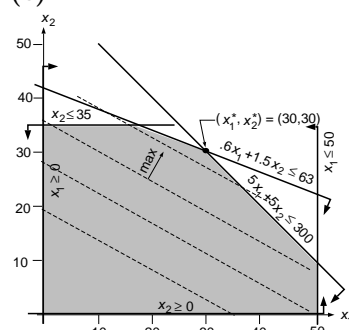


## Chapter 2 Solutions

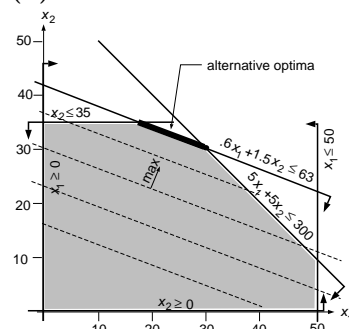
**2-1. (a)**  $\max 200x_1 + 350x_2$  (max total profit), s.t.  $5x_1 + 5x_2 \leq 300$  (legs),  $0.6x_1 + 1.5x_2 \leq 63$  (assembly hours),  $x_1 \leq 50$  (wood tops),  $x_2 \leq 35$  (glass tops),  $x_1 \geq 0$ ,  $x_2 \geq 0$

**(b)**  $x_1^*$ =basic=30,  $x_2^*$ =deluxe=30

**(c)**



**(d)**

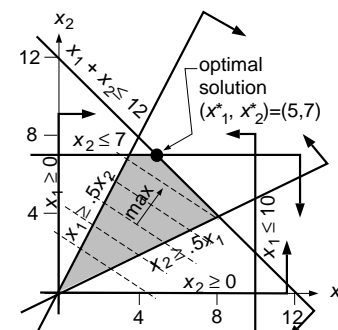


All optimal from  $\mathbf{x} = (30, 30)$  to  $\mathbf{x} = (17.5, 35)$ .

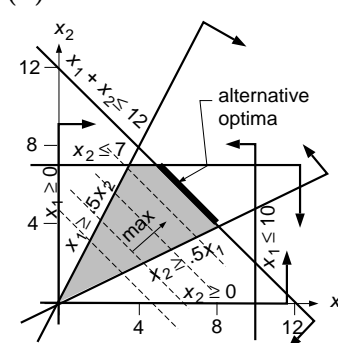
**2-2. (a)**  $\max .11x_1 + .17x_2$  (max total return), s.t.  $x_1 + x_2 \leq 12$  (\$12 million investment),  $x_1 \leq 10$  (max \$10 million domestic),  $x_2 \leq 7$  (max \$7 million foreign),  $x_1 \geq .5x_2$  (domestic at least half foreign),  $x_2 \geq .5x_1$  (foreign at least half domestic),  $x_1 \geq 0$ ,  $x_2 \geq 0$  **(b)**  $x_1^*$ =domestic=\$5 million,  $x_2^*$ =foreign=\$7 million

<sup>1</sup>Supplement to the 2nd edition of *Optimization in Operations Research*, by Ronald L. Rardin, Pearson Higher Education, Hoboken NJ, ©2017.

<sup>2</sup>As of September 24, 2015



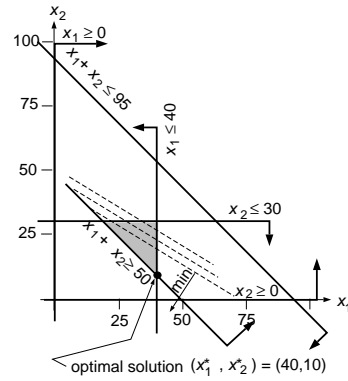
**(d)**



All optimal from  $\mathbf{x} = (5, 7)$  to  $\mathbf{x} = (8, 4)$ .

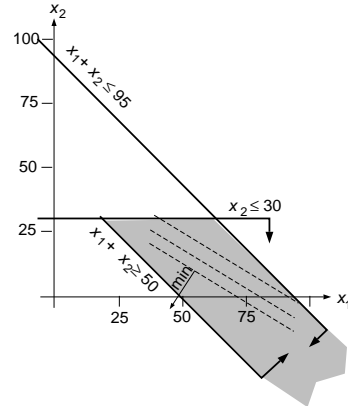
**2-3. (a)**  $\min 3x_1 + 5x_2$  (min total cost), s.t.  $x_1 + x_2 \geq 50$  (at least 50 thousand acres),  $x_1 \leq 40$  (at most 40 thousand from Squawking Eagle),  $x_2 \leq 30$  (at most 30 thousand from Crooked Creek),  $x_1 \geq 0$ ,  $x_2 \geq 0$  **(b)**  $x_1^*$ =Squawking Eagle=40 thousand,  $x_2^*$ =Crooked Creek=10 thousand

(c)



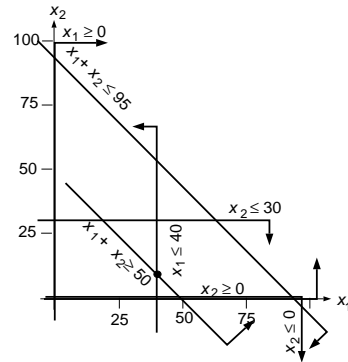
$x_1 + x_2 \geq 125$  (weight at least 125),  
 $2.5x_1 + 1.8x_2 \leq 350$  (calories at most 350),  
 $0.2x_1 + 0.1x_2 \leq 15$  (fat at most 15),  
 $3.5x_1 + 2.5x_2 \leq 360$  (sodium at most 360),  
 $x_1 \geq 0, x_2 \geq 0$  (b)  $x_1^*$ =beef=25g,  
 $x_2^*$ =chicken=100g

(d)



Improves forever in direction  $\Delta x_1 = 1$ ,  
 $\Delta x_2 = -1$ .

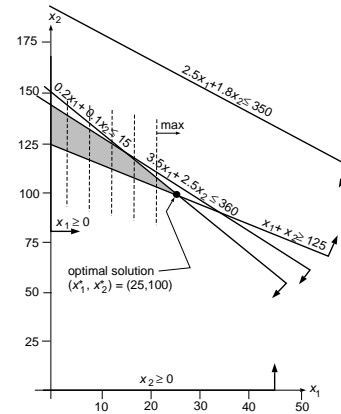
(e)



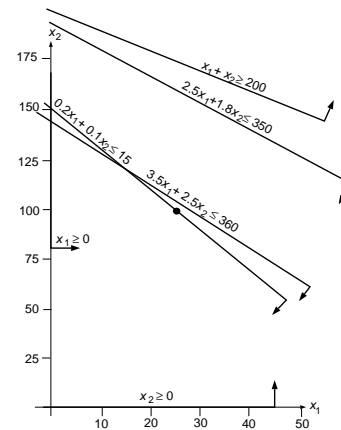
$x_2 = 0$  leaves no feasible.

2-4. (a)  $\max x_1$  (max beef content), s.t.

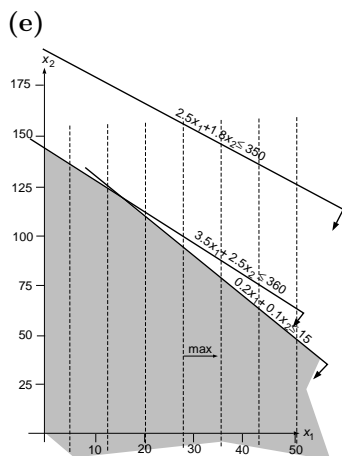
(c)



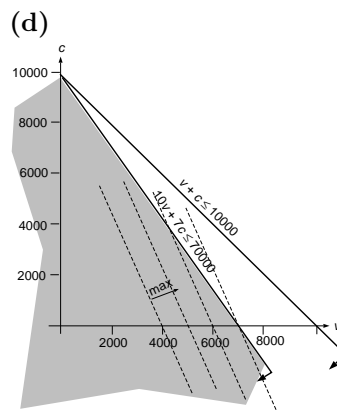
(d)



$x_1 + x_2 \geq 200$  leaves no feasible.

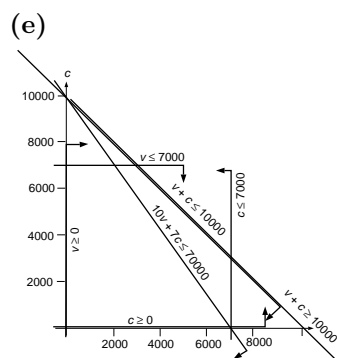


Improve forever in direction  $\Delta x_1 = 1$ ,  
 $\Delta x_2 = -2$ .

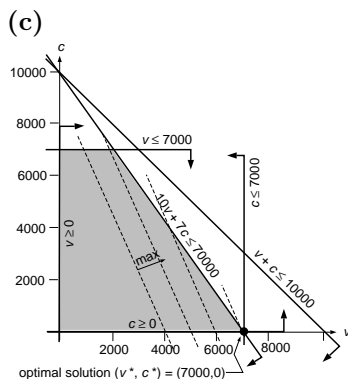


Improves forever in direction  $\Delta v = 10$ ,  
 $\Delta c = -7$ .

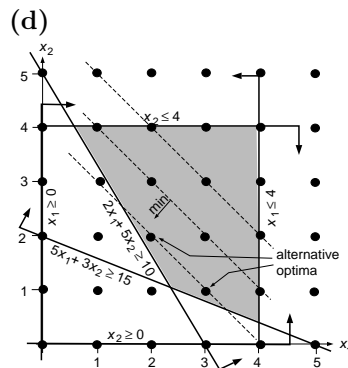
**2-5. (a)** max  $450v + 200c$  (max total profit),  
 s.t.  $10v + 7c \leq 70000$  (water at most 70000  
 units),  $v + c \leq 10000$  (total acreage 10000),  
 $v \leq 7000$  (at most 70% vegetables),  $c \leq 7000$   
 (at most 70% cotton),  $v \geq 0$ ,  $c \geq 0$  **(b)**  
 $v^* = 7000$ ,  $c^* = 0$



No solution with  $v + c = 10000$ .

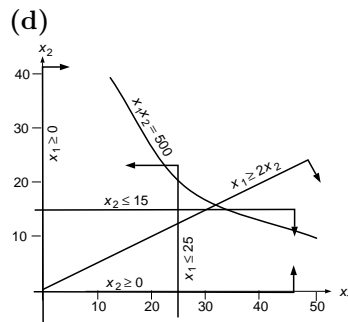
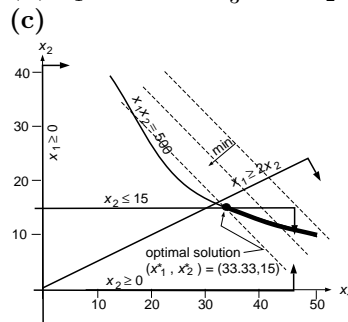


**2-6. (a)** min  $x_1 + x_2$  (min used stock), s.t.  
 $5x_1 + 3x_2 \geq 15$  (cut at least 15 long rolls),  
 $2x_1 + 5x_2 \geq 10$  (cut at least 10 short rolls),  
 $x_1 \leq 4$  (at most 4 times on pattern 1),  $x_2 \leq 4$   
 (at most 4 times on pattern 2),  $x_1, x_2 \geq 0$  and  
 integer. **(b)** Partial cuts make no physical  
 sense because all unused material is scrap. **(c)**  
 Either  $x_1^* = x_2^* = 2$ , or  $x_1^* = 3$ ,  $x_2^* = 1$



(e) Both (2, 2) and (3, 1) are feasible and lie on the best contour of the objective.

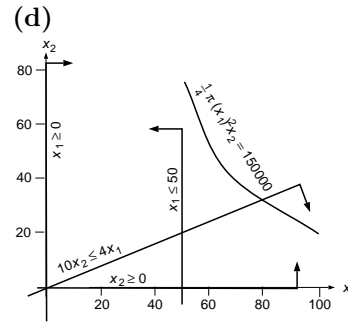
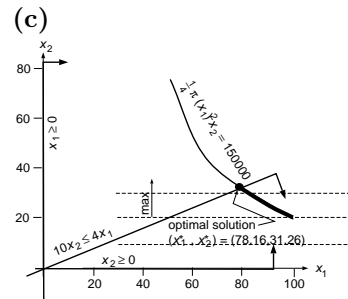
**2-7. (a)** min  $16x_1 + 16x_2$  (min total wall area), s.t.  $x_1x_2 = 500$  (500 sqft pool),  $x_1 \geq 2x_2$  (length at least twice width),  $x_2 \leq 15$  (width at most 15 ft),  $x_1 \geq 0$ ,  $x_2 \geq 0$   
**(b)**  $x_1^*$ =length= $33\frac{1}{3}$  feet,  $x_2^*$ =width=15 feet



$x_1 \leq 25$  leaves no feasible.

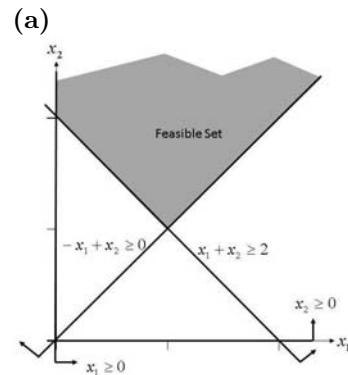
**2-8. (a)** max  $x_2$  (max number of floors), s.t.  $\pi/4(x_1)^2x_2 = 150000$  (150000 sqft floor space),  $10x_2 \leq 4x_1$  (height at most 4 times diameter),  $x_1 \geq 0$ ,  $x_2 \geq 0$  **(b)**  $x_1^*$  = diameter

= 78.16 feet,  $x_2^*$  = floors = 31.26



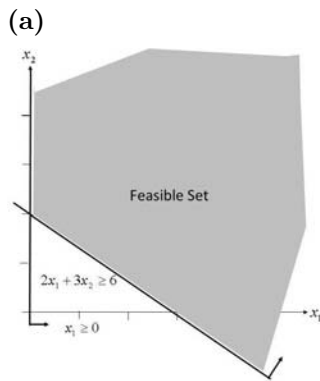
$x_1 \leq 50$  leaves no feasible.

**2-9.**



**(b)** min  $x_2$  **(c)** min  $x_1 + x_2$  **(d)** max  $x_2$  **(e)**  $x_2 \leq 1/2$

**2-10.**



(b)  $\min x_1 + x_2$  (c)  $\min x_1$  (d)  $\max x_1$  (e)  $x_1 + x_2 \leq 1$

2-11. (a)  $\min \sum_{i=3}^4 i \sum_{j=1}^2 y_{i,j}$

(b)  $\max \sum_{i=1}^4 i y_{i,3}$

(c)  $\max \sum_{i=1}^p \alpha_i y_{i,4}$

(d)  $\min \sum_{i=1}^t \delta_i y_i$

(e)  $\sum_{j=1}^4 y_{i,j} = s_i, i = 1, \dots, 3$

(f)  $\sum_{j=1}^4 a_{j,i} y_j = c_i, i = 1, \dots, 3$

2-12. (a)  $\sum_{i=1}^{17} x_{i,j,t} \leq 200, j = 1, \dots, 5; t = \dots, 7; 35$  constraints

(b)  $\sum_{j=1}^5 \sum_{t=1}^7 x_{5,j,t} \leq 4000; 1$  constraint

(c)  $\sum_{j=1}^5 x_{i,j,t} \geq 100, i = 1, \dots, 17; t = 1, \dots, 7; 119$  constraints

2-13. model; param m; param n; param p; set products := 1 .. m; set lines := 1 .. n; set weeks := 1 .. p; var x{i in products, j in lines, t in weeks} >= 0; subject to

# part (a)

linecap {j in lines, t in weeks}: sum {i in products} x[i,j,t] <= 200;

# part (b)

prod5lim: sum {j in lines, t in weeks} x[5,j,t] <= 4000;

# part (c)

minprodn{i in products, t in weeks}: sum {j in lines} x[i,j,t] >= 100;

#

data; param m := 17; param n := 5; param p := 7;

2-14. (a)

$\sum_{j=1}^9 x_{i,j,t} \leq p_i, i = 1, \dots, 47; t = 1, \dots, 10; 470$  constraints

(b)  $0.25 \sum_{i=1}^{47} \sum_{j=1}^9 x_{i,j,t} \leq \sum_{i=1}^{47} x_{i,4,t}; t = 1, \dots, 5; 5$  constraints

(c)  $x_{i,1,t} \geq x_{i,j,t} i = 1, \dots, 47; j = 1, \dots, 9; t = 1, \dots, 10; 4230$  constraints

2-15. model; param m; param n; param

q; set plots := 1 .. m; set crops := 1 .. n; set years := 1 .. q; param p {i in plots}; var x{i in plots, j in crops, t in years} >= 0; subject to

# part (a)

acrelims {i in plots, t in years}: sum {j in crops} x[i,j,t] <= p[i];

# part (b)

crop4min {t in years: t <= 5}:

$0.25 * \sum \{i \text{ in plots, } j \text{ in crops}\}$

$x[i,j,t] <= \sum \{i \text{ in plots}\}$

$x[i,4,t];$

# part (c)

beam1st {i in plots, j in crops, t in years}:  $x[i,1,t] >= x[i,j,t];$

#

data; param m := 47; param n := 9;

param q := 10;

2-16. (a)  $f(y_1, y_2, y_3) \triangleq (y_1)^2 y_2 / y_3,$

$g_1(y_1, y_2, y_3) \triangleq y_1 + y_2 + y_3, b_1 = 13,$

$g_2(y_1, y_2, y_3) \triangleq 2y_1 - y_2 + 9y_3, b_2 = 0,$

$g_3(y_1, y_2, y_3) \triangleq y_1, b_3 = 0, g_4(y_1, y_2, y_3) \triangleq y_3, b_4 = 0$

(b)  $f(y_1, y_2, y_3) \triangleq 13y_1 + 22y_2 + 10y_2 y_3 + 100,$

$g_1(y_1, y_2, y_3) \triangleq y_1 - y_2 + 9y_3, b_1 = -5,$

$g_2(y_1, y_2, y_3) \triangleq 8y_2 - 4y_3, b_2 = 0, g_3(y_1, y_2, y_3)$

$\triangleq y_1, b_3 = 0, g_4(y_1, y_2, y_3) \triangleq y_2, b_4 = 0,$

$g_5(y_1, y_2, y_3) \triangleq y_3, b_5 = 0,$

2-17. (a) Linear because LHS is a weighted

sum of the decision variables. (b) Linear

because both LHS and RHS are weighted

sums of the decision variables. (c) Nonlinear

because LHS has reciprocal  $1/x_9$ . (d) Linear

because LHS is a weighted sum of the decision

variables. (e) Nonlinear because LHS has

$(x_j)^2$  terms. (f) Nonlinear because LHS has

$\log(x_1)$  term, and RHS has a product of

variables. **(g)** Nonlinear because LHS has max operator. **(h)** Linear because LHS is a weighted sum of the decision variables.

**2-18.** **(a)** LP because the objective and all constraints are linear. **(b)** NLP because of the nonlinear objective function with reciprocal of  $w_2$ . **(c)** NLP because of the nonlinear first constraint. **(d)** LP because the objective and all constraints are linear.

**2-19.** **(a)** Continuous because fractions make sense. **(b)** Discrete because they either closed or not. **(c)** Discrete because a specific process must be used. **(d)** Continuous because fractions can probably be ignored.

**2-20.** **(a)**  $\sum_{j=1}^8 x_j = 3$  **(b)**  
 $x_1 + x_2 + x_4 + x_5 \geq 2$  **(c)**  $x_3 + x_8 \leq 1$  **(d)**  
 $x_4 \geq x_1$

**2-21.** **(a)** max  $85x_1 + 70x_2 + 62x_3 + 93x_4$   
(max total score), s.t.

$700x_1 + 400x_2 + 300x_3 + 600x_4 \leq 1000$  (\$1 million available),  $x_j = 0$  or 1,  $j = 1, \dots, 4$

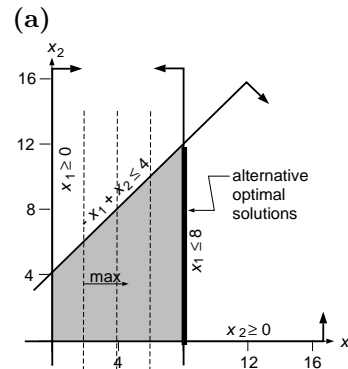
**(b)** Fund 2 and 4, i.e.  $x_1^* = x_3^* = 0$ ,  
 $x_2^* = x_4^* = 1$

**2-22.** **(a)** min  $43y_1 + 175y_2 + 60y_3 + 35y_4$   
(min total land cost), s.t.  $y_2 + y_4 \geq 1$  (service NW),  
 $y_1 + y_2 + y_4 \geq 1$  (service SW),  
 $y_2 + y_3 \geq 1$  (service capital),  $y_1 + y_4 \geq 1$   
(service NE),  $y_1 + y_2 + y_3 \geq 1$  (service SE),  
 $y_j = 0$  or 1,  $j = 1, \dots, 4$  **(b)** Build 3 and 4,  
i.e.  $y_1^* = y_2^* = 0$ ,  $y_3^* = y_4^* = 1$

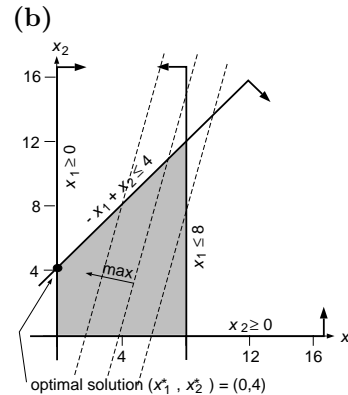
**2-23.** **(a)** ILP because the objective and all constraints are linear, but variables are discrete. **(b)** NLP because the objective is nonlinear and all variables are continuous. **(c)** INLP because the objective is nonlinear and variables are discrete. **(d)** LP because the objective and all constraints are linear, and all variables are continuous. **(e)** INLP because the one constraint is nonlinear, and  $z_3$  are discrete. **(f)** ILP because the objective and all constraints are linear, but variables  $z_1$  and  $z_3$  are discrete. **(g)** LP because the objective and all constraints are linear, and all variables are continuous. **(h)** INLP because the objective is nonlinear and  $z_3$  is discrete.

**2-24.** **(a)** Model (d) because LP's are generally more tractable than ILP's. **(b)** Model (d) because LP's are generally more tractable than NLP's. **(c)** Model (d) because LP's are generally more tractable than INLP's. **(d)** Model (f) because ILP's are generally more tractable than INLP's. **(e)** Model (g) because LP's are generally more tractable than ILP's.

**2-25.**

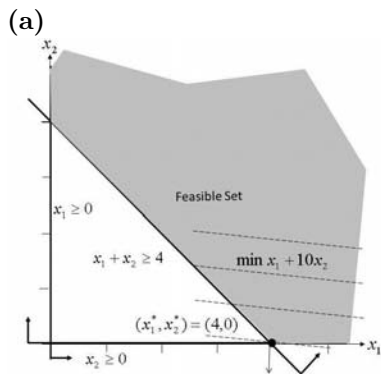


Alternative optima from  $x_1^* = 8$ ,  $x_2^* = 0$  to  $x_1^* = 8$ ,  $x_2^* = 12$

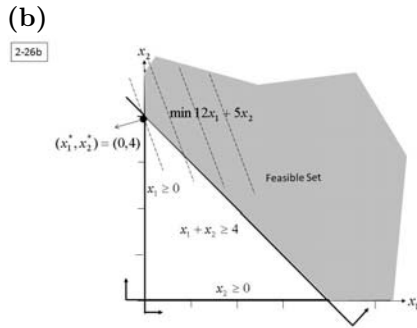


Unique optimum  $x_1^* = 0$ ,  $x_2^* = 4$  **(c)** Helping one can hurt the other.

**2-26.**



Unique optimum  $x_1^* = 4$ ,  $x_2^* = 0$



Unique optimum  $x_1^* = 0$ ,  $x_2^* = 4$  (c) Helping one can hurt the other.

**2-27. (a)** min  
 $.092x_4 + .112x_5 + .141x_6 + .420x_9 + .719x_{12}$   
 (min total cost),  
 s.t.  $x_4 + x_5 + x_6 + x_9 + x_{12} = 16000$  (16000m line),  
 $.279x_4 + .160x_5 + .120x_6 + .065x_9 + .039x_{12} \leq 1600$  (at most 1600 Ohms resistance),  
 $.00175x_4 + .00130x_5 + .00161x_6 + .00095x_9 + .00048x_{12} \leq 8.5$  (at most 8.5 dBell attenuation),  
 $x_4, x_5, x_6, x_9, x_{12} \geq 0$

(b) Nonzeros:  $x_5^* = 1000$ ,  $x_{12}^* = 15000$

**2-28. (a)** Pump rates are the decisions to be made.

(b)  $u_j \triangleq$  the capacity of pump  $j$ ,  $c_j \triangleq$  the pumping cost of pump  $j$

(c)  $\min \sum_{j=1}^{10} c_j x_j$

(d)  $x_1 + x_4 + x_7 \leq 3000$  (well 1),

$x_2 + x_5 + x_8 \leq 2500$  (well 2),

$x_3 + x_6 + x_9 + x_{10} \leq 7000$  (well 3)

(e)  $x_j \leq u_j$ ,  $j = 1, \dots, 10$

(f)  $\sum_{j=1}^{10} x_j \geq 10000$

(g)  $x_j \geq 0$ ,  $j = 1, \dots, 10$

(h) A single objective LP because the one objective and all constraints are linear, and all variables are continuous.

(i)  $x_1^* = x_2^* = x_3^* = 1100$ ,  $x_4^* = x_6^* = 1500$ ,  
 $x_5^* = 1400$ ,  $x_7^* = 400$ ;  $x_8^* = x_{10}^* = 0$ ,  $x_9^* = 1900$

**2-29. (a)** The decisions to be made are which projects to undertake.

(b)  $p_j \triangleq$  the profit for project  $j$ ,  $m_j \triangleq$  the man-days required on project  $j$ , and  $t_j \triangleq$  the CPU time required on project  $j$ .

(c)  $\max \sum_{j=1}^8 p_j x_j$

(d)  $7 \leq \left( \sum_{j=1}^8 m_j x_j \right) / 240 \leq 10$

(e)  $\sum_{j=1}^8 t_j x_j \leq 1000$  (computer time),

$\sum_{j=1}^8 x_j \geq 3$  (select at least 3);

$x_3 + x_4 + x_5 + x_8 \geq 1$  (include at least 1 of director's favorites)

(f)  $x_j = 0$  or 1,  $j = 1, \dots, 8$

(g) A single objective ILP because the one objective and all constraints are linear, but variables are discrete.

(h)  $x_1^* = x_3^* = x_6^* = x_7^* = 1$ , others = 0

**2-30. (a)** We must decide what quantities to move from surplus sites to fulfill each need.

(b)  $s_i \triangleq$  the supply available at  $i$ ,  $r_j \triangleq$  the quantity needed at  $j$ ,  $d_{i,j} \triangleq$  the distance from  $i$  to  $j$ .

(c)  $\min \sum_{i=1}^4 \sum_{j=1}^7 d_{i,j} x_{i,j}$

(d)  $\sum_{j=1}^7 x_{i,j} = s_i$ ,  $i = 1, \dots, 4$

(e)  $\sum_{i=1}^4 x_{i,j} = r_j$ ,  $j = 1, \dots, 7$

(f)  $x_{i,j} \geq 0$ ,  $i = 1, \dots, 4$ ,  $j = 1, \dots, 7$

(g) A single objective LP because the one objective and all constraints are linear, and all variables are continuous.

(h) Nonzeros:  $x_{1,1}^* = 81$ ,  $x_{1,2}^* = 93$ ,  
 $x_{1,3}^* = 166$ ,  $x_{1,5}^* = 90$ ,  $x_{1,6}^* = 85$ ,  $x_{1,7}^* = 145$ ,  
 $x_{2,2}^* = 301$ ,  $x_{3,1}^* = 166$ ,  $x_{3,4}^* = 105$ ,  $x_{4,3}^* = 99$

**2-31. (a)** The values to be chosen are the

coefficients in the estimating relationship.

(b)  $\min \sum_{j=1}^n (c_j - k/(1 + e^{a+bf_j}))^2$  (min total squared error)

(c) Single objective NLP because the objective is quadratic, there are no constraints, and all variables are continuous.

**2-32.** (a) The decisions to be made are where to assign each teacher.

(b)  $\min \sum_{i=1}^{22} \sum_{j=1}^{22} c_{i,j} x_{i,j}$  (min total cost),  
 $\max \sum_{i=1}^{22} \sum_{j=1}^{22} t_{i,j} x_{i,j}$  (max total teacher preference),  $\max \sum_{i=1}^{22} \sum_{j=1}^{22} s_{i,j} x_{i,j}$  (max total supervisor preference),  $\max \sum_{i=1}^{22} \sum_{j=1}^{22} p_{i,j} x_{i,j}$  (max total principal preference)

(c)  $\sum_{j=1}^{22} x_{i,j} = 1, i = 1, \dots, 22$  (each teacher  $i$ )

(d)  $\sum_{i=1}^{22} x_{i,j} = 1, j = 1, \dots, 22$  (each school  $j$ )

(e)  $x_{i,j} = 0$  or  $1, i, j = 1, \dots, 22$

(f) A multiobjective ILP because the 4 objectives and all constraints are linear, but variables are discrete.

**2-33.** (a) Each task must go to Assistant 0 or Assistant 1.

(b)  $\max 100(1 - x_1) + 80x_1 + 85(1 - x_2) + 70x_2 + 40(1 - x_3) + 90x_3 + 45(1 - x_4) + 85x_4 + 70(1 - x_5) + 80x_5 + 82(1 - x_6) + 65x_6$

(c)  $\sum_{j=1}^6 x_j = 3$

(d)  $x_5 = x_6$

(e)  $x_j = 0$  or  $1, j = 1, \dots, 6$

(f) A single objective ILP because the one objective and all constraints are linear, but variables are discrete.

(g)  $x_2^* = x_3^* = x_4^* = 1$ , others  $= 0$

**2-34.** (a) Batch sizes are the decisions to be made.

(b)  $\min x_j/d_j, j = 1, \dots, 4$  (each burger  $j$ )

(c)  $\sum_{j=1}^4 t_j d_j / x_j \leq 60$

(d)  $0 \leq x_j \leq u_j, j = 1, \dots, 4$

(e) Multiobjective NLP because the first constraint is nonlinear and all variables are continuous.

**2-35.** (a) The issue is how many cars to move from where to where.

(b) Relatively large values can be rounded if fractional without much loss, and continuous is more tractable.

(c)  $c_{i,j} \triangleq$  the cost of moving a car from  $i$  to  $j$ ,  
 $p_j \triangleq$  the number of cars presently at  $j$ ,  $n_j \triangleq$  the number of cars required at  $j$

(d)  $\min \sum_{i=1}^5 \sum_{j=1, j \neq i}^5 c_{i,j} x_{i,j}$

(e)  $\sum_{i=1, i \neq k}^5 x_{i,k} - \sum_{j=1, j \neq k}^5 x_{k,j} = n_k - p_k$ ,  
 $k = 1, \dots, 5$  (each region  $k$ )

(f)  $x_{i,j} \geq 0, i, j = 1, \dots, 5, i \neq j$

(g) A single objective LP because the one objective and all constraints are linear, and all variables are continuous.

(h) Nonzero values:  $x_{4,2}^* = 115, x_{4,3}^* = 165, x_{5,1}^* = 85, x_{5,3}^* = 225$

**2-36.** (a) We must decide how much of what fuel to burn at each plant.

(b)  $\min \sum_{f=1}^4 \sum_{p=1}^{23} c_{f,p} x_{f,p}$

(c)  $\min \sum_{f=1}^4 s_f \sum_{p=1}^{23} x_{f,p}$

(d)  $\sum_{f=1}^4 e_f x_{f,p} \geq r_p, p = 1, \dots, 23$  (each plant  $p$ ); 23 constraints

(e)  $x_{f,p} \geq 0, f = 1, \dots, 4, p = 1, \dots, 23$ ; 92 constraints

(f) A multiobjective LP because the 2 objectives and all constraints are linear, and all variables are continuous.

**2-37.** (a) The available options are to buy whole logs or green lumber.

(b) Relatively large magnitudes can be rounded without much loss, and continuous is more tractable.

(c)  $\min$

$70x_{10} + 200x_{15} + 620x_{20} + 1.55y_1 + 1.30y_2$

(d)  $100(.09)x_{10} + 240(.09)x_{15} + 400(.09)x_{20} + .10y_1 + .08y_2 \geq 2350$

(e)  $x_{10} + x_{15} + x_{20} \leq 1500$  (sawing capacity),  
 $100x_{10} + 240x_{15} + 400x_{20} + y_1 + y_2 \leq 26500$  (drying capacity)

(f)  $x_{10} \leq 50$  (size 10 log availability),

$x_{15} \leq 25$  (size 15 log availability),  $x_{20} \leq 10$  (size 20 log availability),  $y_1 \leq 5000$  (grade 1 green lumber availability)

(g)  $x_{10}, x_{15}, x_{20}, y_1, y_2 \geq 0$

(h) A single objective LP because the one



objective and all constraints are linear, and all variables are continuous.

(i)  $x_{10}^* = 50$ ,  $x_{15}^* = 25$ ,  $x_{20}^* = 5$ ,  $y_1^* = 5000$ ,  $y_2^* = 8500$

**2-38. (a)** Decisions to be made are when to schedule each film.

(b)  $\min \sum_{j=1}^{m-1} \sum_{j'=j+1}^m a_{j,j'} \sum_{t=1}^n x_{j,t} x_{j',t}$   
 (c)  $\sum_{t=1}^n x_{j,t} = 1$ ,  $j = 1, \dots, m$  (each film  $j$ )  
 (d)  $\sum_{j=1}^m x_{j,t} \leq 4$ ,  $t = 1, \dots, n$  (each time  $t$ )  
 (e)  $x_{j,t} = 0$  or  $1$ ,  $j = 1, \dots, m$ ;  $t = 1, \dots, n$   
 (f) A single objective INLP because the one objective is nonlinear, and variables are discrete. (g) model; param  $m$ ; param  $n$ ; set films := 1 ..  $m$ ; set slots := 1 ..  $n$ ; var  $x\{j \text{ in films}, t \text{ in slots}\}$  binary; param  $a\{j \text{ in films}, jp \text{ in films}\}$ ; minimize totconflict:  $\sum\{j \text{ in films}, jp \text{ in films}: j < m \text{ and } jp > j\} a[j,jp] * \sum\{t \text{ in slots}\} x[j,t] * x[jp,t]$ ; subject to allin  $\{j \text{ in films}\}$ :  $\sum\{t \text{ in slots}\} x[j,t] = 1$ ; max4  $\{t \text{ in slots}\}$ :  $\sum\{j \text{ in films}\} x[j,t] \leq 4$ ;

**2-39. (a)** We need to decide both which offices to open and how to service customers from them.

(b) Offices must either be opened or not.

(c)  $f_i \triangleq$  fixed cost of site  $i$ ,  $c_{i,j} \triangleq$  unit cost of audits at  $j$  from  $i$ ,  $r_j \triangleq$  required number of audits in state  $j$

(d)  $\min \sum_{i=1}^5 \sum_{j=1}^5 c_{i,j} r_j x_{i,j} + \sum_{i=1}^5 f_i y_i$

(e)  $\sum_{i=1}^5 x_{i,j} = 1$ ,  $j = 1, \dots, 5$  (each location  $j$ )

(f)  $x_{i,j} \leq y_i$ ,  $i, j = 1, \dots, 5$  (each site  $i$ , location  $j$  combination)

(g)  $x_{i,j} \geq 0$ ,  $i, j = 1, \dots, 5$ ,  $y_i = 0$  or  $1$ ,  $i = 1, \dots, 5$

(h) A single objective ILP because the one objective and all constraints are linear, but the  $y_i$  variables are discrete.

(i) Nonzeros:

$x_{2,2}^* = x_{2,4}^* = x_{3,1}^* = x_{3,3}^* = x_{5,5}^* = 1$ ,

$y_2^* = y_3^* = y_5^* = 1$  (j) model; param  $m$ ; param  $n$ ; set sites := 1 ..  $m$ ; set

states := 1 ..  $n$ ; var  $x\{i \text{ in sites}, j \text{ in states}\} \geq 0$ ; var  $y\{i \text{ in sites}\}$  binary; param  $c\{i \text{ in sites}, j \text{ in states}\}$ ; param  $f\{i \text{ in sites}\}$  binary; param  $r\{j \text{ in states}\}$ ; minimize totcost:  $\sum\{i \text{ in sites}, j \text{ in states}\} c[i,j] * r[j] * x[i,j] + \sum\{i \text{ in sites}\} f[i] * y[i]$ ;  $x[j,t] * x[jp,t]$ ; subject to doeach  $\{j \text{ in states}\}$ :  $\sum\{i \text{ in sites}\} x[i,j] = 1$ ; switch  $\{i \text{ in sites}, j \text{ in states}\}$ :  $x[i,j] \leq y[i]$ ; data; param  $m := 5$ ; param  $n := 5$ ; param  $f := 1 \ 160 \ 2 \ 49 \ 3 \ 246 \ 4 \ 86 \ 4 \ 100$ ; param  $r := 1 \ 200 \ 2 \ 100 \ 3 \ 300 \ 4 \ 100 \ 5 \ 200$ ; param  $c$ :  $\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ 0.0 & 0.4 & 0.8 & 0.4 & 0.8 \\ 0.4 & 3 & 0.6 & 0.4 & 0.0 \\ 0.9 & 0.0 & 0.4 & 5 & 0.9 \end{matrix}$   $\begin{matrix} 0.8 & 2 & 0.7 & 0.0 & 0.8 \\ 0.4 & 4 & 0.6 & 0.4 & 0.0 \\ 0.4 & 0.7 & 0.4 & 0.0 & 0.0 \end{matrix}$ ;

**2-40. (a)**  $\max \sum_{j=1}^8 r_j x_j$ , subject to,

$\sum_{j=1}^8 x_j \leq 4$ ,  $x_1 + x_2 + x_3 \geq 2$ ,

$x_4 + x_5 + x_6 + x_7 + x_8 \geq 1$ ,

$x_2 + x_3 + x_4 + x_8 \geq 2$ ,  $x_1 \dots x_8 = 0$  or  $1$  (b)

model; param  $n$ ; set games := 1 ..  $n$ ; #ratings param  $r\{j \text{ in games}\}$ ; #home? param  $h\{j \text{ in games}\}$ ; #state? param  $s\{j \text{ in games}\}$ ; #cover? var  $x\{j \text{ in games}\}$  binary; maximize totat:  $\sum\{j \text{ in games}\} r[j] * x[j]$ ; subject to capacity:  $\sum\{j \text{ in games}\} x[j] \leq 4$ ; home:  $\sum\{j \text{ in games}\} h[j] * x[j] \geq 2$ ; away:  $\sum\{j \text{ in games}\} (1-h[j]) * x[j] \geq 1$ ; state:  $\sum\{j \text{ in games}\} s[j] * x[j] \geq 2$ ; data; param  $n := 8$ ; param  $r := 1 \ 3.0 \ 2 \ 3.7 \ 3 \ 2.6 \ 4 \ 1.8 \ 5 \ 1.5 \ 6 \ 1.3 \ 7 \ 1.6 \ 8 \ 2.0$ ; param  $h := 1 \ 1 \ 2 \ 1 \ 3 \ 1 \ 4 \ 0 \ 5 \ 0 \ 6 \ 0 \ 7 \ 0 \ 8 \ 0$ ; param  $s := 1 \ 0 \ 2 \ 1 \ 3 \ 1 \ 4 \ 1 \ 5 \ 0 \ 6 \ 0 \ 7 \ 0 \ 8 \ 1$ ; (c) The model is an ILP because all constraints and the objective are linear, but decision variables are binary.

**2-41. (a)** How to divide funds is the issue.

(b)  $\max \sum_{j=1}^n v_j x_j$

(c)  $\min \sum_{j=1}^n r_j x_j$

(d)  $\sum_{j=1}^n x_j = 1$

(e)  $x_j \geq \ell_j$ ,  $j = 1, \dots, n$  (each category  $j$ )

(f)  $x_j \leq u_j, j = 1, \dots, n$  (each category  $j$ )

(g) A multiobjective LP because the 2 objectives and all constraints are linear, and all variables are continuous.

$$31x_{2,4} + 18x_{3,4} \leq 7777, x_{1,1} + x_{2,1} + x_{3,1} \geq 200, \\ x_{1,2} + x_{2,2} + x_{3,2} \geq 300, x_{1,3} + x_{2,3} + x_{3,3} \geq 250, x_{1,4} + x_{2,4} + x_{3,4} \geq 500, x_{j,t} \geq 0, j = 1, \dots, 3, t = 1, \dots, 4.$$

**2-42.** (a) The issue is which module goes to which site.

(b) If  $x_{i,j}x_{i',j'} = 1$  the  $i$  is at  $j$  and  $i'$  is at  $j'$ , so wire  $d_{j,j'}$  will be required. Summing over all possible location pairs captures the wire requirements for  $i$  and  $i'$ .

(c) min

$$\sum_{i=1}^{m-1} \sum_{i'=i+1}^m a_{i,i'} \sum_{j=1}^n \sum_{j'=1}^n d_{j,j'} x_{i,j} x_{i',j'}$$

(d)  $\sum_{j=1}^n x_{i,j} = 1, i = 1, \dots, m$  (each module  $i$ )

(e)  $\sum_{i=1}^m x_{i,j} \leq 1, j = 1, \dots, n$  (each site  $j$ )

(f)  $x_{i,j} = 0$  or  $1, i = 1, \dots, m, j = 1, \dots, n$

(g) Single objective INLP because the one objective is nonlinear and variables are

discrete. (h) `model; param m; param n; set modules := 1 .. m; set sites := 1 .. n; var x{i in modules, j in sites} binary; param a{ i in modules, ip in modules }; param d{ j in sites, jp in sites }; minimize totdist: sum{ i in modules, ip in modules: i < m and ip > i } a[i,ip] sum{ j in sites, jp in sites : j < n and jp > j } d[j,jp]*x[i,j]*x[ip,jp]; subject to alli {i in modules }: sum{ j in sites } x[i,j] = 1; allj { j in sites }: sum { i in modules } x[i,j] <= 1;`

**2-43.** max  $199x_1 + 229x_2 + 188x_3 + 205x_4 - 180y_1 - 224y_2 - 497y_3$ , subject to,  
 $23x_3 + 41x_4 \leq 2877y_1, 14x_1 + 29x_2 \leq 2333y_2,$   
 $11x_3 + 27x_4 \leq 3011y_3,$   
 $x_1 + x_2 + x_3 + x_4 \geq 205, y_1 + y_2 + y_3 \leq 2,$   
 $x_1, \dots, x_4 \geq 0, y_1, \dots, y_3 = 0$  or  $1$

**2-44.** max  $11x_{1,1} + 15x_{1,2} + 19x_{1,3} + 10x_{1,4} + 19x_{2,1} + 23x_{2,2} + 44x_{2,3} + 67x_{2,4} + 17x_{3,1} + 18x_{3,2} + 24x_{3,3} + 55x_{3,4}$ , subject to,  $15x_{1,1} + 24x_{2,1} + 17x_{3,1} \leq 7600, 19x_{1,2} + 26x_{2,2} + 13x_{3,2} \leq 8200, 23x_{1,3} + 18x_{2,3} + 16x_{3,3} \leq 6015, 14x_{1,4} + 33x_{2,4} + 14x_{3,4} \leq 5000, 31x_{1,1} + 26x_{2,1} + 21x_{3,1} \leq 6600, 25x_{1,2} + 28x_{2,2} + 17x_{3,2} \leq 7900, 39x_{1,3} + 22x_{2,3} + 20x_{3,2} \leq 5055, 29x_{1,4} +$