

CHAPTER

3

Project Management

DISCUSSION QUESTIONS

1. There are many possible answers. Project management is needed in large construction jobs, in implementing new information systems, in new product development/marketing, in creating a new assembly line, and so on.
2. Project organizations make sure existing programs continue to run smoothly while new projects are successfully completed.
3. The three phases involved in managing a large project are *planning*, *scheduling*, and *controlling*.
4. PERT and CPM help answer questions relating to which task elements are on (or likely to be on) the critical path and to probable completion times for the overall project. Some specific questions include:
 - When will the entire project be completed?
 - Which are the critical activities or tasks in the project; that is, the activities that will delay the entire project if completed behind schedule?
 - Which are the noncritical activities; that is, those that can run behind schedule without delaying the whole project? How far behind schedule can these activities run without disrupting the completion time?
 - What is the probability that the project will be completed by a specific date?
 - At any particular date, is the project on schedule, behind schedule, or ahead of schedule?
 - On any given date, is the money spent equal to, less than, or greater than the budgeted amount?
 - Are there enough resources available to finish the project on time?
 - If the project is required to be finished in a shorter amount of time, what is the least-cost way to accomplish this?
5. WBS is a hierarchical subdivision of effort required to achieve an objective. It defines a project by breaking it down into manageable parts and even finer subdivisions.
6. A Gantt chart is a visual device that shows the duration of tasks in a project. It is a low-cost means of ensuring that (1) all activities are planned for, (2) their order of performance is planned for, (3) the activity times are recorded, and (4) the overall project time is developed.
7. The difference between AOA and AON is that activities are shown on arrows in the former and on the node in the latter. We primarily use AON in this chapter.

8. Any late start or extension of an activity on the critical path will delay the completion of the project.

9. To crash an activity, the project manager would pay money to add resources (overtime, extra help).

10. Activity times used in PERT are assumed to be described by a Beta probability distribution. Given optimistic (a), pessimistic (b), and most likely (m), completion times, average or expected time is given by:

$$t = \frac{a + 4m + b}{6}$$

and the variance by:

$$\text{Variance} = \left[\frac{(b - a)}{6} \right]^2$$

11. Early start (ES) of an activity is the latest of the early finish times of all its predecessors. Early finish (EF) is the early start of an activity plus its duration. Late finish (LF) of an activity is the earliest of the late start times of all successor activities. Late start (LS) of an activity is its late finish less its duration.

12. The critical path is the *shortest time possible for the completion of a series of activities*, but that shortest time is the longest path through the network. Only the longest path allows time for all activities in the series; any smaller amount will leave activities unfinished.

13. Dummy activities have no time duration. They are inserted into a AOA network to maintain the logic of the network, such as when two activities have exactly the same beginning and ending events. A dummy activity is inserted with one of them so that the computer software can handle the problem.

14. They are (1) optimistic time estimate (a), an estimate of the minimum time an activity will require; (2) most likely time estimate (m), an estimate of the normal time an activity will require; and (3) pessimistic time estimate (b), an estimate of the maximum time an activity will require.

15. No. In networks, there is no possibility that crashing a non-critical task can reduce the project duration. Only critical tasks offer the possibility of reducing path length. However, other criteria for crashing may exist: for instance, skills required in one of the activities may also be needed elsewhere.

16. Total PERT project *variance* is computed as the sum of the variances of all activities on the critical path.

17. Slack: the amount of time an activity can be delayed and not affect the overall completion time of the whole project. Slack can be determined by finding the difference between the earliest start time and the latest start time, or the earliest finish time and the latest finish time for a given activity.

18. If there are a sufficient number of tasks along the critical path, we can assume that project completion time is described by a normal probability distribution with mean equal to the sum of the expected times of all activities on the critical path and variance equal to the sum of the variances of all activities on the critical path.

The fundamental assumption required is that the number of activities on the critical path is large enough that the mean of the sum of the Beta distributions is distributed approximately as the normal distribution.

19. Widely used *project management software* include's MS Project, MacProject, Primavera, Mind View, HP Project, and Fast Track.

ETHICAL DILEMMA

Large projects with time/cost overruns are not uncommon situations in the world of project management. Why do MIS projects commonly sport 200–300% cost overruns and completion times twice those projected? Why do massive construction projects run so late and so overbudget?

Students are expected to read about such projects and come up with explanations, especially related to ethics. In the case of MIS projects, long software development tasks are almost doomed to failure because of the changes in technology and staff that take place. It's a necessity to break large projects down into smaller 3- to 6-month modules or pieces that are self-contained. This protects the organization from a *total* loss should the massive project never be completed.

In every case, quality project management means open communication, realistic timetables, good staff, and use of software like MS Project to build and maintain a schedule. Bidding on a contract with a schedule that is not feasible may be unethical as well as poor business.

ACTIVE MODEL EXERCISE*

ACTIVE MODEL 3.1: Gantt Chart

1. Both A and H are critical activities. Describe the difference between what happens on the graph when you increase A vs. increasing H.

When you increase H, it is the only task to change on the chart. However, when you increase A then all critical tasks move to the right and the slack for the noncritical tasks increases.

2. Activity F is not critical. By how many weeks can you increase activity F until it becomes critical?

6 weeks

3. Activity B is not critical. By how many weeks can you increase activity B until it becomes critical? What happens when B becomes critical?

1 week. Activity D also becomes critical.

4. What happens when you increase B by 1 more week after it becomes critical?

Activities A, C, and E become noncritical, and the project takes 1 additional week.

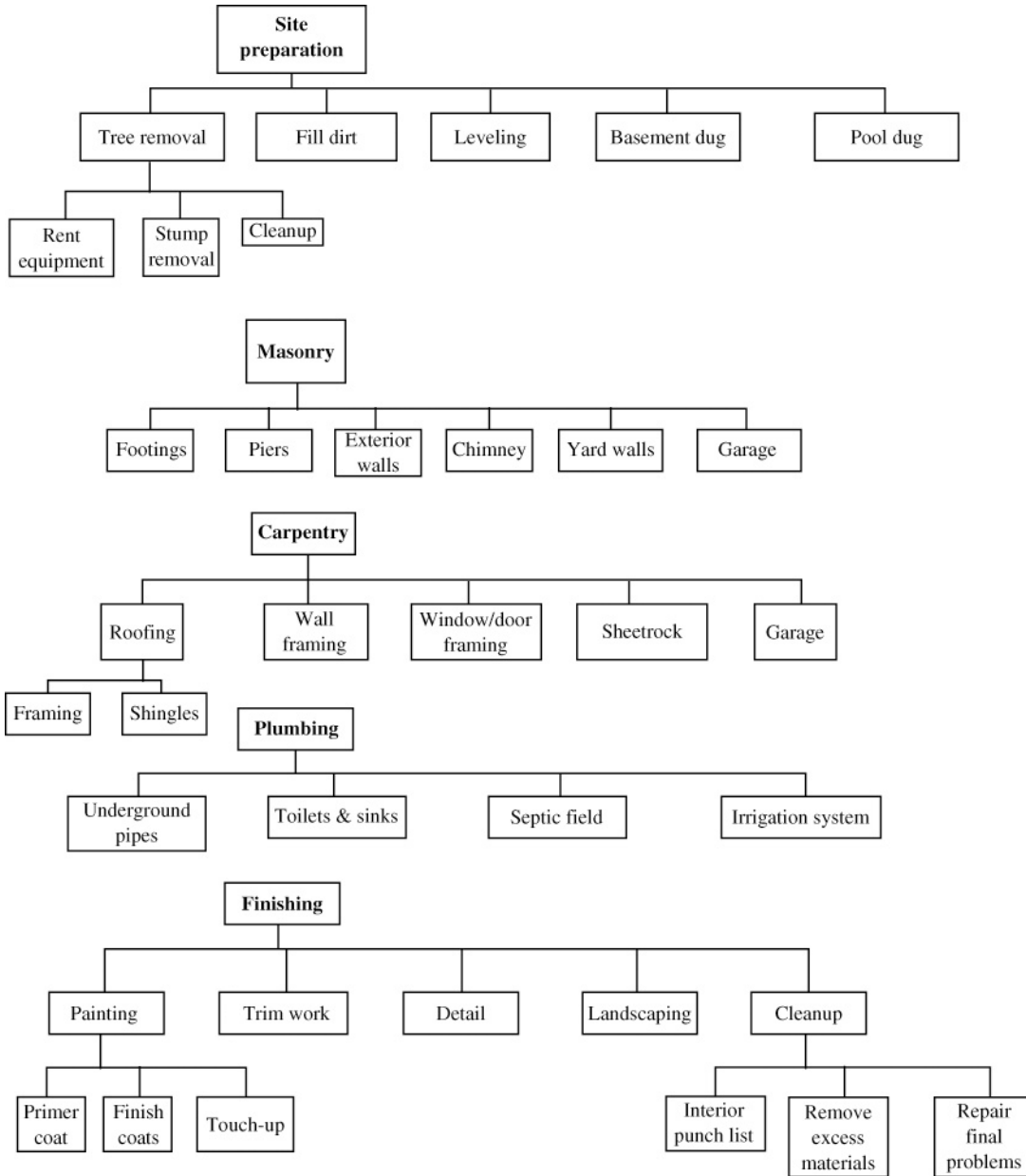
5. Suppose that building codes may change and, as a result, activity B would have to be completed before activity C could be started. How would this affect the project?

Activity B becomes critical, and the project takes 1 additional week.

* Active Model 3.1 appears on our Web sites, at www.pearsonhighered.com/heizer and www.myomlab.com.

END-OF-CHAPTER PROBLEMS

3.1 Some possible Level 3[(a)] and Level 4[(b)] activities for the house appear for each Level 2 activity below.



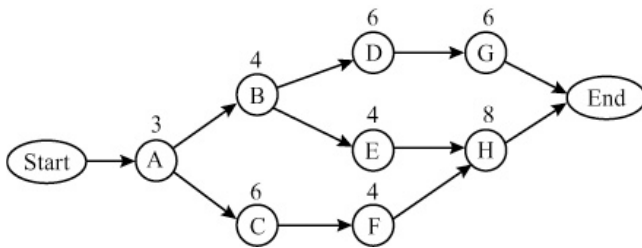
3.2 Here are some detailed activities to add to Day's WBS:*

- 1.11 Set initial goals for fundraising
- 1.12 Set strategy including identifying sources and solicitation
- 1.13 Raise the funds
- 1.21 Identify voters' concerns
- 1.22 Analyze competitor's voting record
- 1.23 Establish positions on issues
- 1.31 Hire campaign manager and political advisor
- 1.32 Get volunteers
- 1.33 Hire a staff
- 1.34 Hire media consultants
- 1.41 Identify filing deadlines
- 1.42 File for candidacy
- 1.51 Train staff for audit planning

Students could make many other choices.

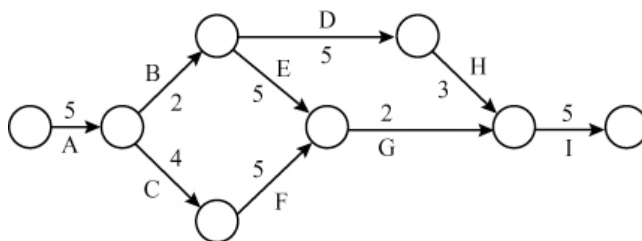
*Source: Modified from an example found in M. Hanna and W. Newman, *Operations Management*: Prentice Hall, Upper Saddle River, NJ (2001): p. 722.

3.3



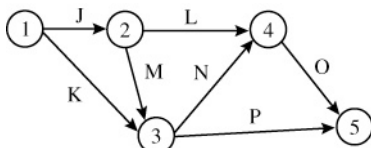
Critical path is A-C-F-H. Time = 21 days.
This is an AON network.

3.4

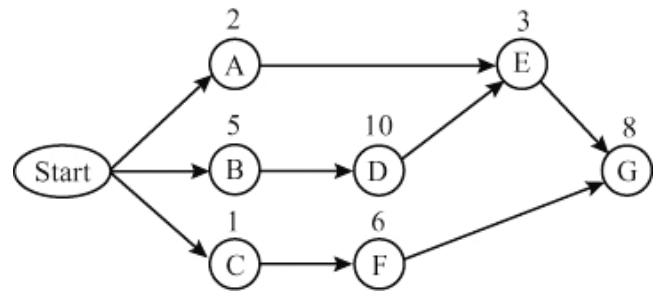


Critical path is A-C-F-G-I. Time = 21 days.
This is an AOA network.

3.5 The paths through this network are J-L-O, J-M-P, J-M-N-O, K-P, and K-N-O. Their path durations are 23, 18, 22, 13, and 17. J-L-O is the critical path; its duration is 23.



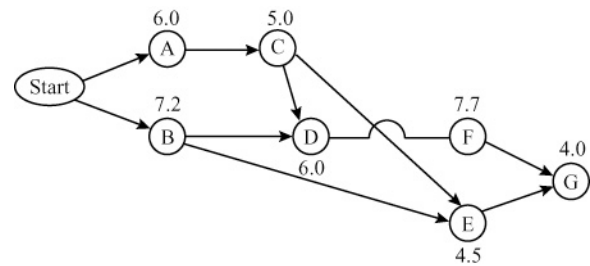
3.6 (a)



(b) Critical path is B-D-E-G

| Activity | Time | ES | EF | LS | LF | Slack | Critical |
|----------|------|----|----|----|----|-------|----------|
| A | 2 | 0 | 2 | 13 | 15 | 13 | No |
| B | 5 | 0 | 5 | 0 | 5 | 0 | Yes |
| C | 1 | 0 | 1 | 11 | 12 | 11 | No |
| D | 10 | 5 | 15 | 5 | 15 | 0 | Yes |
| E | 3 | 15 | 18 | 15 | 18 | 0 | Yes |
| F | 6 | 1 | 7 | 12 | 18 | 11 | No |
| G | 8 | 18 | 26 | 18 | 26 | 0 | Yes |

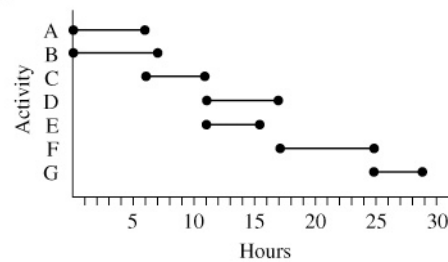
3.7 (a)



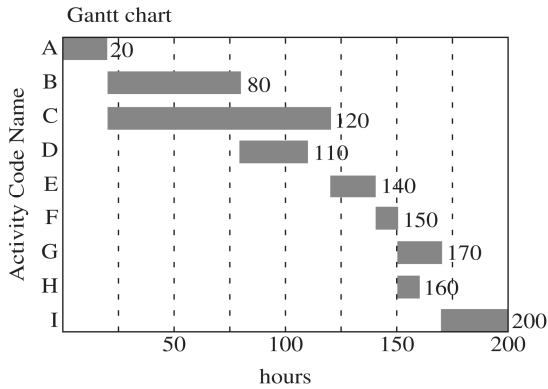
(b, c) There are four paths:

| Path | Time (hours) |
|-----------|-----------------|
| A-C-E-G | 19.5 |
| B-D-F-G | 24.9 |
| A-C-D-F-G | 28.7 (critical) |
| B-E-G | 15.7 |

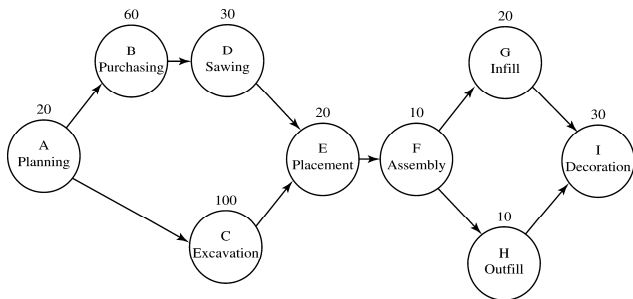
(d) Gantt Chart



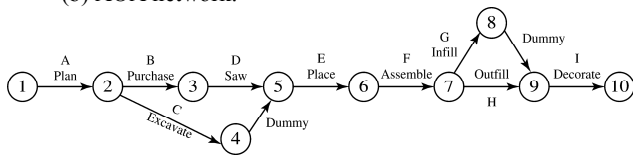
3.8



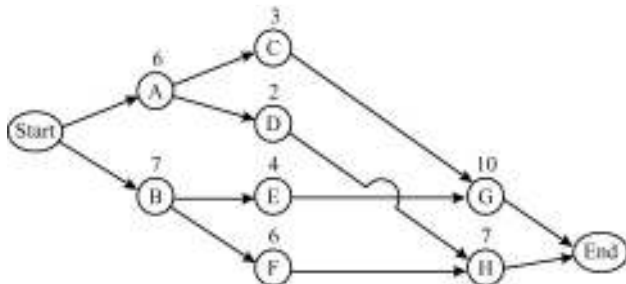
3.9 (a) AON network:



(b) AOA network:



3.10



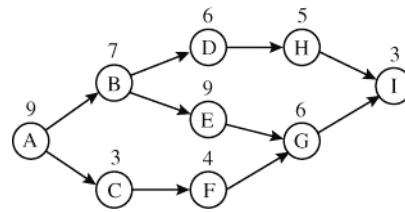
Note: Activity times are shown as an aid for Problem 3.11. They are not required in the solution to Problem 3.10.

3.11

| Activity | Time | ES | EF | LS | LF | Slack | Critical |
|----------|------|----|----|----|----|-------|----------|
| A | 6 | 0 | 6 | 2 | 8 | 2 | No |
| B | 7 | 0 | 7 | 0 | 7 | 0 | Yes |
| C | 3 | 6 | 9 | 8 | 11 | 2 | No |
| D | 2 | 6 | 8 | 12 | 14 | 6 | No |
| E | 4 | 7 | 11 | 7 | 11 | 0 | Yes |
| F | 6 | 7 | 13 | 8 | 14 | 1 | No |
| G | 10 | 11 | 21 | 11 | 21 | 0 | Yes |
| H | 7 | 13 | 20 | 14 | 21 | 1 | No |

The critical path is given for activities B, E, G.
Total project completion time is 21 weeks.

3.12 (a)



(b, c)

| Task | Time | ES | EF | LS | LF | Slack |
|------|------|----|----|----|----|-------|
| A | 9 | 0 | 9 | 0 | 9 | 0 |
| B | 7 | 9 | 16 | 9 | 16 | 0 |
| C | 3 | 9 | 12 | 18 | 21 | 9 |
| D | 6 | 16 | 22 | 20 | 26 | 4 |
| E | 9 | 16 | 25 | 16 | 25 | 0 |
| F | 4 | 12 | 16 | 21 | 25 | 9 |
| G | 6 | 25 | 31 | 25 | 31 | 0 |
| H | 5 | 22 | 27 | 26 | 31 | 4 |
| I | 3 | 31 | 34 | 31 | 34 | 0 |

Activities on the critical path: A, B, E, G, I
Project completion time = 34

3.13

| Activity | a | m | b | $t = \frac{a + 4m + b}{6}$ | $\sigma = \frac{b-a}{6}$ | Variance |
|----------|----|----|----|----------------------------|--------------------------|----------|
| A | 11 | 15 | 19 | 15 | 1.33 | 1.77 |
| B | 27 | 31 | 41 | 32 | 2.33 | 5.44 |
| C | 18 | 18 | 18 | 18 | 0 | 0 |
| D | 8 | 13 | 19 | 13.17 | 1.83 | 3.36 |
| E | 17 | 18 | 20 | 18.17 | 0.5 | 0.25 |
| F | 16 | 19 | 22 | 19 | 1 | 1 |

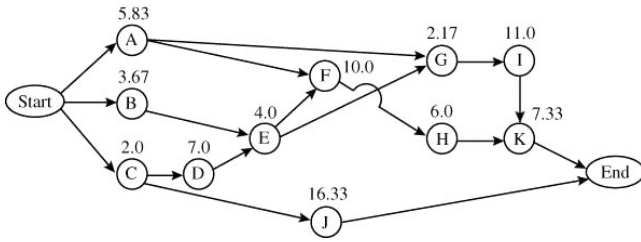
3.14 (a)

| Activity | a | m | b | Expected | Variance |
|----------|----|----|----|----------|----------|
| A | 3 | 6 | 8 | 5.83 | 0.69 |
| B | 2 | 4 | 4 | 3.67 | 0.11 |
| C | 1 | 2 | 3 | 2.00 | 0.11 |
| D | 6 | 7 | 8 | 7.00 | 0.11 |
| E | 2 | 4 | 6 | 4.00 | 0.44 |
| F | 6 | 10 | 14 | 10.00 | 1.78 |
| G | 1 | 2 | 4 | 2.17 | 0.25 |
| H | 3 | 6 | 9 | 6.00 | 1.00 |
| I | 10 | 11 | 12 | 11.00 | 0.11 |
| J | 14 | 16 | 20 | 16.33 | 1.00 |
| K | 2 | 8 | 10 | 7.33 | 1.78 |

(b, c)

| Activity | Time | ES | EF | LS | LF | Slack | Critical |
|----------|-------|-------|-------|-------|-------|-------|----------|
| A | 5.83 | 0.00 | 5.83 | 7.17 | 13.00 | 7.17 | No |
| B | 3.67 | 0.00 | 3.67 | 5.33 | 9.00 | 5.33 | No |
| C | 2.00 | 0.00 | 2.00 | 0.00 | 2.00 | 0.00 | Yes |
| D | 7.00 | 2.00 | 9.00 | 2.00 | 9.00 | 0.00 | Yes |
| E | 4.00 | 9.00 | 13.00 | 9.00 | 13.00 | 0.00 | Yes |
| F | 10.00 | 13.00 | 23.00 | 13.00 | 23.00 | 0.00 | Yes |
| G | 2.17 | 13.00 | 15.17 | 15.83 | 18.00 | 2.83 | No |
| H | 6.00 | 23.00 | 29.00 | 23.00 | 29.00 | 0.00 | Yes |
| I | 11.00 | 15.17 | 26.17 | 18.00 | 29.00 | 2.83 | No |
| J | 16.33 | 2.00 | 18.33 | 20.00 | 36.33 | 18.00 | No |
| K | 7.33 | 29.00 | 36.33 | 29.00 | 36.33 | 0.00 | Yes |

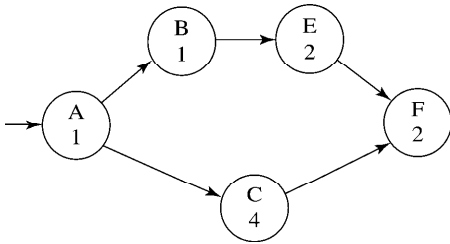
The critical path is given by activities C, D, E, F, H, K. Average project completion time is 36.33 days.



- (d) Expected completion time for the project is 36.33 days. Project variance = Sum of variances of activities on critical path = $0.11 + 0.11 + 0.44 + 1.78 + 1.00 + 1.78 = 5.22$. Standard deviation = 2.28

$$P(t \leq 40) = P\left[z \leq \frac{40 - 36.33}{2.28}\right] = P[z \leq 1.61] = 0.946$$

3.15 (a) AON diagram of the project:

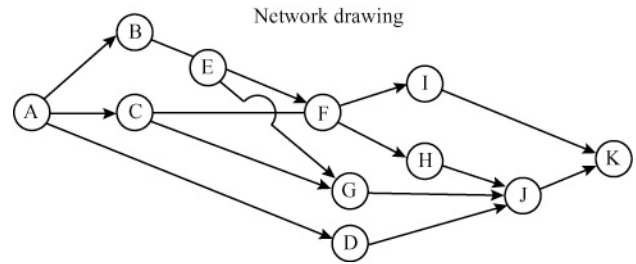


- (b) The critical path, listing all critical activities in chronological order:
 $A \rightarrow B \rightarrow E \rightarrow F$ $1 + 1 + 2 + 2 = 6$ (not CP)
 $A \rightarrow C \rightarrow F$ $1 + 4 + 2 = 7$. This is the CP.
- (c) The project duration (in weeks):
 7 (This is the length of CP.)
- (d) The slack (in weeks) associated with any and all non-critical paths through the project: Look at the paths that aren't critical—only 1 here—so from above:
 $A \rightarrow B \rightarrow E \rightarrow F$ $7 - 6 = 1$ week slack.

3.16 Helps to modify the AON with the lowest costs to crash:

- CP is $A \rightarrow C \rightarrow F$; C is cheapest to crash, so take it to 3 wks at \$200 (and $\$200 < \250)
- Now both paths through are critical. We would need to shorten A or F, or shorten C and either B/E. This is not worth it, so we would not bother to crash any further.

3.17



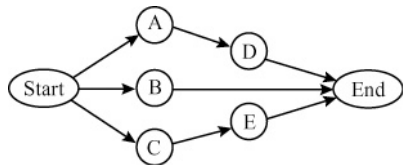
- (a) Estimated (expected) time for C = $[8 + (4 \times 12) + 16]/6 = 72/6 = 12$ weeks
- (b) Variance for C is $\left[\frac{(16 - 8)}{6}\right]^2 = \frac{16}{9} = 1.78$
- (c) Critical path is A-C-F-H-J-K
- (d) Time on critical path = $7.67 + 12 + 9.67 + 2 + 6.67 + 2.17 = 40.18$ weeks (rounded)
- (e) Variance on critical path = $1 + 1.78 + 5.44 + 0 + 1.78 + 0.03 = 10.03$
- (f) $Z = \frac{36 - 40.18}{3.17} = -1.32$, which is about 9.6% chance (.096 probability) of completing project before week 36.

Note that based on possible rounding in part (d)—where time on critical path could be 40.3—the probability can be as low as 8.7%. So a student answer between 8.7% and 9.6% is valid.

Summary table for Problem 3.17 follows:

| Activity | Activity Time | Early Start | Early Finish | Late Start | Late Finish | Slack | Standard Deviation |
|----------|---------------|-------------|--------------|------------|-------------|-------|--------------------|
| A | 7.66 | 0 | 7.66 | 0.0 | 7.66 | 0 | 1 |
| B | 9.66 | 7.66 | 17.33 | 8 | 17.66 | 0.33 | 3.66 |
| C | 12 | 7.66 | 19.66 | 7.66 | 19.66 | 0 | 1.33 |
| D | 6.33 | 7.66 | 14 | 25 | 31.33 | 17.33 | 1 |
| E | 2 | 17.33 | 19.33 | 17.66 | 19.66 | 0.33 | 0.33 |
| F | 9.66 | 19.66 | 29.33 | 19.66 | 29.33 | 0 | 2.33 |
| G | 3 | 19.66 | 22.66 | 28.33 | 31.33 | 8.66 | 0.33 |
| H | 2 | 29.33 | 31.33 | 29.33 | 31.33 | 0 | 0 |
| I | 6 | 29.33 | 35.33 | 32 | 38 | 2.66 | 0 |
| J | 6.66 | 31.33 | 38 | 31.33 | 38 | 0 | 1.33 |
| K | 2.16 | 38 | 40.17 | 38 | 40.17 | 0 | 0.17 |

3.18



Critical path C-E at 12 days.

| Activity | Daily Crash Costs | Maximum Crash |
|----------|-------------------|---------------|
| A | \$100 | 1 day |
| B | 50 | 2 days |
| C | 100 | 1 day |
| D | 150 | 2 days |
| E | 200 | 3 days |

To crash by 4 days, from 12 days to 8 days:

- Crash C by 1 day (\$100) to 11 days total
- Now crash E by 1 day (\$200) and A by 1 day (\$100) to 10 days total.
- Now crash E by 2 days (\$400) and D by 2 days (\$300) to 8 days total.
- Total additional cost to crash 4 days = \$1,100.

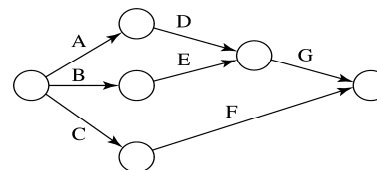
3.19 Crash costs per unit time are \$600 for A, \$900 for B, and \$1,000 for C. (a) A offers the cheapest path to a single day reduction. (b) A cannot supply a second reduction, so the next best choice is B, which adds \$900. (c) The total for both days is \$1,500.

3.20 (a) Project completion time = 16 (Activities A-D-G)

| Activity | Norm. Time-Crash Time | Crash \$-Normal \$ | \$/time |
|----------|-----------------------|--------------------|---------|
| A | 1 | \$600 | \$600 |
| B | 1 | 600 | 600 |
| C | 0 | 0 | — |
| D | 4 | 300 | 75 |
| E | 3 | 300 | 100 |
| F | 1 | 1,200 | 1,200 |
| G | 2 | 600 | 300 |

- (b) Total cost = \$12,300.
- (c) Crash D 1 week at an additional cost of \$75.

Figure for Problem 3.20



(d)

| Activity | Crash | Cost |
|----------|-------|---------|
| D | 4 | \$300 |
| G | 2 | 600 |
| A | 1 | 600 |
| E | 1 | 100 |
| 7 weeks | | \$1,600 |

3.21 (a)

| Activity | a | m | b | t _e | Variance |
|----------|---|----|----|----------------|----------|
| A | 9 | 10 | 11 | 10 | 0.11 |
| B | 4 | 10 | 16 | 10 | 4 |
| C | 9 | 10 | 11 | 10 | 0.11 |
| D | 5 | 8 | 11 | 8 | 1 |

(b) Critical path is A-C with mean (t_e) completion time of 20 weeks. The other path is B-D, with mean completion time of 18 weeks.

(c) Variance of A-C = (Variance of A) + (Variance of C) = 0.11 + 0.11 = 0.22

Variance of B-D = (Variance of B) + (Variance of D) = 4 + 1 = 5

(d) Probability A-C is finished in 22 weeks or less =

$$P\left(Z \leq \frac{22 - 20}{\sqrt{0.22}}\right) = P(Z \leq 4.26) \cong 1.00$$

(e) Probability B-D is finished in 22 weeks or less =

$$P\left(Z \leq \frac{22 - 18}{\sqrt{5}}\right) = P(Z \leq 1.79) = 0.963$$

(f) The critical path has a relatively small variance and will almost certainly be finished in 22 weeks or less. Path B-D has a relatively high variance. Due to this, the probability B-D is finished in 22 weeks or less is only about 0.96. Since the project is not finished until all activities (and paths) are finished, the probability that the project will be finished in 22 weeks or less is not 1.00 but is approximately 0.96.

3.22 (a)

| Activity | a | m | b | Expected Time | Variance |
|----------|---|---|----|---------------|-------------------|
| A | 4 | 6 | 7 | 5.83 | 0.25 ² |
| B | 1 | 2 | 3 | 2.00 | 0.11 |
| C | 6 | 6 | 6 | 6.00 | 0.00 ² |
| D | 5 | 8 | 11 | 8.00 | 1.00 |
| E | 1 | 9 | 18 | 9.17 | 8.03 ² |
| F | 2 | 3 | 6 | 3.33 | 0.44 |
| G | 1 | 7 | 8 | 6.17 | 1.36 |
| H | 4 | 4 | 6 | 4.33 | 0.11 ² |
| I | 1 | 6 | 8 | 5.50 | 1.36 ² |
| J | 2 | 5 | 7 | 4.83 | 0.69 |
| K | 8 | 9 | 11 | 9.17 | 0.25 ² |
| L | 2 | 4 | 6 | 4.00 | 0.44 |
| M | 1 | 2 | 3 | 2.00 | 0.11 ² |
| N | 6 | 8 | 10 | 8.00 | 0.44 ² |

| Activity | Time | ES | EF | LS | LF | Slack | Critical |
|----------|------|-------|-------|-------|-------|-------|----------|
| A | 5.83 | 0.00 | 5.83 | 0.00 | 5.83 | 0.00 | Yes |
| B | 2.00 | 0.00 | 2.00 | 9.83 | 11.83 | 9.83 | No |
| C | 6.00 | 5.83 | 11.83 | 5.83 | 11.83 | 0.00 | Yes |
| D | 8.00 | 5.83 | 13.83 | 9.67 | 17.67 | 3.84 | No |
| E | 9.17 | 11.83 | 21.00 | 11.83 | 21.00 | 0.00 | Yes |
| F | 3.33 | 13.83 | 17.16 | 17.67 | 21.00 | 3.84 | No |
| G | 6.17 | 13.83 | 20.00 | 19.16 | 25.33 | 5.33 | No |
| H | 4.33 | 21.00 | 25.33 | 21.00 | 25.33 | 0.00 | Yes |
| I | 5.50 | 25.33 | 30.83 | 25.33 | 30.83 | 0.00 | Yes |
| J | 4.83 | 30.83 | 35.66 | 33.17 | 38.00 | 2.34 | No |
| K | 9.17 | 30.83 | 40.00 | 30.83 | 40.00 | 0.00 | Yes |
| L | 4.00 | 35.66 | 39.66 | 38.00 | 42.00 | 2.34 | No |
| M | 2.00 | 40.00 | 42.00 | 40.00 | 42.00 | 0.00 | Yes |
| N | 8.00 | 42.00 | 50.00 | 42.00 | 50.00 | 0.00 | Yes |

*The critical path is given by activities A, C, E, H, I, K, M, N. Completion time is 50 days.

(b) $P(\text{Completion in 53 days}) = \Sigma \text{Variances on critical path} = 10.55$ so, $\sigma_{cp} = 3.25$.

$$P(t \leq 53) = P\left[z \leq \frac{53 - 50}{3.25}\right] = P[z \leq 0.92] = 0.821 = 82.1\%$$

(c)

$$P\left[z \leq \frac{x - 50}{3.25}\right]$$

where $z = 2.33$ for 99% probability.

so $2.33 = \frac{x - 50}{3.25}$. Then

$$x = 50 + (2.33)(3.25) = 57.57 \approx 58 \text{ days}$$

3.23 (a) This project management problem can be solved using PERT. The results are below. As you can see, the total project completion time is about 32 weeks. The critical path consists of activities C, H, M, and O.

Project completion time = 32.05

$$\text{Variance (C)} = \left(\frac{13 - 10}{6}\right)^2 = \left(\frac{3}{6}\right)^2 = \frac{9}{36}$$

$$\text{Variance (H)} = \left(\frac{9 - 5}{6}\right)^2 = \frac{16}{36}$$

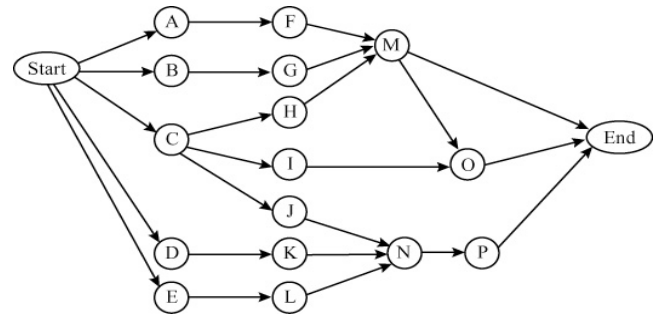
$$\text{Variance (M)} = \left(\frac{6.5 - 5}{6}\right)^2 = \frac{2.25}{36}$$

$$\text{Variance (O)} = \left(\frac{8 - 5}{6}\right)^2 = \frac{9}{36}$$

$$\text{Project variance} = \frac{9}{36} + \frac{16}{36} + \frac{2.25}{36} + \frac{9}{36} \approx 1.00$$

Project standard deviation = 1.00

| Activity | Activity Time | Early Start | Early Finish | Late Start | Late Finish | Slack |
|----------|---------------|-------------|--------------|------------|-------------|-------|
| A | 2.16 | 0 | 2.16 | 10.13 | 12.3 | 10.13 |
| B | 3.5 | 0 | 3.5 | 11.88 | 15.38 | 11.88 |
| C | 11.83 | 0 | 11.83 | 0 | 11.83 | 0 |
| D | 5.16 | 0 | 5.16 | 14.65 | 19.82 | 14.65 |
| E | 3.83 | 0 | 3.83 | 15.98 | 19.82 | 15.98 |
| F | 7 | 2.17 | 9.16 | 12.3 | 19.3 | 10.13 |
| G | 3.92 | 3.5 | 7.42 | 15.38 | 19.3 | 11.88 |
| H | 7.47 | 11.83 | 19.3 | 11.83 | 19.3 | 0 |
| I | 10.32 | 11.83 | 22.15 | 14.9 | 25.22 | 3.06 |
| J | 3.83 | 11.83 | 15.66 | 19.98 | 23.82 | 8.15 |
| K | 4 | 5.16 | 9.16 | 19.82 | 23.82 | 14.65 |
| L | 4 | 3.83 | 7.83 | 19.82 | 23.82 | 15.98 |
| M | 5.92 | 19.3 | 25.22 | 19.3 | 25.22 | 0 |
| N | 1.23 | 15.66 | 16.9 | 23.82 | 22.05 | 8.15 |
| O | 6.83 | 25.22 | 32.05 | 25.22 | 32.05 | 0 |
| P | 7 | 16.9 | 23.9 | 25.05 | 32.05 | 8.15 |



(b) As can be seen in the following analysis, the changes do not have any impact on the critical path or the total project completion time. A summary of the analysis is below.

Project completion time = 32.05

Project standard deviation = 1.00

| Activity | Activity Time | Early Start | Early Finish | Late Start | Late Finish | Slack |
|----------|---------------|-------------|--------------|------------|-------------|-------|
| A | 2.16 | 0 | 2.16 | 10.13 | 12.3 | 10.13 |
| B | 3.5 | 0 | 3.5 | 11.88 | 15.38 | 11.88 |
| C | 11.83 | 0 | 11.83 | 0 | 11.83 | 0 |
| D | 5.16 | 0 | 5.16 | 14.65 | 19.82 | 14.65 |
| E | 3.83 | 0 | 3.83 | 15.98 | 19.82 | 15.98 |
| F | 7 | 2.16 | 9.16 | 12.3 | 19.3 | 10.13 |
| G | 3.92 | 3.5 | 7.42 | 15.38 | 19.3 | 11.88 |
| H | 7.46 | 11.83 | 19.3 | 11.83 | 19.3 | 0 |
| I | 0 | 11.83 | 11.83 | 25.22 | 25.22 | 13.38 |
| J | 0 | 11.83 | 11.83 | 23.82 | 23.82 | 11.98 |
| K | 4 | 5.16 | 9.16 | 19.82 | 23.82 | 14.65 |
| L | 4 | 3.83 | 7.83 | 19.82 | 23.82 | 15.98 |
| M | 5.92 | 19.3 | 25.22 | 19.3 | 25.22 | 0 |
| N | 1.23 | 11.83 | 13.06 | 23.82 | 22.05 | 11.98 |
| O | 6.83 | 25.22 | 32.05 | 25.22 | 32.05 | 0 |
| P | 7 | 13.06 | 20.06 | 25.05 | 32.05 | 11.98 |

3.24 (a) Probability of completion is 17 months or less:

$$P(t \leq 17) = P\left[z \leq \frac{17-21}{2}\right] = P[z \leq -2.0]$$

$$= 1 - P[z \geq 2.0] = 1 - 0.97725 = 0.0228$$

(b) Probability of completion in 20 months or less:

$$P(t \leq 20) = P\left[z \leq \frac{20-21}{2}\right] = P[z \leq -0.5]$$

$$= 1 - P[z \geq 0.5] = 1 - 0.69146 = 0.3085$$

(c) Probability of completion in 23 months or less:

$$P(t \leq 23) = P\left[z \leq \frac{23-21}{2}\right] = P[z \leq 1.0] = 0.84134$$

(d) Probability of completion in 25 months or less:

$$P(t \leq 25) = P\left[z \leq \frac{25-21}{2}\right] = P[z \leq 2.0] = 0.97725$$

(e)

$$P\left[z \leq \frac{x-21}{2}\right] = 1.645 \text{ for a 95\% chance of completion}$$

by the x date.

$$\text{Then } x = 21 + 2(1.645)$$

$$= 24.29, \text{ or } 24 \text{ months.}$$

3.25 (a)

| Project completion time = 14 weeks | | | | | | |
|------------------------------------|------|----|----|----|----|-------|
| Task | Time | ES | EF | LS | LF | Slack |
| A | 3 | 0 | 3 | 0 | 3 | 0 |
| B | 2 | 0 | 2 | 2 | 4 | 2 |
| C | 1 | 0 | 1 | 11 | 12 | 11 |
| D | 7 | 3 | 10 | 3 | 10 | 0 |
| E | 6 | 2 | 8 | 4 | 10 | 2 |
| F | 2 | 1 | 3 | 12 | 14 | 11 |
| G | 4 | 10 | 14 | 10 | 14 | 0 |

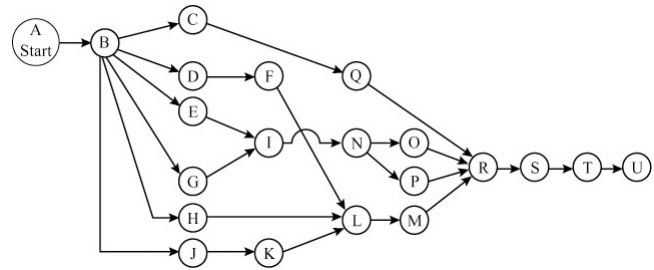
(b) To crash to 10 weeks, we follow 2 steps:

1. Crash D by 2 weeks (\$150).
 2. Crash D and E by 2 weeks each (\$100 + \$150).
- Total crash cost = \$400 additional

(c) Using POM for Windows software, *minimum* project completion time = 7. Additional crashing cost = \$1,550.

| | Normal Time | Crash Time | Normal Cost | Crash Cost | Crash Cost/Pd | Crash By | Crashing Cost |
|---|-------------|------------|-------------|------------|---------------|----------|---------------|
| A | 3 | 2 | 1,000 | 1,600 | 600 | 1 | 600 |
| B | 2 | 1 | 2,000 | 2,700 | 700 | 0 | 0 |
| C | 1 | 1 | 300 | 300 | 0 | 0 | 0 |
| D | 7 | 3 | 1,300 | 1,600 | 75 | 4 | 300 |
| E | 6 | 3 | 850 | 1,000 | 50 | 3 | 150 |
| F | 2 | 1 | 4,000 | 5,000 | 1,000 | 0 | 0 |
| G | 4 | 2 | 1,500 | 2,000 | 250 | 2 | 500 |

3.26 (a)



3.26 (b)

| Task | Time | ES | EF | LS | LF | Slack |
|------|------|------|------|------|------|-------|
| A | 0.0* | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| B | 8.0 | 0.0 | 8.0 | 0.0 | 8.0 | 0.0 |
| C | 0.1 | 8.0 | 8.1 | 10.4 | 10.5 | 2.4 |
| D | 1.0 | 8.0 | 9.0 | 12.0 | 13.0 | 4.0 |
| E | 1.0 | 8.0 | 9.0 | 10.0 | 11.0 | 2.0 |
| F | 1.0 | 9.0 | 10.0 | 13.0 | 14.0 | 4.0 |
| G | 2.0 | 8.0 | 10.0 | 9.0 | 11.0 | 1.0 |
| H | 3.0 | 8.0 | 11.0 | 11.0 | 14.0 | 3.0 |
| I | 1.0 | 10.0 | 11.0 | 11.0 | 12.0 | 1.0 |
| J | 4.0 | 8.0 | 12.0 | 8.0 | 12.0 | 0.0 |
| K | 2.0 | 12.0 | 14.0 | 12.0 | 14.0 | 0.0 |
| L | 1.0 | 14.0 | 15.0 | 14.0 | 15.0 | 0.0 |
| M | 0.5 | 15.0 | 15.5 | 15.0 | 15.5 | 0.0 |
| N | 2.0 | 11.0 | 13.0 | 12.0 | 14.0 | 1.0 |
| O | 1.0 | 13.0 | 14.0 | 14.5 | 15.5 | 1.5 |
| P | 1.5 | 13.0 | 14.5 | 14.0 | 15.5 | 1.0 |
| Q | 5.0 | 8.1 | 13.1 | 10.5 | 15.5 | 2.4 |
| R | 1.0 | 15.5 | 16.5 | 15.5 | 16.5 | 0.0 |
| S | 0.5 | 16.5 | 17.0 | 16.5 | 17.0 | 0.0 |
| T | 1.0 | 17.0 | 18.0 | 17.0 | 18.0 | 0.0 |
| U | 0.0* | 18.0 | 18.0 | 18.0 | 18.0 | 0.0 |

*Note: Start (A) and Finish (U) are assigned times of zero.

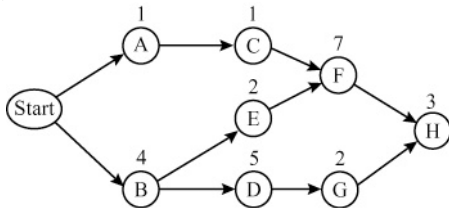
Critical path is A-B-J-K-L-M-R-S-T-U, for 18 days.

- (c) (i) no, transmissions and drivetrains are not on the critical path.
- (ii) no, halving engine building time will reduce the critical path by only one day.
- (iii) no, it is not on the critical path.
- (d) Reallocating workers not involved with critical path activities to activities along the critical path will reduce the critical path length.

ADDITIONAL HOMEWORK PROBLEMS*

Problems 3.27 to 3.33 appear at www.myomlab.com and www.pearsonhighered.com/heizer.

3.27 (a)

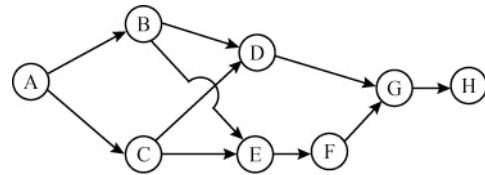


- (b) Critical path is B-E-F-H.
- (c) Time = 16 weeks

- 3.28 (a) Expected times for individual activities (using $(a + 4m + b)/6$). A = 5, B = 6, C = 7, D = 6, E = 3. Expected project completion time = 15 (Activities A-C-E).
- (b) Variance for individual activities (using $[(b - a)/6]^2$). A = 1; B = 1; C = 1; D = 4; E = 0. Project variance = Σ variances on critical path = 1 + 1 + 0 = 2.

* Note to instructor: To broaden the selection of homework problems, these seven problems are also available to you and your students.

3.29 (a)



(b)

| Activity | Time | ES | EF | LS | LF | Slack | σ | σ^2 |
|----------|------|----|----|----|----|-------|----------|------------|
| A | 7 | 0 | 7 | 0 | 7 | 0 | 2 | 4* |
| B | 3 | 7 | 10 | 13 | 16 | 6 | 1 | 1 |
| C | 9 | 7 | 16 | 7 | 16 | 0 | 3 | 9* |
| D | 4 | 16 | 20 | 25 | 29 | 9 | 1 | 1 |
| E | 5 | 16 | 21 | 16 | 21 | 0 | 1 | 1* |
| F | 8 | 21 | 29 | 21 | 29 | 0 | 2 | 4* |
| G | 8 | 29 | 37 | 29 | 37 | 0 | 1 | 1* |
| H | 6 | 37 | 43 | 37 | 43 | 0 | 2 | 4* |

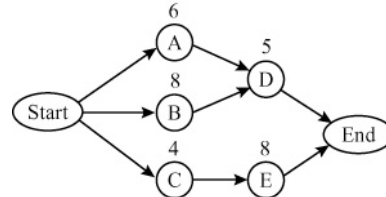
*Activities on the critical path: A, C, E, F, G, H. Project Completion time = 43.

(b) $\sigma = 4.8$
 $z = \frac{49 - 43}{4.8} = 1.25$

$P(t \leq 49) = .89435$

$P(t \geq 49) = (1 - .89435) = 0.10565$

3.30 AON network



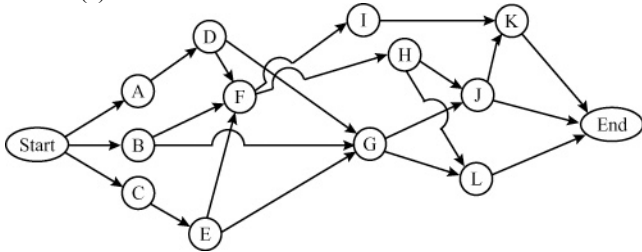
Critical path is B-D, at 13 days

| Activity | Daily Crash Costs | Maximum Crash (days) |
|----------|-------------------|----------------------|
| A | \$100 | 1 day |
| B | 50 | 2 days |
| C | 100 | 1 day |
| D | 150 | 2 days |
| E | 200 | 3 days |

To crash by 4 days, from 13 days to 9 days,

- Crash B by 1 day (\$50) to reach 12 days
- Crash B by a second day (\$50) and C by 1 day (\$100) to reach 11 days.
- Crash D by 2 days (\$300) and E by 2 days (\$400) to reach 9 days total.
- Total cost to crash 4 days = \$900

3.31 (a)



(b) Expected times and variances:

| Activity | a | m | b | Expected Time | Variance |
|----------|----|----|----|---------------|----------|
| A | 8 | 10 | 12 | 10.0 | 0.44 |
| B | 6 | 7 | 9 | 7.2 | 0.25 |
| C | 3 | 3 | 4 | 3.2 | 0.03 |
| D | 10 | 20 | 30 | 20.0 | 11.11 |
| E | 6 | 7 | 8 | 7.0 | 0.11 |
| F | 9 | 10 | 11 | 10.0 | 0.11 |
| G | 6 | 7 | 10 | 7.3 | 0.44 |
| H | 14 | 15 | 16 | 15.0 | 0.11 |
| I | 10 | 11 | 13 | 11.2 | 0.25 |
| J | 6 | 7 | 8 | 7.0 | 0.11 |
| K | 4 | 7 | 8 | 6.7 | 0.44 |
| L | 1 | 2 | 4 | 2.2 | 0.25 |

(c) ES, EF, LS, LF, and slack times:

| Activity | Time | ES | EF | LS | LF | Slack | Critical |
|----------|------|------|------|------|------|-------|----------|
| A | 10.0 | 0.0 | 10.0 | 0.0 | 10.0 | 0.0 | Yes |
| B | 7.2 | 0.0 | 7.2 | 22.8 | 30.0 | 22.8 | No |
| C | 3.2 | 0.0 | 3.2 | 19.8 | 23.0 | 19.8 | No |
| D | 20.0 | 10.0 | 30.0 | 10.0 | 30.0 | 0.0 | Yes |
| E | 7.0 | 3.2 | 10.2 | 23.0 | 30.0 | 19.8 | No |
| F | 10.0 | 30.0 | 40.0 | 30.0 | 40.0 | 0.0 | Yes |
| G | 7.3 | 30.0 | 37.3 | 47.7 | 55.0 | 17.7 | No |
| H | 15.0 | 40.0 | 55.0 | 40.0 | 55.0 | 0.0 | Yes |
| I | 11.2 | 40.0 | 51.2 | 50.8 | 62.0 | 10.8 | No |
| J | 7.0 | 55.0 | 62.0 | 55.0 | 62.0 | 0.0 | Yes |
| K | 6.7 | 62.0 | 68.7 | 62.0 | 68.7 | 0.0 | Yes |
| L | 2.2 | 55.0 | 57.2 | 66.5 | 68.7 | 11.5 | No |

(d) The critical path is given by the activities

A, D, F, H, J, K

Expected project completion time is 68.7 days.

(e) Probability of completion in 70 days or less:

$$\begin{aligned} \text{Project variance} &= \text{Sum of variances of activities} \\ &\quad \text{on critical path} \\ &= 0.44 + 11.11 + 0.11 + 0.11 + \\ &\quad 0.44 = 12.32 \\ \text{Standard deviation} &= 3.51 \end{aligned}$$

$$P(t \leq 70) = P\left[z \leq \frac{70 - 68.7}{3.51}\right] = P[z \leq 0.37] = 0.644$$

(f) Probability of completion in 80 days or less:

$$P(t \leq 80) = P\left[z \leq \frac{80 - 68.7}{3.51}\right] = P[z \leq 3.22] = 0.99934$$

(g) Probability of completion in 90 days or less:

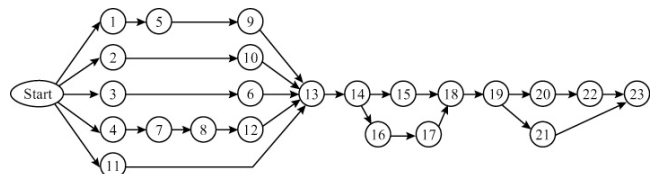
$$P(t \leq 90) = P\left[z \leq \frac{90 - 68.7}{3.51}\right] = P[z \leq 6.07] = 0.99999$$

3.32 The overall purpose of Problem 3.32 is to have students use a network to solve a problem that almost all students face. The first step is for students to list all courses that they must take, including possible electives, to get a degree from their particular college or university. For every course, students should list all the immediate predecessors. Then students are asked to attempt to develop a network diagram that shows these courses and their immediate predecessors or prerequisite courses.

This problem can also point out some of the limitations of the use of project management. As students try to solve this problem they may run into several difficulties. First, it is difficult to incorporate a minimum or maximum number of courses that a student can take during a given semester. In addition, it is difficult to schedule elective courses. Some elective courses have prerequisites, while others may not. Even so, some of the overall approaches of network analysis can be helpful in terms of laying out the courses that are required and their prerequisites.

Students can also be asked to think about other techniques that can be used in solving this problem. One of the most appropriate approaches would be to use linear programming to incorporate many of the constraints, such as minimum and maximum number of credit hours per semester, that are difficult or impossible to incorporate in a project network.

3.33 The construction company problem involves 23 separate activities. These activities, their immediate predecessors, and time estimates were given in the problem. One of the most difficult aspects of this problem is to take the data in the table given in the problem and to construct a network diagram. This network diagram is necessary in order to determine beginning and ending node numbers that can be used in the computer program to solve this particular problem. The network diagram for this problem is as follows:



Once this diagram has been developed, activity numbers, starting and finishing node numbers, and the three time estimates for each activity can be entered into the computer program. The computer program calculates the expected time and variance estimates for each activity, the expected project length, variance, and data for all activities. Like the other network problems, these data include the earliest start time, earliest finish time, latest start time, latest time, and slack for all activities. The data are as follows:

Table for Problem 3.33

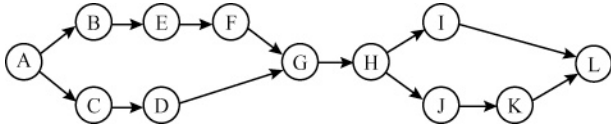
| Activity | Times | | | $E(t)$ | σ | ES | EF | LS | LF | Slack |
|----------|-------|-------|-------|--------|----------|-------|-------|-------|-------|-------|
| | Opt | Most | Pess | | | | | | | |
| 1 | 1.0 | 4.00 | 5.00 | 3.67 | 0.67 | 0.00 | 3.67 | 9.00 | 12.67 | 9.00 |
| 2 | 2.0 | 3.00 | 4.00 | 3.00 | 0.33 | 0.00 | 3.00 | 16.50 | 19.50 | 16.50 |
| 3 | 3.0 | 4.00 | 5.00 | 4.00 | 0.33 | 0.00 | 4.00 | 14.50 | 18.50 | 14.50 |
| 4 | 7.0 | 8.00 | 9.00 | 8.00 | 0.33 | 0.00 | 8.00 | 3.50 | 11.50 | 3.50 |
| 5 | 4.0 | 4.00 | 5.00 | 4.17 | 0.17 | 3.67 | 7.83 | 12.67 | 16.83 | 9.00 |
| 6 | 1.0 | 2.00 | 4.00 | 2.17 | 0.50 | 4.00 | 6.17 | 18.50 | 20.67 | 14.50 |
| 7 | 4.0 | 5.00 | 6.00 | 5.00 | 0.33 | 8.00 | 13.00 | 11.50 | 16.50 | 3.50 |
| 8 | 1.0 | 2.00 | 4.00 | 2.17 | 0.50 | 13.00 | 15.17 | 16.50 | 18.67 | 3.50 |
| 9 | 3.0 | 4.00 | 4.00 | 3.83 | 0.17 | 7.83 | 11.67 | 16.83 | 20.67 | 9.00 |
| 10 | 1.0 | 1.00 | 2.00 | 1.17 | 0.17 | 3.00 | 4.17 | 19.50 | 20.67 | 16.5 |
| 11 | 18.0 | 20.00 | 26.00 | 20.67 | 1.33 | 0.00 | 20.67 | 0.00 | 20.67 | 0.00 |
| 12 | 1.0 | 2.00 | 3.00 | 2.00 | 0.33 | 15.17 | 17.17 | 18.67 | 20.67 | 3.50 |
| 13 | 1.0 | 1.00 | 2.00 | 1.17 | 0.17 | 20.67 | 21.83 | 20.67 | 21.83 | 0.00 |
| 14 | 0.1 | 0.14 | 0.16 | 0.14 | 1.00 | 21.83 | 21.97 | 21.83 | 21.97 | 0.00 |
| 15 | 0.2 | 0.30 | 0.40 | 0.30 | 0.03 | 21.97 | 22.27 | 24.84 | 25.14 | 2.87 |
| 16 | 1.0 | 1.00 | 2.00 | 1.17 | 0.17 | 21.97 | 23.14 | 21.97 | 23.14 | 0.00 |
| 17 | 1.0 | 2.00 | 3.00 | 2.00 | 0.33 | 23.14 | 25.14 | 23.14 | 25.14 | 0.00 |
| 18 | 3.0 | 5.00 | 7.00 | 5.00 | 0.67 | 25.14 | 30.14 | 25.14 | 30.14 | 0.00 |
| 19 | 0.1 | 0.10 | 0.20 | 0.12 | 0.02 | 30.14 | 30.25 | 30.14 | 30.25 | 0.00 |
| 20 | 0.1 | 0.14 | 0.16 | 0.14 | 0.00 | 30.25 | 30.39 | 33.33 | 33.47 | 3.08 |
| 21 | 2.0 | 3.00 | 6.00 | 3.33 | 0.67 | 30.25 | 33.59 | 30.25 | 33.59 | 0.00 |
| 22 | 0.1 | 0.10 | 0.20 | 0.12 | 0.02 | 30.39 | 30.50 | 33.47 | 33.58 | 3.08 |
| 23 | 0.0 | 0.20 | 0.20 | 0.17 | 0.03 | 33.59 | 33.75 | 33.59 | 33.75 | 0.00 |

As you can see, the expected project length is about 34 weeks. The activities along the critical path are activities 11, 13, 14, 16, 17, 18, 19, 21, and 23.

CASE STUDY

SOUTHWESTERN UNIVERSITY: A

1.



| Activity | Mean | S.D. | Variance |
|----------|------|--------|----------|
| t_A | = 30 | 3.33* | 11.11 |
| t_B | = 60 | 10.00 | 100.00 |
| t_C | = 65 | 8.33* | 69.39 |
| t_D | = 55 | 11.66* | 136.10 |
| t_E | = 30 | 1.67 | 2.78 |
| t_F | = 0 | 0.10 | 0.00 |
| t_G | = 30 | 1.67* | 2.78 |
| t_H | = 20 | 3.33* | 11.11 |
| t_I | = 30 | 6.67* | 44.44 |
| t_J | = 10 | 0.67 | 0.44 |
| t_K | = 0 | 0.10 | 0.00 |
| t_L | = 30 | 6.67* | 44.44 |

*Critical path

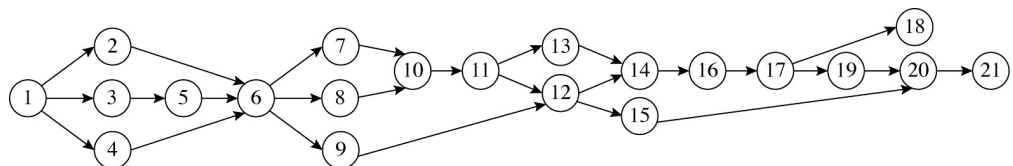
Critical path = A – C – D – G – H – I – L (260 days)

Variance of critical path = 11.11 + 69.39 + 136.1 + 2.78 + 11.11 + 44.44 + 44.44 = 319.37

Standard deviation of critical path = 17.87 days

2. $P(\text{Completion} < 270 \text{ days}) = P(t \leq 270)$

$$P\left[Z \leq \frac{270 - 260}{17.87}\right] = P[Z \leq 0.56] = 0.712 = 71.2\%$$



3. Crash to 250 days and to 240 days

| Activity | Normal Time (days) | Crash Time (days) | Crash Cost/Day |
|----------|--------------------|-------------------|----------------|
| A | 30 | 20 | \$1,500 |
| B | 60 | 20 | \$3,500 |
| C | 65 | 50 | \$4,000 |
| D | 55 | 30 | \$1,900 |
| E | 30 | 25 | \$9,500 |
| F | 0* | 0 | \$0 |
| G | 30 | 25 | \$2,500 |
| H | 20 | 10 | \$2,000 |
| I | 30 | 20 | \$2,000 |
| J | 10 | 8 | \$6,000 |
| K | 0* | 0 | \$0 |
| L | 30 | 20 | \$4,500 |

*Rounded to zero from 0.1

To crash to 250 days (from the current 260 days), select A at \$1,500/day × 10 days = \$15,000.

To crash to 240 days now (from the current 250 days), select D at \$1,900/day × 10 days = \$19,000.

Total cost to crash to 240 days = \$34,000 additional.

VIDEO CASE STUDIES

1 PROJECT MANAGEMENT AT ARNOLD PALMER HOSPITAL

The Arnold Palmer Hospital video for this case (8 minutes) is available from Pearson Prentice Hall. Also note that the *Global Company Profile* in Chapter 6 highlights this hospital.

1. Construction project network:

2. The critical path is Activities 1–3–5–6–8–10–11–12–14–16–17–19–20–21. The project length is 47 months—about four years from start to finish.

3. Building a hospital is much more complex than an office building for several reasons. In this case, hundreds of “users” of the new building had extensive input. Second, the design of the new layout (circular, pod design) is somewhat radical compared to traditional “linear” hospitals. Third, the hospital was built with future expansion in mind. Fourth, the guiding principles impacted on design/construction. Fifth, hospitals, by their very nature, are more complex from a safety, health hazard, security, quiet, serenity perspective than an office building.

4. Since there were 13 months of planning prior to the proposal/review stage (listed as Activity 1) and the project then took 47 months (for a total of 60 months), 22% of the time was spent in planning.

Construction itself started with activity 14 (19.75 months); there were 13 + 19.75 months of work (= 32.75 months) out of 60 months, or 55% of the time was spent prior to building beginning.

- 1.3 Hire operations manager (T)
 - 1.31 Site plan (U)
 - 1.311 Power, etc. (X)
 - 1.32 Security director (V)
 - 1.321 Set police/fire plan (W)

Answers may vary somewhat at the Level 3 and Level 4.

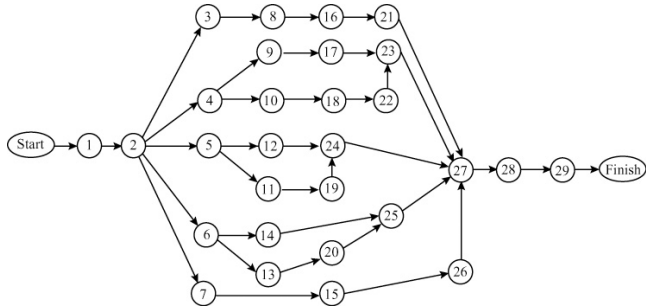
Level 2 activities should be activities B, C, and T.

ADDITIONAL CASE STUDY*

SHALE OIL COMPANY

1. Determine the expected shutdown time and the probability the shutdown will be completed 1 week earlier.
2. What are the probabilities that Shale finishes the maintenance project 1 day, 2 days, 3 days, 4 days, 5 days, or 6 days earlier?

From the precedence data supplied in the problem, we can develop the following AON network:



* This case study is found on our Companion Web sites, at www.pearsonhighered.com/heizer and www.myomlab.com.

The following table indicates the expected times, variances, and slacks needed to complete the rest of the case:

| Activity | Opt | Most Likely | Pess | E(t) | σ | ES | EF | LS | LF | Slack |
|----------|------|-------------|------|-------|----------|-------|-------|-------|-------|--------|
| 1 | 1.0 | 2.0 | 2.5 | 1.92 | 0.25 | 0.00 | 1.92 | 0.00 | 1.92 | 0.000 |
| 2 | 1.5 | 2.0 | 2.5 | 2.00 | 0.17 | 1.92 | 3.92 | 1.92 | 3.92 | 0.000 |
| 3 | 2.0 | 3.0 | 4.0 | 3.00 | 0.33 | 3.92 | 6.92 | 3.92 | 6.92 | 0.000 |
| 4 | 1.0 | 2.0 | 3.0 | 2.00 | 0.33 | 3.92 | 5.92 | 22.50 | 24.50 | 18.580 |
| 5 | 1.0 | 2.0 | 4.0 | 2.17 | 0.50 | 3.92 | 6.08 | 10.25 | 12.42 | 6.333 |
| 6 | 2.0 | 2.5 | 3.0 | 2.50 | 0.17 | 3.92 | 6.42 | 13.42 | 15.92 | 9.500 |
| 7 | 2.0 | 4.0 | 5.0 | 3.83 | 0.50 | 3.92 | 7.75 | 29.58 | 33.42 | 25.670 |
| 8 | 1.0 | 2.0 | 3.0 | 2.00 | 0.33 | 6.92 | 8.92 | 6.92 | 8.92 | 0.000 |
| 9 | 1.0 | 1.5 | 2.0 | 1.50 | 0.17 | 5.92 | 7.42 | 26.67 | 28.17 | 20.750 |
| 10 | 1.0 | 1.5 | 2.0 | 1.50 | 0.17 | 5.92 | 7.42 | 24.50 | 26.00 | 18.580 |
| 11 | 2.0 | 2.5 | 3.0 | 2.50 | 0.17 | 6.08 | 8.58 | 19.92 | 22.42 | 13.830 |
| 12 | 15.0 | 20.0 | 30.0 | 20.83 | 2.50 | 6.08 | 26.92 | 12.42 | 33.25 | 6.330 |
| 13 | 1.0 | 1.5 | 2.0 | 1.50 | 0.17 | 6.42 | 7.92 | 15.92 | 17.42 | 9.500 |
| 14 | 3.0 | 5.0 | 8.0 | 5.17 | 0.83 | 6.42 | 11.58 | 28.08 | 33.25 | 21.670 |
| 15 | 3.0 | 8.0 | 15.0 | 8.33 | 2.00 | 7.75 | 16.08 | 33.42 | 41.75 | 25.670 |
| 16 | 14.0 | 21.0 | 28.0 | 21.00 | 2.33 | 8.92 | 29.92 | 8.92 | 29.92 | 0.000 |
| 17 | 1.0 | 5.0 | 10.0 | 5.17 | 1.50 | 7.42 | 12.58 | 28.17 | 33.33 | 20.750 |
| 18 | 2.0 | 5.0 | 10.0 | 5.33 | 1.33 | 7.42 | 12.75 | 26.00 | 31.33 | 18.580 |
| 19 | 5.0 | 10.0 | 20.0 | 10.83 | 2.50 | 8.58 | 19.42 | 22.42 | 33.25 | 13.830 |
| 20 | 10.0 | 15.0 | 25.0 | 15.83 | 2.50 | 7.92 | 23.75 | 17.42 | 33.25 | 9.500 |
| 21 | 4.0 | 5.0 | 8.0 | 5.33 | 0.67 | 29.92 | 35.25 | 29.92 | 35.25 | 0.000 |
| 22 | 1.0 | 2.0 | 3.0 | 2.00 | 0.33 | 12.75 | 14.75 | 31.33 | 33.33 | 18.580 |
| 23 | 1.0 | 2.0 | 2.5 | 1.92 | 0.25 | 14.75 | 16.67 | 33.33 | 35.25 | 18.580 |
| 24 | 1.0 | 2.0 | 3.0 | 2.00 | 0.33 | 26.92 | 28.92 | 33.25 | 35.25 | 6.330 |
| 25 | 1.0 | 2.0 | 3.0 | 2.00 | 0.33 | 23.75 | 25.75 | 33.25 | 35.25 | 9.500 |
| 26 | 2.0 | 4.0 | 6.0 | 4.00 | 0.67 | 16.08 | 20.08 | 41.75 | 45.75 | 25.670 |
| 27 | 1.5 | 2.0 | 2.5 | 2.00 | 0.17 | 35.25 | 37.25 | 35.25 | 37.25 | 0.000 |
| 28 | 1.0 | 3.0 | 5.0 | 3.00 | 0.67 | 37.25 | 40.25 | 37.25 | 40.25 | 0.000 |
| 29 | 3.0 | 5.0 | 10.0 | 5.50 | 1.17 | 40.25 | 45.75 | 40.25 | 45.75 | 0.000 |

From the table, we can see that the expected shutdown time is 45.75, or 46 days. There are nine activities on the critical path.

| Activity | σ | σ^2 |
|-----------------------------|----------|------------|
| 1 | 0.25 | 0.0625 |
| 2 | 0.17 | 0.0289 |
| 3 | 0.33 | 0.1089 |
| 8 | 0.33 | 0.1089 |
| 16 | 2.33 | 5.4289 |
| 21 | 0.67 | 0.4489 |
| 27 | 0.17 | 0.0289 |
| 28 | 0.67 | 0.4489 |
| 29 | 1.17 | 1.3689 |
| Variance for critical path: | | 8.0337 |

Therefore, $\sigma = \sqrt{8.0337} = 2.834$. As an approximation, we can use the customary equation for the normal distribution:

$$z = \frac{\text{Due date} - E(t)}{\sigma}$$

(Note: This might be a good time to discuss the difference between a continuous and a discrete probability distribution and the appropriate procedure for using a continuous distribution as an approximation to a discrete, if you have not already done so.)

| Finish Time | z | Probability |
|--------------|--------|-------------|
| 1 day early | -0.353 | 36.3 |
| 2 days early | -0.706 | 24.0 |
| 3 days early | -1.058 | 14.5 |
| 4 days early | -1.411 | 7.9 |
| 5 days early | -1.764 | 3.9* |
| 6 days early | -2.117 | 1.7 |
| 7 days early | -2.470 | 0.7 |

*The appropriate procedure for using the normal distribution gives 3.0%—roughly a 30% difference.

There is, by the approximate procedure used, a 3.9% probability of finishing 5 days, or 1 week, early.

3. *Shale Oil is considering increasing the budget to shorten the shutdown. How do you suggest the company proceed?*

In order to shorten the shutdown, Shale Oil would have to determine the costs of decreasing the activities on the critical path. This is the vessel and column branch of the network, which is typically the longest section in a shutdown. The cost of reducing activity time by one time unit for each activity in this branch would have to be calculated. The activity with the lowest of these costs could then be acted upon. Perhaps the repairs to the vessels and columns could be expedited with workers from some of the other branches with high slack time. However, delivery on materials could be an overriding factor.