

**Numerical Analysis 10E  
Chapter 02 Sample Exam**

Name (Print): \_\_\_\_\_

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1. (10 points) The equation  $f(x) = x^2 - 2e^x = 0$  has a solution in the interval  $[-1,1]$ .
    - (a) (5 points) With  $p_0 = -1$  and  $p_1 = 1$  calculate  $p_2$  using the Secant method.
    - (b) (5 points) With  $p_2$  from part (a) calculate  $p_3$  using Newton's method.
  2. (15 points) The equation  $f(x) = 2 - x^2 \sin x = 0$  has a solution in the interval  $[-1,2]$ .
    - (a) (5 points) Verify that the Bisection method can be applied to the function  $f(x)$  on  $[-1,2]$ .
    - (b) (5 points) Using the error formula for the Bisection method find the number of iterations needed for accuracy 0.000001. Do not do the Bisection calculations.
    - (c) (5 points) Compute  $p_3$  for the Bisection method.
  3. (15 points) The following refer to the fixed-point problem
    - (a) (5 points) State the theorem which gives conditions for a fixed-point sequence to converge to a unique fixed point.
    - (b) (5 points) Given  $g(x) = \frac{2 - x^3 + 2x}{3}$ , use the theorem to show that the fixed-point sequence will converge to the unique fixed-point of  $g$  for any  $p_0$  in  $[-1,1.1]$ .
    - (c) (5 points) With  $p_0 = 0.5$  generate  $p_3$ .
  4. (10 points) Suppose the function  $f(x)$  has a unique zero  $p$  in the interval  $[a, b]$ . Further, suppose  $f''(x)$  exists and is continuous on the interval  $[a,b]$ .
    - (a) (5 points) Under what conditions will Newton's Method give a quadratically convergent sequence to  $p$ ?
    - (b) (5 points) Define quadratic convergence.
  5. (10 points) Let  $g(x) = \frac{2 - x^3 + 2x}{3}$  on the interval  $[-1, 1.1]$ . Let the initial value be 0 and compute the result of 2 iterations of Steffensen's Method to approximate the solution of  $x = g(x)$ .