
2 Motion

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Overview

This chapter primarily contains the patterns of motion developed by Isaac Newton (A.D. 1642–1727). Newton made many contributions to science, but his three laws of motion and his law of gravitation are the most famous. The three laws of motion are concerned with (1) what happens to the motion of a single object when no unbalanced forces are involved, (2) the relationship between the force, the mass of an object, and the resulting change of motion when an unbalanced force *is* involved, and (3) the relationship between the force experienced by two objects when they interact. The laws of motion are universal, that is, they apply

throughout the known universe and describe *all* motion. Throughout the universe mass is a measure of inertia, and inertia exists everywhere. A change of motion, acceleration, always results from an unbalanced force everywhere in the known universe. Finally, forces of the universe always come in pairs. Of the two forces one force is always equal in magnitude but opposite in direction to the other. The law of gravitation is also applicable throughout the known universe. All objects in the Solar System — the sun and the planets, the earth and its moon, and all orbiting satellites — obey the law of gravitation. Relativistic considerations should not be mentioned at this time. Concentrate on Newton's laws of motion, not Einstein's modifications of them.

The key to understanding patterns of motion is to understand simultaneously the ideas represented in the three laws of motion. These areas follow:

1. Inertia is the resistance to a change in the state of motion of an object in the absence of an unbalanced force. An object at rest remains at rest and an object moving in a straight line retains its straight-line motion in the absence of an unbalanced force. The analysis of why a ball moving across a smooth floor comes to a stop, as presented in the previous chapter, is an important part of the development of this concept. Newton's first law of motion is also known as the law of inertia.

2. Mass is defined as a measure of inertia, that is, a resistance to a change in the state of motion of an object. Thus the greater the mass the greater the resistance to a change in the state of motion of an object. Acceleration is a change in the state of motion of an object. According to the definition developed in the previous chapter, an object that speeds up, slows down, or changes its direction of travel is undergoing an acceleration. Students who have difficulty accepting the meanings of mass and acceleration often have less difficulty if they are told these are **definitions** of the quantities. A force is a push or a pull that is capable of causing a change in the state of motion of an object, that is, capable of producing an acceleration. The resulting acceleration is always in the same direction as the direction of the applied force. Newton's second law of motion is a relationship between mass, acceleration, and an unbalanced force that becomes clear when the conceptual meaning of these terms is understood. The relationship is that the greater the mass (inertia), the greater the force required to bring about a change in the state of motion (acceleration). In symbol form, the relationship is $a \propto F/m$, or the more familiar $F \propto ma$. Since a newton of force is *defined* in terms of a certain mass (1 kg) and a certain acceleration (1 m/s²), the units are the same on both sides and the relationship becomes an equation, or $F = ma$. This is an example of an equation that defines a concept (see chapter 1).

3. A single force never occurs alone; a force is always produced by the interaction of two or more objects. There is always a matched and opposite force that occurs at the same time, and Newton's second law of motion is a statement of this relationship.

Suggestions

1. The need for precision and exact understanding should be emphasized as the various terms such as speed, velocity, rate, distance, acceleration, and others are presented. Stress the reasoning behind each equation, for example, that velocity is a ratio that describes a property of objects in motion. Likewise, acceleration is a time rate of change of velocity, so $v_f - v_i/t$ not only makes sense but can be reasoned out rather than memorized. Also stress the need to show how units are handled in solving problems. The complete manipulation of units mathematically is stressed throughout this book. Typically students must be shown how unit work serves as a check on problem-solving steps. Students are sometimes confused by the use of the symbol “ v ” for both speed and velocity. Explain that speed is the same quantity as velocity but without direction, so the same symbol is used to simplify things. On the point of simplifying things, avoid the temptation to use calculus in any explanation or discussion.
2. Students are generally interested in “relative to what” questions concerning motion. For example, what is the speed of an insect flying at 5 mph from the front to the back of a bus moving at 50 mph? What do you observe happening to an object dropped inside an airplane moving at 600 mph? What would an observer outside the airplane observe happening to the object?
3. The discussion of what happens to a ball rolling across the floor is an important one, and many students who think from an “Aristotelian framework” are surprised by the analysis. When discussing the role of friction and objects moving on the earth’s surface, it is often interesting to ask why planets do not stop moving around the sun. Spur on the discussion by answering with another question, why should they stop? It might be helpful to review the meaning of vector arrows that represent forces.
4. Another way to consider acceleration is to ask, How fast does “how fast” change? If students have learned the concept of a ratio they will understand the concept of uniform straight-line motion. The acceleration concepts, however, require the use of a ratio within another ratio, that is, a change of velocity (a ratio within) per unit of time (the acceleration ratio). This understanding is necessary (along with some basic math skills) to understand the meaning of such units as m/s^2 .
5. Demonstrations that illustrate the characteristics of projectile motion are illustrated in several devices found in scientific catalogs. Among the most impressive is the “monkey and hunter” demonstration. Students enjoy this demonstration along with the humor that the instructor can induce while performing it.
6. There are many demonstrations and devices available from scientific suppliers that readily illustrate the laws of motion. However, none seems better than the personal experiences of students who have stood in the aisle of a bus as it starts moving, turns a corner, or comes to a stop. Use the three laws of motion to analyze the inertia, forces, and resulting changes of motion of a student standing in such an aisle of a bus.

7. Stress that weight and mass are two entirely different concepts. You will probably have to emphasize more than once that weight is another name for the gravitational force acting on an object, and that weight varies from place to place while mass does not. Use the second law of motion to show how weight can be used to calculate mass. A large demonstration spring scale calibrated in newtons can be used to show that a 1-kg mass weighs 9.8 N. Other masses can be weighed to show that weight and mass are proportional in a given location.
8. In solving problems involving the third law of motion, it is helpful for students to realize that a change in the state of motion always occurs in *the same direction* as the direction of an applied force. If you apply an unbalanced force on a ball toward the North, you would expect the ball to move toward the North. Thus if one starts walking toward the North a force must have been applied in the same direction. The foot pushed on the ground in the opposite direction, so it must be that the equal and opposite force of the ground pushing on the foot is what caused the motion toward the North. It seems almost anthropomorphic to state that the ground pushed on a foot, but no other answer is possible with this analysis. The next step, so to speak, is to realize that since the force of the foot on the ground equals the force of the ground on the foot (third law). Then the mass of the earth times the acceleration of the earth (second law) must equal the mass of the person times the acceleration of the person ($ma = ma$). This means at least two things: (1) that the earth must move when you walk across the surface (earth's acceleration must be greater than zero) and (2) that the earth would move with the same acceleration as the person if both had the same mass. Students are making progress when they can understand or make this kind of analysis.
9. A strong cord attached to a large coffee can half filled with water makes an interesting demonstration of centripetal force and inertia when whirled overhead. Practice this, however, before trying before a class.
10. Additional demonstrations:
 - (a) Show the stroboscopic effect as a means of measuring motion. Use a strobe light or hand stroboscopes, for example, to “stop” the motion of a spinning wheel of an upside-down bicycle.
 - (b) Roll a steel ball down a long ramp and mark the distance at the end of each second. Plot distance vs. time and distance vs. time squared to verify the acceleration equation.
 - (c) Crumple a sheet of paper tightly into a small ball. Drop the crumpled ball and a sheet of uncrumpled paper from the same height. Discuss which is accelerated at 9.8 m/s^2 and the roll of air resistance.
 - (d) Use the commercial apparatus that shoots or moves one ball horizontally and drops another ball vertically at the same time. A single “click” means that both balls hit the floor at the same time. This illustrates the independence of velocities.

- (e) Drop a small steel ball from the highest place practical into a tub of water. Make sure this is done on a day without wind and with no person near the tub. Time the fall with a stopwatch. Measure the vertical distance accurately, then find g from $d = 1/2gt^2$.
- (f) Use a spring scale to show that a 1.0-kg mass weighs 9.8 N. Use other masses to show that the weight of an object is always proportional to the mass in a given location.
- (g) Use an air track to illustrate Newton's first and second law of motion. If an air track is not available, consider a slab of ice or dry ice on a smooth demonstration tabletop. Wood blocks can be set on the ice to add mass.
- (h) Will a jet plane backed up to a brick wall take off faster than one out in the open? Compare the jet plane to a balloon filled with air, that is, a jet of escaping air propels the balloon. Thus, the movement is a consequence of Newton's third law and the brick wall will make no difference—a jet plane backed up to a brick will take off the same as an identical jet plane out in the open.
- (i) Seat yourself on a small cart with a CO₂ fire extinguisher or a bottle of compressed air from the shop. Hold the device between your feet and legs with the escape valve pointed away from your body. With the way clear behind you, carefully discharge a short burst of gas as you accelerate. This attention-grabber affords an opportunity to review all three of Newton's Laws of motion.
- (j) Demonstrate that the acceleration of a freely falling object is independent of weight. Use a commercial "free-fall tube" if one is available. If not, try a large-diameter 1-meter glass or plastic tube with a solid stopper in one end and a one-hole stopper in the other. Place a coin and a feather in the tube, then connect the one-hole stopper to a vacuum pump. Invert the tube to show how the coin and feather fall in air. Pump air from the tube then again invert to show the coin and feather in free fall.

For Class Discussions

1. Neglecting air resistance, a ball in freefall will have
 - a. constant speed and constant acceleration.
 - b. increasing speed and increasing acceleration.
 - c. increasing speed and decreasing acceleration.
 - d. increasing speed and constant acceleration.
 - e. decreasing speed and increasing acceleration.

2. Neglecting air resistance, a ball rolling down the slope of a steep hill will have
- constant speed and constant acceleration.
 - increasing speed and increasing acceleration.
 - increasing speed and decreasing acceleration.
 - increasing speed and constant acceleration.
 - decreasing speed and increasing acceleration.
3. Again neglecting air resistance, a ball thrown straight up will come to a momentary stop at the top of the path. What is the acceleration of the ball during this stop?
- 9.8 m/s^2 .
 - zero.
 - less than 9.8 m/s^2 .
 - more than 9.8 m/s^2 .
4. Again neglecting air resistance, the ball thrown straight up comes to a momentary stop at the top of the path, then falls for 1.0 s. What is speed of the ball after falling 1.0 s?
- 1 m/s
 - 4.9 m/s
 - 9.8 m/s
 - 19.6 m/s
5. Yet again neglecting air resistance, the ball thrown straight up comes to a momentary stop at the top of the path, then falls for 2.0 s. What distance did the ball fall during the 2.0 s?
- 1 m
 - 4.9 m
 - 9.8 m
 - 19.6 m
6. A ball is thrown straight up at the same time a ball is thrown straight down from a bridge, with the same initial speed. Neglecting air resistance, which ball would have a greater speed when it hits the ground?
- The one thrown straight up.
 - The one thrown straight down.
 - Both balls would have the same speed.

7. After being released, a ball thrown straight down from a bridge would have an acceleration of
- 9.8 m/s^2 .
 - zero.
 - less than 9.8 m/s^2 .
 - more than 9.8 m/s^2 .
8. A gun is aimed at an apple hanging from a tree. The instant the gun is fired the apple falls to the ground, and the bullet
- hits the apple.
 - arrives late, missing the apple.
 - may or may not hit the apple, depending on how fast it is moving.
9. You are at rest with a grocery cart at the supermarket, when you see an “opening” in a checkout line. You apply a certain force to the cart for a short time and acquire a certain speed. Neglecting friction, how long would you have to push with *half* the force to acquire the same final speed?
- one-fourth as long.
 - one-half as long.
 - for twice as long.
 - for four times as long.
10. Once again you are at rest with a grocery cart at the supermarket, when you apply a certain force to the cart for a short time and acquire a certain speed. Suppose you had bought more groceries, enough to double the mass of the groceries and cart. Neglecting friction, doubling the mass would have what effect on the resulting final speed if you used the same force for the same length of time? The new final speed would be
- one-fourth.
 - one-half.
 - doubled.
 - quadrupled.
11. You are moving a grocery cart at a constant speed in a straight line down the aisle of a store. The forces on the cart are
- unbalanced, in the direction of the movement.
 - balanced, with a net force of zero.
 - equal to the force of gravity acting on the cart.
 - greater than the frictional forces opposing the motion of the cart.

12. Considering the gravitational attraction between the Earth and Moon, the
- more massive Earth pulls harder on the less massive Moon.
 - less massive Moon pulls harder on the more massive Earth.
 - attraction between the Earth and Moon and the Moon and Earth are equal.
 - attraction varies with the Moon phase, being greatest at a full moon.
13. You are outside a store, moving a loaded grocery cart down the street on a very steep hill. It is difficult, but you are able to pull back on the handle and keep the cart moving down the street in a straight line and at a constant speed. The forces on the cart are
- unbalanced, in the direction of the movement.
 - balanced, with a net force of zero.
 - equal to the force of gravity acting on the cart.
 - greater than the frictional forces opposing the motion of the cart.
14. Which of the following must be true about a horse pulling a buggy?
- According to the third law of motion, the horse pulls on the buggy and the buggy pulls on the horse with an equal and opposite force. Therefore the net force is zero and the buggy cannot move.
 - Since they move forward, this means the horse is pulling harder on the buggy than the buggy is pulling on the horse.
 - The action force from the horse is quicker than the reaction force from the buggy, so the buggy moves forward.
 - The action-reaction force between the horse and buggy are equal, but the resisting frictional force on the buggy is smaller since it is on wheels.
15. Suppose you have a choice of driving your speeding car head on into a massive concrete wall or hitting an identical car head on. Which would produce the greatest change in the momentum of your car?
- The identical car.
 - The concrete wall.
 - Both would be equal.
16. A small, compact car and a large sports utility vehicle collide head on and stick together. Which vehicle had the larger momentum change?
- The small, compact car.
 - The large sports utility vehicle.
 - Both would be equal.

17. Again consider the small, compact car and large sports utility vehicle that collided head on and stuck together. Which experienced the larger deceleration during the collision?

- a. The small, compact car.
- b. The large sports utility vehicle.
- c. Both would be equal.

18. Certain professional football players can throw a football so fast that it moves horizontally in a flat trajectory.

- a. True
- b. False

Answers: 1d, 2c ($a = g$ straight down, but decreases to zero on a level surface), 3a (acceleration is a rate of change of velocity and gravity is acting, $F = ma$, so a must be occurring), 4b (initial speed was zero, average speed is one-half of final speed), 5d, 6c, 7a (after release only gravity acts on ball), 8a (the apple and bullet accelerate downward together, no matter how fast the bullet is moving), 9c, 10b, 11b, 12c, 13b, 14d, 15c, 16c, 17a, 18b.

Answers to Questions for Thought

- 1. The speed of the insect relative to the ground is the 50.0 mi/h of the bus plus the 5.0 mi/h of the insect for a total of 55 mi/h. Relative to the bus alone the speed of the insect is 5.0 mi/h.
- 2. After it leaves the rifle barrel, the force of gravity acting straight down is the only force acting on the bullet.
- 3. Gravity does not depend upon some medium so it can operate in a vacuum.
- 4. Yes, the small car would have to be moving with a much higher velocity, but it can have the same momentum since momentum is mass times velocity.
- 5. A net force of zero is required to maintain a constant velocity. The force from the engine balances the force of friction as a car drives with a constant velocity.
- 6. The action and reaction forces are between two objects that are interacting. An unbalanced force occurs on a single object as the result of one or more interactions with other objects.
- 7. Bending your knees as you hit the ground extends the stopping time. This is important since the change of momentum is equal to the impulse, which is force times the time. A greater time therefore means less force when coming to a stop.
- 8. Your weight can change from place to place because weight is a downward force from gravitational attraction on your mass and the force of gravity can vary from place to place.
- 9. Nothing! There is no force parallel to the motion to increase or decrease Earth's speed, so the speed remains constant.

10. If you have something to throw, such as car keys or a snowball, you can easily get off the frictionless ice. Since the force you apply to the thrown object results in an equal and opposite force (the third law of motion), you will move in the opposite direction as the object is thrown (the second law of motion). The same result can be achieved by blowing a puff of air in a direction opposite to the way you wish to move.
11. Considering everything else to be equal, the two rockets will have the same acceleration. In both cases, the acceleration results as burning rocket fuel escapes the rocket, exerting an unbalanced force on the rocket (third law) and the rocket accelerates during the applied force (second law). The acceleration has nothing to do with the escaping gases having something to “push against.”
12. The astronaut is traveling with the same speed as the spaceship as he or she leaves. If no net force is applied parallel to the direction of motion of either the astronaut or the spaceship, they will both maintain a constant velocity and will stay together.

For Further Analysis

1. Similar – both speed and velocity describe a magnitude of motion, that is, how fast something is moving. Differences – velocity must specify a direction; speed does not.
2. Similar – both velocity and acceleration describe motion. Differences – velocity specifies how fast something is moving in a particular direction; acceleration specified a change of velocity (speed, direction, or both).
3. This requires a comparison of beliefs and an analysis and comparison with new contexts. Answers will vary, but should show understanding of Newton’s three laws of motion.
4. This question requires both clarifying beliefs and comparing perspectives. Answers will vary.
5. Requires refining of understanding. Mass is a measure of inertia, meaning a resistance to a change of motion. Weight is gravitational acceleration acting on a mass. Since gravity can vary from place to place, the weight as a result of gravity will also vary from place to place.
6. Requires clarifying and analyzing several conceptual understandings. Newton’s first law of motion tells us that motion is unchanged in a straight line without an unbalanced force. An object moving on the end of a string in a circular path is pulled out of a straight line by a centripetal force on the string. The object will move off in a straight line if the string breaks. It would move off in some other direction if other forces were involved.

Group B Solutions

1. The distance and time are known and the problem asked for the average velocity. Listing these quantities with their symbols, we have

$$\begin{aligned}d &= 400.0 \text{ km} \\t &= 4.5 \text{ h} \\\bar{v} &= ?\end{aligned}$$

These are the quantities involved in the average speed equation, which is already solved for the unknown average speed:

$$\begin{aligned}\bar{v} &= \frac{d}{t} \\&= \frac{400.0 \text{ km}}{4.5 \text{ h}} \\&= 89 \frac{\text{km}}{\text{h}}\end{aligned}$$

2. Listing the quantities given in this problem, we have

$$\begin{aligned}d &= 16.0 \text{ km} \\t &= 45 \text{ min} \\\bar{v} &= ?\end{aligned}$$

The problem specifies that the answer should be in km/h. We see that 45 minutes is 45/60, or 3/4, or 0.75 of an hour, and the appropriate units are:

$$\begin{aligned}d &= 16.0 \text{ km} \\t &= 0.75 \text{ h} \\\bar{v} &= ?\end{aligned}$$

Substituting the known quantities, we have

$$\begin{aligned}\bar{v} &= \frac{d}{t} \\&= \frac{16.0 \text{ km}}{0.75 \text{ h}} \\&= 21.3333 \frac{\text{km}}{\text{h}} \\&= 21 \frac{\text{km}}{\text{h}}\end{aligned}$$

3. Weight is the gravitational force on an object. Newton's second law of motion is $F = ma$, and since weight (w) is a force (F), then $F = w$ and the second law can be written as $w = ma$. The acceleration (a) is the acceleration due to gravity (g), so the equation for weight is $w = mg$.

(a)

$$\begin{aligned} w &= mg \\ &= (80.0 \text{ kg}) \left(3.93 \frac{\text{m}}{\text{s}^2} \right) \\ &= (80.0) (3.93) \frac{\text{kg} \times \text{m}}{\text{s}^2} \\ &= 314.4 \text{ N} \\ &= 314 \text{ N} \end{aligned}$$

(b)

$$\begin{aligned} w &= mg \\ &= (80.0 \text{ kg}) \left(1.64 \frac{\text{m}}{\text{s}^2} \right) \\ &= (80.0) (1.64) \frac{\text{kg} \times \text{m}}{\text{s}^2} \\ &= 131.2 \text{ N} \\ &= 131 \text{ N} \end{aligned}$$

4. Listing the known and unknown quantities,

$$\begin{aligned} m &= 6000.0 \text{ kg} \\ a &= 2.2 \frac{\text{m}}{\text{s}^2} \\ F &= ? \end{aligned}$$

These are the quantities found in Newton's second law of motion, $F = ma$, which is already solved for force (F). Thus,

$$\begin{aligned} F &= ma \\ &= (6000.0 \text{ kg}) \left(2.2 \frac{\text{m}}{\text{s}^2} \right) \\ &= (6000.0) (2.4) \frac{\text{kg} \times \text{m}}{\text{s}^2} \\ &= 13200 \text{ N} \\ &= 13000 \text{ N} \end{aligned}$$

5. Listing the known and unknown quantities,

$$F = 300 \text{ N}$$

$$m = 3000 \text{ kg}$$

$$a = ?$$

$$\begin{aligned} F &= ma \therefore a = \frac{F}{m} \\ &= \frac{300 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{3000 \text{ kg}} \\ &= \frac{300}{3000} \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \times \frac{1}{\text{kg}} \\ &= 0.1 \frac{\text{m}}{\text{s}^2} \end{aligned}$$

6. We see that 30.0 minutes is 1/2 or 0.50 of an hour, and

$$\begin{aligned} \bar{v} &= \frac{d}{t} \\ &= \frac{15.0 \text{ km}}{0.50 \text{ h}} \\ &= 30.0 \text{ km/h} \end{aligned}$$

7. We see that the distance units are kilometers, but the velocity units are m/s. We need to convert km to m, then

$$\begin{aligned} v &= \frac{d}{t} \therefore t = \frac{d}{v} \\ &= \frac{1.5 \times 10^{11} \text{ m}}{3.0 \times 10^8 \frac{\text{m}}{\text{s}}} \\ &= \frac{1.5 \times 10^{11}}{3.0 \times 10^8} \frac{\text{m}}{1} \times \frac{\text{s}}{\text{m}} \\ &= 5.0 \times 10^2 \text{ s} \\ &= 8.3 \text{ min} \end{aligned}$$

8. The distance that a sound with this velocity travels in the given time is

$$\begin{aligned}v &= \frac{d}{t} \quad \therefore d = vt \\&= (343 \text{ m/s})(0.500 \text{ s}) \\&= 172 \text{ m} \\&\quad \frac{172 \text{ m}}{2} \\&= 86.0 \text{ m}\end{aligned}$$

Since the sound traveled from you to the cliff and then back, the cliff must be $172 \text{ m}/2 = 86.0 \text{ m}$ away.

9. Note that the two speeds given (80.0 km/h and 90.0 km/h) are *average* speeds for two different legs of a trip. They are not the initial and final speeds of an accelerating object, so you cannot add them together and divide by 2. The average speed for the total (entire) trip can be found from definition of average speed, that is, average speed is the *total* distance covered divided by the *total* time elapsed. Therefore, we start by finding the distance covered for each of the two legs of the trip:

$$\begin{aligned}\bar{v} &= \frac{d}{t} \quad \therefore d = \bar{v}t \\ \text{Leg 1 distance} &= \left(80.0 \frac{\text{km}}{\text{h}}\right)(1.00 \text{ h}) \\&= 80.0 \text{ km} \\ \text{Leg 2 distance} &= \left(90.0 \frac{\text{km}}{\text{h}}\right)(2.00 \text{ h}) \\&= 180.0 \text{ km}\end{aligned}$$

Total distance (leg 1 plus leg 2) = 260.0 km

Total time = 3.00 h

$$\bar{v} = \frac{d}{t} = \frac{260.0 \text{ km}}{3.00 \text{ h}} = 86.7 \text{ km/h}$$

10.

$$\begin{aligned}
 a &= \frac{v_f - v_i}{t} \\
 &= \frac{15 \text{ m/s} - 5.0 \text{ m/s}}{6.0 \text{ s}} \\
 &= \frac{10.0}{6.0} \frac{\text{m}}{\text{s}} \times \frac{1}{\text{s}} \\
 &= 1.7 \frac{\text{m}}{\text{s}^2}
 \end{aligned}$$

11.

$$\begin{aligned}
 a &= \frac{v_f - v_i}{t} \quad \therefore \quad t = \frac{v_f - v_i}{a} \\
 t &= \frac{22 \text{ m/s} - 8.0 \text{ m/s}}{3.0 \frac{\text{m}}{\text{s}^2}} \\
 &= \frac{14}{3.0} \frac{\text{m}}{\text{s}} \times \frac{\text{s}^2}{\text{m}} \\
 &= 4.7 \text{ s}
 \end{aligned}$$

12. The relationship between average velocity (\bar{v}), distance (d), and time (t) can be solved for time:

$$\begin{aligned}
 \bar{v} &= \frac{d}{t} \\
 \bar{v}t &= d \\
 t &= \frac{d}{\bar{v}} \\
 t &= \frac{380,000,000 \text{ m}}{11,000 \frac{\text{m}}{\text{s}}} \\
 &= \frac{380,000,000}{11,000} \frac{\text{m}}{1} \times \frac{\text{s}}{\text{m}} \\
 &= 34,545 \text{ s} \\
 &= 35,000 \text{ s (about 9.6 hours)}
 \end{aligned}$$

13. The relationship between average velocity (\bar{v}), distance (d), and time (t) can be solved for distance:

$$\begin{aligned}\bar{v} &= \frac{d}{t} \quad \therefore \quad d = \bar{v}t \\ &= \left(348 \frac{\text{m}}{\text{s}}\right)(4.63 \text{ s}) \\ &= 348 \times 4.63 \frac{\text{m}}{\text{s}} \times \text{s} \\ &= 1611.24 \text{ m} \\ &= 1610 \text{ m}\end{aligned}$$

14. “How many hours...” is a question about time and the distance is given. Since the distance is given in km and the speed in m/s, a unit conversion is needed. The easiest thing to do is to convert km to m. There are 1,000 m in a km, and

$$(6.00 \times 10^9 \text{ km}) \times (1 \times 10^3 \text{ m/km}) = 6.00 \times 10^{12} \text{ m}$$

The relationship between average velocity (\bar{v}), distance (d), and time (t) can be solved for time:

$$\begin{aligned}\bar{v} &= \frac{d}{t} \quad \therefore \quad t = \frac{d}{\bar{v}} \\ t &= \frac{6.00 \times 10^{12} \text{ m}}{3.00 \times 10^8 \frac{\text{m}}{\text{s}}} \\ &= \frac{6.00}{3.00} \times 10^{12-8} \frac{\text{m}}{1} \times \frac{\text{s}}{\text{m}} \\ &= 2.00 \times 10^4 \text{ s} \\ &= \frac{2.00 \times 10^4 \text{ s}}{3,600 \frac{\text{s}}{\text{h}}} = 5.56 \text{ h}\end{aligned}$$

15. The initial velocity (v_i) is given as 724 m/s, the final velocity (v_f) is given as 675 m/s, and the time is given as 5.00 s. Acceleration, including a deceleration or negative acceleration, is found from a change of velocity during a given time. Thus,

$$\begin{aligned}
 a &= \frac{v_f - v_i}{t} \\
 &= \frac{(675 \text{ m/s}) - (724 \text{ m/s})}{5.00 \text{ s}} \\
 &= \frac{-49.0 \text{ m/s}}{5.00 \text{ s}} \\
 &= -9.80 \frac{\text{m}}{\text{s}} \times \frac{1}{\text{s}} \\
 &= -9.80 \frac{\text{m}}{\text{s}^2}
 \end{aligned}$$

(The negative sign means a negative acceleration, or deceleration.)

16. A rock thrown straight up decelerates to a velocity of zero, and then accelerates back to the surface just as a dropped ball would do from the height reached. Thus the time decelerating upward is the same as the time accelerating downward. The ball returns to the surface with the same velocity with which it was thrown (neglecting friction).

Therefore:

$$\begin{aligned}
 a &= \frac{v_f - v_i}{t} \\
 gt &= v_f - v_i \\
 v_f &= gt + v_i \\
 v_f &= \left(9.8 \frac{\text{m}}{\text{s}^2}\right)(2.50 \text{ s}) + 0 \frac{\text{m}}{\text{s}} \\
 &= 9.8 \times 2.50 \frac{\text{m}}{\text{s}^2} \times \text{s} \\
 &= 25 \text{ m/s}
 \end{aligned}$$

17. These three questions are easily answered by using the three sets of relationships, or equations that were presented in this chapter:

$$\begin{aligned}
 \text{(a)} \quad v_f &= at + v_i \\
 v_f &= \left(9.8 \frac{\text{m}}{\text{s}^2}\right)(2.50 \text{ s}) + 0 \frac{\text{m}}{\text{s}} \\
 &= 9.8 \times 2.50 \frac{\text{m}}{\text{s}^2} \times \text{s} \\
 &= 25 \text{ m/s}
 \end{aligned}$$

$$\text{(b)} \quad \bar{v} = \frac{v_f + v_i}{2} = \frac{25 \text{ m/s} + 0}{2} = 13 \frac{\text{m}}{\text{s}}$$

$$\begin{aligned}
 \text{(c)} \quad \bar{v} &= \frac{d}{t} \therefore d = \bar{v}t \\
 d &= \left(13 \frac{\text{m}}{\text{s}}\right)(2.50 \text{ s}) \\
 &= 13 \times 2.50 \frac{\text{m}}{\text{s}} \times \text{s} \\
 &= 33 \text{ m}
 \end{aligned}$$

18. Note that this problem can be solved with a series of three steps as in the previous problem. It can also be solved by the equation that combines all the relationships into one step. Either method is acceptable, but the following example of a one step solution reduces the possibilities of error since fewer calculations are involved:

$$\begin{aligned}
 d &= \frac{1}{2}gt^2 \\
 &= \frac{1}{2}\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(2.00 \text{ s})^2 \\
 &= \frac{1}{2}\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(4.00 \text{ s}^2) \\
 &= \frac{1}{2}(9.8)(4.00) \frac{\text{m}}{\text{s}^2} \times \text{s}^2 \\
 &= 19.6 \text{ m} \\
 &= 2.0 \times 10^1 \text{ m}
 \end{aligned}$$

19.

$$\begin{aligned}
 F = ma \quad \therefore \quad a &= \frac{F}{m} \\
 &= \frac{300 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{3,000 \text{ kg}} \\
 &= \frac{300}{3,000} \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \times \frac{1}{\text{kg}} \\
 &= 0.1 \frac{\text{m}}{\text{s}^2}
 \end{aligned}$$

20.

$$\begin{aligned}
 p &= mv \\
 &= (30.0 \text{ kg}) \left(500 \frac{\text{m}}{\text{s}} \right) \\
 &= 15,000 \frac{\text{kg} \cdot \text{m}}{\text{s}} \\
 &= 20,000 \frac{\text{kg} \cdot \text{m}}{\text{s}} \text{ (one significant figure)}
 \end{aligned}$$

21. Mass of ball:

$$\begin{aligned}
 F = ma \quad \therefore \quad m &= \frac{F}{a} \\
 &= \frac{39.2 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{9.8 \frac{\text{m}}{\text{s}^2}} \\
 &= \frac{39.2}{9.8} \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \times \frac{\text{s}^2}{\text{m}} \\
 &= 4.0 \text{ kg} \\
 p &= mv \\
 &= (4.0 \text{ kg}) \left(7.00 \frac{\text{m}}{\text{s}} \right) \\
 &= 28 \frac{\text{kg} \cdot \text{m}}{\text{s}}
 \end{aligned}$$

22. Listing the known and unknown quantities:

Shell $m = 30.0 \text{ kg}$

Cannon $m = 2,000 \text{ kg}$

Shell $v = 500 \text{ m/s}$

Cannon $v = ? \text{ m/s}$

This is a conservation of momentum question, where the shell and cannon can be considered as a system of interacting objects:

Shell momentum = Cannon momentum

$$(mv)_s = (mv)_c$$

$$(mv)_s - (mv)_c = 0$$

$$(30.0 \text{ kg})\left(500 \frac{\text{m}}{\text{s}}\right) - (2,000 \text{ kg})v_c = 0$$

$$\left(15,000 \text{ kg} \cdot \frac{\text{m}}{\text{s}}\right) - (2,000 \text{ kg} \cdot v_c) = 0$$

$$\left(15,000 \text{ kg} \cdot \frac{\text{m}}{\text{s}}\right) = (2,000 \text{ kg} \cdot v_c)$$

$$\begin{aligned} v_c &= \frac{15,000 \text{ kg} \cdot \frac{\text{m}}{\text{s}}}{2,000 \text{ kg}} \\ &= \frac{15,000}{2,000} \frac{\text{kg}}{1} \times \frac{1}{\text{kg}} \times \frac{\text{m}}{\text{s}} \\ &= 7.5 \frac{\text{m}}{\text{s}} \\ &= 8 \frac{\text{m}}{\text{s}} \end{aligned}$$

23.

Book momentum = Man momentum

$$(mv)_B = (mv)_M$$

$$(mv)_B - (mv)_M = 0$$

$$(2.00 \text{ kg}) \left(10.0 \frac{\text{m}}{\text{s}} \right) - (80.0 \text{ kg}) v_M = 0$$

$$\left(20.0 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \right) - (80.0 \text{ kg} \cdot v_M) = 0$$

$$\begin{aligned} v_M &= \frac{20.0 \text{ kg} \cdot \frac{\text{m}}{\text{s}}}{80.0 \text{ kg}} \\ &= \frac{20.0}{80.0} \frac{\text{kg}}{1} \times \frac{1}{\text{kg}} \times \frac{\text{m}}{\text{s}} \\ &= 0.250 \frac{\text{m}}{\text{s}} \end{aligned}$$

24. (a)

$$w = mg$$

$$= (5.00 \text{ kg}) \left(9.8 \frac{\text{m}}{\text{s}^2} \right) \quad w = mg$$

$$= 49 \text{ N}$$

(b)

$$\begin{aligned} F = ma \quad \therefore \quad a &= \frac{F}{m} \\ &= \frac{10.0 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{5.00 \text{ kg}} \\ &= \frac{10.0}{5.00} \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \times \frac{1}{\text{kg}} \\ &= 2.00 \frac{\text{m}}{\text{s}^2} \end{aligned}$$

25.

$$\begin{aligned}
 F &= ma \\
 &= 20.0 \text{ kg} \times 10.0 \frac{\text{m}}{\text{s}^2} \\
 &= 20.0 \times 10.0 \text{ kg} \times \frac{\text{m}}{\text{s}^2} \\
 &= 2.00 \times 10^2 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \\
 &= 2.00 \times 10^2 \text{ N}
 \end{aligned}$$

26.

$$\begin{aligned}
 F &= ma \\
 &= 60.0 \text{ kg} \times 1.00 \frac{\text{m}}{\text{s}^2} \\
 &= 60.0 \times 1.00 \text{ kg} \times \frac{\text{m}}{\text{s}^2} \\
 &= 60.0 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \\
 &= 60.0 \text{ N}
 \end{aligned}$$

27.

$$\begin{aligned}
 F &= ma \quad \text{and} \quad a = \frac{v_f - v_i}{t} \quad \therefore \quad F = m \times \frac{v_f - v_i}{t} \\
 &= 1,000.0 \text{ kg} \times \frac{(20.0 \text{ m/s} - 10.0 \text{ m/s})}{5.00 \text{ s}} \\
 &= \frac{(1,000.0)(10.0)}{5.00} \frac{\text{kg} \cdot \frac{\text{m}}{\text{s}}}{\text{s}} \\
 &= 2,000 \text{ kg} \cdot \frac{\text{m}}{\text{s}} \times \frac{1}{\text{s}} \\
 &= 2.00 \times 10^3 \text{ N}
 \end{aligned}$$

28.

$$(a) \quad 36.0 \text{ km/h} \times 0.2778 \frac{\text{m/s}}{\text{km/h}} = 10.0 \text{ m/s}$$

$$\begin{aligned} F &= ma \quad \therefore m = \frac{F}{a} \\ \text{and } a &= \frac{v_f - v_i}{t} \quad \therefore m = \frac{Ft}{v_f - v_i} \\ &= \frac{3,000.0 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \times 5.00 \text{ s}}{10.0 \frac{\text{m}}{\text{s}} - 0 \frac{\text{m}}{\text{s}}} \\ &= \frac{(3,000.0)(5.00)}{10.0} \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \times \frac{\text{s}}{1} \times \frac{\text{s}}{\text{m}} \\ &= 1.50 \times 10^3 \text{ kg} \end{aligned}$$

(b)

$$\begin{aligned} w &= mg \\ &= (1.50 \times 10^3 \text{ kg}) \times \left(9.8 \frac{\text{m}}{\text{s}^2} \right) \\ &= 1.5 \times 10^4 \text{ N} \end{aligned}$$

29.

$$\begin{aligned} w &= mg \\ w &= 60.0 \text{ kg} \times 9.8 \frac{\text{m}}{\text{s}^2} \\ &= 588 \text{ N} \\ &= 590 \text{ N} \end{aligned}$$

30.

$$\begin{aligned}
 F = ma \quad \text{and} \quad a = \frac{v^2}{r} \quad \therefore \quad F &= m \left(\frac{v^2}{r} \right) \\
 &= 1.0000 \text{ kg} \times \frac{\left(5.00 \frac{\text{m}}{\text{s}} \right)^2}{0.500 \text{ m}} \\
 &= \frac{(1.0000)(5.00)}{0.500} \frac{\text{kg}}{1} \times \frac{\text{m}^2}{\text{s}^2} \times \frac{1}{\text{m}} \\
 &= 10.0 \text{ N}
 \end{aligned}$$

31.

$$\begin{aligned}
 F = ma \quad \text{and} \quad a = \frac{v_f - v_i}{t} \quad \therefore \quad F = m \times \frac{v_f - v_i}{t} \quad \therefore \quad t &= m \times \frac{v_f - v_i}{F} \\
 t &= 200.0 \text{ kg} \times \frac{\left(0 - 2.00 \frac{\text{m}}{\text{s}} \right)}{-100.0 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}} \\
 &= (200.0)(0.0200) \frac{\text{kg}}{1} \times \frac{\text{m}}{\text{s}} \times \frac{\text{s}^2}{\text{kg} \cdot \text{m}} \\
 &= 4.00 \text{ s}
 \end{aligned}$$

Experiment 2: Ratios

Introduction

The purpose of this introductory laboratory exercise is to investigate how measurement data are simplified in order to generalize and identify trends in the data. Data concerning two quantities will be compared as a **ratio**, which is generally defined as a relationship between numbers or quantities. A ratio is usually simplified by dividing one number by another.

Chalkboard Note: Clean up any spills, please!

Procedure

Part A: Circles and Proportionality Constants

1. Obtain three different sizes of cups, containers, or beakers with circular bases. Trace around the bottoms to make three large but different-sized circles on a blank sheet of paper.

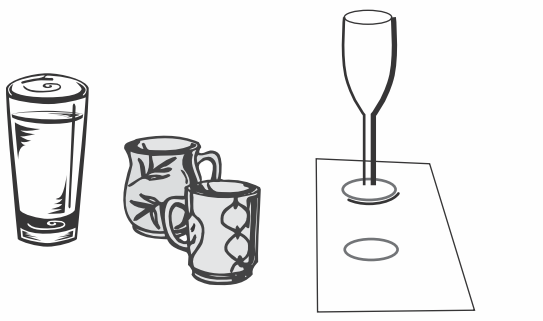


Figure 2.1

2. Mark the diameter on each circle by drawing a straight line across the center. Measure each diameter in mm and record the measurements in Data Table 2.1. Repeat this procedure for each circle for a total of three trials.
3. Measure the circumference of each object by carefully positioning a length of string around the object's base, then grasping the place where the string ends meet. Measure the length in mm and record the measurements for each circle in Data Table 2.1. Repeat the procedure for each circle for a total of three trials. Find the ratio of the circumference of each circle to its diameter. Record the ratio for each trial in Data Table 2.1 on page 23.
4. The ratio of the circumference of a circle to its diameter is known as **pi** (symbol π), which has a value of 3.14... (the periods mean many decimal places). Average all the values of π in Data Table 2.1 and calculate the experimental error.

Part B: Area and Volume Ratios

1. Obtain one cube from the supply of same-sized cubes in the laboratory. Note that a cube has six sides, or six units of surface area. The side of a cube is also called a *face*, so each cube has six identical faces with the same area. The overall surface area of a cube can be found by measuring the length and width of one face (which should have the same value) and then multiplying (length)(width)(number of faces). Use a metric ruler to measure the cube, then calculate the overall surface area and record your finding for this small cube in Data Table 2.2 on page 23.
2. The volume of a cube can be found by multiplying the (length)(width)(height). Measure and calculate the volume of the cube and record your finding for this small cube in Data Table 2.2.
3. Calculate the ratio of surface area to volume and record it in Data Table 2.2.
4. Build a medium-sized cube from eight of the small cubes stacked into one solid cube. Find and record (a) the overall surface area, (b) the volume, and (c) the overall surface area to volume ratio, and record them in Data Table 2.2.
5. Build a large cube from 27 of the small cubes stacked into one solid cube. Again, find and record the overall surface area, volume, and overall surface area to volume ratio and record your findings in Data Table 2.2.
6. Describe a pattern, or generalization, concerning the volume of a cube and its surface area to volume ratio. For example, as the volume of a cube increases, what happens to the surface area to volume ratio? How do these two quantities change together for larger and larger cubes?

As the volume of a cube increases the surface area to volume ratio approaches zero.

Part C: Mass and Volume

1. Obtain at least three straight-sided, rectangular containers. Measure the length, width, and height *inside* the container (you do not want the container material included in the volume). Record these measurements in Data Table 2.3 (page 23) in rows 1, 2, and 3. Calculate and record the volume of each container in row 4 of the data table.

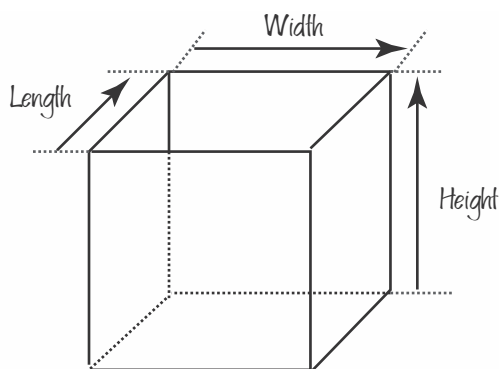


Figure 2.2

2. Measure and record the mass of each container in row 5 of the data table. Measure and record the mass of each container when “level full” of tap water. Record each mass in row 6 of the data table. Calculate and record the mass of the water in each container (mass of container plus water minus mass of empty container, or row 6 minus row 5 for each container). Record the mass of the water in row 7 of the data table.

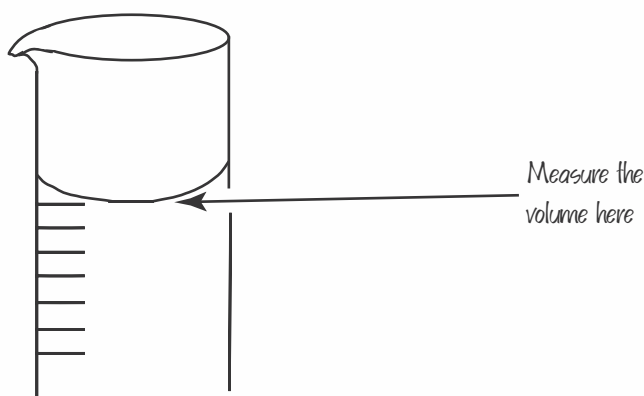


Figure 2.3

3. Use a graduated cylinder to measure the volume of water in each of the three containers. Be sure to get *all* the water into the graduated cylinder. Record the water volume of each container in milliliters (mL) in row 8 of the data table.
4. Calculate the ratio of cubic centimeters (cm^3) to mL for each container by dividing the volume in cubic centimeters (row 4 data) by the volume in milliliters (row 8 data). Record your findings in the data table.
5. Calculate the ratio of mass per unit volume for each container by dividing the mass in grams (row 7 data) by the volume in milliliters (row 8 data). Record your results in the data table.

6. Make a graph of the mass in grams (row 7 data) and the volume in milliliters (row 8 data) to picture the mass per unit volume ratio found in step 5. Put the volume on the x -axis (horizontal axis) and the mass on the y -axis (the vertical axis). The mass and volume data from each container will be a data point, so there will be a total of three data points.
7. Draw a straight line on your graph that is as close as possible to the three data points and the origin (0, 0) as a fourth point. If you wonder why (0, 0) is also a data point, ask yourself about the mass of a zero volume of water!
8. Calculate the slope of your graph. (See appendix II on page 397 for information on calculating a slope.)

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{(800 - 400) \text{ g}}{(800 - 400) \text{ mL}} = 1.0 \text{ g/mL}$$

9. Calculate your experimental error. Use 1.0 g/mL (grams per milliliter) as the accepted value.

You can expect less than 10 percent error, probably less than 5 percent.

10. Density is defined as mass per unit volume, or mass/volume. The slope of a straight line is also a ratio, defined as the ratio of the change in the y -value per the change in the x -value. Discuss why the volume data was placed on the x -axis and mass on the y -axis and not vice versa.

Because if you don't have a volume of water, you do not have a mass. Volume is the independent variable and mass is the dependent variable.

11. Was the purpose of this lab accomplished? Why or why not? (Your answer to this question should show thoughtful analysis and careful, thorough thinking.)

(Student answers will vary.)

Results

1. What is a ratio? Give several examples of ratios in everyday use.

A relationship between numbers or quantities.

Examples: 100 cents per dollar, 60 seconds per minute, 365 days per year.

2. How is the value of π obtained? Why does π not have units?

By taking the ratio of the circumference of a circle to the diameter.

Both circumference and diameter are measured in the same units and when you divide the circumference by the diameter the units cancel out.

3. Describe what happens to the surface area to volume ratio for larger and larger cubes.

Predict if this pattern would also be observed for other geometric shapes such as a sphere.

Explain the reasoning behind your prediction.

Surface area to volume ratio approaches zero for larger and larger cubes.

This pattern would also be true for other shapes because surface area is proportional to length squared and volume is proportional to length cubed so surface area/volume is proportional to $1/\text{length}$ which goes toward zero as the object gets larger.

4. Why does crushed ice melt faster than the same amount of ice in a single block?

There is more surface area for the smaller pieces of ice than the single block, the air is in contact with more of the ice, so it melts faster.

5. Which contains more potato skins: 10 pounds of small potatoes or 10 pounds of large potatoes?

Explain the reasoning behind your answer in terms of this laboratory investigation.

The 10 lbs of small potatoes have more potato skins. There is more total surface area for the same smaller potatoes than the larger potatoes.

6. Using your own words, explain the meaning of the slope of a straight-line graph. What does it tell you about the two graphed quantities?

The slope of a straight-line graph tells you how one quantity changes when the other variable changes. In this case, the slope equals 1.0 g/mL . This tells me that the mass of water in grams equals the volume of the same water in milliliters.

7. Explain why a slope of mass/volume of a particular substance also identifies the density of that substance.

Density is mass/volume. The slope equals the change in mass divided by the change in volume. This is the same as density.

Problems

An aluminum block that is $1\text{ m} \times 2\text{ m} \times 3\text{ m}$ has a mass of 1.62×10^4 kilograms (kg). The following problems concern this aluminum block:

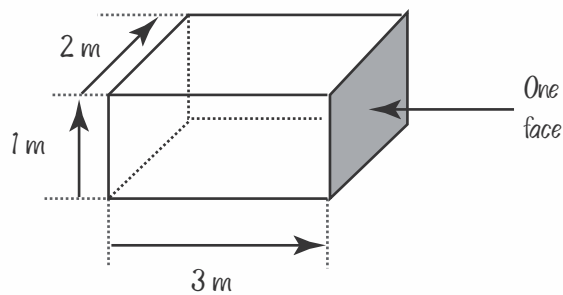


Figure 2.4

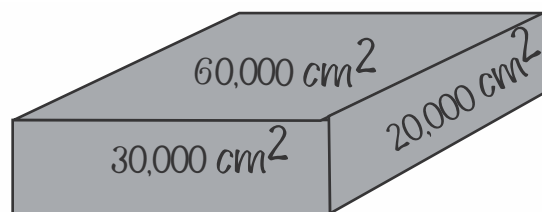
1. What is the volume of the block in cubic meters (m^3)?

$$\text{Volume} = (\text{length})(\text{width})(\text{height}) = (3\text{ m})(2\text{ m})(1\text{ m}) = 6\text{ m}^3.$$

2. What are the dimensions of the block in centimeters (cm)?

300cm by 200cm by 100cm

3. Make a sketch of the aluminum block and show the area of each face in square centimeters (cm^2).



4. What is the volume of the block expressed in cubic centimeters (cm³)?

$$(300\text{ cm})(200\text{ cm})(100\text{ cm}) = 6,000,000\text{ cm}^3.$$

5. What is the mass of the block expressed in grams (g)?

$$1.62 \times 10^4\text{ kg} \times 1000\text{ g/1 kg} = 1.62 \times 10^7\text{ g}$$

6. What is the ratio of mass (g) to volume (cm³) for aluminum?

$$\text{mass/volume} = 1.62 \times 10^7\text{ g}/6 \times 10^6\text{ cm}^3 = 2.7\text{ g/cm}^3$$

7. Under what topic would you look in the index of a reference book to check your answer to question 6? Explain.

Check the value of mass density for aluminum.

Invitation to Inquiry

If you have popped a batch of popcorn, you know that a given batch of kernels might pop into big and fluffy popcorn. But another batch might not be big and fluffy and some of the kernels might not pop. Popcorn pops because each kernel contains moisture that vaporizes into steam, expanding rapidly and causing the kernel to explode, or pop. Here are some questions you might want to consider investigating to find out more about popcorn: Does the ratio of water to kernel mass influence the final fluffy size of popped corn? (Hint: measure mass of kernel before and after popping). Is there an optimum ratio of water to kernel mass for making bigger popped kernels? Is the size of the popped kernels influenced by how rapidly or how slowly you heat the kernels? Can you influence the size of popped kernels by drying or adding moisture to the unpopped kernels? Is a different ratio of moisture to kernel mass better for use in a microwave than in a convention corn popper? Perhaps you can think of more questions about popcorn.



Data Table 2.1 Circles and Ratios									
	Small Circle			Medium Circle			Large Circle		
Trial	1	2	3	1	2	3	1	2	3
Diameter (D)	<u>5.5</u>	<u>5.4</u>	<u>5.5</u>	<u>8.7</u>	<u>8.5</u>	<u>8.6</u>	<u>12.5</u>	<u>12.4</u>	<u>12.3</u>
Circumference (C)	<u>18.0</u>	<u>18.3</u>	<u>17.8</u>	<u>24.7</u>	<u>27.1</u>	<u>27.6</u>	<u>38.9</u>	<u>38.6</u>	<u>38.7</u>
Ratio of C/D	<u>3.27</u>	<u>3.39</u>	<u>3.24</u>	<u>3.15</u>	<u>3.19</u>	<u>3.21</u>	<u>3.11</u>	<u>3.11</u>	<u>3.15</u>
Average $\frac{C}{D} =$ <u>3.20</u> Experimental error: <u>2% from π</u>									

Data Table 2.2 Area and Volume Ratios			
	Small Cube	Medium Cube	Large Cube
Surface Area (cm ²)	<u>24.4</u>	<u>96</u>	<u>386</u>
Volume (cm ³)	<u>8</u>	<u>64</u>	<u>512</u>
Ratio of Area/Volume	<u>3.0 (cm²)/(cm³)</u>	<u>1.5 (cm²)/(cm³)</u>	<u>0.75 (cm²)/(cm³)</u>

Data Table 2.3 Mass and Volume Ratios

Container Number	1	2	3
1. Length of container	<u>6 cm</u>	<u>10 cm</u>	<u>20 cm</u>
2. Width of container	<u>4 cm</u>	<u>5 cm</u>	<u>7.5 cm</u>
3. Height of container	<u>8 cm</u>	<u>4.5 cm</u>	<u>6.5 cm</u>
4. Calculated volume	<u>192 cm³</u>	<u>225 cm³</u>	<u>975 cm³</u>
5. Mass of container	<u>200 g</u>	<u>250 g</u>	<u>400 g</u>
6. Mass of container and water	<u>392 g</u>	<u>475 g</u>	<u>1375 g</u>
7. Mass of water	<u>192 g</u>	<u>225 g</u>	<u>975 g</u>
8. Measured volume of water	<u>192 mL</u>	<u>225 mL</u>	<u>975 mL</u>
9. Ratio of calculated volume to measured volume of water	<u>1.0 cm³/mL</u>	<u>1.0 cm³/mL</u>	<u>1.0 cm³/mL</u>
10. Ratio of mass of water to measured volume of water	<u>1.0 g/mL</u>	<u>1.0 g/mL</u>	<u>1.0 g/mL</u>

