Chapter 2

Solar Energy

Section 2.2 Practice!

1. Find the declination angle and apparent solar time for Richmond, Virginia, longitude 77.47°, latitude 37.53°, on September 21 at 2:00 pm.

<u>Solution</u>

$$\delta = 23.45° \sin\left[\frac{N+284}{365} \times 360°\right]$$

$$= 23.45° \sin[(264 + 284)/365 \times 360°]$$

$$= \underline{\underline{-0.202°}}$$

$$D = \frac{(N-81)}{365} 360°$$

$$= \frac{(264 - 81)(360°)}{365}$$

$$= 180.49°$$

$$ET = 9.87 \sin(2D) - 7.53 \cos(D) - 1.5 \sin(D)$$

$$= 9.87 \sin[(2)(180.49°)] - 7.53 \cos(180.49°) - 1.4 \sin(180.49°)$$

$$= 7.71 \text{ min}$$

$$AST = LST + (4 \text{ min/deg})(LSTM - Long) + ET$$

$$= 840 \text{ min} + (4 \text{ min/deg})(75° - 77.47°) + 7.71 \text{ min}$$

$$= \underline{\underline{837.8 \text{ min}}}$$

2. A collector with a tilt angle of 50° is located in Nashville, Tennessee, longitude 86.78°, latitude 36.17°, and elevation 182 m. The collector faces due south, and the foreground is gravel. Find the total solar irradiance on the collector at 10:30 am on December 31.

Solution

Using equations (2.1) through (2.14) in the text, we obtain the following results:

$D = 270.25°$
$ET = 1.383$ min
$AST = 644.3$ min
$H = -18.93°$
$\beta_1 = 27.75°$
$\alpha_1 = -19.66°$
$\theta = 20.33°$

$I_D = 858.6$ W/m^2
$I_S = 42.87$ W/m^2
$I_R = 12.82$ W/m^2

$I_{tot} = \underline{\underline{914.3 \text{ W/m}^2}}$

3. The collector in Problem 2 has a surface area of 30 m^2. Find the total solar power incident on the collector at 10:30 am. Also, find the insolation and the total solar energy incident on the collector during the day.

Solution

$$\dot{Q}_{solar} = I_{tot} A_{collector}$$

$$= (914.3 \text{ W/m}^2)(30 \text{ m}^2)$$

$$= 2.74 \times 10^4 \text{ W} = \underline{\underline{27.4 \text{ kW}}}$$

Calculating the total solar irradiance at hourly intervals throughout the day and integrating under the curve using the trapezoidal rule, we obtain for the insolation

$$H = \underline{\underline{22.1 \text{ MJ/m}^2}}$$

$$E_{solar} = H A_{collector}$$

$$= (22.1 \text{ MJ/m}^2)(30 \text{ m}^2) = \underline{\underline{663 \text{ MJ}}}$$

Section 2.3 Practice!

1. Find the total solar irradiance for a horizontal photovoltaic solar panel in Santa Fe, New Mexico, at 9:00 am, 1:00 pm and 4:00 pm on October 21. Assume a foreground reflectivity of $\rho = 0.2$.

   Solution

   The results are summarized in the following table:

Quantity	9:00 am	1:00 pm	4:00 pm
D (deg)	210.1	210.1	210.1
ET (min)	15.83	15.83	15.83
AST (min)	552.0	792.0	972.0
H (deg)	-42.00	18.00	63.00
β_1 (deg)	28.18	39.62	14.02
α_1 (deg)	-48.01	23.12	64.04
θ (deg)	61.82	50.38	75.97
I_D (W/m^2)	434	627	173.7
I_S (W/m^2)	67.1	71.8	52.3
I_R (W/m^2)	0	0	0
I_{tot} (W/m^2)	501	698	226

2. The solar panel in Practice Problem 1 has a surface area of 16 m^2. If the efficiency of the cells in the panel is 0.18, find the electrical energy generated by the panel on October 21.

Solution

By plotting the total solar irradiance at hourly intervals and integrating under the curve, we obtain an insolation of $H = 17.78$ MJ/m^2.

$$E_{\text{elect}} = \eta_{\text{cell}}\, H\, A$$

$$= (0.18)(17.78 \text{ MJ/m}^2)(16 \text{ m}^2)$$

$$= 51.2 \text{ MJ} \times \frac{\text{kWh}}{3.6 \text{ MJ}}$$

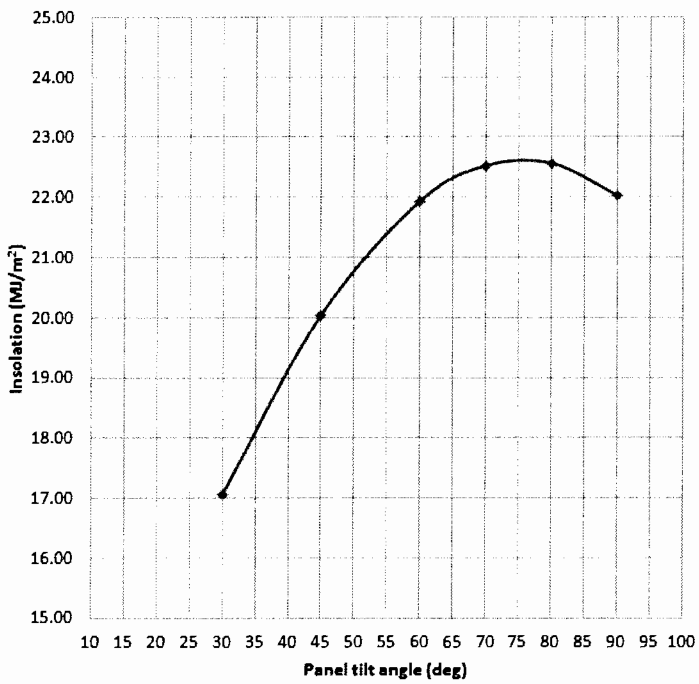
$$= \underline{\underline{14.2 \text{ kWh}}}$$

3. Find the optimum tilt angle for a photovoltaic solar panel in Pittsburgh, Pennsylvania, on December 21. Also, find the insolation corresponding to this tilt angle. The panel faces due south, and the foreground is covered with snow.

Solution

By plotting the insolation as a function of panel tilt angle (see the graph below), we see that the optimum tilt angle is

$$\beta_{2,\text{opt}} \approx \underline{\underline{75^\circ}}$$



As shown in the graph, the insolation corresponding to this tilt angle is

$$H = \underline{\underline{22.6\ \text{MJ/m}^2}}$$

Section 2.4 Practice!

1. A gravel roof-mounted, south-facing flat plate solar collector with a glazing surface measuring 2.5 m × 5.5 m has a tilt angle of 60°. The working fluid is a mixture of ethylene glycol and water with a specific heat of c = 3320 J/kg·°C and a mass flow rate of 0.075 kg/s. Assuming that 55 percent of the total solar irradiance is absorbed by the working fluid and that T_i = 20°C, find the temperature of the working fluid at the outlet of the collector for conditions at 12:00 noon on February 21. The collector is located in Albany, New York, longitude 73.76°, latitude 42.65° and elevation 0 m (Albany borders the Hudson River, a sea level river).

   Solution

   The total solar irradiance for the conditions given is I_{tot} = 1011 W/m^2. Fifty five percent of the solar irradiance is absorbed by the working fluid, so the outlet temperature of the working fluid is

$$T_o = \frac{0.55 I_{tot} A}{\dot{m} c} + T_i$$

$$= \frac{(0.55)(1011\ \text{W/m}^2)(2.5\ \text{m} \times 5.5\ \text{m})}{(0.075\ \text{kg/s})(3320\ \text{J/kg}\cdot°\text{C})} + 20°\text{C}$$

$$= \underline{\underline{50.7°\text{C}}}$$

2. The aperture diameter of a dish concentrator is 1.9 m, and the concentration ratio is 1000. The temperature of the working fluid in the receiver is 1250 °C, and the temperature of the surroundings is 25 °C. If the direct solar irradiance on the dish is 680 W/m^2, find the solar power incident on the concentrator, the heat flux at the receiver and the Carnot efficiency.

Solution

$$\dot{Q}_{solar} = I_D \frac{\pi D^2}{4}$$

$$= (680\ \text{W/m}^2)\pi(1.9\ \text{m})^2/4$$

$$= \underline{\underline{1928\ \text{W}}}$$

$$\dot{Q}''_{rec} = I_D C$$

$$= (680\ \text{W/m}^2)(1000)$$

$$= \underline{\underline{6.80 \times 10^5\ \text{W/m}^2}}$$

$$\eta_{th,ideal} = 1 - \frac{T_L}{T_H}$$

$$= 1 - \frac{(25 + 273)\text{K}}{(1250 + 273)\text{K}}$$

$$= \underline{\underline{0.80}}$$

3. A parabolic trough concentrator has an aperture width and length of 1.6 m and 24 m, respectively. The average direct solar irradiance on the concentrator on a given day is 590 W/m^2. The efficiencies of the concentrator, receiver, and electrical generator are 0.90, 0.82 and 0.94, respectively. Find the maximum possible electrical output power for a working fluid temperature at the receiver of 400 °C. The temperature of the surroundings is 15 °C.

Solution

$$\eta_{th} = 1 - \frac{T_L}{T_H}$$

$$= 1 - \frac{(15 + 273)\text{K}}{(400 + 273)\text{K}}$$

$$= 0.572$$

$$\eta_{overall} = \eta_{th}\,\eta_{con}\,\eta_{rec}\,\eta_{gen}$$

$$= (0.572)(0.90)(0.82)(0.94)$$

$$= 0.397$$

$$P_{elect} = \eta_{overall} I_D A$$

$$= (0.397)(590\ \text{W/m}^2)(1.6\ \text{m} \times 24\ \text{m})$$

$$= \underline{\underline{8991\ \text{W}}}$$

2.1 Problem statement

For Denver, Colorado on December 19 at 10:00 am, find the declination angle and apparent solar time.

Diagram

(Not applicable)

Assumptions

1. None

Governing equations

$$\delta = 23.45° \sin\left[\frac{N+284}{365} \times 360°\right]$$

$$D = \frac{(N-81)}{365} 360°$$

$$ET = 9.87 \sin(2D) - 7.53 \cos(D) - 1.5 \sin(D)$$

$$AST = LST + (4 \text{ min/deg})(LSTM - Long) + ET$$

Calculations

$$\delta = 23.45° \sin\left[\frac{N+284}{365} \times 360°\right]$$

$$= 23.45° \sin[(353 + 284)/365 \times 360°]$$

$$= \underline{\underline{-23.44°}}$$

$$D = \frac{(N-81)}{365} 360°$$

$$= \frac{(353 - 81)}{365} 360°$$

$= 268.27°$

$$ET = 9.87 \sin(2D) - 7.53 \cos(D) - 1.5 \sin(D)$$

$$= 9.87 \sin[2(268.27°)] - 7.53 \cos(268.27°) - 1.5 \sin(268.27°)$$

$$= 2.32 \text{ min}$$

Local standard time, LST, is

$$LST = 10 \text{ h} \times 60 \text{ min/h} = 600 \text{ min}$$

Denver is in the mountain time zone, so $LSTM = 105°$, and the longitude of Denver is 104.88° west.

$$AST = LST + (4 \text{ min/deg})(LSTM - Long) + ET$$

$$= 600 \text{ min} + (4 \text{ min/deg})(105° - 104.88°) + 2.32 \text{ min}$$

$$= \underline{\underline{602.8 \text{ min}}}$$

Solution check

No errors are detected.

Discussion

Apparent solar time differs from local standard time by (602.8 min - 600 min) = 2.8 min.

2.2 Problem statement

For Atlanta, Georgia on March 2 at 2:30 pm, find the declination angle and apparent solar time.

Diagram

(Not applicable)

Assumptions

1. None

Governing equations

$$\delta = 23.45° \sin\left[\frac{N+284}{365} \times 360°\right]$$

$$D = \frac{(N-81)}{365} 360°$$

$$ET = 9.87 \sin(2D) - 7.53 \cos(D) - 1.5 \sin(D)$$

$$AST = LST + (4 \text{ min/deg})(LSTM - Long) + ET$$

Calculations

$$\delta = 23.45° \sin\left[\frac{N+284}{365} \times 360°\right]$$

$$= 23.45° \sin[(61 + 284)/365 \times 360°]$$

$$= \underline{\underline{-7.91°}}$$

$$D = \frac{(N-81)}{365} 360°$$

$$= \frac{(61 - 81)}{365} 360°$$

$$= -19.276°$$

$$ET = 9.87\sin(2D) - 7.53\cos(D) - 1.5\sin(D)$$

$$= 9.87\sin[2(-19.276°)] - 7.53\cos(-19.276°) - 1.5\sin(-19.276°)$$

$$= -12.76 \text{ min}$$

Local standard time, *LST*, is

$$LST = 14.5 \text{ h} \times 60 \text{ min/h} = 870 \text{ min}$$

Atlanta is in the eastern time zone, so $LSTM = 75°$, and the longitude of Atlanta is 84.39° west.

$$AST = LST + (4 \text{ min/deg})(LSTM - Long) + ET$$

$$= 870 \text{ min} + (4 \text{ min/deg})(75° - 84.39°) - 12.76 \text{ min}$$

$$= \underline{\underline{819.6 \text{ min}}}$$

Solution check

No errors are detected.

Discussion

Apparent solar time differs from local standard time by (819.6 min − 870 min) = −50.4 min.

2.3 Problem statement

For Albany, New York on August 20 at 11:00 am, find the declination angle and apparent solar time.

Diagram

(Not applicable)

Assumptions

1. None

Governing equations

$$\delta = 23.45° \sin\left[\frac{N+284}{365} \times 360°\right]$$

$$D = \frac{(N-81)}{365} 360°$$

$ET = 9.87 \sin(2D) - 7.53 \cos(D) - 1.5 \sin(D)$

$AST = LST + (4 \text{ min/deg})(LSTM - Long) + ET$

Calculations

$$\delta = 23.45° \sin\left[\frac{N+284}{365} \times 360°\right]$$

$= 23.45° \sin[(232 + 284)/365 \times 360°]$

$= \underline{\underline{12.10°}}$

$$D = \frac{(N-81)}{365} 360°$$

$$= \frac{(232 - 81)}{365} 360°$$

$= 148.93°$

$$ET = 9.87\sin(2D) - 7.53\cos(D) - 1.5\sin(D)$$

$$= 9.87\sin[2(148.93°)] - 7.53\cos(148.93°) - 1.5\sin(148.93°)$$

$$= -3.05 \text{ min}$$

Local standard time, *LST*, is

$$LST = 11 \text{ h} \times 60 \text{ min/h} = 660 \text{ min}$$

Albany is in the eastern time zone, so $LSTM = 75°$, and the longitude of Albany is 73.757° west.

$$AST = LST + (4 \text{ min/deg})(LSTM - Long) + ET$$

$$= 660 \text{ min} + (4 \text{ min/deg})(75° - 73.757°) - 3.05 \text{ min}$$

$$= \underline{\underline{661.9 \text{ min}}}$$

Solution check

No errors are detected.

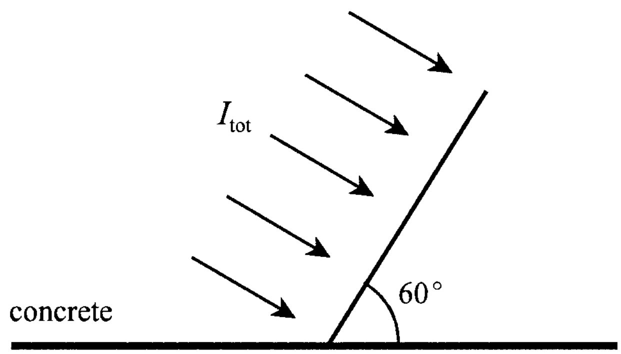
Discussion

Apparent solar time differs from local standard time by (661.9 min − 660 min) = 1.92 min.

2.4 Problem statement

A collector, located in Columbus, Ohio has a tilt angle of 60° and faces due south. If the foreground is concrete, find the total solar irradiance on the collector at 3:00 pm on January 21.

Diagram



Assumptions

1. Sky is clear.
2. All quantities pertinent to the total solar irradiance are constant.

Governing equations

$$\delta = 23.45°\sin\left[\frac{N+284}{365}\times 360°\right]$$

$$D = \frac{(N-81)}{365}360°$$

$$ET = 9.87\sin(2D) - 7.53\cos(D) - 1.5\sin(D)$$

$$AST = LST + (4\,\text{min/deg})(LSTM - Long) + ET$$

$$H = \frac{AST - 720\,\text{min}}{4\,\text{min/deg}}$$

$$\sin(\beta_1) = \cos(L)\cos(\delta)\cos(H) + \sin(L)\sin(\delta)$$

$$\cos(\alpha_1) = \frac{\sin(\beta_1)\sin(L) - \sin(\delta)}{\cos(\beta_1)\cos(L)}$$

$$\cos(\theta) = \sin(\beta_1)\cos(\beta_2) + \cos(\beta_1)\sin(\beta_2)\cos(\alpha_1 - \alpha_2)$$

$$p/p_0 = \exp(-0.1184z)$$

$$I_{DN} = A\exp\left(-\frac{p}{p_0}\frac{B}{\sin(\beta_1)}\right)$$

$$I_D = I_{DN}\cos(\theta)$$

$$I_S = CI_{DN}\left[\frac{1+\cos(\beta_2)}{2}\right]$$

$$I_R = I_{DN}\rho(C+\sin(\beta_1))\left[\frac{1-\cos(\beta_2)}{2}\right]$$

$$I_{tot} = I_D + I_S + I_R$$

<u>Calculations</u>

$$\delta = 23.45°\sin\left[\frac{N+284}{365}\times 360°\right]$$

$$= 23.45°\sin\left[\frac{21+284}{365}\times 360°\right]$$

$$= -20.14°$$

$$D = \frac{(N-81)}{365}360°$$

$$= \frac{(21 - 81)}{365} 360°$$

$$= -59.18°$$

$$ET = 9.87\sin(2D) - 7.53\cos(D) - 1.5\sin(D)$$

$$= 9.87 \sin[(2)(-59.18°)] - 7.53 \cos(-59.18°) - 1.5 \sin(-59.18°)$$

$$= -11.26 \text{ min}$$

Columbus is in the eastern time zone, so $LSTM = 75°$. A time of 3:00 pm gives

$$LST = 15 \text{ h} \times 60 \text{ min/h} = 900 \text{ min.}$$

$$AST = LST + (4 \text{ min/ deg})(LSTM - Long) + ET$$

$$= 900 \text{ min} + (4 \text{ min/deg})(75° - 82.98°) + (-11.26 \text{ min})$$

$$= 856.8 \text{ min}$$

$$H = \frac{AST - 720 \text{ min}}{4 \text{ min/ deg}}$$

$$= \frac{856.8 \text{ min} - 720 \text{ min}}{4 \text{ min/deg}}$$

$$= 34.20°$$

$$\sin(\beta_1) = \cos(L)\cos(\delta)\cos(H) + \sin(L)\sin(\delta)$$

$$= \cos(39.98°) \cos(-20.14°) \cos(34.20°) + \sin(39.98°) \sin(-20.14°)$$

$$= 0.3738$$

$$\beta_1 = \sin^{-1}(0.3738) = 21.95°$$

$$\cos(\alpha_1) = \frac{\sin(\beta_1)\sin(L) - \sin(\delta)}{\cos(\beta_1)\cos(L)}$$

$$= \frac{\sin(21.95°)\sin(39.98°) - \sin(-20.14°)}{\cos(21.95°)\cos(39.98°)}$$

$$= 0.8224$$

$$\alpha_1 = \cos^{-1}(0.8224) = 34.68°$$

$$\cos(\theta) = \sin(\beta_1)\cos(\beta_2) + \cos(\beta_1)\sin(\beta_2)\cos(\alpha_1 - \alpha_2)$$

$$= \sin(21.95°)\cos(60°) + \cos(21.95°)\sin(60°)\cos(34.68° - 0°)$$

$$= 0.8474$$

The elevation of Columbus is $z = 275$ m (0.275 km), so

$$p/p_0 = \exp(-0.1184z)$$

$$= \exp[-(0.1184)(0.275 \text{ km})]$$

$$= 0.9680$$

From Table 2.2 we see that for January 21, $A = 1230$ W/m^2 , $B = 0.142$ and $C = 0.058$.

Hence, the direct normal irradiance is

$$I_{DN} = A\exp\left(-\frac{p}{p_0}\frac{B}{\sin(\beta_1)}\right)$$

$$= (1230 \text{ W/m}^2)\exp[\frac{(-0.9680)(0.142)}{\sin(21.95°)}]$$

$$= 851.5 \text{ W/m}^2$$

The direct irradiance is

$$I_D = I_{DN}\cos(\theta)$$

$$= (851.5 \text{ W/m}^2)(0.8474)$$

$$= 721.6 \text{ W/m}^2$$

Scattered irradiance is

$$I_S = CI_{DN}\left[\frac{1+\cos(\beta_2)}{2}\right]$$

$$= (0.058)(851.5\ \text{W/m}^2)\left[\frac{1 + \cos(60°)}{2}\right]$$

$$= 37.0\ \text{W/m}^2$$

The foreground reflectivity for concrete is $\rho = 0.3$. The reflected irradiance is

$$I_R = I_{DN}\rho(C+\sin(\beta_1))\left[\frac{1-\cos(\beta_2)}{2}\right]$$

$$= (851.5\ \text{W/m}^2)(0.3)[0.058 + \sin(21.95°)]\left[\frac{1 - \cos(60°)}{2}\right]$$

$$= 27.6\ \text{W/m}^2$$

The total irradiance is

$$I_{\text{tot}} = I_{\text{D}} + I_{\text{S}} + I_{\text{R}}$$

$$= (721.6 + 37.0 + 27.6)\ \text{W/m}^2$$

$$= \underline{\underline{786.2\ \text{W/m}^2}}$$

Solution check

No errors are detected.

Discussion

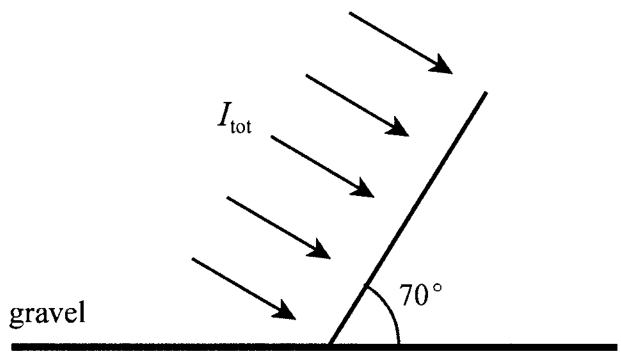
The total irradiances for a range of panel tilt angles are as follows:

β_2 (deg)	I_{tot} (W/m^2)
0	368
30	654
45	743
60	786
75	782

2.5 Problem statement

Consider a south-facing collector in Springfield, Illinois. If the collector tilt angle is 70°, find the total solar irradiance on the collector at 10:30 am on November 21. The foreground is a gravel roof.

Diagram



Assumptions

1. Sky is clear.
2. All quantities pertinent to the total solar irradiance are constant.

Governing equations

The governing equations are the same as those in Problem 2.4.

Calculations

The calculations follow the same order as those in Problem 2.4. The results are summarized below.

Constants:

$Long = 89.62°$
$L = 39.70°$
$z = 0.170$ km
$LSTM = 90°$ (central time zone)
November 21: $N = 325$, $A = 1221$ W/m^2, $B = 0.149$, $C = 0.063$
$\beta_2 = 70°$ (panel tilt angle), $\rho = 0.15$ (gravel foreground)
$\alpha_2 = 0°$ (panel faces due south)

Calculated quantities:

$\delta = -20.44°$
$D = 240.66°$
$ET = 13.43$ min
$LST = 630$ min
$AST = 644.9$ min
$H = -18.76°$
$\beta_1 = 27.38°$
$\alpha_1 = -19.84°$
$\theta = 19.57°$

$I_{DN} = 888.6$ W/m^2
$I_D = 837.3$ W/m^2
$I_S = 37.6$ W/m^2
$I_R = 22.9$ W/m^2

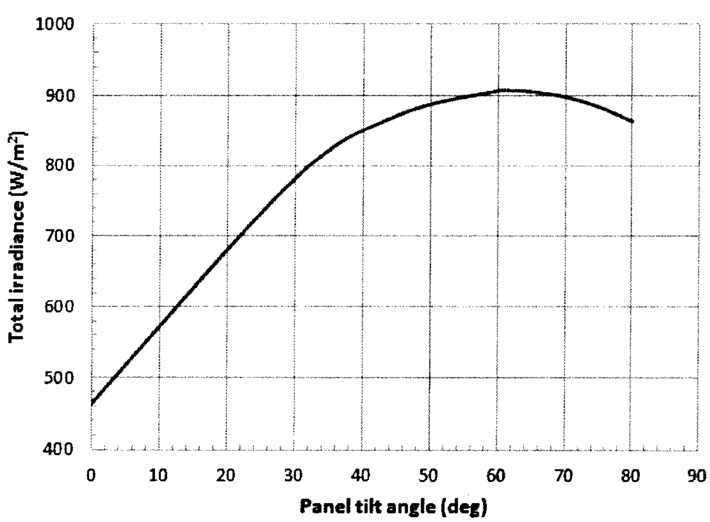
$I_{tot} = \underline{897.8 \text{ W/m}^2}$

Solution check

No errors are detected.

Discussion

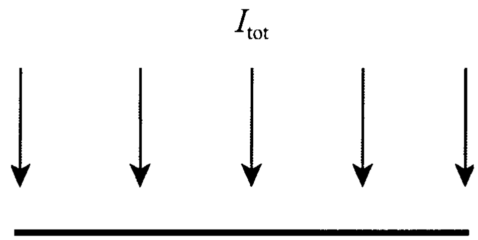
A graph of total irradiance as a function of panel tilt angle reveals an optimum tilt angle of approximately 62°.



2.6 Problem statement

A horizontal collector on a roof top is located in Austin, Texas. Find the total solar irradiance on the collector at 11:30 am on July 12.

Diagram



Assumptions

1. Sky is clear.
2. All quantities pertinent to the total solar irradiance are constant.
3. Given time is local standard time.

Governing equations

The governing equations are the same as those in Problem 2.4.

Calculations

The calculations follow the same order as those in Problem 2.4. The results are summarized below.

Constants:

$Long = 97.75°$
$L = 30.25°$
$z = 0.149$ km
$LSTM = 90°$ (central time zone)
July 12: $N = 193$, $A = 1085$ W/m^2, $B = 0.207$, $C = 0.136$
$\beta_2 = 0°$ (panel tilt angle)

Calculated quantities:

$\delta = 21.97°$
$D = 110.46°$
$ET = -5.24$ min
$LST = 690$ min (if daylight savings time was used, $LST = DST - 1$ h)
$AST = 653.8$ min
$H = -16.56°$
$\beta_1 = 73.01°$
$\alpha_1 = -64.75°$
$\theta = 16.99°$

$I_{DN} = 877.1$ W/m^2
$I_D = 838.9$ W/m^2
$I_S = 119.3$ W/m^2
$I_R = 0.0$ W/m^2 (because panel is horizontal)

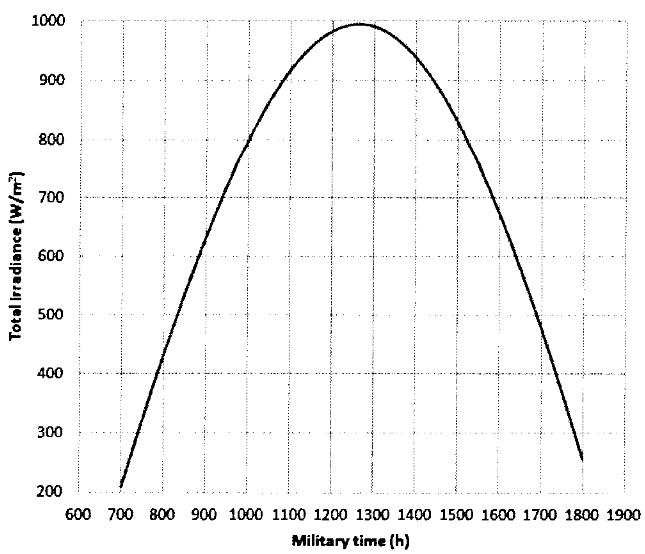
$I_{tot} = \underline{\underline{958.1 \text{ W/m}^2}}$

Solution check

No errors are detected.

Discussion

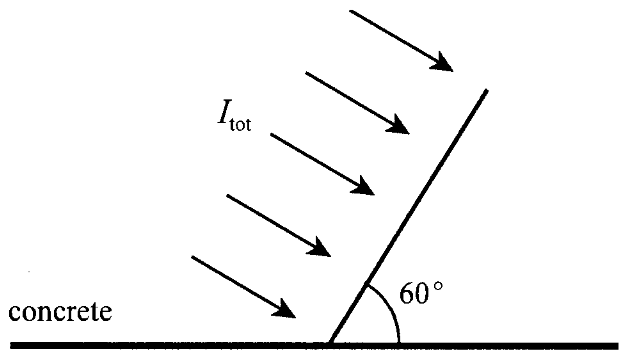
A graph of the total solar irradiance on the panel for July 12 is shown below.



2.7 Problem statement

The collector in Problem 2.4 has a surface area of 24 m^2. Find the total solar power incident on the collector at 1:00 pm. Also, find the insolation and the total solar energy incident on the collector during the day.

Diagram



Assumptions

1. Sky is clear.
2. All quantities pertinent to the total solar irradiance are constant.

Governing equations

The governing equations are the same as those in Problem 2.4 plus the following:

$$\dot{Q}_{solar} = I_{tot} A_{collector}$$

$$H = \int_{t_1}^{t_2} I_{tot} dt$$

$$E_{\text{solar}} = H A_{\text{collector}}$$

Calculations

Using the governing equations in Problem 2.4, the total solar irradiance at 1:00 pm is

$$I_{\text{tot}} = 1009.5 \text{ W/m}^2$$

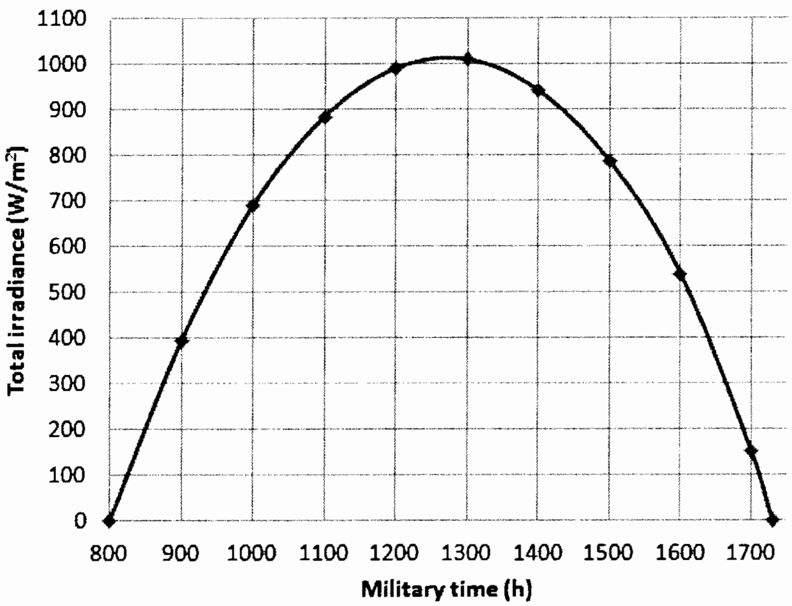
Thus, the total solar power incident on the collector at this time is

$$\dot{Q}_{solar} = I_{tot} A_{collector}$$

$$= (1009.5\ \text{W/m}^2)(24\ \text{m}^2)$$

$$= 2.423 \times 10^4\ \text{W} = \underline{\underline{24.23\ \text{kW}}}$$

The graph below, generated by calculating the total irradiance at hourly intervals, shows the total irradiance throughout the day.



Integrating under this curve using the trapezoidal rule, we obtain the insolation

$$H = \underline{\underline{22.8\ \text{MJ/m}^2}}$$

The total solar energy incident on the collector during this day is the insolation multiplied by the surface area of the collector,

$$E_{\text{solar}} = H\, A_{\text{collector}}$$

$$= (22.8\ \text{MJ/m}^2)(24\ \text{m}^2)$$

$$= \underline{\underline{547.2\ \text{MJ}}}$$

Solution check

No errors are detected.

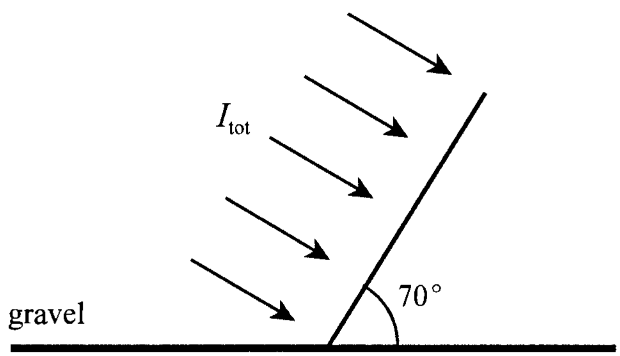
Discussion

The collector cannot generate more than 547.2 MJ of energy during this day.

2.8 Problem statement

The collector in Problem 2.5 has a surface area of 18 m^2. Find the total solar power incident on the collector at 11:00 am. Also, find the insolation and the total solar energy incident on the collector during the day.

Diagram



Assumptions

1. Sky is clear.
2. All quantities pertinent to the total solar irradiance are constant.

Governing equations

The governing equations are the same as those in Problem 2.4 plus the following:

$$\dot{Q}_{solar} = I_{tot} A_{collector}$$

$$H = \int_{t_1}^{t_2} I_{tot} dt$$

$$E_{solar} = H A_{collector}$$

Calculations

Using the governing equations in Problem 2.4, the total solar irradiance at 11:00 am is

$$I_{tot} = 908.4 \text{ W/m}^2$$

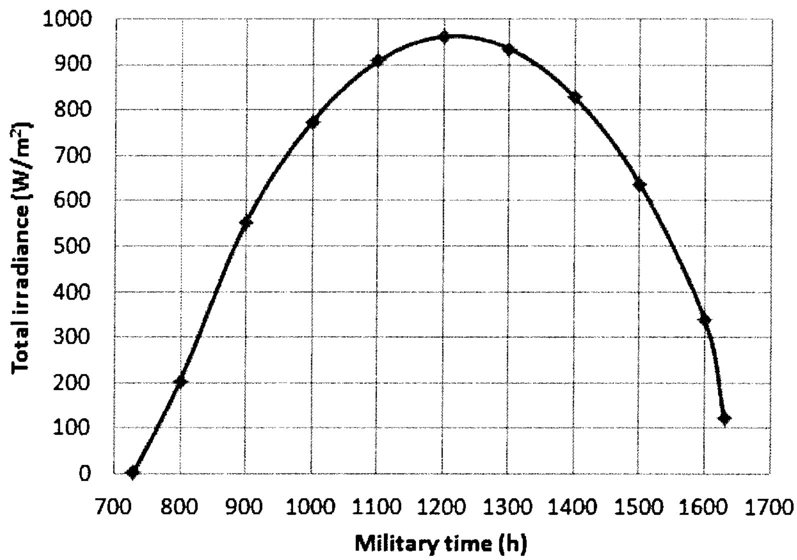
Thus, the total solar power incident on the collector at this time is

$$\dot{Q}_{solar} = I_{tot} A_{collector}$$

$$= (908.4\ \text{W/m}^2)(18\ \text{m}^2)$$

$$= 1.635 \times 10^4\ \text{W} = \underline{\underline{16.35\ \text{kW}}}$$

The graph below, generated by calculating the total irradiance at hourly intervals, shows the total irradiance throughout the day.



Integrating under this curve using the trapezoidal rule, we obtain the insolation

$$H = \underline{\underline{21.5\ \text{MJ/m}^2}}$$

The total solar energy incident on the collector during this day is the insolation multiplied by the surface area of the collector,

$$E_{\text{solar}} = H\, A_{\text{collector}}$$

$$= (21.5\ \text{MJ/m}^2)(18\ \text{m}^2)$$

$$= \underline{\underline{387.0\ \text{MJ}}}$$

Solution check

No errors are detected.

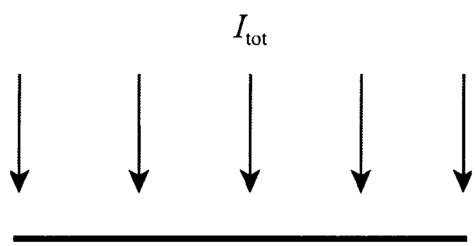
Discussion

The collector cannot generate more than 387.0 MJ of energy during this day.

2.9 Problem statement

The collector in Problem 2.6 has a surface area of 60 m^2. Find the total solar power incident on the collector at 12:00 noon. Also, find the insolation and the total solar energy incident on the collector during the day.

Diagram



Assumptions

1. Sky is clear.
2. All quantities pertinent to the total solar irradiance are constant.

Governing equations

The governing equations are the same as those in Problem 2.4 plus the following:

$$\dot{Q}_{solar} = I_{tot} A_{collector}$$

$$H = \int_{t_1}^{t_2} I_{tot} dt$$

$$E_{\text{solar}} = H A_{\text{collector}}$$

Calculations

Using the governing equations in Problem 2.4, the total solar irradiance at 12:00 noon is

$$I_{\text{tot}} = 983.5 \text{ W/m}^2$$

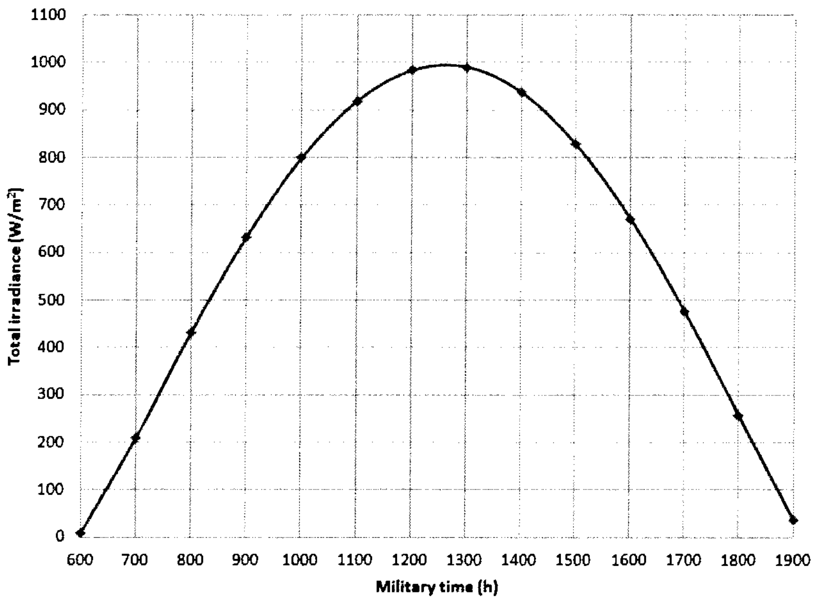
Thus, the total solar power incident on the collector is

$$\dot{Q}_{solar} = I_{tot} A_{collector}$$

$= (983.5 \text{ W/m}^2)(60 \text{ m}^2)$

$= 5.901 \times 10^4 \text{ W} = \underline{\underline{59.01 \text{ kW}}}$

The graph below, generated by calculating the total irradiance at hourly intervals, shows the total irradiance throughout the day.



Integrating under this curve using the trapezoidal rule, we obtain the insolation

$H = \underline{\underline{29.4 \text{ MJ/m}^2}}$

The total solar energy incident on the collector during this day is the insolation multiplied by the surface area of the collector,

$E_{\text{solar}} = H\, A_{\text{collector}}$

$= (29.4 \text{ MJ/m}^2)(60 \text{ m}^2)$

$= \underline{\underline{1764 \text{ MJ}}}$

Solution check

No errors are detected.

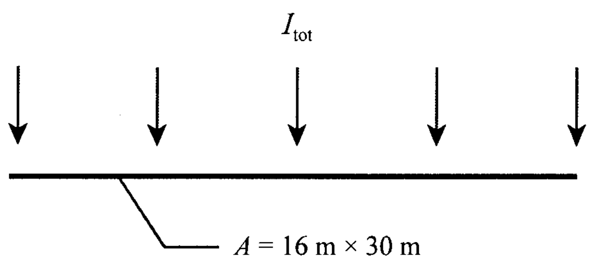
Discussion

The collector cannot generate more than 1764 MJ of energy during this day.

2.10 Problem statement

A manufacturing plant in San Jose, California has a roof-mounted photovoltaic solar panel that measures 16 m × 30 m. The panel is horizontal and has an efficiency of 0.18. How much electrical energy does the panel generate on December 21?

Diagram



Assumptions

1. Sky is clear.
2. All quantities pertinent to the total solar irradiance are constant.

Governing equations

The governing equations are the same as those in Problem 2.4 plus the following:

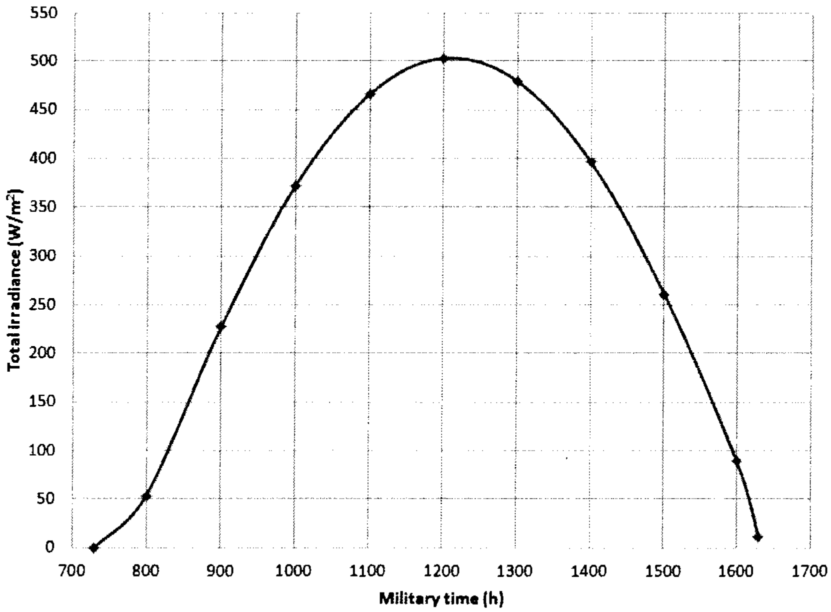
$$H = \int_{t_1}^{t_2} I_{tot}\,dt$$

$$E_{\text{solar}} = H\,A_{\text{collector}}$$

$$E_{\text{elect}} = \eta_{\text{cell}}\,E_{\text{solar}}$$

Calculations

The graph below, generated by calculating the total irradiance at hourly intervals, shows the total irradiance throughout the day.



Integrating under this curve using the trapezoidal rule, we obtain the insolation

$$H = 10.11 \text{ MJ/m}^2$$

The total solar energy incident on the collector during this day is the insolation multiplied by the surface area of the collector,

$$E_{\text{solar}} = H\,A_{\text{collector}}$$

$$= (10.11 \text{ MJ/m}^2)(16 \text{ m} \times 30 \text{ m})$$

$$= 4853 \text{ MJ}$$

The total electrical energy generated by the solar panel during the day is

$$E_{\text{elect}} = \eta_{\text{cell}}\,E_{\text{solar}}$$

$$= (0.18)(4853 \text{ MJ})$$

$$= 873.5 \text{ MJ} \times \frac{1 \text{ kWh}}{3.6 \text{ MJ}}$$

$$= \underline{\underline{242.6 \text{ kWh}}}$$

Solution check

No errors are detected.

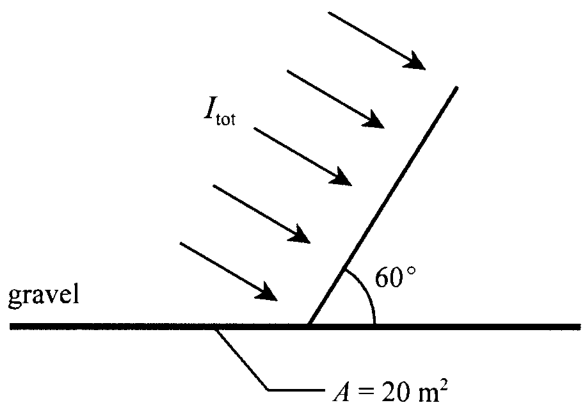
Discussion

The typical daily electrical energy consumption by a home in the US is about 30 kWh. The manufacturing plant has a higher electrical energy demand than a home, but our answer of 243 kWh provides one means of comparison.

2.11 Problem statement

A residential solar system in Montgomery, Alabama incorporates a photovoltaic solar panel with a tilt angle of 60°. The roof-mounted panel, which measures 20 m^2, faces due south and has an efficiency of 0.16. How much electrical energy does the panel generate on October 21?

Diagram



Assumptions

1. Roof is gravel.
2. Sky is clear.
3. All quantities pertinent to the total solar irradiance are constant.

Governing equations

The governing equations are the same as those in Problem 2.4 plus the following:

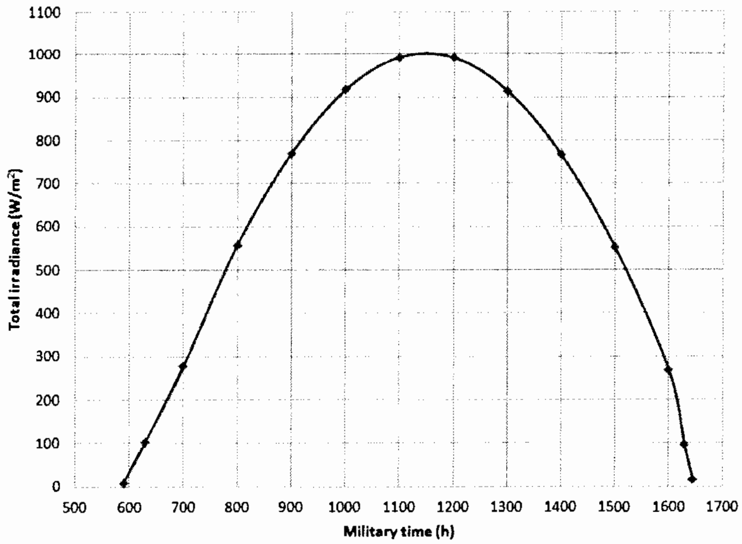
$$H = \int_{t_1}^{t_2} I_{tot}\,dt$$

$$E_{\text{solar}} = H\,A_{\text{collector}}$$

$$E_{\text{elect}} = \eta_{\text{cell}}\,E_{\text{solar}}$$

Calculations

The graph below, generated by calculating the total irradiance at hourly intervals, shows the total irradiance throughout the day.



Integrating under this curve using the trapezoidal rule, we obtain the insolation

$$H = 25.02\ \text{MJ/m}^2$$

The total solar energy incident on the collector during this day is the insolation multiplied by the surface area of the collector,

$$E_{\text{solar}} = H\,A_{\text{collector}}$$

$$= (25.02\ \text{MJ/m}^2)(20\ \text{m}^2)$$

$$= 500.4\ \text{MJ}$$

The total electrical energy generated by the solar panel during the day is

$$E_{\text{elect}} = \eta_{\text{cell}}\,E_{\text{solar}}$$

$$= (0.16)(500.4\ \text{MJ})$$

$$= 80.06\ \text{MJ} \times \frac{1\ \text{kWh}}{3.6\ \text{MJ}}$$

$$= \underline{\underline{22.24\ \text{kWh}}}$$

Solution check

No errors are detected.

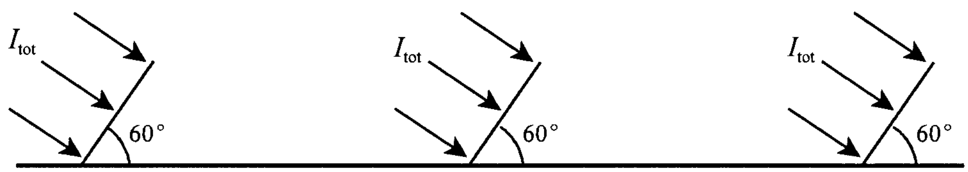
Discussion

The typical daily electrical energy consumption by a home in the US is about 30 kWh. Hence, this photovoltaic system supplies over two-thirds of this amount.

2.12 Problem statement

A classroom building at a university in Raleigh, North Carolina has an array of photovoltaic solar panels on its roof for augmenting electrical power from the grid. The panels face due south and have a tilt angle and efficiency of 60° and 0.22, respectively. How much electrical energy do the panels generate on December 21 for a total panel surface area of 600 m^2?

Diagram



Assumptions

1. Roof is gravel.
2. Sky is clear.
3. No panel blocks incident solar radiation on any other panel.
4. All quantities pertinent to the total solar irradiance are constant.

Governing equations

The governing equations are the same as those in Problem 2.4 plus the following:

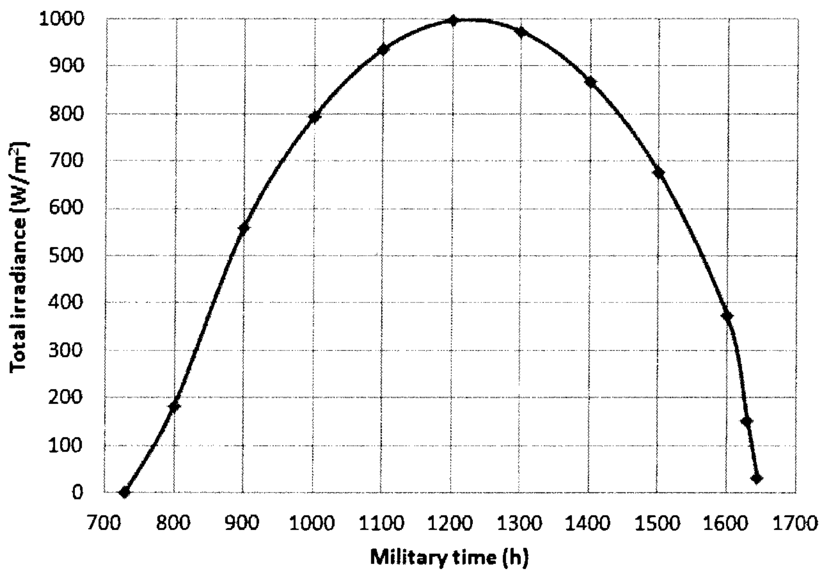
$$H = \int_{t_1}^{t_2} I_{tot}\, dt$$

$$E_{\text{solar}} = H\, A_{\text{collector}}$$

$$E_{\text{elect}} = \eta_{\text{cell}}\, E_{\text{solar}}$$

Calculations

The graph below, generated by calculating the total irradiance at hourly intervals, shows the total irradiance throughout the day.



Integrating under this curve using the trapezoidal rule, we obtain the insolation

$$H = 22.4 \text{ MJ/m}^2$$

The total solar energy incident on the panels during this day is the insolation multiplied by the total surface area of the panels,

$$E_{\text{solar}} = H\, A_{\text{collector}}$$

$$= (22.4 \text{ MJ/m}^2)(600 \text{ m}^2)$$

$$= 1.344 \times 10^4 \text{ MJ}$$

The total electrical energy generated by the solar panels during the day is

$$E_{\text{elect}} = \eta_{\text{cell}}\, E_{\text{solar}}$$

$$= (0.22)(1.344 \times 10^4 \text{ MJ})$$

$$= 2957 \text{ MJ} \times \frac{1 \text{ kWh}}{3.6 \text{ MJ}}$$

$$= \underline{\underline{821 \text{ kWh}}}$$

Solution check

No errors are detected.

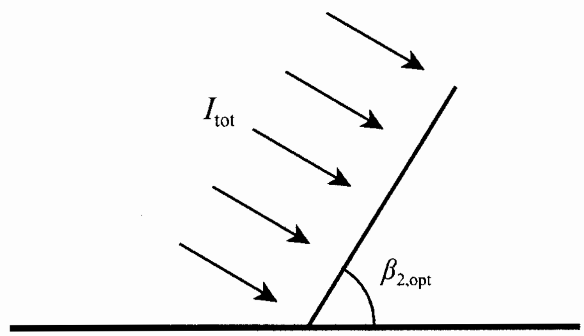
Discussion

Photovoltaic solar panels on roof tops of college and university buildings are becoming more common as these institutions incorporate green policies into their operations.

2.13 Problem statement

Derive Equation (2.18).

Diagram



Assumptions

1. Sky is clear.
2. All quantities pertinent to the total solar irradiance are constant.

Governing equations

$$I_{tot} = I_D + I_S + I_R$$

$$I_D = I_{DN}\left[\sin\beta_1 \cos\beta_s + \cos\beta_1 \sin\beta_2 \cos(\alpha_1 - \alpha_2)\right]$$

$$I_S = C\, I_{DN}\left[\frac{1+\cos\beta_2}{2}\right]$$

$$I_R = I_{DN}\rho(C+\sin\beta_1)\left(\frac{1-\cos\beta_2}{2}\right)$$

Calculations

To find the optimum panel tilt angle, $\beta_{2,\mathrm{opt}}$, we perform a maximization of the total solar irradiance, I_{tot}. Substituting the last three equations into the first equation and taking the derivative of I_{tot} with respect to β_2, and setting the result to zero, we obtain

$$\frac{dI_{tot}}{d\beta_2} = \cos\beta_1 \cos\beta_2 \cos(\alpha_1 - \alpha_2) - \sin\beta_1 \sin\beta_2 - \frac{C}{2}\sin\beta_2 + \frac{\rho}{2}(C + \sin\beta_1)\sin\beta_2 = 0$$

where the term I_{DN} has divided out. After simplification, this relation reduces to

$$\tan\beta_2 = \frac{\cos\beta_1 \cos(\alpha_1 - \alpha_2)}{\frac{C}{2} + \sin\beta_1 - \frac{\rho}{2}(C + \sin\beta_1)}$$

Solving for β_2, we obtain

$$\beta_2 = \tan^{-1}\left[\frac{2\cos\beta_1 \cos(\alpha_1 - \alpha_2)}{C(1-\rho) + (2-\rho)\sin\beta_1}\right]$$

which is Equation (2.18).

Solution check

No errors are detected.

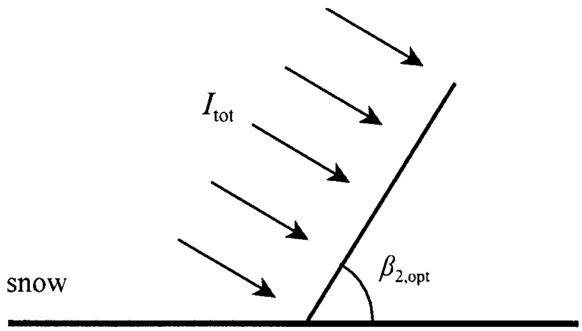
Discussion

It is important to note that this result applies only at an instant of time when the solar altitude angle, β_1, and solar azimuth angle, α_1, have unique values at a given time of day. The optimum panel tilt angle is also a function of the azimuth angle, α_2, the ratio of diffuse radiation on a horizontal surface to direct normal radiation, C, and foreground reflectivity, ρ.

2.14 Problem statement

Using Equation (2.18), calculate the optimum tilt angle for a solar panel in Salt Lake City, Utah at 1:00 pm on January 21. The panel faces a south-southwest direction, and the foreground is snow.

Diagram



Assumptions

1. Sky is clear.
2. All quantities pertinent to the total solar irradiance are constant.

Governing equations

The governing equations are those needed to find β_1 (see Problem 2.4) plus the following:

$$\beta_{2,opt} = \tan^{-1}\left[\frac{2\cos\beta_1\cos(\alpha_1-\alpha_2)}{C(1-\rho)+(2-\rho)\sin\beta_1}\right]$$

Calculations

The results are summarized below.

Constants:

$Long = 111.88°$
$L = 40.75°$
$z = 1.288$ km
$LSTM = 105°$ (mountain time zone)
January 21: $N = 21$, $C = 0.058$

$\rho = 0.80$ (snow foreground)
$\alpha_2 = +22.50°$ (panel faces south-southwest)

Calculated quantities:

$\delta = -20.14°$
$D = -59.18°$
$ET = -11.26$ min
$LST = 780$ min
$AST = 741.2$ min
$H = 5.306°$
$\theta = 14.66°$

$\beta_1 = 28.91°$
$\alpha_1 = 5.692°$

Using Equation (2.18), the optimum panel tilt angle is

$$\beta_{2,opt} = \tan^{-1}\left[\frac{2\cos\beta_1\cos(\alpha_1 - \alpha_2)}{C(1-\rho) + (2-\rho)\sin\beta_1}\right]$$

$$= \tan^{-1}\left[\frac{2\cos(28.91°)\cos(5.692° - 22.50°)}{0.058(1 - 0.80) + (2 - 0.80)\sin(28.91°)}\right]$$

$$= \underline{\underline{70.6°}}$$

Solution check

No errors are detected.

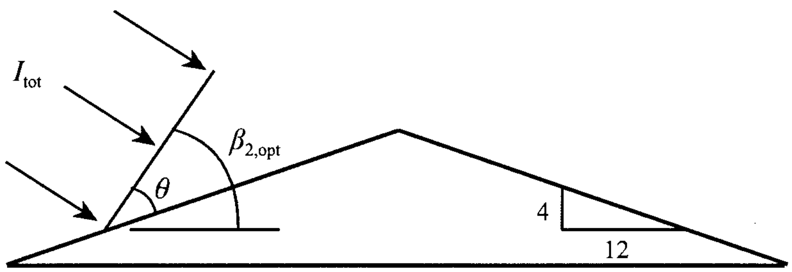
Discussion

The answer seems reasonable because the sun is low in the sky in January. This tilt angle applies only to 1:00 pm on January 21. The optimum tilt angle would vary throughout the day as the sun moves across the sky.

2.15 Problem statement

A photovoltaic solar panel is to be installed on the roof of a home in Phoenix, Arizona. The roof has a 4 on 12 pitch, and one side of the roof faces south-southeast. For November 21, find the optimum angle of the solar panel with respect to the roof.

Diagram



Assumptions

1. Roof is gravel.
2. Sky is clear.
3. All quantities pertinent to the total solar irradiance are constant.

Governing equations

The governing equations are the same as those in Problem 2.4 plus the following:

$$H = \int_{t_1}^{t_2} I_{tot}\, dt$$

Calculations

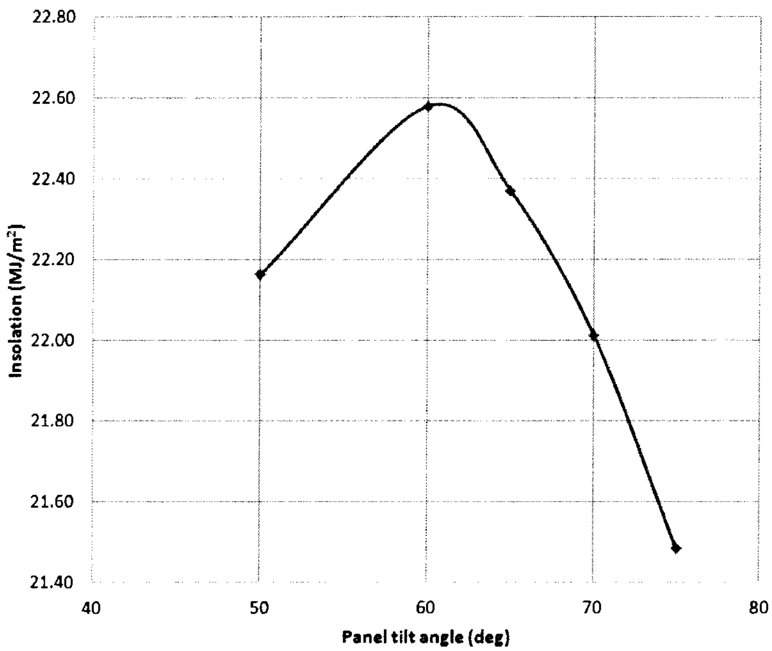
The results are summarized below.

Constants:

$Long = 112.1°$
$L = 33.45°$
$z = 0.331$ km
$LSTM = 105°$ (mountain time zone)
November 21: $N = 325$, $C = 0.063$
$\rho = 0.15$ (gravel foreground, assumed)

$\alpha_2 = -22.50°$ (panel faces south-southeast)

In order to find the optimum panel tilt angle for November 21, the total solar insolation as a function of tilt angle is graphed below.



As shown in the graph, the optimum tilt angle is approximately 61°, but this angle is measured with respect to the horizontal. The roof has a 4 on 12 pitch, which means that the angle of the roof with respect to the horizontal is $\tan^{-1}(4/12) = 18.4°$. Thus, the optimum angle of the solar panel with respect to the roof is

$$\theta = 61° - 18.4° = \underline{\underline{42.6°}}$$

Solution check

No errors are detected.

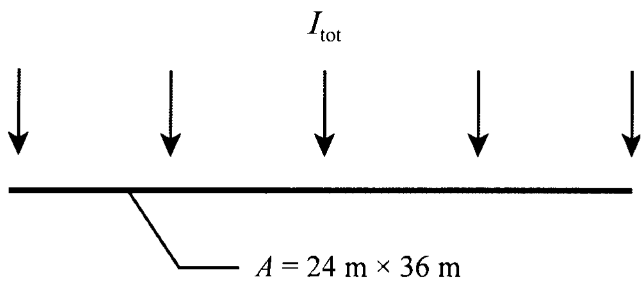
Discussion

As indicated in the graph, the insolation is a weak function of panel tilt angle. For example, a tilt angle of 75° yields an insolation of 21.5 MJ/m^2, about 5 percent lower than the insolation at a tilt angle of 61°.

2.16 Problem statement

An office complex in San Antonio, Texas has a roof-mounted photovoltaic solar panel that measures 24 m × 36 m. The panel is horizontal and has an efficiency of 0.22. How much electrical energy does the panel generate on July 21?

Diagram



Assumptions

1. Sky is clear.
2. All quantities pertinent to the total solar irradiance are constant.
3. Given time is local standard time.

Governing equations

The governing equations are the same as those in Problem 2.4 plus the following:

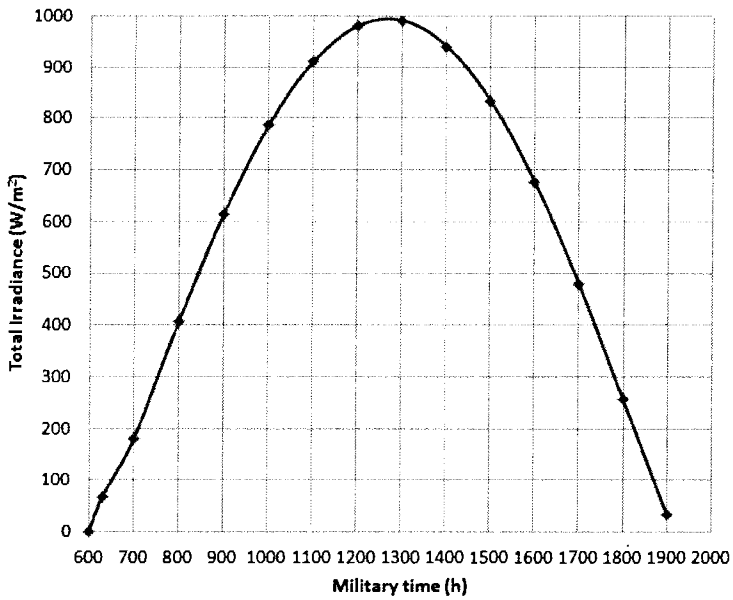
$$H = \int_{t_1}^{t_2} I_{tot}\,dt$$

$$E_{\text{solar}} = H\,A_{\text{collector}}$$

$$E_{\text{elect}} = \eta_{\text{cell}}\,E_{\text{solar}}$$

Calculations

The graph below, generated by calculating the total irradiance at hourly intervals, shows the total irradiance throughout the day.



Integrating under this curve using the trapezoidal rule, we obtain the insolation

$$H = 29.1 \text{ MJ/m}^2$$

The total solar energy incident on the panel during this day is the insolation multiplied by the total surface area of the panel,

$$E_{\text{solar}} = H\, A_{\text{collector}}$$

$$= (29.1 \text{ MJ/m}^2)(24 \text{ m} \times 36 \text{ m})$$

$$= 2.514 \times 10^4 \text{ MJ}$$

The total electrical energy generated by the solar panel during the day is

$$E_{\text{elect}} = \eta_{\text{cell}}\, E_{\text{solar}}$$

$$= (0.22)(\ 2.514 \times 10^4 \text{ MJ})$$

$$= 5531 \text{ MJ} \times \frac{1 \text{ kWh}}{3.6 \text{ MJ}}$$

$$= \underline{\underline{1536 \text{ kWh}}}$$

Solution check

No errors are detected.

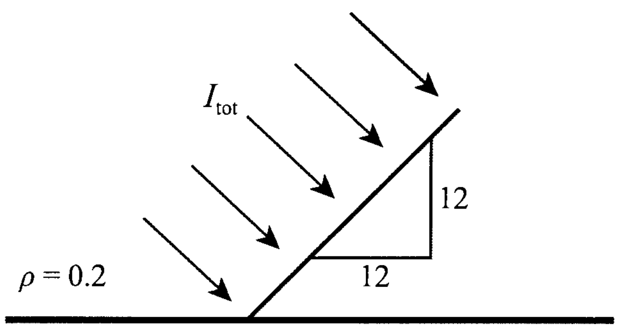
Discussion

It is becoming more common for office buildings to incorporate photovoltaic solar panels into their roof tops to augment electrical power from the grid.

2.17 Problem statement

A home owner in Sacramento, California wishes to install photovoltaic solar panels on his south-facing roof. The pitch and reflectivity of the roof are 12 on 12 and 0.2, respectively. If the efficiency of the panels is 0.18, how much electrical energy can the home owner expect to generate on December 31 if he mounts the panels flat on the roof? The total surface area of the panels is 70 m^2.

Diagram



Assumptions

1. Sky is clear.
2. All quantities pertinent to the total solar irradiance are constant.

Governing equations

The governing equations are the same as those in Problem 2.4 plus the following:

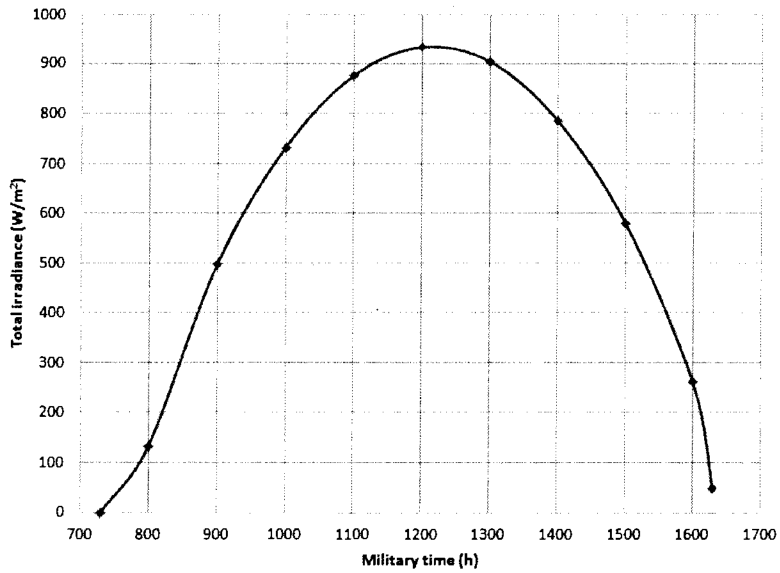
$$H = \int_{t_1}^{t_2} I_{tot} dt$$

$$E_{solar} = H A_{collector}$$

$$E_{elect} = \eta_{cell} E_{solar}$$

Calculations

The graph below, generated by calculating the total irradiance at hourly intervals, shows the total irradiance throughout the day.



Integrating under this curve using the trapezoidal rule, we obtain the insolation

$$H = 20.1\ \text{MJ/m}^2$$

The total solar energy incident on the panels during this day is the insolation multiplied by the total surface area of the panels,

$$E_{\text{solar}} = H\,A_{\text{collector}}$$

$$= (20.1\ \text{MJ/m}^2)(70\ \text{m}^2)$$

$$= 1407\ \text{MJ}$$

The total electrical energy generated by the solar panel during the day is

$$E_{\text{elect}} = \eta_{\text{cell}}\,E_{\text{solar}}$$

$$= (0.18)(1407\ \text{MJ})$$

$$= 253.3\ \text{MJ} \times \frac{1\ \text{kWh}}{3.6\ \text{MJ}}$$

$$= \underline{\underline{70.4\ \text{kWh}}}$$

Solution check

No errors are detected.

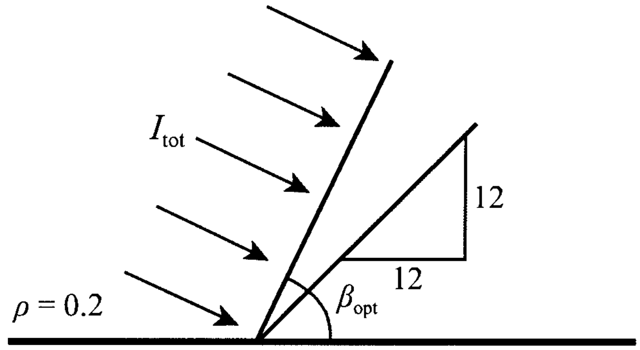
Discussion

A 12 on 12 roof pitch constitutes a 45° tilt angle for the solar panels. While this tilt angle is not optimum, the panels blend in with the roof line creating an aesthetically pleasing installation.

2.18 Problem statement

Work Problem 2.17 for the optimum panel tilt angle for December 31.

Diagram



Assumptions

1. Sky is clear.
2. All quantities pertinent to the total solar irradiance are constant.

Governing equations

The governing equations are the same as those in Problem 2.4 plus the following:

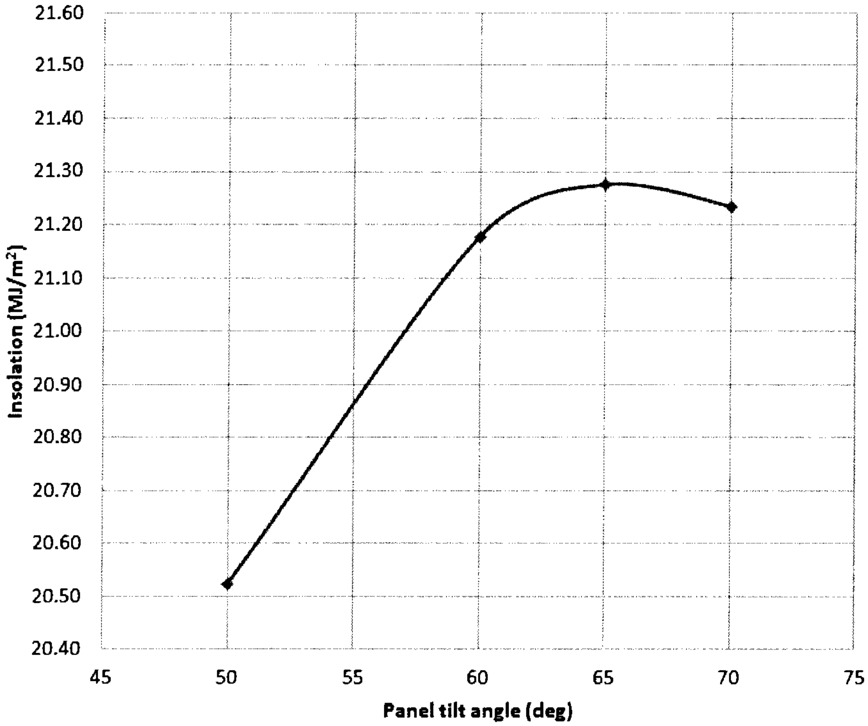
$$H = \int_{t_1}^{t_2} I_{tot} dt$$

$$E_{\text{solar}} = H A_{\text{collector}}$$

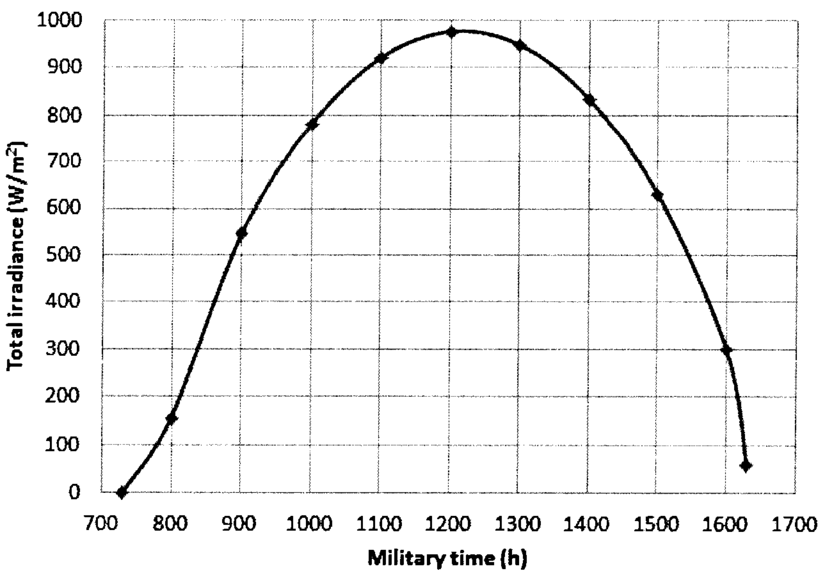
$$E_{\text{elect}} = \eta_{\text{cell}} E_{\text{solar}}$$

Calculations

We find the optimum panel tilt angle by graphing insolation as a function of tilt angle. As seen in the graph below, the optimum panel tilt angle is approximately 66°.



The graph below, generated by calculating the total irradiance at hourly intervals for the optimum tilt angle computed above, shows the total irradiance throughout the day.



Integrating under this curve using the trapezoidal rule, we obtain the insolation

$$H = 21.5 \text{ MJ/m}^2$$

The total solar energy incident on the panel during this day is the insolation multiplied by the total surface area of the panel,

$$E_{\text{solar}} = H\,A_{\text{collector}}$$

$$= (21.5 \text{ MJ/m}^2)(70 \text{ m}^2)$$

$$= 1505 \text{ MJ}$$

The total electrical energy generated by the solar panel during the day is

$$E_{\text{elect}} = \eta_{\text{cell}}\,E_{\text{solar}}$$

$$= (0.18)(1505 \text{ MJ})$$

$$= 270.9 \text{ MJ} \times \frac{1 \text{ kWh}}{3.6 \text{ MJ}}$$

$$= \underline{\underline{75.3 \text{ kWh}}}$$

Solution check

No errors are detected.

Discussion

As shown in the first graph, solar insolation is a fairly weak function of panel tilt angle, but a maximum is clearly indicated.

2.19 Problem statement

After cost incentives totaling \$6500 by state and local governments, the initial cost of a photovoltaic system for a residence is \$11,000. If the annual energy production of the solar panels is 5750 kWh/y, and the energy cost is \$0.10/kWh, what is the simple payback?

Diagram

(Not applicable)

Assumptions

1. All quantities are constant.

Governing equations

$$SP = \frac{IC}{(AEP)(EC)}$$

Calculations

The simple payback is

$$SP = \frac{IC}{(AEP)(EC)}$$

$$= \frac{\$11{,}000}{(5750\ \text{kWh/y})(\$0.10/\text{kWh})}$$

$$= \underline{\underline{19.1\ \text{y}}}$$

Solution check

No errors are detected.

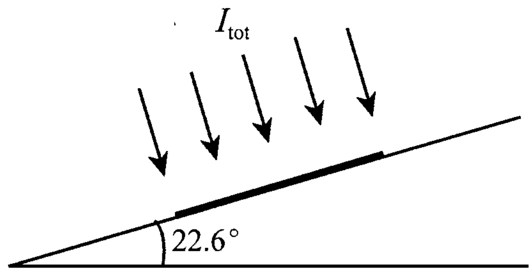
Discussion

The cost incentives of \$6500 were not used in the calculation because they are not part of the initial cost paid by the homeowner.

2.20 Problem statement

Design a photovoltaic system for a residence in Tempe, Arizona. The south-facing roof has a surface area of 45 m^2 and a 5 on 12 pitch. State all your assumptions.

Diagram



Assumptions

1. Solar panel is mounted flat on roof, giving a tilt angle of $\tan^{-1}(5/12) = 22.6°$.
2. Sky is clear.
3. All quantities pertinent to the total solar irradiance are constant.
4. Foreground reflectivity is 0.2.
5. Solar panel efficiency is 0.18.
6. Solar panel occupies 85 percent of south-facing roof.
7. Design day is March 21.

Governing equations

The governing equations are the same as those in Problem 2.4 plus the following:

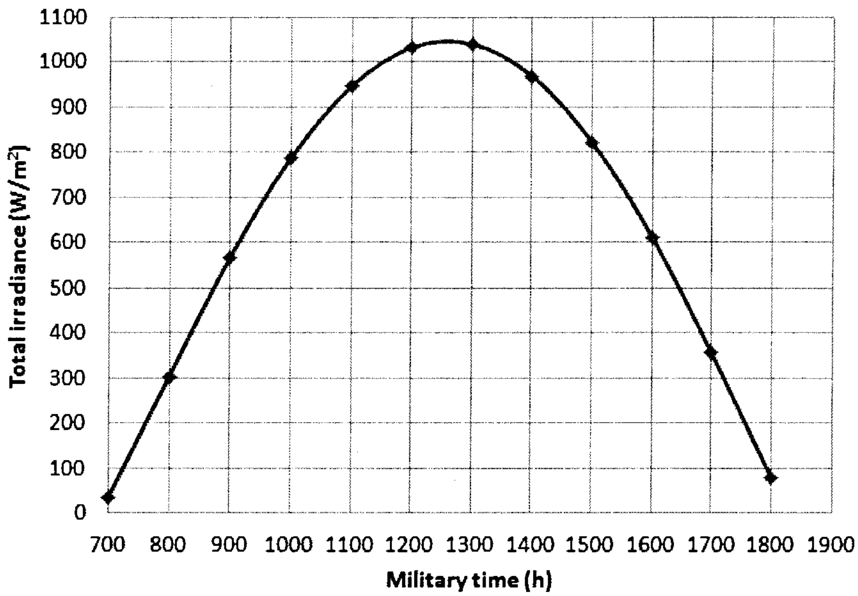
$$H = \int_{t_1}^{t_2} I_{tot} dt$$

$$E_{\text{solar}} = H A_{\text{collector}}$$

$$E_{\text{elect}} = \eta_{\text{cell}} E_{\text{solar}}$$

Calculations

Using the assumptions above and the governing equations, we calculate the solar irradiation at hourly intervals for March 21, as shown in the graph below.



From this graph, we obtain the solar insolation

$$H = 26.9 \text{ MJ/m}^2$$

The total solar energy incident on the panel during this day is the insolation multiplied by the total surface area of the panel,

$$E_{\text{solar}} = H\, A_{\text{collector}}$$

$$= (26.9 \text{ MJ/m}^2)(0.85 \times 45 \text{ m}^2)$$

$$= 1029 \text{ MJ}$$

The total electrical energy generated by the solar panel during the day is

$$E_{\text{elect}} = \eta_{\text{cell}}\, E_{\text{solar}}$$

$$= (0.18)(1029 \text{ MJ})$$

$$= 185.2 \text{ MJ} \times \frac{1 \text{ kWh}}{3.6 \text{ MJ}}$$

$$= \underline{\underline{51.4 \text{ kWh}}}$$

March is a "transition" month in terms of solar radiation, so this result may be taken as an approximate daily electrical energy generation for any given day during the year.

Solution check

No errors are detected.

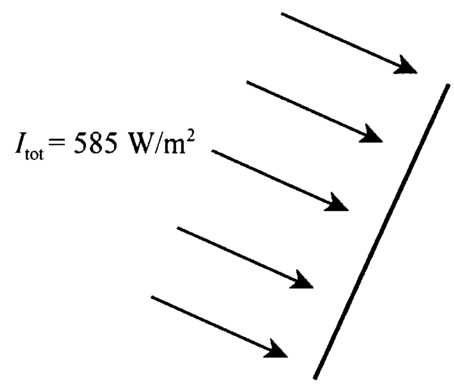
Discussion

To maintain a clean roof line, the solar panel was mounted flush with the roof. If the solar panel was mounted at a steep tilt angle, the insolation would be higher than the value calculated here.

2.21 Problem Statement

A rover for gathering geological data on Mars has a photovoltaic solar panel that supplies electrical power to its sensors, motors, and other electrical components. The panel faces directly into the sun, measures 2.6 m × 1.2 m, and has an efficiency of 0.18. If the solar constant for Mars is 585 W/m^2, what is the electrical power capacity of this system?

Diagram



Assumptions

1. Mars does not have an atmosphere, so I_{tot} = 545 W/m^2, the solar constant for Mars.

Governing equations

$$P_{solar} = I_{tot} A_{collector}$$

$$P_{elect} = \eta_{cell} P_{solar}$$

Calculations

The solar power incident on the panel is

$$P_{solar} = I_{tot} A_{collector}$$

$$= (545 \text{ W/m}^2)(2.6 \text{ m} \times 1.2 \text{ m})$$

$$= 1700 \text{ W}$$

The electrical power generation of the panel is

$$P_{\text{elect}} = \eta_{\text{cell}}\, P_{\text{solar}}$$

$$= (0.18)(1700\text{ W})$$

$$= \underline{\underline{306\text{ W}}}$$

Solution check

No errors are detected.

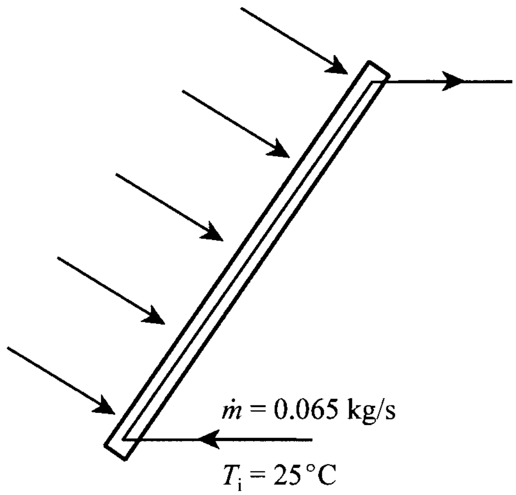
Discussion

Like Earth, Mars rotates on its axis. (One day on Mars is 24 hours, 39 minutes, 35 seconds in duration). Hence, the solar irradiation calculated here applies only to the moment when the rover's solar panel faces directly into the sun.

2.22 Problem statement

In a domestic water heating system, an ethylene-glycol/water mixture flows through the collector at a mass flow rate of 0.065 kg/s. The glazing surface measures 3 m × 5 m, and 550 W/m^2 of solar power is absorbed by the fluid mixture. If the inlet temperature of the fluid mixture is 25 °C, what is the outlet temperature?

Diagram



Assumptions

1. Fluid mixture absorbs 550 W/m^2 of solar radiation.
2. Flow rate of fluid mixture is steady.

Governing equations

$$\dot{Q} = \dot{m}c(T_o - T_i)$$

Calculations

The specific heat of ethylene glycol/water mixture is approximately c = 3320 J/kg·°C. Solving for the outlet temperature of the fluid, we obtain

$$T_o = \frac{\dot{Q}}{\dot{m}c} + T_i$$

$$= \frac{(550\ \mathrm{W/m^2})(3\ \mathrm{m} \times 5\ \mathrm{m})}{(0.065\ \mathrm{kg/s})(3320\ \mathrm{J/kg{\cdot}{}^\circ C})} + 25^\circ\mathrm{C}$$

$$= \underline{\underline{63.2^\circ\mathrm{C}}}$$

Solution check

No errors are detected.

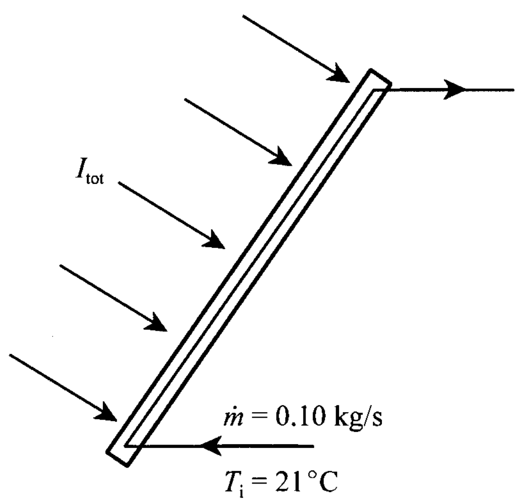
Discussion

The outlet temperature of the ethylene glycol/water mixture is 63.2°C (145.8°F), so the temperature of the water exiting the tank is limited by this temperature. Accounting for heat transfer effects, the water temperature at the tank exit is perhaps around 130°F, hot enough for domestic use, such as a dishwasher.

2.23 Problem statement

A domestic water heating system in Los Angeles, California, incorporates a roof-mounted flat plate solar collector with a tilt angle of 60°. The collector faces south-southeast, and the glazing surface measures 4.0 m × 6.2 m. the working fluid, which has a specific heat of c = 3150 J/kg·°C, enters the collector at a temperature of 21 °C and flows through the collector at a mass flow rate of 0.10 kg/s. If 50 percent of the total solar irradiance is absorbed by the working fluid, find the temperature of the working fluid at the outlet of the collector at 12:00 noon on April 21.

Diagram



Assumptions

1. Sky is clear.
2. All quantities pertinent to the total solar irradiance are constant.
3. Flow rate of working fluid is constant.
4. Foreground is a gravel roof.

Governing equations

The governing equations are the same as those in Problem 2.4 plus the following:

$$\dot{Q}_{solar} = I_{tot} A_{collector}$$

$$\dot{Q} = \dot{m}c(T_o - T_i)$$

Using the computational procedure illustrated in Problem 2.4, the total solar irradiation is

$$I_{tot} = 810 \text{ W/m}^2$$

Thus, the solar radiation incident on the collector is

$$\dot{Q}_{solar} = I_{tot} A_{collector}$$

$$= (810 \text{ W/m}^2)(4.0 \text{ m} \times 6.2 \text{ m})$$

$$= 2.01 \times 10^4 \text{ W}$$

Only 50 percent of this radiation is absorbed by the working fluid, so we have

$$T_o = \frac{\dot{Q}}{\dot{m}c} + T_i$$

$$= \frac{0.50 \times 2.01 \times 10^4 \text{ W}}{(0.10 \text{ kg/s})(3150 \text{ J/kg}\cdot{}^\circ\text{C})} + 21\,^\circ\text{C}$$

$$= \underline{\underline{52.9\,^\circ\text{C}}}$$

Solution check

No errors are detected.

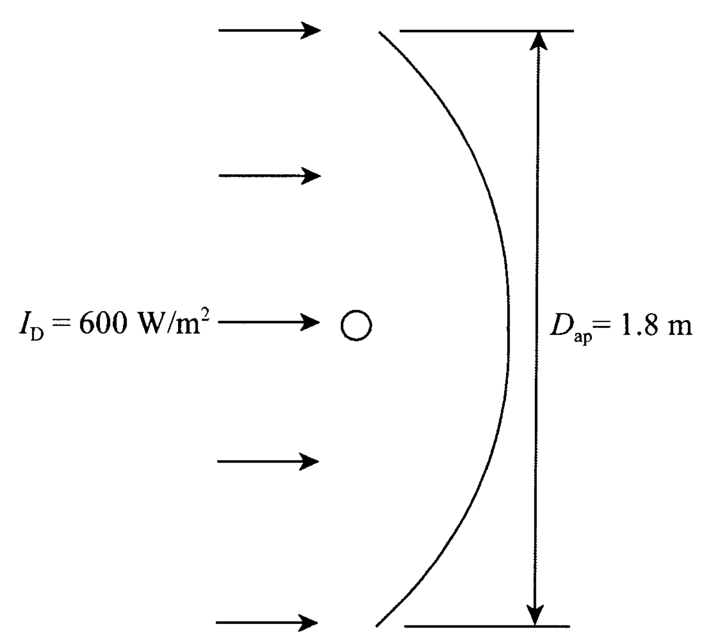
Discussion

This temperature represents an instantaneous value for the conditions given.

2.24 Problem statement

The direct solar irradiance on a dish concentrator is 600 W/m^2, and the dish aperture diameter is 1.80 m. The concentration ratio is 2500, the temperature of the working fluid in the receiver is 950°C, and the temperature of the surroundings is 25°C. Find the solar power incident on the concentrator, the heat flux at the receiver, and the Carnot efficiency.

Diagram



Assumptions

1. Steady operation.

Governing equations

$$\dot{Q}_{con} = I_D A_{ap}$$

$$\dot{Q}''_{rec} = I_D C$$

$$\eta_{th,ideal} = 1 - \frac{T_L}{T_H}$$

Solar power incident on the concentrator is

$$\dot{Q}_{con} = I_D A_{ap}$$

$$= (600 \text{ W/m}^2)\pi(1.8 \text{ m})^2/4$$

$$= \underline{\underline{1527 \text{ W}}}$$

The heat flux at the receiver is

$$\dot{Q}''_{rec} = I_D C$$

$$= (600 \text{ W/m}^2)(2500)$$

$$= 1.50 \times 10^6 \text{ W/m}^2 = \underline{\underline{1.50 \text{ MW/m}^2}}$$

The Carnot efficiency is

$$\eta_{th,ideal} = 1 - \frac{T_L}{T_H}$$

$$= 1 - \frac{(25 + 273) \text{ K}}{(950 + 273) \text{ K}}$$

$$= \underline{\underline{0.756}}$$

Solution check

No errors are detected.

Discussion

In order to calculate the maximum electrical power capacity of this system, we would have to know the efficiencies of the concentrator, receiver and generator and use the relation

$$P_{elect} = \eta_{overall} \dot{Q}_{con}$$

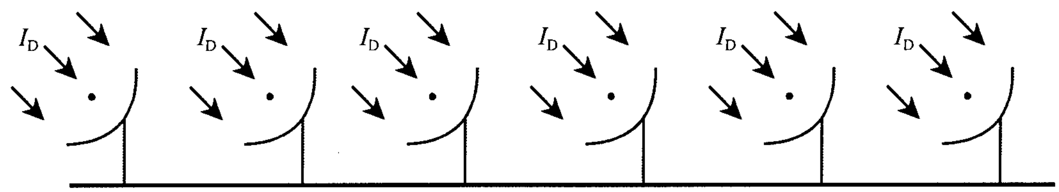
where

$$\eta_{overall} = \eta_{th,ideal} \eta_{con} \eta_{rec} \eta_{gen}$$

2.25 Problem statement

Consider a CSP plant with an array of 400 dish concentrators in Santa Fe, New Mexico. The concentrators, which have an aperture diameter of 2.75 m, are installed in a gravel field and have dual axis tracking. For 12:00 noon on January 21, find the total solar power incident on the concentrators and the heat flux at the receivers if the concentration ratio is 3200.

Diagram



Assumptions

1. Sky is clear.
2. All quantities pertinent to the direct solar irradiance are constant.

Governing equations

The governing equations are the same as those in Problem 2.4 plus the following:

$$\dot{Q}_{con} = I_D A_{ap}$$

$$\dot{Q}''_{rec} = I_D C$$

Calculations

Using the computational procedure illustrated in Problem 2.4, the direct solar irradiance is

$$I_D = 1010 \text{ W/m}^2$$

This value is based on a concentrator tilt angle of 56°, the angle that maximizes the direct solar irradiance. An azimuth angle of 0° was assumed. Thus, the solar radiation incident on one concentrator is

$$\dot{Q}_{con} = I_D A_{ap}$$

$= (1010 \text{ W/m}^2)\pi(2.75 \text{ m})^2/4$

$= 5999 \text{ W}$

so the total solar radiation incident on all concentrators is

$(400)(5999 \text{ W}) = 2.40 \times 10^6 \text{ W} = \underline{\underline{2.40 \text{ MW}}}$

The heat flux at the receivers is

$$\dot{Q}''_{rec} = I_D C$$

$= (1010 \text{ W/m}^2)(3200)$

$= 3.23 \times 10^6 \text{ W/m}^2 = \underline{\underline{3.23 \text{ MW/m}^2}}$

Solution check

No errors are detected.

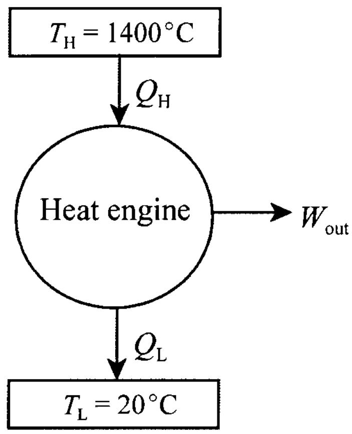
Discussion

Because the CSP system has dual axis tracking, the tilt and azimuth angles change throughout the day providing a maximum direct solar irradiance at all times.

2.26 Problem statement

What is the maximum possible thermal efficiency of a concentrating solar power plant if the temperature of the working fluid at the receiver is 1400°C and the ambient air temperature is 20°C?

Diagram



Assumptions

1. All parameters given in problem are constant.

Governing equations

$$\eta_{th,ideal} = 1 - \frac{T_L}{T_H}$$

Calculations

The maximum thermal efficiency of the concentrating solar power plant is

$$\eta_{th,ideal} = 1 - \frac{T_L}{T_H}$$

$$= 1 - \frac{(20 + 273)\text{ K}}{(1400 + 273)\text{ K}}$$

$$= \underline{\underline{0.825}}$$

Solution check

No errors are detected.

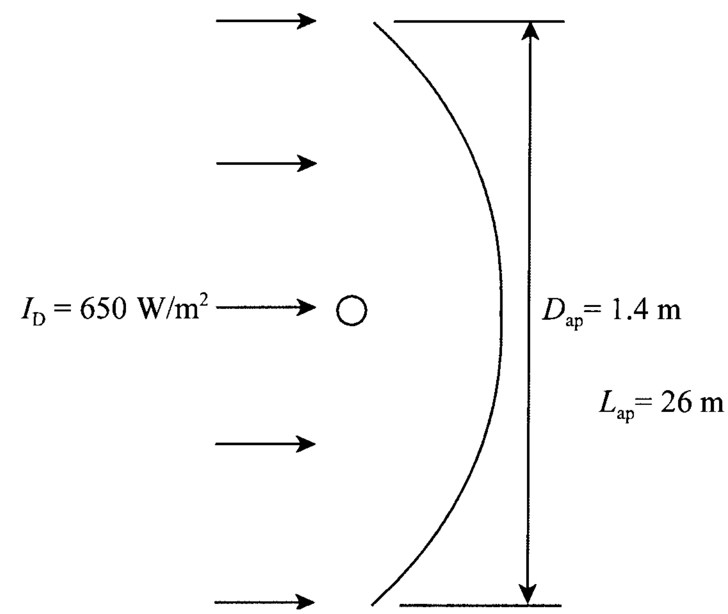
Discussion

As shown in the diagram, a concentrating solar power plant is basically a heat engine that converts solar energy to electrical energy. Thermal efficiency is one efficiency of four efficiencies, concentrator, receiver and generator efficiency, that apply to concentrating solar plants.

2.27 Problem statement

Consider a parabolic trough concentrator with an aperture width and length of 1.4 m and 26 m, respectively. During a given day, the average direct solar irradiance on the concentrator is 650 W/m^2. The efficiencies of the concentrator, receiver, and electrical generator are 0.91, 0.83, and 0.95, respectively. If the temperature of the working fluid in the receiver is 600°C, find the maximum possible electrical output power. The temperature of the surroundings is 10°C.

Diagram



Assumptions

1. All parameters given in problem are constant.

Governing equations

$$\dot{Q}_{con} = I_D A_{ap}$$

$$\eta_{th,ideal} = 1 - \frac{T_L}{T_H}$$

$$\eta_{overall} = \eta_{th,ideal}\,\eta_{con}\,\eta_{rec}\,\eta_{gen}$$

$$P_{elect} = \eta_{overall}\dot{Q}_{con}$$

Calculations

The solar power incident on the concentrator is

$$\dot{Q}_{con} = I_D A_{ap}$$

$$= (650\ \text{W/m}^2)(1.4\ \text{m})(26\ \text{m})$$

$$= 2.37 \times 10^4\ \text{W}$$

The Carnot efficiency is

$$\eta_{th,ideal} = 1 - \frac{T_L}{T_H}$$

$$= 1 - \frac{(10 + 273)\ \text{K}}{(600 + 273)\ \text{K}}$$

$$= 0.676$$

so the overall efficiency is

$$\eta_{overall} = \eta_{th,ideal}\eta_{con}\eta_{rec}\eta_{gen}$$

$$= (0.676)(0.91)(0.83)(0.95)$$

$$= 0.481$$

Because the overall efficiency is based on Carnot thermal efficiency, the maximum electrical output power is

$$P_{elect} = \eta_{overall}\dot{Q}_{con}$$

$$= (0.481)(2.37 \times 10^4\ \text{W})$$

$$= 1.14 \times 10^4\ \text{W} = \underline{\underline{11.4\ \text{kW}}}$$

Solution check

No errors are detected.

Discussion

The actual thermal efficiency might be approximately 0.40, giving an overall efficiency of 0.287 and an electrical output power of 6.8 kW.

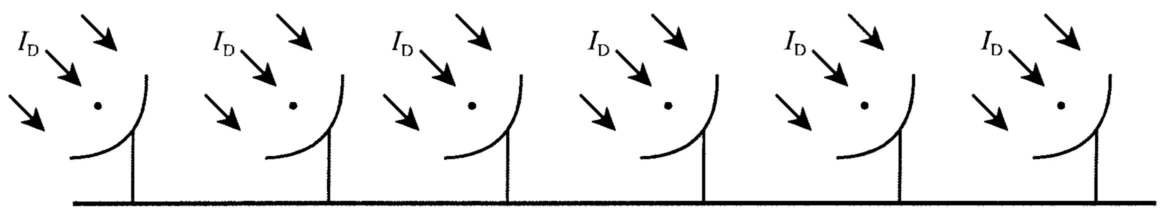
2.28 Problem statement

A concentrating solar power plant is being designed to augment electrical power from the main grid to a small residential community. The plant is to consist of an array of parabolic dish concentrators with an aperture diameter of 2.2 m. The efficiencies of the system are estimated to be:

$$\eta_{th} = 0.30,\ \ \eta_{con} = 0.90,\ \eta_{rec} = 0.82,\ \eta_{gen} = 0.95$$

As a starting point in the design, an engineer uses 800 W/m^2 as a direct solar irradiance averaged over a 12-hour period during which the sun shines on a typical day. The average electrical power consumption of a home in the United States is approximately 30 kWh per day. If the community has 1200 homes, how many dish concentrators are required to provide 25 percent of the power? Also, estimate the amount of land required for the concentrators.

Diagram



Assumptions

1. All parameters given in problem are constant.

Governing equations

$$\dot{Q}_{con} = I_D A_{ap}$$

$$\eta_{overall} = \eta_{th}\eta_{con}\eta_{rec}\eta_{gen}$$

$$P_{elect} = \eta_{overall}\dot{Q}_{con}$$

$$E_{elect} = P_{elect}\Delta t$$

Calculations

The solar power incident on one concentrator is

$$\dot{Q}_{con} = I_D A_{ap}$$

$$= (800 \text{ W/m}^2)\pi(2.2 \text{ m})^2/4$$

$$= 3041 \text{ W}$$

The overall efficiency is

$$\eta_{overall} = \eta_{th}\eta_{con}\eta_{rec}\eta_{gen}$$

$$= (0.30)(0.90)(0.82)(0.95)$$

$$= 0.210$$

The electrical output power of one dish is

$$P_{elect} = \eta_{overall}\dot{Q}_{con}$$

$$= (0.210)(3041 \text{ W})$$

$$= 638.6 \text{ W}$$

so the electrical output energy of one dish for a 12-hour period is

$$E_{elect} = P_{elect}\Delta t$$

$$= (638.6 \text{ W})(3600 \text{ s/h} \times 12 \text{ h})$$

$$= 2.759 \times 10^7 \text{ J} = 27.59 \text{ MJ}$$

$$= 27.59 \text{ MJ} \times \frac{1 \text{ kWh}}{3.6 \text{ MJ}}$$

$$= 7.66 \text{ kWh}$$

Hence, the number of dishes required to provide 25 percent of the electricity for 1200 homes is

$$N = \frac{0.25 \times 1200 \text{ home} \times 30 \text{ kWh/day·home}}{7.66 \text{ kWh/day}}$$

$$= \underline{\underline{1175}}$$

To estimate the land requirement for the dishes, let's assume that each dish requires a space that is three times the aperture area of the dish so that no dish blocks the solar irradiance of adjacent dishes and to make room for maintenance. So, the total land area required is

$$A_{\text{land}} = 1175 \times 3 \times \pi(2.2\text{ m})^2/4$$

$$= 1.340 \times 10^4\text{ m}^2 \times \frac{1\text{ acre}}{4047\text{ m}^2}$$

$$= \underline{\underline{3.31\text{ acre}}}$$

Solution check

No errors are detected.

Discussion

If the concentrating solar power plant provided all the electrical power to the community, 4700 dishes would be needed, taking up a land area of 13.2 acre.

2.29 Problem statement

A concentrating solar power plant with a generation-capacity of 1.75 MW costs \$2.50 million to install. The fixed charge rate is 7.5 percent, the annual operation and maintenance cost is assumed to be 1.0 percent of the initial cost, and the plant capacity factor is 0.20. If the levelized replacement cost is averaged over an expected 25-year lifetime, what is the cost of energy?

Diagram

(Not applicable)

Assumptions

1. All quantities are constant.

Governing equations

$$COE = \frac{(IC)(FCR) + LRC + AOM + AFC}{AEP}$$

Calculations

The concentrating solar power plant has a power capacity of 1.75 MW, so the amount of energy that the plant can generate in one year is

$$E_{\max} = (1.75 \text{ MW})(8760 \text{ h/y})$$

$$= 1.533 \times 10^4 \text{ MWh/y}$$

The capacity factor is 0.20, so the actual annual energy production is

$$AEP = (0.20)(1.533 \times 10^4 \text{ MWh/y})$$

$$= 3066 \text{ MWh/y}$$

For a concentrating solar power plant, the annual fuel cost, AFC, is zero. The LRC and AOM are

$$LRC = (\$2.50 \times 10^6)/(25 \text{ y})$$

$$= \$1.00 \times 10^5/\text{y}$$

$$AOM = (0.01/\text{y})(\$2.50 \times 10^6)$$

$$= \$2.50 \times 10^4/\text{y}$$

Hence, the cost of energy is

$$COE = \frac{(IC)(FCR) + LRC + AOM + AFC}{AEP}$$

$$= \frac{(\$2.50 \times 10^6)(0.075/\text{y}) + \$1.00 \times 10^5/\text{y} + \$2.50 \times 10^4/\text{y}}{3066 \text{ Mwh/y}}$$

$$= \$101.92/\text{MWh}$$

which, when converted to units of \$/kWh is approximately

$$COE = \underline{\underline{\$0.102/\text{kWh}}}$$

Solution check

No errors are detected.

Discussion

The average cost of electricity in the United States is approximately \$0.12/kWh, so this power plant is competitive with respect to this value.