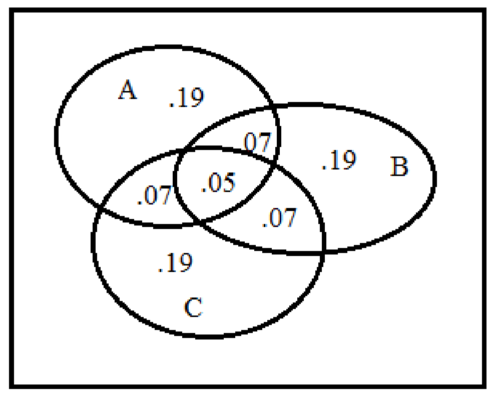
Chapter 2. Probability

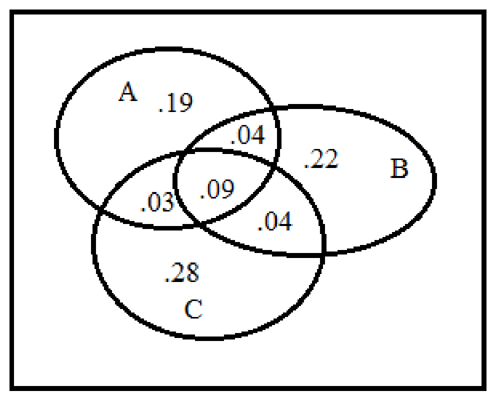
2.1 a. There are 330 blues songs out of 27,333. 330/27,333 = 0.0121 **b.** With 330 blues, 537 jazz, and 8286 rock, we have (330 + 537 + 8286)/27,333 = 9153/27,333 = 0.3349. **c.** No, these types of music are mutually exclusive. **d.** $P(A^C)$ = 1 – 9153/27,333 = 18,180/27,333 = 0.6651

2.2 There are 14 possible routes, 11 of which are not extreme. *P*(not extreme) = 11/14 = 0.7857.

2.3 $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$ $-P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$, so, $P(A \cup B \cup C) =$ 3(0.38) – 3(0.12) + 0.05 = 0.83. You could also use a Venn diagram, filling from the inside out and subtracting what has already been taken into account. Finally, add all the disjoint pieces.

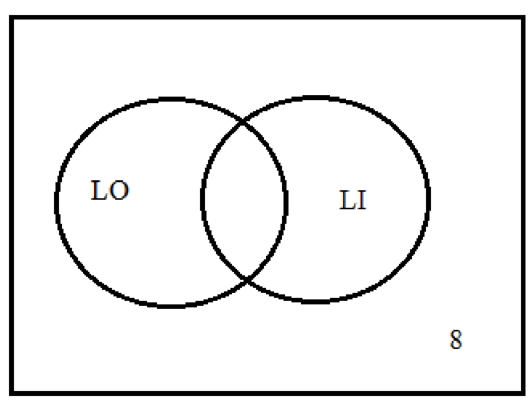


2.4 $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$ $-P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$, so, 0.89 = $P(A)$ + 0.39 + 0.44 – 0.13 – 0.12 – 0.13 + 0.09 = $P(A)$ + 0.54. $P(A)$ = 0.89 – 0.54 = 0.35. Using the Venn diagram, $P(A)$ = 0.19 + 0.04 + 0.09 + 0.03 = 0.35.



2.5 a. The woman has 30 pairs of shoes. 13/30 = 0.4333 = 43.33% will make her taller. **b.** 1 – 0.4333 = 0.5667 = 56.67% will not make her taller. **c.** Answers will vary. Shoes she could wear to the beach (flip flops and sneakers), and ones she could not?

2.6 Because eight children do not like either, 22 children like lollypops, licorice, or both. 22 = LO+ LI + (LO $\cap$ LI) = 19 + 10 – (LO and LI). 22 = 29 – P(LO $\cap$ LI), so 7/30 like both.



2.7 a. Answers will vary. **b.** Answers will vary. One possibility is $S = \{x \mid x \geq 0\}$. **c.** Answers will vary. One possibility might be drought = [0,12], normal = (12,30], flooding = $[30,\infty)$.

2.8 Because each flip has a 0.5 probability of being either a head or a tail, P (HHHHT) = $(0.5)^5$ = 0.03125.

2.9 a. There were a total of 12 pizzas. Five had bacon (the two bacon and 3 meat lovers), so 5/12 = 0.4167. **b.** Nine had pepperoni (3 pepperoni, 3 sausage pepperoni, and 3 meat lovers), so 9/12 = 3/4 = 0.75. **c.** Six had sausage (3 sausage pepperoni and 3 meat lovers), so 6/12 = 0.5.

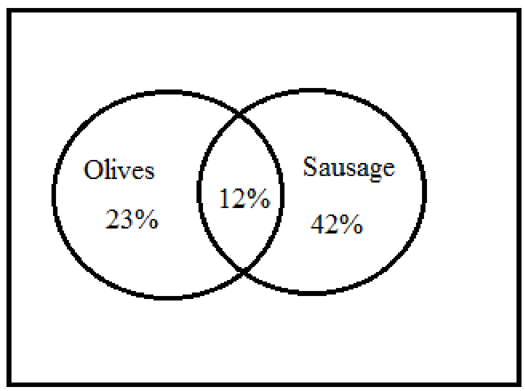
2.10 math + physics – (math and physics) = 100 = 60 + 75 – (math and physics) = 135 – (math and physics), so the probability of a double major is 0.35.

2.11 Each letter has a 1/101 chance to be the correct one, so $(1/101)^7 = 0.0001$.

2.12 a. 2/6 = 1/3 = 0.3333 **b.** $2/6 \times 1/5$ = 2/30 = 1/15 = 0.0667

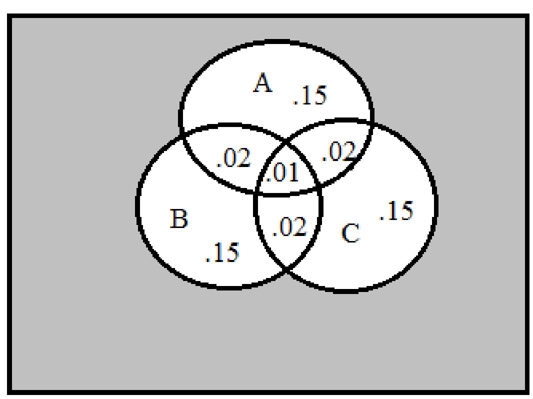
2.13 a. 20/2000 = 0.01 **b.** 200/2000 = 0.1 **c.** 220/2000 = 0.11 **d.** 1 – 0.11 = 0.89

2.14 Note in the Venn diagram that the 35% who like olives is made up of those who like olives but not sausage and the 12% who like both. P (neither sausage nor olives) = 1 – 0.77 = 0.23, or 23%.



2.15 a. $A \cup B \cup C = \{1,3,4,5,6\}$. $P(A \cup B \cup C) = 4/6 = 2/3$. Using Theorem 2.23, we would have 3/6 + 2/6 + 2/6 – 2/6 – 1/6 – 1/6 + 1/6 = 7/6 – 4/6 + 1/6 = 4/6 = 2/3. **b.** $A \cap B \cap C = \{6\}$. The probability is 1/6.

2.16 DeMorgan's first law says $(\cup A_i)^c = \cap A_i^c$. This is the area shaded on the Venn diagram. That area is 1 – 0.2 – 0.15 – 0.02 – 0.15 = 0.48.



2.17 P (P) = 1/3. P ({G, P), (Y, P)}) = $1/3\times1/2+1/3\times1/2$ = 1/3. $P((G,P),(G,Y,P)) =$ $1/3\times1/2+1/3\times1/2\times1=1/3$. P ({(Y, G, P), (G, Y, P)}) = 1/3 and P ((P), (Y, P)}) = 1/2.

2.18 With three dice, there are $6\times6\times6\times6\times6=6^5$ = 7776 possible combinations. Six of these (1, 1, 1, 1, 1) through (6, 6, 6, 6, 6) will have the same number. The desired probability is 6/7776 = 0.00077.

2.19 There are 2 possibilities for the first number, 10 for the second, and 46 for the third. $2\times10\times46=920$ possible combinations.

2.20 89 = M + S + A – (M and S) – (M and A) – (S and A) + (M and S and A).
89 = M + 39 + 44 – 13 – 12 – 13 + 9 = M + 54, so 89 – 54 = 35 students are majoring in at least math.

2.21 **a.** Because each person could be either male or female, 2^7 = 128. **b.** $\binom{7}{2}$ = 21. **c.** $|A_0|=1$, $|A_1|=7$, $|A_2|=21$, $|A_3|=35$, $|A_4|=35$, $|A_5|=21$, $|A_6|=7$, $|A_7|=1$, or $\binom{7}{j}$. **d.** Because each person is equally likely to be male or female, $P(A_j)=\binom{7}{j}(0.5)^7$.

2.22 $P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$, so 0.48 = 0.17 + 0.37 + 0.19 – 0.07 – $P(A\cap C)$ – 0.11 + 0.03. 0.48 = 0.58 – $P(A\cap C)$, giving $P(A\cap C)$ = 0.10.

2.23
$P(A\cup B\cup C)^c=1-(P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C))$.
$P(A\cup B\cup C)^c$ = 1 – (0.20 + 0.10 + 0.40 – 0.05 – 0.10 – 0.03 + 0.1) = 1 – 0.53 = 0.47.

2.24 Proof:
$P(A\cup B\cup C)=P(A\cup(B\cup C))=P(A)+(P(B)+P(C)-P(B\cap C))-P(A\cap(B\cup C))=$
$P(A)+P(B)+P(C)-P(B\cap C)-P((A\cap B)\cup(A\cap C))=P(A)+P(B)+P(C)-P(B\cap C)-$
$P(A\cap B)-P(A\cap C))+P(A\cap B\cap C)$.

2.25 Proof:
$P(A\cup B\cup C\cup D)=P((A\cup B\cup C)\cup D))=P(A\cup B\cup C)+P(D)-P((A\cup B\cup C)\cap D)=$
$P(A)+P(B)+P(C)+P(D)-P(B\cap C)-P((A\cap B)\cup(A\cap C))=P(A)+P(B)+P(C)-$
$P(B\cap C)-P(A\cap B-P(A\cap C))+P(A\cap B\, capC)-P((A\cup B\cup C)\cap D)=$
$P(A)+P(B)+P(C)+P(D)-P(B\cap C)-P((A\cap B)\cup(A\cap C))=P(A)+P(B)+P(C)-$
$P(B\cap C)-P(A\cap B)-P(A\cap C))+P(A\cap B\cap C)-P((A\cap D)\cup(B\cap D)\cup(C\cap D)=$

From here, continue as in the solution to Exercise 2.24 above.

2.26 This is an extension of the previous two exercises. You have proved this is true for i = 3 and 4 events; use mathematical induction.

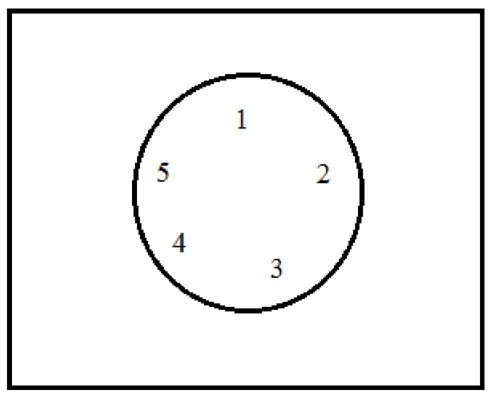
2.27 a. Yes. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, and $P(A \cap B) \geq 0$. **b.** Yes. $P(A \cup B \cup C) = P(A) + (P(B) + P(C) - P(B \cap C)) - P(A \cap B) - P(B \cap C)) + P(A \cap B \cap C)$. If the events are mutually disjoint, $P(B \cap C)) = P(A \cap B) = P(B \cap C)) = P(A \cap B \cap C) = 0$. If they are not disjoint, $P(A \cap B \cap C) \geq 0$ and $P(A \cap B \cap C) \leq P(A \cap B)$, $P(A \cap B \cap C) \leq P(A \cap C)$, and $P(A \cap B \cap C) \leq P(B \cap C)$.

2.28 Because each pen works 25% of the time (i.e., P (pen works) = 0.25 for all pens), P (one that works within the first four) = $0.25 + 0.75(0.25) + 0.75^2(0.25) + 0.75^3(0.25)$ = 0.6836.

2.29 For the sum of the first two to equal the third, you must have (1,1,2), (1, 2,3), (2,1,3), (1,3,4), (3,1,4), (2,2,4), ..., (1,5,6), (5,1,6), (2,4,6), (4,2,6), or (3,3,6). There are 15 possibilities, so the probability is 15/216.

2.30 a. (9/20)(8/19) = 0.1895 **b.** This is the complement of both chocolate chip, so 1 – (9/20)(8/19) = 1 – 0.1895 = 0.8105 **c.** Eating all the rest and leaving two chocolate chip is the same as eating two chocolate chip and leaving the rest. The probability is 0.1895.

2.31 a. Fix Alice at position 1 (because rotations do not count). There are four people who can sit in position 2. For no couple to sit together, we must have one of the other couple in position 3; there are two ways to do this. Position 4 must be the mate of the person in position 2 and the person in position 5 must be the mate of the person in position 3. This means the probability no couple sits together is $P(A_0)$ = 8/24 = 1/3. **b.** With Alice at position 1, there are four ways to seat a person in position 2. The person in position 3 must not be that person's mate (or else we'd have two couples sitting together), so one married couple can sit in positions 3 and 4 (there are two ways to seat them there), and the mate of the person in position 2 will set at 5. This means the probability exactly one couple sits together is $P(A_1)$ = 8/24 = 1/3. **c.** $P(A_2)$ = 8/24 = 1/3. The probabilities must total 1. (See the solution to Exercise 1.14 for more explanation.)



2.32 There are 22 total socks in the drawer. P (matching pair) = P (both white) + P (both black) + P (both red) + P (both purple) = (10/22)(9/21) + (6/22)(5/21) + (4/22)(3/21) + (2/22)(1/21) = 0.2900.

2.33 $P(B_1) = 1/216$ because this must be (1,1,1). $P(B_2) = 8/216$ because this must be (1,1,1), (2,2,2), (1,2,2), (2,1,2), (2,2,1), (1,1,2), (1,2,1), or (2,1,1). For the maximum equal or less than 3, we have the eight ways already found plus (3,3,3), (1,1,3), (1,3,1), (3,1,1), (1,3,3), (3,1,3), (3,3,1), (3,2,2), (2,3,2), (2,2,3), (3,3,2), (3,2,3), (2,3,3). (1,2,3), (1,3,2), (2,1,3), (3,2,1), (3,1,2), or (2,3,1). This makes $P(B_1) = 27/216.$ In general, $P(B_k) = (k/6)^3$.

2.34 This is a generalization of Exercise 2.27. It is true. Equality only holds if all the A_k are disjoint.

2.35 Let A_g be the event that person g draws his/her number. The probability there is no winner is the intersection N of the complement of this for all g. By the inclusion-exclusion principle, $1 - P(N) = \sum_{k\geq 1} (-1)^{k-1}(1/k!)$. So, $P(N) = \sum_{k\geq 0} (-1)^{k-1}(1/k!)$. (b) As $n \to \infty$, $P(N) \to e^{-1}$.