Chapter 2
An Introduction to Linear Programming

Learning Objectives

1. Obtain an overview of the kinds of problems linear programming has been used to solve.

2. Learn how to develop linear programming models for simple problems.

3. Be able to identify the special features of a model that make it a linear programming model.

4. Learn how to solve two variable linear programming models by the graphical solution procedure.

5. Understand the importance of extreme points in obtaining the optimal solution.

6. Know the use and interpretation of slack and surplus variables.

7. Be able to interpret the computer solution of a linear programming problem.

8. Understand how alternative optimal solutions, infeasibility and unboundedness can occur in linear programming problems.

9. Understand the following terms:

problem formulation	feasible region
constraint function	slack variable
objective function	standard form
solution	redundant constraint
optimal solution	extreme point
nonnegativity constraints	surplus variable
mathematical model	alternative optimal solutions
linear program	infeasibility
linear functions	unbounded
feasible solution	

Solutions:

1. a, b, and e, are acceptable linear programming relationships.

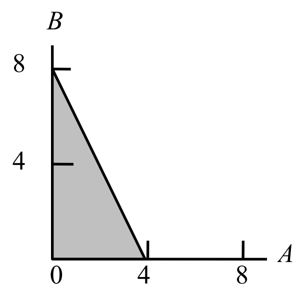
   c is not acceptable because of $-2B^2$

   d is not acceptable because of $3\sqrt{A}$

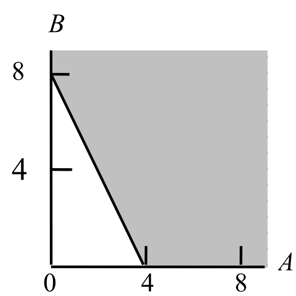
   f is not acceptable because of *1AB*

   c, d, and f could not be found in a linear programming model because they have the above nonlinear terms.

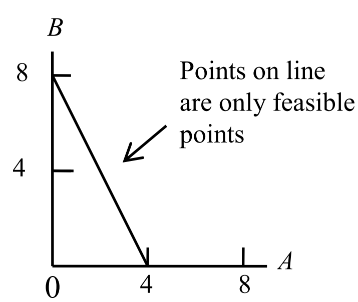
2. a.



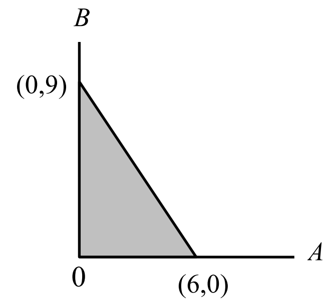
b.



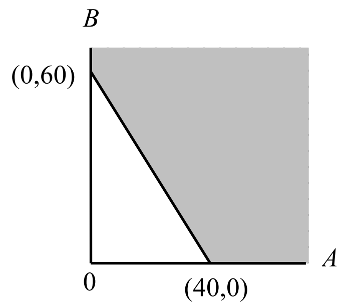
c.



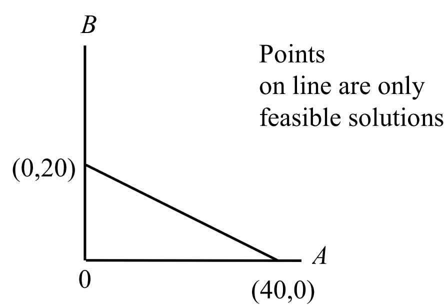
3. a.


B
(0,9)
A
0
(6,0)

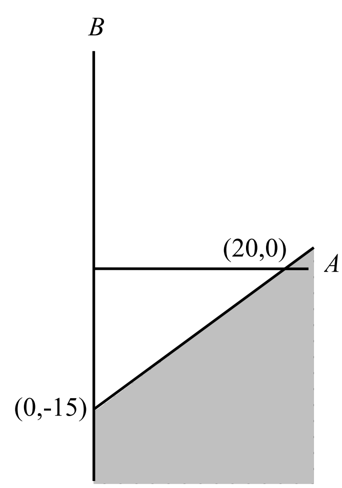
b.


B
(0,60)
A
0
(40,0)

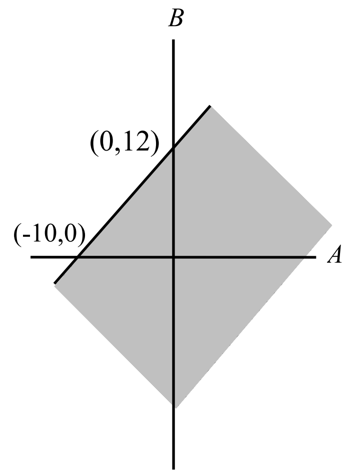
c.


B
Points
on line are only
feasible solutions
(0,20)
A
0
(40,0)

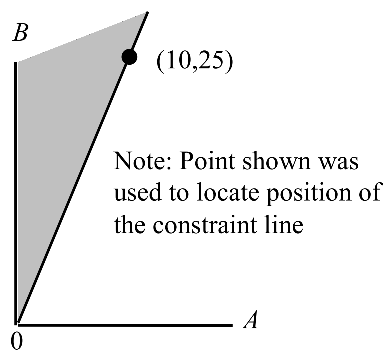
4. a.


B
(20,0)
A
(0,-15)

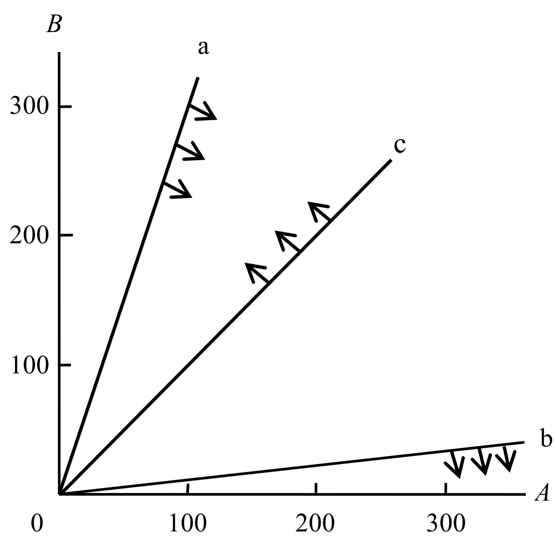
b.


B
(0,12)
(-10,0)
A

c.


B
(10,25)
Note: Point shown was used to locate position of the constraint line
0
A

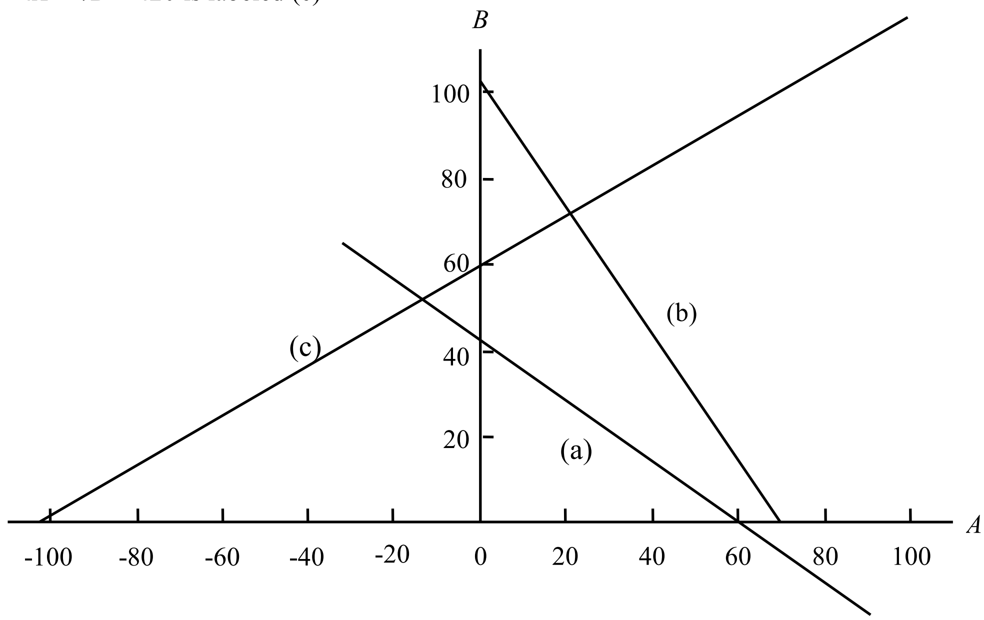
5.


B
a
c
b
300
200
100
0
100
200
300
A

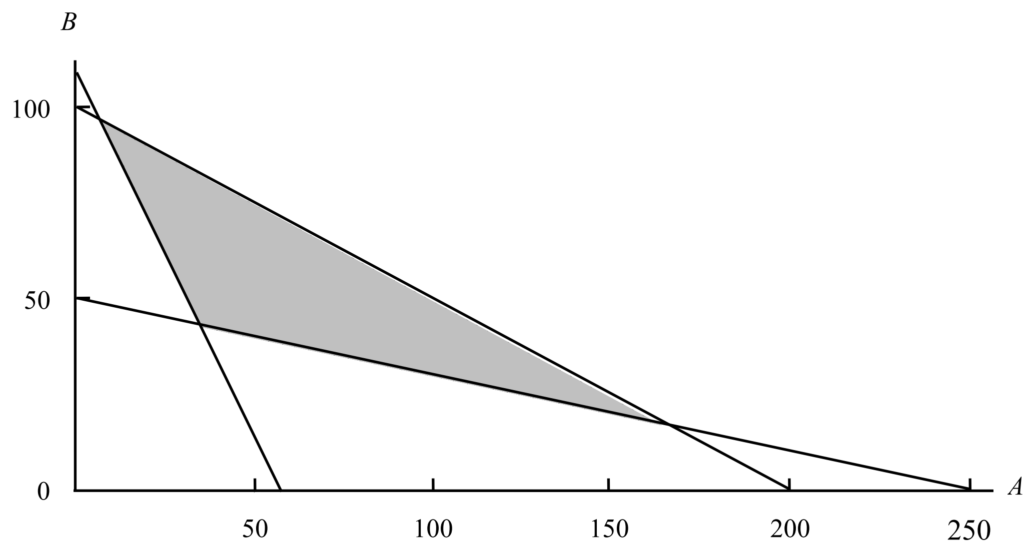
6. $7A + 10B = 420$ is labeled (a)

$6A + 4B = 420$ is labeled (b)

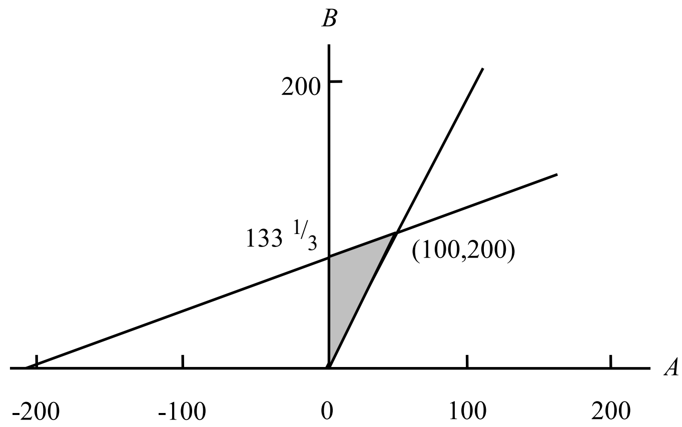
$-4A + 7B = 420$ is labeled (c)



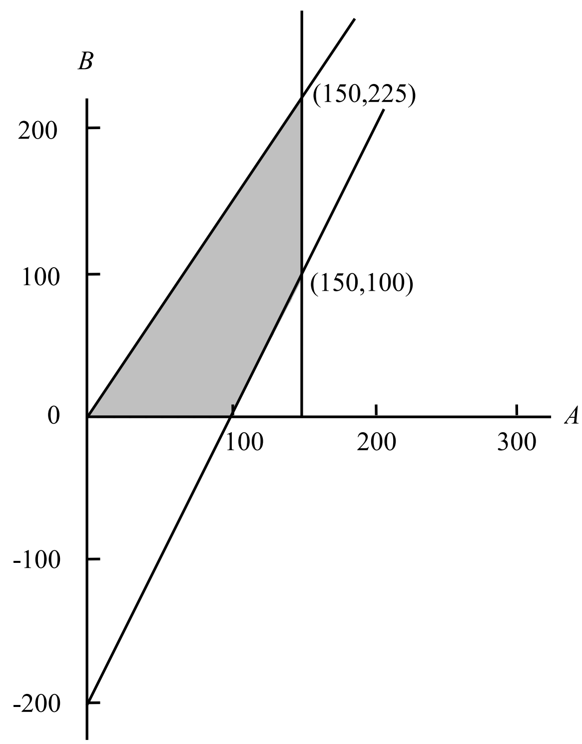
7.



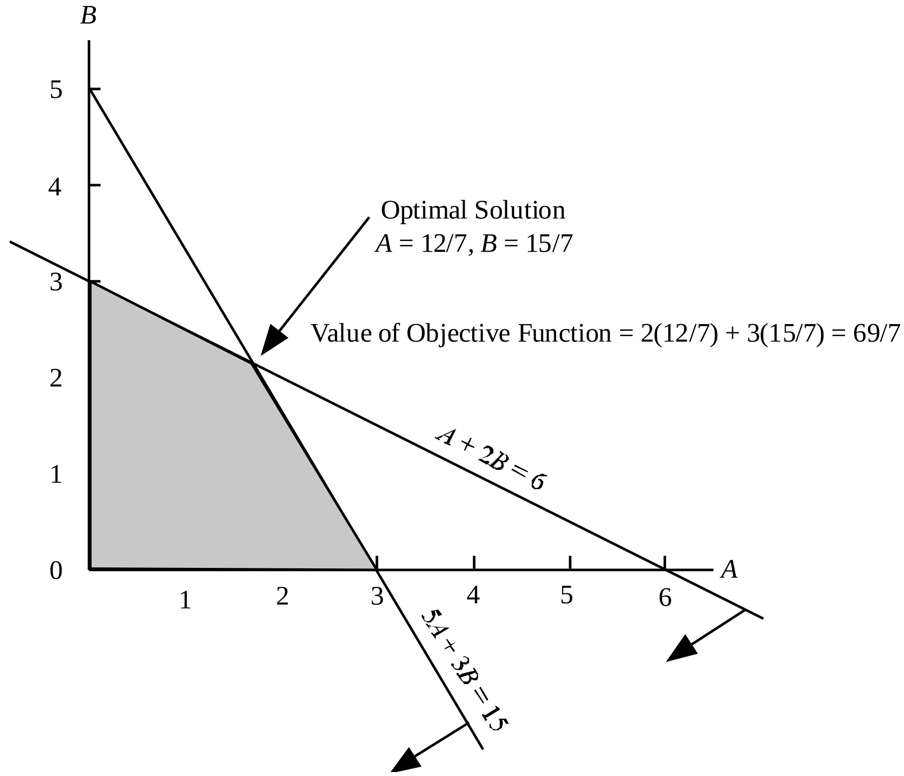
8.


B
200
133 1/3
(100,200)
A
-200
-100
0
100
200

9.


B
(150,225)
200
100
(150,100)
0
A
100
200
300
-100
-200

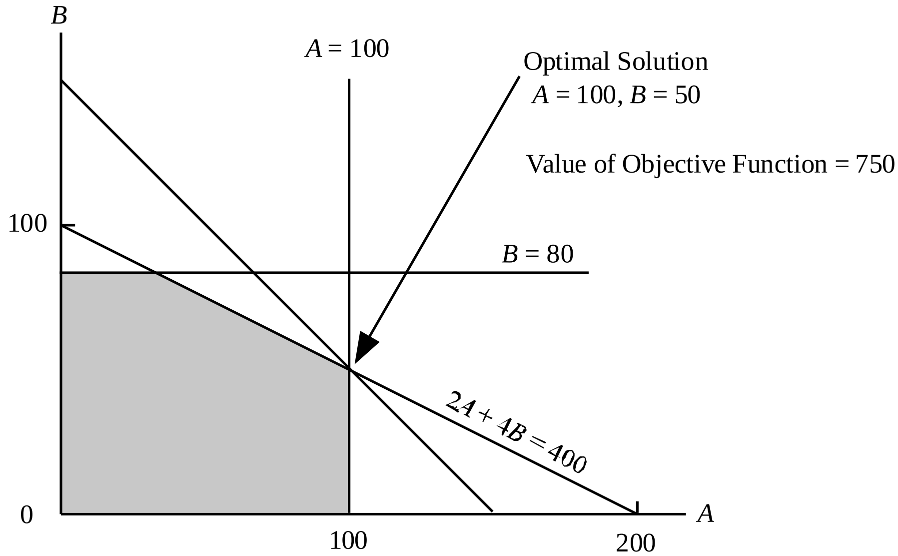
10.



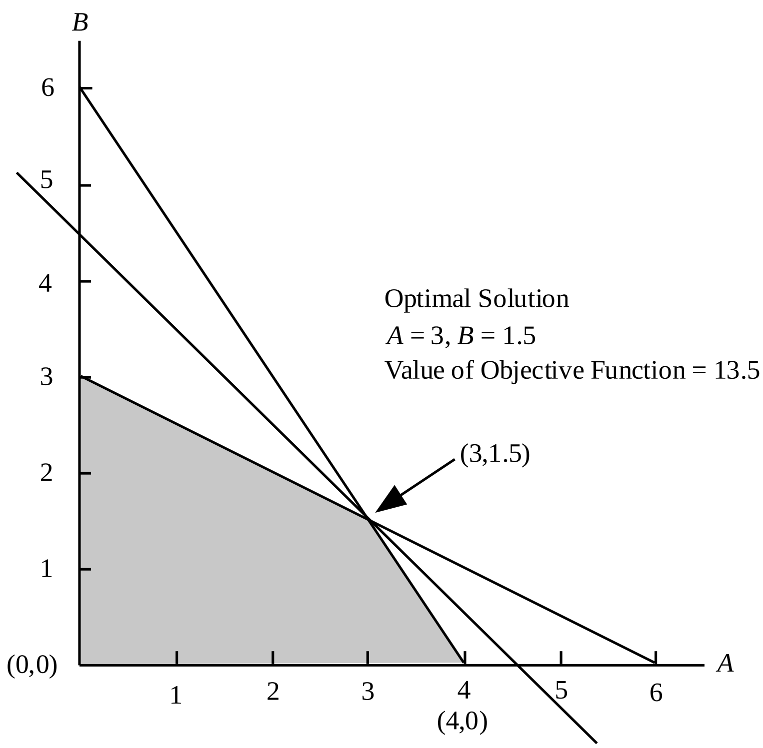
	A	+	$2B$	=	6	(1)
	$5A$	+	$3B$	=	15	(2)
(1) × 5	$5A$	+	$10B$	=	30	(3)
(2) - (3)		-	$7B$	=	-15	
			B	=	15/7	

From (1), $A = 6 - 2(15/7) = 6 - 30/7 = 12/7$

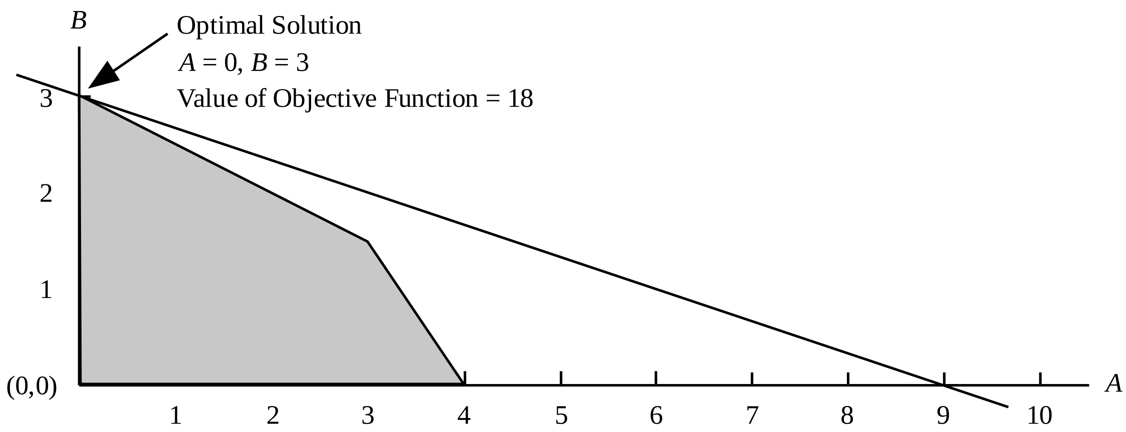
11.


B
A = 100
Optimal Solution
A = 100, B = 50
Value of Objective Function = 750
100
B = 80
2A + 4B = 400
0
100
200
A

12. a.

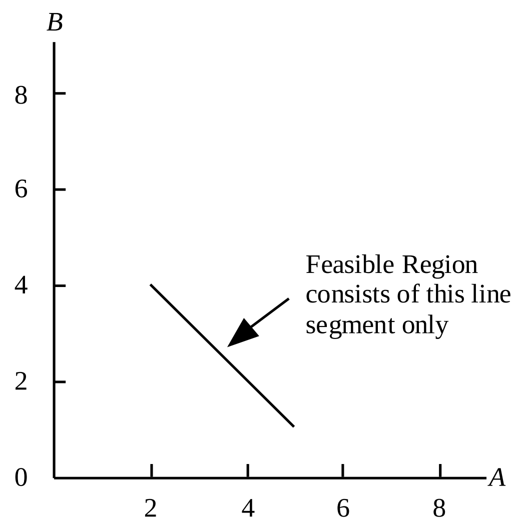

B
6
5
4
3
2
1
Optimal Solution
A = 3, B = 1.5
Value of Objective Function = 13.5
(3,1.5)
(0,0)
1
2
3
4
5
6
(4,0)
A

b.



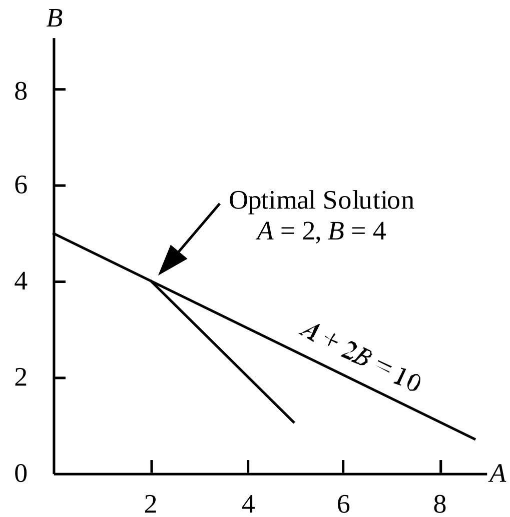
c. There are four extreme points: (0,0), (4,0), (3,1,5), and (0,3).

13. a.



b. The extreme points are (5, 1) and (2, 4).

c.

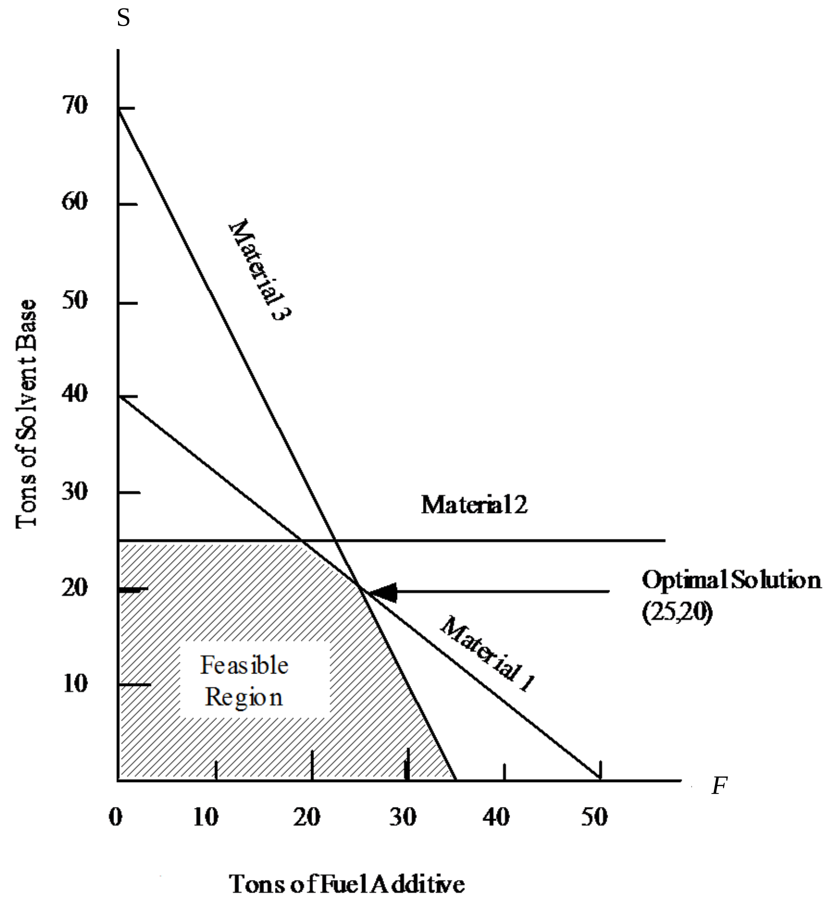


14. a. Let F = number of tons of fuel additive
S = number of tons of solvent base

Max	$40F$	+	$30S$			
s.t.						
	$2/5F$	+	$^1/_2\,S$	$\leq$	200	Material 1
			$^1/_5\,S$	$\leq$	5	Material 2
	$^3/_5\,F$	+	$^3/_{10}\,S$	$\leq$	21	Material 3

$F,\ S \geq 0$

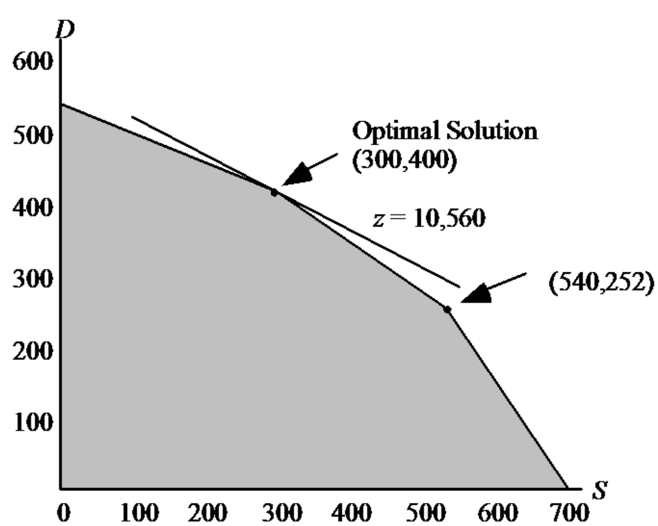
b.



c. Material 2: 4 tons are used, 1 ton is unused.

d. No redundant constraints.

15. a.



b. Similar to part (a): the same feasible region with a different objective function. The optimal solution occurs at (708, 0) with a profit of $z = 20(708) + 9(0) = 14,160$.

c. The sewing constraint is redundant. Such a change would not change the optimal solution to the original problem.

16. a. A variety of objective functions with a slope greater than -4/10 (slope of I & P line) will make extreme point (0, 540) the optimal solution. For example, one possibility is $3S + 9D$.

b. Optimal Solution is $S = 0$ and $D = 540$.

c.

Department	Hours Used	Max. Available	Slack
Cutting and Dyeing	1(540) = 540	630	90
Sewing	$^5/_6(540) = 450$	600	150
Finishing	$^2/_3(540) = 360$	708	348
Inspection and Packaging	$^1/_4(540) = 135$	135	0

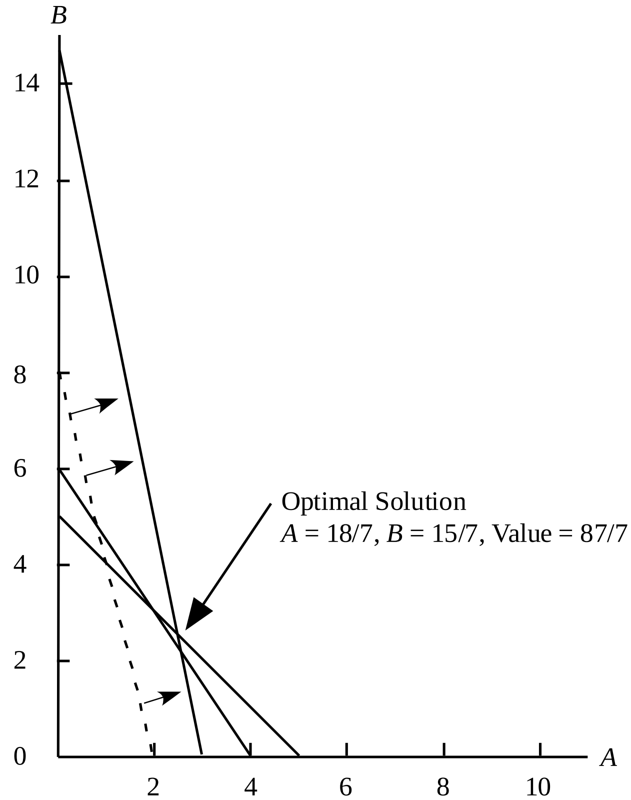
17.

$$\begin{array}{llllllll} \text{Max} & 5A & + 2B & + 0S_1 & + 0S_2 & + 0S_3 & & \\ \text{s.t.} & & & & & & & \\ & 1A & - 2B & + 1S_1 & & & = & 420 \\ & 2A & + 3B & & + 1S_2 & & = & 610 \\ & 6A & - 1B & & & + 1S_3 & = & 125 \\ & & & & A, B, S_1, S_2, S_3 \geq 0 & & & \end{array}$$

18. a.

$$\begin{array}{llllllll} \text{Max} & 4A & + 1B & + 0S_1 & + 0S_2 & + 0S_3 & & \\ \text{s.t.} & & & & & & & \\ & 10A & + 2B & + 1S_1 & & & = & 30 \\ & 3A & + 2B & & + 1S_2 & & = & 12 \\ & 2A & + 2B & & & + 1S_3 & = & 10 \\ & & & & A, B, S_1, S_2, S_3 \geq 0 & & & \end{array}$$

b.



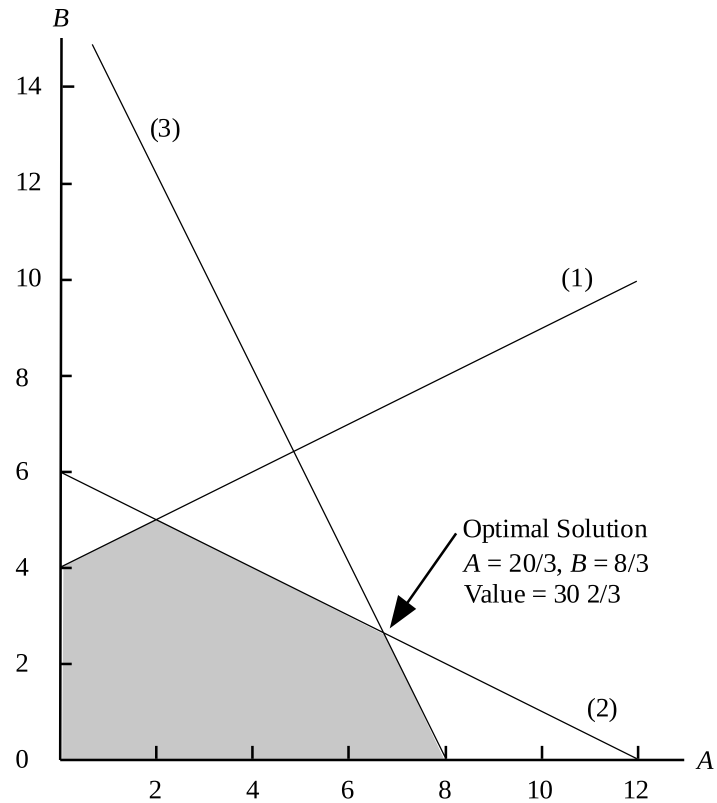
c. $S_1 = 0$, $S_2 = 0$, $S_3 = 4/7$

19. a.

$$
\begin{array}{llllllll}
\text{Max} & 3A & + & 4B & + 0S_1 & + 0S_2 & + 0S_3 & \\
\text{s.t.} & & & & & & & \\
& -1A & + & 2B & + 1S_1 & & & = 8 \quad (1) \\
& 1A & + & 2B & & + 1S_2 & & = 12 \quad (2) \\
& 2A & + & 1B & & & + 1S_3 & = 16 \quad (3) \\
\end{array}
$$

$$A, B, S_1, S_2, S_3 \geq 0$$

b.



c. $S_1 = 8 + A - 2B = 8 + 20/3 - 16/3 = 28/3$

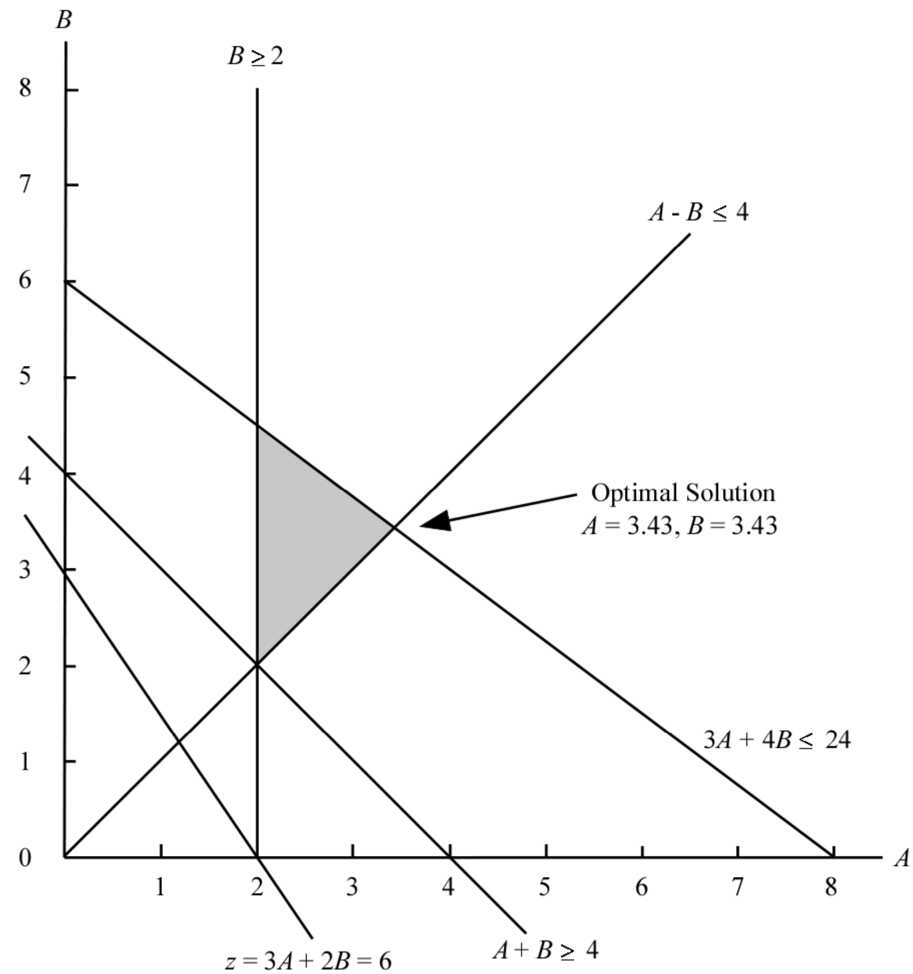
$S_2 = 12 - A - 2B = 12 - 20/3 - 16/3 = 0$

$S_3 = 16 - 2A - B = 16 - 40/3 - 8/3 = 0$

20. a.

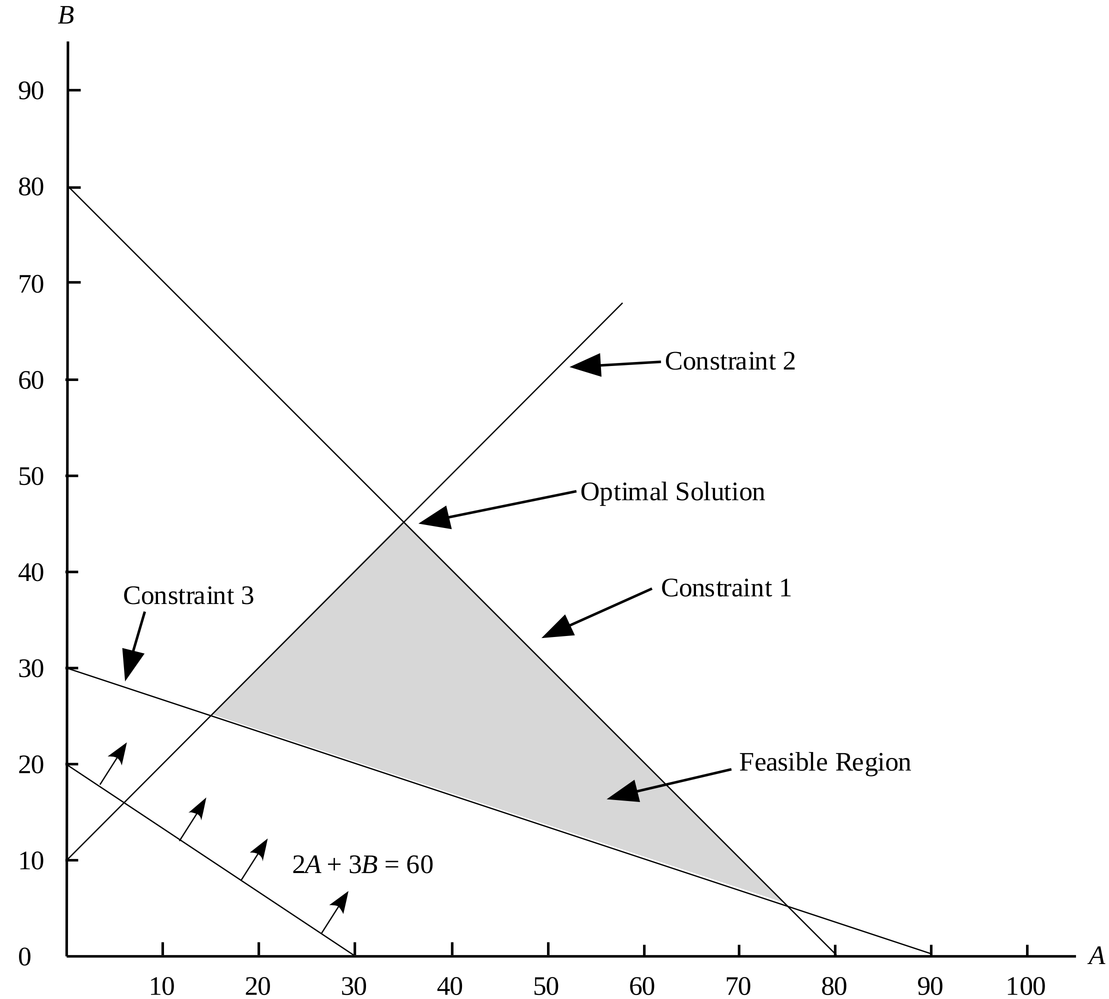
$$
\begin{array}{lrcrcccccl}
\text{Max} & 3A & + & 2B & & & & & & \\
\text{s.t.} & & & & & & & & & \\
 & A & + & B & - S_1 & & & & = & 4 \\
 & 3A & + & 4B & & + S_2 & & & = & 24 \\
 & A & & & & & - S_3 & & = & 2 \\
 & A & - & B & & & & - S_4 & = & 0 \\
 & & & & & & & & & A, B, S_1, S_2, S_3, S_4 \geq 0
\end{array}
$$

b.



c. $S_1 = (3.43 + 3.43) - 4 = 2.86$
$S_2 = 24 - [3(3.43) + 4(3.43)] = 0$
$S_3 = 3.43 - 2 = 1.43$
$S_4 = 0 - (3.43 - 3.43) = 0$

21. a. and b.



c. Optimal solution occurs at the intersection of constraints 1 and 2. For constraint 2,

$$B = 10 + A$$

Substituting for B in constraint 1 we obtain

$$\begin{aligned} 5A + 5(10 + A) &= 400 \\ 5A + 50 + 5A &= 400 \\ 10A &= 350 \\ A &= 35 \end{aligned}$$

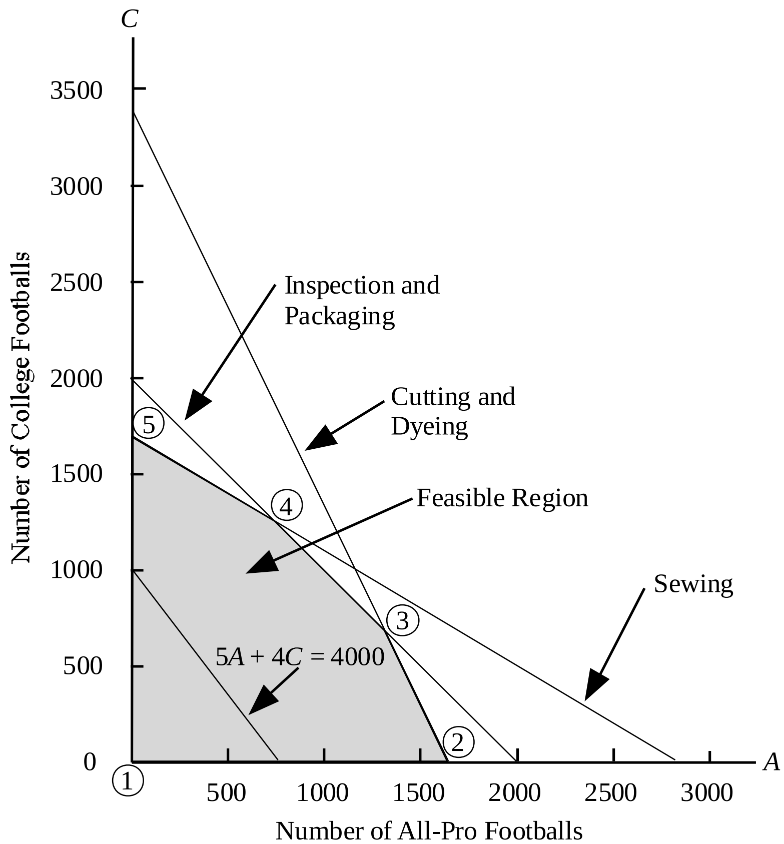
$$B = 10 + A = 10 + 35 = 45$$

Optimal solution is $A = 35$, $B = 45$

d. Because the optimal solution occurs at the intersection of constraints 1 and 2, these are binding constraints.

e. Constraint 3 is the nonbinding constraint. At the optimal solution $1A + 3B = 1(35) + 3(45) = 170$. Because 170 exceeds the right-hand side value of 90 by 80 units, there is a surplus of 80 associated with this constraint.

22. a.



b.

Extreme Point	Coordinates	Profit
1	(0, 0)	5(0) + 4(0) = 0
2	(1700, 0)	5(1700) + 4(0) = 8500
3	(1400, 600)	5(1400) + 4(600) = 9400
4	(800, 1200)	5(800) + 4(1200) = 8800
5	(0, 1680)	5(0) + 4(1680) = 6720

Extreme point 3 generates the highest profit.

c. Optimal solution is $A = 1400$, $C = 600$

d. The optimal solution occurs at the intersection of the cutting and dyeing constraint and the inspection and packaging constraint. Therefore these two constraints are the binding constraints.

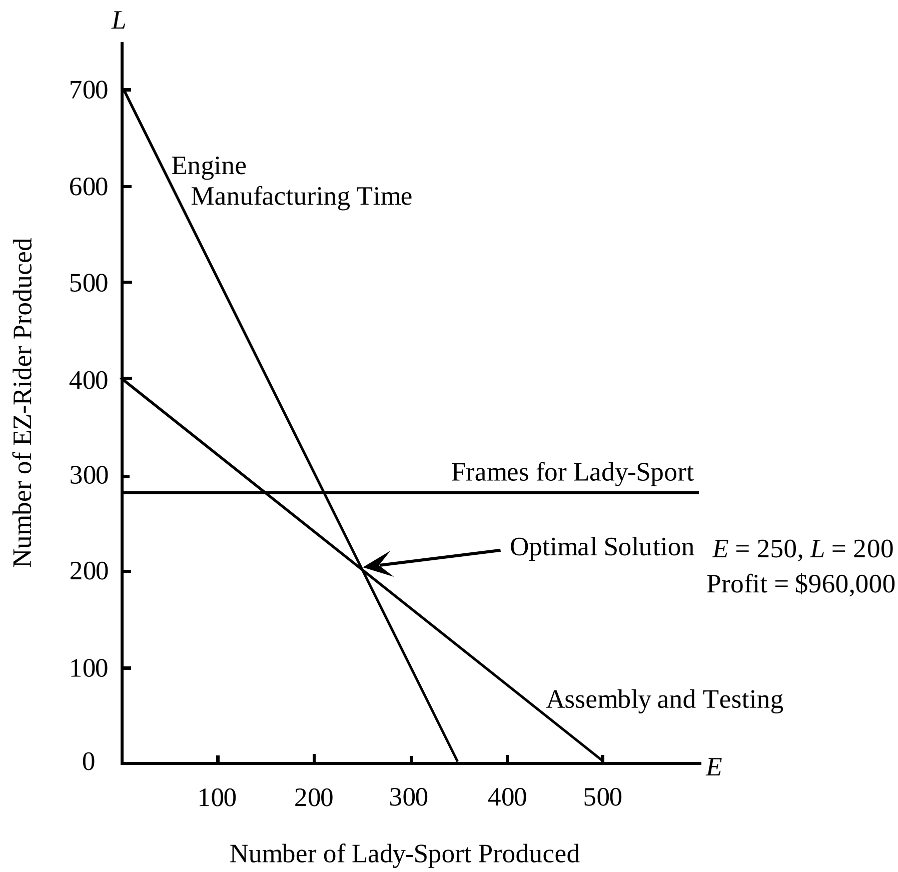
e. New optimal solution is $A = 800$, $C = 1200$

Profit = 4(800) + 5(1200) = 9200

23. a. Let E = number of units of the EZ-Rider produced
L = number of units of the Lady-Sport produced

$$
\begin{array}{llrcrl}
\text{Max} & 2400E + 1800L & & & & \\
\text{s.t.} & & & & & \\
 & 6E + 3L & \leq & 2100 & & \text{Engine time} \\
 & L & \leq & 280 & & \text{Lady-Sport maximum} \\
 & 2E + 2.5L & \leq & 1000 & & \text{Assembly and testing} \\
 & E, L \geq 0 & & & &
\end{array}
$$

b.

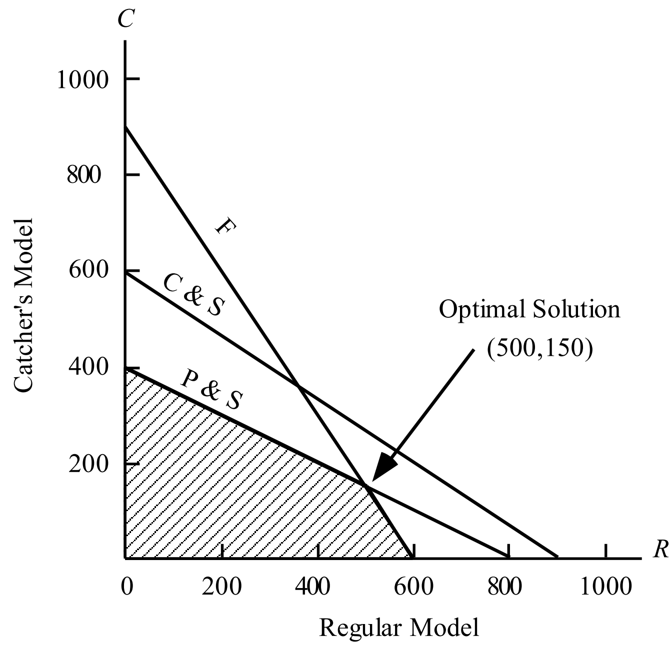


c. The binding constraints are the manufacturing time and the assembly and testing time.

24. a. Let R = number of units of regular model.
C = number of units of catcher's model.

$$
\begin{array}{llclcl}
\text{Max} & 5R & + & 8C & & \\
\text{s.t.} & & & & & \\
 & 1R & + & \frac{3}{2}C & \leq 900 & \text{Cutting and sewing} \\
 & \frac{1}{2}R & + & \frac{1}{3}C & \leq 300 & \text{Finishing} \\
 & \frac{1}{8}R & + & \frac{1}{4}C & \leq 100 & \text{Packing and Shipping} \\
 & R, C \geq 0 & & & &
\end{array}
$$

b.



c. 5(500) + 8(150) = \$3,700

d. C & S $1(500) + {}^{3}/_{2}(150) = 725$

F $^{1}/_{2}(500) + {}^{1}/_{3}(150) = 300$

P & S $^{1}/_{8}(500) + {}^{1}/_{4}(150) = 100$

e.

Department	Capacity	Usage	Slack
C & S	900	725	175 hours
F	300	300	0 hours
P & S	100	100	0 hours

25. a. Let B = percentage of funds invested in the bond fund
S = percentage of funds invested in the stock fund

$$\begin{aligned} \text{Max} \quad & 0.06B + 0.10S \\ \text{s.t.} \quad & \\ & B \geq 0.3 \quad \text{Bond fund minimum} \\ & 0.06B + 0.10S \geq 0.075 \quad \text{Minimum return} \\ & B + S = 1 \quad \text{Percentage requirement} \end{aligned}$$

b. Optimal solution: $B = 0.3$, $S = 0.7$

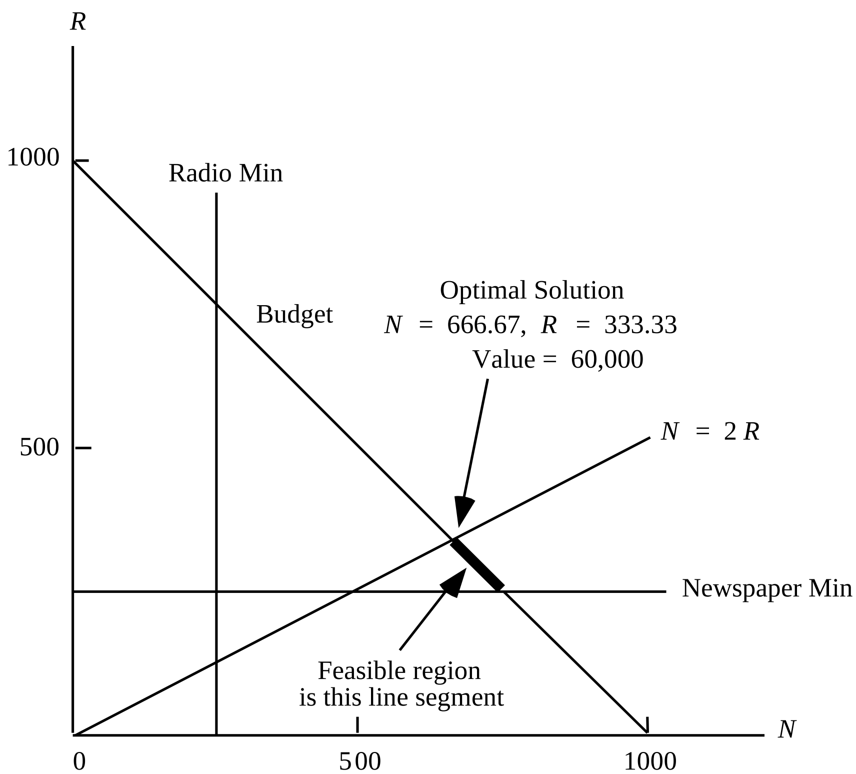
Value of optimal solution is 0.088 or 8.8%

26. a. a. Let N = amount spent on newspaper advertising
R = amount spent on radio advertising

Max	$50N$	+	$80R$			
s.t.						
	N	+	R	=	1000	Budget
	N			≥	250	Newspaper min.
			R	≥	250	Radio min.
	N		$-2R$	≥	0	News ≥ 2 Radio

$N, R \geq 0$

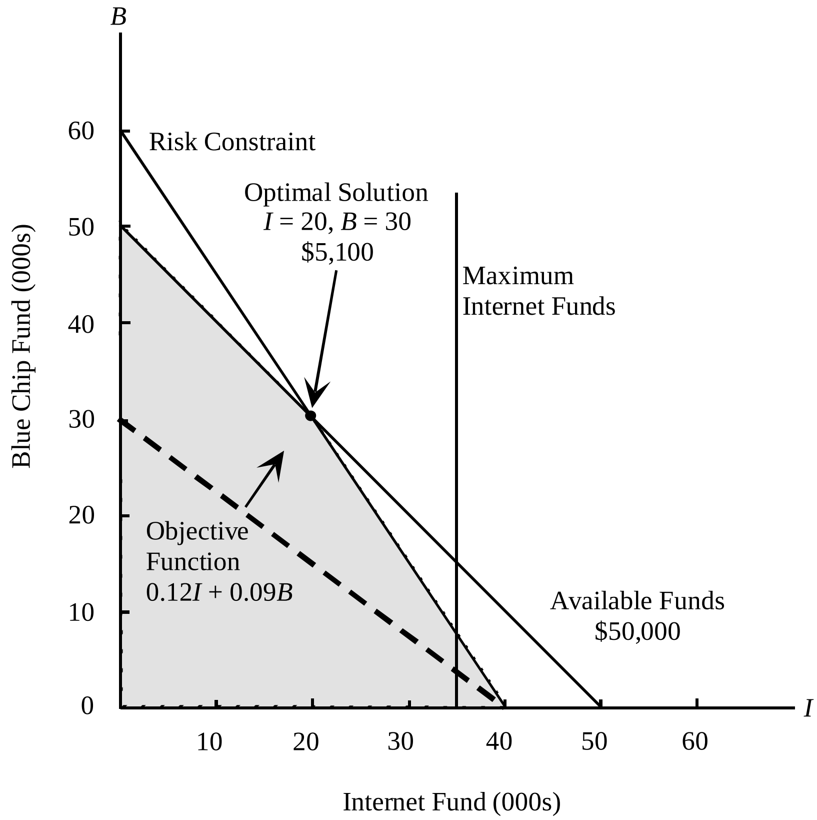
b.



27. Let I = Internet fund investment in thousands
B = Blue Chip fund investment in thousands

Max	$0.12I$	+	$0.09B$			
s.t.						
	$1I$	+	$1B$	≤	50	Available investment funds
	$1I$			≤	35	Maximum investment in the internet fund
	$6I$	+	$4B$	≤	240	Maximum risk for a moderate investor

$I, B \geq 0$



Internet fund	$20,000
Blue Chip fund	$30,000
Annual return	$ 5,100

b. The third constraint for the aggressive investor becomes

$$6I + 4B \leq 320$$

This constraint is redundant; the available funds and the maximum Internet fund investment constraints define the feasible region. The optimal solution is:

Internet fund	$35,000
Blue Chip fund	$15,000
Annual return	$ 5,550

The aggressive investor places as much funds as possible in the high return but high risk Internet fund.

c. The third constraint for the conservative investor becomes

$$6I + 4B \leq 160$$

This constraint becomes a binding constraint. The optimal solution is

Internet fund	$0
Blue Chip fund	$40,000
Annual return	$ 3,600

The slack for constraint 1 is \$10,000. This indicates that investing all \$50,000 in the Blue Chip fund is still too risky for the conservative investor. \$40,000 can be invested in the Blue Chip fund. The remaining \$10,000 could be invested in low-risk bonds or certificates of deposit.

28. a. Let W = number of jars of Western Foods Salsa produced
M = number of jars of Mexico City Salsa produced

Max	$1W$	+	$1.25M$			
s.t.						
	$5W$		$7M$	$\leq$	4480	Whole tomatoes
	$3W$	+	$1M$	$\leq$	2080	Tomato sauce
	$2W$	+	$2M$	$\leq$	1600	Tomato paste
	$W, M \geq 0$					

Note: units for constraints are ounces

b. Optimal solution: $W = 560$, $M = 240$

Value of optimal solution is 860

29. a. Let B = proportion of Buffalo's time used to produce component 1
D = proportion of Dayton's time used to produce component 1

	Maximum Daily Production	
	Component 1	Component 2
Buffalo	2000	1000
Dayton	600	1400

Number of units of component 1 produced: $2000B + 600D$

Number of units of component 2 produced: $1000(1 - B) + 600(1 - D)$

For assembly of the ignition systems, the number of units of component 1 produced must equal the number of units of component 2 produced.

Therefore,

$$2000B + 600D = 1000(1 - B) + 1400(1 - D)$$

$$2000B + 600D = 1000 - 1000B + 1400 - 1400D$$

$$3000B + 2000D = 2400$$

Note: Because every ignition system uses 1 unit of component 1 and 1 unit of component 2, we can maximize the number of electronic ignition systems produced by maximizing the number of units of subassembly 1 produced.

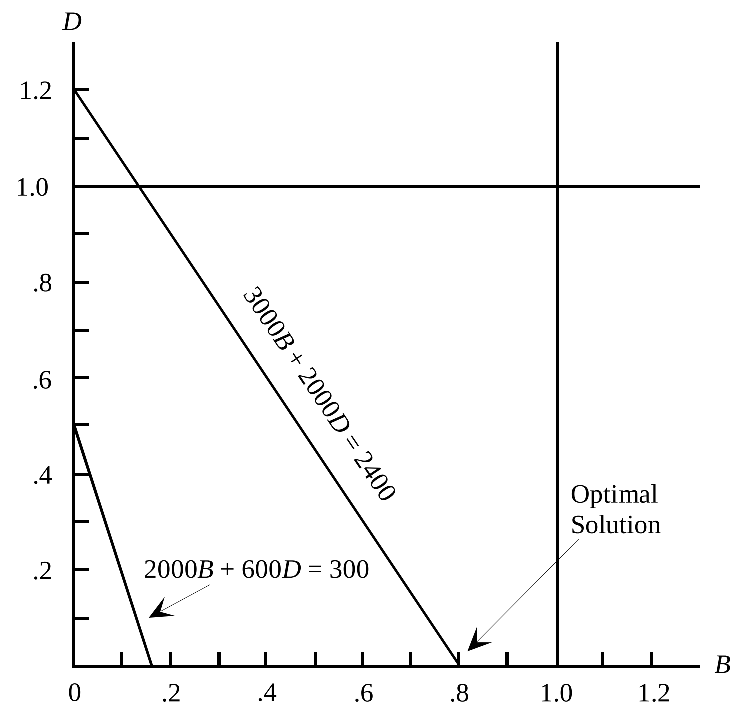
$$\text{Max } 2000B + 600D$$

In addition, $B \leq 1$ and $D \leq 1$.

The linear programming model is:

$$
\begin{aligned}
\text{Max} \quad & 2000B + 600D \\
\text{s.t.} \quad & \\
& 3000B + 2000D = 2400 \\
& B \leq 1 \\
& D \leq 1 \\
& B, D \geq 0
\end{aligned}
$$

The graphical solution is shown below.



Optimal Solution: B = .8, D = 0

Optimal Production Plan

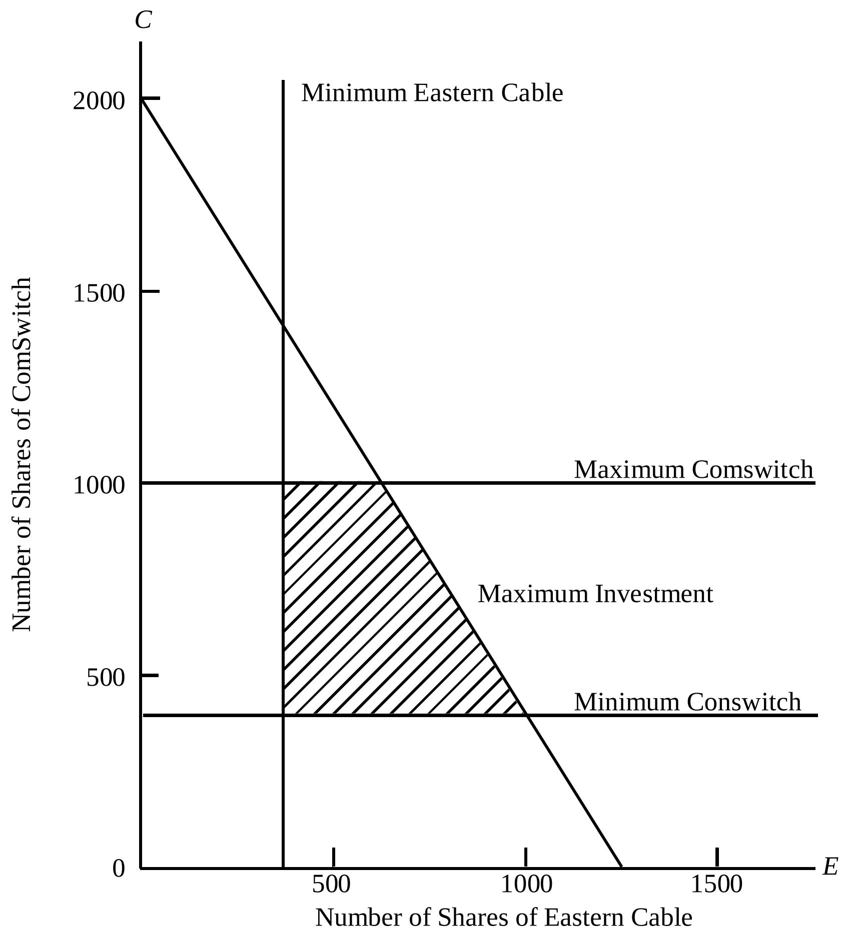
Buffalo - Component 1	.8(2000) = 1600
Buffalo - Component 2	.2(1000) = 200
Dayton - Component 1	0(600) = 0
Dayton - Component 2	1(1400) = 1400

Total units of electronic ignition system = 1600 per day.

30. a. Let E = number of shares of Eastern Cable
C = number of shares of ComSwitch

$$\begin{array}{llll} \text{Max} & 15E + 18C & & \\ \text{s.t.} & & & \\ & 40E + 25C \leq 50{,}000 & & \text{Maximum Investment} \\ & 40E \geq 15{,}000 & & \text{Eastern Cable Minimum} \\ & 25C \geq 10{,}000 & & \text{ComSwitch Minimum} \\ & 25C \leq 25{,}000 & & \text{ComSwitch Maximum} \\ & E, C \geq 0 & & \end{array}$$

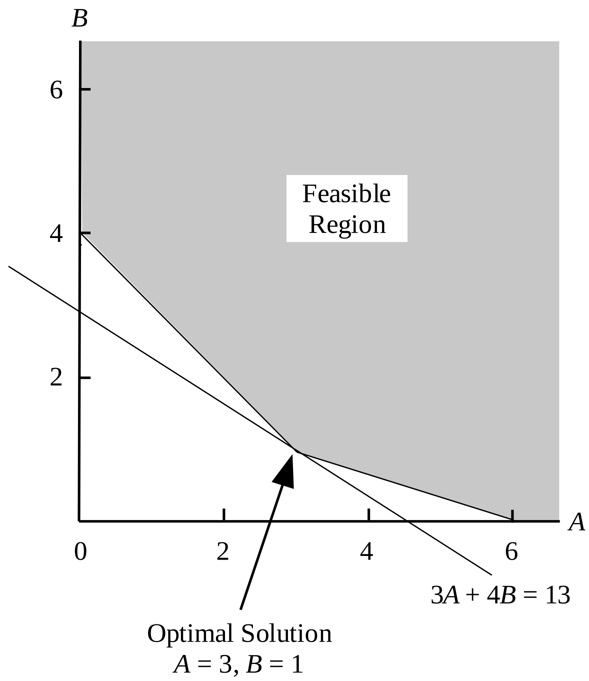
b.



c. There are four extreme points: (375,400); (1000,400);(625,1000); (375,1000)

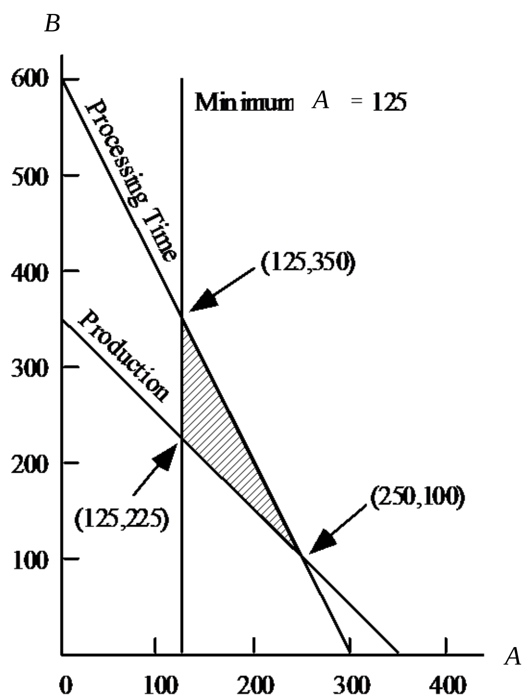
d. Optimal solution is $E = 625$, $C = 1000$
Total return = \$27,375

31.



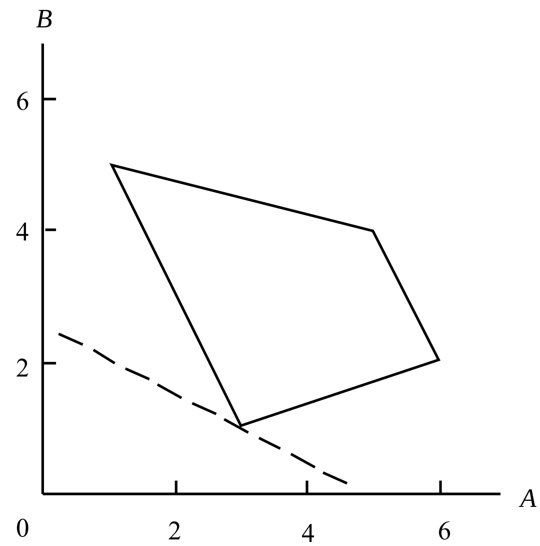
Objective Function Value = 13

32.



Extreme Points	Objective Function Value	Surplus Demand	Surplus Total Production	Slack Processing Time
(*A* = 250, *B* = 100)	800	125	—	—
(*A* = 125, *B* = 225)	925	—	—	125
(*A* = 125, *B* = 350)	1300	—	125	—

33. a.

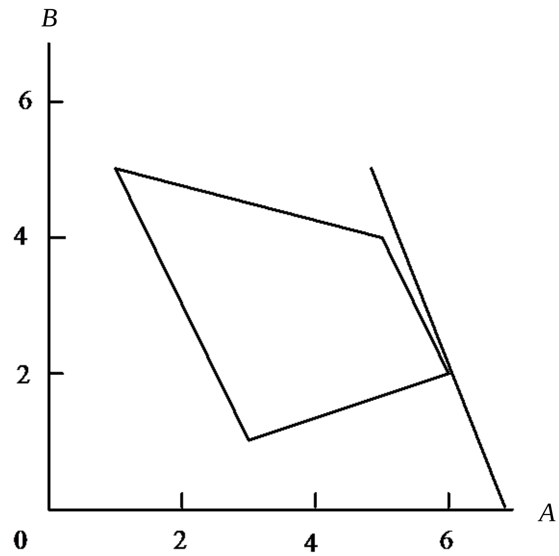


Optimal Solution: *A* = 3, *B* = 1, value = 5

b.

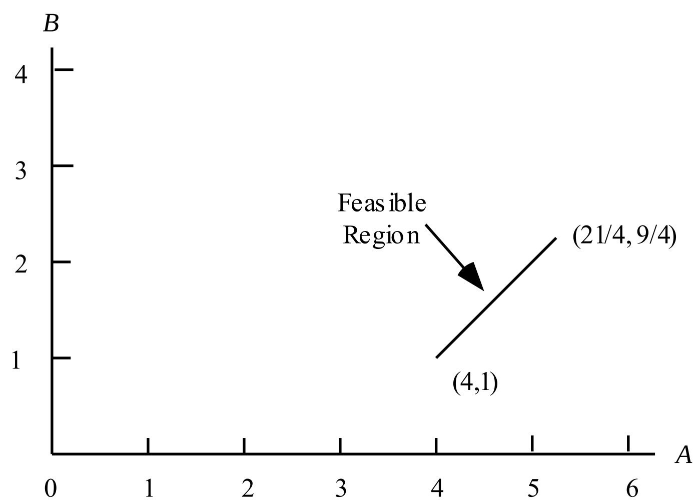
(1)	3 + 4(1) = 7	Slack = 21 - 7 = 14
(2)	2(3) + 1 = 7	Surplus = 7 - 7 = 0
(3)	3(3) + 1.5 = 10.5	Slack = 21 - 10.5 = 10.5
(4)	-2(3) +6(1) = 0	Surplus = 0 - 0 = 0

c.



Optimal Solution: $A = 6$, $B = 2$, value = 34

34. a.



b. There are two extreme points: ($A = 4$, $B = 1$) and ($A = 21/4$, $B = 9/4$)

c. The optimal solution is $A = 4$, $B = 1$

35. a.

$$\begin{array}{lcccccccccccc} \text{Min} & 6A & + & 4B & + & 0S_1 & + & 0S_2 & + & 0S_3 & & \\ \text{s.t.} & & & & & & & & & & & \\ & 2A & + & 1B & - & S_1 & & & & & = & 12 \\ & 1A & + & 1B & & & - & S_2 & & & = & 10 \\ & & & 1B & & & & & + & S_3 & = & 4 \end{array}$$

$$A, B, S_1, S_2, S_3 \geq 0$$

b. The optimal solution is $A = 6$, $B = 4$.

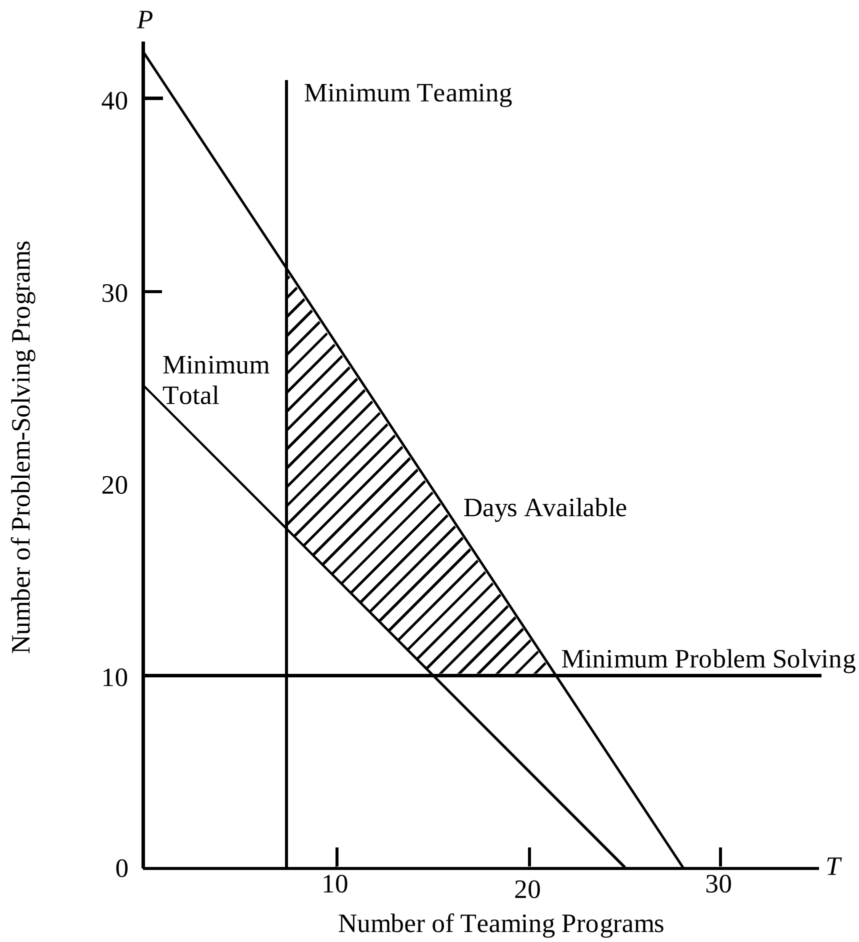
c. $S_1 = 4$, $S_2 = 0$, $S_3 = 0$.

36. a. Let T = number of training programs on teaming
P = number of training programs on problem solving

$$\begin{array}{llcrcrl} \text{Max} & 10{,}000T & + & 8{,}000P & & & \\ \text{s.t.} & & & & & & \\ & T & & & \geq & 8 & \text{Minimum Teaming} \\ & & & P & \geq & 10 & \text{Minimum Problem Solving} \\ & T & + & P & \geq & 25 & \text{Minimum Total} \\ & 3T & + & 2P & \leq & 84 & \text{Days Available} \end{array}$$

$$T, P \geq 0$$

b.



c. There are four extreme points: (15,10); (21.33,10); (8,30); (8,17)

d. The minimum cost solution is $T = 8$, $P = 17$
Total cost = $216,000

37.

	Regular	Zesty	
Mild	80%	60%	8100
Extra Sharp	20%	40%	3000

Let R = number of containers of Regular
Z = number of containers of Zesty

Each container holds 12/16 or 0.75 pounds of cheese

Pounds of mild cheese used $= 0.80\ (0.75)\ R + 0.60\ (0.75)\ Z$
$= 0.60\ R + 0.45\ Z$

Pounds of extra sharp cheese used $= 0.20\ (0.75)\ R + 0.40\ (0.75)\ Z$
$= 0.15\ R + 0.30\ Z$

Cost of Cheese	=	Cost of mild + Cost of extra sharp
	=	1.20 (0.60 R + 0.45 Z) + 1.40 (0.15 R + 0.30 Z)
	=	0.72 R + 0.54 Z + 0.21 R + 0.42 Z
	=	0.93 R + 0.96 Z
Packaging Cost	=	0.20 R + 0.20 Z
Total Cost	=	(0.93 R + 0.96 Z) + (0.20 R + 0.20 Z)
	=	1.13 R + 1.16 Z
Revenue	=	1.95 R + 2.20 Z
Profit Contribution	=	Revenue - Total Cost
	=	(1.95 R + 2.20 Z) - (1.13 R + 1.16 Z)
	=	0.82 R + 1.04 Z

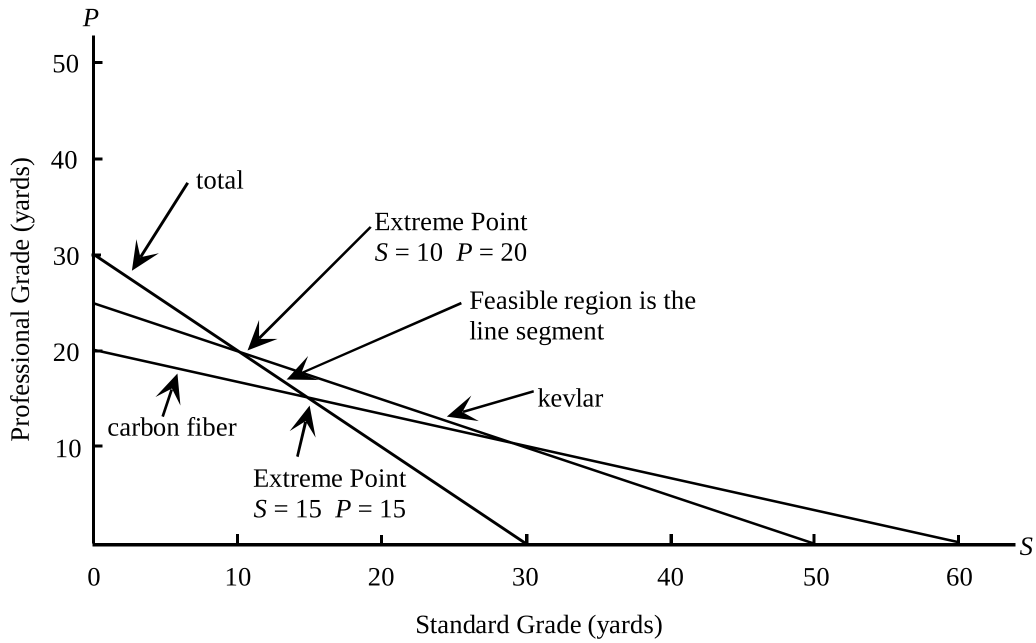
Max	0.82 R	+	1.04 Z			
s.t.						
	0.60 R	+	0.45 Z	≤	8100	Mild
	0.15 R	+	0.30 Z	≤	3000	Extra Sharp
	R, Z ≥ 0					

Optimal Solution: R = 9600, Z = 5200, profit = 0.82(9600) + 1.04(5200) = $13,280

38. a. Let S = yards of the standard grade material per frame
P = yards of the professional grade material per frame

Min	7.50S	+	9.00P			
s.t.						
	0.10S	+	0.30P	≥	6	carbon fiber (at least 20% of 30 yards)
	0.06S	+	0.12P	≤	3	kevlar (no more than 10% of 30 yards)
	S	+	P	=	30	total (30 yards)
	S, P ≥ 0					

b.



c.

Extreme Point	Cost
(15, 15)	7.50(15) + 9.00(15) = 247.50
(10, 20)	7.50(10) + 9.00(20) = 255.00

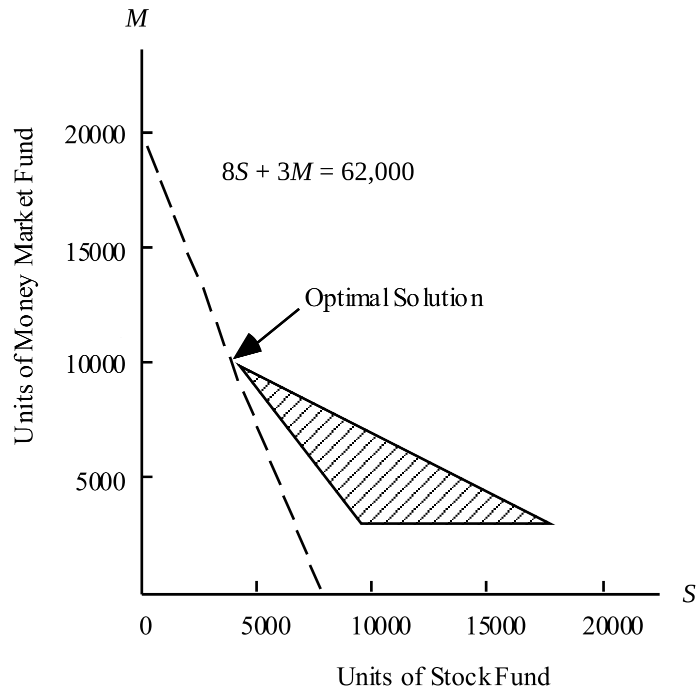
The optimal solution is $S = 15$, $P = 15$

d. Optimal solution does not change: $S = 15$ and $P = 15$. However, the value of the optimal solution is reduced to 7.50(15) + 8(15) = $232.50.

e. At $7.40 per yard, the optimal solution is $S = 10$, $P = 20$. The value of the optimal solution is reduced to 7.50(10) + 7.40(20) = $223.00. A lower price for the professional grade will not change the $S = 10$, $P = 20$ solution because of the requirement for the maximum percentage of kevlar (10%).

39. a. Let S = number of units purchased in the stock fund
M = number of units purchased in the money market fund

$$
\begin{array}{llrcrcrl}
\text{Min} & & 8S & + & 3M & & & \\
\text{s.t.} & & & & & & & \\
& & 50S & + & 100M & \leq & 1{,}200{,}000 & \text{Funds available} \\
& & 5S & + & 4M & \geq & 60{,}000 & \text{Annual income} \\
& & & & M & \geq & 3{,}000 & \text{Minimum units in money market} \\
& & & & S, M, & \geq & 0 &
\end{array}
$$



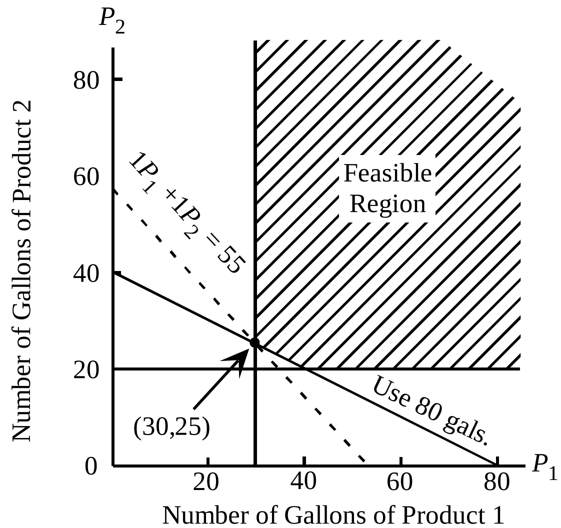
Optimal Solution: S = 4000, M = 10000, value = 62000

b. Annual income = 5(4000) + 4(10000) = 60,000

c. Invest everything in the stock fund.

40. Let P_1 = gallons of product 1
P_2 = gallons of product 2

Min	$1P_1$	+	$1P_2$			
s.t.						
	$1P_1$	+		$\geq$	30	Product 1 minimum
			$1P_2$	$\geq$	20	Product 2 minimum
	$1P_1$	+	$2P_2$	$\geq$	80	Raw material
		$P_1, P_2 \geq 0$				



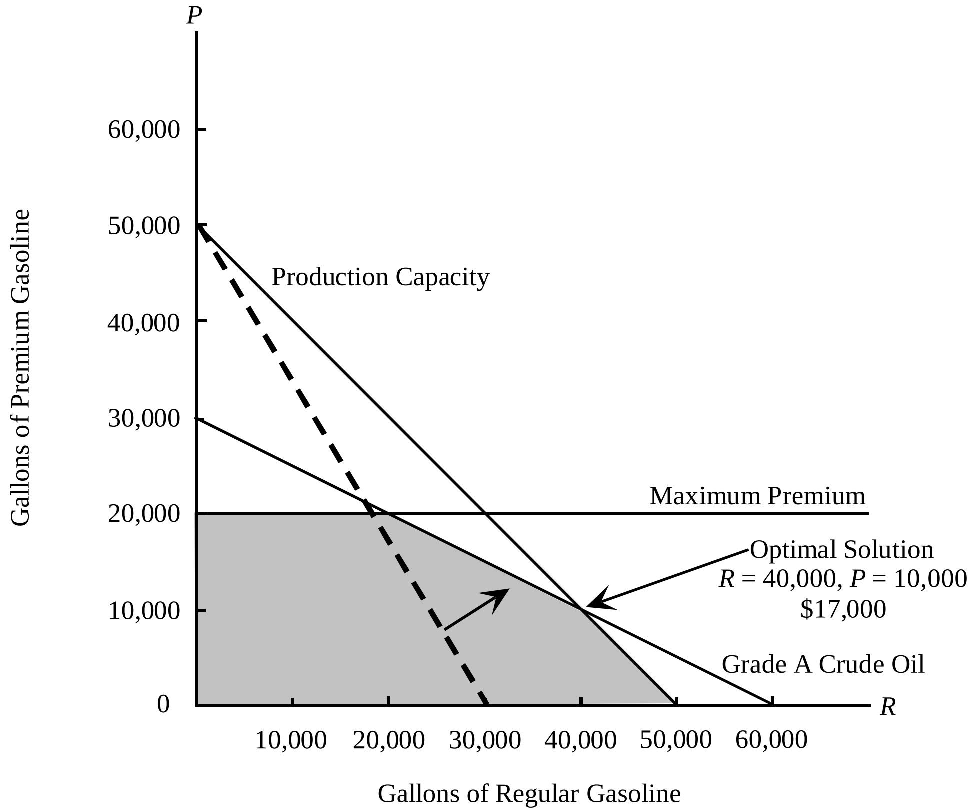
Optimal Solution: $P_1 = 30$, $P_2 = 25$ Cost = $55

41. a. Let R = number of gallons of regular gasoline produced
P = number of gallons of premium gasoline produced

Max	$0.30R$	+	$0.50P$			
s.t.						
	$0.30R$	+	$0.60P$	$\leq$	18,000	Grade A crude oil available
	$1R$	+	$1P$	$\leq$	50,000	Production capacity
			$1P$	$\leq$	20,000	Demand for premium

$R,\ P \geq 0$

b.



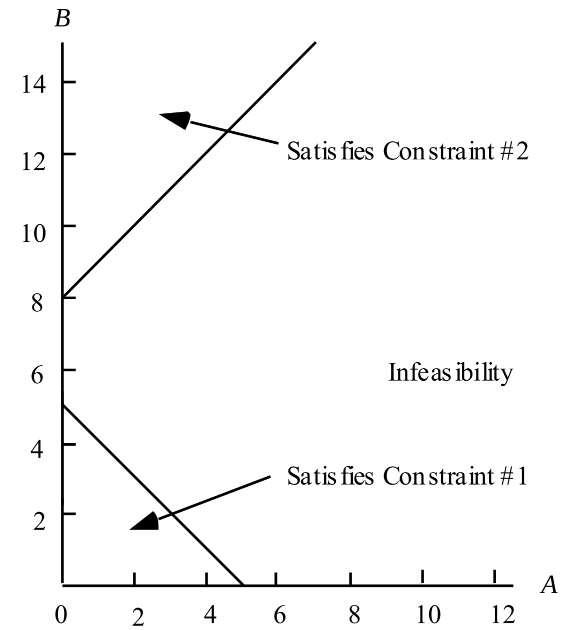
Optimal Solution:
40,000 gallons of regular gasoline
10,000 gallons of premium gasoline
Total profit contribution = $17,000

c.

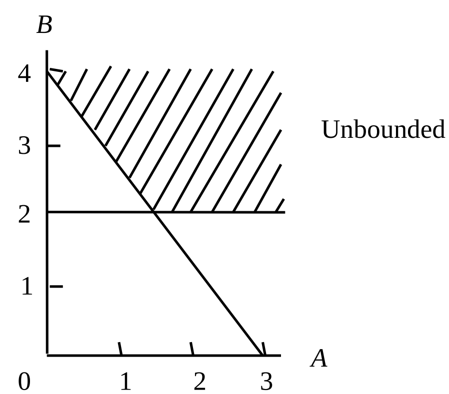
Constraint	Value of Slack Variable	Interpretation
1	0	All available grade A crude oil is used
2	0	Total production capacity is used
3	10,000	Premium gasoline production is 10,000 gallons less than the maximum demand

d. Grade A crude oil and production capacity are the binding constraints.

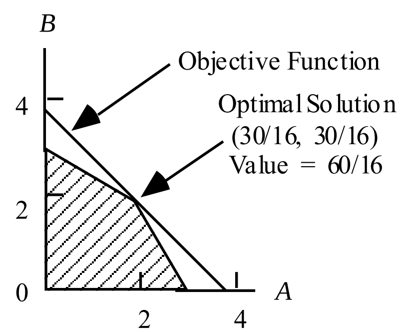
42.



43.

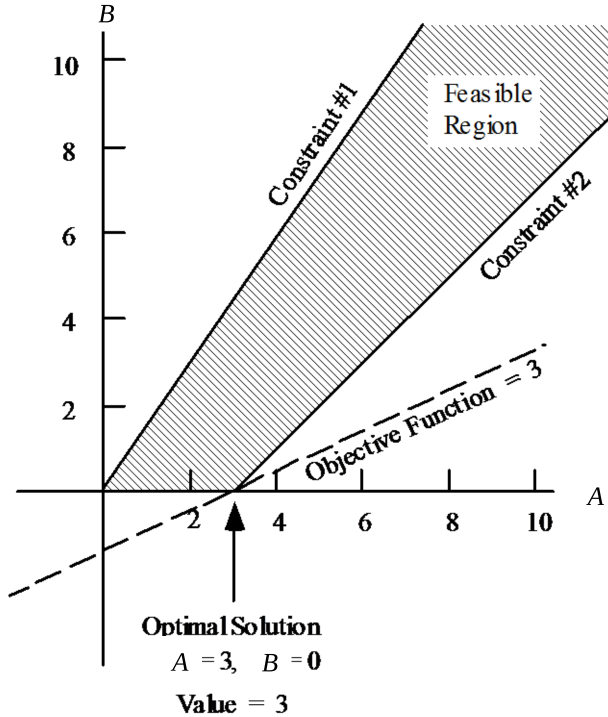


44. a.



b. New optimal solution is $A = 0$, $B = 3$, value = 6.

45. a.



b. Feasible region is unbounded.

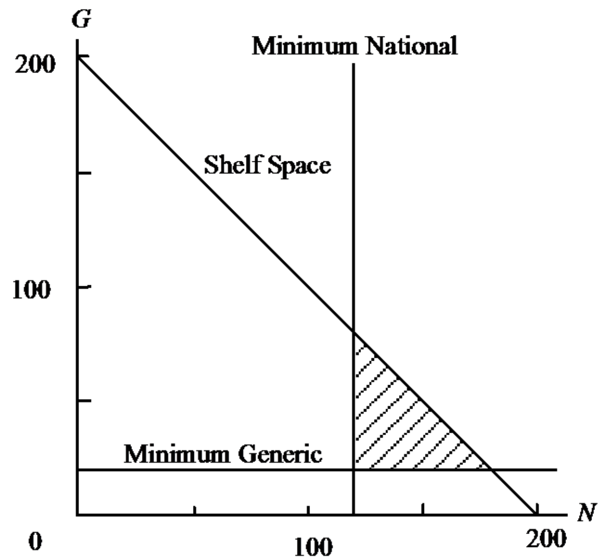
c. Optimal Solution: $A = 3$, $B = 0$, $z = 3$.

d. An unbounded feasible region does not imply the problem is unbounded. This will only be the case when it is unbounded in the direction of improvement for the objective function.

46. Let N = number of sq. ft. for national brands
G = number of sq. ft. for generic brands

Problem Constraints:

N	+	G	$\leq$	200	Space available
N			$\geq$	120	National brands
		G	$\geq$	20	Generic



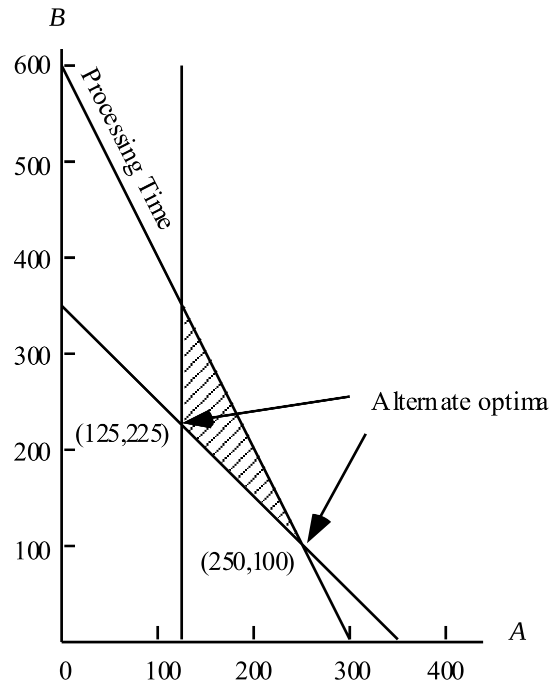
Extreme Point	*N*	*G*
1	120	20
2	180	20
3	120	80

a. Optimal solution is extreme point 2; 180 sq. ft. for the national brand and 20 sq. ft. for the generic brand.

b. Alternative optimal solutions. Any point on the line segment joining extreme point 2 and extreme point 3 is optimal.

c. Optimal solution is extreme point 3; 120 sq. ft. for the national brand and 80 sq. ft. for the generic brand.

47.



Alternative optimal solutions exist at extreme points (A = 125, B = 225) and (A = 250, B = 100).

Cost = 3(125) + 3(225) = 1050

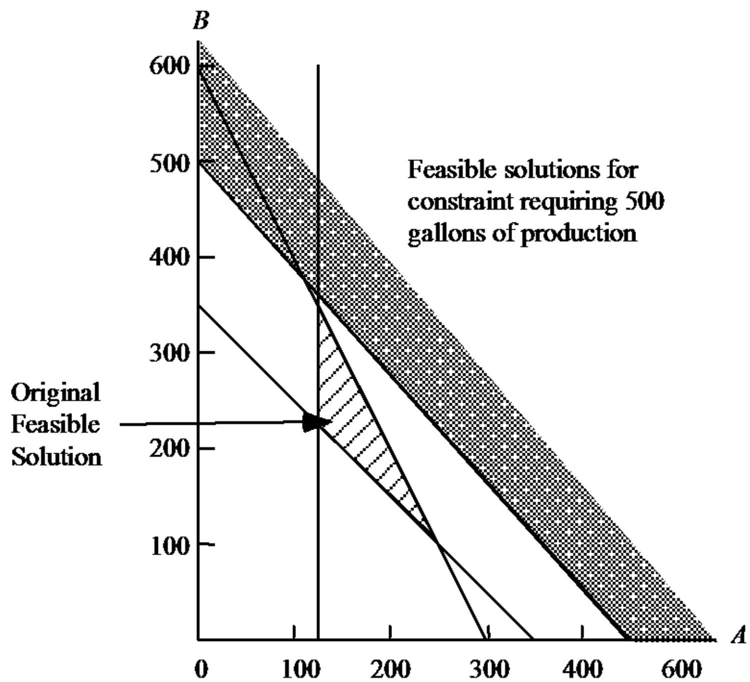
or

Cost = 3(250) + 3(100) = 1050

The solution (A = 250, B = 100) uses all available processing time. However, the solution (A = 125, B = 225) uses only 2(125) + 1(225) = 475 hours.

Thus, (A = 125, B = 225) provides 600 - 475 = 125 hours of slack processing time which may be used for other products.

48.



Possible Actions:

i. Reduce total production to $A = 125$, $B = 350$ on 475 gallons.

ii. Make solution $A = 125$, $B = 375$ which would require $2(125) + 1(375) = 625$ hours of processing time. This would involve 25 hours of overtime or extra processing time.

iii. Reduce minimum A production to 100, making $A = 100$, $B = 400$ the desired solution.

49. a. Let P = number of full-time equivalent pharmacists
T = number of full-time equivalent physicians

The model and the optimal solution are shown below:

```
MIN 40P+10T

     S.T.
       1)   P+T >=250
       2)   2P-T>=0
       3)   P>=90
```

Optimal Objective Value
5200.00000

Variable	Value	Reduced Cost
P	90.00000	0.00000
T	160.00000	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	10.00000
2	20.00000	0.00000
3	0.00000	30.00000

The optimal solution requires 90 full-time equivalent pharmacists and 160 full-time equivalent technicians. The total cost is $5200 per hour.

b.

	Current Levels	**Attrition**	**Optimal Values**	**New Hires Required**
Pharmacists	85	10	90	15
Technicians	175	30	160	15

The payroll cost using the current levels of 85 pharmacists and 175 technicians is 40(85) + 10(175) = $5150 per hour.

The payroll cost using the optimal solution in part (a) is $5200 per hour.

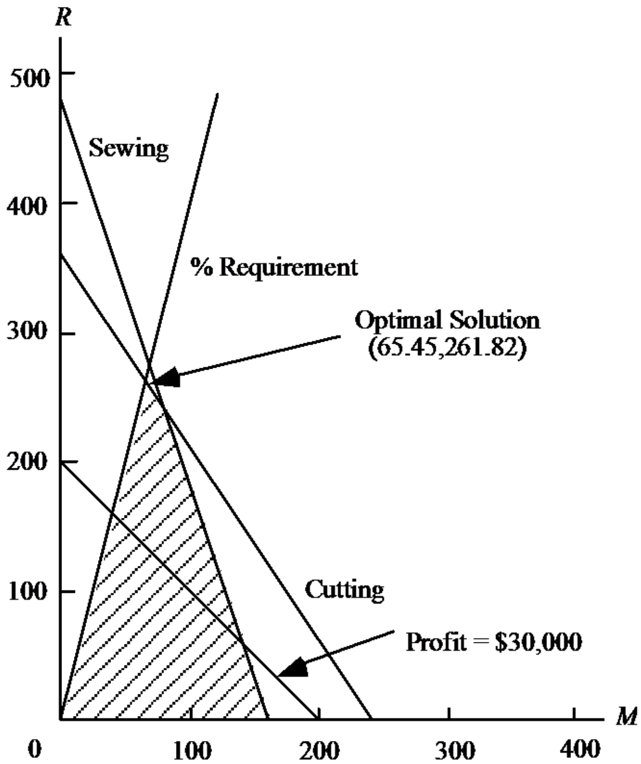
Thus, the payroll cost will go up by $50

50. Let M = number of Mount Everest Parkas
R = number of Rocky Mountain Parkas

$$\begin{array}{lrcrcrl} \text{Max} & 100M & + & 150R & & & \\ \text{s.t.} & & & & & & \\ & 30M & + & 20R & \leq & 7200 & \text{Cutting time} \\ & 45M & + & 15R & \leq & 7200 & \text{Sewing time} \\ & 0.8M & - & 0.2R & \geq & 0 & \text{\% requirement} \end{array}$$

Note: Students often have difficulty formulating constraints such as the % requirement constraint. We encourage our students to proceed in a systematic step-by-step fashion when formulating these types of constraints. For example:

M must be at least 20% of total production
$M \geq 0.2$ (total production)
$M \geq 0.2\,(M + R)$
$M \geq 0.2M + 0.2R$
$0.8M - 0.2R \geq 0$



The optimal solution is M = 65.45 and R = 261.82; the value of this solution is z = 100(65.45) + 150(261.82) = $45,818. If we think of this situation as an on-going continuous production process, the fractional values simply represent partially completed products. If this is not the case, we can approximate the optimal solution by rounding down; this yields the solution M = 65 and R = 261 with a corresponding profit of $45,650.

51. Let C = number sent to current customers
N = number sent to new customers

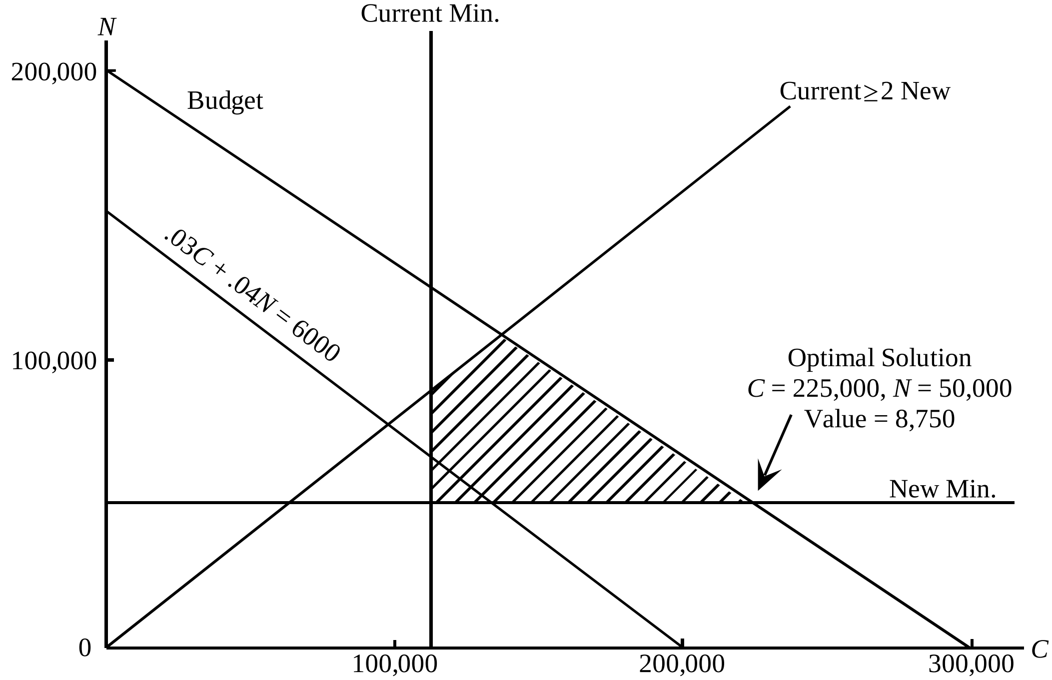
Note:

Number of current customers that test drive = .25 C

Number of new customers that test drive = .20 N

Number sold = .12 (.25 C) + .20 (.20 N)
= .03 C + .04 N

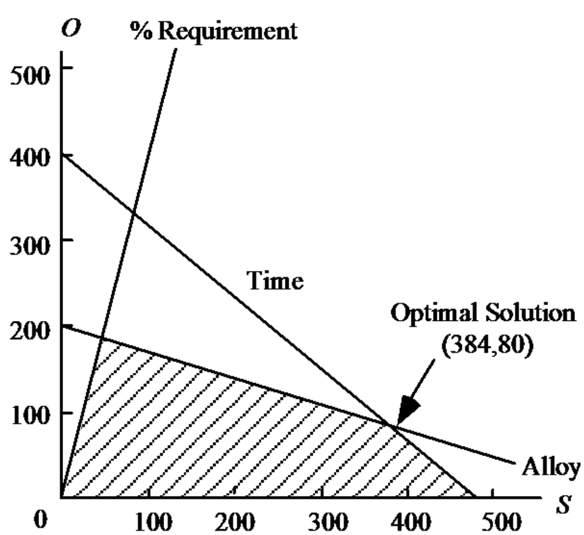
$$
\begin{array}{llrlrl}
\text{Max} & .03C & + & .04N & & \\
\text{s.t.} & & & & & \\
& .25\,C & & & \geq 30{,}000 & \text{Current Min} \\
& & & .20\,N & \geq 10{,}000 & \text{New Min} \\
& .25\,C & - & .40\,N & \geq 0 & \text{Current vs. New} \\
& 4\,C & + & 6\,N & \leq 1{,}200{,}000 & \text{Budget} \\
& C,\ N,\ \geq 0 & & & &
\end{array}
$$



52. Let S = number of standard size rackets
O = number of oversize size rackets

Max	$10S$	+	$15O$			
s.t.						
	$0.8S$	-	$0.2O$	$\geq$	0	% standard
	$10S$	+	$12O$	$\leq$	4800	Time
	$0.125S$	+	$0.4O$	$\leq$	80	Alloy

$S, O, \geq 0$



53. a. Let R = time allocated to regular customer service
N = time allocated to new customer service

$$\begin{array}{lrcrcr} \text{Max} & 1.2R & + & N & & \\ \text{s.t.} & & & & & \\ & R & + & N & \leq & 80 \\ & 25R & + & 8N & \geq & 800 \\ & -0.6R & + & N & \geq & 0 \end{array}$$

$$R,\ N,\ \geq 0$$

b.

Optimal Objective Value
90.00000

Variable	Value	Reduced Cost
R	50.00000	0.00000
N	30.00000	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	1.12500
2	690.00000	0.00000
3	0.00000	-0.12500

Optimal solution: $R = 50$, $N = 30$, value = 90

HTS should allocate 50 hours to service for regular customers and 30 hours to calling on new customers.

54. a. Let M_1 = number of hours spent on the M-100 machine
M_2 = number of hours spent on the M-200 machine

Total Cost
$$6(40)M_1 + 6(50)M_2 + 50M_1 + 75M_2 = 290M_1 + 375M_2$$

Total Revenue
$$25(18)M_1 + 40(18)M_2 = 450M_1 + 720M_2$$

Profit Contribution
$$(450 - 290)M_1 + (720 - 375)M_2 = 160M_1 + 345M_2$$

$$\begin{array}{llllll}
\text{Max} & 160\, M_1 & + & 345 M_2 & & \\
\text{s.t.} & & & & & \\
& M_1 & & & \leq 15 & \text{M-100 maximum} \\
& & & M_2 & \leq 10 & \text{M-200 maximum} \\
& M_1 & & & \geq 5 & \text{M-100 minimum} \\
& & & M_2 & \geq 5 & \text{M-200 minimum} \\
& 40\, M_1 & + & 50\, M_2 & \leq 1000 & \text{Raw material available}
\end{array}$$

$$M_1,\ M_2 \geq 0$$

b.

Optimal Objective Value
5450.00000

Variable	Value	Reduced Cost
M1	12.50000	0.00000
M2	10.00000	145.00000

Constraint	Slack/Surplus	Dual Value
1	2.50000	0.00000
2	0.00000	145.00000
3	7.50000	0.00000
4	5.00000	0.00000
5	0.00000	4.00000

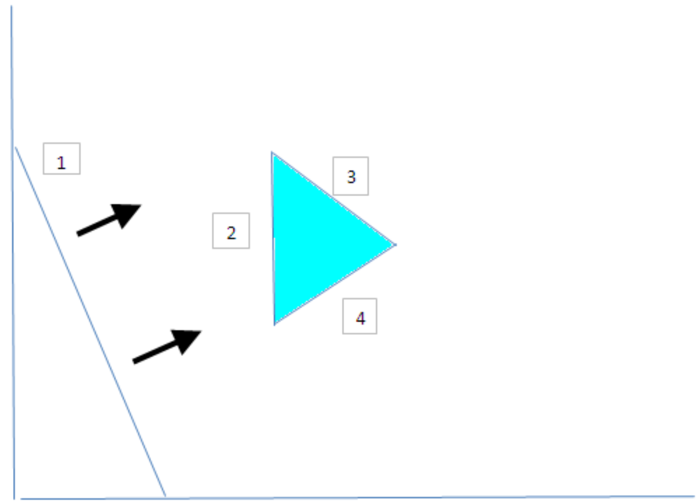
The optimal decision is to schedule 12.5 hours on the M-100 and 10 hours on the M-200.

55. Mr. Krtick's solution cannot be optimal. Every department has unused hours, so there are no binding constraints. With unused hours in every department, clearly some more product can be made.

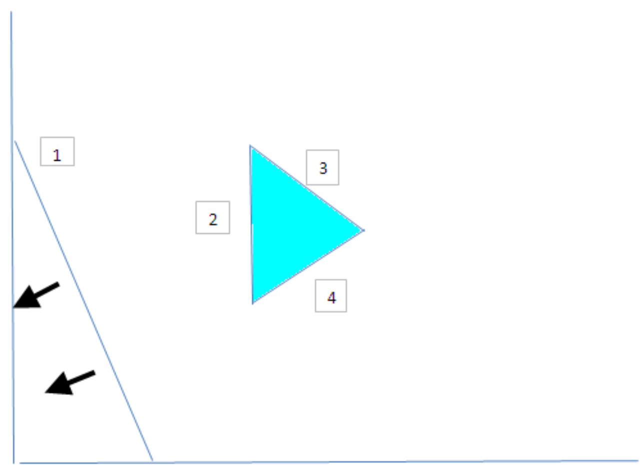
56. No, it is not possible that the problem is now infeasible. Note that the original problem was feasible (it had an optimal solution). Every solution that was feasible is still feasible when we change the constraint to less-than-or-equal-to, since the new constraint is satisfied at equality (as well as inequality). In summary, we have relaxed the constraint so that the previous solutions are feasible (and possibly more satisfying the constraint as strict inequality).

57. Yes, it is possible that the modified problem is infeasible. To see this, consider a redundant greater-than-or-equal to constraint as shown below. Constraints 2,3, and 4 form the feasible region and constraint 1 is redundant. Change constraint 1 to less-than-or-equal-to and the modified problem is infeasible.

Original Problem:



Modified Problem:



58. It makes no sense to add this constraint. The objective of the problem is to minimize the number of products needed so that everyone's top three choices are included. There are only two possible outcomes relative to the boss' new constraint. First, suppose the minimum number of products is <= 15, then there was no need for the new constraint. Second, suppose the minimum number is > 15. Then the new constraint makes the problem infeasible.