Chapter 2: Set Theory: Using Mathematics to Classify Objects

Section 2.1: The Language of Sets

1. {10, 11, 12, 13, 14, 15}
2. $\{f, g, h, i, j\}$
3. {17, 18, 19, 20, 21, 22, 23, 24, 25}
4. {- 4, - 3, - 2, - 1, 0, 1, 2, 3, 4}
5. {4, 8, 12, 16, 20, 24, 28}
6. {7, 9, 11, 13, 15, 17, 19}
7. {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}
8. {New Hampshire, New Jersey, New Mexico, New York}
9. Æ
10. {11, 13, 15, 17, 19, 21, 23, 25}
11. Æ
12. {Alaska, Hawaii}
13. Answers may vary. Possible answers include {x : x is a multiple of 3 between 3 and 12 inclusive}.
14. {x : x is a color of the rainbow}
15. {28, 29, 30, 31}
16. {- 1, - 2, - 3, ...}
17. {January, February, March, April, May, June, July, August, September, October, November, December}
18. {x : x is a sign of the Zodiac}
19. Answers may vary. Possible answers include {101, 102, 103, ...}.
20. Answers may vary. Possible answers include {x : x is an even natural number}.
21. Answers may vary. Possible answers include {x : x is an even natural number between 1 and 101}.
22. Answers may vary. Possible answers include {3, 6, 9, 12, 15, ...}.
23. well defined
24. well defined
25. not well defined
26. not well defined
27. not well defined
28. well defined
29. well defined
30. not well defined
31. Ï
32. I
33. I
34. Ï
35. Ï
36. Ï
37. I
38. I
39. Ï
40. Ï
41. I
42. Ï; {Florida} is a subset, not an element.
43. 6
44. 11
45. 0
46. 48
47. 4
48. 5
49. 2 elements; {1, 2}, {1, 2, 3}
50. 4 elements; {1}, Æ, 0, {0}

51. 1 element; {{Æ}}

52. 4 elements; {1}, {2}, {3}, {1, 2, 3}

53. finite

54. finite

55. infinite

56. finite

57. Answers may vary. Possible answers include 4.5.

58. Answers may vary. Possible answers include the King(Queen) of England.

59. Answers may vary. Possible answers include Sony.

60. Answers may vary. Possible answers include a frog.

61. Answers may vary. Possible answers include Angela Merkel.

62. Answers may vary. Possible answers include Kia.

63. Answers may vary. Possible answers include Sunday.

64. Answers may vary. Possible answers include California.

65. $\{x : x \text{ is a humanities elective}\}$

66. $\{x : x \text{ does not satisfy a world culture requirement}\}$

67. {History012, History223, Geography115, Anthropology111}

68. {History012, English010, English220, Psychology200}

69. $L =$ {AZ, FL, GA, LA, NJ, NM, TX, VA}

70. $G =$ {CA, MN, NY, PA}

71. $\{x : x \text{ is a state with the price of gasoline above \$2.35}\}$

72. $\{x : x \text{ is a state with price of gasoline below \$2.10}\}$

73. $M =$ {jogging, jumping rope}

74. $L =$ {calisthenics, leisure cycling, slow walking}

75. $\{x : x \text{ burns less than 140 calories per one-half hour}\}$

76. $\{x : x \text{ burns at least 300 calories per one-half hour}\}$

77. Answers will vary.

78. When the set is too large or to complicated to list all the elements.

79. Æ is the empty set, it contains no elements. {Æ} is not empty, it contains 1 element, Æ.

80. a) n stands for the word *number*.
    b) A stands for the set A. Set names are always capitalized.
    c) $n(A)$ is the number of elements in set A.

81. – 84. Answers will vary.

85. If the barber shaves himself, then he (the barber) does not shave himself. If the barber does not shave himself, then he (the barber) does shave himself. Conclusion: This is a paradox.

86. If the sentence is true, then the sentence is false. If the sentence is false, then the sentence is true. Conclusion: This is a paradox.

87. If $S \in S$, then $S \notin S$. If $S \notin S$, then $S \in S$. Conclusion: This is a paradox.

Section 2.2: Comparing Sets

1. These two sets are equal. They have the same elements arranged in a different order.

2. These two sets are not equal. The first set is the set of vowels. The second set contains "b" along with other letters. Since "b" is not a vowel, these two sets cannot be equal.

3. These two sets are not equal. The second set contains (infinitely many) elements that don't appear in the first set.

4. These two sets are equal. They are both $\{3, 4, 5, 6, 7, 8, 9, 10\}$.

5. These two sets are equal. They are both $\{1, 3, 5, \ldots, 99\}$.

6. These two sets are not equal. The first set contains the first 5 multiples of 3 that are counting numbers. The second set contains all multiples of 3 that are counting numbers.

7. These two sets are equal. Common sense dictates that nobody born before 1800 should be living.

8. These two sets are not equal. The null set contains no elements. The set, $\{\emptyset\}$, contains one element, namely the null set.

9. true; All the elements of the first set are understood to be elements of the second set and moreover the first set is not equal to the second set.

10. true; All the elements of the first set are also elements of the second set.

11. false; The letter "y" is an element of the first set and not an element of the second set.

12. false; The set on the left and the set on the right are equal. They both represent the set $\{r, u, t, h\}$. The first set cannot be a <u>proper</u> subset of the second set.

13. true; The null set is a subset of all sets.

14. false; Although the null set is a subset of all sets, it is not a *proper* subset because it is equal to itself.

15. The first set is equivalent to the second set because they both have the same number of elements.

16. The first set is not equivalent to the second set. The first set has 6 elements while the second set is understood to have 11 elements.

17. The first set is equivalent to the second set because they both have that same number of elements, namely 4.

18. The first set is not equivalent to the second set. The first set has 7 elements while the second set has 6 elements.

19. The first set is not equivalent to the second set. The first set has 0 elements while the second set has 1 element.

20. The first set is equivalent to the second set. They both have 1 element.

21. The first set is equivalent to the second set. They both have 8 elements.

22. The first set is not equivalent to the second set. The first set has 26 elements while the second set has 24 elements.

23. The first set is not equivalent to the second set. The first set has 366 elements while the second set has 365 elements.

24. The first set is equivalent to the second set. The starting number of players for both teams is the same.

25. $\{1, 2\}, \{1, 3\}, \{2, 3\}$

26. $\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}$

27. $\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$

28. $\{1, 2, 3\}$, $\{1, 2, 4\}$, $\{1, 2, 5\}$, $\{1, 3, 4\}$, $\{1, 3, 5\}$, $\{1, 4, 5\}$, $\{2, 3, 4\}$, $\{2, 3, 5\}$, $\{2, 4, 5\}$, $\{3, 4, 5\}$

29. There are $2^5 = 32$ subsets and $2^5 - 1 = 31$ proper subsets.

30. There are $2^7 = 128$ subsets and $2^7 - 1 = 127$ proper subsets.

31. T; $V = \{\text{Carmen, Frank, Ivana}\} = T$

32. Answers may vary. Note: the **boxed** values have the same cardinality as S. The set of students majoring in art $= A$. The set of students that are involved in drama $= D$.
$n(U) = 4,\ n(L) = 6,\ \boxed{n(S) = 2,}\ n(V) = 3,\ \boxed{n(A) = 2,}\ n(T) = 3,$ and $\boxed{n(D) = 2}$

33. The set of lowerclassmen $= L$; Note: the **boxed** value indicates the set with the largest cardinality.
$$n(U) = 4,\ \boxed{n(L) = 6,}\ n(S) = 2,\ n(V) = 3,\ n(A) = 2,\ n(T) = 3,\ \text{and}\ n(D) = 2$$

34. Answers may vary. Note: the **boxed** values are the sets with the smallest cardinality. The set of students that are science majors $= S$. The set of students that are art majors $= A$. The set of students that are involved in drama $= D$.
$n(U) = 4,\ n(L) = 6,\ \boxed{n(S) = 2,}\ n(V) = 3,\ \boxed{n(A) = 2,}\ n(T) = 3,$ and $\boxed{n(D) = 2}$

35. $2^6 = 64$

36. $2^1 = 2$

37. $2^4 = 16$

38. $2^4 = 16$

39. $2^7 = 128$

40. 9; $2^9 = 512$

41. $2^8 = 256$

42. 10; $2^{10} = 1024$

43. 7

44. 6

45. $\{5\text{P}, 10\text{P}, 25\text{D}\}$

46. $\{1\text{S}, 25\text{S}, 50\text{D}\}$

47. $\{5\text{P}, 10\text{P}, 25\text{D}\}$ or $\{5\text{P}, 10\text{P}, 25\text{S}\}$

48. $\{5\text{P}, 10\text{S}, 50\text{D}\}$

49. 8

50. a) branch 2
    b) branch 7

51. He didn't understand that the order of elements in a set does not matter.

52. He didn't understand that repetition of elements in a set does not matter.

53. a) 25 is not a power of 2.

    b) He confused 5^2 with 2^5.

    c) $2^5 = 32$

54. There are 2^n ways to flip n coins and also to answer an n question true-false test.

55. Answers will vary.

56. $2^{30} = 1{,}073{,}741{,}824$

57. Over 34 years; $\dfrac{1{,}073{,}741{,}824}{365 \times 24 \times 60 \times 60} = \dfrac{1{,}073{,}741{,}824}{31{,}536{,}000} \approx 34.05$

58. $\dfrac{2^{100} \text{ subsets}}{1,000,000,000 \text{ subsets/sec}} \approx 1.26765 \times 10^{21}$ seconds; $\dfrac{1.26765 \times 10^{21}}{365 \times 24 \times 60 \times 60} \approx 4.0197 \times 10^{13}$ years

59. The fifth line counts the number of subsets of sizes 0, 1, 2, 3, 4, and 5 of a five-element set.

60. The sixth line, which is 1 6 15 20 15 6 1, counts the number of subsets of sizes 0, 1, 2, 3, 4, 5, and 6 of a six-element set.

61. The sixth, seventh, eighth, and ninth lines are:

1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 **84** 126 126 84 36 9 1

So Tyra Banks can choose the three contestants in 84 different ways.

62. The sixth, seventh, eighth, ninth, and tenth lines are:

1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 **210** 252 210 120 45 10 1

So the four dancers can be chosen in 210 different ways.

63.

1 → 1
1 1 → 2
1 2 1 → 4
1 3 3 1 → 8
1 4 6 4 1 → 16
1 5 10 10 5 1 → 32
1 6 15 20 15 6 1 → 64
1 7 21 35 35 21 7 1 → 128
1 8 28 56 70 56 28 8 1 → 256
1 9 36 84 126 126 84 36 9 1 → 512
1 10 45 120 210 252 210 120 45 10 1 → 1024

The sum across the rows is always a power of 2, specifically 2^n, where n is the number of the row that is being summed. Note: Recall we start counting these lines with 0, not 1.

64. Answers may vary. When we consider two numbers that are the same in a row of Pascal's triangle, one number is counting subsets of a certain size and the other number is counting the complements of those subsets.

65. 16; We are choosing 3, 4, or 5 senior partners from the five possible, so add the last three elements of the 5th row of Pascal's triangle. (Remember we begin numbering the rows with 0.)

66. 60; Using the 3rd element of the 5th row of Pascal's triangle, there are 10 ways to choose the senior partners. Using the 2nd element of the 4th row of Pascal's triangle, there are 6 ways to choose the associates. There will be $6 \times 10 = 60$ ways to form the committee.

67. Corresponding property: If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$. If $x \in A$, then $x \in B$. Since $x \in B$, then $x \in C$. Therefore, if we have $x \in A$, we must also have $x \in C$. Note: We use capital letters for sets and lower case letters for elements.

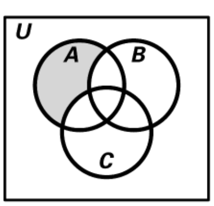
68. Corresponding property: If $A \subseteq B$ and $B \subseteq A$, then $A = B$. Since $A \subseteq B$, then each $x \in A$ is also an element of B. Also, since $B \subseteq A$ then each $x \in B$ is also an element of A. Therefore, each element of each set belongs to the other set, so the two sets must be equal. Note: We use capital letters for sets and lower case letters for elements.

69. Examples will vary. A three-element set has $3! = 6$ correspondences, a four element set has $4! = 24$, and so on. So, a set with n elements will have $n!$ one-to-one correspondences.

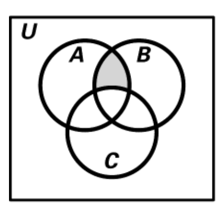
70. Answers will vary.

Section 2.3: Set Operations

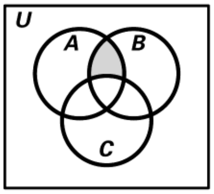
1. $A \cap B = \{1, 3, 5\}$
2. $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 9\}$
3. $B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$
4. $B \cap C = \{2, 4, 6\}$
5. $A \cup \emptyset = \{1, 3, 5, 7, 9\} = A$
6. $A \cap \emptyset = \emptyset$
7. $A \cup U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} = U$
8. $A \cap U = \{1, 3, 5, 7, 9\} = A$
9. $A \cap (B \cup C) = A \cap \{1, 2, 3, 4, 5, 6, 7, 8\} = \{1, 3, 5, 7\}$
10. $A' \cap (B \cup C') = A' \cap (B \cup \{1, 3, 5, 9, 10\}) = A' \cap \{1, 2, 3, 4, 5, 6, 9, 10\}$
    $= \{2, 4, 6, 8, 10\} \cap \{1, 2, 3, 4, 5, 6, 9, 10\} = \{2, 4, 6, 10\}$
11. $(A - B) \cap (A - C) = \{7, 9\} \cap \{1, 3, 5, 9\} = \{9\}$
12. $A - (B \cup C) = A - \{1, 2, 3, 4, 5, 6, 7, 8\} = \{9\}$
13. $M \cap E = \{\text{potato chip, bread, pizza}\}$
14. $M - E = \{\text{flat-screen TV, hat, satellite radio, sofa, hybrid automobile, hammer}\}$
15. $E - M = \{\text{apple, fish, banana}\}$
16. $E' = \{\text{flat-screen TV, hat, satellite radio, sofa, hybrid automobile, hammer}\}$
17. $M' \cap G' = \{\text{fish}\}$
18. $G \cap (M' \cap E) = G \cap (\{\text{apple, fish, banana}\} \cap E) = G \cap \{\text{apple, fish, banana}\} = \{\text{apple, banana}\} = G$; Note: Since all things that grow on a plant from our universal set are edible, the outcome of G should be readily understood.
19. 7
20. 2
21. 5
22. 1
23. $A - (B \cup C)$



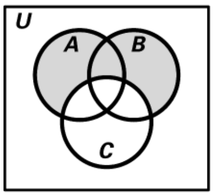
24. $A \cap (B - C)$



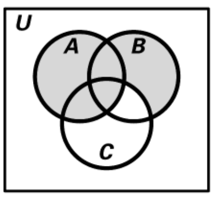
25. $(A \cap B) - C$



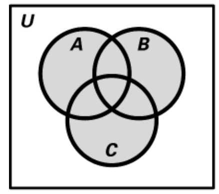
26. $(A \cup B) - C$



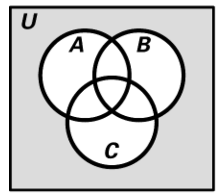
27. $A \cup (B - C)$



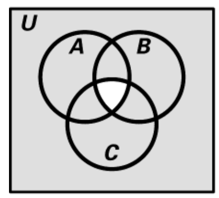
28. $A \cup (B \cup C)$



29. $(A \cup (B \cup C))'$



30. $(A \cap (B \cap C))'$



31. $B - A$

32. $(A \cup B) - (A \cap B)$

33. $(A \cup B)'$

34. $(A \cup B)' \cup (A \cap B)$

35. $A \cap B \cap C$

36. $(A \cap B) \cup (A \cap C)$

37. $(A \cup C) - B$

38. $(A \cup B \cup C) - (A \cap B \cap C)$

39. equal; Using the diagrams from Example 2, $(A \cup B')'$ consists of region r_4. $A' \cap B$ also consists of region r_4. so $(A \cup B')' = A' \cap B$.

40. unequal; Using the diagrams from Example 2, $(A' \cap B)'$ consists of regions r_1, r_2, and r_3, $A \cap B'$ consists of region r_2. so $(A' \cap B)' \neq A \cap B'$

41. 30

42. 50

43. 28

44. 10

45. 20

46. 11

47. 27

48. 6

49. $P \cap C =$ the set of cars whose price is above \$20,000 and is compact $= \{d, f, g\}$

50. $A \cup G =$ the set of cars that have an antitheft package or have a good safety rating $= \{a, b, c, g, h\}$; Note: This is an inclusive "or". The car can have both features.

51. $W \cap G' =$ the set of cars that have a warranty of at least three years and don't have a good safety rating $= \{c, d, f, g\}$

52. $G - A =$ the set of cars that don't have an antitheft package but do have a good safety rating $= \{b, h\}$

53. $P \cap (G \cup W) =$ the set of cars that have a price above $20,000, and a good safety rating or a warranty of at least three years $= \{b, d, f, g, h\} = P$

54. $G' \cap C' =$ the set of cars that don't have a good safety rating and are not compact $= \{c, e\}$

55. $P - (G \cup A) =$ the set of cars that have a price above $20,000 and don't have a good safety rating nor have an antitheft package $= \{d, f\}$

56. $P' - (G \cup C) =$ the set of cars that don't have a price above $20,000 and don't have a good safety rating and aren't compact $= \{c, e\}$

57. $P \cap (B \cup A) = P \cap \{m, mc, hc\} = \{m, mc, hc\}$

58. $(P \cup C) \cap (B \cup A) = \{m, mc, bc, c, hc\} \cap \{m, mc, hc\} = \{m, mc, hc\}$

59. $P \cup C \cup B = \{m, mc, bc, c, hc\}$

60. $P \cap C \cap B = \{mc, hc\}$

61. $H \cap B = \{a, b, e\}$

62. $B' = \{c, d, f, h, j\}$

63. $H - B = \{c, d\}$

64. $L \cap (H \cup B) = \{g, i\}$

65. $M \cap L =$ the set of movies that earned more than $1.5 billion and also earned less than $2 billion $= \{c\}$

66. $M \cup B =$ the set of movies that earned more than $1.5 billion or were made before 2015 $= \{a, b, c, d, g, h, i\}$

67. $M - L =$ the set of movies that earned more than $1.5 billion and did not earn less than $2 billion $= \{a, b\}$

68. $B' =$ the set of movies that were not made before 2015 $= \{c, e, f, j\}$

69. $L - (M \cup B) =$ set of movies that earned less than $2 billion, but neither made more than $1.5 billion nor were made before 2015 $= \{e, f, j\}$

70. $L \cap (M \cup B) =$ set of movies that earned less than $2 billion, and either made more than $1.5 billion or were made before 2015 $= \{c, d, g, h, i\}$

71. "Union" implies joining together . "Intersection" implies overlapping.

72. Set differences are found by removing the elements common to both sets.

73. Answers will vary. Possible answers include confusing DeMorgan's laws with the distributive property.

74. – 76. Answers will vary.

77. false; It is possible that $A = B$ and hence $n(A) = n(B)$. Counterexamples may vary.

78. true

79. true

80. false; Sample counterexample: Let $X = \{1, 2, 3, 4\}$ and $Y = \{3, 4, 5, 6\}$
$X - Y = \{1, 2\}$, $n(X - Y) = 2$, $n(X) = 4$ and $n(Y) = 4$, $2 \neq 4 - 4$

81. A

82. $\emptyset$

83. B

84. B'

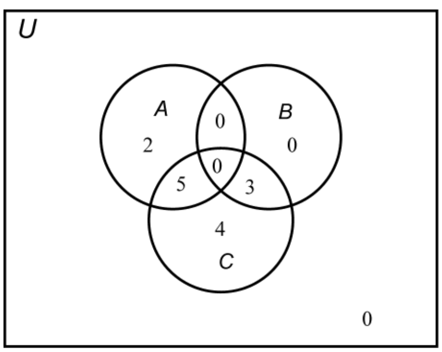
85. a) $A \cap B = B \cap A$ is true.

b) $A \cup (B \cup C) = (A \cup B) \cup C$ is true.

86. yes; Using the diagram from Example 8, $A \cup (B \cap C)$ consists of regions $r_2,\ r_3,\ r_5,\ r_6,$ and r_7. $(A \cup B) \cap (A \cup C)$ also consists of regions $r_2,\ r_3,\ r_5,\ r_6,$ and r_7, so $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Section 2.4: Survey Problems

1. r_2, r_3
2. r_3
3. r_2, r_3, r_4
4. r_4
5. r_2, r_3, r_5, r_6
6. r_5, r_6
7. r_4, r_7
8. r_2; $\{r_2, r_3, r_5, r_6\} - \{r_3, r_4, r_5, r_6, r_7, r_8\} = \{r_2\}$
9. r_7; $\{r_6, r_7\} \cap \{r_1, r_4, r_7, r_8\} = \{r_7\}$
10. r_6; $\{r_3, r_4, r_6, r_7\} \cap \{r_5, r_6, r_7, r_8\} \cap \{r_2, r_3, r_5, r_6\} = \{r_6, r_7\} \cap \{r_2, r_3, r_5, r_6\} = \{r_6\}$
11. 25
12. 12
13. 18
14. 31
15. 16
16. 0
17. 20
18. 29
19. 19
20. 16
21. $n(A) = 7, n(B) = 3,$ and $n(C) = 12$



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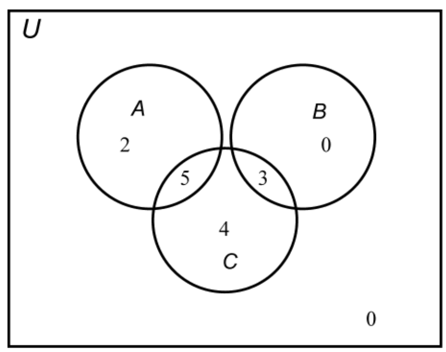
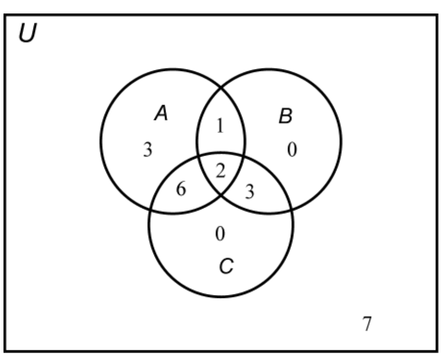
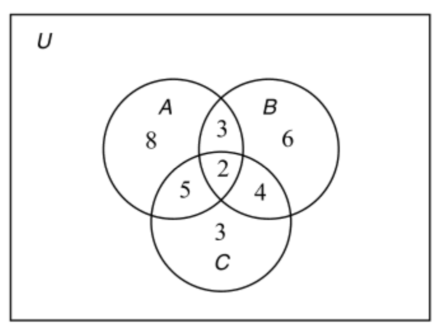


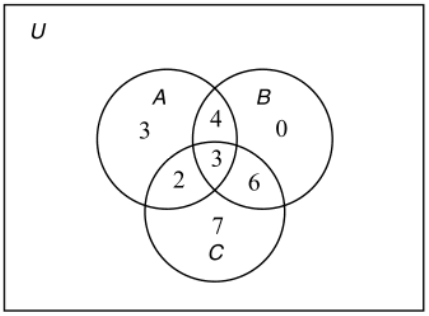
22. $n(A) = 12, n(B) = 6,$ and $n(C) = 11$



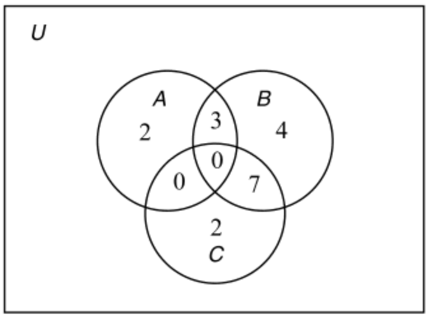
23. $n(A) = 18, n(B) = 15,$ and $n(C) = 14$



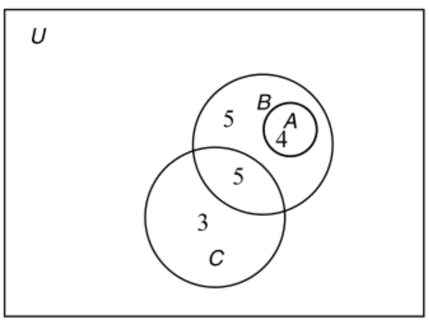
24. $n(A) = 12, n(B) = 13,$ and $n(C) = 18$



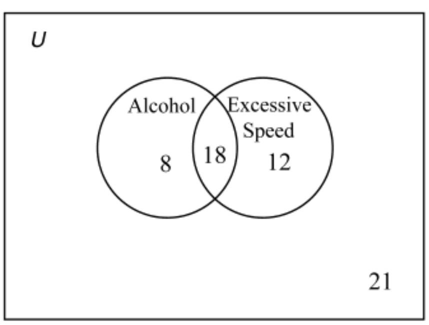
25. $n(A) = 5, n(B) = 14,$ and $n(C) = 9$



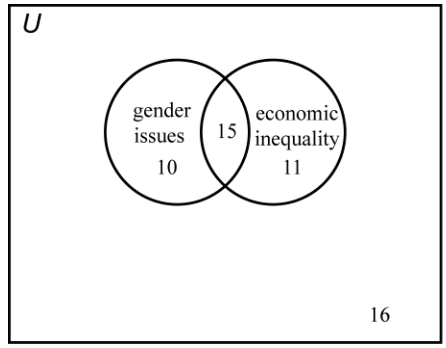
26. $n(A) = 4, n(B) = 14,$ and $n(C) = 8$



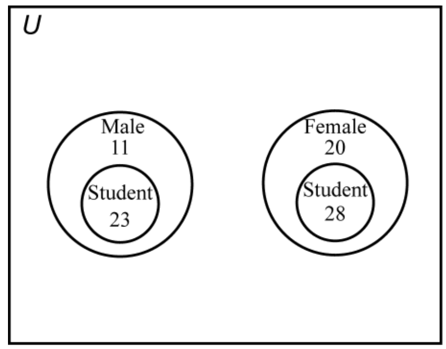
27. 59



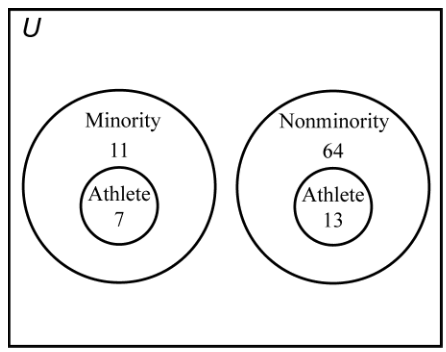
28. 26



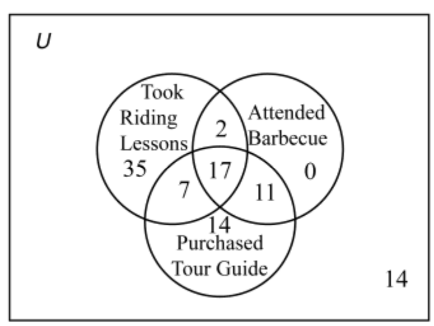
29. 28



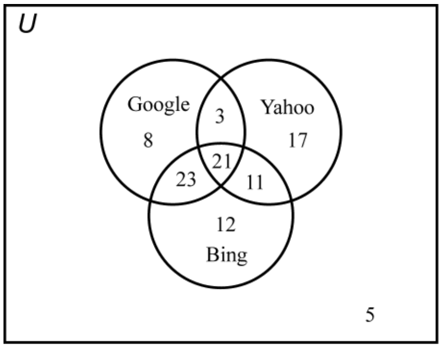
30. 64



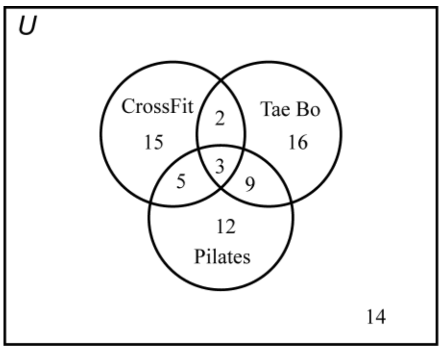
31. 30 attended the barbecue and 49 purchased a tour guide.



32. 55 use Google and 33 do not use Bing.

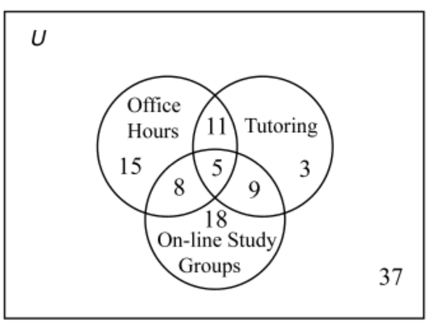


33.



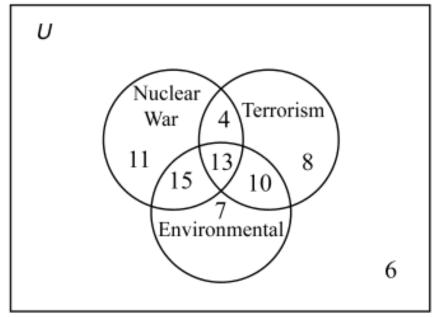
a) 76
b) 16
c) 21

34.



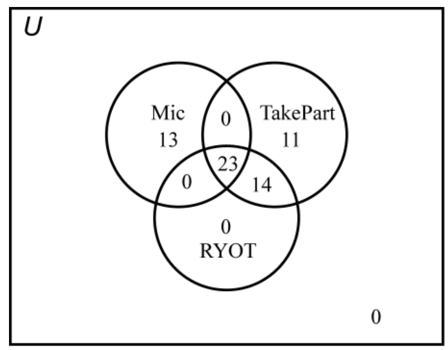
a) 106
b) 15
c) 40

35.

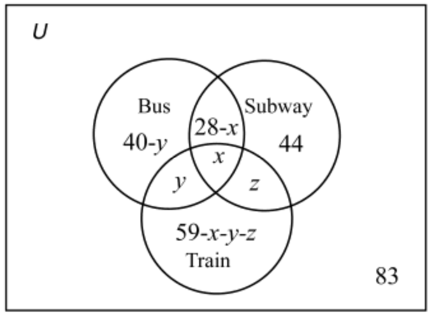


a) 74
b) 45
c) 8

36. 23



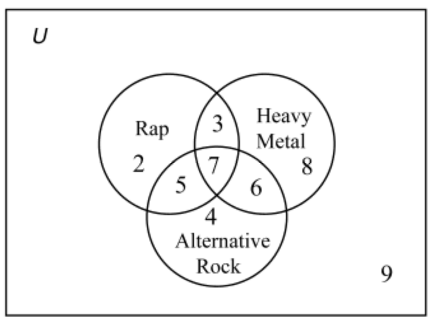
37. Answers may vary. If none use both the bus and the train, then $68+59=127$ use either the bus or train. If we add the 44 who use only the subway and the 83 that use none, then this total exceeds 200. Moreover, from the given information, we can set up the following Venn diagram:



$$(40-y)+(28-x)+44+x+y+z+(59-x-y-z)+83=200$$
$$254-(x+y)=200$$
$$-(x+y)=-54$$
$$x+y=54$$

Thus, there are 54 people that use both the bus and the train, so the intersection of bus and train is not empty.

38.



a) 44

b) 27

c) 8

39. $n(A \cup P)=95+63=158$

40. $n(S \cap D)=5$

41. $n(A-(M \cup S))=95-(44+31)=95-75=20$

42. $n(Y \cap (A \cup D))=20+9=29$

43. A^-

44. O^-

45. A^-, B^-, or AB^-

46. B^+

47. $A \cap B \cap Rh$

48. $(A \cup B \cup Rh)'$

49. B^-, O^-

50. A^+, A^-, O^+, O^-

51. They forget that r_2 excludes the elements in $A \cap B$.

52. They forget that r_3 excludes the elements in $A \cap B \cap C$.

53. – 55. Answers will vary.

56. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

57. 8

58. 16; $A \cap B' \cap C' \cap D$, $A' \cap B \cap C \cap D'$

59. Answers will vary.

60. $r_1 = A' \cap B' \cap C \cap D$; $r_2 = A' \cap B \cap C' \cap D$; $r_3 = A \cap B \cap C' \cap D'$

Section 2.5: Looking Deeper: Infinite Sets

1.

1	2	3	4	5 . . .	*n* . . .
↕	↕	↕	↕	↕	↕
4	8	12	16	20 . . .	4*n* . . .

2.

1	2	3	4	5 . . .	*n* . . .
↕	↕	↕	↕	↕	↕
5	10	15	20	25 . . .	5*n* . . .

3. Each term is 3 more than the previous term. The general term is represented by 8 plus 3, added $n - 1$ times. So the general term would be $8 + (n - 1) \cdot 3 = 8 + 3n - 3 = 3n + 5$.

1	2	3	4	5 . . .	*n* . . .
↕	↕	↕	↕	↕	↕
8	11	14	17	20 . . .	3*n* + 5 . . .

4. Each term is 4 more than the previous term. The general term is represented by 7 plus 4, added $n - 1$ times. So the general term would be $7 + (n - 1) \cdot 4 = 7 + 4n - 4 = 4n + 3$.

1	2	3	4	5 . . .	*n* . . .
↕	↕	↕	↕	↕	↕
7	11	15	19	23 . . .	4*n* + 3 . . .

5.

1	2	3	4	5 . . .	*n* . . .
↕	↕	↕	↕	↕	↕
2	4	8	16	32 . . .	2^n . . .

6.

1	2	3	4	5 . . .	*n* . . .
↕	↕	↕	↕	↕	↕
3	9	27	81	243. . .	3^n . . .

7.

1	2	3	4	5 . . .	*n* . . .
↕	↕	↕	↕	↕	↕
1	1/2	1/3	1/4	1/5. . .	1/*n*. . .

8.

1	2	3	4	5 . . .	*n* . . .
↕	↕	↕	↕	↕	↕
1/2	2/3	3/4	4/5	5/6. . .	*n*/(*n*+1). . .

9. $3 \cdot 1 = 3$; $3 \cdot 2 = 6$; $3 \cdot 3 = 9$; $3 \cdot 4 = 12$; $3 \cdot 5 = 15$

1	2	3	4	5 . . .	*n* . . .
↕	↕	↕	↕	↕	↕
3	6	9	12	15. . .	3*n*. . .

10. $2 \cdot 1 + 3 = 5$; $2 \cdot 2 + 3 = 7$; $2 \cdot 3 + 3 = 9$; $2 \cdot 4 + 3 = 11$; $2 \cdot 5 + 3 = 13$

1	2	3	4	5 . . .	*n* . . .
↕	↕	↕	↕	↕	↕
5	7	9	11	13 . . .	2*n* + 3 . . .

11. $3\times1-2=1;\ 3\times2-2=4;\ 3\times3-2=7;\ 3\times4-2=10;\ 3\times5-2=13$

1	2	3	4	5	. . .	n	. . .
↕	↕	↕	↕	↕		↕	
1	4	7	10	13	. . .	$3n-2$	. . .

12. $4\times1+5=9;\ 4\times2+5=13;\ 4\times3+5=17;\ 4\times4+5=21;\ 4\times5+5=25$

1	2	3	4	5	. . .	n	. . .
↕	↕	↕	↕	↕		↕	
9	13	17	21	25	. . .	$4n+5$	. . .

13. Match $\{2, 4, 6, 8, 10, ...\}$ with $\{4, 6, 8, 10, 12, ...\}$; in general, match $2n$ with $2(n+1)=2n+2$.

14. Match $\{5, 10, 15, 20, 25, ...\}$ with $\{10, 15, 20, 25, 30, ...\}$; in general, match $5n$ with $5(n+1)=5n+5$.

15. Since each term of $\{7, 10, 13, 16, 19, ...\}$ is 3 more than the previous, the general term is $7+3(n-1)=7+3n-3=3n+4$. Match $\{7, 10, 13, 16, 19, ...\}$ with $\{10, 13, 16, 19, 22, ...\}$; in general, match $3n+4$ with $3(n+1)+4=3n+3+4=3n+7$.

16. Since each term of $\{6, 9, 12, 15, 18, ...\}$ is 3 more than the previous, the general term is $6+3(n-1)=6+3n-3=3n+3$. Match $\{6, 9, 12, 15, 18, ...\}$ with $\{9, 12, 15, 18, 21, ...\}$; in general, match $3n+3$ with $3(n+1)+3=3n+3+3=3n+6$.

17. Match $\{2, 4, 8, 16, 32, ...\}$ with $\{4, 8, 16, 32, 64, ...\}$; in general, match 2^n with 2^{n+1}.

18. Match $\{3, 9, 27, 81, 243, ...\}$ with $\{9, 27, 81, 243, 729, ...\}$; in general, match 3^n with 3^{n+1}.

19. Match $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, ...\}$ with $\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, ...\}$; in general, match $\frac{1}{n}$ with $\frac{1}{n+1}$.

20. Match $\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, ...\}$ with $\{\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, ...\}$; in general, match $\frac{n}{n+1}$ with $\frac{n+1}{(n+1)+1}=\frac{n+1}{n+2}$.

21. Match $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, ...\}$ with $\{\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}, ...\}$; in general, match $\frac{1}{2n}$ with $\frac{1}{2(n+1)}=\frac{1}{2n+2}$.

22. Match $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, ...\}$ with $\{\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, ...\}$; in general, match $\frac{1}{2^n}$ with $\frac{1}{2^{n+1}}$.

For Exercises 23 – 26, refer to the diagram on the next page.

23. 6; We skip 2/2, 2/4, 3/3, and 4/2.

24. 3/4; We skip 2/2, 2/4, 3/3, and 4/2.

25. 25; We skip 2/2, 2/4, 3/3, 4/2, 2/6, 4/4, 6/2, and 6/3.

26. 17; We skip 2/2, 2/4, 3/3, and 4/2.

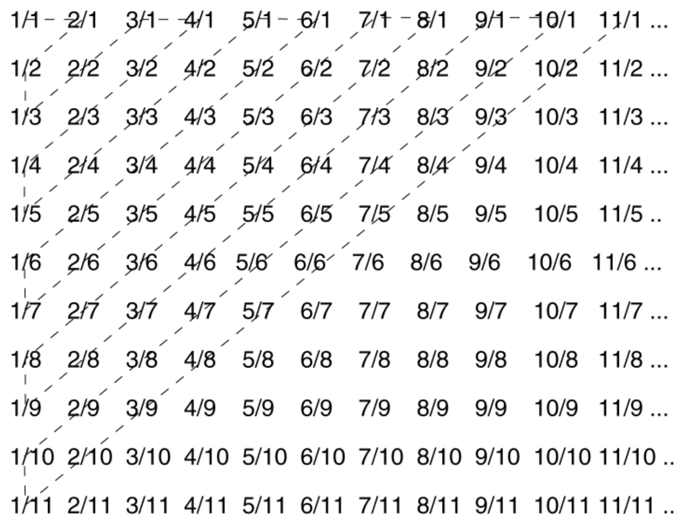
27. It was shown that the set of positive rational numbers has a one-to-one correspondence with the natural numbers.

28. It was shown that if it was assumed that it were possible to put the real numbers between 0 and 1 in a one-to-one correspondence with the natural numbers, there would always be a number between 0 and 1 that we had omitted from the one-to-one correspondence.

29. $\{1, 2, 3, 4, 5\}$ does not have a one-to-one correspondence with any of its 31 proper subsets.

30. $\{2, 4, 6, 8, \ldots\}$ has a one-to-one correspondence with the proper subset $\{2, 4, 6, 10, 14, \ldots\}$.

For Exercises 23 – 26, refer to the following diagram.

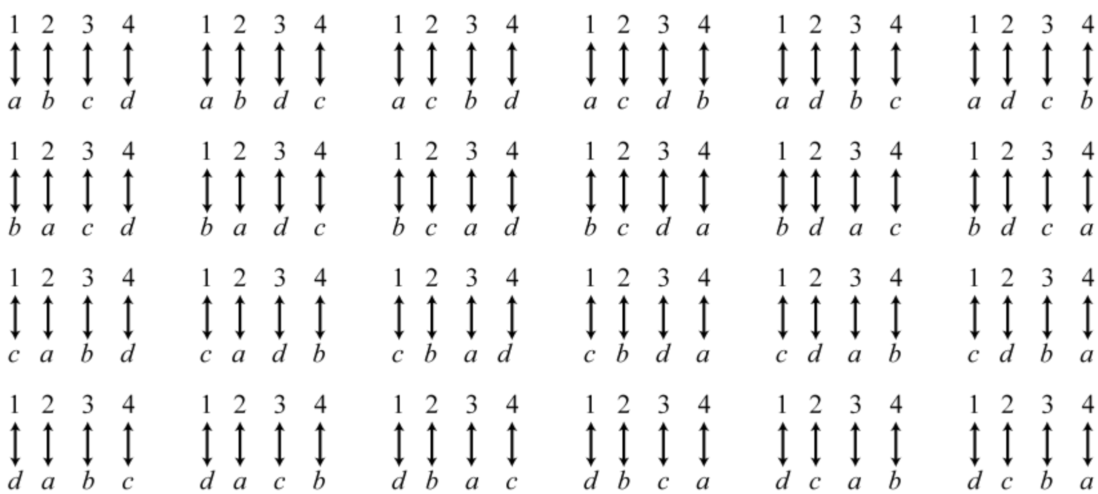


31. They had already been matched as $1, \frac{1}{2}, 1,$ and 2.

32. Change the digit in the 99th decimal place of the 99th number in the list.

33. 6

34. 24; Suppose the two sets were $\{1, 2, 3, 4\}$ and $\{a, b, c, d\}$. The 24 correspondences would be



35. Answers may vary. If we take the union of $\{1\}$, which has cardinal number 1, and $\{2, 3, 4, ...\}$, which has cardinal number À$_0$, we get $\{1, 2, 3, 4, ...\}$, which has cardinal number À$_0$.

36. Answers may vary. Take the union of $\{1, 3, 5, ...\}$ and $\{2, 4, 6, ...\}$, both of which have cardinal number À$_0$ to get À$_0$ $\{1, 2, 3, 4, 5, 6, ...\}$, which has cardinal number À$_0$.

37. From this figure we see that every point on the semi-circle matches with exactly one point on the line and vice versa.

38. No solution provided.

Chapter Review Exercises

1. a) Answers may vary. Possible answers include: $\{x : x$ is an even natural number between 1 and 19$\}$.

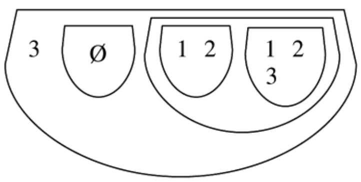
   b) $\{x : x$ a month of the year$\}$

   c) {New Hampshire, New Jersey, New Mexico, New York}

   d) Æ

2. Æ is the empty set; it contains no elements. {Æ} is not empty; it contains Æ.

3.



4. a) 7

   b) 0

   c) 4

5. a) Yes, these two sets are equal. They have the same elements arranged in a different order.

   b) Yes, these two sets are equal. Duplicate elements do not count as distinct (different) elements.

   c) No, these two sets are not equal. The first set has elements, such as 1002, that are not elements in the second set.

6. a) True, it is understood that all of the elements of the first set are also elements of the second set.

   b) True, it is understood that all of the elements of the first set are also elements of the second set.

   c) True, it is understood that all of the elements of the first set are also elements of the second set.

   d) True, the null set is a subset of all sets. We cannot find an element of the empty set that fails to be in $\{1, 2, 3\}$.

   e) False, In order to be a proper subset, the first set cannot be the same as the second set.

7. a) not equivalent

   b) equivalent

   c) equivalent

   d) equivalent

8. a) There are $2^3 = 8$ subsets. They are Æ, $\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$.

   b) $2^7 = 128$

9. a) A Ç $B = \{5, 7, 9\}$

   b) B È $C = \{2, 3, 4, 5, 7, 8, 9, 10\}$

   c) C¢$= \{1, 4, 6, 7, 10\}$

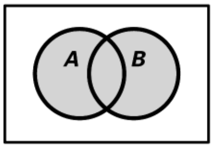
   d) $A - C = \{7\}$

10. a) $(A$ È $B)$¢$= \{1, 6\}$

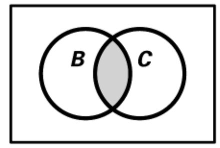
    b) $(A - C)$ È $(A - B) = \{2, 7, 8\}$

    c) A¢Ç $(B$¢È $C) = \{1, 3, 6\}$

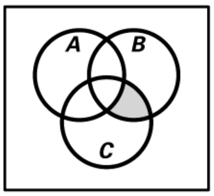
11. a) $A \cup B$



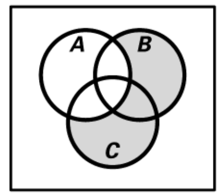
b) $B \cap C$



c) $A' \cap B \cap C$



d) $(B \cup C) - A$



12. $A' \cap B'$

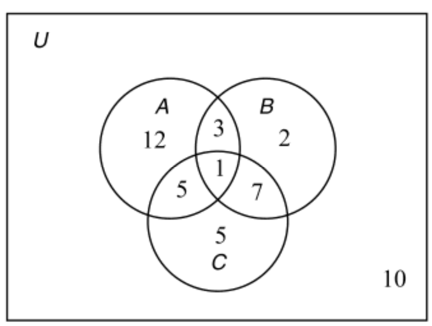
13. a) Any three of the following four are valid responses.
   1) Closure
   2) Commutativity; $A \cup B = B \cup A$
   3) Associativity; $(A \cup B) \cup C = A \cup (B \cup C)$
   4) Identity; $A \cap U = A$

   b) Any three of the following four are valid responses.
   1) Closure
   2) Commutativity; $A \cap B = B \cap A$
   3) Associativity; $(A \cap B) \cap C = A \cap (B \cap C)$
   4) Identity; $A \cup \emptyset = A$

   c) Union distributes over intersection; $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and intersection distributes over union; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

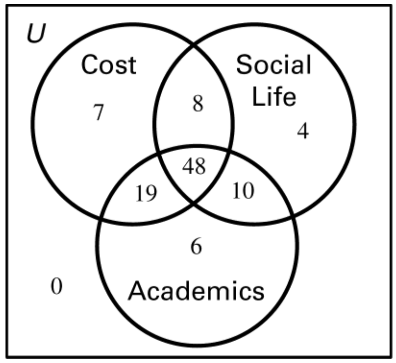
14. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$; We sometimes forget to subtract $n(A \cap B)$.

15.



a) $C - B = 5 + 5 = 10$
b) $10 + 2 + 7 + 5 = 24$

16. 83 said academics. 58 said both academics and social life.



17. a) B^-

b) O^-

18. An infinite set can be put in a one-to-one correspondence with one of its proper subsets.

19. $\{1, 2, 3, 4, \ldots\}$ can be put in a one-to-one correspondence with $\{2, 4, 6, 8, \ldots\}$.

20. $\frac{3}{2}$

21. We chose a digit that was different from the third decimal place of the third number in the list.

Chapter Test

1. a) $\{x : x$ is a natural number greater than 100$\}$

b) {January, February, March, … , December}

c) Æ

2. a) Equal, the order of elements does not matter.

b) Not equal, the set {1} is not the same as the number 1.

c) Equal, the sets contains the same elements.

3. a) not equivalent

b) equivalent

c) equivalent

4. a) C¢= {1,3,5,7,9}

b) B - C = {3,5}

c) (A Ç B)¢ = {1,3,4,6,7,8,9,10}

d) (A¢Ç B¢)È C = {2,4,6,7,8,10}

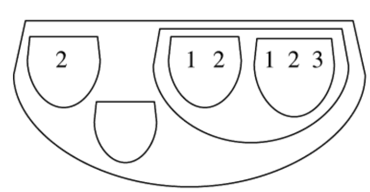
5. Æ contains no elements, but {Æ} contains one element, namely Æ.

6. a) 8

b) 1

7. $2^8 = 256$

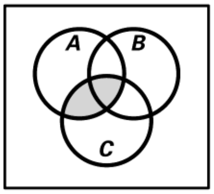
8.



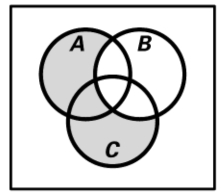
9. a) True; every element of the first set is a member of the second.
   b) True; every element of the first set is a member of the second.
   c) False; $\emptyset$ is not one of the numbers 1, 2, 3.

10. $(A \cap B)' = A' \cup B'$

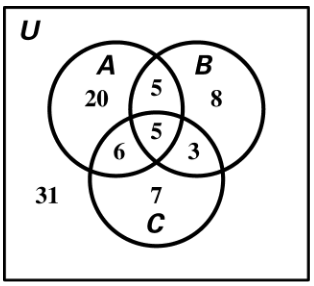
11. a) $A \cap C$



b) $(A \cup C) - B$

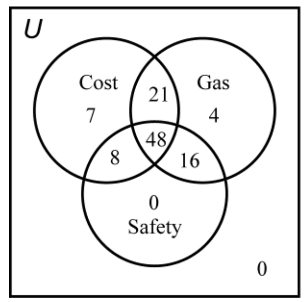


12.



a) 10
b) 64

13.



gas mileage: 89

both gas mileage and safety: 64

14. An infinite set can be put in a one-to-one correspondence with one of its proper subsets.

15. $\{1, 2, 3, 4, \ldots\}$ can be put in a one-to-one correspondence with $\{2, 4, 6, 8, \ldots\}$.

16. $\frac{1}{4}$

17. We chose a digit that was different from the fifth decimal place of the fifth number in the list.

18. a) $A \cap B \cap Rh'$
    b) $A' \cap B' \cap Rh$