Solutions for Sections 2.1 – 2.4.2

1. The pressure on the lower surface of the cylinder is 20 kPa. If the variation of pressure in the vertical direction is given by $\partial p/\partial z = -2400$ Pa/m, the pressure on the upper surface is nearest:

   (A) 19.52 kPa

   Because the variation is constant, the change in pressure can be written as

$$\Delta p = \frac{\partial p}{\partial z}\Delta z = -2400 \text{ Pa/m} \times 0.2 \text{ m} = -480 \text{ Pa}. \quad \therefore p_{\text{upper}} = 20 - 0.48 = \underline{19.52 \text{ kPa}}$$

2. A water-well driller measures the water level in a well to be 20 ft below the surface. The point on the well is 250 ft below the surface. The pressure at the point is estimated to be:

   (C) 688 kPa

   Using Eq. 2.4.4, the pressure is

$$p = \gamma h = 9810 \text{ N/m}^3 \times (250 - 20) \text{ ft} \times 0.3048 \text{ m/ft} = 688\,000 \text{ N/m}^2 \quad \text{or} \quad \underline{688 \text{ kPa}}$$

   Or, we could use English units as follows:

$$p = \gamma h = 62.4 \times (250 - 20) = 14{,}350 \text{ lb/ft}^2. \quad \text{Then,} \quad \frac{14{,}350 \times 101.3}{14.7 \times 144} = \underline{685 \text{ kPa}}$$

   The difference between the two numbers is due to the accuracy of the numbers used: 14.7, 9810, 62.4, and 101.3.

3. To calculate the pressure in the standard atmosphere at 6000 m, the lapse rate is used (see Eq. 2.4.8). The percentage error, considering the pressure from Table B.3 in the Appendix to be more accurate, is nearest:

   (B) 0.13 %

   Equation 2.4.8 yields

$$p = p_{\text{atm}}\left(\frac{T_0 - \alpha z}{T_0}\right)^{g/\alpha R} = 101.3\left(\frac{288.2 - 0.0065 \times 6000}{288.2}\right)^{9.81/0.0065\times 287} = 47.15 \text{ kPa}$$

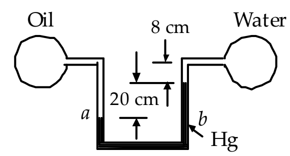
$$\% \text{ error} = \frac{47.21 - 47.15}{47.21} \times 100 = \underline{0.13 \text{ \%}}$$

Solutions for Sections 2.4.3 – 2.4.6

1. The pressure in the oil ($S = 0.86$) pipe is 12 kPa. The pressure in the water pipe is nearest:

   (D) −13.1 kPa

   Place "a" and "b" as shown. Then $p_a = p_b$ since a and b are at the same elevation in the same fluid. The manometer then tells us that



$$p_a = p_b$$

$$p_{\text{oil}} + (0.2 + 0.08) \times 9810 \times 0.86 = p_{\text{water}} + 0.08 \times 9810 + 0.2 \times 9810 \times 13.6$$

$$\therefore p_{\text{water}} = 12\,000 + 2362 - 785 - 26\,683 = -13\,110 \text{ Pa} \quad \text{or} \quad \underline{-13.11 \text{ kPa}}$$

2. A horizontal 80-cm-diameter hatch is located on the top of a submersible designed for a diver to escape. If the submersible is 100 m below the surface of the ocean, estimate the minimum force required to open the hatch. Assume $S_{\text{saltwater}} = 1.02$.

   (C) 500 kN

   The pressure at a depth of 100 m is $p = \gamma h = (9810 \times 1.02) \times 100 = 10^6$ N. The force due to the saltwater acts at the centroid of the circular hatch since it is horizontal. Moments about the hinge, which is assumed to be on the circumference of the circular hatch with the force F on the opposite side, provides

$$0.8P = F \times 0.4 = 10^6 \times 0.4. \quad \therefore P = 500\,000 \text{ N} \quad \text{or} \quad \underline{500 \text{ kN}}$$

3. A 1.2-m high by 2.8-m wide vertical rectangular gate is used to control the flow of water from a reservoir to a grinding mill. A frictionless hinge runs along the bottom of the gate. The minimum force acting normal to and at the very top of the gate needed to hold the gate shut in the vertical position if the water is at the very top of the gate is nearest:

   (A) 6590 N

   The pressure distribution on the gate is linear, increasing from zero at the top to $p = \gamma h = 9810 \times 1.2 = 11\,772$ Pa at the bottom. Hence, the average pressure over the gate is 5886 Pa providing a force of $5886 \times 1.2 \times 2.8 = 19\,780$ N acting on the gate. This force acts through the centroid of the triangular pressure distribution (see Fig. 2.9), 0.8 m from the top. The force on the top of the gate (1.2 m above the hinge and 0.8 m above the force) is found by taking moments about the hinge:

$$1.2P = F \times d = 19\,780 \times 0.4. \quad \therefore P = \underline{6590 \text{ N}}$$

4. A gate automatically opens when the water gets too high. How high above the hinge must the water be for the gate to just open?

(B) 1.2 m

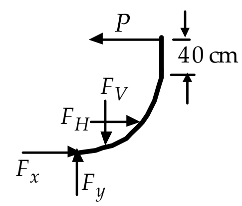
The force of the water on the vertical gate acts at the hinge just as the gate is about to open. If the force moves above the hinge, the gate will open. The distance 0.6 m below the hinge means that the water surface is 2 × 0.6 = 1.2 m above the hinge for the gate to open. See Fig. 2.10.

5. The force P needed to hold the 2-m-radius, 3-m-wide circular arc gate in the position shown is nearest:

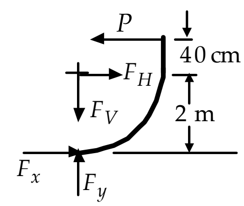
(B) 49 kN

Since the force due to the pressure on each area element of the circular arc acts through the center of the arc, we can move the horizontal and vertical components to that center of the arc. Sketch a free-body diagram of the water above the gate, as in Fig. 2.11b. (Combine F_H and F_V to make **F**, then **F** will pass through the center of the arc at which point it can be decomposed into F_H and F_V.) They produce the same moments when located at the center as if they were located where they actually act. Take moments about the hinge so that only P and F_H produce moments:

$$2.4P = 2F_H = 2(\gamma \bar{h} A) = 2 \times 9810 \times 1 \times (2 \times 3)$$
$$\therefore P = \underline{49\ 050\ \text{N}}$$



Forces in normal position



Move forces to center

For practice, maintain F_H and F_V in their normal positions and solve for P. We use Eq. 2.4.28 to help find the distance d_H above the hinge where F_H acts:

$$d_H = 2 - y_p = 2 - \left(\bar{y} + \frac{\bar{I}}{A\bar{y}} \right) = 2 - 1 - \frac{3 \times 2^3/12}{(3 \times 2) \times 1} = 0.667 \text{ m}$$

F_V acts through the center of gravity of the quarter circle. The distance d_V from the hinge to F_V is:

$$d_V = \frac{4r}{3\pi} = \frac{4 \times 2}{3\pi} = 0.849 \text{ m}$$

Finally, moments about the hinge results in

$$2.4P = 0.667 \times [9810 \times 1 \times 6] + 0.849 \times [9810 \times (\pi \times 2^2/4) \times 3]$$
$$\therefore P = \underline{49\ 060\ \text{N}}$$

Obviously, it's much simpler to move F_H and F_V to the center of the arc.

6. An object requires a force of 30 N to hold it under water. The object weighs 90 N in air. Its density is nearest:

(C) 750 kg/m^3

A free-body diagram would show 30 N and 90 N acting down and the buoyant force F_B acting up (see Eq. 2.4.36):

$$F_B = 30 + 90 = 120 = \gamma_{\text{water}} \not{V} = 9810 \not{V}. \quad \therefore \not{V} = \frac{120}{9810} \text{ m}^3$$

The weight is expressed as $W = \rho g \not{V}$ so that

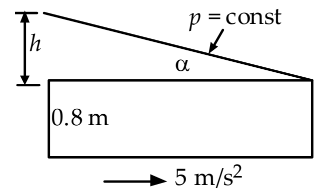
$$90 = \rho_x g \not{V} = \rho_x \times 9.81 \times \frac{120}{9810}. \quad \therefore \rho_x = \underline{750 \text{ kg/m}^3}$$

Solutions for Sections 2.5 and 2.6

1. A rectangular tank 80 cm high and 180 cm long is filled with water and pressurized to 20 kPa at the top. If it is accelerated at the rate of 5 m/s^2 on a horizontal plane in the direction of its longer side, what is the highest pressure in the tank?

   (D) 36.8 kPa

   With $a_z = 0$, Eq. 2.5.3 gives h as



$$h = L\tan\alpha = 1.8 \times \frac{a_x}{g}. \qquad \therefore h = \frac{1.8 \times 5}{9.81} = 0.917 \text{ m}$$

   The pressure at the bottom left corner is then

$$p = \gamma H = 9810 \times (0.917 + 0.8) = 16\,840 \text{ Pa}$$

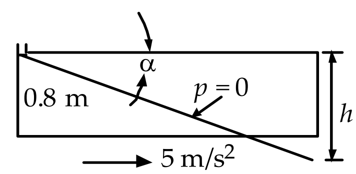
   Since it was pressurized to 20 kPa, we simply add 20 kPa and obtain

$$p_{\max} = 16.84 + 20 = \underline{36.84 \text{ kPa}}$$

2. The tank in Problem 1 has a small hole positioned in its top at the very rear of the tank which is accelerated at a_x = 5 m/s^2 in the direction of the longer side. The lowest pressure in the tank, which is filled with water, is nearest:

   (C) −9 kPa

   Sketch the tank showing the zero pressure line emanating from the left upper corner. It slopes down to the right. Eq. 2.5.3 gives h as



$$h = L\tan\alpha = 1.8 \times \frac{a_x}{g}. \qquad \therefore h = \frac{1.8 \times 5}{9.81} = 0.917 \text{ m}$$

   The minimum pressure exists at the top of the front of the tank:

$$p_{\min} = -\gamma h = -9810 \times 0.917 = \underline{-9000 \text{ Pa}}$$

   Note: The water will not flow out of the hole at the rear since the vacuum will hold it in the tank.

3. The force acting on the bottom of the cylinder of Example 2.12 is nearest:

(A) 55.6 N

The pressure distribution on the bottom of the cylinder, using $p = 0$ at $r = 0$, is (see Eq. 2.6.4)

$$p - 0 = \frac{\rho\omega^2}{2}(r^2 - 0) \quad \text{or} \quad p = \frac{1000}{2}26.6^2 r^2 = 353\,800 r^2$$

Integrate over the bottom area:

$$F_{\text{bottom}} = \int_0^{0.1} 353\,800 r^2 \times 2\pi r\, dr = 2\pi \times \frac{353\,800 \times 0.1^4}{4} = \underline{55.6 \text{ N}}$$

4. The U-tube shown is rotated about the left vertical leg at 100 rpm. The highest pressure in the U-tube is nearest:

(B) 4.2 kPa

The highest pressure occurs at the right-hand corner of the bottom where $r_2 = 0.2$ m. Select $p_1 = 0$ at the top of the left leg. Using $\omega = 100 \times 2\pi/60 = 10.47$ rad/s, Eq.2.6.4 provides

$$p_2 - \cancel{p_1} = \frac{\rho\omega^2}{2}(r_2^2 - \cancel{r_1^2}) - \gamma(z_2 - \cancel{z_1})$$

$$p_2 = \frac{1000}{2}10.47^2 \times 0.2^2 - 9810 \times (-0.2) = \underline{4154 \text{ Pa}}$$

Careful: Point 2 is at (0.2, –0.2) m. (The units will work out if we use kg, N, m, and s as the units. Check them if you're uncertain. Angular velocity must be in rad/s, never rpm).

5. If the U-tube in Problem 4 is rotated about the right leg, what maximum rpm will result in the 20°C water being thrown from the tube?

(B) 672 rpm

The water will leave the tube when the lowest pressure in the tube, the pressure at the top of the right leg, reaches the vapor pressure which is 2.34 kPa abs (see Table B.1 in the Appendix). The pressure there is given by Eq. 2.6.4:

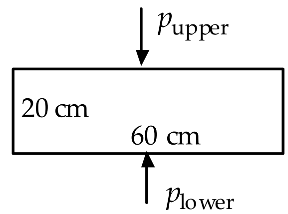
$$p_2 - p_1 = \frac{\rho\omega^2}{2}(r_2^2 - \cancel{r_1^2}) - \gamma(\cancel{z_2} - \cancel{z_1})$$

$$101\,300 - 2340 = \frac{1000}{2}\omega^2 \times 0.2^2. \qquad \therefore \omega = 70.34 \text{ rad/s} \quad \text{or} \quad \underline{672 \text{ rpm}}$$

We used $p_1 = 101\,300$ Pa since the pressure at point 2 is in absolute pressure. The pressure must be in Pa not kPa.

Sections 2.1 – 2.4.2

1. The pressure on the lower surface of the cylinder is 20 kPa. If the variation of pressure in the vertical direction is given by $\partial p/\partial z = -2400$ Pa/m, the pressure on the upper surface is nearest:



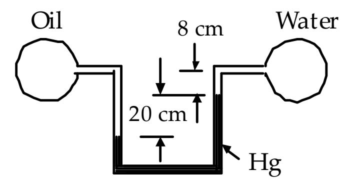
(A) 19.52 kPa
(B) 20.48 kPa
(C) –28.0 kPa
(D) 31.20 kPa

2. A water-well driller measures the water level in a well to be 20 ft below the surface. The point on the well is 250 ft below the surface. The pressure at the point, in SI units, is estimated to be:
(A) 632 kPa
(B) 652 kPa
(C) 688 kPa
(D) 698 kPa

3. To calculate the pressure in the standard atmosphere at 6000 m, the lapse rate is used (see Eq. 2.4.8). The percentage error, considering the pressure from Table B.3 in the Appendix to be more accurate, is nearest:
(A) 0.08%
(B) 0.13%
(C) 0.24%
(D) 0.31%

Sections 2.4.3 – 2.4.6

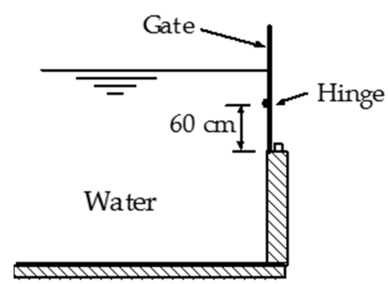
1. The pressure in the oil ($S = 0.86$) pipe is 12 kPa. The pressure in the water pipe is nearest:
   (A) -34.8 kPa
   (B) -28.8 kPa
   (C) -18.8 kPa
   (D) -13.1 kPa



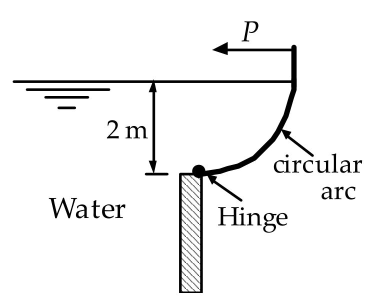
2. A horizontal 80-cm-diameter hatch is located on the top of a submersible designed for a diver to escape. If the submersible is 100 m below the surface of the ocean, estimate the minimum force required to open the hatch. Assume $S_{saltwater} = 1.02$.
   (A) 300 kN
   (B) 400 kN
   (C) 500 kN
   (D) 600 kN

3. A 1.2-m high by 2.8-m wide vertical rectangular gate is used to control the flow of water from a reservoir to a grinding mill. A frictionless hinge runs along the bottom of the gate. The minimum force acting normal to and at the very top of the gate needed to hold the gate shut if the water is at the very top of the gate is nearest:
   (A) 6590 N
   (B) 9800 N
   (C) 10 000 N
   (D) 13 300 N

4. A gate automatically opens when the water gets too high. How high above the hinge must the water be for the gate to just open?
   (A) 0.6 m
   (B) 1.2 m
   (C) 1.8 m
   (D) 2.4 m



5. The force P needed to hold the 2-m-radius circular arc gate in the position shown is nearest:
   (A) 58 kN
   (B) 49 kN
   (C) 39 kN
   (D) 27 kN



6. An object requires a force of 30 N to hold it under water. The object weighs 90 N in air. Its density is nearest:
   (A) 850 kg/m^3
   (B) 800 kg/m^3
   (C) 750 kg/m^3
   (D) 700 kg/m^3

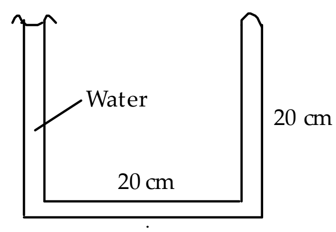
Sections 2.5 and 2.6

1. A rectangular tank 80 cm high and 180 cm long is filled with water and pressurized to 20 kPa at the top. If it is accelerated at the rate of 5 m/s^2 on a horizontal plane in the direction of its longer side, what is the highest pressure in the tank?
   (A) 49.1 kPa
   (B) 44.9 kPa
   (C) 41.2 kPa
   (D) 36.8 kPa

2. The tank in Problem 1 has a small hole positioned in its top at the very rear of the tank which is accelerated at a_x = 5 m/s^2 in the direction of the longer side. The lowest pressure in the tank, which is filled with water, is nearest:
   (A) 12 kPa
   (B) 9 kPa
   (C) −9 kPa
   (D) −18 kPa

3. The force acting on the bottom of the cylinder of Example 2.12 is nearest:
   (A) 55.6 N
   (B) 50.4 N
   (C) 45.2 N
   (D) 38.4 N

4. The U-tube shown is rotated about the left vertical leg at 100 rpm. The highest pressure in the U-tube is nearest:
   (A) 5.6 kPa
   (B) 4.2 kPa
   (C) 2.2 kPa
   (D) 0.8 kPa



5. If the U-tube in Problem 4 is rotated about the right leg, what maximum rpm will result in the 20°C water being thrown from the tube?
   (A) 741 rpm
   (B) 672 rpm
   (C) 569 rpm
   (D) 432 rpm