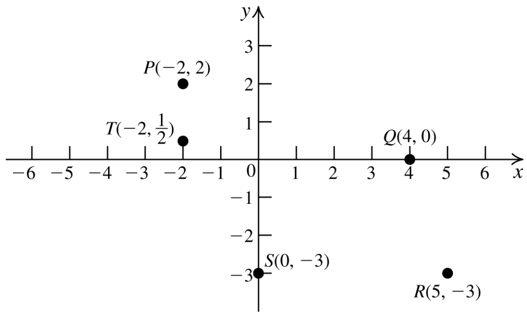
Chapter 2 Graphs and Functions

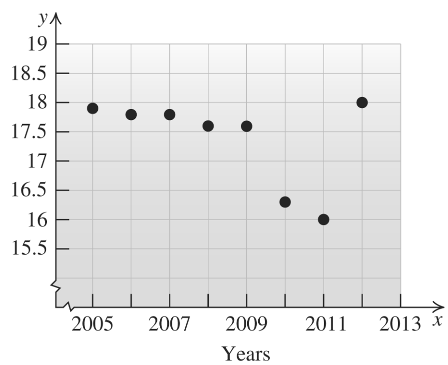
2.1 The Coordinate Plane

2.1 Practice Problems

1.



2. (2005, 17.9), (2006, 17.8), (2007, 17.8), (2008, 17.6), (2009, 17.6), (2010, 16.3), (2011, 16.0), (2012, 18.0)



3. $(x_1, y_1) = (-5, 2);\ (x_2, y_2) = (-4, 1)$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-4 - (-5))^2 + (1 - 2)^2} \\ &= \sqrt{1^2 + (-1)^2} = \sqrt{2} \approx 1.4 \end{aligned}$$

4. $(x_1, y_1) = (6, 2);\ (x_2, y_2) = (-2, 0)$
$(x_3, y_3) = (1, 5)$

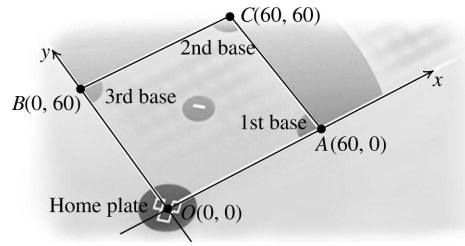
$$\begin{aligned} d_1 &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2 - 6)^2 + (0 - 2)^2} \\ &= \sqrt{(-8)^2 + (-2)^2} = \sqrt{68} \end{aligned}$$

$$\begin{aligned} d_2 &= \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2} \\ &= \sqrt{(1 - 6)^2 + (5 - 2)^2} \\ &= \sqrt{(-5)^2 + (3)^2} = \sqrt{34} \end{aligned}$$

$$\begin{aligned} d_3 &= \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2} \\ &= \sqrt{(1 - (-2))^2 + (5 - 0)^2} \\ &= \sqrt{(3)^2 + (5)^2} = \sqrt{34} \end{aligned}$$

Yes, the triangle is an isosceles triangle.

5.



We are asked to find the distance between the points $A(60, 0)$ and $B(0, 60)$.

$$\begin{aligned} d(A, B) &= \sqrt{(60 - 0)^2 + (0 - 60)^2} \\ &= \sqrt{(60)^2 + (-60)^2} = \sqrt{2(60)^2} \\ &= 60\sqrt{2} \approx 84.85 \end{aligned}$$

6. $M = \left(\dfrac{5+6}{2}, \dfrac{-2+(-1)}{2}\right) = \left(\dfrac{11}{2}, -\dfrac{3}{2}\right)$

2.1 Basic Concepts and Skills

1. A point with a negative first coordinate and a positive second coordinate lies in the second quadrant.

2. Any point on the x-axis has second coordinate 0.

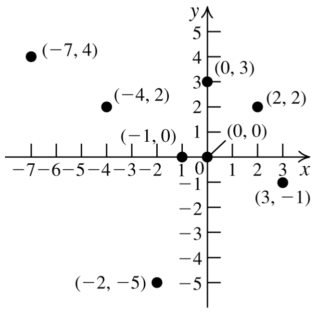
3. The distance between the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the formula $d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

4. The coordinates of the midpoint $M(x, y)$ of the line segment joining $P(x_1, y_1)$ and $Q(x_2, y_2)$ are given by $(x, y) = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

5. True

6. False. The point (7, −4) is 4 units to the right and 6 units below the point (3, 2).

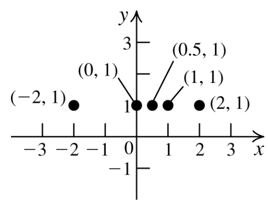
7.



(2, 2): Q1; (3, −1): Q4; (−1, 0): *x*-axis
(−2, −5): Q3; (0, 0): origin; (−7, 4): Q2
(0, 3): *y*-axis; (−4, 2): Q2

8. a. Answers will vary. Sample answer:
$(-2,0),\ (-1,0),(0,0),(1,0),(2,0)$
The *y*-coordinate is 0.

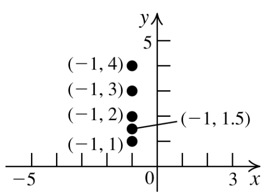
b.



The set of all points of the form (*x*, 1) is a horizontal line that intersects the *y*-axis at 1.

9. a. If the *x*-coordinate of a point is 0, the point lies on the *y*-axis.

b.



The set of all points of the form (–1, *y*) is a vertical line that intersects the *x*-axis at –1.

10. a. A vertical line that intersects the *x*-axis at –3.

b. A horizontal line that intersects the *y*-axis at 4.

11. a. $y > 0$ **b.** $y < 0$

c. $x < 0$ **d.** $x > 0$

12. a. Quadrant III **b.** Quadrant I

c. Quadrant IV **d.** Quadrant II

In Exercises 13–22, use the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ and the midpoint formula, $(x, y) = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

13. a. $d = \sqrt{(2-2)^2 + (5-1)^2} = \sqrt{4^2} = 4$

b. $M = \left(\dfrac{2+2}{2}, \dfrac{1+5}{2}\right) = (2,3)$

14. a. $d = \sqrt{(-2-3)^2 + (5-5)^2} = \sqrt{(-5)^2} = 5$

b. $M = \left(\dfrac{3+(-2)}{2}, \dfrac{5+5}{2}\right) = (0.5,5)$

15. a. $d = \sqrt{(2-(-1))^2 + (-3-(-5))^2}$
$= \sqrt{3^2 + 2^2} = \sqrt{13}$

b. $M = \left(\dfrac{-1+2}{2}, \dfrac{-5+(-3)}{2}\right) = (0.5,-4)$

16. a. $d = \sqrt{(-7-(-4))^2 + (-9-1)^2}$
$= \sqrt{(-3)^2 + (-10)^2} = \sqrt{109}$

b. $M = \left(\dfrac{-4+(-7)}{2}, \dfrac{1+(-9)}{2}\right) = (-5.5,-4)$

17. a. $d = \sqrt{(3-(-1))^2 + (-6.5-1.5)^2}$
$= \sqrt{4^2 + (-8)^2} = \sqrt{80} = 4\sqrt{5}$

b. $M = \left(\dfrac{-1+3}{2}, \dfrac{1.5+(-6.5)}{2}\right) = (1,-2.5)$

18. a. $d = \sqrt{(1-0.5)^2 + (-1-0.5)^2}$
$= \sqrt{(0.5)^2 + (-1.5)^2} = \sqrt{2.5} = \sqrt{\dfrac{5}{2}} = \dfrac{\sqrt{10}}{2}$

b. $M = \left(\dfrac{0.5+1}{2}, \dfrac{0.5+(-1)}{2}\right) = (0.75,-0.25)$

19. a. $d = \sqrt{\left(\sqrt{2}-\sqrt{2}\right)^2 + (5-4)^2} = \sqrt{1^2} = 1$

b. $M = \left(\dfrac{\sqrt{2}+\sqrt{2}}{2}, \dfrac{4+5}{2}\right) = \left(\sqrt{2},4.5\right)$

20. a. $d = \sqrt{((v+w)-(v-w))^2 + (t-t)^2}$
$= \sqrt{(2w)^2} = 2|w|$

b. $M = \left(\frac{(v-w)+(v+w)}{2}, \frac{t+t}{2} \right) = (v, t)$

21. a. $d = \sqrt{(k-t)^2 + (t-k)^2}$
$= \sqrt{\left(k^2 - 2tk + t^2\right) + \left(t^2 - 2kt + k^2\right)}$
$= \sqrt{2t^2 - 4tk + 2k^2} = \sqrt{2\left(t^2 - 2tk + k^2\right)}$
$= \sqrt{2(t-k)^2} = |t-k|\sqrt{2}$

b. $M = \left(\frac{t+k}{2}, \frac{k+t}{2} \right)$

22. a. $d = \sqrt{(-n-m)^2 + (-m-n)^2}$
$= \sqrt{\left(n^2 + 2mn + m^2\right) + \left(m^2 + 2mn + n^2\right)}$
$= \sqrt{2m^2 + 4mn + 2n^2}$
$= \sqrt{2\left(m^2 + 2mn + n^2\right)}$
$= \sqrt{2(m+n)^2} = \sqrt{2}\,|m+n|$

b. $M = \left(\frac{m+(-n)}{2}, \frac{n+(-m)}{2} \right)$
$= \left(\frac{m-n}{2}, \frac{n-m}{2} \right)$

23. $P = (-1, -2)$, $Q = (0, 0)$, $R = (1, 2)$
$d(P,Q) = \sqrt{(0-(-1))^2 + (0-(-2))^2} = \sqrt{5}$
$d(Q,R) = \sqrt{(1-0)^2 + (2-0)^2} = \sqrt{5}$
$d(P,R) = \sqrt{(1-(-1))^2 + (2-(-2))^2}$
$= \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$
Because $d(P, Q) + d(Q, R) = d(P, R)$, the points are collinear.

24. $P = (-3, -4)$, $Q = (0, 0)$, $R = (3, 4)$
$d(P,Q) = \sqrt{(0-(-3))^2 + (0-(-4))^2} = \sqrt{25} = 5$
$d(Q,R) = \sqrt{(3-0)^2 + (4-0)^2} = \sqrt{25} = 5$
$d(P,R) = \sqrt{(3-(-3))^2 + (4-(-4))^2}$
$= \sqrt{6^2 + 8^2} = \sqrt{100} = 10$
Because $d(P, Q) + d(Q, R) = d(P, R)$, the points are collinear.

25. $P = (4, -2)$, $Q = (1, 3)$, $R = (-2, 8)$
$d(P,Q) = \sqrt{(1-4)^2 + (3-(-2))^2} = \sqrt{34}$
$d(Q,R) = \sqrt{(-2-1)^2 + (8-3)^2} = \sqrt{34}$
$d(P,R) = \sqrt{(-2-4)^2 + (8-(-2))^2}$
$= \sqrt{(-6)^2 + 10^2} = \sqrt{136} = 2\sqrt{34}$
Because $d(P, Q) + d(Q, R) = d(P, R)$, the points are collinear.

26. It is not possible to arrange the points in such a way so that $d(P, Q) + d(Q, R) = d(P, R)$, so the points are not collinear.

27. $P = (-1, 4)$, $Q = (3, 0)$, $R = (11, -8)$
$d(P,Q) = \sqrt{(3-(-1))^2 + (0-4)^2} = 4\sqrt{2}$
$d(Q,R) = \sqrt{(11-3)^2 + ((-8)-0)^2} = 8\sqrt{2}$
$d(P,R) = \sqrt{(11-(-1))^2 + (-8-4)^2}$
$= \sqrt{(12)^2 + (-12)^2} = \sqrt{288} = 12\sqrt{2}$
Because $d(P, Q) + d(Q, R) = d(P, R)$, the points are collinear.

28. It is not possible to arrange the points in such a way so that $d(P, Q) + d(Q, R) = d(P, R)$, so the points are not collinear.

29. It is not possible to arrange the points in such a way so that $d(P, Q) + d(Q, R) = d(P, R)$, so the points are not collinear.

30. $P = (1, 7)$, $Q = (-3, 7.5)$, $R = (-7, 8)$
$d(P,Q) = \sqrt{(-3-1)^2 + (7.5-7)^2} = \sqrt{16.25}$
$d(Q,R) = \sqrt{(-7-(-3))^2 + (8-7.5)^2}$
$= \sqrt{16.25}$
$d(P,R) = \sqrt{(-7-1)^2 + (8-7)^2}$
$= \sqrt{(-8)^2 + 1^2} = \sqrt{65} = 2\sqrt{16.25}$
Because $d(P, Q) + d(Q, R) = d(P, R)$, the points are collinear.

31. First, find the midpoint M of PQ.
$M = \left(\frac{-4+0}{2}, \frac{0+8}{2} \right) = (-2, 4)$
Now find the midpoint R of PM.
$R = \left(\frac{-4+(-2)}{2}, \frac{0+4}{2} \right) = (-3, 2)$
Finally, find the midpoint S of MQ.
$S = \left(\frac{-2+0}{2}, \frac{4+8}{2} \right) = (-1, 6)$
Thus, the three points are (–3, 2), (–2, 4), and (–1, 6).

32. First, find the midpoint M of PQ.

$$M = \left(\frac{-8+16}{2}, \frac{4+(-12)}{2}\right) = (4, -4)$$

Now find the midpoint R of PM.

$$R = \left(\frac{-8+4}{2}, \frac{4+(-4)}{2}\right) = (-2, 0)$$

Finally, find the midpoint S of MQ.

$$S = \left(\frac{4+16}{2}, \frac{-4+(-12)}{2}\right) = (10, -8)$$

Thus, the three points are (−2, 0), (4, −4), and (10, −8).

33. $d(P,Q) = \sqrt{(-1-(-5))^2 + (4-5)^2} = \sqrt{17}$

$d(Q,R) = \sqrt{(-4-(-1))^2 + (1-4)^2} = 3\sqrt{2}$

$d(P,R) = \sqrt{(-4-(-5))^2 + (1-5)^2} = \sqrt{17}$

The triangle is isosceles.

34. $d(P,Q) = \sqrt{(6-3)^2 + (6-2)^2} = 5$

$d(Q,R) = \sqrt{(-1-6)^2 + (5-6)^2} = 5\sqrt{2}$

$d(P,R) = \sqrt{(-1-3)^2 + (5-2)^2} = 5$

The triangle is an isosceles triangle.

35. $d(P,Q) = \sqrt{(0-(-4))^2 + (7-8)^2} = \sqrt{17}$

$d(Q,R) = \sqrt{(-3-0)^2 + (5-7)^2} = \sqrt{13}$

$d(P,R) = \sqrt{(-3-(-4))^2 + (5-8)^2} = \sqrt{10}$

The triangle is scalene.

36. $d(P,Q) = \sqrt{(-1-6)^2 + (-1-6)^2} = 7\sqrt{2}$

$d(Q,R) = \sqrt{(-5-(-1))^2 + (3-(-1))^2} = 4\sqrt{2}$

$d(P,R) = \sqrt{(-5-6)^2 + (3-6)^2} = \sqrt{130}$

The triangle is scalene.

37. $d(P,Q) = \sqrt{(9-0)^2 + (-9-(-1))^2} = \sqrt{145}$

$d(Q,R) = \sqrt{(5-9)^2 + (1-(-9))^2} = 2\sqrt{29}$

$d(P,R) = \sqrt{(5-0)^2 + (1-(-1))^2} = \sqrt{29}$

The triangle is scalene.

38. $d(P,Q) = \sqrt{(4-(-4))^2 + (5-4)^2} = \sqrt{65}$

$d(Q,R) = \sqrt{(0-4)^2 + (-2-5)^2} = \sqrt{65}$

$d(P,R) = \sqrt{(0-(-4))^2 + (-2-4)^2} = 2\sqrt{13}$

The triangle is isosceles.

39. $d(P,Q) = \sqrt{(-1-1)^2 + (1-(-1))^2} = 2\sqrt{2}$

$$\begin{aligned} d(Q,R) &= \sqrt{\left(-\sqrt{3}-(-1)\right)^2 + \left(-\sqrt{3}-1\right)^2} \\ &= \sqrt{\left(3-2\sqrt{3}+1\right) + \left(3+2\sqrt{3}+1\right)} \\ &= \sqrt{8} = 2\sqrt{2} \end{aligned}$$

$$\begin{aligned} d(P,R) &= \sqrt{\left(-\sqrt{3}-1\right)^2 + \left(-\sqrt{3}-(-1)\right)^2} \\ &= \sqrt{\left(3+2\sqrt{3}+1\right) + \left(3-2\sqrt{3}+1\right)} \\ &= \sqrt{8} = 2\sqrt{2} \end{aligned}$$

The triangle is equilateral.

40.
$$\begin{aligned} d(P,Q) &= \sqrt{(-1.5-(-0.5))^2 + (1-(-1))^2} \\ &= \sqrt{5} \end{aligned}$$

$$\begin{aligned} d(Q,R) &= \sqrt{\left(\left(\sqrt{3}-1\right)-(-1.5)\right)^2 + \left(\frac{\sqrt{3}}{2}-1\right)^2} \\ &= \sqrt{\left(\left(\sqrt{3}-1\right)^2 + 3\left(\sqrt{3}-1\right) + 2.25\right) + \left(\frac{3}{4}-\sqrt{3}+1\right)} \\ &= \sqrt{\left(3-2\sqrt{3}+1+3\sqrt{3}-3+2.25\right) + \left(1.75-\sqrt{3}\right)} \\ &= \sqrt{5} \end{aligned}$$

$$\begin{aligned} d(P,R) &= \sqrt{\left(\left(\sqrt{3}-1\right)-(-0.5)\right)^2 + \left(\frac{\sqrt{3}}{2}-(-1)\right)^2} \\ &= \sqrt{\left(\left(\sqrt{3}-1\right)^2 + \left(\sqrt{3}-1\right) + 0.25\right) + \left(\frac{3}{4}+\sqrt{3}+1\right)} \\ &= \sqrt{\left(3-2\sqrt{3}+1+\sqrt{3}-1+0.25\right) + \left(1.75+\sqrt{3}\right)} \\ &= \sqrt{5} \end{aligned}$$

The triangle is equilateral.

41. First find the lengths of the sides:

$d(P,Q) = \sqrt{(-1-7)^2 + (3-(-12))^2} = 17$

$d(Q,R) = \sqrt{(14-(-1))^2 + (11-3)^2} = 17$

$d(R,S) = \sqrt{(22-14)^2 + (-4-11)^2} = 17$

$d(S,P) = \sqrt{(22-7)^2 + (-4-(-12))^2} = 17$

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(*continued*)

All the sides are equal, so the quadrilateral is either a square or a rhombus. Now find the length of the diagonals:

$d(P,R)=\sqrt{(14-7)^2+(11-(-12))^2}=17\sqrt{2}$

$d(Q,S)=\sqrt{(22-(-1))^2+(-4-3)^2}=17\sqrt{2}$

The diagonals are equal, so the quadrilateral is a square.

42. First find the lengths of the sides:

$d(P,Q)=\sqrt{(9-8)^2+(-11-(-10))^2}=\sqrt{2}$

$d(Q,R)=\sqrt{(8-9)^2+(-12-(-11))^2}=\sqrt{2}$

$d(R,S)=\sqrt{(7-8)^2+(-11-(-12))^2}=\sqrt{2}$

$d(S,P)=\sqrt{(8-7)^2+(-10-(-11))^2}=\sqrt{2}$

All the sides are equal, so the quadrilateral is either a square or a rhombus. Now find the length of the diagonals.

$d(P,R)=\sqrt{(8-8)^2+(-12-(-10))^2}=2$

$d(Q,S)=\sqrt{(7-9)^2+(-11-(-11))^2}=2$

The diagonals are equal, so the quadrilateral is a square.

43.
$$\begin{aligned}5&=\sqrt{(x-2)^2+(2-(-1))^2}\\&=\sqrt{x^2-4x+4+9}\Rightarrow\end{aligned}$$
$$5=\sqrt{x^2-4x+13}\Rightarrow 25=x^2-4x+13\Rightarrow$$
$$0=x^2-4x-12\Rightarrow 0=(x-6)(x+2)\Rightarrow$$
$$x=-2 \text{ or } x=6$$

44.
$$\begin{aligned}13&=\sqrt{(2-(-10))^2+(y-(-3))^2}\\&=\sqrt{144+y^2+6y+9}\\&=\sqrt{y^2+6y+153}\Rightarrow\end{aligned}$$
$$169=y^2+6y+153$$
$$0=y^2+6y-16\Rightarrow 0=(y+8)(y-2)\Rightarrow$$
$$y=-8 \text{ or } y=2$$

45. $P=(-5,2)$, $Q=(2,3)$, $R=(x,0)$ (R is on the x-axis, so the y-coordinate is 0).

$d(P,R)=\sqrt{(x-(-5))^2+(0-2)^2}$

$d(Q,R)=\sqrt{(x-2)^2+(0-3)^2}$

$$\begin{aligned}\sqrt{(x-(-5))^2+(0-2)^2}&=\sqrt{(x-2)^2+(0-3)^2}\\(x+5)^2+(0-2)^2&=(x-2)^2+(0-3)^2\\x^2+10x+25+4&=x^2-4x+4+9\\10x+29&=-4x+13\\14x&=-16\\x&=-\frac{8}{7}\end{aligned}$$

The coordinates of R are $\left(-\frac{8}{7},0\right)$.

46. $P=(7,-4)$, $Q=(8,3)$, $R=(0,y)$ (R is on the y-axis, so the x-coordinate is 0).

$d(P,R)=\sqrt{(0-7)^2+(y-(-4))^2}$

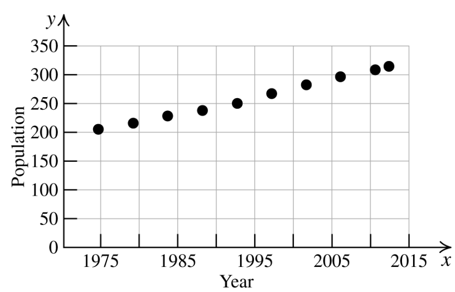
$d(Q,R)=\sqrt{(0-8)^2+(y-3)^2}$

$$\begin{aligned}\sqrt{(0-7)^2+(y-(-4))^2}&\\=\sqrt{(0-8)^2+(y-3)^2}&\\49+(y-(-4))^2&=64+(y-3)^2\\49+y^2+8y+16&=64+y^2-6y+9\\8y+65&=-6y+73\\14y&=8\\y&=\frac{4}{7}\end{aligned}$$

The coordinates of R are $\left(0,\frac{4}{7}\right)$.

2.1 Applying the Concepts

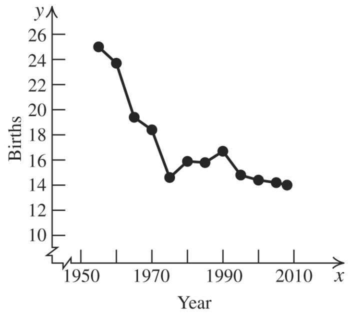
47.



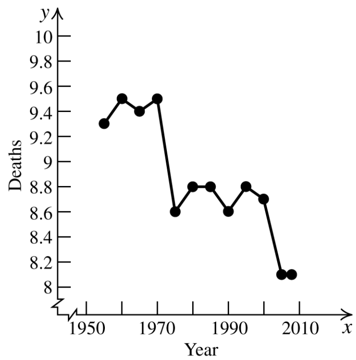
48.
$$\begin{aligned}M&=\left(\frac{2000+2012}{2},\ \frac{282+314}{2}\right)\\&=(2006,\ 298)\end{aligned}$$

The midpoint of the segment gives a good approximation to the actual value, about 300 million.

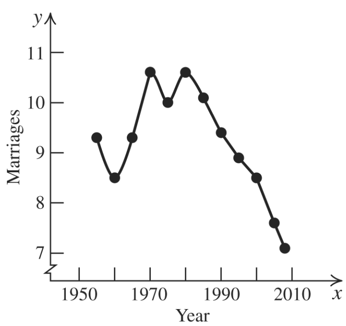
49.



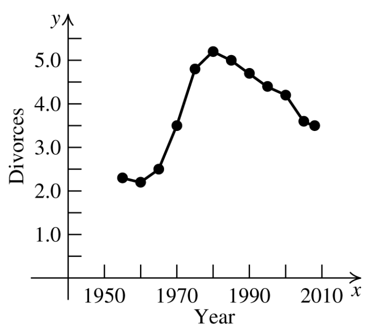
50.



51.



52.



53. $M = \dfrac{16{,}929 + 14{,}612}{2} = 15{,}770.5$

There were about 15,771 murders in 2009.

54. 2009 is the midpoint of the initial range, so

$M_{2009} = \dfrac{228 + 320}{2} = 274$.

2008 is the midpoint of the range

[2007, 2009], so $M_{2008} = \dfrac{228 + 274}{2} = 251$.

2010 is the midpoint of the range

[2009, 2011], so $M_{2010} = \dfrac{274 + 320}{2} = 297$.

So, in 2008, \$251 billion was spent; in 2009, \$274 billion was spent, and \$297 was spent in 2010.

55. 2008 is the midpoint of the initial range, so

$M_{2008} = \dfrac{548 + 925}{2} = 736.5$.

2006 is the midpoint of the range [2004, 2008], so

$M_{2006} = \dfrac{548 + 736.5}{2} = 642.25$.

2005 is the midpoint of the range [2004, 2006], so

$M_{2005} = \dfrac{548 + 642.25}{2} = 595.125$.

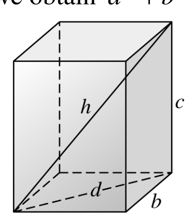
2007 is the midpoint of the range [2006, 2008], so

$M_{2007} = \dfrac{642.25 + 736.5}{2} = 689.375$

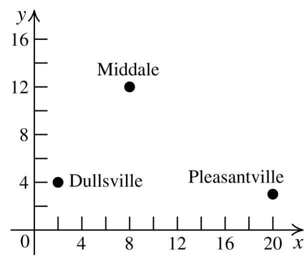
Use similar reasoning to find the amounts for 2009, 2010, and 2011. Defense spending was as follows:

Year	Amount spent
2004	\$548 billion
2005	\$595.125 billion
2006	\$642.25 billion
2007	\$689.375 billion
2008	\$736.5 billion
2009	\$783.625 billion
2010	\$830.75 billion
2011	\$877.875 billion
2012	\$925 billion

56. Denote the diagonal connecting the endpoints of the edges *a* and *b* by *d*. Then *a*, *b*, and *d* form a right triangle. By the Pythagorean theorem, $a^2 + b^2 = d^2$. The edge *c* and the diagonals *d* and *h* also form a right triangle, so $c^2 + d^2 = h^2$. Substituting d^2 from the first equation, we obtain $a^2 + b^2 + c^2 = h^2$.



57. a.



b. $d(D,M) = \sqrt{(800-200)^2 + (1200-400)^2}$
$= 1000$
$d(M, P) = \sqrt{(2000-800)^2 + (300-1200)^2}$
$= 1500$
The distance traveled by the pilot
= 1000 + 1500 = 2500 miles.

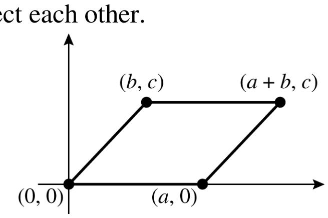
c. $d(D,P) = \sqrt{(2000-200)^2 + (300-400)^2}$
$= \sqrt{3,250,000} = 500\sqrt{13}$
≈ 1802.78 miles

58. First, find the initial length of the rope using the Pythagorean theorem:
$c = \sqrt{24^2 + 10^2} = 26$.
After t seconds, the length of the rope is $26 - 3t$. Now find the distance from the boat to the dock, x, using the Pythagorean theorem again and solving for x:

$$(26-3t)^2 = x^2 + 10^2$$
$$676 - 156t + 9t^2 = x^2 + 100$$
$$576 - 156t + 9t^2 = x^2$$
$$\sqrt{576 - 156t + 9t^2} = x$$

2.1 Beyond the Basics

59. The midpoint of the diagonal connecting (0, 0) and $(a + b, c)$ is $\left(\frac{a+b}{2}, \frac{c}{2}\right)$. The midpoint of the diagonal connecting $(a, 0)$ and (b, c) is also $\left(\frac{a+b}{2}, \frac{c}{2}\right)$. Because the midpoints of the two diagonals are the same, the diagonals bisect each other.



60. a. If AB is one of the diagonals, then DC is the other diagonal, and both diagonals have the same midpoint. The midpoint of AB is $\left(\frac{2+5}{2}, \frac{3+4}{2}\right) = (3.5, 3.5)$. The midpoint of $DC = (3.5, 3.5) = \left(\frac{x+3}{2}, \frac{y+8}{2}\right)$.

So we have $3.5 = \frac{x+3}{2} \Rightarrow x = 4$ and $3.5 = \frac{y+8}{2} \Rightarrow y = -1$.
The coordinates of D are (4, –1).

b. If AC is one of the diagonals, then DB is the other diagonal, and both diagonals have the same midpoint. The midpoint of AC is $\left(\frac{2+3}{2}, \frac{3+8}{2}\right) = (2.5, 5.5)$. The midpoint of $DB = (2.5, 5.5) = \left(\frac{x+5}{2}, \frac{y+4}{2}\right)$.

So we have $2.5 = \frac{x+5}{2} \Rightarrow x = 0$ and $5.5 = \frac{y+4}{2} \Rightarrow y = 7$.
The coordinates of D are (0, 7).

c. If BC is one of the diagonals, then DA is the other diagonal, and both diagonals have the same midpoint. The midpoint of BC is $\left(\frac{5+3}{2}, \frac{4+8}{2}\right) = (4, 6)$. The midpoint of DA is $(4, 6) = \left(\frac{x+2}{2}, \frac{y+3}{2}\right)$. So we have

$4 = \frac{x+2}{2} \Rightarrow x = 6$ and $6 = \frac{y+3}{2} \Rightarrow y = 9$.
The coordinates of D are (6, 9).

61. The midpoint of the diagonal connecting (0, 0) and (x, y) is $\left(\frac{x}{2}, \frac{y}{2}\right)$. The midpoint of the diagonal connecting $(a, 0)$ and (b, c) is $\left(\frac{a+b}{2}, \frac{c}{2}\right)$. Because the diagonals bisect each other, the midpoints coincide. So $\frac{x}{2} = \frac{a+b}{2} \Rightarrow x = a + b$, and $\frac{y}{2} = \frac{c}{2} \Rightarrow y = c$.
Therefore, the quadrilateral is a parallelogram.

62. a. The midpoint of the diagonal connecting (1, 2) and (5, 8) is $\left(\frac{1+5}{2}, \frac{2+8}{2}\right) = (3, 5).$ The midpoint of the diagonal connecting (–2, 6) and (8, 4) is $\left(\frac{-2+8}{2}, \frac{6+4}{2}\right) = (3, 5).$ Because the midpoints are the same, the figure is a parallelogram.

b. The midpoint of the diagonal connecting (3, 2) and (*x*, *y*) is $\left(\frac{3+x}{2}, \frac{2+y}{2}\right).$ The midpoint of the diagonal connecting (6, 3) and (6, 5) is (6, 4). So $\frac{3+x}{2} = 6 \Rightarrow x = 9$ and $\frac{2+y}{2} = 4 \Rightarrow y = 6$.

63. Let *P*(0, 0), *Q*(*a*, 0), *R*(*a* + *b*, *c*), and *S*(*b*, *c*) be the vertices of the parallelogram. $PQ = RS = \sqrt{(a-0)^2 + (0-0)^2} = a.$ $QR = PS = \sqrt{((a+b)-a)^2 + (c-0)^2} = \sqrt{b^2 + c^2}.$
The sum of the squares of the lengths of the sides $= 2(a^2 + b^2 + c^2).$

$$d(P, R) = \sqrt{(a+b)^2 + c^2}.$$

$$d(Q, S) = \sqrt{(a-b)^2 + (0-c)^2}.$$

The sum of the squares of the lengths of the diagonals is

$$\left((a+b)^2 + c^2\right) + \left((a-b)^2 + c^2\right) =$$
$$a^2 + 2ab + b^2 + c^2 + a^2 - 2ab + b^2 + c^2 =$$
$$2a^2 + 2b^2 + 2c^2 = 2(a^2 + b^2 + c^2).$$

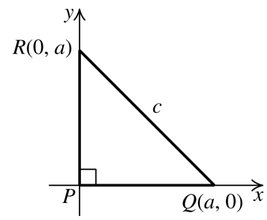
64. Let *P*(0, 0), *Q*(*a*, 0), and *R*(0,*b*) be the vertices of the right triangle. The midpoint *M* of the hypotenuse is $\left(\frac{a}{2}, \frac{b}{2}\right).$

$$d(Q, M) = \sqrt{\left(a - \frac{a}{2}\right)^2 + \left(0 - \frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2} = \frac{\sqrt{a^2 + b^2}}{2}$$

$$d(R, M) = \sqrt{\left(0 - \frac{a}{2}\right)^2 + \left(b - \frac{b}{2}\right)^2} = \sqrt{\left(-\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2} = \frac{\sqrt{a^2 + b^2}}{2}$$

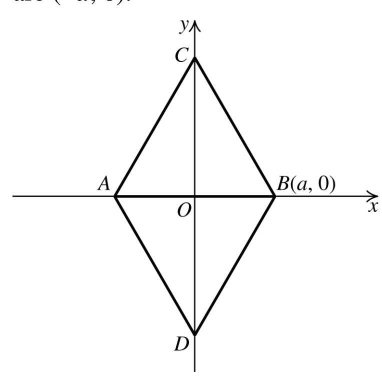
$$d(P, M) = \sqrt{\left(0 - \frac{a}{2}\right)^2 + \left(0 - \frac{b}{2}\right)^2} = \sqrt{\left(-\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2} = \frac{\sqrt{a^2 + b^2}}{2}$$

65. Let *P*(0, 0), *Q*(*a*, 0), and *R*(0, *a*) be the vertices of the triangle.



Using the Pythagorean theorem, we have
$c^2 = a^2 + a^2 \Rightarrow c^2 = 2a^2 \Rightarrow c = \sqrt{2}a \Rightarrow$
$a = \frac{1}{\sqrt{2}}c = \frac{\sqrt{2}}{2}c$

66. Since *ABC* is an equilateral triangle and *O* is the midpoint of *AB*, then the coordinates of *A* are (–*a*, 0).



AB = *AC* = *AB* = 2*a*. Using triangle *BOC* and the Pythagorean theorem, we have
$BC^2 = OB^2 + OC^2 \Rightarrow (2a)^2 = a^2 + OC^2 \Rightarrow$
$4a^2 = a^2 + OC^2 \Rightarrow 3a^2 = OC^2 \Rightarrow OC = \sqrt{3}a$
Thus, the coordinates of *C* are $\left(0, \sqrt{3}a\right)$ and the coordinates of *D* are $\left(0, -\sqrt{3}a\right)$.

67. a. $d(A,C) = \sqrt{\left(x_1 - \frac{2x_1 + x_2}{3}\right)^2 + \left(y_1 - \frac{2y_1 + y_2}{3}\right)^2} = \sqrt{\left(\frac{x_1 - x_2}{3}\right)^2 + \left(\frac{y_1 - y_2}{3}\right)^2}$

$d(C,B) = \sqrt{\left(\frac{2x_1 + x_2}{3} - x_2\right)^2 + \left(\frac{2y_1 + y_2}{3} - y_2\right)^2} = \sqrt{\left(\frac{2x_1 - 2x_2}{3}\right)^2 + \left(\frac{2y_1 - 2y_2}{3}\right)^2}$

$$d(A,C) + d(C,B) = \frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{3} + \frac{\sqrt{(2x_1 - 2x_2)^2 + (2y_1 - 2y_2)^2}}{3}$$
$$= \frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{3} + \frac{2\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{3} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$d(A,B) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

So A, B, and C are collinear.

$$d(A,C) = \sqrt{\left(\frac{x_1 - x_2}{3}\right)^2 + \left(\frac{y_1 - y_2}{3}\right)^2} = \frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{3} = \frac{1}{3}d(A,B).$$

b. $d(A,D) = \sqrt{\left(x_1 - \frac{x_1 + 2x_2}{3}\right)^2 + \left(y_1 - \frac{y_1 + 2y_2}{3}\right)^2} = \sqrt{\left(\frac{2x_1 + 2x_2}{3}\right)^2 + \left(\frac{2y_1 + 2y_2}{3}\right)^2}$

$d(D,B) = \sqrt{\left(\frac{x_1 + 2x_2}{3} - x_2\right)^2 + \left(\frac{y_1 + 2y_2}{3} - y_2\right)^2} = \sqrt{\left(\frac{x_1 - x_2}{3}\right)^2 + \left(\frac{y_1 - y_2}{3}\right)^2}$

$$d(A,D) + d(D,B) = \frac{\sqrt{(2x_1 + 2x_2)^2 + (2y_1 + 2y_2)^2}}{3} + \frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{3}$$
$$= \frac{2\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{3} + \frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{3}$$
$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = d(A,\ B)$$

So A, B, and C are collinear.

$$d(A,D) = \sqrt{\left(\frac{2x_1 + 2x_2}{3}\right)^2 + \left(\frac{2y_1 - 2y_2}{3}\right)^2} = \frac{2\sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2}}{3} = \frac{2}{3}d(A,B).$$

c. $\frac{2x_1 + x_2}{3} = \frac{2(-1) + 4}{3} = \frac{2}{3}$

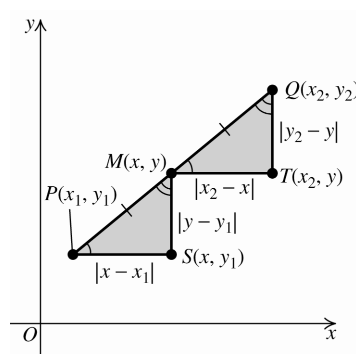
$\frac{2y_1 + y_2}{3} = \frac{2(2) + 1}{3} = \frac{5}{3}$

$\frac{x_1 + 2x_2}{3} = \frac{-1 + 2(4)}{3} = \frac{7}{3}$

$\frac{y_1 + 2y_2}{3} = \frac{2 + 2(1)}{3} = \frac{4}{3}$

The points of trisection are $\left(\frac{2}{3}, \frac{5}{3}\right)$ and $\left(\frac{7}{3}, \frac{4}{3}\right)$.

68.



To show that M is the midpoint of the line segment PQ, we need to show that the distance between M and Q is the same as the distance between M and P and that this distance is half the distance from P to Q.

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(*continued*)

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$MP = \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2}$$
$$= \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2}$$
$$= \sqrt{\frac{(x_2 - x_1)^2}{4} + \frac{(y_2 - y_1)^2}{4}}$$
$$= \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Thus, we have $MP = \frac{1}{2}PQ$.

Similarly, we can show that $MQ = \frac{1}{2}PQ$.

Thus, M is the midpoint of PQ, and
$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

2.1 Critical Thinking/Discussion/Writing

69. a. y-axis

b. x-axis

70. a. The union of the x- and y-axes

b. The plane without the x- and y-axes

71. a. Quadrants I and III

b. Quadrants II and IV

72. a. The origin

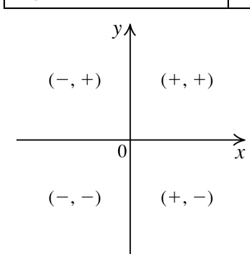
b. The plane without the origin

73. a. Right half-plane

b. Upper half-plane

74. Let (x, y) be the point.

The point lies in	if
Quadrant I	$x > 0$ and $y > 0$
Quadrant II	$x < 0$ and $y > 0$
Quadrant III	$x < 0$ and $y < 0$
Quadrant IV	$x > 0$ and $y < 0$



2.1 Maintaining Skills

75. a. $x^2 + y^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

b. $x^2 + y^2 = \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} + \frac{2}{4} = 1$

76. a. $(x-1)^2 + (y+2)^2 = [(-1)-1]^2 + (1+2)^2$
$= (-2)^2 + 3^2 = 4 + 9 = 13$

b. $(x-1)^2 + (y+2)^2 = (4-1)^2 + (2+2)^2$
$= 3^2 + 4^2 = 9 + 16 = 25$

77. a. $\frac{x}{|x|} + \frac{|y|}{y} = \frac{2}{|2|} + \frac{|-3|}{-3} = \frac{2}{2} + \frac{3}{-3} = 1 - 1 = 0$

b. $\frac{x}{|x|} + \frac{|y|}{y} = \frac{-4}{|-4|} + \frac{|3|}{3} = \frac{-4}{4} + \frac{3}{3} = -1 + 1 = 0$

78. a. $\frac{|x|}{x} + \frac{|y|}{y} = \frac{|-1|}{-1} + \frac{|-2|}{-2} = \frac{1}{-1} + \frac{2}{-2}$
$= -1 + (-1) = -2$

b. $\frac{|x|}{x} + \frac{|y|}{y} = \frac{|3|}{3} + \frac{|2|}{2} = \frac{3}{3} + \frac{2}{2} = 1 + 1 = 2$

79. $x^2 - 6x + \left(\frac{-6}{2}\right)^2 = x^2 - 6x + 3^2$
$= x^2 - 6x + \underline{9}$

80. $x^2 - 8x + \left(\frac{-8}{2}\right)^2 = x^2 - 8x + 4^2$
$= x^2 - 8x + \underline{16}$

81. $y^2 + 3y = y^2 + 3y + \left(\frac{3}{2}\right)^2 = y^2 + 3y + \underline{\frac{9}{4}}$

82. $y^2 + 5y + \left(\frac{5}{2}\right)^2 = y^2 + 5y + \underline{\frac{25}{4}}$

83. $x^2 - ax + \left(\frac{-a}{2}\right)^2 = x^2 - ax + \underline{\frac{a^2}{4}}$

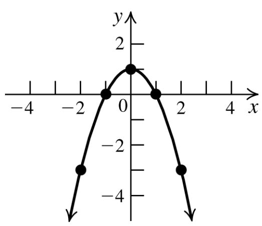
84. $x^2 + xy + \left(\frac{y}{2}\right)^2 = x^2 + xy + \underline{\frac{y^2}{4}}$

Section 2.2 Graphs of Equations

2.2 Practice Problems

1. $y = -x^2 + 1$

x	$y = -x^2 + 1$	(x, y)
−2	$y = -(-2)^2 + 1$	(−2, −3)
−1	$y = -(-1)^2 + 1$	(−1, 0)
0	$y = -(0)^2 + 1$	(0, 1)
1	$y = -(1)^2 + 1$	(1, 0)
2	$y = -(2)^2 + 1$	(2, −3)



2. To find the x-intercept, let $y = 0$, and solve the equation for x: $0 = 2x^2 + 3x - 2 \Rightarrow$ $0 = (2x-1)(x+2) \Rightarrow x = \frac{1}{2}$ or $x = -2$. To find the y-intercept, let $x = 0$, and solve the equation for y:
$y = 2(0)^2 + 3(0) - 2 \Rightarrow y = -2.$
The x-intercepts are $\frac{1}{2}$ and −2; the y-intercept is −2.

3. To test for symmetry about the y-axis, replace x with $-x$ to determine if $(-x, y)$ satisfies the equation.
$(-x)^2 - y^2 = 1 \Rightarrow x^2 - y^2 = 1$, which is the same as the original equation. So the graph is symmetric about the y-axis.

4. x-axis: $x^2 = (-y)^3 \Rightarrow x^2 = -y^3$, which is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
y-axis: $(-x)^2 = y^3 \Rightarrow x^2 = y^3$, which is the same as the original equation, so the equation is symmetric with respect to the y-axis.
origin: $(-x)^2 = (-y)^3 \Rightarrow x^2 = -y^3$, which is not the same as the original equation, so the equation is not symmetric with respect to the origin.

5. $y = -t^4 + 77t^2 + 324$

a. First, find the intercepts. If $t = 0$, then $y = 324$, so the y-intercept is (0, 324). If $y = 0$, then we have

$$0 = -t^4 + 77t^2 + 324$$
$$t^4 - 77t^2 - 324 = 0$$
$$(t^2 - 81)(t^2 + 4) = 0$$
$$(t+9)(t-9)(t^2+4) = 0 \Rightarrow t = -9, 9, \pm 2i$$

So, the t-intercepts are (−9, 0) and (9, 0). Next, check for symmetry.
t-axis: $-y = -t^4 + 77t^2 + 324$ is not the same as the original equation, so the equation is not symmetric with respect to the t-axis.
y-axis: $y = -(-t)^4 + 77(-t)^2 + 324 \Rightarrow$
$y = -t^4 + 77t^2 + 324$, which is the same as the original equation. So the graph is symmetric with respect to the y-axis.
origin: $-y = -(-t)^4 + 77(-t)^2 + 324 \Rightarrow$
$-y = -t^4 + 77t^2 + 324$, which is not the same as the original equation. So the graph is not symmetric with respect to the origin. Now, make a table of values. Since the graph is symmetric with respect to the y-axis, if (t, y) is on the graph, then so is $(-t, y)$. However, the graph pertaining to the physical aspects of the problem consists only of those values for $t \geq 0$.

t	$y = -t^4 + 77t^2 + 324$	(t, y)
0	324	(0, 324)
1	400	(1, 400)
2	616	(2, 616)
3	936	(3, 936)
4	1300	(4, 1300)
5	1624	(5, 1624)
6	1800	(6, 1800)
7	1696	(7, 1696)
8	1156	(8, 1156)
9	0	(9, 0)

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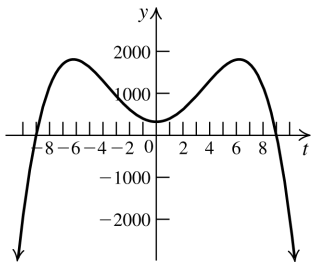
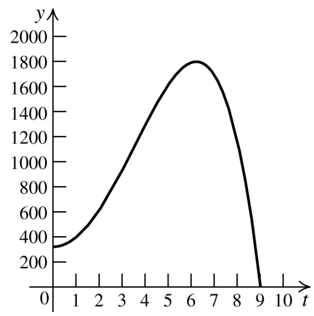


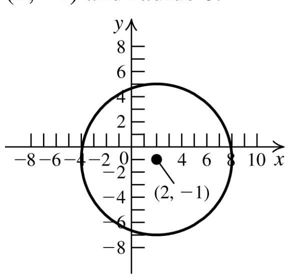
b.



c. The population becomes extinct after 9 years.

6. The standard form of the equation of a circle is $(x-h)^2+(y-k)^2=r^2$
$(h, k) = (3, -6)$ and $r = 10$
The equation of the circle is
$(x-3)^2+(y+6)^2=100.$

7. $(x-2)^2+(y+1)^2=36 \Rightarrow (h,k)=(2,-1),\ r=6$
This is the equation of a circle with center $(2, -1)$ and radius 6.



8. $x^2+y^2+4x-6y-12=0 \Rightarrow$
$x^2+4x+y^2-6y=12$
Now complete the square:
$x^2+4x+4+y^2-6y+9=12+4+9 \Rightarrow$
$(x+2)^2+(y-3)^2=25$
This is a circle with center $(-2, 3)$ and radius 5.

2.2 Basic Concepts and Skills

1. The graph of an equation in two variables, such as x and y, is the set of all ordered pairs (a, b) that satisfy the equation.

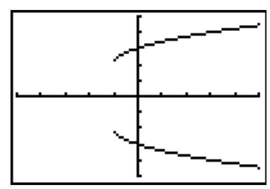
2. If $(-2, 4)$ is a point on a graph that is symmetric with respect to the y-axis, then the point $(2, 4)$ is also on the graph.

3. If $(0, -5)$ is a point of a graph, then -5 is a y- intercept of the graph.

4. An equation in standard form of a circle with center $(1, 0)$ and radius 2 is $(x-1)^2+y^2=4$.

5. False. The equation of a circle has both an x^2-term and a y^2-term. The given equation does not have a y^2-term.

6. False. The graph below is an example of a graph that is symmetric about the x-axis, but does not have an x-intercept.



In exercises 7–14, to determine if a point lies on the graph of the equation, substitute the point's coordinates into the equation to see if the resulting statement is true.

7. on the graph: $(-3, -4)$, $(1, 0)$, $(4, 3)$; not on the graph: $(2, 3)$

8. on the graph: $(-1, 1)$, $(1, 4)$, $\left(-\frac{5}{3}, 0\right)$; not on the graph: $(0, 2)$

9. on the graph: $(3, 2)$, $(0, 1)$, $(8, 3)$; not on the graph: $(8, -3)$

10. on the graph: $(1, 1)$, $\left(2, \frac{1}{2}\right)$; not on the graph: $(0, 0)$, $\left(-3, \frac{1}{3}\right)$

11. on the graph: $(1, 0)$, $\left(2, \sqrt{3}\right), \left(2, -\sqrt{3}\right)$; not on the graph: $(0, -1)$

12. Each point is on the graph.

13. a. x-intercepts: $-3, 3$; no y-intercepts

b. Symmetric about the x-axis, y-axis, and origin.

14. a. No x-intercepts; y-intercepts: –2, 2

b. Symmetric about the x-axis, y-axis, and origin.

15. a. x-intercepts: –3, 3; y-intercepts: –2, 2

b. Symmetric about the x-axis, y-axis, and origin.

16. a. x-intercepts: –3, 3; y-intercepts: –3, 3

b. Symmetric about the x-axis, y-axis, and origin.

17. a. x-intercept: 0; y-intercept: 0

b. Symmetric about the x-axis.

18. a. x-intercepts: $-\pi, 0, \pi$; y-intercept: 0

b. Symmetric about the origin.

19. a. x-intercepts: –3, 3; y-interceps: 2

b. Symmetric about the y-axis.

20. a. x-intercepts: $-\dfrac{3\pi}{2}, -\dfrac{\pi}{2}, \dfrac{\pi}{2}, \dfrac{3\pi}{2}$; y-intercept: 2

b. Symmetric about the y-axis.

21. a. No x-intercept; y-intercept: 1

b. No symmetries

22. a. x-intercept: 1; no y-intercepts

b. No symmetries

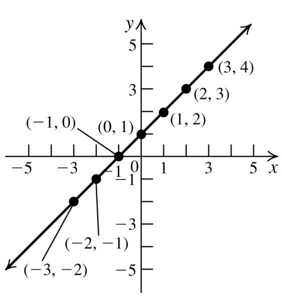
23. a. x-intercepts: 0, 3; y-intercept: 0

b. Symmetric about the x-axis.

24. a. x-intercept: 0; y-intercept: 0

b. Symmetric about the x-axis, y-axis, and origin.

25.



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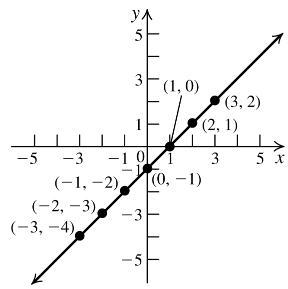

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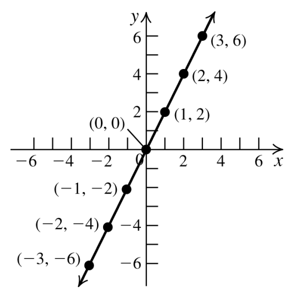

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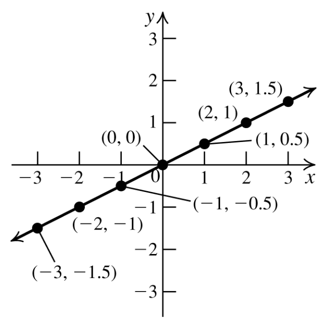

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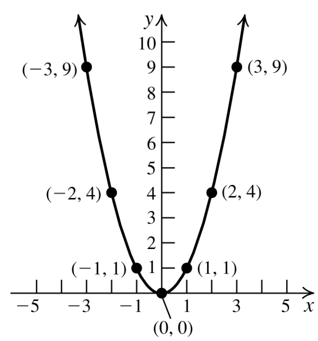

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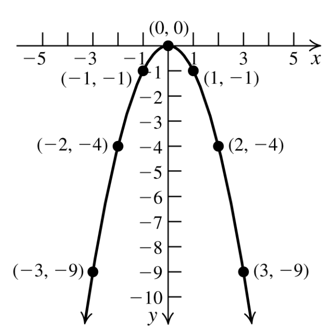

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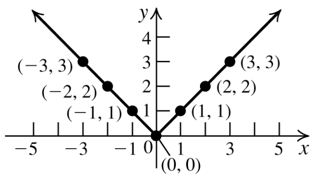

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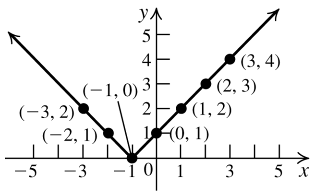

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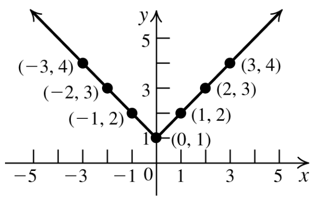

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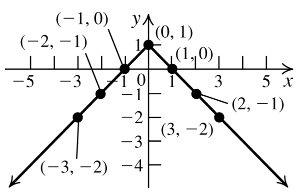

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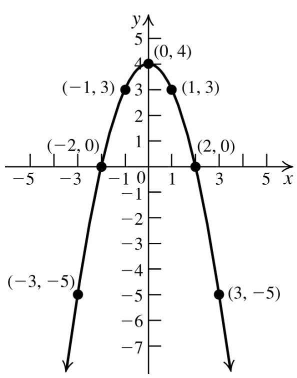

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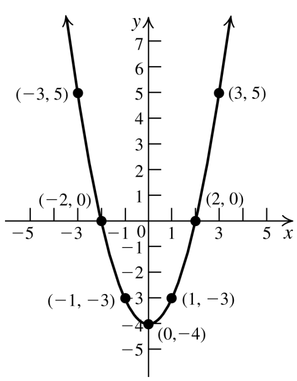

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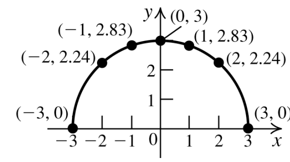

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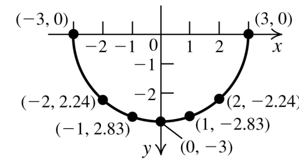

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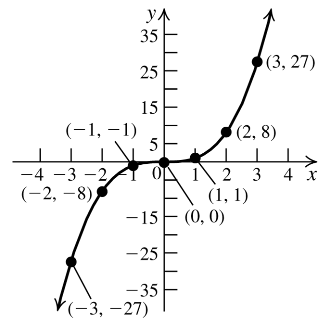

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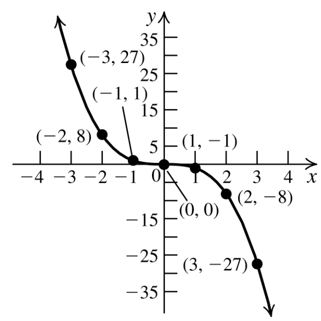

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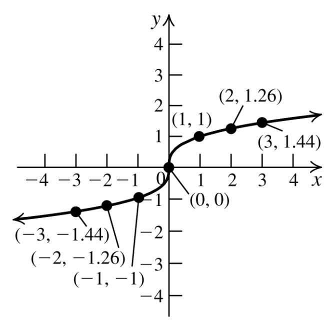

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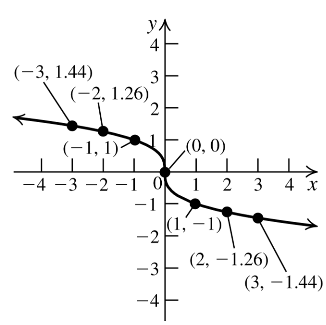

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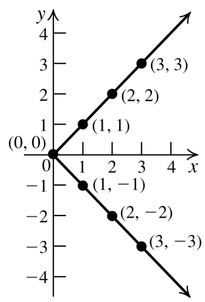

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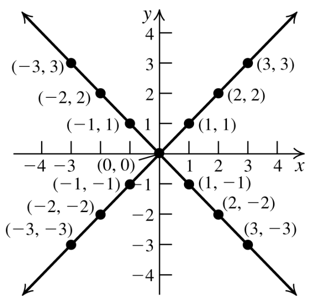

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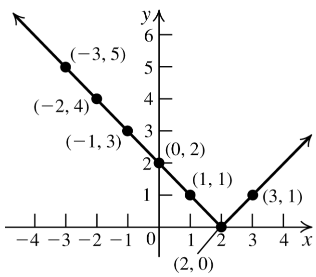

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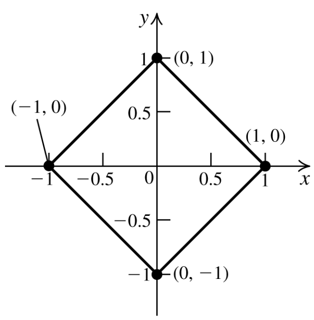


47. To find the x-intercept, let $y = 0$, and solve the equation for x: $3x + 4(0) = 12 \Rightarrow x = 4$. To find the y-intercept, let $x = 0$, and solve the equation for y: $3(0) + 4y = 12 \Rightarrow y = 3$. The x-intercept is 4; the y-intercept is 3.

48. To find the x-intercept, let $y = 0$, and solve the equation for x: $\frac{x}{5} + \frac{0}{3} = 1 \Rightarrow x = 5$. To find the y-intercept, let $x = 0$, and solve the equation for y: $\frac{0}{5} + \frac{y}{3} = 1 \Rightarrow y = 3$. The x-intercept is 5; the y-intercept is 3.

49. To find the x-intercept, let $y = 0$, and solve the equation for x: $0 = x^2 - 6x + 8 \Rightarrow x = 4$ or $x = 2$. To find the y-intercept, let $x = 0$, and solve the equation for y: $y = 0^2 - 6(0) + 8 \Rightarrow y = 8$. The x-intercepts are 2 and 4; the y-intercept is 8.

50. To find the x-intercept, let $y = 0$, and solve the equation for x: $x = 0^2 - 5(0) + 6 \Rightarrow x = 6$. To find the y-intercept, let $x = 0$, and solve the equation for y: $0 = y^2 - 5y + 6 \Rightarrow y = 2$ or $y = 3$. The x-intercept is 6; the y-intercepts are 2 and 3.

51. To find the x-intercept, let $y = 0$, and solve the equation for x: $x^2 + 0^2 = 4 \Rightarrow x = \pm 2$. To find the y-intercept, let $x = 0$, and solve the equation for y: $0^2 + y^2 = 4 \Rightarrow y = \pm 2$. The x-intercepts are –2 and 2; the y-intercepts are –2 and 2.

52. To find the x-intercept, let $y = 0$, and solve the equation for x: $0 = \sqrt{9 - x^2} \Rightarrow x = \pm 3$. To find the y-intercept, let $x = 0$, and solve the equation for y: $y = \sqrt{9 - 0^2} \Rightarrow y = 3$. The x-intercepts are –3 and 3; the y-intercept is 3.

53. To find the x-intercept, let $y = 0$, and solve the equation for x: $0 = \sqrt{x^2 - 1} \Rightarrow x = \pm 1$. To find the y-intercept, let $x = 0$, and solve the equation for y: $y = \sqrt{0^2 - 1} \Rightarrow$ no solution. The x-intercepts are –1 and 1; there is no y-intercept.

54. To find the x-intercept, let $y = 0$, and solve the equation for x: $x(0) = 1 \Rightarrow$ no solution. To find the y-intercept, let $x = 0$, and solve the equation for y: $(0)y = 1 \Rightarrow$ no solution. There is no x-intercept; there is no y-intercept.

55. To find the x-intercept, let $y = 0$, and solve the equation for x: $0 = x^2 + x + 1 \Rightarrow$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{-3}}{2},$$ which

are not real solutions. Therefore, there are no x-intercepts. To find the y-intercept, let $x = 0$, and solve the equation for y: $y = 0^2 + 0 + 1 = 1$. There are no x-intercepts; the y-intercept is 1.

56. To find the x-intercept, let $y = 0$, and solve the equation for x:
$x^3 + 3x(0) + 0^3 = 1 \Rightarrow x^3 = 1 \Rightarrow x = 1.$ To find the y-intercept, let $x = 0$, and solve the equation for y:
$0^3 + 3y(0) + y^3 = 1 \Rightarrow y^3 = 1 \Rightarrow y = 1.$
The x-intercept is 1; the y-intercept is 1.

In exercises 57–66, to test for symmetry with respect to the x-axis, replace y with $-y$ to determine if $(x, -y)$ satisfies the equation. To test for symmetry with respect to the y-axis, replace x with $-x$ to determine if $(-x, y)$ satisfies the equation. To test for symmetry with respect to the origin, replace x with $-x$ and y with $-y$ to determine if $(-x, -y)$ satisfies the equation.

57. $-y = x^2 + 1$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$y = (-x)^2 + 1 \Rightarrow y = x^2 + 1$, so the equation is symmetric with respect to the y-axis.
$-y = (-x)^2 + 1 \Rightarrow -y = x^2 + 1$, is not the same as the original equation, so the equation is not symmetric with respect to the origin.

58. $x = (-y)^2 + 1 \Rightarrow x = y^2 + 1$**,** so the equation is symmetric with respect to the x-axis.
$-x = y^2 + 1$ is not the same as the original equation, so the equation is not symmetric with respect to the y-axis.
$-x = (-y)^2 + 1 \Rightarrow -x = y^2 + 1$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

59. $-y = x^3 + x$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$y = (-x)^3 - x \Rightarrow y = -x^3 - x \Rightarrow$
$y = -(x^3 + x)$ is not the same as the original equation, so the equation is not symmetric with respect to the y-axis.
$-y = (-x)^3 - x \Rightarrow -y = -x^3 - x \Rightarrow$
$-y = -(x^3 + x) \Rightarrow y = x^3 + x,$ so the equation is symmetric with respect to the origin.

60. $-y = 2x^3 - x$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$y = 2(-x)^3 - (-x) \Rightarrow y = -2x^3 + x \Rightarrow$
$y = -2(x^3 - x)$ is not the same as the original equation, so the equation is not symmetric with respect to the y-axis.
$-y = 2(-x)^3 - (-x) \Rightarrow -y = -2x^3 + x \Rightarrow$
$-y = -2(x^3 - x) \Rightarrow y = 2x^3 - x$, so the equation is symmetric with respect to the origin.

61. $-y = 5x^4 + 2x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$y = 5(-x)^4 + 2(-x)^2 \Rightarrow y = 5x^4 + 2x^2$, so the equation is symmetric with respect to the y-axis.
$-y = 5(-x)^4 + 2(-x) \Rightarrow -y = 5x^4 + 2x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

62. $-y = -3x^6 + 2x^4 + x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$y = -3(-x)^6 + 2(-x)^4 + (-x)^2 \Rightarrow$
$y = -3x^6 + 2x^4 + x^2$, so the equation is symmetric with respect to the y-axis.
$-y = -3(-x)^6 + 2(-x)^4 + (-x)^2 \Rightarrow$
$-y = -3x^6 + 2x^4 + x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

63. $-y = -3x^5 + 2x^3$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$y = -3(-x)^5 + 2(-x)^3 \Rightarrow y = 3x^5 - 2x^3$ is not the same as the original equation, so the equation is not symmetric with respect to the y-axis.
$-y = -3(-x)^5 + 2(-x)^3 \Rightarrow -y = 3x^5 - 2x^3 \Rightarrow$
$-y = -(-3x^5 + 2x^3) \Rightarrow y = -3x^5 + 2x^3$, so the equation is symmetric with respect to the origin.

64. $-y = 2x^2 - |x|$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$y = 2(-x)^2 - |-x| \Rightarrow y = 2x^2 - |x|$, so the equation is symmetric with respect to the y-axis.
$-y = 2(-x)^2 - |-x| \Rightarrow -y = 2x^2 - |x|$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

65. $x^2(-y)^2 + 2x(-y) = 1 \Rightarrow x^2y^2 - 2xy = 1$ is not the same as the original equation, so the equation is not symmetric with respect to the x-axis.
$(-x)^2 y^2 + 2(-x)y = 1 \Rightarrow x^2y^2 - 2xy = 1$ is not the same as the original equation, so the equation is not symmetric with respect to the y-axis.
$(-x)^2(-y)^2 + 2(-x)(-y) = 1 \Rightarrow$
$x^2y^2 + 2xy = 1$, so the equation is symmetric with respect to the origin.

66. $x^2 + (-y)^2 = 16 \Rightarrow x^2 + y^2 = 16$, so the equation is symmetric with respect to the x-axis.
$(-x)^2 + y^2 = 16 \Rightarrow x^2 + y^2 = 16$, so the equation is symmetric with respect to the y-axis.
$(-x)^2 + (-y)^2 = 16 \Rightarrow x^2 + y^2 = 16$, so the equation is symmetric with respect to the origin.

For exercises 67–80, use the standard form of the equation of a circle, $(x-h)^2 + (y-k)^2 = r^2$.

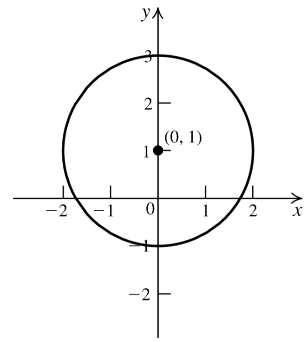
67. Center (2, 3); radius = 6

68. Center (–1, 3); radius = 4

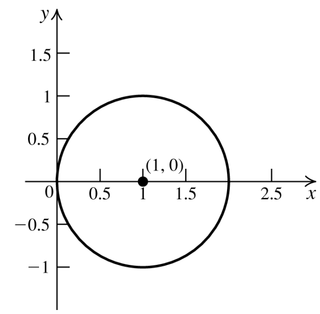
69. Center (–2, –3); radius = $\sqrt{11}$

70. Center $\left(\frac{1}{2}, -\frac{3}{2}\right)$; radius = $\frac{\sqrt{3}}{2}$

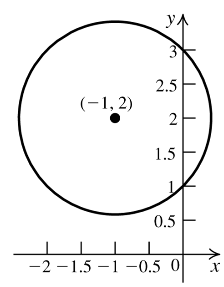
71. $x^2 + (y-1)^2 = 4$



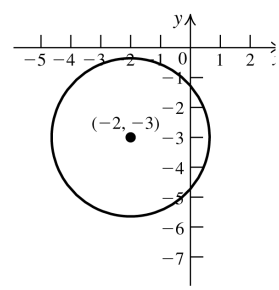
72. $(x-1)^2 + y^2 = 1$



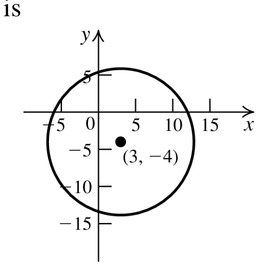
73. $(x+1)^2 + (y-2)^2 = 2$



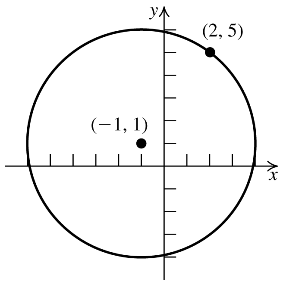
74. $(x+2)^2 + (y+3)^2 = 7$



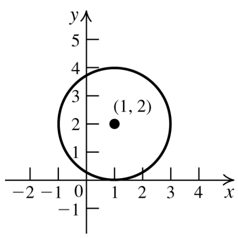
75. Find the radius by using the distance formula:
$d = \sqrt{(-1-3)^2 + (5-(-4))^2} = \sqrt{97}$.
The equation of the circle is
$(x-3)^2 + (y+4)^2 = 97$.



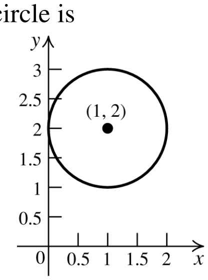
76. Find the radius by using the distance formula: $d = \sqrt{(-1-2)^2 + (1-5)^2} = \sqrt{25} = 5$. The equation of the circle is $(x+1)^2 + (y-1)^2 = 25.$



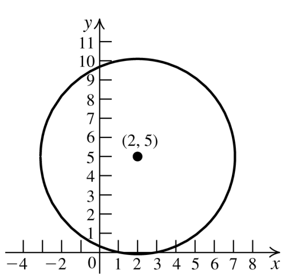
77. The circle touches the x-axis, so the radius is 2. The equation of the circle is $(x-1)^2 + (y-2)^2 = 4.$



78. The circle touches the y-axis, so the radius is 1. The equation of the circle is $(x-1)^2 + (y-2)^2 = 1$.



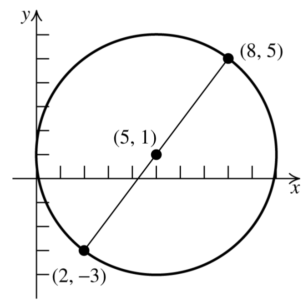
79. Find the diameter by using the distance formula: $d = \sqrt{(-3-7)^2 + (6-4)^2} = \sqrt{104} = 2\sqrt{26}$. So the radius is $\sqrt{26}$. Use the midpoint formula to find the center: $M = \left(\frac{7+(-3)}{2}, \frac{4+6}{2}\right) = (2, 5)$. The equation of the circle is $(x-2)^2 + (y-5)^2 = 26$.



80. Find the center by finding the midpoint of the diameter: $C = \left(\frac{2+8}{2}, \frac{-3+5}{2}\right) = (5, 1)$

Find the length of the radius by finding the length of the diameter and dividing that by 2. $d = \sqrt{(2-8)^2 + (-3-5)^2} = \sqrt{100} = 10$

Thus, the length of the radius is 5, and the equation of the circle is $(x-5)^2 + (y-1)^2 = 25.$



81. a. $x^2 + y^2 - 2x - 2y - 4 = 0 \Rightarrow$
$x^2 - 2x + y^2 - 2y = 4$
Now complete the square:
$x^2 - 2x + 1 + y^2 - 2y + 1 = 4 + 1 + 1 \Rightarrow$
$(x-1)^2 + (y-1)^2 = 6$. This is a circle with center (1, 1) and radius $\sqrt{6}$.

b. To find the x-intercepts, let $y = 0$ and solve for x:
$(x-1)^2 + (0-1)^2 = 6 \Rightarrow (x-1)^2 + 1 = 6 \Rightarrow$
$(x-1)^2 = 5 \Rightarrow x - 1 = \pm\sqrt{5} \Rightarrow x = 1 \pm \sqrt{5}$
Thus, the x-intercepts are $(1+\sqrt{5}, 0)$ and $(1-\sqrt{5}, 0)$.
To find the y-intercepts, let $x = 0$ and solve for y:
$(0-1)^2 + (y-1)^2 = 6 \Rightarrow 1 + (y-1)^2 = 6 \Rightarrow$
$(y-1)^2 = 5 \Rightarrow y - 1 = \pm\sqrt{5} \Rightarrow y = 1 \pm \sqrt{5}$
Thus, the y-intercepts are $(0, 1+\sqrt{5})$ and $(0, 1-\sqrt{5})$.

82. a. $x^2 + y^2 - 4x - 2y - 15 = 0 \Rightarrow$

$x^2 - 4x + y^2 - 2y = 15$

Now complete the square:

$x^2 - 4x + 4 + y^2 - 2y + 1 = 15 + 4 + 1 \Rightarrow$

$(x-2)^2 + (y-1)^2 = 20.$ This is a circle with center (2, 1) and radius $2\sqrt{5}$.

b. To find the *x*-intercepts, let *y* = 0 and solve for *x*: $(x-2)^2 + (0-1)^2 = 20 \Rightarrow$

$(x-2)^2 + 1 = 20 \Rightarrow (x-2)^2 = 19 \Rightarrow$

$x - 2 = \pm\sqrt{19} \Rightarrow x = 2 \pm \sqrt{19}$

Thus, the *x*-intercepts are $(2+\sqrt{19}, 0)$ and $(2-\sqrt{19}, 0)$.

To find the *y*-intercepts, let *x* = 0 and solve for *y*: $(0-2)^2 + (y-1)^2 = 20 \Rightarrow$

$4 + (y-1)^2 = 20 \Rightarrow (y-1)^2 = 16 \Rightarrow$

$y - 1 = \pm 4 \Rightarrow y = -3, 5$

Thus, the *y*-intercepts are (0, −3) and (0, 5).

83. a. $2x^2 + 2y^2 + 4y = 0 \Rightarrow$

$2(x^2 + y^2 + 2y) = 0 \Rightarrow x^2 + y^2 + 2y = 0.$

Now complete the square:

$x^2 + y^2 + 2y + 1 = 0 + 1 \Rightarrow x^2 + (y+1)^2 = 1.$

This is a circle with center (0, –1) and radius 1.

b. To find the *x*-intercepts, let *y* = 0 and solve for *x*: $x^2 + (0+1)^2 = 1 \Rightarrow x^2 = 0 \Rightarrow x = 0$

Thus, the *x*-intercept is (0, 0).

To find the *y*-intercepts, let *x* = 0 and solve for *y*:

$0^2 + (y+1)^2 = 1 \Rightarrow y + 1 = \pm 1 \Rightarrow y = 0, -2$

Thus, the *y*-intercepts are (0, 0) and (0, −2).

84. a. $3x^2 + 3y^2 + 6x = 0 \Rightarrow$

$3(x^2 + y^2 + 2x) = 0 \Rightarrow x^2 + 2x + y^2 = 0.$

Now complete the square:

$x^2 + 2x + 1 + y^2 = 0 + 1 \Rightarrow (x+1)^2 + y^2 = 1.$

This is a circle with center (–1, 0) and radius 1.

b. To find the *x*-intercepts, let *y* = 0 and solve for *x*:

$(x+1)^2 + 0^2 = 1 \Rightarrow x + 1 = \pm 1 \Rightarrow x = 0, -2$

Thus, the *x*-intercepts are (0, 0) and (−2, 0).

To find the *y*-intercepts, let *x* = 0 and solve for *y*: $(0+1)^2 + y^2 = 1 \Rightarrow y^2 = 0 \Rightarrow y = 0$

Thus, the *y*-intercept is (0, 0).

85. a. $x^2 + y^2 - x = 0 \Rightarrow x^2 - x + y^2 = 0.$

Now complete the square:

$x^2 - x + \frac{1}{4} + y^2 = 0 + \frac{1}{4} \Rightarrow$

$\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4}.$ This is a circle with center $\left(\frac{1}{2}, 0\right)$ and radius $\frac{1}{2}$.

b. To find the *x*-intercepts, let *y* = 0 and solve for *x*: $\left(x - \frac{1}{2}\right)^2 + 0^2 = \frac{1}{4} \Rightarrow x - \frac{1}{2} = \pm\frac{1}{2} \Rightarrow$

$x = 0, 1.$ Thus, the *x*-intercepts are (0, 0) and (1, 0). To find the *y*-intercepts, let *x* = 0 and solve for *y*:

$\left(0 - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4} \Rightarrow y^2 + \frac{1}{4} = \frac{1}{4} \Rightarrow$

$y^2 = 0 \Rightarrow y = 0.$

Thus, the *y*-intercept is (0, 0).

86. a. $x^2 + y^2 + 1 = 0 \Rightarrow x^2 + y^2 = -1.$ The radius cannot be negative, so there is no graph.

b. There are no intercepts.

2.2 Applying the Concepts

87. The distance from *P*(*x*, *y*) to the *x*-axis is $|x|$ while the distance from *P* to the *y*-axis is $|y|$. So the equation of the graph is $|x| = |y|$.

88. The distance from *P*(*x*, *y*) to (1, 2) is $\sqrt{(x-1)^2 + (y-2)^2}$ while the distance from *P* to (3, –4) is $\sqrt{(x-3)^2 + (y+4)^2}$.

So the equation of the graph is

$\sqrt{(x-1)^2 + (y-2)^2} = \sqrt{(x-3)^2 + (y+4)^2} \Rightarrow$

$(x-1)^2 + (y-2)^2 = (x-3)^2 + (y+4)^2 \Rightarrow$

(*continued on next page*)

(*continued*)

$$x^2 - 2x + 1 + y^2 - 4y + 4 =$$
$$x^2 - 6x + 9 + y^2 + 8y + 16 \Rightarrow$$
$$-2x - 4y + 5 = -6x + 8y + 25 \Rightarrow$$
$$4x - 20 = 12y \Rightarrow y = \frac{1}{3}x - \frac{5}{3}.$$

89. The distance from $P(x, y)$ to (2, 0) is $\sqrt{(x-2)^2 + y^2}$ while the distance from P to the y-axis is $|x|$. So the equation of the graph is

$$\sqrt{(x-2)^2 + y^2} = |x| \Rightarrow (x-2)^2 + y^2 = x^2 \Rightarrow .$$
$$x^2 - 4x + 4 + y^2 = x^2 \Rightarrow y^2 = 4x - 4 \Rightarrow$$
$$\frac{y^2 + 4}{4} = \frac{y^2}{4} + 1 = x$$

90. 2004 is the midpoint of the initial range, so $M_{2004} = \frac{136 + 234}{2} = 185.$

2002 is the midpoint of the range [2000, 2004], so $M_{2002} = \frac{136 + 185}{2} = 160.5.$

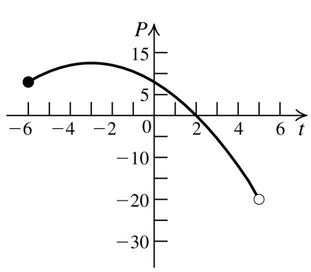
2006 is the midpoint of the range [2004, 2008], so $M_{2006} = \frac{185 + 234}{2} = 209.5.$

So, in 2002, \$160.5 billion was spent, in 2004, \$185 billion was spent and in 2006, \$209.50 was spent.

91. a. Since July 2012 is represented by $t = 0$, March 2012 is represented by $t = -4$. So the monthly profit for March is determined by $P = -0.5(-4)^2 - 3(-4) + 8 = \$12$ million.

b. Since July 2012 is represented by $t = 0$, October 2012 is represented by $t = 3$. So the monthly profit for October is determined by $P = -0.5(3)^2 - 3(3) + 8 = -\$5.5$ million. This is a loss.

c.



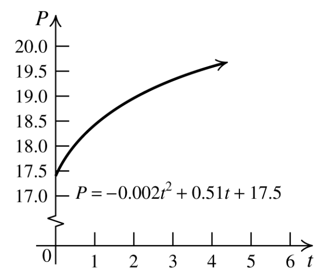
d. To find the t-intercept, set $P = 0$ and solve for t: $0 = -0.5t^2 - 3t + 8 \Rightarrow$

$$t = \frac{3 \pm \sqrt{(-3)^2 - 4(-0.5)(8)}}{2(-0.5)} = \frac{3 \pm \sqrt{25}}{-1}$$
$$= 2 \text{ or } -8$$

The t-intercepts represent the months with no profit and no loss. In this case, $t = -8$ makes no sense in terms of the problem, so we disregard this solution. $t = 2$ represents Sept 2012.

e. To find the P-intercept, set $t = 0$ and solve to P: $P = -0.5(0)^2 - 3(0) + 8 \Rightarrow P = 8.$ The P-intercept represents the profit in July 2012.

92. a.

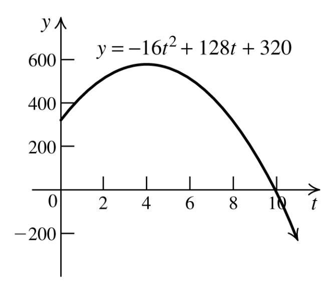


b. To find the P-intercept, set $t = 0$ and solve to P: $P = -0.002(0)^2 + 0.51(0) + 17.5 \Rightarrow$ $P = 17.5$. The P-intercept represents the number of female college students in 2005.

93. a.

t	Height = $-16t^2 + 128t + 320$
0	320 feet
1	432 feet
2	512 feet
3	560 feet
4	576 feet
5	560 feet
6	512 feet

b.



c. $0 \le t \le 10$

d. To find the t-intercept, set $y = 0$ and solve for t:

$0 = -16t^2 + 128t + 320 \Rightarrow$

$0 = -16(t^2 - 8t - 20) \Rightarrow$

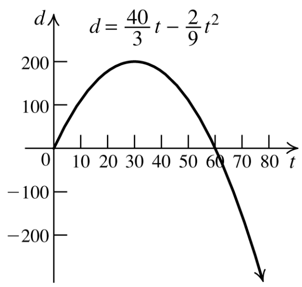
$0 = (t - 10)(t + 2) \Rightarrow\ t = 10$ or $t = -2.$

The graph does not apply if $t < 0$, so the t-intercept is 10. This represents the time when the object hits the ground. To find the y-intercept, set $t = 0$ and solve for y:

$y = -16(0)^2 + 128(0) + 320 \Rightarrow\ y = 320.$

This represents the height of the building.

94. a.

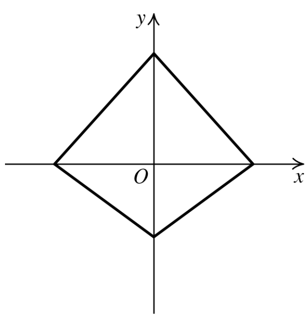


b. $0 \le t \le 60$

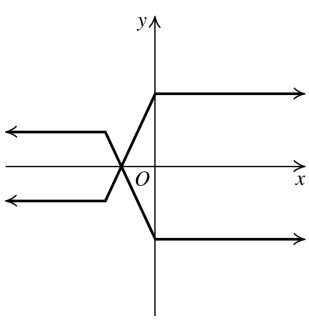
c. The total time of the experiment is 60 minutes or 1 hour.

2.2 Beyond the Basics

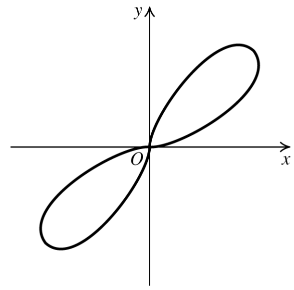
95.



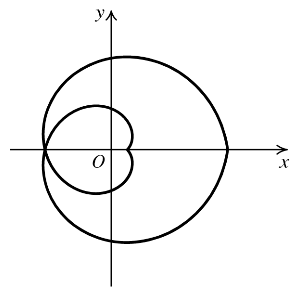
96.



97.



98.



99. $x^2 + y^2 - 4x + 2y - 20 = 0 \Rightarrow$

$x^2 - 4x + y^2 + 2y = 20 \Rightarrow$

$x^2 - 4x + 4 + y^2 + 2y + 1 = 20 + 4 + 1 \Rightarrow$

$(x - 2)^2 + (y + 1)^2 = 25$

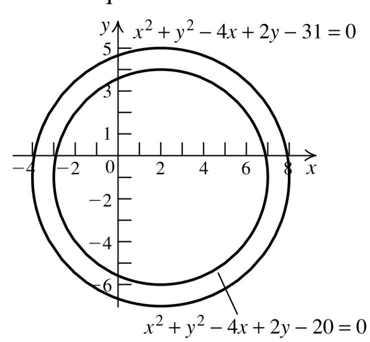
So this is the graph of a circle with center (2, –1) and radius 5. The area of this circle is $25\pi.$ $x^2 + y^2 - 4x + 2y - 31 = 0 \Rightarrow$

$x^2 - 4x + y^2 + 2y = 31 \Rightarrow$

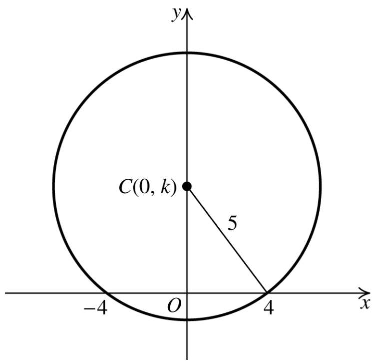
$x^2 - 4x + 4 + y^2 + 2y + 1 = 31 + 4 + 1 \Rightarrow$

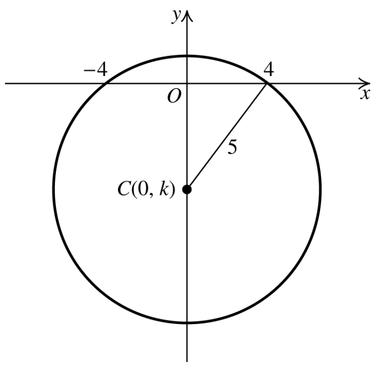
$(x - 2)^2 + (y + 1)^2 = 36$

So, this is the graph of a circle with center (2, –1) and radius 6. The area of this circle is $36\pi.$ Both circles have the same center, so the area of the region bounded by the two circles equals $36\pi - 25\pi = 11\pi.$



100. Using the hint, we know that the center of the circle will have coordinates (0, *k*).

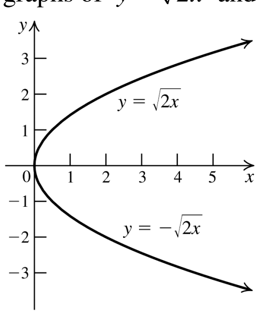




Use the Pythagorean theorem to find *k*.
$k^2 + 4^2 = 5^2 \Rightarrow k^2 + 16 = 25 \Rightarrow k^2 = 9 \Rightarrow$
$k = \pm 3$
The equations of the circles are
$x^2 + (y \pm 3)^2 = 5^2.$

2.2 Critical Thinking/Discussion/Writing

101. The graph of $y^2 = 2x$ is the union of the graphs of $y = \sqrt{2x}$ and $y = -\sqrt{2x}$.



102. Let (*x*, *y*) be a point on the graph. Since the graph is symmetric with regard to the *x*-axis, then the point (*x*, –*y*) is also on the graph. Because the graph is symmetric with regard to the *y*-axis, the point (–*x*, *y*) is also on the graph. Therefore the point (–*x*, –*y*) is on the graph, and the graph is symmetric with respect to the origin. The graph of $y = x^3$ is an example of a graph that is symmetric with respect to the origin but is not symmetric with respect to the *x*- and *y*-axes.

103. a. First find the radius of the circle:

$$d(A, B) = \sqrt{(6-0)^2 + (8-1)^2} = \sqrt{85} \Rightarrow$$
$$r = \frac{\sqrt{85}}{2}.$$

The center of the circle is

$$\left(\frac{6+0}{2}, \frac{1+8}{2}\right) = \left(3, \frac{9}{2}\right).$$

So the equation of the circle is

$$(x-3)^2 + \left(y - \frac{9}{2}\right)^2 = \frac{85}{4}.$$

To find the *x*-intercepts, set *y* = 0, and solve for *x*:

$$(x-3)^2 + \left(0 - \frac{9}{2}\right)^2 = \frac{85}{4} \Rightarrow$$
$$(x-3)^2 + \frac{81}{4} = \frac{85}{4} \Rightarrow x^2 - 6x + 9 = 1 \Rightarrow$$
$$x^2 - 6x + 8 = 0$$

The *x*-intercepts are the roots of this equation.

b. First find the radius of the circle:

$$d(A, B) = \sqrt{(a-0)^2 + (b-1)^2}$$
$$= \sqrt{a^2 + (b-1)^2} \Rightarrow$$
$$r = \frac{\sqrt{a^2 + (b-1)^2}}{2}.$$

The center of the circle is

$$\left(\frac{a+0}{2}, \frac{b+1}{2}\right) = \left(\frac{a}{2}, \frac{b+1}{2}\right)$$

So the equation of the circle is

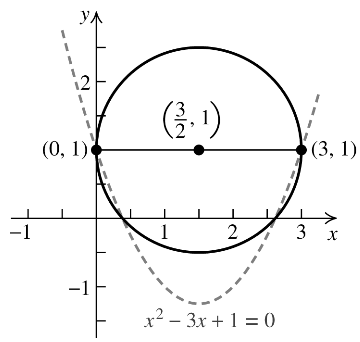
$$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b+1}{2}\right)^2 = \frac{a^2 + (b-1)^2}{4}.$$

To find the *x*-intercepts, set *y* = 0 and solve for *x*:

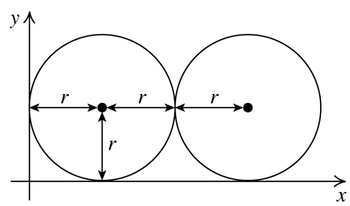
$$\left(x - \frac{a}{2}\right)^2 + \left(0 - \frac{b+1}{2}\right)^2 = \frac{a^2 + (b-1)^2}{4}$$
$$x^2 - ax + \frac{a^2}{4} + \frac{(b+1)^2}{4} = \frac{a^2 + (b-1)^2}{4}$$
$$4x^2 - 4ax + a^2 + b^2 + 2b + 1 = a^2 + b^2 - 2b + 1$$
$$4x^2 - 4ax + 4b = 0$$
$$x^2 - ax + b = 0$$

The *x*-intercepts are the roots of this equation.

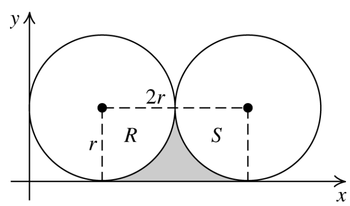
c. $a = 3$ and $b = 1$. Approximate the roots of the equation by drawing a circle whose diameter has endpoints $A(0, 1)$ and $B(3, 1)$. The center of the circle is $\left(\frac{3}{2}, 1\right)$ and the radius is $\frac{3}{2}$. The roots are approximately (0.4, 0) and (2.6, 0).



104. a. The coordinates of the center of each circle are (r, r) and $(3r, r)$.



b. To find the area of the shaded region, first find the area of the rectangle shown in the figure below, and then subtract the sum of the areas of the two sectors, A and B.



$$A_{\text{rect}} = r(2r) = 2r^2$$
$$A_{\text{sector } R} = A_{\text{sector } S} = \frac{1}{4}\pi r^2$$
$$A_{\text{shaded region}} = 2r^2 - \left(\frac{1}{4}\pi r^2 + \frac{1}{4}\pi r^2\right) = 2r^2 - \frac{1}{2}\pi r^2 = \left(2 - \frac{\pi}{2}\right)r^2$$

2.2 Maintaining Skills

105. $\frac{5-3}{6-2} = \frac{2}{4} = \frac{1}{2}$

106. $\frac{1-2}{-2-2} = \frac{-1}{-4} = \frac{1}{4}$

107. $\frac{2-(-3)}{3-13} = \frac{5}{-10} = -\frac{1}{2}$

108. $\frac{3-1}{-2-(-6)} = \frac{2}{4} = \frac{1}{2}$

109. $\frac{\frac{1}{2}-\frac{1}{4}}{\frac{3}{8}-\left(-\frac{1}{4}\right)} = \frac{\frac{1}{4}}{\frac{5}{8}} = \frac{1}{4}\cdot\frac{8}{5} = \frac{2}{5}$

110. $\frac{\frac{3}{4}-1}{\frac{1}{2}-\frac{1}{6}} = \frac{-\frac{1}{4}}{\frac{1}{3}} = \left(-\frac{1}{4}\right)(3) = -\frac{3}{4}$

111. $A = P + Prt = P(1+rt) \Rightarrow P = \frac{A}{1+rt}$

112. $2x + 3y = 6 \Rightarrow 3y = 6 - 2x \Rightarrow$
$y = \frac{6-2x}{3} = 2 - \frac{2}{3}x$

113. $\frac{x}{2} - \frac{y}{5} = 3 \Rightarrow \frac{x}{2} - 3 = \frac{y}{5} \Rightarrow \frac{5}{2}x - 15 = y$

114. $y - 2 - \frac{2}{3}(x+1) = 0 \Rightarrow$
$y = 2 + \frac{2}{3}(x+1) = 2 + \frac{2}{3}x + \frac{2}{3} = \frac{2}{3}x + \frac{8}{3}$

115. $ax + by + c = 0 \Rightarrow by = -ax - c \Rightarrow$
$y = -\frac{a}{b}x - \frac{c}{b}$

116. $0.1x + 0.2y - 1 = 0 \Rightarrow x + 2y - 10 = 0 \Rightarrow$
$2y = -x + 10 \Rightarrow y = -0.5x + 5$

117. $-\frac{1}{2}$

118. $\frac{1}{3}$

119. $\frac{3}{2}$

120. $-\frac{3}{4}$

121. $-\frac{2+\frac{3}{4}}{1-\frac{1}{2}} = -\frac{\frac{11}{4}}{\frac{1}{2}} = -\frac{11}{4}\cdot 2 = -\frac{11}{2}$

122. $-\frac{\frac{5}{6}-\left(-\frac{3}{4}\right)}{\frac{2}{3}-\frac{1}{4}} = -\frac{\frac{19}{12}}{\frac{5}{12}} = -\frac{19}{5}$

2.3 Lines

2.3 Practice Problems

1. $m = \dfrac{5-(-3)}{-7-6} = -\dfrac{8}{13}$

A slope of $-\frac{8}{13}$ means that the value of y decreases 8 units for every 13 units increase in x.

2. $P(-2,-3)$, $m = -\dfrac{2}{3}$

$$y-(-3) = -\frac{2}{3}[x-(-2)]$$
$$y+3 = -\frac{2}{3}(x+2)$$
$$y+3 = -\frac{2}{3}x - \frac{4}{3} \Rightarrow y = -\frac{2}{3}x - \frac{13}{3}$$

3. $m = \dfrac{-4-6}{-3-(-1)} = \dfrac{-10}{-2} = 5$

Use either point to determine the equation of the line. Using (−3, −4), we have

$y-(-4) = 5[x-(-3)] \Rightarrow y+4 = 5(x+3) \Rightarrow$
$y+4 = 5x+15 \Rightarrow y = 5x+11$

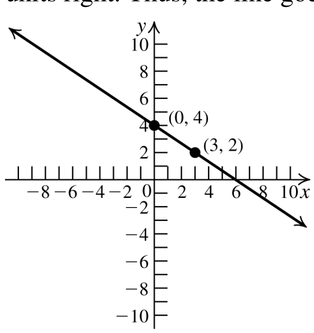
Using (−1, 6), we have

$y-6 = 5[x-(-1)] \Rightarrow y-6 = 5(x+1) \Rightarrow$
$y-6 = 5x+5 \Rightarrow y = 5x+11$

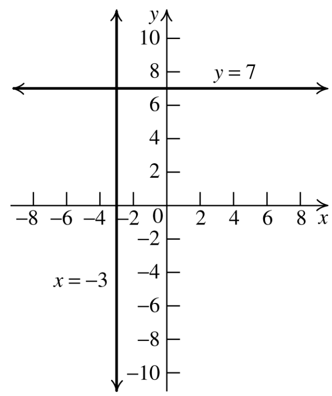
4. $y - y_1 = m(x - x_1) \Rightarrow y-(-3) = 2(x-0)$
point-slope form

$y-(-3) = 2(x-0) \Rightarrow$
$y+3 = 2x \Rightarrow y = 2x-3$

5. The slope is $-\dfrac{2}{3}$ and the y-intercept is 4. The line goes through (0, 4), so locate a second point by moving two units down and three units right. Thus, the line goes through (3, 2).



6. $x = -3$. The slope is undefined, and there is no y-intercept. The x-intercept is −3.
$y = 7$. The slope is 0, and the y-intercept is 7.

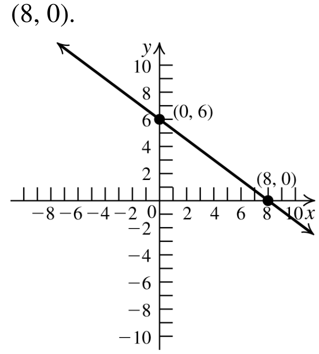


7. First, solve for y to write the equation in slope-intercept form:

$3x+4y = 24 \Rightarrow 4y = -3x+24 \Rightarrow$

$y = -\dfrac{3}{4}x+6$. The slope is $-\dfrac{3}{4}$, and the y-intercept is 6. Find the x-intercept by setting $y = 0$ and solving the equation for x:

$0 = -\dfrac{3}{4}x+6 \Rightarrow 6 = \dfrac{3}{4}x \Rightarrow 8 = x$. Thus, the graph passes through the points (0, 6) and (8, 0).



8. Use the equation $H = 2.6x + 65$.

$H_1 = 2.6(43) + 65 = 176.8$
$H_2 = 2.6(44) + 65 = 179.4$

The person is between 176.8 cm and 179.4 cm tall, or 1.768 m and 1.794 m.

9. a. Parallel lines have the same slope, so the slope of the line is $m = \dfrac{3-7}{2-5} = \dfrac{-4}{-3} = \dfrac{4}{3}$.

Using the point-slope form, we have

$y-5 = \dfrac{4}{3}[x-(-2)] \Rightarrow 3y-15 = 4(x+2) \Rightarrow$
$3y-15 = 4x+8 \Rightarrow 4x-3y+23 = 0$

b. The slopes of perpendicular lines are negative reciprocals. Write the equation $4x+5y+1=0$ in slope-intercept form to find its slope: $4x+5y+1=0 \Rightarrow$

$5y=-4x-1 \Rightarrow y=-\frac{4}{5}x-\frac{1}{5}$. The slope of a line perpendicular to this lines is $\frac{5}{4}$.

Using the point-slope form, we have

$y-(-4)=\frac{5}{4}(x-3) \Rightarrow 4(y+4)=5(x-3) \Rightarrow$
$4y+16=5x-15 \Rightarrow 5x-4y-31=0$

10. Since 2014 is 8 years after 2006, set $x = 8$. Then $y = 0.44(8) + 6.70 = 10.22$
There were 10.22 million registered motorcycles in the U.S. in 2014.

2.3 Basic Concepts and Skills

1. The number that measure the "steepness" of a line is called the slope.

2. In the slope formula, $m=\frac{y_2-y_1}{x_2-x_1}$, the number y_2-y_1 is called the rise.

3. If two line have the same slope, then they are parallel.

4. The y-intercept of the line with equation $y=3x-5$ is -5.

5. A line perpendicular to a line with slope $-\frac{1}{5}$ has slope 5.

6. True

7. False. The slope of the line through (1, 2) and (2, 4) is $\frac{4-2}{2-1}=2$.

8. False. The slope of the line through (1, 3) and $(2, y)$ is $\frac{y-3}{2-1}=y-3$. If $y > 3$, then the slope is positive.

9. $m=\frac{7-3}{4-1}=\frac{4}{3}$; the graph is rising.

10. $m=\frac{0-4}{2-0}=\frac{-4}{2}=-2$; the graph is falling.

11. $m=\frac{-2-(-2)}{-2-6}=\frac{0}{-8}=0$; the graph is horizontal.

12. $m=\frac{7-(-4)}{-3-(-3)}=\frac{11}{0} \Rightarrow$ slope is undefined; the graph is vertical.

13. $m=\frac{-3.5-2}{3-0.5}=\frac{-5.5}{2.5}=-2.2$; the graph is falling.

14. $m=\frac{-3-(-2)}{2-3}=\frac{-1}{-1}=1$; the graph is rising.

15. $m=\frac{5-1}{\left(1+\sqrt{2}\right)-\sqrt{2}}=\frac{4}{1}=4$; the graph is rising.

16. $m=\frac{3\sqrt{3}-0}{\left(1+\sqrt{3}\right)-\left(1-\sqrt{3}\right)}=\frac{3\sqrt{3}}{2\sqrt{3}}=\frac{3}{2}$; the graph is rising.

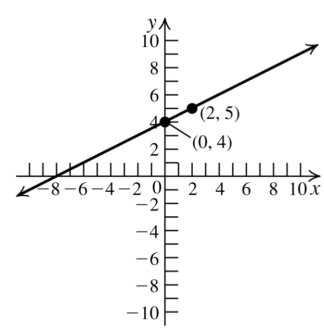
17. a. ℓ_3 **b.** ℓ_2 **c.** ℓ_4 **d.** ℓ_1

18. ℓ_1 passes through the points (2, 3) and (−1, 0). $m_{\ell_1}=\frac{0-3}{-1-2}=1.$
ℓ_2 is a horizontal line, so it has slope 0
ℓ_3 passes through the points (2, 3) and (0, −1). $m_{\ell_3}=\frac{-1-3}{0-2}=2.$
ℓ_4 passes through the points (−6, 7) and (0, −1). $m_{\ell_4}=\frac{-1-7}{0-(-6)}=-\frac{4}{3}.$

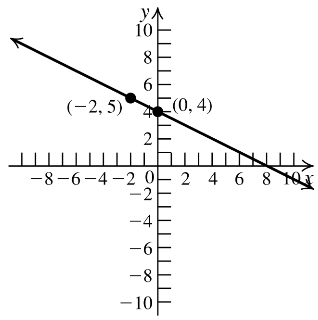
19. a. $y=0$ **b.** $x=0$

20. a. $y=-4$ **b.** $x=5$

21. $y=\frac{1}{2}x+4$

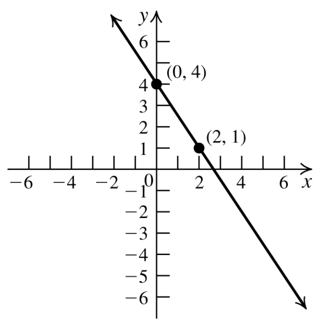


22. $y = -\frac{1}{2}x + 4$

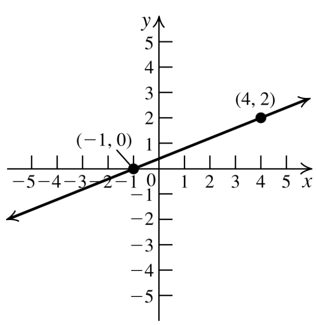


23. $y - 1 = -\frac{3}{2}(x - 2) \Rightarrow y - 1 = -\frac{3}{2}x + 3 \Rightarrow$

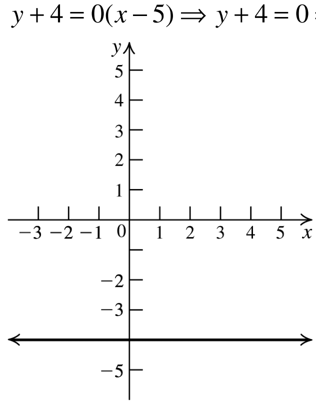
$y = -\frac{3}{2}x + 4$



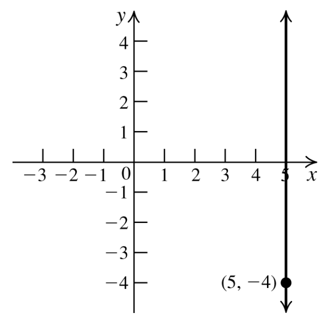
24. $y = \frac{2}{5}(x + 1) \Rightarrow y = \frac{2}{5}x + \frac{2}{5}$



25. $y + 4 = 0(x - 5) \Rightarrow y + 4 = 0 \Rightarrow y = -4$



26. Because the slope is undefined, the graph is vertical. The equation is $x = 5$.



27. $m = \frac{0 - 1}{1 - 0} = -1$. The y-intercept is (0, 1), so the equation is $y = -x + 1$.

28. $m = \frac{3 - 1}{1 - 0} = 2$. The y-intercept is (0, 1), so the equation is $y = 2x + 1$.

29. $m = \frac{3 - 3}{3 - (-1)} = 0$ Because the slope = 0, the line is horizontal. Its equation is $y = 3$.

30. $m = \frac{7 - 1}{2 - (-5)} = \frac{6}{7}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.

$y - 1 = \frac{6}{7}(x + 5) \Rightarrow y - 1 = \frac{6}{7}x + \frac{30}{7} \Rightarrow$

$y = \frac{6}{7}x + \frac{37}{7}$

31. $m = \frac{1 - (-1)}{1 - (-2)} = \frac{2}{3}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.

$y + 1 = \frac{2}{3}(x + 2) \Rightarrow y + 1 = \frac{2}{3}x + \frac{4}{3} \Rightarrow$

$y = \frac{2}{3}x + \frac{1}{3}$

32. $m = \frac{-9 - (-3)}{6 - (-1)} = -\frac{6}{7}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.

$y + 3 = -\frac{6}{7}(x + 1) \Rightarrow y + 3 = -\frac{6}{7}x - \frac{6}{7} \Rightarrow$

$y = -\frac{6}{7}x - \frac{27}{7}$

33. $m = \dfrac{2-\frac{1}{4}}{0-\frac{1}{2}} = \dfrac{\frac{7}{4}}{-\frac{1}{2}} = -\dfrac{7}{2}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form. $y-2=-\dfrac{7}{2}x \Rightarrow y=-\dfrac{7}{2}x+2$

34. $m = \dfrac{3-(-7)}{4-4} = \dfrac{10}{0} \Rightarrow$ the slope is undefined. So the graph is a vertical line. The equation is $x = 4$.

35. $x=5$

36. $y=1.5$

37. $y=0$

38. $x=0$

39. $y=14$

40. $y=2x+5$

41. $y=-\dfrac{2}{3}x-4$

42. $y=-6x-3$

43. $m=\dfrac{4-0}{0-(-3)}=\dfrac{4}{3};\ \ y=\dfrac{4}{3}x+4$

44. $m=\dfrac{-2-0}{0-(-5)}=-\dfrac{2}{5};\ \ y=-\dfrac{2}{5}x-2$

45. $y=7$

46. $x=4$

47. $y=-5$

48. $x=-3$

49. Two lines are parallel if their slopes are equal. The lines are perpendicular if the slope of one is the negative reciprocal of the slope of the other.

a. $m_{\ell_2}=\dfrac{1-5}{3-1}=\dfrac{-4}{2}=-2 \Rightarrow \ell_2 \parallel \ell_1$

b. $m_{\ell_2}=\dfrac{4-3}{5-7}=-\dfrac{1}{2}$. The slope of ℓ_2 is neither equal to the slope of ℓ_1 nor the negative reciprocal of the slope of ℓ_1. Therefore, the lines are neither parallel nor perpendicular.

c. $m_{\ell_2}=\dfrac{4-3}{4-2}=\dfrac{1}{2} \Rightarrow \ell_2 \perp \ell_1$

50. ℓ_1 passes through the points (2, 3) and (−1, 0) and has slope 1. Its equation is $y-0=x-(-1) \Rightarrow y=x+1$

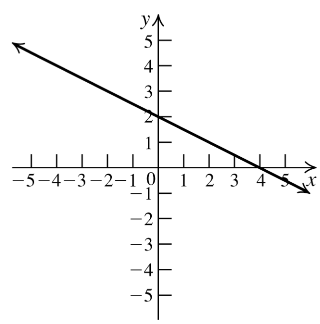
ℓ_2 has slope 0 and passes through ((−5, −4), so its equation is $y = -4$.

ℓ_3 passes through the points (2, 3) and (0, −1) and has slope 2. Its equation is $y=2x-1$.

ℓ_4 passes through the points (−6, 7) and (0, −1) and has slope $-\dfrac{4}{3}$. Its equation is $y=-\dfrac{4}{3}x-1$.

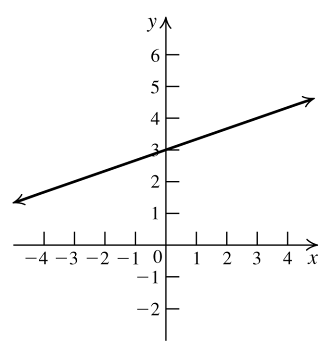
51. $x+2y-4=0 \Rightarrow 2y=-x+4 \Rightarrow y=-\dfrac{1}{2}x+2$.

The slope is $-1/2$, and the y-intercept is (0, 2). To find the x-intercept, set $y = 0$ and solve for x: $x+2(0)-4=0 \Rightarrow x=4$.



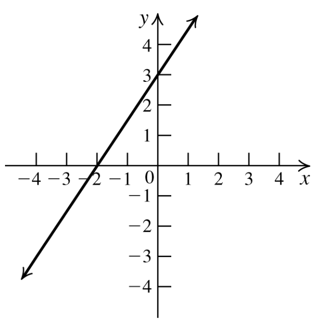
52. $x=3y-9 \Rightarrow x+9=3y \Rightarrow y=\dfrac{1}{3}x+3$

The slope is $1/3$, and the y-intercept is (0, 3). To find the x-intercept, set $y = 0$ and solve for x: $x=3(0)-9 \Rightarrow x=-9$.



53. $3x - 2y + 6 = 0 \Rightarrow 3x + 6 = 2y \Rightarrow \frac{3}{2}x + 3 = y$.

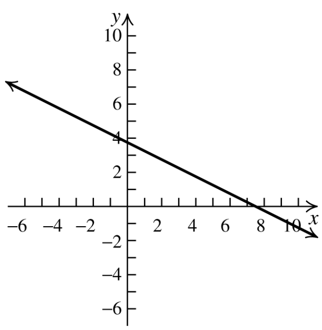
The slope is $3/2$, and the y-intercept is (0, 3). To find the x-intercept, set $y = 0$ and solve for x: $3x - 2(0) + 6 = 0 \Rightarrow 3x = -6 \Rightarrow x = -2$.



54. $2x = -4y + 15 \Rightarrow 2x - 15 = -4y \Rightarrow$

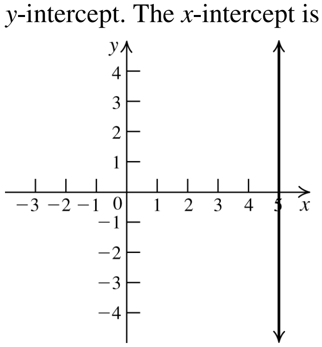
$-\frac{1}{2}x + \frac{15}{4} = y$

The slope is $-1/2$, and the y-intercept is $15/4$. To find the x-intercept, set $y = 0$ and solve for x: $2x = -4(0) + 15 \Rightarrow x = 15/2$.



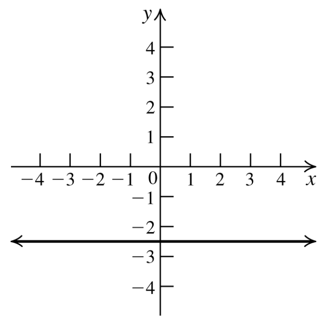
55. $x - 5 = 0 \Rightarrow x = 5$

The slope is undefined, and there is no y-intercept. The x-intercept is 5.



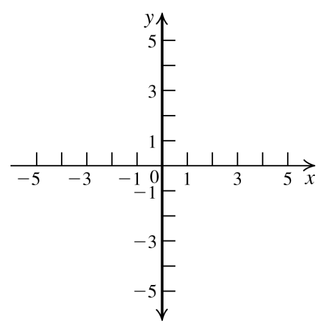
56. $2y + 5 = 0 \Rightarrow y = -\frac{5}{2}$

The slope is 0, and the y-intercept is $-5/2$. This is a horizontal line, so there is no x-intercept.



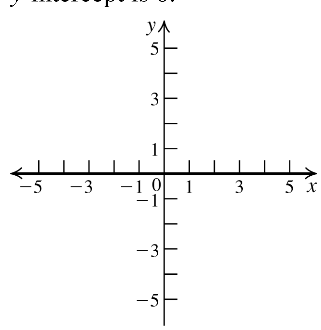
57. $x = 0$

The slope is undefined, and the y-intercepts are the y-axis. This is a vertical line whose x-intercept is 0.



58. $y = 0$

The slope is 0, and the x-intercepts are the x-axis. This is a horizontal line whose y-intercept is 0.



59. The slope of the line through (a, 0) and (0, b) is $\frac{b-0}{0-a} = -\frac{b}{a}$. The equation of the line can be written as

$$y - b = -\frac{b}{a}x \Rightarrow ay - ab = -bx \Rightarrow$$

$$ay + bx = ab \Rightarrow \frac{ay}{ab} + \frac{bx}{ab} = \frac{ab}{ab} \Rightarrow \frac{y}{b} + \frac{x}{a} = 1$$

60. $\frac{x}{4}+\frac{y}{3}=1$

61. $2x+3y=6 \Rightarrow \frac{2x}{6}+\frac{3y}{6}=\frac{6}{6} \Rightarrow \frac{x}{3}+\frac{y}{2}=1;$
x-intercept = 3; y-intercept = 2

62. $3x-4y+12=0 \Rightarrow 3x-4y=-12 \Rightarrow$
$\frac{3x}{-12}-\frac{4y}{-12}=\frac{-12}{-12} \Rightarrow -\frac{x}{4}+\frac{y}{3}=1;$
x-intercept = –4; y-intercept = 3

63. Let the intercepts be $(a, 0)$ and $(0, a)$. Then the equation of the line is $\frac{x}{a}+\frac{y}{a}=1.$ Now substitute $x = 3$ and $y = –5$ into the equation to solve for a: $\frac{3}{a}-\frac{5}{a}=1 \Rightarrow 3-5=a \Rightarrow -2=a.$
So the equation of the line is $-\frac{x}{2}-\frac{y}{2}=1 \Rightarrow$
$-x-y=2 \Rightarrow y=-x-2.$

64. Let the intercepts be $(a, 0)$ and $(0, –a)$. Then the equation of the line is $\frac{x}{a}-\frac{y}{a}=1.$ Now substitute $x = –5$ and $y = –8$ into the equation to solve for a:
$-\frac{5}{a}+\frac{8}{a}=1 \Rightarrow -5+8=a \Rightarrow 3=a.$ So the equation of the line is $\frac{x}{3}-\frac{y}{3}=1 \Rightarrow$
$x-y=3 \Rightarrow y=x-3.$

65. $m=\frac{9-4}{7-2}=\frac{5}{5}=1.$ The equation of the line through (2, 4) and (7, 9) is $y-4=1(x-2) \Rightarrow$ $y = x + 2$. Check to see if (–1, 1) satisfies the equation by substituting $x = –1$ and $y = 1$: $1=-1+2 \Rightarrow 1=1.$ So (–1, 1) also lies on the line. The points are collinear.

66. $m=\frac{-3-2}{2-7}=\frac{-5}{-5}=1.$ The equation of the line through (7, 2) and (2, –3) is $y-2=1(x-7) \Rightarrow y=x-5.$ Check to see if (5, 1) satisfies the equation by substituting $x = 5$ and $y = 1$:
$1=5-5 \Rightarrow 1 \neq 0.$
So (5, 1) does not lie on the line. The points are not collinear.

67. The line with x-intercept –4 and passing through $\left(\frac{1}{2}, 9\right)$ has slope
$m=\frac{9-0}{\frac{1}{2}-(-4)}=\frac{9}{9/2}=2.$ Then the line perpendicular to this line has slope $-\frac{1}{2}.$ Since the line also passes through $\left(\frac{1}{2}, 9\right),$ its equation is
$y-9=-\frac{1}{2}\left(x-\frac{1}{2}\right) \Rightarrow y=-\frac{1}{2}x+\frac{37}{4}.$

68. The line through the origin has slope $\frac{4}{2}=2.$ Then the line parallel to this line also has slope 2. The line passes through (–2, 0), so its equation is $y-0=2\left(x-(-2)\right) \Rightarrow y=2x+4$

69. Both lines are vertical lines. The lines are parallel.

70. $x=0$ is the equation of the y-axis. $y=0$ is the equation of the x-axis. The lines are perpendicular.

71. The slope of $2x+3y=7$ is $-2/3$, while $y=2$ is a horizontal line. The lines are neither parallel nor perpendicular.

72. The slope of $y=3x+1$ is 3. The slope of $6y+2x=0$ is $-1/3$. The lines are perpendicular.

73. The slope of $x=4y+8$ is $1/4$. The slope of $y=-4x+1$ is -4, so the lines are perpendicular.

74. The slope of $4x+3y=1$ is $-4/3$, while the slope of $3+y=2x$ is 2. The lines are neither parallel nor perpendicular.

75. The slope of $3x+8y=7$ is $-3/8$, while the slope of $5x-7y=0$ is $5/7$. The lines are neither parallel nor perpendicular.

76. The slope of $10x+2y=3$ is -5. The slope of $y+1=-5x$ is also -5, so the lines are parallel.

77. a. Use the point-slope form.
$y-(-3)=3(x-2) \Rightarrow y+3=3x-6 \Rightarrow$
$y=3x-9$

b. The slope of the line we are seeking is 2. Using the point-slope form, we have
$y-2=2(x-(-1)) \Rightarrow y-2=2(x+1) \Rightarrow$
$y-2=2x+2 \Rightarrow y=2x+4$

78. a. The slope of the line we are seeking is
$\frac{-2-2}{1-(-3)}=\frac{-4}{4}=-1.$
Using the point-slope form, we have
$y-(-5)=-(x-(-2)) \Rightarrow y+5=-(x+2) \Rightarrow$
$y+5=-x-2 \Rightarrow y=-x-7$

b. The slope of the given line is
$\frac{2-(-1)}{-3-(-4)}=\frac{3}{1}=3,$ so the slope of the line we are seeking is $-\frac{1}{3}.$ Using the point-slope form, we have
$y-(-2)=-\frac{1}{3}(x-1) \Rightarrow y+2=-\frac{1}{3}x+\frac{1}{3} \Rightarrow$
$y=-\frac{1}{3}x-\frac{5}{3}$

79. The slope of $x + y = 1$ is -1. The lines are parallel, so they have the same slope. The equation of the line through (1, 1) with slope -1 is $y-1=-(x-1) \Rightarrow y-1=-x+1 \Rightarrow$
$y=-x+2$.

80. The slope of $y=6x+5$ is 6. The lines are parallel, so they have the same slope. The equation of the line with slope 6 and *y*-intercept -2 is $y=6x-2$.

81. The slope of $3x-9y=18$ is $1/3$. The lines are perpendicular, so the slope of the new line is -3. The equation of the line through (–2, 4) with slope -3 is $y-4=-3(x-(-2)) \Rightarrow$
$y-4=-3x-6 \Rightarrow y=-3x-2$.

82. The slope of $-2x+y=14$ is 2. The lines are perpendicular, so the slope of the new line is $-1/2$. The equation of the line through (0, 0) with slope $-1/2$ is $y-0=-\frac{1}{2}(x-0) \Rightarrow$
$y=-\frac{1}{2}x$.

83. The slope of the line $y=6x+5$ is 6. The lines are perpendicular, so the slope of the new line is $-1/6$. The equation of the line with slope $-1/6$ and *y*-intercept 4 is
$y=-\frac{1}{6}x+4$.

84. The slope of $-2x+3y-7=0$ is $2/3$. The lines are parallel, so they have the same slope. The equation of the line through (1, 0) with slope $2/3$ is $y-0=\frac{2}{3}(x-1) \Rightarrow y=\frac{2}{3}x-\frac{2}{3}$.

85. The slope of *AB* is $\frac{3-5}{2-(-1)}=-\frac{2}{3},$ so the slope of its perpendicular bisector is $\frac{3}{2}.$ The midpoint of *AB* is
$\left(\frac{2+(-1)}{2}, \frac{3+5}{2}\right)=\left(\frac{1}{2}, 4\right).$
Using the point-slope form, the equation of the perpendicular bisector is
$y-4=\frac{3}{2}\left(x-\frac{1}{2}\right) \Rightarrow y-4=\frac{3}{2}x-\frac{3}{4} \Rightarrow$
$y=\frac{3}{2}x+\frac{13}{4}$

86. The slope of *AB* is $\frac{b-a}{a-b}=-1,$ so the slope of its perpendicular bisector is 1. The midpoint of *AB* is $\left(\frac{a+b}{2}, \frac{b+a}{2}\right).$
Using the point-slope form, the equation of the perpendicular bisector is
$y-\frac{b+a}{2}=x-\frac{a+b}{2} \Rightarrow y=x$

2.3 Applying the Concepts

87. slope = $\frac{\text{rise}}{\text{run}} \Rightarrow \frac{4}{40}=\frac{1}{10}$

88. 4 miles = 21,120 feet. slope = $\frac{\text{rise}}{\text{run}} \Rightarrow$
$\frac{2000}{21{,}120}=\frac{25}{264}$

89. a. x = the number of weeks; y = the amount of money in the account after x weeks;
$y=7x+130$

b. The slope is the amount of money deposited each week; the y-intercept is the initial deposit.

90. a. x = the number of sessions of golf; y = the yearly payment to the club; $y = 35x + 1000$

b. The slope is the cost per golf session; the y-intercept is the yearly membership fee.

91. a. x = the number of hours worked per week; y = the amount earned per week;

$$y = \begin{cases} 11x & x \le 40 \\ 16.5x - 220 & x > 40 \end{cases}$$

To compute the salary when $x > 40$, use the following steps: For 40 hours, Judy earns 40(11) = \$440. The number of overtime hours is $x - 40$. For those hours, she earns $(1.5)(11)(x - 40) = 16.5x - 660$. So her total wage is $440 + 16.50x - 660 = 16.5x - 220$.

b. The slope is the hourly wage; the y-intercept is the wage for 0 hours of work.

92. a. x = the number of months owed to pay off the refrigerator; y = the amount owed; $y = -15x + 600$

b. The slope is the amount paid each month; the y-intercept is the down payment.

93. a. x = the number of rupees; y = the number of dollars equal to x rupees.

$$y = \frac{1}{53.87}x \approx .0186x.$$

b. The slope is the number of dollars per rupee. When $x = 0$, $y = 0$.

94. a. x = the number of years after 2010; y = the life expectancy of a female born in the year $2010 + x$; $y = 0.17x + 80.8$

b. The slope is the rate of increase in life expectancy; the y-intercept is the current life expectancy of a female born in the U.S. in 2010.

95. a. The y-intercept represents the initial value of the machine, \$9000.

b. The point (10, 1) gives the value of the machine after 10 years as \$1000.

c. The value of the machine decreased from \$9000 to \$1000 over 10 years. This is a decrease of $\frac{9000-1000}{10} = \frac{8000}{10} = \$800$ per year.

d. Using the points (0, 9) and (10, 1), the slope is $\frac{9-1}{0-10} = -0.8$.

$$y - 9 = -0.8(x - 0) \Rightarrow y = -0.8x + 9$$

e. The slope gives the machine's yearly depreciation, $-0.8(1000) = -800$.

96. a. $v = -1400(2) + 14{,}000 = \$11{,}200$

b. $v = -1400(6) + 14{,}000 = \$5600$

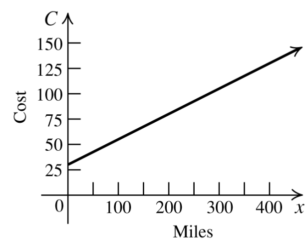
To find when the tractor will have no value, set $v = 0$ and solve the equation for t:

$0 = -1400t + 14{,}000 \Rightarrow t = 10$

97. $y = 5x + 40{,}000$

98. a. $C = 0.25x + 30$

b.



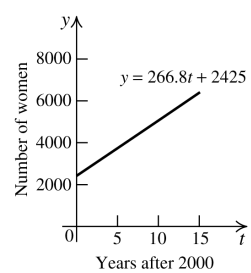
c. $y = 0.25(60) + 30 = \$45$

d. $47.75 = 0.25x + 30 \Rightarrow x = 71$ miles

99. a. The year 2005 is represented by $t = 0$, and the year 2011 is represented by $t = 6$. The points are (0, 2425) and (6, 4026). So the slope is $\frac{4026-2425}{6} \approx 266.8$

The equation is $y - 2425 = 266.8(t - 0) \Rightarrow$ $y = 266.8t + 2425$

b.



c. The year 2008 is represented by $t = 3$. So $y = 266.8(3) + 2425 \Rightarrow y = 3225.4$.

Note that there cannot be a fraction of a person, so. there were 3225 women prisoners in 2008.

d. The year 2017 is represented by $t = 12$. So $y = 266.8(12) + 2425 \Rightarrow y = 5626.6$. There will be 5627 women prisoners in 2017.

100. a. The two points are (100, 212) and (0, 32). So the slope is $\frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}$. The equation is

$$F - 32 = \frac{9}{5}(C - 0) \Rightarrow F = \frac{9}{5}C + 32$$

b. One degree Celsius change in the temperature equals $9/5$ degrees change in degrees Fahrenheit.

c.

C	$F = \frac{9}{5}C + 32$
40°*C*	104°*F*
25°*C*	77°*F*
–5°*C*	23°*F*
–10°*C*	14°*F*

d. $100°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 37.78 \approx 38°\text{C}$

$90°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 32.22 \approx 32°\text{C}$

$75°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 23.89 \approx 24°\text{C}$

$-10°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = -23.33 \approx -23°\text{C}$

$-20°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = -28.89 \approx -29°\text{C}$

e. $97.6°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 36.44°\text{C}$;

$99.6°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 37.56°\text{C}$

Normal body temperature ranges from 36.44°*C* to 37.56°*C*.

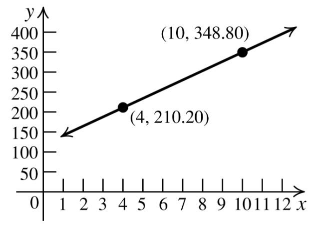
f. Let $x = °F = °C$. Then $x = \frac{9}{5}x + 32 \Rightarrow$

$-\frac{4}{5}x = 32 \Rightarrow x = -40$. At –40°, $°F = °C$.

101. a. The two points are (4, 210.20) and (10, 348.80). So the slope is

$$\frac{348.80 - 210.20}{10 - 4} = \frac{138.6}{6} = 23.1.$$

The equation is $y - 348.8 = 23.1(x - 10) \Rightarrow$
$y = 23.1x + 117.8$



b. The slope represents the cost of producing one modem. The *y*-intercept represents the fixed cost.

c. $y = 23.1(12) + 117.8 \Rightarrow y = \$395$

102. a. The two points are (5, 5.73) and (8, 6.27). The slope is $\frac{6.27 - 5.73}{8 - 5} = \frac{0.54}{3} = 0.18$. The equation is $y - 5.73 = 0.18(x - 5) \Rightarrow$ $y = 0.18x + 4.83$.

b. The slope represents the monthly change in the number of viewers. The *y*-intercept represents the number of viewers when the show first started.

c. $y = 0.18(11) + 4.83 \Rightarrow y = 6.81$ million

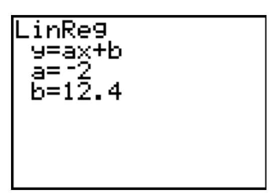
103. The independent variable *t* represents the number of years after 2005, with $t = 0$ representing 2005. The two points are (0, 12.7) and (3, 11.68). So the slope is $\frac{12.7 - 11.68}{-3} = -0.34$. The equation is

$p - 12.7 = -0.34(t - 0) \Rightarrow p = -0.34t + 12.7.$

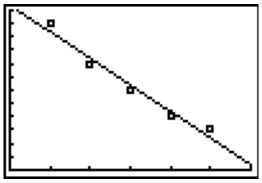
The year 2013 is represented by $t = 8$.
$p = -0.34(8) + 12.7 \Rightarrow p = 9.98\%.$

104. The year 2004 is represented by $t = 0$, so the year 2009 is represented by $t = 5$. The two points are (0, 82.7) and (5, 84.2). So the slope is $\frac{84.2 - 82.7}{5} = \frac{1.5}{5} = 0.3$.
The equation is $y = 0.3t + 82.7$.

105. a.



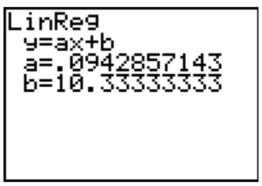
$y = -2x + 12.4$

b. 

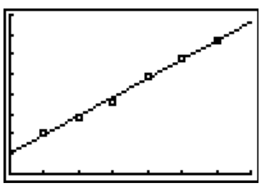
[0, 6, 1] by [0, 12, 1]

c. The price in the table is given as the number of nickels. 35¢ = 7 nickels, so let $x = 7.$ $y = -2(7) + 12.4 = -1.6$

Thus, no newspapers will be sold if the price per copy is 35¢. Note that this is also clear from the graph, which appears to cross the x-axis at approximately $x = 6$.

106. a. 

$y \approx 0.09x + 10.3$

b. 

[0, 700, 100] by [0, 80, 10]

c. The advertising expenses in the table are given as thousands of dollars, so let $x = 700.$ $y \approx 0.09(700) + 10.3 = 73.3$

Sales are given in thousands, so approximately $73.3 \times 1000 = 73,300$ computers will be sold.

2.3 Beyond the Basics

107. a. Let $A = (0, 1)$, $B = (1, 3)$, $C = (-1, -1)$.

$$m_{AB} = \frac{3-1}{1-0} = 2; m_{BC} = \frac{-1-3}{-1-1} = \frac{-4}{-2} = 2$$

$m_{AC} = \frac{-1-1}{-1-0} = 2$. The slopes of the three segments are the same, so the points are collinear.

b. $d(A, B) = \sqrt{(1-0)^2 + (3-1)^2} = \sqrt{5}$

$$d(B, C) = \sqrt{(-1-1)^2 + (-1-3)^2} = 2\sqrt{5}$$

$$d(A, C) = \sqrt{(-1-0)^2 + (-1-1)^2} = \sqrt{5}$$

Because $d(B, C) = d(A, B) + d(A, C)$, the three points are collinear.

108. a. Let $A = (1, 0.5)$, $B = (2, 0)$, $C = (0.5, 0.75)$.

$$m_{AB} = \frac{0-0.5}{2-1} = -0.5; m_{BC} = \frac{0.75-0}{0.5-2} = -0.5$$

$$m_{AC} = \frac{0.75-0.5}{0.5-1} = -0.5$$

The slopes of the three segments are the same, so the points are collinear.

b. $d(A, B) = \sqrt{(1-2)^2 + \left(\frac{1}{2}-0\right)^2} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$

$$d(B, C) = \sqrt{\left(\frac{1}{2}-2\right)^2 + \left(\frac{3}{4}-0\right)^2} = \sqrt{\frac{45}{16}} = \frac{3\sqrt{5}}{4}$$

$$d(A, C) = \sqrt{\left(\frac{1}{2}-1\right)^2 + \left(\frac{3}{4}-\frac{1}{2}\right)^2} = \sqrt{\frac{5}{16}} = \frac{\sqrt{5}}{4}$$

Because $d(B, C) = d(A, B) + d(A, C)$, the three points are collinear.

109. Since the points are collinear, the slope is the same no matter which two points are used to determine the slope. So we have

$$\frac{c-1}{1-(-5)} = \frac{-2-1}{4-(-5)} \Rightarrow \frac{c-1}{6} = \frac{-3}{9} \Rightarrow$$
$$9(c-1) = -18 \Rightarrow 9c - 9 = -18 \Rightarrow$$
$$9c = -9 \Rightarrow c = -1$$

110. Since the points are collinear, the slope is the same no matter which two points are used to determine the slope. So we have

$$\frac{\frac{11}{2}-5}{c-2} = \frac{6-5}{4-2} \Rightarrow \frac{\frac{1}{2}}{c-2} = \frac{1}{2} \Rightarrow$$
$$\frac{1}{2(c-2)} = \frac{1}{2} \Rightarrow 2(c-2) = 2 \Rightarrow$$
$$2c - 4 = 2 \Rightarrow 2c = 6 \Rightarrow c = 3$$

111. a. $m_{AB} = \frac{4-1}{-1-1} = -\frac{3}{2}; m_{BC} = \frac{8-4}{5-(-1)} = \frac{2}{3}$.

The product of the slopes $= -1$, so $AB \perp BC$, and the triangle is a right triangle.

b. $d(A,B)=\sqrt{(-1-1)^2+(4-1)^2}=\sqrt{13}$
$d(B,C)=\sqrt{(5-(-1))^2+(8-4)^2}=\sqrt{52}$
$d(A,C)=\sqrt{(5-1)^2+(8-1)^2}=\sqrt{65}$
$(d(A,B))^2+(d(B,C))^2=(d(A,C))^2$, so the triangle is a right triangle.

112. $m_{AB}=\dfrac{2-(-1)}{1-(-4)}=\dfrac{3}{5}; m_{BC}=\dfrac{1-2}{3-1}=-\dfrac{1}{2}$
$m_{CD}=\dfrac{-2-1}{-2-3}=\dfrac{3}{5}; m_{AD}=\dfrac{-2-(-1)}{-2-(-4)}=-\dfrac{1}{2}$
So, $AB \parallel CD$ and $BC \parallel AD$, and $ABCD$ is a parallelogram.

113. The equation of ℓ_1 is $y=m_1x+b_1$ and the equation of ℓ_2 is $y=m_2x+b_2$. Let (x_1,y_1) and (x_2,y_2) be on ℓ_1. If $\ell_1 \parallel \ell_2$, then the distance between them is b_1-b_2. In other words, $(x_1, y_1-(b_1-b_2))$ and $(x_2, y_2-(b_1-b_2))$ are on ℓ_2. So,
$y_1-(b_1-b_2)=m_2x_1+b_2 \Rightarrow y_1-b_1=m_2x_1 \Rightarrow$
$y_1=m_2x_1+b_1$. However, (x_1,y_1) lies on ℓ_1.
So $y_1=m_2x_1+b_1=m_1x_1+b_1 \Rightarrow m_2=m_1$.

114. $d(A,O)=\sqrt{x^2+(m_1x)^2}=\sqrt{x^2+m_1{}^2x^2}$.
$d(B,O)=\sqrt{x^2+(m_2x)^2}=\sqrt{x^2+m_2{}^2x^2}$.
$d(A,B)=\sqrt{(x-x)^2+(m_2x-m_1x)^2}$
$=\sqrt{(m_2x-m_1x)^2}$.
Apply the Pythagorean theorem to obtain

$$\left(\sqrt{(m_2x-m_1x)^2}\right)^2=\left(\sqrt{x^2+m_1{}^2x^2}\right)^2+\left(\sqrt{x^2+m_2{}^2x^2}\right)^2$$

$$(m_2x-m_1x)^2=x^2+m_1{}^2x^2+x^2+m_2{}^2x^2$$

$$m_2{}^2x^2-2m_1m_2x^2+m_1{}^2x^2=x^2+m_1{}^2x^2+x^2+m_2{}^2x^2$$

$$x^2(m_2-m_1)^2=x^2(m_1{}^2+1)+x^2(m_2{}^2+1)$$

$$m_2{}^2-2m_1m_2+m_1{}^2=m_1{}^2+m_2{}^2+2$$

$$-2m_1m_2=2 \Rightarrow m_1m_2=-1$$

115. Let (x, y) be the coordinates of point B. Then
$d(A,B)=12.5=\sqrt{(x-2)^2+(y-2)^2} \Rightarrow$
$(x-2)^2+(y-2)^2=156.25$ and
$m_{AB}=\dfrac{4}{3}=\dfrac{y-2}{x-2} \Rightarrow 4(x-2)=3(y-2) \Rightarrow$
$y=\dfrac{4}{3}x-\dfrac{2}{3}$. Substitute this into the first equation and solve for x:

$$(x-2)^2+\left(\left(\frac{4}{3}x-\frac{2}{3}\right)-2\right)^2=156.25$$

$$(x-2)^2+\left(\frac{4}{3}x-\frac{8}{3}\right)^2=156.25$$

$$x^2-4x+4+\frac{16}{9}x^2-\frac{64}{9}x+\frac{64}{9}=156.25$$

$$9x^2-36x+36+16x^2-64x+64=1406.25$$

$$25x^2-100x-1306.25=0$$

Solve this equation using the quadratic formula:

$$x=\frac{100\pm\sqrt{100^2-4(25)(-1306.25)}}{2(25)}$$

$$=\frac{100\pm\sqrt{10,000+130,625}}{50}$$

$$=\frac{100\pm\sqrt{140,625}}{50}=\frac{100\pm 375}{50}=9.5 \text{ or } -5.5$$

Now find y by substituting the x-values into the slope formula: $\dfrac{4}{3}=\dfrac{y-2}{9.5-2} \Rightarrow y=12$ or
$\dfrac{4}{3}=\dfrac{y-2}{-5.5-2} \Rightarrow y=-8$. So the coordinates of B are (9.5, 12) or (–5.5, –8).

116. Let (x, y) be a point on the circle with (x_1, y_1) and (x_2, y_2) as the endpoints of a diameter. Then the line that passes through (x, y) and (x_1, y_1) is perpendicular to the line that passes through (x, y) and (x_2, y_2), and their slopes are negative reciprocals. So
$\dfrac{y-y_1}{x-x_1}=-\dfrac{x-x_2}{y-y_2} \Rightarrow$
$(y-y_1)(y-y_2)=-(x-x_1)(x-x_2) \Rightarrow$
$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0.$

117. Write the equations of each circle in standard form to find the centers.
$(x^2+6x+9)+(y^2-14y+49)=1+9+49 \Rightarrow$
$(x+3)^2+(y-7)^2=59$

(*continued on next page*)

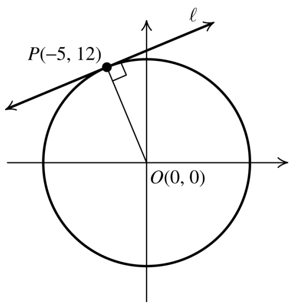
(*continued*)

$$\left(x^2-4x+4\right)+\left(y^2+10y+25\right)=2+4+25$$
$$(x-2)^2+(y+5)^2=31$$

The centers are (−3, 7) and (2, −5).
Using the result from exercise 116, we have

$$(x-(-3))(x-2)+(y-7)(y-(-5))=0 \Rightarrow$$
$$(x+3)(x-2)+(y-7)(y+5)=0 \Rightarrow$$
$$x^2+x-6+y^2-2y-35=0 \Rightarrow$$
$$x^2+y^2+x-2y-41=0$$

118.



$$m_{\overline{OP}}=\frac{12-0}{-5-0}=-\frac{12}{5}$$

Since the tangent line ℓ is perpendicular to $\overline{OP}$, the slope of ℓ is the negative reciprocal of $-\frac{12}{5}$ or $\frac{5}{12}$. Using the point-slope form, we have

$$y-12=\frac{5}{12}\left[x-(-5)\right] \Rightarrow y-12=\frac{5}{12}(x+5) \Rightarrow$$
$$y-12=\frac{5}{12}x+\frac{25}{12} \Rightarrow y=\frac{5}{12}x+\frac{169}{12}$$

119. The tangent line at a point is perpendicular to the radius drawn to that point. The center of $x^2+y^2=25$ is (0, 0), so the slope of the radius is $-\frac{3}{4}$ and the slope of the tangent is $\frac{4}{3}$. Using the point-slope form, the equation of the tangent is

$$y-(-3)=\frac{4}{3}(x-4) \Rightarrow y+3=\frac{4}{3}x-\frac{16}{3} \Rightarrow$$
$$y=\frac{4}{3}x-\frac{25}{3}.$$

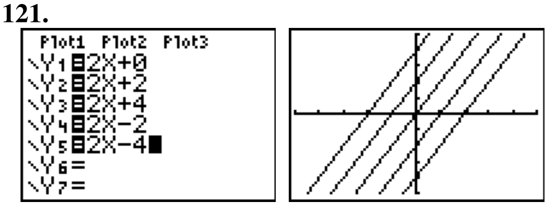
120. The tangent line at a point is perpendicular to the radius drawn to that point. First, find the center of $x^2+y^2+4x-6y-12=0$.

$$x^2+y^2+4x-6y-12=0 \Rightarrow$$
$$\left(x^2+4x+4\right)+\left(y^2-6y+9\right)=12+4+9 \Rightarrow$$
$$(x+2)^2+(y-3)^2=25$$

The center of the circle is (−2, 3). The slope of the radius is $\frac{7-3}{1-(-2)}=\frac{4}{3}$ and the slope of the tangent is $-\frac{3}{4}$. Using the point-slope form, the equation of the tangent is

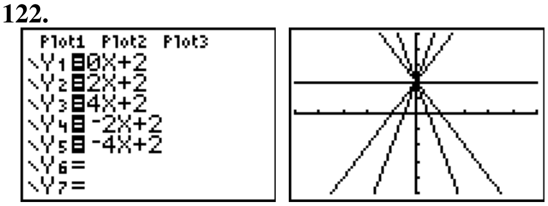
$$y-7=-\frac{3}{4}(x-1) \Rightarrow y-7=-\frac{3}{4}x+\frac{3}{4} \Rightarrow$$
$$y=-\frac{3}{4}x+\frac{31}{4}.$$

121.



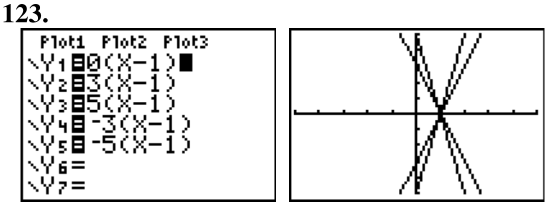
The family of lines has slope 2. The lines have different *y*-intercepts.

122.



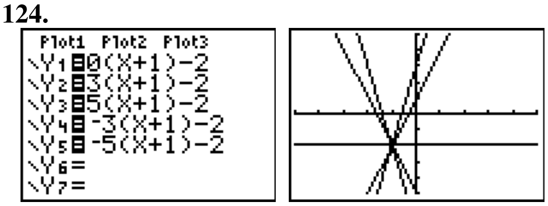
The family of lines has *y*-intercept 2. The lines have different slopes.

123.



The lines pass through (1, 0). The lines have different slopes.

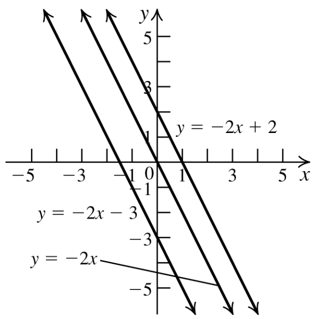
124.



The lines pass through (−1, −2). The lines have different slopes.

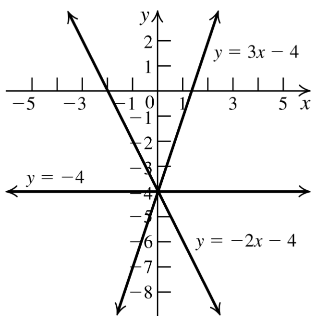
2.3 Critical Thinking/Discussion/Writing

125. a.



This is a family of lines parallel to the line $y = -2x$. They all have slope –2.

b.



This is a family of lines that passes through the point (0, –4).

126. $\left.\begin{array}{l} y = m_1x + b_1 \\ y = m_2x + b_2 \end{array}\right\} \Rightarrow m_1x + b_1 = m_2x + b_2 \Rightarrow$
$m_1x - m_2x = b_2 - b_1 \Rightarrow x(m_1 - m_2) = b_2 - b_1 \Rightarrow$
$x = \frac{b_2 - b_1}{m_1 - m_2}$

a. If $m_1 > m_2 > 0$ and $b_1 > b_2$, then
$x = \frac{b_2 - b_1}{m_1 - m_2} = -\frac{b_1 - b_2}{m_1 - m_2}.$

b. If $m_1 > m_2 > 0$ and $b_1 < b_2$, then
$x = \frac{b_2 - b_1}{m_1 - m_2}.$

c. If $m_1 < m_2 < 0$ and $b_1 > b_2$, then
$x = \frac{b_2 - b_1}{m_1 - m_2} = \frac{b_1 - b_2}{m_2 - m_1}.$

d. If $m_1 < m_2 < 0$ and $b_1 < b_2$, then
$x = \frac{b_2 - b_1}{m_1 - m_2} = -\frac{b_2 - b_1}{m_2 - m_1}.$

2.3 Maintaining Skills

127. $x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0 \Rightarrow$
$x = 2, -1$
Solution set: $\{-1, 2\}$

128. $x^2 + 2x - 3 = 0 \Rightarrow (x+3)(x-1) = 0 \Rightarrow$
$x = -3, 1$
Solution set: $\{-3, 1\}$

129. $\frac{x^2 + 5x + 6}{x - 1} = 0 \Rightarrow x^2 + 5x + 6 = 0 \Rightarrow$
$(x+2)(x+3) = 0 \Rightarrow x = -2, -3$
Solution set: $\{-3, -2\}$

130. $\frac{6x^2 - x - 1}{x^2 - x - 12} = 0 \Rightarrow \frac{(3x+1)(2x-1)}{(x-4)(x+3)} = 0 \Rightarrow$
$(3x+1)(2x-1) = 0 \Rightarrow x = -\frac{1}{3}, \frac{1}{2}$
Solution set: $\left\{-\frac{1}{3}, \frac{1}{2}\right\}$

131. $x^2 - 5x + 6 \geq 0 \Rightarrow (x-3)(x-2) \geq 0$
Now solve the associated equation:
$(x-3)(x-2) = 0 \Rightarrow x = 3$ or $x = 2$.
So, the intervals are
$(-\infty, 2], [2, 3],$ and $[3, \infty)$.

Interval	Test point	Value of $x^2 - 5x + 6$	Result
$(-\infty, 3]$	0	6	+
$[2, 3]$	$\frac{5}{2}$	$-\frac{1}{4}$	–
$[3, \infty)$	5	6	+

The solution set is $(-\infty, 2] \cup [3, \infty)$.

132. $\frac{x-1}{x+2} + 1 \geq 0 \Rightarrow \frac{x - 1 + x + 2}{x+2} \geq 0 \Rightarrow$
$\frac{2x+1}{x+2} \geq 0$
Now solve $2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$ and
$x + 2 = 0 \Rightarrow x = -2.$
So the intervals are
$(-\infty, -2), \left(-2, -\frac{1}{2}\right],$ and $\left[-\frac{1}{2}, \infty\right)$. The original fraction is not defined if $x = -2$, so -2 is not included in the intervals.

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Interval	Test point	Value of $\frac{-2x+5}{x-2}$	Result
$(-\infty, -2)$	-3	5	+
$\left(-2, -\frac{1}{2}\right]$	-1	-1	+
$\left[-\frac{1}{2}, \infty\right)$	0	$\frac{1}{2}$	+

The solution set is $(-\infty, -2)\cup\left[-\frac{1}{2}, \infty\right)$.

133. $$\frac{1}{h}\left[\frac{1}{x+h}-\frac{1}{x}\right]=\frac{1}{h}\left[\frac{x}{x(x+h)}-\frac{x+h}{x(x+h)}\right]$$
$$=\frac{1}{h}\left[\frac{x-(x+h)}{x(x+h)}\right]$$
$$=\frac{1}{h}\left[\frac{-h}{x(x+h)}\right]=-\frac{1}{x(x+h)}$$

134. $$\frac{-(x+h)^2+x^2}{h}=\frac{-\left(x^2+2xh+h^2\right)+x^2}{h}$$
$$=\frac{-2xh-h^2}{h}=-2x-h$$

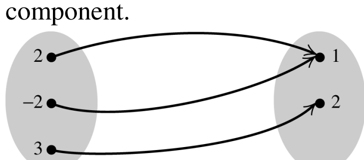
135. $$\frac{\sqrt{5}-\sqrt{2}}{3}\cdot\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}=\frac{5-2}{3\left(\sqrt{5}+\sqrt{2}\right)}$$
$$=\frac{3}{3\left(\sqrt{5}+\sqrt{2}\right)}=\frac{1}{\sqrt{5}+\sqrt{2}}$$

136. $$\frac{\sqrt{x+2}-\sqrt{x}}{2}\cdot\frac{\sqrt{x+2}+\sqrt{x}}{\sqrt{x+2}+\sqrt{x}}=\frac{(x+2)-x}{2\left(\sqrt{x+2}+\sqrt{x}\right)}$$
$$=\frac{2}{2\left(\sqrt{x+2}+\sqrt{x}\right)}$$
$$=\frac{1}{\sqrt{x+2}+\sqrt{x}}$$

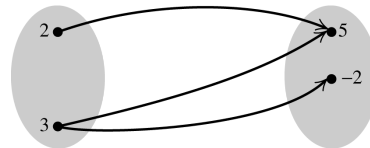
2.4 Functions

2.4 Practice Problems

1.a. The domain of R is {2, −2, 3} and its range is {1, 2}. The relation R is a function because no two ordered pairs in R have the same first component.



b. The domain of S is {2, 3} and its range is {5, −2}. The relation S is not a function because the ordered paired (3, −2) and (3, 5) have the same first component.



2. Solve each equation for y.

a. $2x^2-y^2=1 \Rightarrow 2x^2-1=y^2 \Rightarrow$ $\pm\sqrt{2x^2-1}=y$; not a function

b. $x-2y=5 \Rightarrow x-5=2y \Rightarrow \frac{1}{2}(x-5)=y$; a function

3.a. $g(0)=-2(0)^2+5(0)=0$

b. $g(-1)=-2(-1)^2+5(-1)=-7$

c. $$g(x+h)=-2(x+h)^2+5(x+h)$$
$$=-2\left(x^2+2xh+h^2\right)+5x+5h$$
$$=-2x^2-4hx+5x-2h^2+5h$$

4. $$A_{TLMS}=(\text{length})(\text{height})=(|3-1|)(22)$$
$$=(2)(22)=44 \text{ sq. units}$$

5.a. $f(x)=\frac{1}{\sqrt{1-x}}$ is not defined when $1-x=0 \Rightarrow x=1$ or when $1-x<0 \Rightarrow 1<x.$. Thus, the domain of f is $(-\infty,1)$.

b. $f(x)=\sqrt{\frac{x+2}{x-3}}$ is not defined when the denominator equals 0 or when $\frac{x+2}{x-3}<0.$

$\frac{x+2}{x-3}<0$ for (−2, 3), so this interval is not in the domain of f. The denominator $x-3\le 0 \Rightarrow x\le 3,$ so all numbers less than or equal to 3 are not in the domain of f. Thus, the domain of f is $(-\infty, -2]\cup(3, \infty)$.

6. $f(x)=x^2,$ domain X = [−3, 3]

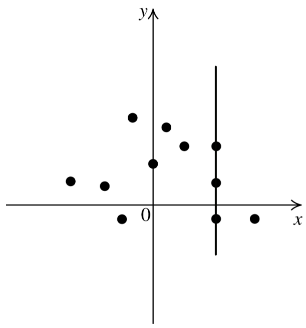
a. $f(x)=10 \Rightarrow x^2=10 \Rightarrow x=\pm\sqrt{10}\approx\pm3.16$

Since $\sqrt{10}>3$ and $-\sqrt{10}<-3,$ neither solution is in the interval X = [−3, 3]. Therefore, 10 is not in the range of f.

b. $f(x) = 4 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$
Since $-3 < -2 < 2 < 3$, 4 is in the range of *f*.

c. The range of *f* is the interval [0, 9] because for each number *y* in this interval, the number $x = \sqrt{y}$ is in the interval [−3, 3].

7.



The graph is not a function because a vertical line can be drawn through three points, as shown.

8. $y = f(x) = x^2 + 4x - 5$

a. Check whether the ordered pair (2, 7) satisfies the equation:

$7 \stackrel{?}{=} 2^2 + 4(2) - 5$
$7 = 7$ ✓

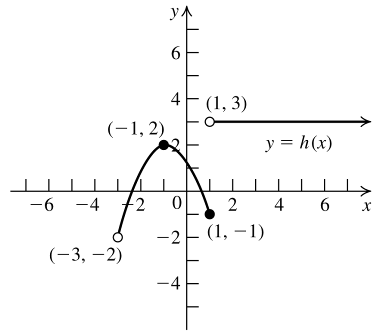
The point (2, 7) is on the graph.

b. Let $y = -8$, then solve for *x*:
$-8 = x^2 + 4x - 5 \Rightarrow 0 = x^2 + 4x + 3 \Rightarrow$
$0 = (x+3)(x+1) \Rightarrow x = -3$ or $x = -1$
The points (−3, −8) and (−1, −8) lie on the graph.

c. Let $x = 0$, then solve for *y*:
$y = 0^2 + 4(0) - 5 = -5$
The *y*-intercept is −5.

d. Let $y = 0$, then solve for *x*:
$0 = x^2 + 4x - 5 \Rightarrow 0 = (x+5)(x-1) \Rightarrow$
$x = -5$ or $x = 1$
The *x*-intercepts are −5 and 1.

9.



Domain: $(-3, \infty)$; range: $(-2, 2] \cup \{3\}$

10. The range of *C*(*t*) is [6, 12).
$C(11) = \frac{1}{2}C(10) + 6 = \frac{1}{2}(11.988) + 6 = 11.994$.

11. From Example 11, we have $AP = \sqrt{500^2 + x^2}$ and $\overline{PD} = 1200 - x$ feet. If *c* = the cost on land, the total cost *C* is given by
$C = 1.3c(PD) + c(AP)$
$= 1.3c\sqrt{500^2 + x^2} + c(1200 - x)$

12.a. $C(x) = 1200x + 100{,}000$

b. $R(x) = 2500x$

c. $P(x) = R(x) - C(x)$
$= 2500x - (1200x + 100{,}000)$
$= 1300x - 100{,}000$

d. The break-even point occurs when $C(x) = R(x)$.
$1200x + 100{,}000 = 2500x$
$100{,}000 = 1300x \Rightarrow x \approx 77$
Metro needs 77 shows to break even.

2.4 Basic Concepts and Skills

1. In the functional notation $y = f(x)$, *x* is the <u>independent</u> variable.
2. If $f(-2) = 7$, then −2 is in the <u>domain</u> of the function *f*, and 7 is in the <u>range</u> of *f*.
3. If the point (9, −14) is on the graph of a function *f*, then <u>$f(9) = -14$</u>.
4. If (3, 7) and (3, 0) ar both points on a graph, then the graph cannot be the graph of a <u>function</u>.
5. To find the *x*-intercepts of the graph of an equation in *x* and *y*, we solve the equation <u>$y = 0$</u>.

6. False. For example, if $f(x) = \frac{1}{x}$, then $a = 1$ and $b = -1$ are both in the domain of f. However, $a + b = 0$ is not in the domain of f.

7. True. $-x = 7$ and the square root function is defined for all positive numbers.

8. False. The domain of f is all real x for $x > -2$. Values of $x \le -2$ make the square root undefined.

9. Domain: $\{a, b, c\}$; range: $\{d, e\}$; function

10. Domain: $\{a, b, c\}$; range: $\{d, e, f\}$; function

11. Domain: $\{a, b, c\}$; range: $\{1, 2\}$; function

12. Domain: $\{1, 2, 3\}$; range: $\{a, b, c, d\}$; not a function

13. Domain: $\{0, 3, 8\}$; range: $\{-3, -2, -1, 1, 2\}$; not a function

14. Domain: $\{-3, -1, 0, 1, 2, 3\}$; range: $\{-8, -3, 0, 1\}$; function

15. $x + y = 2 \Rightarrow y = -x + 2$; a function

16. $x = y - 1 \Rightarrow y = x + 1$; a function

17. $y = \frac{1}{x}$; a function

18. $xy = -1 \Rightarrow y = -\frac{1}{x}$; a function

19. $y^2 = x^2 \Rightarrow y = \pm\sqrt{x^2} \Rightarrow y = \pm x$; not a function

20. $x = |y| \Rightarrow y = x$ or $y = -x$; not a function

21. $y = \frac{1}{\sqrt{2x-5}}$; a function

22. $y = \frac{1}{\sqrt{x^2-1}}$; a function

23. $2 - y = 3x \Rightarrow y = 2 - 3x$; a function

24. $3x - 5y = 15 \Rightarrow y = \frac{3}{5}x - 3$; a function

25. $x + y^2 = 8 \Rightarrow y = \pm\sqrt{8-x}$; not a function

26. $x = y^2 \Rightarrow y = \sqrt{x}$ or $y = -\sqrt{x}$; not a function

27. $x^2 + y^3 = 5 \Rightarrow y = \sqrt[3]{5-x^2}$; a function

28. $x + y^3 = 8 \Rightarrow y = \sqrt[3]{8-x}$; a function

In exercises 29–32, $f(x) = x^2 - 3x + 1$, $g(x) = \frac{2}{\sqrt{x}}$, and $h(x) = \sqrt{2-x}$.

29. $f(0) = 0^2 - 3(0) + 1 = 1$

$g(0) = \frac{2}{\sqrt{0}} \Rightarrow g(0)$ is undefined

$; h(0) = \sqrt{2-0} = \sqrt{2}$

$f(a) = a^2 - 3a + 1$

$f(-x) = (-x)^2 - 3(-x) + 1 = x^2 + 3x + 1$

30. $f(1) = 1^2 - 3(1) + 1 = -1; g(1) = \frac{2}{\sqrt{1}} = 2;$

$h(1) = \sqrt{2-1} = 1; g(a) = \frac{2}{\sqrt{a}};$

$g(x^2) = \frac{2}{\sqrt{x^2}} = \frac{2}{|x|}$

31. $f(-1) = (-1)^2 - 3(-1) + 1 = 5;$

$g(-1) = \frac{2}{\sqrt{-1}} \Rightarrow g(-1)$ is undefined;

$h(-1) = \sqrt{2-(-1)} = \sqrt{3}; h(c) = \sqrt{2-c};$

$h(-x) = \sqrt{2-(-x)} = \sqrt{2+x}$

32. $f(4) = 4^2 - 3(4) + 1 = 5; g(4) = \frac{2}{\sqrt{4}} = 1;$

$h(4) = \sqrt{2-4} = \sqrt{-2} \Rightarrow h(4)$ is undefined;

$g(2+k) = \frac{2}{\sqrt{2+k}};$

$$f(a+k) = (a+k)^2 - 3(a+k) + 1$$
$$= a^2 + 2ak + k^2 - 3a - 3k + 1$$

33.a. $f(0) = \frac{2(0)}{\sqrt{4-0^2}} = 0$

b. $f(1) = \frac{2(1)}{\sqrt{4-1^2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

c. $f(2) = \frac{2(2)}{\sqrt{4-2^2}} = \frac{4}{0} \Rightarrow f(2)$ is undefined

d. $f(-2) = \frac{2(-2)}{\sqrt{4-(-2)^2}} = \frac{-4}{0} \Rightarrow f(-2)$ is undefined

e. $f(-x) = \dfrac{2(-x)}{\sqrt{4-(-x)^2}} = \dfrac{-2x}{\sqrt{4-x^2}}$

34.a. $g(0) = 2(0) + \sqrt{0^2 - 4} \Rightarrow g(0)$ is undefined

b. $g(1) = 2(1) + \sqrt{1^2 - 4} \Rightarrow g(1)$ is undefined

c. $g(2) = 2(2) + \sqrt{2^2 - 4} = 4$

d. $g(-3) = 2(-3) + \sqrt{(-3)^2 - 4} = -6 + \sqrt{5}$

e. $g(-x) = 2(-x) + \sqrt{(-x)^2 - 4}$
$= -2x + \sqrt{x^2 - 4}$

35. The width of each rectangle is 1. The height of the left rectangle is $f(1) = 1^2 + 2 = 3.$ The height of the right rectangle is $f(2) = 2^2 + 2 = 6.$

$A = (1)(f(1)) + (1)(f(2))$
$= 1(3) + (1)(6) = 9$ sq. units

36. The width of each rectangle is 1. The height of the left rectangle is $f(0) = 0^2 + 2 = 2.$ The height of the right rectangle is $f(1) = 1^2 + 2 = 3.$

$A = (1)(f(0)) + (1)(f(1))$
$= 1(2) + (1)(3) = 5$ sq. units

37. $(-\infty, \infty)$

38. $(-\infty, \infty)$

39. The denominator is not defined for $x = 9$. The domain is $(-\infty, 9) \cup (9, \infty)$

40. The denominator is not defined for $x = -9$. The domain is $(-\infty, -9) \cup (-9, \infty)$

41. The denominator is not defined for $x = -1$ or $x = 1$. The domain is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

42. The denominator is not defined for $x = -2$ or $x = 2$. The domain is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

43. The numerator is not defined for $x < 3$, and the denominator is not defined for $x = -2$. The domain is $[3, \infty)$

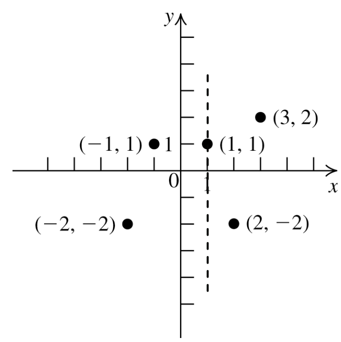
44. The denominator is not defined for $x \geq 4$. The domain is $(-\infty, 4)$

45. The denominator equals 0 if $x = -1$ or $x = -2$. The domain is $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$.

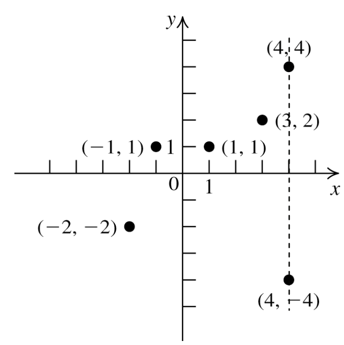
46. The denominator equals 0 if $x = -2$ or $x = -3$. The domain is $(-\infty, -3) \cup (-3, -2) \cup (-2, \infty)$.

47. The denominator is not defined for $x = 0$. The domain is $(-\infty, 0) \cup (0, \infty)$

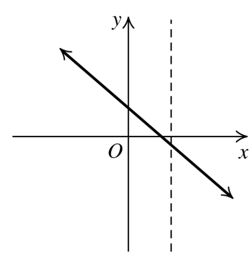
48. The denominator is defined for all values of x. The domain is $(-\infty, \infty)$ **.**

49.


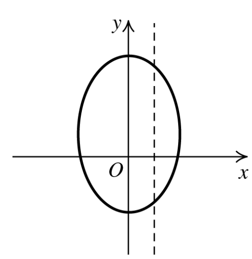
a function

50.


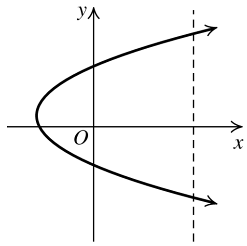
not a function

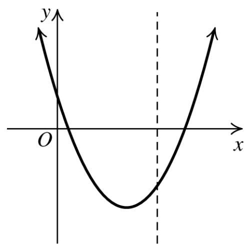
51.


a function

52.


not a function

53.  not a function

54.  a function

55. $f(-4) = -2; f(-1) = 1; f(3) = 5; f(5) = 7$

56. $g(-2) = 5; g(1) = -4; g(3) = 0; g(4) = 5$

57. $h(-2) = -5; h(-1) = 4; h(0) = 3; h(1) = 4$

58. $f(-1) = 4; f(0) = 0; f(1) = -4$

59. $h(x) = 7$, so solve the equation $7 = x^2 - x + 1$.
$x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0 \Rightarrow x = -2$ or $x = 3$.

60. $H(x) = 7$, so solve the equation $7 = x^2 + x + 8$.
$x^2 + x + 1 = 0 \Rightarrow x = \dfrac{-1 \pm \sqrt{1 - 4(1)(1)}}{2(1)} \Rightarrow$
$x = \dfrac{-1 \pm \sqrt{-3}}{2} \Rightarrow$ there is no real solution.

61.a. $1 = -2(1+1)^2 + 7 \Rightarrow 1 = -1,$ which is false. Therefore, (1, 1) does not lie on the graph of *f*.

b. $1 = -2(x+1)^2 + 7 \Rightarrow 2(x+1)^2 = 6 \Rightarrow$
$(x+1)^2 = 3 \Rightarrow x + 1 = \pm\sqrt{3} \Rightarrow x = -1 \pm \sqrt{3}$
The points $\left(-1-\sqrt{3}, 1\right)$ and $\left(-1+\sqrt{3}, 1\right)$ lie on the graph of *f*.

c. $y = -2(0+1)^2 + 7 \Rightarrow y = 5$
The *y*-intercept is (0, 5).

d. $0 = -2(x+1)^2 + 7 \Rightarrow -7 = -2(x+1)^2 \Rightarrow$
$\dfrac{7}{2} = (x+1)^2 \Rightarrow \pm\sqrt{\dfrac{7}{2}} = \pm\dfrac{\sqrt{14}}{2} = x + 1 \Rightarrow$
$x = -1 \pm \dfrac{\sqrt{14}}{2}$
The *x*-intercepts are $\left(-1 - \dfrac{\sqrt{14}}{2}, 0\right)$ and $\left(-1 + \dfrac{\sqrt{14}}{2}, 0\right)$.

62.a. $10 = -3(-2)^2 - 12(-2) \Rightarrow 10 = 12,$ which is false. Therefore, (–2, 10) does not lie on the graph of *f*.

b. $f(x) = 12$, so solve the equation $-3x^2 - 12x = 12$.
$-3x^2 - 12x = 12 \Rightarrow x^2 + 4x = -4 \Rightarrow$
$x^2 + 4x + 4 = 0 \Rightarrow (x+2)^2 = 0 \Rightarrow$
$x + 2 = 0 \Rightarrow x = -2$

c. $y = -3(0)^2 - 12(0) \Rightarrow y = 0$
The *y*-intercept is (0, 0).

d. $0 = -3x^2 - 12x \Rightarrow 0 = -3x(x+4) \Rightarrow$
$x = 0$ or $x = -4$
The *x*-intercepts are (0, 0) and (–4, 0).

63. Domain: [–3, 2]; range: [–3, 3]

64. Domain: [–1, 3]; range: [–2, 4]

65. Domain: $[-4, \infty)$; range: [–2, 3]

66. Domain: $(-\infty, 4]$; range: [–1, 3]

67. Domain: $[-3, \infty)$; range: $[-1, 4] \cup \{-3\}$

68. Domain: $(-\infty, -1) \cup [1, 4)$
Range: (–2, 4]

69. Domain: $(-\infty, 4] \cup [-2, 2] \cup [4, \infty)$
Range: $[-2, 2] \cup \{3\}$

70. Domain: $(-\infty, -2) \cup [-1, \infty)$
Range: $(-\infty, \infty)$

71. $[-9, \infty)$

72. [–1, 7]

73. –3, 4, 7, 9

74. 6

75. $f(-7) = 4,\ f(1) = 5,\ f(5) = 2$

76. $f(-4) = 4,\ f(-1) = 7,\ f(3) = 3$

77. $\{-3.75, -2.25, 3\} \cup [12, \infty)$

78. $\varnothing$

79. $[-9, \infty)$ **80.** $\{-4\} \cup [-2, 6]$

81. $g(-4) = -1,\ g(1) = 3,\ g(3) = 4$

82. $|g(-5) - g(5)| = |-2 - 6| = 8$

83. $[-9, -5)$ **84.** $[5, \infty)$

2.4 Applying the Concepts

85. A function because there is only one high temperature per day.

86. A function because there is only one cost of a first-class stamp on January 1 each year.

87. Not a function because there are several states that begin with N (i.e., New York, New Jersey, New Mexico, Nevada, North Carolina, North Dakota); there are also several states that begin with T and S.

88. Not a function because people with a different name may have the same birthday.

89. $A(x) = x^2; A(4) = 16;\ A(4)$ represents the area of a tile with side 4.

90. $V(x) = x^3; V(3) = 27 \text{ in.}^3;\ V(3)$ represents the volume of a cube with edge 3.

91. It is a function. $S(x) = 6x^2; S(3) = 54$

92. $f(x) = \dfrac{x}{39.37}; f(59) \approx 1.5$ meters

93.a. The domain is [0, 8].

b. $h(2) = 128(2) - 16(2^2) = 192$

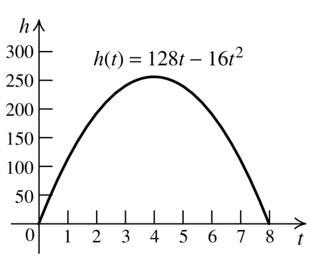
$h(4) = 128(4) - 16(4^2) = 256$

$h(6) = 128(6) - 16(6^2) = 192$

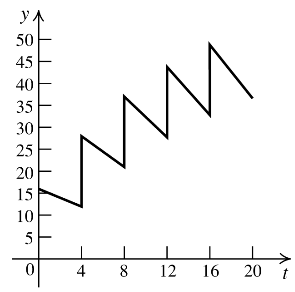
c. $0 = 128t - 16t^2 \Rightarrow 0 = 16t(8 - t) \Rightarrow$

$t = 0$ or $t = 8$. It will take 8 seconds for the stone to hit the ground.

d.



94. After 4 hours, there are (0.75)(16) = 12 ml of the drug.
After 8 hours, there are (0.75)(12 + 16) = 21 ml.
After 12 hours, there are
(0.75)(21 + 16) = 27.75 ml.
After 16 hours, there are
(0.75)(27.75 + 16) = 32.81 ml.
After 20 hours, there are
(0.75)(32.81 + 16) = 36.61 ml.



95. $x + y = 28 \Rightarrow y = 28 - x$

$P = x(28 - x) = 28x - x^2$

96. $P = 60 = 2(x + y) \Rightarrow 30 = x + y \Rightarrow y = 30 - x$

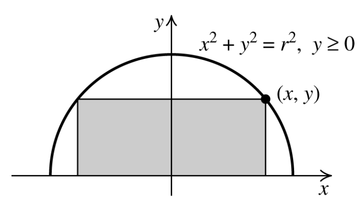
$A = x(30 - x) = 30x - x^2$

97. Note that the length of the base = the width of the base = x.

$V = lwh = x^2h = 64 \Rightarrow h = \dfrac{64}{x^2}$

$$S = 2lw + 2lh + 2wh$$
$$= 2x^2 + 2x\left(\frac{64}{x^2}\right) + 2x\left(\frac{64}{x^2}\right)$$
$$= 2x^2 + \frac{128}{x} + \frac{128}{x} = 2x^2 + \frac{256}{x}$$

98.



a. $x^2 + y^2 = r^2 \Rightarrow y^2 = r^2 - x^2 \Rightarrow$

$y = \sqrt{r^2 - x^2}$

The length of the rectangle is $2x$ and its height is $y = \sqrt{r^2 - x^2}$.

$$P = 2l + 2w = 2(2x) + 2\sqrt{r^2 - x^2}$$
$$= 4x + 2\sqrt{r^2 - x^2}$$

b. $A = lw = 2x\sqrt{r^2 - x^2}$

99. The piece with length x is formed into a circle, so $C = x = 2\pi r \Rightarrow r = \frac{x}{2\pi}$. Thus, the area of the circle is $A = \pi r^2 = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}$.

The piece with length $20 - x$ is formed into a square, so $P = 20 - x = 4s \Rightarrow s = \frac{1}{4}(20 - x)$.

Thus, the area of the square is

$s^2 = \left[\frac{1}{4}(20 - x)\right]^2 = \frac{1}{16}(20 - x)^2$.

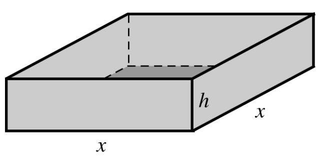
The sum of the areas is $A = \frac{x^2}{4\pi} + \frac{1}{16}(20 - x)^2$

100. The volume of the tank is $V = 64 = \pi r^2 h$, so $h = \frac{64}{\pi r^2}$. The top is open, so the surface area is given by

$$\pi r^2 + 2\pi r h = \pi r^2 + 2\pi r\left(\frac{64}{\pi r^2}\right) = \pi r^2 + \frac{128}{r}.$$

101. The volume of the pool is

$V = 288 = x^2 h \Rightarrow h = \frac{288}{x^2}$.



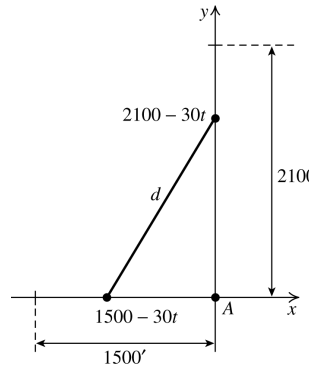
The total area to be tiled is

$4xh = 4x\left(\frac{288}{x^2}\right) = \frac{1152}{x}$

The cost of the tile is $6\left(\frac{1152}{x}\right) = \frac{6912}{x}$.

The area of the bottom of the pool is x^2, so the cost of the cement is $2x^2$. Therefore, the total cost is $C = 2x^2 + \frac{6912}{x}$.

102.

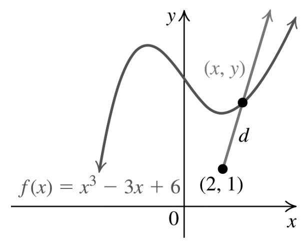


Using the Pythagorean theorem, we have

$d^2 = (1500 - 30t)^2 + (2100 - 30t)^2 \Rightarrow$

$d = \left[(1500 - 30t)^2 + (2100 - 30t)^2\right]^{1/2}$

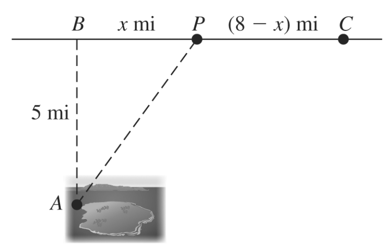
103.



Using the distance formula we have

$$\begin{aligned} d &= \sqrt{(x-2)^2 + (y-1)^2} \\ &= \sqrt{(x-2)^2 + \left[\left(x^3 - 3x + 6\right) - 1\right]^2} \\ &= \sqrt{(x-2)^2 + \left(x^3 - 3x + 5\right)^2} \\ &= \left[(x-2)^2 + \left(x^3 - 3x + 5\right)^2\right]^{1/2} \end{aligned}$$

104.

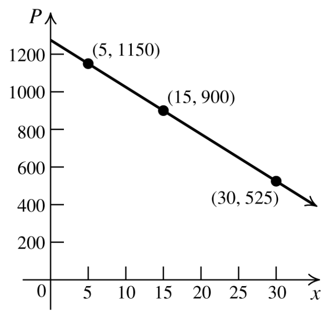


The distance from A to P is $\sqrt{x^2 + 5^2} = \sqrt{x^2 + 25}$ mi. At 4 mi/hr, it will take Julio $\frac{\sqrt{x^2 + 25}}{4}$ hr to row that distance. The distance from P to C is $(8 - x)$ mi, so it will take Julio $\frac{8 - x}{5}$ hr to walk that distance. The total time it will take him to travel is

$T = \frac{\sqrt{x^2 + 25}}{4} + \frac{8 - x}{5}$.

105.a. $p(5) = 1275 - 25(5) = 1150$. If 5000 TVs can be sold, the price per TV is \$1150.
$p(15) = 1275 - 25(15) = 900$. If 15,000 TVs can be sold, the price per TV is \$900.
$p(30) = 1275 - 25(30) = 525$. If 30,000 TVs can be sold, the price per TV is \$525.

b.

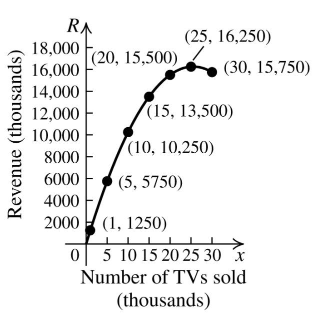


c. $650 = 1275 - 25x \Rightarrow -625 = -25x \Rightarrow x = 25$
25,000 TVs can be sold at \$650 per TV.

106.a. $R(x) = (1275 - 25x)x = 1275x - 25x^2$
domain [1, 30]

b. $R(1) = 1275(1) - 25(1^2) = 1250$
$R(5) = 1275(5) - 25(5^2) = 5750$
$R(10) = 1275(10) - 25(10^2) = 10,250$
$R(15) = 1275(15) - 25(15^2) = 13,500$
$R(20) = 1275(20) - 25(20^2) = 15,500$
$R(25) = 1275(25) - 25(25^2) = 16,250$
$R(30) = 1275(30) - 25(30^2) = 15,750$
This is the amount of revenue (in thousands of dollars) for the given number of TVs sold (in thousands).

c.



d. $4700 = 1275x - 25x^2 \Rightarrow$
$x^2 - 51x + 188 = 0 \Rightarrow$
$\dfrac{51 \pm \sqrt{51^2 - 4(1)(188)}}{2(1)} = x \Rightarrow$
$x = 4$ or $x = 47$
47 is not in the domain, so 4000 TVs must be sold in order to generate revenue of 4.7 million dollars.

107.a. $C(x) = 5.5x + 75,000$

b. $R(x) = 0.6(15)x = 9x$

c. $P(x) = R(x) - C(x) = 9x - (5.5x + 75,000)$
$= 3.5x - 75,000$

d. The break-even point is when the profit is zero: $3.5x - 75,000 = 0 \Rightarrow x = 21,429$

e. $P(46,000) = 3.5(46,000) - 75,000$
$= \$86,000$
The company's profit is \$86,000 when 46,000 copies are sold.

108.a. $C(x) = 0.5x + 500,000; R(x) = 5x$. The break-even point is when the profit is zero (when the revenue equals the cost):
$5x = 0.5x + 500,000 \Rightarrow 4.5x = 500,000 \Rightarrow$
$x = 111,111.11$. Because a fraction of a CD cannot be sold, 111,111 CD's must be sold.

b. $P(x) = R(x) - C(x)$
$750,000 = 5x - (0.5x + 500,000)$
$1,250,000 = 4.5x \Rightarrow x = 277,778$
The company must sell 277,778 CDs in order to make a profit of \$750,000.

2.4 Beyond the Basics

109. $x = \dfrac{2}{y-4} \Rightarrow xy - 4x = 2 \Rightarrow xy = 2 + 4x \Rightarrow$
$y = \dfrac{4x+2}{x} \Rightarrow f(x) = \dfrac{4x+2}{x};$
Domain: $(-\infty, 0) \cup (0, \infty)$. $f(4) = \dfrac{9}{2}$.

110. $xy - 3 = 2y \Rightarrow 2y - xy = -3 \Rightarrow$
$y(2-x) = -3 \Rightarrow y = -\dfrac{3}{2-x} \Rightarrow f(x) = \dfrac{3}{x-2}$
Domain: $(-\infty, 2) \cup (2, \infty)$. $f(4) = \dfrac{3}{2}$

111. $\left(x^2+1\right)y + x = 2 \Rightarrow y = \dfrac{2-x}{x^2+1} \Rightarrow$
$f(x) = \dfrac{2-x}{x^2+1}$; Domain: $(-\infty, \infty)$; $f(4) = -\dfrac{2}{17}$

112. $yx^2 - \sqrt{x} = -2y \Rightarrow yx^2 + 2y = \sqrt{x} \Rightarrow$
$y\left(x^2+2\right) = \sqrt{x} \Rightarrow y = \dfrac{\sqrt{x}}{x^2+2} \Rightarrow f(x) = \dfrac{\sqrt{x}}{x^2+2}$
Domain: $[0, \infty)$; $f(4) = \dfrac{1}{9}$

113. $f(x) \neq g(x)$ because they have different domains.

114. $f(x) \neq g(x)$ because they have different domains.

115. $f(x) \neq g(x)$ because they have different domains. $g(x)$ is not defined for $x = -1$, while $f(x)$ is defined for all real numbers.

116. $f(x) \neq g(x)$ because they have different domains. $g(x)$ is not defined for $x = 3$, while $f(x)$ is not defined for $x = 3$ or $x = -2$.

117. $f(x) = g(x)$ because $f(3) = 10 = g(3)$ and $f(5) = 26 = g(5)$.

118. $f(x) \neq g(x)$ because $f(2) = 16$ while $g(2) = 13$.

119. $f(2) = 15 = a(2^2) + 2a - 3 \Rightarrow 15 = 6a - 3 \Rightarrow a = 3$.

120. $g(6) = 28 = 6^2 + 6b + b^2 \Rightarrow b^2 + 6b + 8 = 0 \Rightarrow (b+2)(b+4) = 0 \Rightarrow b = -2$ or $b = -4$.

121. $h(6) = 0 = \dfrac{3(6) + 2a}{2(6) - b} \Rightarrow 0 = 18 + 2a \Rightarrow a = -9$

$h(3)$ is undefined $\Rightarrow \dfrac{3(3) + 2(-9)}{2(3) - b}$ has a zero in the denominator. So $6 - b = 0 \Rightarrow b = 6$.

122. $f(x) = 2x - 3 \Rightarrow f\left(x^2\right) = 2x^2 - 3$

$\left(f(x)\right)^2 = (2x-3)^2 = 4x^2 - 12x + 9$

123. $g(x) = x^2 - \dfrac{1}{x^2} \Rightarrow g\left(\dfrac{1}{x}\right) = \dfrac{1}{x^2} - \dfrac{1}{\frac{1}{x^2}} = \dfrac{1}{x^2} - x^2$

$$g(x) + g\left(\frac{1}{x}\right) = \left(x^2 - \frac{1}{x^2}\right) + \left(\frac{1}{x^2} - x^2\right) = 0$$

124. $f(x) = \dfrac{x-1}{x+1} \Rightarrow f\left(\dfrac{x-1}{x+1}\right) = \dfrac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1}$

$$= \frac{\frac{(x-1)-(x+1)}{x+1}}{\frac{(x-1)+(x+1)}{x+1}} = -\frac{2}{2x} = -\frac{1}{x}$$

125. $f(x) = \dfrac{x+3}{4x-5} \Rightarrow$

$$f(t) = \frac{\frac{3+5x}{4x-1} + 3}{4\left(\frac{3+5x}{4x-1}\right) - 5} = \frac{\frac{(3+5x) + 3(4x-1)}{4x-1}}{\frac{(12+20x) - (5(4x-1))}{4x-1}}$$

$$= \frac{(3+5x) + (12x-3)}{(12+20x) - (20x-5)} = \frac{17x}{17} = x$$

2.4 Critical Thinking/Discussion/Writing

126. Answers may vary. Sample answers are given

a. $y = \sqrt{x-2}$ **b.** $y = \dfrac{1}{\sqrt{x-2}}$

c. $y = \sqrt{2-x}$ **d.** $y = \dfrac{1}{\sqrt{2-x}}$

127.a. $ax^2 + bx + c = 0$

b. $y = c$

c. The equation will have no x-intercepts if $b^2 - 4ac < 0$.

d. It is not possible for the equation to have no y-intercepts because $y = f(x)$.

128.a. $f(x) = |x|$ **b.** $f(x) = 0$

c. $f(x) = x$

d. $f(x) = \sqrt{-x^2}$ (Note: the point is the origin.)

e. $f(x) = 1$

f. A vertical line is not a function.

129.a. $\{(a, 1), (b, 1)\}$ $\{(a, 2), (b, 1)\}$
$\{(a, 1), (b, 2)\}$ $\{(a, 2), (b, 2)\}$
$\{(a, 1), (b, 3)\}$ $\{(a, 2), (b, 3)\}$

$\{(a, 3), (b, 1)\}$
$\{(a, 3), (b, 2)\}$
$\{(a, 3), (b, 3)\}$

There are nine functions from X to Y.

b. $\{(1, a)\}, \{(2, a)\}, \{(3, a)\}$
$\{(1, a)\}, \{(2, a)\}, \{(3, b)\}$
$\{(1, a)\}, \{(2, b\}, \{(3, a)\}$
$\{(1, b)\}, \{(2, a)\}, \{(3, a)\}$
$\{(1, b)\}, \{(2, a)\}, \{(3, a)\}$
$\{(1, b)\}, \{(2, b\}, \{(3, a)\}$
$\{(1, b)\}, \{(2, a)\}, \{(3, b)\}$
$\{(1, b)\}, \{(2, b)\}, \{(3, b)\}$

There are eight functions from Y to X.

130. If a set X has m elements and a set of Y has n elements, there are n^m functions that can be defined from X to Y. This is true since a function assigns each element of X to an element of Y. There are m possibilities for each element of X, so there are $\underbrace{n \cdot n \cdot n \cdot \cdots \cdot n}_{m} = n^m$ possible functions.

2.4 Maintaining Skills

131. $m = \dfrac{-2-0}{2-0} = -1$

$y - 0 = -1(x - 0) \Rightarrow y = -x$

132. $m = \dfrac{-2-3}{4-(-1)} = \dfrac{-5}{5} = -1$

$y - 3 = -(x - (-1)) \Rightarrow y - 3 = -x - 1 \Rightarrow$
$y = -x + 2$

133. $m = \dfrac{4-2}{1-(-3)} = \dfrac{2}{4} = \dfrac{1}{2}$

$y - 2 = \dfrac{1}{2}(x - (-3)) \Rightarrow y - 2 = \dfrac{1}{2}(x + 3) \Rightarrow$
$y - 2 = \dfrac{1}{2}x + \dfrac{3}{2} \Rightarrow y = \dfrac{1}{2}x + \dfrac{7}{2}$

134. $m = \dfrac{-3-(-5)}{8-3} = \dfrac{2}{5}$

$y - (-5) = \dfrac{2}{5}(x - 3) \Rightarrow y + 5 = \dfrac{2}{5}x - \dfrac{6}{5} \Rightarrow$
$y = \dfrac{2}{5}x - \dfrac{31}{5}$

135. $f(x) = 2x^2 - 3x$

a. $f(-x) = 2(-x)^2 - 3(-x) = 2x^2 + 3x$

b. $-f(x) = -(2x^2 - 3x) = -2x^2 + 3x$

136. $f(x) = 3x^4 - 7x^2 + 5$

a. $f(-x) = 3(-x)^4 - 7(-x)^2 + 5$
$= 3x^4 - 7x^2 + 5$

b. $-f(x) = -(3x^4 - 7x^2 + 5) = -3x^4 + 7x^2 - 5$

137. $f(x) = x^3 - 2x$

a. $f(-x) = (-x)^3 - 2(-x) = -x^3 + 2x$

b. $-f(x) = -(x^3 - 2x) = -x^3 + 2x$

138. $f(x) = 2x^3 - 5x^2 + x$

a. $f(-x) = 2(-x)^3 - 5(-x)^2 + (-x)$
$= -2x^3 - 5x^2 - x$

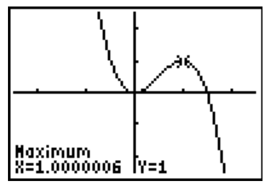
b. $-f(x) = -(2x^3 - 5x^2 + x) = -2x^3 + 5x^2 - x$

2.5 Properties of Functions

2.5 Practice Problems

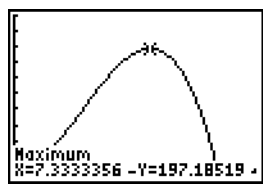
1. f is increasing on $(2, \infty)$, decreasing on $(-\infty, -3)$, and constant on $(-3, 2)$.

2.



Relative minimum: (0, 0)
Relative maximum: (1, 1)

3. $v = (11 - r)r^2$



[0, 13, 1] by [0, 250, 25]

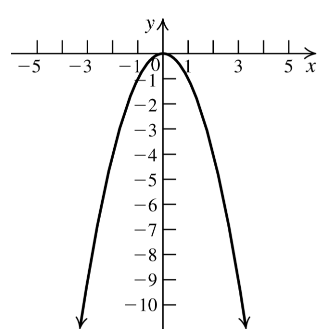
Mrs. Osborn's windpipe should be contracted to a radius of 7.33 mm for maximizing the airflow velocity.

4. $f(x) = -x^2$

Replace x with $-x$:

$f(-x) = -(-x)^2 = -x^2 = f(x)$

Thus, the function is even.

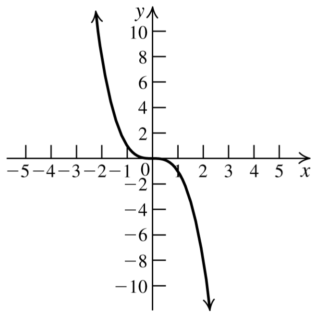


5. $f(x) = -x^3$

Replace x with $-x$:

$f(-x) = -(-x)^3 = x^3 = -f(x)$

Thus, the function is odd.



6.a. $g(-x) = 3(-x)^4 - 5(-x)^2$
$= 3x^4 - 5x^2 = f(x) \Rightarrow$
$g(x)$ is even.

b. $f(-x) = 4(-x)^5 + 2(-x)^3 = -4x^5 - 2x^3$
$= -\left(4x^5 + 2x^3\right) = -f(x) \Rightarrow$
$f(x)$ is odd.

c. $h(-x) = 2(-x) + 1 = -2x + 1$
$\neq h(x)$
$\neq h(-x) \Rightarrow h$ is neither even nor odd.

7. $f(x) = 1 - x^2;\ a = 2, b = 4$

$f(2) = 1 - 2^2 = -3;\ f(4) = 1 - 4^2 = -15$

$$\frac{f(b) - f(a)}{b - a} = \frac{-15 - (-3)}{4 - 2} = \frac{-12}{2} = -6$$

The average rate of change is −6.

8. $f(t) = 1 - t;\ a = 2,\ b = x,\ x \neq 2$

$f(a) = f(2) = 1 - 2 = -1$

$f(b) = f(x) = 1 - x$

$$\frac{f(b) - f(a)}{b - a} = \frac{(1 - x) - (-1)}{x - 2} = \frac{2 - x}{x - 2} = \frac{-1(x - 2)}{x - 2} = -1$$

The average rate of change is −1.

9. $f(x) = -x^2 + x - 3$

$f(x + h) = -(x + h)^2 + (x + h) - 3$
$= -x^2 - 2xh - h^2 + x + h - 3$

$$\frac{f(x + h) - f(x)}{h} = \frac{\left(-x^2 - 2xh - h^2 + x + h - 3\right) - \left(-x^2 + x - 3\right)}{h} = \frac{-2xh - h^2 + h}{h} = -2x - h + 1$$

2.5 Basic Concepts and Skills

1. A function *f* is decreasing if $x_1 < x_2$ implies that $\underline{f(x_1) > f(x_2)}$.
2. $f(a)$ is a relative maximum of *f* if there is an interval $(x_1,\ x_2)$ containing *a* such that $\underline{f(a) \geq f(x)}$ for every *x* in the interval $(x_1,\ x_2)$.
3. A function *f* is even if $\underline{f(-x) = f(x)}$ for all *x* in the domain of *f*.
4. The average rate of change of *f* as *x* changes from $x = a$ to $x = b$ is $\underline{\frac{f(b) - f(a)}{b - a}}$, $a \neq b$.
5. True
6. False. A relative maximum or minimum could occur at and endpoint of the domain of the function.
7. Increasing on $(-\infty,\ \infty)$
8. Decreasing on $(-\infty,\ \infty)$
9. Increasing on $(-\infty,\ 2)$, decreasing on $(2,\ \infty)$
10. Decreasing on $(-\infty,\ 3)$, increasing on $(3,\ \infty)$
11. Increasing on $(-\infty,\ -2)$, constant on $(-2,\ 2)$, increasing on $(2,\ \infty)$
12. Decreasing on $(-\infty,\ -1)$, constant on $(-1,\ 4)$, decreasing on $(4,\ \infty)$
13. Increasing on $(-\infty,\ -3)$ and $\left(-\frac{1}{2},\ 2\right)$, decreasing on $\left(-3,\ -\frac{1}{2}\right)$ and $(2,\ \infty)$
14. Increasing on $(-3,\ -1)$, $(0,\ 1)$, and $(2,\ \infty)$. Decreasing on $(-\infty,\ -3)$, $(-1,\ 0)$, and $(1,\ 2)$.
15. No relative extrema
16. No relative extrema

17. (2, 10) is a relative maximum point and a turning point.

18. (3, 2) is a relative minimum point and a turning point.

19. Any point on (x, 2) is a relative maximum and a relative minimum point on the interval (–2, 2). Relative maximum at (–2, 2); relative minimum at (2, 2). None of these points are turning points.

20. Any point on (x, 3) is a relative maximum and a relative minimum point on the interval (–1, 4). Relative maximum at (4, 3); relative minimum at (−1, 3). None of these points are turning points.

21. (–3, 4) and (2, 5) are relative maxima points and turning points. $\left(-\frac{1}{2}, -2\right)$ is a relative minimum and a turning point.

22. (–3, –2), (0, 0), and (2, –3) are relative minimum points and turning points. (–1, 1) and (1, 2) are relative maximum points and turning points.

For exercises 23−32, recall that the graph of an even function is symmetric about the y-axis, and the graph of an odd function is symmetric about the origin.

23. The graph is symmetric with respect to the origin. The function is odd.

24. The graph is symmetric with respect to the origin. The function is odd.

25. The graph has no symmetries, so the function is neither odd nor even.

26. The graph has no symmetries, so the function is neither odd nor even.

27. The graph is symmetric with respect to the origin. The function is odd.

28. The graph is symmetric with respect to the origin. The function is odd.

29. The graph is symmetric with respect to the y-axis. The function is even.

30. The graph is symmetric with respect to the y-axis. The function is even.

31. The graph is symmetric with respect to the origin. The function is odd.

32. The graph is symmetric with respect to the origin. The function is odd.

For exercises 33–46, $f(-x) = f(x) \Rightarrow f(x)$ is even and $f(-x) = -f(x) \Rightarrow f(x)$ is odd.

33. $f(-x) = 2(-x)^4 + 4 = 2x^4 + 4 = f(x) \Rightarrow$
$f(x)$ is even.

34. $g(-x) = 3(-x)^4 - 5 = 3x^4 - 5 = g(x) \Rightarrow$
$g(x)$ is even.

35. $f(-x) = 5(-x)^3 - 3(-x) = -5x^3 + 3x$
$= -(5x^3 - 3x) = -f(x) \Rightarrow$
$f(x)$ is odd.

36. $g(-x) = 2(-x)^3 + 4(-x)$
$= -2x^3 - 4x = -g(x) \Rightarrow$
$g(x)$ is odd.

37. $f(-x) = 2(-x) + 4 = -2x + 4$
$\neq -f(x) \neq f(x) \Rightarrow$
$f(x)$ is neither even nor odd.

38. $g(-x) = 3(-x) + 7 = -3x + 7$
$\neq -g(x) \neq g(x) \Rightarrow$
$g(x)$ is neither even nor odd.

39. $f(-x) = \dfrac{1}{(-x)^2 + 4} = \dfrac{1}{x^2 + 4} = f(x) \Rightarrow$
$f(x)$ is even.

40. $g(-x) = \dfrac{(-x)^2 + 2}{(-x)^4 + 1} = \dfrac{x^2 + 2}{x^4 + 1} = g(x) \Rightarrow$
$g(x)$ is even.

41. $f(-x) = \dfrac{(-x)^3}{(-x)^2 + 1} = -\dfrac{x^3}{x^2 + 1} = -f(x) \Rightarrow$
$f(x)$ is odd.

42. $g(-x) = \dfrac{(-x)^4 + 3}{2(-x)^3 - 3(-x)} = \dfrac{x^4 + 3}{-2x^3 + 3x}$
$= -\dfrac{x^4 + 3}{2x^3 - 3x} = -f(x) \Rightarrow f(x)$ is odd.

43. $f(-x) = \dfrac{-x}{(-x)^5 - 3(-x)^3} = \dfrac{-x}{-x^5 + 3x^3}$
$= \dfrac{(-1)(-x)}{(-1)\left(-x^5 + 3x^3\right)} = \dfrac{x}{x^5 - 3x^3} = f(x)$
Thus, $f(x)$ is even.

44. $g(-x) = \dfrac{(-x)^3 + 2(-x)}{2(-x)^5 - 3(-x)} = \dfrac{-x^3 - 2x}{-2x^5 + 3x}$
$= \dfrac{(-1)\left(-x^3 - 2x\right)}{(-1)\left(-2x^5 + 3x\right)} = \dfrac{x^3 + 2x}{2x^5 - 3x} = f(x)$

Thus, $f(x)$ is even.

45. $f(-x) = \dfrac{(-x)^2 - 2(-x)}{5(-x)^4 + 4(-x)^2 + 7} = \dfrac{x^2 + 2x}{5x^4 + 4x^2 + 7}$
$\neq -f(x) \neq f(x)$

Thus, $f(x)$ is neither even nor odd.

46. $g(-x) = \dfrac{3(-x)^2 + 7}{(-x) - 3} = \dfrac{3x^2 + 7}{-x - 3} \neq -g(x) \neq g(x)$

Thus, $g(x)$ is neither even nor odd.

47.a. domain: $(-\infty, \infty)$; range: $(-\infty, 3]$

b. x-intercepts: $(-3, 0)$, $(3, 0)$
y-intercept: $(0, 3)$

c. increasing on $(-\infty, 0)$, decreasing on $(0, \infty)$

d. relative maximum at $(0, 3)$

e. even

48.a. domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

b. x-intercepts: $(-4, 0)$, $(0, 0)$, $(4, 0)$
y-intercept: $(0, 0)$

c. decreasing on $(-\infty, -2)$ and $(2, \infty)$,
increasing on $(-2, 2)$

d. relative maximum at $(2,3)$; relative minimum at $(-2, -3)$

e. odd

49.a. domain: $(-3, 4)$; range: $[-2, 2]$

b. x-intercept: $(1, 0)$; y-intercept: $(0, -1)$

c. constant on $(-3, -1)$ and $(3, 4)$
increasing on $(-1, 3)$

d. Since the function is constant on $(-3, -1)$, any point $(x, -2)$ is both a relative maximum and a relative minimum on that interval. Since the function is constant on $(3, 4)$, any point $(x, 2)$ is both a relative maximum and a relative minimum on that interval.

e. neither even not odd

50.a. domain: $(-3, 3)$; range: $\{-2, 0, 2\}$

b. x-intercept: $(0, 0)$; y-intercept: $(0, 0)$

c. constant on $(-3, 0)$ and $(0, 3)$

d. Since the function is constant on $(-3, 0)$, any point $(x, 2)$ is both a relative maximum and a relative minimum on that interval. Since the function is constant on $(0, 3)$, any point $(x, -2)$ is both a relative maximum and a relative minimum on that interval.

e. odd

51.a. domain: $(-2, 4)$; range: $(-2, 3)$

b. x-intercept: $(0, 0)$; y-intercept: $(0, 0)$

c. decreasing on $(-2, -1)$ and $(3, 4)$
increasing on $(-1, 3)$

d. relative maximum: $(3, 3)$
relative minimum: $(-1, -2)$

e. neither even nor odd

52.a. domain: $(-\infty, \infty)$
range: $(-\infty, \infty)$

b. x-intercepts: $(2, 0)$, $(3, 0)$
y-intercept: $(0, 3)$

c. decreasing on $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$

d. no relative minimum
relative maximum: $(0, 3)$

e. neither even nor odd

53.a. domain: $(-\infty, \infty)$; range: $(0, \infty)$

b. no x-intercept; y-intercept: $(0, 1)$

c. increasing on $(-\infty, \infty)$

d. no relative minimum or relative maximum

e. neither even nor odd

54.a. domain: $(-\infty, 0) \cup (0, \infty)$
range: $(-\infty, \infty)$

b. x-intercepts: $(-1.5, 0)$, $(1.5, 0)$
no y-intercept

c. decreasing on $(-\infty, 0)$
increasing on $(0, \infty)$

d. no relative minimum or relative maximum

e. even

55. $f(x) = -2x + 7;\ a = -1,\ b = 3$
$f(3) = -2(3) + 7 = 1;\ f(-1) = -2(-1) + 7 = 9$
$$\text{average rate of change} = \frac{f(3) - f(-1)}{3 - (-1)} = \frac{1 - 9}{4} = -2$$

56. $f(x) = 4x - 9;\ a = -2,\ b = 2$
$f(2) = 4(2) - 9 = -1;\ f(-2) = 4(-2) - 9 = -17$
$$\text{average rate of change} = \frac{f(2) - f(-2)}{2 - (-2)} = \frac{-1 - (-17)}{4} = 4$$

57. $f(x) = 3x + c;\ a = 1,\ b = 5$
$f(5) = 3 \cdot 5 + c = 15 + c;\ f(1) = 3 \cdot 1 + c = 3 + c$
$$\text{average rate of change} = \frac{f(5) - f(1)}{5 - 1} = \frac{15 + c - (3 + c)}{4} = \frac{12}{4} = 3$$

58. $f(x) = mx + c;\ a = -1,\ b = 7$
$f(7) = 7m + c;\ f(-1) = -m + c$
$$\text{average rate of change} = \frac{f(7) - f(-1)}{7 - (-1)} = \frac{7m + c - (-m + c)}{8} = \frac{8m}{8} = m$$

59. $h(x) = x^2 - 1;\ a = -2,\ b = 0$
$h(0) = 0^2 - 1 = -1;\ h(-2) = (-2)^2 - 1 = 3$
$$\text{average rate of change} = \frac{h(0) - h(-2)}{0 - (-2)} = \frac{-1 - 3}{2} = -2$$

60. $h(x) = 2 - x^2;\ a = 3,\ b = 4$
$h(4) = 2 - 4^2 = -14;\ h(3) = 2 - 3^2 = -7$
$$\text{average rate of change} = \frac{h(4) - h(3)}{4 - 3} = \frac{-14 - (-7)}{1} = -7$$

61. $f(x) = (3 - x)^2;\ a = 1,\ b = 3$
$f(4) = (3 - 3)^2 = 0;\ f(1) = (3 - 1)^2 = 4$
$$\text{average rate of change} = \frac{f(3) - f(1)}{3 - 1} = \frac{0 - 4}{2} = -2$$

62. $f(x) = (x - 2)^2;\ a = -1,\ b = 5$
$f(5) = (5 - 2)^2 = 9;\ f(-1) = (-1 - 2)^2 = 9$
$$\text{average rate of change} = \frac{f(5) - f(-1)}{5 - (-1)} = \frac{9 - 9}{6} = 0$$

63. $g(x) = x^3;\ a = -1,\ b = 3$
$g(3) = 3^3 = 27;\ g(-1) = (-1)^3 = -1$
$$\text{average rate of change} = \frac{g(3) - g(-1)}{3 - (-1)} = \frac{27 - (-1)}{4} = 7$$

64. $g(x) = -x^3;\ a = -1,\ b = 3$
$g(3) = -3^3 = -27;\ g(-1) = -(-1)^3 = 1$
$$\text{average rate of change} = \frac{g(3) - g(-1)}{3 - (-1)} = \frac{-27 - 1}{4} = -7$$

65. $h(x) = \frac{1}{x};\ a = 2,\ b = 6$
$h(2) = \frac{1}{2};\ h(6) = \frac{1}{6}$
$$\text{average rate of change} = \frac{h(6) - h(2)}{6 - 2} = \frac{\frac{1}{6} - \frac{1}{2}}{4} = -\frac{1}{12}$$

66. $h(x) = \frac{4}{x + 3};\ a = -2,\ b = 4$
$h(4) = \frac{4}{4 + 3} = \frac{4}{7};\ h(-2) = \frac{4}{-2 + 3} = 4$
$$\text{average rate of change} = \frac{h(4) - h(-2)}{4 - (-2)} = \frac{\frac{4}{7} - 4}{6} = -\frac{4}{7}$$

67. $f(x) = 2x,\ a = 3 \Rightarrow f(a) = 2 \cdot 3 = 6$
$$\frac{f(x) - f(a)}{x - a} = \frac{2x - 6}{x - 3} = \frac{2(x - 3)}{x - 3} = 2$$

68. $f(x)=3x+2,\ a=2 \Rightarrow f(a)=3\cdot 2+2=8$

$$\frac{f(x)-f(a)}{x-a}=\frac{3x+2-8}{x-2}=\frac{3x-6}{x-2}=\frac{3(x-2)}{x-2}=3$$

69. $f(x)=-x^2,\ a=1 \Rightarrow f(a)=-1$

$$\frac{f(x)-f(a)}{x-a}=\frac{-x^2-(-1)}{x-1}=\frac{-x^2+1}{x-1}=\frac{-(x-1)(x+1)}{x-1}=-x-1$$

70. $f(x)=2x^2,\ a=-1 \Rightarrow f(a)=2$

$$\frac{f(x)-f(a)}{x-a}=\frac{2x^2-2}{x+1}=\frac{2(x^2-1)}{x+1}=\frac{2(x-1)(x+1)}{x+1}=2x-2$$

71. $f(x)=3x^2+x,\ a=2 \Rightarrow$

$f(a)=3(2)^2+2=14$

$$\frac{f(x)-f(a)}{x-a}=\frac{3x^2+x-14}{x-2}=\frac{(3x+7)(x-2)}{x-2}=3x+7$$

72. $f(x)=-2x^2+x,\ a=3 \Rightarrow$

$f(a)=-2(3)^2+3=-15$

$$\frac{f(x)-f(a)}{x-a}=\frac{-2x^2+x+15}{x-3}=\frac{-(2x^2-x-15)}{x-3}=\frac{-(2x+5)(x-3)}{x-3}=-2x-5$$

73. $f(x)=\frac{4}{x},\ a=1 \Rightarrow f(a)=4$

$$\frac{f(x)-f(a)}{x-a}=\frac{\frac{1}{x}-4}{x-1}=\frac{\frac{4-4x}{x}}{x-1}=\frac{-4(x-1)}{x(x-1)}=-\frac{4}{x}$$

74. $f(x)=-\frac{4}{x},\ a=1 \Rightarrow f(a)=-4$

$$\frac{f(x)-f(a)}{x-a}=\frac{-\frac{4}{x}+4}{x-1}=\frac{\frac{4x-4}{x}}{x-1}=\frac{4(x-1)}{x(x-1)}=\frac{4}{x}$$

75. $f(x+h)=x+h$

$f(x+h)-f(x)=x+h-x=h$

$$\frac{f(x+h)-f(x)}{h}=\frac{h}{h}=1$$

76. $f(x+h)=3(x+h)+2=3x+3h+2$

$f(x+h)-f(x)=3x+3h+2-(3x+2)=3h$

$$\frac{f(x+h)-f(x)}{h}=\frac{3h}{h}=3$$

77. $f(x+h)=-2(x+h)+3=-2x-2h+3$

$f(x+h)-f(x)=-2x-2h+3-(-2x+3)=-2h$

$$\frac{f(x+h)-f(x)}{h}=\frac{-2h}{h}=-2$$

78. $f(x+h)=-5(x+h)-6=-5x-5h-6$

$f(x+h)-f(x)=-5x-5h-6-(-5x-6)=-5h$

$$\frac{f(x+h)-f(x)}{h}=\frac{-5h}{h}=-5$$

79. $f(x+h)=m(x+h)+b=mx+mh+b$

$f(x+h)-f(x)=mx+mh+b-(mx+b)=mh$

$$\frac{f(x+h)-f(x)}{h}=\frac{mh}{h}=m$$

80. $f(x+h)=-2a(x+h)+c=-2ax-2ah+c$

$f(x+h)-f(x)=-2ax-2ah+c-(-2ax+c)=-2ah$

$$\frac{f(x+h)-f(x)}{h}=\frac{-2ah}{h}=-2a$$

81. $f(x+h)=(x+h)^2=x^2+2xh+h^2$

$f(x+h)-f(x)=x^2+2xh+h^2-x^2=2xh+h^2$

$$\frac{f(x+h)-f(x)}{h}=\frac{2xh+h^2}{h}=2x+h$$

82. $f(x+h)=(x+h)^2-(x+h)=x^2+2xh+h^2-x-h=x^2+2xh-x+h^2-h$

$f(x+h)-f(x)$

$=x^2+2xh-x+h^2-h-(x^2-x)$

$=2xh+h^2-h$

$$\frac{f(x+h)-f(x)}{h}=\frac{2xh+h^2-h}{h}=2x+h-1$$

83. $f(x+h) = 2(x+h)^2 + 3(x+h)$
$= 2x^2 + 4xh + 2h^2 + 3x + 3h$
$= 2x^2 + 4xh + 3x + 2h^2 + 3h$

$f(x+h) - f(x)$
$= 2x^2 + 4xh + 3x + 2h^2 + 3h - (2x^2 + 3x)$
$= 4xh + 2h^2 + 3h$

$$\frac{f(x+h)-f(x)}{h} = \frac{4xh + 2h^2 + 3h}{h} = 4x + 2h + 3$$

84. $f(x+h) = 3(x+h)^2 - 2(x+h) + 5$
$= 3x^2 + 6xh + 3h^2 - 2x - 2h + 5$
$= 3x^2 + 6xh - 2x + 3h^2 - 2h + 5$

$f(x+h) - f(x) = 3x^2 + 6xh - 2x + 3h^2 - 2h + 5 - (3x^2 - 2x + 5)$
$= 6xh + 3h^2 - 2h$

$$\frac{f(x+h)-f(x)}{h} = \frac{6xh + 3h^2 - 2h}{h} = 6x + 3h - 2$$

85. $f(x+h) = 4$
$f(x+h) - f(x) = 4 - 4 = 0$
$$\frac{f(x+h)-f(x)}{h} = \frac{0}{h} = 0$$

86. $f(x+h) = -3$
$f(x+h) - f(x) = -3 - (-3) = 0$
$$\frac{f(x+h)-f(x)}{h} = \frac{0}{h} = 0$$

87. $f(x+h) = \dfrac{1}{x+h}$

$$f(x+h) - f(x) = \frac{1}{x+h} - \frac{1}{x} = \frac{x}{x(x+h)} - \frac{x+h}{x(x+h)} = -\frac{h}{x(x+h)}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{-\frac{h}{x(x+h)}}{h} = -\frac{1}{x(x+h)}$$

88. $f(x+h) = -\dfrac{1}{x+h}$

$$f(x+h) - f(x) = -\frac{1}{x+h} - \left(-\frac{1}{x}\right) = -\frac{x}{x(x+h)} + \frac{x+h}{x(x+h)} = \frac{h}{x(x+h)}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{h}{x(x+h)}}{h} = \frac{1}{x(x+h)}$$

2.5 Applying the Concepts

89. domain: $[0, \infty)$
The particle's motion is tracked indefinitely from time $t = 0$.

90. range: $[-7, 5]$
The particle takes on all velocities between –7 an 5. Note that a negative velocity indicates that the particle is moving backward.

91. The graph is above the t-axis on the intervals (0, 9) and (21, 24). This means that the particle was moving forward between 0 and 9 seconds and between 21 and 24 seconds.

92. The graph is below the t-axis on the interval (11, 19). This means that the particle is moving backward between 11 and 19 seconds.

93. The function is increasing on (0, 3), (5, 6), (16, 19), and (21, 23). However, the speed $|v|$ of the particle is increasing on (0, 3), (5, 6), (11, 15), and (21, 23). Note that the particle is moving forward on (0, 3), (5, 6), and (21, 23), and moving backward on (11, 15).

94. The function is decreasing on (6, 9), (11, 15), and (23, 24). However, the speed $|v|$ of the particle is decreasing on (6, 9), (16, 19), and (23, 24). Note that the particle is moving forward on (6, 9) and (23, 24), and moving backward on (16, 19).

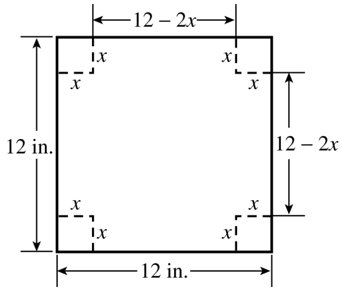
95. The maximum speed is between times $t = 15$ and $t = 16$.

96. The minimum speed is 0 on the intervals (9, 11), (19,21), and (24, ∞).

97. The particle is moving forward with increasing velocity.

98. The particle is moving backward with decreasing speed.

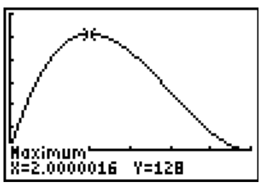
99.



a. $V = lwh = (12-2x)(12-2x)x$
$= \left(144 - 48x + 4x^2\right)x$
$= 4x^3 - 48x^2 + 144x$

b. The length of the squares in the corners must be greater than 0 and less than 6, so the domain of V is (0, 6).

c.

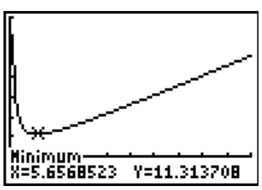


[0, 6, 1] by [–25, 150, 25]
range: [0, 128]

d. V is at its maximum when $x = 2$.

100.a. Let x = one of the numbers. Then $\frac{32}{x} =$ the other number. The sum of the numbers is $S = x + \frac{32}{x}$.

b.



[0, 50, 10] by [–10, 70, 10]
The minimum value of approximately 11.31 occurs at $x \approx 5.66$.

101.a. $C(x) = 210x + 10{,}500$

b. $C(50) = 210(50) + 10{,}500 = \$21{,}000$
It costs \$21,000 to produce 50 notebooks per day.

c. average cost $= \frac{\$21{,}000}{50} = \$420$

d. $\frac{210x + 10{,}500}{x} = 315$
$210x + 10{,}500 = 315x$
$10{,}500 = 105x \Rightarrow x = 100$
The average cost per notebook will be \$315 when 100 notebooks are produced.

102. $f(x) = -2x^2 + 3x + 4$
$f(1) = -2(1)^2 + 3(1) + 4 = 5$
$f(3) = -2(3)^2 + 3(3) + 4 = -5$
The secant passes through the points (1, 5) and (3, −5).
$m = \frac{-5-5}{3-1} = \frac{-10}{2} = -5$
The equation of the secant is
$y - 5 = -5(x-1) \Rightarrow y - 5 = -5x + 5 \Rightarrow$
$y = -5x + 10$

103.a. $f(0) = 0^2 + 3(0) + 4 = 4$
The particle is 4 ft to the right from the origin.

b. $f(4) = 4^2 + 3(4) + 4 = 32$
The particle started 4 ft from the origin, so it traveled 32 − 4 = 28 ft in four seconds.

c. $f(3) = 3^2 + 3(3) + 4 = 22$
The particle started 4 ft from the origin, so it traveled 22 − 4 = 18 ft in three seconds. The average velocity is $18/3 = 6$ ft/sec

d. $f(2) = 2^2 + 3(2) + 4 = 14$
$f(5) = 5^2 + 3(5) + 4 = 44$
The particle traveled 44 − 14 = 30 ft between the second and fifth seconds. The average velocity is $30/(5-2) = 10$ ft/sec

104.a. $P(0) = 0.01(0)^2 + 0.2(0) + 50 = 50$
$P(4) = 0.01(4)^2 + 0.2(4) + 50 = 50.96$
The population of Sardonia was 50 million in 2000 and 50.96 million in 2004.

b. $P(10) = 0.01(10)^2 + 0.2(10) + 50 = 53$
The average rate of growth from 2000 to 2010 was $\frac{53-50}{10} = .3$ million per year.

2.5 Beyond the Basics

105. $f(x)=\dfrac{x-1}{x+1}$

$f(2x)=\dfrac{2x-1}{2x+1}$

$$\frac{3f(x)+1}{f(x)+3}=\frac{3\left(\frac{x-1}{x+1}\right)+1}{\frac{x-1}{x+1}+3}=\frac{\frac{3x-3}{x+1}+1}{\frac{x-1+3(x+1)}{x+1}}$$

$$=\frac{\frac{3x-3+x+1}{x+1}}{\frac{x-1+3(x+1)}{x+1}}=\frac{4x-2}{4x+2}$$

$$=\frac{2(2x-1)}{2(2x+1)}=\frac{2x-1}{2x+1}=f(2x)$$

106. $f(x)=0$

107. In order to find the relative maximum, first observe that the relative maximum of $-(x+1)^2\le 0.$ Then $-(x+1)^2\le 0\Rightarrow$

$(x+1)^2\ge 0\Rightarrow x\ge -1.$

Thus, the *x*-coordinate of the relative maximum is $-1.$ $f(-1)=-(-1+1)^2+5=5$

The relative maximum is (−1, 5).
There is no relative minimum.

108. $f(x)=\begin{cases}x+10 & \text{if } x<-5\\ 5 & \text{if } -5\le x\le 5\\ -x & \text{if } x>5\end{cases}$

The point (0, 5) is a relative maximum, but not a turning point.

109. $f(x)=\sqrt{x}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$=\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

110. $f(x)=\sqrt{x-1}$

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{\sqrt{x+h-1}-\sqrt{x-1}}{h}\cdot\frac{\sqrt{x+h-1}+\sqrt{x-1}}{\sqrt{x+h-1}+\sqrt{x-1}}$$

$$=\frac{x+h-1-(x-1)}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}$$

$$=\frac{h}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}=\frac{1}{\sqrt{x+h-1}+\sqrt{x-1}}$$

111. $f(x)=-\dfrac{1}{\sqrt{x}}$

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{-\frac{1}{\sqrt{x+h}}+\frac{1}{\sqrt{x}}}{h}=\frac{\frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x}\sqrt{x+h}}}{h}$$

$$=\frac{\frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x(x+h)}}}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h\sqrt{x(x+h)}}$$

$$=\frac{\sqrt{x+h}-\sqrt{x}}{h\sqrt{x(x+h)}}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$=\frac{(x+h)-x}{h\sqrt{x(x+h)}\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=\frac{1}{\sqrt{x(x+h)}\left(\sqrt{x}+\sqrt{x+h}\right)}$$

112. $f(x)=\dfrac{1}{x^2}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$$

$$=\frac{\frac{x^2-(x+h)^2}{x^2(x+h)^2}}{h}$$

$$=\frac{x^2-(x+h)^2}{hx^2(x+h)^2}$$

$$=\frac{x^2-\left(x^2+2xh+h^2\right)}{hx^2(x+h)^2}$$

$$=\frac{-2xh-h^2}{hx^2(x+h)^2}=-\frac{2x+h}{x^2(x+h)^2}$$

2.5 Critical Thinking/Discussion/Writing

113. *f* has a relative maximum at $x = a$ if there is an interval $[a, x_1)$ with $a < x_1 < b$ such that $f(a) \geq f(x)$, or $f(x) \leq f(a)$, for every *x* in the interval $(x_1, b]$.

114. *f* has a relative minimum at $x = b$ if there is x_1 in $[a, b]$ such that $f(x) \geq f(b)$ for every *x* in the interval $(x_1, b]$.

115. Answers will vary. Sample answers are given.

a. $f(x) = x$ on the interval [−1, 1]

b. $f(x) = \begin{cases} x & \text{if } 0 \leq x < 1 \\ 0 & \text{if } x = 1 \end{cases}$

c. $f(x) = \begin{cases} x & \text{if } 0 < x \leq 1 \\ 1 & \text{if } x = 0 \end{cases}$

d. $f(x) = \begin{cases} 0 & \text{if } x = 0 \text{ or } x = 1 \\ 1 & \text{if } 0 < x < 1 \text{ and } x \text{ is rational} \\ -1 & \text{if } 0 < x < 1 \text{ and } x \text{ is irrational} \end{cases}$

2.5 Maintaining Skills

116. $m = \dfrac{-2-3}{4-(-1)} = \dfrac{-5}{5} = -1$

$y - 3 = -(x - (-1)) \Rightarrow y - 3 = -(x+1) \Rightarrow$
$y - 3 = -x - 1 \Rightarrow y = -x + 2$

117. $m = \dfrac{-1-2}{7-6} = \dfrac{-3}{1} = -3$

$y - 2 = -3(x-6) \Rightarrow y - 2 = -3x + 18 \Rightarrow$
$y = -3x + 20$

118. $m = \dfrac{-3-(-5)}{6-3} = \dfrac{2}{3}$

$y - (-5) = \dfrac{2}{3}(x-3) \Rightarrow y + 5 = \dfrac{2}{3}x - 2 \Rightarrow$
$y = \dfrac{2}{3}x - 7$

119. $f(x) = x^{3/2}$

(i) $f(2) = 2^{3/2} = \left(\sqrt{2}\right)^3 = \sqrt{8} = 2\sqrt{2}$

(ii) $f(4) = 4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

(iii) $f(-4) = (-4)^{3/2} = \left(\sqrt{-4}\right)^3 = (2i)^3 = -8i$

120. $f(x) = x^{2/3}$

(i) $f(2) = 2^{2/3} = 4^{1/3} = \sqrt[3]{4}$

(ii) $f(8) = 8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$

(iii) $f(-8) = 8^{2/3} = \left(\sqrt[3]{-8}\right)^2 = (-2)^2 = 4$

2.6 A Library of Functions

2.6 Practice Problems

1. Since $g(-2) = 2$ and $g(1) = 8$, the line passes through the points (−2, 2) and (1, 8).

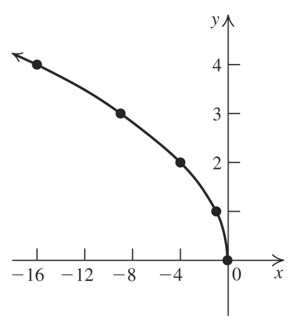
$m = \dfrac{8-2}{1-(-2)} = \dfrac{6}{3} = 2$

Use the point-slope form:

$y - 8 = 2(x-1) \Rightarrow y - 8 = 2x - 2 \Rightarrow$
$y = 2x + 6 \Rightarrow g(x) = 2x + 6$

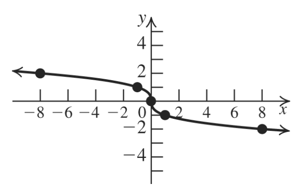
2. Using the formula
Shark length = (0.96)(tooth height) − 0.22, gives:
Shark length = (0.96)(16.4) − 0.22 = 15.524 m

3.



Domain: $(-\infty, 0]$; range: $[0, \infty)$

4.



Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

5. $f(x) = \begin{cases} x^2 & \text{if } x \leq -1 \\ 2x & \text{if } x > -1 \end{cases}$

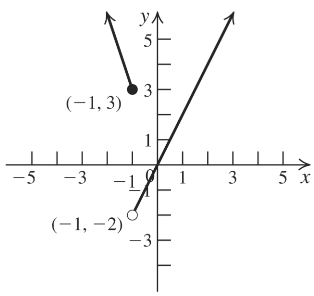
$f(-2) = (-2)^2 = 4;\quad f(3) = 2(3) = 6$

6.a. $f(x) = \begin{cases} 50 + 4(x - 55) & 56 \le x < 75 \\ 200 + 5(x - 75) & x \ge 75 \end{cases}$

b. The fine for driving 60 mph is $50 + 4(60 - 55) = \$70$.

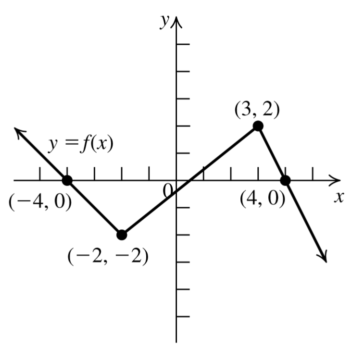
c. The fine for driving 90 mph is $200 + 5(90 - 75) = \$275$.

7. $f(x) = \begin{cases} -3x & \text{if } x \le -1 \\ 2x & \text{if } x > -1 \end{cases}$



Graph $f(x) = -3x$ on the interval $(-\infty, -1]$, and graph $f(x) = 2x$ on the interval $(-1, \infty)$.

8.



The graph of f is made up of three parts.
For $x \le -2$, the graph is made up of the half-line passing through the points (–4, 0) and (–2, –2).

$$m = \frac{-2 - 0}{-2 - (-4)} = \frac{-2}{2} = -1$$

$$y - 0 = -1(x - (-4)) \Rightarrow y = -x - 4$$

For $-2 < x < 3$, the graph is a line segment passing through the points (–2, –2) and (3, 2).

$$m = \frac{2 - (-2)}{3 - (-2)} = \frac{4}{5}$$

$$y - 2 = \frac{4}{5}(x - 3) \Rightarrow y - 2 = \frac{4}{5}x - \frac{12}{5} \Rightarrow$$

$$y = \frac{4}{5}x - \frac{2}{5}$$

For $x \ge 3$, the graph is a half-line passing through (3, 2) and (4, 0).

$$m = \frac{0 - 2}{4 - 3} = -2$$

$$y - 0 = -2(x - 4) \Rightarrow y = -2x + 8$$

Combining the three parts, we have

$$f(x) = \begin{cases} -x - 4 & \text{if } x \le -2 \\ \frac{4}{5}x - \frac{2}{5} & \text{if } -2 < x < 3 \\ -2x + 8 & \text{if } x \ge 3 \end{cases}$$

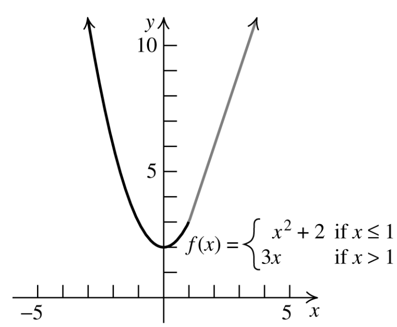
9. $f(x) = [\![x]\!]$
$f(-3.4) = -4$; $f(4.7) = 4$

2.6 Basic Concepts and Skills

1. The graph of the linear function $f(x) = b$ is a <u>horizontal</u> line.

2. The absolute value function can be expressed as a piecewise function by writing
$f(x) = |x| = \underline{\begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \ge 0 \end{cases}}$.

3. The graph of the function
$f(x) = \begin{cases} x^2 + 2 & \text{if } x \le 1 \\ ax & \text{if } x > 1 \end{cases}$
will have a break at $x = 1$ unless $a = \underline{3}$.



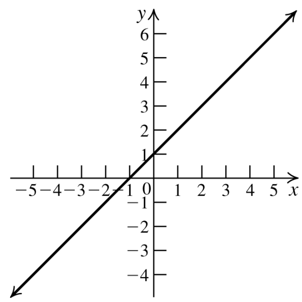
4. False. The function is constant on [0, 1), [1, 2), and [2, 3).

In exercises 5–14, first find the slope of the line using the two points given. Then substitute the coordinates of one of the points into the point-slope form of the equation to solve for b.

5. The two points are (0, 1) and (–1, 0).

$m = \dfrac{0-1}{-1-0} = 1.\;\; 1 = 1(0) + b \Rightarrow b = 1.$

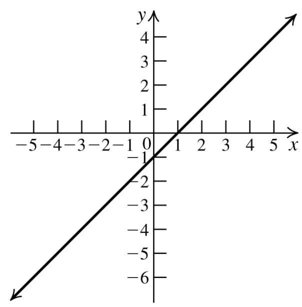
$f(x) = x + 1$



6. The two points are (1, 0) and (2, 1).

$m = \dfrac{1-0}{2-1} = 1.\;\; 0 = 1 + b \Rightarrow b = -1.$

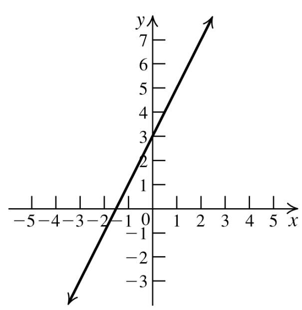
$f(x) = x - 1$



7. The two points are (–1, 1) and (2, 7).

$m = \dfrac{7-1}{2-(-1)} = 2.\;\; 1 = 2(-1) + b \Rightarrow 3 = b.$

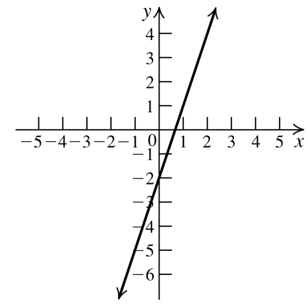
$f(x) = 2x + 3$



8. The two points are (–1, –5) and (2, 4).

$m = \dfrac{4-(-5)}{2-(-1)} = 3.\;\; 4 = 3(2) + b \Rightarrow b = -2.$

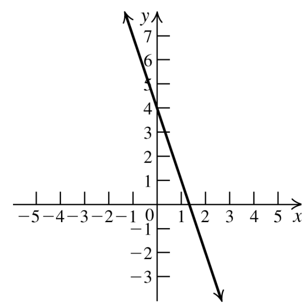
$f(x) = 3x - 2.$



9. The two points are (1, 1) and (2, –2).

$m = \dfrac{-2-1}{2-1} = -3.\;\; 1 = -3(1) + b \Rightarrow b = 4.$

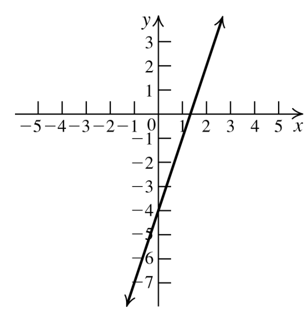
$f(x) = -3x + 4.$



10. The two points are (1, –1) and (3, 5).

$m = \dfrac{5-(-1)}{3-1} = 3.\;\; -1 = 3(1) + b \Rightarrow b = -4.$

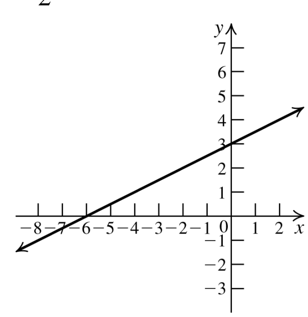
$f(x) = 3x - 4.$



11. The two points are (–2, 2) and (2, 4).

$m = \dfrac{4-2}{2-(-2)} = \dfrac{1}{2}.\;\; 4 = \dfrac{1}{2}(2) + b \Rightarrow b = 3.$

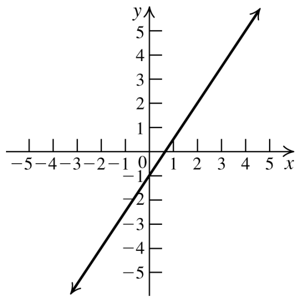
$f(x) = \dfrac{1}{2}x + 3.$



12. The two points are (2, 2) and (4, 5).

$m = \dfrac{5-2}{4-2} = \dfrac{3}{2}$. $2 = \dfrac{3}{2}(2) + b \Rightarrow b = -1$.

$f(x) = \dfrac{3}{2}x - 1$.

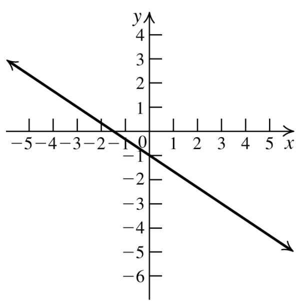


13. The two points are (0, –1) and (3, –3).

$m = \dfrac{-3-(-1)}{3-0} = -\dfrac{2}{3}$.

$-1 = -\dfrac{2}{3}(0) + b \Rightarrow b = -1$.

$f(x) = -\dfrac{2}{3}x - 1$.

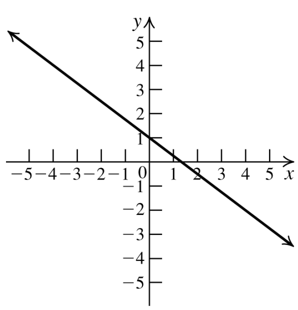


14. The two points are (1, 1/4) and (4, –2).

$m = \dfrac{-2-1/4}{4-1} = \dfrac{-9/4}{3} = -\dfrac{3}{4}$.

$-2 = -\dfrac{3}{4}(4) + b \Rightarrow b = 1$.

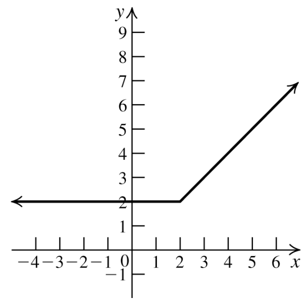
$f(x) = -\dfrac{3}{4}x + 1$.



15. $f(x) = \begin{cases} x & \text{if } x \ge 2 \\ 2 & \text{if } x < 2 \end{cases}$

a. $f(1) = 2;\ f(2) = 2;\ f(3) = 3$

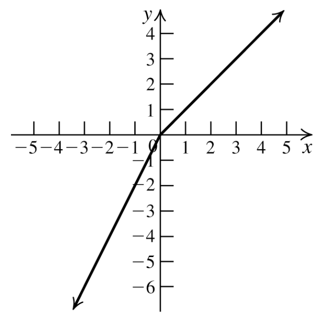
b.



16. $g(x) = \begin{cases} 2x & \text{if } x < 0 \\ x & \text{if } x \ge 0 \end{cases}$

a. $g(-1) = -2;\ g(0) = 0;\ g(1) = 1$

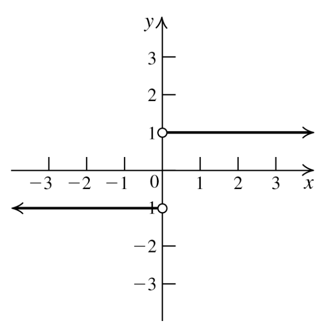
b.



17. $g(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$

a. $f(-15) = -1;\ f(12) = 1$

b.

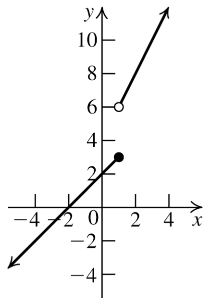


c. domain: $(-\infty, 0) \cup (0, \infty)$
range: $\{-1, 1\}$

18. $g(x) = \begin{cases} 2x+4 & \text{if } x > 1 \\ x+2 & \text{if } x \le 1 \end{cases}$

a. $g(-3) = -1;\ g(1) = 3;\ g(3) = 10$

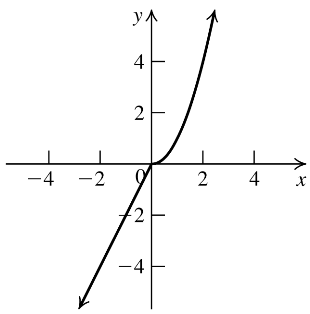
b.



c. domain: $(-\infty, \infty)$

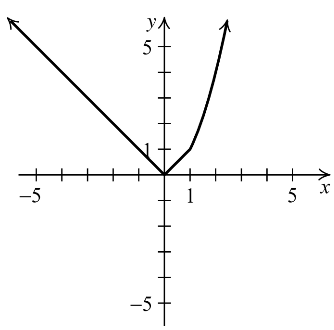
range: $(-\infty, 3] \cup (6, \infty)$

19. $f(x) = \begin{cases} 2x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$



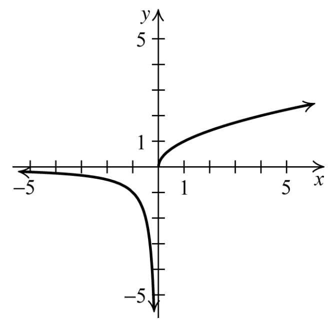
Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

20. $f(x) = \begin{cases} |x| & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$



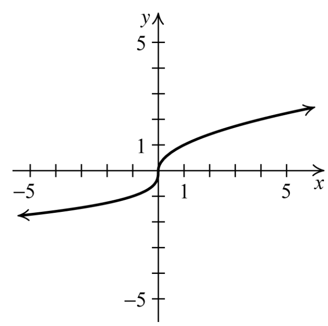
Domain: $(-\infty, \infty)$; range: $[0, \infty)$

21. $g(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \geq 0 \end{cases}$



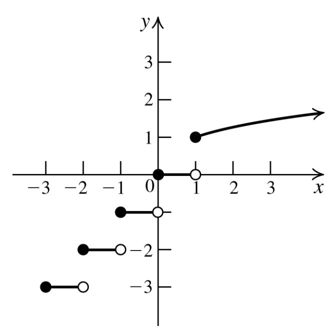
Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

22. $h(x) = \begin{cases} \sqrt[3]{x} & \text{if } x < 1 \\ \sqrt{x} & \text{if } x \geq 1 \end{cases}$



Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

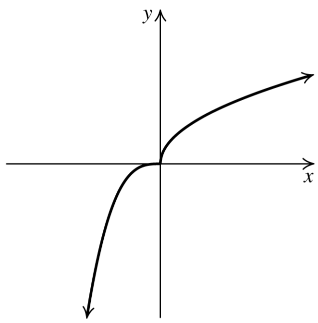
23. $f(x) = \begin{cases} [\![x]\!] & \text{if } x < 1 \\ \sqrt[3]{x} & \text{if } x \geq 1 \end{cases}$



Domain: $(-\infty, \infty)$;

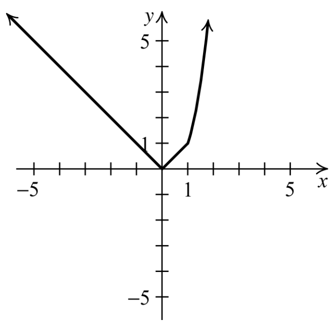
range: $\{\ldots, -3, -2, -1, 0\} \cup [1, \infty)$

24. $g(x)=\begin{cases} x^3 & \text{if } x<0 \\ \sqrt{x} & \text{if } x \ge 0 \end{cases}$



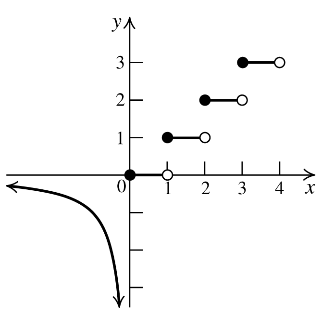
Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

25. $g(x)=\begin{cases} |x| & \text{if } x<1 \\ x^3 & \text{if } x \ge 1 \end{cases}$



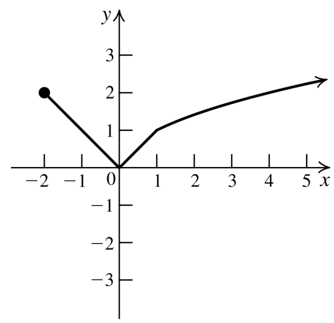
Domain: $(-\infty, \infty)$; range: $[0, \infty)$

26. $f(x)=\begin{cases} \dfrac{1}{x} & \text{if } x<0 \\ [\![x]\!] & \text{if } x \ge 0 \end{cases}$



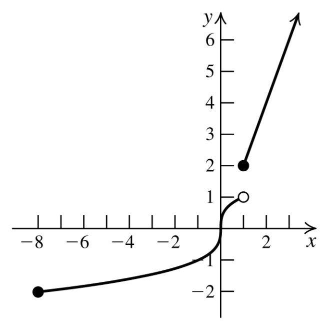
Domain: $(-\infty, \infty)$; range: $(-\infty, 0] \cup \{1, 2, 3, \ldots\}$

27. $f(x)=\begin{cases} |x| & \text{if } -2 \le x < 1 \\ \sqrt{x} & \text{if } x \ge 1 \end{cases}$



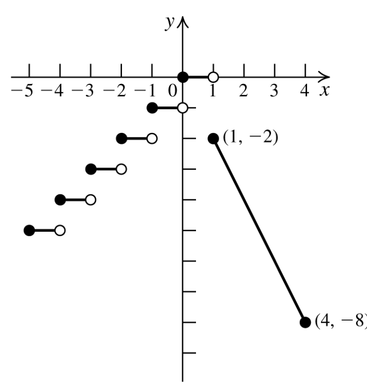
Domain: $[-2, \infty)$; range: $[0, \infty)$

28. $g(x)=\begin{cases} \sqrt[3]{x} & \text{if } -8 \le x < 1 \\ 2x & \text{if } x \ge 1 \end{cases}$



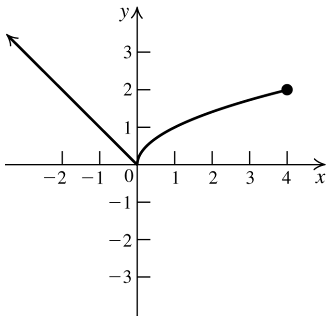
Domain: $[-8, \infty)$; range: $[-2, 1) \cup [2, \infty)$

29. $f(x)=\begin{cases} [\![x]\!] & \text{if } x<1 \\ -2x & \text{if } 1 \le x \le 4 \end{cases}$



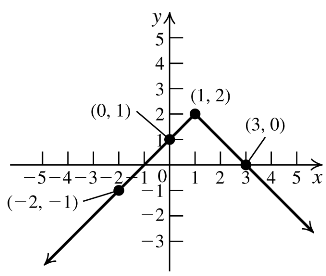
Domain: $(-\infty, 4]$;
range: $\{\ldots, -3, -2, -1, 0\} \cup [-8, -2]$

30. $h(x) = \begin{cases} |x| & \text{if } x < 0 \\ \sqrt{x} & \text{if } 0 \le x \le 4 \end{cases}$



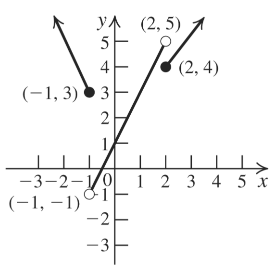
Domain: $(-\infty, 4]$; range: $[0, \infty)$

31. $f(x) = \begin{cases} 2x+3 & \text{if } x < -2 \\ x+1 & -2 \le x < 1 \\ -x+3 & \text{if } x \ge 1 \end{cases}$



Domain: $(-\infty, \infty)$; range: $(-\infty, 2]$

32. $f(x) = \begin{cases} -2x+1 & \text{if } x \le -1 \\ 2x+1 & -1 < x < 2 \\ x+2 & \text{if } x \ge 2 \end{cases}$



Domain: $(-\infty, \infty)$; range: $(-1, \infty)$

33. The graph of *f* is made up of two parts.
For $x < 2$, the graph is made up of the half-line passing through the points (−1, 0) and (2, 3).

$$m = \frac{3-0}{2-(-1)} = \frac{3}{3} = 1$$
$$y - 0 = x - (-1) \Rightarrow y = x + 1$$

For $x \ge 2$, the graph is a line segment passing through the points (2, 3) and (3, 0).

$$m = \frac{0-3}{3-2} = -3$$
$$y - 0 = -3(x-3) \Rightarrow y = -3x + 9$$

Combining the two parts, we have

$$f(x) = \begin{cases} x+1 & \text{if } x < -2 \\ -3x+9 & \text{if } x \ge 2 \end{cases}$$

34. The graph of *f* is made up of two parts.
For $x < 2$, the graph is made up of the half-line passing through the points (2, −1) and (0, 3).

$$m = \frac{3-(-1)}{0-2} = \frac{4}{-2} = -2$$
$$y = -2x + 3$$

For $x \ge 2$, the graph is a line segment passing through the points (2, −1) and (4, 0).

$$m = \frac{-1-0}{2-4} = \frac{1}{2}$$
$$y - 0 = \frac{1}{2}(x-4) \Rightarrow y = \frac{1}{2}x - 2$$

Combining the two parts, we have

$$f(x) = \begin{cases} -2x+3 & \text{if } x < -2 \\ \frac{1}{2}x - 2 & \text{if } x \ge 2 \end{cases}$$

35. The graph of *f* is made up of three parts.
For $x < -2$, the graph is the half-line passing through the points (−2, 2) and (−3, 0).

$$m = \frac{0-2}{-3-(-2)} = \frac{-2}{-1} = 2$$
$$y - 0 = 2(x-(-3)) \Rightarrow y = 2(x+3) \Rightarrow$$
$$y = 2x + 6$$

For $-2 \le x < 2$, the graph is a horizontal line segment passing through the points (−2, 4) and (2, 4), so the equation is $y = 4$.
For $x \ge 2$, the graph is the half-line passing through the points (2, 1) and (3, 0).

$$m = \frac{0-1}{3-2} = -1$$
$$y - 0 = -(x-3) \Rightarrow y = -x + 3$$

Combining the three parts, we have

$$f(x) = \begin{cases} 2x+6 & \text{if } x < -2 \\ 4 & \text{if } -2 \le x < 2 \\ -x+3 & \text{if } x \ge 2 \end{cases}$$

36. The graph of *f* is made up of four parts.
For $x \le -2$, the graph is the half-line passing through the points (−2, 0) and (−4, 3).

$$m = \frac{3-0}{-4-(-2)} = -\frac{3}{2}$$

$$y - 0 = -\frac{3}{2}(x-(-2)) \Rightarrow y = -\frac{3}{2}(x+2) \Rightarrow$$

$$y = -\frac{3}{2}x - 3$$

For $-2 < x \le 0$, the graph is a line segment passing through the points (−2, 0) and (0, 3).

$$m = \frac{3-0}{0-(-2)} = \frac{3}{2}$$

$$y = \frac{3}{2}x + 3$$

For $0 < x \le 2$, the graph is a line segment passing through the points (0, 3) and (2, 0).

$$m = \frac{0-3}{2-0} = -\frac{3}{2}$$

$$y = -\frac{3}{2}x + 3$$

For $x \ge 2$, the graph is the half-line passing through the points (2, 0) and (4, 3).

$$m = \frac{3-0}{4-2} = \frac{3}{2}$$

$$y - 0 = \frac{3}{2}(x-2) \Rightarrow y = \frac{3}{2}x - 3$$

Combining the four parts, we have

$$f(x) = \begin{cases} y = -\frac{3}{2}x - 3 & \text{if } x \le -2 \\ y = \frac{3}{2}x + 3 & \text{if } -2 < x \le 0 \\ y = -\frac{3}{2}x + 3 & \text{if } 0 < x \le 2 \\ y = \frac{3}{2}x - 3 & \text{if } x > 2 \end{cases}$$

2.6 Applying the Concepts

37.a. $f(x) = \frac{x}{33.81}$
Domain: $[0, \infty)$; range: $[0, \infty)$.

b. $f(3) = \frac{3}{33.81} \approx 0.0887$
This means that 3 oz ≈ 0.0887 liters.

c. $f(12) = \frac{12}{33.81} \approx 0.3549$ liters.

38.a. $B(0) = -1.8(0) + 212 = 212.$
The *y*-intercept is 212. This means that water boils at 212°F at sea level.
$0 = -1.8h + 212 \Rightarrow h \approx 117.8$
The *h*-intercept is approximately 117.80. This means that water boils at 0°F at approximately 117,800 feet above sea level.

b. Domain: closed interval from 0 to the end of the atmosphere, in thousands of feet.

c. $98.6 = -1.8h + 212 \Rightarrow h = 63.$ Water boils at 98.6°F at 63,000 feet. It is dangerous because 98.6°F is the temperature of human blood.

39.a. $P(0) = \frac{1}{33}(0) + 1 = 1.$ The *y*-intercept is 1.
This means that the pressure at sea level ($d = 0$) is 1 atm.
$0 = \frac{1}{33}d + 1 \Rightarrow d = -33.$
d can't be negative, so there is no *d*-intercept.

b. $P(0) = 1$ atm; $P(10) = \frac{1}{33}(10) + 1 \approx 1.3$ atm;
$P(33) = \frac{1}{33}(33) + 1 = 2$ atm;
$P(100) = \frac{1}{33}(100) + 1 \approx 4.03$ atm.

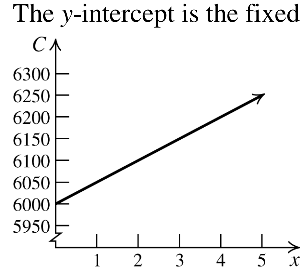
c. $5 = \frac{1}{33}d + 1 \Rightarrow d = 132$ feet
The pressure is 5 atm at 132 feet.

40.a. $V(90) = 1055 + 1.1(90) = 1154$ ft/sec
The speed of sound at 90ºF is 1154 feet per second.

b. $1100 = 1055 + 1.1T \Rightarrow T \approx 40.91$°F
The speed of sound is 1100 feet per second at approximately 40.91ºF.

41.a. $C(x) = 50x + 6000$

b. The *y*-intercept is the fixed overhead cost.



c. $11,500 = 50x + 6000 \Rightarrow 110$
110 printers were manufactures on a day when the total cost was $11,500.

42.a. The rate of change (slope) is 100. Find the y-intercept by using the point (10, 750): $750 = 100(10) + b \Rightarrow b = -250.$ The equation is $f(p) = 100p - 250.$

b. $f(15) = 100(15) - 250 = 1250$
When the price is \$15 per unit, there are 1250 units.

c. $1750 = 100p - 250 \Rightarrow p = \$20.$
1750 units can be supplied at \$20 per unit.

43.a. $R = 900 - 30x$

b. $R(6) = 900 - 30(6) = 720$
If you move in 6 days after the first of the month, the rent is \$720.

c. $600 = 900 - 30x \Rightarrow x = 10$
You moved in ten days after first of the month.

44.a. Let $t = 0$ represent the year 2009. The rate of change (slope) is $\frac{995-976}{0-2} = -9.5.$ The y-intercept is 995, so the equation is $f(t) = -9.5t + 995.$

b. $f(4) = -9.5(4) + 995 = 957$
The average SAT score will be 957 in 2013.

c. $-9.5t + 995 = 900 \Rightarrow -9.5t = -95 \Rightarrow t = 10$
2009 + 10 = 2019.
The average SAT score will be 900 in 2019.

45. The rate of change (slope) is $\frac{100-40}{20-80} = -1.$
Use the point (20, 100) to find the equation of the line: $100 = -20 + b \Rightarrow b = 120.$ The equation of the line is $y = -x + 120.$ Now solve $50 = -x + 120 \Rightarrow x = 70.$
Age 70 corresponds to 50% capacity.

46.a. $y = \frac{2}{25}(5)(60) = 24$
The dosage for a five-year-old child is 24 mg.

b. $60 = \frac{2}{25}(60)a \Rightarrow a = 12.5$
A child would have to be 12.5 years old to be prescribed an adult dosage.

47.a. The rate of change (slope) is $\frac{50-30}{420-150} = \frac{2}{27}.$
The equation of the line is $y - 30 = \frac{2}{27}(x-150) \Rightarrow$
$y = \frac{2}{27}(x-150) + 30.$

b. $y = \frac{2}{27}(350-150) + 30 \Rightarrow y = \frac{1210}{27} \approx 44.8$
There can't be a fractional number of deaths, so round up. There will be about 45 deaths when x = 350 milligrams per cubic meter.

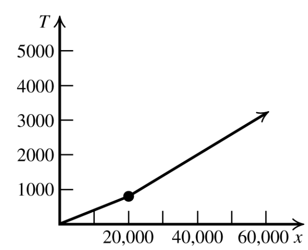
c. $70 = \frac{2}{27}(x-150) + 30 \Rightarrow x = 690$
If the number of deaths per month is 70, the concentration of sulfur dioxide in the air is 690 mg/m^3.

48.a. The rate of change is $\frac{1}{3}$. The y-intercept is $\frac{47}{12}$, so the equation is $y = L(S) = \frac{1}{3}S + \frac{47}{12}.$

b. $L(4) = \frac{1}{3}(4) + \frac{47}{12} = 5.25$
A child's size 4 shoe has insole length 5.25 inches.

c. $\frac{61}{10} = \frac{1}{3}x + \frac{47}{12} \Rightarrow x = 6.55 \approx 6.5$
A child whose insole length is 6.1 inches wears a size 6.5 shoe.

49.a.



b.(i) $T(12{,}000) = 0.04(12{,}000) = \$480$

(ii) $T(20{,}000) = 800 + 0.06(20{,}000 - 20{,}000) = \$800$

(iii) $T(50{,}000) = 800 + 0.06(50{,}000 - 20{,}000) = \$2600$

c. (i) $600 = 0.04x \Rightarrow x = \$15{,}000$

(ii) $1200 = 0.04x \Rightarrow x = \$30{,}000$, which is outside of the domain. Try $1200 = 800 + 0.06(x - 20{,}000) \Rightarrow$ $x \approx \$26{,}667$

(iii) $2300 = 800 + 0.06(x - 20,000) \Rightarrow x = \$45,000$

50.a. If $0 < x \le 8350,\ f(x) = 0.1x$
If $8350 < x \le 33,950,\ f(x) = 835 + 0.15(x - 8350) = 0.15x - 417.50$
If $33,950 < x \le 82,250,\ f(x) = 4675 + 0.25(x - 33,950) = 0.25x - 3812.50$
If $82,250 < x \le 171,550,\ f(x) = 16,750 + 0.28(x - 82,250) = 0.28x - 6280$
If $171,550 < x \le 372,950,\ f(x) = 41,754 + 0.33(x - 171,550) = 0.33x - 14,857.50$
If $372,950 < x,\ f(x) = 108,216 + 0.35(x - 372,950) = 0.35x - 22,316.50$

Write the equation as:

$$f(x) = \begin{cases} 0.1x & \text{if } 0 < x \le 8350 \\ 0.15x - 417.50 & \text{if } 8350 < x \le 33,950 \\ 0.25x - 3812.50 & \text{if } 33,950 < x \le 82,250 \\ 0.28x - 6280 & \text{if } 82,250 < x \le 171,550 \\ 0.33x - 14,857.50 & \text{if } 171,550 < x \le 372,950 \\ 0.35x - 22,316.50 & \text{if } x > 372,950 \end{cases}$$

b. (i) $f(35,000) = 0.25(35,000) - 3812.50 = \$4937.50$

(ii) $f(100,000) = 0.28(100,000) - 6280 = \$21,720$

(iii) $f(500,000) = 0.35(500,000) - 22,316.50 = \$152,683.50$

c. (i) $3500 = 0.15x - 417.50 \Rightarrow x = \$26,116.67$

(ii) $12,700 = 0.25x - 3812.50 \Rightarrow x = \$66,050.00$

(iii) $35,000 = 0.28x - 6280 \Rightarrow x = \$147,428.57$

2.6 Beyond the Basics

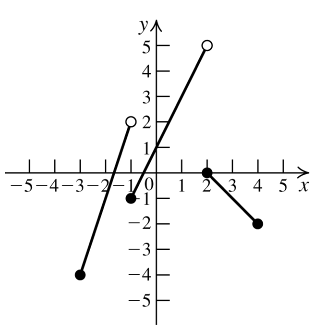
51.a. (i) $f(-2) = 3(-2) + 5 = -1$

(ii) $f(-1) = 2(-1) + 1 = -1$

(iii) $f(3) = 2 - 3 = -1$

b. Try the first rule: $2 = 3x + 5 \Rightarrow x = -1$, which is not in the domain for that rule. Now try the second rule: $2 = 2x + 1 \Rightarrow x = \frac{1}{2}$, which is in the domain for that rule.

c.



52.a. (i) $g(-2) = 3$

(ii) $g\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right) - 3 = -\frac{3}{2}$

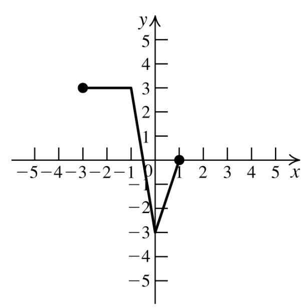
(iii) $g(0) = 3(0) - 3 = -3$

(iv) $g(-1) = -6(-1) - 3 = 3$

b. (i) $0 = -6x - 3 \Rightarrow x = -\frac{1}{2}$ or $0 = 3x - 3 \Rightarrow x = 1$

(ii)
$$2g(x) + 3 = 0$$
$$2(-6x - 3) + 3 = 0$$
$$-12x - 6 + 3 = 0 \Rightarrow x = -\frac{1}{4} \text{ or}$$
$$2(3x - 3) + 3 = 0$$
$$6x - 6 + 3 = 0 \Rightarrow x = \frac{1}{2}$$

c.



53.a. Domain: $(-\infty, \infty)$; range: $[0,1)$

b. The function is increasing on $(n, n + 1)$ for every integer n.

c. $f(-x) = -x - [\![-x]\!] \ne -f(x) \ne f(x)$, so the function is neither even nor odd.

54.a. Domain: $(-\infty, 0) \cup [1, \infty)$

range: $\left\{\frac{1}{n} : n \ne 0, n \text{ an integer}\right\}$

b. The function is constant on $(n, n+1)$ for every nonzero integer n.

c. $f(-x) = \dfrac{1}{[\![-x]\!]} \neq -f(x) \neq f(x)$, so the function is neither even nor odd.

55. $\left|f(x) - f(-x)\right| = \left|\dfrac{|x|}{x} - \dfrac{|-x|}{-x}\right| = \left|\dfrac{|x|}{x} + \dfrac{|x|}{x}\right|$
$= |1+1| = 2$ or $|-1+(-1)| = 2$

Thus, $\left|f(x) - f(-x)\right| = 2$

56.a. **(i)** $\text{WCI}(2) = 40$

(ii) $\text{WCI}(16) = 91.4 + (91.4 - 40)\cdot (0.0203(16) - 0.304\sqrt{16} - 0.474) \approx 21$

(iii) $\text{WCI}(50) = 1.6(40) - 55 = 9$

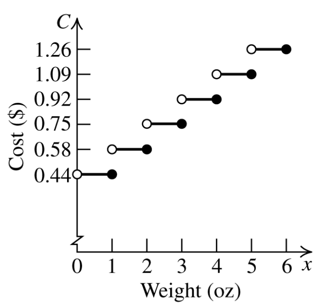
b. **(i)** $-58 = 91.4 + (91.4 - T)\cdot (0.0203(36) - 0.304\sqrt{36} - 0.474$
$-58 = 91.4 + (91.4 - T)(-1.5672)$
$-58 = 91.4 - 143.24 + 1.5672T$
$-58 = -51.84 + 1.5672T \Rightarrow T \approx -4°\text{F}$

(ii) $-10 = 1.6T - 55 \Rightarrow T \approx 28°\text{F}$

2.6 Critical Thinking/Discussion/Writing

57.a. $C(x) = 17\left(f(x) - 1\right) + 44$
$= -17\left([\![-x]\!] + 1\right) + 44$

b.

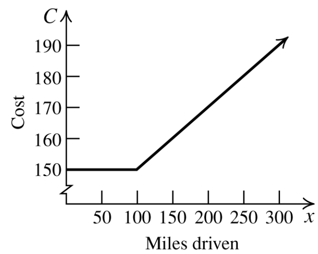


c. Domain: $(0, \infty)$
range: $\{17n + 44 : n \text{ a nonnegative integer}\}$

58. $C(x) = 2[\![x]\!] + 4$

59.a. $C(x) = \begin{cases} 150 & \text{if } x \le 100 \\ -0.2[\![x - 100]\!] + 150 & \text{if } x > 100 \end{cases}$

b.



c. $190 = 0.2[\![x - 99]\!] + 150$
$40 = 0.2[\![x - 99]\!] \Rightarrow 200 = [\![x - 99]\!] \Rightarrow$
$x = (299, 300]$ miles

60.a. $f(x) = [\![x]\!] + [\![-x]\!] \Rightarrow$
$f(x) = \begin{cases} -1 & \text{if } x \text{ is not an integer} \\ 0 & \text{if } x \text{ is an integer} \end{cases}$

For example, $\left[\!\!\left[\dfrac{3}{2}\right]\!\!\right] + \left[\!\!\left[-\dfrac{3}{2}\right]\!\!\right] = 1 + (-2) = -1$ and $[\![3]\!] + [\![-3]\!] = 3 + (-3) = 0.$

b. $g(x) = [\![x]\!] - [\![-x]\!] \Rightarrow$
$g(x) = \begin{cases} 2[\![x]\!] + 1 & \text{if } x \text{ is not an integer} \\ 2[\![x]\!] & \text{if } x \text{ is an integer} \end{cases}$

For example, $\left[\!\!\left[\dfrac{3}{2}\right]\!\!\right] - \left[\!\!\left[-\dfrac{3}{2}\right]\!\!\right] = 1 - (-2) = 3,$

while $2\left[\!\!\left[\dfrac{3}{2}\right]\!\!\right] + 1 = 2 + 1 = 3$ and

$[\![3]\!] - [\![-3]\!] = 3 - (-3) = 6 = 2[\![3]\!].$

c. $h(x) = x - |x| \Rightarrow$
$h(x) = \begin{cases} 2x & \text{if } x < 0 \\ 0 & \text{if } x \ge 0 \end{cases}$

For example, $-3 - |-3| = -3 - 3 = -6 = 2(-3)$ and $3 - |3| = 3 - 3 = 0.$

d. $F(x) = x|x| \Rightarrow$
$F(x) = \begin{cases} -x^2 & \text{if } x < 0 \\ x^2 & \text{if } x \ge 0 \end{cases}$

For example, $-4|-4| = -4(4) = -16 = -4^2$ and $4|4| = 4(4) = 16 = 4^2.$

e. $G(x) = |x - 1| + |x - 2| \Rightarrow$
$G(x) = \begin{cases} -2x + 3 & \text{if } x < 1 \\ 1 & \text{if } 1 \le x < 2 \\ 2x - 3 & \text{if } x \ge 2 \end{cases}$

For example, if $x = 0$, then $G(x) = -2(0) + 3 = 3$. If $x = 5$, then $G(x) = 2(5) - 3 = 7$.

2.6 Maintaining Skills

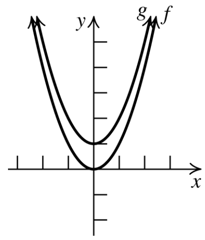
61. If we add 3 to each y-coordinate of the graph of f, we will obtain the graph of $y =$ <u>$f(x) + 3$.</u>

62. If we subtract 2 from each x-coordinate of the graph of f, we will obtain the graph of $y =$ <u>$f(x + 2)$.</u>

63. If we replace each x-coordinate with its opposite in the graph of f, we will obtain the graph of $y =$ <u>$f(-x)$.</u>

64. If we replace each y-coordinate with its opposite in the graph of f, we will obtain the graph of $y =$ <u>$-f(x)$.</u>

65.



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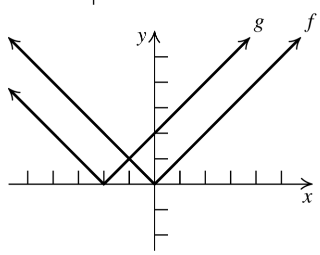

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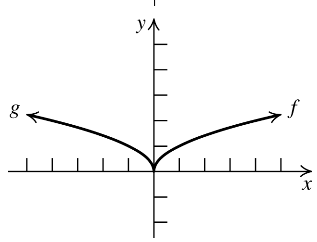

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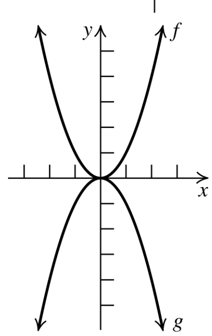
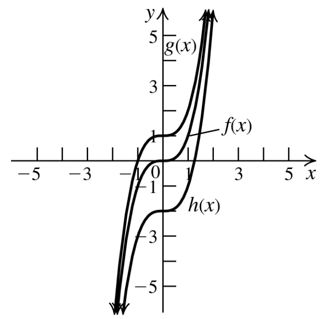


2.7 Transformations of Functions

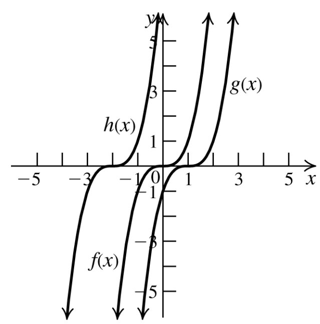
2.7 Practice Problems

1.



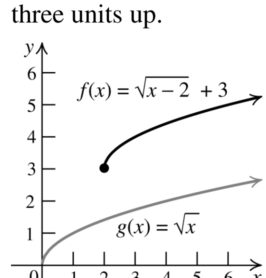
The graph of g is the graph of f shifted one unit up. The graph of h is the graph of f shifted two units down.

2.



The graph of g is the graph of f shifted one unit to the right. The graph of h is the graph of f shifted two units to the left.

3. The graph of $f(x) = \sqrt{x-2} + 3$ is the graph of $g(x) = \sqrt{x}$ shifted two units to the right and three units up.

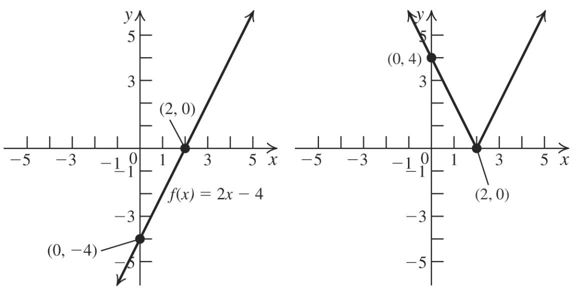


4. The graph of $y = -(x-1)^2 + 2$ can be obtained from the graph of $y = x^2$ by first shifting the graph of $y = x^2$ one unit to the right. Reflect the resulting graph about the x-axis, and then shift the graph two units up

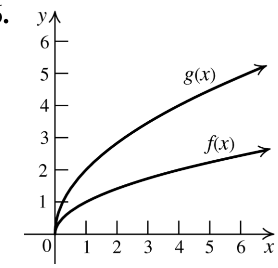
5. The graph of $y = 2x - 4$ is obtained from the graph of $y = 2x$ by shifting it down by four units. We know that

$$|y| = \begin{cases} y & \text{if } y \geq 0 \\ -y & \text{if } y < 0. \end{cases}$$

This means that the portion of the graph on or above the x-axis $(y \geq 0)$ is unchanged while the portion of the graph below the x-axis $(y < 0)$ is reflected above the x-axis.

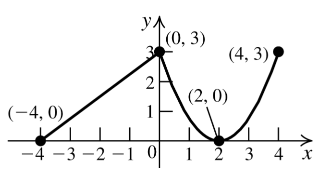


6.

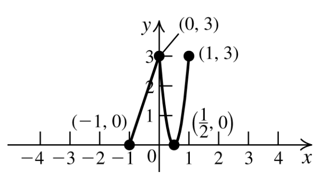


The graph of g is the graph of f stretched vertically by multiplying each of its y-coordinates by 2.

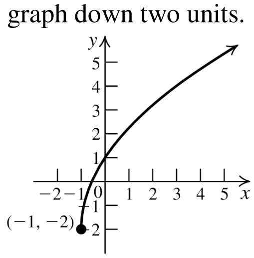
7.a.



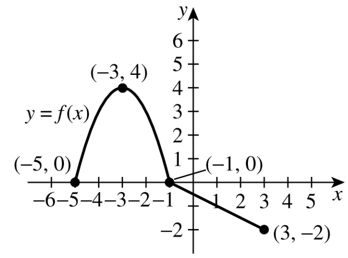
b.



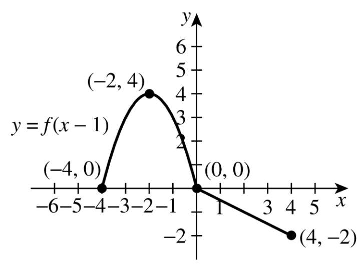
8. Start with the graph of $y = \sqrt{x}$. Shift the graph one unit to the left, then stretch the graph vertically by a factor of three. Shift the resulting graph down two units.



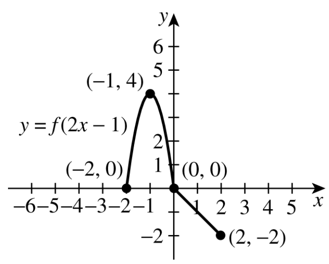
9.



Shift the graph one unit right to graph $y = f(x-1)$.

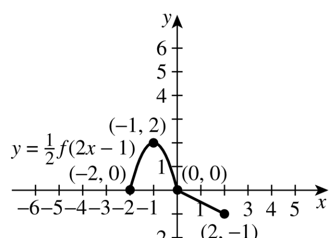


Compress horizontally by a factor of 2. Multiply each x-coordinate by $\frac{1}{2}$ to graph $y = f(2x-1)$.



Compress vertically by a factor of $\frac{1}{2}$.

Multiply each y-coordinate by $\frac{1}{2}$ to graph $y = \frac{1}{2} f(2x-1)$.



Shift the graph up three units to graph $y = \frac{1}{2} f(2x-1) + 3$.

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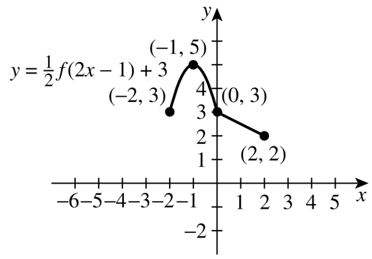
(*continued*)



2.7 Basic Concepts and Skills

1. The graph of $y = f(x) - 3$ is found by vertically shifting the graph of $y = f(x)$ three units down.

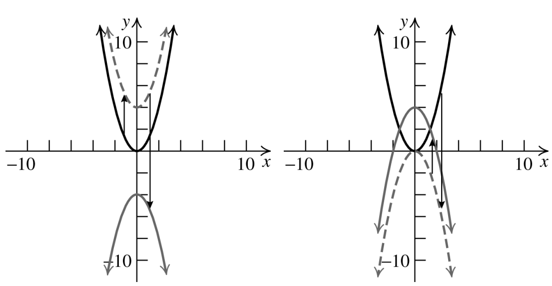
2. The graph of $y = f(x+5)$ is found by horizontally shifting the graph of $y = f(x)$ five units to the left.

3. The graph of $y = f(bx)$ is a horizontal compression of the graph of $y = f(x)$ is b is greater than 1.

4. The graph of $y = f(-x)$ is found by reflecting the graph of $y = f(x)$ about the y-axis.

5. False. The graphs are the same if the function is an even function.

6. False. The graph on the left shows $y = x^2$ first shifted up two units and then reflected about the x-axis, while the graph on the right shows $y = x^2$ reflected about the x-axis and then shifted up two units.



7.a. The graph of g is the graph of f shifted two units up.

b. The graph of h is the graph of f shifted one unit down.

8.a. The graph of g is the graph of f shifted one unit up.

b. The graph of h is the graph of f shifted two units down.

9.a. The graph of g is the graph of f shifted one unit to the left.

b. The graph of h is the graph of f shifted two units to the right.

10.a. The graph of g is the graph of f shifted two units to the left.

b. The graph of h is the graph of f shifted three units to the right.

11.a. The graph of g is the graph of f shifted one unit left and two units down.

b. The graph of h is the graph of f shifted one unit right and three units up.

12.a. The graph of g is the graph of f reflected about the x-axis.

b. The graph of h is the graph of f reflected about the y-axis.

13.a. The graph of g is the graph of f reflected about the x-axis.

b. The graph of h is the graph of f reflected about the y-axis.

14.a. The graph of g is the graph of f stretched vertically by a factor of 2.

b. The graph of h is the graph of f compressed horizontally by a factor of 2.

15.a. The graph of g is the graph of f vertically stretched by a factor of 2.

b. The graph of h is the graph of f horizontally compressed by a factor of 2.

16.a. The graph of g is the graph of f shifted two units to the right and one unit up.

b. The graph of h is the graph of f shifted one unit to the left, reflected about the x-axis, and then shifted two units up.

17.a. The graph of g is the graph of f reflected about the x-axis and then shifted one unit up.

b. The graph of h is the graph of f reflected about the y-axis and then shifted one unit up.

18.a. The graph of g is the graph of f shifted one unit to the right and then shifted two units up.

b. The graph of h is the graph of f stretched vertically by a factor of three and then shifted one unit down.

19.a. The graph of g is the graph of f shifted one unit up.

b. The graph of h is the graph of f shifted one unit to the left.

20.a. The graph of g is the graph of f shifted one unit left, vertically stretched by a factor of 2, reflected about the y-axis, and then shifted 4 units up.

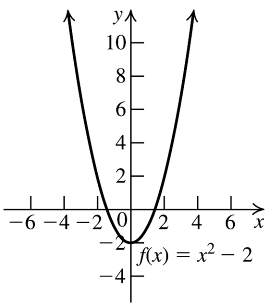
b. The graph of h is the graph of f shifted one unit to the right, reflected about the x-axis, and then shifted three units up.

21. e **22.** c **23.** g **24.** h

25. i **26.** a **27.** b **28.** k

29. l **30.** f **31.** d **32.** j

33.



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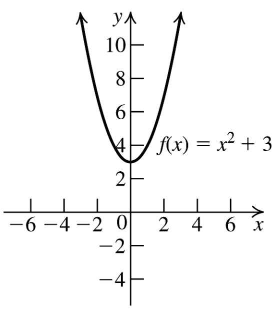

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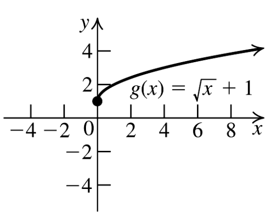

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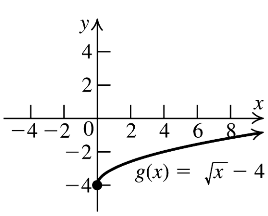

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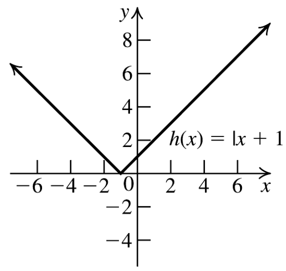

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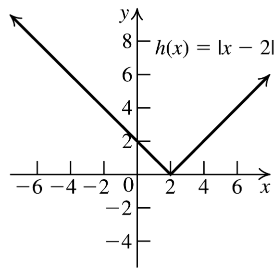

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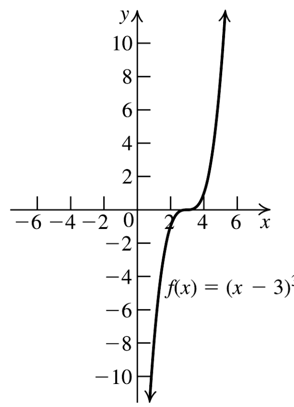

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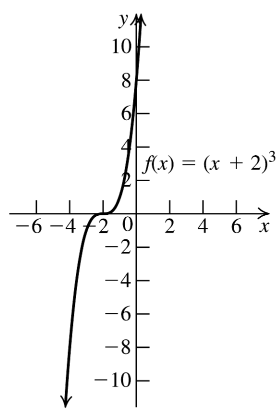

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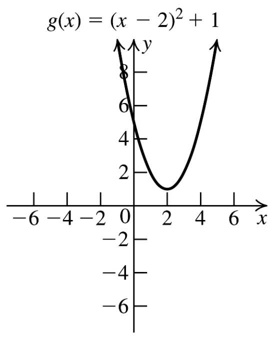

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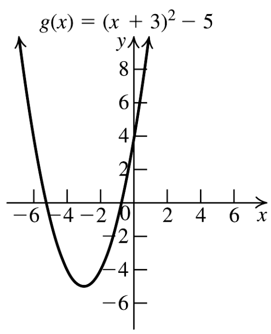

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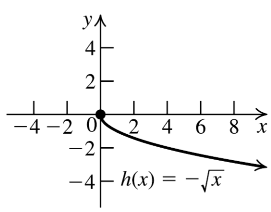

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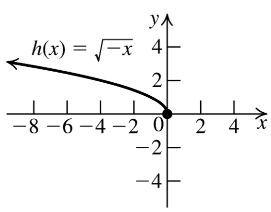

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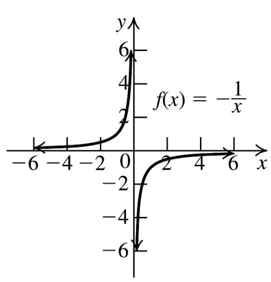

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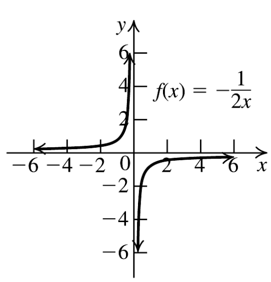

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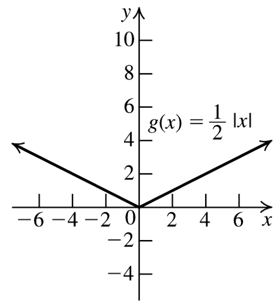

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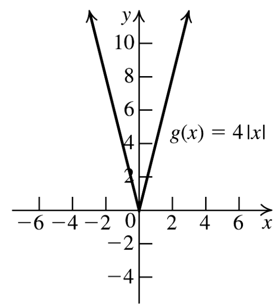

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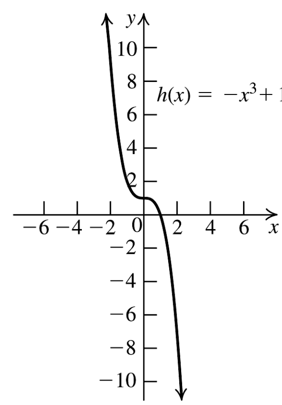

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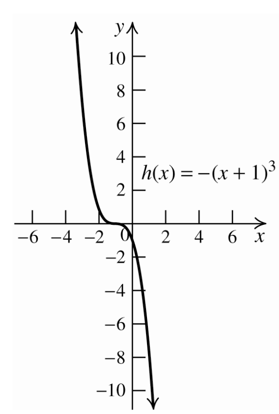

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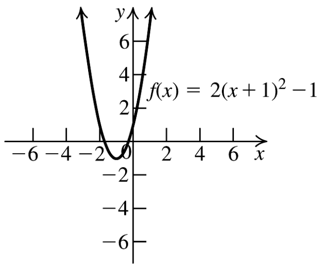

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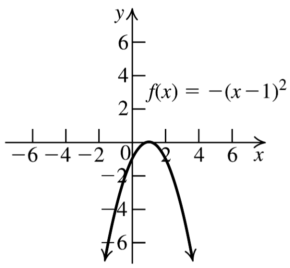
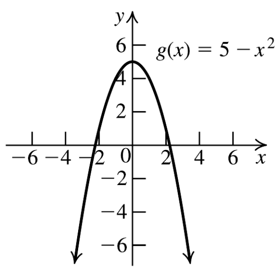


53.



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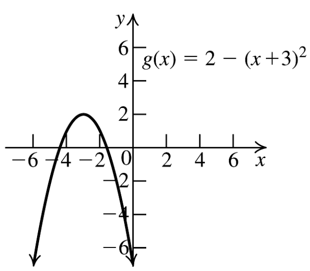

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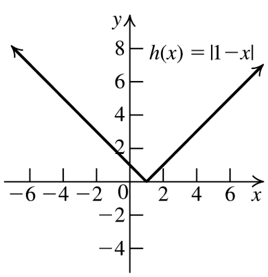

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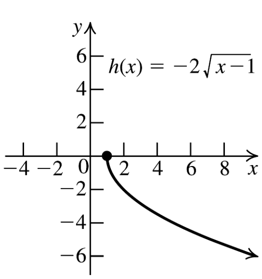

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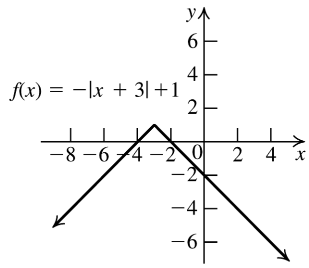

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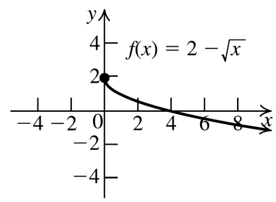

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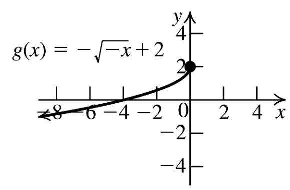

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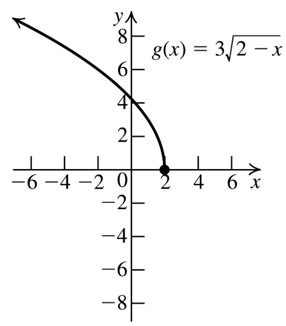

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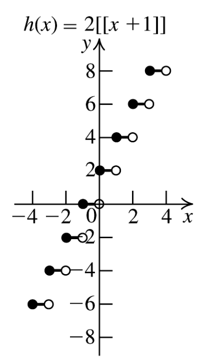

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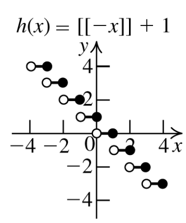


63. $y = x^3 + 2$

64. $y = \sqrt{x+3}$

65. $y = -|x|$

66. $y = \sqrt{-x}$

67. $y = (x-3)^2 + 2$

68. $y = -(x+2)^2$

69. $y = -\sqrt{x+3} - 2$

70. $y = -\frac{1}{2}\left(\sqrt{x} - 2\right)$

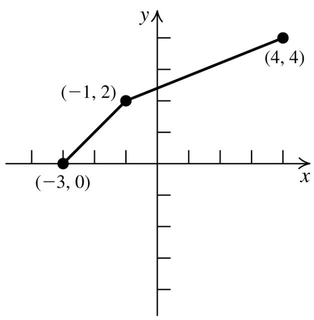
71. $y = 3(-x+4)^3 + 2$

72. $y = -(-x+1)^3 + 1$

73. $y = -2|x-4| - 3$

74. $y = \frac{1}{2}|-x-2| - 3$

75.



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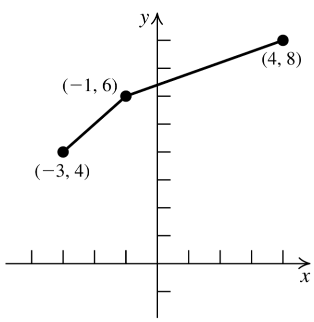

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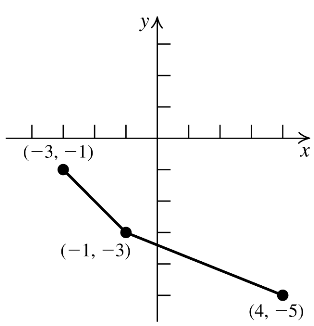

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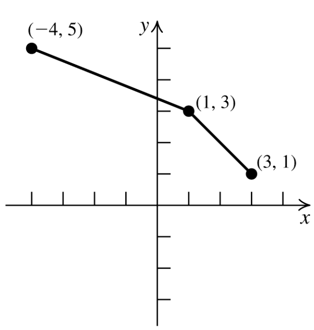

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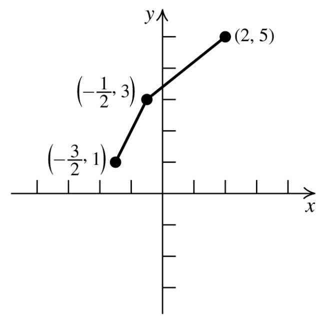

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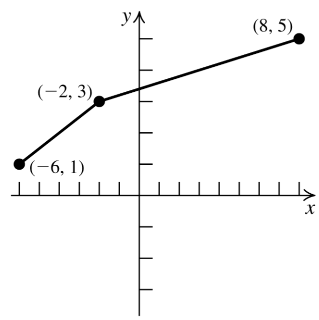

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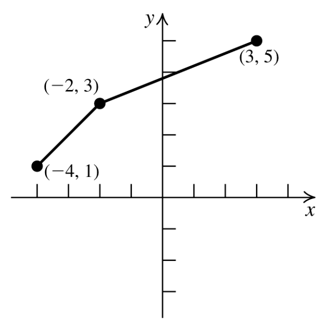

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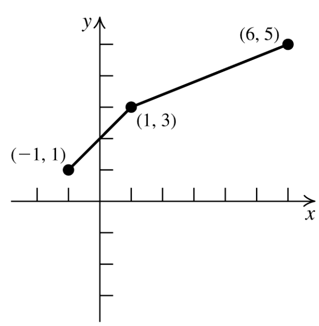

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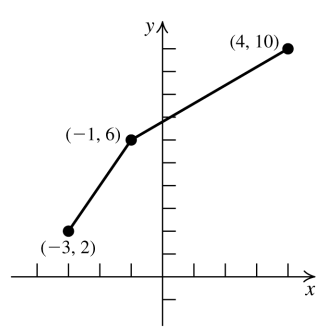

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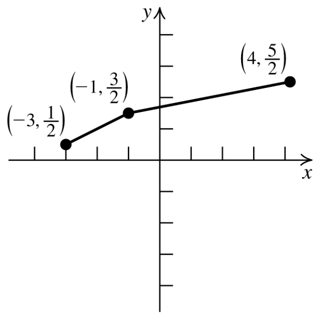

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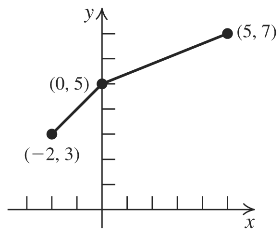

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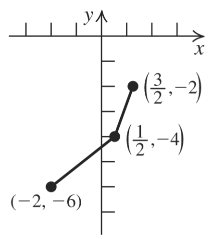

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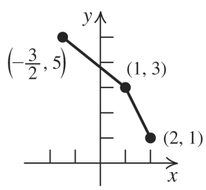

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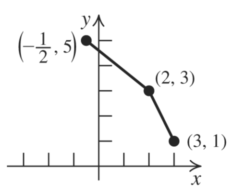
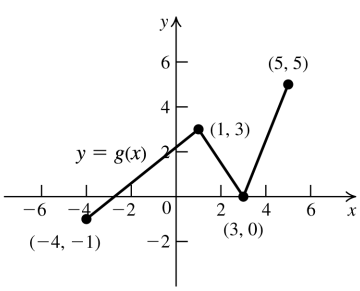


89.a.



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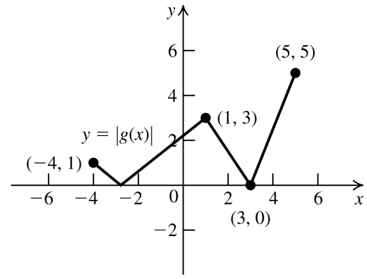
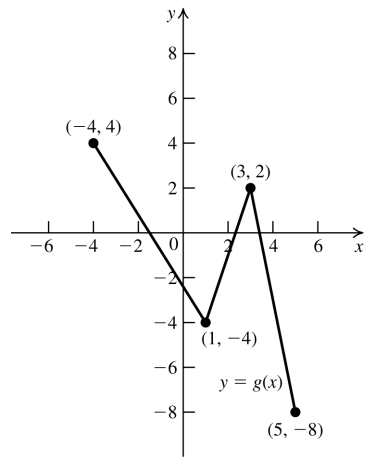


90.a.



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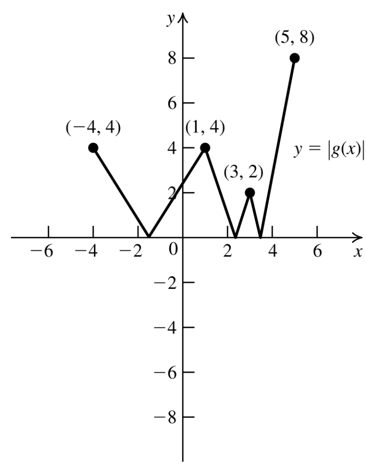
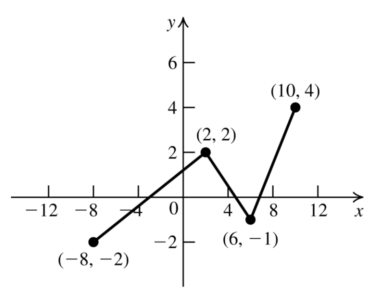


91.a.



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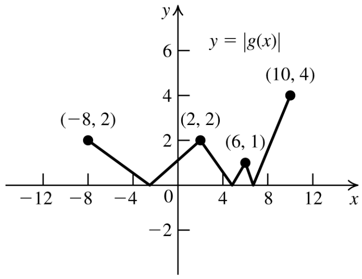
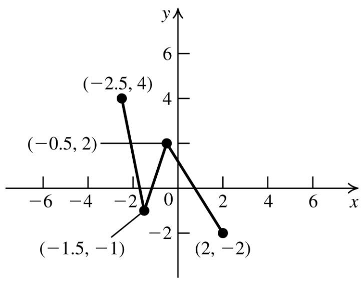


92.a.



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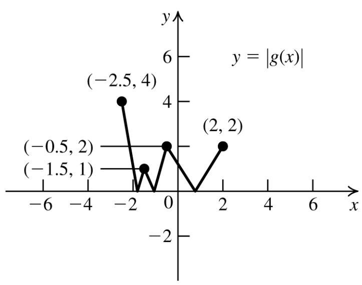
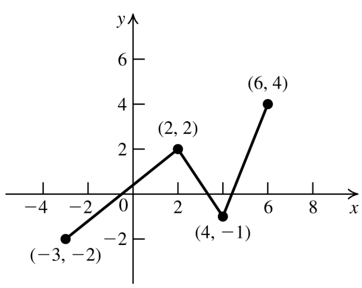


93.a.



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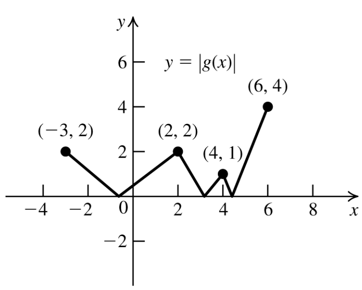
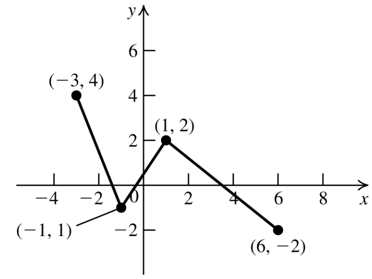


94.a.



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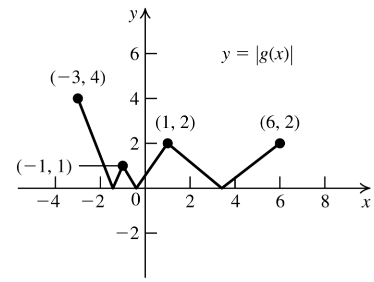
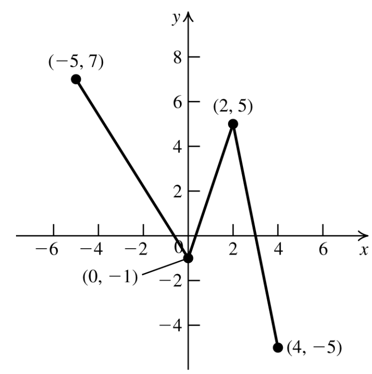
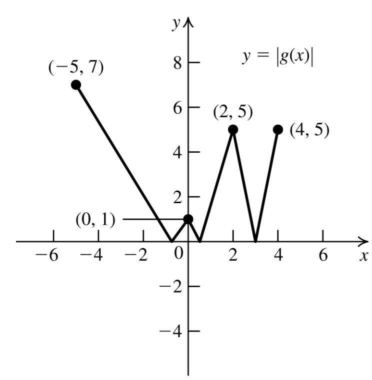


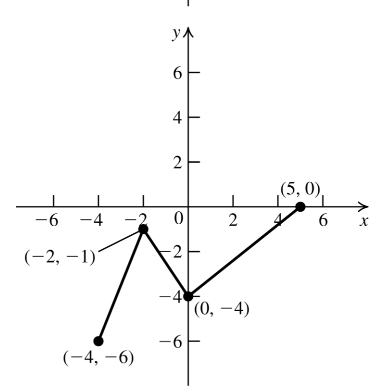
95.a.



b.



96.a.



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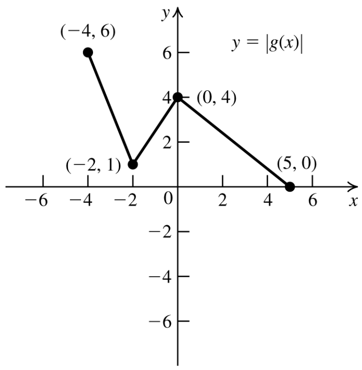
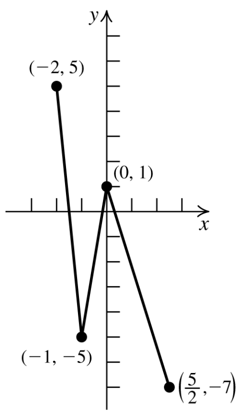


97.a.



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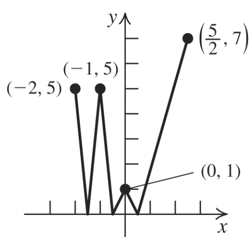
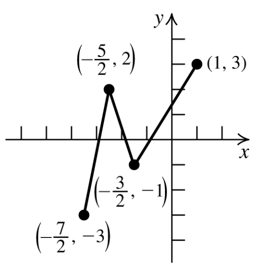
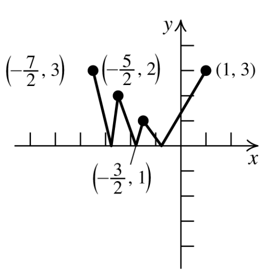


98.a.



b.



2.7 Applying the Concepts

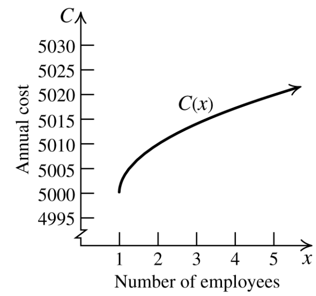
99. $g(x) = f(x) + 800$

100. $h(x) = 1.05 f(x)$

101. $p(x) = 1.02\left(f(x) + 500\right)$

102. $g(x) = \begin{cases} 1.1 f(x) & \text{if } f(x) < 30{,}000 \\ 1.02 f(x) & \text{if } f(x) \ge 30{,}000 \end{cases}$

103.a. Shift one unit right, stretch vertically by a factor of 10, and shift 5000 units up.



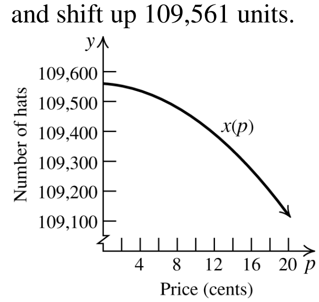
b. $C(400) = 5000 + 10\sqrt{400 - 1} = \$5199.75$

104.a. For the center of the artery, $R = 3$ mm and $r = 0$.
$v = 1000\left(3^2 - 0^2\right) = 9000$ mm/minute

b. For the inner linings of the artery, $R = 3$ mm and $r = 3$ mm
$v = 1000\left(3^2 - 3^2\right) = 0$ mm/minute

c. Midway between the center and the inner linings, $R = 3$ mm and $r = 1.5$ mm
$v = 1000\left(3^2 - 1.5^2\right) = 6750$ mm/minute

105.a. Shift one unit left, reflect across the *x*-axis, and shift up 109,561 units.



b.
$$69{,}160 = 109{,}561 - (p + 1)^2$$
$$40{,}401 = (p + 1)^2$$
$$201 = p + 1 \Rightarrow p = 200¢ = \$2.00$$

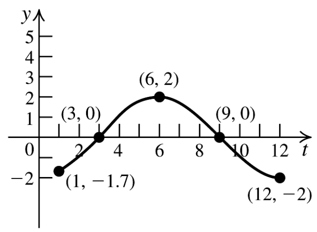
c.
$$0 = 109{,}561 - (p + 1)^2$$
$$109{,}561 = (p + 1)^2$$
$$331 = p + 1 \Rightarrow p = 330¢ = \$3.30$$

106. Write $R(p)$ in the form $-3(p-h)^2+k$:

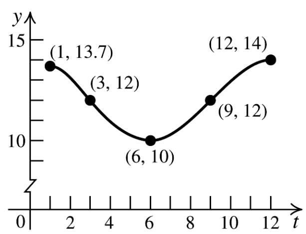
$$R(p) = -3p^2 + 600p = -3(p^2 - 200p)$$
Complete the square
$$= -3(p^2 - 200p + 10{,}000) + 30{,}000$$
$$= -3(p-100)^2 + 30{,}000$$

To graph this, shift $R(p)$ 100 units to the right, stretch by a factor of 3, reflect about the x-axis, and shift by 30,000 units up.

107. The first coordinate gives the month; the second coordinate gives the hours of daylight. From March to September, there is daylight more than half of the day each day. From September to March, more than half of the day is dark each day.



108. The graph shows the number of hours of darkness.

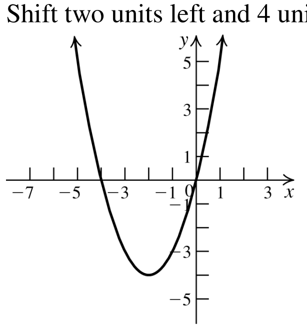


2.7 Beyond the Basics

109. The graph is shifted one unit right then reflected about the x-axis, and finally reflected about the y-axis. The equation is $g(x) = -\sqrt{1-x}$.

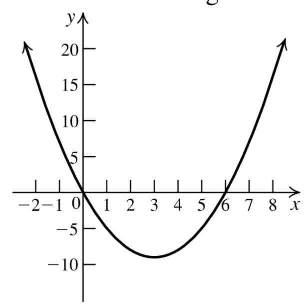
110. The graph is shifted two units left and then reflected about the y-axis.
The equation is $g(x) = f(-2-x)$.

111. Shift two units left and 4 units down.



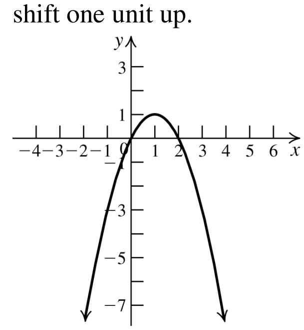
112. $f(x) = x^2 - 6x = \left(x^2 - 6x + 9\right) - 9$
$= (x-3)^2 - 9$

Shift three units right and 9 units down.



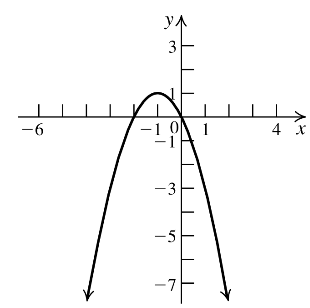
113. $f(x) = -x^2 + 2x = -\left(x^2 - 2x + 1\right) + 1$
$= -(x-1)^2 + 1$

Shift one unit right, reflect about the x-axis, shift one unit up.



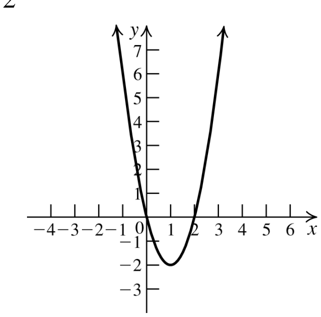
114. $f(x) = -x^2 - 2x = -\left(x^2 + 2x + 1\right) + 1$
$= -(x+1)^2 + 1$

Shift one unit left, reflect about the x-axis, shift one unit up.



115. $f(x) = 2x^2 - 4x = 2\left(x^2 - 2x + 1\right) - 2$
$= 2(x-1)^2 - 2$

Shift one unit right, stretch vertically by a factor of 2, shift two units down.

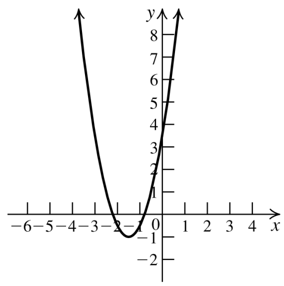


116. $f(x) = 2x^2 + 6x + 3.5$

$$= 2\left(x^2 + 3x + 1.75 + 0.5\right) - 1$$

$$= 2(x + 1.5)^2 - 1$$

Shift 1.5 units left, stretch vertically by a factor of 2, shift one unit down.

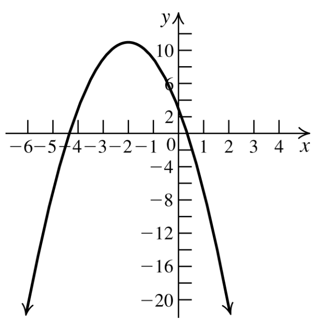


117. $f(x) = -2x^2 - 8x + 3 = -2(x^2 + 4x - 1.5)$

$$= -2\left(x^2 + 4x - 1.5 + 5.5\right) + 11$$

$$= -2(x + 2)^2 + 11$$

Shift two units left, stretch vertically by a factor of 2, reflect across the *x*-axis, shift eleven units up.



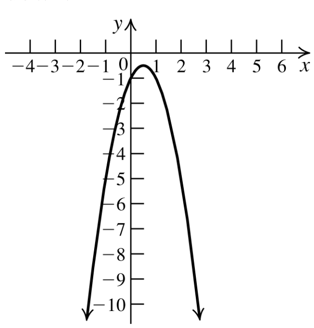
118. $f(x) = -2x^2 + 2x - 1 = -2(x^2 - x + 0.5)$

$$= -2\left(x^2 - x + 0.5 - 0.25\right) - 0.5$$

$$= -2\left(x^2 - x + 0.25\right) - 0.5$$

$$= -2(x - 0.5)^2 - 0.5$$

Shift 0.5 unit right, stretch vertically by a factor of 2, reflect across the *x*-axis, shift 0.5 unit down.



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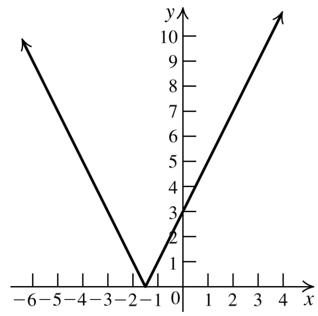
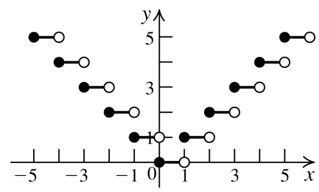
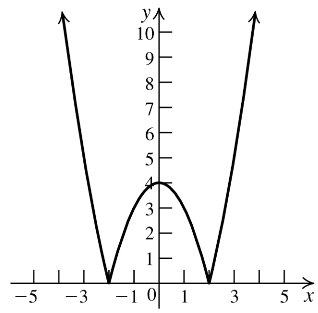


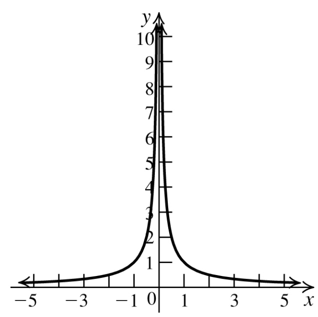
120.



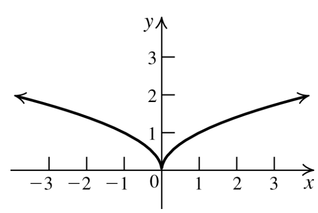
121.



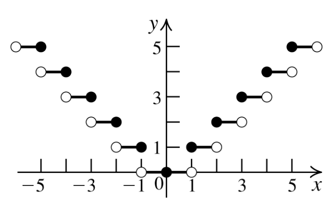
122.



123.



124.



2.7 Critical Thinking/Discussion/Writing

125.a. The function is shifted three units to the right, so the x-intercept is $-2 + 3 = 1$. The y-intercept cannot be determined.

b. The function is stretched horizontally by a factor of 5, so the x-intercept is not affected and remains at -2, while the y-intercept is $5 \cdot 3 = 15$.

c. The function is reflected about the x-axis, so the x-intercept is not affected and remains at -2, while the y-intercept is -3.

d. The function is reflected about the y-axis, so the x-intercept is 2, while the y-intercept is not changed and remains at 3.

126.a. The function is shifted two units to the left, so the x-intercept is $4 - 2 = 2$. The y-intercept cannot be determined.

b. The function is stretched horizontally by a factor of 2, so the x-intercept is not affected and remains at 4, while the y-intercept is $2(-1) = -2$.

c. The function is reflected about the x-axis, so the x-intercept is not affected and remains at 4, while the y-intercept is 1.

d. The function is reflected in the y-axis, so the x-intercept is -4, while the y-intercept is not changed and remains at -1.

127.a. $g(x) = h(x-3)+3$

The graph of g is the graph of h shifted three units to the right and three units up.

b. $g(x) = h(x-1)-1.$

The graph of g is the graph of h shifted one unit to the left and one unit down.

c. $g(x) = 2h\left(\frac{1}{2}x\right).$

The graph of g is the graph of h stretched horizontally and vertically by a factor of 2.

d. $g(x) = -3h\left(-\frac{1}{3}x\right).$

The graph of g is the graph of h stretched horizontally by a factor of 3, reflected about the y-axis, stretched vertically by a factor of 3, and reflected about the x-axis.

128. $y = f(x) = f\left(-\frac{1}{4}(-4x)\right).$

Stretch the graph of $y = f(-4x)$ horizontally by a factor of 4 and reflect it about the y-axis.

2.7 Maintaining Skills

129. $\left(5x^2+5x+7\right)+\left(x^2+9x-4\right)=6x^2+14x+3$

130. $\left(x^2+2x\right)+\left(6x^3-2x+5\right)=6x^3+x^2+5$

131. $\left(5x^2+6x-2\right)-\left(3x^2-9x+1\right)=2x^2+15x-3$

132. $\left(x^3+2\right)-\left(2x^3+5x-3\right)=-x^3-5x+5$

133. $(x-2)\left(x^2+2x+4\right)$
$= x^3+2x^2+4x-2x^2-4x-8$
$= x^3-8$

134. $\left(x^2+x+1\right)\left(x^2-x+1\right)$
$= x^4-x^3+x^2+x^3-x^2+x+x^2-x+1$
$= x^4+x^2+1$

135. $f(x)=\frac{2x-3}{x^2-5x+6}$

The function is not defined when the denominator is zero.

$x^2-5x+6=0 \Rightarrow (x-2)(x-3)=0 \Rightarrow x=2,\ 3$

The domain is $(-\infty,\ 2)\cup(2,\ 3)\cup(3,\ \infty)$.

136. $f(x)=\frac{x-2}{x^2-4}$

The function is not defined when the denominator is zero.

$x^2-4=0 \Rightarrow (x+2)(x-2)=0 \Rightarrow x=-2,\ 2$

The domain is $(-\infty,\ -2)\cup(-2,\ 2)\cup(2,\ \infty)$.

137. $f(x)=\sqrt{2x-3}$

The function is defined only if $2x-3 \ge 0$.

$2x-3\ge 0 \Rightarrow 2x \ge 3 \Rightarrow x \ge \frac{3}{2}$

The domain is $\left[\frac{3}{2},\ \infty\right)$.

138. $f(x)=\dfrac{1}{\sqrt{5-2x}}$

The function is defined only if $5-2x>0$.

$5-2x>0 \Rightarrow -2x>-5 \Rightarrow x<\dfrac{5}{2}$

The domain is $\left(-\infty, \dfrac{5}{2}\right)$.

2.8 Combining Functions; Composite Functions

2.8 Practice Problems

1. $f(x)=3x-1,\ g(x)=x^2+2$

$$\begin{aligned}(f+g)(x)&=f(x)+g(x)\\&=3x-1+x^2+2=x^2+3x+1\\(f-g)(x)&=f(x)-g(x)\\&=(3x-1)-\left(x^2+2\right)=-x^2+3x-3\\(fg)(x)&=f(x)\cdot g(x)\\&=(3x-1)\left(x^2+2\right)=3x^3-x^2+6x-2\end{aligned}$$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{3x-1}{x^2+2}$$

2. $f(x)=\sqrt{x-1},\ g(x)=\sqrt{3-x}$

The domain of *f* is $[1, \infty)$ and the domain of *g* is $(-\infty, 3]$. The intersection of D_f and D_g, $D_f \cap D_g=[1, 3]$.

The domain of *fg* is [1, 3].

The domain of $\dfrac{f}{g}$ is [1, 3).

The domain of $\dfrac{g}{f}$ is (1, 3].

3. $f(x)=-5x,\ g(x)=x^2+1$

a. $$\begin{aligned}(f\circ g)(0)&=f(g(0))\\&=f\left(0^2+1\right)=f(1)=-5\end{aligned}$$

b. $$\begin{aligned}(g\circ f)(0)&=g(f(0))\\&=g(-5\cdot 0)=g(0)=1\end{aligned}$$

4. $f(x)=2-x,\ g(x)=2x^2+1$

a. $$\begin{aligned}(g\circ f)(x)&=g(f(x))=g(2-x)\\&=2(2-x)^2+1\\&=2\left(4-4x+x^2\right)+1\\&=8-8x+2x^2+1=2x^2-8x+9\end{aligned}$$

b. $$\begin{aligned}(f\circ g)(x)&=f(g(x))=f\left(2x^2+1\right)\\&=2-\left(2x^2+1\right)=1-2x^2\end{aligned}$$

c. $$\begin{aligned}(g\circ g)(x)&=g(g(x))=g\left(2x^2+1\right)\\&=2\left(2x^2+1\right)^2+1\\&=2\left(4x^4+4x^2+1\right)+1\\&=8x^4+8x^2+3\end{aligned}$$

5. $f(x)=\sqrt{x+1},\ g(x)=\dfrac{2}{x-3}$

Let $A=\{x \mid g(x) \text{ is defined}\}$.

$g(x)$ is not defined if $x=3$, so

$A=(-\infty, 3)\cup(3, \infty)$.

Let $B=\{x \mid f(g(x)) \text{ is defined}\}$.

$$f(g(x))=\sqrt{\frac{2}{x-3}+1}=\sqrt{\frac{2+x-3}{x-3}}=\sqrt{\frac{x-1}{x-3}}$$

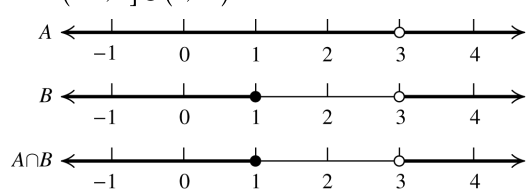
$f(g(x))$ is not defined if $x=3$ or if $\dfrac{x-1}{x-3}<0$.

$x-1=0 \Rightarrow x=1$

Interval	Test point	Value of $\dfrac{x-1}{x-3}$	Result
$(-\infty, 1]$	0	$\frac{1}{3}$	+
$[1, 3)$	2	−1	−
$(3, \infty)$	4	3	+

$f(g(x))$ is not defined for $[1, 3)$, so

$B=(-\infty, 1]\cup(3, \infty)$.



The domain of $f\circ g$ is

$A\cap B=(-\infty, 1]\cup(3, \infty)$.

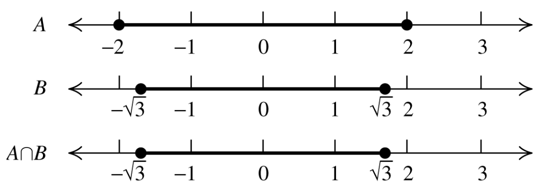
6. $f(x)=\sqrt{x-1}, \quad g(x)=\sqrt{4-x^2}$

a. $(f\circ g)(x)=f(g(x))=f\left(\sqrt{4-x^2}\right)$
$=\sqrt{\sqrt{4-x^2}-1}$

The function $g(x)=\sqrt{4-x^2}$ is defined for $4-x^2\ge 0\Rightarrow x^2\le 4\Rightarrow -2\le x\le 2$. So, $A=[-2, 2]$.

The function $f(g(x))$ is defined for $\sqrt{4-x^2}-1\ge 0\Rightarrow \sqrt{4-x^2}\ge 1\Rightarrow$
$4-x^2\ge 1\Rightarrow -x^2\ge -3\Rightarrow x^2\le 3\Rightarrow$
$-\sqrt{3}\le x\le \sqrt{3}$

So, $B=\left[-\sqrt{3}, \sqrt{3}\right]$.

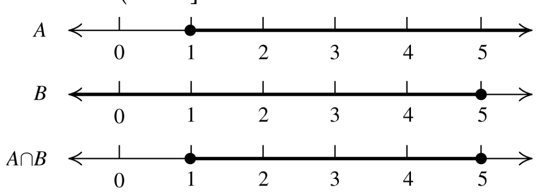


The domain of $f\circ g$ is $A\cap B=\left[-\sqrt{3}, \sqrt{3}\right]$.

b. $(g\circ f)(x)=g(f(x))=\sqrt{4-\left(\sqrt{x-1}\right)^2}$
$=\sqrt{4-(x-1)}=\sqrt{5-x}$

The function $f(x)=\sqrt{x-1}$ is defined for $x-1\ge 0\Rightarrow x\ge 1$. So, $A=[1, \infty)$.

The function $g(f(x))$ is defined for $5-x\ge 0\Rightarrow 5\ge x$, or $x\le 5$. So, $B=(-\infty, 5]$.



The domain of $g\circ f$ is $A\cap B=[1, 5]$.

7. $H(x)=\dfrac{1}{\sqrt{2x^2+1}}, \quad g(x)=\sqrt{2x^2+1}$

If $f(x)=\dfrac{1}{x}$, then

$$H(x)=(f\circ g)(x)=f\left(\sqrt{2x^2+1}\right)=\frac{1}{\sqrt{2x^2+1}}.$$

8.a. $A=f(g(t))=f(g(3))=f(3t)$
$=\pi(3t)^2=9\pi t^2$

b. $A=9\pi t^2=9\pi(6)^2=324\pi$

The area covered by the oil slick is $324\pi\approx 1018$ square miles.

9.a. $r(x)=x-4500$

b. $d(x)=x-0.06x=0.94x$

c. i. $(r\circ d)(x)=r(0.94x)=0.94x-4500$

ii. $(d\circ r)(x)=d(x-4500)=0.94(x-4500)$
$=0.94x-4230$

d. $(d\circ r)(x)-(r\circ d)(x)$
$=(0.94x-4230)-(0.94x-4500)$
$=270$

2.8 Basic Concepts and Skills

1. $(g-f)(2)=4-12=-8$

2. $g(x)=[f(x)+g(x)]-f(x)$
$=0-(2x-1)=1-2x$

3. $(f\circ g)(x)=f(g(x))=f(1)=1$

4. $(g\circ f)(1)=g(f(1))=g(4)=7$

5. False. $(f\circ g)(x)=f(g(x))=f\left(\dfrac{1}{2x}\right)=\dfrac{1}{x}$

6. False. For example, if $f(x)=2x$ and $g(x)=\dfrac{1}{2}x$, then

$(f\circ g)(x)=f\left(\dfrac{1}{2}x\right)=\dfrac{1}{2}\cdot 2x=x$ and

$(g\circ f)(x)=g(f(x))=g(2x)=\dfrac{1}{2}\cdot 2x=x$.

7.a. $(f+g)(-1)=f(-1)+g(-1)$
$=2(-1)+-(-1)=-2+1=-1$

b. $(f-g)(0)=f(0)-g(0)=2(0)-(-0)=0$

c. $(f\cdot g)(2)=f(2)\cdot g(2)=2(2)\cdot(-2)=-8$

d. $\left(\dfrac{f}{g}\right)(1)=\dfrac{f(1)}{g(1)}=\dfrac{2(1)}{-1}=-2$

8.a. $(f+g)(-1) = f(-1) + g(-1)$
$= \left(1-(-1)^2\right) + \left(-1+1\right) = 0$

b. $(f-g)(0) = f(0) - g(0)$
$= (1-0^2) - (0+1) = 0$

c. $(f\cdot g)(2) = f(2)\cdot g(2)$
$= (1-2^2)\cdot(2+1) = -9$

d. $\left(\dfrac{f}{g}\right)(1) = \dfrac{f(1)}{g(1)} = \dfrac{1-1^2}{1+1} = 0$

9.a. $(f+g)(-1) = f(-1) + g(-1)$
$= \dfrac{1}{\sqrt{-1+2}} + \left(2(-1)+1\right) = 0$

b. $(f-g)(0) = f(0) - g(0)$
$= \dfrac{1}{\sqrt{0+2}} - \left(2(0)+1\right) = \dfrac{\sqrt{2}}{2} - 1$

c. $(f\cdot g)(2) = f(2)\cdot g(2)$
$= \dfrac{1}{\sqrt{2+2}}\cdot\left(2(2)+1\right) = \dfrac{5}{2}$

d. $\left(\dfrac{f}{g}\right)(1) = \dfrac{f(1)}{g(1)} = \dfrac{\frac{1}{\sqrt{1+2}}}{2(1)+1} = \dfrac{1}{3\sqrt{3}} = \dfrac{\sqrt{3}}{9}$

10.a. $(f+g)(-1) = f(-1) + g(-1)$
$= \dfrac{-1}{(-1)^2 - 6(-1) + 8} + \left(3-(-1)\right)$
$= -\dfrac{1}{15} + 4 = \dfrac{59}{15}$

b. $(f-g)(0) = f(0) - g(0)$
$= \dfrac{0}{0^2 - 6(0) + 8} - (3-0) = -3$

c. $(f\cdot g)(2) = f(2)\cdot g(2)$
$= \dfrac{2}{2^2 - 6(2) + 8}\cdot(3-2) = \dfrac{2}{0}\cdot 1 \Rightarrow$
the product does not exist.

d. $\left(\dfrac{f}{g}\right)(1) = \dfrac{f(1)}{g(1)} = \dfrac{\frac{1}{1^2 - 6(1) + 8}}{3-1} = \dfrac{\frac{1}{3}}{2} = \dfrac{1}{6}$

11.a. $f + g = x^2 + x - 3$; domain: $(-\infty, \infty)$

b. $f - g = x - 3 - x^2 = -x^2 + x - 3$;
domain: $(-\infty, \infty)$

c. $f\cdot g = (x-3)x^2 = x^3 - 3x^2$;
domain: $(-\infty, \infty)$

d. $\dfrac{f}{g} = \dfrac{x-3}{x^2}$; domain: $(-\infty, 0)\cup(0, \infty)$

e. $\dfrac{g}{f} = \dfrac{x^2}{x-3}$; domain: $(-\infty, 3)\cup(3, \infty)$

12.a. $f + g = x^2 + 2x - 1$; domain: $(-\infty, \infty)$

b. $f - g = 2x - 1 - x^2 = -x^2 + 2x - 1$;
domain: $(-\infty, \infty)$

c. $f\cdot g = (2x-1)x^2 = 2x^3 - x^2$;
domain: $(-\infty, \infty)$

d. $\dfrac{f}{g} = \dfrac{2x-1}{x^2}$; domain: $(-\infty, 0)\cup(0, \infty)$

e. $\dfrac{g}{f} = \dfrac{x^2}{2x-1}$; domain: $\left(-\infty, \dfrac{1}{2}\right)\cup\left(\dfrac{1}{2}, \infty\right)$

13.a. $f + g = \left(x^3 - 1\right) + \left(2x^2 + 5\right) = x^3 + 2x^2 + 4$;
domain: $(-\infty, \infty)$

b. $f - g = \left(x^3 - 1\right) - \left(2x^2 + 5\right) = x^3 - 2x^2 - 6$;
domain: $(-\infty, \infty)$

c. $f\cdot g = \left(x^3 - 1\right)\left(2x^2 + 5\right)$
$= 2x^5 + 5x^3 - 2x^2 - 5$;
domain: $(-\infty, \infty)$

d. $\dfrac{f}{g} = \dfrac{x^3 - 1}{2x^2 + 5}$; domain: $(-\infty, \infty)$

e. $\dfrac{g}{f} = \dfrac{2x^2 + 5}{x^3 - 1}$; domain: $(-\infty, 1)\cup(1, \infty)$

14.a. $f + g = \left(x^2 - 4\right) + \left(x^2 - 6x + 8\right)$
$= 2x^2 - 6x + 4$; domain: $(-\infty, \infty)$

b. $f - g = \left(x^2 - 4\right) - \left(x^2 - 6x + 8\right) = 6x - 12$;
domain: $(-\infty, \infty)$

c. $f\cdot g = \left(x^2 - 4\right)\left(x^2 - 6x + 8\right)$
$= x^4 - 6x^3 + 4x^2 + 24x - 32$;
domain: $(-\infty, \infty)$

d. $\frac{f}{g}=\frac{x^2-4}{x^2-6x+8}=\frac{(x+2)(x-2)}{(x-4)(x-2)}=\frac{x+2}{x-4}$
$g(x)=0$ if $x=2$ or $x=4$, so the domain is $(-\infty,2)\cup(2,4)\cup(4,\infty)$.

e. $\frac{g}{f}=\frac{x^2-6x+8}{x^2-4}=\frac{(x-4)(x-2)}{(x+2)(x-2)}=\frac{x-4}{x+2}$
$f(x)=0$ if $x=-2$ or $x=2$, so the domain is $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$.

15.a. $f+g=2x+\sqrt{x}-1$; domain: $[0,\infty)$

b. $f-g=2x-\sqrt{x}-1$; domain: $[0,\infty)$

c. $f\cdot g=(2x-1)\sqrt{x}=2x\sqrt{x}-\sqrt{x}$;
domain: $[0,\infty)$

d. $\frac{f}{g}=\frac{2x-1}{\sqrt{x}}$; domain: $(0,\infty)$

e. $\frac{g}{f}=\frac{\sqrt{x}}{2x-1}$; the numerator is defined only for $x\ge 0$, while the denominator = 0 when $x=\frac{1}{2}$, so the domain is $\left[0,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$.

16.a. $f+g=\left(1-\frac{1}{x}\right)+\frac{1}{x}=1$
Neither *f* nor *g* is defined for $x=0$, so the domain is $(-\infty,0)\cup(0,\infty)$.

b. $f-g=\left(1-\frac{1}{x}\right)-\frac{1}{x}=1-\frac{2}{x}$;
domain: $(-\infty,0)\cup(0,\infty)$.

c. $f\cdot g=\left(1-\frac{1}{x}\right)\left(\frac{1}{x}\right)=\frac{1}{x}-\frac{1}{x^2}=\frac{x-1}{x^2}$;
domain: $(-\infty,0)\cup(0,\infty)$.

d. $\frac{f}{g}=\frac{1-\frac{1}{x}}{\frac{1}{x}}=\frac{\frac{x-1}{x}}{\frac{1}{x}}=x-1$
Neither *f* nor *g* is defined for $x=0$, so the domain is $(-\infty,0)\cup(0,\infty)$.

e. $\frac{g}{f}=\frac{\frac{1}{x}}{1-\frac{1}{x}}=\frac{\frac{1}{x}}{\frac{x-1}{x}}=\frac{1}{x-1}$
Neither *f* nor *g* is defined for $x=0$, and *g*/*f* is not defined for $x=1$, so the domain is $(-\infty,0)\cup(0,1)\cup(1,\infty)$.

17.a. $f+g=\frac{2}{x+1}+\frac{x}{x+1}=\frac{2+x}{x+1}$
Neither *f* nor *g* is defined for $x=-1$, so the domain is $(-\infty,-1)\cup(-1,\infty)$.

b. $f-g=\frac{2}{x+1}-\frac{x}{x+1}=\frac{2-x}{x+1}$.
domain: $(-\infty,-1)\cup(-1,\infty)$.

c. $f\cdot g=\left(\frac{2}{x+1}\right)\left(\frac{x}{x+1}\right)=\frac{2x}{(x+1)^2}$.
domain: $(-\infty,-1)\cup(-1,\infty)$.

d. $\frac{f}{g}=\frac{\frac{2}{x+1}}{\frac{x}{x+1}}=\frac{2}{x}$
Neither *f* nor *g* is defined for $x=-1$, and f/g is not defined for $x=0$, so the domain is $(-\infty,-1)\cup(-1,0)\cup(0,\infty)$.

e. $\frac{f}{g}=\frac{\frac{x}{x+1}}{\frac{2}{x+1}}=\frac{x}{2}$
Neither *f* nor *g* is defined for $x=-1$, so the domain is $(-\infty,-1)\cup(-1,\infty)$.

18. $f(x)=\frac{5x-1}{x+1}$; $g(x)=\frac{4x+10}{x+1}$

a. $f+g=\frac{5x-1}{x+1}+\frac{4x+10}{x+1}=\frac{9x+9}{x+1}$
$=\frac{9(x+1)}{x+1}=9$
Neither *f* nor *g* is defined for $x=-1$, so the domain is $(-\infty,-1)\cup(-1,\infty)$.

b. $f-g=\frac{5x-1}{x+1}-\frac{4x+10}{x+1}=\frac{x-11}{x+1}$
Neither *f* nor *g* is defined for $x=-1$, so the domain is $(-\infty,-1)\cup(-1,\infty)$.

c. $f\cdot g=\dfrac{5x-1}{x+1}\cdot\dfrac{4x+10}{x+1}=\dfrac{20x^2+46x-10}{x^2+2x+1}$

Neither f nor g is defined for $x=-1$, so the domain is $(-\infty,-1)\cup(-1,\infty)$.

d. $\dfrac{f}{g}=\dfrac{\frac{5x-1}{x+1}}{\frac{4x+10}{x+1}}=\dfrac{5x-1}{4x+10}$

Neither f nor g is defined for $x=-1$ and f/g is not defined for $x=-5/2$, so the domain is $\left(-\infty,-\frac{5}{2}\right)\cup\left(-\frac{5}{2},-1\right)\cup(-1,\infty)$.

e. $\dfrac{g}{f}=\dfrac{\frac{4x+10}{x+1}}{\frac{5x-1}{x+1}}=\dfrac{4x+10}{5x-1}$

Neither f nor g is defined for $x=-1$ and g/f is not defined for $x=1/5$, so the domain is $(-\infty,-1)\cup\left(-1,\frac{1}{5}\right)\cup\left(\frac{1}{5},\infty\right)$.

19. $f(x)=\dfrac{x^2}{x+1};\ g(x)=\dfrac{2x}{x^2-1}$

a. $f+g=\dfrac{x^2}{x+1}+\dfrac{2x}{x^2-1}=\dfrac{x^2(x-1)}{x^2-1}+\dfrac{2x}{x^2-1}$
$=\dfrac{x^3-x^2+2x}{x^2-1}$

f is not defined for $x=-1$, g is not defined for $x=\pm1$, and $f+g$ is not defined for either -1 or 1, so the domain is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

b. $f-g=\dfrac{x^2}{x+1}-\dfrac{2x}{x^2-1}=\dfrac{x^2(x-1)}{x^2-1}-\dfrac{2x}{x^2-1}$
$=\dfrac{x^3-x^2-2x}{x^2-1}=\dfrac{x\left(x^2-x-2\right)}{x^2-1}$
$=\dfrac{x(x-2)(x+1)}{(x-1)(x+1)}=\dfrac{x^2-2x}{x-1}$

f is not defined for $x=-1$, g is not defined for $x=\pm1$, and $f-g$ is not defined for 1, so the domain is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

c. $f\cdot g=\dfrac{x^2}{x+1}\cdot\dfrac{2x}{x^2-1}=\dfrac{2x^3}{x^3+x^2-x-1}$

f is not defined for $x=-1$, g is not defined for $x=\pm1$, and fg is not defined for either -1 or 1, so the domain is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

d. $\dfrac{f}{g}=\dfrac{\frac{x^2}{x+1}}{\frac{2x}{x^2-1}}=\dfrac{x^2}{x+1}\cdot\dfrac{x^2-1}{2x}=\dfrac{x(x-1)}{2}$

f is not defined for $x=-1$, g is not defined for $x=\pm1$, and f/g is not defined for either -1 or 1, so the domain is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

e. $\dfrac{g}{f}=\dfrac{\frac{2x}{x^2-1}}{\frac{x^2}{x+1}}=\dfrac{2x}{x^2-1}\cdot\dfrac{x+1}{x^2}=\dfrac{2}{x(x-1)}$

Neither f nor g is defined for $x=-1$ and g/f is not defined for $x=0$ or $x=1$, so the domain is $(-\infty,-1)\cup(-1,0)\cup(0,1)\cup(1,\infty)$.

20. $f(x)=\dfrac{x-3}{x^2-25};\ g(x)=\dfrac{x-3}{x^2+9x+20}$

a. $f+g=\dfrac{x-3}{x^2-25}+\dfrac{x-3}{x^2+9x+20}$
$=\dfrac{x-3}{(x-5)(x+5)}+\dfrac{x-3}{(x+4)(x+5)}$
$=\dfrac{(x-3)(x+4)+(x-3)(x-5)}{(x-5)(x+5)(x+4)}$
$=\dfrac{x^2+x-12+x^2-8x+15}{x^3+4x^2-25x-100}$
$=\dfrac{2x^2-7x+3}{x^3+4x^2-25x-100}$

f is not defined for $x=-5$ and $x=5$, g is not defined for $x=-4$ and $x=-5$, and $f+g$ is not defined for -5, 5 or -4, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,5)\cup(5,\infty)$.

b. $f-g=\dfrac{x-3}{x^2-25}-\dfrac{x-3}{x^2+9x+20}$
$=\dfrac{x-3}{(x-5)(x+5)}-\dfrac{x-3}{(x+4)(x+5)}$
$=\dfrac{(x-3)(x+4)-(x-3)(x-5)}{(x-5)(x+5)(x+4)}$
$=\dfrac{x^2+x-12-(x^2-8x+15)}{x^3+4x^2-25x-100}$
$=\dfrac{9x-27}{x^3+4x^2-25x-100}$

f is not defined for $x=-5$ and $x=5$, g is not defined for $x=-4$ and $x=-5$, and $f+g$ is not defined for -5, 5, or -4, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,5)\cup(5,\infty)$.

c. $f\cdot g=\dfrac{x-3}{x^2-25}\cdot\dfrac{x-3}{x^2+9x+20}$
$=\dfrac{(x-3)^2}{(x^2-25)(x^2+9x+20)}$
$=\dfrac{x^2-6x+9}{x^4+9x^3-5x^2-225x-500}$

f is not defined for $x=-5$ and $x=5$, g is not defined for $x=-4$ and $x=-5$, and fg is not defined for -5, 5, or -4, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,5)\cup(5,\infty)$.

d. $\dfrac{f}{g}=\dfrac{\frac{x-3}{x^2-25}}{\frac{x-3}{x^2+9x+20}}$
$=\dfrac{x-3}{(x-5)(x+5)}\cdot\dfrac{(x+5)(x+4)}{x-3}=\dfrac{x+4}{x-5}$

f is not defined for $x=-5$ and $x=5$, g is not defined for $x=-4$ and $x=-5$, and f/g is not defined for $x=5$ or $x=3$, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,3)\cup(3,5)\cup(5,\infty)$.

e. $\dfrac{g}{f}=\dfrac{\frac{x-3}{x^2+9x+20}}{\frac{x-3}{x^2-25}}$
$=\dfrac{x-3}{(x+5)(x+4)}\cdot\dfrac{(x-5)(x+5)}{x-3}=\dfrac{x-5}{x+4}$

f is not defined for $x=-5$ and $x=5$, g is not defined for $x=-4$ and $x=-5$, and g/f is not defined for $x=-4$ or $x=3$, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,3)\cup(3,5)\cup(5,\infty)$.

21. $f(x)=\dfrac{2x}{x^2-16}$; $g(x)=\dfrac{2x-7}{x^2-7x+12}$

a. $f+g=\dfrac{2x}{x^2-16}+\dfrac{2x-7}{x^2-7x+12}$
$=\dfrac{2x}{(x-4)(x+4)}+\dfrac{2x-7}{(x-4)(x-3)}$
$=\dfrac{2x(x-3)+(2x-7)(x+4)}{(x-4)(x+4)(x-3)}$
$=\dfrac{2x^2-6x+2x^2+x-28}{x^3-3x^2-16x+48}$
$=\dfrac{4x^2-5x-28}{x^3-3x^2-16x+48}$

f is not defined for $x=-4$ and $x=4$, g is not defined for $x=3$ and $x=4$, and $f+g$ is not defined for -4, 4, or 3, so the domain is $(-\infty,-4)\cup(-4,3)\cup(3,4)\cup(4,\infty)$.

b. $f-g=\dfrac{2x}{x^2-16}-\dfrac{2x-7}{x^2-7x+12}$
$=\dfrac{2x}{(x-4)(x+4)}-\dfrac{2x-7}{(x-4)(x-3)}$
$=\dfrac{2x(x-3)-(2x-7)(x+4)}{(x-4)(x+4)(x-3)}$
$=\dfrac{2x^2-6x-(2x^2+x-28)}{(x-4)(x+4)(x-3)}$
$=\dfrac{-7x+28}{(x-4)(x+4)(x-3)}$
$=\dfrac{-7(x-4)}{(x-4)(x+4)(x-3)}$
$=-\dfrac{7}{(x+4)(x-3)}$

f is not defined for $x=-4$ and $x=4$, g is not defined for $x=3$ and $x=4$, and $f+g$ is not defined for -4, 4, or 3, so the domain is $(-\infty,-4)\cup(-4,3)\cup(3,4)\cup(4,\infty)$.

c. $f\cdot g=\dfrac{2x}{x^2-16}\cdot\dfrac{2x-7}{x^2-7x+12}$
$=\dfrac{4x^2-14x}{x^4-7x^3-4x^2+112x-192}$

f is not defined for $x=-4$ and $x=4$, g is not defined for $x=3$ and $x=4$, and fg is not defined for -4, 4 or 3, so the domain is $(-\infty,-4)\cup(-4,3)\cup(3,4)\cup(4,\infty)$.

d. $\frac{f}{g} = \frac{\frac{2x}{x^2-16}}{\frac{2x-7}{x^2-7x+12}}$

$= \frac{2x}{(x-4)(x+4)} \cdot \frac{(x-4)(x-3)}{2x-7}$

$= \frac{2x^2-6x}{2x^2+x-28}$

f is not defined for $x = -4$ and $x = 4$, g is not defined for $x = 3$ and $x = 4$, and f/g is not defined for $x = -4$ and $x = 7/2$, so the domain is

$(-\infty, -4) \cup (-4, 3) \cup \left(3, \frac{7}{2}\right) \cup \left(\frac{7}{2}, 4\right)$
$\cup (4, \infty)$.

e. $\frac{g}{f} = \frac{\frac{2x-7}{x^2-7x+12}}{\frac{2x}{x^2-16}}$

$= \frac{2x-7}{(x-4)(x-3)} \cdot \frac{(x-4)(x+4)}{2x}$

$= \frac{(2x-7)(x+4)}{2x(x-3)} = \frac{2x^2+x-28}{2x^2-6x}$

f is not defined for $x = -4$ and $x = 4$, g is not defined for $x = 3$ and $x = 4$, and g/f is not defined for $x = 0$ and $x = 3$ so the domain is $(-\infty, -4) \cup (-4, 0) \cup (0, 3) \cup (3, 4)$
$\cup (4, \infty)$.

22.a. $f + g = \frac{x^2+3x+2}{x^3+4x} + \frac{2x^3+8x}{x^2+x-2}$

$= \frac{(x+2)(x+1)}{x(x^2+4)} + \frac{2x(x^2+4)}{(x-1)(x+2)}$

$= \frac{2x^6+17x^4+4x^3+35x^2-4x-4}{x(x^2+4)(x-1)(x+2)}$

$= \frac{2x^6+17x^4+4x^3+35x^2-4x-4}{x^5+x^4+2x^3+4x^2-8x}$

f is not defined for $x = 0$ and g is not defined for $x = -2$ and $x = 1$, so the domain is $(-\infty, -2) \cup (-2, 0) \cup (0, 1) \cup (1, \infty)$.

b. $f - g = \frac{x^2+3x+2}{x^3+4x} - \frac{2x^3+8x}{x^2+x-2}$

$= \frac{(x+2)(x+1)}{x(x^2+4)} - \frac{2x(x^2+4)}{(x-1)(x+2)}$

$= \frac{-2x^6-15x^4+4x^3-29x^2-4x-4}{x(x^2+4)(x-1)(x+2)}$

$= -\frac{2x^6+15x^4-4x^3+29x^2+4x+4}{x^5+x^4+2x^3+4x^2-8x}$

f is not defined for $x = 0$ and g is not defined for $x = -2$ and $x = 1$, so the domain is $(-\infty, -2) \cup (-2, 0) \cup (0, 1) \cup (1, \infty)$.

c. $f \cdot g = \left(\frac{x^2+3x+2}{x^3+4x}\right)\left(\frac{2x^3+8x}{x^2+x-2}\right)$

$= \left(\frac{(x+2)(x+1)}{x(x^2+4)}\right)\left(\frac{2x(x^2+4)}{(x-1)(x+2)}\right)$

$= \frac{2x+2}{x-1}$

f is not defined for $x = 0$, g is not defined for $x = -2$ and $x = 1$, and fg is not defined for $x = 1$ so the domain is $(-\infty, -2) \cup (-2, 0) \cup (0, 1) \cup (1, \infty)$.

d. $\frac{f}{g} = \frac{\frac{x^2+3x+2}{x^3+4x}}{\frac{2x^3+8x}{x^2+x-2}} = \frac{\frac{(x+2)(x+1)}{x(x^2+4)}}{\frac{2x(x^2+4)}{(x-1)(x+2)}}$

$= \frac{(x+2)^2(x-1)(x+1)}{2x^2(x^2+4)^2}$

$= \frac{x^4+4x^3+3x^2-4x-4}{2x^6+16x^4+32x^2}$

f is not defined for $x = 0$, g is not defined for $x = -2$ and $x = 1$, and f/g is not defined for $x = 0$, so the domain is $(-\infty, -2) \cup (-2, 0) \cup (0, 1) \cup (1, \infty)$.

e. $\frac{g}{f} = \frac{\frac{2x^3+8x}{x^2+x-2}}{\frac{x^2+3x+2}{x^3+4x}} = \frac{\frac{2x\left(x^2+4\right)}{(x-1)(x+2)}}{\frac{(x+2)(x+1)}{x\left(x^2+4\right)}}$

$= \frac{2x^2\left(x^2+4\right)^2}{(x+2)^2(x-1)(x+1)}$

$= \frac{2x^6+16x^4+32x^2}{x^4+4x^3+3x^2-4x-4}$

f is not defined for $x = 0$, g is not defined for $x = -2$ and $x = 1$, and g/f is not defined for $x = -2$, $x = -1$, and $x = 1$, so the domain is $(-\infty, -2)\cup(-2, -1)\cup(-1, 0)\cup(0, 1)\cup(1, \infty)$.

23. $f(x) = \sqrt{x-1}$, $g(x) = \sqrt{5-x}$

$f(x)$ is defined if $x - 1 \ge 0 \Rightarrow x \ge 1$. Thus, $D_1 = [1, \infty)$. $g(x)$ is defined if $5 - x \ge 0 \Rightarrow -x \ge -5 \Rightarrow x \le 5$. Thus, $D_2 = (-\infty, 5]$.

a. The domain of fg is $D_1 \cap D_2 = [1, 5]$.

b. f/g is not defined for $\sqrt{5-x} = 0 \Rightarrow x = 5$, so, the domain of f/g is $[1, 5)$.

24. $f(x) = \sqrt{x-2}$, $g(x) = \sqrt{x+2}$

$f(x)$ is defined if $x - 2 \ge 0 \Rightarrow x \ge 2$. Thus, $D_1 = [2, \infty)$. $g(x)$ is defined if $x + 2 \ge 0 \Rightarrow x \ge -2$. Thus, $D_2 = [-2, \infty)$.

a. The domain of fg is $D_1 \cap D_2 = [2, \infty)$.

b. f/g is not defined for $\sqrt{x+2} = 0 \Rightarrow x = -2$. Since $-2 \notin D_1 \cap D_2$, the domain of f/g is $D_1 \cap D_2 = [2, \infty)$.

25. $f(x) = \sqrt{x+2}$, $g(x) = \sqrt{9-x^2}$

$f(x)$ is defined if $x + 2 \ge 0 \Rightarrow x \ge -2$. Thus, $D_1 = [-2, \infty)$. $g(x)$ is defined if $9 - x^2 \ge 0 \Rightarrow 9 \ge x^2 \Rightarrow -3 \le x \le 3$. Thus, $D_2 = [-3, 3]$.

a. The domain of fg is $D_1 \cap D_2 = [-2, 3]$.

b. f/g is not defined for $\sqrt{9-x^2} = 0 \Rightarrow x = \pm 3$. Thus, the domain of f/g is $[-2, 3)$.

26. $f(x) = \sqrt{x^2-4}$, $g(x) = \sqrt{25-x^2}$

$f(x)$ is defined if $x^2 - 4 \ge 0 \Rightarrow x^2 \ge 4 \Rightarrow (-\infty, -2]\cup[2, \infty)$.

Thus, $D_1 = (-\infty, -2]\cup[2, \infty)$. $g(x)$ is defined if $25 - x^2 \ge 0 \Rightarrow 25 \ge x^2 \Rightarrow -5 \le x \le 5$. Thus, $D_2 = [-5, 5]$.

a. The domain of fg is $D_1 \cap D_2 = [-5, -2]\cup[2, 5]$.

b. f/g is not defined for $\sqrt{25-x^2} = 0 \Rightarrow x = \pm 5$. Thus, the domain of f/g is $(-5, -2]\cup[2, 5)$.

27. $(g \circ f)(x) = 2\left(x^2-1\right)+3 = 2x^2+1;$

$(g \circ f)(2) = 2\left(2^2-1\right)+3 = 9;$

$(g \circ f)(-3) = 2\left((-3)^2-1\right)+3 = 19$

28. $(g \circ f)(x) = 3|x+1|^2 - 1 = 3\left(x^2+2x+1\right)-1$

$= 3x^2+6x+2$

$(g \circ f)(2) = 3|2+1|^2 - 1 = 26$

$(g \circ f)(-3) = 3|(-3)+1|^2 - 1 = 11$

29. $(f \circ g)(2) = 2\left(2(2^2)-3\right)+1 = 11$

30. $(g \circ f)(2) = 2(2(2)+1)^2 - 3 = 47$

31. $(f \circ g)(-3) = 2\left(2(-3)^2-3\right)+1 = 31$

32. $(g \circ f)(-5) = 2(2(-5)+1)^2 - 3 = 159$

33. $(g \circ f)(a) = 2(2a+1)^2 - 3$

$= 2\left(4a^2+4a+1\right)-3$

$= 8a^2+8a-1$

34. $(g \circ f)(-a) = 2(2(-a)+1)^2 - 3$

$= 2\left(4a^2-4a+1\right)-3$

$= 8a^2-8a-1$

35. $(f \circ f)(1) = 2(2(1)+1)+1 = 7$

36. $(g \circ g)(-1) = 2\left(2(-1)^2-3\right)^2 - 3 = -1$

37. $(f \circ g)(x) = \dfrac{2}{\frac{1}{x}+1} = \dfrac{2}{\frac{x+1}{x}} = \dfrac{2x}{x+1}$

The domain of g is $(-\infty, 0) \cup (0, \infty)$. Since -1 is not in the domain of f, we must exclude those values of x that make $g(x) = -1$.

$\dfrac{1}{x} = -1 \Rightarrow x = -1$

Thus, the domain of $f \circ g$ is $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$.

38. $(f \circ g)(x) = \dfrac{1}{\frac{2}{x+3}-1} = \dfrac{1}{\frac{2-(x+3)}{x+3}}$
$= \dfrac{x+3}{-x-1} = -\dfrac{x+3}{x+1}.$

The domain of g is $(-\infty, -3) \cup (0, -3)$. Since 1 is not in the domain of f, we must exclude those values of x that make $g(x) = 1$.

$\dfrac{2}{x+3} = 1 \Rightarrow 2 = x+3 \Rightarrow x = -1$

Thus, the domain of $f \circ g$ is $(-\infty, -3) \cup (-3, -1) \cup (-1, \infty)$.

39. $(f \circ g)(x) = \sqrt{(2-3x)-3} = \sqrt{-1-3x}.$

The domain of g is $(-\infty, \infty)$. Since f is not defined for $(-\infty, 3)$, we must exclude those values of x that make $g(x) > 3$:

$2 - 3x < 3 \Rightarrow -3x < 1 \Rightarrow x > -\dfrac{1}{3}$

Thus, the domain of $f \circ g$ is $\left(-\infty, -\dfrac{1}{3}\right]$.

40. $(f \circ g)(x) = \dfrac{2+5x}{(2+5x)-1} = \dfrac{2+5x}{1+5x}$

The domain of g is $(-\infty, \infty)$. Since f is not defined for $x = 1$ we must exclude those values of x that make $g(x) = 1$.

$2 + 5x = 1 \Rightarrow 5x = -1 \Rightarrow x = -\dfrac{1}{5}$

Thus, the domain of $f \circ g$ is $\left(-\infty, -\dfrac{1}{5}\right) \cup \left(-\dfrac{1}{5}, \infty\right)$.

41. $(f \circ g)(x) = \left|x^2 - 1\right|$; domain: $(-\infty, \infty)$

42. $(f \circ g)(x) = 3|x-1| - 2$; domain: $(-\infty, \infty)$

43. $f(x) = 2x - 3,\ g(x) = x + 4$

Domain of $f = (-\infty, \infty)$

Domain of $g = (-\infty, \infty)$

a. $(f \circ g)(x) = 2(x+4) - 3 = 2x + 5$;
Domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = (2x-3) + 4 = 2x + 1$;
Domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 2(2x-3) - 3 = 4x - 9$;
Domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = (x+4) + 4 = x + 8$;
Domain: $(-\infty, \infty)$

44. $f(x) = x - 3,\ g(x) = 3x - 5$

Domain of $f = (-\infty, \infty)$

Domain of $g = (-\infty, \infty)$

a. $(f \circ g)(x) = (3x-5) - 3 = 3x - 8$;
Domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 3(x-3) - 5 = 3x - 14$;
Domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = (x-3) - 3 = x - 6$;
Domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 3(3x-5) - 5 = 9x - 20$;
Domain: $(-\infty, \infty)$

45. $f(x) = 1 - 2x,\ g(x) = 1 + x^2$

Domain of $f = (-\infty, \infty)$

Domain of $g = (-\infty, \infty)$

a. $(f \circ g)(x) = 1 - 2(1 + x^2) = -2x^2 - 1$;
Domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 1 + (1-2x)^2 = 4x^2 - 4x + 2$;
Domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 1 - 2(1-2x) = 4x - 1$;
Domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 1 + (1 + x^2)^2 = x^4 + 2x^2 + 2$;
Domain: $(-\infty, \infty)$

46. $f(x) = 2x - 3,\ \ g(x) = 2x^2$

Domain of $f = (-\infty, \infty)$

Domain of $g = (-\infty, \infty)$

a. $(f \circ g)(x) = 2(2x^2) - 3 = 4x^2 - 3;$
Domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 2(2x - 3)^2 = 8x^2 - 24x + 18;$
Domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 2(2x - 3) - 3 = 4x - 9;$
Domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 2\left(2x^2\right)^2 = 8x^4;$
Domain: $(-\infty, \infty)$

47. $f(x) = x^2,\ \ g(x) = \sqrt{x}$

Domain of $f = (-\infty, \infty)$

Domain of $g = [0, \infty)$

7.a. $(f \circ g)(x) = \left(\sqrt{x}\right)^2 = x;$ domain: $[0, \infty)$

b. $(g \circ f)(x) = \sqrt{x^2} = |x|;$ domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = \left(x^2\right)^2 = x^4;$ domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = \sqrt{\sqrt{x}} = \sqrt[4]{x};$ domain: $[0, \infty)$

48. $f(x) = x^2 + 2x,\ \ g(x) = \sqrt{x + 2}$

Domain of $f = (-\infty, \infty)$

Domain of $g = [-2, \infty)$

a. $(f \circ g)(x) = \left(\sqrt{x + 2}\right)^2 + 2\sqrt{x + 2}$
$= x + 2 + 2\sqrt{x + 2};$ domain: $[-2, \infty)$

b. $(g \circ f)(x) = \sqrt{x^2 + 2x + 2};$ domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = (x^2 + 2x)^2 + 2(x^2 + 2x)$
$= x^4 + 4x^3 + 4x^2 + 2x^2 + 4x$
$= x^4 + 4x^3 + 6x^2 + 4x;$
Domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = \sqrt{\sqrt{x + 2} + 2};$ domain: $[-2, \infty)$

49. $f(x) = \dfrac{1}{2x - 1},\ \ g(x) = \dfrac{1}{x^2}$

Domain of $f = \left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right).$

Domain of $g = (-\infty, 0) \cup (0, \infty).$

a. $(f \circ g)(x) = \dfrac{1}{2\left(\frac{1}{x^2}\right) - 1} = \dfrac{1}{\frac{2 - x^2}{x^2}}$
$= \dfrac{x^2}{2 - x^2} = -\dfrac{x^2}{x^2 - 2}.$

The domain of g is $(-\infty, 0) \cup (0, \infty).$ Since $\frac{1}{2}$ is not in the domain of f, we must find those values of x that make $g(x) = \frac{1}{2}.$

$\dfrac{1}{x^2} = \dfrac{1}{2} \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

Thus, the domain of $f \circ g$ is
$\left(-\infty, -\sqrt{2}\right) \cup \left(-\sqrt{2}, 0\right) \cup \left(0, \sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right).$

b. $(g \circ f) = \dfrac{1}{\left(\frac{1}{2x - 1}\right)^2} = (2x - 1)^2$

The domain of f is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right).$ Since 0 is not in the domain of g, we must find those values of x that make $f(x) = 0.$ However, there are no such values, so the domain of $g \circ f$ is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right).$

c. $(f \circ f)(x) = \dfrac{1}{2\left(\frac{1}{2x - 1}\right) - 1} = \dfrac{1}{\frac{2 - 2x + 1}{2x - 1}}$
$= \dfrac{2x - 1}{3 - 2x} = -\dfrac{2x - 1}{2x - 3}$

The domain of f is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right).$

$-\dfrac{2x - 1}{2x - 3}$ is defined for $\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right),$

so the domain of $f \circ f$ is
$\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right).$

d. $(g \circ g)(x) = \dfrac{1}{\left(\dfrac{1}{x^2}\right)^2} = x^4.$

The domain of g is $(-\infty, 0) \cup (0, \infty)$, while $g \circ g = x^4$ is defined for all real numbers. Thus, the domain of $g \circ g$ is $(-\infty, 0) \cup (0, \infty)$.

50. $f(x) = x - 1,\ g(x) = \dfrac{x}{x+1}$

Domain of $f = (-\infty, \infty)$.

Domain of $g = (-\infty, -1) \cup (-1, \infty)$.

a. $(f \circ g)(x) = \dfrac{x}{x+1} - 1 = \dfrac{x - (x+1)}{x+1} = -\dfrac{1}{x+1}$

The domain of g is $(-\infty, -1) \cup (-1, \infty)$. Since f is defined for all real numbers, there are no values that must be excluded. Thus, the domain of $f \circ g$ is $(-\infty, -1) \cup (1, \infty)$.

b. $(g \circ f)(x) = \dfrac{x-1}{(x-1)+1} = \dfrac{x-1}{x}$

The domain of f is all real numbers. Since g is not defined for $x = -1$, we must exclude those values of x that make $f(x) = -1$.

$x - 1 = -1 \Rightarrow x = 0$

Thus, the domain of $g \circ f$ is $(-\infty, 0) \cup (0, \infty)$.

c. $(f \circ f)(x) = (x-1) - 1 = x - 2;$ domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = \dfrac{\dfrac{x}{x+1}}{\dfrac{x}{x+1} + 1} = \dfrac{\dfrac{x}{x+1}}{\dfrac{x + x + 1}{x+1}} = \dfrac{x}{2x+1}$

The domain of g is $(-\infty, -1) \cup (-1, \infty)$, while $\dfrac{x}{2x+1}$ is defined for $\left(-\infty, -\dfrac{1}{2}\right) \cup \left(-\dfrac{1}{2}, \infty\right)$.

The domain of $g \circ g$ is $(-\infty, -1) \cup \left(-1, -\dfrac{1}{2}\right) \cup \left(-\dfrac{1}{2}, \infty\right)$.

51. $f(x) = \sqrt{x-1},\ g(x) = \sqrt{4-x}$

Domain of $f = [1, \infty)$.

Domain of $g = (-\infty, 4]$.

a. $(f \circ g)(x) = \sqrt{\sqrt{4-x} - 1};$ domain: $(-\infty, 3]$

b. $(g \circ f)(x) = \sqrt{4 - \sqrt{x-1}};$ domain: $[1, 17]$

c. $(f \circ f)(x) = \sqrt{\sqrt{x-1} - 1};$ domain: $[2, \infty)$

d. $(g \circ g)(x) = \sqrt{4 - \sqrt{4-x}};$ domain: $[-12, 4]$

52. $f(x) = x^2 - 4,\ g(x) = \sqrt{4 - x^2}$

Domain of $f = (-\infty, \infty)$.

Domain of $g = [-2, 2]$.

a. $(f \circ g)(x) = \left(\sqrt{4 - x^2}\right)^2 - 4 = -x^2$

Domain: $[-2, 2]$

b. $(g \circ f)(x) = \sqrt{4 - \left(x^2 - 4\right)^2}$

Domain: $\left[-\sqrt{6}, -\sqrt{2}\right] \cup \left[\sqrt{2}, \sqrt{6}\right]$

c.
$$\begin{aligned}(f \circ f)(x) &= \left(x^2 - 4\right)^2 - 4 \\ &= \left(x^4 - 8x^2 + 16\right) - 4 \\ &= x^4 - 8x^2 + 14\end{aligned}$$

Domain: $(-\infty, \infty)$

d.
$$\begin{aligned}(g \circ g)(x) &= \sqrt{4 - \left(\sqrt{4 - x^2}\right)^2} \\ &= \sqrt{4 - \left(4 - x^2\right)} \\ &= \sqrt{4 - 4 + x^2} \\ &= \sqrt{x^2} = |x|\end{aligned}$$

Domain: $[-2, 2]$

53. $f(x) = \sqrt{x^2 - 1},\ g(x) = \sqrt{4 - x^2}$

Domain of $f = (-\infty, -1] \cup [1, \infty)$.

Domain of $g = [-2, 2]$

a.
$$\begin{aligned}(f \circ g)(x) &= \sqrt{\left(\sqrt{4 - x^2}\right)^2 - 1} \\ &= \sqrt{4 - x^2 - 1} = \sqrt{3 - x^2}\end{aligned}$$

Domain: $\left[-\sqrt{3}, \sqrt{3}\right]$

b.
$$\begin{aligned}(g \circ f)(x) &= \sqrt{4 - \left(\sqrt{x^2 - 1}\right)^2} \\ &= \sqrt{4 - x^2 + 1} = \sqrt{5 - x^2}\end{aligned}$$

Domain: $\left[-\sqrt{5}, -1\right] \cup \left[1, \sqrt{5}\right]$

c. $(f \circ f)(x) = \sqrt{\left(\sqrt{x^2-1}\right)^2 - 1} = \sqrt{x^2-2}$

Domain: $f = \left(-\infty, -\sqrt{2}\right] \cup \left[\sqrt{2}, \infty\right)$.

d. $(g \circ g)(x) = \sqrt{4 - \left(\sqrt{4-x^2}\right)^2}$
$= \sqrt{4-4+x^2} = \sqrt{x^2} = |x|$

Domain: $[-2, 2]$

54. $f(x) = \sqrt{x^2-9},\ g(x) = \sqrt{9-x^2}$

Domain of $f = (-\infty, -3] \cup [3, \infty)$.

Domain of $g = [-3, 3]$

a. $(f \circ g)(x) = \sqrt{\left(\sqrt{9-x^2}\right)^2 - 9}$
$= \sqrt{9-x^2-9} = \sqrt{-x^2}$

Domain: $\{0\}$

b. $(g \circ f)(x) = \sqrt{9 - \left(\sqrt{x^2-9}\right)^2}$
$= \sqrt{9-x^2+9} = \sqrt{18-x^2}$

Domain: $\left[-3\sqrt{2}, -3\right] \cup \left[3, 3\sqrt{2}\right]$

c. $(f \circ f)(x) = \sqrt{\left(\sqrt{x^2-9}\right)^2 - 9} = \sqrt{x^2-18}$

Domain: $f = \left(-\infty, -3\sqrt{2}\right] \cup \left[3\sqrt{2}, \infty\right)$.

d. $(g \circ g)(x) = \sqrt{9 - \left(\sqrt{9-x^2}\right)^2}$
$= \sqrt{9-9+x^2} = \sqrt{x^2} = |x|$

Domain: $[-3, 3]$

55. $f(x) = 1 + \frac{1}{x},\ g(x) = \frac{1+x}{1-x}$

Domain of $f = (-\infty, 0) \cup (0, \infty)$.

Domain of $g = (-\infty, 1) \cup (1, \infty)$.

a. $(f \circ g)(x) = 1 + \frac{1}{\frac{1+x}{1-x}} = 1 + \frac{1-x}{1+x}$
$= \frac{1+x+1-x}{1+x} = \frac{2}{1+x}$.

The domain of g is $(-\infty,1) \cup (1,\infty)$. Since 0 is not in the domain of f, we must find those values of x that make $g(x) = 0$.

$\frac{1+x}{1-x} = 0 \Rightarrow 1+x = 0 \Rightarrow x = -1$

Thus, the domain of $f \circ g$ is $(-\infty,-1) \cup (-1,1) \cup (1,\infty)$.

b. $(g \circ f)(x) = \dfrac{1 + \left(1 + \frac{1}{x}\right)}{1 - \left(1 + \frac{1}{x}\right)} = \dfrac{2 + \frac{1}{x}}{-\frac{1}{x}} \cdot \dfrac{x}{x}$
$= \frac{2x+1}{-1} = -2x - 1$

The domain of f is $(-\infty,0) \cup (0,\infty)$. Since 1 is not in the domain of g, we must find those values of x that make $f(x) = 1$.

$1 + \frac{1}{x} = 1 \Rightarrow \frac{1}{x} = 0$

There are no values of x that make this true, so there are no additional values to be excluded from the domain of $g \circ f$. Thus, the domain of $g \circ f$ is $(-\infty,0) \cup (0,\infty)$.

c. $(f \circ f)(x) = 1 + \dfrac{1}{1 + \frac{1}{x}} = 1 + \dfrac{1}{\frac{x+1}{x}} = 1 + \dfrac{x}{x+1}$
$= \frac{2x+1}{x+1}$

The domain of f is $(-\infty,0) \cup (0,\infty)$.

$\frac{2x+1}{x+1}$ is defined for $(-\infty,-1) \cup (-1,\infty)$, so the domain of $f \circ f$ is $(-\infty,-1) \cup (-1,0) \cup (0,\infty)$.

d. $(g \circ g)(x) = \dfrac{1 + \frac{1+x}{1-x}}{1 - \frac{1+x}{1-x}} = \dfrac{\frac{2}{1-x}}{\frac{-2x}{1-x}} = -\dfrac{1}{x}$.

The domain of g is $(-\infty,1) \cup (1,\infty)$, while $-\frac{1}{x}$ is defined for $(-\infty,0) \cup (0,\infty)$.

The domain of $g \circ g$ is $(-\infty,0) \cup (0,1) \cup (1,\infty)$.

56. $f(x) = \sqrt[3]{x+1},\ g(x) = x^3 + 1$

Domain of $f = (-\infty, \infty)$.

Domain of $g = (-\infty, \infty)$.

a. $(f \circ g)(x) = \sqrt[3]{(x^3+1)+1} = \sqrt[3]{x^3+2}$;
domain: $(-\infty,\infty)$

b. $(g \circ f)(x) = \left(\sqrt[3]{x+1}\right)^3 + 1 = x + 2$;
domain: $(-\infty,\infty)$

c. $(f \circ f)(x) = \sqrt[3]{\sqrt[3]{x+1}+1}$; domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = (x^3+1)^3+1$; domain: $(-\infty, \infty)$

In exercises 57–66, sample answers are given. Other answers are possible.

57. $H(x) = \sqrt{x+2} \Rightarrow f(x) = \sqrt{x}, g(x) = x+2$

58. $H(x) = |3x+2| \Rightarrow f(x) = |x|, g(x) = 3x+2$

59. $H(x) = \left(x^2-3\right)^{10} \Rightarrow f(x) = x^{10}, g(x) = x^2-3$

60. $H(x) = \sqrt{3x^2}+5 \Rightarrow f(x) = \sqrt{x}+5, g(x) = 3x^2$

61. $H(x) = \dfrac{1}{3x-5} \Rightarrow f(x) = \dfrac{1}{x}, g(x) = 3x-5$

62. $H(x) = \dfrac{5}{2x+3} \Rightarrow f(x) = \dfrac{5}{x}, g(x) = 2x+3$

63. $H(x) = \sqrt[3]{x^2-7} \Rightarrow f(x) = \sqrt[3]{x}, g(x) = x^2-7$

64. $H(x) = \sqrt[4]{x^2+x+1} \Rightarrow f(x) = \sqrt[4]{x}$,
$g(x) = x^2+x+1$

65. $H(x) = \dfrac{1}{|x^3-1|} \Rightarrow f(x) = \dfrac{1}{|x|}, g(x) = x^3-1$

66. $H(x) = \sqrt[3]{1+\sqrt{x}} \Rightarrow f(x) = \sqrt[3]{x}, g(x) = 1+\sqrt{x}$

2.8 Applying the Concepts

67.a. $f(x)$ is the cost function.

b. $g(x)$ is the revenue function.

c. $h(x)$ is the selling price of x shirts including sales tax.

d. $P(x)$ is the profit function.

68.a.
$$\begin{aligned} C(p) &= C(5000-5p) \\ &= 4(5000-5p)+12,000 \\ &= 20,000-20p+12,000 \\ &= 32,000-20p \end{aligned}$$

b. $R(p) = px = p(5000-5p) = 5000p-5p^2$

c.
$$\begin{aligned} P(p) &= R(p)-C(p) \\ &= 5000p-5p^2-(32,000-20p) \\ &= -5p^2+5020p-32,000 \end{aligned}$$

69.a.
$$\begin{aligned} P(x) &= R(x)-C(x) = 25x-(350+5x) \\ &= 20x-350 \end{aligned}$$

b. $P(20) = 20(20)-350 = 50$. This represents the profit when 20 radios are sold.

c. $P(x) = 20x-350; 500 = 20x-350 \Rightarrow x = 43$

d. $C = 350+5x \Rightarrow x = \dfrac{C-350}{5} = x(C)$.

$(R \circ x)(C) = 25\left(\dfrac{C-350}{5}\right) = 5C-1750.$

This function represents the revenue in terms of the cost C.

70.a. $g(x) = 0.04x$

b. $h(x)$ is the after tax selling price of merchandise worth x dollars.

c. $f(x) = 0.02h(x)+3$

d. $T(x)$ represents the total price of merchandise worth x dollars, including the shipping and handling fee.

71.a. $f(x) = 0.7x$

b. $g(x) = x-5$

c. $(g \circ f)(x) = 0.7x-5$

d. $(f \circ g)(x) = 0.7(x-5)$

e.
$$\begin{aligned} (f \circ g)-(g \circ f) &= 0.7(x-5)-(0.7x-5) \\ &= 0.7x-3.5-0.7x+5 \\ &= \$1.50 \end{aligned}$$

72.a. $f(x) = 0.8x$ **b.** $g(x) = 0.9x$

c. $(g \circ f)(x) = 0.9(0.8x) = 0.72x$

d. $(f \circ g)(x) = 0.8(0.9x) = 0.72x$

e. They are the same.

73.a. $f(x) = 1.1x; g(x) = x+8$

b. $(f \circ g)(x) = 1.1(x+8) = 1.1x+8.8$

. This represents a final test score computed by first adding 8 points to the original score and then increasing the total by 10%.

c. $(g \circ f)(x) = 1.1x+8$

This represents a final test score computed by first increasing the original score by 10% and then adding 8 points.

d. $(f \circ g)(70) = 1.1(70 + 8) = 85.8;$
$(g \circ f)(70) = 1.1(70) + 8 = 85.0;$

e. $(f \circ g)(x) \neq (g \circ f)(x)$

f. **(i)** $(f \circ g)(x) = 1.1x + 8.8 \geq 90 \Rightarrow x \geq 73.82$

(ii) $(g \circ f)(x) = 1.1x + 8 \geq 90 \Rightarrow x \geq 74.55$

74.a. $f(x)$ is a function that models 3% of an amount *x*.

b. $g(x)$ represents the amount of money that qualifies for a 3% bonus.

c. Her bonus is represented by $(f \circ g)(x)$.

d. $200 + 0.03(17,500 - 8000) = \$485$

e. $521 = 200 + 0.03(x - 8000) \Rightarrow x = \$18,700$

75.a. $f(x) = \pi x^2$

b. $g(x) = \pi(x + 30)^2$

c. $g(x) - f(x)$ represents the area between the fountain and the fence.

d. The circumference of the fence is $2\pi(x + 30)$.
$10.5(2\pi(x + 30)) = 4200 \Rightarrow$
$\pi(x + 30) = 200 \Rightarrow$
$\pi x + 30\pi = 200 \Rightarrow \pi x = 200 - 30\pi.$

$$g(x) - f(x) = \pi(x + 30)^2 - \pi x^2 = \pi(x^2 + 60x + 900) - \pi x^2 = 60\pi x + 900\pi.$$

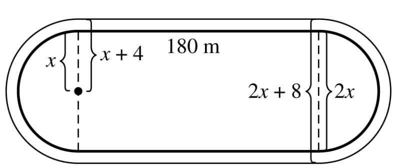
Now substitute $200 - 30\pi$ for πx to compute the estimate:
$$1.75[60(200 - 30\pi) + 900\pi] = 1.75(12,000 - 900\pi) \approx \$16,052.$$

76.a. $f(x) = 180(2x + 8) + \pi(x + 4)^2$
$= 1440 + 360x + \pi(x + 4)^2$

b. $g(x) = 2x(180) + \pi x^2 = 360x + \pi x^2$

c. $f(x) - g(x)$ represents the area of the track.

d.



(i) First find the radius of the inner track: $900 = 2\pi x + 360 \Rightarrow \dfrac{270}{\pi} = x$. Use this value to compute $f(x) - g(x)$.

$$f\left(\frac{270}{\pi}\right) - g\left(\frac{270}{\pi}\right)$$
$$= \left(1440 + 360\left(\frac{270}{\pi}\right) + \pi\left(\frac{270}{\pi} + 4\right)^2\right) - \left(360\left(\frac{270}{\pi}\right) + \pi\left(\frac{270}{\pi}\right)^2\right)$$
$$= 1440 + 360\left(\frac{270}{\pi}\right) + \frac{270^2}{\pi} + 2160 + 16\pi - 360\left(\frac{270}{\pi}\right) - \frac{270^2}{\pi}$$
$$= 3600 + 16\pi \approx 3650.27 \text{ square meters}$$

(ii) The outer perimeter
$$= 360 + 2\pi\left(\frac{270}{\pi} + 4\right) \approx 925.13 \text{ meters}$$

77.a. $(f \circ g)(t) = \pi(2t + 1)^2$

b. $A(t) = f(2t + 1) = \pi(2t + 1)^2$

c. They are the same.

78.a. $(f \circ g)(t) = \dfrac{4}{3}\pi(2t)^3 = \dfrac{32}{3}\pi t^3$

b. $V(t) = \dfrac{4}{3}\pi(2t)^3 = \dfrac{32}{3}\pi t^3$

c. They are the same.

2.8 Beyond the Basics

79.a. When you are looking for the domain of the sum of two functions that are given as sets, you are looking for the intersection of their domains. Since the *x*-values that *f* and *g* have in common are −2, 1, and 3, the domain of $f + g$ is $\{-2, 1, 3\}$. Now add the *y*-values.
$(f + g)(-2) = 3 + 0 = 3$
$(f + g)(1) = 2 + (-2) = 0$
$(f + g)(3) = 0 + 2 = 2$
Thus, $f + g = \{(-2, 3), (1, 0), (3, 2)\}$.

b. When you are looking for the domain of the product of two functions that are given as sets, you are looking for the intersection of their domains. Since the x-values that f and g have in common are −2, 1, and 3, the domain of $f + g$ is {−2, 1, 3}.Now multiply the y-values.

$(fg)(-2) = 3 \cdot 0 = 0$

$(fg)(1) = 2 \cdot (-2) = -4$

$(fg)(3) = 0 \cdot 2 = 0$

Thus, fg = {(−2, 0), (1, −4), (3, 0)}.

c. When you are looking for the domain of the quotient of two functions that are given as sets, you are looking for the intersection of their domains and values of x that do not cause the denominator to equal zero. The x-values that f and g have in common are −2, 1, and 3; however, $g(-2) = 0$, so the domain is {1, 3}. Now divide the y-values.

$$\left(\frac{f}{g}\right)(1) = \frac{2}{-2} = -1$$

$$\left(\frac{f}{g}\right)(3) = \frac{0}{2} = 0$$

Thus, $\frac{f}{g} = \{(1, -1), (3, 0)\}$.

d. When you are looking for the domain of the composition of two functions that are given as sets, you are looking for values that come from the domain of the inside function and when you plug those values of x into the inside function, the output is in the domain of the outside function.

$f(g(-2)) = f(0)$, which is undefined

$f(g(0)) = f(2) = 1,$

$f(g(1)) = f(-2) = 3,$

$f(g(3)) = f(2) = 1$

Thus, $f \circ g = \{(0, 1), (1, 3), (3, 1)\}$.

80. When you are looking for the domain of the sum of two functions, you are looking for the intersection of their domains. The domain of f is [−2, 3], while the domain of g is [−3, 3]. The intersection of the two domains is [−2, 3], so the domain of $f + g$ is [−2, 3].
For the interval [−2, 1],
$f + g = 2x + (x + 1) = 3x + 1$.
For the interval (1, 2)
$f + g = (x + 1) + (x + 1) = 2x + 2$.
For the interval [2, 3],
$f + g = (x + 1) + (2x - 1) = 3x$.

Thus,

$$(f+g)(x) = \begin{cases} 3x+1 & \text{if } -2 \le x \le 1 \\ 2x+2 & \text{if } 1 < x < 2 \\ 3x & \text{if } 2 \le x \le 3. \end{cases}$$

81.a. $f(-x) = h(-x) + h(-(-x)) = h(-x) + h(x)$
$= f(x) \Rightarrow f(x)$ is an even function.

b. $g(-x) = h(-x) - h(-(-x)) = h(-x) - h(x)$
$= -g(x) \Rightarrow g(x)$ is an odd function.

c. $\begin{cases} f(x) = h(x) + h(-x) \\ g(x) = h(x) - h(-x) \Rightarrow \end{cases}$

$f(x) + g(x) = 2h(x) \Rightarrow$

$h(x) = \dfrac{f(x)+g(x)}{2} = \dfrac{f(x)}{2} + \dfrac{g(x)}{2} \Rightarrow$

$h(x)$ is the sum of an even function and an odd function.

82.a. $h(x) = x^2 - 2x + 3 \Rightarrow f(x) = x^2$ (even),
$g(x) = -2x + 3$ (odd) or $f(x) = x^2 + 3$ (even),
$g(x) = -2x$ (odd)

b. $h(x) = [\![x]\!] + x \Rightarrow f(x) = \dfrac{[\![x]\!] + [\![-x]\!]}{2}$ (even),

$g(x) = x + \dfrac{[\![x]\!] - [\![-x]\!]}{2}$ (odd)

83. $f(x) = \sqrt{\dfrac{1-|x|}{2-|x|}}$

$f(x)$ is defined if $\dfrac{1-|x|}{2-|x|} \ge 0$ and $2 - |x| \ne 0$.

$2 - |x| = 0 \Rightarrow 2 = |x| \Rightarrow x = \pm 2$

Thus, the values −2 and 2 are not in the domain of f.

$\dfrac{1-|x|}{2-|x|} \ge 0$ if $1 - |x| \ge 0$ and $2 - |x| > 0$, or if $1 - |x| \le 0$ and $2 - |x| < 0$.

Case 1: $1 - |x| \ge 0$ and $2 - |x| > 0$.

$1 - |x| \ge 0 \Rightarrow 1 \ge |x| \Rightarrow -1 \le x \le 1$

$2 - |x| > 0 \Rightarrow 2 > |x| \Rightarrow -2 < x < 2$

Thus, $1 - |x| \ge 0$ and $2 - |x| > 0 \Rightarrow -1 \le x \le 1$.

(*continued on next page*)

(*continued*)

Case 2: $1-|x|\le 0$ and $2-|x|<0$.

$1-|x|\le 0 \Rightarrow 1\le |x| \Rightarrow (-\infty, -1]\cup[1, \infty)$

$2-|x|<0 \Rightarrow 3\le |x| \Rightarrow (-\infty, -2)\cup(2, \infty)$

Thus, $1-|x|\le 0$ and $2-|x|<0 \Rightarrow$ $(-\infty, -2)\cup(2, \infty)$.

The domain of *f* is $(-\infty, -2)\cup[-1, 1]\cup(2, \infty)$.

84. $f(x)=\begin{cases}-1 & \text{if } -2\le x\le 0\\ x-1 & \text{if } 0<x\le 2\end{cases}$

$f(|x|)=|x|-1,\ -2\le x\le 2$

$|f(x)|=\begin{cases}1 & \text{if } -2\le x\le 0\\ |x-1|=1-x & \text{if } 0<x<1\\ x-1 & \text{if } 1\le x\le 2\end{cases}$

$g(x)=f(|x|)+|f(x)|$

If $-2\le x\le 0$, then $g(x)=|x|-1+1=|x|=-x$.

If $0<x<1$, then $g(x)=(1-x)+(x-1)=0$.

If $1\le x\le 2$, then $g(x)=(x-1)+(x-1)=2(x-1)$.

Writing *g* as a piecewise function, we have

$g(x)=\begin{cases}|x|=-x & \text{if } -2\le x\le 0\\ 0 & \text{if } 0<x<1\\ 2(x-1) & \text{if } 1\le x\le 2\end{cases}$

2.8 Critical Thinking/Discussion/Writing

85.a. The domain of $f(x)$ is $(-\infty,0)\cup[1,\infty)$.

b. The domain of $g(x)$ is [0, 2].

c. The domain of $f(x)+g(x)$ is [1, 2].

d. The domain of $\dfrac{f(x)}{g(x)}$ is [1, 2).

86.a. The domain of *f* is $(-\infty,0)$. The domain of $f\circ f$ is $\varnothing$ because $f\circ f=\dfrac{1}{\sqrt{-\dfrac{1}{\sqrt{-x}}}}$ and the denominator is the square root of a negative number.

b. The domain of *f* is $(-\infty,1)$. The domain of $f\circ f$ is $(-\infty,0)$ because $f\circ f=\dfrac{1}{\sqrt{1-\dfrac{1}{\sqrt{1-x}}}}$ and the denominator must be greater than 0. If $x = 0$, then the denominator = 0.

87.a. The sum of two even functions is an even function. $f(x)=f(-x)$ and $g(x)=g(-x)\Rightarrow$ $(f+g)(x)=f(x)+g(x)=f(-x)+g(-x)$ $=(f+g)(-x)$.

b. The sum of two odd functions is an odd function.
$f(-x)=-f(x)$ and $g(-x)=-g(x)\Rightarrow$
$(f+g)(-x)=f(-x)+g(-x)=-f(x)-g(x)$ $=-(f+g)(x)$.

c. The sum of an even function and an odd function is neither even nor odd.
$f(x)$ even $\Rightarrow f(x)=f(-x)$ and $g(x)$ odd $\Rightarrow$ $g(-x)=-g(x)\Rightarrow f(-x)+g(-x)=$ $f(x)+(-g(x))$, which is neither even nor odd.

d. The product of two even functions is an even function. $f(x)=f(-x)$ and $g(x)=g(-x)\Rightarrow$ $(f\cdot g)(x)=f(x)\cdot g(x)=f(-x)\cdot(g(-x))$ $=(f\cdot g)(-x)$.

e. The product of two odd functions is an even function.
$f(-x)=-f(x)$ and $g(-x)=-g(x)\Rightarrow$
$(f\cdot g)(-x)=f(-x)\cdot g(-x)=-f(x)\cdot(-g(x))$ $=(f\cdot g)(x)$.

f. The product of an even function and an odd function is an odd function.
$f(x)$ even $\Rightarrow f(x)=f(-x)$ and $g(x)$ odd $\Rightarrow$ $g(-x)=-g(x)\Rightarrow$
$f(-x)\cdot g(-x)=f(x)\cdot(-g(x))=-(f\cdot g)(x)$

88.a. $f(-x)=-f(x)$ and $g(-x)=-g(x)\Rightarrow$
$(f\circ g)(-x)=f(g(-x))=f(-g(x))=$ $-f(g(x))\Rightarrow(f\circ g)(x)$ is odd.

b. $f(x)=f(-x)$ and $g(x)=g(-x)\Rightarrow$
$(f\circ g)(-x)=f(g(-x))=f(g(x))\Rightarrow$ $(f\circ g)(x)$ is even.

c. $f(x)$ odd $\Rightarrow f(-x) = -f(x)$ and
$g(x)$ even $\Rightarrow g(x) = g(-x) \Rightarrow (f \circ g)(-x)$
$f(g(x)) = f(g(-x)) \Rightarrow (f \circ g)(x)$ is even.

d. $f(x)$ even $\Rightarrow f(x) = f(-x)$ and $g(x)$ odd $\Rightarrow$
$g(-x) = -g(x) \Rightarrow (f \circ g)(-x) = f(-g(x))$
$= f(g(x)) = (f \circ g)(-x) \Rightarrow (f \circ g)(x)$ is even.

2.8 Maintaining Skills

89.a. Yes, R defines a function.

b. $S = \{(2, -3), (1, -1), (3, 1), (1, 2)\}$
No, S does not define a function since the first value 1 maps to two different second values, −1 and 2.

90. The slope of $PP' = \frac{2-5}{5-2} = -1,$ while the slope of $y = x$ is 1. Since the slopes are the negative reciprocals, the lines are perpendicular. The midpoint of PP' is $\left(\frac{2+5}{2}, \frac{5+2}{2}\right) = \left(\frac{7}{2}, \frac{7}{2}\right),$ which lies on the line $y = x$. Thus, $y = x$ is the perpendicular bisector of PP'.

91. $x = 2y + 3 \Rightarrow x - 3 = 2y \Rightarrow \frac{x-3}{2} = y$

92. $x = y^2 + 1,\ y \ge 0 \Rightarrow x - 1 = y^2 \Rightarrow \sqrt{x-1} = y$

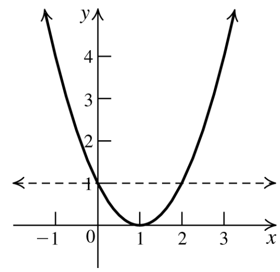
93. $x^2 + y^2 = 4,\ x \le 0 \Rightarrow x^2 = 4 - y^2 \Rightarrow$
$x = -\sqrt{4 - y^2}$

94. $2x - \frac{1}{y} = 3 \Rightarrow -\frac{1}{y} = 3 - 2x \Rightarrow \frac{1}{y} = -3 + 2x \Rightarrow$
$y = \frac{1}{2x-3}$

2.9 Inverse Functions

2.9 Practice Problems

1. $f(x) = (x-1)^2$ is not one-to-one because the horizontal line $y = 1$ intersects the graph at two different points.

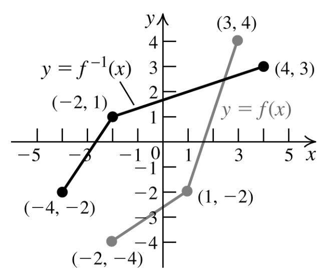


2.a. $f^{-1}(12) = -3$

b. $f(9) = 4$

3. $f(x) = 3x - 1,\ g(x) = \frac{x+1}{3}$
$(f \circ g)(x) = f\left(\frac{x+1}{3}\right) = 3\left(\frac{x+1}{3}\right) - 1 = x$
$(g \circ f)(x) = g(3x-1) = \frac{3x-1+1}{3} = x$
Since $f(g(x)) = g(f(x)) = x,$ the two functions are inverses.

4. The graph of f^{-1} is the reflection of the graph of f about the line $y = x$.



5. $f(x) = -2x + 3$ is a one-to-one function, so the function has an inverse. Interchange the variables and solve for y:
$f(x) = y = -2x + 3 \Rightarrow x = -2y + 3 \Rightarrow$
$\frac{x-3}{-2} = y \Rightarrow y = f^{-1}(x) = \frac{3-x}{2}.$

6. Interchange the variables and solve for y:
$f(x) = y = \frac{x}{x+3}, x \ne 3$
$x = \frac{y}{y+3} \Rightarrow xy + 3x = y \Rightarrow 3x = y - xy \Rightarrow$
$3x = y(1-x) \Rightarrow \frac{3x}{1-x} = y \Rightarrow$
$f^{-1}(x) = \frac{3x}{1-x},\ x \ne 1$

7. $f(x)=\frac{x}{x+3}$
The function is not defined if the denominator is zero, so the domain is $(-\infty,-3)\cup(-3,\infty)$. The range of the function is the same as the domain of the inverse (see practice problem 6), thus the range is $(-\infty,1)\cup(1,\infty)$.

8. G is one-to-one since the domain is restricted, so an inverse exists.
$G(x)=y=x^2-1, x\le 0$. Interchange the variables and solve for y:
$x=y^2-1,\ y\le 0\Rightarrow y=G^{-1}(x)=-\sqrt{x+1}.$

9. From the text, we have $d=\frac{11p}{5}-33$.
$d=\frac{11\cdot 1650}{5}-33=3597$
The bell was 3597 feet below the surface when the gauge failed.

2.9 Basic Concepts and Skills

1. If no horizontal line intersects the graph of a function *f* in more than one point, the *f* is a <u>one-to-one</u> function.

2. A function *f* is one-to-one if different *x*-values correspond to <u>different *y*-values</u>.

3. If $f(x)=3x$, then $f^{-1}(x)=\underline{\frac{1}{3}}x$.

4. The graphs of a function *f* and its inverse f^{-1} are symmetric about the line <u>$y=x$</u>.

5. True

6. True. For example, the inverse of $f(x)=x$ is $f^{-1}(x)=x$.

7. One-to-one

8. Not one-to-one

9. Not one-to-one

10. One-to-one

11. Not one-to-one

12. Not one-to-one

13. One-to-one

14. Not one-to-one

15. $f(2)=7\Rightarrow f^{-1}(7)=2$

16. $f^{-1}(4)=-7\Rightarrow f(-7)=4$

17. $f(-1)=2\Rightarrow f^{-1}(2)=-1$

18. $f^{-1}(-3)=5\Rightarrow f(5)=-3$

19. $f(a)=b\Rightarrow f^{-1}(b)=a$

20. $f^{-1}(c)=d\Rightarrow f(d)=c$

21. $\left(f^{-1}\circ f\right)(337)=f^{-1}\left(f(337)\right)=337$

22. $\left(f\circ f^{-1}\right)(25\pi)=f\left(f^{-1}(25\pi)\right)=25\pi$

23. $\left(f\circ f^{-1}\right)(-1580)=f\left(f^{-1}(-1580)\right)=-1580$

24. $\left(f^{-1}\circ f\right)(9728)=f^{-1}\left(f(9728)\right)=9728$

25.a. $f(3)=2(3)-3=3$

b. Using the result from part (a), $f^{-1}(3)=3$.

c. $\left(f\circ f^{-1}\right)(19)=f\left(f^{-1}(19)\right)=19$

d. $\left(f\circ f^{-1}\right)(5)=f\left(f^{-1}(5)\right)=5$

26.a. $f(2)=2^3=8$

b. Using the result from part (a), $f^{-1}(8)=2$.

c. $\left(f\circ f^{-1}\right)(15)=f\left(f^{-1}(15)\right)=15$

d. $\left(f\circ f^{-1}\right)(27)=f\left(f^{-1}(27)\right)=27$

27.a. $f(1)=1^3+1=2$

b. Using the result from part (a), $f^{-1}(2)=1$.

c. $\left(f\circ f^{-1}\right)(269)=f\left(f^{-1}(269)\right)=269$

28.a. $g(1)=\sqrt[3]{2\left(1^3\right)-1}=\sqrt[3]{1}=1$

b. Using the result from part (a), $g^{-1}(1)=1$.

c. $\left(g^{-1}\circ g\right)(135)=g^{-1}\left(g(135)\right)=135$

29. $f\left(g(x)\right)=3\left(\frac{x-1}{3}\right)+1=x-1+1=x$
$g\left(f(x)\right)=\frac{(3x+1)-1}{3}=\frac{3x}{3}=x$

30. $f\left(g(x)\right)=2-3\left(\frac{2-x}{3}\right)=2-2+x=x$
$g\left(f(x)\right)=\frac{2-(2-3x)}{3}=\frac{3x}{3}=x$

31. $f\left(g\left(x\right)\right)=\left(\sqrt[3]{x}\right)^3=x$

$g\left(f\left(x\right)\right)=\sqrt[3]{x^3}=x$

32. $f\left(g\left(x\right)\right)=g\left(f\left(x\right)\right)=\dfrac{1}{\frac{1}{x}}=x$

33. $$f\left(g\left(x\right)\right)=\frac{\frac{1+2x}{1-x}-1}{\frac{1+2x}{1-x}+2}=\frac{\frac{1+2x-(1-x)}{1-x}}{\frac{1+2x+2(1-x)}{1-x}}=\frac{3x}{3}=x$$

$$g\left(f\left(x\right)\right)=\frac{1+2\left(\frac{x-1}{x+2}\right)}{1-\frac{x-1}{x+2}}=\frac{1+\frac{2x-2}{x+2}}{1-\frac{x-1}{x+2}}$$

$$=\frac{\frac{x+2+2x-2}{x+2}}{\frac{x+2-(x-1)}{x+2}}=\frac{3x}{3}=x$$

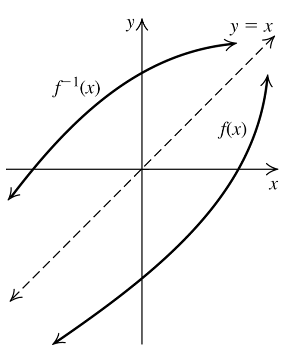
34. $$f\left(g\left(x\right)\right)=\frac{3\left(\frac{x+2}{x-3}\right)+2}{\frac{x+2}{x-3}-1}=\frac{\frac{3x+6}{x-3}+\frac{2(x-3)}{x-3}}{\frac{x+2}{x-3}-\frac{1(x-3)}{x-3}}$$

$$=\frac{\frac{3x+6+2x-6}{x-3}}{\frac{x+2-x+3}{x-3}}=\frac{5x}{5}=x$$

$$g\left(f\left(x\right)\right)=\frac{\frac{3x+2}{x-1}+2}{\frac{3x+2}{x-1}-3}=\frac{\frac{3x+2}{x-1}+\frac{2(x-1)}{x-1}}{\frac{3x+2}{x-1}-\frac{3(x-1)}{x-1}}$$

$$=\frac{\frac{3x+2+2x-2}{x-1}}{\frac{3x+2-3x+3}{x-1}}=\frac{5x}{5}=x$$

35.



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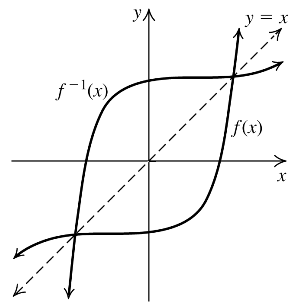

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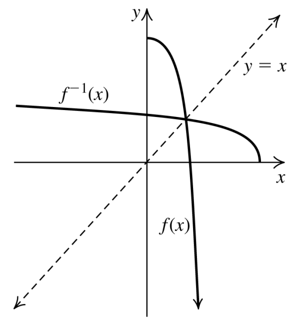

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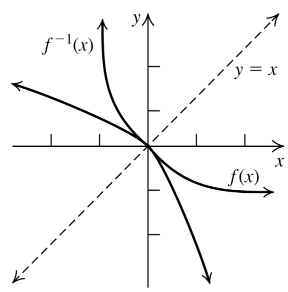

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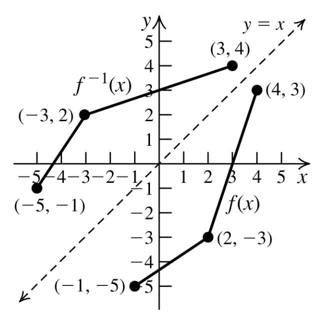

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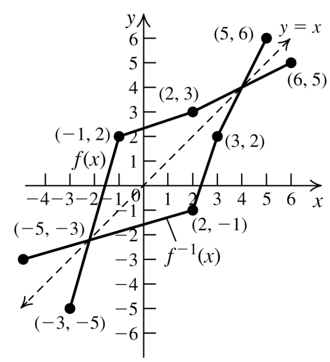
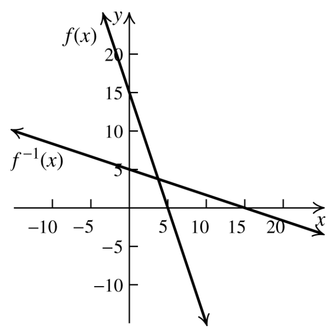


41.a. One-to-one

b. $f(x) = y = 15 - 3x$. Interchange the variables and solve for $y: x = 15 - 3y \Rightarrow$
$y = f^{-1}(x) = \dfrac{15 - x}{3} = 5 - \dfrac{1}{3}x.$

c.

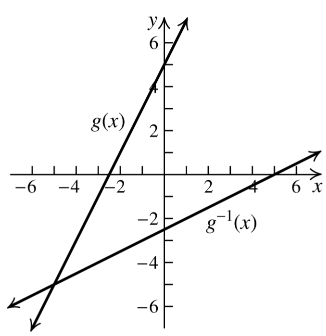


d. Domain of *f*: $(-\infty, \infty)$; *x*-intercept of *f*: 5;
y-intercept of *f*: 15
domain of $f^{-1}: (-\infty, \infty)$; *x*-intercept of
f^{-1}: 15; *y*-intercept of f^{-1}: 5

42.a. One-to-one

b. $g(x) = y = 2x + 5$. Interchange the variables and solve for $y: x = 2y + 5 \Rightarrow$
$y = g^{-1}(x) = \dfrac{x - 5}{2} = \dfrac{1}{2}x - \dfrac{5}{2}.$

c.



d. Domain of *g*: $(-\infty, \infty)$
x-intercept of *g*: $-\dfrac{5}{2}$
y-intercept of *g*: 5
domain of $g^{-1}: (-\infty, \infty)$; *x*-intercept of
g^{-1}: 5; *y*-intercept of $g^{-1}: -\dfrac{5}{2}$

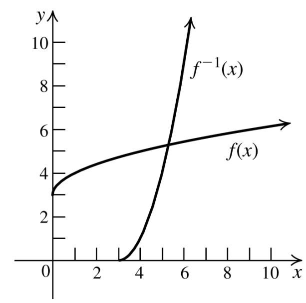
43.a. Not one-to-one

44.a. Not one-to-one

45.a. One-to-one

b. $f(x) = y = \sqrt{x} + 3$ Interchange the variables and solve for $y: x = \sqrt{y} + 3 \Rightarrow$
$x - 3 = \sqrt{y} \Rightarrow y = f^{-1}(x) = (x-3)^2.$

c.

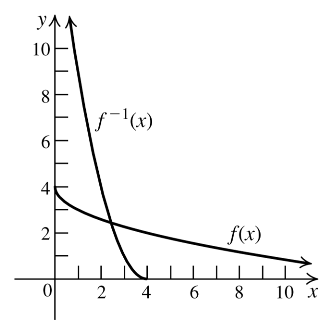


d. Domain of *f*: $[0, \infty)$; *x*-intercept of *f*: none;
y-intercept of *f*: 3
domain of $f^{-1}: [3, \infty)$; *x*-intercept of
$f^{-1}: 3$; *y*-intercept of f^{-1}: none

46.a. One-to-one

b. $f(x) = y = 4 - \sqrt{x}$. Interchange the variables and solve for $y: x = 4 - \sqrt{y} \Rightarrow$
$-4 + x = -\sqrt{y} \Rightarrow y = f^{-1}(x) = (x-4)^2$

c.

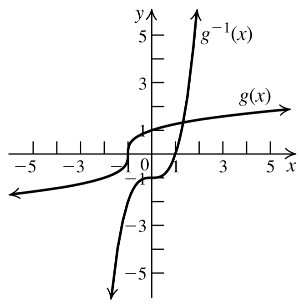


d. Domain of *f*: $[0, \infty)$; *x*-intercept of *f*: 16;
y-intercept of *f*: 4
domain of $f^{-1}: (-\infty, 4]$; *x*-intercept of
f^{-1}: 4; *y*-intercept of f^{-1}: 16

47.a. One-to-one

b. $g(x) = y = \sqrt[3]{x+1}$. Interchange the variables and solve for $y: x = \sqrt[3]{y+1} \Rightarrow$
$x^3 = y + 1 \Rightarrow y = g^{-1}(x) = x^3 - 1$

c.

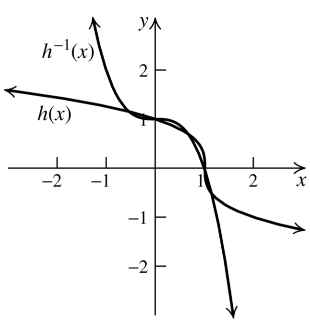


d. Domain of g: $(-\infty, \infty)$; x-intercept of g: -1;
y-intercept of g: 1
domain of g^{-1}: $(-\infty, \infty)$; x-intercept of g^{-1}: 1; y-intercept of g^{-1}: -1

48.a. One-to-one

b. $h(x) = y = \sqrt[3]{1-x}$. Interchange the variables and solve for y: $x = \sqrt[3]{1-y} \Rightarrow$
$x^3 = 1 - y \Rightarrow y = g^{-1}(x) = 1 - x^3$.

c.

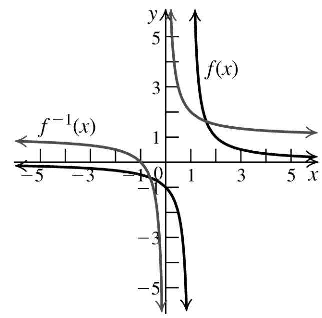


d. Domain of h: $(-\infty, \infty)$; x-intercept of h: 1;
y-intercept of h: 1
domain of h^{-1}: $(-\infty, \infty)$; x-intercept of h^{-1}: 1; y-intercept of h^{-1}: 1

49.a. One-to-one

b. $f(x) = y = \dfrac{1}{x-1}$. Interchange the variables and solve for y: $x = \dfrac{1}{y-1} \Rightarrow x(y-1) = 1 \Rightarrow$
$\dfrac{1}{x} = y - 1 \Rightarrow y = f^{-1}(x) = \dfrac{1}{x} + 1 = \dfrac{1+x}{x}$.

c.

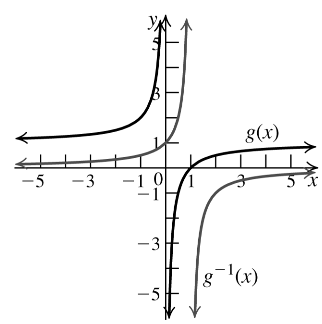


d. Domain of f: $(-\infty, 1) \cup (1, \infty)$
x-intercept of f: none; y-intercept of f: -1
domain of f^{-1}: $(-\infty, 0) \cup (0, \infty)$
x-intercept of f^{-1}: -1
y-intercept of f^{-1}: none

50.a. One-to-one

b. $g(x) = y = 1 - \dfrac{1}{x}$. Interchange the variables and solve for y: $x = 1 - \dfrac{1}{y} \Rightarrow x = \dfrac{y-1}{y} \Rightarrow$
$xy = y - 1 \Rightarrow xy - y = -1 \Rightarrow y(x-1) = -1 \Rightarrow$
$y = g^{-1}(x) = -\dfrac{1}{x-1} = \dfrac{1}{1-x}$.

c.

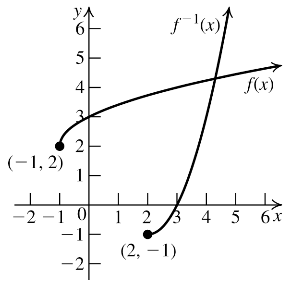


d. Domain of g: $(-\infty, 0) \cup (0, \infty)$
x-intercept of g: 1; y-intercept of g: none
domain of g^{-1}: $(-\infty, 1) \cup (1, \infty)$
x-intercept of g^{-1}: none
y-intercept of g^{-1}: 1

51.a. One-to-one

b. $f(x) = y = 2 + \sqrt{x+1}$. Interchange the variables and solve for y: $x = 2 + \sqrt{y+1} \Rightarrow$
$x - 2 = \sqrt{y+1} \Rightarrow (x-2)^2 = y + 1 \Rightarrow$
$y = f^{-1}(x) = (x-2)^2 - 1 = x^2 - 4x + 3$

c.



d. Domain of *f*: $[-1, \infty)$; *x*-intercept of *f*: none; *y*-intercept of *f*: 3

Domain of f^{-1}: $[2, \infty)$;

x-intercept of f^{-1}: 3

y-intercept of f^{-1}: none

52.a. One-to-one

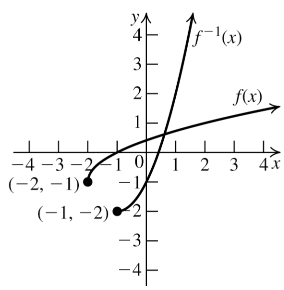
b. $f(x) = y = -1 + \sqrt{x+2}$. Interchange the variables and solve for y:

$x = -1 + \sqrt{y+2} \Rightarrow x + 1 = \sqrt{y+2} \Rightarrow$

$(x+1)^2 = y + 2 \Rightarrow$

$y = f^{-1}(x) = (x+1)^2 - 2 = x^2 + 2x - 1$

c.



d. Domain of *f*: $[-2, \infty)$; *x*-intercept of *f*: -1;

y-intercept of *f*: $-1+\sqrt{2}$

Domain of f^{-1}: $[-1, \infty)$

x-intercept of f^{-1}: $-1+\sqrt{2}$

y-intercept of f^{-1}: -1

In exercises 53 and 54, use the fact that the range of *f* is the same as the domain of f^{-1}.

53. Domain: $(-\infty, -2) \cup (-2, \infty)$
Range: $(-\infty, 1) \cup (1, \infty)$

54. Domain: $(-\infty, 1) \cup (1, \infty)$
Range: $(-\infty, 3) \cup (3, \infty)$

55. $f(x) = y = \dfrac{x+1}{x-2}$. Interchange the variables and solve for y: $x = \dfrac{y+1}{y-2} \Rightarrow xy - 2x = y + 1 \Rightarrow$

$xy - y = 2x + 1 \Rightarrow y(x-1) = 2x + 1 \Rightarrow$

$y = f^{-1}(x) = \dfrac{2x+1}{x-1}$.

Domain of *f*: $(-\infty, 2) \cup (2, \infty)$

Range of *f*: $(-\infty, 1) \cup (1, \infty)$.

56. $g(x) = y = \dfrac{x+2}{x+1}$. Interchange the variables and solve for y: $x = \dfrac{y+2}{y+1} \Rightarrow xy + x = y + 2 \Rightarrow$

$xy - y = -x + 2 \Rightarrow y(x-1) = -x + 2 \Rightarrow$

$y = g^{-1}(x) = \dfrac{-x+2}{x-1} = \dfrac{x-2}{1-x}$.

Domain of *g*: $(-\infty, -1) \cup (-1, \infty)$

Range of *g*: $(-\infty, 1) \cup (1, \infty)$.

57. $f(x) = y = \dfrac{1-2x}{1+x}$. Interchange the variables and solve for y: $x = \dfrac{1-2y}{1+y} \Rightarrow$

$x + xy = 1 - 2y \Rightarrow xy + 2y = 1 - x \Rightarrow$

$y(x+2) = 1 - x \Rightarrow y = f^{-1}(x) = \dfrac{1-x}{x+2}$.

Domain of *f*: $(-\infty, -1) \cup (-1, \infty)$

Range of *f*: $(-\infty, -2) \cup (-2, \infty)$.

58. $h(x) = y = \dfrac{x-1}{x-3}$. Interchange the variables and solve for y: $x = \dfrac{y-1}{y-3} \Rightarrow xy - 3x = y - 1 \Rightarrow$

$xy - y = 3x - 1 \Rightarrow y(x-1) = 3x - 1 \Rightarrow$

$y = h^{-1}(x) = \dfrac{3x-1}{x-1}$.

Domain of *h*: $(-\infty, 3) \cup (3, \infty)$

Range of *h*: $(-\infty, 1) \cup (1, \infty)$.

59. *f* is one-to-one since the domain is restricted, so an inverse exists.

$f(x) = y = -x^2, x \ge 0$. Interchange the variables and solve for y:

$x = -y^2 \Rightarrow y = \sqrt{-x},\ x \le 0$.

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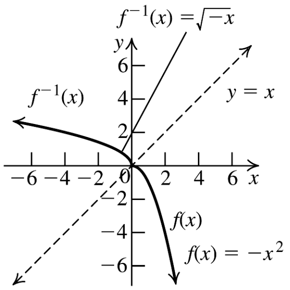
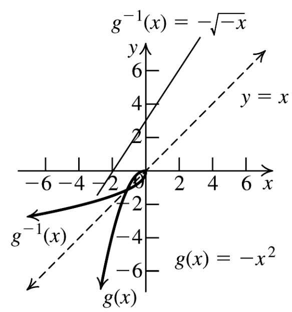


60. g is one-to-one since the domain is restricted, so an inverse exists.

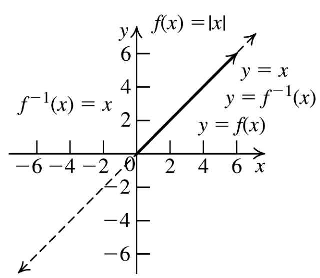
$g(x) = y = -x^2,\ x \le 0.$ Interchange the variables and solve for y:

$x = -y^2 \Rightarrow y = -\sqrt{-x},\ x \le 0.$



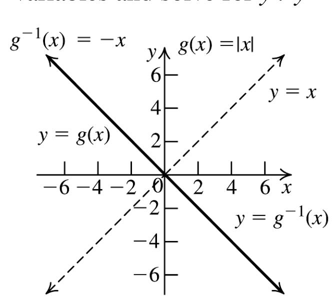
61. f is one-to-one since the domain is restricted, so an inverse exists.

$f(x) = y = |x| = x,\ x \ge 0.$ Interchange the variables and solve for y: $y = x,\ x \ge 0.$



62. g is one-to-one since the domain is restricted, so an inverse exists.

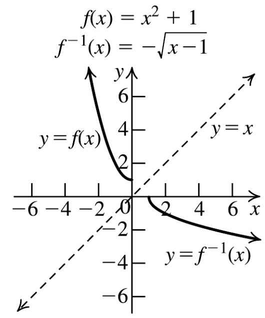
$g(x) = y = |x| = -x,\ x \le 0.$ Interchange the variables and solve for y: $y = -x,\ x \ge 0.$



63. f is one-to-one since the domain is restricted, so an inverse exists.

$f(x) = y = x^2 + 1,\ x \le 0.$ Interchange the variables and solve for y:

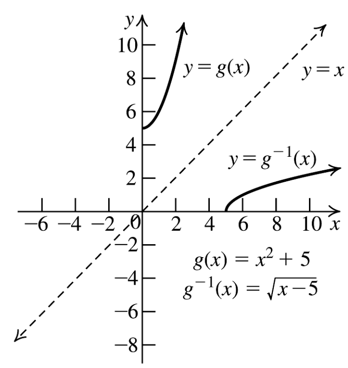
$x = y^2 + 1 \Rightarrow y = -\sqrt{x-1},\ x \ge 1.$



64. g is one-to-one since the domain is restricted, so an inverse exists.

$g(x) = y = x^2 + 5,\ x \ge 0.$ Interchange the variables and solve for y:

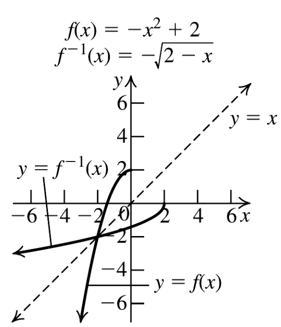
$x = y^2 + 5 \Rightarrow y = \sqrt{x-5},\ x \ge 5.$



65. f is one-to-one since the domain is restricted, so an inverse exists.

$f(x) = y = -x^2 + 2,\ x \le 0.$ Interchange the variables and solve for y:

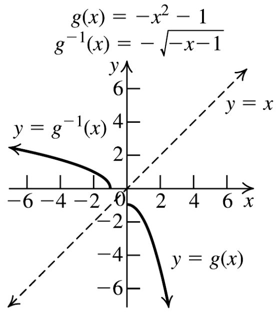
$x = -y^2 + 2 \Rightarrow y = -\sqrt{2-x},\ x \le 2.$



66. g is one-to-one since the domain is restricted, so an inverse exists.

$g(x) = y = -x^2 - 1,\ x \ge 0$. Interchange the variables and solve for y:

$x = -y^2 - 1 \Rightarrow y = \sqrt{-x-1},\ x \le -1.$



2.9 Applying the Concepts

67.a. $K(C) = C + 273 \Rightarrow$

$C(K) = K - 273 = K^{-1}(C).$

This represents the Celsius temperature corresponding to a given Kelvin temperature.

b. $C(300) = 300 - 273 = 27°\text{C}$

c. $K(22) = 22 + 273 = 295°\text{K}$

68.a. The two points are (212, 373) and (32, 273).

The rate of change is $\dfrac{373-273}{212-32} = \dfrac{100}{180} = \dfrac{5}{9}.$

$273 = \dfrac{5}{9}(32) + b \Rightarrow b = \dfrac{2297}{9} \Rightarrow$

$K(F) = \dfrac{5}{9}F + \dfrac{2297}{9}.$

b. $K = \dfrac{5}{9}F + \dfrac{2297}{9} \Rightarrow K - \dfrac{2297}{9} = \dfrac{5}{9}F \Rightarrow$

$9K - 2297 = 5F \Rightarrow F(K) = \dfrac{9}{5}K - \dfrac{2297}{5}$

This represents the Fahrenheit temperature corresponding to a given Kelvin temperature.

c. $K(98.6) = \dfrac{5}{9}(98.6) + \dfrac{2297}{9} = 310°\text{K}$

69.a.
$$\begin{aligned} F(K(C)) &= \frac{9}{5}(C+273) - \frac{2297}{5} \\ &= \frac{9}{5}C + \frac{9(273)}{5} - \frac{2297}{5} \\ &= \frac{9}{5}C + \frac{160}{5} = \frac{9}{5}C + 32 \end{aligned}$$

b.
$$\begin{aligned} C(K(F)) &= \frac{5}{9}F + \frac{2297}{9} - 273 \\ &= \frac{5}{9}F + \frac{2297-2457}{9} \\ &= \frac{5}{9}F - \frac{160}{9} \end{aligned}$$

70.
$$\begin{aligned} F(C(x)) &= \frac{9}{5}\left(\frac{5}{9}x - \frac{160}{9}\right) + 32 \\ &= x - 32 + 32 = x \\ C(F(x)) &= \frac{5}{9}\left(\frac{9}{5}x + 32\right) - \frac{160}{9} \\ &= x + \frac{160}{9} - \frac{160}{9} = x \end{aligned}$$

Therefore, F and C are inverses of each other.

71.a. $E(x) = 0.75x$ where x represents the number of dollars; $D(x) = 1.25x$ where x represents the number of euros.

b. $E(D(x)) = 0.75(1.25x) = 0.9375x \ne x.$

Therefore, the two functions are not inverses.

c. She loses money either way.

72.a. $w = 4 + 0.05x \Rightarrow w - 4 = 0.05x \Rightarrow$

$x = 20w - 80.$

This represents the food sales in terms of his hourly wage.

b. $x = 20(12) - 80 = \$160$

73.a. $7 = 4 + 0.05x \Rightarrow x = \$60$. This means that if food sales $\le$ \$60, he will receive the minimum hourly wage. If food sales > \$60, his wages will be based on food sales.

$$w = \begin{cases} 4 + 0.05x & \text{if } x > 60 \\ 7 & \text{if } x \le 60 \end{cases}$$

b. The function does not have an inverse because it is constant on (0, 60), and it is not one-to-one.

c. If the domain is restricted to $[60, \infty)$, the function has an inverse.

74.a. $T = 1.11\sqrt{l} \Rightarrow l = \left(\dfrac{T}{1.11}\right)^2$. This shows the length as the function of the period.

b. $l = \left(\dfrac{2}{1.11}\right)^2 \approx 3.2$ ft

c. $T = 1.11\sqrt{70} \approx 9.3$ sec

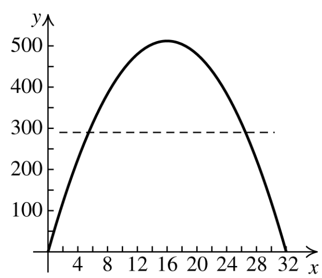
75.a. $V = 8\sqrt{x} \Rightarrow \frac{V}{8} = \sqrt{x} \Rightarrow \frac{1}{64}V^2 = x = V^{-1}(x)$

This represents the height of the water in terms of the velocity.

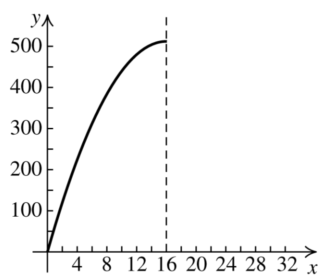
b. **(i)** $x = \frac{1}{64}\left(30^2\right) = 14.0625$ ft

(ii) $x = \frac{1}{64}\left(20^2\right) = 6.25$ ft

76.a. $y = 64x - 2x^2$ has no inverse because it is not one-to-one across its domain, [0, 32]. (It fails the horizontal line test.)



However, if the domain is restricted to [0, 16], the function is one-to-one, and it has an inverse.



$y = 64x - 2x^2 \Rightarrow 2x^2 - 64x + y = 0 \Rightarrow$

$x = \frac{64 \pm \sqrt{64^2 - 8y}}{4} \Rightarrow$

$x = \frac{64 \pm \sqrt{4096 - 8y}}{4} = \frac{64 \pm 2\sqrt{1024 - 2y}}{4}$

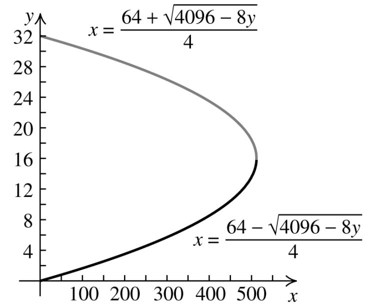
$= \frac{32 \pm \sqrt{1024 - 2y}}{2}$

$1024 - 2y \ge 0 \Rightarrow 0 \le y \le 512.$

(Because y is a number of feet, it cannot be negative.) This is the range of the original function. The domain of the original function is [0, 16], which is the range of the inverse.

The range of $x = \frac{32 + \sqrt{1024 - 2y}}{2}$ is [16, 32], so this is not the inverse.

The range of $x = \frac{32 - \sqrt{1024 - 2y}}{2}$, $0 \le y \le 512$, is [0, 16], so this is the inverse.



Note that the bottom half of the graph is the inverse.

b. **(i)** $x = \frac{64 - \sqrt{4096 - 8(32)}}{4} \approx 0.51$ ft

(ii) $x = \frac{64 - \sqrt{4096 - 8(256)}}{4} \approx 4.69$ ft

(iii) $x = \frac{64 - \sqrt{4096 - 8(512)}}{4} \approx 16$ ft

77.a. The function represents the amount she still owes after x months.

b. $y = 36{,}000 - 600x$. Interchange the variables and solve for y: $x = 36{,}000 - 600y \Rightarrow$

$600y = 36{,}000 - x \Rightarrow y = 60 - \frac{x}{600} \Rightarrow$

$f^{-1}(x) = 60 - \frac{1}{600}x$. This represents the number of months that have passed from the first payment until the balance due is \$$x$.

c. $y = 60 - \frac{1}{600}(22{,}000) = 23.33 \approx 24$ months

There are 24 months remaining.

78.a. To find the inverse, solve $x = 8p^2 - 32p + 1200$ for p:

$8p^2 - 32p + 1200 - x = 0 \Rightarrow$

$p = \frac{32 \pm \sqrt{(-32)^2 - 4(8)(1200 - x)}}{2(8)}$

$= \frac{32 \pm \sqrt{1024 - 38{,}400 + 32x}}{16}$

$= \frac{32 \pm \sqrt{32x - 37376}}{16} = \frac{32 \pm 4\sqrt{2x - 2336}}{16}$

$= 2 \pm \frac{1}{4}\sqrt{2x - 2336}$

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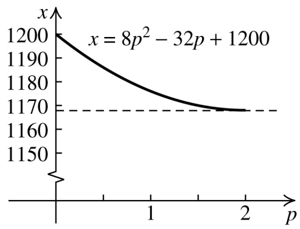
Because the domain of the original function is (0, 2], its range is [1168, 1200).

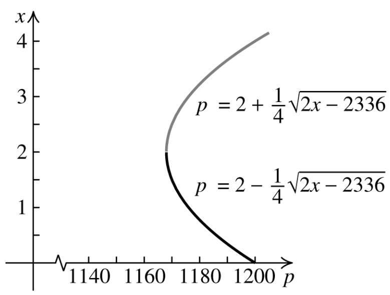


So the domain of the inverse is [1168, 1200), and its range is (0, 2]. The range of

$p = 2 + \frac{1}{4}\sqrt{2x - 2336}$ is (2, 4], so it is not the inverse. The range of

$p = 2 - \frac{1}{4}\sqrt{2x - 2336}, 1168 \le x < 1200,$ is

(0, 2], so it is the inverse. This gives the price of computer chips in terms of the demand x.



Note that the bottom half of the graph is the inverse.

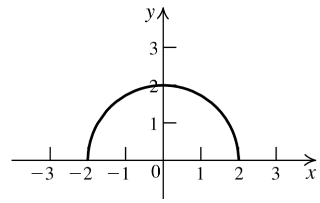
b. $p = 2 - \frac{1}{4}\sqrt{2(1180.5) - 2336} = \$0.75$

2.9 Beyond the Basics

79. $f(g(3)) = f(1) = 3, f(g(5)) = f(3) = 5,$ and $f(g(2)) = f(4) = 2 \Rightarrow f(g(x)) = x$ for each x. $g(f(1)) = g(3) = 1, g(f(3)) = g(5) = 3,$ and $g(f(4)) = g(2) = 4 \Rightarrow g(f(x)) = x$ for each x. So, f and g are inverses.

80. $f(g(-2)) = f(1) = -2, f(g(0)) = f(2) = 0,$ $f(g(-3)) = f(3) = -3,$ and $f(g(-2)) = f(1) = -2 \Rightarrow f(g(x)) = x$ for each x. $g(f(1)) = g(-2) = 1, g(f(2)) = g(0) = 2,$ $g(f(3)) = g(-3) = 3,$ and $g(f(4)) = g(1) = 4$ $\Rightarrow g(f(x)) = x$ for each x. So f and g are inverses.

81.a.



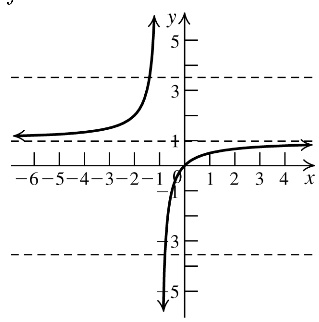
b. f is not one-to-one

c. Domain: [–2, 2]; range: [0, 2]

82.a. Domain: $(-\infty, 2) \cup [3, \infty)$. Note that the domain is not $(-\infty, 2) \cup (2, \infty)$ because $[\![x]\!] = 2$ for $2 \le x < 3$.

b. The function is not one-to-one. The function is constant on each interval $[n, n + 1)$, n an integer.

83.a. f satisfies the horizontal line test.



b. $y = 1 - \frac{1}{x+1}$. Interchange the variables and solve for y: $x = 1 - \frac{1}{y+1} \Rightarrow$

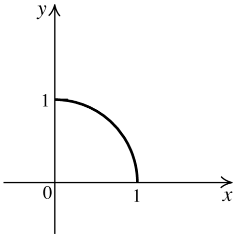
$\frac{1}{y+1} = 1 - x \Rightarrow 1 = y + 1 - xy - x \Rightarrow$

$xy - y = -x \Rightarrow y(x-1) = -x \Rightarrow$

$y = f^{-1}(x) = -\frac{x}{x-1} = \frac{x}{1-x}$

c. Domain of f: $(-\infty, -1) \cup (-1, \infty)$; range of f: $(-\infty, 1) \cup (1, \infty)$.

84.a. g satisfies the horizontal line test.



b. $y = \sqrt{1-x^2}$. Interchange the variables and solve for $y: x = \sqrt{1-y^2} \Rightarrow$
$x^2 = 1-y^2 \Rightarrow y^2 = 1-x^2 \Rightarrow$
$y = g^{-1}(x) = \sqrt{1-x^2}$

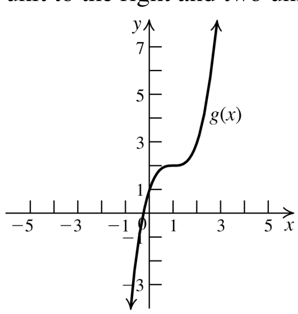
c. Domain of f = range of f: [0, 1]

85.a. $M = \left(\frac{3+7}{2}, \frac{7+3}{2}\right) = (5,5).$
Since the coordinates of M satisfy the equation $y = x$, it lies on the line.

b. The slope of $y = x$ is 1, while the slope of $\overline{PQ}$ is $\frac{3-7}{7-3} = -1.$ So, $y = x$ is perpendicular to $\overline{PQ}$.

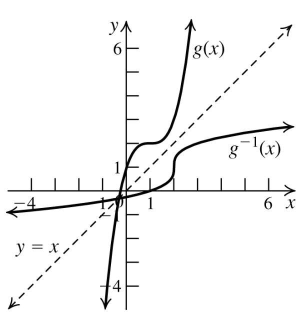
86. $M = \left(\frac{a+b}{2}, \frac{b+a}{2}\right).$
Since the coordinates of M satisfy the equation $y = x$, it lies on the line. The slope of the line segment between the two points is $\frac{b-a}{a-b} = -1,$ while the slope of $y = x$ is 1. So the two lines are perpendicular, and the points (a, b) and (b, a) are symmetric about the line $y = x$.

87.a. The graph of g is the graph of f shifted one unit to the right and two units up.



b. $g(x) = y = (x-1)^3 + 2$
Interchange the variables and solve for y.
$x = (y-1)^3 + 2 \Rightarrow y = g^{-1}(x) = \sqrt[3]{x-2} + 1$

c.



88.a. **(i)** $f(x) = y = 2x-1$. Interchange the variables and solve for y: $x = 2y-1 \Rightarrow$
$y = f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$

(ii) $g(x) = y = 3x+4$. Interchange the variables and solve for y: $x = 3y+4 \Rightarrow$
$y = g^{-1}(x) = \frac{1}{3}x - \frac{4}{3}$

(iii) $(f \circ g)(x) = 2(3x+4) - 1 = 6x+7$

(iv) $(g \circ f)(x) = 3(2x-1) + 4 = 6x+1$

(v) $(f \circ g)(x) = y = 6x+7$. Interchange the variables and solve for y:
$x = 6y+7 \Rightarrow (f \circ g)^{-1}(x) = \frac{1}{6}x - \frac{7}{6}$

(vi) $(g \circ f)(x) = y = 6x+1$. Interchange the variables and solve for y:
$x = 6y+1 \Rightarrow (f \circ g)^{-1}(x) = \frac{1}{6}x - \frac{1}{6}$

(vii) $\left(f^{-1} \circ g^{-1}\right)(x) = \frac{1}{2}\left(\frac{1}{3}x - \frac{4}{3}\right) + \frac{1}{2}$
$= \frac{1}{6}x - \frac{2}{3} + \frac{1}{2} = \frac{1}{6}x - \frac{1}{6}$

(viii) $\left(g^{-1} \circ f^{-1}\right)(x) = \frac{1}{3}\left(\frac{1}{2}x + \frac{1}{2}\right) - \frac{4}{3}$
$= \frac{1}{6}x + \frac{1}{6} - \frac{4}{3} = \frac{1}{6}x - \frac{7}{6}$

b. **(i)** $(f \circ g)^{-1}(x) = \frac{1}{6}x - \frac{7}{6}$
$= \left(g^{-1} \circ f^{-1}\right)(x)$

(ii) $(g \circ f)^{-1}(x) = \frac{1}{6}x - \frac{1}{6}$
$= \left(f^{-1} \circ g^{-1}\right)(x)$

89.a. **(i)** $f(x) = y = 2x+3$. Interchange the variables and solve for y: $x = 2y+3 \Rightarrow$
$y = f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}$

(ii) $g(x) = y = x^3 - 1$. Interchange the variables and solve for y: $x = y^3 - 1 \Rightarrow$
$y = g^{-1}(x) = \sqrt[3]{x+1}$

(iii) $(f \circ g)(x) = 2\left(x^3 - 1\right) + 3 = 2x^3 + 1$

(iv) $(g \circ f)(x) = (2x+3)^3 - 1$
$= 8x^3 + 36x^2 + 54x + 26$

(v) $(f \circ g)(x) = y = 2x^3 + 1$. Interchange the variables and solve for y:
$x = 2y^3 + 1 \Rightarrow (f \circ g)^{-1}(x) = \sqrt[3]{\frac{x-1}{2}}$

(vi) $(g \circ f)(x) = y$
$= 8x^3 + 36x^2 + 54x + 26$
Interchange the variables and solve for y:
$x = 8y^3 + 36y^2 + 54y + 26 \Rightarrow$
$x + 1 = 8y^3 + 36y^2 + 54y + 27 \Rightarrow$
$x + 1 = (2y+3)^3 \Rightarrow \sqrt[3]{x+1} = 2y + 3 \Rightarrow$
$y = (g \circ f)^{-1}(x) = \frac{1}{2}\sqrt[3]{x+1} - \frac{3}{2}$

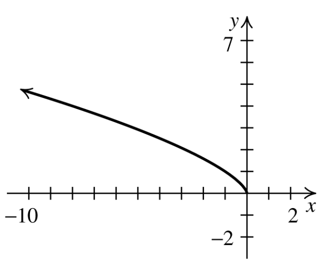
(vii) $\left(f^{-1} \circ g^{-1}\right)(x) = \frac{1}{2}\left(\sqrt[3]{x+1}\right) - \frac{3}{2}$

(viii) $\left(g^{-1} \circ f^{-1}\right)(x) = \sqrt[3]{\frac{1}{2}x - \frac{3}{2} + 1}$
$= \sqrt[3]{\frac{1}{2}x - \frac{1}{2}} = \sqrt[3]{\frac{x-1}{2}}$

b. (i) $(f \circ g)^{-1}(x) = \sqrt[3]{\frac{1}{2}x - \frac{1}{2}} = \sqrt[3]{\frac{x-1}{2}}$
$= \left(g^{-1} \circ f^{-1}\right)(x)$

(ii) $(g \circ f)^{-1}(x) = \frac{1}{2}\left(\sqrt[3]{x+1}\right) - \frac{3}{2}$
$= \left(f^{-1} \circ g^{-1}\right)(x)$

90. $f(x) = x^{2/3},\ x \le 0$



It is clear from the graph that the function satisfies the horizontal line test. Thus, the function is one-to-one. To find the inverse, interchange the variables and solve for *y*.
$f(x) = y = x^{2/3} \Rightarrow x = y^{2/3} \Rightarrow x^3 = y^2$

Now we must take the square root of each side. We choose the negative square root because *x* is restricted to those values less than or equal to zero. $x^3 = y^2 \Rightarrow -\sqrt{x^3} = y \Rightarrow$
$y = f^{-1}(x) = -x^{3/2},\ x \ge 0$

2.9 Critical Thinking/Discussion/Writing

91. No. For example, $f(x) = x^3 - x$ is odd, but it does not have an inverse, because $f(0) = f(1)$, so it is not one-to-one.

92. Yes. The function $f = \{(0,1)\}$ is even, and it has an inverse: $f^{-1} = \{(1,0)\}$.

93. Yes, because increasing and decreasing functions are one-to-one.

94.a. $R = \{(-1,1), (0,0), (1,1)\}$

b. $R = \{(-1,1), (0,0), (1,2)\}$

2.9 Maintaining Skills

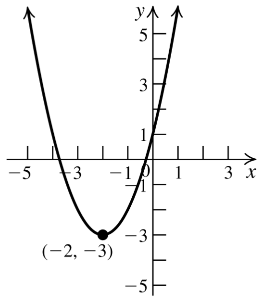
95. $x^2 - 7x + 12 = 0 \Rightarrow (x-3)(x-4) = 0 \Rightarrow$
$x = 3,\ 4$
Solution set: $\{3, 4\}$

96. $6x^2 + x - 2 = 0 \Rightarrow (3x+2)(2x-1) = 0 \Rightarrow$
$x = -\frac{2}{3},\ \frac{1}{2}$
Solution set: $\left\{-\frac{2}{3}, \frac{1}{2}\right\}$

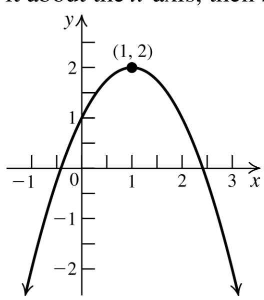
97. $12 - 3(x-1)^2 = 0 \Rightarrow 3\left(4 - (x-1)^2\right) = 0 \Rightarrow$
$4 - (x-1)^2 = 0 \Rightarrow$
$[2 + (x-1)][2 - (x-1)] = 0 \Rightarrow$
$(1+x)(3-x) = 0 \Rightarrow x = -1,\ 3$
Solution set: $\{-1, 3\}$

98. $x^2 - 4x + 1 = 0$
$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{12}}{2}$
$= 2 \pm \sqrt{3}$
Solution set: $\left\{2 \pm \sqrt{3}\right\}$

99. Shift the graph of $y = x^2$ two units left and three units down.



100. Shift the graph of $y = x^2$ one unit right, reflect it about the x-axis, then shift it two units up.



Chapter 2 Review Exercises

Basic Concepts and Skills

1. False. The midpoint is $\left(\frac{-3+3}{2}, \frac{1+11}{2}\right) = (0, 6)$.

2. False. The equation is a circle with center $(-2, -3)$ and radius $\sqrt{5}$.

3. True

4. False. A graph that is symmetric with respect to the origin is the graph of an odd function. A graph that is symmetric with respect to the y-axis is the graph of an even function.

5. False.
The slope is 4/3 and the y-intercept is 3.

6. False. The slope of a line that is perpendicular to a line with slope 2 is –1/2.

7. True

8. False. There is no graph because the radius cannot be negative.

9.a. $d(P, Q) = \sqrt{(-1-3)^2 + (3-5)^2} = 2\sqrt{5}$

b. $M = \left(\frac{3+(-1)}{2}, \frac{5+3}{2}\right) = (1, 4)$

c. $m = \frac{3-5}{-1-3} = \frac{1}{2}$

10.a. $d(P, Q) = \sqrt{(3-(-3))^2 + (-1-5)^2} = 6\sqrt{2}$

b. $M = \left(\frac{-3+3}{2}, \frac{5+(-1)}{2}\right) = (0, 2)$

c. $m = \frac{-1-5}{3-(-3)} = -1$

11.a. $d(P, Q) = \sqrt{(9-4)^2 + (-8-(-3))^2} = 5\sqrt{2}$

b. $M = \left(\frac{4+9}{2}, \frac{-3+(-8)}{2}\right) = \left(\frac{13}{2}, -\frac{11}{2}\right)$

c. $m = \frac{-8-(-3)}{9-4} = -1$

12.a. $d(P, Q) = \sqrt{(-7-2)^2 + (-8-3)^2} = \sqrt{202}$

b. $M = \left(\frac{2+(-7)}{2}, \frac{3+(-8)}{2}\right) = \left(-\frac{5}{2}, -\frac{5}{2}\right)$

c. $m = \frac{-8-3}{-7-2} = \frac{11}{9}$

13.a. $D(P, Q) = \sqrt{(5-2)^2 + (-2-(-7))^2} = \sqrt{34}$

b. $M = \left(\frac{2+5}{2}, \frac{-7+(-2)}{2}\right) = \left(\frac{7}{2}, -\frac{9}{2}\right)$

c. $m = \frac{-2-(-7)}{5-2} = \frac{5}{3}$

14.a. $d(P, Q) = \sqrt{(10-(-5))^2 + (-3-4)^2} = \sqrt{274}$

b. $M = \left(\frac{-5+10}{2}, \frac{4+(-3)}{2}\right) = \left(\frac{5}{2}, \frac{1}{2}\right)$

c. $m = \frac{-3-4}{10-(-5)} = -\frac{7}{15}$

15. $d(A, B) = \sqrt{(-2-0)^2 + (-3-5)^2} = \sqrt{68}$
$d(A, C) = \sqrt{(3-0)^2 + (0-5)^2} = \sqrt{34}$
$d(B, C) = \sqrt{(3-(-2))^2 + (0-(-3))^2} = \sqrt{34}$
Using the Pythagorean theorem, we have

$$AC^2 + BC^2 = \left(\sqrt{34}\right)^2 + \left(\sqrt{34}\right)^2 = 68 = \left(\sqrt{68}\right)^2 = AB^2$$

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Alternatively, we can show that AC and CB are perpendicular using their slopes.

$m_{AC} = \frac{0-5}{3-0} = -\frac{5}{3}; m_{CB} = \frac{0-(-3)}{3-(-2)} = \frac{3}{5}$

$m_{AC} \cdot m_{CB} = -1 \Rightarrow AC \perp CB,$ so $\triangle ABC$ is a right triangle.

16. $d(A,B) = \sqrt{(4-1)^2 + (8-2)^2} = 3\sqrt{5}$

$d(C,D) = \sqrt{(10-7)^2 + (5-(-1))^2} = 3\sqrt{5}$

$d(A,C) = \sqrt{(7-1)^2 + (-1-2)^2} = 3\sqrt{5}$

$d(B,D) = \sqrt{(10-4)^2 + (5-8)^2} = 3\sqrt{5}$

The four sides are equal, so the quadrilateral is a rhombus.

17. $A = (-6,3), B = (4,5)$

$d(A,O) = \sqrt{(-6-0)^2 + (3-0)^2} = \sqrt{45}$

$d(B,O) = \sqrt{(4-0)^2 + (5-0)^2} = \sqrt{41}$

(4, 5) is closer to the origin.

18. $A = (-6,4), B = (5,10), C = (2,3)$

$d(A,C) = \sqrt{(2-(-6))^2 + (3-4)^2} = \sqrt{65}$

$d(B,C) = \sqrt{(2-5)^2 + (3-10)^2} = \sqrt{58}$

(5, 10) is closer to (2, 3).

19. $A = (-5,3), B = (4,7), C = (x,0)$

$d(A,C) = \sqrt{(x-(-5))^2 + (0-3)^2} = \sqrt{(x+5)^2 + 9}$

$d(B,C) = \sqrt{(x-4)^2 + (0-7)^2} = \sqrt{(x-4)^2 + 49}$

$d(A,C) = d(B,C) \Rightarrow$

$\sqrt{(x+5)^2 + 9} = \sqrt{(x-4)^2 + 49}$

$(x+5)^2 + 9 = (x-4)^2 + 49$

$x^2 + 10x + 34 = x^2 - 8x + 65$

$x = \frac{31}{18} \Rightarrow$ The point is $\left(\frac{31}{18}, 0\right)$.

20. $A = (-3,-2), B(2,-1), C(0,y)$

$d(A,C) = \sqrt{(0-(-3))^2 + (y-(-2))^2} = \sqrt{(y+2)^2 + 9}$

$d(B,C) = \sqrt{(0-(2))^2 + (y-(-1))^2} = \sqrt{(y+1)^2 + 4}$

$d(A,C) = d(B,C) \Rightarrow$

$\sqrt{(y+2)^2 + 9} = \sqrt{(y+1)^2 + 4}$

$(y+2)^2 + 9 = (y+1)^2 + 4$

$y^2 + 4y + 13 = y^2 + 2y + 5$

$y = -4 \Rightarrow$ The point is $(0,-4)$.

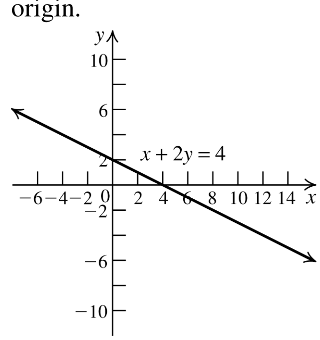
21. Not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.

22. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.

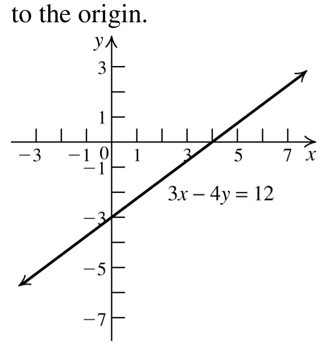
23. Symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.

24. Symmetric with respect to the x-axis; symmetric with respect to the y-axis; symmetric with respect to the origin.

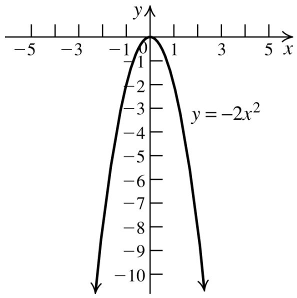
25. x-intercept: 4; y-intercept: 2; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.



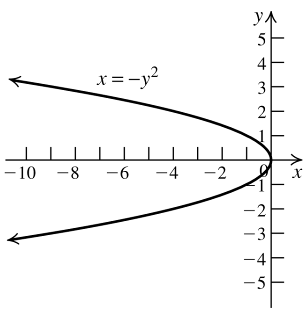
26. x-intercept: 4; y-intercept: –3; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.



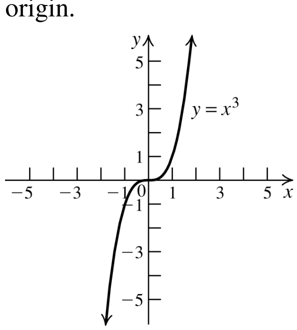
27. x-intercept: 0; y-intercept: 0; not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.



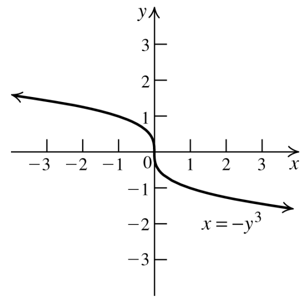
28. x-intercept: 0; y-intercept: 0; symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.



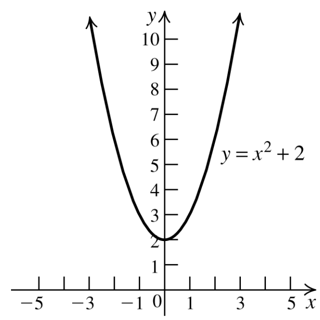
29. x-intercept: 0; y-intercept: 0; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.



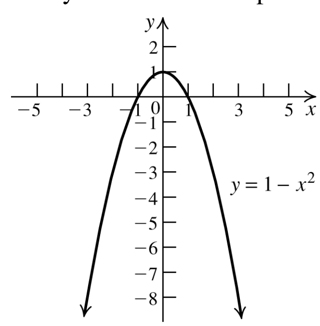
30. x-intercept: 0; y-intercept: 0; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.



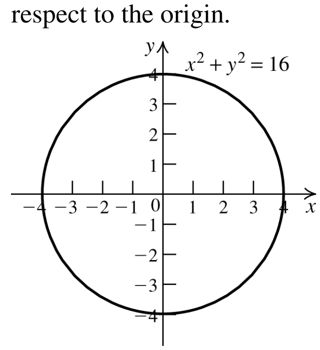
31. No x-intercept; y-intercept: 2; not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.



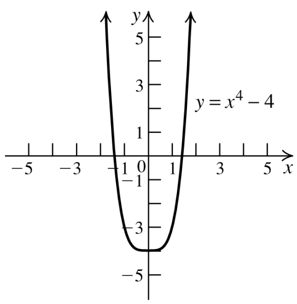
32. x-intercepts: –1, 1; y-intercept: 1;
not symmetric with respect to the x-axis;
symmetric with respect to the y-axis;
not symmetric with respect to the origin.



33. x-intercepts: –4, 4; y-intercepts: –4, 4; symmetric with respect to the x-axis; symmetric with respect to the y-axis; symmetric with respect to the origin.



34. x-intercepts: $-\sqrt{2}, \sqrt{2}$; y-intercept: -4
not symmetric with respect to the x-axis
symmetric with respect to the y-axis
not symmetric with respect to the origin.

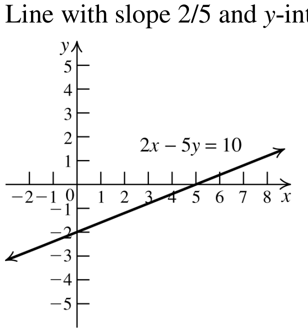


35. $(x-2)^2+(y+3)^2=25$

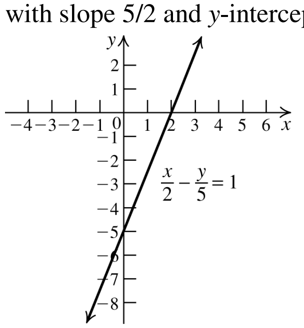
36. The center of the circle is the midpoint of the diameter. $M=\left(\frac{5+(-5)}{2}+\frac{2+4}{2}\right)=(0,3)$. The length of the radius is the distance from the center to one of the endpoints of the diameter = $\sqrt{(5-0)^2+(2-3)^2}=\sqrt{26}$. The equation of the circle is $x^2+(y-3)^2=26$.

37. The radius is 2, so the equation of the circle is $(x+2)^2+(y+5)^2=4$.

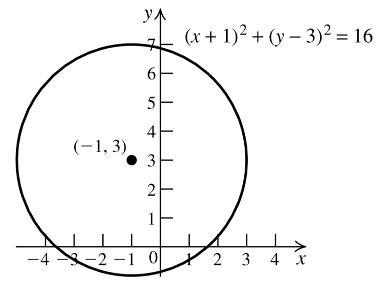
38. $2x-5y=10 \Rightarrow \frac{2}{5}x-2=y$.
Line with slope 2/5 and y-intercept –2.



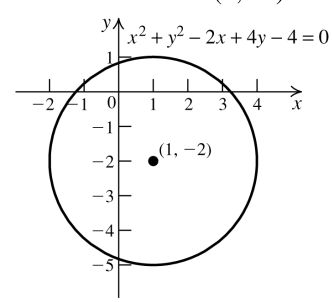
39. $\frac{x}{2}-\frac{y}{5}=1 \Rightarrow 5x-2y=10 \Rightarrow \frac{5}{2}x-5=y$. Line with slope 5/2 and y-intercept –5.



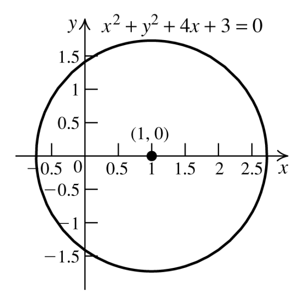
40. Circle with center (–1, 3) and radius 4.



41. $x^2+y^2-2x+4y-4=0 \Rightarrow$
$x^2-2x+1+y^2+4y+4=4+1+4 \Rightarrow$
$(x-1)^2+(y+2)^2=9$.
Circle with center (1, –2) and radius 3.



42. $3x^2+3y^2-6x-6=0 \Rightarrow x^2-2x+y^2=2 \Rightarrow$
$x^2-2x+1+y^2=2+1 \Rightarrow (x-1)^2+y^2=3$.
Circle with center (1, 0) and radius $\sqrt{3}$.



43. $y-2=-2(x-1) \Rightarrow y=-2x+4$

44. $m=\frac{5-0}{0-2}=-\frac{5}{2}$; $y=-\frac{5}{2}x+5$

45. $m=\frac{7-3}{-1-1}=-2; 3=-2(1)+b \Rightarrow 5=b \Rightarrow$
$y=-2x+5$

46. $x=1$

47.a. $y=3x-2 \Rightarrow m=3; y=3x+2 \Rightarrow m=3$ The slopes are equal, so the lines are parallel.

b. $3x - 5y + 7 \Rightarrow m = 3/5;$

$5x - 3y + 2 = 0 \Rightarrow m = 5/3$

The slopes are neither equal nor negative reciprocals, so the lines are neither parallel nor perpendicular.

c. $ax + by + c = 0 \Rightarrow m = -a/b;$

$bx - ay + d = 0 \Rightarrow m = b/a$

The slopes are negative reciprocals, so the lines are perpendicular.

d. $y + 2 = \frac{1}{3}(x - 3) \Rightarrow m = \frac{1}{3};$

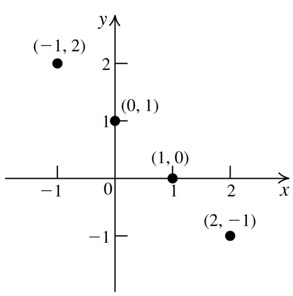
$y - 5 = 3(x - 3) \Rightarrow m = 3$

The slopes are neither equal nor negative reciprocals, so the lines are neither parallel nor perpendicular.

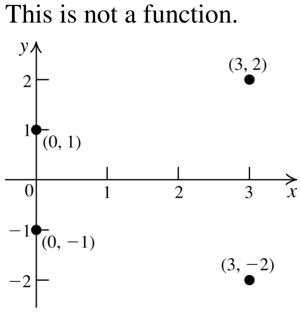
48.a. The equation with x-intercept 4 passes through the points (0, 2) and (4, 0), so its slope is $\frac{0-2}{4-0} = -\frac{1}{2}$. Thus, the slope of the line we are seeking is also $-\frac{1}{2}$. The line passes through (0, 1), so its equation is $y - 1 = -\frac{1}{2}(x - 0) \Rightarrow y = -\frac{1}{2}x + 1.$

b. The slope of the line we are seeking is 2 and the line passes through the origin, so its equation is $y - 0 = 2(x - 0)$, or $y = 2x$.

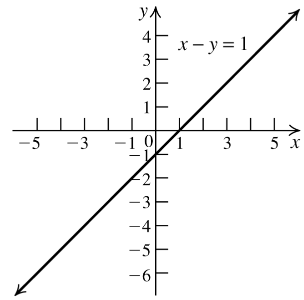
49. Domain: {–1, 0, 1, 2}; range: {–1, 0, 1, 2}. This is a function.



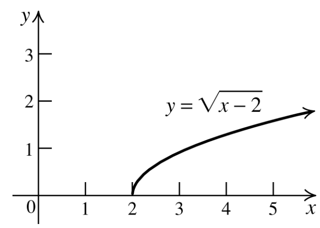
50. Domain: {0,3}; range: {–2, –1, 1, 2}. This is not a function.



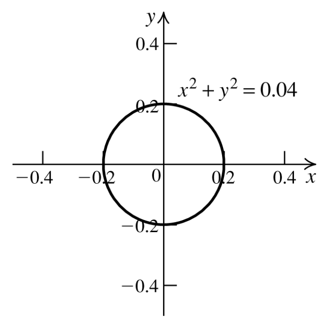
51. Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$. This is a function.



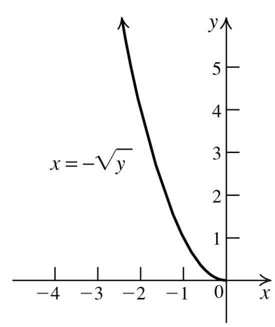
52. Domain: $[2, \infty)$; range: $[0, \infty)$. This is a function.



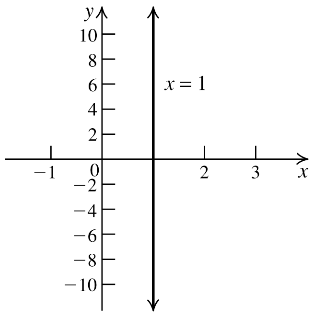
53. Domain: $[-0.2, 0.2]$; range: $[-0.2, 0.2]$. This is not a function.



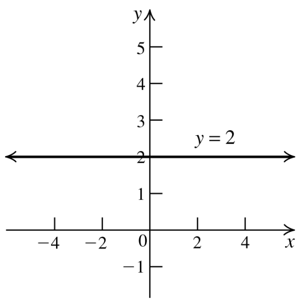
54. Domain: $(-\infty, 0]$; range: $[0, \infty)$. This is a function.



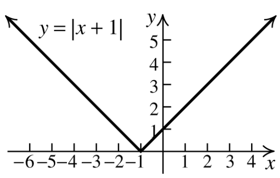
55. Domain: $\{1\}$; range: $(-\infty, \infty)$.
This is not a function.



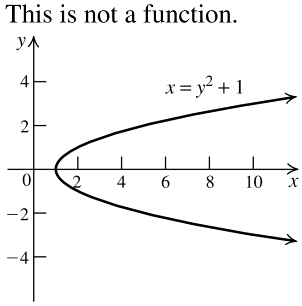
56. Domain: $(-\infty, \infty)$; range: $\{2\}$.
This is a function.



57. Domain: $(-\infty, \infty)$; range: $[0, \infty)$.
This is a function.



58. Domain: $[1, \infty)$; range: $(-\infty, \infty)$.
This is not a function.



59. $f(-2) = 3(-2) + 1 = -5$

60. $g(-2) = (-2)^2 - 2 = 2$

61. $f(x) = 4 \Rightarrow 3x + 1 = 4 \Rightarrow x = 1$

62. $g(x) = 2 \Rightarrow x^2 - 2 = 2 \Rightarrow x = \pm 2$

63. $$(f+g)(1) = f(1) + g(1) = (3(1)+1) + (1^2 - 2) = 3$$

64. $$(f-g)(-1) = f(-1) - g(-1) = (3(-1)+1) - ((-1)^2 - 2) = -1$$

65. $$(f \cdot g)(-2) = f(-2) \cdot g(-2) = (3(-2)+1) \cdot ((-2)^2 - 2) = -10$$

66. $$(g \cdot f)(0) = g(0) \cdot f(0) = (0^2 - 2) \cdot (3(0)+1) = -2$$

67. $(f \circ g)(3) = 3(3^2 - 2) + 1 = 22$

68. $(g \circ f)(-2) = (3(-2)+1)^2 - 2 = 23$

69. $(f \circ g)(x) = 3(x^2 - 2) + 1 = 3x^2 - 5$

70. $(g \circ f)(x) = (3x+1)^2 - 2 = 9x^2 + 6x - 1$

71. $(f \circ f)(x) = 3(3x+1) + 1 = 9x + 4$

72. $(g \circ g)(x) = (x^2 - 2)^2 - 2 = x^4 - 4x^2 + 2$

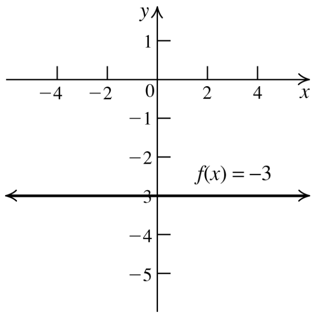
73. $f(a+h) = 3(a+h) + 1 = 3a + 3h + 1$

74. $g(a-h) = (a-h)^2 - 2 = a^2 - 2ah + h^2 - 2$

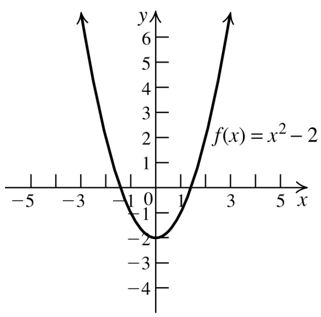
75. $$\frac{f(x+h) - f(x)}{h} = \frac{(3(x+h)+1) - (3x+1)}{h} = \frac{3x + 3h + 1 - 3x - 1}{h} = \frac{3h}{h} = 3$$

76. $$\frac{g(x+h) - g(x)}{h} = \frac{((x+h)^2 - 2) - (x^2 - 2)}{h} = \frac{x^2 + 2xh + h^2 - 2 - x^2 + 2}{h} = \frac{h^2 + 2xh}{h} = h + 2x$$

77. Domain: $(-\infty, \infty)$; range: $\{-3\}$.
Constant on $(-\infty, \infty)$.

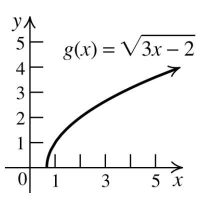


78. Domain: $(-\infty, \infty)$; range: $[-2, \infty)$.
Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$.

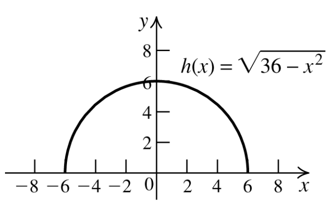


79. Domain: $\left[\frac{2}{3}, \infty\right)$; range: $[0, \infty)$

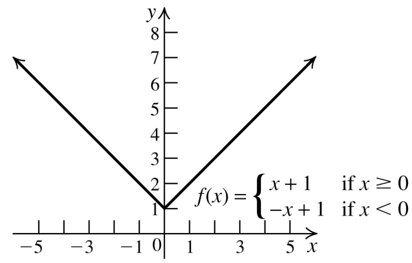
Increasing on $\left(\frac{2}{3}, \infty\right)$.



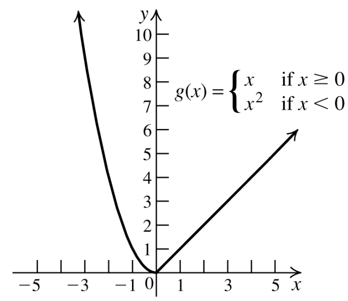
80. Domain: $[-6, 6]$; range: $[0, 6]$. Increasing on $(-6, 0)$; decreasing on $(0, 6)$.



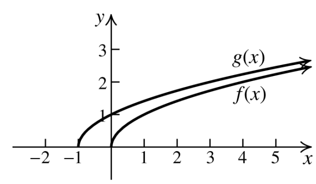
81. Domain: $(-\infty, \infty)$; range: $[1, \infty)$. Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$.



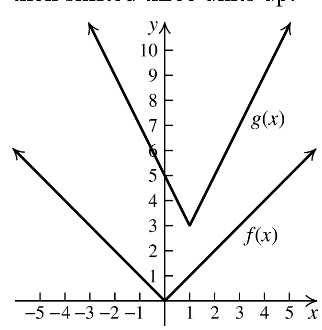
82. Domain: $(-\infty, \infty)$; range: $[0, \infty)$. Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$.



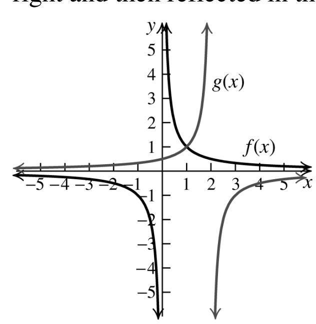
83. The graph of *g* is the graph of *f* shifted one unit left.



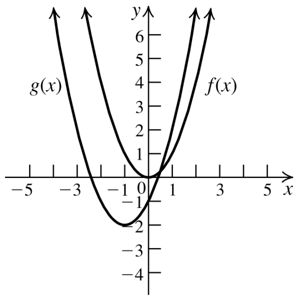
84. The graph of *g* is the graph of *f* shifted one unit right, stretched vertically by a factor of 2, and then shifted three units up.



85. The graph of *g* is the graph of *f* shifted two units right and then reflected in the *x*-axis.



86. The graph of g is the graph of f shifted one unit left and two units down.



87. $f(-x) = (-x)^2 - (-x)^4 = x^2 - x^4 = f(x) \Rightarrow$ $f(x)$ is even. Not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.

88. $f(-x) = (-x)^3 + (-x) = -x^3 - x = -f(x) \Rightarrow$ $f(x)$ is odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.

89. $f(-x) = |-x| + 3 = |x| + 3 = f(x) \Rightarrow$ $f(x)$ is even. Not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.

90. $f(-x) = -3x + 5 \neq f(x)$ or $f(-x) \Rightarrow f(x)$ is neither even nor odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.

91. $f(-x) = \sqrt{-x} \neq f(x)$ or $f(-x) \Rightarrow f(x)$ is neither even nor odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.

92. $f(-x) = -\dfrac{2}{x} = -f(x) \Rightarrow f(x)$ is odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.

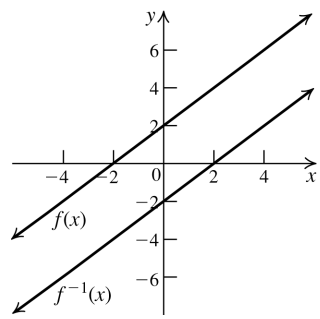
93. $f(x) = \sqrt{x^2 - 4} \Rightarrow f(x) = (g \circ h)(x)$ where $g(x) = \sqrt{x}$ and $h(x) = x^2 - 4$.

94. $g(x) = \left(x^2 - x + 2\right)^{50} \Rightarrow g(x) = (f \circ h)(x)$ where $f(x) = x^{50}$ and $h(x) = x^2 - x + 2$.

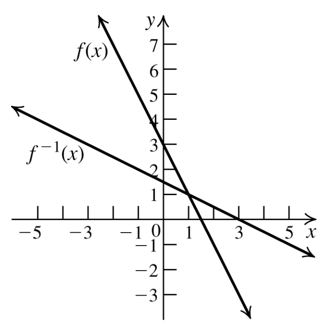
95. $h(x) = \sqrt{\dfrac{x-3}{2x+5}} \Rightarrow h(x) = (f \circ g)(x)$ where $f(x) = \sqrt{x}$ and $g(x) = \dfrac{x-3}{2x+5}$.

96. $H(x) = (2x-1)^3 + 5 \Rightarrow H(x) = (f \circ g)(x)$ where $f(x) = x^3 + 5$ and $g(x) = 2x - 1$.

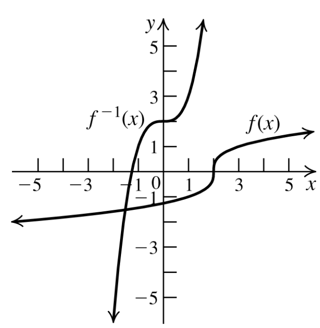
97. $f(x)$ is one-to-one. $f(x) = y = x + 2$. Interchange the variables and solve for y: $x = y + 2 \Rightarrow y = x - 2 = f^{-1}(x)$.



98. $f(x)$ is one-to-one. $f(x) = y = -2x + 3$. Interchange the variables and solve for y: $x = -2y + 3 \Rightarrow y = -\dfrac{1}{2}x + \dfrac{3}{2} = f^{-1}(x)$.



99. $f(x)$ is one-to-one. $f(x) = y = \sqrt[3]{x-2}$. Interchange the variables and solve for y: $x = \sqrt[3]{y-2} \Rightarrow y = x^3 + 2 = f^{-1}(x)$.

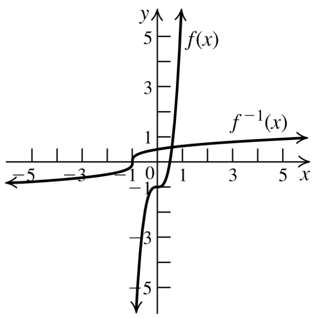


100. $f(x)$ is one-to-one. $f(x) = y = 8x^3 - 1.$

Interchange the variables and solve for y:

$x = 8y^3 - 1 \Rightarrow y = \sqrt[3]{\dfrac{x+1}{8}} \Rightarrow$

$y = \dfrac{1}{2}\sqrt[3]{x+1} = f^{-1}(x).$



101. $f(x) = y = \dfrac{x-1}{x+2}, x \neq 2.$

Interchange the variables and solve for y.

$x = \dfrac{y-1}{y+2} \Rightarrow xy + 2x = y - 1 \Rightarrow$

$xy - y = -2x - 1 \Rightarrow y(x-1) = -2x - 1 \Rightarrow$

$y = \dfrac{-2x-1}{x-1} \Rightarrow y = f^{-1}(x) = \dfrac{2x+1}{1-x}$

Domain of f: $(-\infty, -2) \cup (-2, \infty)$

Range of f: $(-\infty, 1) \cup (1, \infty)$

102. $f(x) = y = \dfrac{2x+3}{x-1}, x \neq 1.$

Interchange the variables and solve for y.

$x = \dfrac{2y+3}{y-1} \Rightarrow xy - x = 2y + 3 \Rightarrow$

$xy - 2y = x + 3 \Rightarrow y(x-2) = x + 3 \Rightarrow$

$y = f^{-1}(x) = \dfrac{x+3}{x-2}$

Domain of f: $(-\infty, 1) \cup (1, \infty)$

Range of f: $(-\infty, 2) \cup (2, \infty)$

103.a. $A = (-3, -3), B = (-2, 0), C = (0, 1), D = (3, 4).$

Find the equation of each segment:

$m_{AB} = \dfrac{0-(-3)}{-2-(-3)} = 3.0 = 3(-2) + b \Rightarrow b = 6.$

The equation of AB is $y = 3x + 6.$

$m_{BC} = \dfrac{1-0}{0-(-2)} = \dfrac{1}{2}; b = 1.$

The equation of BC is $y = \dfrac{1}{2}x + 1.$

$m_{CD} = \dfrac{4-1}{3-3} = 1; b = 1.$

The equation of CD is $y = x + 1.$

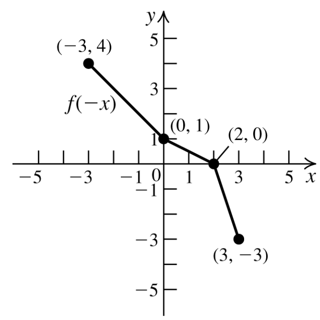
So,

$$f(x) = \begin{cases} 3x+6 & \text{if } -3 \le x \le -2 \\ \dfrac{1}{2}x + 1 & \text{if } -2 < x < 0 \\ x+1 & \text{if } 0 \le x \le 3 \end{cases}$$

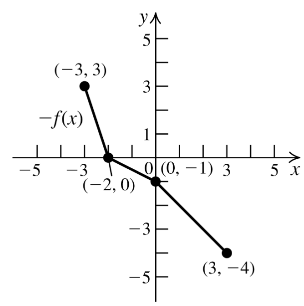
b. Domain: [–3,3]; range: [–3,4]

c. x-intercept: –2; y-intercept: 1

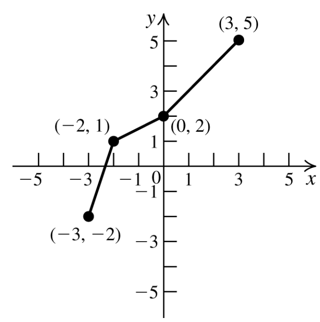
d.



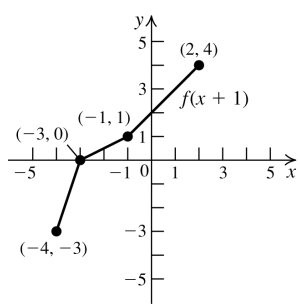
e.



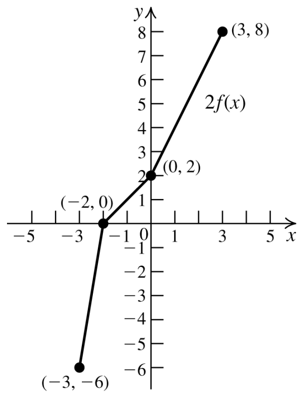
f.



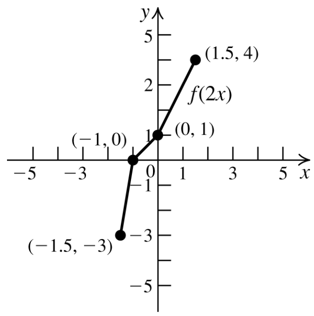
g.



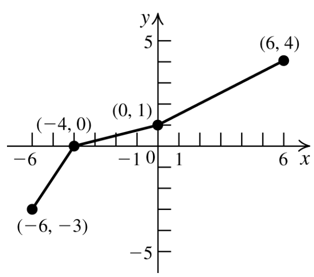
h.



i.

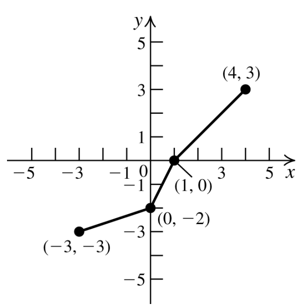


j.



k. *f* is one-to-one because it satisfies the horizontal line test.

l.



Applying the Concepts

104.a. rate of change (slope) = $\frac{25.95-19.2}{25-10}=0.45.$

$19.2=0.45(10)+b \Rightarrow b=14.7.$

The equation is $P=0.45d+14.7.$

b. The slope represents the amount of increase in pressure (in pounds per square inch) as the diver descends one foot deeper. The *y*-intercept represents the pressure at the surface of the sea.

c. $P=0.45(160)+14.7=86.7$ lb/in.^{2}

d. $104.7=0.45d+14.7 \Rightarrow 200$ feet

105.a. rate of change $(\text{slope})=\frac{173,000-54,000}{223,000-87,000}=0.875$

$54,000=0.875(87,000)+b \Rightarrow b=-22,125.$

The equation is $C=0.875w-22,125.$

b. The slope represents the cost to dispose of one pound of waste. The *x*-intercept represents the amount of waste that can be disposed with no cost. The *y*-intercept represents the fixed cost.

c. $C=0.875(609,000)-22,125=\$510,750$

d. $1,000,000=0.875w-22,125 \Rightarrow$
$w=1,168,142.86$ pounds

106.a. At 60 mph = 1 mile per minute, so if the speedometer is correct, the number of minutes elapsed is equal to the number of miles driven.

b. The odometer is based on the speedometer, so if the speedometer is incorrect, so is the odometer.

107.a. $f(2)=100+55(2)-3(2)^2=\$198.$

She started with \$100, so she won \$98.

b. She was winning at a rate of \$49/hour.

c. $0=100+55t-3t^2 \Rightarrow (-t+20)(3t+5) \Rightarrow$

$t=20,\ t=-5/3.$ Since *t* represents the amount of time, we reject $t=-5/3.$

Chloe will lose all her money after playing for 20 hours.

d. \$100/20 = \$5/hour.

108. If $100<x\le 500$, then the sales price per case is \$4 – 0.2(4) = \$3.20. The first 100 cases cost \$400.

$$f(x)=\begin{cases}4x & \text{if } 0\le x\le 100\\ 3.2x+80 & \text{if } 100<x\le 500\\ 3x+180 & \text{if } x>500\end{cases}$$

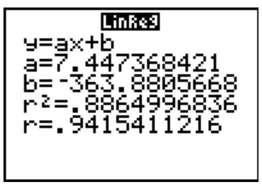
109.a. $(L \circ x)(t) = 0.5\sqrt{\left(1+0.002t^2\right)^2+4}$
$= 0.5\sqrt{0.000004t^4+0.004t^2+5}$

b. $(L \circ x)(5) = 0.5\sqrt{\left(1+0.002(5^2)\right)^2+4}$
$= 0.5\sqrt{(1.05)^2+4} = 0.5\sqrt{5.1025}$
≈ 1.13

110.a. Revenue = number of units × price per unit:
$x \bullet p = (5000+50t+10t^2)(10+0.5t)$
$= 5t^3+125t^2+3000t+50,000$

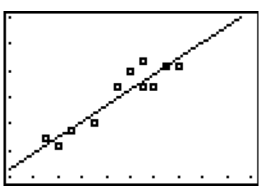
b. $p = 10+0.5t \Rightarrow t = 2p-20.$
$x(t) = x(2p-20)$
$= 5000+50(2p-20)+10(2p-20)^2$
$= 40p^2-700p+8000,$ which is the number of toys made at price p. The revenue is $p\left(40p^2-700p+8000\right) =$
$40p^3-700p^2+8000p.$

111.a.



$y \approx 7.4474x - 363.88$

b.



[70, 90, 2] by [150, 300, 25]

c. $y \approx 7.4474(76) - 363.88 \approx 202$
A player whose height is 76 inches weighs about 202 pounds.

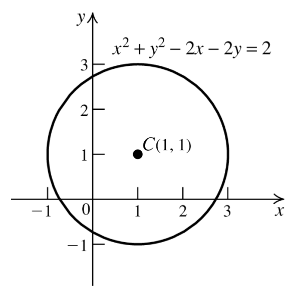
Chapter 2 Practice Test A

1. The endpoints of the diameter are (−2, 3) and (−4, 5), so the center of the circle is
$C = \left(\frac{-2+(-4)}{2}, \frac{3+5}{2}\right) = (-3, 4).$
The length of the diameter is
$\sqrt{\left(-4-(-2)\right)^2+(5-3)^2} = \sqrt{8} = 2\sqrt{2}.$
Therefore, the length of the radius is $\sqrt{2}.$
The equation of the circle is
$(x+3)^2+(y-4)^2 = 2.$

2. To test if the graph is symmetric with respect to the y-axis, replace x with $-x$:
$3(-x)+2(-x)y^2 = 1 \Rightarrow -3x-2xy^2 = 1,$ which is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. To test if the graph is symmetric with respect to the x-axis, replace y with $-y$:
$3x+2x(-y)^2 = 1 \Rightarrow 3x+2xy^2 = 1,$ which is the same as the original equation, so the graph is symmetric with respect to the x-axis. To test if the graph is symmetric with respect to the origin, replace x with $-x$ and y with $-y$:
$3(-x)+2(-x)(-y)^2 = 1 \Rightarrow -3x-2xy^2 = 1,$ which is not the same as the original equation, so the graph is not symmetric with respect to the origin.

3. $0 = x^2(x-3)(x+1) \Rightarrow x = 0$ or $x = 3$ or $x = -1$
$y = 0^2(0-3)(0+1) \Rightarrow y = 0.$ The x-intercepts are 0, 3, and −1; the y-intercept is 0.

4.



Intercepts:
$y^2-2y = 2 \Rightarrow y = 1 \pm \sqrt{3}$
$x^2-2x = 2 \Rightarrow x = 1 \pm \sqrt{3}$

5. $7 = -1(2)+b \Rightarrow 9 = b$
The equation is $y = -x + 9$.

6. $8x-2y = 7 \Rightarrow y = 4x - \frac{7}{2} \Rightarrow$ the slope of the line is 4. $-1 = 4(2)+b \Rightarrow b = -9.$ So the equation is $y = 4x-9.$

7. $(fg)(2) = f(2)\cdot g(2)$
$= \left(-2(2)+1\right)\left(2^2+3(2)+2\right)$
$= (-3)(12) = -36$

8. $g(f(2)) = g(2(2)-3) = g(1) = 1-2(1)^2 = -1$

9. $\left(f \circ f\right)(x) = (x^2 - 2x)^2 - 2(x^2 - 2x)$
$= x^4 - 4x^3 + 4x^2 - 2x^2 + 4x$
$= x^4 - 4x^3 + 2x^2 + 4x$

10.a. $f(-1) = (-1)^3 - 2 = -3$

b. $f(0) = 0^3 - 2 = -2$

c. $f(1) = 1 - 2(1)^2 = -1$

11. $1 - x > 0 \Rightarrow x < 1;\, x$ must also be greater than or equal to 0, so the domain is [0, 1).

12. $\dfrac{f(4) - f(1)}{4 - 1} = \dfrac{\left(2(4)+7\right) - \left(2(1)+7\right)}{3} = 2$

13. $f(-x) = 2(-x)^4 - \dfrac{3}{(-x)^2} = 2x^4 - \dfrac{3}{x^2} = f(x) \Rightarrow$
$f(x)$ is even.

14. Increasing on $(-\infty, 0)$ and $(2, \infty)$; decreasing on $(0, 2)$.

15. Shift the graph of $y = \sqrt{x}$ three units to the right, then stretch the graph vertically by a factor of 2, and then shift the resulting graph four units up.

16. $25 = 25 - (2t - 5)^2 \Rightarrow 0 = -(2t - 5)^2 \Rightarrow$
$0 = 2t - 5 \Rightarrow t = 5/2 = 2.5$ seconds

17. $f\left(2\right) = 7 \Rightarrow f^{-1}\left(7\right) = 2$

18. $f(x) = y = \dfrac{2x}{x - 1}$. Interchange the variables and solve for y: $x = \dfrac{2y}{y - 1} \Rightarrow$
$xy - x = 2y \Rightarrow xy - 2y = x \Rightarrow$
$y(x - 2) = x \Rightarrow y = f^{-1}(x) = \dfrac{x}{x - 2}$

19. $A(x) = 100x + 1000$

20.a. $C(230) = 0.25(230) + 30 = \$87.50$

b. $57.50 = 0.25m + 30 \Rightarrow m = 110$ miles

Chapter 2 Practice Test B

1. To test if the graph is symmetric with respect to the y-axis, replace x with $-x$:
$\left|-x\right| + 2\left|y\right| = 2 \Rightarrow \left|x\right| + 2\left|y\right| = 2,$ which is the same as the original equation, so the graph is symmetric with respect to the y-axis. To test if the graph is symmetric with respect to the x-axis, replace y with $-y$:
$\left|x\right| + 2\left|-y\right| = 2 \Rightarrow \left|x\right| + 2\left|y\right| = 2,$ which is the same as the original equation, so the graph is symmetric with respect to the x-axis. To test if the graph is symmetric with respect to the origin, replace x with $-x$, and y with $-y$:
$\left|-x\right| + 2\left|-y\right| = 2 \Rightarrow \left|x\right| + 2\left|y\right| = 2,$ which is the same as the original equation, so the graph is symmetric with respect to the origin. The answer is D.

2. $0 = x^2 - 9 \Rightarrow x = \pm 3;\, y = 0^2 - 9 \Rightarrow y = -9.$ The x-intercepts are ±3; the y-intercept is –9. The answer is B.

3. D **4.** D **5.** C

6. Suppose the coordinates of the second point are (a, b). Then $-\dfrac{1}{2} = \dfrac{b - 2}{a - 3}$. Substitute each of the points given into this equation to see which makes it true. The answer is C.

7. Find the slope of the original line:
$6x - 3y = 5 \Rightarrow y = 2x - \dfrac{5}{3}$. The slope is 2. The equation of the line with slope 2, passing through (–1, 2) is $y - 2 = 2(x + 1)$.
The answer is D.

8. $\left(f \circ g\right)(x) = 3\left(2 - x^2\right) - 5 = 1 - 3x^2.$
The answer is B.

9. $\left(f \circ f\right)(x) = 2\left(2x^2 - x\right)^2 - \left(2x^2 - x\right)$
$= 8x^4 - 8x^3 + x$
The answer is A.

10. $g(a - 1) = \dfrac{1 - (a - 1)}{1 + (a - 1)} = \dfrac{2 - a}{a}.$
The answer is C.

11. $1 - x \geq 0 \Rightarrow x \leq 1;$ x must also be greater than or equal to 0, so the domain is [0, 1].
The answer is A.

12. $x^2+3x-4=6 \Rightarrow x^2+3x-10=0 \Rightarrow$
$(x+5)(x-2)=0 \Rightarrow x=-5,\ 2$
The answer is D.

13. A **14.** A **15.** B

16. D **17.** C

18. $f(x)=y=\dfrac{x}{3x+2}$. Interchange the variables and solve for y: $x=\dfrac{y}{3y+2} \Rightarrow$
$3xy+2x=y \Rightarrow 3xy-y=-2x \Rightarrow$
$y(3x-1)=-2x \Rightarrow y=f^{-1}(x)=-\dfrac{2x}{3x-1} \Rightarrow$
$f^{-1}(x)=\dfrac{2x}{1-3x}$
The answer is C.

19. $w=5x-190; w=5(70)-190=160.$
The answer is B.

20. $50=0.2m+25 \Rightarrow m=125.$ The answer is A.

Cumulative Review Exercises (Chapters P–2)

1.a. $\left(\dfrac{x^3}{y^2}\right)^2\left(\dfrac{y^2}{x^3}\right)^3=\left(\dfrac{x^6}{y^4}\right)\left(\dfrac{y^6}{x^9}\right)=\dfrac{y^2}{x^3}$

b. $\dfrac{x^{-1}y^{-1}}{x^{-1}+y^{-1}}=\dfrac{\frac{1}{x}\cdot\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}}=\dfrac{\frac{1}{xy}}{\frac{y+x}{xy}}=\dfrac{1}{x+y}$

2.a. $2x^2+x-15=(2x-5)(x+3)$

b. $x^3-2x^2+4x-8=x^2(x-2)+4(x-2)$
$=(x^2+4)(x-2)$

3.a. $\sqrt{75}+\sqrt{108}-\sqrt{192}=5\sqrt{3}+6\sqrt{3}-8\sqrt{3}$
$=3\sqrt{3}$

b. $\dfrac{x-1}{x+1}-\dfrac{x-2}{x+2}=\dfrac{(x-1)(x+2)-(x-2)(x+1)}{(x+1)(x+2)}$
$=\dfrac{(x^2+x-2)-(x^2-x-2)}{(x+1)(x+2)}$
$=\dfrac{2x}{(x+1)(x+2)}$

4.a. $\dfrac{1}{2+\sqrt{3}}=\dfrac{1}{2+\sqrt{3}}\cdot\dfrac{2-\sqrt{3}}{2-\sqrt{3}}=\dfrac{2-\sqrt{3}}{4-3}=2-\sqrt{3}$

b. $\dfrac{1}{\sqrt{5}-2}=\dfrac{1}{\sqrt{5}-2}\cdot\dfrac{\sqrt{5}+2}{\sqrt{5}+2}=\dfrac{\sqrt{5}+2}{5-4}=\sqrt{5}+2$

5.a. $3x-7=5 \Rightarrow 3x=12 \Rightarrow x=4$

b. $\dfrac{1}{x-1}=\dfrac{3}{x-1} \Rightarrow$ There is no solution.

6.a. $x^2-3x=0 \Rightarrow x(x-3)=0 \Rightarrow$
$x=0$ or $x=3$

b. $x^2+3x-10=0 \Rightarrow (x+5)(x-2)=0 \Rightarrow$
$x=-5$ or $x=2$

7.a. $2x^2-x+3=0 \Rightarrow x=\dfrac{1\pm\sqrt{1-4(2)(3)}}{2(2)} \Rightarrow$
$x=\dfrac{1\pm\sqrt{-23}}{4} \Rightarrow x=\dfrac{1\pm i\sqrt{23}}{4}$

b. $4x^2-12x+9=0 \Rightarrow (2x-3)^2=0 \Rightarrow x=\dfrac{3}{2}$

8.a. $x-6\sqrt{x}+8=0 \Rightarrow \left(\sqrt{x}-4\right)\left(\sqrt{x}-2\right)=0 \Rightarrow$
$\sqrt{x}=4 \Rightarrow x=16$ or $\sqrt{x}=2 \Rightarrow x=4$

b. $\left(x-\dfrac{1}{x}\right)^2-10\left(x-\dfrac{1}{x}\right)+21=0.$
Let $u=x-\dfrac{1}{x}$.
$u^2-10u+21=0 \Rightarrow$
$(u-7)(u-3)=0 \Rightarrow u=7 \cup u=3;$
$x-\dfrac{1}{x}=7 \Rightarrow x^2-1=7x \Rightarrow$
$x^2-7x-1=0 \Rightarrow x=\dfrac{7\pm\sqrt{7^2-(4)(-1)}}{2} \Rightarrow$
$x=\dfrac{7\pm\sqrt{53}}{2};\ x-\dfrac{1}{x}=3 \Rightarrow x^2-1=3x \Rightarrow$
$x^2-3x-1=0 \Rightarrow x=\dfrac{3\pm\sqrt{3^2-4(-1)}}{2} \Rightarrow$
$x=\dfrac{3\pm\sqrt{13}}{2}$
The solution set is
$\left\{\dfrac{7-\sqrt{53}}{2},\dfrac{7+\sqrt{53}}{2},\dfrac{3-\sqrt{13}}{2},\dfrac{3+\sqrt{13}}{2}\right\}.$

9.a. $\sqrt{3x-1}=2x-1 \Rightarrow 3x-1=(2x-1)^2 \Rightarrow$
$3x-1=4x^2-4x+1 \Rightarrow 4x^2-7x+2=0 \Rightarrow$
$x=\dfrac{7\pm\sqrt{(-7)^2-4(4)(2)}}{2(4)}=\dfrac{7\pm\sqrt{17}}{8}$. If
$x=\dfrac{7-\sqrt{17}}{8}, \sqrt{3\left(\dfrac{7-\sqrt{17}}{8}\right)-1}\approx 0.281$ while
$2\left(\dfrac{7-\sqrt{17}}{8}\right)-1\approx -0.281$, so the solution set
is $\left\{\dfrac{7+\sqrt{17}}{8}\right\}$.

b.
$$\sqrt{1-x}=2-\sqrt{2x+1}$$
$$\left(\sqrt{1-x}\right)^2=\left(2-\sqrt{2x+1}\right)^2$$
$$1-x=4-4\sqrt{2x+1}+2x+1$$
$$-4-3x=-4\sqrt{2x+1}$$
$$(-4-3x)^2=\left(-4\sqrt{2x+1}\right)^2$$
$$16+24x+9x^2=16(2x+1)$$
$$16+24x+9x^2=32x+16$$
$$9x^2-8x=0 \Rightarrow x(9x-8)=0$$
$$x=0 \text{ or } x=\frac{8}{9}.$$
Check to make sure that neither solution is extraneous. The solution set is $\{0, 8/9\}$.

10.a. $2x-5<11 \Rightarrow x<8 \Rightarrow (-\infty, 8)$

b. $-3x+4>-5 \Rightarrow x<3 \Rightarrow (-\infty, 3)$

11.a. $-3<2x-3<5 \Rightarrow 0<2x<8 \Rightarrow 0<x<4.$
The solution set is (0, 4).

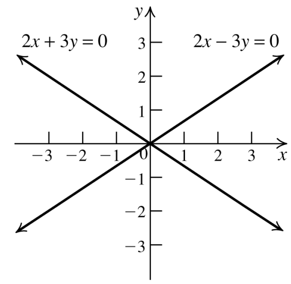
b. $5\le 1-2x\le 7 \Rightarrow 4\le -2x\le 6 \Rightarrow -2\ge x\ge -3.$
The solution set is [–3, –2].

12.a. $|2x-1|\le 7 \Rightarrow 2x-1\le 7 \Rightarrow x\le 4$
or $2x-1\ge -7 \Rightarrow x\ge -3.$
The solution set is [–3, 4].

b. $|2x-3|\ge 5 \Rightarrow 2x-3\ge 5 \Rightarrow x\ge 4$ or
$2x-3\le -5 \Rightarrow x\le -1.$
The solution set is $(-\infty,-1]\cup[4,\infty)$.

13. $d(A,C)=\sqrt{(2-5)^2+(2-(-2))^2}=5$
$d(B,C)=\sqrt{(2-6)^2+(2-5)^2}=5$
Since the lengths of the two sides are equal, the triangle is isosceles.

14.



15. First, find the equation of the circle with center (2, –1) and radius determined by (2, –1) and (–3, –1): $r=\sqrt{2-(-3))^2+(-1-(-1))^2}=5$.
The equation is $(x-2)^2+(y+1)^2=5^2$. Now check to see if the other three points satisfy the equation: $(2-2)^2+(4+1)^2=5^2 \Rightarrow 5^2=5^2$,
$(5-2)^2+(3+1)^2=5^2 \Rightarrow 3^2+4^2=5^2$ (true because 3, 4, 5 is a Pythagorean triple), and
$(6-2)^2+(2+1)^2=5^2 \Rightarrow 4^2+3^2=5^2.$
Since all the points satisfy the equation, they lie on the circle.

16. $x^2+y^2-6x+4y+9=0 \Rightarrow$
$x^2-6x+y^2+4y=-9.$
Now complete both squares:
$x^2-6x+9+y^2+4y+4=-9+9+4 \Rightarrow$
$(x-3)^2+(y+2)^2=4.$
The center is (3, –2) and the radius is 2.

17. $y=-3x+5$

18. The *x*-intercept is 4, so (4, 0) satisfies the equation. To write the equation in slope-intercept form, find the *y*-intercept:
$0=2(4)+b \Rightarrow -8=b$
The equation is $y=2x-8$.

19. The slope of the perpendicular line is the negative reciprocal of the slope of the original line. The slope of the original line is 2, so the slope of the perpendicular is –1/2. Now find the *y*-intercept of the perpendicular:
$-1=-\dfrac{1}{2}(2)+b \Rightarrow b=0.$ The equation of the perpendicular is $y=-\dfrac{1}{2}x.$

20. The slope of the parallel line is the same as the slope of the original line, 2. Now find the y-intercept of the parallel line: $-1 = 2(2) + b \Rightarrow b = -5$. The equation of the parallel line is $y = 2x - 5$.

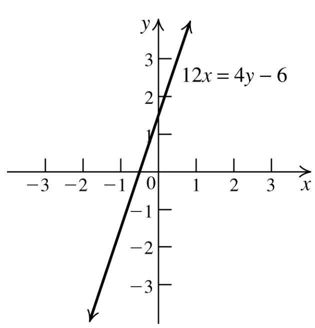
21. The slope of the perpendicular line is the negative reciprocal of the slope of the original line. The slope of the original line is $\frac{7-(-1)}{5-3} = 4$, so the slope of the perpendicular is –1/4. The perpendicular bisector passes through the midpoint of the original segment. The midpoint is $\left(\frac{3+5}{2}, \frac{-1+7}{2}\right) = (4, 3)$. Use this point and the slope to find the y-intercept: $3 = -\frac{1}{4}(4) + b \Rightarrow b = 4$. The equation of the perpendicular bisector is $y = -\frac{1}{4}x + 4$.

22. The slope is undefined because the line is vertical. Because it passes through (5, 7), the equation of the line is $x = 5$.

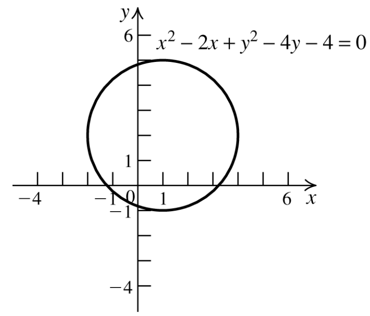
23. Use the slope formula to solve for x:
$2 = \frac{5-11}{x-5} \Rightarrow 2(x-5) = -6 \Rightarrow 2x - 10 = -6 \Rightarrow$
$x = 2$

24. The line through (x, 3) and (3, 7) has slope –2 because it is perpendicular to a line with slope 1/2. Use the slope formula to solve for x:
$-2 = \frac{3-7}{x-3} \Rightarrow -2(x-3) = -4 \Rightarrow x - 3 = 2 \Rightarrow$
$x = 5$

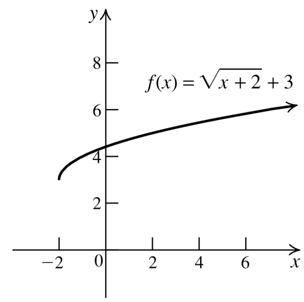
25.



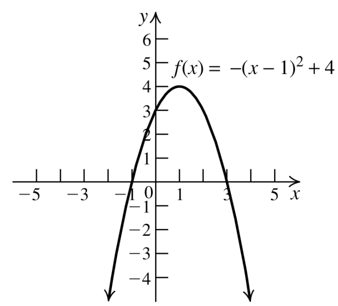
26.



27.



28.



29. Let x = the number of books initially purchased, and $\frac{1650}{x} =$ the cost of each book. Then x – 16 = the number of books sold, and $\frac{1650}{x-16} =$ the selling price of each book. The profit = the selling price – the cost, so
$\frac{1650}{x-16} - \frac{1650}{x} = 10 \Rightarrow$
$1650x - 1650(x-16) = 10x(x-16) \Rightarrow$
$1650x - 1650x + 26,400 = 10x^2 - 160x \Rightarrow$
$10x^2 - 160x - 26,400 = 0 \Rightarrow$
$x^2 - 16x - 2640 = 0 \Rightarrow (x-60)(x+44) = 0 \Rightarrow$
$x = 60,\ x = -44$. Reject –44 because there cannot be a negative number of books. So she bought 60 books.

30. Let x = the monthly note on the 1.5 year lease, and $1.5(12)x = 18x$ = the total expense for the 1.5 year lease. Then $x - 250$ = the monthly note on the 2 year lease, and $2(12)(x - 250) = 24x - 6000$ the total expenses for the 2 year lease. Then $18x + 24x - 6000 = 21,000 \Rightarrow$ $42x = 27,000 \Rightarrow x = 642.86.$ So the monthly note for the 1.5 year lease is \$642.86, and the monthly note for the 2 year lease is \$642.86 – 250 = \$392.86.

31.a. The domain of f is the set of all values of x which make $x+1 \geq 0$ (because the square root of a negative number is not a real value.) So $x \geq -1$ or $[-1, \infty)$ in interval notation is the domain.

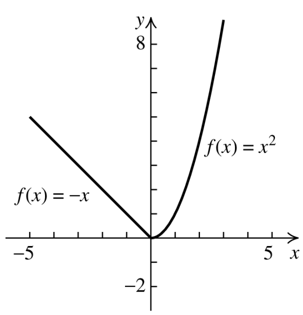
b. $y = \sqrt{0+1} - 3 \Rightarrow y = -2; 0 = \sqrt{x+1} - 3 \Rightarrow$ $3 = \sqrt{x+1} \Rightarrow 9 = x+1 \Rightarrow 8 = x.$ The x-intercept is 8, and the y-intercept is –2.

c. $f(-1) = \sqrt{-1+1} - 3 = -3$

d. $f(x) > 0 \Rightarrow \sqrt{x+1} - 3 > 0 \Rightarrow \sqrt{x+1} > 3 \Rightarrow$ $x+1 > 9 \Rightarrow x > 8.$ In interval notation, this is $(8, \infty).$

32.a. $f(-2) = -(-2) = 2; f(0) = 0^2 = 0;$ $f(2) = 2^2 = 4$

b. f decreases on $(-\infty, 0)$ and increases on $(0, \infty).$



33.a. $(f \circ g)(x) = \dfrac{1}{\frac{2}{x} - 2} = \dfrac{1}{\frac{2-2x}{x}} = \dfrac{x}{2-2x}.$

Because 0 is not in the domain of g, it must be excluded from the domain of $(f \circ g)$.

Because 2 is not in the domain of f, any values of x for which $g(x) = 2$ must also be excluded from the domain of

$(f \circ g): \dfrac{2}{x} = 2 \Rightarrow x = 1,$ so 1 is excluded also. The domain of $(f \circ g)$ is $(-\infty, 0) \cup (0, 1) \cup (1, \infty).$

b. $(g \circ f)(x) = \dfrac{2}{\frac{1}{x-2}} = 2(x-2) = 2x - 4.$

Because 2 is not in the domain of f, it must be excluded from the domain of $(g \circ f)$.

Because 0 is not in the domain of g, any values of x for which $f(x) = 0$ must also be excluded from the domain of $(g \circ f)$.

However, there is no value for x which makes $f(x) = 0$. So the domain of $(g \circ f)$ is $(-\infty, 2) \cup (2, \infty).$