Precalculus Mathematics for Calculus 7th Edition Stewart Solutions Manual
Full Download: http://testbanklive.com/download/precalculus-mathematics-for-calculus-7th-edition-stewart-solutions-manual/

2 FUNCTIONS

2.1 FUNCTIONS

1. If $f(x) = x^3 + 1$, then

(a) the value of f at $x = -1$ is $f(-1) = (-1)^3 + 1 = 0$.

(b) the value of f at $x = 2$ is $f(2) = 2^3 + 1 = 9$.

(c) the net change in the value of f between $x = -1$ and $x = 2$ is $f(2) - f(-1) = 9 - 0 = 9$.

2. For a function f, the set of all possible inputs is called the *domain* of f, and the set of all possible outputs is called the *range* of f.

3. (a) $f(x) = x^2 - 3x$ and $g(x) = \dfrac{x-5}{x}$ have 5 in their domain because they are defined when $x = 5$. However, $h(x) = \sqrt{x-10}$ is undefined when $x = 5$ because $\sqrt{5-10} = \sqrt{-5}$, so 5 is not in the domain of h.

(b) $f(5) = 5^2 - 3(5) = 25 - 15 = 10$ and $g(5) = \dfrac{5-5}{5} = \dfrac{0}{5} = 0$.

4. (a) *Verbal:* "Subtract 4, then *square* and *add* 3."

(b) *Numerical*:

x	$f(x)$
0	19
2	7
4	3
6	7

5. A function f is a rule that assigns to each element x in a set A exactly *one* element called $f(x)$ in a set B. Table (i) defines y as a function of x, but table (ii) does not, because $f(1)$ is not uniquely defined.

6. (a) Yes, it is possible that $f(1) = f(2) = 5$. [For instance, let $f(x) = 5$ for all x.]

(b) No, it is not possible to have $f(1) = 5$ and $f(1) = 6$. A function assigns each value of x in its domain exactly one value of $f(x)$.

7. Multiplying x by 3 gives $3x$, then subtracting 5 gives $f(x) = 3x - 5$.

8. Squaring x gives x^2, then adding two gives $f(x) = x^2 + 2$.

9. Subtracting 1 gives $x - 1$, then squaring gives $f(x) = (x-1)^2$.

10. Adding 1 gives $x + 1$, taking the square root gives $\sqrt{x+1}$, then dividing by 6 gives $f(x) = \dfrac{\sqrt{x+1}}{6}$.

11. $f(x) = 2x + 3$: Multiply by 2, then add 3.

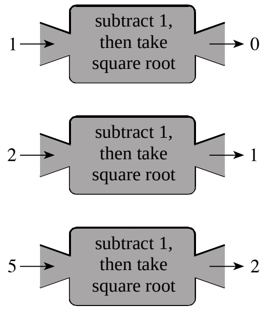
12. $g(x) = \dfrac{x+2}{3}$: Add 2, then divide by 3.

13. $h(x) = 5(x+1)$: Add 1, then multiply by 5.

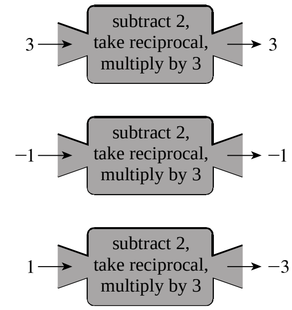
14. $k(x) = \dfrac{x^2-4}{3}$: Square, then subtract 4, then divide by 3.

Full download all chapters instantly please go to Solutions Manual, Test Bank site: testbanklive.com

15. Machine diagram for $f(x) = \sqrt{x-1}$.



16. Machine diagram for $f(x) = \dfrac{3}{x-2}$.



17. $f(x) = 2(x-1)^2$

x	$f(x)$
-1	$2(-1-1)^2 = 8$
0	$2(-1)^2 = 2$
1	$2(1-1)^2 = 0$
2	$2(2-1)^2 = 2$
3	$2(3-1)^2 = 8$

18. $g(x) = |2x+3|$

x	$g(x)$
-3	$\lvert 2(-3)+3\rvert = 3$
-2	$\lvert 2(-2)+3\rvert = 1$
0	$\lvert 2(0)+3\rvert = 3$
1	$\lvert 2(1)+3\rvert = 5$
3	$\lvert 2(3)+3\rvert = 9$

19. $f(x) = x^2 - 6$; $f(-3) = (-3)^2 - 6 = 9 - 6 = 3$; $f(3) = 3^2 - 6 = 9 - 6 = 3$; $f(0) = 0^2 - 6 = -6$; $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 6 = \frac{1}{4} - 6 = -\frac{23}{4}$.

20. $f(x) = x^3 + 2x$; $f(-2) = (-2)^3 + 2(-2) = -8 - 4 = -12$; $f(-1) = (-1)^3 + 2(-1) = -1 - 2 = -3$; $f(0) = 0^3 + 2(0) = 0$; $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 2\left(\frac{1}{2}\right) = \frac{1}{8} + 1 = \frac{9}{8}$.

21. $f(x) = \dfrac{1-2x}{3}$; $f(2) = \dfrac{1-2(2)}{3} = -1$; $f(-2) = \dfrac{1-2(-2)}{3} = \dfrac{5}{3}$; $f\left(\frac{1}{2}\right) = \dfrac{1-2\left(\frac{1}{2}\right)}{3} = 0$; $f(a) = \dfrac{1-2a}{3}$; $f(-a) = \dfrac{1-2(-a)}{3} = \dfrac{1+2a}{3}$; $f(a-1) = \dfrac{1-2(a-1)}{3} = \dfrac{3-2a}{3}$.

22. $h(x) = \dfrac{x^2+4}{5}$; $h(2) = \dfrac{2^2+4}{5} = \dfrac{8}{5}$; $h(-2) = \dfrac{(-2)^2+4}{5} = \dfrac{8}{5}$; $h(a) = \dfrac{a^2+4}{5}$; $h(-x) = \dfrac{(-x)^2+4}{5} = \dfrac{x^2+4}{5}$; $h(a-2) = \dfrac{(a-2)^2+4}{5} = \dfrac{a^2-4a+8}{5}$; $h\left(\sqrt{x}\right) = \dfrac{\left(\sqrt{x}\right)^2+4}{5} = \dfrac{x+4}{5}$.

23. $f(x) = x^2 + 2x$; $f(0) = 0^2 + 2(0) = 0$; $f(3) = 3^2 + 2(3) = 9 + 6 = 15$; $f(-3) = (-3)^2 + 2(-3) = 9 - 6 = 3$; $f(a) = a^2 + 2(a) = a^2 + 2a$; $f(-x) = (-x)^2 + 2(-x) = x^2 - 2x$; $f\left(\dfrac{1}{a}\right) = \left(\dfrac{1}{a}\right)^2 + 2\left(\dfrac{1}{a}\right) = \dfrac{1}{a^2} + \dfrac{2}{a}$.

24. $h(x) = x + \dfrac{1}{x}$; $h(-1) = (-1) + \frac{1}{-1} = -1 - 1 = -2$; $h(2) = 2 + \frac{1}{2} = \frac{5}{2}$; $h\left(\frac{1}{2}\right) = \dfrac{1}{2} + \dfrac{1}{\frac{1}{2}} = \dfrac{1}{2} + 2 = \dfrac{5}{2}$; $h(x-1) = x - 1 + \dfrac{1}{x-1}$; $h\left(\dfrac{1}{x}\right) = \dfrac{1}{x} + \dfrac{1}{\frac{1}{x}} = \dfrac{1}{x} + x$.

25. $g(x)=\dfrac{1-x}{1+x}$; $g(2)=\dfrac{1-(2)}{1+(2)}=\dfrac{-1}{3}=-\dfrac{1}{3}$; $g(-1)=\dfrac{1-(-1)}{1+(-1)}$, which is undefined; $g\left(\dfrac{1}{2}\right)=\dfrac{1-\left(\frac{1}{2}\right)}{1+\left(\frac{1}{2}\right)}=\dfrac{\frac{1}{2}}{\frac{3}{2}}=\dfrac{1}{3}$;

$g(a)=\dfrac{1-(a)}{1+(a)}=\dfrac{1-a}{1+a}$; $g(a-1)=\dfrac{1-(a-1)}{1+(a-1)}=\dfrac{1-a+1}{1+a-1}=\dfrac{2-a}{a}$; $g\left(x^2-1\right)=\dfrac{1-\left(x^2-1\right)}{1+\left(x^2-1\right)}=\dfrac{2-x^2}{x^2}$.

26. $g(t)=\dfrac{t+2}{t-2}$; $g(-2)=\dfrac{-2+2}{-2-2}=0$; $g(2)=\dfrac{2+2}{2-2}$, which is undefined; $g(0)=\dfrac{0+2}{0-2}=-1$; $g(a)=\dfrac{a+2}{a-2}$;

$g\left(a^2-2\right)=\dfrac{a^2-2+2}{a^2-2-2}=\dfrac{a^2}{a^2-4}$; $g(a+1)=\dfrac{a+1+2}{a+1-2}=\dfrac{a+3}{a-1}$.

27. $k(x)=-x^2-2x+3$; $k(0)=-0^2-2(0)+3=3$; $k(2)=-2^2-2(2)+3=-5$; $k(-2)=-(-2)^2-2(-2)+3=3$;

$k\left(\sqrt{2}\right)=-\left(\sqrt{2}\right)^2-2\left(\sqrt{2}\right)+3=1-2\sqrt{2}$; $k(a+2)=-(a+2)^2-2(a+2)+3=-a^2-6a-5$;

$k(-x)=-(-x)^2-2(-x)+3=-x^2+2x+3$; $k\left(x^2\right)=-\left(x^2\right)^2-2\left(x^2\right)+3=-x^4-2x^2+3$.

28. $k(x)=2x^3-3x^2$; $k(0)=2(0)^3-3(0)^2=0$; $k(3)=2(3)^3-3(3)^2=27$; $k(-3)=2(-3)^3-3(-3)^2=-81$;

$k\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3-3\left(\frac{1}{2}\right)^2=-\frac{1}{2}$; $k\left(\frac{a}{2}\right)=2\left(\frac{a}{2}\right)^3-3\left(\frac{a}{2}\right)^2=\dfrac{a^3-3a^2}{4}$; $k(-x)=2(-x)^3-3(-x)^2=-2x^3-3x^2$;

$k\left(x^3\right)=2\left(x^3\right)^3-3\left(x^3\right)^2=2x^9-3x^6$.

29. $f(x)=2|x-1|$; $f(-2)=2|-2-1|=2(3)=6$; $f(0)=2|0-1|=2(1)=2$;

$f\left(\frac{1}{2}\right)=2\left|\frac{1}{2}-1\right|=2\left(\frac{1}{2}\right)=1$; $f(2)=2|2-1|=2(1)=2$; $f(x+1)=2|(x+1)-1|=2|x|$;

$f\left(x^2+2\right)=2\left|\left(x^2+2\right)-1\right|=2\left|x^2+1\right|=2x^2+2$ (since $x^2+1>0$).

30. $f(x)=\dfrac{|x|}{x}$; $f(-2)=\dfrac{|-2|}{-2}=\dfrac{2}{-2}=-1$; $f(-1)=\dfrac{|-1|}{-1}=\dfrac{1}{-1}=-1$; $f(x)$ is not defined at $x=0$;

$f(5)=\dfrac{|5|}{5}=\dfrac{5}{5}=1$; $f\left(x^2\right)=\dfrac{\left|x^2\right|}{x^2}=\dfrac{x^2}{x^2}=1$ since $x^2>0$, $x\neq 0$; $f\left(\dfrac{1}{x}\right)=\dfrac{|1/x|}{1/x}=\dfrac{x}{|x|}$.

31. Since $-2<0$, we have $f(-2)=(-2)^2=4$. Since $-1<0$, we have $f(-1)=(-1)^2=1$. Since $0\geq 0$, we have $f(0)=0+1=1$. Since $1\geq 0$, we have $f(1)=1+1=2$. Since $2\geq 0$, we have $f(2)=2+1=3$.

32. Since $-3\leq 2$, we have $f(-3)=5$. Since $0\leq 2$, we have $f(0)=5$. Since $2\leq 2$, we have $f(2)=5$. Since $3>2$, we have $f(3)=2(3)-3=3$. Since $5>2$, we have $f(5)=2(5)-3=7$.

33. Since $-4\leq -1$, we have $f(-4)=(-4)^2+2(-4)=16-8=8$. Since $-\frac{3}{2}\leq -1$, we have $f\left(-\frac{3}{2}\right)=\left(-\frac{3}{2}\right)^2+2\left(-\frac{3}{2}\right)=\frac{9}{4}-3=-\frac{3}{4}$. Since $-1\leq -1$, we have $f(-1)=(-1)^2+2(-1)=1-2=-1$. Since $-1<0\leq 1$, we have $f(0)=0$. Since $25>1$, we have $f(25)=-1$.

34. Since $-5<0$, we have $f(-5)=3(-5)=-15$. Since $0\leq 0\leq 2$, we have $f(0)=0+1=1$. Since $0\leq 1\leq 2$, we have $f(1)=1+1=2$. Since $0\leq 2\leq 2$, we have $f(2)=2+1=3$. Since $5>2$, we have $f(5)=(5-2)^2=9$.

35. $f(x+2)=(x+2)^2+1=x^2+4x+4+1=x^2+4x+5$; $f(x)+f(2)=x^2+1+(2)^2+1=x^2+1+4+1=x^2+6$.

36. $f(2x)=3(2x)-1=6x-1$; $2f(x)=2(3x-1)=6x-2$.

37. $f\left(x^2\right)=x^2+4$; $\left[f(x)\right]^2=[x+4]^2=x^2+8x+16$.

38. $f\left(\dfrac{x}{3}\right)=6\left(\dfrac{x}{3}\right)-18=2x-18$; $\dfrac{f(x)}{3}=\dfrac{6x-18}{3}=\dfrac{3(2x-6)}{3}=2x-6$.

39. $f(x)=3x-2$, so $f(1)=3(1)-2=1$ and $f(5)=3(5)-2=13$. Thus, the net change is $f(5)-f(1)=13-1=12$.

40. $f(x)=4-5x$, so $f(3)=4-5(3)=-11$ and $f(5)=4-5(5)=-21$. Thus, the net change is $f(5)-f(3)=-21-(-11)=-10$.

41. $g(t) = 1 - t^2$, so $g(-2) = 1 - (-2)^2 = 1 - 4 = -3$ and $g(5) = 1 - 5^2 = -24$. Thus, the net change is $g(5) - g(-2) = -24 - (-3) = -21$.

42. $h(t) = t^2 + 5$, so $h(-3) = (-3)^2 + 5 = 14$ and $h(6) = 6^2 + 5 = 41$. Thus, the net change is $h(6) - h(-3) = 41 - 14 = 27$.

43. $f(a) = 5 - 2a$; $f(a+h) = 5 - 2(a+h) = 5 - 2a - 2h$;

$$\frac{f(a+h) - f(a)}{h} = \frac{5 - 2a - 2h - (5 - 2a)}{h} = \frac{5 - 2a - 2h - 5 + 2a}{h} = \frac{-2h}{h} = -2.$$

44. $f(a) = 3a^2 + 2$; $f(a+h) = 3(a+h)^2 + 2 = 3a^2 + 6ah + 3h^2 + 2$;

$$\frac{f(a+h) - f(a)}{h} = \frac{\left(3a^2 + 6ah + 3h^2 + 2\right) - \left(3a^2 + 2\right)}{h} = \frac{6ah + 3h^2}{h} = 6a + 3h.$$

45. $f(a) = 5$; $f(a+h) = 5$; $\dfrac{f(a+h) - f(a)}{h} = \dfrac{5-5}{h} = 0$.

46. $f(a) = \dfrac{1}{a+1}$; $f(a+h) = \dfrac{1}{a+h+1}$;

$$\frac{f(a+h) - f(a)}{h} = \frac{\dfrac{1}{a+h+1} - \dfrac{1}{a+1}}{h} = \frac{\dfrac{a+1}{(a+1)(a+h+1)} - \dfrac{a+h+1}{(a+1)(a+h+1)}}{h}$$

$$= \frac{\dfrac{-h}{(a+1)(a+h+1)}}{h} = \frac{-1}{(a+1)(a+h+1)}.$$

47. $f(a) = \dfrac{a}{a+1}$; $f(a+h) = \dfrac{a+h}{a+h+1}$;

$$\frac{f(a+h) - f(a)}{h} = \frac{\dfrac{a+h}{a+h+1} - \dfrac{a}{a+1}}{h} = \frac{\dfrac{(a+h)(a+1)}{(a+h+1)(a+1)} - \dfrac{a(a+h+1)}{(a+h+1)(a+1)}}{h}$$

$$= \frac{\dfrac{(a+h)(a+1) - a(a+h+1)}{(a+h+1)(a+1)}}{h} = \frac{a^2 + a + ah + h - \left(a^2 + ah + a\right)}{h(a+h+1)(a+1)}$$

$$= \frac{1}{(a+h+1)(a+1)}$$

48. $f(a) = \dfrac{2a}{a-1}$; $f(a+h) = \dfrac{2(a+h)}{a+h-1}$;

$$\frac{f(a+h) - f(a)}{h} = \frac{\dfrac{2(a+h)}{a+h-1} - \dfrac{2a}{a-1}}{h} = \frac{\dfrac{(2a+2h)(a-1)}{(a+h-1)(a-1)} - \dfrac{2a(a+h-1)}{(a+h-1)(a-1)}}{h}$$

$$= \frac{\dfrac{2(a+h)(a-1) - 2a(a+h-1)}{(a+h-1)(a-1)}}{h} = \frac{2a^2 + 2ah - 2a - 2h - 2a^2 - 2ah + 2a}{h(a+h-1)(a-1)}$$

$$= \frac{-2h}{h(a+h-1)(a-1)} = -\frac{2}{(a+h-1)(a-1)}$$

49. $f(a) = 3 - 5a + 4a^2$;

$$f(a+h) = 3 - 5(a+h) + 4(a+h)^2 = 3 - 5a - 5h + 4\left(a^2 + 2ah + h^2\right)$$
$$= 3 - 5a - 5h + 4a^2 + 8ah + 4h^2;$$

$$\frac{f(a+h) - f(a)}{h} = \frac{\left(3 - 5a - 5h + 4a^2 + 8ah + 4h^2\right) - \left(3 - 5a + 4a^2\right)}{h}$$

$$= \frac{3 - 5a - 5h + 4a^2 + 8ah + 4h^2 - 3 + 5a - 4a^2}{h} = \frac{-5h + 8ah + 4h^2}{h}$$

$$= \frac{h(-5 + 8a + 4h)}{h} = -5 + 8a + 4h.$$

50. $f(a) = a^3$; $f(a+h) = (a+h)^3 = a^3 + 3a^2h + 3ah^2 + h^3$;

$$\frac{f(a+h) - f(a)}{h} = \frac{\left(a^3 + 3a^2h + 3ah^2 + h^3\right) - \left(a^3\right)}{h} = \frac{3a^2h + 3ah^2 + h^3}{h}$$
$$= \frac{h\left(3a^2 + 3ah + h^2\right)}{h} = 3a^2 + 3ah + h^2.$$

51. $f(x) = 3x$. Since there is no restriction, the domain is all real numbers, $(-\infty, \infty)$. Since every real number y is three times the real number $\frac{1}{3}y$, the range is all real numbers $(-\infty, \infty)$.

52. $f(x) = 5x^2 + 4$. Since there is no restriction, the domain is all real numbers, $(-\infty, \infty)$. Since $5x^2 \geq 0$ for all x, $5x^2 + 4 \geq 4$ for all x, so the range is $[4, \infty)$.

53. $f(x) = 3x$, $-2 \leq x \leq 6$. The domain is $[-2, 6]$, $f(-2) = 3(-2) = -6$, and $f(6) = 3(6) = 18$, so the range is $[-6, 18]$.

54. $f(x) = 5x^2 + 4$, $0 \leq x \leq 2$. The domain is $[0, 2]$, $f(0) = 5(0)^2 + 4 = 4$, and $f(2) = 5(2)^2 + 4 = 24$, so the range is $[4, 24]$.

55. $f(x) = \dfrac{1}{x-3}$. Since the denominator cannot equal 0 we have $x - 3 \neq 0 \Leftrightarrow x \neq 3$. Thus the domain is $\{x \mid x \neq 3\}$. In interval notation, the domain is $(-\infty, 3) \cup (3, \infty)$.

56. $f(x) = \dfrac{1}{3x-6}$. Since the denominator cannot equal 0, we have $3x - 6 \neq 0 \Leftrightarrow 3x \neq 6 \Leftrightarrow x \neq 2$. In interval notation, the domain is $(-\infty, 2) \cup (2, \infty)$.

57. $f(x) = \dfrac{x+2}{x^2-1}$. Since the denominator cannot equal 0 we have $x^2 - 1 \neq 0 \Leftrightarrow x^2 \neq 1 \Rightarrow x \neq \pm 1$. Thus the domain is $\{x \mid x \neq \pm 1\}$. In interval notation, the domain is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

58. $f(x) = \dfrac{x^4}{x^2+x-6}$. Since the denominator cannot equal 0, $x^2 + x - 6 \neq 0 \Leftrightarrow (x+3)(x-2) \neq 0 \Rightarrow x \neq -3$ or $x \neq 2$. In interval notation, the domain is $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$.

59. $f(x) = \sqrt{x+1}$. We must have $x + 1 \geq 0 \Leftrightarrow x \geq -1$. Thus, the domain is $[-1, \infty)$.

60. $g(x) = \sqrt{x^2+9}$. The argument of the square root is positive for all x, so the domain is $(-\infty, \infty)$.

61. $f(t) = \sqrt[3]{t-1}$. Since the odd root is defined for all real numbers, the domain is the set of real numbers, $(-\infty, \infty)$.

62. $g(x) = \sqrt{7-3x}$. For the square root to be defined, we must have $7 - 3x \geq 0 \Leftrightarrow 7 \geq 3x \Leftrightarrow \frac{7}{3} \geq x$. Thus the domain is $\left(-\infty, \frac{7}{3}\right]$.

63. $f(x) = \sqrt{1-2x}$. Since the square root is defined as a real number only for nonnegative numbers, we require that $1 - 2x \geq 0 \Leftrightarrow x \leq \frac{1}{2}$. So the domain is $\{x \mid x \leq \frac{1}{2}\}$. In interval notation, the domain is $\left(-\infty, \frac{1}{2}\right]$.

64. $g(x) = \sqrt{x^2-4}$. We must have $x^2 - 4 \geq 0 \Leftrightarrow (x-2)(x+2) \geq 0$. We make a table:

	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
Sign of $x - 2$	$-$	$-$	$+$
Sign of $x + 2$	$-$	$+$	$+$
Sign of $(x-2)(x+2)$	$+$	$-$	$+$

Thus the domain is $(-\infty, -2] \cup [2, \infty)$.

65. $g(x) = \dfrac{\sqrt{2+x}}{3-x}$. We require $2 + x \geq 0$, and the denominator cannot equal 0. Now $2 + x \geq 0 \Leftrightarrow x \geq -2$, and $3 - x \neq 0 \Leftrightarrow x \neq 3$. Thus the domain is $\{x \mid x \geq -2 \text{ and } x \neq 3\}$, which can be expressed in interval notation as $[-2, 3) \cup (3, \infty)$.

66. $g(x) = \dfrac{\sqrt{x}}{2x^2 + x - 1}$. We must have $x \geq 0$ for the numerator and $2x^2 + x - 1 \neq 0$ for the denominator. So $2x^2 + x - 1 \neq 0$ $\Leftrightarrow (2x - 1)(x + 1) \neq 0 \Rightarrow 2x - 1 \neq 0$ or $x + 1 \neq 0 \Leftrightarrow x \neq \frac{1}{2}$ or $x \neq -1$. Thus the domain is $\left[0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

67. $g(x) = \sqrt[4]{x^2 - 6x}$. Since the input to an even root must be nonnegative, we have $x^2 - 6x \geq 0 \Leftrightarrow x(x - 6) \geq 0$. We make a table:

	$(-\infty, 0)$	$(0, 6)$	$(6, \infty)$
Sign of x	$-$	$+$	$+$
Sign of $x - 6$	$-$	$-$	$+$
Sign of $x(x - 6)$	$+$	$-$	$+$

Thus the domain is $(-\infty, 0] \cup [6, \infty)$.

68. $g(x) = \sqrt{x^2 - 2x - 8}$. We must have $x^2 - 2x - 8 \geq 0 \Leftrightarrow (x - 4)(x + 2) \geq 0$. We make a table:

	$(-\infty, -2)$	$(-2, 4)$	$(4, \infty)$
Sign of $x - 4$	$-$	$-$	$+$
Sign of $x + 2$	$-$	$+$	$+$
Sign of $(x - 4)(x + 2)$	$+$	$-$	$+$

Thus the domain is $(-\infty, -2] \cup [4, \infty)$.

69. $f(x) = \dfrac{3}{\sqrt{x - 4}}$. Since the input to an even root must be nonnegative and the denominator cannot equal 0, we have $x - 4 > 0 \Leftrightarrow x > 4$. Thus the domain is $(4, \infty)$.

70. $f(x) = \dfrac{x^2}{\sqrt{6 - x}}$. Since the input to an even root must be nonnegative and the denominator cannot equal 0, we have $6 - x > 0 \Leftrightarrow 6 > x$. Thus the domain is $(-\infty, 6)$.

71. $f(x) = \dfrac{(x + 1)^2}{\sqrt{2x - 1}}$. Since the input to an even root must be nonnegative and the denominator cannot equal 0, we have $2x - 1 > 0 \Leftrightarrow x > \frac{1}{2}$. Thus the domain is $\left(\frac{1}{2}, \infty\right)$.

72. $f(x) = \dfrac{x}{\sqrt[4]{9 - x^2}}$. Since the input to an even root must be nonnegative and the denominator cannot equal 0, we have $9 - x^2 > 0 \Leftrightarrow (3 - x)(3 + x) > 0$. We make a table:

Interval	$(-\infty, -3)$	$(-3, 3)$	$(3, \infty)$
Sign of $3 - x$	$+$	$+$	$-$
Sign of $3 + x$	$-$	$+$	$+$
Sign of $(x - 4)(x + 2)$	$-$	$+$	$-$

Thus the domain is $(-3, 3)$.

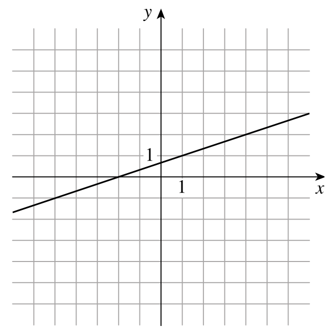
73. To evaluate $f(x)$, divide the input by 3 and add $\frac{2}{3}$ to the result.

(a) $f(x) = \dfrac{x}{3} + \dfrac{2}{3}$

(b)

x	$f(x)$
2	$\frac{4}{3}$
4	2
6	$\frac{8}{3}$
8	$\frac{10}{3}$

(c)



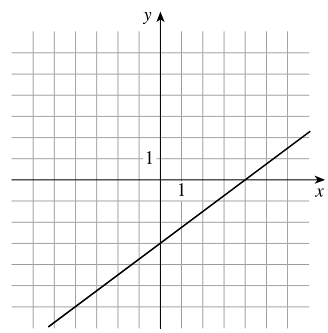
74. To evaluate $g(x)$, subtract 4 from the input and multiply the result by $\frac{3}{4}$.

(a) $g(x) = (x-4)\cdot\frac{3}{4} = \frac{3}{4}(x-4)$

(b)

x	$g(x)$
2	$-\frac{3}{2}$
4	0
6	$\frac{3}{2}$
8	3

(c)



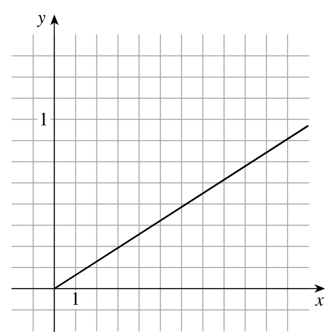
75. Let $T(x)$ be the amount of sales tax charged in Lemon County on a purchase of x dollars. To find the tax, take 8% of the purchase price.

(a) $T(x) = 0.08x$

(b)

x	$T(x)$
2	0.16
4	0.32
6	0.48
8	0.64

(c)



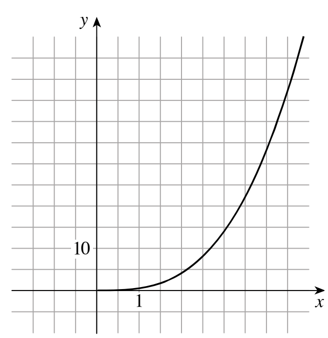
76. Let $V(d)$ be the volume of a sphere of diameter d. To find the volume, take the cube of the diameter, then multiply by π and divide by 6.

(a) $V(d) = d^3 \cdot \pi/6 = \frac{\pi}{6}d^3$

(b)

x	$f(x)$
2	$\frac{4\pi}{3} \approx 4.2$
4	$\frac{32\pi}{3} \approx 33.5$
6	$36\pi \approx 113$
8	$\frac{256\pi}{3} \approx 268$

(c)



77. $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 5 & \text{if } x \text{ is irrational} \end{cases}$ The domain of f is all real numbers, since every real number is either rational or irrational; and the range of f is $\{1, 5\}$.

78. $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 5x & \text{if } x \text{ is irrational} \end{cases}$ The domain of f is all real numbers, since every real number is either rational or irrational. If x is irrational, then $5x$ is also irrational, and so the range of f is $\{x \mid x = 1 \text{ or } x \text{ is irrational}\}$.

79. (a) $V(0) = 50\left(1 - \frac{0}{20}\right)^2 = 50$ and $V(20) = 50\left(1 - \frac{20}{20}\right)^2 = 0$.

(b) $V(0) = 50$ represents the volume of the full tank at time $t = 0$, and $V(20) = 0$ represents the volume of the empty tank twenty minutes later.

(c)

x	$V(x)$
0	50
5	28.125
10	12.5
15	3.125
20	0

(d) The net change in V as t changes from 0 minutes to 20 minutes is $V(20) - V(0) = 0 - 50 = -50$ gallons.

80. (a) $S(2) = 4\pi(2)^2 = 16\pi \approx 50.27$, $S(3) = 4\pi(3)^2 = 36\pi \approx 113.10$.

(b) $S(2)$ represents the surface area of a sphere of radius 2, and $S(3)$ represents the surface area of a sphere of radius 3.

81. (a) $L(0.5c) = 10\sqrt{1 - \frac{(0.5c)^2}{c^2}} \approx 8.66$ m, $L(0.75c) = 10\sqrt{1 - \frac{(0.75c)^2}{c^2}} \approx 6.61$ m, and

$L(0.9c) = 10\sqrt{1 - \frac{(0.9c)^2}{c^2}} \approx 4.36$ m.

(b) It will appear to get shorter.

82. (a) $R(1) = \sqrt{\frac{13 + 7(1)^{0.4}}{1 + 4(1)^{0.4}}} = \sqrt{\frac{20}{5}} = 2$ mm,

$R(10) = \sqrt{\frac{13 + 7(10)^{0.4}}{1 + 4(10)^{0.4}}} \approx 1.66$ mm, and

$R(100) = \sqrt{\frac{13 + 7(100)^{0.4}}{1 + 4(100)^{0.4}}} \approx 1.48$ mm.

(b)

x	$R(x)$
1	2
10	1.66
100	1.48
200	1.44
500	1.41
1000	1.39

(c) The net change in R as x changes from 10 to 100 is $R(100) - R(10) \approx 1.48 - 1.66 = -0.18$ mm.

83. (a) $\upsilon(0.1) = 18500\left(0.25 - 0.1^2\right) = 4440$,

$\upsilon(0.4) = 18500\left(0.25 - 0.4^2\right) = 1665$.

(b) They tell us that the blood flows much faster (about 2.75 times faster) 0.1 cm from the center than 0.1 cm from the edge.

(c)

r	$\upsilon(r)$
0	4625
0.1	4440
0.2	3885
0.3	2960
0.4	1665
0.5	0

(d) The net change in V as r changes from 0.1 cm to 0.5 cm is $V(0.5) - V(0.1) = 0 - 4440 = -4440$ cm/s.

84. (a) $D(0.1) = \sqrt{2(3960)(0.1) + (0.1)^2} = \sqrt{792.01} \approx 28.1$ miles

$D(0.2) = \sqrt{2(3960)(0.2) + (0.2)^2} = \sqrt{1584.04} \approx 39.8$ miles

(b) 1135 feet $= \frac{1135}{5280}$ miles ≈ 0.215 miles. $D(0.215) = \sqrt{2(3960)(0.215)+(0.215)^2} = \sqrt{1702.846} \approx 41.3$ miles

(c) $D(7) = \sqrt{2(3960)(7)+(7)^2} = \sqrt{55489} \approx 235.6$ miles

(d) The net change in D as h changes from 1135 ft (or 0.215 mi) to 7 mi is $D(7) - D(0.215) \approx 235.6 - 41.3 = 194.3$ miles.

85. (a) Since $0 \le 5{,}000 \le 10{,}000$ we have $T(5{,}000) = 0$. Since $10{,}000 < 12{,}000 \le 20{,}000$ we have $T(12{,}000) = 0.08(12{,}000) = 960$. Since $20{,}000 < 25{,}000$ we have $T(25{,}000) = 1600 + 0.15(25{,}000) = 5350$.

(b) There is no tax on \$5000, a tax of \$960 on \$12,000 income, and a tax of \$5350 on \$25,000.

86. (a) $C(75) = 75 + 15 = \$90$; $C(90) = 90 + 15 = \$105$; $C(100) = \$100$; and $C(105) = \$105$.

(b) The total price of the books purchased, including shipping.

87. (a) $T(x) = \begin{cases} 75x & \text{if } 0 \le x \le 2 \\ 150 + 50(x-2) & \text{if } x > 2 \end{cases}$

(b) $T(2) = 75(2) = 150$; $T(3) = 150 + 50(3-2) = 200$; and $T(5) = 150 + 50(5-2) = 300$.

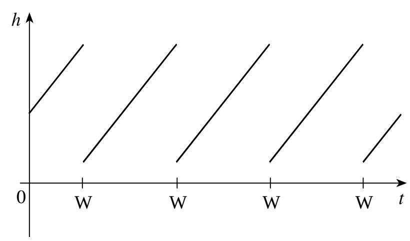
(c) The total cost of the lodgings.

88. (a) $F(x) = \begin{cases} 15(40-x) & \text{if } 0 < x < 40 \\ 0 & \text{if } 40 \le x \le 65 \\ 15(x-65) & \text{if } x > 65 \end{cases}$

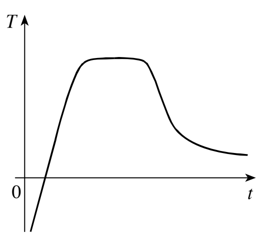
(b) $F(30) = 15(40-10) = 15 \cdot 10 = \$150$; $F(50) = \$0$; and $F(75) = 15(75-65)\, 15 \cdot 10 = \$150$.

(c) The fines for violating the speed limits on the freeway.

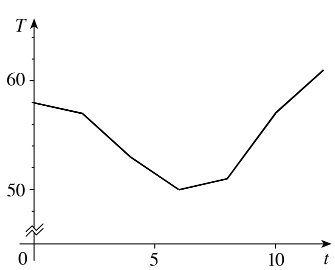
89. We assume the grass grows linearly.



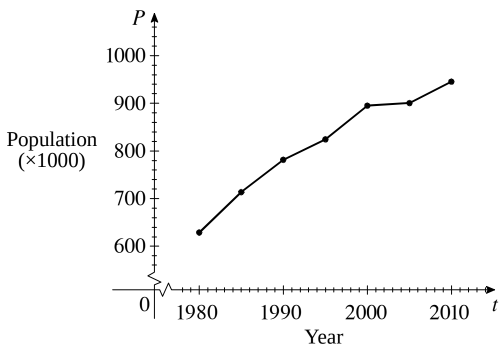
90.



91.



92.



93. Answers will vary.

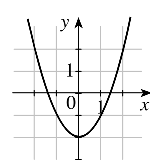
94. Answers will vary.

95. Answers will vary.

2.2 GRAPHS OF FUNCTIONS

1. To graph the function f we plot the points $(x, f(x))$ in a coordinate plane. To graph $f(x) = x^2 - 2$, we plot the points $\left(x, x^2 - 2\right)$. So, the point $\left(3, 3^2 - 2\right) = (3, 7)$ is on the graph of f. The height of the graph of f above the x-axis when $x = 3$ is 7.

x	$f(x)$	(x, y)
-2	2	$(-2, 2)$
-1	-1	$(-1, -1)$
0	-2	$(0, -2)$
1	-1	$(1, -1)$
2	2	$(2, 2)$



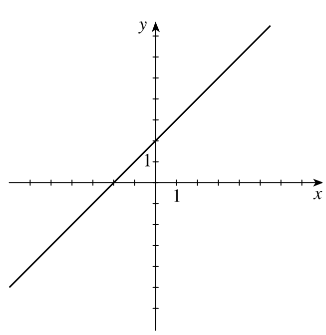
2. If $f(4) = 10$ then the point $(4, 10)$ is on the graph of f.

3. If the point $(3, 7)$ is on the graph of f, then $f(3) = 7$.

4. **(a)** $f(x) = x^2$ is a power function with an even exponent. It has graph IV.
(b) $f(x) = x^3$ is a power function with an odd exponent. It has graph II.
(c) $f(x) = \sqrt{x}$ is a root function. It has graph I.
(d) $f(x) = |x|$ is an absolute value function. It has graph III.

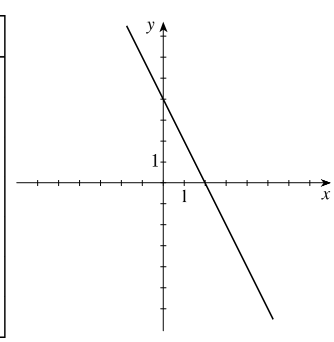
5.

x	$f(x) = x + 2$
-6	-4
-4	-2
-2	0
0	2
2	4
4	6
6	8



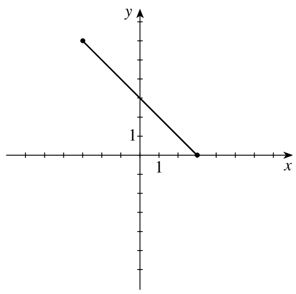
6.

x	$f(x) = 4 - 2x$
-2	8
-1	6
0	4
1	2
2	0
3	-2
4	-4



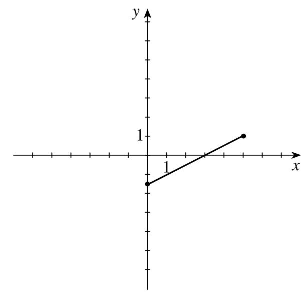
7.

x	$f(x) = -x + 3$, $-3 \le x \le 3$
-3	6
-2	5
0	3
1	2
2	1
3	0



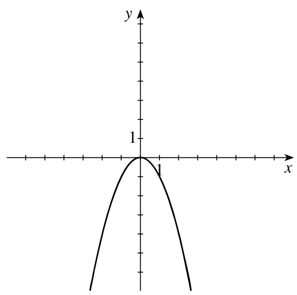
8.

x	$f(x) = \dfrac{x-3}{2}$, $0 \le x \le 5$
0	-1.5
1	-1
2	-0.5
3	0
4	0.5
5	1



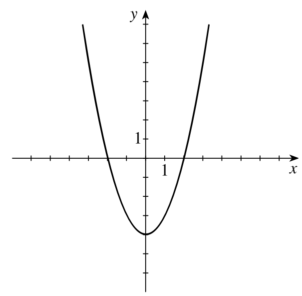
9.

x	$f(x) = -x^2$
± 4	-16
± 3	-9
± 2	-4
± 1	-1
0	0



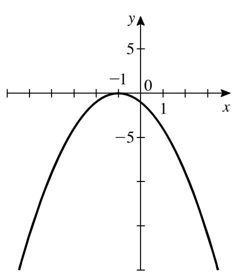
10.

x	$f(x) = x^2 - 4$
± 5	21
± 4	12
± 3	5
± 2	0
± 1	-3
0	-4



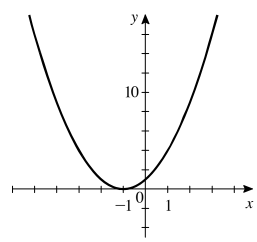
11.

x	$g(x) = -(x+1)^2$
-5	-16
-3	-4
-2	-1
-1	0
0	-1
1	-4
3	-16



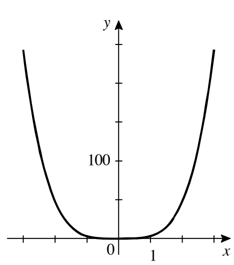
12.

x	$g(x) = x^2 + 2x + 1$
-5	16
-3	4
-2	1
-1	0
0	1
1	4
3	16



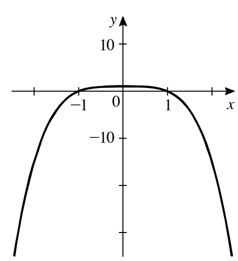
13.

x	$r(x) = 3x^4$
-3	243
-2	48
-1	3
0	0
1	3
2	48
3	243



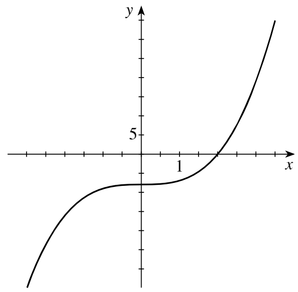
14.

x	$r(x) = 1 - x^4$
-3	-80
-2	-15
-1	0
0	1
1	0
2	-15
3	-80



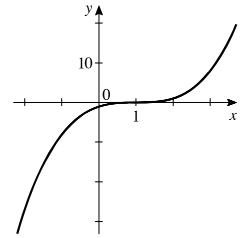
15.

x	$g(x) = x^3 - 8$
-3	-35
-2	-16
-1	-9
0	-8
1	-7
2	0
3	19



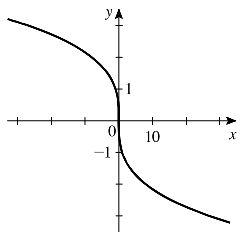
16.

x	$g(x) = (x-1)^3$
-2	-27
-1	-8
0	-1
1	0
2	1
3	8
4	27



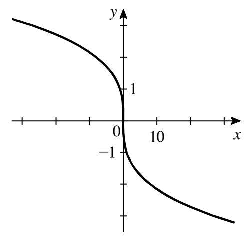
17.

x	$k(x) = \sqrt[3]{-x}$
-27	3
-8	2
-1	1
0	0
1	-1
8	-2
27	-3



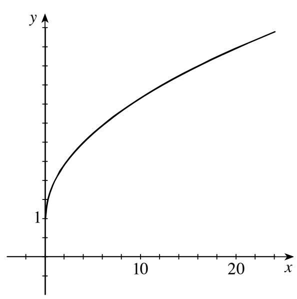
18.

x	$k(x) = -\sqrt[3]{x}$
-27	3
-8	2
-1	1
0	0
1	-1
8	-2
27	-3



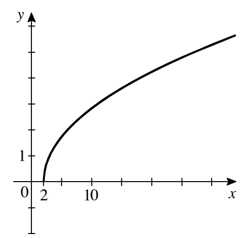
19.

x	$f(x) = 1 + \sqrt{x}$
0	1
1	2
4	3
9	4
16	5
25	6



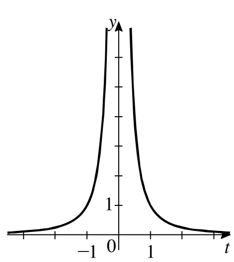
20.

x	$f(x) = \sqrt{x-2}$
2	0
3	1
6	2
11	3
18	4
27	5
38	6



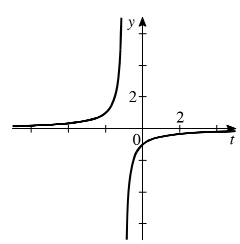
21.

x	$C(t) = \dfrac{1}{t^2}$
-2	$\frac{1}{4}$
-1	1
$-\frac{1}{2}$	4
$-\frac{1}{4}$	16
0	$-$
$\frac{1}{4}$	16
$\frac{1}{2}$	4
1	1
2	$\frac{1}{4}$



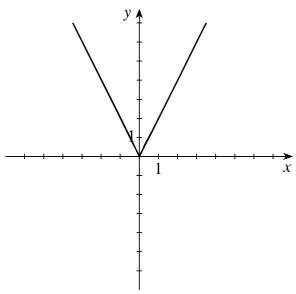
22.

x	$C(t) = -\dfrac{1}{t+1}$
-3	$\frac{1}{2}$
-2	1
$-\frac{3}{2}$	2
-1	$-$
$-\frac{1}{2}$	-2
0	-1
1	$-\frac{1}{2}$
2	$-\frac{1}{3}$



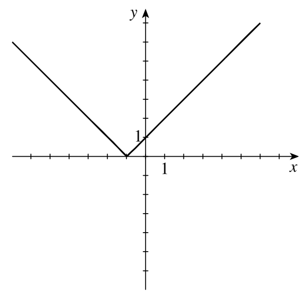
23.

x	$H(x) = \lvert 2x \rvert$
± 5	10
± 4	8
± 3	6
± 2	4
± 1	2
0	0



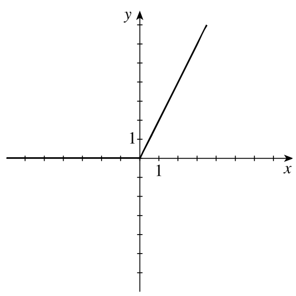
24.

x	$H(x) = \lvert x + 1 \rvert$
-5	4
-4	3
-3	2
-2	1
-1	0
0	1
1	2



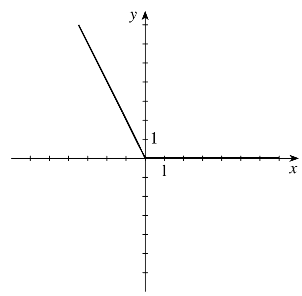
25.

x	$G(x) = \lvert x \rvert + x$
-5	0
-2	0
0	0
1	2
2	4
5	10



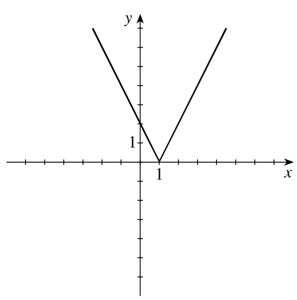
26.

x	$G(x) = \lvert x \rvert - x$
-5	10
-2	4
-1	2
0	0
1	0
3	0



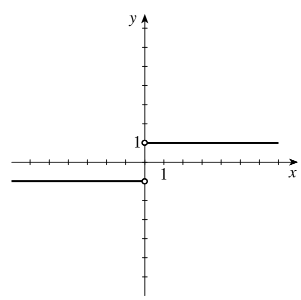
27.

x	$f(x) = \lvert 2x - 2 \rvert$
-5	12
-2	8
0	2
1	0
2	2
5	8



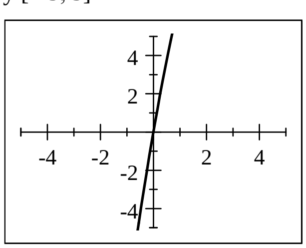
28.

x	$f(x) = \dfrac{x}{\lvert x \rvert}$
-3	-1
-2	-1
-1	-1
0	undefined
1	1
2	1
3	1

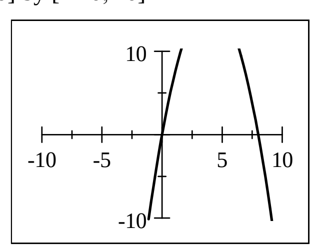


29. $f(x) = 8x - x^2$

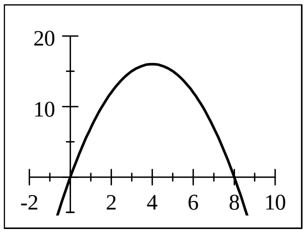
(a) $[-5, 5]$ by $[-5, 5]$



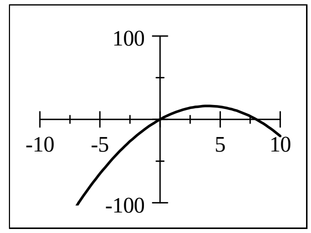
(b) $[-10, 10]$ by $[-10, 10]$



(c) $[-2, 10]$ by $[-5, 20]$



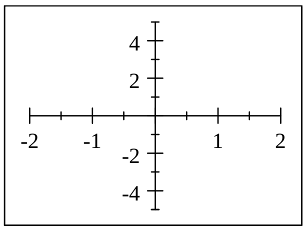
(d) $[-10, 10]$ by $[-100, 100]$



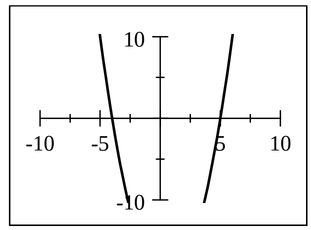
The viewing rectangle in part (c) produces the most appropriate graph of the equation.

30. $g(x) = x^2 - x - 20$

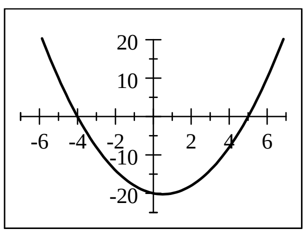
(a) $[-2, 2]$ by $[-5, 5]$



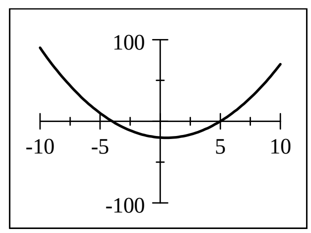
(b) $[-10, 10]$ by $[-10, 10]$



(c) $[-7, 7]$ by $[-25, 20]$



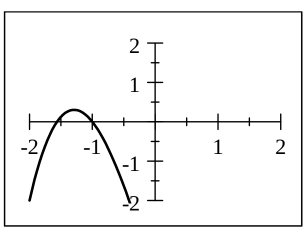
(d) $[-10, 10]$ by $[-100, 100]$



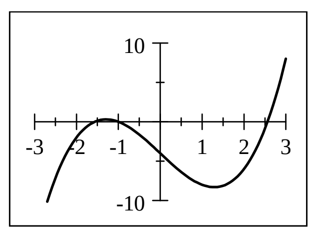
The viewing rectangle in part (c) produces the most appropriate graph of the equation.

31. $h(x) = x^3 - 5x - 4$

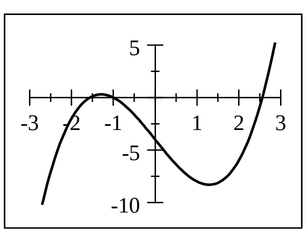
(a) $[-2, 2]$ by $[-2, 2]$



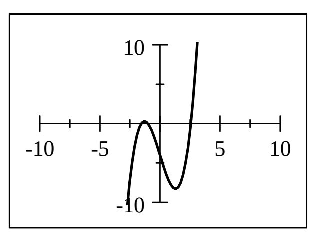
(b) $[-3, 3]$ by $[-10, 10]$



(c) $[-3, 3]$ by $[-10, 5]$



(d) $[-10, 10]$ by $[-10, 10]$

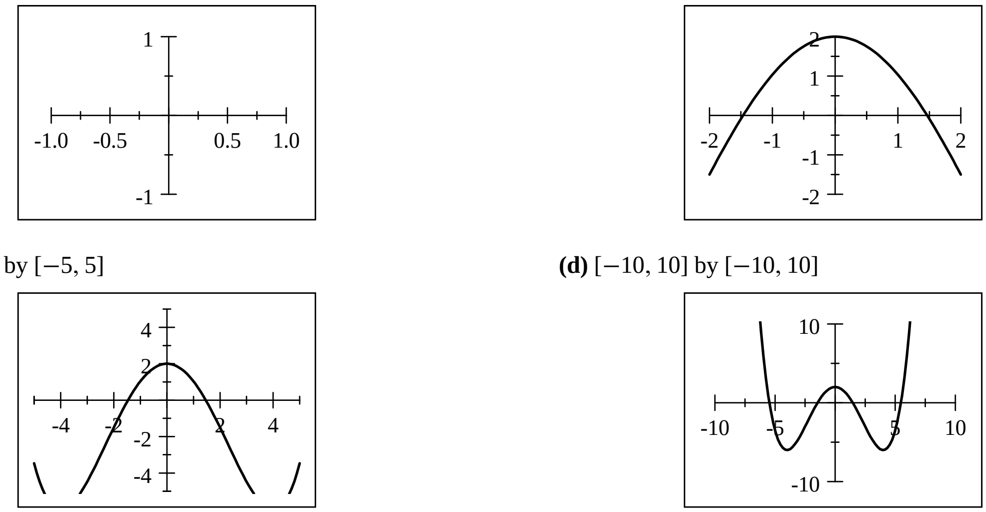


The viewing rectangle in part (c) produces the most appropriate graph of the equation.

32. $k(x) = \frac{1}{32}x^4 - x^2 + 2$

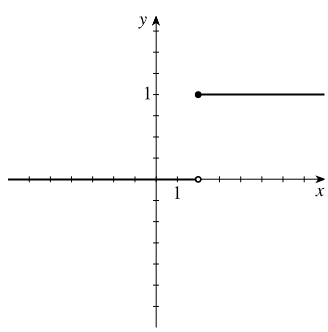
(a) $[-1, 1]$ by $[-1, 1]$ **(b)** $[-2, 2]$ by $[-2, 2]$

(c) $[-5, 5]$ by $[-5, 5]$ **(d)** $[-10, 10]$ by $[-10, 10]$

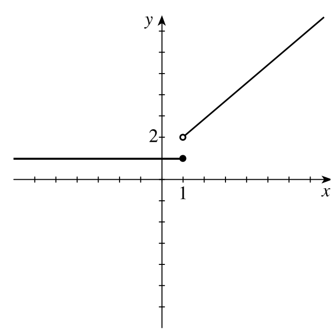


The viewing rectangle in part (d) produces the most appropriate graph of the equation.

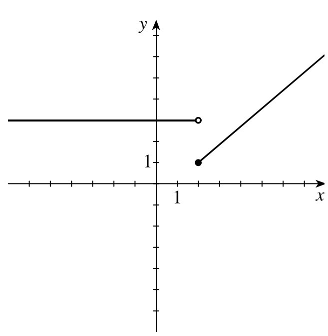
33. $f(x) = \begin{cases} 0 & \text{if } x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$



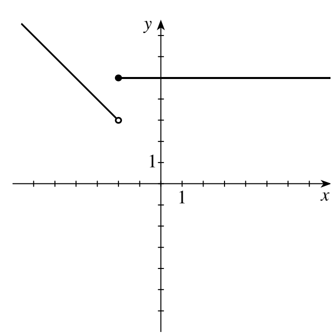
34. $f(x) = \begin{cases} 1 & \text{if } x \leq 1 \\ x + 1 & \text{if } x > 1 \end{cases}$



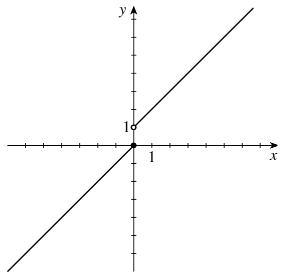
35. $f(x) = \begin{cases} 3 & \text{if } x < 2 \\ x - 1 & \text{if } x \geq 2 \end{cases}$



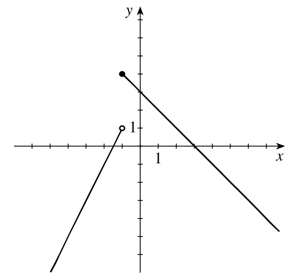
36. $f(x) = \begin{cases} 1 - x & \text{if } x < -2 \\ 5 & \text{if } x \geq -2 \end{cases}$



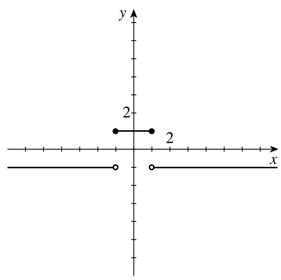
37. $f(x) = \begin{cases} x & \text{if } x \le 0 \\ x+1 & \text{if } x > 0 \end{cases}$



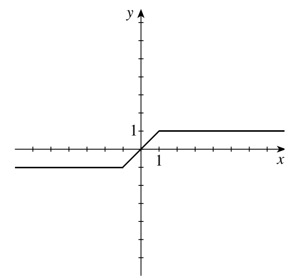
38. $f(x) = \begin{cases} 2x+3 & \text{if } x < -1 \\ 3-x & \text{if } x \ge -1 \end{cases}$



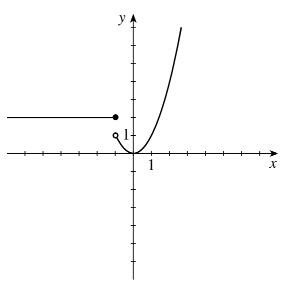
39. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ 1 & \text{if } -1 \le x \le 1 \\ -1 & \text{if } x > 1 \end{cases}$



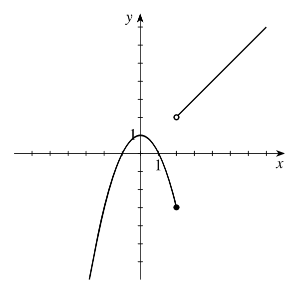
40. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ x & \text{if } -1 \le x \le 1 \\ 1 & \text{if } x > 1 \end{cases}$



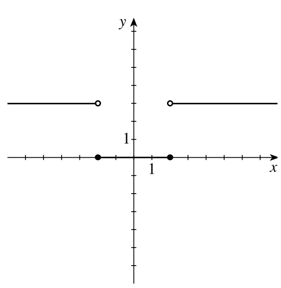
41. $f(x) = \begin{cases} 2 & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$



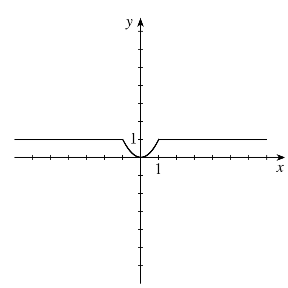
42. $f(x) = \begin{cases} 1-x^2 & \text{if } x \le 2 \\ x & \text{if } x > 2 \end{cases}$



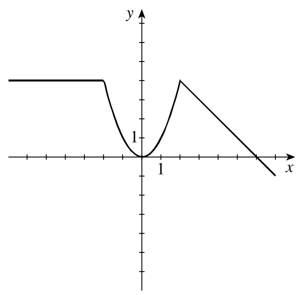
43. $f(x) = \begin{cases} 0 & \text{if } |x| \le 2 \\ 3 & \text{if } |x| > 2 \end{cases}$



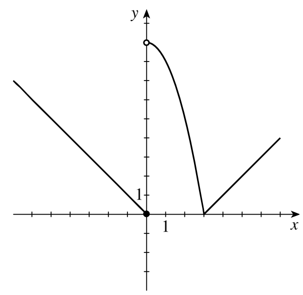
44. $f(x) = \begin{cases} x^2 & \text{if } |x| \le 1 \\ 1 & \text{if } |x| > 1 \end{cases}$



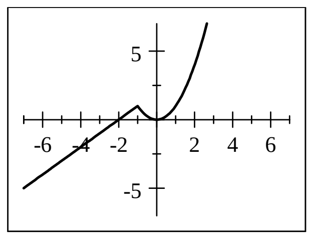
45. $f(x) = \begin{cases} 4 & \text{if } x < -2 \\ x^2 & \text{if } -2 \le x \le 2 \\ -x+6 & \text{if } x > 2 \end{cases}$



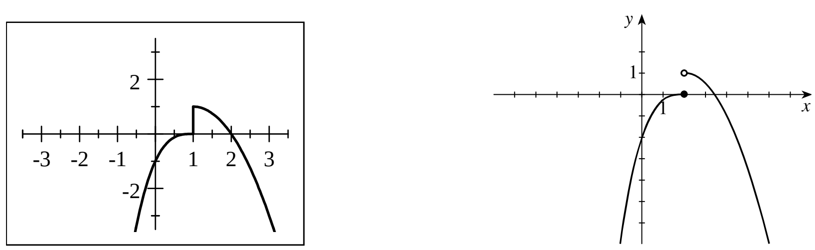
46. $f(x) = \begin{cases} -x & \text{if } x \le 0 \\ 9-x^2 & \text{if } 0 < x \le 3 \\ x-3 & \text{if } x > 3 \end{cases}$



47. $f(x) = \begin{cases} x+2 & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$



48. $f(x) = \begin{cases} 2x-x^2 & \text{if } x > 1 \\ (x-1)^3 & \text{if } x \le 1 \end{cases}$ The first graph shows the output of a typical graphing device. However, the actual graph of this function is also shown, and its difference from the graphing device's version should be noted.



49. $f(x) = \begin{cases} -2 & \text{if } x < -2 \\ x & \text{if } -2 \le x \le 2 \\ 2 & \text{if } x > 2 \end{cases}$

50. $f(x) = \begin{cases} 1 & \text{if } x \le -1 \\ 1-x & \text{if } -1 < x \le 2 \\ -2 & \text{if } x > 2 \end{cases}$

51. The curves in parts (a) and (c) are graphs of a function of x, by the Vertical Line Test.

52. The curves in parts (b) and (c) are graphs of functions of x, by the Vertical Line Test.

53. The given curve is the graph of a function of x, by the Vertical Line Test. Domain: $[-3, 2]$. Range: $[-2, 2]$.

54. No, the given curve is not the graph of a function of x, by the Vertical Line Test.

55. No, the given curve is not the graph of a function of x, by the Vertical Line Test.

56. The given curve is the graph of a function of x, by the Vertical Line Test. Domain: $[-3, 2]$. Range: $\{-2\} \cup (0, 3]$.

57. Solving for y in terms of x gives $3x - 5y = 7 \Leftrightarrow y = \frac{3}{5}x - \frac{7}{5}$. This defines y as a function of x.

58. Solving for y in terms of x gives $3x^2 - y = 5 \Leftrightarrow y = 3x^2 - 5$. This defines y as a function of x.

59. Solving for y in terms of x gives $x = y^2 \Leftrightarrow y = \pm\sqrt{x}$. The last equation gives two values of y for a given value of x. Thus, this equation does not define y as a function of x.

60. Solving for y in terms of x gives $x^2 + (y-1)^2 = 4 \Leftrightarrow (y-1)^2 = 4 - x^2 \Leftrightarrow y - 1 = \pm\sqrt{4-x^2} \Leftrightarrow y = 1 \pm \sqrt{4-x^2}$. The last equation gives two values of y for a given value of x. Thus, this equation does not define y as a function of x.

61. Solving for y in terms of x gives $2x - 4y^2 = 3 \Leftrightarrow 4y^2 = 2x - 3 \Leftrightarrow y = \pm\frac{1}{2}\sqrt{2x-3}$. The last equation gives two values of y for a given value of x. Thus, this equation does not define y as a function of x.

62. Solving for y in terms of x gives $2x^2 - 4y^2 = 3 \Leftrightarrow 4y^2 = 2x^2 - 3 \Leftrightarrow y = \pm\frac{1}{2}\sqrt{2x^2-3}$. The last equation gives two values of y for a given value of x. Thus, this equation does not define y as a function of x.

63. Solving for y in terms of x using the Quadratic Formula gives $2xy - 5y^2 = 4 \Leftrightarrow 5y^2 - 2xy + 4 = 0 \Leftrightarrow$ $y = \dfrac{-(-2x) \pm \sqrt{(-2x)^2 - 4(5)(4)}}{2(5)} = \dfrac{2x \pm \sqrt{4x^2-80}}{10} = \dfrac{x \pm \sqrt{x^2-20}}{5}$. The last equation gives two values of y for a given value of x. Thus, this equation does not define y as a function of x.

64. Solving for y in terms of x gives $\sqrt{y} - 5 = x \Leftrightarrow y = (x+5)^2$. This defines y as a function of x.

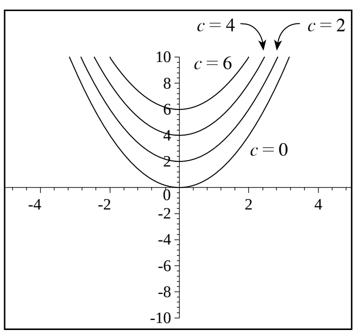
65. Solving for y in terms of x gives $2|x| + y = 0 \Leftrightarrow y = -2|x|$. This defines y as a function of x.

66. Solving for y in terms of x gives $2x + |y| = 0 \Leftrightarrow |y| = -2x$. Since $|a| = |-a|$, the last equation gives two values of y for a given value of x. Thus, this equation does not define y as a function of x.

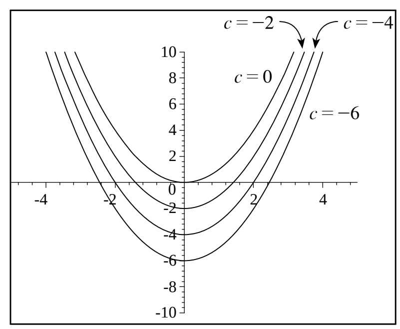
67. Solving for y in terms of x gives $x = y^3 \Leftrightarrow y = \sqrt[3]{x}$. This defines y as a function of x.

68. Solving for y in terms of x gives $x = y^4 \Leftrightarrow y = \pm\sqrt[4]{x}$. The last equation gives two values of y for any positive value of x. Thus, this equation does not define y as a function of x.

69. **(a)** $f(x) = x^2 + c$, for $c = 0, 2, 4$, and 6.

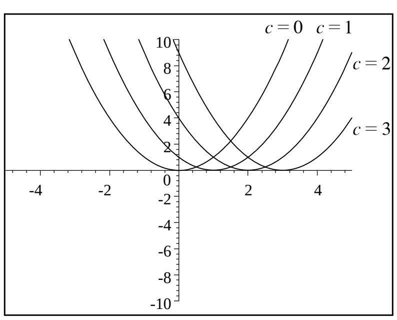


(b) $f(x) = x^2 + c$, for $c = 0, -2, -4$, and -6.

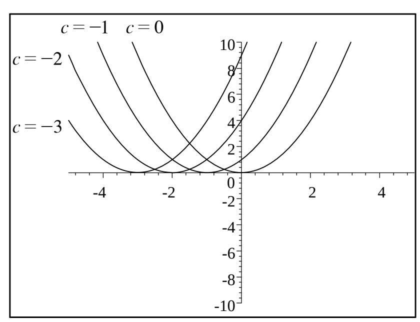


(c) The graphs in part (a) are obtained by shifting the graph of $f(x) = x^2$ upward c units, $c > 0$. The graphs in part (b) are obtained by shifting the graph of $f(x) = x^2$ downward c units.

70. **(a)** $f(x) = (x-c)^2$, for $c = 0, 1, 2$, and 3.

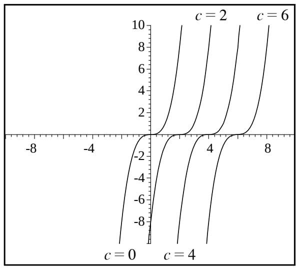


(b) $f(x) = (x-c)^2$, for $c = 0, -1, -2$, and -3.

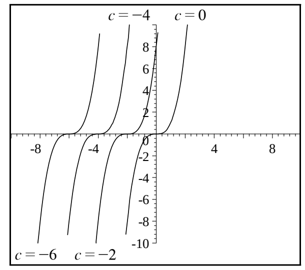


(c) The graphs in part (a) are obtained by shifting the graph of $y = x^2$ to the right 1, 2, and 3 units, while the graphs in part (b) are obtained by shifting the graph of $y = x^2$ to the left 1, 2, and 3 units.

71. (a) $f(x) = (x - c)^3$, for $c = 0, 2, 4$, and 6.

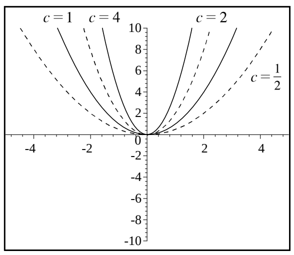


(b) $f(x) = (x - c)^3$, for $c = 0, -2, -4$, and -6.

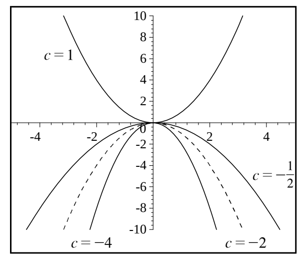


(c) The graphs in part (a) are obtained by shifting the graph of $f(x) = x^3$ to the right c units, $c > 0$. The graphs in part (b) are obtained by shifting the graph of $f(x) = x^3$ to the left $|c|$ units, $c < 0$.

72. (a) $f(x) = cx^2$, for $c = 1, \frac{1}{2}, 2$, and 4.

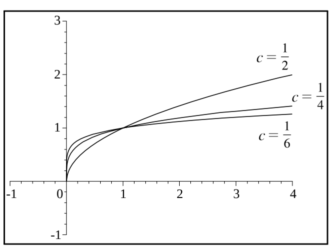


(b) $f(x) = cx^2$, for $c = 1, -1, -\frac{1}{2}$, and -2.

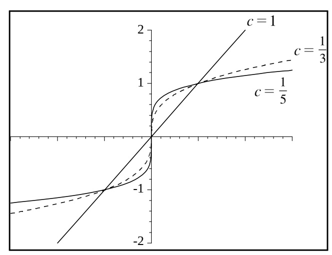


(c) As $|c|$ increases, the graph of $f(x) = cx^2$ is stretched vertically. As $|c|$ decreases, the graph of f is flattened. When $c < 0$, the graph is reflected about the x-axis.

73. (a) $f(x) = x^c$, for $c = \frac{1}{2}, \frac{1}{4}$, and $\frac{1}{6}$.

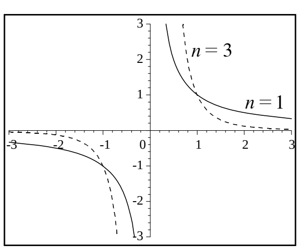


(b) $f(x) = x^c$, for $c = 1, \frac{1}{3}$, and $\frac{1}{5}$.

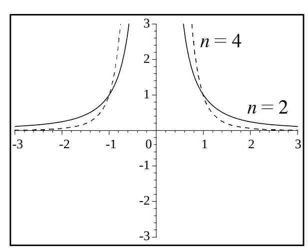


(c) Graphs of even roots are similar to $y = \sqrt{x}$, graphs of odd roots are similar to $y = \sqrt[3]{x}$. As c increases, the graph of $y = \sqrt[c]{x}$ becomes steeper near $x = 0$ and flatter when $x > 1$.

74. (a) $f(x) = \dfrac{1}{x^n}$, for $n = 1$ and 3.



(b) $f(x) = \dfrac{1}{x^n}$, for $n = 2$ and 4.



(c) As n increases, the graphs of $y = 1/x^n$ go to zero faster for x large. Also, as n increases and x goes to 0, the graphs of $y = 1/x^n$ go to infinity faster. The graphs of $y = 1/x^n$ for n odd are similar to each other. Likewise, the graphs for n even are similar to each other.

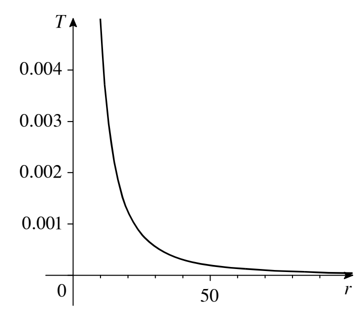
75. The slope of the line segment joining the points $(-2, 1)$ and $(4, -6)$ is $m = \dfrac{-6-1}{4-(-2)} = -\frac{7}{6}$. Using the point-slope form, we have $y - 1 = -\frac{7}{6}(x+2) \Leftrightarrow y = -\frac{7}{6}x - \frac{7}{3} + 1 \Leftrightarrow y = -\frac{7}{6}x - \frac{4}{3}$. Thus the function is $f(x) = -\frac{7}{6}x - \frac{4}{3}$ for $-2 \le x \le 4$.

76. The slope of the line containing the points $(-3, -2)$ and $(6, 3)$ is $m = \dfrac{-2-3}{-3-6} = \dfrac{-5}{-9} = \frac{5}{9}$. Using the point-slope equation of the line, we have $y - 3 = \frac{5}{9}(x-6) \Leftrightarrow y = \frac{5}{9}x - \frac{10}{3} + 3 = \dfrac{5}{9}x - \frac{1}{3}$. Thus the function is $f(x) = \frac{5}{9}x - \frac{1}{3}$, for $-3 \le x \le 6$.

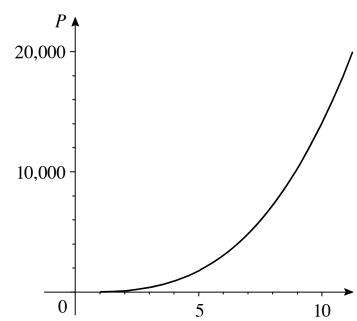
77. First solve the circle for y: $x^2 + y^2 = 9 \Leftrightarrow y^2 = 9 - x^2 \Rightarrow y = \pm\sqrt{9-x^2}$. Since we seek the top half of the circle, we choose $y = \sqrt{9-x^2}$. So the function is $f(x) = \sqrt{9-x^2}$, $-3 \le x \le 3$.

78. First solve the circle for y: $x^2 + y^2 = 9 \Leftrightarrow y^2 = 9 - x^2 \Rightarrow y = \pm\sqrt{9-x^2}$. Since we seek the bottom half of the circle, we choose $y = -\sqrt{9-x^2}$. So the function is $f(x) = -\sqrt{9-x^2}$, $-3 \le x \le 3$.

79. We graph $T(r) = \dfrac{0.5}{r^2}$ for $10 \le r \le 100$. As the balloon is inflated, the skin gets thinner, as we would expect.

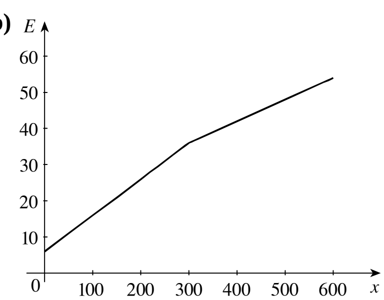


80. We graph $P(v) = 14.1v^3$ for $1 \le v \le 10$. As wind speed increases, so does power output, as expected.

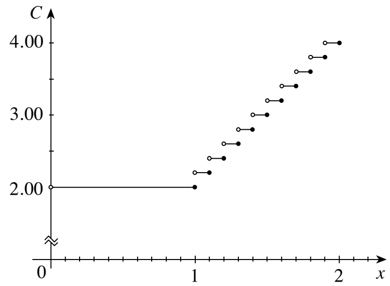


81. (a) $E(x) = \begin{cases} 6.00 + 0.10x & \text{if } 0 \le x \le 300 \\ 36.00 + 0.06(x - 300) & \text{if } 300 < x \end{cases}$

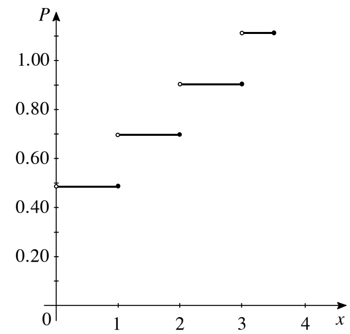
(b)



82. $C(x) = \begin{cases} 2.00 & \text{if } 0 < x \le 1 \\ 2.20 & \text{if } 1 < x \le 1.1 \\ 2.40 & \text{if } 1.1 < x \le 1.2 \\ \vdots & \\ 4.00 & \text{if } 1.9 < x < 2 \end{cases}$



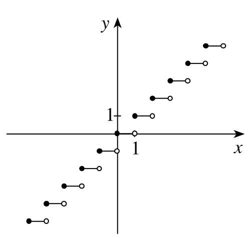
83. $P(x) = \begin{cases} 0.49 & \text{if } 0 < x \le 1 \\ 0.70 & \text{if } 1 < x \le 2 \\ 0.91 & \text{if } 2 < x \le 3 \\ 1.12 & \text{if } 3 < x \le 3.5 \end{cases}$



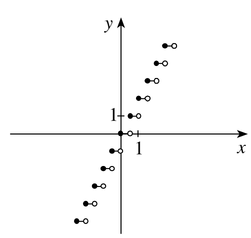
84. The graph of $x = y^2$ is not the graph of a function because both $(1, 1)$ and $(-1, 1)$ satisfy the equation $x = y^2$. The graph of $x = y^3$ is the graph of a function because $x = y^3 \Leftrightarrow x^{1/3} = y$. If n is even, then both $(1, 1)$ and $(-1, 1)$ satisfies the equation $x = y^n$, so the graph of $x = y^n$ is not the graph of a function. When n is odd, $y = x^{1/n}$ is defined for all real numbers, and since $y = x^{1/n} \Leftrightarrow x = y^n$, the graph of $x = y^n$ is the graph of a function.

85. Answers will vary. Some examples are almost anything we purchase based on weight, volume, length, or time, for example gasoline. Although the amount delivered by the pump is continuous, the amount we pay is rounded to the penny. An example involving time would be the cost of a telephone call.

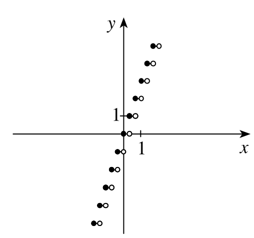
86.



$f(x) = [\![x]\!]$



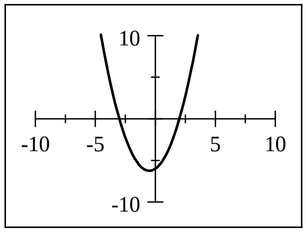
$g(x) = [\![2x]\!]$

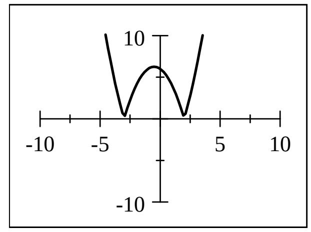


$h(x) = [\![3x]\!]$

The graph of $k(x) = [\![nx]\!]$ is a step function whose steps are each $\dfrac{1}{n}$ wide.

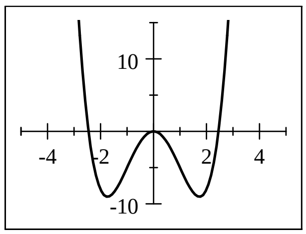
87. **(a)** The graphs of $f(x) = x^2 + x - 6$ and $g(x) = \left|x^2 + x - 6\right|$ are shown in the viewing rectangle $[-10, 10]$ by $[-10, 10]$.

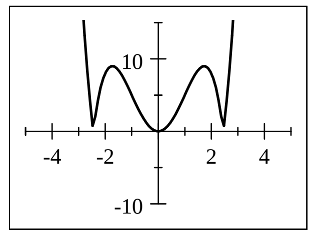




For those values of x where $f(x) \geq 0$, the graphs of f and g coincide, and for those values of x where $f(x) < 0$, the graph of g is obtained from that of f by reflecting the part below the x-axis about the x-axis.

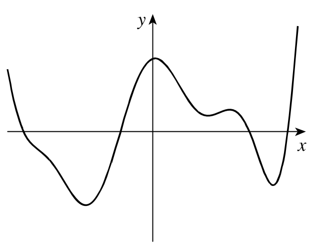
(b) The graphs of $f(x) = x^4 - 6x^2$ and $g(x) = \left|x^4 - 6x^2\right|$ are shown in the viewing rectangle $[-5, 5]$ by $[-10, 15]$.



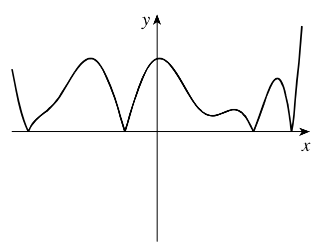


For those values of x where $f(x) \geq 0$, the graphs of f and g coincide, and for those values of x where $f(x) < 0$, the graph of g is obtained from that of f by reflecting the part below the x-axis above the x-axis.

(c) In general, if $g(x) = |f(x)|$, then for those values of x where $f(x) \geq 0$, the graphs of f and g coincide, and for those values of x where $f(x) < 0$, the graph of g is obtained from that of f by reflecting the part below the x-axis above the x-axis.



$y = f(x)$



$y = g(x)$

2.3 GETTING INFORMATION FROM THE GRAPH OF A FUNCTION

1. To find a function value $f(a)$ from the graph of f we find the height of the graph above the x-axis at $x = a$. From the graph of f we see that $f(3) = 4$ and $f(1) = 0$. The net change in f between $x = 1$ and $x = 3$ is $f(3) - f(1) = 4 - 0 = 4$.

2. The domain of the function f is all the x-values of the points on the graph, and the range is all the corresponding y-values. From the graph of f we see that the domain of f is the interval $(-\infty, \infty)$ and the range of f is the interval $(-\infty, 7]$.

3. **(a)** If f is increasing on an interval, then the y-values of the points on the graph *rise* as the x-values increase. From the graph of f we see that f is increasing on the intervals $(-\infty, 2)$ and $(4, 5)$.

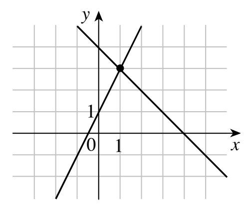
(b) If f is decreasing on an interval, then y-values of the points on the graph *fall* as the x-values increase. From the graph of f we see that f is decreasing on the intervals $(2, 4)$ and $(5, \infty)$.

4. (a) A function value $f(a)$ is a local maximum value of f if $f(a)$ is the *largest* value of f on some interval containing a. From the graph of f we see that there are two local maximum values of f: one maximum is 7, and it occurs when $x = 2$; the other maximum is 6, and it occurs when $x = 5$.

(b) A function value $f(a)$ is a local minimum value of f if $f(a)$ is the *smallest* value of f on some interval containing a. From the graph of f we see that there is one local minimum value of f. The minimum value is 2, and it occurs when $x = 4$.

5. The solutions of the equation $f(x) = 0$ are the x-intercepts of the graph of f. The solution of the inequality $f(x) \geq 0$ is the set of x-values at which the graph of f is on or above the x-axis. From the graph of f we find that the solutions of the equation $f(x) = 0$ are $x = 1$ and $x = 7$, and the solution of the inequality $f(x) \geq 0$ is the interval $[1, 7]$.

6. (a) To solve the equation $2x + 1 = -x + 4$ graphically we graph the functions $f(x) = 2x + 1$ and $g(x) = -x + 4$ on the same set of axes and determine the values of x at which the graphs of f and g intersect. From the graph, we see that the solution is $x = 1$.



(b) To solve the inequality $2x + 1 < -x + 4$ graphically we graph the functions $f(x) = 2x + 1$ and $g(x) = -x + 4$ on the same set of axes and find the values of x at which the graph of g is *higher* than the graph of f. From the graphs in part (a) we see that the solution of the inequality is $(-\infty, 1)$.

7. (a) $h(-2) = 1$, $h(0) = -1$, $h(2) = 3$, and $h(3) = 4$.

(b) Domain: $[-3, 4]$. Range: $[-1, 4]$.

(c) $h(-3) = 3$, $h(2) = 3$, and $h(4) = 3$, so $h(x) = 3$ when $x = -3$, $x = 2$, or $x = 4$.

(d) The graph of h lies below or on the horizontal line $y = 3$ when $-3 \leq x \leq 2$ or $x = 4$, so $h(x) \leq 3$ for those values of x.

(e) The net change in h between $x = -3$ and $x = 3$ is $h(3) - h(-3) = 4 - 3 = 1$.

8. (a) $g(-4) = 3$, $g(-2) = 2$, $g(0) = -2$, $g(2) = 1$, and $g(4) = 0$.

(b) Domain: $[-4, 4]$. Range: $[-2, 3]$.

(c) $g(-4) = 3$. [Note that $g(2) = 1$ not 3.]

(d) It appears that $g(x) \leq 0$ for $-1 \leq x \leq 1.8$ and for $x = 4$; that is, for $\{x \mid -1 \leq x \leq 1.8\} \cup \{4\}$.

(e) $g(-1) = 0$ and $g(2) = 1$, so the net change between $x = -1$ and $x = 2$ is $1 - 0 = 1$.

9. (a) $f(0) = 3 > \frac{1}{2} = g(0)$. So $f(0)$ is larger.

(b) $f(-3) \approx -1 < 2.5 = g(-3)$. So $g(-3)$ is larger.

(c) $f(x) = g(x)$ for $x = -2$ and $x = 2$.

(d) $f(x) \leq g(x)$ for $-4 \leq x \leq -2$ and $2 \leq x \leq 3$; that is, on the intervals $[-4, -2]$ and $[2, 3]$.

(e) $f(x) > g(x)$ for $-2 < x < 2$; that is, on the interval $(-2, 2)$.

10. (a) The graph of g is higher than the graph of f at $x = 6$, so $g(6)$ is larger.

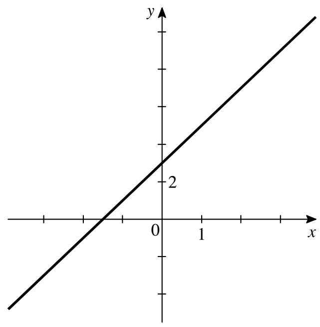
(b) The graph of f is higher than the graph of g at $x = 3$, so $f(3)$ is larger.

(c) The graphs of f and g intersect at $x = 2$, $x = 5$, and $x \approx 7$, so $f(x) = g(x)$ for these values of x.

(d) $f(x) \leq g(x)$ for $1 \leq x \leq 2$ and approximately $5 \leq x \leq 7$; that is, on $[1, 2]$ and $[5, 7]$.

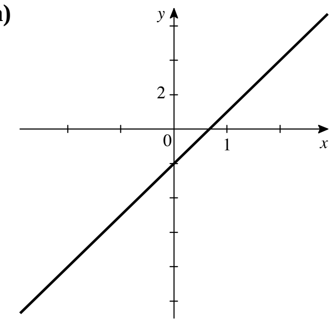
(e) $f(x) > g(x)$ for $2 < x < 5$ and approximately $7 < x \leq 8$; that is, on $[2, 5)$ and $(7, 8]$.

11. (a)



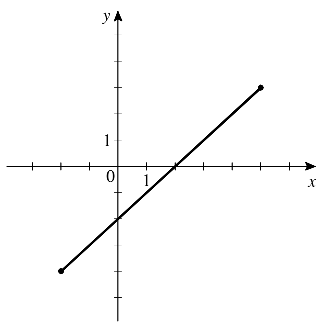
(b) Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$

12. (a)



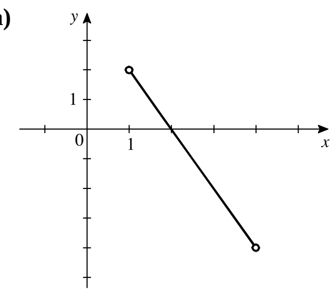
(b) Domain: $(-\infty, \infty)$; Range $(-\infty, \infty)$

13. (a)



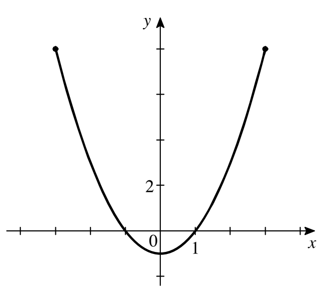
(b) Domain: $[-2, 5]$; Range $[-4, 3]$

14. (a)



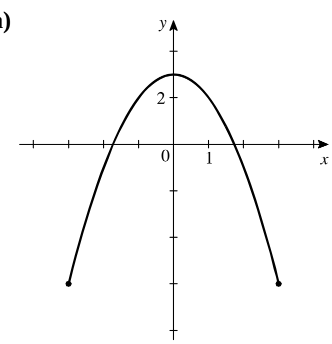
(b) Domain: $(1, 4)$; Range $(-4, 2)$

15. (a)



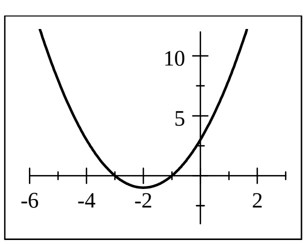
(b) Domain: $[-3, 3]$; Range: $[-1, 8]$

16. (a)



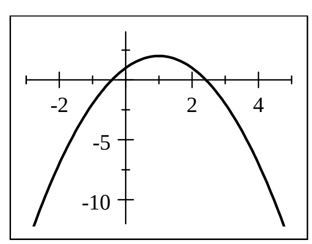
(b) Domain: $[-3, 3]$; Range $[-6, 3]$

17. (a)



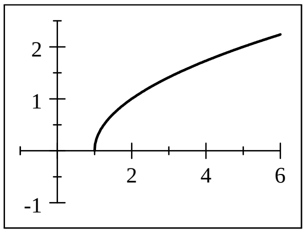
(b) Domain: $(-\infty, \infty)$; Range: $[-1, \infty)$

18. (a)



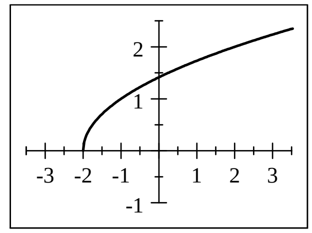
(b) Domain: $(-\infty, \infty)$; Range: $(-\infty, 2]$

19. (a)



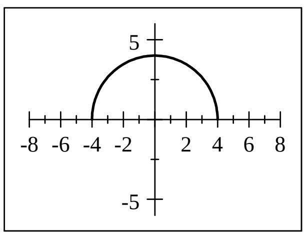
(b) Domain: $[1, \infty)$; Range: $[0, \infty)$

20. (a)



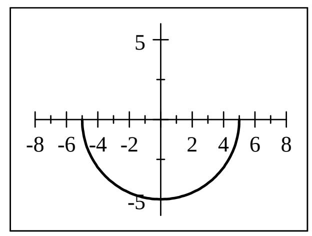
(b) Domain: $[-2, \infty)$; Range: $[0, \infty)$

21. (a)



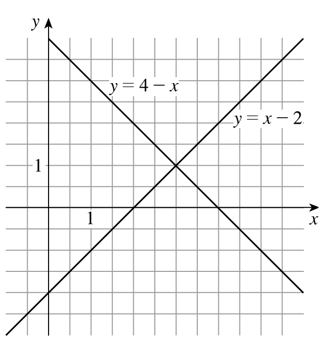
(b) Domain: $[-4, 4]$; Range: $[0, 4]$

22. (a)



(b) Domain: $[-5, 5]$; Range: $[-5, 0]$

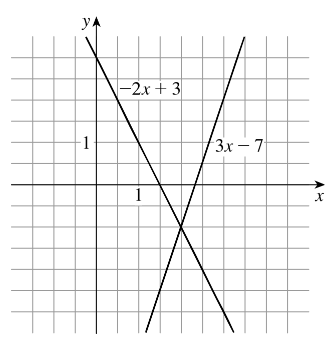
23.



(a) From the graph, we see that $x - 2 = 4 - x$ when $x = 3$.

(b) From the graph, we see $x - 2 > 4 - x$ when $x > 3$.

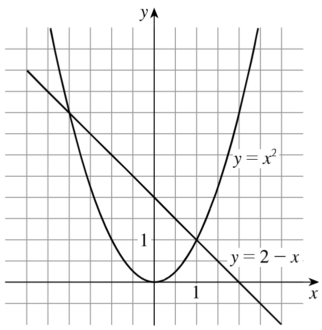
24.



(a) From the graph, we see that $-2x + 3 = 3x - 7$ when $x = 2$.

(b) From the graph, we see that $-2x + 3 \leq 3x - 7$ when $x \geq 2$.

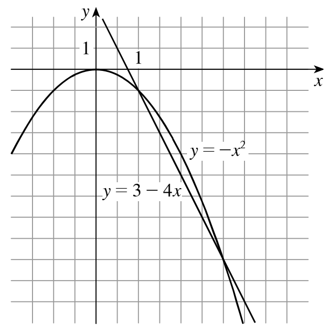
25.



(a) From the graph, we see that $x^2 = 2 - x$ when $x = -2$ or $x = 1$.

(b) From the graph, we see that $x^2 \le 2 - x$ when $-2 \le x \le 1$.

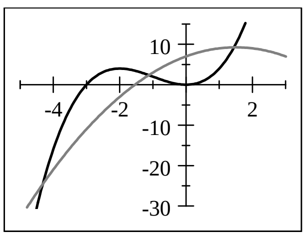
26.



(a) From the graph, we see that $-x^2 = 3 - 4x$ when $x = 1$ or $x = 3$.

(b) From the graph, we see that $-x^2 \ge 3 - 4x$ when $1 \le x \le 3$.

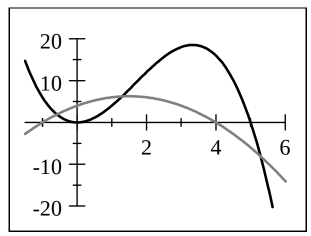
27.



(a) We graph $y = x^3 + 3x^2$ (black) and $y = -x^2 + 3x + 7$ (gray). From the graph, we see that the graphs intersect at $x \approx -4.32$, $x \approx -1.12$, and $x \approx 1.44$.

(b) From the graph, we see that $x^3 + 3x^2 \ge -x^2 + 3x + 7$ on approximately $[-4.32, -1.12]$ and $[1.44, \infty)$.

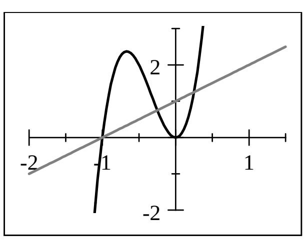
28.



(a) We graph $y = 5x^2 - x^3$ (black) and $y = -x^2 + 3x + 4$ (gray). From the graph, we see that the graphs intersect at $x \approx -0.58$, $x \approx 1.29$, and $x \approx 5.29$.

(b) From the graph, we see that $5x^2 - x^3 \le -x^2 + 3x + 4$ on approximately $[-0.58, 1.29]$ and $[5.29, \infty)$.

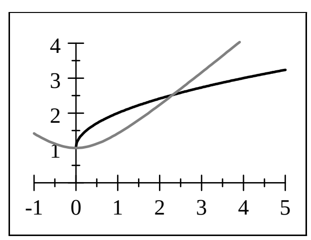
29.



(a) We graph $y = 16x^3 + 16x^2$ (black) and $y = x + 1$ (gray). From the graph, we see that the graphs intersect at $x = -1$, $x = -\frac{1}{4}$, and $x = \frac{1}{4}$.

(b) From the graph, we see that $16x^3 + 16x^2 \ge x + 1$ on $\left[-1, -\frac{1}{4}\right]$ and $\left[\frac{1}{4}, \infty\right)$.

30.



(a) We graph $y = 1 + \sqrt{x}$ (black) and $y = \sqrt{x^2 + 1}$ (gray). From the graph, we see that the solutions are $x = 0$ and $x \approx 2.31$.

(b) From the graph, we see that $1 + \sqrt{x} > \sqrt{x^2 + 1}$ on approximately $(0, 2.31)$.

31. **(a)** The domain is $[-1, 4]$ and the range is $[-1, 3]$.

(b) The function is increasing on $(-1, 1)$ and $(2, 4)$ and decreasing on $(1, 2)$.

32. **(a)** The domain is $[-2, 3]$ and the range is $[-2, 3]$.

(b) The function is increasing on $(0, 1)$ and decreasing on $(-2, 0)$ and $(1, 3)$.

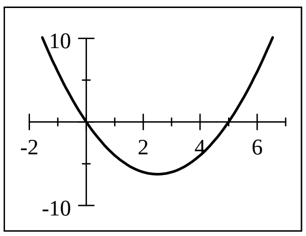
33. **(a)** The domain is $[-3, 3]$ and the range is $[-2, 2]$.

(b) The function is increasing on $(-2, -1)$ and $(1, 2)$ and decreasing on $(-3, -2)$, $(-1, 1)$, and $(2, 3)$.

34. **(a)** The domain is $[-2, 2]$ and the range is $[-2, 2]$.

(b) The function is increasing on $(-1, 1)$ and decreasing on $(-2, -1)$ and $(1, 2)$.

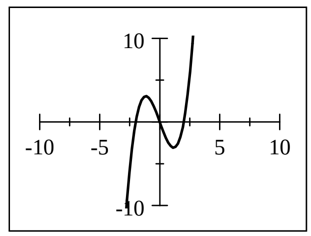
35. **(a)** $f(x) = x^2 - 5x$ is graphed in the viewing rectangle $[-2, 7]$ by $[-10, 10]$.



(b) The domain is $(-\infty, \infty)$ and the range is $[-6.25, \infty)$.

(c) The function is increasing on $(2.5, \infty)$. It is decreasing on $(-\infty, 2.5)$.

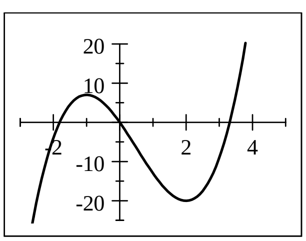
36. **(a)** $f(x) = x^3 - 4x$ is graphed in the viewing rectangle $[-10, 10]$ by $[-10, 10]$.



(b) The domain and range are $(-\infty, \infty)$.

(c) The function is increasing on $(-\infty, -1.15)$ and $(1.15, \infty)$. It is decreasing on $(-1.15, 1.15)$.

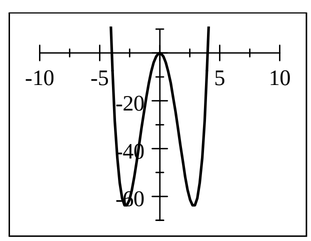
37. **(a)** $f(x) = 2x^3 - 3x^2 - 12x$ is graphed in the viewing rectangle $[-3, 5]$ by $[-25, 20]$.



(b) The domain and range are $(-\infty, \infty)$.

(c) The function is increasing on $(-\infty, -1)$ and $(2, \infty)$. It is decreasing on $(-1, 2)$.

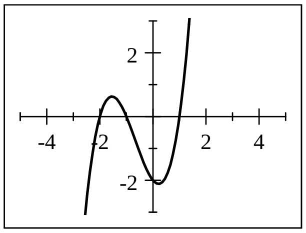
38. **(a)** $f(x) = x^4 - 16x^2$ is graphed in the viewing rectangle $[-10, 10]$ by $[-70, 10]$.



(b) The domain is $(-\infty, \infty)$ and the range is $[-64, \infty)$.

(c) The function is increasing on $(-2.83, 0)$ and $(2.83, \infty)$. It is decreasing on $(-\infty, -2.83)$ and $(0, 2.83)$.

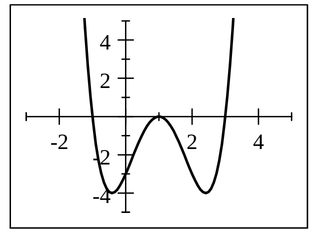
39. (a) $f(x) = x^3 + 2x^2 - x - 2$ is graphed in the viewing rectangle $[-5, 5]$ by $[-3, 3]$.



(b) The domain and range are $(-\infty, \infty)$.

(c) The function is increasing on $(-\infty, -1.55)$ and $(0.22, \infty)$. It is decreasing on $(-1.55, 0.22)$.

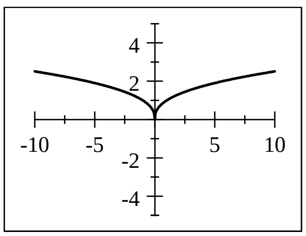
40. (a) $f(x) = x^4 - 4x^3 + 2x^2 + 4x - 3$ is graphed in the viewing rectangle $[-3, 5]$ by $[-5, 5]$.



(b) The domain is $(-\infty, \infty)$ and the range is $[-4, \infty)$.

(c) The function is increasing on $(-0.4, 1)$ and $(2.4, \infty)$. It is decreasing on $(-\infty, -0.4)$ and $(1, 2.4)$.

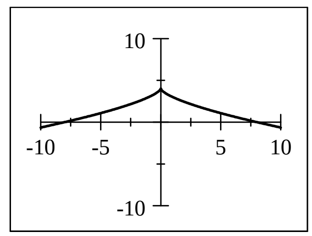
41. (a) $f(x) = x^{2/5}$ is graphed in the viewing rectangle $[-10, 10]$ by $[-5, 5]$.



(b) The domain is $(-\infty, \infty)$ and the range is $[0, \infty)$.

(c) The function is increasing on $(0, \infty)$. It is decreasing on $(-\infty, 0)$.

42. (a) $f(x) = 4 - x^{2/3}$ is graphed in the viewing rectangle $[-10, 10]$ by $[-10, 10]$.



(b) The domain is $(-\infty, \infty)$ and the range is $(-\infty, 4]$.

(c) The function is increasing on $(-\infty, 0)$. It is decreasing on $(0, \infty)$.

43. (a) Local maximum: 2 at $x = 0$. Local minimum: -1 at $x = -2$ and 0 at $x = 2$.

(b) The function is increasing on $(-2, 0)$ and $(2, \infty)$ and decreasing on $(-\infty, -2)$ and $(0, 2)$.

44. (a) Local maximum: 2 at $x = -2$ and 1 at $x = 2$. Local minimum: -1 at $x = 0$.

(b) The function is increasing on $(-\infty, -2)$ and $(0, 2)$ and decreasing on $(-2, 0)$ and $(2, \infty)$.

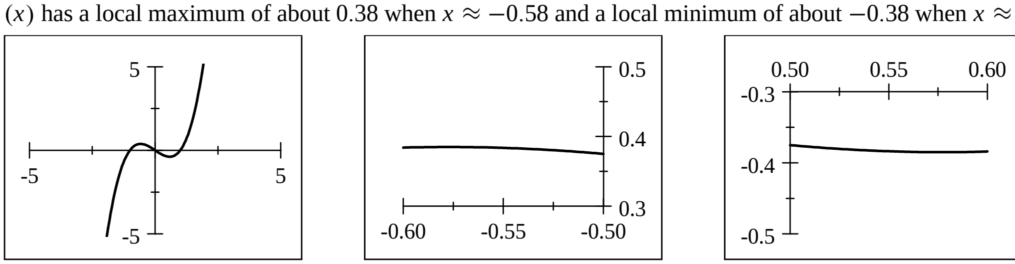
45. (a) Local maximum: 0 at $x = 0$ and 1 at $x = 3$. Local minimum: -2 at $x = -2$ and -1 at $x = 1$.

(b) The function is increasing on $(-2, 0)$ and $(1, 3)$ and decreasing on $(-\infty, -2)$, $(0, 1)$, and $(3, \infty)$.

46. (a) Local maximum: 3 at $x = -2$ and 2 at $x = 1$. Local minimum: 0 at $x = -1$ and -1 at $x = 2$.

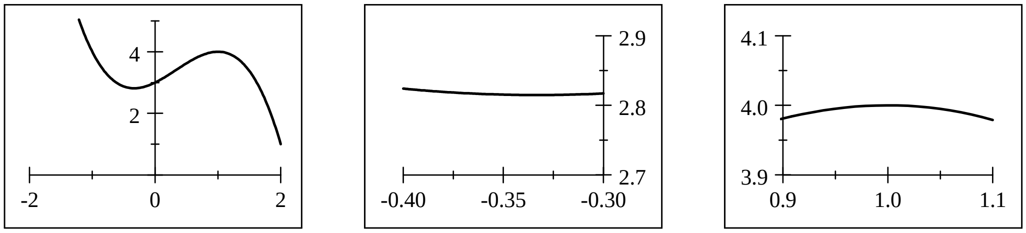
(b) The function is increasing on $(-\infty, -2)$, $(-1, 1)$, and $(2, \infty)$ and decreasing on $(-2, -1)$ and $(1, 2)$.

47. (a) In the first graph, we see that $f(x) = x^3 - x$ has a local minimum and a local maximum. Smaller x- and y-ranges show that $f(x)$ has a local maximum of about 0.38 when $x \approx -0.58$ and a local minimum of about -0.38 when $x \approx 0.58$.



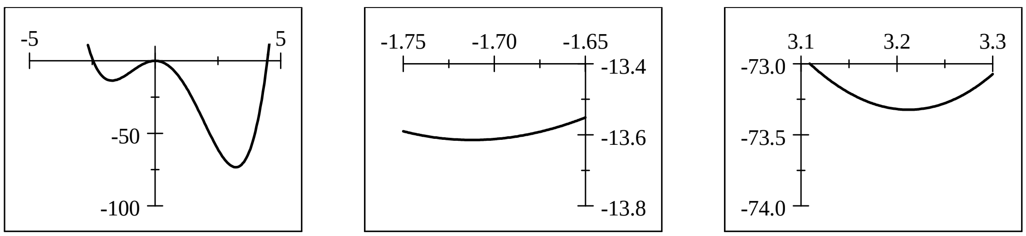
(b) The function is increasing on $(-\infty, -0.58)$ and $(0.58, \infty)$ and decreasing on $(-0.58, 0.58)$.

48. (a) In the first graph, we see that $f(x) = 3 + x + x^2 - x^3$ has a local minimum and a local maximum. Smaller x- and y-ranges show that $f(x)$ has a local maximum of about 4.00 when $x \approx 1.00$ and a local minimum of about 2.81 when $x \approx -0.33$.



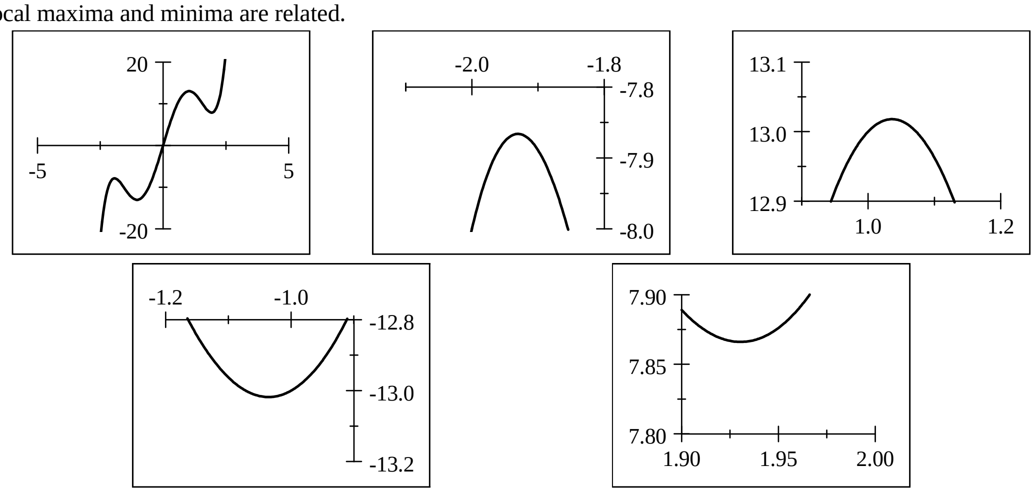
(b) The function is increasing on $(-0.33, 1.00)$ and decreasing on $(-\infty, -0.33)$ and $(1.00, \infty)$.

49. (a) In the first graph, we see that $g(x) = x^4 - 2x^3 - 11x^2$ has two local minimums and a local maximum. The local maximum is $g(x) = 0$ when $x = 0$. Smaller x- and y-ranges show that local minima are $g(x) \approx -13.61$ when $x \approx -1.71$ and $g(x) \approx -73.32$ when $x \approx 3.21$.



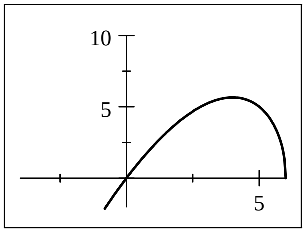
(b) The function is increasing on $(-1.71, 0)$ and $(3.21, \infty)$ and decreasing on $(-\infty, -1.71)$ and $(0, 3.21)$.

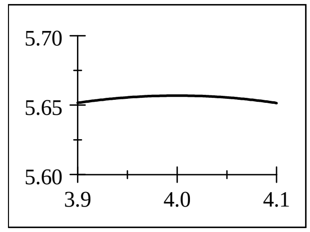
50. (a) In the first graph, we see that $g(x) = x^5 - 8x^3 + 20x$ has two local minimums and two local maximums. The local maximums are $g(x) \approx -7.87$ when $x \approx -1.93$ and $g(x) \approx 13.02$ when $x = 1.04$. Smaller x- and y-ranges show that local minimums are $g(x) \approx -13.02$ when $x = -1.04$ and $g(x) \approx 7.87$ when $x \approx 1.93$. Notice that since $g(x)$ is odd, the local maxima and minima are related.



(b) The function is increasing on $(-\infty, -1.93)$, $(-1.04, 1.04)$, and $(1.93, \infty)$ and decreasing on $(-1.93, -1.04)$ and $(1.04, 1.93)$.

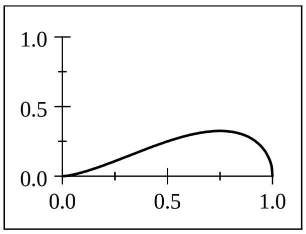
51. (a) In the first graph, we see that $U(x) = x\sqrt{6-x}$ has only a local maximum. Smaller x- and y-ranges show that $U(x)$ has a local maximum of about 5.66 when $x \approx 4.00$.

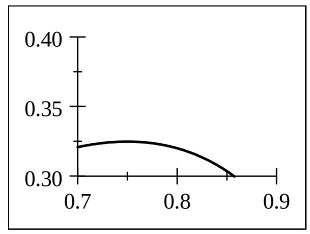




(b) The function is increasing on $(-\infty, 4.00)$ and decreasing on $(4.00, 6)$.

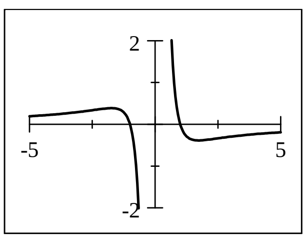
52. (a) In the first viewing rectangle below, we see that $U(x) = x\sqrt{x-x^2}$ has only a local maximum. Smaller x- and y-ranges show that $U(x)$ has a local maximum of about 0.32 when $x \approx 0.75$.

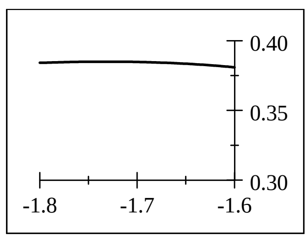


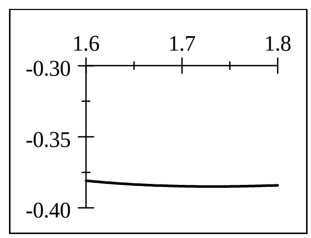


(b) The function is increasing on $(0, 0.75)$ and decreasing on $(0.75, 1)$.

53. (a) In the first graph, we see that $V(x) = \dfrac{1-x^2}{x^3}$ has a local minimum and a local maximum. Smaller x- and y-ranges show that $V(x)$ has a local maximum of about 0.38 when $x \approx -1.73$ and a local minimum of about -0.38 when $x \approx 1.73$.

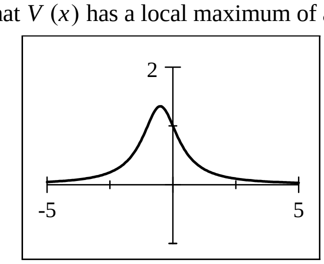


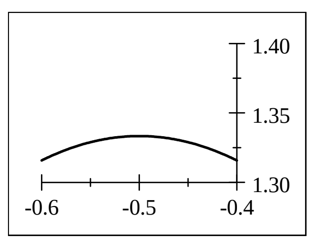




(b) The function is increasing on $(-\infty, -1.73)$ and $(1.73, \infty)$ and decreasing on $(-1.73, 0)$ and $(0, 1.73)$.

54. (a) In the first viewing rectangle below, we see that $V(x) = \dfrac{1}{x^2+x+1}$ has only a local maximum. Smaller x- and y-ranges show that $V(x)$ has a local maximum of about 1.33 when $x \approx -0.50$.





(b) The function is increasing on $(-\infty, -0.50)$ and decreasing on $(-0.50, \infty)$.

55. (a) At 6 A.M. the graph shows that the power consumption is about 500 megawatts. Since $t = 18$ represents 6 P.M., the graph shows that the power consumption at 6 P.M. is about 725 megawatts.

(b) The power consumption is lowest between 3 A.M. and 4 A.M..

(c) The power consumption is highest just before 12 noon.

(d) The net change in power consumption from 9 A.M. to 7 P.M. is $P(19) - P(9) \approx 690 - 790 \approx -100$ megawatts.

56. (a) The first noticeable movements occurred at time $t = 5$ seconds.

(b) It seemed to end at time $t = 30$ seconds.

(c) Maximum intensity was reached at $t = 17$ seconds.

57. (a) This person appears to be gaining weight steadily until the age of 21 when this person's weight gain slows down. The person continues to gain weight until the age of 30, at which point this person experiences a sudden weight loss. Weight gain resumes around the age of 32, and the person dies at about age 68. Thus, the person's weight W is increasing on $(0, 30)$ and $(32, 68)$ and decreasing on $(30, 32)$.

(b) The sudden weight loss could be due to a number of reasons, among them major illness, a weight loss program, etc.

(c) The net change in the person's weight from age 10 to age 20 is $W(20) - W(10) = 150 - 50 = 100$ lb.

58. (a) Measuring in hours since midnight, the salesman's distance from home D is increasing on $(8, 9)$, $(10, 12)$, and $(15, 17)$, constant on $(9, 10)$, $(12, 13)$, and $(17, 18)$, and decreasing on $(13, 15)$ and $(18, 19)$.

(b) The salesman travels away from home and stops to make a sales call between 9 A.M. and 10 A.M., and then travels further from home for a sales call between 12 noon and 1 P.M. Next he travels along a route that takes him closer to home before taking him further away from home. He then makes a final sales call between 5 P.M. and 6 P.M. and then returns home.

(c) The net change in the distance D from noon to 1 P.M. is $D(1\text{ P.M.}) - D(\text{noon}) = 0$.

59. (a) The function W is increasing on $(0, 150)$ and $(300, \infty)$ and decreasing on $(150, 300)$.

(b) W has a local maximum at $x = 150$ and a local minimum at $x = 300$.

(c) The net change in the depth W from 100 days to 300 days is $W(300) - W(100) = 25 - 75 = -50$ ft.

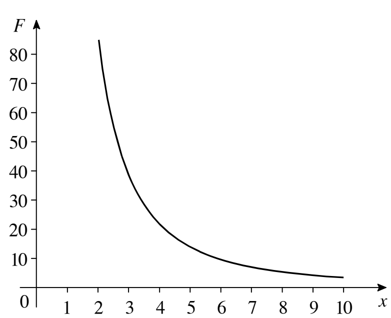
60. (a) The function P is increasing on $(0, 25)$ and decreasing on $(25, 50)$.

(b) The maximum population was 50,000, and it was attained at $x = 25$ years, which represents the year 1975.

(c) The net change in the population P from 1970 to 1990 is $P(40) - P(20) = 40 - 40 = 0$.

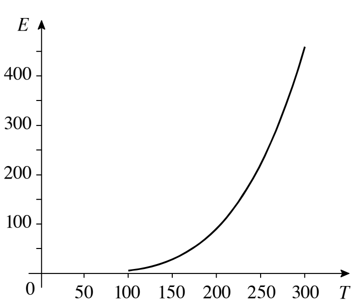
61. Runner A won the race. All runners finished the race. Runner B fell, but got up and finished the race.

62. (a)



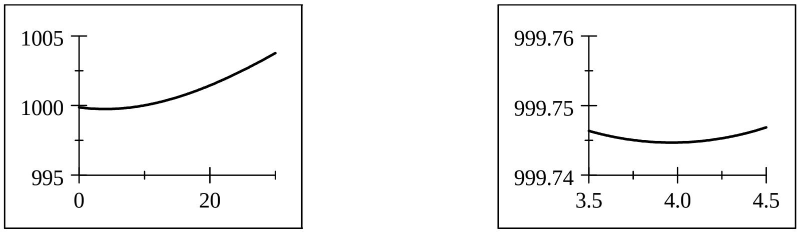
(b) As the distance x increases, the gravitational attraction F decreases. The rate of decrease is rapid at first, and slows as the distance increases.

63. (a)

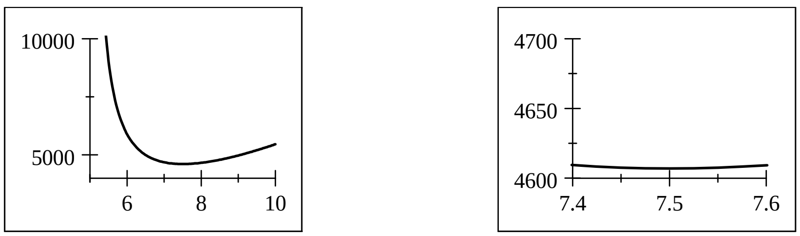


(b) As the temperature T increases, the energy E increases. The rate of increase gets larger as the temperature increases.

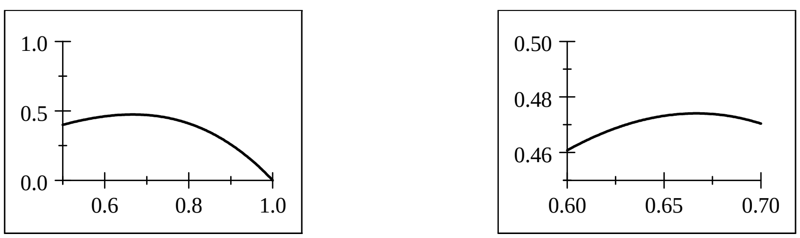
64. In the first graph, we see the general location of the minimum of $V = 999.87 - 0.06426T + 0.0085043T^2 - 0.0000679T^3$ is around $T = 4$. In the second graph, we isolate the minimum, and from this graph, we see that the minimum volume of 1 kg of water occurs at $T \approx 3.96^\circ$ C.



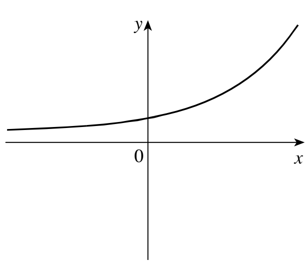
65. In the first graph, we see the general location of the minimum of $E(v) = 2.73v^3\frac{10}{v-5}$. In the second graph, we isolate the minimum, and from this graph, we see that energy is minimized when $v \approx 7.5$ mi/h.



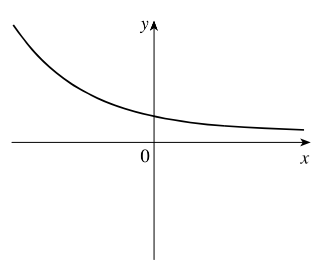
66. In the first graph, we see the general location of the maximum of $v(r) = 3.2(1-r)r^2$ is around $r = 0.7$ cm. In the second graph, we isolate the maximum, and from this graph we see that at the maximum velocity is approximately 0.47 when $r \approx 0.67$ cm.



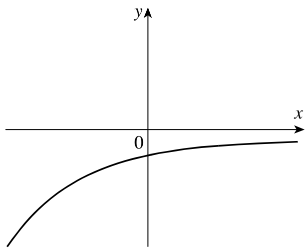
67. (a) $f(x)$ is always increasing, and $f(x) > 0$ for all x.



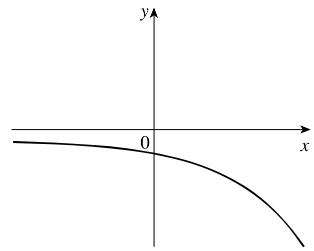
(b) $f(x)$ is always decreasing, and $f(x) > 0$ for all x.



(c) $f(x)$ is always increasing, and $f(x) < 0$ for all x.



(d) $f(x)$ is always decreasing, and $f(x) < 0$ for all x.



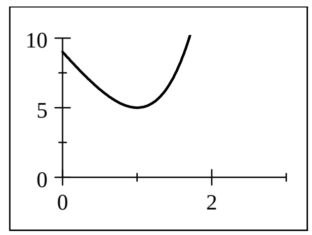
68. Numerous answers are possible.

69. **(a)** If $x = a$ is a local maximum of $f(x)$ then $f(a) \geq f(x) \geq 0$ for all x around $x = a$. So $[g(a)]^2 \geq [g(x)]^2$ and thus $g(a) \geq g(x)$. Similarly, if $x = b$ is a local minimum of $f(x)$, then $f(x) \geq f(b) \geq 0$ for all x around $x = b$. So $[g(x)]^2 \geq [g(b)]^2$ and thus $g(x) \geq g(b)$.

(b) Using the distance formula,

$$g(x) = \sqrt{(x-3)^2 + (x^2 - 0)^2} = \sqrt{x^4 + x^2 - 6x + 9}$$

(c) Let $f(x) = x^4 + x^2 - 6x + 9$. From the graph, we see that $f(x)$ has a minimum at $x = 1$. Thus $g(x)$ also has a minimum at $x = 1$ and this minimum value is $g(1) = \sqrt{1^4 + 1^2 - 6(1) + 9} = \sqrt{5}$.



2.4 AVERAGE RATE OF CHANGE OF A FUNCTION

1. If you travel 100 miles in two hours then your average speed for the trip is average speed $= \dfrac{100 \text{ miles}}{2 \text{ hours}} = 50$ mi/h.

2. The average rate of change of a function f between $x = a$ and $x = b$ is average rate of change $= \dfrac{f(b) - f(a)}{b - a}$.

3. The average rate of change of the function $f(x) = x^2$ between $x = 1$ and $x = 5$ is average rate of change $= \frac{f(5)-f(1)}{5-1} = \dfrac{5^2 - 1^2}{4} = \dfrac{25-1}{4} = \dfrac{24}{4} = 6$.

4. **(a)** The average rate of change of a function f between $x = a$ and $x = b$ is the slope of the *secant* line between $(a, f(a))$ and $(b, f(b))$.

(b) The average rate of change of the linear function $f(x) = 3x + 5$ between any two points is 3.

5. **(a)** Yes, the average rate of change of a function between $x = a$ and $x = b$ is the slope of the secant line through $(a, f(a))$ and $(b, f(b))$; that is, $\dfrac{f(b) - f(a)}{b - a}$.

(b) Yes, the average rate of change of a linear function $y = mx + b$ is the same (namely m) for all intervals.

6. **(a)** No, the average rate of change of an increasing function is positive over any interval.

(b) No, just because the average rate of change of a function between $x = a$ and $x = b$ is negative, it does not follow that the function is decreasing on that interval. For example, $f(x) = x^2$ has negative average rate of change between $x = -2$ and $x = 1$, but f is increasing for $0 < x < 1$.

7. **(a)** The net change is $f(4) - f(1) = 5 - 3 = 2$.

(b) We use the points $(1, 3)$ and $(4, 5)$, so the average rate of change is $\dfrac{5-3}{4-1} = \dfrac{2}{3}$.

8. (a) The net change is $f(5) - f(1) = 2 - 4 = -2$.

(b) We use the points $(1, 4)$ and $(5, 2)$, so the average rate of change is $\dfrac{2-4}{5-1} = \dfrac{-2}{4} = -\dfrac{1}{2}$.

9. (a) The net change is $f(5) - f(0) = 2 - 6 = -4$.

(b) We use the points $(0, 6)$ and $(5, 2)$, so the average rate of change is $\dfrac{2-6}{5-0} = \dfrac{-4}{5}$.

10. (a) The net change is $f(5) - f(-1) = 4 - 0 = 4$.

(b) We use the points $(-1, 0)$ and $(5, 4)$, so the average rate of change is $\dfrac{4-0}{5-(-1)} = \dfrac{4}{6} = \dfrac{2}{3}$.

11. (a) The net change is $f(3) - f(2) = [3(3) - 2] - [3(2) - 2] = 7 - 4 = 3$.

(b) The average rate of change is $\dfrac{f(3) - f(2)}{3-2} = \dfrac{3}{1} = 3$.

12. (a) The net change is $r(6) - r(3) = \left[3 - \frac{1}{3}(6)\right] - \left[3 - \frac{1}{3}(3)\right] = 1 - 2 = -1$.

(b) The average rate of change is $\dfrac{r(6) - r(3)}{6-3} = -\dfrac{1}{3}$.

13. (a) The net change is $h(1) - h(-4) = \left[-1 + \frac{3}{2}\right] - \left[-(-4) + \frac{3}{2}\right] = \frac{1}{2} - \frac{11}{2} = -5$.

(b) The average rate of change is $\dfrac{h(1) - h(-4)}{1-(-4)} = \dfrac{-5}{5} = -1$.

14. (a) The net change is $g(2) - g(-3) = \left[2 - \frac{2}{3}(2)\right] - \left[2 - \frac{2}{3}(-3)\right] = \frac{2}{3} - 4 = -\frac{10}{3}$.

(b) The average rate of change is $\dfrac{g(2) - g(-3)}{2-(-3)} = \dfrac{-\frac{10}{3}}{5} = -\dfrac{2}{3}$.

15. (a) The net change is $h(6) - h(3) = \left[2(6)^2 - 6\right] - \left[2(3)^2 - 3\right] = 66 - 15 = 51$.

(b) The average rate of change is $\dfrac{h(6) - h(3)}{6-3} = \dfrac{51}{3} = 17$.

16. (a) The net change is $f(0) - f(-2) = \left[1 - 3(0)^2\right] - \left[1 - 3(-2)^2\right] = 1 - (-11) = 12$.

(b) The average rate of change is $\dfrac{f(0) - f(-2)}{0-(-2)} = \dfrac{12}{2} = 6$.

17. (a) The net change is $f(10) - f(0) = \left[10^3 - 4\left(10^2\right)\right] - \left[0^3 - 4\left(0^2\right)\right] = 600 - 0 = 600$.

(b) The average rate of change is $\dfrac{f(10) - f(0)}{10-0} = \dfrac{600}{10} = 60$.

18. (a) The net change is $g(2) - g(-2) = \left[2^4 - 2^3 + 2^2\right] - \left[(-2)^4 - (-2)^3 + (-2)^2\right] = 12 - 28 = -16$.

(b) The average rate of change is $\dfrac{g(2) - g(-2)}{2-(-2)} = \dfrac{-16}{4} = -4$.

19. (a) The net change is $f(3+h) - f(3) = \left[5(3+h)^2\right] - \left[5(3)^2\right] = 45 + 30h + 5h^2 - 45 = 5h^2 + 30h$.

(b) The average rate of change is $\dfrac{f(3+h) - f(3)}{(3+h)-3} = \dfrac{5h^2 + 30h}{h} = 5h + 30$.

20. (a) The net change is $f(2+h) - f(2) = \left[1 - 3(2+h)^2\right] - \left[1 - 3(2)^2\right] = \left(-3h^2 - 12h - 11\right) - (-11) = -3h^2 - 12h$.

(b) The average rate of change is $\dfrac{f(2+h) - f(2)}{(2+h)-2} = \dfrac{-3h^2 - 12h}{h} = -3h - 12$.

21. **(a)** The net change is $g(a) - g(1) = \frac{1}{a} - \frac{1}{1} = \frac{1-a}{a}$.

(b) The average rate of change is $\frac{g(a) - g(1)}{a-1} = \frac{\frac{1-a}{a}}{a-1} = \frac{1-a}{a(a-1)} = -\frac{1}{a}$.

22. **(a)** The net change is $g(h) - g(0) = \frac{2}{h+1} - \frac{2}{0+1} = -\frac{2h}{h+1}$.

(b) The average rate of change is $\frac{g(h) - g(0)}{h-0} = \frac{-\frac{2h}{h+1}}{h} = \frac{-2h}{h(h+1)} = -\frac{2}{h+1}$.

23. **(a)** The net change is $f(a+h) - f(a) = \frac{2}{a+h} - \frac{2}{a} = -\frac{2h}{a(a+h)}$.

(b) The average rate of change is

$$\frac{f(a+h) - f(a)}{(a+h)-a} = \frac{-\frac{2h}{a(a+h)}}{h} = -\frac{2h}{ah(a+h)} = -\frac{2}{a(a+h)}.$$

24. **(a)** The net change is $f(a+h) - f(a) = \sqrt{a+h} - \sqrt{a}$.

(b) The average rate of change is

$$\frac{f(a+h) - f(a)}{(a+h)-a} = \frac{\sqrt{a+h} - \sqrt{a}}{h} \cdot \frac{\sqrt{a+h} + \sqrt{a}}{\sqrt{a+h} + \sqrt{a}} = \frac{(a+h)-a}{h\left(\sqrt{a+h} + \sqrt{a}\right)} = \frac{h}{h\left(\sqrt{a+h} + \sqrt{a}\right)} = \frac{1}{\sqrt{a+h} + \sqrt{a}}.$$

25. **(a)** The average rate of change is

$$\frac{f(a+h) - f(a)}{(a+h)-a} = \frac{\left[\frac{1}{2}(a+h)+3\right] - \left[\frac{1}{2}a+3\right]}{h} = \frac{\frac{1}{2}a + \frac{1}{2}h + 3 - \frac{1}{2}a - 3}{h} = \frac{\frac{1}{2}h}{h} = \frac{1}{2}.$$

(b) The slope of the line $f(x) = \frac{1}{2}x + 3$ is $\frac{1}{2}$, which is also the average rate of change.

26. **(a)** The average rate of change is

$$\frac{g(a+h) - g(a)}{(a+h)-a} = \frac{[-4(a+h)+2] - [-4a+2]}{h} = \frac{-4a - 4h + 2 + 4a - 2}{h} = \frac{-4h}{h} = -4.$$

(b) The slope of the line $g(x) = -4x + 2$ is -4, which is also the average rate of change.

27. The function f has a greater average rate of change between $x = 0$ and $x = 1$. The function g has a greater average rate of change between $x = 1$ and $x = 2$. The functions f and g have the same average rate of change between $x = 0$ and $x = 1.5$.

28. The average rate of change of f is constant, that of g increases, and that of h decreases.

29. The average rate of change is $\frac{W(200) - W(100)}{200 - 100} = \frac{50 - 75}{200 - 100} = \frac{-25}{100} = -\frac{1}{4}$ ft/day.

30. **(a)** The average rate of change is $\frac{P(40) - P(20)}{40 - 20} = \frac{40 - 40}{40 - 20} = \frac{0}{20} = 0$.

(b) The population increased and decreased the same amount during the 20 years.

31. **(a)** The average rate of change of population is $\frac{1{,}591 - 856}{2001 - 1998} = \frac{735}{3} = 245$ persons/yr.

(b) The average rate of change of population is $\frac{826 - 1{,}483}{2004 - 2002} = \frac{-657}{2} = -328.5$ persons/yr.

(c) The population was increasing from 1997 to 2001.

(d) The population was decreasing from 2001 to 2006.

32. **(a)** The average speed is $\dfrac{800-400}{152-68}=\dfrac{400}{84}=\dfrac{100}{21}\approx 4.76$ m/s.

(b) The average speed is $\dfrac{1{,}600-1{,}200}{412-263}=\dfrac{400}{149}\approx 2.68$ m/s.

(c)

Lap	Length of time to run lap	Average speed of lap.
1	32	6.25 m/s
2	36	5.56 m/s
3	40	5.00 m/s
4	44	4.55 m/s
5	51	3.92 m/s
6	60	3.33 m/s
7	72	2.78 m/s
8	77	2.60 m/s

The man is slowing down throughout the run.

33. **(a)** The average rate of change of sales is $\dfrac{635-495}{2013-2003}=\dfrac{140}{10}=14$ players/yr.

(b) The average rate of change of sales is $\dfrac{513-495}{2004-2003}=\dfrac{18}{1}=18$ players/yr.

(c) The average rate of change of sales is $\dfrac{410-513}{2005-2004}=\dfrac{-103}{1}=-103$ players/yr.

(d)

Year	DVD players sold	Change in sales from previous year
2003	495	—
2004	513	18
2005	410	−103
2006	402	−8
2007	520	118
2008	580	60
2009	631	51
2010	719	88
2011	624	−95
2012	582	−42
2013	635	53

Sales increased most quickly between 2006 and 2007, and decreased most quickly between 2004 and 2005.

34.

Year	Number of books
1980	420
1981	460
1982	500
1985	620
1990	820
1992	900
1995	1020
1997	1100
1998	1140
1999	1180
2000	1220

35. The average rate of change of the temperature of the soup over the first 20 minutes is $\dfrac{T(20)-T(0)}{20-0} = \dfrac{119-200}{20-0} = \dfrac{-81}{20} = -4.05^\circ$ F/min. Over the next 20 minutes, it is $\dfrac{T(40)-T(20)}{40-20} = \dfrac{89-119}{40-20} = -\dfrac{30}{20} = -1.5^\circ$ F/min. The first 20 minutes had a higher average rate of change of temperature (in absolute value).

36. (a) (i) Between 1860 and 1890, the average rate of change was $\dfrac{y(1890)-y(1860)}{1890-1860} \approx \dfrac{4570-2040}{30} \approx 84$, a gain of about 84 farms per year.

(ii) Between 1950 and 1970, the average rate of change was $\dfrac{y(1970)-y(1950)}{1970-1950} \approx \dfrac{2780-5390}{20} \approx -131$, a loss of about 131 farms per year.

(b) From the graph, it appears that the steepest rate of decline was during the period from 1950 to 1960.

37. (a) For all three skiers, the average rate of change is $\dfrac{d(10)-d(0)}{10-0} = \dfrac{100}{10} = 10$.

(b) Skier A gets a great start, but slows at the end of the race. Skier B maintains a steady pace. Runner C is slow at the beginning, but accelerates down the hill.

38. (a) Skater B won the race, because he travels 500 meters before Skater A.

(b) Skater A's average speed during the first 10 seconds is $\dfrac{A(10)-A(0)}{10-0} \approx \dfrac{200-0}{10} = 20$ m/s.

Skater B's average speed during the first 10 seconds is $\dfrac{B(10)-B(0)}{10-0} \approx \dfrac{100-0}{10} = 10$ m/s.

(c) Skater A's average speed during his last 15 seconds is $\dfrac{A(40)-A(25)}{40-25} \approx \dfrac{500-395}{15} = 7$ m/s.

Skater B's average speed during his last 15 seconds is $\dfrac{B(35)-B(20)}{35-20} \approx \dfrac{500-200}{15} = 20$ m/s.

39.

$t = a$	$t = b$	Average Speed $= \dfrac{f(b) - f(a)}{b - a}$
3	3.5	$\dfrac{16(3.5)^2 - 16(3)^2}{3.5 - 3} = 104$
3	3.1	$\dfrac{16(3.1)^2 - 16(3)^2}{3.1 - 3} = 97.6$
3	3.01	$\dfrac{16(3.01)^2 - 16(3)^2}{3.01 - 3} = 96.16$
3	3.001	$\dfrac{16(3.001)^2 - 16(3)^2}{3.001 - 3} = 96.016$
3	3.0001	$\dfrac{16(3.0001)^2 - 16(3)^2}{3.0001 - 3} = 96.0016$

From the table it appears that the average speed approaches 96 ft/s as the time intervals get smaller and smaller. It seems reasonable to say that the speed of the object is 96 ft/s at the instant $t = 3$.

2.5 LINEAR FUNCTIONS AND MODELS

1. If f is a function with constant rate of change, then
(a) f is a linear function of the form $f(x) = ax + b$.
(b) The graph of f is a line.

2. If $f(x) = -5x + 7$, then
(a) The rate of change of f is -5.
(b) The graph of f is a line with slope -5 and y-intercept 7.

3. From the graph, we see that $y(2) = 50$ and $y(0) = 20$, so the slope of the graph is $m = \dfrac{y(2) - y(0)}{2 - 0} = \dfrac{50 - 20}{2} = 15$ gal/min.

4. From Exercise 3, we see that the pool is being filled at the rate of 15 gallons per minute.

5. If a linear function has positive rate of change, its graph slopes upward.

6. $f(x) = 3$ is a linear function because it is of the form $f(x) = ax + b$, with $a = 0$ and $b = 3$. Its slope (and hence its rate of change) is 0.

7. $f(x) = 3 + \frac{1}{3}x = \frac{1}{3}x + 3$ is linear with $a = \frac{1}{3}$ and $b = 3$.

8. $f(x) = 2 - 4x = -4x + 2$ is linear with $a = -4$ and $b = 2$.

9. $f(x) = x(4 - x) = 4x - x^2$ is not of the form $f(x) = ax + b$ for constants a and b, so it is not linear.

10. $f(x) = \sqrt{x} + 1$ is not linear.

11. $f(x) = \dfrac{x + 1}{5} = \frac{1}{5}x + \frac{1}{5}$ is linear with $a = \frac{1}{5}$ and $b = \frac{1}{5}$.

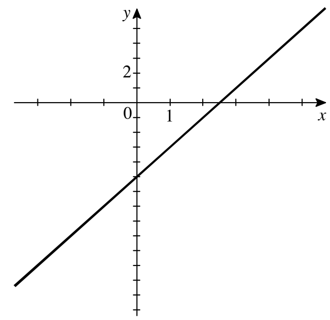
12. $f(x) = \dfrac{2x - 3}{x} = 2 - \dfrac{3}{x}$ is not linear.

13. $f(x) = (x + 1)^2 = x^2 + 2x + 1$ is not of the form $f(x) = ax + b$ for constants a and b, so it is not linear.

14. $f(x) = \frac{1}{2}(3x - 1) = \frac{3}{2}x - \frac{1}{2}$ is linear with $a = \frac{3}{2}$ and $b = -\frac{1}{2}$.

15.

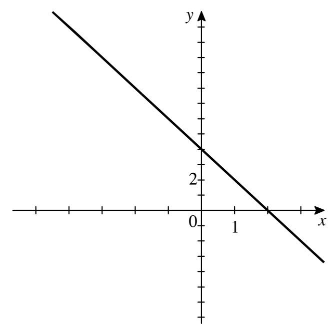
x	$f(x) = 2x - 5$
-1	-7
0	-5
1	-3
2	-1
3	1
4	3



The slope of the graph of $f(x) = 2x - 5$ is 2.

16.

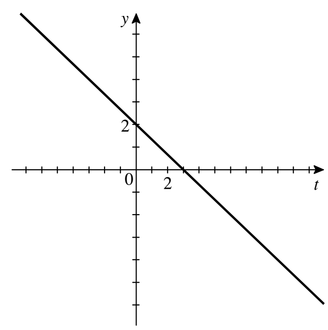
x	$g(x) = 4 - 2x$
-1	6
0	4
1	2
2	0
3	-2
4	-4



The slope of the graph of $g(x) = 4 - 2x = -2x + 4$ is -2.

17.

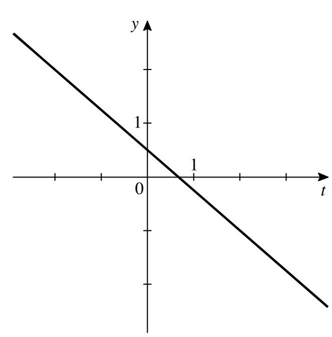
t	$r(t) = -\frac{2}{3}t + 2$
-1	2.67
0	2
1	1.33
2	0.67
3	0
4	-0.67



The slope of the graph of $r(t) = -\frac{2}{3}t + 2$ is $-\frac{2}{3}$.

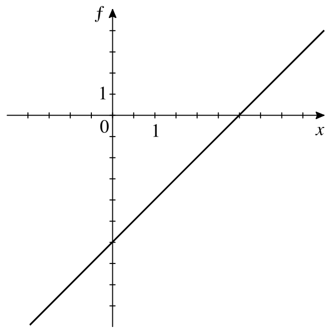
18.

t	$h(t) = \frac{1}{2} - \frac{3}{4}t$
-2	2
-1	1.25
0	0.5
1	-0.25
2	-1
3	-1.75



The slope of the graph of $h(t) = \frac{1}{2} - \frac{3}{4}t$ is $-\frac{3}{4}$.

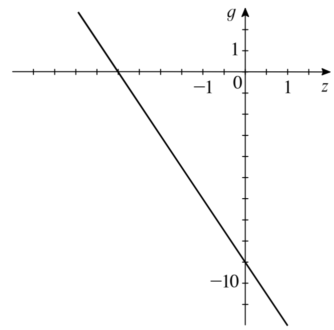
19. (a)



(b) The graph of $f(x) = 2x - 6$ has slope 2.

(c) $f(x) = 2x - 6$ has rate of change 2.

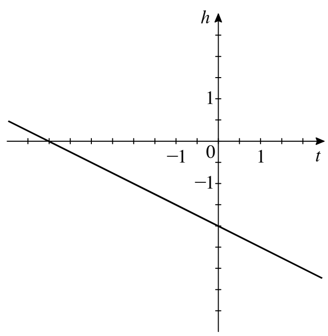
20. (a)



(b) The graph of $g(z) = -3z - 9$ has slope -3.

(c) $g(z) = -3z - 9$ has rate of change -3.

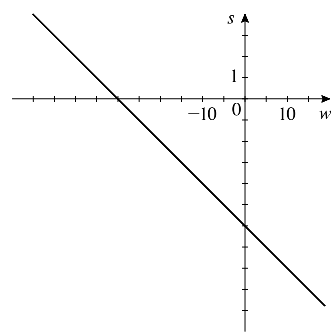
21. (a)



(b) The graph of $h(t) = -0.5t - 2$ has slope -0.5.

(c) $h(t) = -0.5t - 2$ has rate of change -0.5.

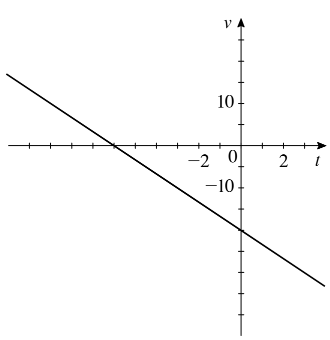
22. (a)



(b) The graph of $s(w) = -0.2w - 6$ has slope -0.2.

(c) $s(w) = -0.2w - 6$ has rate of change -0.2.

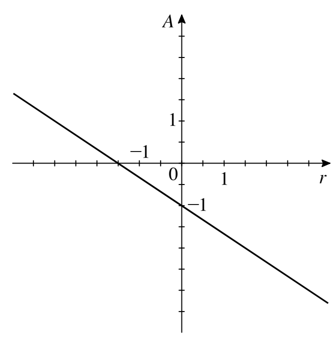
23. (a)



(b) The graph of $v(t) = -\frac{10}{3}t - 20$ has slope $-\frac{10}{3}$.

(c) $v(t) = -\frac{10}{3}t - 20$ has rate of change $-\frac{10}{3}$.

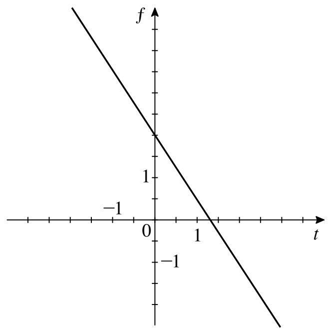
24. (a)



(b) The graph of $A(r) = -\frac{2}{3}r - 1$ has slope $-\frac{2}{3}$.

(c) $A(r) = -\frac{2}{3}r - 1$ has rate of change $-\frac{2}{3}$.

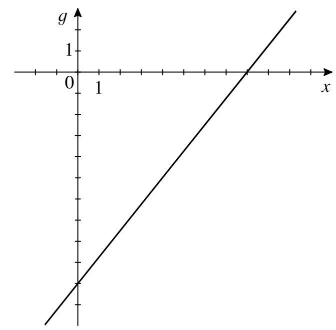
25. (a)



(b) The graph of $f(t) = -\frac{3}{2}t + 2$ has slope $-\frac{3}{2}$.

(c) $f(t) = -\frac{3}{2}t + 2$ has rate of change $-\frac{3}{2}$.

26. (a)



(b) The graph of $g(x) = \frac{5}{4}x - 10$ has slope $\frac{5}{4}$.

(c) $g(x) = \frac{5}{4}x - 10$ has rate of change $\frac{5}{4}$.

27. The linear function f with rate of change 3 and initial value -1 has equation $f(x) = 3x - 1$.

28. The linear function g with rate of change -12 and initial value 100 has equation $g(x) = -12x + 100$.

29. The linear function h with slope $\frac{1}{2}$ and y-intercept 3 has equation $h(x) = \frac{1}{2}x + 3$.

30. The linear function k with slope $-\frac{4}{5}$ and y-intercept -2 has equation $k(x) = -\frac{4}{5}x - 2$.

31. (a) From the table, we see that for every increase of 2 in the value of x, $f(x)$ increases by 3. Thus, the rate of change of f is $\frac{3}{2}$.

(b) When $x = 0$, $f(x) = 7$, so $b = 7$. From part (a), $a = \frac{3}{2}$, and so $f(x) = \frac{3}{2}x + 7$.

32. (a) From the table, we see that $f(-3) = 11$ and $f(0) = 2$. Thus, when x increases by 3, $f(x)$ decreases by 9, and so the rate of change of f is -3.

(b) When $x = 0$, $f(x) = 2$, so $b = 2$. From part (a), $a = -3$, and so $f(x) = -3x + 2$.

33. (a) From the graph, we see that $f(0) = 3$ and $f(1) = 4$, so the rate of change of f is $\dfrac{4-3}{1-0} = 1$.

(b) From part (a), $a = 1$, and $f(0) = b = 3$, so $f(x) = x + 3$.

34. (a) From the graph, we see that $f(0) = 4$ and $f(2) = 0$, so the rate of change of f is $\dfrac{0-4}{2-0} = -2$.

(b) From part (a), $a = -2$, and $f(0) = b = 4$, so $f(x) = -2x + 4$.

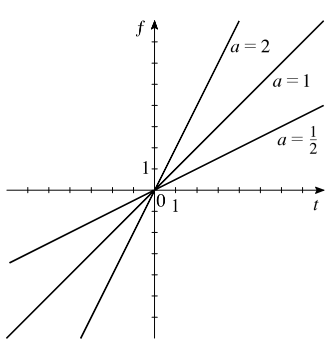
35. (a) From the graph, we see that $f(0) = 2$ and $f(4) = 0$, so the rate of change of f is $\dfrac{0-2}{4-0} = -\dfrac{1}{2}$.

(b) From part (a), $a = -\frac{1}{2}$, and $f(0) = b = 2$, so $f(x) = -\frac{1}{2}x + 2$.

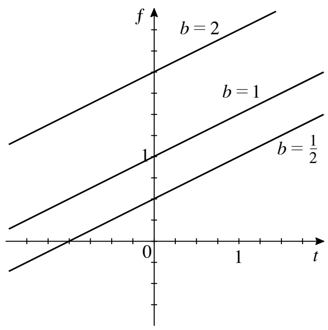
36. (a) From the graph, we see that $f(0) = -1$ and $f(2) = 0$, so the rate of change of f is $\dfrac{0-(-1)}{2-0} = \dfrac{1}{2}$.

(b) From part (a), $a = \frac{1}{2}$, and $f(0) = b = -1$, so $f(x) = \frac{1}{2}x - 1$.

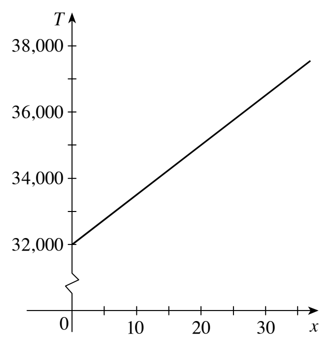
37.



Increasing the value of a makes the graph of f steeper. In other words, it increases the rate of change of f.

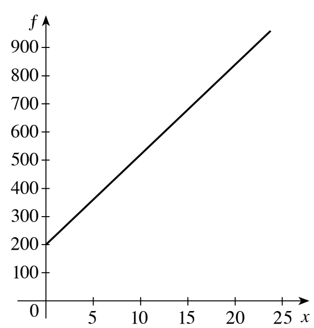
38. 

Increasing the value of b moves the graph of f upward, but does not affect the rate of change of f.

39. (a) 

(b) The slope of $T(x) = 150x + 32{,}000$ is the value of a, 150.

(c) The amount of trash is changing at a rate equal to the slope of the graph, 150 thousand tons per year.

40. (a) 

(b) The slope of the graph of $f(x) = 200 + 32x$ is 32.

(c) Ore is being produced at a rate equal to the slope of the graph, 32 thousand tons per year.

41. (a) Let $V(t) = at + b$ represent the volume of hydrogen. The balloon is being filled at the rate of 0.5 ft^3/s, so $a = 0.5$, and initially it contains 2 ft^3, so $b = 2$. Thus, $V(t) = 0.5t + 2$.

(b) We solve $V(t) = 15 \Leftrightarrow 0.5t + 2 = 15 \Leftrightarrow 0.5t = 13 \Leftrightarrow t = 26$. Thus, it takes 26 seconds to fill the balloon.

42. (a) Let $V(t) = at + b$ represent the volume of water. The pool is being filled at the rate of 10 gal/min, so $a = 10$, and initially it contains 300 gal, so $b = 300$. Thus, $V(t) = 10t + 300$.

(b) We solve $V(t) = 1300 \Leftrightarrow 10t + 300 = 1300 \Leftrightarrow 10t = 1000 \Leftrightarrow t = 100$. Thus, it takes 100 minutes to fill the pool.

43. (a) Let $H(x) = ax + b$ represent the height of the ramp. The maximum rise is 1 inch per 12 inches, so $a = \frac{1}{12}$. The ramp starts on the ground, so $b = 0$. Thus, $H(x) = \frac{1}{12}x$.

(b) We find $H(150) = \frac{1}{12}(150) = 12.5$. Thus, the ramp reaches a height of 12.5 inches.

44. Meilin descends 1200 vertical feet over 15,000 feet, so the grade of her road is $\dfrac{-1200}{15{,}000} = -0.075$, or -7.5%.

Brianna descends 500 vertical feet over 10,000 feet, so the grade of her road is $\dfrac{-500}{10{,}000} = -0.05$, or -5%.

45. (a) From the graph, we see that the slope of Jari's trip is steeper than that of Jade. Thus, Jari is traveling faster.

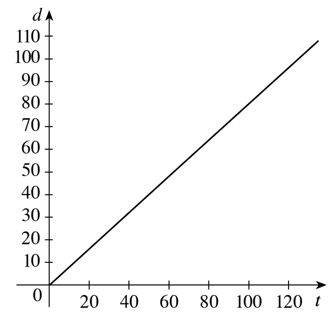
(b) The points (0, 0) and (6, 7) are on Jari's graph, so her speed is $\dfrac{7-0}{6-0} = \dfrac{7}{6}$ miles per minute or $60\left(\dfrac{7}{6}\right) = 70$ mi/h.

The points (0, 10) and (6, 16) are on Jade's graph, so her speed is $60 \cdot \dfrac{16-10}{6-0} = 60$ mi/h.

(c) t is measured in minutes, so Jade's speed is 60 mi/h $\cdot\ \frac{1}{60}$ h/min $=$ 1 mi/min and Jari's speed is 70 mi/h $\cdot\ \frac{1}{60}$ h/min $= \frac{7}{6}$ mi/min. Thus, Jade's distance is modeled by $f(t) = 1(t-0)+10 = t+10$ and Jari's distance is modeled by $g(t) = \frac{7}{6}(t-0)+0 = \frac{7}{6}t$.

46. **(a)** Let $d(t)$ represent the distance traveled. When $t=0$, $d=0$, and when $t=50$, $d=40$. Thus, the slope of the graph is $\dfrac{40-0}{50-0} = 0.8$. The y-intercept is 0, so $d(t) = 0.8t$.

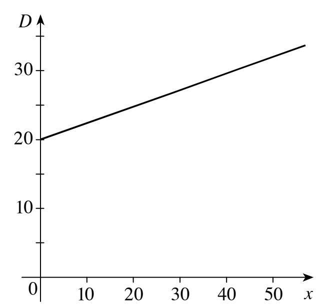
(b)



(c) Jacqueline's speed is equal to the slope of the graph of d, that is, 0.8 mi/min or $0.8(60) = 48$ mi/h.

47. Let x be the horizontal distance and y the elevation. The slope is $-\frac{6}{100}$, so if we take (0, 0) as the starting point, the elevation is $y = -\frac{6}{100}x$. We have descended 1000 ft, so we substitute $y = -1000$ and solve for x: $-1000 = -\frac{6}{100}x \Leftrightarrow x \approx 16{,}667$ ft. Converting to miles, the horizontal distance is $\frac{1}{5280}(16{,}667) \approx 3.16$ mi.

48. **(a)**

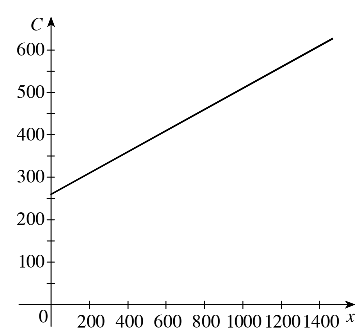


(b) The slope of the graph of $D(x) = 20 + 0.24x$ is 0.24.

(c) The rate of sedimentation is equal to the slope of the graph, 0.24 cm/yr or 2.4 mm/yr.

49. **(a)** Let $C(x) = ax + b$ be the cost of driving x miles. In May Lynn drove 480 miles at a cost of \$380, and in June she drove 800 miles at a cost of \$460. Thus, the points (480, 380) and (800, 460) are on the graph, so the slope is $a = \dfrac{460-380}{800-480} = \dfrac{1}{4}$. We use the point (480, 380) to find the value of b: $380 = \frac{1}{4}(480) + b \Leftrightarrow b = 260$. Thus, $C(x) = \frac{1}{4}x + 260$.

(b)

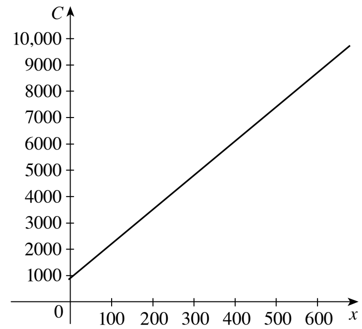


The slope of the graph of $C(x) = \frac{1}{4}x + 260$ is the value of a, $\frac{1}{4}$.

(c) The rate at which her cost increases is equal to the slope of the line, that is $\frac{1}{4}$. So her cost increases by \$0.25 for every additional mile she drives.

50. **(a)** Let $C(x) = ax + b$ be the cost of producing x chairs in one day. The first day, it cost \$2200 to produce 100 chairs, and the other day it cost \$4800 to produce 300 chairs.. Thus, the points (100, 2200) and (300, 4800) are on the graph, so the slope is $a = \dfrac{4800 - 2200}{300 - 100} = 13$. We use the point (100, 2200) to find the value of b: $2200 = 13(100) + b \Leftrightarrow b = 900$. Thus, $C(x) = 13x + 900$.

(b)



The slope of the graph of $C(x) = 13x + 900$ is the value of a, 13.

(c) The rate at which the factory's cost increases is equal to the slope of the line, that is \$13/chair.

51. **(a)** By definition, the average rate of change between x_1 and x_2 is $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{(ax_2 + b) - (ax_1 + b)}{x_2 - x_1} = \dfrac{ax_2 - ax_1}{x_2 - x_1}$.

(b) Factoring the numerator and cancelling, the average rate of change is $\dfrac{ax_2 - ax_1}{x_2 - x_1} = \dfrac{a(x_2 - x_1)}{x_2 - x_1} = a$.

52. **(a)** The rate of change between any two points is c. In particular, between a and x, the rate of change is $\dfrac{f(x) - f(a)}{x - a} = c$.

(b) Multiplying the equation in part (a) by $x - a$, we obtain $f(x) - f(a) = c(x - a)$. Rearranging and adding $f(a)$ to both sides, we have $f(x) = cx + (f(a) - ca)$, as desired. Because this equation is of the form $f(x) = Ax + B$ with constants $A = c$ and $B = f(a) - ca$, it represents a linear function with slope c and y-intercept $f(a) - ca$.

2.6 TRANSFORMATIONS OF FUNCTIONS

1. **(a)** The graph of $y = f(x) + 3$ is obtained from the graph of $y = f(x)$ by shifting *upward* 3 units.

(b) The graph of $y = f(x + 3)$ is obtained from the graph of $y = f(x)$ by shifting *left* 3 units.

2. **(a)** The graph of $y = f(x) - 3$ is obtained from the graph of $y = f(x)$ by shifting *downward* 3 units.

(b) The graph of $y = f(x - 3)$ is obtained from the graph of $y = f(x)$ by shifting *right* 3 units.

3. **(a)** The graph of $y = -f(x)$ is obtained from the graph of $y = f(x)$ by reflecting in the *x-axis*.

(b) The graph of $y = f(-x)$ is obtained from the graph of $y = f(x)$by reflecting in the *y-axis*.

4. **(a)** The graph of $f(x) + 2$ is obtained from that of $y = f(x)$ by shifting upward 2 units, so it has graph II.

(b) The graph of $f(x + 3)$ is obtained from that of $y = f(x)$ by shifting to the left 3 units, so it has graph I.

(c) The graph of $f(x - 2)$ is obtained from that of $y = f(x)$ by shifting to the right 2 units, so it has graph III.

(d) The graph of $f(x) - 4$ is obtained from that of $y = f(x)$ by shifting downward 4 units, so it has graph IV.

5. If f is an even function, then $f(-x) = f(x)$ and the graph of f is symmetric about the y-axis.

6. If f is an odd function, then $f(-x) = -f(x)$ and the graph of f is symmetric about the origin.

7. **(a)** The graph of $y = f(x) - 1$ can be obtained by shifting the graph of $y = f(x)$ downward 1 unit.

(b) The graph of $y = f(x - 2)$ can be obtained by shifting the graph of $y = f(x)$ to the right 2 units.

8. **(a)** The graph of $y = f(x + 4)$ can be obtained by shifting the graph of $y = f(x)$ to the left 5 units.

(b) The graph of $y = f(x) + 4$ can be obtained by shifting the graph of $y = f(x)$ upward 4 units.

9. **(a)** The graph of $y = f(-x)$ can be obtained by reflecting the graph of $y = f(x)$ in the y-axis.

(b) The graph of $y = 3f(x)$ can be obtained by stretching the graph of $y = f(x)$ vertically by a factor of 3.

10. **(a)** The graph of $y = -f(x)$ can be obtained by reflecting the graph of $y = f(x)$ about the x-axis.

(b) The graph of $y = \frac{1}{3} f(x)$ can be obtained by shrinking the graph of $y = f(x)$ vertically by a factor of $\frac{1}{3}$.

11. **(a)** The graph of $y = f(x - 5) + 2$ can be obtained by shifting the graph of $y = f(x)$ to the right 5 units and upward 2 units.

(b) The graph of $y = f(x + 1) - 1$ can be obtained by shifting the graph of $y = f(x)$ to the left 1 unit and downward 1 unit.

12. **(a)** The graph of $y = f(x + 3) + 2$ can be obtained by shifting the graph of $y = f(x)$ to the left 3 units and upward 2 units.

(b) The graph of $y = f(x - 7) - 3$ can be obtained by shifting the graph of $y = f(x)$ to the right 7 units and downward 3 units.

13. **(a)** The graph of $y = -f(x) + 5$ can be obtained by reflecting the graph of $y = f(x)$ in the x-axis, then shifting the resulting graph upward 5 units.

(b) The graph of $y = 3f(x) - 5$ can be obtained by stretching the graph of $y = f(x)$ vertically by a factor of 3, then shifting the resulting graph downward 5 units.

14. **(a)** The graph of $y = 1 - f(-x)$ can be obtained by reflect the graph of $y = f(x)$ about the x-axis, then reflecting about the y-axis, then shifting upward 1 unit.

(b) The graph of $y = 2 - \frac{1}{5} f(x)$ can be obtained by shrinking the graph of $y = f(x)$ vertically by a factor of $\frac{1}{5}$, then reflecting about the x-axis, then shifting upward 2 units.

15. **(a)** The graph of $y = 2f(x + 5) - 1$ can be obtained by shifting the graph of $y = f(x)$ to the left 5 units, stretching vertically by a factor of 2, then shifting downward 1 unit.

(b) The graph of $y = \frac{1}{4} f(x - 3) + 5$ can be obtained by shifting the graph of $y = f(x)$ to the right 3 units, shrinking vertically by a factor of $\frac{1}{4}$, then shifting upward 5 units.

16. **(a)** The graph of $y = \frac{1}{3} f(x - 2) + 5$ can be obtained by shifting the graph of $y = f(x)$ to the right 2 units, shrinking vertically by a factor of $\frac{1}{3}$, then shifting upward 5 units.

(b) The graph of $y = 4f(x + 1) + 3$ can be obtained by shifting the graph of $y = f(x)$ to the left 1 unit, stretching vertically by a factor of 4, then shifting upward 3 units.

17. **(a)** The graph of $y = f(4x)$ can be obtained by shrinking the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{4}$.

(b) The graph of $y = f\left(\frac{1}{4}x\right)$ can be obtained by stretching the graph of $y = f(x)$ horizontally by a factor of 4.

18. **(a)** The graph of $y = f(2x) - 1$ can be obtained by shrinking the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{2}$, then shifting it downward 1 unit.

(b) The graph of $y = 2f\left(\frac{1}{2}x\right)$ can be obtained by stretching the graph of $y = f(x)$ horizontally by a factor of 2 and stretching it vertically by a factor of 2.

19. **(a)** The graph of $g(x) = (x + 2)^2$ is obtained by shifting the graph of $f(x)$ to the left 2 units.

(b) The graph of $g(x) = x^2 + 2$ is obtained by shifting the graph of $f(x)$ upward 2 units.

20. **(a)** The graph of $g(x) = (x - 4)^3$ is obtained by shifting the graph of $f(x)$ to the right 4 units.

(b) The graph of $g(x) = x^3 - 4$ is obtained by shifting the graph of $f(x)$ downward 4 units.

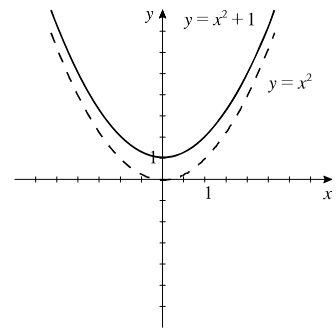
21. **(a)** The graph of $g(x) = |x + 2| - 2$ is obtained by shifting the graph of $f(x)$ to the left 2 units and downward 2 units.

(b) The graph of $g(x) = g(x) = |x - 2| + 2$ is obtained from by shifting the graph of $f(x)$ to the right 2 units and upward 2 units.

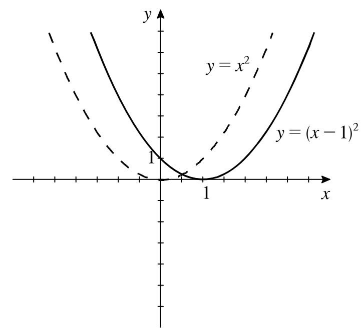
22. **(a)** The graph of $g(x) = -\sqrt{x} + 1$ is obtained by reflecting the graph of $f(x)$ in the x-axis, then shifting the resulting graph upward 1 unit.

(b) The graph of $g(x) = \sqrt{-x} + 1$ is obtained by reflecting the graph of $f(x)$ in the y-axis, then shifting the resulting graph upward 1 unit.

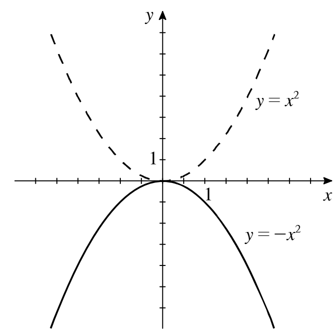
23. **(a)**



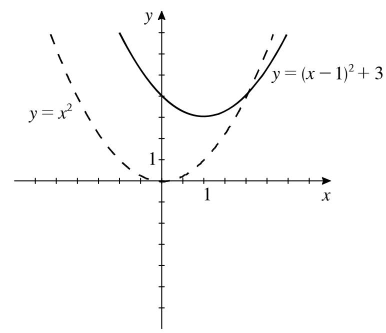
(b)



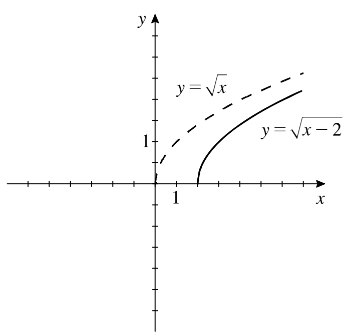
(c)



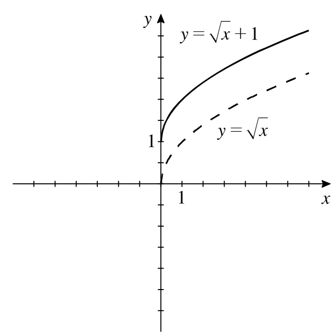
(d)



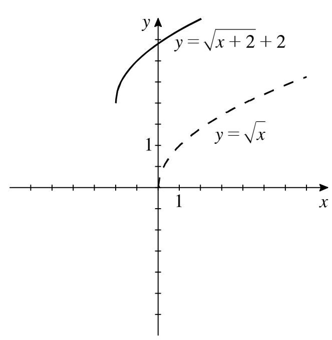
24. **(a)**



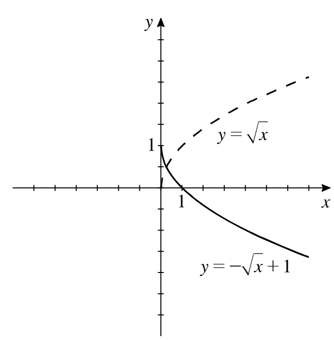
(b)



(c)



(d)



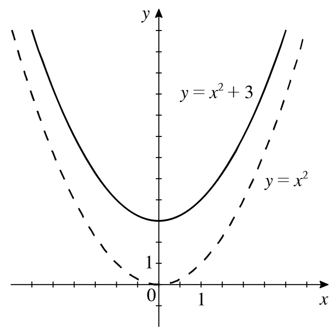
25. The graph of $y = |x + 1|$ is obtained from that of $y = |x|$ by shifting to the left 1 unit, so it has graph II.

26. $y = |x - 1|$ is obtained from that of $y = |x|$ by shifting to the right 1 unit, so it has graph IV.

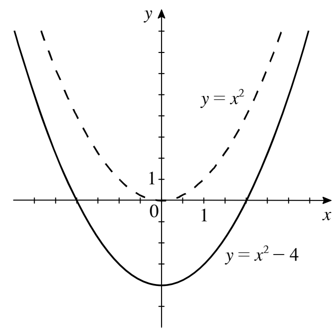
27. The graph of $y = |x| - 1$ is obtained from that of $y = |x|$ by shifting downward 1 unit, so it has graph I.

28. The graph of $y = -|x|$ is obtained from that of $y = |x|$ by reflecting in the x-axis, so it has graph III.

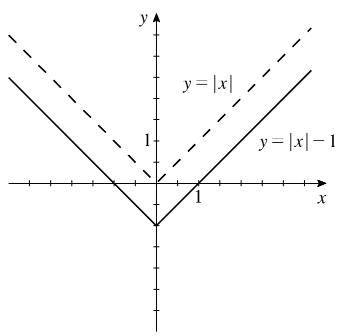
29. $f(x) = x^2 + 3$. Shift the graph of $y = x^2$ upward 3 units.



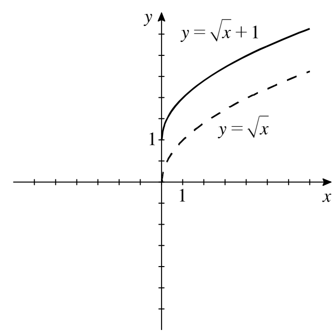
30. $f(x) = x^2 - 4$. Shift the graph of $y = x^2$ downward 4 units.



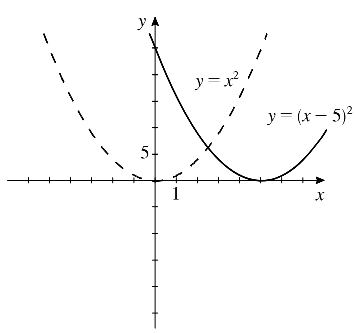
31. $f(x) = |x| - 1$. Shift the graph of $y = |x|$ downward 1 unit.



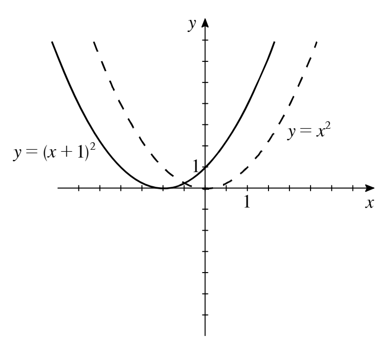
32. $f(x) = \sqrt{x} + 1$. Shift the graph of $y = \sqrt{x}$ upward 1 unit.



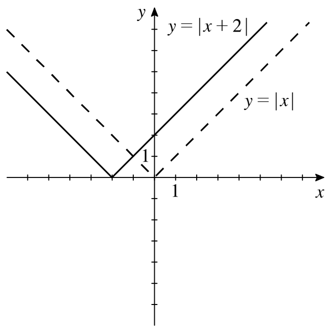
33. $f(x) = (x - 5)^2$. Shift the graph of $y = x^2$ to the right 5 units.



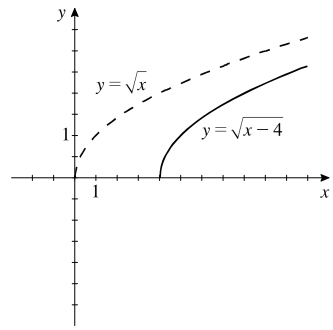
34. $f(x) = (x + 1)^2$. Shift the graph of $y = x^2$ to the left 1 unit.



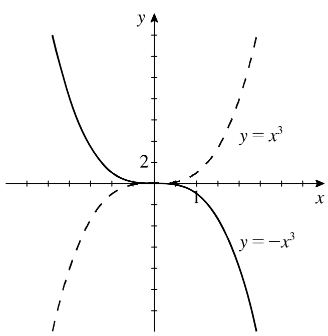
35. $f(x) = |x + 2|$. Shift the graph of $y = |x|$ to the left 2 units.



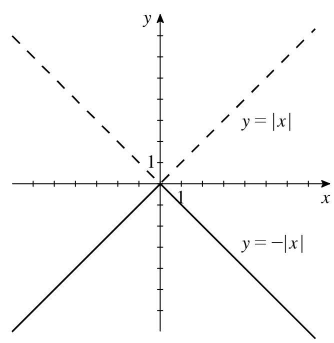
36. $f(x) = \sqrt{x - 4}$. Shift the graph of $y = \sqrt{x}$ to the right 4 units.



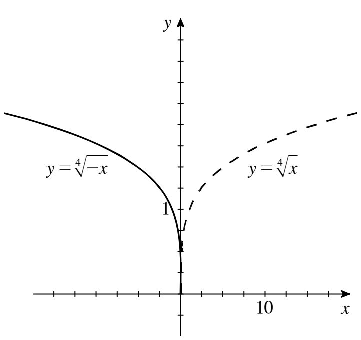
37. $f(x) = -x^3$. Reflect the graph of $y = x^3$ in the x-axis.



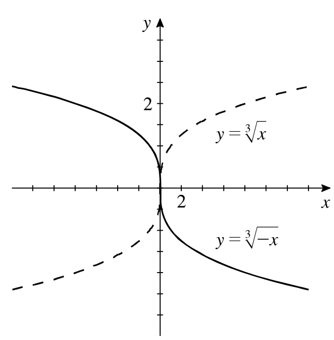
38. $f(x) = -|x|$. Reflect the graph of $y = |x|$ in the x-axis.



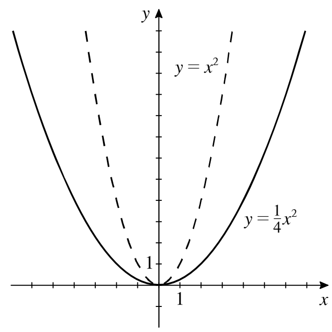
39. $y = \sqrt[4]{-x}$. Reflect the graph of $y = \sqrt[4]{x}$ in the y-axis.



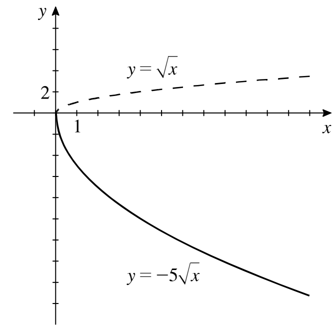
40. $y = \sqrt[3]{-x}$. Reflect the graph of $y = \sqrt[3]{x}$ in the y-axis.



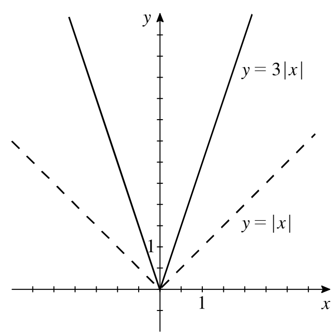
41. $y = \frac{1}{4}x^2$. Shrink the graph of $y = x^2$ vertically by a factor of $\frac{1}{4}$.



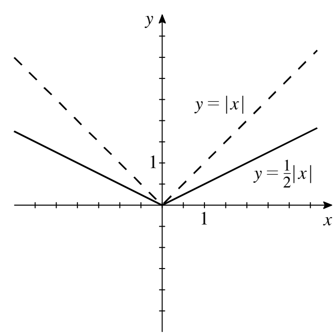
42. $y = -5\sqrt{x}$. Stretch the graph of $y = \sqrt{x}$ vertically by a factor of 5, then reflect it in the x-axis.



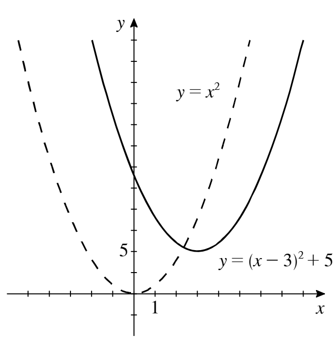
43. $y = 3|x|$. Stretch the graph of $y = |x|$ vertically by a factor of 3.



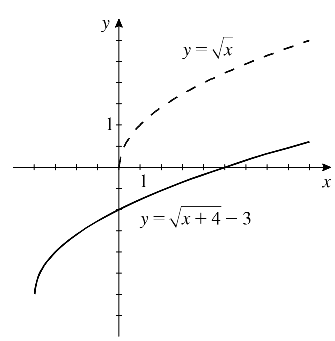
44. $y = \frac{1}{2}|x|$. Shrink the graph of $y = |x|$ vertically by a factor of $\frac{1}{2}$.



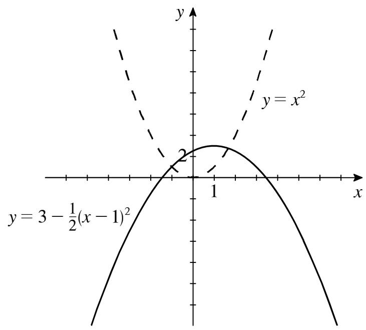
45. $y = (x-3)^2 + 5$. Shift the graph of $y = x^2$ to the right 3 units and upward 5 units.



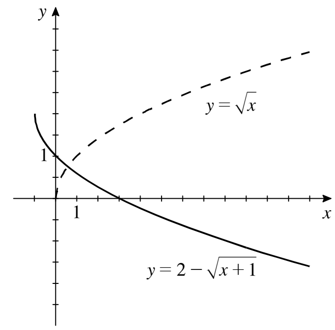
46. $y = \sqrt{x+4} - 3$. Shift the graph of $y = \sqrt{x}$ to the left 4 units and downward 3 units.



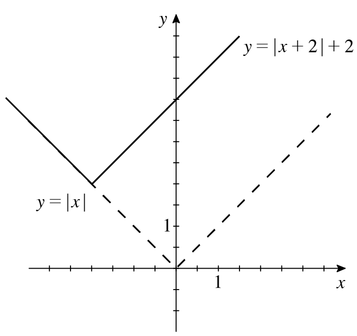
47. $y = 3 - \frac{1}{2}(x-1)^2$. Shift the graph of $y = x^2$ to the right one unit, shrink vertically by a factor of $\frac{1}{2}$, reflect in the x-axis, then shift upward 3 units.



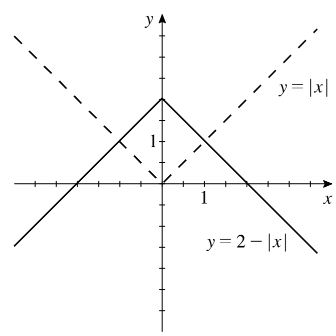
48. $y = 2 - \sqrt{x+1}$. Shift the graph of $y = \sqrt{x}$ to the left 1 unit, reflect the result in the x-axis, then shift upward 2 units.



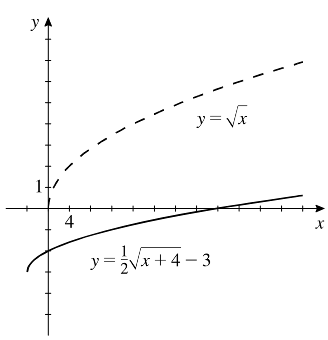
49. $y = |x+2| + 2$. Shift the graph of $y = |x|$ to the left 2 units and upward 2 units.



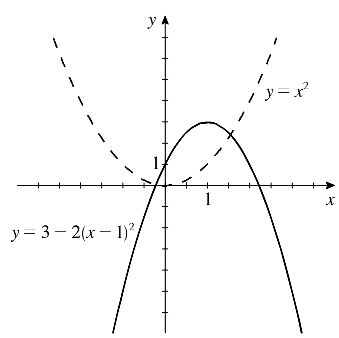
50. $y = 2 - |x|$. Reflect the graph of $y = |x|$ in the x-axis, then shift upward 2 units.



51. $y = \frac{1}{2}\sqrt{x+4} - 3$. Shrink the graph of $y = \sqrt{x}$ vertically by a factor of $\frac{1}{2}$, then shift the result to the left 4 units and downward 3 units.



52. $y = 3 - 2(x-1)^2$. Stretch the graph of $y = x^2$ vertically by a factor of 2, reflect the result in the x-axis, then shift the result to the right 1 unit and upward 3 units.



53. $y = f(x) - 3$. When $f(x) = x^2$, $y = x^2 - 3$.

54. $y = f(x) + 5$. When $f(x) = x^3$, $y = x^3 + 5$.

55. $y = f(x+2)$. When $f(x) = \sqrt{x}$, $y = \sqrt{x+2}$.

56. $y = f(x-1)$. When $f(x) = \sqrt[3]{x}$, $y = \sqrt[3]{x-1}$.

57. $y = f(x+2) - 5$. When $f(x) = |x|$, $y = |x+2| - 5$.

58. $y = -f(x-4) + 3$. When $f(x) = |x|$, $y = -|x-4| + 3$.

59. $y = f(-x) + 1$. When $f(x) = \sqrt[4]{x}$, $y = \sqrt[4]{-x} + 1$.

60. $y = -f(x+2)$. When $f(x) = x^2$, $y = -(x+2)^2$.

61. $y = 2f(x-3) - 2$. When $f(x) = x^2$,
$y = 2(x-3)^2 - 2$.

62. $y = \frac{1}{2} f(x+1) + 3$. When $f(x) = |x|$,
$y = \frac{1}{2}|x+1| + 3$.

63. $g(x) = f(x-2) = (x-2)^2 = x^2 - 4x + 4$

64. $g(x) = f(x) + 3 = x^3 + 3$

65. $g(x) = f(x+1) + 2 = |x+1| + 2$

66. $g(x) = 2f(x) = 2|x|$

67. $g(x) = -f(x+2) = -\sqrt{x+2}$

68. $g(x) = -f(x-2) + 1 = -(x-2)^2 + 1 = -x^2 + 4x - 3$

69. **(a)** $y = f(x-4)$ is graph #3.

(b) $y = f(x) + 3$ is graph #1.

(c) $y = 2f(x+6)$ is graph #2.

(d) $y = -f(2x)$ is graph #4.

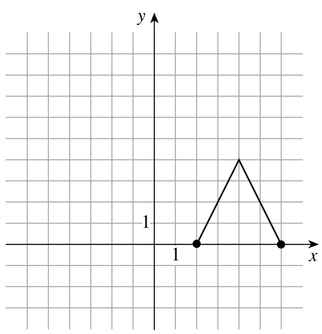
70. **(a)** $y = \frac{1}{3} f(x)$ is graph #2.

(b) $y = -f(x+4)$ is graph #3.

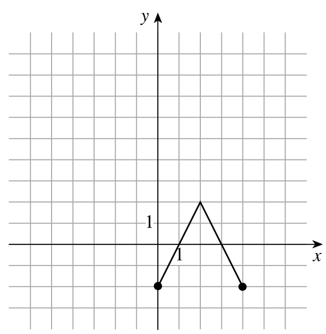
(c) $y = f(x-5) + 3$ is graph #1.

(d) $y = f(-x)$ is graph #4.

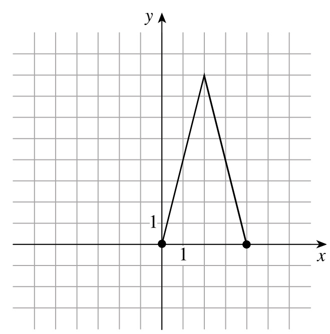
71. **(a)** $y = f(x-2)$



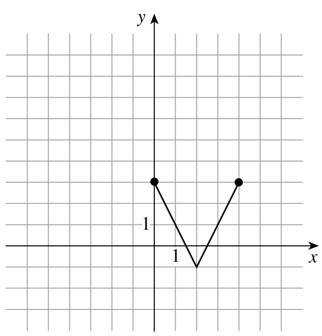
(b) $y = f(x) - 2$



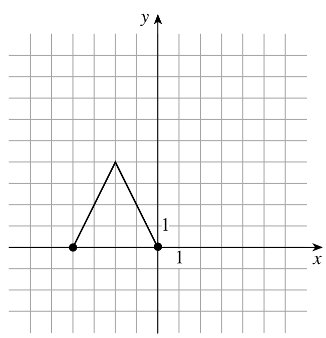
(c) $y = 2f(x)$



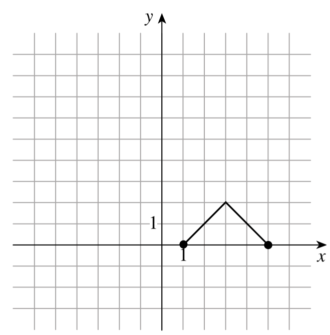
(d) $y = -f(x) + 3$



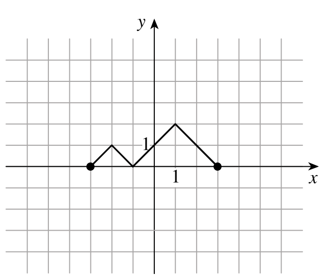
(e) $y = f(-x)$



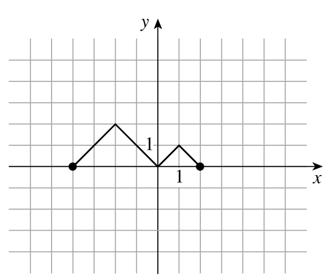
(f) $y = \frac{1}{2} f(x-1)$



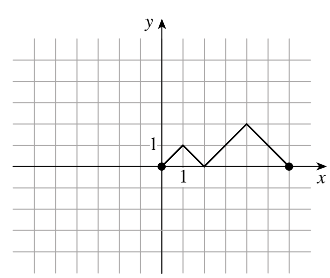
72. **(a)** $y = g(x+1)$



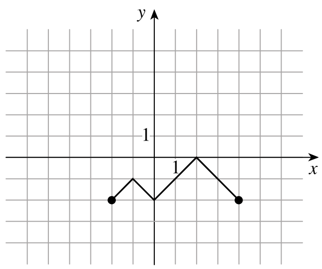
(b) $y = g(-x)$



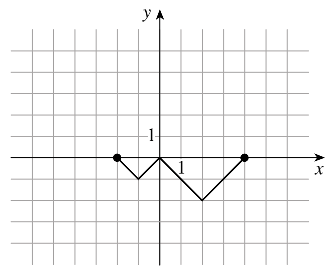
(c) $y = g(x-2)$



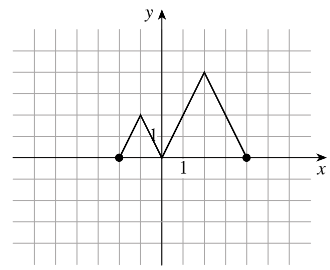
(d) $y = g(x) - 2$



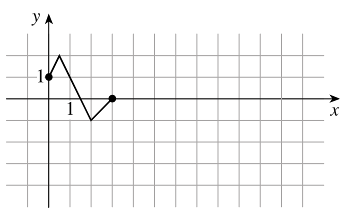
(e) $y = -g(x)$



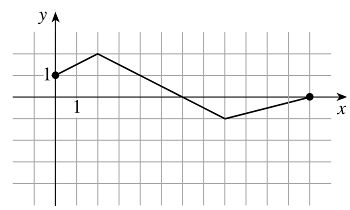
(f) $y = 2g(x)$



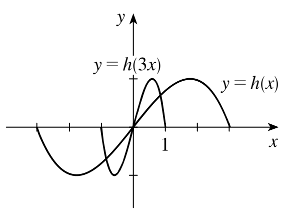
73. (a) $y = g(2x)$



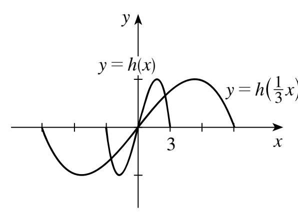
(b) $y = g\left(\frac{1}{2}x\right)$



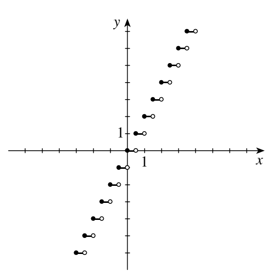
74. (a) $y = h(3x)$



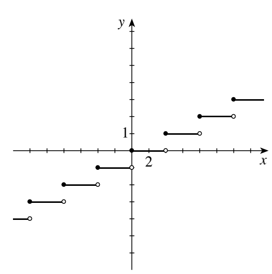
(b) $y = h\left(\frac{1}{3}x\right)$



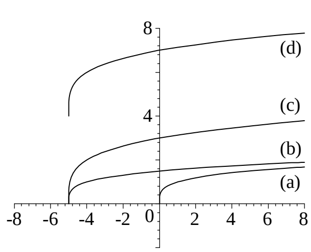
75. $y = [\![2x]\!]$



76. $y = \left[\!\left[\frac{1}{4}x\right]\!\right]$

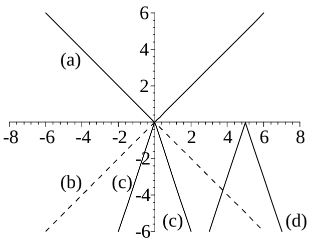


77.



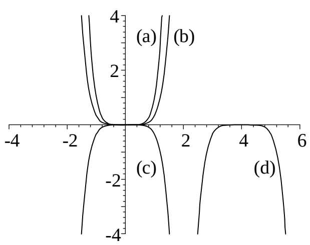
For part (b), shift the graph in (a) to the left 5 units; for part (c), shift the graph in (a) to the left 5 units, and stretch it vertically by a factor of 2; for part (d), shift the graph in (a) to the left 5 units, stretch it vertically by a factor of 2, and then shift it upward 4 units.

78.



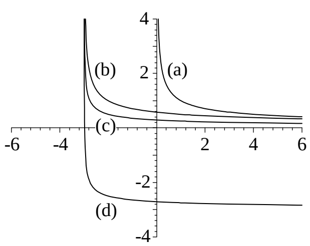
For (b), reflect the graph in (a) in the x-axis; for (c), stretch the graph in (a) vertically by a factor of 3 and reflect in the x-axis; for (d), shift the graph in (a) to the right 5 units, stretch it vertically by a factor of 3, and reflect it in the x-axis. The order in which each operation is applied to the graph in (a) is not important to obtain the graphs in part (c) and (d).

79.



For part (b), shrink the graph in (a) vertically by a factor of $\frac{1}{3}$; for part (c), shrink the graph in (a) vertically by a factor of $\frac{1}{3}$, and reflect it in the x-axis; for part (d), shift the graph in (a) to the right 4 units, shrink vertically by a factor of $\frac{1}{3}$, and then reflect it in the x-axis.

80.

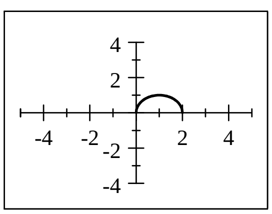


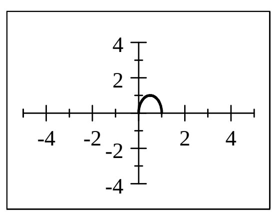
For (b), shift the graph in (a) to the left 3 units; for (c), shift the graph in (a) to the left 3 units and shrink it vertically by a factor of $\frac{1}{2}$; for (d), shift the graph in (a) to the left 3 units, shrink it vertically by a factor of $\frac{1}{2}$, and then shift it downward 3 units. The order in which each operation is applied to the graph in (a) is not important to sketch (c), while it is important in (d).

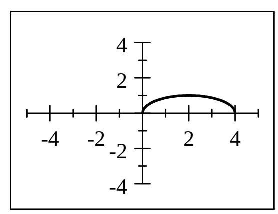
81. (a) $y = f(x) = \sqrt{2x - x^2}$

(b) $y = f(2x) = \sqrt{2(2x) - (2x)^2} = \sqrt{4x - 4x^2}$

(c) $y = f\left(\frac{1}{2}x\right) = \sqrt{2\left(\frac{1}{2}x\right) - \left(\frac{1}{2}x\right)^2} = \sqrt{x - \frac{1}{4}x^2}$





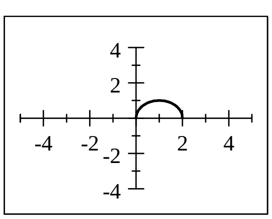


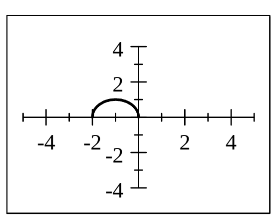
The graph in part (b) is obtained by horizontally shrinking the graph in part (a) by a factor of $\frac{1}{2}$ (so the graph is half as wide). The graph in part (c) is obtained by horizontally stretching the graph in part (a) by a factor of 2 (so the graph is twice as wide).

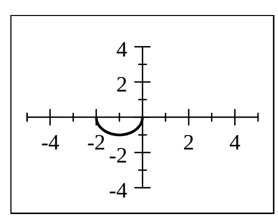
82. (a) $y = f(x) = \sqrt{2x - x^2}$

(b) $y = f(-x) = \sqrt{2(-x) - (-x)^2} = \sqrt{-2x - x^2}$

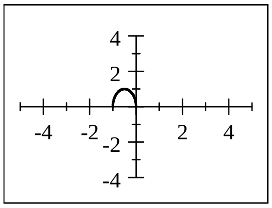
(c) $y = -f(-x) = -\sqrt{2(-x) - (-x)^2} = -\sqrt{-2x = x^2}$



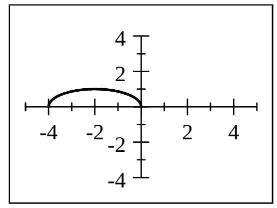




(d) $y = f(-2x) = \sqrt{2(-2x) - (-2x)^2}$
$= -\sqrt{-2x - x^2} = \sqrt{-4x - 4x^2}$

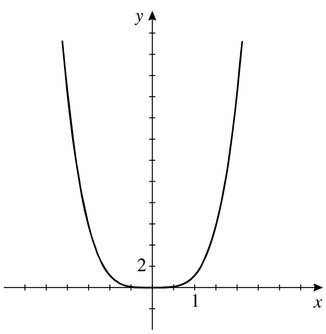


(e) $y = f\left(-\frac{1}{2}x\right) = \sqrt{2\left(-\frac{1}{2}x\right) - \left(-\frac{1}{2}x\right)^2}$
$= \sqrt{-x - \frac{1}{4}x^2}$

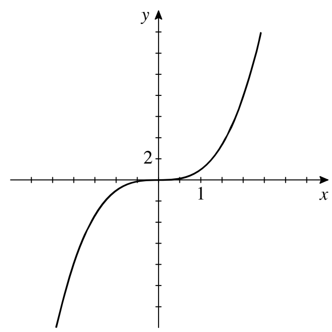


The graph in part (b) is obtained by reflecting the graph in part (a) in the y-axis. The graph in part (c) is obtained by rotating the graph in part (a) through 180° about the origin [or by reflecting the graph in part (a) first in the x-axis and then in the y-axis]. The graph in part (d) is obtained by reflecting the graph in part (a) in the y-axis and then horizontally shrinking the graph by a factor of $\frac{1}{2}$ (so the graph is half as wide). The graph in part (e) is obtained by reflecting the graph in part (a) in the y-axis and then horizontally stretching the graph by a factor of 2 (so the graph is twice as wide).

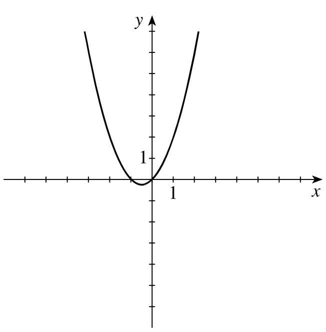
83. $f(x) = x^4$. $f(-x) = (-x)^4 = x^4 = f(x)$. Thus $f(x)$ is even.



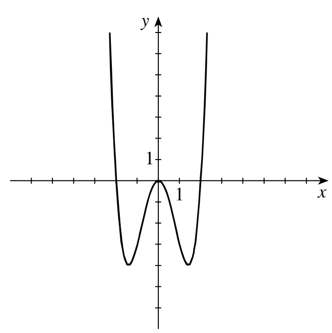
84. $f(x) = x^3$. $f(-x) = (-x)^3 = -x^3 = -f(x)$. Thus $f(x)$ is odd.



85. $f(x) = x^2 + x$. $f(-x) = (-x)^2 + (-x) = x^2 - x$. Thus $f(-x) \neq f(x)$. Also, $f(-x) \neq -f(x)$, so $f(x)$ is neither odd nor even.



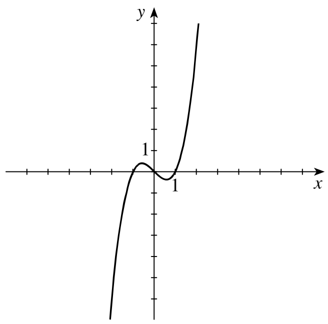
86. $f(x) = x^4 - 4x^2$.
$f(-x) = (-x)^4 - 4(-x)^2 = x^4 - 4x^2 = f(x)$. Thus $f(x)$ is even.



87. $f(x) = x^3 - x$.

$$f(-x) = (-x)^3 - (-x) = -x^3 + x$$
$$= -\left(x^3 - x\right) = -f(x).$$

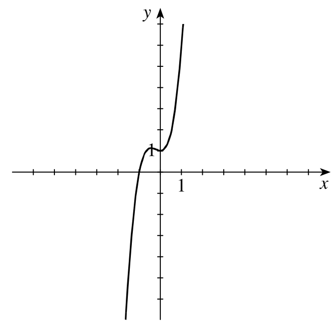
Thus $f(x)$ is odd.



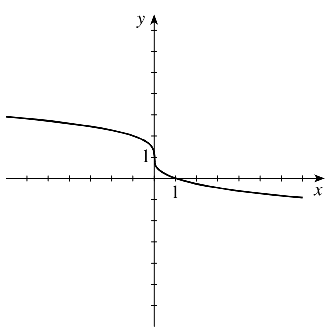
88. $f(x) = 3x^3 + 2x^2 + 1$.

$f(-x) = 3(-x)^3 + 2(-x)^2 + 1 = -3x^3 + 2x^2 + 1$.

Thus $f(-x) \neq f(x)$. Also $f(-x) \neq -f(x)$, so $f(x)$ is neither odd nor even.



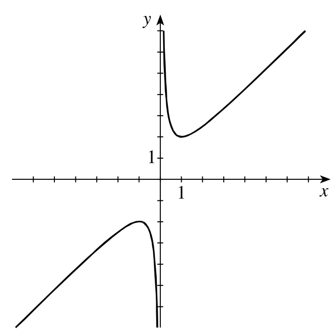
89. $f(x) = 1 - \sqrt[3]{x}$. $f(-x) = 1 - \sqrt[3]{(-x)} = 1 + \sqrt[3]{x}$. Thus $f(-x) \neq f(x)$. Also $f(-x) \neq -f(x)$, so $f(x)$ is neither odd nor even.



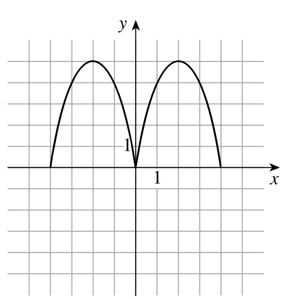
90. $f(x) = x + 1/x$.

$$f(-x) = (-x) + 1/(-x) = -x - 1/x$$
$$= -(x + 1/x) = -f(x).$$

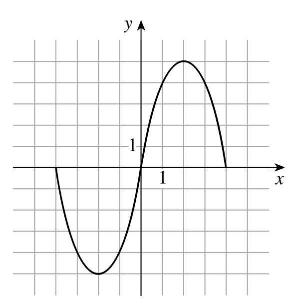
Thus $f(x)$ is odd.



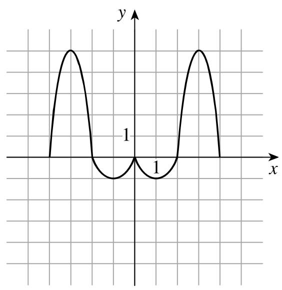
91. (a) Even



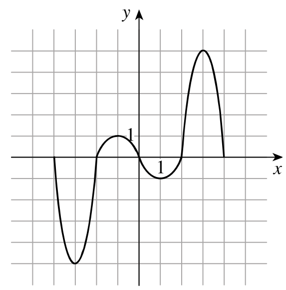
(b) Odd



92. (a) Even

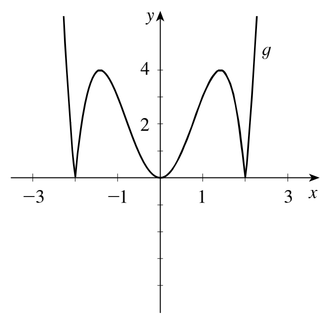


(b) Odd

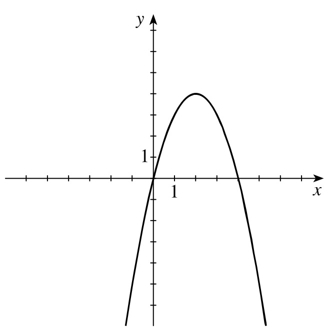


93. Since $f(x) = x^2 - 4 < 0$, for $-2 < x < 2$, the graph of $y = g(x)$ is found by sketching the graph of $y = f(x)$ for $x \le -2$ and $x \ge 2$, then reflecting in the x-axis the part of the graph of $y = f(x)$ for $-2 < x < 2$.

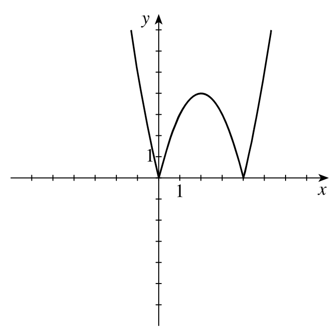
94. $g(x) = \left|x^4 - 4x^2\right|$



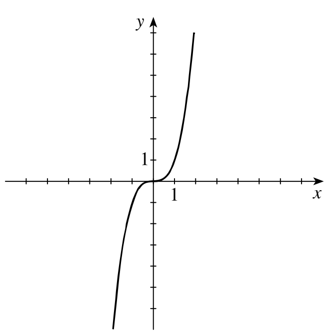
95. (a) $f(x) = 4x - x^2$



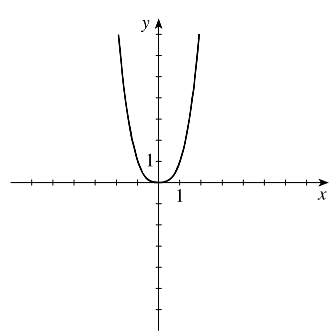
(b) $f(x) = \left|4x - x^2\right|$



96. (a) $f(x) = x^3$

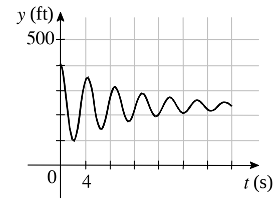


(b) $g(x) = \left|x^3\right|$



97. **(a)** Luisa drops to a height of 200 feet, bounces up and down, then settles at 350 feet.

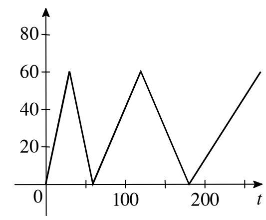
(b)



(c) To obtain the graph of H from that of h, we shift downward 100 feet. Thus, $H(t) = h(t) - 100$.

98. **(a)** Miyuki swims two and a half laps, slowing down with each successive lap. In the first 30 seconds she swims 50 meters, so her average speed is $\frac{50}{30} \approx 1.67$ m/s.

(b)

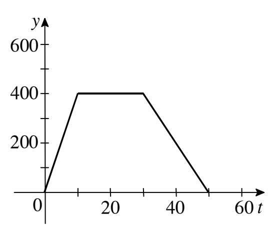


This graph is obtained by stretching the original graph vertically by a factor of 1.2.

(c) Here Miyuki swims 60 meters in 30 seconds, so her average speed is $\frac{60}{30} = 2$ m/s.

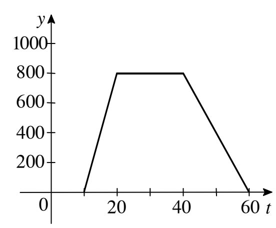
99. **(a)** The trip to the park corresponds to the first piece of the graph. The class travels 800 feet in 10 minutes, so their average speed is $\frac{800}{10} = 80$ ft/min. The second (horizontal) piece of the graph stretches from $t = 10$ to $t = 30$, so the class spends 20 minutes at the park. The park is 800 feet from the school.

(b)



The new graph is obtained by shrinking the original graph vertically by a factor of 0.50. The new average speed is 40 ft/min, and the new park is 400 ft from the school.

(c)



This graph is obtained by shifting the original graph to the right 10 minutes. The class leaves ten minutes later than it did in the original scenario.

100. To obtain the graph of $g(x) = (x-2)^2 + 5$ from that of $f(x) = (x+2)^2$, we shift to the right 4 units and upward 5 units.

101. To obtain the graph of $g(x)$ from that of $f(x)$, we reflect the graph about the y-axis, then reflect about the x-axis, then shift upward 6 units.

102. f even implies $f(-x) = f(x)$; g even implies $g(-x) = g(x)$; f odd implies $f(-x) = -f(x)$; and g odd implies $g(-x) = -g(x)$

If f and g are both even, then $(f+g)(-x) = f(-x) + g(-x) = f(x) + g(x) = (f+g)(x)$ and $f+g$ is even.

If f and g are both odd, then $(f+g)(-x) = f(-x) + g(-x) = -f(x) - g(x) = -(f+g)(x)$ and $f+g$ is odd.

If f odd and g even, then $(f+g)(-x) = f(-x) + g(-x) = -f(x) + g(x)$, which is neither odd nor even.

103. f even implies $f(-x) = f(x)$; g even implies $g(-x) = g(x)$; f odd implies $f(-x) = -f(x)$; and g odd implies $g(-x) = -g(x)$.

If f and g are both even, then $(fg)(-x) = f(-x) \cdot g(-x) = f(x) \cdot g(x) = (fg)(x)$. Thus fg is even.

If f and g are both odd, then $(fg)(-x) = f(-x) \cdot g(-x) = -f(x) \cdot (-g(x)) = f(x) \cdot g(x) = (fg)(x)$. Thus fg is even

If f if odd and g is even, then $(fg)(-x) = f(-x) \cdot g(-x) = f(x) \cdot (-g(x)) = -f(x) \cdot g(x) = -(fg)(x)$. Thus fg is odd.

104. $f(x) = x^n$ is even when n is an even integer and $f(x) = x^n$ is odd when n is an odd integer.
These names were chosen because polynomials with only terms with odd powers are odd functions, and polynomials with only terms with even powers are even functions.

2.7 COMBINING FUNCTIONS

1. From the graphs of f and g in the figure, we find $(f+g)(2) = f(2) + g(2) = 3 + 5 = 8$, $(f-g)(2) = f(2) - g(2) = 3 - 5 = -2$, $(fg)(2) = f(2)\,g(2) = 3 \cdot 5 = 15$, and $\left(\dfrac{f}{g}\right)(2) = \dfrac{f(2)}{g(2)} = \dfrac{3}{5}$.

2. By definition, $f \circ g(x) = f(g(x))$. So, if $g(2) = 5$ and $f(5) = 12$, then $f \circ g(2) = f(g(2)) = f(5) = 12$.

3. If the rule of the function f is "add one" and the rule of the function g is "multiply by 2" then the rule of $f \circ g$ is "*multiply by* 2*, then add one*" and the rule of $g \circ f$ is "*add one, then multiply by* 2."

4. We can express the functions in Exercise 3 algebraically as $f(x) = x + 1$, $g(x) = 2x$, $(f \circ g)(x) = 2x + 1$, and $(g \circ f)(x) = 2(x+1)$.

5. **(a)** The function $(f+g)(x)$ is defined for all values of x that are in the domains of both f and g.
(b) The function $(fg)(x)$ is defined for all values of x that are in the domains of both f and g.
(c) The function $(f/g)(x)$ is defined for all values of x that are in the domains of both f and g, and $g(x)$ is not equal to 0.

6. The composition $(f \circ g)(x)$ is defined for all values of x for which x is in the domain of g and $g(x)$ is in the domain of f.

7. $f(x) = x$ has domain $(-\infty, \infty)$. $g(x) = 2x$ has domain $(-\infty, \infty)$. The intersection of the domains of f and g is $(-\infty, \infty)$.
$(f+g)(x) = x + 2x = 3x$, and the domain is $(-\infty, \infty)$. $(f-g)(x) = x - 2x = -x$, and the domain is $(-\infty, \infty)$.
$(fg)(x) = x(2x) = 2x^2$, and the domain is $(-\infty, \infty)$. $\left(\dfrac{f}{g}\right)(x) = \dfrac{x}{2x} = \dfrac{1}{2}$, and the domain is $(-\infty, 0) \cup (0, \infty)$.

8. $f(x) = x$ has domain $(-\infty, \infty)$. $g(x) = \sqrt{x}$ has domain $[0, \infty)$. The intersection of the domains of f and g is $[0, \infty)$.
$(f+g)(x) = x + \sqrt{x}$, and the domain is $[0, \infty)$. $(f-g)(x) = x - \sqrt{x}$, and the domain is $[0, \infty)$.
$(fg)(x) = x\sqrt{x} = x^{3/2}$, and the domain is $[0, \infty)$. $\left(\dfrac{f}{g}\right)(x) = \dfrac{x}{\sqrt{x}} = \sqrt{x}$, and the domain is $(0, \infty)$.

9. $f(x) = x^2 + x$ and $g(x) = x^2$ each have domain $(-\infty, \infty)$. The intersection of the domains of f and g is $(-\infty, \infty)$.
$(f+g)(x) = 2x^2 + x$, and the domain is $(-\infty, \infty)$. $(f-g)(x) = x$, and the domain is $(-\infty, \infty)$.
$(fg)(x) = x^4 + x^3$, and the domain is $(-\infty, \infty)$. $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2 + x}{x^2} = 1 + \dfrac{1}{x}$, and the domain is $(-\infty, 0) \cup (0, \infty)$.

10. $f(x) = 3 - x^2$ and $g(x) = x^2 - 4$ each have domain $(-\infty, \infty)$. The intersection of the domains of f and g is $(-\infty, \infty)$.
$(f+g)(x) = -1$, and the domain is $(-\infty, \infty)$. $(f-g)(x) = -2x^2 + 7$, and the domain is $(-\infty, \infty)$.
$(fg)(x) = \left(3 - x^2\right)\left(x^2 - 4\right) = -x^4 + 7x^2 - 12$, and the domain is $(-\infty, \infty)$. $\left(\dfrac{f}{g}\right)(x) = \dfrac{3 - x^2}{x^2 - 4} = \dfrac{3 - x^2}{(x-2)(x+2)}$, and the domain is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

11. $f(x) = 5 - x$ and $g(x) = x^2 - 3x$ each have domain $(-\infty, \infty)$. The intersection of the domains of f and g is $(-\infty, \infty)$.
$(f+g)(x) = (5 - x) + \left(x^2 - 3x\right) = x^2 - 4x + 5$, and the domain is $(-\infty, \infty)$.
$(f-g)(x) = (5 - x) - \left(x^2 - 3x\right) = -x^2 + 2x + 5$, and the domain is $(-\infty, \infty)$.
$(fg)(x) = (5 - x)\left(x^2 - 3x\right) = -x^3 + 8x^2 - 15x$, and the domain is $(-\infty, \infty)$.
$\left(\dfrac{f}{g}\right)(x) = \dfrac{5 - x}{x^2 - 3x} = \dfrac{5 - x}{x(x-3)}$, and the domain is $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$.

12. $f(x) = x^2 + 2x$ has domain $(-\infty, \infty)$. $g(x) = 3x^2 - 1$ has domain $(-\infty, \infty)$. The intersection of the domains of f and g is $(-\infty, \infty)$.

$(f + g)(x) = x^2 + 2x + \left(3x^2 - 1\right) = 4x^2 + 2x - 1$, and the domain is $(-\infty, \infty)$.

$(f - g)(x) = x^2 + 2x - \left(3x^2 - 1\right) = -2x^2 + 2x + 1$, and the domain is $(-\infty, \infty)$.

$(fg)(x) = \left(x^2 + 2x\right)\left(3x^2 - 1\right) = 3x^4 + 6x^3 - x^2 - 2x$, and the domain is $(-\infty, \infty)$.

$\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2 + 2x}{3x^2 - 1}$, $3x^2 - 1 \neq 0 \Rightarrow x \neq \pm\frac{\sqrt{3}}{3}$, and the domain is $\left\{x \mid x \neq \pm\frac{\sqrt{3}}{3}\right\}$.

13. $f(x) = \sqrt{25 - x^2}$, has domain $[-5, 5]$. $g(x) = \sqrt{x + 3}$, has domain $[-3, \infty)$. The intersection of the domains of f and g is $[-3, 5]$.

$(f + g)(x) = \sqrt{25 - x^2} + \sqrt{x + 3}$, and the domain is $[-3, 5]$.

$(f - g)(x) = \sqrt{25 - x^2} - \sqrt{x + 3}$, and the domain is $[-3, 5]$.

$(fg)(x) = \sqrt{\left(25 - x^2\right)(x + 3)}$, and the domain is $[-3, 5]$.

$\left(\dfrac{f}{g}\right)(x) = \sqrt{\dfrac{25 - x^2}{x + 3}}$, and the domain is $(-3, 5]$.

14. $f(x) = \sqrt{16 - x^2}$ has domain $[-4, 4]$. $g(x) = \sqrt{x^2 - 1}$ has domain $(-\infty, -1] \cup [1, \infty)$. The intersection of the domains of f and g is $[-4, -1] \cup [1, 4]$.

$(f + g)(x) = \sqrt{16 - x^2} + \sqrt{x^2 - 1}$, and the domain is $[-4, -1] \cup [1, 4]$.

$(f - g)(x) = \sqrt{16 - x^2} - \sqrt{x^2 - 1}$, and the domain is $[-4, -1] \cup [1, 4]$.

$(fg)(x) = \sqrt{\left(16 - x^2\right)\left(x^2 - 1\right)}$, and the domain is $[-4, -1] \cup [1, 4]$.

$\left(\dfrac{f}{g}\right)(x) = \sqrt{\dfrac{16 - x^2}{x^2 - 1}}$, and the domain is $[-4, -1) \cup (1, 4]$.

15. $f(x) = \dfrac{2}{x}$ has domain $x \neq 0$. $g(x) = \dfrac{4}{x + 4}$, has domain $x \neq -4$. The intersection of the domains of f and g is $\{x \mid x \neq 0, -4\}$; in interval notation, this is $(-\infty, -4) \cup (-4, 0) \cup (0, \infty)$.

$(f + g)(x) = \dfrac{2}{x} + \dfrac{4}{x + 4} = \dfrac{2}{x} + \dfrac{4}{x + 4} = \dfrac{2(3x + 4)}{x(x + 4)}$, and the domain is $(-\infty, -4) \cup (-4, 0) \cup (0, \infty)$.

$(f - g)(x) = \dfrac{2}{x} - \dfrac{4}{x + 4} = -\dfrac{2(x - 4)}{x(x + 4)}$, and the domain is $(-\infty, -4) \cup (-4, 0) \cup (0, \infty)$.

$(fg)(x) = \dfrac{2}{x} \cdot \dfrac{4}{x + 4} = \dfrac{8}{x(x + 4)}$, and the domain is $(-\infty, -4) \cup (-4, 0) \cup (0, \infty)$.

$\left(\dfrac{f}{g}\right)(x) = \dfrac{\dfrac{2}{x}}{\dfrac{4}{x + 4}} = \dfrac{x + 4}{2x}$, and the domain is $(-\infty, -4) \cup (-4, 0) \cup (0, \infty)$.

16. $f(x)=\dfrac{2}{x+1}$ has domain $x\neq -1$. $g(x)=\dfrac{x}{x+1}$ has domain $x\neq -1$. The intersection of the domains of f and g is $\{x \mid x\neq -1\}$; in interval notation, this is $(-\infty,-1)\cup(-1,\infty)$.

$(f+g)(x)=\dfrac{2}{x+1}+\dfrac{x}{x+1}=\dfrac{x+2}{x+1}$, and the domain is $(-\infty,-1)\cup(-1,\infty)$.

$(f-g)(x)=\dfrac{2}{x+1}-\dfrac{x}{x+1}=\dfrac{2-x}{x+1}$, and the domain is $(-\infty,-1)\cup(-1,\infty)$.

$(fg)(x)=\dfrac{2}{x+1}\cdot\dfrac{x}{x+1}=\dfrac{2x}{(x+1)^2}$, and the domain is $(-\infty,-1)\cup(-1,\infty)$.

$\left(\dfrac{f}{g}\right)(x)=\dfrac{\frac{2}{x+1}}{\frac{x}{x+1}}=\dfrac{2}{x}$, so $x\neq 0$ as well. Thus the domain is $(-\infty,-1)\cup(-1,0)\cup(0,\infty)$.

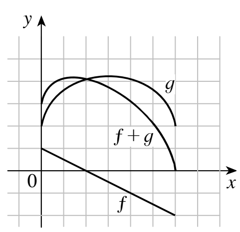
17. $f(x)=\sqrt{x}+\sqrt{3-x}$. The domain of $\sqrt{x}$ is $[0,\infty)$, and the domain of $\sqrt{3-x}$ is $(-\infty,3]$. Thus, the domain of f is $(-\infty,3]\cap[0,\infty)=[0,3]$.

18. $f(x)=\sqrt{x+4}-\dfrac{\sqrt{1-x}}{x}$. The domain of $\sqrt{x+4}$ is $[-4,\infty)$, and the domain of $\dfrac{\sqrt{1-x}}{x}$ is $(-\infty,0)\cup(0,1]$. Thus, the domain of f is $[-4,\infty)\cap\{(-\infty,0)\cup(0,1]\}=[-4,0)\cup(0,1]$.

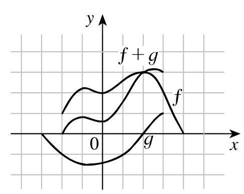
19. $h(x)=(x-3)^{-1/4}=\dfrac{1}{(x-3)^{1/4}}$. Since $1/4$ is an even root and the denominator can not equal 0, $x-3>0\Leftrightarrow x>3$. So the domain is $(3,\infty)$.

20. $k(x)=\dfrac{\sqrt{x+3}}{x-1}$. The domain of $\sqrt{x+3}$ is $[-3,\infty)$, and the domain of $\dfrac{1}{x-1}$ is $x\neq 1$. Since $x\neq 1$ is $(-\infty,1)\cup(1,\infty)$, the domain is $[-3,\infty)\cap\{(-\infty,1)\cup(1,\infty)\}=[-3,1)\cup(1,\infty)$.

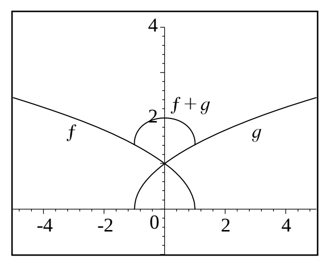
21.



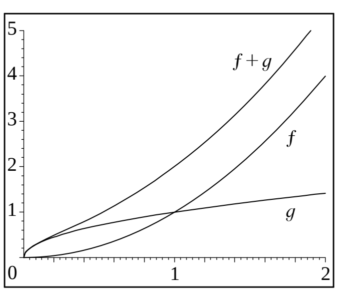
22.



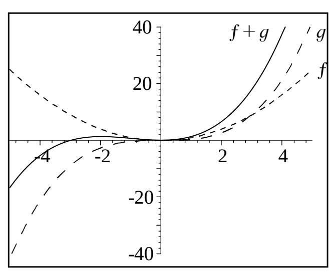
23.



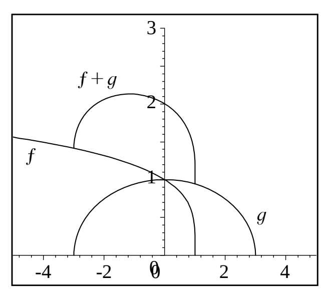
24.



25.



26.



27. $f(x)=2x-3$ and $g(x)=4-x^2$.

(a) $f(g(0))=f\left(4-(0)^2\right)=f(4)=2(4)-3=5$

(b) $g(f(0))=g(2(0)-3)=g(-3)=4-(-3)^2=-5$

28. **(a)** $f(f(2))=f(2(2)-3)=f(1)=2(1)-3=-1$

(b) $g(g(3))=g\left(4-3^2\right)=g(-5)=4-(-5)^2=-21$

29. (a) $(f \circ g)(-2) = f(g(-2)) = f\left(4 - (-2)^2\right) = f(0) = 2(0) - 3 = -3$

(b) $(g \circ f)(-2) = g(f(-2)) = g(2(-2) - 3) = g(-7) = 4 - (-7)^2 = -45$

30. (a) $(f \circ f)(-1) = f(f(-1)) = f(2(-1) - 3) = f(-5) = 2(-5) - 3 = -13$

(b) $(g \circ g)(-1) = g(g(-1)) = g\left(4 - (-1)^2\right) = g(3) = 4 - 3^2 = -5$

31. (a) $(f \circ g)(x) = f(g(x)) = f\left(4 - x^2\right) = 2\left(4 - x^2\right) - 3 = 8 - 2x^2 - 3 = 5 - 2x^2$

(b) $(g \circ f)(x) = g(f(x)) = g(2x - 3) = 4 - (2x - 3)^2 = 4 - \left(4x^2 - 12x + 9\right) = -4x^2 + 12x - 5$

32. (a) $(f \circ f)(x) = f(f(x)) = f(3x - 5) = 3(3x - 5) - 5 = 9x - 15 - 5 = 9x - 20$

(b) $(g \circ g)(x) = g(g(x)) = g\left(2 - x^2\right) = 2 - \left(2 - x^2\right)^2 = 2 - \left(4 - 4x^2 + x^4\right) = -x^4 + 4x^2 - 2$

33. $f(g(2)) = f(5) = 4$

34. $f(0) = 0$, so $g(f(0)) = g(0) = 3$.

35. $(g \circ f)(4) = g(f(4)) = g(2) = 5$

36. $g(0) = 3$, so $(f \circ g)(0) = f(3) = 0$.

37. $(g \circ g)(-2) = g(g(-2)) = g(1) = 4$

38. $f(4) = 2$, so $(f \circ f)(4) = f(2) = -2$.

39. From the table, $g(2) = 5$ and $f(5) = 6$, so $f(g(2)) = 6$.

40. From the table, $f(2) = 3$ and $g(3) = 6$, so $g(f(2)) = 6$.

41. From the table, $f(1) = 2$ and $f(2) = 3$, so $f(f(1)) = 3$.

42. From the table, $g(2) = 5$ and $g(5) = 1$, so $g(g(2)) = 1$.

43. From the table, $g(6) = 4$ and $f(4) = 1$, so $(f \circ g)(6) = 1$.

44. From the table, $f(2) = 3$ and $g(3) = 6$, so $(g \circ f)(2) = 6$.

45. From the table, $f(5) = 6$ and $f(6) = 3$, so $(f \circ f)(5) = 3$.

46. From the table, $g(2) = 5$ and $g(5) = 1$, so $(g \circ g)(5) = 1$.

47. $f(x) = 2x + 3$, has domain $(-\infty, \infty)$; $g(x) = 4x - 1$, has domain $(-\infty, \infty)$.
$(f \circ g)(x) = f(4x - 1) = 2(4x - 1) + 3 = 8x + 1$, and the domain is $(-\infty, \infty)$.
$(g \circ f)(x) = g(2x + 3) = 4(2x + 3) - 1 = 8x + 11$, and the domain is $(-\infty, \infty)$.
$(f \circ f)(x) = f(2x + 3) = 2(2x + 3) + 3 = 4x + 9$, and the domain is $(-\infty, \infty)$.
$(g \circ g)(x) = g(4x - 1) = 4(4x - 1) - 1 = 16x - 5$, and the domain is $(-\infty, \infty)$.

48. $f(x) = 6x - 5$ has domain $(-\infty, \infty)$. $g(x) = \dfrac{x}{2}$ has domain $(-\infty, \infty)$.
$(f \circ g)(x) = f\left(\dfrac{x}{2}\right) = 6\left(\dfrac{x}{2}\right) - 5 = 3x - 5$, and the domain is $(-\infty, \infty)$.
$(g \circ f)(x) = g(6x - 5) = \dfrac{6x - 5}{2} = 3x - \frac{5}{2}$, and the domain is $(-\infty, \infty)$.
$(f \circ f)(x) = f(6x - 5) = 6(6x - 5) - 5 = 36x - 35$, and the domain is $(-\infty, \infty)$.
$(g \circ g)(x) = g\left(\dfrac{x}{2}\right) = \dfrac{\frac{x}{2}}{2} = \dfrac{x}{4}$, and the domain is $(-\infty, \infty)$.

49. $f(x) = x^2$, has domain $(-\infty, \infty)$; $g(x) = x + 1$, has domain $(-\infty, \infty)$.
$(f \circ g)(x) = f(x + 1) = (x + 1)^2 = x^2 + 2x + 1$, and the domain is $(-\infty, \infty)$.
$(g \circ f)(x) = g\left(x^2\right) = \left(x^2\right) + 1 = x^2 + 1$, and the domain is $(-\infty, \infty)$.
$(f \circ f)(x) = f\left(x^2\right) = \left(x^2\right)^2 = x^4$, and the domain is $(-\infty, \infty)$.
$(g \circ g)(x) = g(x + 1) = (x + 1) + 1 = x + 2$, and the domain is $(-\infty, \infty)$.

50. $f(x) = x^3 + 2$ has domain $(-\infty, \infty)$. $g(x) = \sqrt[3]{x}$ has domain $(-\infty, \infty)$.
$(f \circ g)(x) = f\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 + 2 = x + 2$, and the domain is $(-\infty, \infty)$.
$(g \circ f)(x) = g\left(x^3 + 2\right) = \sqrt[3]{x^3 + 2}$ and the domain is $(-\infty, \infty)$.
$(f \circ f)(x) = f\left(x^3 + 2\right) = \left(x^3 + 2\right)^3 + 2 = x^9 + 6x^6 + 12x^3 + 8 + 2 = x^9 + 6x^6 + 12x^3 + 10$, and the domain is $(-\infty, \infty)$.
$(g \circ g)(x) = g\left(\sqrt[3]{x}\right) = \sqrt[3]{\sqrt[3]{x}} = \left(x^{1/3}\right)^{1/3} = x^{1/9}$, and the domain is $(-\infty, \infty)$.

51. $f(x) = \dfrac{1}{x}$, has domain $\{x \mid x \neq 0\}$; $g(x) = 2x + 4$, has domain $(-\infty, \infty)$.
$(f \circ g)(x) = f(2x + 4) = \dfrac{1}{2x + 4}$. $(f \circ g)(x)$ is defined for $2x + 4 \neq 0 \Leftrightarrow x \neq -2$. So the domain is $\{x \mid x \neq -2\} = (-\infty, -2) \cup (-2, \infty)$.
$(g \circ f)(x) = g\left(\dfrac{1}{x}\right) = 2\left(\dfrac{1}{x}\right) + 4 = \dfrac{2}{x} + 4$, the domain is $\{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty)$.
$(f \circ f)(x) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{\left(\dfrac{1}{x}\right)} = x$. $(f \circ f)(x)$ is defined whenever both $f(x)$ and $f(f(x))$ are defined; that is, whenever $\{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty)$.
$(g \circ g)(x) = g(2x + 4) = 2(2x + 4) + 4 = 4x + 8 + 4 = 4x + 12$, and the domain is $(-\infty, \infty)$.

52. $f(x) = x^2$ has domain $(-\infty, \infty)$. $g(x) = \sqrt{x - 3}$ has domain $[3, \infty)$.
$(f \circ g)(x) = f\left(\sqrt{x - 3}\right) = \left(\sqrt{x - 3}\right)^2 = x - 3$, and the domain is $[3, \infty)$.
$(g \circ f)(x) = g\left(x^2\right) = \sqrt{x^2 - 3}$. For the domain we must have $x^2 \geq 3 \Rightarrow x \leq -\sqrt{3}$ or $x \geq \sqrt{3}$. Thus the domain is $\left(-\infty, -\sqrt{3}\right] \cup \left[\sqrt{3}, \infty\right)$.
$(f \circ f)(x) = f\left(x^2\right) = \left(x^2\right)^2 = x^4$, and the domain is $(-\infty, \infty)$.
$(g \circ g)(x) = g\left(\sqrt{x - 3}\right) = \sqrt{\sqrt{x - 3} - 3}$. For the domain we must have $\sqrt{x - 3} \geq 3 \Rightarrow x - 3 \geq 9 \Rightarrow x \geq 12$, so the domain is $[12, \infty)$.

53. $f(x) = |x|$, has domain $(-\infty, \infty)$; $g(x) = 2x + 3$, has domain $(-\infty, \infty)$
$(f \circ g)(x) = f(2x + 4) = |2x + 3|$, and the domain is $(-\infty, \infty)$.
$(g \circ f)(x) = g(|x|) = 2|x| + 3$, and the domain is $(-\infty, \infty)$.
$(f \circ f)(x) = f(|x|) = ||x|| = |x|$, and the domain is $(-\infty, \infty)$.
$(g \circ g)(x) = g(2x + 3) = 2(2x + 3) + 3 = 4x + 6 + 3 = 4x + 9$. Domain is $(-\infty, \infty)$.

54. $f(x) = x - 4$ has domain $(-\infty, \infty)$. $g(x) = |x + 4|$ has domain $(-\infty, \infty)$.
$(f \circ g)(x) = f(|x + 4|) = |x + 4| - 4$, and the domain is $(-\infty, \infty)$.
$(g \circ f)(x) = g(x - 4) = |(x - 4) + 4| = |x|$, and the domain is $(-\infty, \infty)$.
$(f \circ f)(x) = f(x - 4) = (x - 4) - 4 = x - 8$, and the domain is $(-\infty, \infty)$.
$(g \circ g)(x) = g(|x + 4|) = ||x + 4| + 4| = |x + 4| + 4$ ($|x + 4| + 4$ is always positive). The domain is $(-\infty, \infty)$.

55. $f(x) = \dfrac{x}{x+1}$, has domain $\{x \mid x \neq -1\}$; $g(x) = 2x - 1$, has domain $(-\infty, \infty)$

$(f \circ g)(x) = f(2x-1) = \dfrac{2x-1}{(2x-1)+1} = \dfrac{2x-1}{2x}$, and the domain is $\{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty)$.

$(g \circ f)(x) = g\left(\dfrac{x}{x+1}\right) = 2\left(\dfrac{x}{x+1}\right) - 1 = \dfrac{2x}{x+1} - 1$, and the domain is $\{x \mid x \neq -1\} = (-\infty, -1) \cup (-1, \infty)$

$(f \circ f)(x) = f\left(\dfrac{x}{x+1}\right) = \dfrac{\dfrac{x}{x+1}}{\dfrac{x}{x+1}+1} \cdot \dfrac{x+1}{x+1} = \dfrac{x}{x+x+1} = \dfrac{x}{2x+1}$. $(f \circ f)(x)$ is defined whenever both $f(x)$ and $f(f(x))$ are defined; that is, whenever $x \neq -1$ and $2x+1 \neq 0 \Rightarrow x \neq -\frac{1}{2}$, which is $(-\infty, -1) \cup \left(-1, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$.

$(g \circ g)(x) = g(2x-1) = 2(2x-1) - 1 = 4x - 2 - 1 = 4x - 3$, and the domain is $(-\infty, \infty)$.

56. $f(x) = \dfrac{1}{\sqrt{x}}$ has domain $\{x \mid x > 0\}$; $g(x) = x^2 - 4x$ has domain $(-\infty, \infty)$.

$(f \circ g)(x) = f\left(x^2 - 4x\right) = \dfrac{1}{\sqrt{x^2 - 4x}}$. $(f \circ g)(x)$ is defined whenever $0 < x^2 - 4x = x(x-4)$. The product of two numbers is positive either when both numbers are negative or when both numbers are positive. So the domain of $f \circ g$ is $\{x \mid x < 0 \text{ and } x < 4\} \cup \{x \mid x > 0 \text{ and } x > 4\}$ which is $(-\infty, 0) \cup (4, \infty)$.

$(g \circ f)(x) = g\left(\dfrac{1}{\sqrt{x}}\right) = \left(\dfrac{1}{\sqrt{x}}\right)^2 - 4\left(\dfrac{1}{\sqrt{x}}\right) = \dfrac{1}{x} - \dfrac{4}{\sqrt{x}}$. $(g \circ f)(x)$ is defined whenever both $f(x)$ and $g(f(x))$ are defined, that is, whenever $x > 0$. So the domain of $g \circ f$ is $(0, \infty)$.

$(f \circ f)(x) = f\left(\dfrac{1}{\sqrt{x}}\right) = \dfrac{1}{\sqrt{\dfrac{1}{\sqrt{x}}}} = x^{1/4}$. $(f \circ f)(x)$ is defined whenever both $f(x)$ and $f(f(x))$ are defined, that is, whenever $x > 0$. So the domain of $f \circ f$ is $(0, \infty)$.

$(g \circ g)(x) = g\left(x^2 - 4x\right) = \left(x^2 - 4x\right)^2 - 4\left(x^2 - 4x\right) = x^4 - 8x^3 + 16x^2 - 4x^2 + 16x = x^4 - 8x^3 + 12x^2 + 16x$, and the domain is $(-\infty, \infty)$.

57. $f(x) = \dfrac{x}{x+1}$, has domain $\{x \mid x \neq -1\}$; $g(x) = \dfrac{1}{x}$ has domain $\{x \mid x \neq 0\}$.

$(f \circ g)(x) = f\left(\dfrac{1}{x}\right) = \dfrac{\frac{1}{x}}{\frac{1}{x}+1} = \dfrac{1}{x\left(\frac{1}{x}+1\right)} = \dfrac{1}{x+1}$. $(f \circ g)(x)$ is defined whenever both $g(x)$ and $f(g(x))$ are defined, so the domain is $\{x \mid x \neq -1, 0\}$.

$(g \circ f)(x) = g\left(\dfrac{x}{x+1}\right) = \dfrac{1}{\frac{x}{x+1}} = \dfrac{x+1}{x}$. $(g \circ f)(x)$ is defined whenever both $f(x)$ and $g(f(x))$ are defined, so the domain is $\{x \mid x \neq -1, 0\}$.

$(f \circ f)(x) = f\left(\dfrac{x}{x+1}\right) = \dfrac{\frac{x}{x+1}}{\frac{x}{x+1}+1} = \dfrac{x}{(x+1)\left(\frac{x}{x+1}+1\right)} = \dfrac{x}{2x+1}$. $(f \circ f)(x)$ is defined whenever both $f(x)$ and $f(f(x))$ are defined, so the domain is $\left\{x \mid x \neq -1, -\frac{1}{2}\right\}$.

$(g \circ g)(x) = g\left(\dfrac{1}{x}\right) = \dfrac{1}{\frac{1}{x}} = x$. $(g \circ g)(x)$ is defined whenever both $g(x)$ and $g(g(x))$ are defined, so the domain is $\{x \mid x \neq 0\}$.

58. $f(x)=\dfrac{2}{x}$ has domain $\{x \mid x\neq 0\}$; $g(x)=\dfrac{x}{x+2}$ has domain $\{x \mid x\neq -2\}$.

$(f\circ g)(x)=f\left(\dfrac{x}{x+2}\right)=\dfrac{2}{\frac{x}{x+2}}=\dfrac{2x+4}{x}$. $(f\circ g)(x)$ is defined whenever both $g(x)$ and $f(g(x))$ are defined; that is, whenever $x\neq 0$ and $x\neq -2$. So the domain is $\{x \mid x\neq 0,-2\}$.

$(g\circ f)(x)=g\left(\dfrac{2}{x}\right)=\dfrac{\frac{2}{x}}{\frac{2}{x}+2}=\dfrac{2}{2+2x}=\dfrac{1}{1+x}$. $(g\circ f)(x)$ is defined whenever both $f(x)$ and $g(f(x))$ are defined; that is, whenever $x\neq 0$ and $x\neq -1$. So the domain is $\{x \mid x\neq 0,-1\}$.

$(f\circ f)(x)=f\left(\dfrac{2}{x}\right)=\dfrac{2}{\frac{2}{x}}=x$. $(f\circ f)(x)$ is defined whenever both $f(x)$ and $f(f(x))$ are defined; that is, whenever $x\neq 0$. So the domain is $\{x \mid x\neq 0\}$.

$(g\circ g)(x)=g\left(\dfrac{x}{x+2}\right)=\dfrac{\frac{x}{x+2}}{\frac{x}{x+2}+2}=\dfrac{x}{x+2(x+2)}=\dfrac{x}{3x+4}$. $(g\circ g)(x)$ is defined whenever both $g(x)$ and $g(g(x))$ are defined; that is whenever $x\neq -2$ and $x\neq -\frac{4}{3}$. So the domain is $\left\{x \mid x\neq -2,-\frac{4}{3}\right\}$.

59. $(f\circ g\circ h)(x)=f(g(h(x)))=f(g(x-1))=f\left(\sqrt{x-1}\right)=\sqrt{x-1}-1$

60. $(g\circ h)(x)=g\left(x^2+2\right)=\left(x^2+2\right)^3=x^6+6x^4+12x^2+8$.

$(f\circ g\circ h)(x)=f\left(x^6+6x^4+12x^2+8\right)=\dfrac{1}{x^6+6x^4+12x^2+8}$.

61. $(f\circ g\circ h)(x)=f(g(h(x)))=f\left(g\left(\sqrt{x}\right)\right)=f\left(\sqrt{x}-5\right)=\left(\sqrt{x}-5\right)^4+1$

62. $(g\circ h)(x)=g\left(\sqrt[3]{x}\right)=\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}$. $(f\circ g\circ h)(x)=f\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)=\sqrt{\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}}$.

For Exercises 63–72, many answers are possible.

63. $F(x)=(x-9)^5$. Let $f(x)=x^5$ and $g(x)=x-9$, then $F(x)=(f\circ g)(x)$.

64. $F(x)=\sqrt{x}+1$. If $f(x)=x+1$ and $g(x)=\sqrt{x}$, then $F(x)=(f\circ g)(x)$.

65. $G(x)=\dfrac{x^2}{x^2+4}$. Let $f(x)=\dfrac{x}{x+4}$ and $g(x)=x^2$, then $G(x)=(f\circ g)(x)$.

66. $G(x)=\dfrac{1}{x+3}$. If $f(x)=\dfrac{1}{x}$ and $g(x)=x+3$, then $G(x)=(f\circ g)(x)$.

67. $H(x)=\left|1-x^3\right|$. Let $f(x)=|x|$ and $g(x)=1-x^3$, then $H(x)=(f\circ g)(x)$.

68. $H(x)=\sqrt{1+\sqrt{x}}$. If $f(x)=\sqrt{1+x}$ and $g(x)=\sqrt{x}$, then $H(x)=(f\circ g)(x)$.

69. $F(x)=\dfrac{1}{x^2+1}$. Let $f(x)=\dfrac{1}{x}$, $g(x)=x+1$, and $h(x)=x^2$, then $F(x)=(f\circ g\circ h)(x)$.

70. $F(x)=\sqrt[3]{\sqrt{x}-1}$. If $g(x)=x-1$ and $h(x)=\sqrt{x}$, then $(g\circ h)(x)=\sqrt{x}-1$, and if $f(x)=\sqrt[3]{x}$, then $F(x)=(f\circ g\circ h)(x)$.

71. $G(x)=\left(4+\sqrt[3]{x}\right)^9$. Let $f(x)=x^9$, $g(x)=4+x$, and $h(x)=\sqrt[3]{x}$, then $G(x)=(f\circ g\circ h)(x)$.

72. $G(x)=\dfrac{2}{\left(3+\sqrt{x}\right)^2}$. If $g(x)=3+x$ and $h(x)=\sqrt{x}$, then $(g\circ h)(x)=3+\sqrt{x}$, and if $f(x)=\dfrac{2}{x^2}$, then $G(x)=(f\circ g\circ h)(x)$.

73. Yes. If $f(x) = m_1 x + b_1$ and $g(x) = m_2 x + b_2$, then $(f \circ g)(x) = f(m_2 x + b_2) = m_1(m_2 x + b_2) + b_1 = m_1 m_2 x + m_1 b_2 + b_1$, which is a linear function, because it is of the form $y = mx + b$. The slope is $m_1 m_2$.

74. $g(x) = 2x + 1$ and $h(x) = 4x^2 + 4x + 7$.

Method 1: Notice that $(2x+1)^2 = 4x^2 + 4x + 1$. We see that adding 6 to this quantity gives $(2x+1)^2 + 6 = 4x^2 + 4x + 1 + 6 = 4x^2 + 4x + 7$, which is $h(x)$. So let $f(x) = x^2 + 6$, and we have $(f \circ g)(x) = (2x+1)^2 + 6 = h(x)$.

Method 2: Since $g(x)$ is linear and $h(x)$ is a second degree polynomial, $f(x)$ must be a second degree polynomial, that is, $f(x) = ax^2 + bx + c$ for some a, b, and c. Thus $f(g(x)) = f(2x+1) = a(2x+1)^2 + b(2x+1) + c \Leftrightarrow 4ax^2 + 4ax + a + 2bx + b + c = 4ax^2 + (4a+2b)x + (a+b+c) = 4x^2 + 4x + 7$. Comparing this with $f(g(x))$, we have $4a = 4$ (the x^2 coefficients), $4a + 2b = 4$ (the x coefficients), and $a + b + c = 7$ (the constant terms) $\Leftrightarrow a = 1$ and $2a + b = 2$ and $a + b + c = 7 \Leftrightarrow a = 1, b = 0, c = 6$. Thus $f(x) = x^2 + 6$.

$f(x) = 3x + 5$ and $h(x) = 3x^2 + 3x + 2$.

Note since $f(x)$ is linear and $h(x)$ is quadratic, $g(x)$ must also be quadratic. We can then use trial and error to find $g(x)$. Another method is the following: We wish to find g so that $(f \circ g)(x) = h(x)$. Thus $f(g(x)) = 3x^2 + 3x + 2 \Leftrightarrow 3(g(x)) + 5 = 3x^2 + 3x + 2 \Leftrightarrow 3(g(x)) = 3x^2 + 3x - 3 \Leftrightarrow g(x) = x^2 + x - 1$.

75. The price per sticker is $0.15 - 0.000002x$ and the number sold is x, so the revenue is $R(x) = (0.15 - 0.000002x)x = 0.15x - 0.000002x^2$.

76. As found in Exercise 75, the revenue is $R(x) = 0.15x - 0.000002x^2$, and the cost is $0.095x - 0.0000005x^2$, so the profit is $P(x) = 0.15x - 0.000002x^2 - \left(0.095x - 0.0000005x^2\right) = 0.055x - 0.0000015x^2$.

77. **(a)** Because the ripple travels at a speed of 60 cm/s, the distance traveled in t seconds is the radius, so $g(t) = 60t$.

(b) The area of a circle is πr^2, so $f(r) = \pi r^2$.

(c) $f \circ g = \pi(g(t))^2 = \pi(60t)^2 = 3600\pi t^2$ cm^2. This function represents the area of the ripple as a function of time.

78. **(a)** Let $f(t)$ be the radius of the spherical balloon in centimeters. Since the radius is increasing at a rate of 1 cm/s, the radius is $f(t) = t$ after t seconds.

(b) The volume of the balloon can be written as $g(r) = \frac{4}{3}\pi r^3$.

(c) $g \circ f = \frac{4}{3}\pi(t)^3 = \frac{4}{3}\pi t^3$. $g \circ f$ represents the volume as a function of time.

79. Let r be the radius of the spherical balloon in centimeters. Since the radius is increasing at a rate of 2 cm/s, the radius is $r = 2t$ after t seconds. Therefore, the surface area of the balloon can be written as $S = 4\pi r^2 = 4\pi(2t)^2 = 4\pi\left(4t^2\right) = 16\pi t^2$.

80. **(a)** $f(x) = 0.80x$

(b) $g(x) = x - 50$

(c) $(f \circ g)(x) = f(x - 50) = 0.80(x - 50) = 0.80x - 40$. $f \circ g$ represents applying the \$50 coupon, then the 20% discount. $(g \circ f)(x) = g(0.80x) = 0.80x - 50$. $g \circ f$ represents applying the 20% discount, then the \$50 coupon. So applying the 20% discount, then the \$50 coupon gives the lower price.

81. **(a)** $f(x) = 0.90x$

(b) $g(x) = x - 100$

(c) $(f \circ g)(x) = f(x - 100) = 0.90(x - 100) = 0.90x - 90$. $f \circ g$ represents applying the \$100 coupon, then the 10% discount. $(g \circ f)(x) = g(0.90x) = 0.90x - 100$. $g \circ f$ represents applying the 10% discount, then the \$100 coupon. So applying the 10% discount, then the \$100 coupon gives the lower price.

82. Let t be the time since the plane flew over the radar station.

(a) Let s be the distance in miles between the plane and the radar station, and let d be the horizontal distance that the plane has flown. Using the Pythagorean theorem, $s = f(d) = \sqrt{1 + d^2}$.

(b) Since distance = rate × time, we have $d = g(t) = 350t$.

(c) $s(t) = (f \circ g)(t) = f(350t) = \sqrt{1 + (350t)^2} = \sqrt{1 + 122{,}500t^2}$.

83. $A(x) = 1.05x$. $(A \circ A)(x) = A(A(x)) = A(1.05x) = 1.05(1.05x) = (1.05)^2 x$.
$(A \circ A \circ A)(x) = A(A \circ A(x)) = A\left((1.05)^2 x\right) = 1.05\left[(1.05)^2 x\right] = (1.05)^3 x$.
$(A \circ A \circ A \circ A)(x) = A(A \circ A \circ A(x)) = A\left((1.05)^3 x\right) = 1.05\left[(1.05)^3 x\right] = (1.05)^4 x$. A represents the amount in the account after 1 year; $A \circ A$ represents the amount in the account after 2 years; $A \circ A \circ A$ represents the amount in the account after 3 years; and $A \circ A \circ A \circ A$ represents the amount in the account after 4 years. We can see that if we compose n copies of A, we get $(1.05)^n x$.

84. If $g(x)$ is even, then $h(-x) = f(g(-x)) = f(g(x)) = h(x)$. So yes, h is always an even function.
If $g(x)$ is odd, then h is not necessarily an odd function. For example, if we let $f(x) = x - 1$ and $g(x) = x^3$, g is an odd function, but $h(x) = (f \circ g)(x) = f\left(x^3\right) = x^3 - 1$ is not an odd function.
If $g(x)$ is odd and f is also odd, then $h(-x) = (f \circ g)(-x) = f(g(-x)) = f(-g(x)) = -f(g(x)) = -(f \circ g)(x) = -h(x)$. So in this case, h is also an odd function.
If $g(x)$ is odd and f is even, then $h(-x) = (f \circ g)(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = (f \circ g)(x) = h(x)$, so in this case, h is an even function.

2.8 ONE-TO-ONE FUNCTIONS AND THEIR INVERSES

1. A function f is one-to-one if different inputs produce *different* outputs. You can tell from the graph that a function is one-to-one by using the *Horizontal Line* Test.

2. (a) For a function to have an inverse, it must be *one-to-one*. $f(x) = x^2$ is not one-to-one, so it does not have an inverse. However $g(x) = x^3$ is one-to-one, so it has an inverse.

(b) The inverse of $g(x) = x^3$ is $g^{-1}(x) = \sqrt[3]{x}$.

3. (a) Proceeding backward through the description of f, we can describe f^{-1} as follows: "Take the third root, subtract 5, then divide by 3."

(b) $f(x) = (3x + 5)^3$ and $f^{-1}(x) = \dfrac{\sqrt[3]{x} - 5}{3}$.

4. Yes, the graph of f is one-to-one, so f has an inverse. Because $f(4) = 1$, $f^{-1}(1) = 4$, and because $f(5) = 3$, $f^{-1}(3) = 5$.

5. If the point $(3, 4)$ is on the graph of f, then the point $(4, 3)$ is on the graph of f^{-1}. [This is another way of saying that $f(3) = 4 \Leftrightarrow f^{-1}(4) = 3$.]

6. (a) False. For instance, if $f(x) = x$, then $f^{-1}(x) = x$, but $\dfrac{1}{f(x)} = \dfrac{1}{x} \neq f^{-1}(x)$.

(b) This is true, by definition.

7. By the Horizontal Line Test, f is not one-to-one.

8. By the Horizontal Line Test, f is one-to-one.

9. By the Horizontal Line Test, f is one-to-one.

10. By the Horizontal Line Test, f is not one-to-one.

11. By the Horizontal Line Test, f is not one-to-one.

12. By the Horizontal Line Test, f is one-to-one.

13. $f(x) = -2x + 4$. If $x_1 \neq x_2$, then $-2x_1 \neq -2x_2$ and $-2x_1 + 4 \neq -2x_2 + 4$. So f is a one-to-one function.

14. $f(x) = 3x - 2$. If $x_1 \neq x_2$, then $3x_1 \neq 3x_2$ and $3x_1 - 2 \neq 3x_2 - 2$. So f is a one-to-one function.

15. $g(x) = \sqrt{x}$. If $x_1 \neq x_2$, then $\sqrt{x_1} \neq \sqrt{x_2}$ because two different numbers cannot have the same square root. Therefore, g is a one-to-one function.

16. $g(x) = |x|$. Because every number and its negative have the same absolute value (for example, $|-1| = 1 = |1|$), g is not a one-to-one function.

17. $h(x) = x^2 - 2x$. Because $h(0) = 0$ and $h(2) = (2) - 2(2) = 0$ we have $h(0) = h(2)$. So f is not a one-to-one function.

18. $h(x) = x^3 + 8$. If $x_1 \neq x_2$, then $x_1^3 \neq x_2^3$ and $x_1^3 + 8 \neq x_2^3 + 8$. So f is a one-to-one function.

19. $f(x) = x^4 + 5$. Every nonzero number and its negative have the same fourth power. For example, $(-1)^4 = 1 = (1)^4$, so $f(-1) = f(1)$. Thus f is not a one-to-one function.

20. $f(x) = x^4 + 5$, $0 \le x \le 2$. If $x_1 \neq x_2$, then $x_1^4 \neq x_2^4$ because two different positive numbers cannot have the same fourth power. Thus, $x_1^4 + 5 \neq x_2^4 + 5$. So f is a one-to-one function.

21. $r(t) = t^6 - 3$, $0 \le t \le 5$. If $t_1 \neq t_2$, then $t_1^6 \neq t_2^6$ because two different positive numbers cannot have the same sixth power. Thus, $t_1^6 - 3 \neq t_2^6 - 3$. So r is a one-to-one function.

22. $r(t) = t^4 - 1$. Every nonzero number and its negative have the same fourth power. For example, $(-1)^4 = 1 = (1)^4$, so $r(-1) = r(1)$. Thus r is not a one-to-one function.

23. $f(x) = \dfrac{1}{x^2}$. Every nonzero number and its negative have the same square. For example, $\dfrac{1}{(-1)^2} = 1 = \dfrac{1}{(1)^2}$, so $f(-1) = f(1)$. Thus f is not a one-to-one function.

24. $f(x) = \dfrac{1}{x}$. If $x_1 \neq x_2$, then $\dfrac{1}{x_1} \neq \dfrac{1}{x_2}$. So f is a one-to-one function.

25. **(a)** $f(2) = 7$. Since f is one-to-one, $f^{-1}(7) = 2$.
(b) $f^{-1}(3) = -1$. Since f is one-to-one, $f(-1) = 3$.

26. **(a)** $f(5) = 18$. Since f is one-to-one, $f^{-1}(18) = 5$.
(b) $f^{-1}(4) = 2$. Since f is one-to-one, $f(2) = 4$.

27. $f(x) = 5 - 2x$. Since f is one-to-one and $f(1) = 5 - 2(1) = 3$, then $f^{-1}(3) = 1$. (Find 1 by solving the equation $5 - 2x = 3$.)

28. To find $g^{-1}(5)$, we find the x value such that $g(x) = 5$; that is, we solve the equation $g(x) = x^2 + 4x = 5$. Now $x^2 + 4x = 5 \Leftrightarrow x^2 + 4x - 5 = 0 \Leftrightarrow (x-1)(x+5) = 0 \Leftrightarrow x = 1$ or $x = -5$. Since the domain of g is $[-2, \infty)$, $x = 1$ is the only value where $g(x) = 5$. Therefore, $g^{-1}(5) = 1$.

29. **(a)** Because $f(6) = 2$, $f^{-1}(2) = 6$. **(b)** Because $f(2) = 5$, $f^{-1}(5) = 2$. **(c)** Because $f(0) = 6$, $f^{-1}(6) = 0$.

30. **(a)** Because $g(4) = 2$, $g^{-1}(2) = 4$. **(b)** Because $g(7) = 5$, $g^{-1}(5) = 7$. **(c)** Because $g(8) = 6$, $g^{-1}(6) = 8$.

31. From the table, $f(4) = 5$, so $f^{-1}(5) = 4$.

32. From the table, $f(5) = 0$, so $f^{-1}(0) = 5$.

33. $f^{-1}(f(1)) = 1$

34. $f\left(f^{-1}(6)\right) = 6$

35. From the table, $f(6) = 1$, so $f^{-1}(1) = 6$. Also, $f(2) = 6$, so $f^{-1}(6) = 1$. Thus, $f^{-1}\left(f^{-1}(1)\right) = f^{-1}(6) = 1$.

36. From the table, $f(5) = 0$, so $f^{-1}(0) = 5$. Also, $f(4) = 5$, so $f^{-1}(5) = 4$. Thus, $f^{-1}\left(f^{-1}(0)\right) = f^{-1}(5) = 4$.

37. $f(g(x)) = f(x+6) = (x+6) - 6 = x$ for all x.
$g(f(x)) = g(x-6) = (x-6) + 6 = x$ for all x. Thus f and g are inverses of each other.

38. $f(g(x)) = f\left(\dfrac{x}{3}\right) = 3\left(\dfrac{x}{3}\right) = x$ for all x.

$g(f(x)) = g(3x) = \dfrac{3x}{3} = x$ for all x. Thus f and g are inverses of each other.

39. $f(g(x)) = f\left(\dfrac{x-4}{3}\right) = 3\left(\dfrac{x-4}{3}\right) + 4 = x - 4 + 4 = x$ for all x.

$g(f(x)) = g(3x+4) = \dfrac{(3x+4)-4}{3} = x$ for all x. Thus f and g are inverses of each other.

40. $f(g(x)) = f\left(\dfrac{2-x}{5}\right) = 2 - 5\left(\dfrac{2-x}{5}\right) = 2 - (2-x) = x$ for all x.

$g(f(x)) = g(2-5x) = \dfrac{2-(2-5x)}{5} = \dfrac{5x}{5} = x$ for all x. Thus f and g are inverses of each other.

41. $f(g(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = x$ for all $x \neq 0$. Since $f(x) = g(x)$, we also have $g(f(x)) = x$ for all $x \neq 0$. Thus f and g are inverses of each other.

42. $f(g(x)) = f\left(\sqrt[5]{x}\right) = \left(\sqrt[5]{x}\right)^5 = x$ for all x.

$g(f(x)) = g\left(x^5\right) = \sqrt[5]{x^5} = x$ for all x. Thus f and g are inverses of each other.

43. $f(g(x)) = f\left(\sqrt{x+9}\right) = \left(\sqrt{x+9}\right)^2 - 9 = x + 9 - 9 = x$ for all $x \geq -9$.

$g(f(x)) = g\left(x^2 - 9\right) = \sqrt{\left(x^2-9\right)+9} = \sqrt{x^2} = x$ for all $x \geq 0$. Thus f and g are inverses of each other.

44. $f(g(x)) = f\left((x-1)^{1/3}\right) = \left((x-1)^{1/3}\right)^3 + 1 = x - 1 + 1 = x$ for all x.

$g(f(x)) = g\left(x^3+1\right) = \left[(x-1)^{1/3}\right]^3 + 1 = x - 1 + 1 = x$ for all x. Thus f and g are inverses of each other.

45. $f(g(x)) = f\left(\dfrac{1}{x} + 1\right) = \dfrac{1}{\left(\dfrac{1}{x}+1\right) - 1} = x$ for all $x \neq 0$.

$g(f(x)) = g\left(\dfrac{1}{x-1}\right) = \dfrac{1}{\left(\dfrac{1}{x-1}\right)} + 1 = (x-1) + 1 = x$ for all $x \neq 1$. Thus f and g are inverses of each other.

46. $f(g(x)) = f\left(\sqrt{4-x^2}\right) = \sqrt{4 - \left(\sqrt{4-x^2}\right)^2} = \sqrt{4-4+x^2} = \sqrt{x^2} = x$, for all $0 \leq x \leq 2$. (Note that the last equality is possible since $x \geq 0$.)

$g(f(x)) = g\left(\sqrt{4-x^2}\right) = \sqrt{4 - \left(\sqrt{4-x^2}\right)^2} = \sqrt{4-4+x^2} = \sqrt{x^2} = x$, for all $0 \leq x \leq 2$. (Again, the last equality is possible since $x \geq 0$.) Thus f and g are inverses of each other.

47. $f(g(x)) = f\left(\dfrac{2x+2}{x-1}\right) = \dfrac{\frac{2x+2}{x-1} + 2}{\frac{2x+2}{x-1} - 2} = \dfrac{2x+2+2(x-1)}{2x+2-2(x-1)} = \dfrac{4x}{4} = x$ for all $x \neq 1$.

$g(f(x)) = g\left(\dfrac{x+2}{x-2}\right) = \dfrac{2\left(\frac{x+2}{x-2}\right) + 2}{\frac{x+2}{x-2} - 1} = \dfrac{2(x+2)+2(x-2)}{x+2-1(x-2)} = \dfrac{4x}{4} = x$ for all $x \neq 2$. Thus f and g are inverses of each other.

48. $f(g(x)) = f\left(\dfrac{5+4x}{1-3x}\right) = \dfrac{\frac{5+4x}{1-3x} - 5}{3\left(\frac{5+4x}{1-3x}\right) + 4} = \dfrac{5+4x-5(1-3x)}{3(5+4x)+4(1-3x)} = \dfrac{19x}{19} = x$ for all $x \neq \frac{1}{3}$.

$g(f(x)) = g\left(\dfrac{x-5}{3x+4}\right) = \dfrac{5 + 4\left(\frac{x-5}{3x+4}\right)}{1 - 3\left(\frac{x-5}{3x+4}\right)} = \dfrac{5(3x+4)+4(x-5)}{3x+4-3(x-5)} = \dfrac{19x}{19}$ for all $x \neq -\frac{4}{3}$. Thus f and g are inverses of each other.

49. $f(x) = 3x + 5$. $y = 3x + 5 \Leftrightarrow 3x = y - 5 \Leftrightarrow x = \frac{1}{3}(y-5) = \frac{1}{3}y - \frac{5}{3}$. So $f^{-1}(x) = \frac{1}{3}x - \frac{5}{3}$.

50. $f(x) = 7 - 5x$. $y = 7 - 5x \Leftrightarrow 5x = 7 - y \Leftrightarrow x = \frac{1}{5}(7-y) = -\frac{1}{5}y + \frac{7}{5}$. So $f^{-1}(x) = -\frac{1}{5}x + \frac{7}{5}$.

51. $f(x) = 5 - 4x^3$. $y = 5 - 4x^3 \Leftrightarrow 4x^3 = 5 - y \Leftrightarrow x^3 = \frac{1}{4}(5 - y) \Leftrightarrow x = \sqrt[3]{\frac{1}{4}(5 - y)}$. So $f^{-1}(x) = \sqrt[3]{\frac{1}{4}(5 - x)}$.

52. $f(x) = 3x^3 + 8$. $y = 3x^3 + 8 \Leftrightarrow 3x^3 = y - 8 \Leftrightarrow x^3 = \frac{1}{3}y - \frac{8}{3} \Leftrightarrow x = \sqrt[3]{\frac{1}{3}y - \frac{8}{3}}$. So $f^{-1}(x) = \sqrt[3]{\frac{1}{3}(x - 8)}$.

53. $f(x) = \dfrac{1}{x+2}$. $y = \dfrac{1}{x+2} \Leftrightarrow x + 2 = \dfrac{1}{y} \Leftrightarrow x = \dfrac{1}{y} - 2$. So $f^{-1}(x) = \dfrac{1}{x} - 2$.

54. $f(x) = \dfrac{x-2}{x+2}$. $y = \dfrac{x-2}{x+2} \Leftrightarrow y(x+2) = x - 2 \Leftrightarrow xy + 2y = x - 2 \Leftrightarrow xy - x = -2 - 2y \Leftrightarrow x(y-1) = -2(y+1)$ $\Leftrightarrow x = \dfrac{-2(y+1)}{y-1}$. So $f^{-1}(x) = \dfrac{-2(x+1)}{x-1}$.

55. $f(x) = \dfrac{x}{x+4}$. $y = \dfrac{x}{x+4} \Leftrightarrow y(x+4) = x \Leftrightarrow xy + 4y = x \Leftrightarrow x - xy = 4y \Leftrightarrow x(1-y) = 4y \Leftrightarrow x = \dfrac{4y}{1-y}$. So $f^{-1}(x) = \dfrac{4x}{1-x}$.

56. $f(x) = \dfrac{3x}{x-2}$. $y = \dfrac{3x}{x-2} \Leftrightarrow y(x-2) = 3x \Leftrightarrow xy - 2y = 3x \Leftrightarrow xy - 3x = 2y \Leftrightarrow x(y-3) = 2y \Leftrightarrow x = \dfrac{2y}{y-3}$. So $f^{-1}(x) = \dfrac{2x}{x-3}$.

57. $f(x) = \dfrac{2x+5}{x-7}$. $y = \dfrac{2x+5}{x-7} \Leftrightarrow y(x-7) = 2x + 5 \Leftrightarrow xy - 7y = 2x + 5 \Leftrightarrow xy - 2x = 7y + 5 \Leftrightarrow x(y-2) = 7y + 5$ $\Leftrightarrow x = \dfrac{7y+5}{y-2}$. So $f^{-1}(x) = \dfrac{7x+5}{x-2}$.

58. $f(x) = \dfrac{4x-2}{3x+1}$. $y = \dfrac{4x-2}{3x+1} \Leftrightarrow y(3x+1) = 4x - 2 \Leftrightarrow 3xy + y = 4x - 2 \Leftrightarrow 4x - 3xy = y + 2 \Leftrightarrow x(4-3y) = y + 2$ $\Leftrightarrow x = \dfrac{y+2}{4-3y}$. So $f^{-1}(x) = \dfrac{x+2}{4-3x}$.

59. $f(x) = \dfrac{2x+3}{1-5x}$. $y = \dfrac{2x+3}{1-5x} \Leftrightarrow y(1-5x) = 2x + 3 \Leftrightarrow y - 5xy = 2x + 3 \Leftrightarrow 2x + 5xy = y - 3 \Leftrightarrow x(2+5y) = y - 3$ $\Leftrightarrow x = \dfrac{y-3}{5y+2}$. So $f^{-1}(x) = \dfrac{x-3}{5x+2}$.

60. $f(x) = \dfrac{3-4x}{8x-1}$. $y = \dfrac{3-4x}{8x-1} \Leftrightarrow y(8x-1) = 3 - 4x \Leftrightarrow 8xy - y = 3 - 4x \Leftrightarrow 4x(2y+1) = y + 3 \Leftrightarrow x = \dfrac{y+3}{4(2y+1)}$. So $f^{-1}(x) = \dfrac{x+3}{4(2x+1)}$.

61. $f(x) = 4 - x^2$, $x \ge 0$. $y = 4 - x^2 \Leftrightarrow x^2 = 4 - y \Leftrightarrow x = \sqrt{4-y}$. So $f^{-1}(x) = \sqrt{4-x}$, $x \le 4$. [Note that $x \ge 0 \Rightarrow f(x) \le 4$.]

62. $f(x) = x^2 + x = \left(x^2 + x + \frac{1}{4}\right) - \frac{1}{4} = \left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$, $x \ge -\dfrac{1}{2}$. $y = \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} \Leftrightarrow \left(x + \frac{1}{2}\right)^2 = y + \frac{1}{4} \Leftrightarrow$ $x + \frac{1}{2} = \sqrt{y + \frac{1}{4}} \Leftrightarrow x = \sqrt{y + \frac{1}{4}} - \frac{1}{2}$, $y \ge -\frac{1}{4}$. So $f^{-1}(x) = \sqrt{x + \frac{1}{4}} - \frac{1}{2}$, $x \ge -\frac{1}{4}$. (Note that $x \ge -\frac{1}{2}$, so that $x + \frac{1}{2} \ge 0$, and hence $\left(x + \frac{1}{2}\right)^2 = y + \frac{1}{4} \Leftrightarrow x + \frac{1}{2} = \sqrt{y + \frac{1}{4}}$. Also, since $x \ge -\frac{1}{2}$, $y = \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} \ge -\frac{1}{4}$ so that $y + \frac{1}{4} \ge 0$, and hence $\sqrt{y + \frac{1}{4}}$ is defined.)

63. $f(x) = x^6$, $x \ge 0$. $y = x^6 \Leftrightarrow x = \sqrt[6]{y}$ for $x \ge 0$. The range of f is $\{y \mid y \ge 0\}$, so $f^{-1}(x) = \sqrt[6]{x}$, $x \ge 0$.

64. $f(x) = \dfrac{1}{x^2}$, $x > 0$. $y = \dfrac{1}{x^2} \Leftrightarrow x^2 = \dfrac{1}{y} \Leftrightarrow x = \dfrac{1}{\sqrt{y}}$. The range of f is $\{y \mid y > 0\}$, so $f^{-1}(x) = \dfrac{1}{\sqrt{x}}$, $x > 0$.

65. $f(x) = \dfrac{2-x^3}{5}$. $y = \dfrac{2-x^3}{5} \Leftrightarrow 5y = 2 - x^3 \Leftrightarrow x^3 = 2 - 5y \Leftrightarrow x = \sqrt[3]{2-5y}$. Thus, $f^{-1}(x) = \sqrt[3]{2-5x}$.

66. $f(x) = \left(x^5 - 6\right)^7$. $y = \left(x^5 - 6\right)^7 \Leftrightarrow \sqrt[7]{y} = x^5 - 6 \Leftrightarrow x^5 = \sqrt[7]{y} + 6 \Leftrightarrow x = \sqrt[5]{\sqrt[7]{y} + 6}$. Thus, $f^{-1}(x) = \sqrt[5]{\sqrt[7]{x} + 6}$.

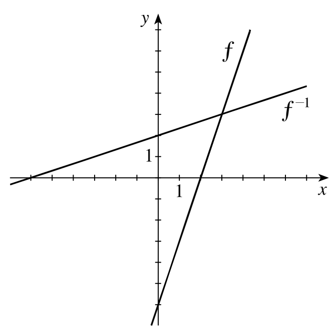
67. $f(x) = \sqrt{5+8x}$. Note that the range of f (and thus the domain of f^{-1}) is $[0,\infty)$. $y = \sqrt{5+8x} \Leftrightarrow y^2 = 5+8x \Leftrightarrow 8x = y^2 - 5 \Leftrightarrow x = \dfrac{y^2-5}{8}$. Thus, $f^{-1}(x) = \dfrac{x^2-5}{8}$, $x \geq 0$.

68. $f(x) = 2 + \sqrt{3+x}$. The range of f is $[2,\infty)$. $y = 2 + \sqrt{3+x} \Leftrightarrow y - 2 = \sqrt{3+x} \Leftrightarrow (y-2)^2 = 3+x \Leftrightarrow x = (y-2)^2 - 3$. Thus, $f^{-1}(x) = (x-2)^2 - 3$, $x \geq 2$.

69. $f(x) = 2 + \sqrt[3]{x}$. $y = 2 + \sqrt[3]{x} \Leftrightarrow y - 2 = \sqrt[3]{x} \Leftrightarrow x = (y-2)^3$. Thus, $f^{-1}(x) = (x-2)^3$.

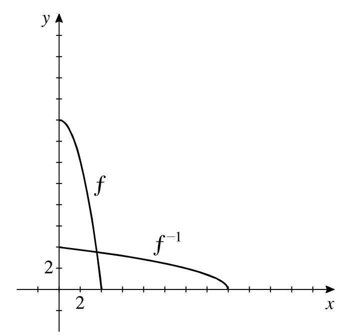
70. $f(x) = \sqrt{4-x^2}$, $0 \leq x \leq 2$. The range of f is $[0,2]$. $y = \sqrt{4-x^2} \Leftrightarrow y^2 = 4 - x^2 \Leftrightarrow x^2 = 4 - y^2 \Leftrightarrow x = \sqrt{4-y^2}$. Thus, $f^{-1}(x) = \sqrt{4-x^2}$, $0 \leq x \leq 2$.

71. (a), (b) $f(x) = 3x - 6$



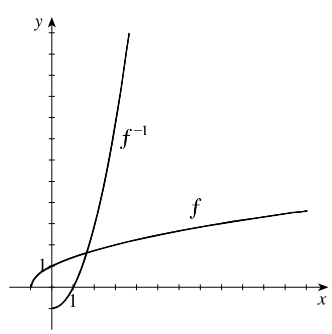
(c) $f(x) = 3x - 6$. $y = 3x - 6 \Leftrightarrow 3x = y + 6 \Leftrightarrow x = \frac{1}{3}(y+6)$. So $f^{-1}(x) = \frac{1}{3}(x+6)$.

72. (a), (b) $f(x) = 16 - x^2$, $x \geq 0$



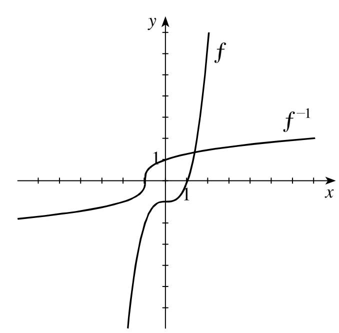
(c) $f(x) = 16 - x^2$, $x \geq 0$. $y = 16 - x^2 \Leftrightarrow x^2 = 16 - y \Leftrightarrow x = \sqrt{16-y}$. So $f^{-1}(x) = \sqrt{16-x}$, $x \leq 16$. (*Note:* $x \geq 0 \Rightarrow f(x) = 16 - x^2 \leq 16$.)

73. (a), (b) $f(x) = \sqrt{x+1}$



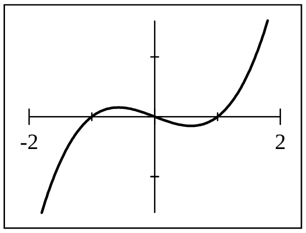
(c) $f(x) = \sqrt{x+1}$, $x \geq -1$. $y = \sqrt{x+1}$, $y \geq 0 \Leftrightarrow y^2 = x + 1 \Leftrightarrow x = y^2 - 1$ and $y \geq 0$. So $f^{-1}(x) = x^2 - 1$, $x \geq 0$.

74. (a), (b) $f(x) = x^3 - 1$

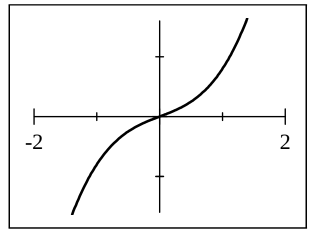


(c) $f(x) = x^3 - 1 \Leftrightarrow y = x^3 - 1 \Leftrightarrow x^3 = y + 1 \Leftrightarrow x = \sqrt[3]{y+1}$. So $f^{-1}(x) = \sqrt[3]{x+1}$.

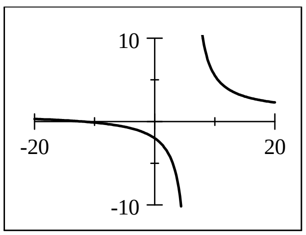
75. $f(x) = x^3 - x$. Using a graphing device and the Horizontal Line Test, we see that f is not a one-to-one function. For example, $f(0) = 0 = f(-1)$.



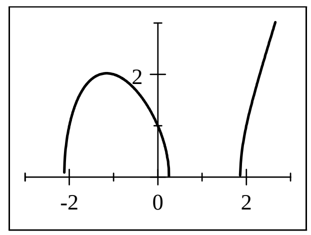
76. $f(x) = x^3 + x$. Using a graphing device and the Horizontal Line Test, we see that f is a one-to-one function.



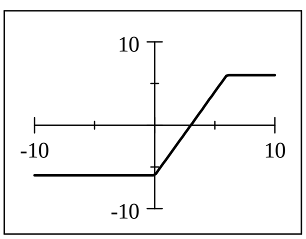
77. $f(x) = \dfrac{x+12}{x-6}$. Using a graphing device and the Horizontal Line Test, we see that f is a one-to-one function.



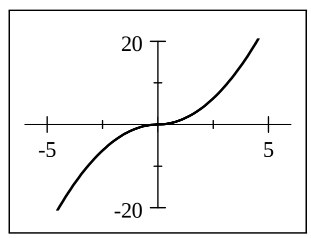
78. $f(x) = \sqrt{x^3 - 4x + 1}$. Using a graphing device and the Horizontal Line Test, we see that f is not a one-to-one function. For example, $f(0) = 1 = f(2)$.



79. $f(x) = |x| - |x - 6|$. Using a graphing device and the Horizontal Line Test, we see that f is not a one-to-one function. For example $f(0) = -6 = f(-2)$.

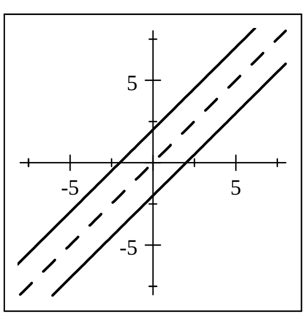


80. $f(x) = x \cdot |x|$. Using a graphing device and the Horizontal Line Test, we see that f is a one-to-one function.



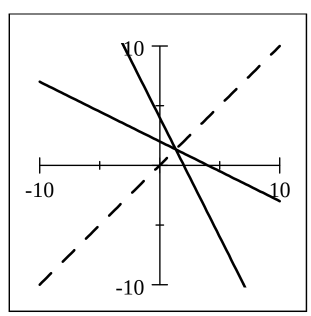
81. **(a)** $y = f(x) = 2 + x \Leftrightarrow x = y - 2$. So $f^{-1}(x) = x - 2$.

(b)



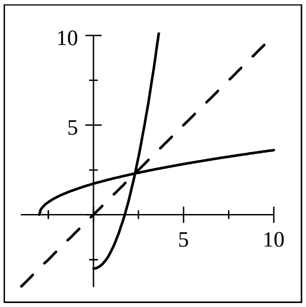
82. **(a)** $y = f(x) = 2 - \frac{1}{2}x \Leftrightarrow \frac{1}{2}x = 2 - y \Leftrightarrow x = 4 - 2y$. So $f^{-1}(x) = 4 - 2x$.

(b)



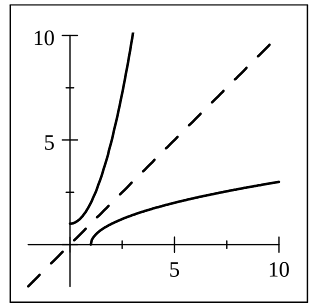
83. **(a)** $y = g(x) = \sqrt{x+3},\ y \geq 0 \Leftrightarrow x + 3 = y^2,\ y \geq 0$ $\Leftrightarrow x = y^2 - 3,\ y \geq 0$. So $g^{-1}(x) = x^2 - 3,\ x \geq 0$.

(b)



84. **(a)** $y = g(x) = x^2 + 1,\ x \geq 0 \Leftrightarrow x^2 = y - 1,\ x \geq 0 \Leftrightarrow$ $x = \sqrt{y-1}$. So $g^{-1}(x) = \sqrt{x-1}$.

(b)



85. If we restrict the domain of $f(x)$ to $[0, \infty)$, then $y = 4 - x^2 \Leftrightarrow x^2 = 4 - y \Rightarrow x = \sqrt{4-y}$ (since $x \geq 0$, we take the positive square root). So $f^{-1}(x) = \sqrt{4-x}$.

If we restrict the domain of $f(x)$ to $(-\infty, 0]$, then $y = 4 - x^2 \Leftrightarrow x^2 = 4 - y \Rightarrow x = -\sqrt{4-y}$ (since $x \leq 0$, we take the negative square root). So $f^{-1}(x) = -\sqrt{4-x}$.

86. If we restrict the domain of $g(x)$ to $[1, \infty)$, then $y = (x-1)^2 \Rightarrow x - 1 = \sqrt{y}$ (since $x \geq 1$ we take the positive square root) $\Leftrightarrow x = 1 + \sqrt{y}$. So $g^{-1}(x) = 1 + \sqrt{x}$.

If we restrict the domain of $g(x)$ to $(-\infty, 1]$, then $y = (x-1)^2 \Rightarrow x - 1 = -\sqrt{y}$ (since $x \leq 1$ we take the negative square root) $\Leftrightarrow x = 1 - \sqrt{y}$. So $g^{-1}(x) = 1 - \sqrt{x}$.

87. If we restrict the domain of $h(x)$ to $[-2, \infty)$, then $y = (x+2)^2 \Rightarrow x + 2 = \sqrt{y}$ (since $x \geq -2$, we take the positive square root) $\Leftrightarrow x = -2 + \sqrt{y}$. So $h^{-1}(x) = -2 + \sqrt{x}$.

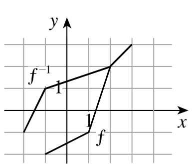
If we restrict the domain of $h(x)$ to $(-\infty, -2]$, then $y = (x+2)^2 \Rightarrow x + 2 = -\sqrt{y}$ (since $x \leq -2$, we take the negative square root) $\Leftrightarrow x = -2 - \sqrt{y}$. So $h^{-1}(x) = -2 - \sqrt{x}$.

88. $k(x) = |x - 3| = \begin{cases} -(x-3) & \text{if } x - 3 < 0 \Leftrightarrow x < 3 \\ x - 3 & \text{if } x - 3 \geq 0 \Leftrightarrow x \geq 3 \end{cases}$

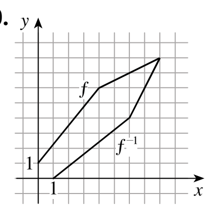
If we restrict the domain of $k(x)$ to $[3, \infty)$, then $y = x - 3 \Leftrightarrow x = 3 + y$. So $k^{-1}(x) = 3 + x$.

If we restrict the domain of $k(x)$ to $(-\infty, 3]$, then $y = -(x-3) \Leftrightarrow y = -x + 3 \Leftrightarrow x = 3 - y$. So $k^{-1}(x) = 3 - x$.

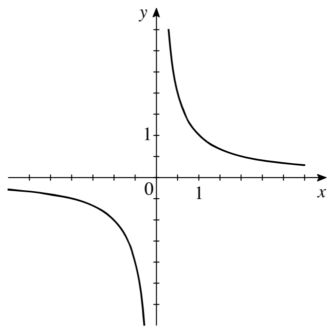
89.



90.



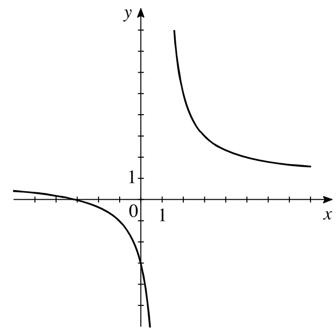
91. (a)



(b) Yes, the graph is unchanged upon reflection about the line $y = x$.

(c) $y = \frac{1}{x} \Leftrightarrow x = \frac{1}{y}$, so $f^{-1}(x) = \frac{1}{x}$.

92. (a)



(b) Yes, the graph is unchanged upon reflection about the line $y = x$.

(c) $y = \dfrac{x+3}{x-1} \Leftrightarrow y(x-1) = x+3 \Leftrightarrow$ $x(y-1) = y+3 \Leftrightarrow x = \dfrac{y+3}{y-1}$. Thus, $f^{-1}(x) = \dfrac{x+3}{x-1}$.

93. (a) The price of a pizza with no toppings (corresponding to the y-intercept) is \$16, and the cost of each additional topping (the rate of change of cost with respect to number of toppings) is \$1.50. Thus, $f(n) = 16 + 1.5n$.

(b) $p = f(n) = 16 + 1.5n \Leftrightarrow p - 16 = 1.5n \Leftrightarrow n = \frac{2}{3}(p-16)$. Thus, $n = f^{-1}(p) = \frac{2}{3}(p-16)$. This function represents the number of toppings on a pizza that costs x dollars.

(c) $f^{-1}(25) = \frac{2}{3}(25-16) = \frac{2}{3}(9) = 6$. Thus, a \$25 pizza has 6 toppings.

94. (a) $f(x) = 500 + 80x$.

(b) $p = f(x) = 500 + 80x$. $p = 500 + 80x \Leftrightarrow 80x = p - 500 \Leftrightarrow x = \dfrac{p-500}{80}$. So $x = f^{-1}(p) = \dfrac{p-500}{80}$. f^{-1} represents the number of hours the investigator spends on a case for x dollars.

(c) $f^{-1}(1220) = \dfrac{1220-500}{80} = \dfrac{720}{80} = 9$. If the investigator charges \$1220, he spent 9 hours investigating the case.

95. (a) $V = f(t) = 100\left(1 - \frac{t}{40}\right)^2$, $0 \le t \le 40$. $V = 100\left(1 - \frac{t}{40}\right)^2 \Leftrightarrow \frac{V}{100} = \left(1 - \frac{t}{40}\right)^2 \Rightarrow 1 - \frac{t}{40} = \pm\sqrt{\frac{V}{100}} \Leftrightarrow$ $\frac{t}{40} = 1 \pm \frac{\sqrt{V}}{10} \Leftrightarrow t = 40 \pm 4\sqrt{V}$. Since $t \le 40$, we must have $t = f^{-1}(V) = 40 - 4\sqrt{V}$. f^{-1} represents time that has elapsed since the tank started to leak.

(b) $f^{-1}(15) = 40 - 4\sqrt{15} \approx 24.5$ minutes. In 24.5 minutes the tank has drained to just 15 gallons of water.

96. (a) $v = g(r) = 18{,}500\left(0.25 - r^2\right)$. $v = 18{,}500\left(0.25 - r^2\right) \Leftrightarrow v = 4625 - 18{,}500r^2 \Leftrightarrow 18{,}500r^2 = 4625 - v \Leftrightarrow$ $r^2 = \dfrac{4625 - v}{18{,}500} \Rightarrow r = \pm\sqrt{\dfrac{4625 - v}{18{,}500}}$. Since r represents a distance, $r \ge 0$, so $g^{-1}(v) = \sqrt{\dfrac{4625 - v}{18{,}500}}$. $g^{-1}(v)$ represents the radial distance from the center of the vein at which the blood has velocity v.

(b) $g^{-1}(30) = \sqrt{\dfrac{4625 - 30}{18{,}500}} \approx 0.498$ cm. The velocity is 30 cm/s at a distance of 0.498 cm from the center of the artery or vein.

97. **(a)** $D = f(p) = -3p + 150$. $D = -3p + 150 \Leftrightarrow 3p = 150 - D \Leftrightarrow p = 50 - \frac{1}{3}D$. So $f^{-1}(D) = 50 - \frac{1}{3}D$. $f^{-1}(D)$ represents the price that is associated with demand D.

(b) $f^{-1}(30) = 50 - \frac{1}{3}(30) = 40$. So when the demand is 30 units, the price per unit is \$40.

98. **(a)** $F = g(C) = \frac{9}{5}C + 32$. $F = \frac{9}{5}C + 32 \Leftrightarrow \frac{9}{5}C = F - 32 \Leftrightarrow C = \frac{5}{9}(F - 32)$. So $g^{-1}(F) = \frac{5}{9}(F - 32)$. $g^{-1}(F)$ represents the Celsius temperature that corresponds to the Fahrenheit temperature of F.

(b) $F^{-1}(86) = \frac{5}{9}(86 - 32) = \frac{5}{9}(54) = 30$. So 86° Fahrenheit is the same as 30° Celsius.

99. **(a)** $f^{-1}(U) = 1.02396U$.

(b) $U = f(x) = 0.9766x$. $U = 0.9766x \Leftrightarrow x = 1.0240U$. So $f^{-1}(U) = 1.0240U$. $f^{-1}(U)$ represents the value of U US dollars in Canadian dollars.

(c) $f^{-1}(12{,}250) = 1.0240(12{,}250) = 12{,}543.52$. So \$12,250 in US currency is worth \$12,543.52 in Canadian currency.

100. **(a)** $f(x) = \begin{cases} 0.1x, & \text{if } 0 \le x \le 20{,}000 \\ 2000 + 0.2(x - 20{,}000) & \text{if } x > 20{,}000 \end{cases}$

(b) We will find the inverse of each piece of the function f.

$f_1(x) = 0.1x$. $T = 0.1x \Leftrightarrow x = 10T$. So $f_1^{-1}(T) = 10T$.

$f_2(x) = 2000 + 0.2(x - 20{,}000) = 0.2x - 2000$. $T = 0.2x - 2000 \Leftrightarrow 0.2x = T + 2000 \Leftrightarrow x = 5T + 10{,}000$. So $f_2^{-1}(T) = 5T + 10{,}000$.

Since $f(0) = 0$ and $f(20{,}000) = 2000$ we have $f^{-1}(T) = \begin{cases} 10T, & \text{if } 0 \le T \le 2000 \\ 5T + 10{,}000 & \text{if } T > 2000 \end{cases}$ This represents the taxpayer's income.

(c) $f^{-1}(10{,}000) = 5(10{,}000) + 10{,}000 = 60{,}000$. The required income is €60,000.

101. **(a)** $f(x) = 0.85x$.

(b) $g(x) = x - 1000$.

(c) $H(x) = (f \circ g)x = f(x - 1000) = 0.85(x - 1000) = 0.85x - 850$.

(d) $P = H(x) = 0.85x - 850$. $P = 0.85x - 850 \Leftrightarrow 0.85x = P + 850 \Leftrightarrow x = 1.176P + 1000$. So $H^{-1}(P) = 1.176P + 1000$. The function H^{-1} represents the original sticker price for a given discounted price P.

(e) $H^{-1}(13{,}000) = 1.176(13{,}000) + 1000 = 16{,}288$. So the original price of the car is \$16,288 when the discounted price (\$1000 rebate, then 15% off) is \$13,000.

102. $f(x) = mx + b$. Notice that $f(x_1) = f(x_2) \Leftrightarrow mx_1 + b = mx_2 + b \Leftrightarrow mx_1 = mx_2$. We can conclude that $x_1 = x_2$ if and only if $m \ne 0$. Therefore f is one-to-one if and only if $m \ne 0$. If $m \ne 0$, $f(x) = mx + b \Leftrightarrow y = mx + b \Leftrightarrow mx = y - b \Leftrightarrow x = \dfrac{y - b}{m}$. So, $f^{-1}(x) = \dfrac{x - b}{m}$.

103. **(a)** $f(x) = \dfrac{2x + 1}{5}$ is "multiply by 2, add 1, and then divide by 5 ". So the reverse is "multiply by 5, subtract 1, and then divide by 2 " or $f^{-1}(x) = \dfrac{5x - 1}{2}$. Check: $f \circ f^{-1}(x) = f\left(\dfrac{5x - 1}{2}\right) = \dfrac{2\left(\dfrac{5x - 1}{2}\right) + 1}{5} = \dfrac{5x - 1 + 1}{5} = \dfrac{5x}{5} = x$ and $f^{-1} \circ f(x) = f^{-1}\left(\dfrac{2x + 1}{5}\right) = \dfrac{5\left(\dfrac{2x + 1}{5}\right) - 1}{2} = \dfrac{2x + 1 - 1}{2} = \dfrac{2x}{2} = x$.

(b) $f(x) = 3 - \frac{1}{x} = \frac{-1}{x} + 3$ is "take the negative reciprocal and add 3 ". Since the reverse of "take the negative reciprocal" is "take the negative reciprocal ", $f^{-1}(x)$ is "subtract 3 and take the negative reciprocal ", that is, $f^{-1}(x) = \frac{-1}{x-3}$. Check: $f \circ f^{-1}(x) = f\left(\frac{-1}{x-3}\right) = 3 - \frac{1}{\frac{-1}{x-3}} = 3 - \left(1 \cdot \frac{x-3}{-1}\right) = 3 + x - 3 = x$ and $f^{-1} \circ f(x) = f^{-1}\left(3 - \frac{1}{x}\right) = \frac{-1}{\left(3 - \frac{1}{x}\right) - 3} = \frac{-1}{-\frac{1}{x}} = -1 \cdot \frac{x}{-1} = x$.

(c) $f(x) = \sqrt{x^3 + 2}$ is "cube, add 2, and then take the square root". So the reverse is "square, subtract 2, then take the cube root " or $f^{-1}(x) = \sqrt[3]{x^2 - 2}$. Domain for $f(x)$ is $\left[-\sqrt[3]{2}, \infty\right)$; domain for $f^{-1}(x)$ is $[0, \infty)$. Check: $f \circ f^{-1}(x) = f\left(\sqrt[3]{x^2 - 2}\right) = \sqrt{\left(\sqrt[3]{x^2 - 2}\right)^3 + 2} = \sqrt{x^2 - 2 + 2} = \sqrt{x^2} = x$ (on the appropriate domain) and $f^{-1} \circ f(x) = f^{-1}\left(\sqrt{x^3 + 2}\right) = \sqrt[3]{\left(\sqrt{x^3 + 2}\right)^2 - 2} = \sqrt[3]{x^3 + 2 - 2} = \sqrt[3]{x^3} = x$ (on the appropriate domain).

(d) $f(x) = (2x - 5)^3$ is "double, subtract 5, and then cube". So the reverse is "take the cube root, add 5, and divide by 2" or $f^{-1}(x) = \frac{\sqrt[3]{x} + 5}{2}$ Domain for both $f(x)$ and $f^{-1}(x)$ is $(-\infty, \infty)$. Check: $f \circ f^{-1}(x) = f\left(\frac{\sqrt[3]{x} + 5}{2}\right) = \left[2\left(\frac{\sqrt[3]{x} + 5}{2}\right) - 5\right]^3 = \left(\sqrt[3]{x} + 5 - 5\right)^3 = \left(\sqrt[3]{x}\right)^3 = \sqrt[3]{x^3} = x$ and $f^{-1} \circ f(x) = f^{-1}\left((2x - 5)^3\right) = \frac{\sqrt[x]{(2x - 5)^3} + 5}{2} = \frac{(2x - 5) + 5}{2} = \frac{2x}{2} = x$.

In a function like $f(x) = 3x - 2$, the variable occurs only once and it easy to see how to reverse the operations step by step. But in $f(x) = x^3 + 2x + 6$, you apply two different operations to the variable x (cubing and multiplying by 2) and then add 6, so it is not possible to reverse the operations step by step.

104. $f(I(x)) = f(x)$; therefore $f \circ I = f$. $I(f(x)) = f(x)$; therefore $I \circ f = f$.
By definition, $f \circ f^{-1}(x) = x = I(x)$; therefore $f \circ f^{-1} = I$. Similarly, $f^{-1} \circ f(x) = x = I(x)$; therefore $f^{-1} \circ f = I$.

105. (a) We find $g^{-1}(x)$: $y = 2x + 1 \Leftrightarrow 2x = y - 1 \Leftrightarrow x = \frac{1}{2}(y - 1)$. So $g^{-1}(x) = \frac{1}{2}(x - 1)$. Thus $f(x) = h \circ g^{-1}(x) = h\left(\frac{1}{2}(x - 1)\right) = 4\left[\frac{1}{2}(x - 1)\right]^2 + 4\left[\frac{1}{2}(x - 1)\right] + 7 = x^2 - 2x + 1 + 2x - 2 + 7 = x^2 + 6$.

(b) $f \circ g = h \Leftrightarrow f^{-1} \circ f \circ g = f^{-1} \circ h \Leftrightarrow I \circ g = f^{-1} \circ h \Leftrightarrow g = f^{-1} \circ h$. Note that we compose with f^{-1} on the left on each side of the equation. We find f^{-1}: $y = 3x + 5 \Leftrightarrow 3x = y - 5 \Leftrightarrow x = \frac{1}{3}(y - 5)$. So $f^{-1}(x) = \frac{1}{3}(x - 5)$. Thus $g(x) = f^{-1} \circ h(x) = f^{-1}\left(3x^2 + 3x + 2\right) = \frac{1}{3}\left[\left(3x^2 + 3x + 2\right) - 5\right] = \frac{1}{3}\left[3x^2 + 3x - 3\right] = x^2 + x - 1$.

CHAPTER 2 REVIEW

1. "Square, then subtract 5" can be represented by the function $f(x) = x^2 - 5$.

2. "Divide by 2, then add 9" can be represented by the function $g(x) = \frac{x}{2} + 9$.

3. $f(x) = 3(x + 10)$: "Add 10, then multiply by 3."

4. $f(x) = \sqrt{6x - 10}$: "Multiply by 6, then subtract 10, then take the square root."

5. $g(x) = x^2 - 4x$

x	$g(x)$
-1	5
0	0
1	-3
2	-4
3	-3

6. $h(x) = 3x^2 + 2x - 5$

x	$h(x)$
-2	3
-1	-4
0	-5
1	0
2	11

7. $C(x) = 5000 + 30x - 0.001x^2$

(a) $C(1000) = 5000 + 30(1000) - 0.001(1000)^2 = \$34{,}000$ and
$C(10{,}000) = 5000 + 30(10{,}000) - 0.001(10{,}000)^2 = \$205{,}000$.

(b) From part (a), we see that the total cost of printing 1000 copies of the book is \$34,000 and the total cost of printing 10,000 copies is \$205,000.

(c) $C(0) = 5000 + 30(0) - 0.001(0)^2 = \$5000$. This represents the fixed costs associated with getting the print run ready.

(d) The net change in C as x changes from 1000 to 10,000 is $C(10{,}000) - C(1000) = 205{,}000 - 34{,}000 = \$171{,}000$, and the average rate of change is $\dfrac{C(10{,}000) - C(1000)}{10{,}000 - 1000} = \dfrac{171{,}000}{9000} = \$19/\text{copy}$.

8. $E(x) = 400 + 0.03x$

(a) $E(2000) = 400 + 0.03(2000) = \$460$ and $E(15{,}000) = 400 + 0.03(15{,}000) = \$850$.

(b) From part (a), we see that if Reynalda sells \$2000 worth of goods, she makes \$460, and if she sells \$15,000 worth of goods, she makes \$850.

(c) $E(0) = 400 + 0.03(0) = \$400$ is Reynalda's base weekly salary.

(d) The net change in E as x changes from 2000 to 15,000 is $E(15{,}000) - E(2000) = 850 - 460 = \$390$, and the average rate of change is $\dfrac{E(15{,}000) - E(2000)}{15{,}000 - 2000} = \dfrac{390}{13{,}000} = \$0.03$ per dollar.

(e) Because the value of goods sold x is multiplied by 0.03 or 3%, we see that Reynalda earns a percentage of 3% on the goods that she sells.

9. $f(x) = x^2 - 4x + 6$; $f(0) = (0)^2 - 4(0) + 6 = 6$; $f(2) = (2)^2 - 4(2) + 6 = 2$; $f(-2) = (-2)^2 - 4(-2) + 6 = 18$; $f(a) = (a)^2 - 4(a) + 6 = a^2 - 4a + 6$; $f(-a) = (-a)^2 - 4(-a) + 6 = a^2 + 4a + 6$; $f(x+1) = (x+1)^2 - 4(x+1) + 6 = x^2 + 2x + 1 - 4x - 4 + 6 = x^2 - 2x + 3$; $f(2x) = (2x)^2 - 4(2x) + 6 = 4x^2 - 8x + 6$.

10. $f(x) = 4 - \sqrt{3x-6}$; $f(5) = 4 - \sqrt{15-6} = 1$; $f(9) = 4 - \sqrt{27-6} = 4 - \sqrt{21}$; $f(a+2) = 4 - \sqrt{3a+6-6} = 4 - \sqrt{3a}$; $f(-x) = 4 - \sqrt{3(-x)-6} = 4 - \sqrt{-3x-6}$; $f\left(x^2\right) = 4 - \sqrt{3x^2-6}$.

11. By the Vertical Line Test, figures (b) and (c) are graphs of functions. By the Horizontal Line Test, figure (c) is the graph of a one-to-one function.

12. (a) $f(-2) = -1$ and $f(2) = 2$.

(b) The net change in f from -2 to 2 is $f(2) - f(-2) = 2 - (-1) = 3$, and the average rate of change is $\dfrac{f(2) - f(-2)}{2 - (-2)} = \dfrac{3}{4}$.

(c) The domain of f is $[-4, 5]$ and the range of f is $[-4, 4]$.

(d) f is increasing on $(-4, -2)$ and $(-1, 4)$; f is decreasing on $(-2, -1)$ and $(4, 5)$.

(e) f has local maximum values of -1 (at $x = -2$) and 4 (at $x = 4$).

(f) f is not a one-to-one, for example, $f(-2) = -1 = f(0)$. There are many more examples.

13. Domain: We must have $x + 3 \geq 0 \Leftrightarrow x \geq -3$. In interval notation, the domain is $[-3, \infty)$.
Range: For x in the domain of f, we have $x \geq -3 \Leftrightarrow x + 3 \geq 0 \Leftrightarrow \sqrt{x+3} \geq 0 \Leftrightarrow f(x) \geq 0$. So the range is $[0, \infty)$.

14. $F(t) = t^2 + 2t + 5 = \left(t^2 + 2t + 1\right) + 5 - 1 = (t+1)^2 + 4$. Therefore $F(t) \geq 4$ for all t. Since there are no restrictions on t, the domain of F is $(-\infty, \infty)$, and the range is $[4, \infty)$.

15. $f(x) = 7x + 15$. The domain is all real numbers, $(-\infty, \infty)$.

16. $f(x) = \dfrac{2x+1}{2x-1}$. Then $2x - 1 \neq 0 \Leftrightarrow x \neq \frac{1}{2}$. So the domain of f is $\left\{x \mid x \neq \frac{1}{2}\right\}$.

17. $f(x) = \sqrt{x+4}$. We require $x + 4 \geq 0 \Leftrightarrow x \geq -4$. Thus the domain is $[-4, \infty)$.

18. $f(x) = 3x - \dfrac{2}{\sqrt{x+1}}$. The domain of f is the set of x where $x + 1 > 0 \Leftrightarrow x > -1$. So the domain is $(-1, \infty)$.

19. $f(x) = \dfrac{1}{x} + \dfrac{1}{x+1} + \dfrac{1}{x+2}$. The denominators cannot equal 0, therefore the domain is $\{x \mid x \neq 0, -1, -2\}$.

20. $g(x) = \dfrac{2x^2 + 5x + 3}{2x^2 - 5x - 3} = \dfrac{2x^2 + 5x + 3}{(2x+1)(x-3)}$. The domain of g is the set of all x where the denominator is not 0. So the domain is $\{x \mid 2x + 1 \neq 0 \text{ and } x - 3 \neq 0\} = \left\{x \mid x \neq -\frac{1}{2} \text{ and } x \neq 3\right\}$.

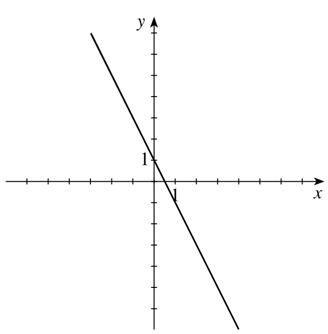
21. $h(x) = \sqrt{4-x} + \sqrt{x^2 - 1}$. We require the expression inside the radicals be nonnegative. So $4 - x \geq 0 \Leftrightarrow 4 \geq x$; also $x^2 - 1 \geq 0 \Leftrightarrow (x-1)(x+1) \geq 0$. We make a table:

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
Sign of $x - 1$	$-$	$-$	$+$
Sign of $x + 1$	$-$	$+$	$+$
Sign of $(x-1)(x+1)$	$+$	$-$	$+$

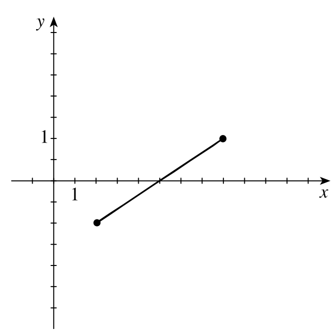
Thus the domain is $(-\infty, 4] \cap \{(-\infty, -1] \cup [1, \infty)\} = (-\infty, -1] \cup [1, 4]$.

22. $f(x) = \dfrac{\sqrt[3]{2x+1}}{\sqrt[3]{2x}+2}$. Since we have an odd root, the domain is the set of all x where the denominator is not 0. Now $\sqrt[3]{2x} + 2 \neq 0 \Leftrightarrow \sqrt[3]{2x} \neq -2 \Leftrightarrow 2x \neq -8 \Leftrightarrow x \neq -4$. Thus the domain of f is $\{x \mid x \neq -4\}$.

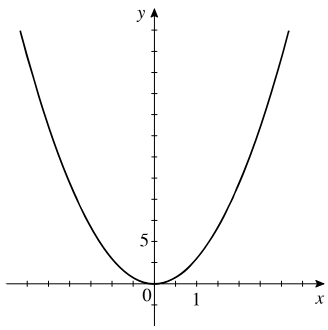
23. $f(x) = 1 - 2x$



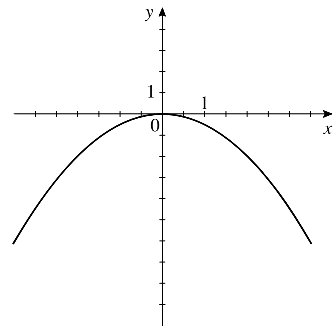
24. $f(x) = \frac{1}{3}(x-5)$, $2 \leq x \leq 8$



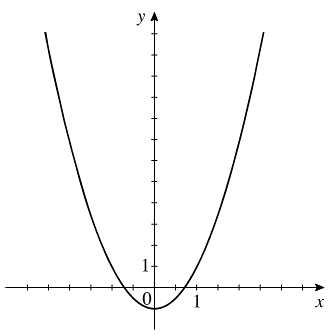
25. $f(x) = 3x^2$



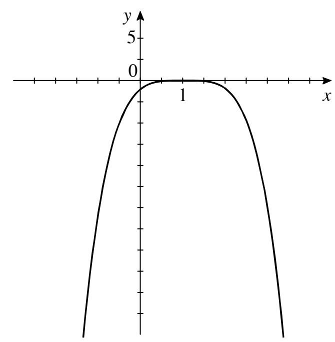
26. $f(x) = -\frac{1}{4}x^2$



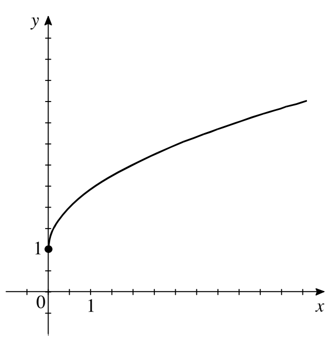
27. $f(x) = 2x^2 - 1$



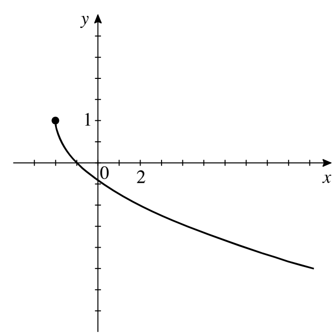
28. $f(x) = -(x-1)^4$



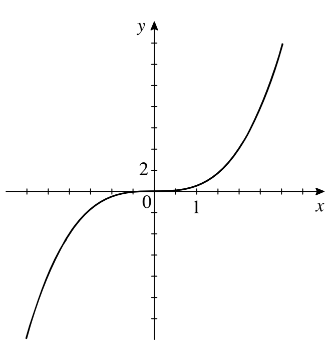
29. $f(x) = 1 + \sqrt{x}$



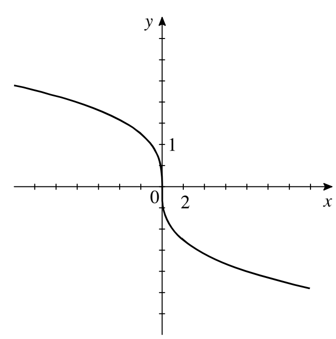
30. $f(x) = 1 - \sqrt{x+2}$



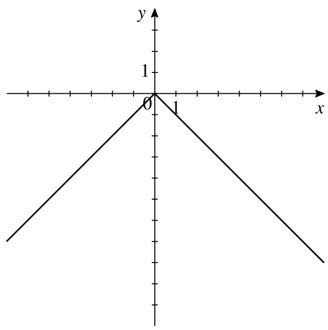
31. $f(x) = \frac{1}{2}x^3$



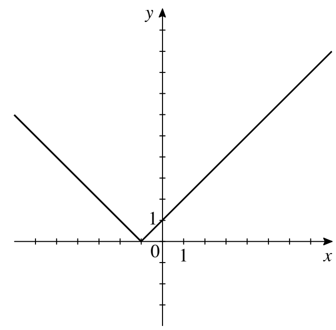
32. $f(x) = \sqrt[3]{-x}$



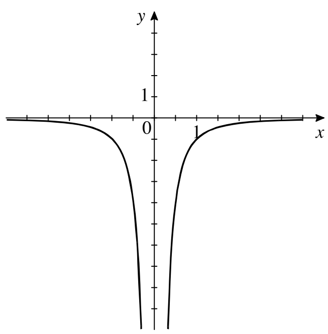
33. $f(x) = -|x|$



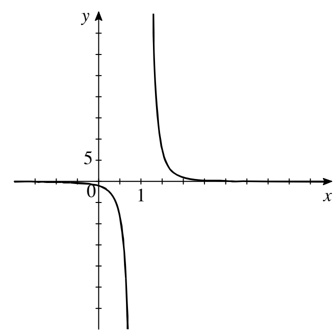
34. $f(x) = |x+1|$



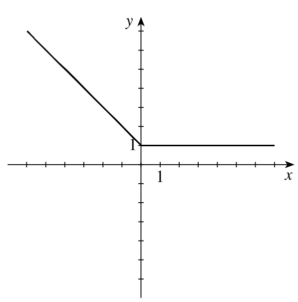
35. $f(x) = -\dfrac{1}{x^2}$



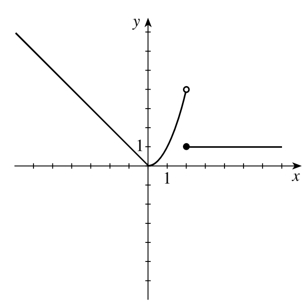
36. $f(x) = \dfrac{1}{(x-1)^3}$



37. $f(x) = \begin{cases} 1-x & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$



38. $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x^2 & \text{if } 0 \leq x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$



39. $x + y^2 = 14 \Rightarrow y^2 = 14 - x \Rightarrow y = \pm\sqrt{14-x}$, so the original equation does not define y as a function of x.

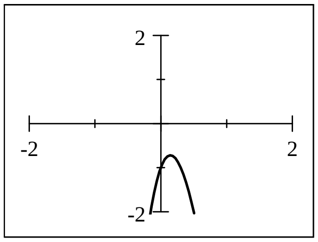
40. $3x - \sqrt{y} = 8 \Rightarrow \sqrt{y} = 3x - 8 \Rightarrow y = (3x-8)^2$, so the original equation defines y as a function of x.

41. $x^3 - y^3 = 27 \Leftrightarrow y^3 = x^3 - 27 \Leftrightarrow y = \left(x^3 - 27\right)^{1/3}$, so the original equation defines y as a function of x (since the cube root function is one-to-one).

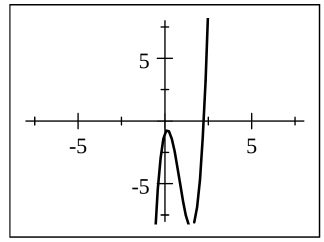
42. $2x = y^4 - 16 \Leftrightarrow y^4 = 2x + 16 \Leftrightarrow y = \pm\sqrt[4]{2x+16}$, so the original equation does not define y as a function of x.

43. $f(x) = 6x^3 - 15x^2 + 4x - 1$

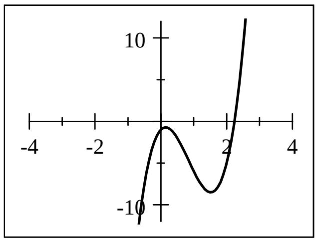
(i) $[-2, 2]$ by $[-2, 2]$



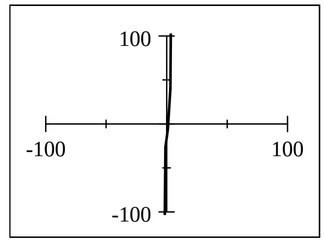
(ii) $[-8, 8]$ by $[-8, 8]$



(iii) $[-4, 4]$ by $[-12, 12]$



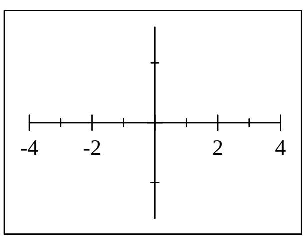
(iv) $[-100, 100]$ by $[-100, 100]$



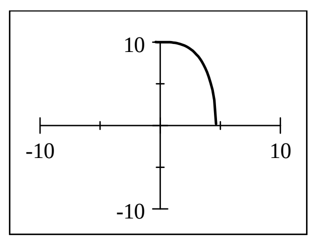
From the graphs, we see that the viewing rectangle in (iii) produces the most appropriate graph.

44. $f(x) = \sqrt{100 - x^3}$

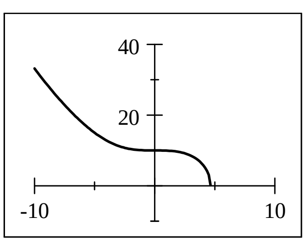
(i) $[-4, 4]$ by $[-4, 4]$



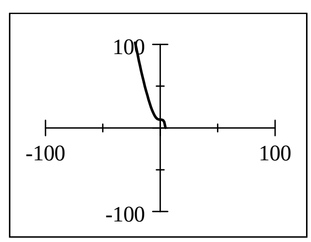
(ii) $[-10, 10]$ by $[-10, 10]$



(iii) $[-10, 10]$ by $[-10, 40]$

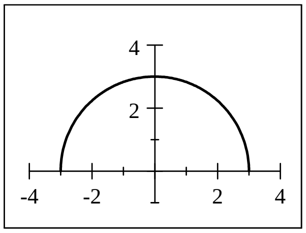


(iv) $[-100, 100]$ by $[-100, 100]$



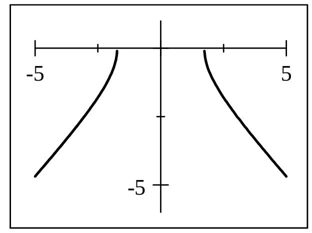
From the graphs, we see that the viewing rectangle in (iii) produces the most appropriate graph of f.

45. **(a)** We graph $f(x) = \sqrt{9 - x^2}$ in the viewing rectangle $[-4, 4]$ by $[-1, 4]$.



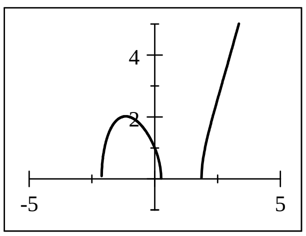
(b) From the graph, the domain of f is $[-3, 3]$ and the range of f is $[0, 3]$.

46. **(a)** We graph $f(x) = -\sqrt{x^2 - 3}$ in the viewing rectangle $[-5, 5]$ by $[-6, 1]$.



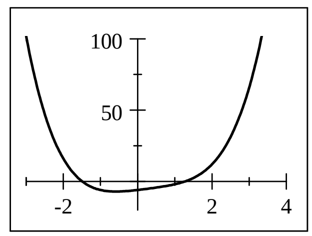
(b) From the graph, the domain of f is $(-\infty, -1.73] \cup [1.73, \infty)$ and the range of f is $(-\infty, 0]$.

47. **(a)** We graph $f(x) = \sqrt{x^3 - 4x + 1}$ in the viewing rectangle $[-5, 5]$ by $[-1, 5]$.



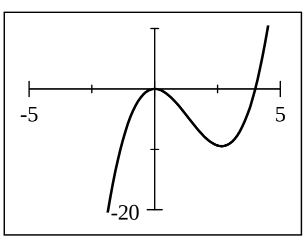
(b) From the graph, the domain of f is approximately $[-2.11, 0.25] \cup [1.86, \infty)$ and the range of f is $[0, \infty)$.

48. **(a)** We graph $f(x) = x^4 - x^3 + x^2 + 3x - 6$ in the viewing rectangle $[-3, 4]$ by $[-20, 100]$.

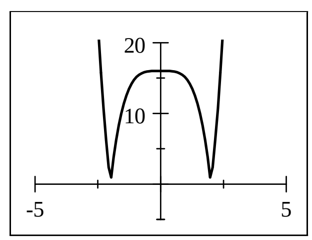


(b) From the graph, the domain of f is $(-\infty, \infty)$ and the range of f is approximately $[-7.10, \infty)$.

49. $f(x) = x^3 - 4x^2$ is graphed in the viewing rectangle $[-5, 5]$ by $[-20, 10]$. $f(x)$ is increasing on $(-\infty, 0)$ and $(2.67, \infty)$. It is decreasing on $(0, 2.67)$.



50. $f(x) = \left|x^4 - 16\right|$ is graphed in the viewing rectangle $[-5, 5]$ by $[-5, 20]$. $f(x)$ is increasing on $(-2, 0)$ and $(2, \infty)$. It is decreasing on $(-\infty, -2)$ and $(0, 2)$.



51. The net change is $f(8) - f(4) = 8 - 12 = -4$ and the average rate of change is $\dfrac{f(8) - f(4)}{8 - 4} = \dfrac{-4}{4} = -1$.

52. The net change is $g(30) - g(10) = 30 - (-5) = 35$ and the average rate of change is $\dfrac{g(30) - g(10)}{30 - 10} = \dfrac{35}{20} = \dfrac{7}{4}$.

53. The net change is $f(2) - f(-1) = 6 - 2 = 4$ and the average rate of change is $\dfrac{f(2) - f(-1)}{2 - (-1)} = \dfrac{4}{3}$.

54. The net change is $f(3) - f(1) = -1 - 5 = -6$ and the average rate of change is $\dfrac{f(3) - f(1)}{3 - 1} = \dfrac{-6}{2} = -3$.

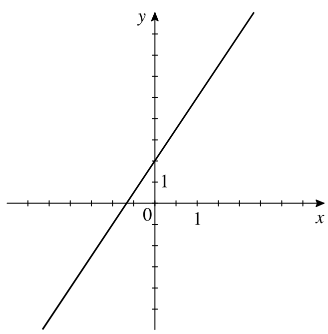
55. The net change is $f(4) - f(1) = \left[4^2 - 2(4)\right] - \left[1^2 - 2(1)\right] = 8 - (-1) = 9$ and the average rate of change is $\dfrac{f(4) - f(1)}{4 - 1} = \dfrac{9}{3} = 3$.

56. The net change is $g(a+h) - g(a) = (a+h+1)^2 - (a+1)^2 = 2ah + 2h + h^2$ and the average rate of change is $\dfrac{g(a+h) - g(a)}{a+h-a} = \dfrac{2ah + 2h + h^2}{h} = 2a + 2 + h$.

57. $f(x) = (2+3x)^2 = 9x^2 + 12x + 4$ is not linear. It cannot be expressed in the form $f(x) = ax + b$ with constant a and b.

58. $g(x) = \dfrac{x+3}{5} = \frac{1}{5}x + \frac{3}{5}$ is linear with $a = \frac{1}{5}$ and $b = \frac{3}{5}$.

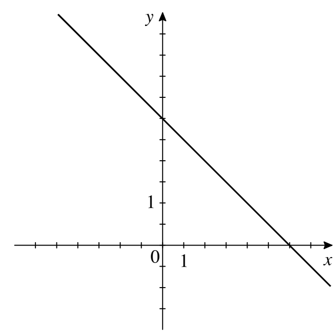
59. (a)



(b) The slope of the graph is the value of a in the equation $f(x) = ax + b = 3x + 2$; that is, 3.

(c) The rate of change is the slope of the graph, 3.

60. (a)



(b) The slope of the graph is the value of a in the equation $f(x) = ax + b = -\frac{1}{2}x + 3$; that is, $-\frac{1}{2}$.

(c) The rate of change is the slope of the graph, $-\frac{1}{2}$.

61. The linear function with rate of change -2 and initial value 3 has $a = -2$ and $b = 3$, so $f(x) = -2x + 3$.

62. The linear function whose graph has slope $\frac{1}{2}$ and y-intercept -1 has $a = \frac{1}{2}$ and $b = -1$, so $f(x) = \frac{1}{2}x - 1$.

63. Between $x = 0$ and $x = 1$, the rate of change is $\dfrac{f(1) - f(0)}{1 - 0} = \dfrac{5 - 3}{1} = 2$. At $x = 0$, $f(x) = 3$. Thus, an equation is $f(x) = 2x + 3$.

64. Between $x = 0$ and $x = 2$, the rate of change is $\dfrac{f(2) - f(0)}{2 - 0} = \dfrac{5.5 - 6}{2} = -\frac{1}{4}$. At $x = 0$, $f(x) = 6$. Thus, an equation is $f(x) = -\frac{1}{4}x + 6$.

65. The points $(0, 4)$ and $(8, 0)$ lie on the graph, so the rate of change is $\dfrac{0 - 4}{8 - 0} = -\dfrac{1}{2}$. At $x = 0$, $y = 4$. Thus, an equation is $y = -\frac{1}{2}x + 4$.

66. The points $(0, -4)$ and $(2, 0)$ lie on the graph, so the rate of change is $\dfrac{0 - (-4)}{2 - 0} = 2$. At $x = 0$, $y = -4$. Thus, an equation is $y = 2x - 4$.

67. $P(t) = 3000 + 200t + 0.1t^2$

(a) $P(10) = 3000 + 200(10) + 0.1(10)^2 = 5010$ represents the population in its 10th year (that is, in 1995), and $P(20) = 3000 + 200(20) + 0.1(20)^2 = 7040$ represents its population in its 20th year (in 2005).

(b) The average rate of change is $\dfrac{P(20) - P(10)}{20 - 10} = \dfrac{7040 - 5010}{10} = \dfrac{2030}{10} = 203$ people/year. This represents the average yearly change in population between 1995 and 2005.

68. $D(t) = 3500 + 15t^2$

(a) $D(0) = 3500 + 15(0)^2 = \$3500$ represents the amount deposited in 1995 and $D(15) = 3500 + 15(15)^2 = \$6875$ represents the amount deposited in 2010.

(b) Solving the equation $D(t) = 17{,}000$, we get $17{,}000 = 3500 + 15t^2 \Leftrightarrow 15t^2 = 13{,}500 \Leftrightarrow t^2 = \dfrac{13500}{15} = 900 \Leftrightarrow t = 30$, so thirty years after 1995 (that is, in the year 2025) she will deposit \$17,000.

(c) The average rate of change is $\dfrac{D(15) - D(0)}{15 - 0} = \dfrac{6875 - 3500}{15} = \$225/\text{year}$. This represents the average annual increase in contributions between 1995 and 2010.

69. $f(x) = \frac{1}{2}x - 6$

(a) The average rate of change of f between $x = 0$ and $x = 2$ is

$\dfrac{f(2) - f(0)}{2 - 0} = \dfrac{\left[\frac{1}{2}(2) - 6\right] - \left[\frac{1}{2}(0) - 6\right]}{2} = \dfrac{-5 - (-6)}{2} = \dfrac{1}{2}$, and the average rate of change of f between $x = 15$ and $x = 50$ is

$\dfrac{f(50) - f(15)}{50 - 15} = \dfrac{\left[\frac{1}{2}(50) - 6\right] - \left[\frac{1}{2}(15) - 6\right]}{35} = \dfrac{19 - \frac{3}{2}}{35} = \dfrac{1}{2}$.

(b) The rates of change are the same.

(c) Yes, f is a linear function with rate of change $\frac{1}{2}$.

70. $f(x) = 8 - 3x$

(a) The average rate of change of f between $x = 0$ and $x = 2$ is $\dfrac{f(2) - f(0)}{2 - 0} = \dfrac{[8 - 3(2)] - [8 - 3(0)]}{2} = \dfrac{2 - 8}{2} = -3$, and the average rate of change of f between $x = 15$ and $x = 50$ is

$\dfrac{f(50) - f(15)}{50 - 15} = \dfrac{[8 - 3(50)] - [8 - 3(15)]}{35} = \dfrac{-142 - (-37)}{35} = -3$.

(b) The rates of change are the same.

(c) Yes, f is a linear function with rate of change -3.

71. **(a)** $y = f(x) + 8$. Shift the graph of $f(x)$ upward 8 units.

(b) $y = f(x + 8)$. Shift the graph of $f(x)$ to the left 8 units.

(c) $y = 1 + 2f(x)$. Stretch the graph of $f(x)$ vertically by a factor of 2, then shift it upward 1 unit.

(d) $y = f(x - 2) - 2$. Shift the graph of $f(x)$ to the right 2 units, then downward 2 units.

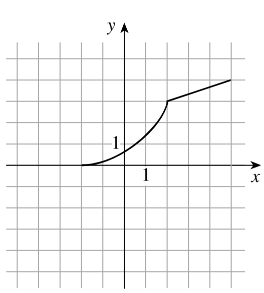
(e) $y = f(-x)$. Reflect the graph of $f(x)$ about the y-axis.

(f) $y = -f(-x)$. Reflect the graph of $f(x)$ first about the y-axis, then reflect about the x-axis.

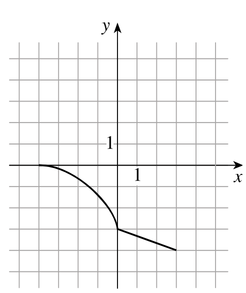
(g) $y = -f(x)$. Reflect the graph of $f(x)$ about the x-axis.

(h) $y = f^{-1}(x)$. Reflect the graph of $f(x)$ about the line $y = x$.

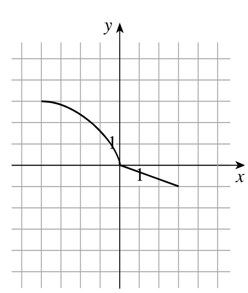
72. **(a)** $y = f(x - 2)$



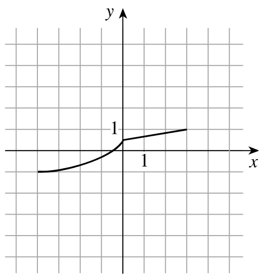
(b) $y = -f(x)$



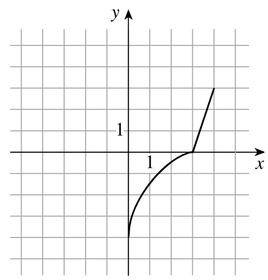
(c) $y = 3 - f(x)$



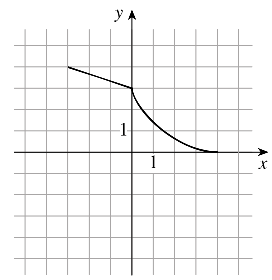
(d) $y = \frac{1}{2} f(x) - 1$



(e) $y = f^{-1}(x)$



(f) $y = f(-x)$



73. **(a)** $f(x) = 2x^5 - 3x^2 + 2$. $f(-x) = 2(-x)^5 - 3(-x)^2 + 2 = -2x^5 - 3x^2 + 2$. Since $f(x) \neq f(-x)$, f is not even. $-f(x) = -2x^5 + 3x^2 - 2$. Since $-f(x) \neq f(-x)$, f is not odd.

(b) $f(x) = x^3 - x^7$. $f(-x) = (-x)^3 - (-x)^7 = -\left(x^3 - x^7\right) = -f(x)$, hence f is odd.

(c) $f(x) = \dfrac{1 - x^2}{1 + x^2}$. $f(-x) = \dfrac{1 - (-x)^2}{1 + (-x)^2} = \dfrac{1 - x^2}{1 + x^2} = f(x)$. Since $f(x) = f(-x)$, f is even.

(d) $f(x) = \dfrac{1}{x + 2}$. $f(-x) = \dfrac{1}{(-x) + 2} = \dfrac{1}{2 - x}$. $-f(x) = -\dfrac{1}{x + 2}$. Since $f(x) \neq f(-x)$, f is not even, and since $f(-x) \neq -f(x)$, f is not odd.

74. **(a)** This function is odd.

(b) This function is neither even nor odd.

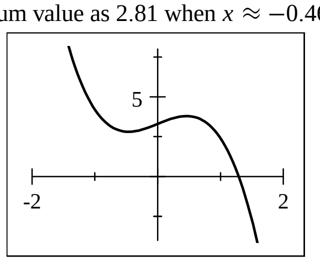
(c) This function is even.

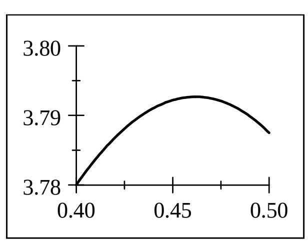
(d) This function is neither even nor odd.

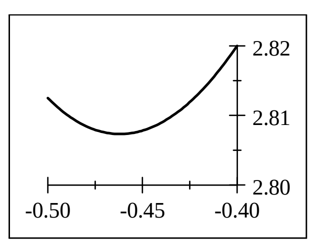
75. $g(x) = 2x^2 + 4x - 5 = 2\left(x^2 + 2x\right) - 5 = 2\left(x^2 + 2x + 1\right) - 5 - 2 = 2(x + 1)^2 - 7$. So the local minimum value -7 when $x = -1$.

76. $f(x) = 1 - x - x^2 = -\left(x^2 + x\right) + 1 = -\left(x^2 + x + \frac{1}{4}\right) + 1 + \frac{1}{4} = -\left(x + \frac{1}{2}\right)^2 + \frac{5}{4}$. So the local maximum value is $\frac{5}{4}$ when $x = -\frac{1}{2}$.

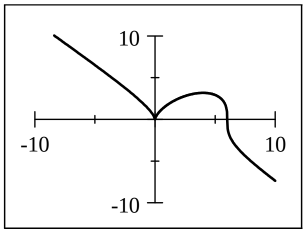
77. $f(x) = 3.3 + 1.6x - 2.5x^3$. In the first viewing rectangle, $[-2, 2]$ by $[-4, 8]$, we see that $f(x)$ has a local maximum and a local minimum. In the next viewing rectangle, $[0.4, 0.5]$ by $[3.78, 3.80]$, we isolate the local maximum value as approximately 3.79 when $x \approx 0.46$. In the last viewing rectangle, $[-0.5, -0.4]$ by $[2.80, 2.82]$, we isolate the local minimum value as 2.81 when $x \approx -0.46$.

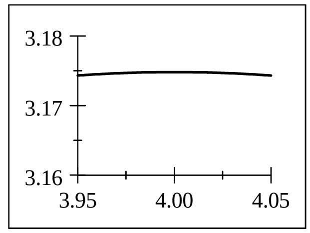






78. $f(x) = x^{2/3}(6-x)^{1/3}$. In the first viewing rectangle, $[-10, 10]$ by $[-10, 10]$, we see that $f(x)$ has a local maximum and a local minimum. The local minimum is 0 at $x = 0$ (and is easily verified). In the next viewing rectangle, $[3.95, 4.05]$ by $[3.16, 3.18]$, we isolate the local maximum value as approximately 3.175 when $x \approx 4.00$.





79. $h(t) = -16t^2 + 48t + 32 = -16\left(t^2 - 3t\right) + 32 = -16\left(t^2 - 3t + \frac{9}{4}\right) + 32 + 36$

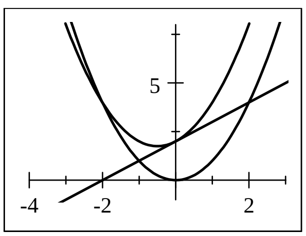
$= -16\left(t^2 - 3t + \frac{9}{4}\right) + 68 = -16\left(t - \frac{3}{2}\right)^2 + 68$

The stone reaches a maximum height of 68 feet.

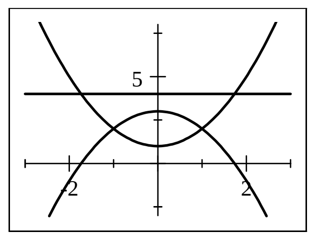
80. $P(x) = -1500 + 12x - 0.0004x^2 = -0.0004\left(x^2 - 30{,}000x\right) - 1500 = -0.0004\left(x^2 - 30{,}000x + 225{,}000{,}000\right) - 1500 + 90{,}000 = -0.0004(x - 15{,}000)^2 + 88{,}500$

The maximum profit occurs when 15,000 units are sold, and the maximum profit is \$88,500.

81. $f(x) = x + 2$, $g(x) = x^2$



82. $f(x) = x^2 + 1$, $g(x) = 3 - x^2$



83. $f(x) = x^2 - 3x + 2$ and $g(x) = 4 - 3x$.

(a) $(f+g)(x) = \left(x^2 - 3x + 2\right) + (4 - 3x) = x^2 - 6x + 6$

(b) $(f-g)(x) = \left(x^2 - 3x + 2\right) - (4 - 3x) = x^2 - 2$

(c) $(fg)(x) = \left(x^2 - 3x + 2\right)(4 - 3x) = 4x^2 - 12x + 8 - 3x^3 + 9x^2 - 6x = -3x^3 + 13x^2 - 18x + 8$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2 - 3x + 2}{4 - 3x}$, $x \neq \frac{4}{3}$

(e) $(f \circ g)(x) = f(4 - 3x) = (4 - 3x)^2 - 3(4 - 3x) + 2 = 16 - 24x + 9x^2 - 12 + 9x + 2 = 9x^2 - 15x + 6$

(f) $(g \circ f)(x) = g\left(x^2 - 3x + 2\right) = 4 - 3\left(x^2 - 3x + 2\right) = -3x^2 + 9x - 2$

84. $f(x) = 1 + x^2$ and $g(x) = \sqrt{x - 1}$. (Remember that the proper domains must apply.)

(a) $(f \circ g)(x) = f\left(\sqrt{x - 1}\right) = 1 + \left(\sqrt{x - 1}\right)^2 = 1 + x - 1 = x$

(b) $(g \circ f)(x) = g\left(1 + x^2\right) = \sqrt{\left(1 + x^2\right) - 1} = \sqrt{x^2} = |x|$

(c) $(f \circ g)(2) = f(g(2)) = f\left(\sqrt{(2) - 1}\right) = f(1) = 1 + (1)^2 = 2.$

(d) $(f \circ f)(2) = f(f(2)) = f\left(1 + (2)^2\right) = f(5) = 1 + (5)^2 = 26.$

(e) $(f \circ g \circ f)(x) = f((g \circ f)(x)) = f(|x|) = 1 + (|x|)^2 = 1 + x^2$. Note that $(g \circ f)(x) = |x|$ by part (b).

(f) $(g \circ f \circ g)(x) = g((f \circ g)(x)) = g(x) = \sqrt{x - 1}$. Note that $(f \circ g)(x) = x$ by part (a).

85. $f(x) = 3x - 1$ and $g(x) = 2x - x^2$.

$(f \circ g)(x) = f\left(2x - x^2\right) = 3\left(2x - x^2\right) - 1 = -3x^2 + 6x - 1$, and the domain is $(-\infty, \infty)$.

$(g \circ f)(x) = g(3x - 1) = 2(3x - 1) - (3x - 1)^2 = 6x - 2 - 9x^2 + 6x - 1 = -9x^2 + 12x - 3$, and the domain is $(-\infty, \infty)$

$(f \circ f)(x) = f(3x - 1) = 3(3x - 1) - 1 = 9x - 4$, and the domain is $(-\infty, \infty)$.

$(g \circ g)(x) = g\left(2x - x^2\right) = 2\left(2x - x^2\right) - \left(2x - x^2\right)^2 = 4x - 2x^2 - 4x^2 + 4x^3 - x^4 = -x^4 + 4x^3 - 6x^2 + 4x$, and domain is $(-\infty, \infty)$.

86. $f(x) = \sqrt{x}$, has domain $\{x \mid x \geq 0\}$. $g(x) = \dfrac{2}{x - 4}$, has domain $\{x \mid x \neq 4\}$.

$(f \circ g)(x) = f\left(\dfrac{2}{x - 4}\right) = \sqrt{\dfrac{2}{x - 4}}$. $(f \circ g)(x)$ is defined whenever both $g(x)$ and $f(g(x))$ are defined; that is, whenever $x \neq 4$ and $\dfrac{2}{x - 4} \geq 0$. Now $\dfrac{2}{x - 4} \geq 0 \Leftrightarrow x - 4 > 0 \Leftrightarrow x > 4$. So the domain of $f \circ g$ is $(4, \infty)$.

$(g \circ f)(x) = g\left(\sqrt{x}\right) = \dfrac{2}{\sqrt{x} - 4}$. $(g \circ f)(x)$ is defined whenever both $f(x)$ and $g(f(x))$ are defined; that is, whenever $x \geq 0$ and $\sqrt{x} - 4 \neq 0$. Now $\sqrt{x} - 4 \neq 0 \Leftrightarrow x \neq 16$. So the domain of $g \circ f$ is $[0, 16) \cup (16, \infty)$.

$(f \circ f)(x) = f\left(\sqrt{x}\right) = \sqrt{\sqrt{x}} = x^{1/4}$. $(f \circ f)(x)$ is defined whenever both $f(x)$ and $f(f(x))$ are defined; that is, whenever $x \geq 0$. So the domain of $f \circ f$ is $[0, \infty)$.

$(g \circ g)(x) = g\left(\dfrac{2}{x - 4}\right) = \dfrac{2}{\dfrac{2}{x - 4} - 4} = \dfrac{2(x - 4)}{2 - 4(x - 4)} = \dfrac{x - 4}{9 - 2x}$. $(g \circ g)(x)$ is defined whenever both $g(x)$ and $g(g(x))$ are defined; that is, whenever $x \neq 4$ and $9 - 2x \neq 0$. Now $9 - 2x \neq 0 \Leftrightarrow 2x \neq 9 \Leftrightarrow x \neq \frac{9}{2}$. So the domain of $g \circ g$ is $\left\{x \mid x \neq \frac{9}{2}, 4\right\}$.

87. $f(x) = \sqrt{1 - x}$, $g(x) = 1 - x^2$ and $h(x) = 1 + \sqrt{x}$.

$$\begin{aligned}(f \circ g \circ h)(x) &= f(g(h(x))) = f\left(g\left(1 + \sqrt{x}\right)\right) = f\left(1 - \left(1 + \sqrt{x}\right)^2\right) = f\left(1 - \left(1 + 2\sqrt{x} + x\right)\right) \\ &= f\left(-x - 2\sqrt{x}\right) = \sqrt{1 - \left(-x - 2\sqrt{x}\right)} = \sqrt{1 + 2\sqrt{x} + x} = \sqrt{\left(1 + \sqrt{x}\right)^2} = 1 + \sqrt{x}\end{aligned}$$

88. If $h(x) = \sqrt{x}$ and $g(x) = 1 + x$, then $(g \circ h)(x) = g\left(\sqrt{x}\right) = 1 + \sqrt{x}$. If $f(x) = \dfrac{1}{\sqrt{x}}$, then $(f \circ g \circ h)(x) = f\left(1 + \sqrt{x}\right) = \dfrac{1}{\sqrt{1 + \sqrt{x}}} = T(x)$.

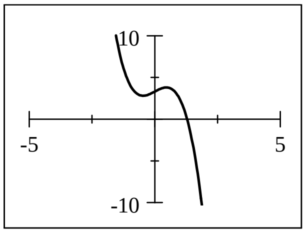
89. $f(x) = 3 + x^3$. If $x_1 \neq x_2$, then $x_1^3 \neq x_2^3$ (unequal numbers have unequal cubes), and therefore $3 + x_1^3 \neq 3 + x_2^3$. Thus f is a one-to-one function.

90. $g(x) = 2 - 2x + x^2 = \left(x^2 - 2x + 1\right) + 1 = (x - 1)^2 + 1$. Since $g(0) = 2 = g(2)$, as is true for all pairs of numbers equidistant from 1, g is not a one-to-one function.

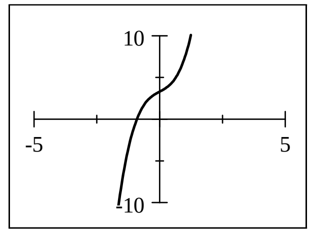
91. $h(x) = \dfrac{1}{x^4}$. Since the fourth powers of a number and its negative are equal, h is not one-to-one. For example, $h(-1) = \dfrac{1}{(-1)^4} = 1$ and $h(1) = \dfrac{1}{(1)^4} = 1$, so $h(-1) = h(1)$.

92. $r(x) = 2 + \sqrt{x + 3}$. If $x_1 \neq x_2$, then $x_1 + 3 \neq x_2 + 3$, so $\sqrt{x_1 + 3} \neq \sqrt{x_2 + 3}$ and $2 + \sqrt{x_1 + 3} \neq 2 + \sqrt{x_2 + 3}$. Thus r is one-to-one.

93. $p(x) = 3.3 + 1.6x - 2.5x^3$. Using a graphing device and the Horizontal Line Test, we see that p is not a one-to-one function.



94. $q(x) = 3.3 + 1.6x + 2.5x^3$. Using a graphing device and the Horizontal Line Test, we see that q is a one-to-one function.



95. $f(x) = 3x - 2 \Leftrightarrow y = 3x - 2 \Leftrightarrow 3x = y + 2 \Leftrightarrow x = \frac{1}{3}(y+2)$. So $f^{-1}(x) = \frac{1}{3}(x+2)$.

96. $f(x) = \dfrac{2x+1}{3}$. $y = \dfrac{2x+1}{3} \Leftrightarrow 2x + 1 = 3y \Leftrightarrow 2x = 3y - 1 \Leftrightarrow x = \frac{1}{2}(3y-1)$. So $f^{-1}(x) = \frac{1}{2}(3x-1)$.

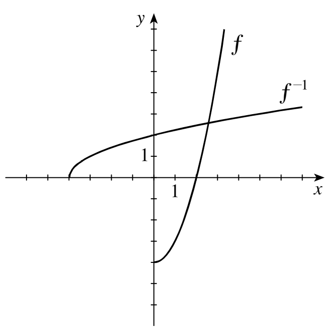
97. $f(x) = (x+1)^3 \Leftrightarrow y = (x+1)^3 \Leftrightarrow x + 1 = \sqrt[3]{y} \Leftrightarrow x = \sqrt[3]{y} - 1$. So $f^{-1}(x) = \sqrt[3]{x} - 1$.

98. $f(x) = 1 + \sqrt[5]{x-2}$. $y = 1 + \sqrt[5]{x-2} \Leftrightarrow y - 1 = \sqrt[5]{x-2} \Leftrightarrow x - 2 = (y-1)^5 \Leftrightarrow x = 2 + (y-1)^5$. So $f^{-1}(x) = 2 + (x-1)^5$.

99. The graph passes the Horizontal Line Test, so f has an inverse. Because $f(1) = 0$, $f^{-1}(0) = 1$, and because $f(3) = 4$, $f^{-1}(4) = 3$.

100. The graph fails the Horizontal Line Test, so f does not have an inverse.

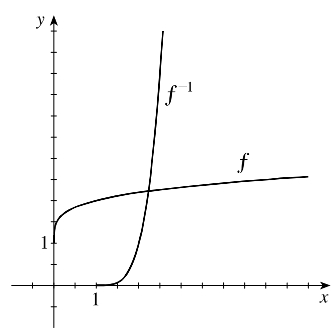
101. **(a), (b)** $f(x) = x^2 - 4$, $x \geq 0$



(c) $f(x) = x^2 - 4$, $x \geq 0 \Leftrightarrow y = x^2 - 4$, $y \geq -4$
$\Leftrightarrow x^2 = y + 4 \Leftrightarrow x = \sqrt{y+4}$. So
$f^{-1}(x) = \sqrt{x+4}$, $x \geq -4$.

102. **(a)** If $x_1 \neq x_2$, then $\sqrt[4]{x_1} \neq \sqrt[4]{x_2}$, and so $1 + \sqrt[4]{x_1} \neq 1 + \sqrt[4]{x_2}$. Therefore, f is a one-to-one function.

(b), (c)



(d) $f(x) = 1 + \sqrt[4]{x}$. $y = 1 + \sqrt[4]{x} \Leftrightarrow \sqrt[4]{x} = y - 1$
$\Leftrightarrow x = (y-1)^4$. So $f^{-1}(x) = (x-1)^4$, $x \geq 1$. Note that the domain of f is $[0, \infty)$, so $y = 1 + \sqrt[4]{x} \geq 1$. Hence, the domain of f^{-1} is $[1, \infty)$.

CHAPTER 2 TEST

1. By the Vertical Line Test, figures (a) and (b) are graphs of functions. By the Horizontal Line Test, only figure (a) is the graph of a one-to-one function.

2. (a) $f(0) = \dfrac{\sqrt{0}}{0+1} = 0$; $f(2) = \dfrac{\sqrt{2}}{2+1} = \dfrac{\sqrt{2}}{3}$; $f(a+2) = \dfrac{\sqrt{a+2}}{a+2+1} = \dfrac{\sqrt{a+2}}{a+3}$.

(b) $f(x) = \dfrac{\sqrt{x}}{x+1}$. Our restrictions are that the input to the radical is nonnegative and that the denominator must not be 0. Thus, $x \geq 0$ and $x + 1 \neq 0 \Leftrightarrow x \neq -1$. (The second restriction is made irrelevant by the first.) In interval notation, the domain is $[0, \infty)$.

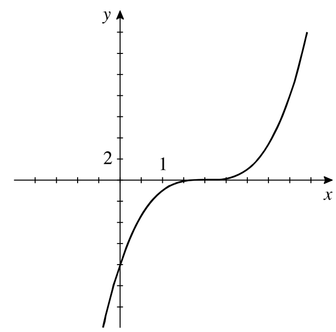
(c) The average rate of change is $\dfrac{f(10) - f(2)}{10 - 2} = \dfrac{\dfrac{\sqrt{10}}{10+1} - \dfrac{\sqrt{2}}{2+1}}{10-2} = \dfrac{3\sqrt{10} - 11\sqrt{2}}{264}$.

3. (a) "Subtract 2, then cube the result" can be expressed algebraically as $f(x) = (x-2)^3$.

(b)

x	$f(x)$
-1	-27
0	-8
1	-1
2	0
3	1
4	8

(c)



(d) We know that f has an inverse because it passes the Horizontal Line Test. A verbal description for f^{-1} is, "Take the cube root, then add 2."

(e) $y = (x-2)^3 \Leftrightarrow \sqrt[3]{y} = x - 2 \Leftrightarrow x = \sqrt[3]{y} + 2$. Thus, a formula for f^{-1} is $f^{-1}(x) = \sqrt[3]{x} + 2$.

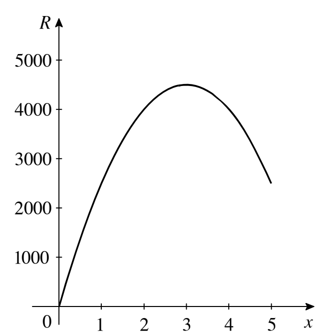
4. (a) f has a local minimum value of -4 at $x = -1$ and local maximum values of -1 at $x = -4$ and 4 at $x = 3$.

(b) f is increasing on $(-\infty, -4)$ and $(-1, 3)$ and decreasing on $(-4, -1)$ and $(3, \infty)$.

5. $R(x) = -500x^2 + 3000x$

(a) $R(2) = -500(2)^2 + 3000(2) = \$4000$ represents their total sales revenue when their price is \$2 per bar and $R(4) = -500(4)^2 + 3000(4) = \$4000$ represents their total sales revenue when their price is \$4 per bar

(b)



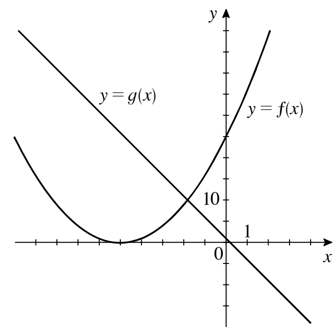
(c) The maximum revenue is \$4500, and it is achieved at a price of $x = \$3$.

6. The net change is $f(2+h) - f(2) = \left[(2+h)^2 - 2(2+h)\right] - \left[2^2 - 2(2)\right] = \left(4 + h^2 + 4h - 4 - 2h\right) - 0 = 2h + h^2$ and the average rate of change is $\dfrac{f(2+h) - f(2)}{2+h-2} = \dfrac{2h + h^2}{h} = 2 + h$.

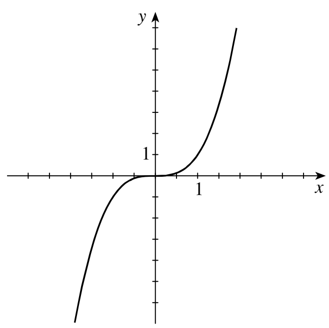
7. (a) $f(x) = (x+5)^2 = x^2 + 10x + 25$ is not linear because it cannot be expressed in the form $f(x) = ax + b$ for constants a and b.
$g(x) = 1 - 5x$ is linear.

(c) $g(x)$ has rate of change -5.

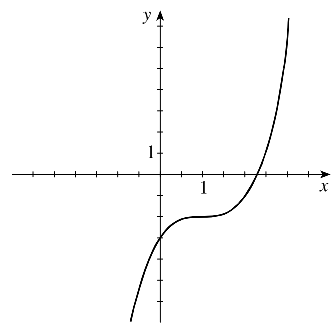
(b)



8. (a) $f(x) = x^3$



(b) $g(x) = (x-1)^3 - 2$. To obtain the graph of g, shift the graph of f to the right 1 unit and downward 2 units.

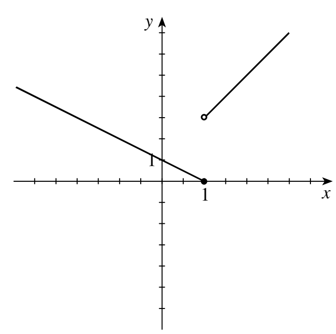


9. (a) $y = f(x-3) + 2$. Shift the graph of $f(x)$ to the right 3 units, then shift the graph upward 2 units.

(b) $y = f(-x)$. Reflect the graph of $f(x)$ about the y-axis.

10. (a) $f(-2) = 1 - (-2) = 1 + 2 = 3$ (since $-2 \le 1$).
$f(1) = 1 - 1 = 0$ (since $1 \le 1$).

(b)



11. $f(x) = x^2 + x + 1$; $g(x) = x - 3$.

(a) $(f+g)(x) = f(x) + g(x) = \left(x^2 + x + 1\right) + (x-3) = x^2 + 2x - 2$

(b) $(f-g)(x) = f(x) - g(x) = \left(x^2 + x + 1\right) - (x-3) = x^2 + 4$

(c) $(f \circ g)(x) = f(g(x)) = f(x-3) = (x-3)^2 + (x-3) + 1 = x^2 - 6x + 9 + x - 3 + 1 = x^2 - 5x + 7$

(d) $(g \circ f)(x) = g(f(x)) = g\left(x^2 + x + 1\right) = \left(x^2 + x + 1\right) - 3 = x^2 + x - 2$

(e) $f(g(2)) = f(-1) = (-1)^2 + (-1) + 1 = 1$. [We have used the fact that $g(2) = 2 - 3 = -1$.]

(f) $g(f(2)) = g(7) = 7 - 3 = 4$. [We have used the fact that $f(2) = 2^2 + 2 + 1 = 7$.]

(g) $(g \circ g \circ g)(x) = g(g(g(x))) = g(g(x-3)) = g(x-6) = (x-6) - 3 = x - 9$. [We have used the fact that $g(x-3) = (x-3) - 3 = x - 6$.]

12. (a) $f(x) = x^3 + 1$ is one-to-one because each real number has a unique cube.

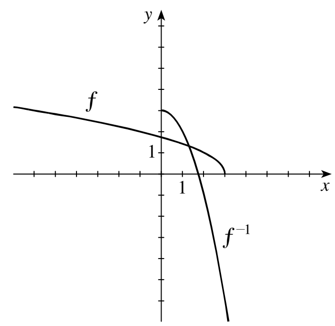
(b) $g(x) = |x + 1|$ is not one-to-one because, for example, $g(-2) = g(0) = 1$.

13. $f(g(x)) = \dfrac{1}{\left(\dfrac{1}{x} + 2\right) - 2} = \dfrac{1}{\dfrac{1}{x}} = x$ for all $x \neq 0$, and $g(f(x)) = \dfrac{1}{\dfrac{1}{x-2}} + 2 = x - 2 + 2 = x$ for all $x \neq -2$. Thus, by the Inverse Function Property, f and g are inverse functions.

14. $f(x) = \dfrac{x-3}{2x+5}$. $y = \dfrac{x-3}{2x+5} \Leftrightarrow (2x+5)y = x - 3 \Leftrightarrow x(2y-1) = -5x - 3 \Leftrightarrow x = -\dfrac{5y+3}{2y-1}$. Thus, $f^{-1}(x) = -\dfrac{5x+3}{2x-1}$.

15. (a) $f(x) = \sqrt{3-x}$, $x \leq 3 \Leftrightarrow y = \sqrt{3-x} \Leftrightarrow y^2 = 3 - x \Leftrightarrow x = 3 - y^2$. Thus $f^{-1}(x) = 3 - x^2$, $x \geq 0$.

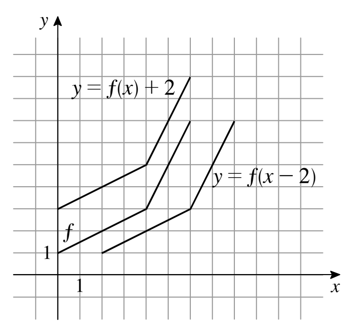
(b) $f(x) = \sqrt{3-x}$, $x \leq 3$ and $f^{-1}(x) = 3 - x^2$, $x \geq 0$



16. The domain of f is $[0, 6]$, and the range of f is $[1, 7]$.

17. The graph passes through the points $(0, 1)$ and $(4, 3)$, so $f(0) = 1$ and $f(4) = 3$.

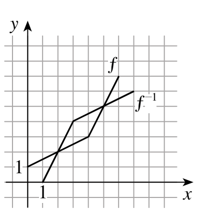
18. The graph of $f(x-2)$ can be obtained by shifting the graph of $f(x)$ to the right 2 units. The graph of $f(x) + 2$ can be obtained by shifting the graph of $f(x)$ upward 2 units.



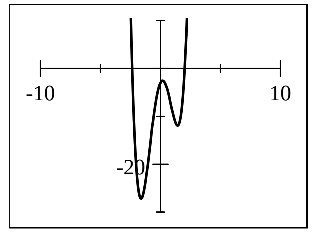
19. The net change of f between $x = 2$ and $x = 6$ is $f(6) - f(2) = 7 - 2 = 5$ and the average rate of change is $\dfrac{f(6) - f(2)}{6 - 2} = \dfrac{5}{4}$.

20. Because $f(0) = 1$, $f^{-1}(1) = 0$. Because $f(4) = 3$, $f^{-1}(3) = 4$.

21.

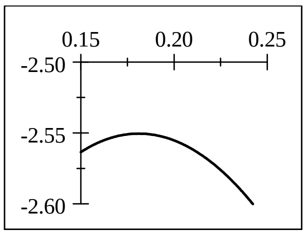


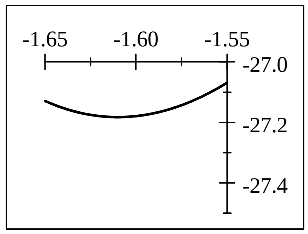
22. **(a)** $f(x) = 3x^4 - 14x^2 + 5x - 3$. The graph is shown in the viewing rectangle $[-10, 10]$ by $[-30, 10]$.

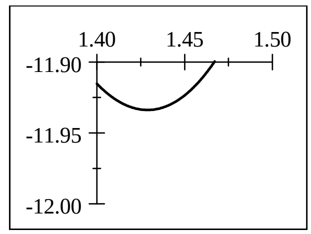


(b) No, by the Horizontal Line Test.

(c) The local maximum is approximately -2.55 when $x \approx 0.18$, as shown in the first viewing rectangle $[0.15, 0.25]$ by $[-2.6, -2.5]$. One local minimum is approximately -27.18 when $x \approx -1.61$, as shown in the second viewing rectangle $[-1.65, -1.55]$ by $[-27.5, -27]$. The other local minimum is approximately -11.93 when $x \approx 1.43$, as shown is the viewing rectangle $[1.4, 1.5]$ by $[-12, -11.9]$.







(d) Using the graph in part (a) and the local minimum, -27.18, found in part (c), we see that the range is $[-27.18, \infty)$.

(e) Using the information from part (c) and the graph in part (a), $f(x)$ is increasing on the intervals $(-1.61, 0.18)$ and $(1.43, \infty)$ and decreasing on the intervals $(-\infty, -1.61)$ and $(0.18, 1.43)$.

FOCUS ON MODELING Modeling with Functions

1. Let w be the width of the building lot. Then the length of the lot is $3w$. So the area of the building lot is $A(w) = 3w^2$, $w > 0$.

2. Let w be the width of the poster. Then the length of the poster is $w + 10$. So the area of the poster is $A(w) = w(w + 10) = w^2 + 10w$.

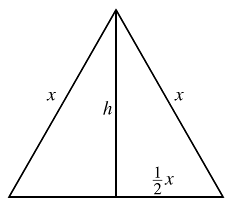
3. Let w be the width of the base of the rectangle. Then the height of the rectangle is $\frac{1}{2}w$. Thus the volume of the box is given by the function $V(w) = \frac{1}{2}w^3$, $w > 0$.

4. Let r be the radius of the cylinder. Then the height of the cylinder is $4r$. Since for a cylinder $V = \pi r^2 h$, the volume of the cylinder is given by the function $V(r) = \pi r^2 (4r) = 4\pi r^3$.

5. Let P be the perimeter of the rectangle and y be the length of the other side. Since $P = 2x + 2y$ and the perimeter is 20, we have $2x + 2y = 20 \Leftrightarrow x + y = 10 \Leftrightarrow y = 10 - x$. Since area is $A = xy$, substituting gives $A(x) = x(10 - x) = 10x - x^2$, and since A must be positive, the domain is $0 < x < 10$.

6. Let A be the area and y be the length of the other side. Then $A = xy = 16 \Leftrightarrow y = \dfrac{16}{x}$. Substituting into $P = 2x + 2y$ gives $P = 2x + 2 \cdot \dfrac{16}{x} = 2x + \dfrac{32}{x}$, where $x > 0$.

7.



Let h be the height of an altitude of the equilateral triangle whose side has length x, as shown in the diagram. Thus the area is given by $A = \frac{1}{2}xh$. By the Pythagorean Theorem, $h^2 + \left(\frac{1}{2}x\right)^2 = x^2 \Leftrightarrow h^2 + \frac{1}{4}x^2 = x^2 \Leftrightarrow h^2 = \frac{3}{4}x^2 \Leftrightarrow h = \frac{\sqrt{3}}{2}x$. Substituting into the area of a triangle, we get $A(x) = \frac{1}{2}xh = \frac{1}{2}x\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$, $x > 0$.

8. Let d represent the length of any side of a cube. Then the surface area is $S = 6d^2$, and the volume is $V = d^3 \Leftrightarrow d = \sqrt[3]{V}$. Substituting for d gives $S(V) = 6\left(\sqrt[3]{V}\right)^2 = 6V^{2/3}$, $V > 0$.

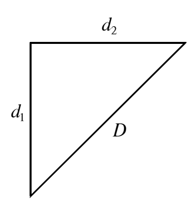
9. We solve for r in the formula for the area of a circle. This gives $A = \pi r^2 \Leftrightarrow r^2 = \dfrac{A}{\pi} \Rightarrow r = \sqrt{\dfrac{A}{\pi}}$, so the model is $r(A) = \sqrt{\dfrac{A}{\pi}}$, $A > 0$.

10. Let r be the radius of a circle. Then the area is $A = \pi r^2$, and the circumference is $C = 2\pi r \Leftrightarrow r = \dfrac{C}{2\pi}$. Substituting for r gives $A(C) = \pi\left(\dfrac{C}{2\pi}\right)^2 = \dfrac{C^2}{4\pi}$, $C > 0$.

11. Let h be the height of the box in feet. The volume of the box is $V = 60$. Then $x^2h = 60 \Leftrightarrow h = \dfrac{60}{x^2}$. The surface area, S, of the box is the sum of the area of the 4 sides and the area of the base and top. Thus $S = 4xh + 2x^2 = 4x\left(\dfrac{60}{x^2}\right) + 2x^2 = \dfrac{240}{x} + 2x^2$, so the model is $S(x) = \dfrac{240}{x} + 2x^2$, $x > 0$.

12. By similar triangles, $\dfrac{5}{L} = \dfrac{12}{L+d} \Leftrightarrow 5(L+d) = 12L \Leftrightarrow 5d = 7L \Leftrightarrow L = \dfrac{5d}{7}$. The model is $L(d) = \frac{5}{7}d$.

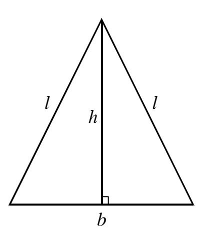
13.



Let d_1 be the distance traveled south by the first ship and d_2 be the distance traveled east by the second ship. The first ship travels south for t hours at 5 mi/h, so $d_1 = 15t$ and, similarly, $d_2 = 20t$. Since the ships are traveling at right angles to each other, we can apply the Pythagorean Theorem to get $D(t) = \sqrt{d_1^2 + d_2^2} = \sqrt{(15t)^2 + (20t)^2} = \sqrt{225t^2 + 400t^2} = 25t$.

14. Let n be one of the numbers. Then the other number is $60 - n$, so the product is given by the function $P(n) = n(60 - n) = 60n - n^2$.

15.



Let b be the length of the base, l be the length of the equal sides, and h be the height in centimeters. Since the perimeter is 8, $2l + b = 8 \Leftrightarrow 2l = 8 - b \Leftrightarrow l = \frac{1}{2}(8 - b)$. By the Pythagorean Theorem, $h^2 + \left(\frac{1}{2}b\right)^2 = l^2 \Leftrightarrow h = \sqrt{l^2 - \frac{1}{4}b^2}$. Therefore the area of the triangle is

$$A = \tfrac{1}{2} \cdot b \cdot h = \tfrac{1}{2} \cdot b\sqrt{l^2 - \tfrac{1}{4}b^2} = \frac{b}{2}\sqrt{\tfrac{1}{4}(8-b)^2 - \tfrac{1}{4}b^2}$$
$$= \frac{b}{4}\sqrt{64 - 16b + b^2 - b^2} = \frac{b}{4}\sqrt{64 - 16b} = \frac{b}{4} \cdot 4\sqrt{4 - b} = b\sqrt{4 - b}$$

so the model is $A(b) = b\sqrt{4 - b}$, $0 < b < 4$.

16. Let x be the length of the shorter leg of the right triangle. Then the length of the other triangle is $2x$. Since it is a right triangle, the length of the hypotenuse is $\sqrt{x^2+(2x)^2}=\sqrt{5x^2}=\sqrt{5}\,x$ (since $x \geq 0$). Thus the perimeter of the triangle is $P(x)=x+2x+\sqrt{5}\,x=\left(3+\sqrt{5}\right)x$.

17. Let w be the length of the rectangle. By the Pythagorean Theorem, $\left(\frac{1}{2}w\right)^2+h^2=10^2 \Leftrightarrow \dfrac{w^2}{4}+h^2=10^2 \Leftrightarrow$ $w^2=4\left(100-h^2\right) \Leftrightarrow w=2\sqrt{100-h^2}$ (since $w>0$). Therefore, the area of the rectangle is $A=wh=2h\sqrt{100-h^2}$, so the model is $A(h)=2h\sqrt{100-h^2}$, $0<h<10$.

18. Using the formula for the volume of a cone, $V=\frac{1}{3}\pi r^2 h$, we substitute $V=100$ and solve for h. Thus $100=\frac{1}{3}\pi r^2 h \Leftrightarrow$ $h(r)=\dfrac{300}{\pi r^2}$.

19. **(a)** We complete the table.

First number	Second number	Product
1	18	18
2	17	34
3	16	48
4	15	60
5	14	70
6	13	78
7	12	84
8	11	88
9	10	90
10	9	90
11	8	88

From the table we conclude that the numbers is still increasing, the numbers whose product is a maximum should both be 9.5.

(b) Let x be one number: then $19-x$ is the other number, and so the product, p, is

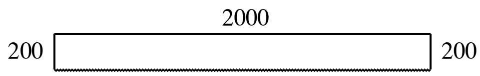
$p(x)=x(19-x)=19x-x^2$.

(c)
$$\begin{aligned} p(x) &= 19x-x^2=-\left(x^2-19x\right) \\ &= -\left[x^2-19x+\left(\tfrac{19}{2}\right)^2\right]+\left(\tfrac{19}{2}\right)^2 \\ &= -(x-9.5)^2+90.25 \end{aligned}$$

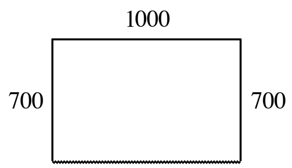
So the product is maximized when the numbers are both 9.5.

20. Let the positive numbers be x and y. Since their sum is 100, we have $x+y=100 \Leftrightarrow y=100-x$. We wish to minimize the sum of squares, which is $S=x^2+y^2=x^2+(100-x)^2$. So $S(x)=x^2+(100-x)^2=x^2+10{,}000-200x+x^2=2x^2-200x+10{,}000=2\left(x^2-100x\right)+10{,}000=2\left(x^2-100x+2500\right)+10{,}000-5000=2(x-50)^2+5000$. Thus the minimum sum of squares occurs when $x=50$. Then $y=100-50=50$. Therefore both numbers are 50.

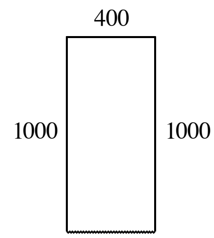
21. (a) Let x be the width of the field (in feet) and l be the length of the field (in feet). Since the farmer has 2400 ft of fencing we must have $2x + l = 2400$.



Area = 2000(200) = 400,000



Area = 1000(700) = 700,000



Area = 400(1000) = 400,000

Width	Length	Area
200	2000	400,000
300	1800	540,000
400	1600	640,000
500	1400	700,000
600	1200	720,000
700	1000	700,000
800	800	640,000

It appears that the field of largest area is about 600 ft × 1200 ft.

(b) Let x be the width of the field (in feet) and l be the length of the field (in feet). Since the farmer has 2400 ft of fencing we must have $2x + l = 2400 \Leftrightarrow l = 2400 - 2x$. The area of the fenced-in field is given by $A(x) = l \cdot x = (2400 - 2x)\,x = -2x^2 + 2400x = -2\left(x^2 - 1200x\right)$.

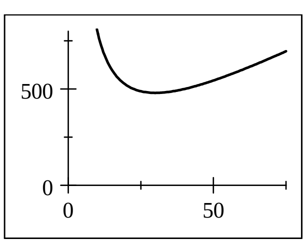
(c) The area is $A(x) = -2\left(x^2 - 1200x + 600^2\right) + 2\left(600^2\right) = -2(x - 600)^2 + 720{,}000$. So the maximum area occurs when $x = 600$ feet and $l = 2400 - 2(600) = 1200$ feet.

22. (a) Let w be the width of the rectangular area (in feet) and l be the length of the field (in feet). Since the farmer has 750 feet of fencing, we must have $5w + 2l = 750 \Leftrightarrow 2l = 750 - 5w \Leftrightarrow l = \frac{5}{2}(150 - w)$. Thus the total area of the four pens is $A(w) = l \cdot w = \frac{5}{2}w(150 - w) = -\frac{5}{2}\left(w^2 - 150w\right)$.

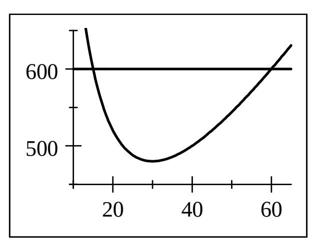
(b) We complete the square to get $A(w) = -\frac{5}{2}\left(w^2 - 150w\right) = -\frac{5}{2}\left(w^2 - 150w + 75^2\right) + \left(\frac{5}{2}\right) \cdot 75^2 = -\frac{5}{2}(w - 75)^2 + 14062.5$. Therefore, the largest possible total area of the four pens is 14,062.5 square feet.

23. (a) Let x be the length of the fence along the road. If the area is 1200, we have $1200 = x \cdot$ width, so the width of the garden is $\dfrac{1200}{x}$. Then the cost of the fence is given by the function $C(x) = 5(x) + 3\left[x + 2 \cdot \dfrac{1200}{x}\right] = 8x + \dfrac{7200}{x}$.

(b) We graph the function $y = C(x)$ in the viewing rectangle $[0, 75] \times [0, 800]$. From this we get the cost is minimized when $x = 30$ ft. Then the width is $\frac{1200}{30} = 40$ ft. So the length is 30 ft and the width is 40 ft.



(c) We graph the function $y = C(x)$ and $y = 600$ in the viewing rectangle $[10, 65] \times [450, 650]$. From this we get that the cost is at most \$600 when $15 \le x \le 60$. So the range of lengths he can fence along the road is 15 feet to 60 feet.



24. (a) Let x be the length of wire in cm that is bent into a square. So $10 - x$ is the length of wire in cm that is bent into the second square. The width of each square is $\dfrac{x}{4}$ and $\dfrac{10-x}{4}$, and the area of each square is $\left(\dfrac{x}{4}\right)^2 = \dfrac{x^2}{16}$ and $\left(\dfrac{10-x}{4}\right)^2 = \dfrac{100-20x+x^2}{16}$. Thus the sum of the areas is $A(x) = \dfrac{x^2}{16} + \dfrac{100-20x+x^2}{16} = \dfrac{100-20x+2x^2}{16} = \frac{1}{8}x^2 - \frac{5}{4}x + \frac{25}{4}$.

(b) We complete the square. $A(x) = \frac{1}{8}x^2 - \frac{5}{4}x + \frac{25}{4} = \frac{1}{8}\left(x^2 - 10x\right) + \frac{25}{4} = \frac{1}{8}\left(x^2 - 10x + 25\right) + \dfrac{25}{4} - \frac{25}{8} = \frac{1}{8}(x-5)^2 + \frac{25}{8}$ So the minimum area is $\frac{25}{8}$ cm^2 when each piece is 5 cm long.

25. (a) Let h be the height in feet of the straight portion of the window. The circumference of the semicircle is $C = \frac{1}{2}\pi x$. Since the perimeter of the window is 30 feet, we have $x + 2h + \frac{1}{2}\pi x = 30$. Solving for h, we get $2h = 30 - x - \frac{1}{2}\pi x \Leftrightarrow h = 15 - \frac{1}{2}x - \frac{1}{4}\pi x$. The area of the window is $A(x) = xh + \frac{1}{2}\pi\left(\frac{1}{2}x\right)^2 = x\left(15 - \frac{1}{2}x - \frac{1}{4}\pi x\right) + \frac{1}{8}\pi x^2 = 15x - \frac{1}{2}x^2 - \frac{1}{8}\pi x^2$.

(b) $A(x) = 15x - \frac{1}{8}(\pi+4)x^2 = -\frac{1}{8}(\pi+4)\left[x^2 - \dfrac{120}{\pi+4}x\right]$

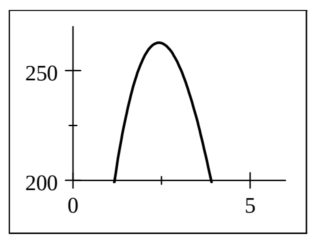
$$= -\tfrac{1}{8}(\pi+4)\left[x^2 - \frac{120}{\pi+4}x + \left(\frac{60}{\pi+4}\right)^2\right] + \frac{450}{\pi+4} = -\tfrac{1}{8}(\pi+4)\left(x - \frac{60}{\pi+4}\right)^2 + \frac{450}{\pi+4}$$

The area is maximized when $x = \dfrac{60}{\pi+4} \approx 8.40$, and hence $h \approx 15 - \frac{1}{2}(8.40) - \frac{1}{4}\pi(8.40) \approx 4.20$.

26. (a) The height of the box is x, the width of the box is $12 - 2x$, and the length of the box is $20 - 2x$. Therefore, the volume of the box is

$$\begin{aligned} V(x) &= x(12-2x)(20-2x) \\ &= 4x^3 - 64x^2 + 240x,\ 0 < x < 6 \end{aligned}$$

(b) We graph the function $y = V(x)$ in the viewing rectangle $[0, 6] \times [200, 270]$.

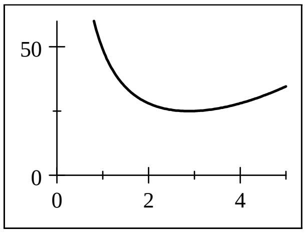


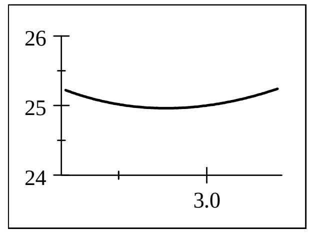
From the calculator we get that the volume of the box is greater than 200 in^3 for $1.174 \le x \le 3.898$ (accurate to 3 decimal places).

(c) From the graph, the volume of the box with the largest volume is 262.682 in^3 when $x \approx 2.427$.

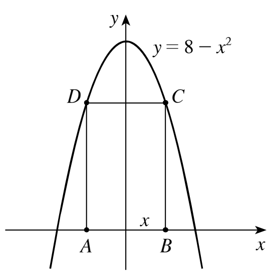
27. (a) Let x be the length of one side of the base and let h be the height of the box in feet. Since the volume of the box is $V = x^2h = 12$, we have $x^2h = 12 \Leftrightarrow h = \dfrac{12}{x^2}$. The surface area, A, of the box is sum of the area of the four sides and the area of the base. Thus the surface area of the box is given by the formula $A(x) = 4xh + x^2 = 4x\left(\dfrac{12}{x^2}\right) + x^2 = \dfrac{48}{x} + x^2$, $x > 0$.

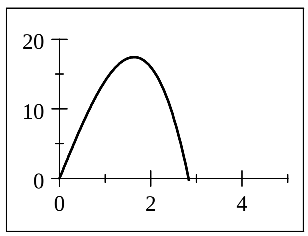
(b) The function $y = A(x)$ is shown in the first viewing rectangle below. In the second viewing rectangle, we isolate the minimum, and we see that the amount of material is minimized when x (the length and width) is 2.88 ft. Then the height is $h = \dfrac{12}{x^2} \approx 1.44$ ft.

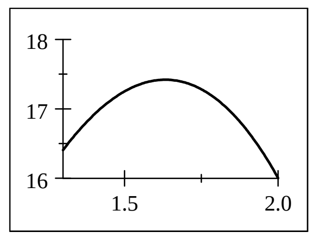




28. Let $A, B, C,$ and D be the vertices of a rectangle with base AB on the x-axis and its other two vertices C and D above the x-axis and lying on the parabola $y = 8 - x^2$. Let C have the coordinates (x, y), $x > 0$. By symmetry, the coordinates of D must be $(-x, y)$. So the width of the rectangle is $2x$, and the length is $y = 8 - x^2$. Thus the area of the rectangle is $A(x) = \text{length} \cdot \text{width} = 2x\left(8 - x^2\right) = 16x - 2x^3$. The graphs of $A(x)$ below show that the area is maximized when $x \approx 1.63$. Hence the maximum area occurs when the width is 3.26 and the length is 5.33.

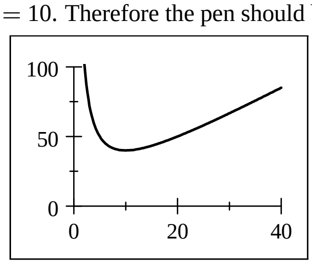


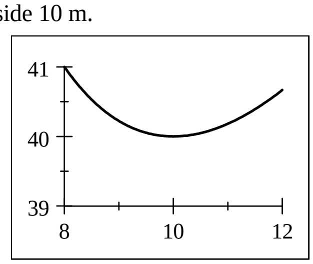




29. **(a)** Let w be the width of the pen and l be the length in meters. We use the area to establish a relationship between w and l. Since the area is 100 m^2, we have $l \cdot w = 100 \Leftrightarrow l = \dfrac{100}{w}$. So the amount of fencing used is $F = 2l + 2w = 2\left(\dfrac{100}{w}\right) + 2w = \dfrac{200 + 2w^2}{w}$.

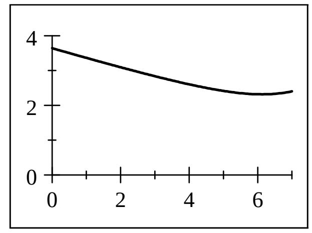
(b) Using a graphing device, we first graph F in the viewing rectangle $[0, 40]$ by $[0, 100]$, and locate the approximate location of the minimum value. In the second viewing rectangle, $[8, 12]$ by $[39, 41]$, we see that the minimum value of F occurs when $w = 10$. Therefore the pen should be a square with side 10 m.





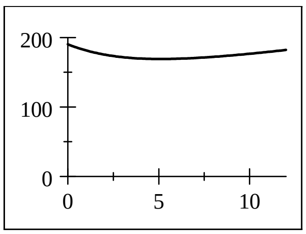
30. **(a)** Let t_1 represent the time, in hours, spent walking, and let t_2 represent the time spent rowing. Since the distance walked is x and the walking speed is 5 mi/h, the time spent walking is $t_1 = \frac{1}{5}x$. By the Pythagorean Theorem, the distance rowed is $d = \sqrt{2^2 + (7 - x)^2} = \sqrt{x^2 - 14x + 53}$, and so the time spent rowing is $t_2 = \frac{1}{2} \cdot \sqrt{x^2 - 14x + 53}$. Thus the total time is $T(x) = \frac{1}{2}\sqrt{x^2 - 14x + 53} + \frac{1}{5}x$.

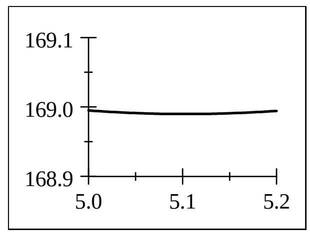
(b) We graph $y = T(x)$. Using the zoom function, we see that T is minimized when $x \approx 6.13$. He should land at a point 6.13 miles from point B.



31. **(a)** Let x be the distance from point B to C, in miles. Then the distance from A to C is $\sqrt{x^2 + 25}$, and the energy used in flying from A to C then C to D is $f(x) = 14\sqrt{x^2 + 25} + 10(12 - x)$.

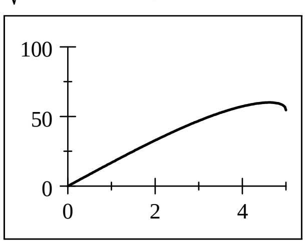
(b) By using a graphing device, the energy expenditure is minimized when the distance from B to C is about 5.1 miles.

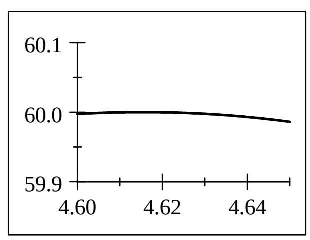




32. **(a)** Using the Pythagorean Theorem, we have that the height of the upper triangles is $\sqrt{25 - x^2}$ and the height of the lower triangles is $\sqrt{144 - x^2}$. So the area of the each of the upper triangles is $\frac{1}{2}x\sqrt{25 - x^2}$, and the area of the each of the lower triangles is $\frac{1}{2}x\sqrt{144 - x^2}$. Since there are two upper triangles and two lower triangles, we get that the total area is $A(x) = 2 \cdot \left[\frac{1}{2}x\sqrt{25 - x^2}\right] + 2 \cdot \left[\frac{1}{2}x\sqrt{144 - x^2}\right] = x\left(\sqrt{25 - x^2} + \sqrt{144 - x^2}\right)$.

(b) The function $y = A(x) = x\left(\sqrt{25 - x^2} + \sqrt{144 - x^2}\right)$ is shown in the first viewing rectangle below. In the second viewing rectangle, we isolate the maximum, and we see that the area of the kite is maximized when $x \approx 4.615$. So the length of the horizontal crosspiece must be $2 \cdot 4.615 = 9.23$. The length of the vertical crosspiece is $\sqrt{5^2 - (4.615)^2} + \sqrt{12^2 - (4.615)^2} \approx 13.00$.





2 FUNCTIONS

2.1 FUNCTIONS

▼ Suggested Time and Emphasis

$\frac{1}{2}$–1 class. Essential material.

▼ Points to Stress

1. The idea of function, viewed as the dependence of one quantity on a different quantity.

2. The notation associated with piecewise-defined functions.

3. Domains and ranges from an algebraic perspective.

4. Four different representations of functions (verbally, algebraically, visually, and numerically).

▼ Sample Questions

- **Text Question:** What is a function?
  Answer: Answers will vary. The hope is that the students will, in their own words, arrive at the idea that a function assigns each element of one set to exactly one element of another set (or the same set).

- **Drill Question:** Let $f(x) = x + \sqrt{x}$. Find $f(0)$ and $f(4)$.
  Answer: $f(0) = 0$, $f(4) = 6$

▼ In-Class Materials

- Discuss the ties between the idea of a function and a calculator key. Keys such as $\sin$, $\cos$, $\tan$, and $\sqrt{}$ represent functions. It is easy to compute and graph functions on a calculator. Contrast this with equations such as $y^3 - x^3 = 2xy$, which have graphs but are not easy to work with, even with a calculator. (Even computer algebra systems have a tough time with some general relations.) Point out that calculators often give approximations to function values—applying the square root function key to the number 2 gives 1.4142136 which is close to, but not equal to, $\sqrt{2}$.

- Most math courses through calculus emphasize functions where both the domain and range sets are numerical. One could give a more abstract definition of function, where D and R can be any set. For example, there is a function mapping each student in the class to his or her birthplace. A nice thing about this point of view is that it can be pointed out that the map from each student to his or her telephone number may *not* be a function, because a student may have more than one telephone number, or none at all.

- Function notation can trip students up. Start with a function such as $f(x) = x^2 - x$ and have your students find $f(0)$, $f(1)$, $f\left(\sqrt{3}\right)$, and $f(-1)$. Then have them find $f(\pi)$, $f(y)$, and (of course) $f(x+h)$. Some students will invariably, some day, assume that $f(a+b) = f(a) + f(b)$ for all functions, but this can be minimized if plenty of examples such as $f(2+3)$ are done at the outset.

- Discuss the usual things to look for when trying to find the domain of a function: zero denominators and negative even roots.

  Then discuss the domain and range of $f(x) = \begin{cases} x^2 & \text{if } x \text{ is an integer} \\ 0 & \text{if } x \text{ is not an integer} \end{cases}$

  If the class seems interested, perhaps let them think about $f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$
- Let $f(x) = \dfrac{x(x-2)}{x-2}$ and $g(x) = x$. Ask students if the functions are the same function. If they say "yes", ask them to compare the domains, or to compute $g(2)$ and $f(2)$. If they say "no", ask them to find a value such that $f(x) \neq g(x)$. [This activity assumes that students know the equation of a circle with radius r. If they do not, this may be a good opportunity to introduce the concept.]

▼ Examples

- Real-world piecewise functions:
  1. The cost of mailing a parcel that weighs w ounces (see Figure 1 in the text)
  2. The cost of making x photocopies (given that there is usually a bulk discount)
  3. The cost of printing x pages from an inkjet printer (at some point the cartridges must be replaced)
- A function with a nontrivial domain: $\sqrt{\dfrac{x^2-5x+6}{x^2-2x+1}}$ has domain $(-\infty, 1) \cup (1, 2] \cup [3, \infty)$.

▼ Group Work 1: Finding a Formula

Make sure that students know the equation of a circle with radius r, and that they remember the notation for piecewise-defined functions. Divide the class into groups of four. In each group, have half of them work on each problem first, and then have them check each other's work. If students find these problems difficult, have them work together on each problem.

Answers: 1. $f(x) = \begin{cases} -x-2 & \text{if } x \leq -2 \\ x+2 & \text{if } -2 < x \leq 0 \\ 2 & \text{if } x > 0 \end{cases}$ **2.** $g(x) = \begin{cases} x+4 & \text{if } x \leq -2 \\ 2 & \text{if } -2 < x \leq 0 \\ \sqrt{4-x^2} & \text{if } 0 < x \leq 2 \\ x-2 & \text{if } x > 2 \end{cases}$

▼ Group Work 2: Rounding the Bases

On the board, review how to compute the percentage error when estimating π by $\frac{22}{7}$. (Answer: 0.04%) Have them work on the problem in groups. If a group finishes early, have them look at $h(7)$ and $h(10)$ to see how fast the error grows. Students have not seen exponential functions before, but Problem 3 is a good foreshadowing of Section 4.1.

Answers: 1. 17.811434627, 17, 4.56% **2.** 220.08649875, 201, 8.67% **3.** 45.4314240633, 32, 29.56%

▼ Homework Problems

Core Exercises: 3, 15, 20, 33, 37, 50, 58, 68, 80, 89, 94

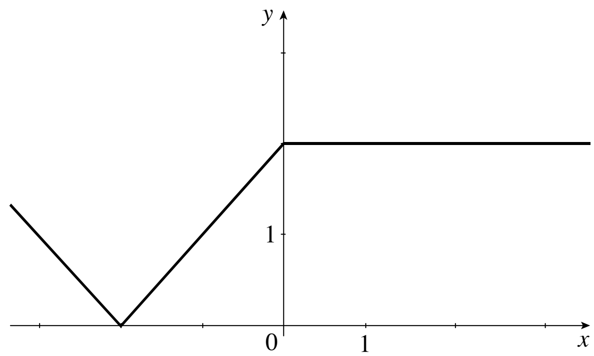
Sample Assignment: 1, 3, 8, 13, 15, 17, 20, 25, 29, 32, 33, 35, 37, 42, 43, 50, 53, 57, 58, 64, 68, 73, 76, 80, 83, 86, 89, 94

GROUP WORK 1, SECTION 2.1

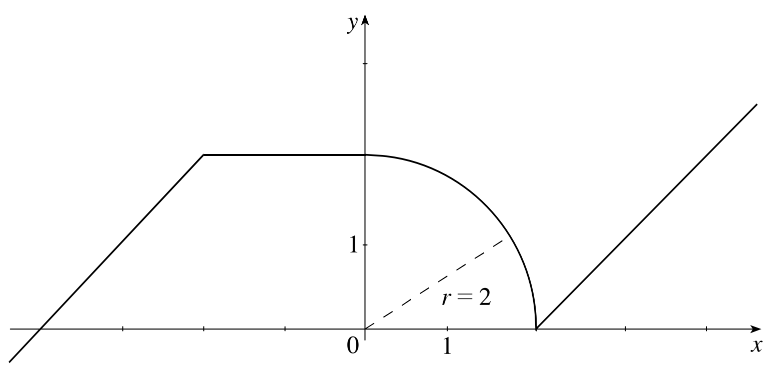
Finding a Formula

Find formulas for the following functions:

1.



2.



GROUP WORK 2, SECTION 2.1

Rounding the Bases

1. For computational efficiency and speed, we often round off constants in equations. For example, consider the linear function

$$f(x) = 3.137619523x + 2.123337012$$

In theory, it is easy and quick to find $f(1)$, $f(2)$, $f(3)$, $f(4)$, and $f(5)$. In practice, most people doing this computation would probably substitute

$$f(x) = 3x + 2$$

unless a very accurate answer is called for. For example, compute $f(5)$ both ways to see the difference.

The actual value of $f(5)$: __________

The "rounding" estimate: __________

The percentage error: __________

2. Now consider

$$g(x) = 1.12755319x^3 + 3.125694x^2 + 1$$

Again, one is tempted to substitute $g(x) = x^3 + 3x^2 + 1$.

The actual value of $g(5)$: __________

The "rounding" estimate: __________

The percentage error: __________

3. It turns out to be dangerous to similarly round off exponential functions, due to the nature of their growth. For example, let's look at the function

$$h(x) = (2.145217198123)^x$$

One may be tempted to substitute $h(x) = 2^x$ for this one. Once again, look at the difference between these two functions.

The actual value of $h(5)$: __________

The "rounding" estimate: __________

The percentage error: __________

2.2 GRAPHS OF FUNCTIONS

▼ Suggested Time and Emphasis

1 class. Essential material.

▼ Points to Stress

1. The Vertical Line Test.

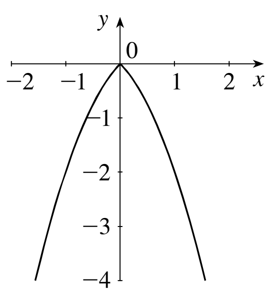
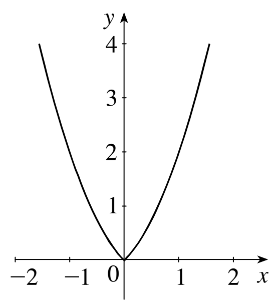
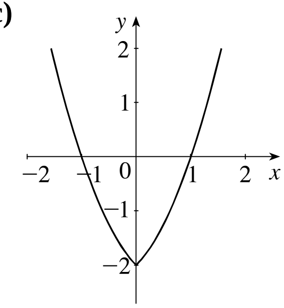
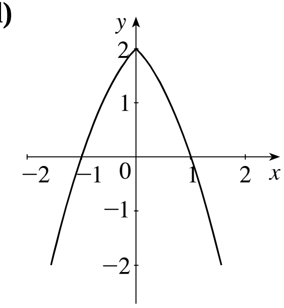
2. Graphs of piecewise-defined functions.

3. The greatest integer function.

▼ Sample Questions

- **Text Question:** Your text discusses the greatest integer function $[\![x]\!]$. Compute $[\![2.6]\!]$, $[\![2]\!]$, $[\![-2.6]\!]$, and $[\![-2]\!]$.
  Answer: $[\![2.6]\!] = 2$, $[\![2]\!] = 2$, $[\![-2.6]\!] = -3$, $[\![-2]\!] = -2$

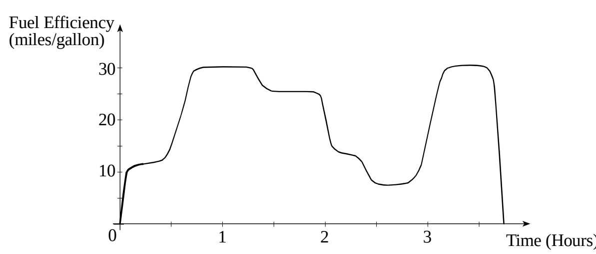
- **Drill Question:** Let $f(x) = x^2 + |x|$. Which of the following is the graph of f? How do you know?

  (a)
  
  (b)
  
  (c)
  
  (d)
  

  Answer: (b) is the graph of f, because $f(x) \geq 0$ for all x.

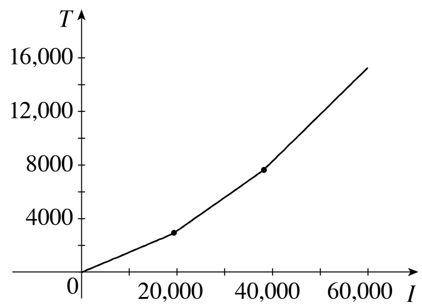
▼ In-Class Materials

- Draw a graph of fuel efficiency versus time on a trip, such as the one below. Lead a discussion of what could have happened on the trip.



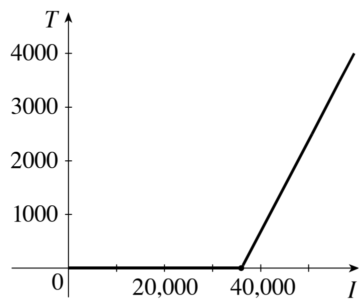
- In 1984, United States President Ronald Reagan proposed a plan to change the personal income tax system. According to his plan, the income tax would be 15% on the first \$19,300 earned, 25% on the next \$18,800, and 35% on all income above and beyond that. Describe this situation to the class, and have them graph tax owed T as a function of income I for incomes ranging from \$0 to \$80,000. Then have them try to come up with equations describing this situation.

Answer:



- In the year 2000, Presidential candidate Steve Forbes proposed a “flat tax” model: The first $\$36,000$ of a taxpayer’s annual income would not be taxed at all, and the rest would be taxed at a rate of 17%. Have your students do the same analysis they did of Reagan’s 1984 plan, and compare the models. As an extension, consider having them look at a current tax table and draw similar graphs.

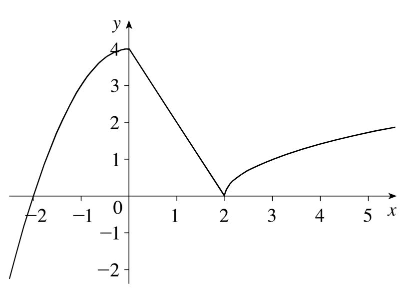
Answer:



- Discuss the shape, symmetries, and general “flatness” near 0 of the power functions x^n for various values of n. Similarly discuss $\sqrt[n]{x}$ for n even and n odd. A blackline master is provided at the end of this section, before the group work handouts.

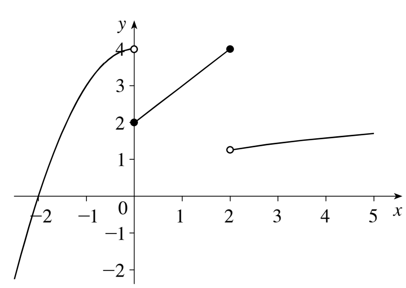
▼ Examples

- A continuous piecewise-defined function



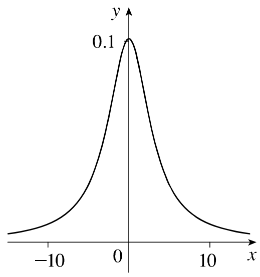
$$f(x) = \begin{cases} 4 - x^2 & \text{if } x < 0 \\ 4 - 2x & \text{if } 0 \le x \le 2 \\ \sqrt{x - 2} & \text{if } x > 2 \end{cases}$$

- A discontinuous piecewise-defined function

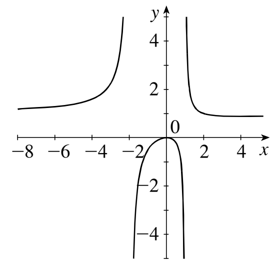


$$f(x) = \begin{cases} 4 - x^2 & \text{if } x < 0 \\ x + 2 & \text{if } 0 \le x \le 2 \\ \sqrt[3]{x} & \text{if } x > 2 \end{cases}$$

• Classic rational functions with interesting graphs



$$h(x) = \frac{1}{x^2 + \pi^2}$$



$$i(x) = \frac{x^2}{x^2 + x - 2}$$

▼ Group Work 1: Every Picture Tells a Story

Put students in groups of four, and have them work on the exercise. If there are questions, encourage them to ask each other before asking you. After going through the correct matching with them, have each group tell their story to the class and see if it fits the remaining graph.

Answers:

1. (b) **2.** (a) **3.** (c)

4. The roast was cooked in the morning and put in the refrigerator in the afternoon.

▼ Group Work 2: Functions in the Classroom

Before starting this one, review the definition of “function”. Some of the problems can be answered only by polling the class after they are finished working. Don’t forget to take leap years into account for the eighth problem. For an advanced class, anticipate Section 2.8 by quickly defining “one-to-one” and “bijection”, then determining which of the functions have these properties.

Answers:

Chairs: Function, one-to-one, bijection (if all chairs are occupied)

Eye color: Function, not one-to-one

Mom & Dad's birthplace: Not a function; mom and dad could have been born in different places

Molecules: Function, one-to-one (with nearly 100% probability); inverse assigns a number of molecules to the appropriate student.

Spleens: Function, one-to-one, bijection. Inverse assigns each spleen to its owner.

Pencils: Not a function; some people may have more than one or (horrors!) none.

Social Security Number: Function, one-to-one; inverse assigns each number to its owner.

February birthday: Not a function; not defined for someone born on February 29.

Birthday: Function, perhaps one-to-one.

Cars: Not a function; some have none, some have more than one.

Cash: Function, perhaps one-to-one.

Middle names: Not a function; some have none, some have more than one.

Identity: Function, one-to-one, bijection. Inverse is the same as the function.

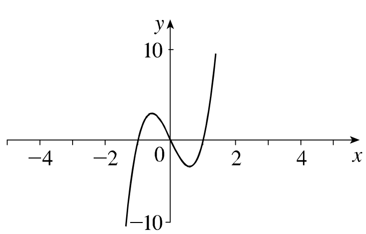
Instructor: Function, not one-to-one.

▼ Group Work 3: Rational Functions

Remind students of the definition of a rational function as a quotient of polynomials. Students should be able to do this activity by plotting points and looking at domains and ranges.

Answers:

1.

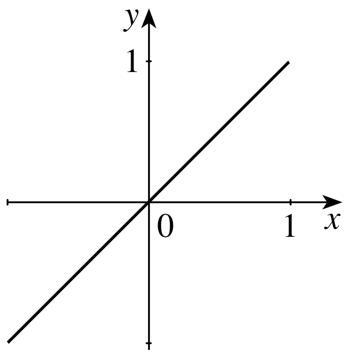


2. (d) **3.** (b) **4.** (e) **5.** (c) **6.** (a)

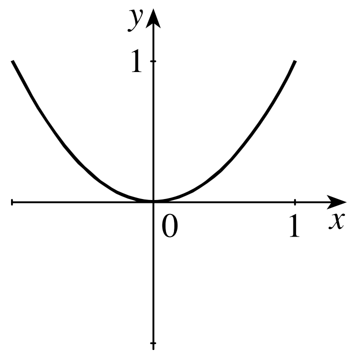
▼ Homework Problems

Core Exercises: 3, 7, 22, 34, 42, 47, 52, 60, 70, 78, 83

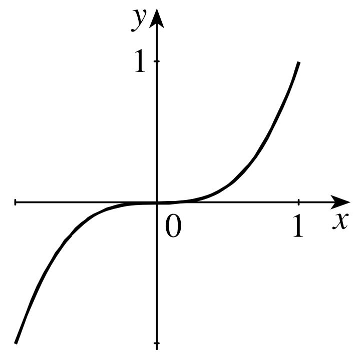
Sample Assignment: 3, 4, 7, 8, 16, 22, 25, 31, 34, 38, 42, 45, 47, 50, 52, 56, 60, 64, 66, 70, 73, 76, 78, 81, 83, 87



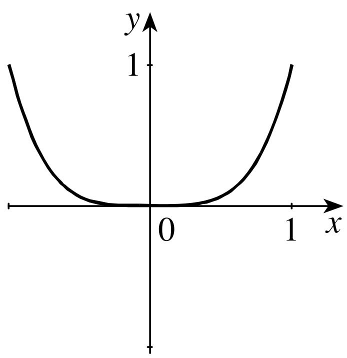
x



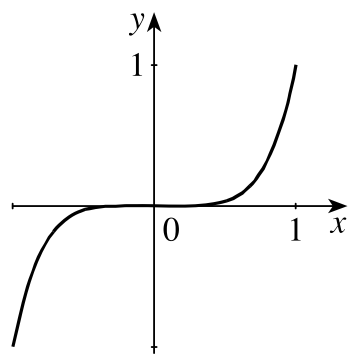
x^2



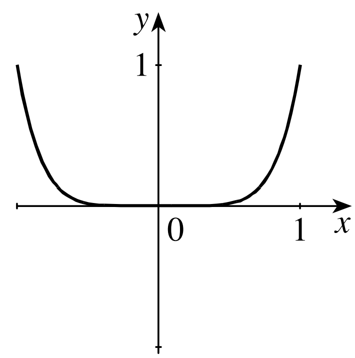
x^3



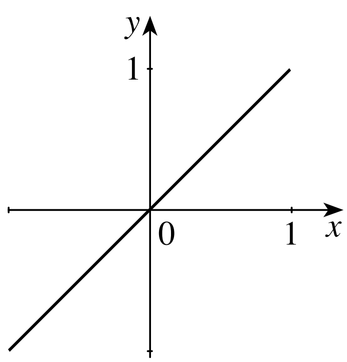
x^4



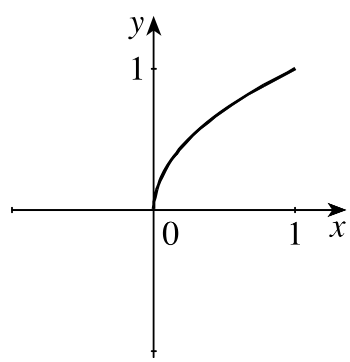
x^5



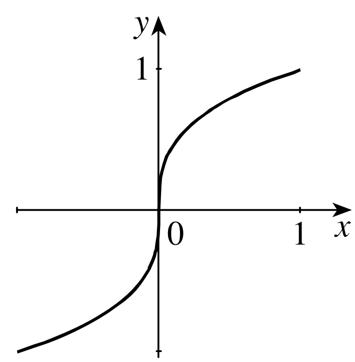
x^6



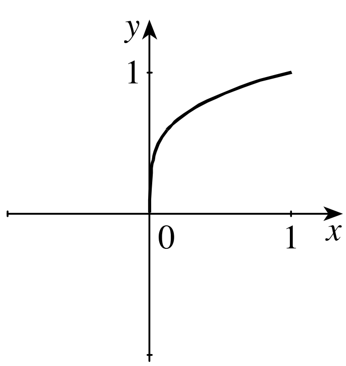
x



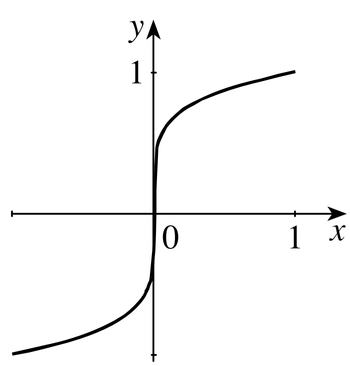
$\sqrt{x}$



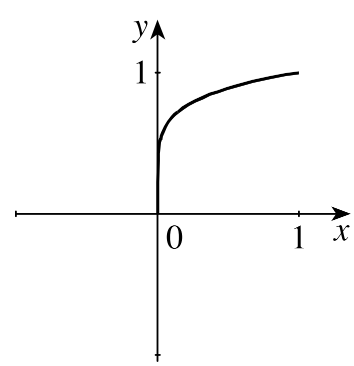
$\sqrt[3]{x}$



$\sqrt[4]{x}$



$\sqrt[5]{x}$

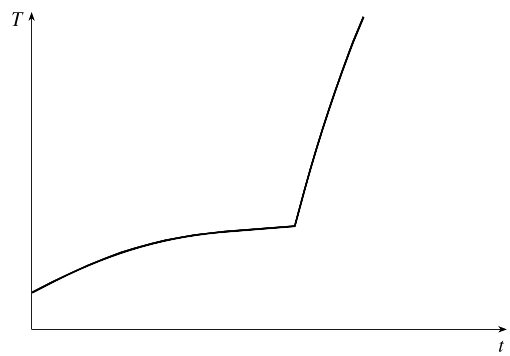


$\sqrt[6]{x}$

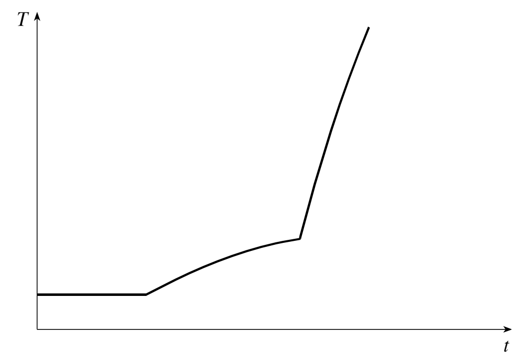
GROUP WORK 1, SECTION 2.2

Every Picture Tells a Story

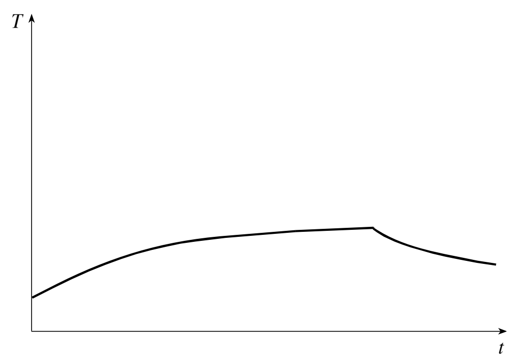
One of the skills you will be learning in this course is the ability to take a description of a real-world occurrence, and translate it into mathematics. Conversely, given a mathematical description of a phenomenon, you will learn how to describe what is happening in plain language. Here follow four graphs of temperature versus time and three stories. Match the stories with the graphs. When finished, write a similar story that would correspond to the final graph.



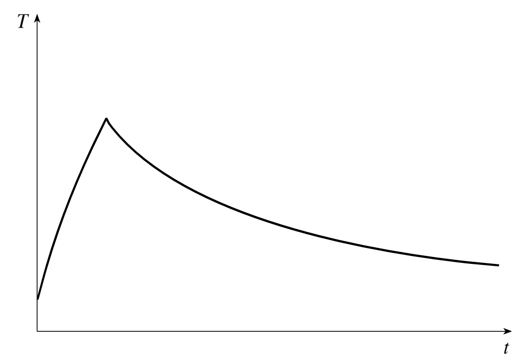
Graph 1



Graph 2



Graph 3



Graph 4

(a) I took my roast out of the freezer at noon, and left it on the counter to thaw. Then I cooked it in the oven when I got home.

(b) I took my roast out of the freezer this morning, and left it on the counter to thaw. Then I cooked it in the oven when I got home.

(c) I took my roast out of the freezer this morning, and left it on the counter to thaw. I forgot about it, and went out for Chinese food on my way home from work. I put it in the refrigerator when I finally got home.

GROUP WORK 2, SECTION 2.2

Functions in the Classroom

Which of the following relations are functions?

Domain	Function Values	Function
All the people in your classroom	Chairs	f (person) $=$ his or her chair
All the people in your classroom	The set $\{$blue, brown, green, hazel$\}$	f (person) $=$ his or her eye color
All the people in your classroom	Cities	f (person) $=$ birthplace of their mom and dad
All the people in your classroom	$\mathbb{R}$, the real numbers	f (person) $=$ number of molecules in their body
All the people in your classroom	Spleens	f (person) $=$ his or her own spleen
All the people in your classroom	Pencils	f (person) $=$ his or her pencil
All the people in the United States	Integers from 0–999999999	f (person) $=$ his or her Social Security number
All the living people born in February	Days in February, 2019	f (person) $=$ his or her birthday in February 2019
All the people in your classroom	Days of the year	f (person) $=$ his or her birthday
All the people in your classroom	Cars	f (person) $=$ his or her car
All the people in your classroom	$\mathbb{R}$, the real numbers	f (person) $=$ how much cash he or she has
All the people in your college	Names	f (person) $=$ his or her middle name
All the people in your classroom	People	f (person) $=$ himself or herself
All the people in your classroom	People	f (person) $=$ his or her mathematics instructor

GROUP WORK 3, SECTION 2.2

Rational Functions

The functions below are sad and lonely because they have lost their graphs! Help them out by matching each function with its graph. One function's graph is not pictured here; when you are done matching, go ahead and sketch that function's graph.

1. $\dfrac{x^3 - x}{0.125}$

2. $\dfrac{x^2 - 1}{x^2 - 4}$

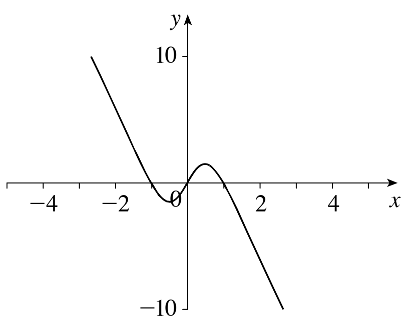
3. $\dfrac{x^2 - 4}{x^2 - 1}$

4. $\dfrac{x^2 - 1}{x + 0.5}$

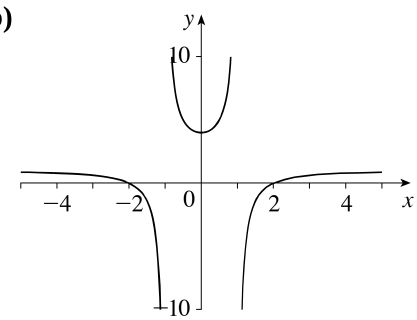
5. $\dfrac{5x\left(x^2 - 1\right)}{x^2 + 1}$

6. $\dfrac{5x\left(x^2 - 1\right)}{-x^2 - 1}$

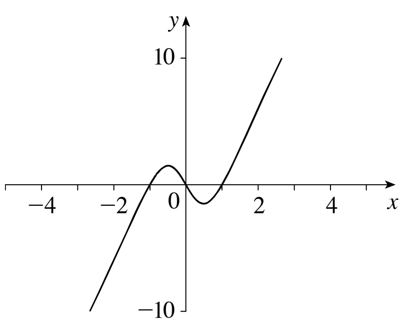
(a)



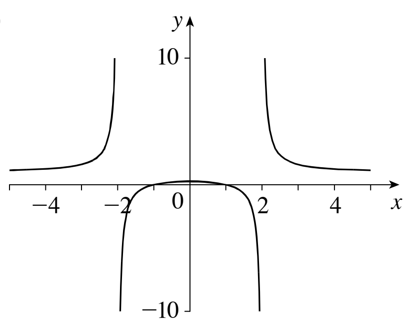
(b)



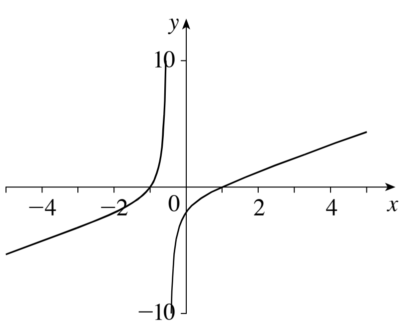
(c)

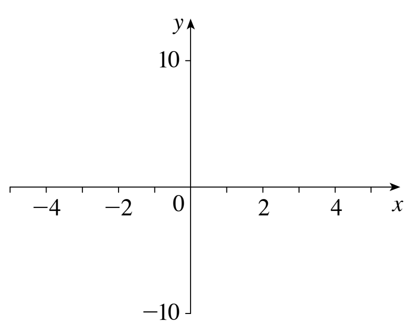


(d)



(e)





2.3 GETTING INFORMATION FROM THE GRAPH OF A FUNCTION

▼ Suggested Time and Emphasis

$\frac{1}{2}$–1 class. Essential material.

▼ Points to Stress

1. Gaining information about a function from its graph, including finding function values, domain and range.

2. Algebraic and geometric definitions of increasing and decreasing.

3. Finding local extrema of a function from its graph.

▼ Sample Questions

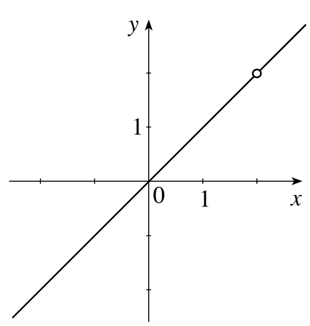
- **Text Question:** Draw a graph of a function with domain $[-10, 10]$ and range $[-2, 2]$. There should be at least one interval where the graph is increasing and at least one interval where the graph is decreasing.

  Answer: Answers will vary.

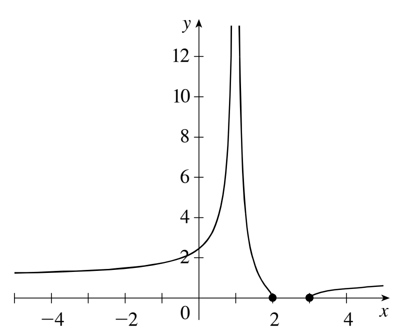
- **Drill Question:** If $f(x) = -x^2 + 9x + 2$, find the extreme value of f. Is it a maximum or a minimum?
  Answer: $f\left(\frac{9}{2}\right) = \frac{89}{4}$ is a maximum.

▼ In-Class Materials

- Explore domain and range with some graphs that have holes, such as the graphs of some of the functions in the previous section.



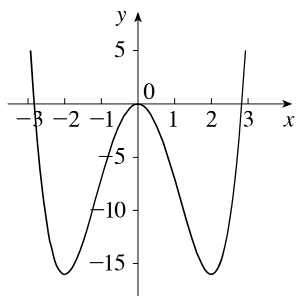
$$f(x) = \frac{x(x-2)}{x-2}$$



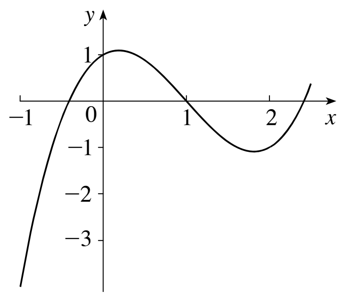
$$g(x) = \sqrt{\frac{x^2-5x+6}{x^2-2x+1}}$$

- Draw a graph of electrical power consumption in the classroom versus time on a typical weekday, pointing out important features throughout, and using the vocabulary of this section as much as possible.

- Notice that it is fairly easy to tell where some functions are increasing and decreasing by looking at their graphs. For example, the graph of $f(x) = x^4 - 8x^2$ makes things clear. Note that in this case, the intervals are not immediately apparent from looking at the formula. However, for many functions such as $g(x) = x^3 - 3x^2 + x + 1$, it is difficult to find the exact intervals where the function is increasing/decreasing. In this example, the endpoints of the intervals will occur at precisely $x = 1 \pm \frac{1}{3}\sqrt{6}$.



$f(x) = x^4 - 8x^2$

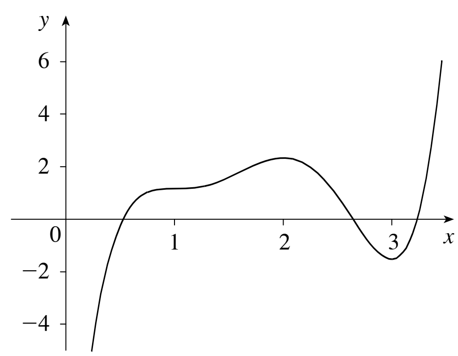


$g(x) = x^3 - 3x^2 + x + 1$

- Examine $f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$ pointing out that it is neither increasing nor decreasing near $x = 0$. Stress that when dealing with new sorts of functions, it becomes important to know the precise mathematical definitions of such terms.

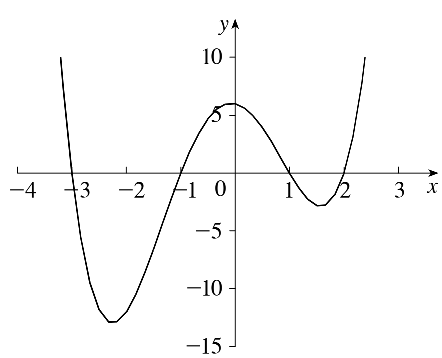
▼ Examples

- A function with two integer turning points and a flat spot:



$\frac{1}{6}\left(12x^5 - 105x^4 + 340x^3 - 510x^2 + 360x - 90\right)$

- A function with several local extrema: $f(x) = x^4 + x^3 - 7x^2 - x + 6 = (x+3)(x+1)(x-1)(x-2)$



The extrema occur at $x \approx -2.254$, $x \approx -0.0705$, and $x \approx 1.5742$.

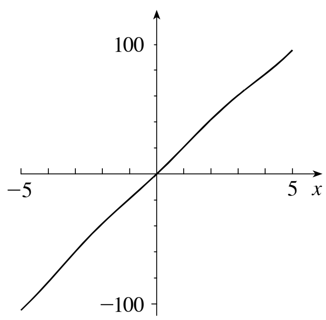
▼ Group Work 1: Calculator Exploration

This gives students a chance to graph things on their calculator and make conclusions. It will also serve as a warning that relying on calculator graphs without understanding the functions can lead one astray.

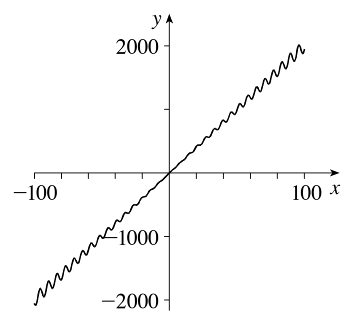
Notice that in calculus, when we say a function is increasing, we are saying it is increasing at every point on its domain. In this context, we are talking about increasing over an interval, which is slightly different. The curve $-1/x$, for example, is increasing at every point in its domain. Can we say it is decreasing over the interval $[-10, 1]$? No, because it is not defined in that interval. So the curve $-1/x$ is increasing over every interval for which it is defined.

Answers:

1. (c) **2.** (a) **3.** (a) (assuming positive intervals) **4.** (c) **5.** (c) **6.** (a) **7.** (b) **8.** (a) **9.** (c)
10. (c)



$f(x) = 20x + x \sin x$



$f(x) = 20x + x \sin x$

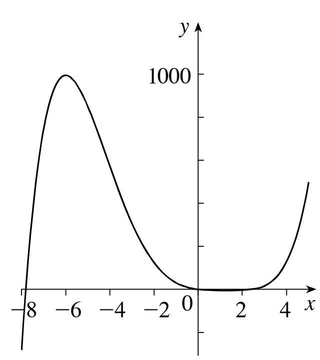
▼ Group Work 2: The Little Dip

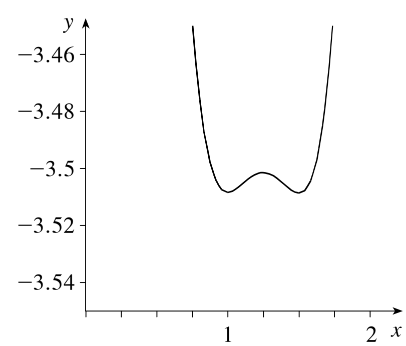
In this exercise students analyze a function with some subtle local extrema. After they have tried, reveal that there are two local maxima and two local minima.

After students have found the extrema, point out that if they take calculus, they will learn a relatively simple way to find the exact coordinates of the extrema.

Answers:

1.





2. There are local maxima at $x = -6$ and $x = \frac{5}{4}$, and local minima at $x = 1$ and $x = \frac{3}{2}$.

▼ Homework Problems

Core Exercises: 3, 8, 13, 22, 32, 37, 45, 48, 52, 59, 66, 67

Sample Assignment: 1, 3, 7, 8, 9, 13, 15, 20, 22, 32, 34, 36, 37, 41, 44, 45, 48, 52, 56, 57, 59, 62, 65, 66, 67, 69

GROUP WORK 1, SECTION 2.3

Calculator Exploration

Graph the following curves on your calculator. For each curve specify which of the following applies.

(a) The graph of f is increasing over every interval (assuming the curve is defined everywhere in that interval).

(b) The graph of f is decreasing over every interval (assuming the curve is defined everywhere in that interval).

(c) The graph of f is increasing over some intervals and decreasing over others.

1. $f(x) = x^2$

2. $f(x) = x^3$

3. $f(x) = \sqrt{x}$

4. $f(x) = \sin x$

5. $f(x) = \cos x$

6. $f(x) = \tan x$

7. $f(x) = e^{-x}$

8. $f(x) = \ln x$

9. $f(x) = 5x^4 - 1.01^x$

10. $f(x) = 20x + x \sin x$

GROUP WORK 2, SECTION 2.3

The Little Dip

Consider $f(x) = \frac{1}{5}x^5 + \frac{9}{16}x^4 - \frac{143}{24}x^3 + \frac{207}{16}x^2 - \frac{45}{4}x$.

1. Draw a graph of f.

2. Estimate the x-values of all local extrema. Make sure your estimates are accurate to three decimal places.

2.4 AVERAGE RATE OF CHANGE OF A FUNCTION

▼ Suggested Time and Emphasis

$\frac{1}{2}$–1 class. Essential material.

▼ Points to Stress

1. Average rate of change.

▼ Sample Questions

- **Text Question:**

  Let $f(t) = 3t + 2$.

  (a) What is the average rate of change of f from $t = 1$ to $t = 3$?

  (b) What is the average rate of change of f from $t = 1$ to $t = \pi$?

  Answer: (a) 3 **(b)** 3

- **Drill Question:** If $f(t) = \left|t^2 - |3t|\right|$, what is the average rate of change between $t = -3$ and $t = -1$?

  Answer: 1

▼ In-Class Materials

- Students should see the geometry of the average rate of change — that the average rate of change from $x = a$ to $x = b$ is the slope of the line from $(a, f(a))$ to $(b, f(b))$. Armed with this knowledge, students now have a way of estimating average rate of change: graph the function (making sure that the x- and y-scales are the same), plot the relevant points, and then estimate the slope of the line between them.
- It is possible, at this point, to foreshadow calculus nicely. Take a simple function such as $l(t) = t^2$ and look at the average rate of change from $t = 1$ to $t = 2$. Then look at the average rate of change from $t = 1$ to $t = \frac{3}{2}$. If students work in parallel, it won't take them long to fill in the following table:

From	To	Average Rate of Change
$t = 1$	2	3
$t = 1$	1.5	2.5
$t = 1$	1.25	2.25
$t = 1$	1.1	2.1
$t = 1$	1.01	2.01
$t = 1$	1.001	2.001

  Note that these numbers seem to be approaching 2. This idea is pursued further in the group work.
- Assume that a car drove for two hours and traversed 120 miles. The average rate of change is clearly 60 miles per hour. Ask the students if it was possible for the car to have gone over 60 mph at some point in the interval, and explain how. Ask the students if it was possible for the car to have stayed under 60 mph the whole time. Ask the students if it was possible for the car never to have gone exactly 60 mph. Their intuition will probably say that the car had to have had traveled exactly 60 mph at one point, but it will be hard for them to justify. The truth of this statement is an example of the Mean Value Theorem from calculus.

▼ Examples

If $f(x) = x^3 - x$, the average rate of change from $x = 1$ to $x = 4$ is

$$\frac{f(4) - f(1)}{4 - 1} = \frac{\left(4^3 - 4\right) - \left(1^3 - 1\right)}{3} = \frac{(64 - 4) - (1 - 1)}{3} = \frac{60}{3} = 20$$

▼ Group Work: Small Intervals

If you have the time, and really wish to foreshadow calculus, have the students find the limit starting with $x = 1$ and then again with $x = 3$. Then see if they can find the pattern, and discover that the average value is going to approach $3a^2$ if we start at a.

Students won't remember every detail of this problem in a year, obviously. So when you close, try to convey the main idea that as we narrow the interval, the average values approach a single number, and that everything blows up if we make the interval consist of a single point. You may want to mention that exploring this phenomenon is a major part of the first semester of calculus.

Answers:

1. 19 **2.** 15.25 **3.** 12.61 **4.** 12.0601 **5.** 12.006001 **6.** 11.9401 **7.** 12 **8.** You get $\frac{0}{0}$, which is undefined.

▼ Homework Problems

Core Exercises: 3, 10, 16, 24, 31, 36, 40

Sample Assignment: 3, 4, 8, 10, 11, 15, 16, 22, 24, 26, 29, 31, 33, 36, 38, 40

GROUP WORK, SECTION 2.4

Small Intervals

Let us consider the curve $y = x^3$. Assume I am interested only in what is happening near $x = 2$. It is clear that the function is getting larger there, but my question is, how quickly is it increasing? One way to find out is to compute average rates of change.

1. Find the average rate of change between $x = 2$ and $x = 3$.

2. The number 2.5 is even closer to the number 2. Remember, I only really care about what is happening very close to $x = 2$. So compute the average rate of change between $x = 2$ and $x = 2.5$.

3. We can get closer still. Compute the average rate of change between $x = 2$ and $x = 2.1$.

4. Can we get closer? Sure! Compute the average rate of change between $x = 2$ and $x = 2.01$.

5. Compute the average rate of change between $x = 2$ and $x = 2.001$.

6. We can also approach 2 from the other side. Compute the average rate of change between $x = 2$ and $x = 1.99$.

7. Your answers should be approaching some particular number as we get closer and closer to 2. What is that number?

8. Hey, the closest number to 2 is 2 itself, right? So go ahead and compute the average rate of change between $x = 2$ and $x = 2$. What happens?

2.5 LINEAR FUNCTIONS AND MODELS

▼ Suggested Time and Emphasis

$\frac{1}{2}$–1 class. Essential material.

▼ Points to Stress

1. Definition of a linear function.

2. The relationship between slope and rate of change, including units.

3. Creating linear models in an applied context.

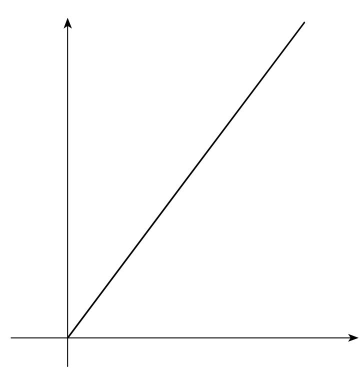
▼ Sample Questions

- **Text Question:** What is the difference between "slope" and "rate of change?"
  Answer: The text describes this as a difference in points of view. Any response that gets at the idea of a slope being a property of a graph and a rate of change being a property of a physical situation should be given full credit.

- **Drill Question:** Which of the following, if any, are linear functions?
  (a) $f(x) = 3x + \sqrt{2}$ **(b)** $f(x) = 3x - 2 + x$ **(c)** $f(x) = \pi x$ **(d)** $f(x) = \sqrt{3x} - 4$ **(e)** $f(x) = \dfrac{\sqrt{5}}{2} - \dfrac{x\sqrt{3}}{4}$
  Answer: (a), (b), (c), (e)

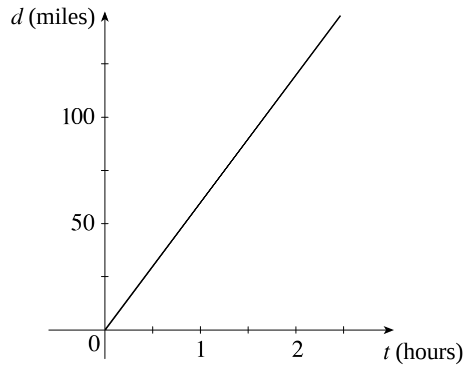
▼ In-Class Materials

- It is important to guide students from the perspective of Section 1.10 (Lines) to the more applied perspective of this section. Start by asking the students to graph $f(x) = 60x$. You should see graphs that look like the one below.



  As you know, students tend not to label their axes, and if the x- and y-axis scales are equal, the slope of the line is nowhere near 60. Ask them for the slope and y-intercept of their line.

Now show them this graph of the distance a car travels as a function of time over a two hour trip.

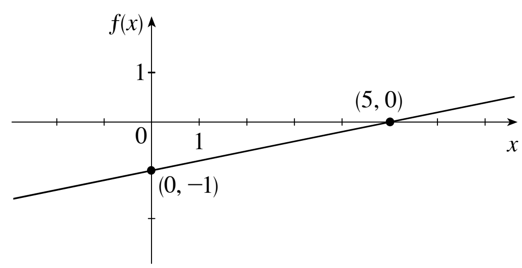


Point out that this graph looks the same as theirs, but now the slope has both units and meaning—the car travelled at 60 miles per hour. The fact that the graph is linear means the car traveled at a constant speed.

- One nice aspect of linear functions is interpolation. For example, assume that between 2012 and 2016, a tree grew at a constant rate. Assume it was 10 feet tall in 2012 and 20 feet tall in 2016. Now we can use this model to figure out how tall it was in 2015: 15 feet. We also can extrapolate to predict its height in 2040, but we should always be careful when we extrapolate. Just because the growth rate was constant between 2012 and 2016, we cannot necessarily assume it will be constant through 2040.
- In many applications, complicated functions are approximated by linear functions. The group work for this section will explore that concept further.

▼ Examples

- A linear function with nice intercepts: $f(x) = \frac{1}{5}x - 1$



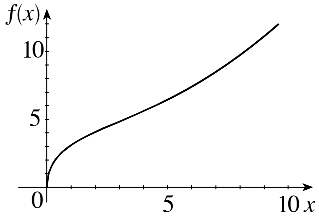
- A real-world example of a linear model: The force exerted by a spring increases linearly as it is stretched. When it is at rest (not stretched at all), it exerts no force. Now assume that when you stretch it 3 inches, it exerts $\frac{1}{10}$ lb of force on your hand. How much will it exert if you stretch it only 1 inch?
  Answer $\frac{1}{30}$ lb

▼ Group Work: Approximating Using Linear Models

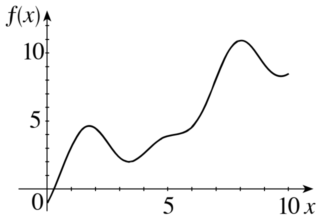
This activity introduces the idea of linear approximation from both geometric and algebraic perspectives, and assumes the students are using graphing technology. These ideas show up again in first semester calculus.

For the second question, if the students are not feeling playful, give each student a different x-value in the interval $[0, 10]$ to zoom in on.

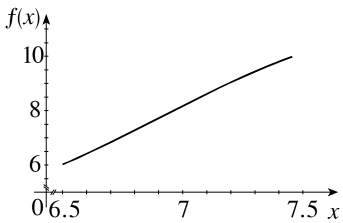
Answers:

1.(a) 

(b) It looks linear. **(c)** It looks linear.

2.(a) 

(b) It looks linear. **(c)** It looks linear.

3.(a) 

(b) $g(6.5) = g(7.5) = L(x)$
$= 4.11x - 20.31$

(c) They are close.

▼ Homework Problems

Core Exercises: 3, 9, 14, 17, 21, 29, 35, 42, 46, 52

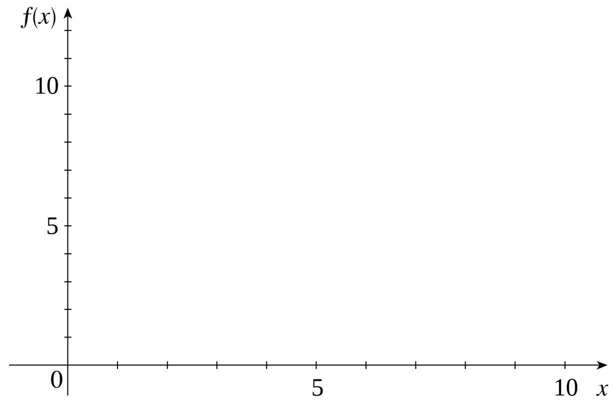
Sample Assignment: 1, 3, 5, 7, 9, 13, 14, 15, 17, 18, 21, 22, 25, 27, 29, 32, 35, 36, 38, 42, 43, 45, 46, 49, 52

GROUP WORK, SECTION 2.5

Approximating Using Linear Models

1. Consider the rather nasty function $f(x) = \dfrac{x^2}{10} - x + 4\sqrt{x}$.

(a) Sketch this function on the interval $[0, 10]$.

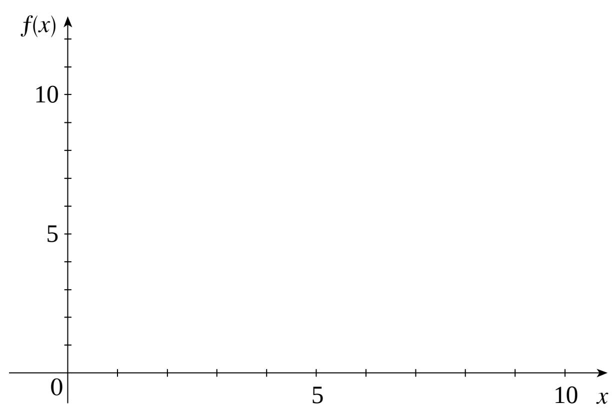


(b) f is clearly not a linear function. But notice that between $x = 8$ and $x = 9$ it looks almost linear. Plot this function on your calculator on the interval $[8, 9]$. Does it look linear?

(c) Now look at your original graph near $x = \frac{1}{2}$. It is clearly curved there. But look what happens when you plot it on the interval $[0.4, 0.6]$. Does it look linear?

2. Let's play with a kind of function you may never have seen yet: $g(x) = x + 2\sin x - \cos 2x$.

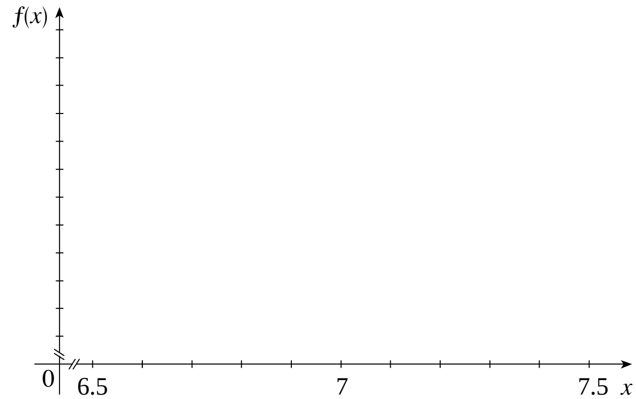
(a) Sketch this function on the interval $[0, 10]$.



(b) Notice how this function is even wigglier than the previous one! But something interesting happens when you zoom in. Pick any value of x and zoom in on it repeatedly. Does it eventually look linear?

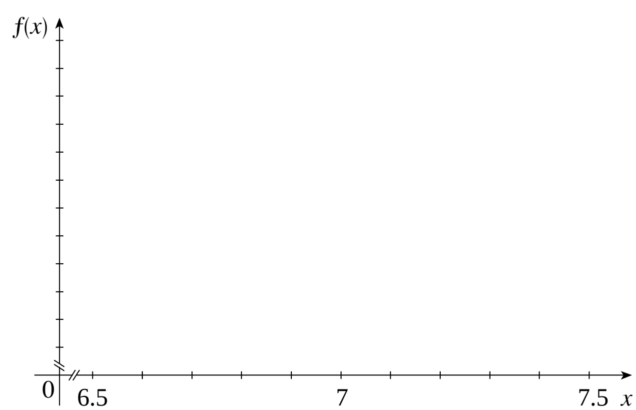
3. This is a key idea that is used in many real-world applications. If a function is "smooth" (that is, its graph has no breaks or sharp corners) then no matter how twisty it is, if you zoom in close enough, it looks linear.

(a) Graph g, the function from Part 2, on the interval $[6.5, 7.5]$.



(b) Compute $g\,(6.5)$ and $g\,(7.5)$, and use this information to find the equation of a straight line that intersects the graph of g at $x = 6.5$ and at $x = 7.5$.

(c) Graph g and your line on the same axes. Are they close?



2.6 TRANSFORMATIONS OF FUNCTIONS

▼ Suggested Time and Emphasis

1 class. Essential material.

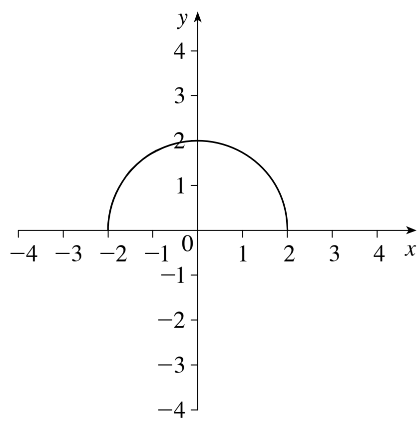
▼ Points to Stress

1. Transforming a given function to a different one by shifting, stretching, and reflection.
2. Using the technique of reflection to better understand the concepts of even and odd functions.

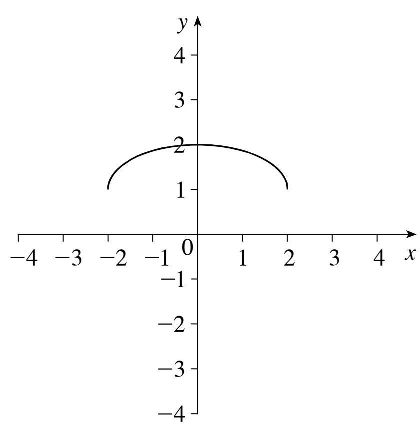
▼ Sample Questions

- **Text Question:** What is the difference between a vertical stretch and a vertical shift?

  Answer: A vertical stretch extends the graph in the vertical direction, changing its shape. A vertical shift moves the graph in the vertical direction, preserving its shape.
- **Drill Question:** Given the graph of $f(x)$ below, sketch the graph of $\frac{1}{2}f(x) + 1$.



Answer:



▼ In-Class Materials

- Students will often view this section as a process of memorizing eight similar formulas. Although it doesn't hurt to memorize how to shift, reflect, or stretch a graph, emphasize to students the importance of understanding what they are doing when they transform a graph. The group work "Discovering the Shift" (in Section 1.9) should help students understand and internalize. Tell students that if worse comes to worst, they can always plot a few points if they forget in which direction the graphs should move.

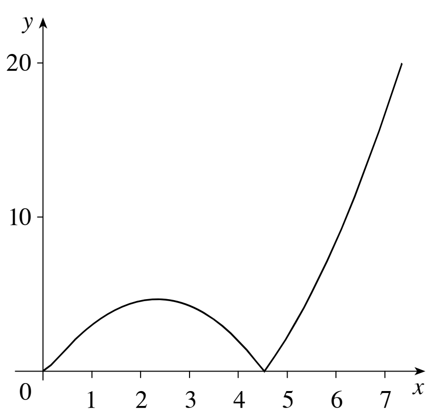
- Show the class a function they have not learned about yet, such as $f(x) = \sin x$. (If students know about $\sin$, then show them $\arctan$ or e^{-x^2}—any function with which they are unfamiliar.) Point out that even though they don't know a lot about $\sin x$, once they've seen the graph, they can graph $\sin x+3$, $\sin(x-1)$, $2\sin x$, $-\sin x$, etc.

- Graph $f(x) = x^2$ with the class. Then anticipate Section 3.2by having students graph $(x-2)^2 - 3$ and $(x+1)^2 + 2$, finally working up to $g(x) = (x-h)^2 + k$. If you point out that any equation of the form $g(x) = ax^2 + bx + c$ can be written in this so-called *standard form*, students will have a good start on the next section in addition to learning this one.

- This is a good time to start discussing parameters. Ask your students to imagine a scientist who knows that a given function will be shaped like a stretched parabola, but has to do some more measurements to find out exactly what the stretching factor is. In other words, she can write $f(x) = -ax^2$, noting that she will have to figure out the a experimentally. The a is not a variable, it is a parameter. Similarly, if we are going to do a bunch of calculations with the function $f(x) = \sqrt[3]{x+2}$, and then do the same calculations with $\sqrt[3]{x+3}$, $\sqrt[3]{x-\pi}$, and $\sqrt[3]{x-\frac{2}{3}}$, it is faster and easier to do the set of calculations just once, with the function $g(x) = \sqrt[3]{x+h}$, and then fill in the different values for h at the end. Again, this letter h is called a parameter. Ask the class how, in the expression $f(t) = t + 3s$, they can tell which is the variable, and which is the parameter—the answer may encourage them to use careful notation.

▼ Examples

A distinctive-looking, asymmetric curve that can be stretched, shifted and reflected:



$f(x) = \left|x^2 - 5x + \sqrt{x}\right|$

▼ Group Work 1: Label Label Label, I Made It Out of Clay

Some of these transformations are not covered in the book. If the students are urged not to give up, and to use the process of elimination and testing individual points, they should be able to successfully complete this activity.

Answers: 1. (d) **2.** (a) **3.** (f) **4.** (e) **5.** (i) **6.** (j) **7.** (b) **8.** (c) **9.** (g) **10.** (h)

▼ Group Work 2: Which is the Original?

The second problem has a subtle difficulty: the function is defined for all x, so some graphs show much more of the behavior of $f(x)$than others do.

Answers: 1. $2f(x+2)$, $2f(x)$, $f(2x)$, $f(x+2)$, $f(x)$ **2.** $2f(x)$, $f(x)$, $f(x+2)$, $f(2x)$, $2f(x+2)$

▼ Homework Problems

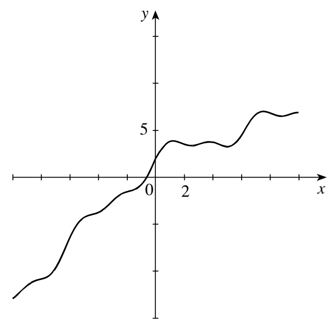
Core Exercises: 3, 9, 14, 17, 21, 29, 35, 42, 46, 52

Sample Assignment: 1, 3, 5, 7, 9, 13, 14, 15, 17, 18, 21, 22, 25, 27, 29, 32, 35, 36, 38, 42, 43, 45, 46, 49, 52

GROUP WORK 1, SECTION 2.6

Label Label Label, I Made it Out of Clay

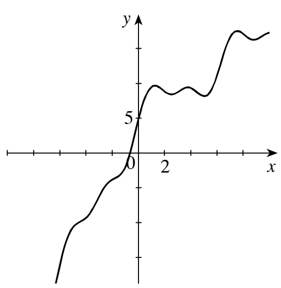
This is a graph of the function $f(x)$:



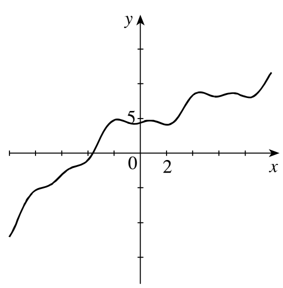
Give each graph below the correct label from the following:

(a) $f(x+3)$ (b) $f(x-3)$ (c) $f(2x)$ (d) $2f(x)$ (e) $|f(x)|$

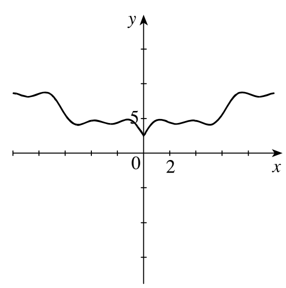
(f) $f(|x|)$ (g) $2f(x)-1$ (h) $f(2x)+2$ (i) $f(x)-x$ (j) $1/f(x)$



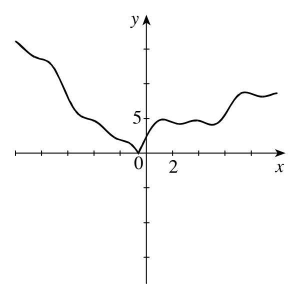
Graph 1



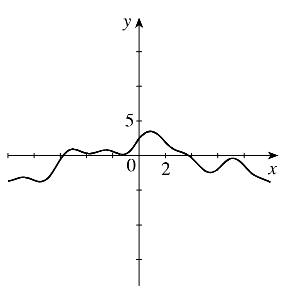
Graph 2



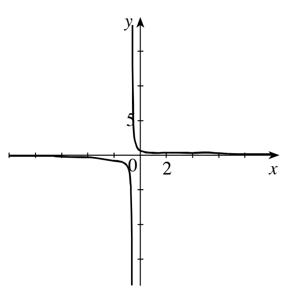
Graph 3



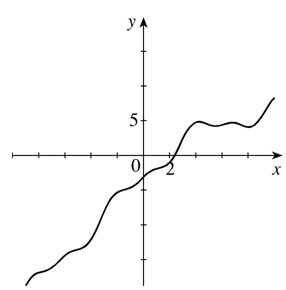
Graph 4



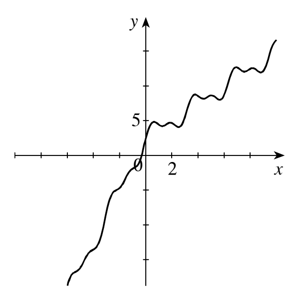
Graph 5



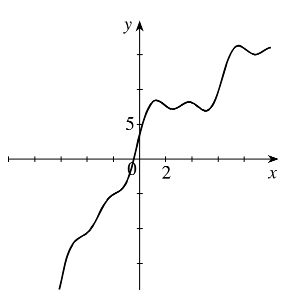
Graph 6



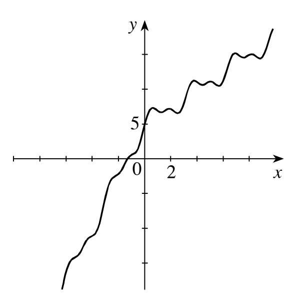
Graph 7



Graph 8



Graph 9



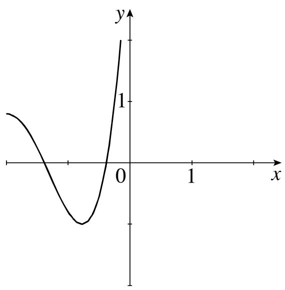
Graph 10

GROUP WORK 2, SECTION 2.6

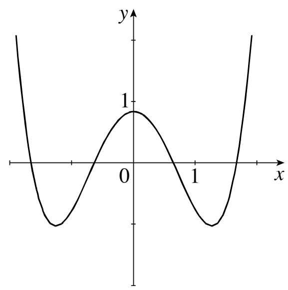
Which is the Original?

Below are five graphs. One is the graph of a function $f(x)$ and the others include the graphs of $2f(x)$, $f(2x)$, $f(x+2)$, and $2f(x+2)$. Determine which is the graph of $f(x)$ and match the other functions with their graphs.

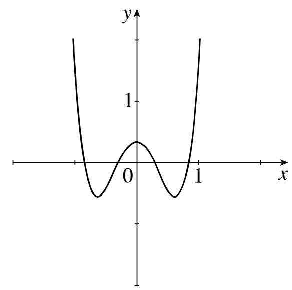
1.



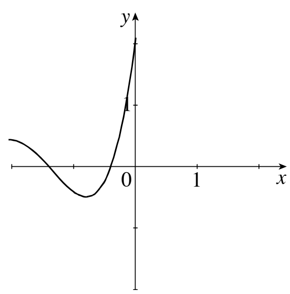
Graph 1



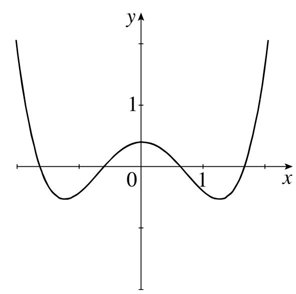
Graph 2



Graph 3

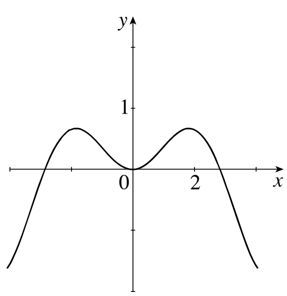


Graph 4

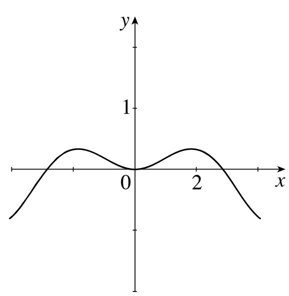


Graph 5

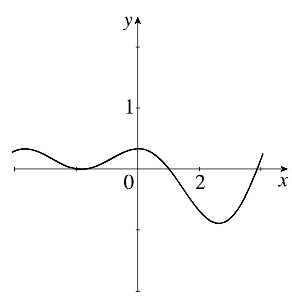
2.



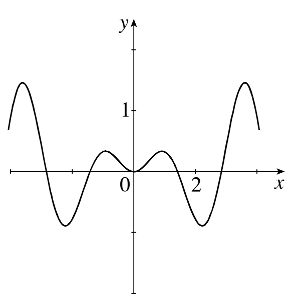
Graph 1



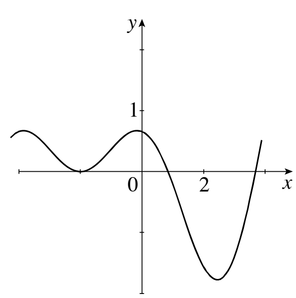
Graph 2



Graph 3



Graph 4



Graph 5

2.7 COMBINING FUNCTIONS

▼ Suggested Time and Emphasis

$\frac{1}{2}$–1 class. Essential material.

▼ Points to Stress

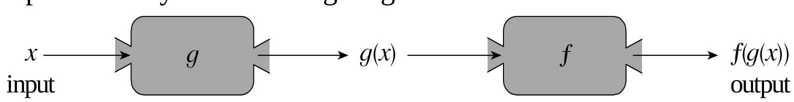
1. Addition, subtraction, multiplication, and division of functions.

2. Composition of functions.

3. Finding the domain of a function based on analysis of the domain of its components.

▼ Sample Questions

- **Text Question:** The text describes addition, multiplication, division, and composition of functions. Which of these operations is represented by the following diagram?



Answer: Composition

- **Drill Question:** Let $f(x) = 4x$ and $g(x) = x^3 + x$.

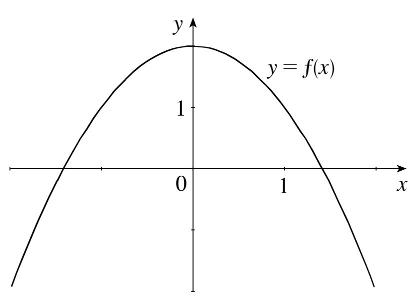
(a) Compute $(f \circ g)(x)$.

(b) Compute $(g \circ f)(x)$.

Answer: (a) $4\left(x^3 + x\right) = 4x^3 + 4x$ **(b)** $(4x)^3 + 4x = 64x^3 + 4x$

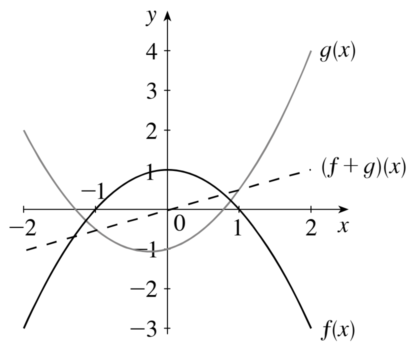
▼ In-Class Materials

- Do the following problem with the class:



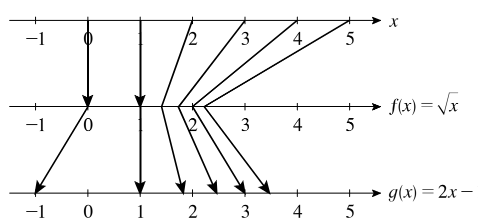
From the graph of $y = f(x) = -x^2 + 2$ shown above, compute $f \circ f$ at $x = -1$, 0, and 1. First do it graphically, then algebraically.

- Show the tie between algebraic addition of functions and graphical addition. For example, let $f(x) = 1 - x^2$ and $g(x) = x^2 + \frac{1}{2}x - 1$. First add the functions graphically, as shown below, and then show how this result can be obtained algebraically: $\left(1 - x^2\right) + \left(x^2 + \frac{1}{2}x - 1\right) = \frac{1}{2}x$.



- Point out that it is important to keep track of domains, especially when doing algebraic simplification. For example, if $f(x) = x + \sqrt{x}$ and $g(x) = 3x^2 + \sqrt{x}$, even though $(f - g)(x) = x - 3x^2$, its domain is not $\mathbb{R}$ but $\{x \mid x \geq 0\}$.

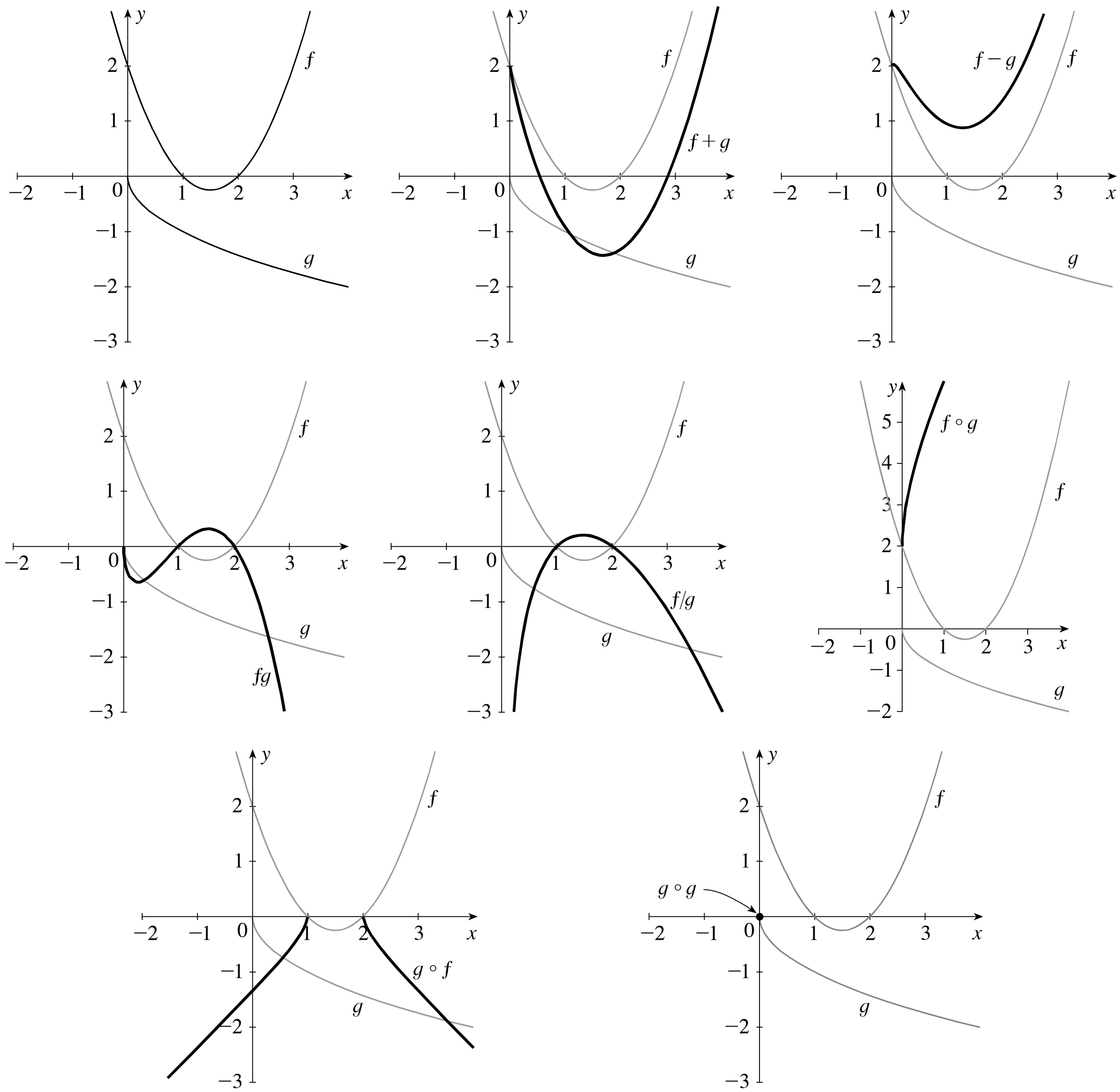
- Function maps are a nice way to explain composition of functions. To demonstrate $g \circ f(x)$, draw three number lines labeled x, $f(x)$, and $g(x)$, and then indicate how each number x goes to $f(x)$ which then goes to $g(f(x))$. For example, if $f(x) = \sqrt{x}$ and $g(x) = 2x - 1$, the diagram looks like this:



- After doing a few basic examples of composition, it is possible to foreshadow the idea of inverses, which will be covered in the next section. Let $f(x) = 2x^3 + 3$ and $g(x) = x^2 - x$. Compute $f \circ g$ and $g \circ f$ for your students. Then ask them to come up with a function $h(x)$ with the property that $(f \circ h)(x) = x$. They may not be used to the idea of coming up with examples for themselves, so the main hints they will need might be "don't give up," "when in doubt, just try something and see what happens," and "I'm not expecting you to get it in fifteen seconds." If the class is really stuck, have them try $f(x) = 2x^3$ to get a feel for how the game is played. Once they have determined that $h(x) = \sqrt[3]{\dfrac{x - 3}{2}}$, have them compute $(h \circ f)(x)$ and have them conjecture whether, in general, if $(f \circ g)(x) = x$ then $(g \circ f)(x)$ must also equal x.

▼ Examples

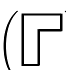
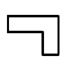
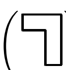
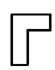
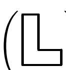
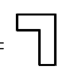
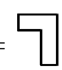
Combined functions with graphs: Let $f(x) = x^2 - 3x + 2$ and $g(x) = -\sqrt{x}$.

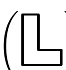
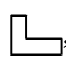
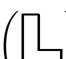
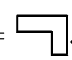


▼ Group Work 1: Transformation of Plane Figures

This tries to remove the composition idea from the numerical context, and introduces the notion of symmetry groups. It is a longer activity than it seems, and can lead to an interesting class discussion of this topic.

Answers:

1. (a) f () =  **(b)** g () =  **(c)** f (f ()) =  **(d)** g (g ()) = 

2. This is false. For example, $(f \circ g)$ () = , but $(g \circ f)$ () = .

3. It is true: reversing something thrice in a mirror gives the same result as reversing it once.

4. It rotates the shape 270° clockwise or, equivalently, 90° counterclockwise.

▼ Group Work 2: Odds and Evens

This is an extension of Exercise 102 in the text. Students may find the third problem difficult to start. You may want to give selected table entries on the board first, before handing the activity out, to make sure students understand what they are trying to do.

Answers:

1.

a	b	$a+b$
even	even	even
odd	even	odd
even	odd	odd
odd	odd	even

2.

a	b	$a \cdot b$
even	even	even
odd	even	even
even	odd	even
odd	odd	odd

3.

f	g	$f+g$	fg	$f \circ g$	$g \circ f$
even	even	even	even	even	even
even	odd	neither	odd	even	even
odd	even	neither	odd	even	even
odd	odd	odd	even	odd	odd
neither	neither	unknown	unknown	unknown	unknown

▼ Group Work 3: It's More Fun to Compute

Each group gets one copy of the graph. During each round, one representative from each group stands, and one of the questions below is asked. The representatives write their answer down, and all display their answers at the same time. Each representative has the choice of consulting with their group or not. A correct solo answer is worth two points, and a correct answer after a consult is worth one point.

1. $(f \circ g)(5)$
2. $(g \circ f)(5)$
3. $(f \circ g)(0)$
4. $(f \circ f)(5)$
5. $(g \circ g)(5)$
6. $(g \circ g)(-3)$
7. $(g \circ g)(-1)$
8. $(f \circ g)(1)$
9. $(g \circ f)(1)$
10. $(f \circ f \circ g)(4)$
11. $(g \circ f \circ f)(4)$
12. $(f \circ g \circ f)(4)$

Answers: **1.** 0 **2.** 0 **3.** 1 **4.** 5 **5.** 1 **6.** 1 **7.** 1 **8.** 0 **9.** 2 **10.** 1 **11.** 1 **12.** 1

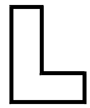
▼ Homework Problems

Core Exercises: 4, 10, 15, 20, 26, 34, 38, 49, 57, 64, 71, 75, 79, 86, 92, 104

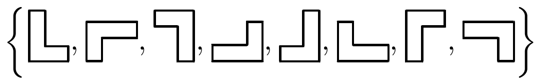
Sample Assignment: 2, 4, 6, 7, 10, 13, 15, 20, 22, 24, 26, 27, 29, 34, 36, 38, 45, 49, 52, 55, 57, 61, 64, 67, 69, 71, 73, 75, 77, 79, 82, 85, 86, 92, 93, 95, 98, 104

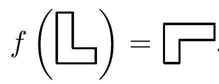
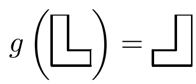
Transformation of Plane Figures

So far, when we have been talking about functions, we have been assuming that their domains and ranges have been sets of numbers. This is not necessarily the case. For example, look at this figure:

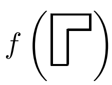
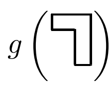
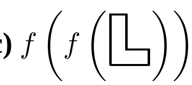
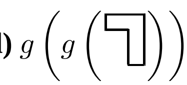


Let's let our domain be all the different ways we can move this figure around, including flipping it over:

$D =$ 

Now let f be the function that rotates the shape $90°$ clockwise: . Let g be the function that flips the shape over a vertical line drawn through the center: 

1. Find the following:

(a)    **(b)**    **(c)**    **(d)** 

2. Is it true that $f \circ g = g \circ f$? Why or why not?

3. Is it true that $g \circ g \circ g = g$? Why or why not?

4. Write, in words, what the function $f \circ f \circ f$ does to a shape.

GROUP WORK 2, SECTION 2.7

Odds and Evens

1. Let a be an odd number, and b be an even number. Fill in the following table (the first row is done for you).

a	b	$a+b$
even	even	even
odd	even	
even	odd	
odd	odd	

2. We can also multiply numbers together. Fill in the corresponding multiplication table:

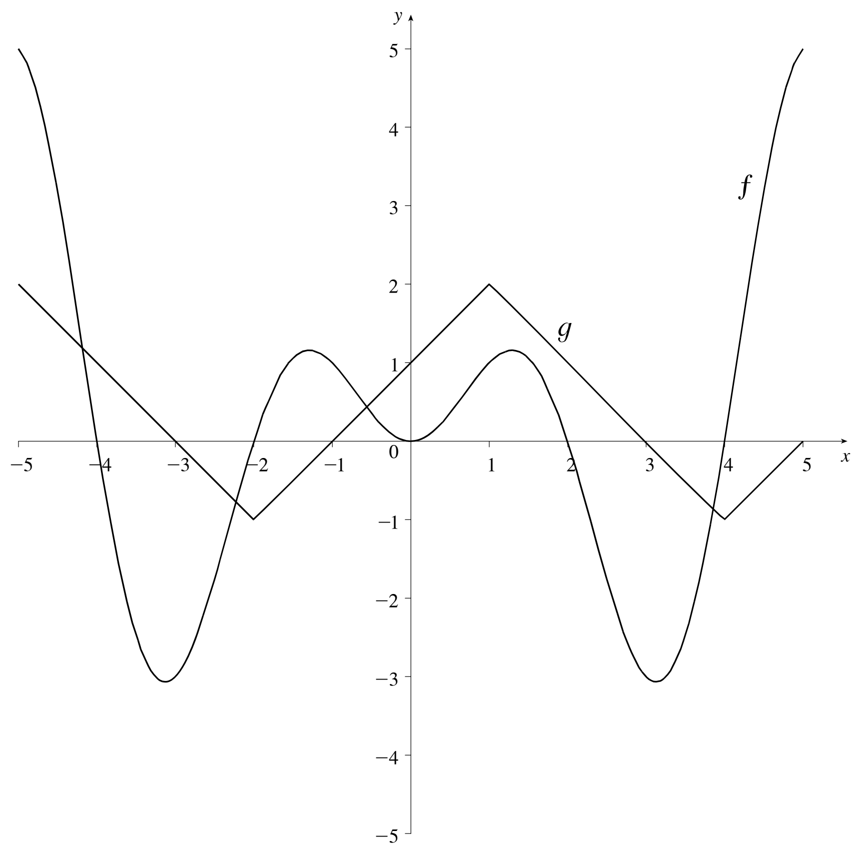
a	b	$a \cdot b$
even	even	
odd	even	
even	odd	
odd	odd	

3. Now we let f and g be (nonzero) functions, not numbers. We are going to think about what happens when we combine these functions. When you fill in the table, you can write "unknown" if the result can be odd *or* even, depending on the functions. You can solve this problem by drawing some pictures, or by using the definition of odd and even functions.

f	g	$f+g$	fg	$f \circ g$	$g \circ f$
even	even				
even	odd				
odd	even				
odd	odd				
neither	neither				

GROUP WORK 3, SECTION 2.7

It's More Fun to Compute



2.8 ONE-TO-ONE FUNCTIONS AND THEIR INVERSES

▼ Suggested Time and Emphasis

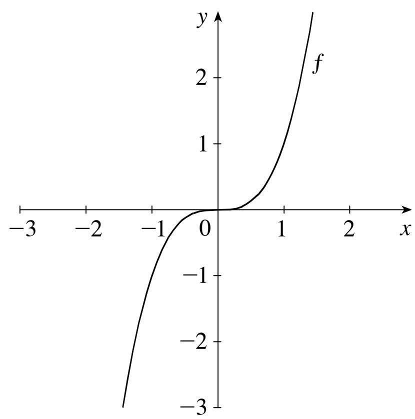
1–2 classes. Essential material.

▼ Points to Stress

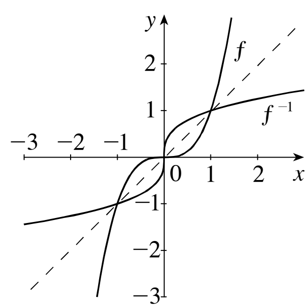
1. One-to-one functions: their definition and the Horizontal Line Test.
2. Algebraic and geometric properties of inverse functions.
3. Finding inverse functions.

▼ Sample Questions

- **Text Question:** The function f is graphed below. Sketch f^{-1}, the inverse function of f.



Answer:



- **Drill Question:** If $f(-2) = 4$, $f(-1) = 3$, $f(0) = 2$, $f(1) = 1$ and $f(2) = 3$, what is $f^{-1}(2)$?
  Answer: 0

▼ In-Class Materials

- Make sure students understand the notation: f^{-1} is not the same thing as $\dfrac{1}{f}$.
- Starting with $f(x) = \sqrt[3]{x-4}$, compute $f^{-1}(-2)$ and $f^{-1}(0)$. Then use algebra to find a formula for $f^{-1}(x)$. Have the class try to repeat the process with $g(x) = x^3 + x - 2$. Note that facts such as $g^{-1}(-2) = 0$, $g^{-1}(0) = 1$, and and $g^{-1}(8) = 2$ can be found by looking at a table of values for $g(x)$ but that algebra fails to give us a general formula for $g^{-1}(x)$. Finally, draw graphs of f, f^{-1}, g, and g^{-1}.

- Pose the question: If f is always increasing, is f^{-1} always increasing? Give students time to try prove their answer.

  Answer: This is true. Proofs may involve diagrams and reflections about $y = x$, or you may try to get them to be more rigorous. This is an excellent opportunity to discuss concavity, noting that if f is concave up and increasing, then f^{-1} is concave down and increasing.

- Point out that the idea of "reversing input and output" permeates the idea of inverse functions, in all four representations of "function". When finding inverse functions algebraically, we explicitly reverse x and y. When drawing the inverse function of a graph, by reflecting across the line $y = x$ we are reversing the y- and x-axes. If $c(x)$ is the cost (in dollars) to make x fruit roll-ups, then $c^{-1}(x)$ is the number of fruit roll-ups that could be made for x dollars—again reversing the input and the output. Finally, show the class how to find the inverse of a function given a numeric data table, and note that again the inputs and outputs are reversed.

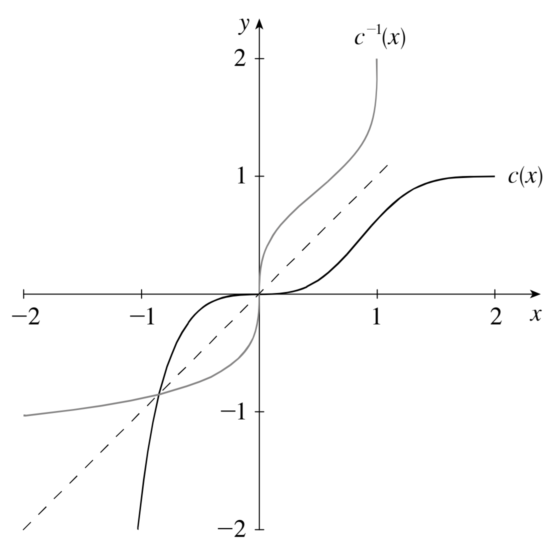
x	$f(x)$
1	3
2	4.2
3	5.7
4	8

x	$f^{-1}(x)$
3	1
4.2	2
5.7	3
8	4

- Make sure to discuss units carefully: when comparing $y = f(x)$ to $y = f^{-1}(x)$, the units of y and x trade places.

▼ Examples

- The graph of a complicated function and its inverse:



▼ Group Work 1: Inverse Functions: Domains and Ranges

While discussing the domains and ranges of inverse functions, this exercise foreshadows later excursions into the maximum and minimum values of functions.

If a group finishes early, ask them this question:

"Now consider the graph of $f(x) = \sqrt{2x-3}+2$. What are the domain and range of $f(x)$? Try to figure out the domain and range of $f^{-1}(x)$ by looking at the graph of f. In general, what information do you need to be able to compute the domain and range of $f^{-1}(x)$ from the graph of a function f?"

Answers:

1. It is one-to-one, because the problem says it climbs steadily.
2. a^{-1} is the time in minutes at which the plane achieves a given altitude.
3. Reverse the data columns in the given table to get the table for the inverse function. The domain and range of a are $0 \leq t \leq 30$ and $0 \leq a \leq 29{,}000$, so the domain and range of a^{-1} are $0 \leq x \leq 29{,}000$ and $0 \leq a^{-1} \leq 30$.
4. You can expect to turn on your computer after about 8.5 minutes.
5. a is no longer 1-1, because heights are now achieved more than once.

Bonus The domain of f^{-1} is the set of all y-values on the graph of f, and the range of f^{-1} is the set of all x-values on the graph of f.

▼ Group Work 2: The Column of Liquid

If the students need a hint, you can mention that the liquid in the mystery device was mercury.

Answers **1.** The liquid is 1 cm high when the temperature is 32 °F. **2.** The liquid is 2 cm high when the temperature is 212 °F **3.** The inverse function takes a height in cm, and gives the temperature. So it is a device for measuring temperature. **4.** A thermometer

▼ Group Work 3: Functions in the Classroom Revisited

This activity starts the same as "Functions in the Classroom" from Section 2.2. At this point, students have learned about one-to-one functions, and they are able to explore this activity in more depth.

Answers

Chairs: Function, one-to-one, bijection (if all chairs are occupied). If one-to-one, the inverse assigns a chair to a person.

Eye color: Function, not one-to-one

Mom & Dad's birthplace: Not a function; mom and dad could have been born in different places

Molecules: Function, one-to-one (with nearly 100% probability); inverse assigns a number of molecules to the appropriate student.

Spleens: Function, one-to-one, bijection. Inverse assigns each spleen to its owner.

Pencils: Not a function; some people may have more than one or (horrors!) none.

Social Security Number: Function, one-to-one; inverse assigns each number to its owner.

February birthday: Not a function; not defined for someone born on February 29.

Birthday: Function, perhaps one-to-one. If one-to-one, the inverse assigns a day to a person.

Cars: Not a function; some have none, some have more than one.

Cash: Function, perhaps one-to-one. If one-to-one, the inverse assigns an amount of money to a person.

Middle names: Not a function; some have none, some have more than one.

Identity: Function, one-to-one, bijection. Inverse is the same as the function.

Instructor: Function, not one-to-one.

▼ Homework Problems

Core Exercises: 3, 8, 14, 20, 24, 35, 48, 54, 57, 64, 67, 79, 71

Sample Assignment: 1, 3, 8, 10, 14, 15, 18, 20, 22, 24, 25, 29, 31, 35, 38, 41, 46, 47, 48, 54, 56, 57, 60, 64, 66, 67, 70, 71, 73, 77, 79, 81, 82

GROUP WORK 1, SECTION 2.8

Inverse Functions: Domains and Ranges

Let $a(t)$ be the altitude in feet of a plane that climbs steadily from takeoff until it reaches its cruising altitude after 30 minutes. We don't have a formula for a, but extensive research has given us the following table of values:

t	$a(t)$
0.1	50
0.5	150
1	500
3	2000
7	8000
10	$12{,}000$
20	$21{,}000$
25	$27{,}000$
30	$29{,}000$

1. Is $a(t)$ a one-to-one function? How do you know?

2. What does the function a^{-1} measure in real terms? Your answer should be descriptive, similar to the way $a(t)$ was described above.

3. We are interested in computing values of a^{-1}. Fill in the following table for as many values of x as you can. What quantity does x represent?

x	$a^{-1}(x)$

What are the domain and range of a? What are the domain and range of a^{-1}?

4. You are allowed to turn on electronic equipment after the plane has reached $10{,}000$ feet. Approximately when can you expect to turn on your laptop computer after taking off?

5. Suppose we consider $a(t)$ from the time of takeoff to the time of touchdown. Is $a(t)$ still one-to-one?

GROUP WORK 2, SECTION 2.8

The Column of Liquid

It is a fact that if you take a tube and fill it partway with liquid, the liquid will rise and fall based on the temperature. Assume that we have a tube of liquid, and we have a function $h(T)$, where h is the height of the liquid in cm at temperature T in $^\circ$F.

1. It is true that $h(32) = 1$. What does that mean in physical terms?

2. It is true that $h(212) = 10$. What does that mean in physical terms?

3. Describe the inverse function h^{-1}. What are its inputs? What are its outputs? What does it measure?

4. Hospitals used to use a device that measured the function h^{-1}. Some people used to have such a device in their homes. What is the name of this device?

GROUP WORK 3, SECTION 2.8

Functions in the Classroom Revisited

Which of the following are functions? Of the ones that are functions, which are one-to-one functions? Describe what the inverses tell you.

Domain	Function Values	Function
All the people in your classroom	Chairs	f (person) $=$ his or her chair
All the people in your classroom	The set $\{$blue, brown, green, hazel$\}$	f (person) $=$ his or her eye color
All the people in your classroom	Cities	f (person) $=$ birthplace of their mom and dad
All the people in your classroom	$\mathbb{R}$, the real numbers	f (person) $=$ number of molecules in their body
All the people in your classroom	Spleens	f (person) $=$ his or her own spleen
All the people in your classroom	Pencils	f (person) $=$ his or her pencil
All the people in the United States	Integers from 0–999999999	f (person) $=$ his or her Social Security number
All the living people born in February	Days in February, 2019	f (person) $=$ his or her birthday in February 2019
All the people in your classroom	Days of the year	f (person) $=$ his or her birthday
All the people in your classroom	Cars	f (person) $=$ his or her car
All the people in your classroom	$\mathbb{R}$, the real numbers	f (person) $=$ how much cash he or she has
All the people in your college	Names	f (person) $=$ his or her middle name
All the people in your classroom	People	f (person) $=$ himself or herself
All the people in your classroom	People	f (person) $=$ his or her instructor

FOCUS ON PROBLEM SOLVING 2 SOLUTIONS

1. The final digit of 947^{362} is determined by 7^{362}. Looking at the first few powers of 7, we have $7^1 = 7, 7^2 = 49, 7^3 = 343, 7^4 = 2401,$ and $7^5 = 16807, 7^6 = 117649, 7^7 = 823543, 7^8 = 5764801$, and so on. The last digit seems to follow a pattern that repeats every fourth power: $7, 9, 3, 1, 7, 9, 3, 1, \ldots$ We divide 362 by 4 and get a remainder 2, which corresponds to 9 (the second term in the pattern). So the final digit of 947^{362} is the number 9.

2. We are given $f_0(x) = x^2$ and $f_{n+1}(x) = f_0(f_n(x))$, for $n = 0, 1, 2, \ldots$ We look at the first three terms to find a pattern.

$$f_0(x) = x^2,$$
$$f_1(x) = f_{0+1}(x) = f_0(f_0(x)) = f_0(x^2) = (x^2)^2 = x^4$$
$$f_2(x) = f_{1+1}(x) = f_0(f_1(x)) = f_0(x^4) = (x^4)^2 = x^8$$
$$f_3(x) = f_{2+1}(x) = f_0(f_2(x)) = f_0(x^8) = (x^8)^2 = x^{16}$$

We can see that the formula is $f_n(x) = x^{2^{n+1}}$, for $n = 0, 1, 2, \ldots$

3. We have $f_0(x) = \dfrac{1}{2-x}$ and $f_{n+1}(x) = f_0(f_n(x))$, for $n = 0, 1, 2, \ldots$ We look at the first three terms to find a pattern.

$$f_0(x) = \frac{1}{2-x},$$
$$f_1(x) = f_{0+1}(x) = f_0(f_0(x)) = f_0\left(\frac{1}{2-x}\right) = \frac{1}{2 - \dfrac{1}{2-x}}$$
$$= \frac{1}{2 - \dfrac{1}{2-x}} \cdot \frac{2-x}{2-x} = \frac{2-x}{2(2-x)-1} = \frac{2-x}{3-2x}$$
$$f_2(x) = f_{1+1}(x) = f_0(f_1(x)) = f_0\left(\frac{2-x}{3-2x}\right) = \frac{1}{2 - \dfrac{2-x}{3-2x}}$$
$$= \frac{1}{2 - \dfrac{2-x}{3-2x}} \cdot \frac{3-2x}{3-2x} = \frac{3-2x}{2(3-2x)-(2-x)} = \frac{3-2x}{4-3x}$$

$$f_3(x) = f_{2+1}(x) = f_0(f_2(x)) = f_0\left(\frac{3-2x}{4-3x}\right) = \frac{1}{2-\frac{3-2x}{4-3x}}$$

$$= \frac{1}{2-\frac{3-2x}{4-3x}} \cdot \frac{4-3x}{4-3x} = \frac{4-3x}{2(4-3x)-(3-2x)} = \frac{4-3x}{5-4x}$$

We can see that the formula is $f_n(x) = \frac{(n+1)-nx}{(n+2)-(n+1)x}$, for $n = 0,1,2,...$ and hence $f_{100}(3) = \frac{101-100(3)}{102-101(3)} = \frac{-199}{-201} = \frac{199}{201}$.

4. Let us see what happens when we square similar numbers with fewer 9's.

$$39^2 = 1521, 399^2 = 159201, 3999^2 = 15992001, 39999^2 = 1599920001$$

We observe that the pattern is the square begins with 15, then there is a number of 9's with the number being one less than the original number of 9's, then there is a 2, then there is a number of 0's with the number being the same as the number of 9's, and then ends with a 1. Note that $3,999,999,999,999$ contains 12 nines, so according to the pattern this number squared begins with 15, then has eleven 9's, then has a 2, then has eleven 0's, and ends with one:

$$3999999999999^2 = 15,999,999,999,992,000,000,000,001$$

This can be verified algebraically as follows:

$$\begin{aligned}3,999,999,999,999^2 &= (4,000,000,000,000-1)^2 \\ &= (4,000,000,000,000)^2 - 2(4,000,000,000,000)+1 \\ &= 16,000,000,000,000,000,000,000,000 - 8,000,000,000,000+1 \\ &= 15,999,999,999,992,000,000,000,001\end{aligned}$$

5. To find the domain of the function $f(x) = \sqrt{1-\sqrt{2-\sqrt{3-x}}}$, we use the fact that the square root function is only defined for positive real values, including 0.

$1-\sqrt{2-\sqrt{3-x}} \ge 0 \Leftrightarrow 0 \le \sqrt{2-\sqrt{3-x}} \le 1 \Leftrightarrow 0 \le 2-\sqrt{3-x} \le 1$

(we can square all the terms in the inequality because the squaring function is increasing for positive real values)

By similar reasoning,

$0 \le 2-\sqrt{3-x} \le 1 \Leftrightarrow -2 \le -\sqrt{3-x} \le -1$

$\Leftrightarrow 1 \le \sqrt{3-x} \le 2 \Leftrightarrow 1 \le 3-x \le 2^2$

Again by similar reasoning,

$1 \le 3-x \le 4 \Leftrightarrow -2 \le -x \le 1 \Leftrightarrow -1 \le x \le 2$.

So the domain of f is $[-1,2]$.

6. We have the following equivalence if $x \geq 0$:

$$f(x) = \left|x^2 - 4|x| + 3\right| = \left|x^2 - 4x + 3\right| = |(x-1)(x-3)|$$

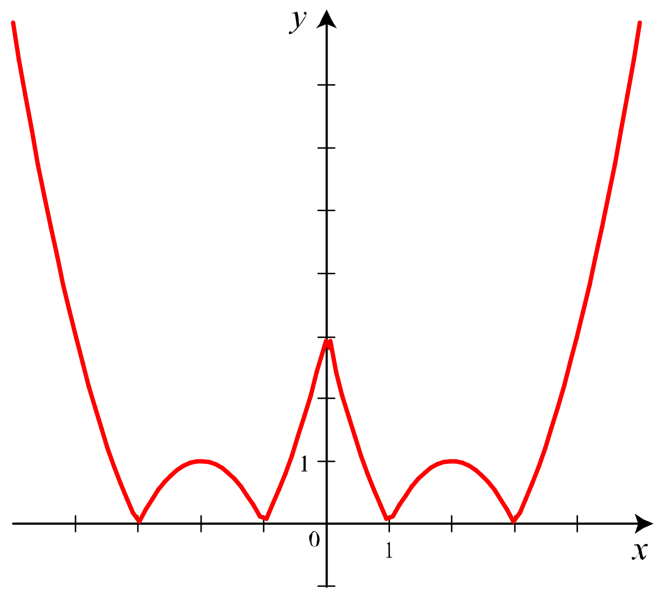
We find the sign of $(x-1)(x-3)$ on the following intervals:

Case (i): $0 \leq x \leq 1 \Rightarrow (x-1)(x-3) \geq 0 \Leftrightarrow f(x) = \left|x^2 - 4x + 3\right| = x^2 - 4x + 3$

Case (ii): $1 \leq x \leq 3 \Rightarrow (x-1)(x-3) \leq 0 \Leftrightarrow f(x) = \left|x^2 - 4x + 3\right| = -(x^2 - 4x + 3)$

Case (iii): $3 \leq x \leq \infty \Rightarrow (x-1)(x-3) \geq 0 \Leftrightarrow f(x) = \left|x^2 - 4x + 3\right| = x^2 - 4x + 3$

We now can sketch the graph of *f* for $x \geq 0$, and then use the fact that *f* is an even function to reflect the graph about the *y*-axis to obtain the graph of *f* for $x < 0$.



7. Note that $f(x) = \left|x^2 - 1\right| - \left|x^2 - 4\right| = |(x-1)(x+1)| - |(x-2)(x+2)|$ we consider the following intervals:

Case (i) $x \leq -2 \Rightarrow (x-1)(x+1) \geq 0$ and $(x-2)(x+2) \geq 0$

$\Rightarrow f(x) = \left(x^2 - 1\right) - \left(x^2 - 4\right) = 3$

Case (ii): $-2 \leq x \leq -1 \Rightarrow (x-1)(x+1) \geq 0$ and $(x-2)(x+2) \leq 0$

$\Rightarrow f(x) = \left(x^2 - 1\right) + \left(x^2 - 4\right) = 2x^2 + 5$

Case (iii) $-1 \leq x \leq 1 \Rightarrow (x-1)(x+1) \leq 0$ and $(x-2)(x+2) \leq 0$

$\Rightarrow f(x) = -\left(x^2 - 1\right) + \left(x^2 - 4\right) = -3$

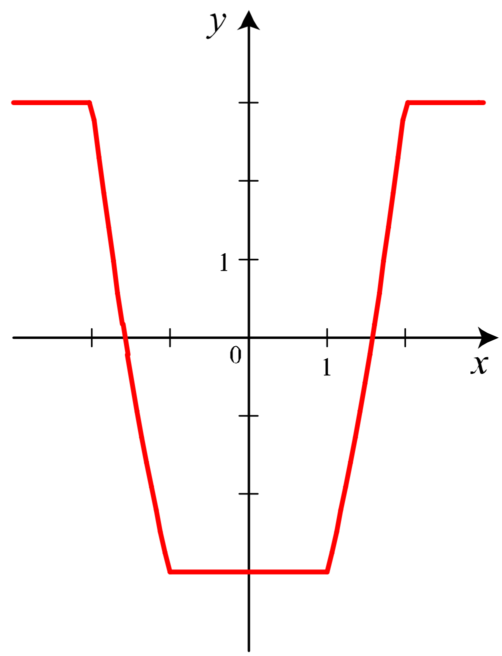
Case (iv): $1 \leq x \leq 2 \Rightarrow (x-1)(x+1) \geq 0$ and $(x-2)(x+2) \leq 0$

$\Rightarrow f(x) = \left(x^2 - 1\right) + \left(x^2 - 4\right) = 2x^2 + 5$

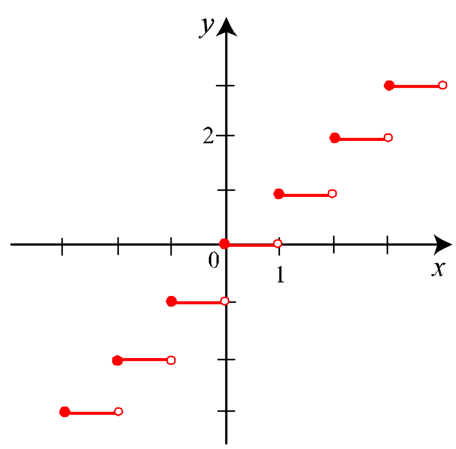
Case (v) $x \geq 2 \Rightarrow (x-1)(x+1) \geq 0$ and $(x-2)(x+2) \geq 0$

$\Rightarrow f(x) = \left(x^2 - 1\right) - \left(x^2 - 4\right) = 3$

We now can sketch the graph of *f* for $x \ge 0$, and then use the fact that *f* is an even function to reflect the graph about the *y*-axis to obtain the graph of *f* for $x < 0$.

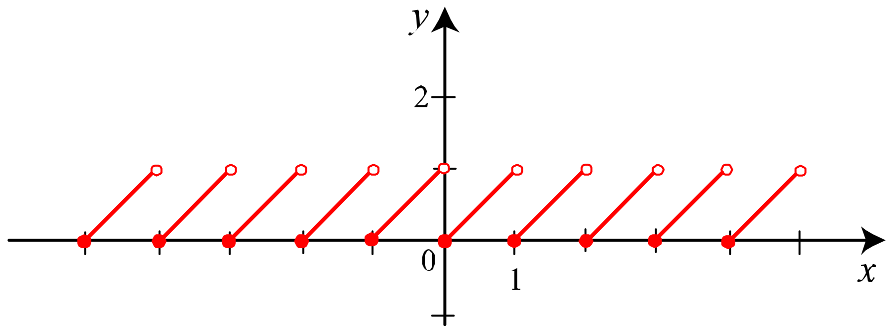


8. **(a)** Notice that $[\![x]\!] = 0$ for $0 \le x < 1$, $[\![x]\!] = 1$ for $1 \le x < 2$, $[\![x]\!] = 2$ for $2 \le x < 3$, $[\![x]\!] = -1$ for $-1 \le x < 0$, and so on. We use these observations to sketch a graph of the greatest integer function.



(b) Notice that $x - [\![x]\!] = x$ for $0 \le x < 1$, $x - [\![x]\!] = x - 1$ for $1 \le x < 2$, $x - [\![x]\!] = x - 2$ for $2 \le x < 3$, $x - [\![x]\!] = -x$ for $-1 \le x < 0$, and so on.

We use these observations to sketch a graph of $f(x) = x - [\![x]\!]$.



9. We consider the function $[\![x]\!]^2 + [\![y]\!]^2 = 1$ and look at the four cases.

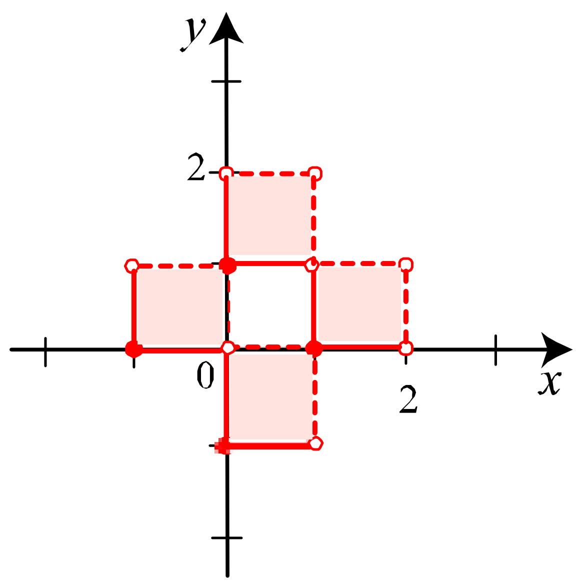
Case (i): $[\![x]\!] = 0$ and $[\![y]\!] = 1 \Rightarrow 0 \le x < 1$ and $1 \le y < 2$

Case (ii): $[\![x]\!] = 1$ and $[\![y]\!] = 0 \Rightarrow 1 \le x < 2$ and $0 \le y < 1$

Case (iii): $[\![x]\!] = 0$ and $[\![y]\!] = -1 \Rightarrow 0 \le x < 1$ and $-1 \le y < 0$

Case (iv): $[\![x]\!] = -1$ and $[\![y]\!] = 0 \Rightarrow 0 \le x < -1$ and $0 \le y < 1$

We use these observations to sketch a graph.



10.

Range	Number	Digits	Total Digits
[1, 9]	9	1	9
[10, 99]	90	2	189
[100, 999]	900	3	2889
[1000, 9999]	9000	4	38,889
[10000, 99999]	90,000	5	488,889

100^{th} position: First we note that the last 1-digit number is in position 9 and that $100-9=91$ positions will be filled by 2-digit numbers, and 45 2-digit numbers will fill 90 positions, so the number in the 100^{th} position is the first digit of the number $10+45=55$. So the number in the 100^{th} position is 5.

1000^{th} position: First we note that the last 2-digit number is in position 189 and that $1000-189=811$ positions will be filled by 3-digit numbers, and 270 3-digit numbers will fill 810 positions, so the number in the 1000^{th} position is the first digit of the number $100+270=370$. So the number in the 1000^{th} position is 3.

$10{,}000^{th}$ position: First we note that the last 3-digit number is in position 2889 and that $10,000-2889=7111$ positions will be filled by 4-digit numbers, and 1777 4-digit numbers will fill 7108 positions, so the number in the $10{,}000^{th}$ position is the third digit of the number $10,000+1777=11,777$. So the number in the $10{,}000^{th}$ position is 7.

$300{,}000^{th}$ position: First we note that the last 4-digit number is in position 38,889 and that $300,000-38,889=261,111$ positions will be filled by 5-digit numbers, and 52,222 3-digit numbers will fill 261,110 positions, so the number in the $300{,}000^{th}$ position is the first digit of the number $10,000+52,222=62,222$. So the number in the $300{,}000^{th}$ position is 6.

11. Let h_1 be the height of the pyramid whose square base length is *a*. Then h_1-h is the height of the pyramid whose square base length is *b*. Thus the volume of the truncated pyramid is $V=\frac{1}{3}h_1a^2-\frac{1}{3}(h_1-h)b^2=\frac{1}{3}h_1a^2-\frac{1}{3}h_1b^2+\frac{1}{3}hb^2=\frac{1}{3}h_1\left(a^2-b^2\right)+\frac{1}{3}hb^2$.

Next we must find a relationship between h_1 and the other variables. Using geometry, the ratio of the height to the base of the two pyramids must be the same. Thus $\frac{h_1}{a}=\frac{h_1-h}{b}\Leftrightarrow bh_1=ah_1-ah\Leftrightarrow h_1(a-b)=ah\Leftrightarrow h_1=\frac{ah}{a-b}$. Substituting for h_1 we have $V=\frac{1}{3}\frac{ah}{a-b}\left(a^2-b^2\right)+\frac{1}{3}hb^2=\frac{1}{3}ah\left(a+b\right)+\frac{1}{3}hb^2=\frac{1}{3}h\left(a^2+ab+b^2\right)$.

Precalculus Mathematics for Calculus 7th Edition Stewart Solutions Manual
Full Download: http://testbanklive.com/download/precalculus-mathematics-for-calculus-7th-edition-stewart-solutions-manual/

CUMULATIVE REVIEW TEST: CHAPTERS 2, 3, and 4

1. $f(x) = x^2 - 4x$, $g(x) = \sqrt{x+4}$

(a) The domain of f is $(-\infty, \infty)$.

(b) The domain of g is the set of all x for which $x + 4 \geq 0 \Leftrightarrow x \geq -4$, that is, $[-4, \infty)$.

(c) $f(-2) = (-2)^2 - 4(-2) = 12$, $f(0) = 0^2 - 4(0) = 0$, $f(4) = 4^2 - 4(4) = 0$, $g(0) = \sqrt{0+4} = 2$, $g(8) = \sqrt{8+4} = 2\sqrt{3}$, $g(-6) = \sqrt{-6+4} = \sqrt{-2}$, which is undefined.

(d) $f(x+2) = (x+2)^2 - 4(x+2) = x^2 + 4x + 4 - 4x - 8 = x^2 - 4$, $g(x+2) = \sqrt{(x+2)+4} = \sqrt{x+6}$, $f(2+h) = (2+h)^2 - 4(2+h) = 4 + 4h + h^2 - 8 - 4h = h^2 - 4$

(e) $\dfrac{g(21) - g(5)}{21 - 5} = \dfrac{\sqrt{21+4} - \sqrt{5+4}}{16} = \dfrac{5-3}{16} = \dfrac{1}{8}$

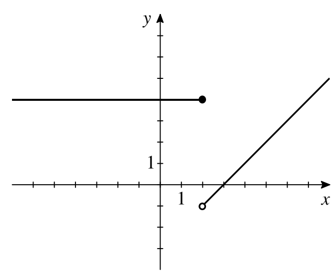
(f) $f \circ g(x) = f(g(x)) = \left(\sqrt{x+4}\right)^2 - 4\left(\sqrt{x+4}\right) = x + 4 - 4\sqrt{x+4}$,
$g \circ f(x) = g(f(x)) = \sqrt{x^2 - 4x + 4} = \sqrt{(x-2)^2} = |x - 2|$, $f(g(12)) = 12 + 4 - 4\sqrt{12+4} = 16 - 4\sqrt{16} = 0$,
$g(f(12)) = 12 - 2 = 10$

(g) $y = \sqrt{x+4} \Rightarrow y^2 = x + 4 \Leftrightarrow x = y^2 - 4$. Reverse x and y: $y = x^2 - 4$. Thus, the inverse of g is $g^{-1}(x) = x^2 - 4$, $x \geq 0$.

2. $f(x) = \begin{cases} 4 & \text{if } x \leq 2 \\ x - 3 & \text{if } x > 2 \end{cases}$

(a) $f(0) = 4$, $f(1) = 4$, $f(2) = 4$, $f(3) = 3 - 3 = 0$, and $f(4) = 4 - 3 = 1$.

(b)

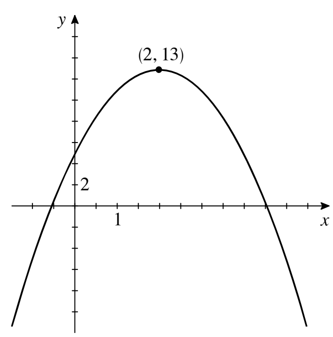


3. $f(x) = -2x^2 + 8x + 5$

(a) $f(x) = -2\left(x^2 - 4x\right) + 5 = -2\left(x^2 - 4x + 4\right) + 8 + 5$
$= -2(x-2)^2 + 13$

(b) Because $a = -2 < 0$, f has a maximum value of 13 at $x = 2$.

(c)



(d) f is increasing on $(-\infty, 2)$ and decreasing on $(2, \infty)$.

(e) $g(x) = -2x^2 + 8x + 10 = f(x) + 5$, so its graph is obtained by shifting that of f upward 5 units.

(f) $h(x) = -2(x+3)^2 + 8(x+3) + 5 = f(x+3)$, so its graph is obtained by shifting that of f to the left 3 units.

Full download all chapters instantly please go to Solutions Manual, Test Bank site: testbanklive.com