Chapter 2 - An Introduction to Linear Programming

True / False

1. Increasing the right-hand side of a nonbinding constraint will not cause a change in the optimal solution.
   a. True
   b. False

ANSWER: False
POINTS: 1
TOPICS: Introduction

2. In a linear programming problem, the objective function and the constraints must be linear functions of the decision variables.
   a. True
   b. False

ANSWER: True
POINTS: 1
TOPICS: Mathematical statement of the RMC Problem

3. In a feasible problem, an equal-to constraint cannot be nonbinding.
   a. True
   b. False

ANSWER: True
POINTS: 1
TOPICS: Graphical solution

4. Only binding constraints form the shape (boundaries) of the feasible region.
   a. True
   b. False

ANSWER: False
POINTS: 1
TOPICS: Graphical solution

5. The constraint $5x_1 - 2x_2 \leq 0$ passes through the point (20, 50).
   a. True
   b. False

ANSWER: True
POINTS: 1
TOPICS: Graphing lines

6. A redundant constraint is a binding constraint.
   a. True
   b. False

ANSWER: False
POINTS: 1
TOPICS: Slack variables

7. Because surplus variables represent the amount by which the solution exceeds a minimum target, they are given positive coefficients in the objective function.

a. True

b. False

ANSWER: False

POINTS: 1

TOPICS: Slack variables

8. Alternative optimal solutions occur when there is no feasible solution to the problem.

a. True

b. False

ANSWER: False

POINTS: 1

TOPICS: Alternative optimal solutions

9. A range of optimality is applicable only if the other coefficient remains at its original value.

a. True

b. False

ANSWER: True

POINTS: 1

TOPICS: Simultaneous changes

10. Because the dual price represents the improvement in the value of the optimal solution per unit increase in right-hand-side, a dual price cannot be negative.

a. True

b. False

ANSWER: False

POINTS: 1

TOPICS: Right-hand sides

11. Decision variables limit the degree to which the objective in a linear programming problem is satisfied.

a. True

b. False

ANSWER: False

POINTS: 1

TOPICS: Introduction

12. No matter what value it has, each objective function line is parallel to every other objective function line in a problem.

a. True

b. False

ANSWER: True

POINTS: 1

TOPICS: Graphical solution

13. The point (3, 2) is feasible for the constraint $2x_1 + 6x_2 \leq 30$.

a. True
b. False

ANSWER: True
POINTS: 1
TOPICS: Graphical solution

14. The constraint $2x_1 - x_2 = 0$ passes through the point (200,100).
a. True
b. False

ANSWER: False
POINTS: 1
TOPICS: A note on graphing lines

15. The standard form of a linear programming problem will have the same solution as the original problem.
a. True
b. False

ANSWER: True
POINTS: 1
TOPICS: Surplus variables

16. An optimal solution to a linear programming problem can be found at an extreme point of the feasible region for the problem.
a. True
b. False

ANSWER: True
POINTS: 1
TOPICS: Extreme points

17. An unbounded feasible region might not result in an unbounded solution for a minimization or maximization problem.
a. True
b. False

ANSWER: True
POINTS: 1
TOPICS: Special cases: unbounded

18. An infeasible problem is one in which the objective function can be increased to infinity.
a. True
b. False

ANSWER: False
POINTS: 1
TOPICS: Special cases: infeasibility

19. A linear programming problem can be both unbounded and infeasible.
a. True
b. False

ANSWER: False
POINTS: 1
TOPICS: Special cases: infeasibility and unbounded

20. It is possible to have exactly two optimal solutions to a linear programming problem.
    a. True
    b. False
ANSWER: False
POINTS: 1
TOPICS: Special cases: alternative optimal solutions

Multiple Choice

21. The maximization or minimization of a quantity is the
    a. goal of management science.
    b. decision for decision analysis.
    c. constraint of operations research.
    d. objective of linear programming.
ANSWER: d
POINTS: 1
TOPICS: Introduction

22. Decision variables
    a. tell how much or how many of something to produce, invest, purchase, hire, etc.
    b. represent the values of the constraints.
    c. measure the objective function.
    d. must exist for each constraint.
ANSWER: a
POINTS: 1
TOPICS: Objective function

23. Which of the following is a valid objective function for a linear programming problem?
    a. Max 5xy
    b. Min $4x + 3y + (2/3)z$
    c. Max $5x^2 + 6y^2$
    d. Min $(x_1 + x_2)/x_3$
ANSWER: b
POINTS: 1
TOPICS: Objective function

24. Which of the following statements is NOT true?
    a. A feasible solution satisfies all constraints.
    b. An optimal solution satisfies all constraints.
    c. An infeasible solution violates all constraints.

d. A feasible solution point does not have to lie on the boundary of the feasible region.

ANSWER: c
POINTS: 1
TOPICS: Graphical solution

25. A solution that satisfies all the constraints of a linear programming problem except the nonnegativity constraints is called
a. optimal.
b. feasible.
c. infeasible.
d. semi-feasible.

ANSWER: c
POINTS: 1
TOPICS: Graphical solution

26. Slack
a. is the difference between the left and right sides of a constraint.
b. is the amount by which the left side of a $\leq$ constraint is smaller than the right side.
c. is the amount by which the left side of a $\geq$ constraint is larger than the right side.
d. exists for each variable in a linear programming problem.

ANSWER: b
POINTS: 1
TOPICS: Slack variables

27. To find the optimal solution to a linear programming problem using the graphical method
a. find the feasible point that is the farthest away from the origin.
b. find the feasible point that is at the highest location.
c. find the feasible point that is closest to the origin.
d. None of the alternatives is correct.

ANSWER: d
POINTS: 1
TOPICS: Extreme points

28. Which of the following special cases does not require reformulation of the problem in order to obtain a solution?
a. alternate optimality
b. infeasibility
c. unboundedness
d. each case requires a reformulation.

ANSWER: a
POINTS: 1
TOPICS: Special cases

29. The improvement in the value of the objective function per unit increase in a right-hand side is the
a. sensitivity value.
b. dual price.

c. constraint coefficient.
d. slack value.

ANSWER: b
POINTS: 1
TOPICS: Right-hand sides

30. As long as the slope of the objective function stays between the slopes of the binding constraints
a. the value of the objective function won't change.
b. there will be alternative optimal solutions.
c. the values of the dual variables won't change.
d. there will be no slack in the solution.

ANSWER: c
POINTS: 1
TOPICS: Objective function

31. Infeasibility means that the number of solutions to the linear programming models that satisfies all constraints is
a. at least 1.
b. 0.
c. an infinite number.
d. at least 2.

ANSWER: b
POINTS: 1
TOPICS: Alternate optimal solutions

32. A constraint that does not affect the feasible region is a
a. non-negativity constraint.
b. redundant constraint.
c. standard constraint.
d. slack constraint.

ANSWER: b
POINTS: 1
TOPICS: Feasible regions

33. Whenever all the constraints in a linear program are expressed as equalities, the linear program is said to be written in
a. standard form.
b. bounded form.
c. feasible form.
d. alternative form.

ANSWER: a
POINTS: 1
TOPICS: Slack variables

34. All of the following statements about a redundant constraint are correct EXCEPT
a. A redundant constraint does not affect the optimal solution.
b. A redundant constraint does not affect the feasible region.

c. Recognizing a redundant constraint is easy with the graphical solution method.

d. At the optimal solution, a redundant constraint will have zero slack.

ANSWER: d
POINTS: 1
TOPICS: Slack variables

35. All linear programming problems have all of the following properties EXCEPT

a. a linear objective function that is to be maximized or minimized.

b. a set of linear constraints.

c. alternative optimal solutions.

d. variables that are all restricted to nonnegative values.

ANSWER: c
POINTS: 1
TOPICS: Problem formulation

36. If there is a maximum of 4,000 hours of labor available per month and 300 ping-pong balls (x_1) or 125 wiffle balls (x_2) can be produced per hour of labor, which of the following constraints reflects this situation?

a. $300x_1 + 125x_2 \geq 4{,}000$

b. $300x_1 + 125x_2 \leq 4{,}000$

c. $425(x_1 + x_2) \leq 4{,}000$

d. $300x_1 + 125x_2 = 4{,}000$

ANSWER: b
POINTS: 1

37. In what part(s) of a linear programming formulation would the decision variables be stated?

a. objective function and the left-hand side of each constraint

b. objective function and the right-hand side of each constraint

c. the left-hand side of each constraint only

d. the objective function only

ANSWER: a
POINTS: 1

38. The three assumptions necessary for a linear programming model to be appropriate include all of the following except

a. proportionality

b. additivity

c. divisibility

d. normality

ANSWER: d
POINTS: 1

39. A redundant constraint results in

a. no change in the optimal solution(s)

b. an unbounded solution

c. no feasible solution

d. alternative optimal solutions

ANSWER: a
POINTS: 1

40. A variable added to the left-hand side of a less-than-or-equal-to constraint to convert the constraint into an equality is
a. a standard variable
b. a slack variable
c. a surplus variable
d. a non-negative variable

ANSWER: b
POINTS: 1

Subjective Short Answer

41. Solve the following system of simultaneous equations.

$6X + 2Y = 50$
$2X + 4Y = 20$
ANSWER: X = 8, Y =1
POINTS: 1
TOPICS: Simultaneous equations

42. Solve the following system of simultaneous equations.

$6X + 4Y = 40$
$2X + 3Y = 20$
ANSWER: X = 4, Y = 4
POINTS: 1
TOPICS: Simultaneous equations

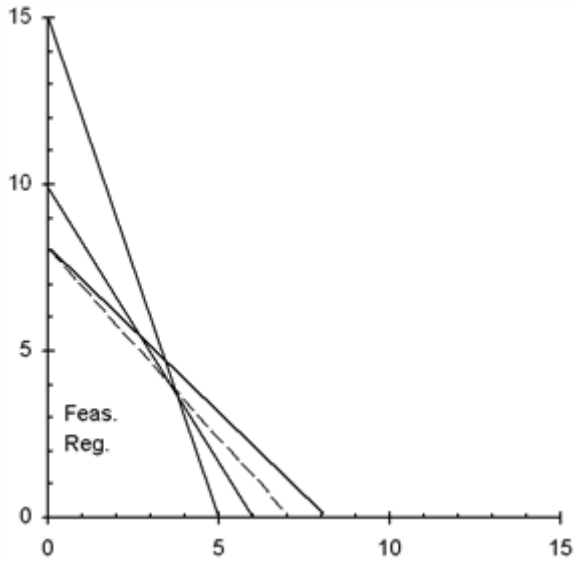
43. Consider the following linear programming problem

$$\begin{aligned} \text{Max} \quad & 8X + 7Y \\ \text{s.t.} \quad & 15X + 5Y \leq 75 \\ & 10X + 6Y \leq 60 \\ & X + Y \leq 8 \\ & X, Y \geq 0 \end{aligned}$$

a. Use a graph to show each constraint and the feasible region.
b. Identify the optimal solution point on your graph. What are the values of X and Y at the optimal solution?
c. What is the optimal value of the objective function?

ANSWER:

a.



b. The optimal solution occurs at the intersection of constraints 2 and 3. The point is X = 3, Y = 5.

c. The value of the objective function is 59.

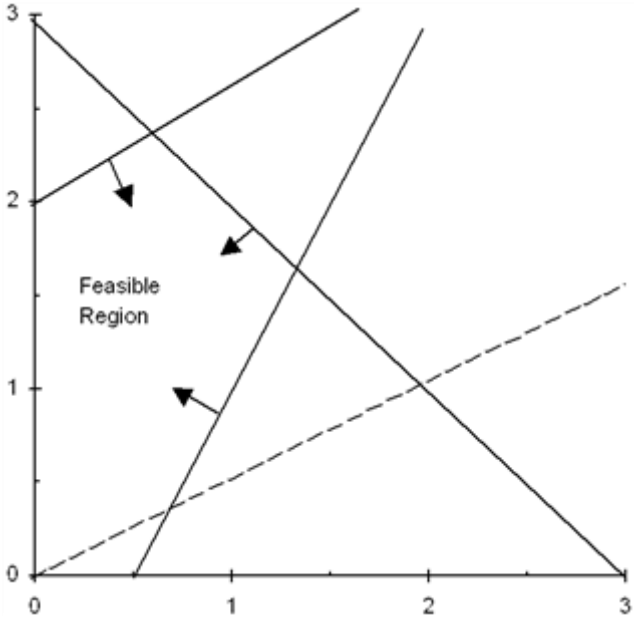
POINTS: 1

TOPICS: Graphical solution

44. For the following linear programming problem, determine the optimal solution by the graphical solution method

Max $-X + 2Y$
s.t. $6X - 2Y \leq 3$
$-2X + 3Y \leq 6$
$X + Y \leq 3$
$X, Y \geq 0$

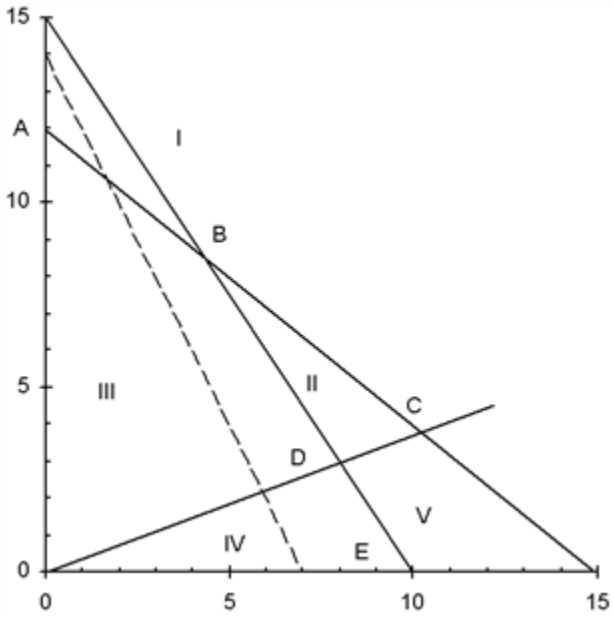
ANSWER: X = 0.6 and Y = 2.4



POINTS: 1

TOPICS: Graphical solution

45. Use this graph to answer the questions.



Max $20X + 10Y$
s.t. $12X + 15Y \leq 180$
$15X + 10Y \leq 150$
$3X - 8Y \leq 0$
$X , Y \geq 0$

a. Which area (I, II, III, IV, or V) forms the feasible region?
b. Which point (A, B, C, D, or E) is optimal?
c. Which constraints are binding?
d. Which slack variables are zero?

ANSWER:

a. Area III is the feasible region
b. Point D is optimal
c. Constraints 2 and 3 are binding
d. S_2 and S_3 are equal to 0

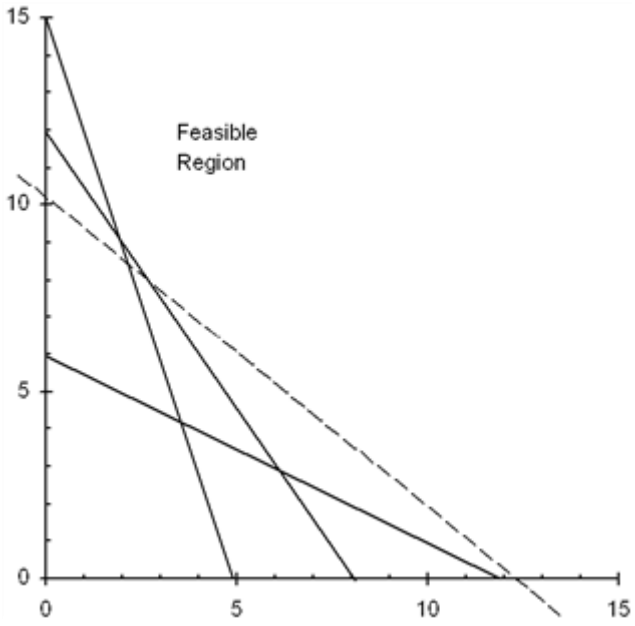
POINTS: 1

TOPICS: Graphical solution

46. Find the complete optimal solution to this linear programming problem.

Min $5X + 6Y$
s.t. $3X + Y \geq 15$
$X + 2Y \geq 12$
$3X + 2Y \geq 24$
$X , Y \geq 0$

ANSWER:



The complete optimal solution is X = 6, Y = 3, Z = 48, S_1 = 6, S_2 = 0, S_3 = 0

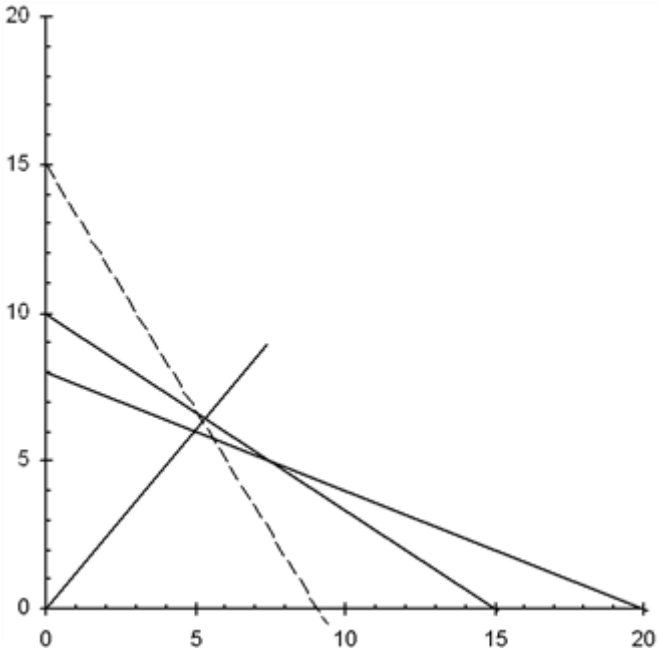
POINTS: 1

TOPICS: Graphical solution

47. Find the complete optimal solution to this linear programming problem.

Max $5X + 3Y$
s.t. $2X + 3Y \leq 30$
$2X + 5Y \leq 40$
$6X - 5Y \leq 0$
$X, Y \geq 0$

ANSWER:



The complete optimal solution is X = 15, Y = 0, Z = 75, S_1 = 0, S_2 = 10, S_3 = 90

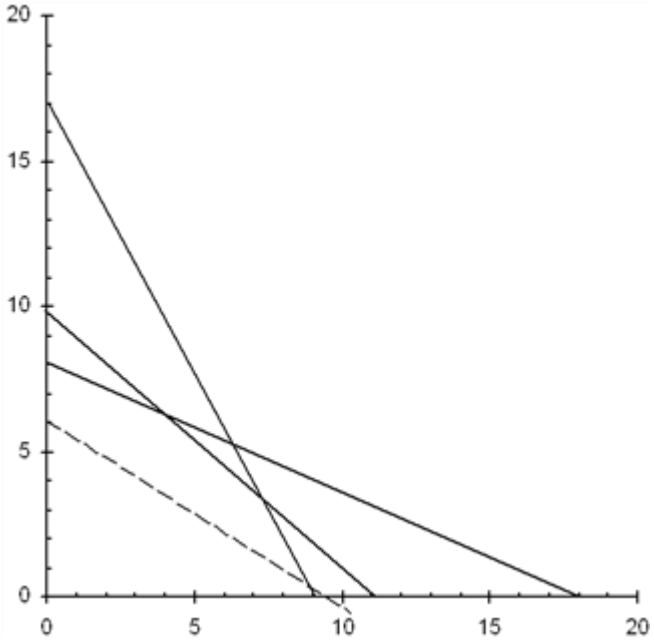
POINTS: 1

TOPICS: Graphical solution

48. Find the complete optimal solution to this linear programming problem.

Max $2X + 3Y$
s.t. $4X + 9Y \leq 72$
$10X + 11Y \leq 110$
$17X + 9Y \leq 153$
$X , Y \geq 0$

ANSWER:



The complete optimal solution is $X = 4.304$, $Y = 6.087$, $Z = 26.87$, $S_1 = 0$, $S_2 = 0$, $S_3 = 25.043$

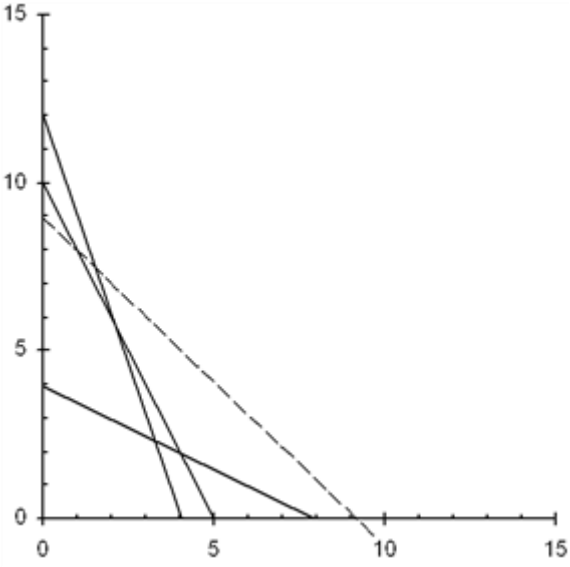
POINTS: 1

TOPICS: Graphical solution

49. Find the complete optimal solution to this linear programming problem.

Min $3X + 3Y$
s.t. $12X + 4Y \geq 48$
$10X + 5Y \geq 50$
$4X + 8Y \geq 32$
$X , Y \geq 0$

ANSWER:



The complete optimal solution is X = 4, Y = 2, Z = 18, S_1 = 8, S_2 = 0, S_3 = 0

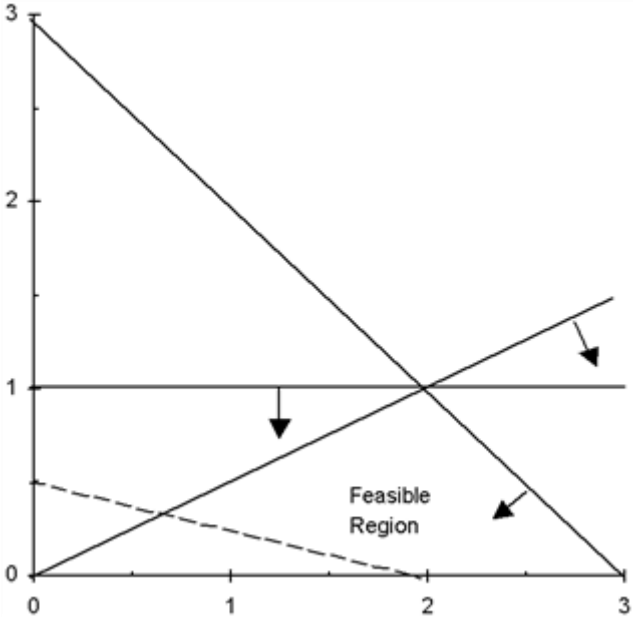
POINTS: 1

TOPICS: Graphical solution

50. For the following linear programming problem, determine the optimal solution by the graphical solution method. Are any of the constraints redundant? If yes, then identify the constraint that is redundant.

Max X + 2Y
s.t. X + Y ≤ 3
X − 2Y ≥ 0
Y ≤ 1
X, Y ≥ 0

ANSWER: X = 2, and Y = 1 Yes, there is a redundant constraint; Y ≤ 1



POINTS: 1

TOPICS: Graphical solution

51. Maxwell Manufacturing makes two models of felt tip marking pens. Requirements for each lot of pens are given below.

	Fliptop Model	Tiptop Model	Available
Plastic	3	4	36
Ink Assembly	5	4	40
Molding Time	5	2	30

The profit for either model is $1000 per lot.

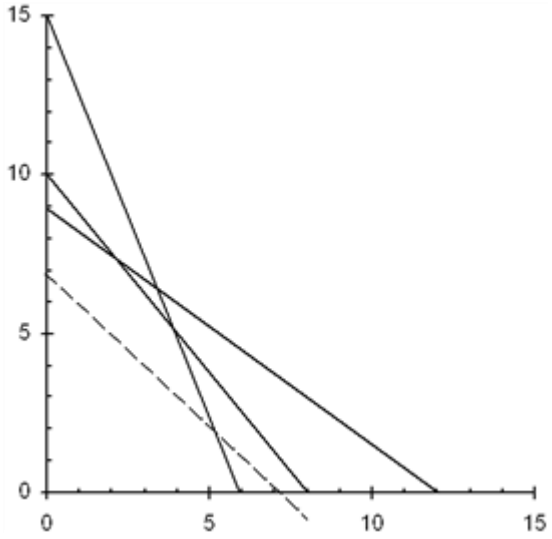
a. What is the linear programming model for this problem?
b. Find the optimal solution.
c. Will there be excess capacity in any resource?

ANSWER:

a. Let F = the number of lots of Fliptop pens to produce
Let T = the number of lots of Tiptop pens to produce

Max 1000F + 1000T
s.t. $3F + 4T \leq 36$
$5F + 4T \leq 40$
$5F + 2T \leq 30$
$F, T \geq 0$

b.



The complete optimal solution is F = 2, T = 7.5, Z = 9500, $S_1 = 0$, $S_2 = 0$, $S_3 = 5$

c. There is an excess of 5 units of molding time available.

POINTS: 1

TOPICS: Modeling and graphical solution

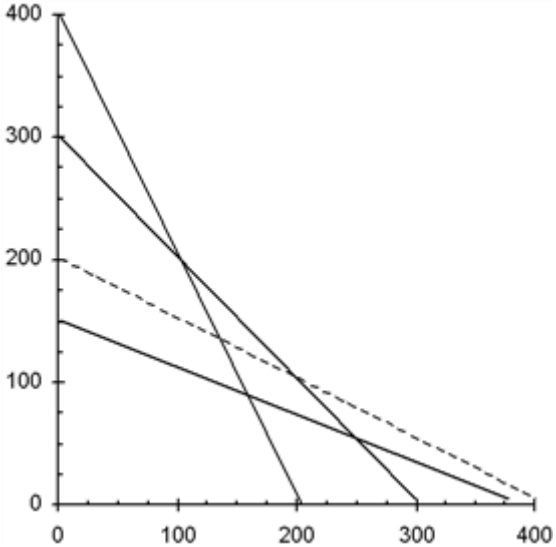
52. The Sanders Garden Shop mixes two types of grass seed into a blend. Each type of grass has been rated (per pound) according to its shade tolerance, ability to stand up to traffic, and drought resistance, as shown in the table. Type A seed costs $1 and Type B seed costs $2. If the blend needs to score at least 300 points for shade tolerance, 400 points for traffic resistance, and 750 points for drought resistance, how many pounds of each seed should be in the blend? Which targets will be exceeded? How much will the blend cost?

	Type A	Type B
Shade Tolerance	1	1
Traffic Resistance	2	1

Drought Resistance	2	5

ANSWER: Let A = the pounds of Type A seed in the blend
Let B = the pounds of Type B seed in the blend

Min 1A + 2B
s.t. 1A + 1B ≥ 300
2A + 1B ≥ 400
2A + 5B ≥ 750
A, B ≥ 0



The optimal solution is at A = 250, B = 50. Constraint 2 has a surplus value of 150. The cost is 350.

POINTS: 1

TOPICS: Modeling and graphical solution

53. Muir Manufacturing produces two popular grades of commercial carpeting among its many other products. In the coming production period, Muir needs to decide how many rolls of each grade should be produced in order to maximize profit. Each roll of Grade X carpet uses 50 units of synthetic fiber, requires 25 hours of production time, and needs 20 units of foam backing. Each roll of Grade Y carpet uses 40 units of synthetic fiber, requires 28 hours of production time, and needs 15 units of foam backing.

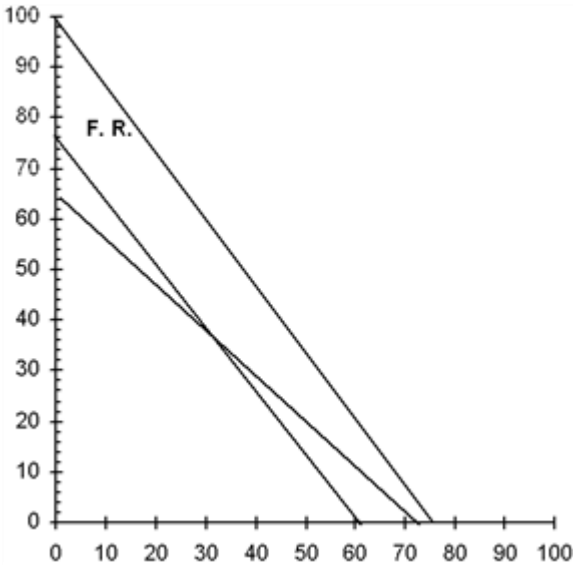
The profit per roll of Grade X carpet is $200 and the profit per roll of Grade Y carpet is $160. In the coming production period, Muir has 3000 units of synthetic fiber available for use. Workers have been scheduled to provide at least 1800 hours of production time (overtime is a possibility). The company has 1500 units of foam backing available for use.

Develop and solve a linear programming model for this problem.

ANSWER: Let X = the number of rolls of Grade X carpet to make
Let Y = the number of rolls of Grade Y carpet to make

Max 200X + 160Y

s.t. 50X + 40Y ≤ 3000
25X + 28Y ≥ 1800
20X + 15Y ≤ 1500
X , Y ≥ 0



The complete optimal solution is X = 30, Y = 37.5, Z = 12000, $S_1 = 0$, $S_2 = 0$, $S_3 = 337.5$

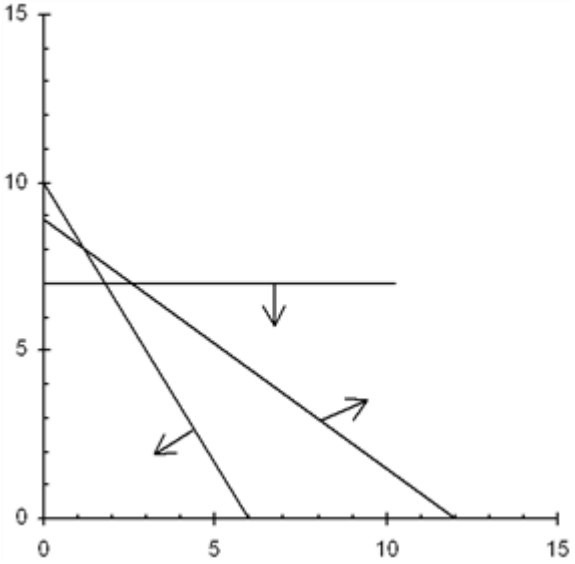
POINTS: 1

TOPICS: Modeling and graphical solution

54. Does the following linear programming problem exhibit infeasibility, unboundedness, or alternate optimal solutions? Explain.

Min $1X + 1Y$
s.t. $5X + 3Y \leq 30$
$3X + 4Y \geq 36$
$Y \leq 7$
$X , Y \geq 0$

ANSWER: The problem is infeasible.



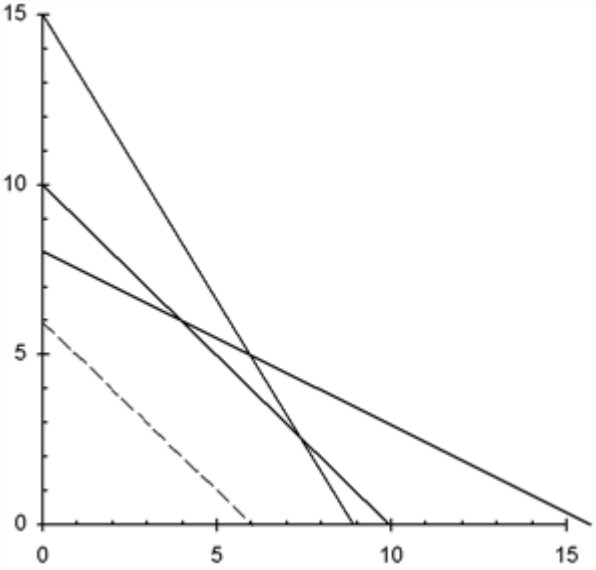
POINTS: 1

TOPICS: Special cases

55. Does the following linear programming problem exhibit infeasibility, unboundedness, or alternate optimal solutions? Explain.

Min $3X + 3Y$
s.t. $1X + 2Y \leq 16$
$1X + 1Y \leq 10$
$5X + 3Y \leq 45$
$X , Y \geq 0$

ANSWER: The problem has alternate optimal solutions.



POINTS: 1

TOPICS: Special cases

56. A businessman is considering opening a small specialized trucking firm. To make the firm profitable, it is estimated that it must have a daily trucking capacity of at least 84,000 cu. ft. Two types of trucks are appropriate for the specialized operation. Their characteristics and costs are summarized in the table below. Note that truck 2 requires 3 drivers for long haul trips. There are 41 potential drivers available and there are facilities for at most 40 trucks. The businessman's objective is to minimize the total cost outlay for trucks.

Truck	Cost	Capacity (Cu. Ft.)	Drivers Needed
Small	$18,000	2,400	1
Large	$45,000	6,000	3

Solve the problem graphically and note there are alternate optimal solutions. Which optimal solution:

a. uses only one type of truck?
b. utilizes the minimum total number of trucks?
c. uses the same number of small and large trucks?

ANSWER:

a. 35 small, 0 large
b. 5 small, 12 large
c. 10 small, 10 large

POINTS: 1

TOPICS: Alternative optimal solutions

57. Consider the following linear program:

Max $60X + 43Y$
s.t. $X + 3Y \geq 9$

$$6X - 2Y = 12$$
$$X + 2Y \leq 10$$
$$X, Y \geq 0$$

a. Write the problem in standard form.
b. What is the feasible region for the problem?
c. Show that regardless of the values of the actual objective function coefficients, the optimal solution will occur at one of two points. Solve for these points and then determine which one maximizes the current objective function.

ANSWER:

a. Max $60X + 43Y$
s.t. $X + 3Y - S_1 = 9$
$6X - 2Y = 12$
$X + 2Y + S_3 = 10$
$X, Y, S_1, S_3 \geq 0$

b. Line segment of $6X - 2Y = 12$ between (22/7,24/7) and (27/10,21/10).
c. Extreme points: (22/7,24/7) and (27/10,21/10). First one is optimal, giving Z = 336.

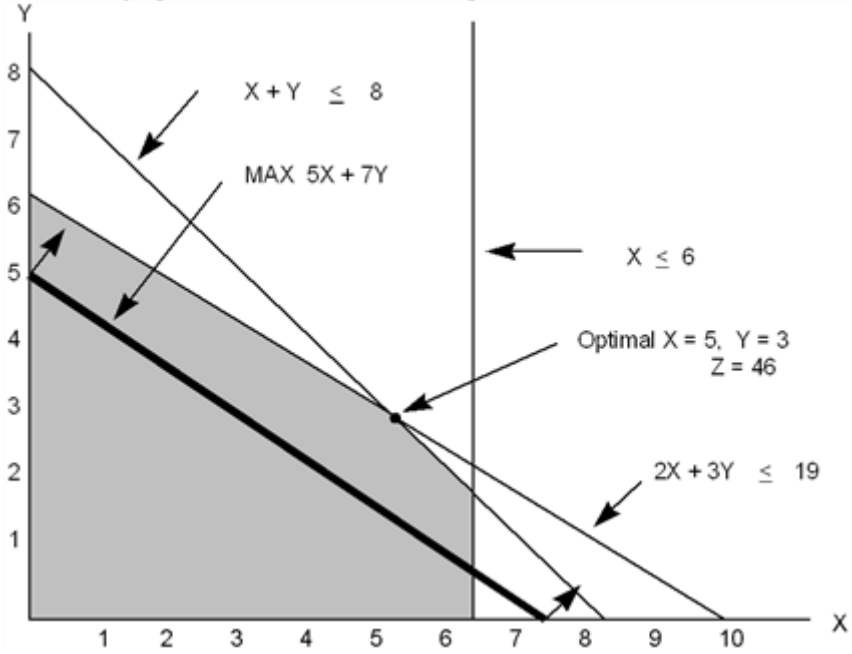
POINTS: 1

TOPICS: Standard form and extreme points

58. Solve the following linear program graphically.

Max $5X + 7Y$
s.t. $X \leq 6$
$2X + 3Y \leq 19$
$X + Y \leq 8$
$X, Y \geq 0$

ANSWER: From the graph below we see that the optimal solution occurs at X = 5, Y = 3, and Z = 46.



POINTS: 1

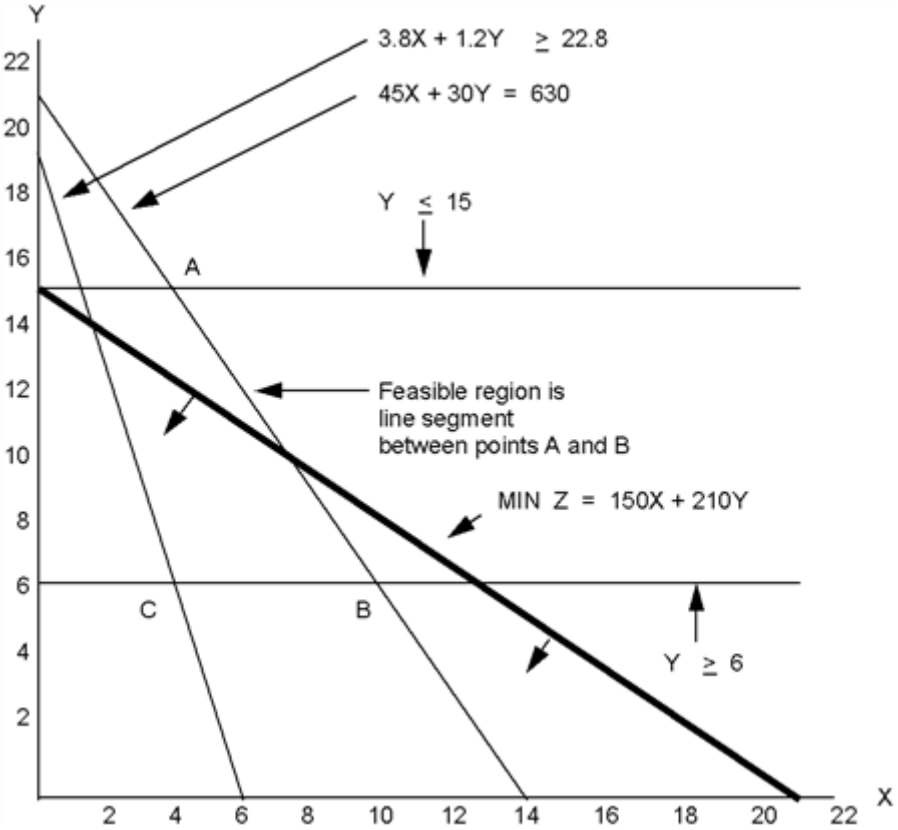
TOPICS: Graphical solution procedure

59. Given the following linear program:

Min 150X + 210Y
s.t. 3.8X + 1.2Y ≥ 22.8
Y ≥ 6
Y ≤ 15
45X + 30Y = 630
X, Y ≥ 0

Solve the problem graphically. How many extreme points exist for this problem?

ANSWER: Two extreme points exist (Points A and B below). The optimal solution is X = 10, Y = 6, and Z = 2760 (Point B).



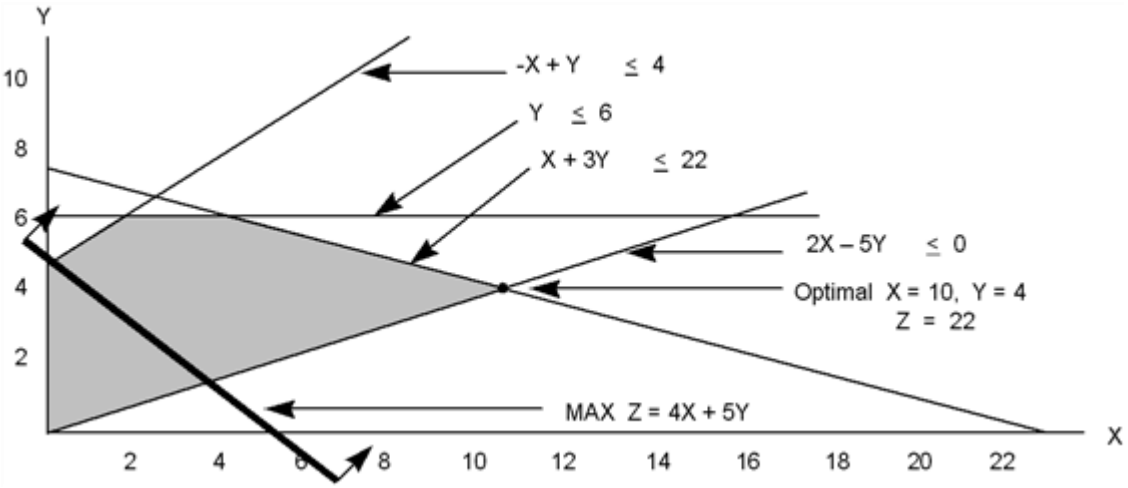
POINTS: 1

TOPICS: Graphical solution procedure

60. Solve the following linear program by the graphical method.

Max 4X + 5Y
s.t. X + 3Y ≤ 22
−X + Y ≤ 4
Y ≤ 6
2X − 5Y ≤ 0
X, Y ≥ 0

ANSWER: Two extreme points exist (Points A and B below). The optimal solution is X = 10, Y = 6, and Z = 2760 (Point B).



POINTS: 1
TOPICS: Graphical solution procedure

Essay

61. Explain the difference between profit and contribution in an objective function. Why is it important for the decision maker to know which of these the objective function coefficients represent?
ANSWER: Answer not provided.
POINTS: 1
TOPICS: Objective function

62. Explain how to graph the line $x_1 - 2x_2 \geq 0$.
ANSWER: Answer not provided.
POINTS: 1
TOPICS: Graphing lines

63. Create a linear programming problem with two decision variables and three constraints that will include both a slack and a surplus variable in standard form. Write your problem in standard form.
ANSWER: Answer not provided.
POINTS: 1
TOPICS: Standard form

64. Explain what to look for in problems that are infeasible or unbounded.
ANSWER: Answer not provided.
POINTS: 1
TOPICS: Special cases

65. Use a graph to illustrate why a change in an objective function coefficient does not necessarily lead to a change in the optimal values of the decision variables, but a change in the right-hand sides of a binding constraint does lead to new values.
ANSWER: Answer not provided.
POINTS: 1
TOPICS: Graphical sensitivity analysis

66. Explain the concepts of proportionality, additivity, and divisibility.
ANSWER: Answer not provided.
POINTS: 1
TOPICS: Notes and comments

67. Explain the steps necessary to put a linear program in standard form.
ANSWER: Answer not provided.
POINTS: 1
TOPICS: Surplus variables

68. Explain the steps of the graphical solution procedure for a minimization problem.
ANSWER: Answer not provided.
POINTS: 1
TOPICS: Graphical solution procedure for minimization problems