4\cos(10\pi t)}_{\text{even}}dt + \int_{-1/20}^{1/20}\underbrace{8\sin(5\pi t)}_{\text{odd}}dt$$

$$\int_{-1/20}^{1/20}\left[4\cos(10\pi t)+8\sin(5\pi t)\right]dt = 8\int_{0}^{1/20}\cos(10\pi t)\,dt = \frac{8}{10\pi}$$

(c) $$\int_{-1/20}^{1/20}4\underbrace{\underbrace{t}_{\text{odd}}\underbrace{\cos(10\pi t)}_{\text{even}}}_{\text{odd}}dt = 0$$

(d) $$\int_{-1/10}^{1/10}\underbrace{\underbrace{t}_{\text{odd}}\underbrace{\sin(10\pi t)}_{\text{odd}}}_{\text{even}}dt = 2\int_{0}^{1/10}t\sin(10\pi t)\,dt = 2\left[-t\frac{\cos(10\pi t)}{10\pi}\bigg|_{0}^{1/10} + \int_{0}^{1/10}\frac{\cos(10\pi t)}{10\pi}dt\right]$$

$$\int_{-1/10}^{1/10}\underbrace{\underbrace{t}_{\text{odd}}\underbrace{\sin(10\pi t)}_{\text{odd}}}_{\text{even}}dt == 2\left[\frac{1}{100\pi} + \frac{\sin(10\pi t)}{(10\pi)^2}\bigg|_{0}^{1/10}\right] = \frac{1}{50\pi}$$

(e) $$\int_{-1}^{1}\underbrace{e^{-|t|}}_{\text{even}}dt = 2\int_{0}^{1}e^{-|t|}\,dt = 2\int_{0}^{1}e^{-t}\,dt = 2\left[-e^{-t}\right]_{0}^{1} = 2\left(1-e^{-1}\right) \approx 1.264$$

(f) $\int_{-1}^{1} \underbrace{\underbrace{t}_{\text{odd}} \underbrace{e^{-|t|}}_{\text{even}}}_{\text{odd}} dt = 0$

Periodic Signals

27. Find the fundamental period and fundamental frequency of each of these functions.

(a) $g(t) = 10\cos(50\pi t)$ $f_0 = 25 \text{ Hz} \; , \; T_0 = 1/25 \text{ s}$

(b) $g(t) = 10\cos(50\pi t + \pi/4)$ $f_0 = 25 \text{ Hz} \; , \; T_0 = 1/25 \text{ s}$

(c) $g(t) = \cos(50\pi t) + \sin(15\pi t)$

The fundamental period of the sum of two periodic signals is the least common multiple (LCM) of their two individual fundamental periods. The fundamental frequency of the sum of two periodic signals is the greatest common divisor (GCD) of their two individual fundamental frequencies.

$$f_0 = \text{GCD}(25, 15/2) = 2.5 \text{ Hz} \; , \; T_0 = 1/2.5 = 0.4 \text{ s}$$

(d) $g(t) = \cos(2\pi t) + \sin(3\pi t) + \cos(5\pi t - 3\pi/4)$

$$f_0 = \text{GCD}(1, 3/2, 5/2) = 1/2 \text{ Hz} \; , \; T_0 = \frac{1}{1/2} = 2 \text{ s}$$

(e) $g(t) = 3\sin(20t) + 8\cos(4t)$

$$f_0 = \text{GCD}\left(\frac{10}{\pi}, \frac{2}{\pi}\right) = \frac{2}{\pi} \text{ Hz} \; , \; T_0 = \pi/2 \text{ s}$$

(f) $g(t) = 10\sin(20t) + 7\cos(10\pi t)$

$$f_0 = \text{GCD}\left(\frac{10}{\pi}, 5\right) = 0 \text{ Hz} \; , \; T_0 \to \infty \; , \; \therefore g(t) \text{ is not periodic}$$

(g) $g(t) = 3\cos(2000\pi t) - 8\sin(2500\pi t)$

$$f_0 = \text{GCD}(1000, 1250) = 250 \text{ Hz} \; , \; T_0 = 4 \text{ ms}$$

(h) $g_1(t)$ is periodic with fundamental period $T_{01} = 15\mu s$
$g_2(t)$ is periodic with fundamental period $T_{02} = 40\mu s$

$$T_0 = \text{LCM}(15\mu s, 40\mu s) = 120\mu s \ , \ f_0 = \frac{1}{120\mu s} = 8333\frac{1}{3} \text{ Hz}$$

28. Find a function of continuous time t for which the two successive transformations $t \to -t$ and $t \to t-1$ leave the function unchanged.

$\cos(2\pi nt)$, $\delta_{1/n}(t)$, n an integer. Any even periodic function with a period of one.

29. One period of a periodic signal $x(t)$ with period T_0 is graphed in Figure E-29. Assuming $x(t)$ has a period T_0, what is the value of $x(t)$ at time, $t = 220\text{ms}$?

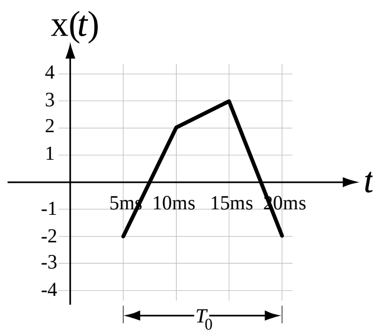


Figure E-29

Since the function is periodic with period 15 ms,
$x(220\text{ms}) = x(220\text{ms} - n \times 15\text{ms})$ where n is any integer. If we choose $n = 14$ we get

$$x(220\text{ms}) = x(220\text{ms} - 14 \times 15\text{ms}) = x(220\text{ms} - 210\text{ms}) = x(10\text{ms}) = 2 \ .$$

Signal Energy and Power of Signals

30. Find the signal energy of each of these signals.

(a) $x(t) = 2\,\text{rect}(t)$ $\qquad E_x = \int_{-\infty}^{\infty} |2\,\text{rect}(t)|^2 \, dt = 4 \int_{-1/2}^{1/2} dt = 4$

(b) $x(t) = A(u(t) - u(t-10))$

$$E_x = \int_{-\infty}^{\infty} \left| A\left(\mathrm{u}(t) - \mathrm{u}(t-10)\right) \right|^2 dt = A^2 \int_0^{10} dt = 10A^2$$

(c) $\mathrm{x}(t) = \mathrm{u}(t) - \mathrm{u}(10-t)$

$$E_x = \int_{-\infty}^{\infty} \left| \mathrm{u}(t) - \mathrm{u}(10-t) \right|^2 dt = \int_{-\infty}^{0} dt + \int_{10}^{\infty} dt \to \infty$$

(d) $\mathrm{x}(t) = \mathrm{rect}(t)\cos(2\pi t)$

$$E_x = \int_{-\infty}^{\infty} \left| \mathrm{rect}(t)\cos(2\pi t) \right|^2 dt = \int_{-1/2}^{1/2} \cos^2(2\pi t)\,dt = \frac{1}{2}\int_{-1/2}^{1/2} \left(1 + \cos(4\pi t)\right) dt$$

$$E_x = \frac{1}{2}\left[\int_{-1/2}^{1/2} dt + \underbrace{\int_{-1/2}^{1/2} \cos(4\pi t)\,dt}_{=0}\right] = \frac{1}{2}$$

(e) $\mathrm{x}(t) = \mathrm{rect}(t)\cos(4\pi t)$

$$E_x = \int_{-\infty}^{\infty} \left| \mathrm{rect}(t)\cos(4\pi t) \right|^2 dt = \int_{-1/2}^{1/2} \cos^2(4\pi t)\,dt = \frac{1}{2}\int_{-1/2}^{1/2} \left(1 + \cos(8\pi t)\right) dt$$

$$\mathrm{E}_x = \frac{1}{2}\left[\int_{-1/2}^{1/2} dt + \underbrace{\int_{-1/2}^{1/2} \cos(8\pi t)\,dt}_{=0}\right] = \frac{1}{2}$$

(f) $\mathrm{x}(t) = \mathrm{rect}(t)\sin(2\pi t)$

$$E_x = \int_{-\infty}^{\infty} \left| \mathrm{rect}(t)\sin(2\pi t) \right|^2 dt = \int_{-1/2}^{1/2} \sin^2(2\pi t)\,dt = \frac{1}{2}\int_{-1/2}^{1/2} \left(1 - \cos(4\pi t)\right) dt$$

$$E_x = \frac{1}{2}\left[\int_{-1/2}^{1/2} dt - \underbrace{\int_{-1/2}^{1/2} \cos(4\pi t)\,dt}_{=0}\right] = \frac{1}{2}$$

(g) $\mathrm{x}(t) = \begin{cases} |t| - 1 & , \quad |t| < 1 \\ 0 & , \quad \text{otherwise} \end{cases}$

$$E_x = \int_{-1}^{1} \left||t| - 1\right|^2 dt = 2\int_{0}^{1} (t-1)^2 \, dt = 2\int_{0}^{1} (t^2 - 2t + 1) dt$$

$$E_x = 2\left[\frac{t^3}{3} - t^2 + t\right]_0^1 = 2\left(\frac{1}{3} - 1 + 1\right) = \frac{2}{3}$$

31. A signal is described by $\mathrm{x}(t) = A\,\mathrm{rect}(t) + B\,\mathrm{rect}(t - 0.5)$. What is its signal energy?

$$E_{\mathrm{x}} = \int_{-\infty}^{\infty} \left|A\,\mathrm{rect}(t) + B\,\mathrm{rect}(t - 0.5)\right|^2 dt$$

Since these are purely real functions,

$$E_{\mathrm{x}} = \int_{-\infty}^{\infty} \left(A\,\mathrm{rect}(t) + B\,\mathrm{rect}(t - 0.5)\right)^2 dt$$

$$E_{\mathrm{x}} = \int_{-\infty}^{\infty} \left(A^2\,\mathrm{rect}^2(t) + B^2\,\mathrm{rect}^2(t - 0.5) + 2AB\,\mathrm{rect}(t)\mathrm{rect}(t - 0.5)\right) dt$$

$$\mathrm{E}_{\mathrm{x}} = A^2 \int_{-1/2}^{1/2} dt + B^2 \int_{0}^{1} dt + 2AB \int_{0}^{1/2} dt = A^2 + B^2 + AB$$

32. Find the average signal power of the periodic signal $\mathrm{x}(t)$ in Figure E-32.

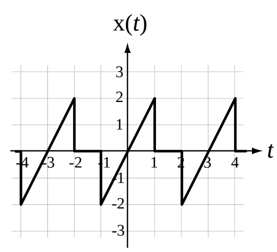


Figure E-32

$$P = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} |\mathrm{x}(t)|^2 \, dt = \frac{1}{3} \int_{-1}^{2} |\mathrm{x}(t)|^2 \, dt = \frac{1}{3} \int_{-1}^{1} |2t|^2 \, dt = \frac{4}{3} \int_{-1}^{1} t^2 \, dt = \frac{4}{3}\left[\frac{t^3}{3}\right]_{-1}^{1} = \frac{8}{9}$$

33. Find the average signal power of each of these signals.

(a) $\mathrm{x}(t) = A$ $\quad \mathrm{P_x} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |A|^2\,dt = \lim_{T\to\infty}\frac{A^2}{T}\int_{-T/2}^{T/2} dt = \lim_{T\to\infty}\frac{A^2}{T}T = A^2$

(b) $\mathrm{x}(t) = \mathrm{u}(t)$ $\quad P_\mathrm{x} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} |\mathrm{u}(t)|^2\,dt = \lim_{T\to\infty}\frac{1}{T}\int_{0}^{T/2} dt = \lim_{T\to\infty}\frac{1}{T}\frac{T}{2} = \frac{1}{2}$

(c) $\mathrm{x}(t) = A\cos(2\pi f_0 t + \theta)$

$$P_\mathrm{x} = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |A\cos(2\pi f_0 t + \theta)|^2\,dt = \frac{A^2}{T_0}\int_{-T_0/2}^{T_0/2}\cos^2(2\pi f_0 t + \theta)\,dt$$

$$P_\mathrm{x} = \frac{A^2}{2T_0}\int_{-T_0/2}^{T_0/2}\left(1+\cos(4\pi f_0 t + 2\theta)\right)dt = \frac{A^2}{2T_0}\left[t + \frac{\sin(4\pi f_0 t + 2\theta)}{4\pi f_0}\right]_{-T_0/2}^{T_0/2}$$

$$P_\mathrm{x} = \frac{A^2}{2T_0}\left[T_0 + \underbrace{\frac{\sin(4\pi f_0 T_0/2 + 2\theta)}{4\pi f_0} - \frac{\sin(-4\pi f_0 T_0/2 + 2\theta)}{4\pi f_0}}_{=0}\right] = \frac{A^2}{2}$$

The average signal power of a periodic power signal is unaffected if it is shifted in time. Therefore we could have found the average signal power of $A\cos(2\pi f_0 t)$ instead, which is somewhat easier algebraically.

(d) $\mathrm{x}(t)$ is periodic with fundamental period four and one fundamental period is described by $\mathrm{x}(t) = t(1-t)$, $1 < t < 5$.

$$P_x = \frac{1}{T_0}\int_{T_0}|\mathrm{x}(t)|^2\,dt = \frac{1}{4}\int_1^5 |t(t-1)|^2\,dt = \frac{1}{4}\int_1^5 (t^4 - 2t^3 + t^2)\,dt$$

$$P_x = \frac{1}{4}\left[\frac{t^5}{5} - \frac{t^4}{2} + \frac{t^3}{3}\right]_1^5 = \frac{1}{4}\left(\frac{3125}{5} - \frac{625}{2} + \frac{125}{3} - \frac{1}{5} + \frac{1}{2} - \frac{1}{3}\right) = 88.5333$$

(e) $\mathrm{x}(t)$ is periodic with fundamental period six. This signal is described over one fundamental period by

$$x(t) = \text{rect}\left(\frac{t-2}{3}\right) - 4\,\text{rect}\left(\frac{t-4}{2}\right)\ ,\ 0 < t < 6.$$

The signal can be described in the time period $0 < t < 6$ by

$$x(t) = \begin{cases} 0 & , \quad 0 < t < 1/2 \\ 1 & , \quad 1/2 < t < 3 \\ -3 & , \quad 3 < t < \frac{7}{2} \\ -4 & , \quad 7/2 < t < 5 \\ 0 & , \quad 5 < t < 6 \end{cases}$$

$$P_x = \frac{1}{6}\left(0^2 \times \frac{1}{2} + 1^2 \times \frac{5}{2} + (-3)^2 \times \frac{1}{2} + (-4)^2 \times \frac{3}{2} + 0^2 \times 1\right) = \frac{2.5 + 4.5 + 24}{6} = 5.167$$

Exercises Without Answers in Text

Signal Functions

34. Let the unit impulse function be represented by the limit,

$$\delta(x) = \lim_{a \to 0} (1/a)\text{rect}(x/a)\ ,\ a > 0\ .$$

The function $(1/a)\text{rect}(x/a)$ has an area of one regardless of the value of *a*.

(a) What is the area of the function $\delta(4x) = \lim_{a \to 0} (1/a)\text{rect}(4x/a)$?

This is a rectangle with the same height as $(1/a)\text{rect}(x/a)$ but 1/4 times the base width. Therefore its area is 1/4 times as great or 1/4.

(b) What is the area of the function $\delta(-6x) = \lim_{a \to 0} (1/a)\text{rect}(-6x/a)$?

This is a rectangle with the same height as $(1/a)\text{rect}(x/a)$ but 1/6 times the base width. (The fact that the factor is "-6" instead of "6" just means that the rectangle is reversed in time which does not change its shape or area.) Therefore its area is 1/6 times as great or 1/6.

(c) What is the area of the function $\delta(bx) = \lim_{a \to 0} (1/a)\text{rect}(bx/a)$ for *b* positive and for *b* negative ?

It is simply $1/|b|$.

35. Using a change of variable and the definition of the unit impulse, prove that

$$\delta\left(a\left(t-t_0\right)\right)=\left(1/|a|\right)\delta\left(t-t_0\right) \quad .$$

$$\delta(x)=0 \quad , \quad x\neq 0 \quad , \quad \int_{-\infty}^{\infty}\delta(x)\,dx=1$$

$$\delta\left[a\left(t-t_0\right)\right]=0 \quad , \quad \text{where } a\left(t-t_0\right)\neq 0 \quad \text{or} \quad t\neq t_0$$

$$\text{Strength}=\int_{-\infty}^{\infty}\delta\left[a\left(t-t_0\right)\right]dt$$

Let

$$a\left(t-t_0\right)=\lambda \quad \text{and} \quad \therefore adt=d\lambda$$

Then, for $a > 0$,

$$\text{Strength}=\int_{-\infty}^{\infty}\delta(\lambda)\frac{d\lambda}{a}=\frac{1}{a}\int_{-\infty}^{\infty}\delta(\lambda)\,d\lambda=\frac{1}{a}=\frac{1}{|a|}$$

and for $a < 0$,

$$\text{Strength}=\int_{\infty}^{-\infty}\delta(\lambda)\frac{d\lambda}{a}=\frac{1}{a}\int_{\infty}^{-\infty}\delta(\lambda)\,d\lambda=-\frac{1}{a}\int_{-\infty}^{\infty}\delta(\lambda)\,d\lambda=-\frac{1}{a}=\frac{1}{|a|}$$

Therefore for $a > 0$ and $a < 0$,

$$\text{Strength}=\frac{1}{|a|} \quad \text{and} \quad \delta\left[a\left(t-t_0\right)\right]=\frac{1}{|a|}\delta\left(t-t_0\right) \quad .$$

36. Using the results of Exercise 35,

(a) Show that $\delta_1(ax)=\left(1/|a|\right)\sum_{n=-\infty}^{\infty}\delta\left(x-n/a\right)$

From the definition of the periodic impulse $\delta_1(ax)=\sum_{-\infty}^{\infty}\delta(ax-n)$.

Then, using the property from Exercise 35

$$\delta_1(ax)=\sum_{-\infty}^{\infty}\delta\left[a\left(x-n/a\right)\right]=\frac{1}{|a|}\sum_{-\infty}^{\infty}\delta\left(x-n/a\right).$$

(b) Show that the average value of $\delta_1(ax)$ is one, independent of the value of a

The period is $1/a$. Therefore

$$\left\langle\delta_1(ax)\right\rangle=\frac{1}{1/a}\int_{t_0}^{t_0+1/a}\delta_1(ax)\,dx=a\int_{-1/2a}^{1/2a}\delta_1(ax)\,dx=a\int_{-1/2a}^{1/2a}\delta(ax)\,dx$$

Letting $\lambda = ax$

$$\langle \delta_1(ax) \rangle = \int_{-1/2}^{1/2} \delta(\lambda) d\lambda = 1$$

(c) Show that even though $\delta(at) = (1/|a|)\delta(t)$, $\delta_1(ax) \neq (1/|a|)\delta_1(x)$

$$\delta_1(ax) = \sum_{n=-\infty}^{\infty} \delta(ax - n) \neq (1/|a|) \sum_{n=-\infty}^{\infty} \delta(x - n) = (1/|a|)\delta_1(x)$$

$$\delta_1(ax) \neq (1/|a|)\delta_1(x) \text{ QED}$$

Scaling and Shifting

37. A signal is zero for all time before $t = -2$, rises linearly from 0 to 3 between $t = -2$ and $t = 4$ and is zero for all time after that. This signal can be expressed in the form $\mathrm{x}(t) = A\,\mathrm{rect}\left(\frac{t - t_{01}}{w_1}\right)\mathrm{tri}\left(\frac{t - t_{02}}{w_2}\right)$. Find the numerical values of the constants.

$$\mathrm{x}(t) = 3\,\mathrm{rect}\left(\frac{t-1}{6}\right)\mathrm{tri}\left(\frac{t-4}{6}\right)$$

38. Let $\mathrm{x}(t) = -3e^{-t/4}\,\mathrm{u}(t-1)$ and let $\mathrm{y}(t) = -4\,\mathrm{x}(5t)$.

(a) What is the smallest value of t for which $\mathrm{y}(t)$ is not zero?

$$\mathrm{y}(t) = -4\,\mathrm{x}(5t) = 12e^{-5t/4}\,\mathrm{u}(5t-1)$$

Smallest t for which $\mathrm{y}(t) \neq 0$ occurs where $5t - 1 = 0$ or $t = 1/5$.

(b) What is the maximum value of $\mathrm{y}(t)$ over all time?

$$\max(\mathrm{y}(t)) = 9.3456$$

(c) What is the minimum value of $\mathrm{y}(t)$ over all time?

$$\min(\mathrm{y}(t)) = 0$$

(d) What is the value of $\mathrm{y}(1)$?

$$\mathrm{y}(t)=12e^{-5t/4}\,\mathrm{u}(5t-1)\Rightarrow \mathrm{y}(1)=12e^{-5/4}\,\mathrm{u}(4)=12e^{-5/4}=3.4381$$

(e) What is the largest value of *t* for which $\mathrm{y}(t)>2$?

$$\mathrm{y}(t)=2=12e^{-5t/4}\Rightarrow e^{-5t/4}=1/6\Rightarrow -5t/4=\ln(1/6)=-1.7918$$

$$t=4(1.7918)/5=1.4334$$

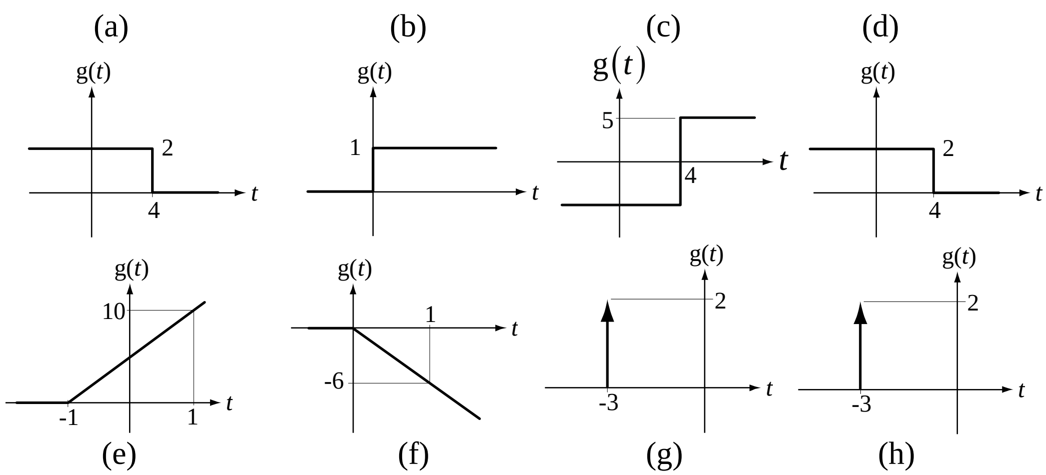
39. Graph these singularity and related functions.

(a) $\mathrm{g}(t)=2\,\mathrm{u}(4-t)$ (b) $\mathrm{g}(t)=\mathrm{u}(2t)$

(c) $\mathrm{g}(t)=5\,\mathrm{sgn}(t-4)$ (d) $\mathrm{g}(t)=1+\mathrm{sgn}(4-t)$

(e) $\mathrm{g}(t)=5\,\mathrm{ramp}(t+1)$ (f) $\mathrm{g}(t)=-3\,\mathrm{ramp}(2t)$

(g) $\mathrm{g}(t)=2\delta(t+3)$ (h) $\mathrm{g}(t)=6\delta(3t+9)$

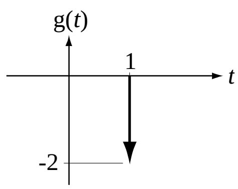


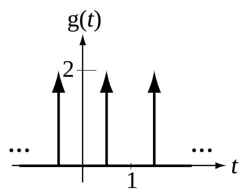
(i) $\mathrm{g}(t)=-4\delta(2(t-1))$ (j) $\mathrm{g}(t)=2\delta_1(t-1/2)$

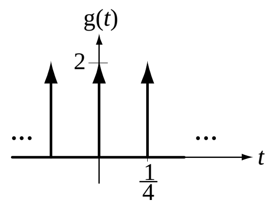
(k) $\mathrm{g}(t)=8\delta_1(4t)$ (l) $\mathrm{g}(t)=-6\delta_2(t+1)$

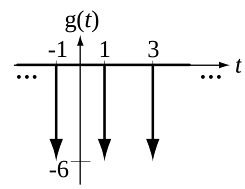
(m) $\mathrm{g}(t)=2\,\mathrm{rect}(t/3)$ (n) $\mathrm{g}(t)=4\,\mathrm{rect}((t+1)/2)$

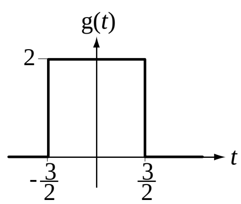
(o) $g(t) = -3\text{rect}(t-2)$ (p) $g(t) = 0.1\text{rect}((t-3)/4)$

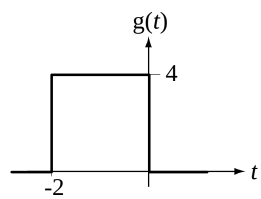


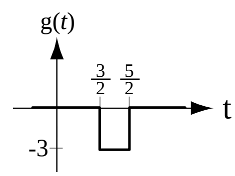


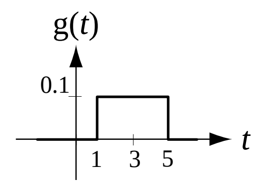












40. Graph these functions.

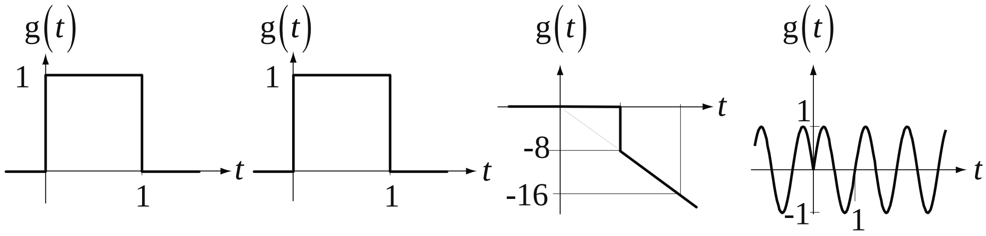
(a) $g(t) = u(t) - u(t-1)$ (b) $g(t) = \text{rect}(t-1/2)$

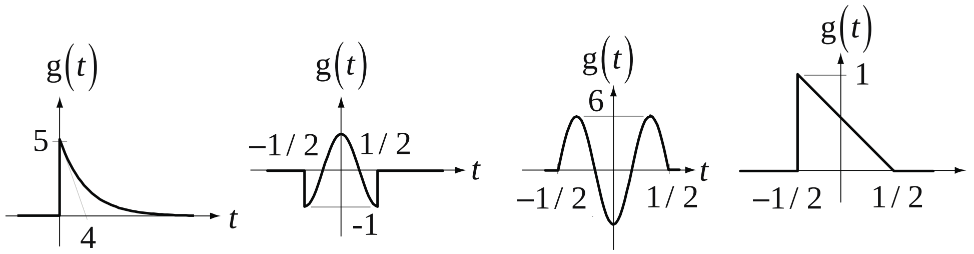
(c) $g(t) = -4\,\text{ramp}(t)u(t-2)$ (d) $g(t) = \text{sgn}(t)\sin(2\pi t)$

(e) $g(t) = 5e^{-t/4}\,u(t)$ (f) $g(t) = \text{rect}(t)\cos(2\pi t)$

(g) $g(t) = -6\,\text{rect}(t)\cos(3\pi t)$ (h) $g(t) = u(t+1/2)\text{ramp}(1/2-t)$

(a) (b) (c) (d)





(e) (f) (g) (h)

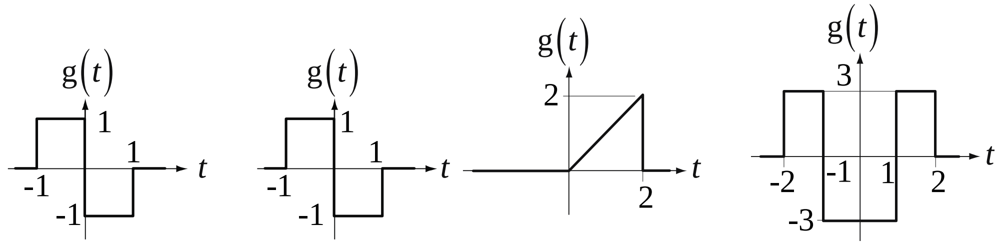
(i) $\mathrm{g}(t) = \mathrm{rect}(t+1/2) - \mathrm{rect}(t-1/2)$

(j) $\mathrm{g}(t) = \left[\int_{-\infty}^{t} \delta(\lambda+1) - 2\delta(\lambda) + \delta(\lambda-1)\right] d\lambda$

(k) $\mathrm{g}(t) = 2\,\mathrm{ramp}(t)\,\mathrm{rect}((t-1)/2)$

(l) $\mathrm{g}(t) = 3\,\mathrm{rect}(t/4) - 6\,\mathrm{rect}(t/2)$

(i) (j) (k) (l)



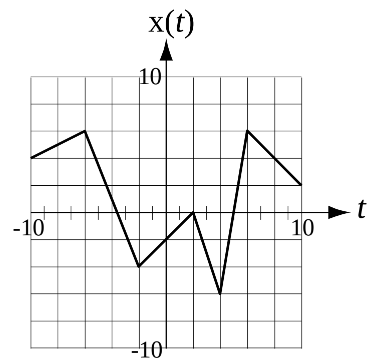
41. A continuous-time signal $\mathrm{x}(t)$ is defined by the graph below. Let $\mathrm{y}(t) = -4\,\mathrm{x}(t+3)$ and let $\mathrm{z}(t) = 8\,\mathrm{x}(t/4)$ Find the numerical values.

(a) $\mathrm{y}(3) = -4\,\mathrm{x}(3+3) = -4\,\mathrm{x}(6) = -4 \times 6 = -24$

(b) $\mathrm{z}(-4) = 8\,\mathrm{x}(-4/4) = 8\,\mathrm{x}(-1) = 8 \times (-3) = -24$

(c) $\left.\frac{d}{dt}(\mathrm{z}(t))\right|_{t=10} = \left.\frac{d}{dt}(8\,\mathrm{x}(t/4))\right|_{t=10} = 8\left.\frac{d}{dt}(\mathrm{x}(t/4))\right|_{t=10}$

$$\frac{d}{dt}\left(\mathrm{z}(t)\right)\bigg|_{t=10} = \frac{8\dfrac{d}{dt}\left(\mathrm{x}(t)\right)\bigg|_{t=10/4}}{4} = \frac{8\times(-3)}{4} = -6$$



42. Find the numerical values of

(a) $\mathrm{ramp}(-3(-2))\times\mathrm{rect}(-2/10) = \mathrm{ramp}(6)\mathrm{rect}(-1/5) = 6\times 1 = 6$

(b) $\int_{-\infty}^{\infty} 3\delta(t-4)\cos(\pi t/10)\,dt = 3\cos(4\pi/10) = 0.9271$

(c) $\left[\frac{d}{dt}\left(2\,\mathrm{sgn}(t/5)\mathrm{ramp}(t-8)\right)\right]_{t=13}$

$$\frac{d}{dt}\left(2\,\mathrm{sgn}(t/5)\mathrm{ramp}(t-8)\right) = 2\,\mathrm{sgn}(t/5)\mathrm{u}(t-8) + 2\,\mathrm{ramp}(t-8)\times 2\delta(t)$$

$$\left[\frac{d}{dt}\left(2\,\mathrm{sgn}(t/5)\mathrm{ramp}(t-8)\right)\right]_{t=13} = 2\underbrace{\mathrm{sgn}(13/5)}_{1}\underbrace{\mathrm{u}(5)}_{1} + 2\underbrace{\mathrm{ramp}(5)}_{5}\times 2\underbrace{\delta(13)}_{0} = 2$$

43. Let a function be defined by $\mathrm{g}(t) = \mathrm{tri}(t)$. Below are four other functions based on this function. All of them are zero for large negative values of *t*.

$$\mathrm{g}_1(t) = -5\,\mathrm{g}\left(\frac{2-t}{6}\right) \qquad \mathrm{g}_2(t) = 7\,\mathrm{g}(3t) - 4\,\mathrm{g}(t-4)$$

$$\mathrm{g}_3(t) = \mathrm{g}(t+2) - 4\,\mathrm{g}\left(\frac{t+4}{3}\right) \qquad \mathrm{g}_4(t) = -5\,\mathrm{g}(t)\,\mathrm{g}\left(t-\frac{1}{2}\right)$$

(a) Which of these transformed functions is the first to become non-zero (becomes non-zero at the earliest time)? $\mathrm{g}_3(t)$

(b) Which of these transformed functions is the last to go back to zero and stay there? $g_1(t)$

(c) Which of these transformed functions has a maximum value that is greater than all the other maximum values of all the other transformed functions? ("Greater than" in the strict mathematical sense of "more positive than". For example, $2 > -5$.) $g_2(t)$

(d) Which of these transformed functions has a minimum value that is less than all the other minimum values of all the other transformed functions? $g_1(t)$

44. (a) Write a functional description of the time-domain energy signal in Figure E-44 as the product of two functions of *t*.

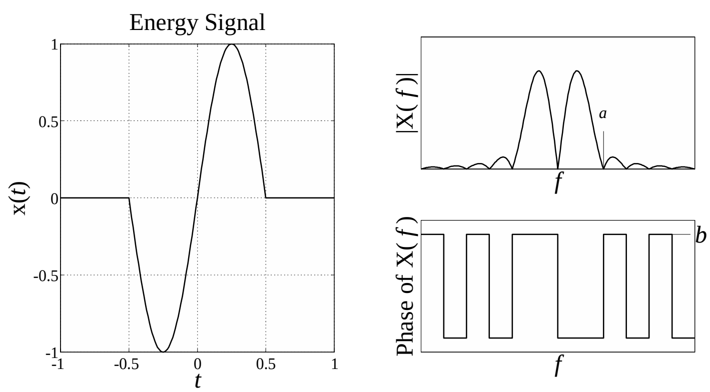


Figure E-44

$$x(t) = \text{rect}(t)\sin(2\pi t)$$

(b) Write a functional description of the frequency-domain signal as the sum of two functions of *f*.

$$X(f) = \text{sinc}(f) * (j/2)[\delta(f+1) - \delta(f-1)]$$
$$X(f) = (j/2)\text{sinc}(f+1) - (j/2)\text{sinc}(f-1)$$

(c) Find the numerical values of *a* and *b*.

a is at the first positive-frequency null of $\text{sinc}(f-1)$ which occurs at $f = 2$. Therefore *a* = 2.

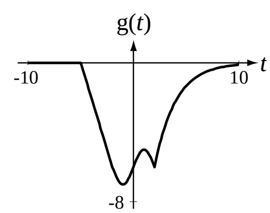
The phase values are all either $\pi / 2$ or $-\pi / 2$ because of the $(j / 2)$ factor. Therefore $b = \pi / 2$.

45. A function $g(t)$ has the following description. It is zero for $t < -5$. It has a slope of –2 in the range $-5 < t < -2$. It has the shape of a sine wave of unit amplitude and with a frequency of $1/4$ Hz plus a constant in the range $-2 < t < 2$. For $t > 2$ it decays exponentially toward zero with a time constant of 2 seconds. It is continuous everywhere.

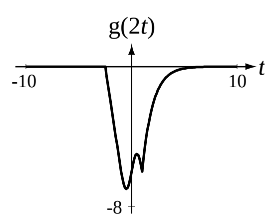
(a) Write an exact mathematical description of this function.

$$g(t) = \begin{cases} 0 & , \; t < -5 \\ -10 - 2t & , \; -5 < t < -2 \\ \sin(\pi t / 2) & , \; -2 < t < 2 \\ -6e^{-t/2} & , \; t > 2 \end{cases}$$

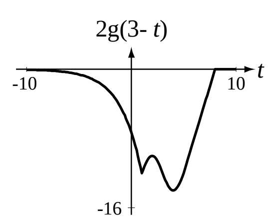
(b) Graph $g(t)$ in the range $-10 < t < 10$.



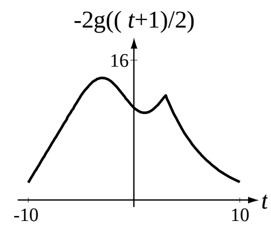
(c) Graph $g(2t)$ in the range $-10 < t < 10$.



(d) Graph $2g(3-t)$ in the range $-10 < t < 10$.



(e) Graph $-2\,\mathrm{g}\left(\left(t+1\right)/2\right)$ in the range $-10 < t < 10$.



46. A signal occurring in a television set is illustrated in Figure E46. Write a mathematical description of it.

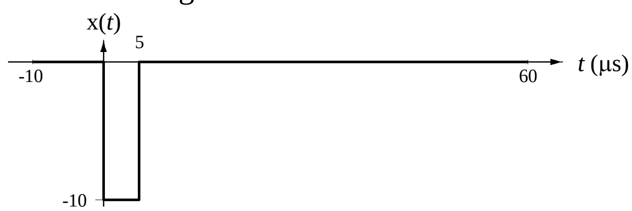


Figure E46 Signal occurring in a television set

$$\mathrm{x}(t) = -10\,\mathrm{rect}\left(\frac{t-2.5\times10^{-6}}{5\times10^{-6}}\right)$$

47. The signal illustrated in Figure E47 is part of a binary-phase-shift-keyed (BPSK) binary data transmission. Write a mathematical description of it.

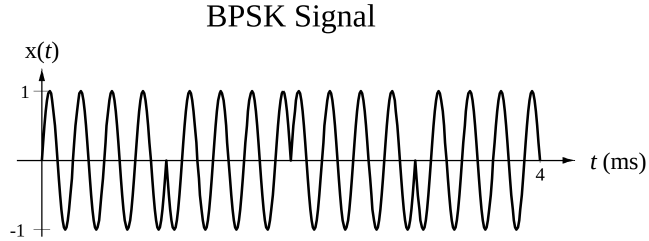


Figure E47 BPSK signal

$$\mathrm{x}(t) = \left[\begin{array}{l} \sin(8000\pi t)\,\mathrm{rect}\left(\dfrac{t-0.5\times10^{-3}}{10^{-3}}\right) - \sin(8000\pi t)\,\mathrm{rect}\left(\dfrac{t-1.5\times10^{-3}}{10^{-3}}\right) \\ +\sin(8000\pi t)\,\mathrm{rect}\left(\dfrac{t-2.5\times10^{-3}}{10^{-3}}\right) - \sin(8000\pi t)\,\mathrm{rect}\left(\dfrac{t-3.5\times10^{-3}}{10^{-3}}\right) \end{array}\right]$$

48. The signal illustrated in Figure E48 is the response of an RC lowpass filter to a sudden change in excitation. Write a mathematical description of it.

On a decaying exponential, a tangent line at any point intersects the final value one time constant later. Theconstant value before the decaying exponential is -4 V and the slope of the tangent line at 4 ns is -2.67V/4 ns or -2/3 V/ns.

RC Filter Signal

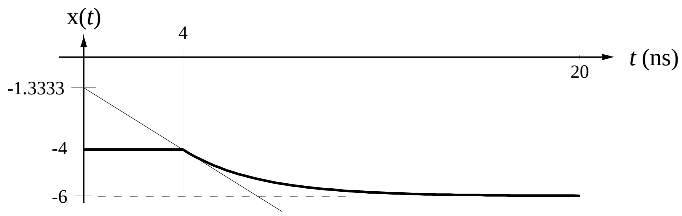


Figure E48 Transient response of an RC filter

$$\mathrm{x}(t) = -4 - 2\left(1 - e^{-(t-4)/3}\right)\mathrm{u}(t-4) \text{ (times in ns)}$$

49. Describe the signal in Figure E49 as a ramp function minus a summation of step functions.

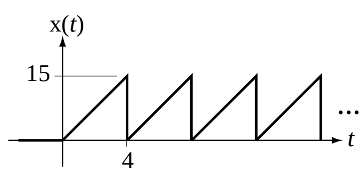


Figure E49

$$\mathrm{x}(t) = 3.75\,\mathrm{ramp}(t) - 15\sum_{n=1}^{\infty}\mathrm{u}(t-4n)$$

50. Mathematically describe the signal in Figure E-50 .

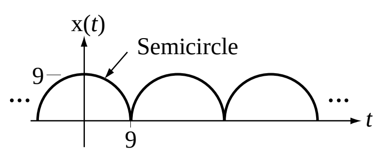


Figure E-50

The semicircle centered at $t = 0$ is the top half of a circle defined by

$$\mathrm{x}^2(t) + t^2 = 81$$

Therefore

$$x(t) = \sqrt{81 - t^2} \quad , \quad -9 < t < 9 \ .$$

This one period of this periodic function. The other periods are just shifted versions.

$$x(t) = \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t-18n}{18}\right)\sqrt{81-(t-18n)^2}$$

(The rectangle function avoids the problem of imaginary values for the square roots of negative numbers.)

51. Let two signals be defined by

$$x_1(t) = \begin{cases} 1 \ , \ \cos(2\pi t) \ge 0 \\ 0 \ , \ \cos(2\pi t) < 0 \end{cases} \quad \text{and} \quad x_2(t) = \sin(2\pi t/10) \ ,$$

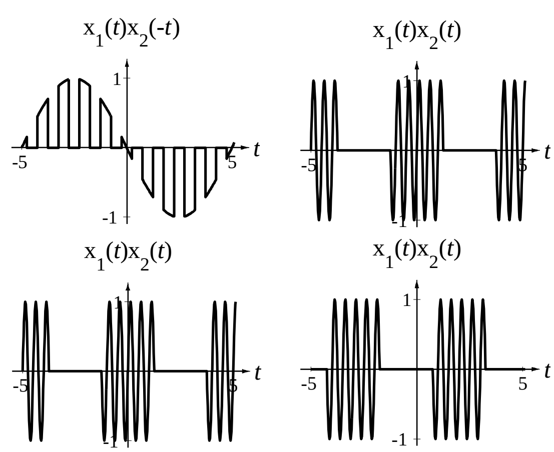
Graph these products over the time range, $-5 < t < 5$.

(a) $x_1(2t)x_2(-t)$

(b) $x_1(t/5)x_2(20t)$

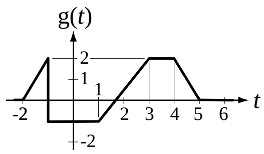
(c) $x_1(t/5)x_2(20(t+1))$

(d) $x_1((t-2)/5)x_2(20t)$



52. Given the graphical definitions of functions in Figure E-52, graph the indicated transformations.

(a)

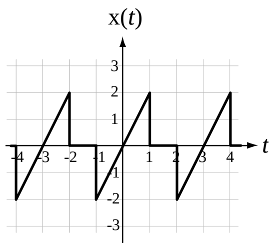


$g(t) \to g(2t)$

$g(t) \to -3g(-t)$

$g(t) = 0$, $t > 6$ or $t < -2$

(b)



$t \to t + 4$

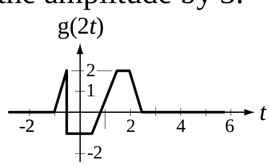
$g(t) \to -2g((t-1)/2)$

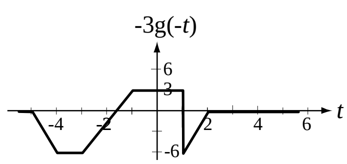
$g(t)$ is periodic with fundamental period, 4

Figure E-52

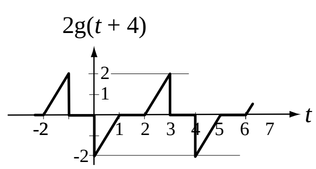
(a)

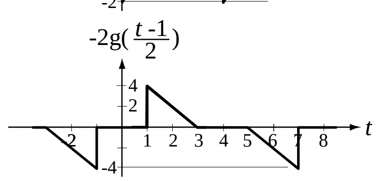
The transformation $g(t) \to g(2t)$ simply compresses the time scale by a factor of 2. The transformation $g(t) \to -3g(-t)$ time inverts the signal, amplitude inverts the signal and then multiplies the amplitude by 3.





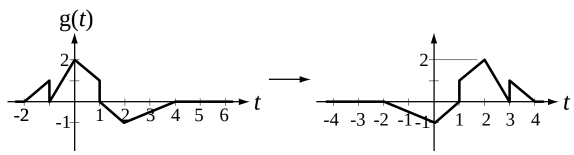
(b)





53. For each pair of functions graphed in Figure E-53 determine what transformation has been done and write a correct functional expression for the transformed function.

(a)



(b)

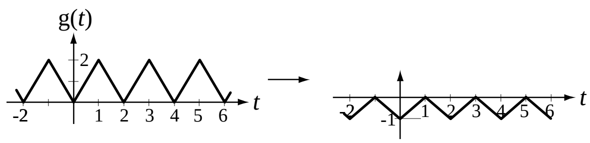


Figure E-53

In (b), assuming $\mathrm{g}(t)$ is periodic with fundamental period 2 find two different transformations which yield the same result

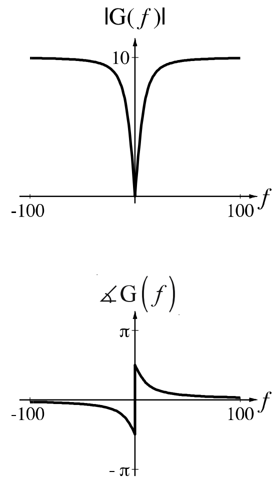
(a)
It should be visually obvious that the transformed signal has been time inverted and time shifted. By identifying a few corresponding points on both curves we see that after the time inversion the shift is to the right by 2. This corresponds to two successive transformations $t \to -t$ followed by $t \to t-2$. The overall effect of the two successive transformations is then $t \to -(t-2) = 2-t$. Therefore the transformation is

$$\mathrm{g}(t) \to \mathrm{g}(2-t)$$

(b) $\mathrm{g}(t) \to -(1/2)\mathrm{g}(t+1)$ or $\mathrm{g}(t) \to -(1/2)\mathrm{g}(t-1)$

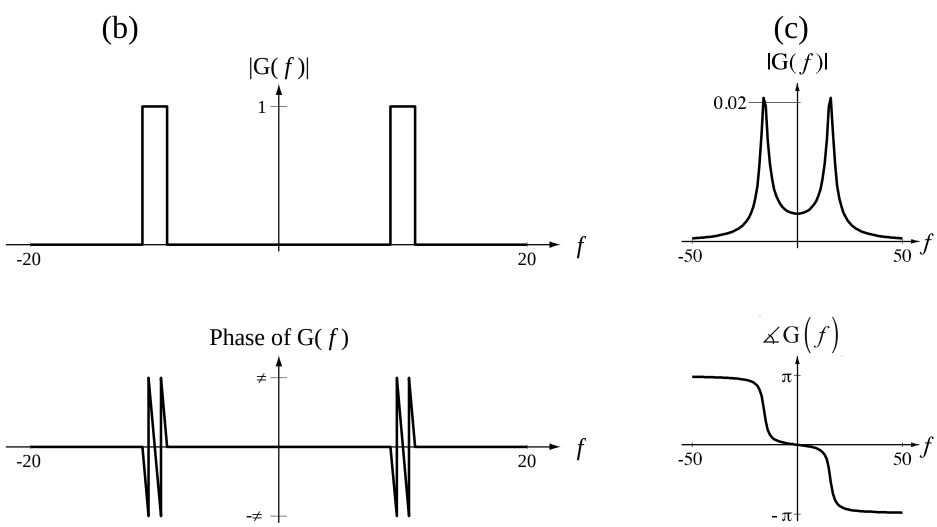
54. Graph the magnitude and phase of each function versus *f*.

(a) $\mathrm{G}(f) = \dfrac{jf}{1 + jf/10}$



(b) $\mathrm{G}(f)=\left[\mathrm{rect}\left(\frac{f-1000}{100}\right)+\mathrm{rect}\left(\frac{f+1000}{100}\right)\right]e^{-j\pi f/500}$

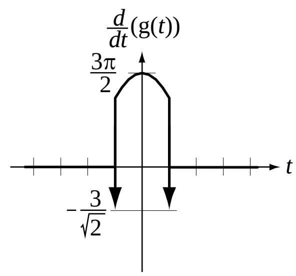
(c) $\mathrm{G}(f)=\frac{1}{250-f^2+j3f}$



Generalized Derivative

55. Graph the generalized derivative of $\mathrm{g}(t)=3\sin(\pi t/2)\mathrm{rect}(t)$.

Except at the discontinuities at $t=\pm 1/2$, the derivative is either zero, for $|t|>1/2$, or it is the derivative of $3\sin(\pi t/2)$, $(3\pi/2)\cos(\pi t/2)$, for $|t|<1/2$. At the discontinuities the generalized derivative is an impulse whose strength is the difference between the limit approached from above and the limit approached from below. In both cases that strength is $-3/\sqrt{2}$.



Alternate solution:

$$g(t) = 3\sin(\pi t/2)\left[u(t+1/2) - u(t-1/2)\right]$$

$$\frac{d}{dt}(g(t)) = \begin{Bmatrix} 3\sin(\pi t/2)\left[\delta(t+1/2) - \delta(t-1/2)\right] \\ +(3\pi/2)\cos(\pi t/2)\left[u(t+1/2) - u(t-1/2)\right] \end{Bmatrix}$$

$$\frac{d}{dt}(g(t)) = \begin{Bmatrix} \left[3\sin(-\pi/4)\delta(t+1/2) - 3\sin(\pi/4)\delta(t-1/2)\right] \\ +(3\pi/2)\cos(\pi t/2)\text{rect}(t) \end{Bmatrix}$$

$$\frac{d}{dt}(g(t)) = \begin{Bmatrix} -3\sqrt{2}/2\left[\delta(t+1/2) + \delta(t-1/2)\right] \\ +(3\pi/2)\cos(\pi t/2)\text{rect}(t) \end{Bmatrix}$$

56. Find the generalized derivative of the function described by $x(t) = \begin{cases} 4 \ , & t < 3 \\ 7t \ , & t \geq 3 \end{cases}$.

$$x'(t) = 17\delta(t-3) + \begin{Bmatrix} 0 \ , & t < 3 \\ 7 \ , & t > 3 \end{Bmatrix} = 17\delta(t-3) + 7\,u(t-3)$$

Derivatives and Integrals of Functions

57. What is the numerical value of each of the following integrals?

(a) $\int_{-\infty}^{\infty} \delta(t)\cos(48\pi t)\,dt = \cos(0) = 1$,

(b) $\int_{-\infty}^{\infty} \delta(t-5)\cos(\pi t)\,dt = \cos(5\pi) = -1$

(c) $\int_{0}^{20} \delta(t-8)\operatorname{rect}(t/16)\,dt = \operatorname{rect}(8/16) = 1/2$

(d) $\int_{-8}^{22} 8e^{4t}\delta(t-2)\,dt = 8e^{8} = 23{,}848$

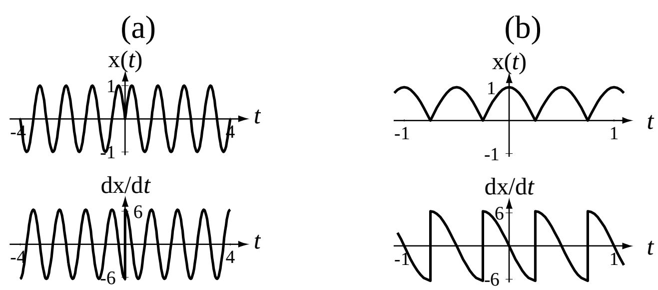
(e) $\int_{11}^{82} 3\sin(200t)\delta(t-7)\,dt = 0$

(f) $\int_{-2}^{10} 39t^{2}\delta_{4}(t-1)\,dt = 39\left(1^{2}+5^{2}+9^{2}\right) = 4173$

58. Graph the time derivatives of these functions.

(a) $\mathrm{g}(t) = \sin(2\pi t)\operatorname{sgn}(t)$ $\qquad$ $\mathrm{g}'(t) = 2\pi \begin{cases} -\cos(2\pi t) & , \ t<0 \\ \cos(2\pi t) & , \ t \ge 0 \end{cases}$

(b) $\mathrm{g}(t) = \left|\cos(2\pi t)\right|$ $\qquad$ $\mathrm{g}'(t) = 2\pi \begin{cases} \sin(2\pi t) & , \ \cos(2\pi t) < 0 \\ -\sin(2\pi t) & , \ \cos(2\pi t) > 0 \end{cases}$

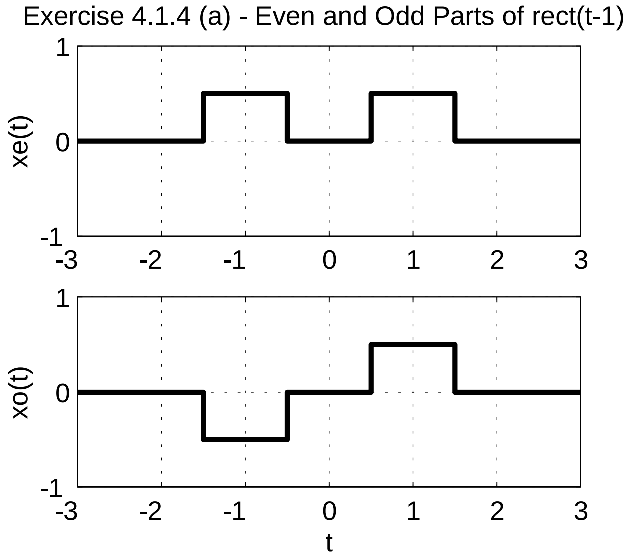


Even and Odd Functions

59. Graph the even and odd parts of these signals.

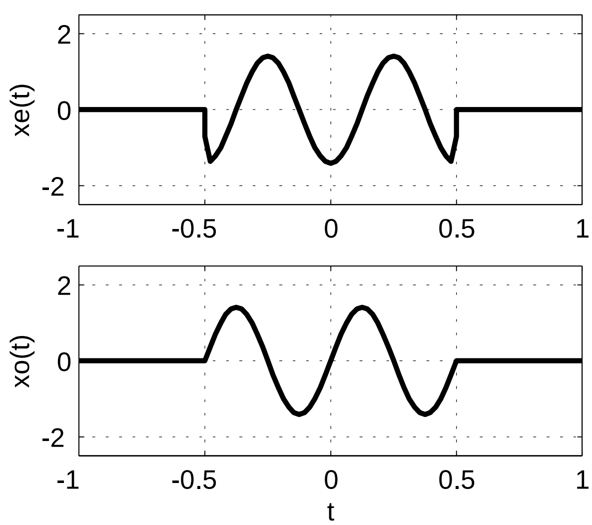
(a) $\mathrm{x}(t) = \operatorname{rect}(t-1)$

$$\mathrm{x}_e(t) = \frac{\operatorname{rect}(t-1)+\operatorname{rect}(t+1)}{2} \quad , \quad \mathrm{x}_o(t) = \frac{\operatorname{rect}(t-1)-\operatorname{rect}(t+1)}{2}$$



(b) $\mathrm{x}(t) = 2\sin(4\pi t - \pi/4)\mathrm{rect}(t)$

$\mathrm{x}_e(t) = -2\sin(\pi/4)\cos(4\pi t)\mathrm{rect}(t)$, $\mathrm{x}_o(t) = 2\cos(\pi/4)\sin(4\pi t)\mathrm{rect}(t)$



60. Is there a function that is both even and odd simultaneously? Discuss.

The only function that can be both odd and even simultaneously is the trivial signal, $\mathrm{x}(t) = 0$. Applying the definitions of even and odd functions,

$$x_e(t) = \frac{0+0}{2} = 0 = x(t) \text{ and } x_o(t) = \frac{0-0}{2} = 0 = x(t)$$

proving that the signal is equal to both its even and odd parts and is therefore both even and odd.

61. Find and graph the even and odd parts of the function $x(t)$ in Figure E-61

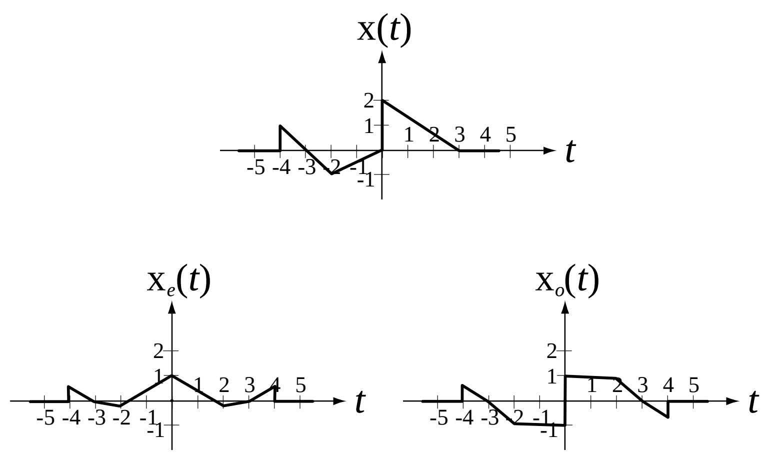


Figure E-61

Periodic Functions

62. For each of the following signals decide whether it is periodic and, if it is, find the fundamental period.

(a) $g(t) = 28\sin(400\pi t)$ Periodic. Fundamental frequency = 200 Hz, Period = 5 ms.

(b) $g(t) = 14 + 40\cos(60\pi t)$ Periodic. Fundamental frequency = 30 Hz Period = 33.33...ms.

(c) $g(t) = 5t - 2\cos(5000\pi t)$ Not periodic.

(d) $g(t) = 28\sin(400\pi t) + 12\cos(500\pi t)$ Periodic. Two sinusoidal components with periods of 5 ms and 4 ms. Least common multiple is 20 ms. Period of the overall signal is 20 ms.

(e) $g(t) = 10\sin(5t) - 4\cos(7t)$ Periodic. The Periods of the two sinusoids are $2\pi/5$ s and $2\pi/7$ s. Least common multiple is 2π. Period of the overall signal is 2π s.

(f) $g(t) = 4\sin(3t) + 3\sin(\sqrt{3}t)$ Not periodic because least common multiple is infinite.

63. Is a constant a periodic signal? Explain why it is or is not periodic and, if it is periodic what is its fundamental period?

A constant is periodic because it repeats for all time. The fundamental period of a periodic signal is defined as the minimum positive time in which it repeats. A constant repeats in any time, no matter how small. Therefore since there is no minimum positive time in which it repeats it does not have a fundamental period.

Signal Energy and Power of Signals

64. Find the signal energy of each of these signals.

(a) $2\,\text{rect}(-t)$, $\text{E} = \int_{-\infty}^{\infty} [2\,\text{rect}(-t)]^2\,dt = 4\int_{-1/2}^{1/2} dt = 4$

(b) $\text{rect}(8t)$, $\text{E} = \int_{-\infty}^{\infty} [\text{rect}(8t)]^2\,dt = \int_{-1/16}^{1/16} dt = \frac{1}{8}$

(c) $3\text{rect}\left(\frac{t}{4}\right)$, $E = \int_{-\infty}^{\infty} \left[3\text{rect}\left(\frac{t}{4}\right)\right]^2 dt = 9\int_{-2}^{2} dt = 36$

(d) $2\sin(200\pi t)$

$$E = \int_{-\infty}^{\infty} [2\sin(200\pi t)]^2\,dt = 4\int_{-\infty}^{\infty} \sin^2(200\pi t)\,dt = 4\int_{-\infty}^{\infty} \left[\frac{1}{2} - \frac{1}{2}\sin(400\pi t)\right] dt$$

$$E = 2\left[t + \frac{\cos(400\pi t)}{400\pi}\right]_{-\infty}^{\infty} \to \infty$$

(e) $\delta(t)$ (Hint: First find the signal energy of a signal which approaches an impulse some limit, then take the limit.)

$$\delta(t) = \lim_{a\to 0}(1/a)\text{rect}(t/a)$$

$$E = \int_{-\infty}^{\infty} \left[\lim_{a\to 0}\frac{1}{a}\text{rect}\left(\frac{t}{a}\right)\right]^2 dt = \lim_{a\to 0}\frac{1}{a^2}\int_{-a/2}^{a/2} \text{rect}\left(\frac{t}{a}\right) dt = \lim_{a\to 0}\frac{a}{a^2} \to \infty$$

(f) $\text{x}(t) = \frac{d}{dt}(\text{rect}(t))$

$$\frac{d}{dt}(\text{rect}(t)) = \delta(t+1/2) - \delta(t-1/2)$$

$$E_{\text{x}} = \int_{-\infty}^{\infty} [\delta(t+1/2) - \delta(t-1/2)]^2\,dt \to \infty$$

(g) $$\mathrm{x}(t)=\int_{-\infty}^{t}\mathrm{rect}(\lambda)\,d\lambda=\mathrm{ramp}(t+1/2)-\mathrm{ramp}(t-1/2)$$

$$E_{\mathrm{x}}=\underbrace{\int_{-1/2}^{1/2}(t+1/2)^2\,dt}_{\text{finite}}+\underbrace{\int_{1/2}^{\infty}dt}_{\text{infinite}}\to\infty$$

(h) $$\mathrm{x}(t)=e^{(-1-j8\pi)t}\,\mathrm{u}(t)$$

$$E_{\mathrm{x}}=\int_{-\infty}^{\infty}|\mathrm{x}(t)|^2\,dt=\int_{-\infty}^{\infty}\left|e^{(-1-j8\pi)t}\,\mathrm{u}(t)\right|^2dt=\int_{0}^{\infty}e^{(-1-j8\pi)t}e^{(-1+j8\pi)t}\,dt$$

$$E_{\mathrm{x}}=\int_{0}^{\infty}e^{-2t}\,dt=\left[\frac{e^{-2t}}{-2}\right]_0^{\infty}=\frac{1}{2}$$

(i) $$\mathrm{x}(t)=2\,\mathrm{rect}(t/4)-3\mathrm{rect}\left(\frac{t-1}{4}\right)$$

$$E_{\mathrm{x}}=\int_{-\infty}^{\infty}|\mathrm{x}(t)|^2\,dt=\int_{-\infty}^{\infty}\left|2\,\mathrm{rect}(t/4)-3\mathrm{rect}\left(\frac{t-1}{4}\right)\right|^2dt$$

The function can be expressed as

$$\mathrm{x}(t)=\begin{cases}2 & , \quad -2<t<-1\\ -1 & , \quad -1<t<2\\ -3 & , \quad 2<t<3\end{cases}$$

$$E_{\mathrm{x}}=4\int_{-2}^{-1}dt+\int_{-1}^{2}dt+9\int_{2}^{3}dt=4+3+9=16$$

Alternate Solution:

Since the function is real, the square of its magnitude equals its square.

$$E_{\mathrm{x}}=\int_{-\infty}^{\infty}\left[2\,\mathrm{rect}(t/4)-3\mathrm{rect}\left(\frac{t-1}{4}\right)\right]^2dt$$

$$E_{\mathrm{x}}=\int_{-\infty}^{\infty}\left[2\,\mathrm{rect}(t/4)\right]^2dt-\int_{-\infty}^{\infty}\left[12\,\mathrm{rect}(t/4)\mathrm{rect}\left(\frac{t-1}{4}\right)\right]dt+\int_{-\infty}^{\infty}\left[-3\mathrm{rect}\left(\frac{t-1}{4}\right)\right]^2dt$$

$$E_x = 4\int_{-\infty}^{\infty} \mathrm{rect}(t/4)\,dt - 12\int_{-\infty}^{\infty} \mathrm{rect}(t/4)\,\mathrm{rect}\left(\frac{t-1}{4}\right)dt + 9\int_{-\infty}^{\infty} \mathrm{rect}\left(\frac{t-1}{4}\right)dt$$

$$E_x = 4\int_{-2}^{2} dt - 12\int_{-1}^{2} dt + 9\int_{-1}^{3} dt = 16 - 36 + 36 = 16$$

(j) $\mathrm{x}(t) = 3\,\mathrm{rect}\left(\frac{t-1}{6}\right)\mathrm{tri}\left(\frac{t-4}{6}\right)$

From a sketch it is easily observed that $\mathrm{x}(t) = \frac{t+2}{2}\ ,\ \ -2 < t < 4$.

$$E_x = \int_{-\infty}^{\infty} |\mathrm{x}(t)|^2\,dt = \int_{-2}^{4}\left(\frac{t+2}{2}\right)^2 dt = (1/4)\int_{-2}^{4}\left(t^2 + 4t + 4\right)dt$$

$$E_x = (1/4)\left[\frac{t^3}{3} + 2t^2 + 4t\right]_{-2}^{4} = (1/4)\left[\frac{64}{3} + 32 + 16 + \frac{8}{3} - 8 + 8\right] = 18$$

(k) $\mathrm{x}(t) = 5e^{-4t}\,\mathrm{u}(t)$

$$E_x = \int_{-\infty}^{\infty} |\mathrm{x}(t)|^2\,dt = 25\int_{0}^{\infty} e^{-8t}\,dt = 25\left[\frac{e^{-8t}}{-8}\right]_{0}^{\infty} = 25/8 \text{ or } 3.125$$

(l) A signal $\mathrm{x}(t)$ has the following description:

1. It is zero for all time $t < -4$.
2. It is a straight line from the point $t = -4, \mathrm{x} = 0$ to the point $t = -4, \mathrm{x} = 4$.
3. It is a straight line from the point $t = -4, \mathrm{x} = 4$ to the point $t = 3, \mathrm{x} = 0$.
4. It is zero for all time $t > 3$.

The only non-zero values of $\mathrm{x}(t)$ lie on a straight line between $t = -4$ and $t = 3$. The signal energy of this signal is therefore the same as this signal shifted to the left so that it starts its non-zero values at $t = -7$ and goes to zero at $t = 0$. Such a signal would be described in its non-zero range by

$$\mathrm{x}(t) = -(4/7)t\ \ ,\ \ -7 \le t < 0$$

Its signal energy is

$$E_x = \int_{-7}^{0} \left|(-4/7)t\right|^2 dt = (16/49)\int_{-7}^{0}(t^3/3)dt = (1/3)(16/49)\times 343 = 37.333$$

65. An even continuous-time energy signal $\mathrm{x}(t)$ is described in positive time by

$$\mathrm{x}(t) = \begin{cases} 3\mathrm{u}(t) + 5\mathrm{u}(t-4) - 11\mathrm{u}(t-7) & , \ 0 \le t < 10 \\ 0 & , \ t \ge 10 \end{cases}.$$

Another continuous-time energy signal $\mathrm{y}(t)$ is described by $\mathrm{y}(t) = -3\mathrm{x}(2t-2)$.

(a) Find the signal energy E_x of $\mathrm{x}(t)$.

$$E_x = \int_{-\infty}^{\infty} |\mathrm{x}(t)|^2 dt = \underbrace{2\int_{0}^{\infty} |\mathrm{x}(t)|^2 dt}_{\text{Because } \mathrm{x}(t) \text{ is even}} = 2\left[3^2 \times 4 + 8^2 \times 3 + (-3)^2 \times 3\right] = 2(36 + 192 + 27) = 510$$

(b) Find the signal energy E_y of $\mathrm{y}(t)$.

$$E_y = \int_{-\infty}^{\infty} |\mathrm{y}(t)|^2 dt = \int_{-\infty}^{\infty} |-3\mathrm{x}(2t-2)|^2 dt = 9\int_{-\infty}^{\infty} \mathrm{x}^2(2t-2)dt$$

Let $\lambda = 2t - 2 \Rightarrow d\lambda = 2dt$.

$$E_y = 9\int_{-\infty}^{\infty} \mathrm{x}^2(\lambda)\frac{d\lambda}{2} = 4.5\int_{-\infty}^{\infty} \mathrm{x}^2(\lambda)d\lambda = 4.5\underbrace{\int_{-\infty}^{\infty} \mathrm{x}^2(t)dt}_{=E_x} = 4.5 \times 510 = 2295$$

66. Find the average signal power of each of these signals:

(a) $\mathrm{x}(t) = 2\sin(200\pi t)$ This is a periodic function. Therefore

$$\mathrm{P_x} = \frac{1}{T}\int_{-T/2}^{T/2} \left[2\sin(200\pi t)\right]^2 dt = \frac{4}{T}\int_{-T/2}^{T/2}\left[\frac{1}{2} - \frac{1}{2}\cos(400\pi t)\right]dt$$

$$\mathrm{P_x} = \frac{2}{T}\left[t - \frac{\sin(400\pi t)}{400\pi}\right]_{-T/2}^{T/2} = \frac{2}{T}\left[\frac{T}{2} - \frac{\sin(200\pi T)}{400\pi} + \frac{T}{2} + \frac{\sin(-200\pi T)}{400\pi}\right] = 2$$

For any sinusoid, the average signal power is half the square of the amplitude.

(b) $\mathrm{x}(t)=\delta_1(t)$ This is a periodic signal whose period, T, is 1. Between $-T/2$ and $+T/2$, there is one impulse whose energy is infinite. Therefore the average power is the energy in one period, divided by the period, or infinite.

(c) $\mathrm{x}(t)=e^{j100\pi t}$ This is a periodic function. Therefore

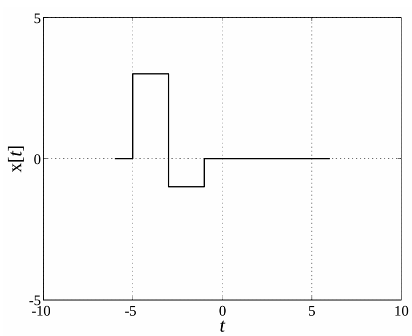
$$P_{\mathrm{x}}=\frac{1}{T_0}\int_{T_0}\left|\mathrm{x}(t)\right|^2 dt=\frac{1}{T_0}\int_{-T_0/2}^{T_0/2}\left|e^{j100\pi t}\right|^2 dt=50\int_{-1/100}^{1/100} e^{j100\pi t}e^{-j100\pi t}\,dt$$

$$P_{\mathrm{x}}=50\int_{-1/100}^{1/100} dt=1$$

(d) A periodic continuous-time signal with fundamental period 12 described over one fundamental period by

$$\mathrm{x}(t)=3\,\mathrm{rect}\left(\frac{t+3}{4}\right)-4\,\mathrm{rect}(t/2+1)\ ,\quad -6<t<6$$

One Fundamental Period



$$P_{\mathrm{x}}=(1/T_0)\int_{T_0}\left|\mathrm{x}(t)\right|^2 dt=(1/12)\int_{-6}^{6}\left|3\,\mathrm{rect}\left(\frac{t+3}{4}\right)-4\,\mathrm{rect}(t/2+1)\right|^2 dt$$

$$P_{\mathrm{x}}=(1/12)\left[9\int_{-5}^{-1} dt+16\int_{-3}^{-1} dt-24\int_{-3}^{-1} dt\right]=\frac{36+32-48}{12}=5/3\cong 1.667$$

(e) $\mathrm{x}(t)=-3\,\mathrm{sgn}(2(t-4))$ $\qquad \left|\mathrm{x}(t)\right|^2=9\Rightarrow P_{\mathrm{x}}=9$

67. A signal x is periodic with fundamental period $T_0=6$. This signal is described over the time period $0<t<6$ by

$$\text{rect}\left(\left(t-2\right)/3\right)-4\,\text{rect}\left(\left(t-4\right)/2\right).$$

What is the signal power of this signal?

The signal x can be described in the time period $0<t<6$ by

$$\text{x}\left(t\right)=\begin{cases}0 & , \quad 0<t<1/2 \\ 1 & , \quad 1/2<t<3 \\ -3 & , \quad 3<t<7/2 \\ -4 & , \quad 7/2<t<5 \\ 0 & , \quad 5<t<6\end{cases}$$

The signal power is the signal energy in one fundamental period divided by the fundamental period.

$$P=\frac{1}{6}\left(0^2\times\frac{1}{2}+1^2\times\frac{5}{2}+\left(-3\right)^2\times\frac{1}{2}+\left(-4\right)^2\times\frac{3}{2}+0^2\times 1\right)=\frac{2.5+4.5+24}{6}=5.167$$