CHAPTER 2

EXERCISE 2.1

Part A

1. $S = 100(1.055)^5 = \$130.70,$ Int. $= \$30.70$

2. $S = 500\left(1 + \frac{0.03}{12}\right)^{24} = \$530.88,$ Int. $= \$30.88$

3. $S = 220\left(1 + \frac{0.088}{4}\right)^{12} = \$285.65,$ Int. $= \$65.65$

4. $S = 1000(1.045)^{12} = \$1695.88,$ Int. $= \$695.88$

5. $S = 50(1.005)^{48} = \$63.52,$ Int. $= \$13.52$

6. $S = 800(1.0775)^{10} = \$1687.57,$ Int. $= \$887.57$

7. $S = 300\left(1 + \frac{0.08}{52}\right)^{156} = \$381.30,$ Int. $= \$81.30$

8. $S = 1000\left(1 + \frac{0.045}{365}\right)^{730} = \$1094.17,$ Int. $= \$94.17$

9. a) $S = 500\left(1 + \frac{0.04}{12}\right)^{12} = \$520.37$
   b) $S = 500\left(1 + \frac{0.08}{12}\right)^{12} = \$541.50$
   c) $S = 500\left(1 + \frac{0.10}{12}\right)^{12} = \$563.41$

10. $S = 2000(1.0175)^{12} = \$2462.88$

11. a) $S = 100(1.08)^5 = \$146.93$
    b) $S = 100(1.04)^{10} = \$148.02$
    c) $S = 100(1.02)^{20} = \$148.59$
    d) $S = 100\left(1 + \frac{0.08}{12}\right)^{60} = \$148.98$
    e) $S = 100\left(1 + \frac{0.12}{365}\right)^{1825} = \$149.18$

12. $S = 1000(1.005)^{216} = \$2936.77$

13. a) $S = 10\,000(1.03)^{522} = 5.02379 \times 10^{10} = \$50.2379$ billion
    b) $S = 10\,000[1 + (0.03)(522)] = \$166\,600$

14. a) $S = 1000(1.06136)^1 = \$1061.36$

b) $S = 1000(1.030225)^2 = \$1061.36$
c) $S = 1000(1.015)^4 = \$1061.36$
d) $S = 1000(1.004975)^{12} = \$1061.36$

15. At the end of 5 years $= 8000\left(1+\frac{0.035}{2}\right)^{10} = \$9515.56$

At the end of 6 years $= 9515.56\left(1+\frac{0.04}{2}\right)^{2} = \$9899.99$

Interest earned $= \$384.43$

EXERCISE 2.1
Part B

1. a) $S = 100\left(1+\frac{0.06}{365}\right)^{365} = \$1061.83$, Interest $= \$61.83$

b)

Period	Interest
January 1-June 30	$1000 \times 0.06 \times \frac{181}{365} = \$29.75$
July 1 – December 31	$1029.75 \times 0.06 \times \frac{184}{365} = \$31.15$
Total interest earned	$= \$60.90$

c)

Period	Interest
January	$1000 \times 0.06 \times \frac{31}{365} = \$5.10$
February	$1005.10 \times 0.06 \times \frac{28}{365} = \$4.63$
March	$1009.73 \times 0.06 \times \frac{31}{365} = \$5.15$
April	$1014.88 \times 0.06 \times \frac{30}{365} = \$5.00$
May	$1019.88 \times 0.06 \times \frac{31}{365} = \$5.20$
June	$1025.08 \times 0.06 \times \frac{30}{365} = \$5.06$
July	$1030.14 \times 0.06 \times \frac{31}{365} = \$5.25$
August	$1035.39 \times 0.06 \times \frac{31}{365} = \$5.28$
September	$1040.67 \times 0.06 \times \frac{30}{365} = \$5.13$
October	$1045.80 \times 0.06 \times \frac{31}{365} = \$5.33$
November	$1051.13 \times 0.06 \times \frac{30}{365} = \$5.18$
December	$1056.31 \times 0.06 \times \frac{31}{365} = \$5.38$
Total interest earned	$= \$61.69$

2.

Growth of $1000

Years	n	$j_{365} = 4\%$	$j_{365} = 7\%$	$j_{365} = 10\%$
5	1825	1221.39	1419.02	1648.61
10	3650	1491.79	2013.62	2717.91
15	5475	1822.06	2857.36	4480.77
20	7300	2225.44	4054.66	7387.03
25	9125	2718.13	5753.63	12 178.32

3.

m	i	n	S	Interest
1	0.054	10	16 920.22	6920.22
2	0.027	20	17 037.62	7037.62
4	0.0135	40	17 098.19	7098.19
12	0.0045	120	17 139.29	7139.29
52	$\frac{0.054}{52}$	520	17 155.26	7155.26
365	$\frac{0.054}{365}$	3650	17 159.38	7159.38

EXERCISE 2.2

Part A

1. a) $j = (1.035)^2 - 1 = 0.071225 = 7.12\%$

   b) $j = \left(1 + \frac{0.03}{4}\right)^4 - 1 = 0.030339191 = 3.03\%$

   c) $j = (1.02)^4 - 1 = 0.08243216 = 8.24\%$

   d) $j = \left(1 + \frac{0.12}{365}\right)^{365} - 1 = 0.127474614 = 12.75\%$

   e) $j = \left(1 + \frac{0.09}{12}\right)^{12} - 1 = 0.093806897 = 9.38\%$

2. a) $(1+i)^2 = 1.06 \ \rightarrow i = (1.06)^{1/2} - 1$

   $$j_2 = 2[(1.06)^{1/2} - 1] = 5.91\%$$

   b) $(1+i)^4 = 1.09 \ \rightarrow i = (1.09)^{1/4} - 1$

   $$j_4 = 4[(1.09)^{1/4} - 1] = 8.71\%$$

   c) $(1+i)^{12} = 1.10 \ \rightarrow i = (1.10)^{1/12} - 1$

   $$j_{12} = 12[(1.10)^{1/12} - 1] = 9.57\%$$

   d) $(1+i)^{365} = 1.17 \ \rightarrow i = (1.17)^{1/365} - 1$

   $$j_{365} = 365[(1.17)^{1/365} - 1] = 15.70\%$$

   e) $(1+i)^{52} = 1.045 \ \rightarrow i = (1.045)^{1/52} - 1$

   $$j_{52} = 52[(1.045)^{1/52} - 1] = 4.40\%$$

3. a) $(1+i)^4 = (1.04)^2 \ \rightarrow i = (1.04)^{1/2} - 1$

   $$j_4 = 4[(1.04)^{1/2} - 1] = 7.92\%$$

   b) $(1+i)^2 = (1.05)^4 \ \rightarrow i = (1.015)^2 - 1$

   $$j_2 = 2[(1.015)^2 - 1] = 6.05\%$$

   c) $[(1+i)^4 = \left(1 + \frac{0.18}{12}\right)^2 \rightarrow i = (1.015)^3 - 1$

   $$j_4 = 4[(1.015)^3 - 1] = 18.27\%$$

   d) $(1+i)^{12} = \left(1 + \frac{0.1}{6}\right)^6 \rightarrow i = \left(1 + \frac{0.10}{6}\right)^{1/2} - 1$

   $$j_{12} = 12\left[\left(1 + \frac{0.10}{6}\right)^{1/2} - 1\right] = 9.96\%$$

   e) $(1+i)^2 = (1.02)^4 \rightarrow i = (1.02)^2 - 1$

   $$j_2 = 2[(1.02)^2 - 1] = 8.08\%$$

   f) $(1+i)^2 = \left(1 + \frac{0.04}{52}\right)^{52} \rightarrow i = \left(1 + \frac{0.04}{52}\right)^{26} - 1$

   $$j_2 = 2[\left(1 + \frac{0.04}{52}\right)^{26} - 1] = 4.04\%$$

g) $(1+i)^{12} = \left(1+\frac{0.0525}{2}\right)^2 \to i = \left(1+\frac{0.0525}{2}\right)^{1/6} - 1$

$$j_{12} = 12[\left(1+\frac{0.0525}{2}\right)^{1/6} - 1] = 5.19\%$$

h) $(1+i)^{365} = \left(1+\frac{0.1279}{4}\right)^4 \to i = \left(1+\frac{0.1279}{4}\right)^{4/365} - 1$

$$j_{365} = 365[\left(1+\frac{0.1279}{4}\right)^{\frac{4}{365}} - 1] = 12.59\%$$

4. $1+2r = (1+\frac{0.057}{12})^{24} \to r = \frac{1}{2}[\left(1+\frac{0.057}{12}\right)^{24} - 1] = 6.02\%$

5. $1+3r = (1+\frac{0.08}{365})^{1095} \to r = \frac{1}{3}[\left(1+\frac{0.08}{365}\right)^{1095} - 1] = 9.04\%$

6. $j = (1.0175)^{12} - 1 = 23.14\%$

7. $j_2 = 4.9\% \to j = \left(1+\frac{0.049}{2}\right)^2 - 1 = 4.96\%$
   $j_1 = 5\% \quad \to j = 5\%$
   Thus $j_1 = 5\%$ yields the higher annual effective rate of interest.

8. a) $j_{12} = 15\% \quad \to j = \left(1+\frac{0.15}{12}\right)^{12} - 1 \quad = 16.08\%$
   $j_2 = 15\frac{1}{2}\% \quad \to j = \left(1+\frac{0.155}{2}\right)^2 - 1 \quad = 16.10\%$ BEST
   $j_{365} = 14.9\% \quad \to j = \left(1+\frac{0.149}{365}\right)^{365} - 1 \quad = 16.06\%$ WORST
   b) $j_{12} = 6\% \quad \to j = (1.005)^{12} - 1 \quad = 6.17\%$
   $j_2 = 6\frac{1}{2}\% \quad \to j = (1.0325)^2 - 1 \quad = 6.61\%$ BEST
   $j_{365} = 5.9\% \quad \to j = \left(1+\frac{0.059}{365}\right)^{365} - 1 \quad = 6.08\%$ WORST

9. Bank A : $j_1 = 0.10 \to$ annual effective rate $= 0.10$
   Bank B : $j_m = 0.0.975 \to$ annual effective rate $= j$
   Calculate m, such that $j = (1+\frac{0.0975}{m})^m - 1 \geq 0.10$
   for $m = 2$, $j = 0.0998766$
   for $m = 4$, $j = 0.1011231$
   The minimum frequency of compounding for Bank B is $m = 4$.
   However, if Bank A offered 5% and Bank B offered 4.75%, $j_m = 4.75\%$
   will never be equivalent to $j_1 = 5\%$, no matter what value of m is chosen.

EXERCISE 2.2
Part B

3. a) $m = 1$: $S = 20\,000(1.06)^5 = \$26\,764.51$
   $m = 2$: $S = 20\,000(1.03)^{10} = \$26\,878.33$
   $m = 4$: $S = 20\,000(1.015)^{20} = \$26\,937.10$
   $m = 12$: $S = 20\,000(1.005)^{60} = \$26\,977.00$
   $m = 365$: $S = 20\,000\left(1+\frac{0.06}{365}\right)^{1825} = \$26\,996.51$

b)

$j_m = 6\%$	$j = (1+i)^m$	$S = 20\,000(1+j)^5$
j_1	$j = (1.06)^1 - 1$	26 764.51
j_2	$j = (1.03)^2 - 1$	26 878.33
j_4	$j = (1.015)^4 - 1$	26 937.10
j_{12}	$j = (1.005)^{12} - 1$	26 977.00
j_{365}	$j = (1+\frac{0.06}{365})^{365} - 1$	26 996.51

c)

$j_m = 6\%$	$j_{12} = 12[\left(1+\frac{j_m}{m}\right)^{m/12} - 1$	$S = 20\,000\left(1+\frac{j_{12}}{12}\right)^{60}$
j_1	0.058410607	26 764.51
j_2	0.059263464	26 878.33
j_4	0.059702475	26 937.10
j_{12}	0.06	26 977.00
j_{365}	0.060145294	26 996.51

4. a) $(1+j)^2 = (1.06)^2 \rightarrow j = 6\%$
   b) $(1+j)^3 = (1.06)^3(1.02) \rightarrow j = [(1.06)^3(1.02)]^{1/3} - 1 = 6.70\%$
   c) $(1+j)^4 = (1.06)^4(1.02) \rightarrow j = [(1.06)^4(1.02)]^{1/4} - 1 = 6.53\%$

5. Annual effective yield = $[(1 + 0.0201)(0.995)]^4 - 1 = 6.14\%$

6. Let $j_2 = 2i$; Then at the present time:
   $100 = 51.50 + 51.50(1+i)^{-1}$
   $(1+i) = \frac{51.50}{48.50} \rightarrow i = 0.06185567 \rightarrow j_2 = 2i = 0.12371134 = 12.37\%$

7. a) Amount of interest during the n-th year $= P[1+rn] - P[1+r(n-1)]$
   $= P + Prn - P - Prn + Pr = Pr$
   Amount of principal at the beginning of the n-th year $= P[1+r(n-1)]$
   Annual effective rate interest $= \frac{Pr}{P[1+r(n-1)]} = \frac{r}{1+r(n-1)}$
   b) Amount of interest during the n–th year $= P(1+i)^n - P(1+i)^{n-1}$
   $= P(1+i)^{n-1}[(1+i) - 1]$
   $= P(1+i)^{n-1}\ i$
   Amount of principal at the beginning of n-th year $= P(1+i)^{n-1}$
   Annual effective rate of interest $= \frac{P(1+i)^{n-1}\ i}{P(1+i)^{n-1}} = i$

EXERCISE 2.3

Part A

1. $P = 100(1.015)^{-12} = \$83.64$
2. $P = 50(1+\frac{0.085}{12})^{-24} = \$42.21$
3. $P = 2000(1.118)^{-10} = \$655.56$
4. $P = 500(1.05)^{-10} = \$306.96$
5. $P = 800(1+\frac{0.05}{365})^{-1095} = \$688.57$
6. $P = 1000(1+\frac{0.08}{4})^{-20} = \$672.97$
7. $P = 2000(1+\frac{0.048}{12})^{-36} = \$1732.27$
8. $P = 250\ 000(1+\frac{0.065}{2})^{-20} = \$131\ 867.81$
9. $P = 10\ 000(1.03)^{-40} = \$3065.57$
10. $P = 2000(1+\frac{0.055}{4})^{-18} = \$1564.14$
11. $P = 800(1.03)^{-24} = \$393.55$
12. Maturity value $S = 250(1+\frac{0.09}{12})^{48} = \$357.85$

    Proceeds $P = 357.85\left(1+\frac{0.075}{4}\right)^{-11} = \$291.71$
13. Maturity value $S = 1000(1.03)^{10} = \$1343.92$

    Proceeds $P = 1343.92(1+\frac{0.07}{4})^{-14} = \$1054.12$
14. Discounted value of the payment plan : $230\ 000 + 200\ 000(1+\frac{0.04}{2})^{-10}$

    $= 230\ 000 + 164\ 069.66 = \$394\ 069.66$

    The payment scheme is cheaper by $400\ 000 - 394\ 069.66 = \$5\ 930.34$
15. Total current value $= 1000(1.045)^{20} + 600(1.045)^{-14}$

    $= 2411.71 + 323.98 = \$2735.69$
16. $P = 3000(1+\frac{0.0575}{2})^{10}(1.05)^{-5} = \$3120.85$

EXERCISE 2.3
Part B

1. Maturity value $S = 2500\left(1 + \frac{0.12}{12}\right)^{40} = \$3722.16$

   Financial Consultants pay: $3722.16\left(1 + \frac{0.1325}{4}\right)^{-12} = \$2517.45$

   Financial Consultants receive: $3722.16(1.13)^{-3} = \$2579.64$

   Financial Consultants profit: $2579.64 - 2517.45 = \$62.19$

2. $S = 1000(1.075)^5 = \$1435.63$

m	i	n	P	Discount
1	0.06	5	1072.79	362.84
2	0.03	10	1068.24	367.39
4	0.015	20	1065.91	369.72
12	$\frac{0.06}{12}$	60	1064.34	371.29
52	$\frac{0.06}{52}$	260	1063.72	371.91
365	$\frac{0.06}{365}$	1825	1063.57	372.06

3. Net present value of proposal A :

   $95\,400(1.14)^{-1} + 39\,000(1.14)^{-2} + 12\,000(1.14)^{-3} - 80\,000$

   $= 83\,684.21 + 30\,009.23 + 8099.66 - 80\,000 = \$41\,793.10$

   Net present value of proposal B:

   $35\,000(1.14)^{-1} + 58\,000(1.14)^{-2} + 80\,000(1.14)^{-3} - 100\,000$

   $= 30\,701.75 + 44\,629.12 + 53\,997.72 - 100\,000 = \$29\,328.59$

   Select proposal A with higher net present value.

EXERCISE 2.4

Part A

1. a) $S = 100\left(1+\frac{0.065}{2}\right)^{11\frac{1}{6}} = \$142.92$

   b) $S = 100\left(1+\frac{0.065}{2}\right)^{11}\left[1+(0.065)\left(\frac{1}{12}\right)\right] = \$142.93$

2. a) $S = 800(1.01)^{18\frac{1}{3}} = \$960.10$

   b) $S = 800(1.01)^{18}\left[1+(0.04)\left(\frac{1}{12}\right)\right] = \$960.11$

3. a) $S = 5000\left(1+\frac{0.074}{2}\right)^{-17\frac{2}{3}} = \$2631.55$

   b) $S = 5000\left(1+\frac{0.074}{2}\right)^{-18}\left[1+(0.074)\left(\frac{2}{12}\right)\right] = \$2631.94$

4. a) $S = 280(1.0175)^{-3\frac{7}{12}} = \$263.12$

   b) $S = 280(1.0175\,)^{-4}\left[1+(0.0175)\left(\frac{5}{12}\right)\right] = \$263.13$

5. Maturity date is October 20, 2019.

   Time = 22 interest periods less 8 days

   $P = 2000(1.03)^{-22}[1+(0.12)\left(\frac{8}{365}\right)] = 1046.53$

6. $S = 1200(1.00525)^{38}[1+(0.063)\left(\frac{11}{365}\right)] = \$1466.97$

7. $S = 4000(1.05)^{10}[1+(0.10)\left(\frac{165}{365}\right)] = \$6810.12$

8. Maturity date is December 8, 2017.

   Time = 7 interest periods less 60 days

   $P = 850\left(1+\frac{0.0525}{2}\right)^{-7}[1+(0.0525)\left(\frac{60}{365}\right)] = \$715.12$

9. Maturity date is August 24, 2015:

   $S = 1200\left(1+\frac{0.0875}{12}\right)^{24} = \$1428.59$

   Proceeds: $P = 1428.59\left(1+\frac{0.095}{4}\right)^{-5}[1+(0.095)\left(\frac{25}{365}\right)] = \$1278.66$

   Compound discount: $S - P = \$149.93$

EXERCISE 2.4

Part B

1. a) From the binomial theorem

$$(1+i)^t = 1 + it + \binom{t}{2} i^2 + \cdots$$

The 3rd term in the series will overshadow all the remaining terms.

If $0 < t < 1$ then $\binom{t}{2} i^2$ is negative

And $(1+i)^t < 1 + it$

If $t > 1$ then $\binom{t}{2} i^2$ is positive

and $(1+i)^t > 1 + it$

c) For $0 < t < 1$: $P(1+i)^k(1+i)^t < P(1+i)^k[1+it]$

$$S(1+i)^{-k}(1+i)^t < S(1+i)^{-k}[1+it]$$

2. $$\begin{aligned}(1-k)(1+i)^n + k(1+i)^{n+1} &= (1-k)(1+i)^n + k(1+i)(1+i)^n \\ &= (1+i)^n[(1-k) + k(1+i)] \\ &= (1+i)^n(1+ki)\end{aligned}$$

3. Maturity value on October 4, 2018:

$S = 2000(1.03)^4\left[1 + (0.03)\left(\frac{182}{365}\right)\right] = \$2284.69$

Proceeds: $P = 2283.69\left(1 + \frac{0.035}{4}\right)^{-14}[1 + (0.035)\left(\frac{64}{365}\right)] = \$2034.77$

Compound discount: $S - P = \$1750.08$

EXERCISE 2.5

Part A

1. $2000(1+i)^{15} = 3000$

$$(1+i)^{15} = 1.5$$
$$1+i = (1.5)^{1/15}$$
$$i = (1.5)^{1/15} - 1$$
$$i = 0.027399659$$
$$j_4 = 0.109598636$$
$$j_4 = 10.96\%$$

2. $100(1+i)^{55} = 150$

$$(1+i)^{55} = 1.5$$
$$1+i = (1.5)^{1/55}$$
$$i = (1.5)^{1/55} - 1$$
$$i = 0.007399334$$
$$j_{12} = 0.088792004$$
$$j_{12} = 8.88\%$$

3. $200(1+i)^{15} = 600$

$$(1+i)^{15} = 3$$
$$1+i = 3^{1/15}$$
$$i = 3^{1/15} - 1$$
$$i = 0.075989625$$
$$j_1 = 7.60\%$$

4. $1000(1+i)^{7} = 1181.72$

$$(1+i)^{7} = 1.18172$$
$$1+i = (1.18172)^{1/7}$$
$$i = (1.18172)^{1/7} - 1$$
$$i = 0.024139759$$
$$j_2 = 0.048279518$$
$$j_2 = 4.83\%$$

5. $2000(1.01)^{n} = 2800$

$$(1.01)^{n} = 1.4$$
$$n\,log(1.01) = log\ 1.4$$
$$n = 33.81518078 \text{ quarters}$$
$$n = 8 \text{ years}, 5 \text{ months}, 14 \text{ days}$$

6. $1000(1.045)^{n} = 130$

$$(1.045)^{n} = 1.3$$
$$n\,log(1.045) = log\ 1.3$$
$$n = 5.96053678 \text{ half years}$$
$$n = 2 \text{ years}, 11 \text{ months}, 23 \text{ days}$$

7. $500(1.005)^n = 800$

$$(1.005)^n = 1.6$$
$$n\, log 1.005 = log\ 1.6$$
$$n = 94.23553231 \text{ months}$$
$$n = 7 \text{ years}, 10 \text{ months}, 8 \text{ days}$$

8. $1800(1.02)^n = 2200$

$$(1.02)^n = \frac{22}{18}$$
$$n\, log 1.02 = \log \frac{22}{18}$$
$$n = 10.13353897 \text{ quarters}$$
$$n = 2 \text{ years}, 6 \text{ months}, 12 \text{ days}$$

9. $(1+i)^{10} = 2$

$$i = 2^{1/10} - 1$$
$$j_1 = 2^{1/10} - 1 = 7.18\%$$

10. $(1+i)^{16} = 1.5$

$$i = (1.5)^{1/16} - 1$$
$$j_4 = 4[(1.5)^{1/16} - 1] = 10.27\%$$

11. $4.71(1+i)^5 = 9.38$

$$(1+i)^5 = \frac{9.38}{4.71}$$
$$i = \left(\frac{9.38}{4.71}\right)^{1/5} - 1 = 14.77\%$$

12. $4000(1+i)^{1095} = 5000$

$$(1+i)^{1095} = 1.25$$
$$i = (1.25)^{1/1095} - 1$$
$$j_{365} = 365[(1.25)^{\frac{1}{1095}} - 1] = 7.44\%$$

13. a) $(1.0456)^n = 2$

$$n \log 1.10456 = \log 2$$
$$n = 15.54459407 \text{ years}$$
$$n = 15 \text{ years}, 199 \text{ days OR } 15 \text{ years}, 6 \text{ months}, 17 \text{ days}$$

Rule of 70

$$n = \frac{70}{4.56}$$
$$n = 15.35087719 \text{ years}$$
$$n = 15 \text{ years}, 129 \text{ days OR } 15 \text{ years}, 4 \text{ months}, 7 \text{ days}$$

b) $\left(1+\frac{0.07}{365}\right)^n = 2$

$$n\log\left(1+\frac{0.07}{365}\right) = \log 2$$
$$n = 3614.614035 \text{ days}$$
$$n = 9 \text{ years}, 330 \text{ days OR } 9 \text{ years}, 10 \text{ months}, 26 \text{ days}$$

Rule of 70

$$n = \frac{70}{\frac{7}{365}}$$
$$n = \frac{70}{0.019178082}$$
$$n = 3650 \text{ days} = 10 \text{ years}$$

14. $800\left(1+\frac{0.098}{2}\right)^n = 1500$

$$(1.049)^n = 1.875$$
$$n\log(1.049) = \log 1.875$$
$$n = 13.14054666 \text{ half} - \text{years}$$
$$n = 6 \text{ years}, 208 \text{ days OR } 6 \text{ years}, 6 \text{ months}, 26 \text{ days}$$

15. $\left(1+\frac{0.05}{365}\right)^n = 1.5$

$$n\log\left(1+\frac{0.05}{365}\right) = \log 1.5$$
$$n = 2960.098047 \text{ days}$$
$$n = 8 \text{ years}, 41 \text{ days OR } 8 \text{ years}, 1 \text{ month}, 10 \text{ days}$$

EXERCISE 2.5

Part B

1. $(1+i)^{16} = 2$

   $(1+i) = 2^{1/6}$

   $1+i = 1.044273782$

   a) $S = 1000(1+i)^{10} = \$1542.21$

   b) $S = 1000(1+i)^{20} = \$2378.41$

2. $(1+i)^{2190} = 2$

   $1+i = 2^{1/2190}$

   $1+i = 1.000316556$

   $(1+i)^n = 3$

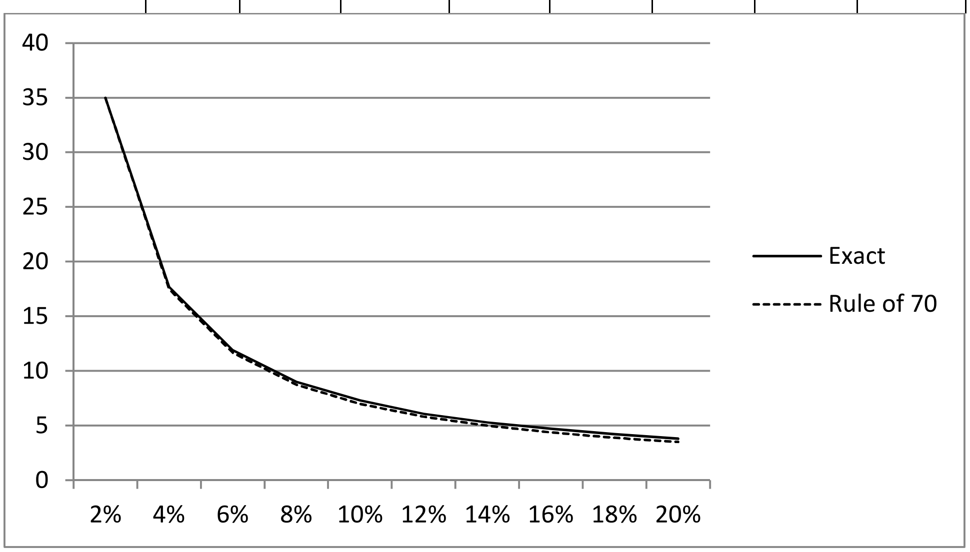
   $n\log(1+i) = \log 3$

   $n = 3471.06782$ days

   $n = 9$ yrs, 186 days OR 9 yrs, 6 mths, 4 days

3.

Rate j_1	2%	4%	6%	8%	10%	12%	14%	16%	18%	20%
Years	35	17.7	11.9	9	7.3	6.1	5.3	4.7	4.2	3.8
Rule of 70	35	17.5	11.67	8.75	7	5.83	5	4.375	3.89	3.5



4. $\left(1+\frac{0.045}{12}\right)^{12n} = 2\left(1+\frac{0.025}{2}\right)^{2n}$

   $\left[\frac{(1.00375)^{12}}{(1.0125)^2}\right]^n = 2$

   $n\log\left[\frac{(1.00375)^{12}}{(1.0125)^2}\right] = \log 2$

   $n = 34.535$ years

5. $(1+j_1^*)^{t/2} = (1+j_1)^t \;\rightarrow\; 1+j_1^* = (1+j_1)^2 \;\rightarrow\; j_1^* = (1+j_1)^2 - 1$

6. $800(1.045)^n = 2(600)(1.035)^n$

$$\left(\frac{1.045}{1.035}\right)^n = \frac{1200}{800}$$

$$n = \frac{\log(1200/800)}{\log(1.045/1.035)}$$

$$n = 42.16804634 \text{ half -years}$$

$$n = 21 \text{ years, } 31 \text{ days OR } 21 \text{ years, } 1 \text{ month, } 1 \text{ day}$$

7. In n years:

$$1.5[100(1.04)^n + 25(1.04)^{n-2}] = 95(1.08)^{n-1}$$

$$1.5(1.04)^n[100 + 25(1.04)^{-2}] = 95(1.08)^n(1.08)^{-1}$$

$$\left(\frac{1.04}{1.08}\right)^n = \frac{95(1.08)^{-1}}{1.5[100+25(1.04)^{-2}]}$$

$$\left(\frac{1.04}{1.08}\right)^n = 0.476322924$$

$$n = \frac{\log 0.476322924}{\log\frac{1.04}{1.08}}$$

$$n = 19.65163748 \text{ years} = 19 \text{ years, } 238 \text{ days}$$

Using simple interest for the last X days we obtain:

$$1.5(1.04)^{19}[100 + 25(1.04)^{-2}]\left[1 + (0.04)\left(\frac{X}{365}\right)\right] = 95(1.08)^{18}[1 + (0.08)\left(\frac{X}{365}\right)]$$

This solves for $X = 233$; It takes 19 years and 233 days

8. $500(1.08)^n + 800(1.08)^{n-3} = 2000$

$$(1.08)^n[500 + 800(1.08)^{-3}] = 2000$$

$$1135.065793(1.08)^n = 2000$$

$$(1.08)^n = 1.762012398$$

$$n = \frac{\log 1.762012398}{\log 1.08}$$

$$n = 7.360302768 \text{ years} = 7 \text{ years, } 132 \text{ days}$$

Using compound interest for 7 years and simple interest for X days we have

$$500(1.08)^7\left[1 + (0.08)\left(\frac{X}{365}\right)\right] + 800(1.08)^4\left[1 + (0.08)\left(\frac{X}{365}\right)\right] = 2000$$

Solving for X we obtain X $\doteq$ 128 days; It takes 7 years, 128 days

Check : $500(1.08)^7[1 + (0.08)\left(\frac{128}{365}\right)] + 800(1.08)^4[1 + (0.08)\left(\frac{128}{365}\right)]$

$= 880.95 + 1118.93 = 1999.88$

EXERCISE 2.6

Part A

1. $X = 1000(1.01)^{36} = \$1709.14$

2. $X = 1800\left(1 + \frac{0.1175}{2}\right)^{-14} = \$809.40$

3. a) $X = 2500\left(1 + \frac{0.09}{12}\right)^{-48} = \$1746.54$

   b) $Y = 2500\left(1 + \frac{0.09}{12}\right)^{36} = \$3271.61$

   Note: $1746.54\left(1 + \frac{0.09}{12}\right)^{84} = \$3271.61$

4. $X = 1000(1.02)^4 + 1500(1.02)^{-4} = 1082.43 + 1385.77 = \$2468.20$

5. a) $X = 800(1.0025)^{-24} + 700(1.0025)^{-72} = \$1338.29$

   b) $Y = 800(1.0025)^{24} + 700(1.00225)^{-24} = \$1508.69$

   c) $Z = 800(1.0025)^{72} + 700(1.0025)^{24} = \$1700.79$

   $X(1.0025)^{48} = Y \qquad 1338.29(1.0025)^{48} = \$1508.69$

   $Y(1.0025)^{48} = Z \qquad 1508.69(1.0025)^{48} = \$1700.79$

6. At the end of 7 years: $\mathrm{X} + 1000(1.035)^8 = 2000(1.035)^{-2}$

   $X + 1316.81 = 1867.02$

   $X = \$550.21$

7. $X = 4000(1.015)^{12} - 1000(1.015)^8 - 2000(1.015)^4$

   $= 4782.47 - 1126.49 - 2122.73 = \$1533.25$

8. $X = 1200(1.015)^{12} - 500(1.015)^6 = 1434.74 - 546.72 = \$888.02$

9. At the end of 4 years:

   $375\left(1 + \frac{0.08}{12}\right)^{36} + X\left(1 + \frac{0.08}{12}\right)^{24} + X\left(1 + \frac{0.08}{12}\right)^{12} = 1000$

   $476.34 + 1.172887932X + 1.082999507X = 1000$

   $2.255887439X = 523.66$

   $X = \$232.13$

10. a) On December 1, 2015:

    $X + X(1.03)^2 + 1200(1.03)^4 + 900(1.03)^7 = 3000(1.03)^9$

    $X + 1.0609X + 1350.61 + 1106.89 = 3914.32$

    $2.0609X = 1456.82$

    $X = \$706.89$

    b) Balance on September 1, 2015:

    $= 3000(1.03)^8 - 900(1.03)^6 - 1200(1.03)^3 - 900(1.03)^2$

    $= 3800.31 - 1074.65 - 1311.27 - 954.81 = \$459.58$

11. $X = 200(1.03)^4 + 150(1.03)^3 - 250(1.03)^2 + 100(1.03)$
$= 225.10 + 163.91 - 265.23 + 103 = \$226.78$

12. At the end of 3 years:

$$X + 2X(1.1)^{-3} = 400(1.1)^{-2} + 300(1.1)^{-7}$$
$$X + 1.502629602X = 330.58 + 153.95$$
$$2.502629602X = 484.53$$
$$X = \$193.61$$

13. At the time of the man's death:

$$X(1.03)^{-4} + X(1.03)^{-12} + X(1.03)^{-16} = 50\ 000$$
$$2.17033867X = 50\ 000$$
$$X = \$22\ 593.42$$

14. $X(1.006)^9 + 2X(1.006)^5 + 2X = 4000(1.006)^{12}$
$$5.11603864X = 4297.70$$
$$X = \$840.04$$

15. Maturity value of original debt:
$S = 3000(1.0075)^{24} = \$3589.24$

Equation of value at time 5:

$$1000(1.03)^4 + 1500(1.03)^3 + X = 3589.24(1.03)$$
$$1125.50881 + 1639.0905 + X = 3696.9172$$
$$X = \$932.32$$

16. $500 + 800(1.08)^{-3} = 2000(1.08)^{-t}$
$$(1.08)^{-t} = \frac{1135.065793}{2000} = 0.567532896$$
$$t = 7.36\ years$$

17. $1000 = 700(1+i)^{-6} + 400(1+i)^{-10}$
By trial and error,

$i = j_4/4$	Right hand side
0.02	949.72
0.015	984.85
0.013	999.33
0.0128	1000.80
0.0129	1000.06

Thus, $j_4 \doteq 4(0.0129) = 0.0516 = 5.16\%$

EXERCISE 2.6
Part B

1. a) Let X and Y be the two dated values due $n_1$and n_2 periods from now.
Let $D_1$and D_2 be two equivalent dated values of the set at $t_1$and t_2 interest periods from now.

	X	D_1	Y	D_2
0	n_1	t_1	n_2	t_2

$$D_1 = X(1+i)^{t_1-n_1} + Y(1+i)^{t_1-n_2}$$
$$D_2 = X(1+i)^{t_2-n_1} + Y(1+i)^{t_2-n_2}$$

Multiplying the first equation by $(1+\mathrm{i})^{\mathrm{t}_2-t_1}$ and simplifying we obtain

$$D_1(1+i)^{t_2-t_1} = X(1+i)^{t_2-n_1} + Y(1+i)^{t_2-n_2} = \mathrm{D}_2$$

which is the condition that $D_1$and D_2 are equivalent

b) Assuming that the times are in years

$$D_1 = X[1+r(t_1-n_1)] + Y[1+r(n_2-t_1)]^{-1}$$
$$D_2 = X[1+r(t_2-n_1)] + Y[1+r(t_2-n_2)]$$

Multiplying the first equation by $[1+r\{t_2-t_1)]$ we obtain

$$\begin{aligned} D_1[1+r(t_2-t_1)] &= X[1+r(t_1-n_1)][1+r(t_2-t_1)] \\ &\quad +Y[1+r(n_2-t_1)]^{-1}[1+r(t_2-t_1)] \\ &\neq X[1+r(t_2-n_1)] + Y[1+r(t_2-n_2)] = D_2 \end{aligned}$$

2. $X = 1000(1.08)^2 + 2000\left(1+\frac{0.125}{2}\right)^8(1.08)^{-2}$
$= 1166.40 + 2784.93 = \$3951.33$

3. On January 1, 2018:

$$\begin{aligned} X + X(1.02)^8 + 500(1.02)^{16} &= 5000(1.0225)^{24} \\ X + 1.171659381X + 686.39 &= 8528.83 \\ 2.171659281X &= 7842.44 \\ X &= \$3611.27 \end{aligned}$$

4. At the present time:

$$\begin{aligned} X + X(1.06)^{-2} &= 3000(1.05)^{-8} + 4000(1.04)^{-10} \\ 1.88999644X &= 2030.52 + 2702.26 \\ 1.88999644X &= 4732.78 \\ X &= \$2504.12 \end{aligned}$$

5. Let i be the interest rate per year.
At the end of year 18:

$$
\begin{aligned}
&(1)\ 240(1+i)^{12} + 200(1+i)^6 + 300 = X \\
&(2)\ \quad 360(1+i)^6 + 700 = X + 100 \\
&(3)\ \quad Y(1+i)^{12} + 600(1+i)^6 = X
\end{aligned}
$$

Let $(1+i)^6 = Z$. Then,

$$
\begin{aligned}
&(1)\ 240Z^2 + 200Z + 300 = X \\
&(2)\ \quad 360Z + 600 = X \\
&(3)\ \quad YZ^2 + 600Z = X
\end{aligned}
$$

From the first two equations:

$$
\begin{aligned}
240Z^2 + 200Z + 300 &= 360Z + 600 \\
240Z^2 - 160Z - 300 &= 0 \\
12Z^2 - 8Z - 15 &= 0 \\
Z &= \frac{8 \pm \sqrt{64+720}}{24} = \frac{36}{24} = 1.5 \\
&\text{or } -\frac{20}{24} \text{ (not applicable)}
\end{aligned}
$$

Substituting $Z = 1.5$ into (1) we obtain

$$
\begin{aligned}
240(1.5)^2 + 200(1.5) + 300 &= X \\
X &= \$1140
\end{aligned}
$$

Substuting $Z = 1.5, X = 1140$ into (3) we obtain

$$
\begin{aligned}
Y(1.5)^2 + 600(1.5) &= 1140 \\
2.25Y &= 1140 - 900 \\
Y &= \frac{240}{2.25} \\
Y &= \$106.67
\end{aligned}
$$

EXERCISE 2.7
Part A

1. $S = 2000\left(1+\frac{0.04}{12}\right)^{72}\left(1+\frac{0.09}{12}\right)^{72} = \$3042.03$

2. $P = 1000(1.07)^{-4}(1.08)^{-2} = 654.06$

3. $S = 500(1.025)^2(1.03)^4(1.0225)^4 = 646.28$
   $500(1+j_1)^5 = 646.28$
   $j_1 = 5.27\%$

4. $S = 2000(1.025)^6(1.02)^{16}(1.005)^{36} = \$3810.26$
   Compound interest $= 3810.26 - 2000 = \$1810.26$
   $200\left(1+\frac{j_2}{2}\right)^{20} = 3810.26$
   $j_2 = 6.55\%$

5. $X = 2000(1.05)^4(1.045)^9 = \$3612.72$

6. At the present time:
   $X + X(1.048)^{-4}(1.061)^{-6} = 5000(1.061)^{-5}$
   $1.581115643X = 3718.72$
   $X = \$2351.96$

7. $Y = 20\,000(1.06)^5 + 30\,000 + 35\,000(1.05)^{-7}$
   $= 26\,764.51 + 30\,000 + 24\,873.85$
   $= \$81\,638.36$

8. Present value of the offer $= 65\,000 + 150\,000(1.02)^{-4} + 150\,000(1.02)^{-4}(1.015)^{-8}$
   $= 65\,000 + 138\,576.81 + 123\,016.18 = \$326\,592.99$
   They should accept the offer.

9. a) Discounted value of the payments option:
   $60\,000 + 60\,000\left(1+\frac{0.072}{12}\right)^{-24} + 60\,000\left(1+\frac{0.072}{12}\right)^{-60}$
   $= 60\,000 + 51\,975.62 + 41\,905.63 = \$153\,881.26$
   The payment option is better.

   b) Discounted value of the payments option:
   $60\,000 + 60\,000\left(1+\frac{0.075}{4}\right)^{-8} + 60\,000\left(1+\frac{0.075}{4}\right)^{-12}\left(1+\frac{0.04}{4}\right)^{-8}$
   $= 60\,000 + 51\,714.26 + 44\,337.25 = \$156\,051.51$
   The cash option is better.

10. $(1+j)^6 = (1.015)^8\left(1+\frac{0.08}{12}\right)^{48}$
    $(1+j)^6 = 1.549677664$
    $1+j = 1.075738955$
    $j = 7.57\%$

EXERCISE 2.7
Part B

1. $(1+i)^n \times (1+j)^n = [(1+i)(1+j)]^n = (1+i+j+ij)^n$

$$\left(1+\frac{i+j}{2}\right)^{2n} - \left[\left(1+\frac{i+j}{2}\right)^2\right]^n = \left[1+\frac{2(i+j)}{2}+\frac{i^2+2ij+j^2}{4}\right]^n$$
$$= \left(1+i+j+\frac{i^2+2ij+j^2}{4}\right)^n$$

Since $ij \neq \frac{i^2+2ij+j^2}{4}$ then $(1+i)^n \times (1+j)^n \neq \left(1+\frac{i+j}{2}\right)^{2n}$

2. $S = 500(1.04)^2(1.02)^4\left(1+\frac{0.08}{12}\right)^{12}\left(1+\frac{0.08}{365}\right)^{365} = \$686.76$
Difference $= 686.76 - 500(1.04)^8 = 686.76 - 684.28 = \$2.48$

3. $X = 1000(1.02)^{14} + 2000(1.01)^{-20}$
$= 1319.48 + 1639.09 = \$2958.57$

4. $P = 20\,000(1.12)^{-3}(1.05)^{-10} + 30\,000(1.12)^{-3}(1.05)^{-12}(1.02)^{-12}(1.0075)^{-36}$
$= 8739.43 + 7164.26 = \$15\,903.69$

5. Amount in the account on April 21, 2014 :
$X = 1000(1.0175)^{11}(1.025)^3 + 2000(1.0175)(1.025)^3$
$= 1303.32 + 2191.47 = \$3494.79$

Calculate $i = \frac{j_{12}}{12}$ such that $1000(1+i)^{51} + 2000(1+i)^{21} = 3494.79$
By trial and error we determine:
at $j_{12} =$ 5%: $1000(1+i)^{51} + 2000(1+i)^{21} = 3418.71$
at $j_{12} =$ 6%: $1000(1+i)^{51} + 2000(1+i)^{21} = 3510.48$

$$\begin{array}{ccc|cc} & & \text{amount} & j_{12} & \\ \hline 91.77\Bigg\{ & 76.08\Big\{ & \begin{matrix}3418.71\\3494.79\end{matrix} & \begin{matrix}5\%\\ j_{12}\end{matrix} & \Big\}\,d\ \Bigg\}\,1\% \\ & & 3510.48 & 6\% & \end{array}$$

$\frac{d}{1\%} = \frac{76.08}{91.77}$
$d \doteq 0.83\%$
$j_{12} = 5.83\%$

Check at $j_{12} = 5.83\%$: $1000(1+i)^{51} + 2000(1+i)^{21} = \$3494.68$

6. $(1+j_1)^3 = \left(1+\frac{0.04}{12}\right)^{12}\left(1+\frac{0.08}{4}\right)^4\left(1+\frac{0.055}{365}\right)^{365}$
$(1+j_1)^3 = 1.190222002$
$j_1 = (1.190222002)^{1/3} - 1 = 0.059764396 = 5.98\%$

7. Let $j_4 = 4i$
$(1+i)^{12} = [1+(0.06)(1)][1-(0.08)(2)]^{-1}$
$(1+i)^{12} = 1.261904762$
$1+i = 1.019574304$
$i = 0.019574304$
and $j_4 = 4i = 0.078297216 = 7.83\%$

EXERCISE 2.8
Part A

1. $0.6(1.08)^n = 1$
   $(1.08)^n = \frac{1}{0.6}$
   $n \log 1.08 = \log \frac{1}{0.6}$
   $n = 6.637457293$ years

2. $S = 320\,000(1.021)^5 = \$355\,041.15$
   Increase $= \$35\,041.15$

3. a) $i_{real} = \frac{0.06-0.02}{1+0.02} = 3.92\%$ $\quad i_{realAT} = \frac{0.06(1-0.26)-0.02}{1+0.02} = 2.39\%$

   b) $i_{real} = \frac{0.08-0.04}{1+0.04} = 3.85\%$ $\quad i_{realAT} = \frac{0.08(1-0.26)-0.04}{1+0.04} = 1.85\%$

   c) $i_{real} = \frac{0.10-0.06}{1+0.06} = 3.77\%$ $\quad i_{realAT} = \frac{0.10(1-0.26)-0.06}{1+0.06} = 1.32\%$

EXERCISE 2.8
Part B

1. Let $X = \$1000$.
   You need $1000(1.03)^{-1}$ U.S. dollars now in U.S. dollars account, which is equivalent to $1000(1.03)^{-1}\left(\frac{1}{0.9717}\right) = \$999.15$ Cdn.
   This amount invested in a Canadian dollar account will accumulate to

   $$\$999.15(1.04) = \$1039.12$$

   The implied exchange rate one year from now is

   $$\$1000 \text{ U.S.} = \$1039.12 \text{ Cdn.} \quad \text{OR} \;\; \$0.9624 \text{ U.S.} = \$1 \text{ Cdn.}$$

2. Present value of $(1+r)^n$ due in n-years at annual effective rate i is:

   $$(1+r)^n(1+i)^{-n} = \left(\frac{1+r}{1+i}\right)^n$$

   Present value of 1 due in n years at annual effective rate $\frac{i-r}{1+r}$ is:

   $$\left(1+\frac{i-r}{1+r}\right)^{-n} = \left(\frac{1+r+i-r}{1+r}\right)^{-n} = \left(\frac{1+i}{1+r}\right)^{-n} = \left(\frac{1+r}{1+i}\right)^n$$

EXERCISE 2.9
Part A

1. $S = 40\,000(1.04)^{20} \doteq 87\,645$

2. Increase = 2% of $15\,000(1.02)^7 \doteq 345$

3. $(1+j)^{11} = 2$
$$j = 2^{1/11} - 1$$
$$j = 0.065041089$$
$$j = 6.50\%$$

4. $S = 48\,000(1.05)^{42} = \$372\,556.20$

5. $P = 0.25$ $S = 10$ $i = 0.10$

$$(0.25)(1.10)^n = 10$$
$$(1.10)^n = 40$$
$$n = \frac{\log 40}{\log 1.10}$$
$$n = 38.70393972 \text{ hours}$$
$$n = 1.61 \text{ days}$$

EXERCISE 2.9
Part B

1. a) Number of flies at 7 a.m. $= 100\,000(1.04)^{27} \doteq 288\,337$
Number of flies at 11 a.m. $= 100\,000(1.04)^{33} \doteq 364\,838$
Increase between 7 a.m. and 10 a.m. $= 76\,501$

b) $(1.04)^n = 2$
$$n = \frac{\log 2}{\log 1.04} = 17.67298769 \text{ periods} \doteq 707 \text{ minutes}$$
At 0:47 a.m. there will be 20 000 flies in the lab.

2. $200\,000(1+i)^{10} = 250\,000$
$$(1+i)^{10} = 1.25$$
$$i = (1.25)^{1/10} - 1$$
$$i = 0.022565183$$
Population in 2014 $= 200\,000(1+i)^{20} = 312\,500$
Population in 2019 $= 200\,000(1+i)^{25} = 349\,386$
Increase in population $= 36\,886$

EXERCISE 2.10

Part A

1. a) $S = 1500(1.09)^{1.5} = \$1706.99$

   b) $S = 1500\left(1 + \frac{0.09}{12}\right)^{18} = \$1715.94$

   c) $S = 1500e^{(0.09)(1.5)} = \$1716.81$

2. a) $P = 8000(1.02)^{-20} = \$5383.77$

   b) $P = 8000\left(1 + \frac{0.08}{365}\right)^{-1825} = \$5362.80$

   c) $P = 8000e^{-(0.08)(5)} = \$5362.56$

3. $e^{j_\infty(5)} = 1.5$

   $5\,j_\infty = \ln 1.5$

   $j_\infty = \frac{\ln 1.5}{5} = 0.081093022 = 8.11\%$

   $j = e^{j_\infty} - 1 = 0.084471771 = 8.45\%$

4. a) $800\left(1 + \frac{0.06}{365}\right)^n = 1200$

   $\left(1 + \frac{0.06}{365}\right)^n = 1.5$

   $n = \frac{\log 1.5}{\log\left(1+\frac{0.06}{365}\right)} = 2466.782157 \ \rightarrow 2467 \text{ days} = 6 \text{ years}, 277 \text{ days}$

   On November 8, 2020 the deposit will be worst at least \$1200.

   b) $800e^{0.06t} = 1200$

   $e^{0.06t} = 1.5$

   $0.06t = \ln 1.5$

   $t = \frac{\ln 1.5}{0.06} - 6.757751802 \text{ years} \ \doteq 6 \text{ years} \ \ 277 \text{ days}$

   On November 8, 2016 the deposit will be worth at least \$1200.

5. $e^{5j_\infty} = 2$

   $5\,j_\infty = \ln 2$

   $j_\infty = \frac{\ln 2}{5}$

   $e^{j_\infty t} = 3$

   $j_\infty\, t = \ln 3$

   $t = \frac{\ln 3}{j_\infty} = \frac{\ln 3}{\frac{\ln 2}{5}} = 7.924812504 \text{ years}$

6. a) $S = 1000e^{0.08(2)} = \$1173.51$

   b) $S = 1000\left(1 + \frac{0.0825}{2}\right)^4 = \$1175.49$

   c) $S = 1000[1 + (0.085)(2)] = \$1170$

   She should accept offer c) as it has the lowest interest charges.

EXERCISE 2.10

Part B

1. $(1 + 5r) = e^{0.07(5)}$

$$5r = e^{0.35} - 1$$
$$r = \frac{e^{0.35}-1}{5}$$
$$r = 0.08381351$$
$$r = 8.38\%$$

2. $e^{j_\infty(25)} = 2$

$$j_\infty = \frac{\ln 2}{25}$$

$$e^{\frac{\ln 2}{25}t} = 1.5$$
$$t\frac{(\ln 2)}{25} = \ln 1.5$$
$$t = 25\left(\frac{\ln 1.5}{\ln\ 2}\right)$$
$$t = 14.62406252 \text{ years}$$

3. At the end of t-years :

$$1000e^{0.10(t-1.25)} + 1500e^{-0.10(6.5-t)} = 2500$$
$$e^{0.10t}(1000e^{-0.125} + 1500e^{-0.65}) = 2500$$
$$e^{0.10t} = 1.500991644$$
$$0.10t = \ln 1.500991644$$
$$t = 4.061259858 \text{ years}$$

4. $250e^{0.07(2)}e^{0.08(n-2)} = 400$

$$e^{0.08(n-2)} = \frac{400}{250}e^{-0.14}$$
$$0.08(n-2) = \ln\left(\frac{400}{250}\right) - 0.14$$
$$n - 2 = \frac{1}{0.08}\left(\ln\frac{400}{250} - 0.14\right)$$
$$n = 6.125045366 \text{ years}$$

5. At the end of 12 months:

$$400e^{0.04(0.75)} + Xe^{0.04(0.5)} + X = 1000e^{0.04}$$
$$412.187 + 1.02020134X + X = 1040.81$$
$$2.02020134X = 628.63$$
$$X = \$311.17$$

6. $1 - 4d = e^{-0.08(4)}$

$$d = \frac{1-e^{-0.32}}{4}$$
$$d = 0.068462741$$
$$d = 6.85\%$$

REVIEW EXERCISE 2.11

1. $X = 1000\left(1+\frac{0.0638}{2}\right)^{9} + 800\left(1+\frac{0.0638}{2}\right)^{-11} = 1326.60 + 566.34 = \$1892.94$

2. $S = 1500\left(1+\frac{0.098}{365}\right)^{3650} = \$3996.16$

3. $S = 1000(1.045)^{20} = \$2411.71$

4. a) Theoretical method : $S = 2000(1.04)^{2\frac{1}{3}} = \$2191.67$
   Practical method : $S = 2000(1.04)^{2}\left[1+(0.08)\left(\frac{2}{12}\right)\right] = \$2192.04$
   b) Theoretical method : $S = 2000(1.04)^{-2\frac{1}{3}} = \$1825.10$
   Practical method : $S = 2000(1.04)^{-3}\left[1+(0.08)\left(\frac{4}{12}\right)\right] = \$1825.41$

5. $S = 680\,000(1.04)^{5} = \$\,827\,323.97$

6. Interest $= 100(1.035)^{20} - 100(1.035)^{10} = 198.98 - 141.06 = \$57.92$

7. $D = 1500 - 500\left(1+\frac{0.21}{12}\right)^{-3} - 600\left(1+\frac{0.21}{12}\right)^{-6} - 300\left(1+\frac{0.21}{12}\right)^{-9}$
   $= 1500 - 474.64 - 540.69 - 256.63 = \$228.04$

8. $j_2 = 6.75\% \;\rightarrow j = \left(1+\frac{0.0675}{2}\right)^{2} - 1 \doteq 6.86\%$ BEST
   $j_4 = 6.25\% \;\rightarrow j = \left(1+\frac{0.0625}{4}\right)^{4} - 1 \doteq 6.40\%$ MIDDLE
   $j_{12} = 6.125\% \;\rightarrow j = \left(1+\frac{0.06125}{12}\right)^{12} - 1 \doteq 6.30\%$ WORST

9. Maturity date is November 21, 2018
   Proceeds $P = 3000(1.015)^{-19}\left[1+(0.06)\left(\frac{41}{365}\right)\right] = \$2276.06$

10. $1000\left(1+\frac{0.06}{365}\right)^{n} = 2500$
    $\left(1+\frac{0.06}{365}\right)^{n} = 2.5$
    $n\log\left(1+\frac{0.06}{365}\right) = \log 2.5$
    $n = 5547.560135$ days
    $n = 15$ years, 100 days OR 15 years, 2 months, 12 days

11. $(1+i)^{60} = 3$
    $i = 3^{1/60} - 1$
    $j_4 = 4\left[3^{\frac{1}{60}} - 1\right] = 7.39\%$

12. Maturity Value of Loan $= 10\ 000\left(1+\frac{0.12}{2}\right)^{12} = \$20\ 121.96$

On January 1, 2019:

$$2000(1.02)^{12} + X(1.02)^4 + X = 20\ 121.96$$
$$2536.48 + 2.08243216X = 20\ 121.96$$
$$X = \$8\ 444.68$$

13. $\left(1+\frac{0.045}{365}\right)^n = 1.25$

$$n = \frac{\log 1.25}{\log\left(1+\frac{0.045}{365}\right)}$$
$$n \doteq 1810 \text{ days}$$

4 years, 350 days from November 20, 2013 is November 5, 2017.

14. At the present time:

$$X(1.0075)^{-2} + 2X(1.0075)-^{5} + 3X(1.0075)^{-10} = 5000$$
$$5.695834944X = 5000$$
$$X = \$877.83$$

15. a) at j_{12}: $\left(1+\frac{j_{12}}{12}\right)^{120} = 3$

$$j_{12} = 12\left[3^{\frac{1}{120}} - 1\right] \doteq 11.04\%$$

b) at j_{365}: $\left(1+\frac{j_{365}}{365}\right)^{3650} = 3$

$$j_{365} = 365\left[3^{\frac{1}{3650}} - 1\right] \doteq 10.99\%$$

c) at $j_\infty \equiv \sigma$: $e^{10\sigma} = 3$

$$10\sigma = \ln 3$$
$$\sigma = \frac{\ln 3}{10} \doteq 10.99\%$$

16. $1000(1.045)^n = 1246.18$

$$(1.045)^n = 1.24618$$
$$n = \frac{\log 1.24618}{\log 1.045}$$
$$n = 5$$
$$S = 1000(1.06)^5 = \$1338.23$$

17. Discounted value of the payments option:

$$P = 20\ 000 + 20\ 000(1.04)^{-4} + 20\ 000(1.04)^{-8}$$
$$= 20\ 000 + 17\ 096.08 + 14\ 613.80 = \$51\ 709.88$$

Cash option is better by \$1709.88

18. Maturity value on October 6, 2012:

$S = 2000\left(1+\frac{0.08}{12}\right)^{24} = \$2345.78$

Proceeds on January 16, 2014:

$P = 2345.78\left(1+\frac{0.09}{4}\right)^{-9}\left[1+(0.09)\left(\frac{10}{365}\right)\right] = \$2199.72$

Compound discount $= 2345.78 - 2199.72 = \$146.06$

19. $S = 500\left(1+\frac{0.053}{2}\right)^{4}\left(1+\frac{0.07}{12}\right)^{36}\left(1+\frac{0.045}{365}\right)^{365} = \$715.95$

$$500(1+j_1)^6 = 715.95$$
$$j_1 = 6.17\%$$

20. $P = 2000(1.02)^{-8}(1.05)^{-7} = \$1213.12$

21. a) $1000(1.06)^5 = \$1338.23$

b) $1000\left(1+\frac{0.06}{12}\right)^{60} = \$1348.85$

c) $1000e^{0.06(5)} = \$1349.86$

22. She will receive $2000(1.05)^5\left[1+(0.05)\left(\frac{3}{12}\right)\right] = \$2584.47$

23. a) $S = 5000\left(1+\frac{0.036}{12}\right)^{22}\left[1+(0.036)\left(\frac{26}{365}\right)\right] = \$5354.30$

b) $S = 5000\left(1+\frac{0.036}{12}\right)^{22+\frac{26}{365}(12)} = \$5354.30$

24. Value on December 13, 2013:

$$2000(1.025)^{-9}\left[1+(0.1)\left(\frac{39}{365}\right)\right] = \$1618.57$$

25. a) Equation of value at 12 months:

$$\begin{aligned} X(1.0075)^9 + 2X(1.0075)^5 + 2X &= 4000(1.0075)^{12} \\ 1.069560839X + 2.076133469X + 2X &= 4375.23 \\ 5.145694308X &= 4375.23 \\ X &= \$850.27 \end{aligned}$$

b) Equation of value at 12 months:

$$\begin{aligned} Xe^{0.09\left(\frac{9}{12}\right)} + 2Xe^{0.09\left(\frac{5}{12}\right)} + 2X &= 4000e^{0.09\left(\frac{12}{12}\right)} \\ 1.0698026X + 2.076423994X + 2X &= 4376.70 \\ 5.146254254X &= 4376.70 \\ X &= \$850.46 \end{aligned}$$

26. a) At $j_{365} = 10\%$

$\left(1 + \frac{0.10}{365}\right)^n = 2$

$n = \frac{\log 2}{\log(1+\frac{0.10}{365})}$

$n \doteq 2530.33 = 2531$ days

$n \doteq 6$ years, 341 days OR 6 years, 11 months, 7 days

b) At $j_\infty = 10\%$

$e^{0.1t} = 2$

$0.1t = \ln 2$

$t = \frac{\ln 2}{0.1}$

$t = 6.931471806$ years

$t \doteq 6$ years, 340 days OR 6 years, 11 months, 6 days

c) At $j_4 = 10\%$

$(1.05)^n = 2$

$n = 28.07103453$ quarters

$n = 7$ years, 0 months, 7 days

d) At $j_2 = 10\%$

$(1.05)^n = 2$

$n = 14.20669908$ half years

$n = 7$ years, 38 days OR 7 years, 1 months, 8 days

Rule of 70

a) $\frac{70}{\frac{10}{365}} = 2555$ days $= 7$ years

c) $\frac{70}{\frac{10}{4}} = 28$ quarters $= 7$ years

d) $\frac{70}{\frac{10}{2}} = 14$ half years $= 7$ years

Case Study I – Payday Loans

a) Calculate j such that:
$(1+j) = (1.25)^{\frac{365}{14}}$
$1+j = 336.188$
$j = 335.2\%$

b) If you are one week late, the penalty is 10% of 1000 or another \$100. Thus you borrow \$800 and pay back \$1100 in 21 days. Thus:
$(1+j) = \left(\frac{1100}{800}\right)^{\frac{365}{21}}$
$1+j = 253.415$
$j = 252.4\%$

If you are two weeks late, you owe \$1200 in 28 days. Thus:
$(1+j) = \left(\frac{1200}{800}\right)^{\frac{365}{28}}$
$1+j = 197.458$
$j = 196.5\%$

c) When the fee is 15%:
$(1+j) = (1.15)^{\frac{365}{14}}$
$1+j = 38.2366$
$j = 37.2\%$

At 20%:
$(1+j) = (1.20)^{\frac{365}{14}}$
$1+j = 115.976$
$j = 114.98\%$

At 30%
$(1+j) = (1.30)^{\frac{365}{14}}$
$1+j = 934.687$
$j = 933.7\%$

Case Study II – Overnight Rates

a) $I = 20{,}000{,}000\left(\frac{0.04}{365}\right) = \$2191.78$
b) $I = 20{,}000{,}000\left(e^{\frac{0.04}{365}} - 1\right) = \$2191.90$
c) $j = \left(\frac{25{,}002{,}568}{25{,}000{,}000}\right)^{\frac{365}{1}} - 1 = 0.0328 = 3.28\%$