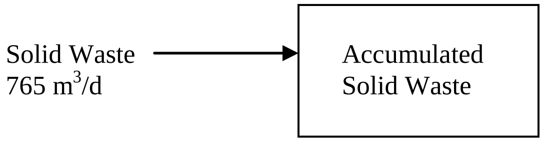
CHAPTER 2
SOLUTIONS

2-1 Expected life of landfill

Given: 16.2 ha at depth of 10 m, 765 m^3 dumped 5 days per week, compacted to twice delivered density

Solution:

a. Mass balance diagram



b. Total volume of landfill

$$(16.2 \text{ ha})(10^4 \text{ m}^2/\text{ha})(10 \text{ m}) = 1.620 \times 10^6 \text{ m}^3$$

c. Volume of solid waste is ½ delivered volume after it is compacted to 2 times its delivered density

$$(765 \text{ m}^3)(0.5) = 382.5 \text{ m}^3$$

d. Annual volume of solid waste placed in landfill

$$(382.5 \text{ m}^3)(5 \text{ d/wk})(52 \text{ wk/y}) = 9.945 \times 10^4 \text{ m}^3/\text{y}$$

e. Estimated expected life

$$\frac{1.620\times10^6 \text{ m}^3}{9.945\times10^4 \text{ m}^3/\text{y}} = 16.29 \text{ or 16 years}$$

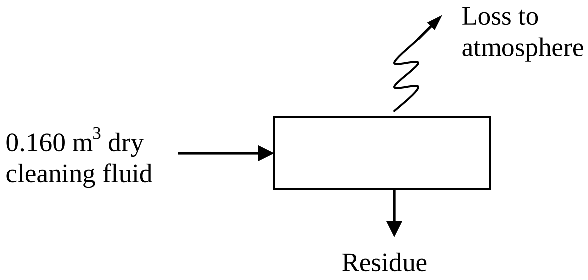
NOTE: the actual life will be somewhat less due to the need to cover the waste with soil each day.

2-2 Estimated emission of dry cleaning fluid

Given: 1 barrel (0.160 m^3) of dry cleaning fluid per month, density = 1.5940 g/mL, 90% lost to atmosphere.

Solution:

a. Mass balance diagram



b. Mass of dry cleaning fluid into tank

$$\frac{\left(0.160\,\mathrm{m^3/mo}\right)\left(1.5940\,\mathrm{g/mL}\right)\left(1000\,\mathrm{mL/L}\right)\left(1000\,\mathrm{L/m^3}\right)}{1000\,\mathrm{g/kg}} = 255.04\,\mathrm{kg/mo}$$

c. Mass emission rate at 90% loss

(0.90)(255.04 kg/month) = 229.54 kg/month

2-3 Estimated emission of a new dry cleaning fluid

Given: Problem 2-2, Volatility = 1/6 of former fluid, Density = 1.622 g/mL

Solution:

a. Mass balance diagram same as problem 2-2

b. Mass of dry cleaning fluid into tank

$$\frac{\left(0.160\,\mathrm{m^3/mo}\right)\left(1.6620\,\mathrm{g/mL}\right)\left(1000\,\mathrm{mL/L}\right)\left(1000\,\mathrm{L/m^3}\right)}{1000\,\mathrm{g/kg}} = 265.92\,\mathrm{kg/mo}$$

c. Mass emission rate at 1/6 volatility

(1/6)(0.90)(265.92 kg/mo) = 39.89 kg/mo

d. Savings in volume (note: 1.0 g/mL = 1000 kg/m^3)

Old dry cleaning fluid (from problem 2-2)

$$\text{Mass}_{\text{out}} = (0.90)(255.04 \text{ kg/mo}) = 229.54 \text{ kg/mo}$$

$$\cancel{V}_{\text{out}} = \frac{229.54\,\text{kg/mo}}{1594\,\text{kg/m}^3} = 0.1440\,\text{m}^3/\text{mo}$$

New dry cleaning volume

$$\cancel{V}_{\text{out}} = \frac{39.89\,\text{kg/mo}}{1622\,\text{kg/m}^3} = 0.0240\,\text{m}^3/\text{mo}$$

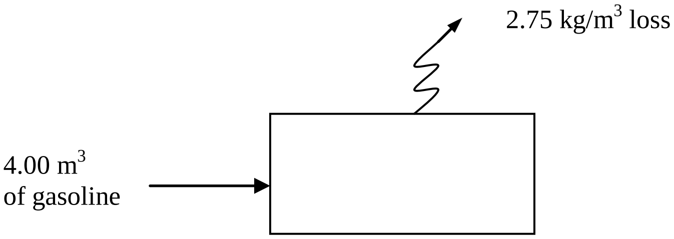
Savings = (0.1440 m^3/mo – 0.0240 m^3/mo)(12 mo/y) = 1.44 m^3/y

2-4 Annual loss of gasoline

Given: Uncontrolled loss = 2.75 kg/m^3 of gasoline
Controlled loss = 0.095 kg/m^3 of gasoline
Refill tank once a week
Tank volume = 4.00 m^3
Specific gravity of gasoline is 0.80
Condensed vapor density = 0.80 g/mL
Cost of gasoline = \$0.80/L

Solution:

a. Mass balance diagram



b. Annual loss with splash fill method

$$\text{Loss} = (4.00 \text{ m}^3/\text{wk})(2.75 \text{ kg/m}^3)(52 \text{ wk/y}) = 572 \text{ kg/y}$$

c. Value of fuel captured with vapor control

Mass captured = (4.00 m^3/wk)(2.75 kg/m^3 – 0.095 kg/m^3)(52 wk/y) = 552.24 kg/y

Value (note: 1.0 g/mL = 1000 kg/m^3)

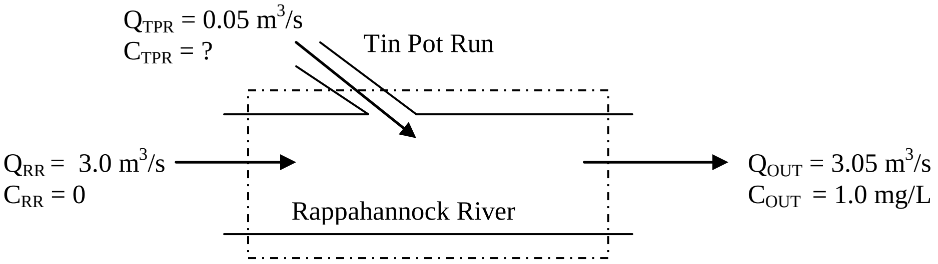
$$\frac{(552.24\,\text{kg/y})(1000\,\text{L/m}^3)}{800\,\text{kg/m}^3}(\$1.06/\text{L}) = \$731.72 \text{ or } \$732/\text{y}$$

2-5 Mass rate of tracer addition

Given: Q_{RR} = 3.00 m^3/s, Q_{TPR} = 0.05 m^3/s, detection limit = 1.0 mg/L

Solution:

a. Mass balance diagram (NOTE: $Q_{out} = Q_{RR} + Q_{TPR}$ = 3.05 m^3/s)



b. Mass balance equation

$$C_{RR}Q_{RR} + C_{TRP}Q_{TPR} = C_{out}Q_{out}$$

Because C_{RR} in = 0 this equation reduces to:

$$C_{TPR}Q_{TPR} = C_{out}Q_{out}$$

c. Note that the quantity $C_{TPR}Q_{TPR}$ is the mass flow rate of the tracer into TPR and substitute values

$$C_{TPR}Q_{TPR} = \frac{1.0\text{mg}}{\text{L}} \times \frac{3.05\text{m}^3}{\text{s}} \times \frac{1000\text{L}}{\text{m}^3} \times \frac{1\text{kg}}{10^6\,\text{mg}} \times \frac{86400\text{s}}{\text{d}} = 264\,\text{kg/d}$$

d. Concentration in Tin Pot Run

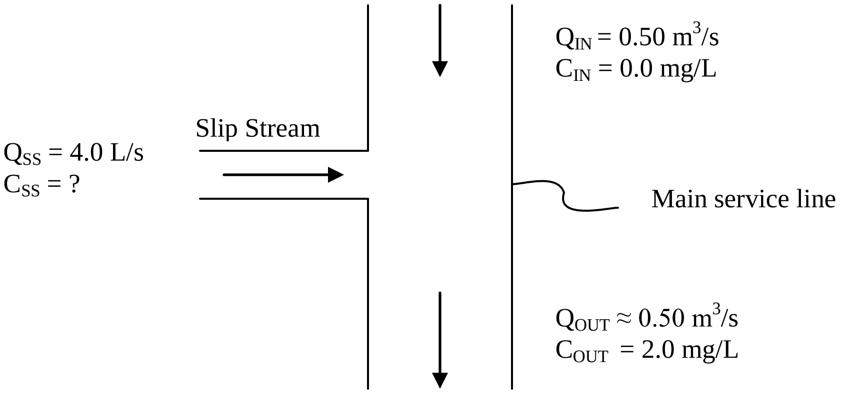
$$C_{TPR} = \frac{C_{TPR}Q_{TPR}}{Q_{TPR}} = \frac{(264\,\text{kg/d})(10^6\,\text{mg/kg})}{(0.05\,\text{m}^3/\text{s})(86400\,\text{s/d})(1000\,\text{L/m}^3)} = 61 \text{ or } 60 \text{ mg/L}$$

2-6 NaOCl pumping rate

Given: NaOCl at 52,000 mg/L
Piping scheme in figure P-2-6
Main service line flow rate = 0.50 m^3/s
Slip stream flow rate 4.0 L/s

Solution:

a. Mass balance at return of slip stream to main service line



b. Calculate C_{SS}

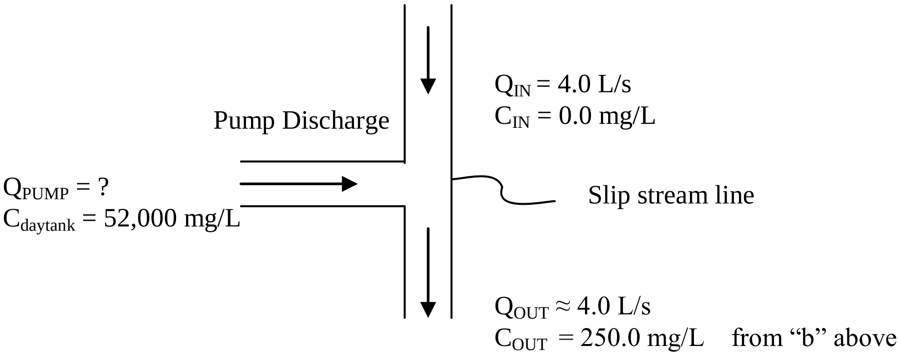
Mass out = Mass in

$(0.50\ m^3/s)(2.0\ mg/L)(1000\ L/m^3) = (4.0\ L/s)(C_{SS})$

$1000\ mg/s = (4.0\ L/s)(C_{SS})$

$$C_{SS} = \frac{1000\, mg/s}{4.0\, L/s} = 250\, mg/L$$

c. Mass balance at the junction of pump discharge and slip stream line



d. Calculate Q_{PUMP}

Mass in = Mass out

$(Q_{PUMP})(52{,}000\ \text{mg/L}) = (4.0\ \text{L/s})(250\ \text{mg/L})$

$$Q_{PUMP} = \frac{(4.0\,\text{L/s})(250\,\text{mg/L})}{52{,}000\,\text{mg/L}} = 0.0192\,\text{L/s}$$

2-7 Dilution of NaOCl in day tank

Given: Pump rated at 1.0 L/s
8 hour shift
NaOCl feed rate 1000 mg/s
Stock solution from Prob 2-6 = 52,000 mg/L

Solution:

a. Mass of NaOCl to be fed in 8 h

$(8\ \text{h})(3600\ \text{s/h})(1000\ \text{mg/s}) = 2.88 \times 10^7\ \text{mg}$

b. Volume of stock solution

$$\frac{2.88 \times 10^7\,\text{mg}}{52{,}000\,\text{mg/L}} = 5.54 \times 10^2\,\text{L} \text{ or } 0.554\ \text{m}^3$$

c. Volume of dilution water

$(8\ \text{h})(3600\ \text{s/h})(1.0\ \text{L/s}) = 2.88 \times 10^4\ \text{L}$ or $28.8\ \text{m}^3$

d. Check

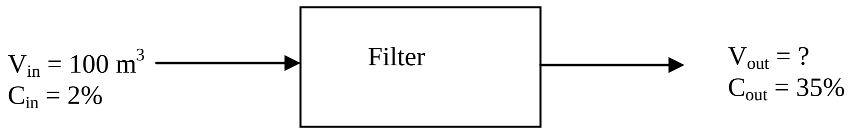
$$28.8 \text{ m}^3 + 0.554 \text{ m}^3 = 29.4 \text{ m}^3 < 30 \text{ m}^3$$

2-8 Volume of sludge after filtration

Given: Sludge concentration of 2%, sludge volume = 100 m^3, sludge concentration after filtration = 35%

Solution:

a. Mass balance diagram



b. Mass balance equation

$$C_{in}\forall_{in} = C_{out}\forall_{out}$$

c. Solve for $\forall_{out}$

$$V_{out} = \frac{C_{in}\forall_{in}}{C_{out}}$$

d. Substituting values

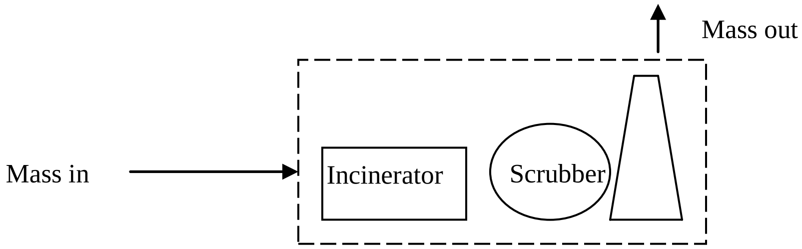
$$V_{out} = \frac{(0.02)(100\text{m}^3)}{0.35} = 5.71\text{m}^3$$

2-9 Hazardous waste incinerator emission

Given: Four nines DRE
Mass flow rate in = 1.0000 g/s
Incinerator is 90% efficient

Solution:

a. Mass balance diagram



b. Allowable quantity in exit stream

Mass out = (1 - DRE)(Mass in)

= (1 – 0.9999)(1.0000 g/s) = 0.00010 g/s

c. Scrubber efficiency

Mass out of incinerator = (1 – 0.90)(1.000 g/s) = 0.10000 g/s

Mass out of scrubber must be 0.00010 g/s from “b”, therefore

$$\eta = \frac{0.1000\,\mathrm{g/s} - 0.00010\,\mathrm{g/s}}{0.1000\,\mathrm{g/s}} = 0.999 \text{ or } 99.9\%$$

2-10 Sampling filter efficiency

Given: First filter captures 1941 particles
Second filter captures 63 particles
Figure P-2-10
Each filter has same efficiency

Solution:

a. Note that

$$\eta = \frac{C_2}{C_1} \text{ and } \eta = \frac{C_3}{C_2}$$

b. The concentration C_2 is

$C_2 = C_1 - 1{,}941$

c. Substitute efficiency for C_1 and C_2

$$\frac{63}{\eta} = \frac{1941}{\eta} - 1941$$

d. Solve for η

$$63 = 1{,}941 - 1941\eta$$

$$-1{,}941\eta = 63 - 1941 = -1{,}878$$

$$\eta = \frac{1878}{1941} = 0.9675$$

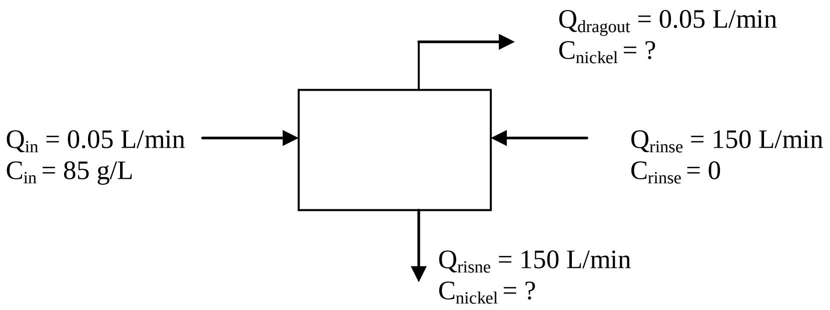
e. The efficiency of the sampling filters is 96.75%

2-11 Concentration of nickel in wastewater stream

Given: Figure P – 2-11, concentration of plating solution = 85 g/L, drag-out rate = 0.05 L/min, flow into rinse tank = 150 L/min, assume no accumulation in tank.

Solution:

a. Mass balance diagram



b. Mass balance equation

$$Q_{in}C_{in} + Q_{rinse}C_{rinse} - Q_{dragout}C_{nickel} - Q_{rinse}C_{nickel} = 0$$

c. Because $C_{rinse} = 0$ this reduces to

$$Q_{in}C_{in} = Q_{dragout}C_{nickel} + Q_{rinse}C_{nickel}$$

d. Solving for C_{nickel}

$$C_{\text{nickel}} = \frac{Q_{\text{in}} C_{\text{in}}}{Q_{\text{dragout}} + Q_{\text{rinse}}}$$

e. Substituting values

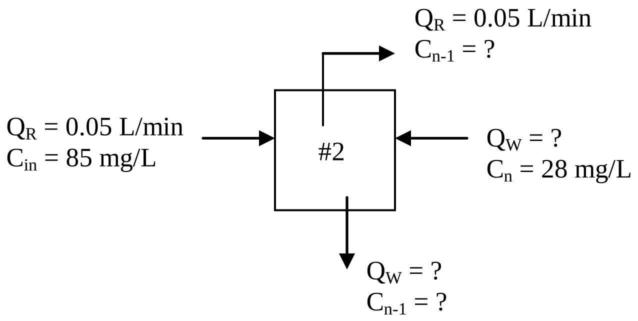
$$C_{\text{nickel}} = \frac{(0.05\,\text{L/min})(85\,\text{g/L})}{0.05\,\text{L/min} + 150\,\text{L/min}} = 28\,\text{mg/L}$$

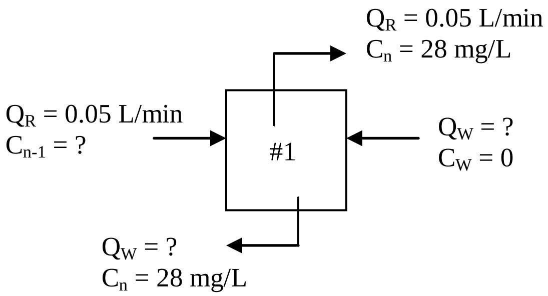
2-12 Counter-current rinse tanks

Given: Figure P-2-12, $C_n = 28$ mg/L, assume no accumulation in tanks

Solution:

a. Because there are two unknowns we must set up two mass balance equations and solve them simultaneously. The mass balance diagrams are:





b. Mass balance equation, starting with the right-hand rinse tank (#1)

$$(C_{n-1})(Q_R) + (C_W)(Q_W) = (C_n)(Q_R) + (C_n)(Q_W)$$

c. Note that $(C_w)(Q_R) = 0$ because $C_W = 0$, then solve for Q_W

$$Q_W = \frac{(C_{n-1})(Q_R) - (C_n)(Q_R)}{C_n}$$

$$Q_W = \frac{Q_R\left[(C_{n-1}) - C_n\right]}{C_n}$$

d. Mass balance equation for tank at the left hand side (#2)

$$(C_{in})(Q_R) + (C_n)(Q_W) = (C_{n-1})(Q_R) + (C_{n-1})(Q_W)$$

e. Solving for C_{n-1}

$$C_{n-1} = \frac{(C_{in})(Q_R) + (C_n)(Q_W)}{Q_R + Q_W}$$

f. Substitute solution for tank #2 into solution for tank #1 and simplify

$$Q_W = \frac{Q_R\left[\dfrac{C_{in}Q_R + C_nQ_W}{Q_R + Q_W} - C_n\right]}{C_n}$$

$$Q_W{}^2 + Q_RQ_W + Q_R{}^2\left(\frac{C_n - C_{in}}{C_n}\right) = 0$$

This equation is a quadratic equation with a = 1, b = Q_R and $c = Q_R{}^2\left(\frac{C_n - C_{in}}{C_n}\right)$

g. The solution to the quadratic equation is

$$Q_W = \frac{-Q_R + \left[Q_R{}^2 - 4Q_R{}^2\left(\dfrac{C_n - C_{in}}{C_n}\right)\right]^{1/2}}{2}$$

h. Substituting the values for the variables, note C_n is in mg/L and C_{in} is in g/L

$$Q_W = \frac{-0.05 + \left[0.05^2 - 4(0.05)^2\left(\dfrac{0.028 - 85}{0.028}\right)\right]^{1/2}}{2}$$

$$Q_W = \frac{-0.05 + 5.51}{2} = 2.73 \text{ or } 3 \text{ L/min}$$

2-13 Multiple countercurrent rinse tanks

Given: EPA equation for multiple tanks; Figure P-2-13

Solution:

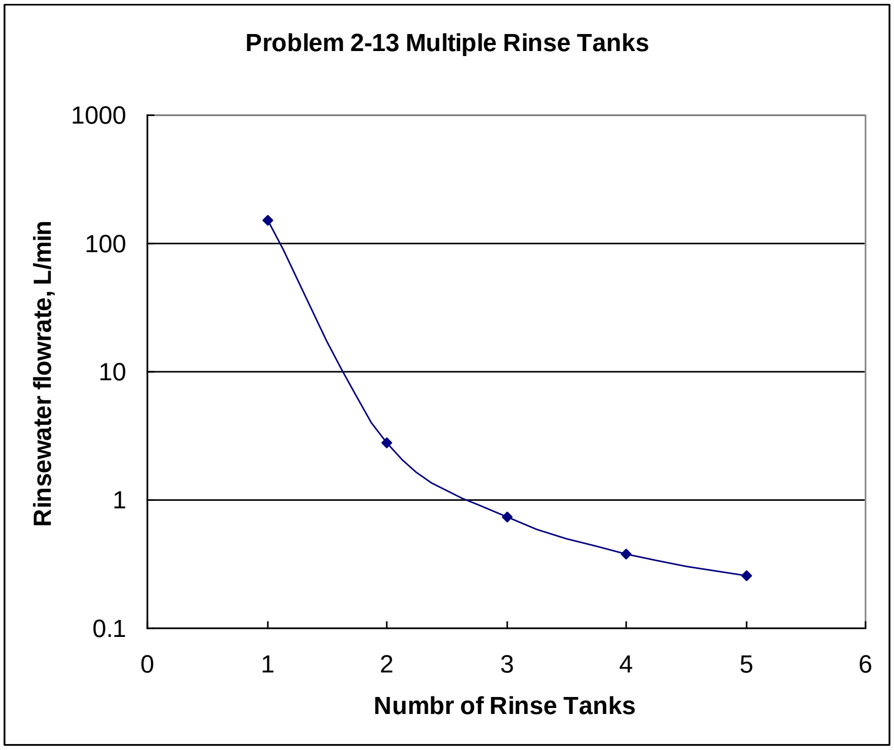


Figure S-2-13: Multiple rinse tanks

2-14 Oxygen concentration in bottle

Given: Starting O_2 concentration = 8 mg/L, rate constant of 0.35 d^{-1}

Solution:

a. General mass balance equation for the bottle is Eqn 2-28

$$C_t = C_o e^{-kt}$$

b. With C_o = 8.0 mg/L and k = 0.35, the plotting points for oxygen remaining are:

Day	Oxygen Remaining, mg/L
1	5.64
2	3.97
3	2.79
4	1.97
5	1.39

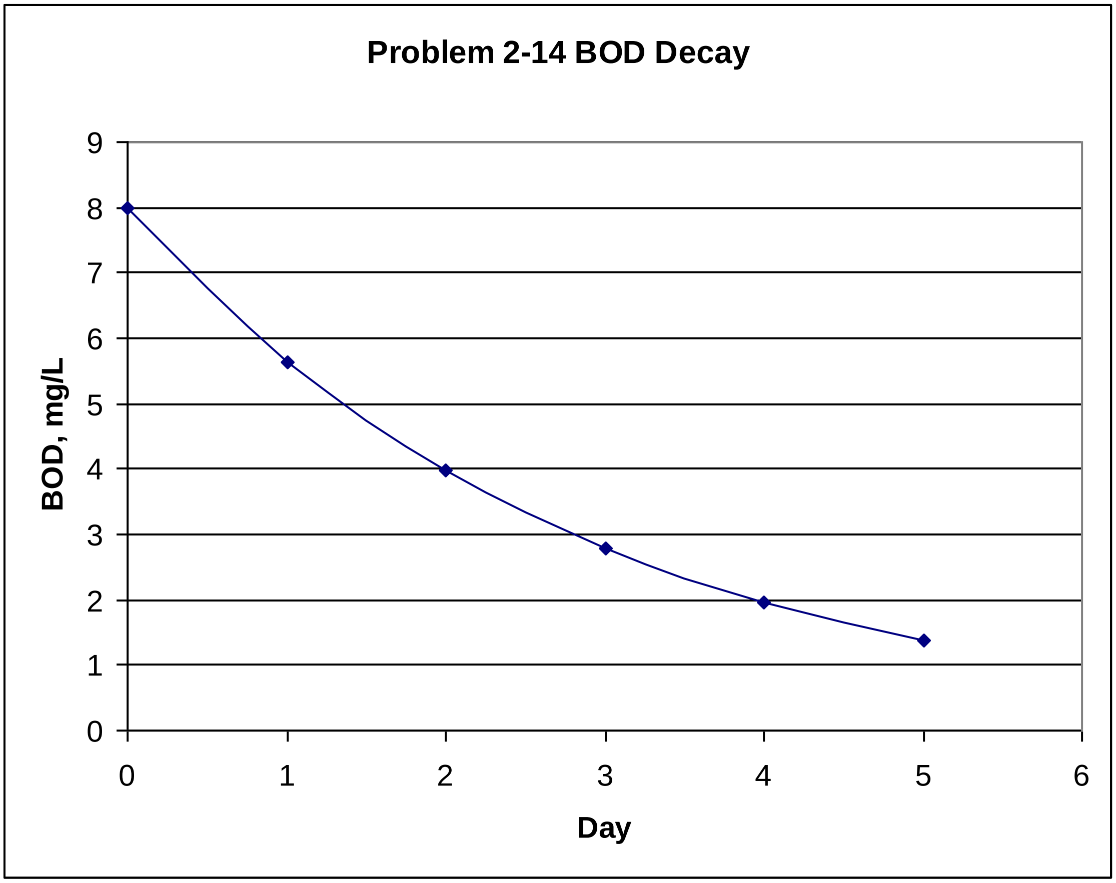


Figure S-2-14: BOD decay

2-15 Decay rate for anthrax die-off

Given: Die-off data points

no./mL	Time, min
398	0
251	30
158	60

Solution:

a. Assume this is 1st order decay (Eqn 2-28)

$$C_t = C_o e^{-kt}$$

b. Using two values (t = 0 and t = 60 min), set $C_o = C_{60}$ and solve for k

$$158 = 398e^{-k(60\ \text{min})}$$

$$\frac{158}{398} = e^{-k(60)}$$

$0.397 = e^{-k(60)}$

Take the natural log of both sides

$\ln(0.397) = \ln[e^{-k(60)}]$

$-0.924 = -k(60)$

$k = 0.0154\ \text{min}^{-1}$ (or $k = 22.18\ \text{d}^{-1}$)

c. Check at t = 30 min

$C_t = 398e^{-(0.0154)(30)} = 250.767$ or 251

2-16 Chlorine decay in water tower

Given: 4000 m^3 water tower
Initial chlorine concentration = 2.0 mg/L
$k = 0.2\ h^{-1}$
Shut down for 8h
Assume completely mixed batch reactor

Solution:

a. Because there is no influent or effluent, the concentration is described by Eqn 2-28.

$$\frac{C_t}{C_o} = e^{-kt}$$

b. Substituting values and solving for C_t
(Note: 8h = 0.33 d)

$C_t = 2.0 \exp[-(1.0\ d^{-1})(0.33\ d)]$

$C_t = 2.0(0.72) = 1.44$ mg/L

c. Mass of chlorine to raise concentration back to 2.0 mg/L

Concentration change required

2.0 mg/L – 1.44 mg/L = 0.56 mg/L

Mass required in kg

$$\frac{(0.56\,\text{mg/L})(4000\text{m}^3)(1000\,\text{L/m}^3)}{10^6\ \text{mg/kg}} = 2.25 \text{ or } 2.3\text{kg}$$

2-17 Expression for half-life

Given: Batch reactor

Solution:

a. Mass balance equation (Eqn 2-16)

$$\frac{dM}{dt} = \frac{d(In)}{dt} - \frac{d(Out)}{dt} - kC\cancel{V}$$

b. Since it is a batch reactor with no "in" or "out", this reduces to

$$\frac{dM}{dt} = -kC\cancel{V}$$

c. Because the reactor volume is constant the change in mass may be written as

$$\frac{dM}{dt} = \cancel{V}\frac{dC}{dt}$$

so,

$$\cancel{V}\frac{dC}{dt} = -kC\cancel{V} \text{ or } \frac{dC}{dt} = -kC$$

d. Integrating

$$C_{out} = C_{in}e^{-kt}$$

e. For half the substance to decay

$$\frac{C_{out}}{C_{in}} = \frac{1}{2}$$

f. So the time for ½ the substance to decay is

$$\frac{1}{2} = e^{-kt}$$

Taking the natural log of both sides,

$\ln(0.5) = \ln(e^{-kt})$

$-0.693 = -kt$

$$t = \frac{0.693}{k}$$

2-18 Amount of substance remaining after half-life

Given: $k = 6 \text{ months}^{-1}$, 1, 2, 3, and 4 half-lives, initial amount = 100%

Solution:

a. Recognizing the half-life concept, then the amount remaining is by observation

Half Life	Amount Remaining, %
0	100
1	50
2	25
3	12.5
4	6.25

b. By equation

$$t_{1/2} = \frac{0.693}{6\text{mo}^{-1}} = 0.1155\text{months}$$

c. For one half life

$C_t = 100\%e^{-(6\text{ /months})(0.1155\text{ months})}$

$C_t = 50\%$

d. For two half lives (2*0.1155 = 0.231 months)

$C_t = 100\%e^{-(6\text{ /months})(0.231\text{ months})}$

$C_t = 25.01\%$ or 25%

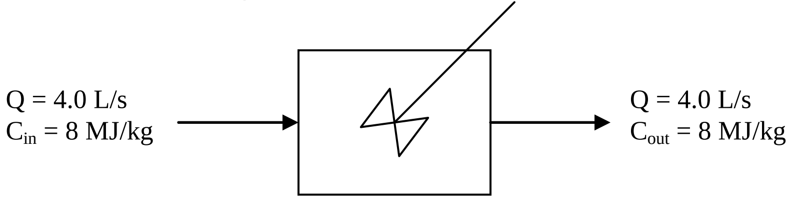
etc.

2-19 Mixing time to achieve desired energy content

Given: CMFR, current waste energy content = 8.0 MJ/kg, new waste energy content = 10.0 MJ/kg, volume of CMFR = 0.20 m^3, flow rate into and out of CMFR = 4.0 L/s, effluent energy content = 9 MJ/kg.

Solution:

a. Mass balance diagram at $t < 0$



b. Step change in influent concentration at $t \geq 0$

$C_{in} = 8$ MJ/kg increases to $C_{in} = 10$ MJ/kg

c. Assuming this is non-reactive then the behavior is as shown in Figure 2-8 and Eqn 2-30 applies. Using the given values:

$$9\frac{MJ}{kg} = 8\frac{MJ}{kg}e^{-t/\theta} + 10\frac{MJ}{kg}\left(1 - e^{-t/\theta}\right)$$

Compute theoretical detention time:

$$\theta = \frac{0.20\text{m}^3}{(4.0\,\text{L/s})(10^{-3}\,\text{m}^3/\text{L})} = 50\text{s}$$

Solving for the exponential term:

$$9 = 8e^{-t/50} + 10 - 10e^{-t/50}$$

$$-1 = (8 - 10)e^{-t/50}$$

$$0.50 = e^{-t/50}$$

Taking the natural log of both sides

$$-0.693 = \frac{-t}{50}$$

t = 34.66 or 35 s

2-20 Repeat Problem 2-19 with new waste at 12 MJ/kg

Given: Data in Problem 2-19

Solution:

a. See Problem 2-19 for initial steps

$$9 = 8e^{-t/50} + 12 - 12e^{-t/50}$$

$$0.75 = e^{-t/50}$$

Taking the natural log of both sides:

$$-0.288 = \frac{-t}{50}$$

t = 14.38 or 14 s

2-21 Time for sample to reach instrument

Given: 2.54 cm diameter sample line
Sample line is 20 m long
Flow rate = 1.0 L/min

Solution:

a. Calculate area of sample line

$$A = \frac{\pi(2.54\text{cm})^2}{4} = 5.07\text{cm}^2$$

In m^2

$$\frac{5.07\,\mathrm{cm}^2}{10^4\ \mathrm{cm}^2/\mathrm{m}^2} = 5.07\times10^{-4}\,\mathrm{m}^2$$

b. Speed of water in the pipe

$$u = \frac{(1.0\,\mathrm{L/min})(10^{-3}\ \mathrm{m}^3/\mathrm{L})}{5.07\times10^{-4}\,\mathrm{m}^2} = 1.97\,\mathrm{m/min}$$

c. Time to reach sample

$$t = \frac{20\mathrm{m}}{1.97\,\mathrm{m/min}} = 10.13\ \text{or 10 min}$$

d. Volume of water (ignoring 10 mL sample size)

$\forall$ = (1.0 L/min)(10 min) = 10 L

2-22 Brine pond dilution

Given: Pond volume = 20,000 m^3, salt concentration = 25,000 mg/L, Atlantic ocean salt concentration = 30,000 mg/L, final salt concentration = 500 mg/L, time to achieve final concentration = 1 year.

Solution:

a. Assuming the pond is completely mixed, treat as a step decrease in CMFR and use Eqn 2-33 and solve for θ.

$$500 = 25000\exp\left(-\frac{1\mathrm{year}}{\theta}\right)$$

$$0.020 = \exp\left(\frac{-1\mathrm{y}}{\theta}\right)$$

Take the natural log of both sides

$$-3.912 = \left(\frac{-1\mathrm{y}}{\theta}\right)$$

$$\theta = \frac{1}{3.912} = 0.2556\mathrm{y}$$

b. Recognize that

$$\theta = \frac{\cancel{V}}{Q}$$

and solve for Q

$$0.2556\text{y} = \frac{20000\text{m}^3}{Q}$$

$$Q = \frac{20000\text{m}^3}{0.2556\text{y}} = 78{,}240\,\text{m}^3/\text{y}$$

c. Convert to units of m^3/s

$$78{,}240\,\text{m}^3/\text{y} \times \frac{1}{365\,\text{d}/\text{y}} \times \frac{1}{86400\,\text{s}/\text{d}} = 0.0025\,\text{m}^3/\text{s}$$

2-23 Venting water tower after disinfection

Given: Volume = 1,900 m^3, chlorine concentration = 15 mg/m^3, allowable concentration = 0.0015 mg/L, air flow = 2.35 m^3/s.

Solution:

a. Assume the water tower behaves as CMFR and apply Eqn 2-33

$$\theta = \frac{1900\text{m}^3}{2.35\,\text{m}^3/\text{s}} = 808.51\text{s}$$

Convert concentration to similar units

$(0.0015\ \text{mg/L})(1{,}000\ \text{L/m}^3) = 1.5\ \text{mg/m}^3$

Now solve Eqn 2-33

$$1.5\,\text{mg}/\text{m}^3 = 15\,\text{mg}\Big/\text{m}^3 \exp\left(-\frac{t}{808.51\text{s}}\right)$$

$$0.10 = \exp\left(\frac{-t}{808.51s}\right)$$

Take the natural log of both sides

$$-2.303 = \left(\frac{-t}{808.51s}\right)$$

t = 1,861.66 s or 31 min or 30 min

2-24 Railroad car derailed and ruptured

Given: Volume of pesticide = 380 m^3
Mud Lake Drain: ν = 0.10 m/s, Q = 0.10 m^3/s, L = 20 km
Mud Lake: ∀ = 40,000m^3
Assume pesticide is non-reactive, assume pulse injection and lake is CMFR, assume drain behaves like PFR

Solution:

a. Treat as two part problem: a PFR followed by a CMFR

b. Time for pulse to reach Mud Lake

$$t = \frac{L}{u} = \frac{(20km)(1000\,m/km)}{0.10\,m/s} = 200{,}000s \text{ or } 2.31 \text{ d}$$

c. Pulse injection into CMFR. If it is completely mixed, then the initial concentration as the pulse enters the lake is C_o. To achieve 99% removal, $C_t = (1\text{-}0.99)C_o = 0.01C_o$.

$$\frac{C_t}{C_o} = \frac{0.01C_o}{C_o} = 0.01$$

d. Using Eqn 2-33 with

$$\theta = \frac{\cancel{V}}{Q} = \frac{40000m^3}{0.10\,m^3/s} = 400{,}000s$$

$$0.01 = \exp\left(-\frac{t}{400{,}000}\right)$$

Taking the natural log of both sides

$$-4.605 = \frac{-t}{400{,}000}$$

t = 1,842,068 s or 30,701 min or 511 h or 21.3 d

2-25 Fluoride feeder failure

Given: Rapid mix tank, $\forall = 2.50\ m^3$
Find concentration = 0.01 mg/L, initial concentration = 1.0 mg/L, $Q = 0.44\ m^3/s$
Pipe, L = 5 km, $\nu = 0.17\ m/s$

Solution:

a. Treat as two part problem: a CMFR followed by PFR

b. Use Eqn 2-27 to find θ and Eqn 2-33 to solve for t

$$\theta = \frac{\cancel{V}}{Q} = \frac{2.5\mathrm{m}^3}{0.44\,\mathrm{m}^3/\mathrm{s}} = 5.68\mathrm{s}$$

$$\frac{0.01}{1.0} = \exp\left(\frac{-t}{5.68\mathrm{s}}\right)$$

t = 26.16 s

c. Assuming pipe behaves as PFR, the time for the last parcel at 0.01 mg/L to travel the length of the pipe is

$$\frac{L}{u} = \frac{(5\mathrm{km})(1000\,\mathrm{m/km})}{0.17\,\mathrm{m/s}} = 29{,}411\mathrm{s}$$

d. Total time

t_{total} = 26.16 + 29,411.76 = 29,437.92 s or 490.6 min or 8.17 h

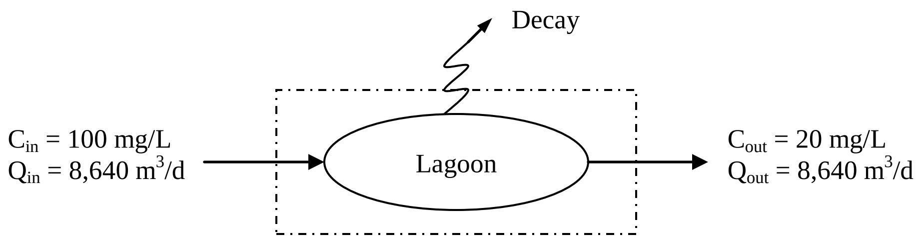
2-26 Rate constant for sewage lagoon

Given: Area = 10 ha, depth = 1 m, flow into lagoon = 8,640 m^3/d, biodegradable material = 100 mg/L, effluent must meet = 20 mg/L, assume 1st order reaction.

Solution:

a. There are two methods to solve this problem: (1) by using mass balance, (2) using equation from Table 2-2

b. First by mass balance



The mass balance equation is

$$\frac{dM}{dt} = C_{in}Q_{in} - C_{out}Q_{out} - kC_{lagoon}\forall$$

Assuming steady state, CMFR then

$$\frac{dM}{dt} = 0 \text{ and } C_{lagoon} = C_{out}$$

So,

$$C_{in}Q_{in} - C_{out}Q_{out} - kC_{out}\forall = 0$$

Solving for k

$$C_{in}Q_{in} - C_{out}Q_{out} = kC_{out}\forall$$

$$k = \frac{C_{in}Q_{in} - C_{out}Q_{out}}{C_{out}\forall}$$

Note that 1 mg/L = 1 g/m^3

$$k = \frac{(100\,g/m^3)(8640\,m^3/d) - (20\,g/m^3)(8640\,m^3/d)}{(20\,g/m^3)(10ha)(10000\,m^2/ha)(1m)}$$

$$k = 0.3456\ d^{-1}$$

c. Repeat using Table 2-2 equation for CMFR and 1st order reaction

$$C_t = \frac{C_o}{1 + k\theta}$$

$$\theta = \frac{\forall}{Q} = \frac{(10\text{ha})(10000\,\text{m}^2/\text{ha})(1\text{m})}{8640\,\text{m}^3/\text{d}} = 11.574\text{d}$$

$$20\,\text{mg/L} = \frac{100\,\text{mg/L}}{1 + k(11.574\text{d})}$$

Solve for k

$$0.20 = \frac{1}{1 + k(11.574\text{d})}$$

5.00 = 1 + k(11.574 d)

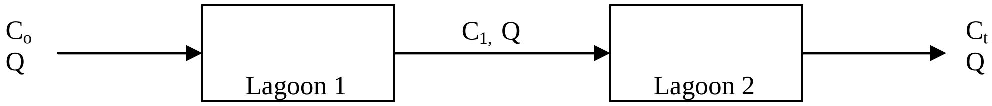
$$k = \frac{4.00}{11.574\text{d}} = 0.3456\text{d}^{-1}$$

2-27 Rate constant for two lagoons in series

Given: Data from Problem 2-26, two lagoons in series, area of each lagoon = 5 ha, depth = 1 m

Solution:

a. Mass balance diagram



Thus, the output from the 1st lagoon is the input to the 2nd lagoon. Solve the problem sequentially.

b. Calculate volume and hydraulic retention time

$\forall$ = (5 ha)(10,000 m^2/ha)(1 m) = 5.0 x 10^4 m^3

$$\theta = \frac{\forall}{Q} = \frac{5.0 \times 10^4\,\text{m}^3}{8640\,\text{m}^3/\text{d}} = 5.787\text{d}$$

c. Using Table 2-2

$$C_1 = \frac{C_o}{1 + k\theta}$$

d. Because $C_1 = C_o$ for the second lagoon and the second lagoon has the same relationship

$$C_t = \frac{C_1}{1 + k\theta}$$

Substituting for C_1

$$C_t = \left(\frac{1}{1 + k\theta}\right)\left(\frac{C_o}{1 + k\theta}\right)$$

$$\frac{C_t}{C_o} = \left(\frac{1}{1 + k\theta}\right)^2$$

$$\left(\frac{C_t}{C_o}\right)^{1/2} = \frac{1}{1 + k\theta}$$

$$1 + k\theta = \left(\frac{C_o}{C_t}\right)^{1/2}$$

k = 0.2136 or 0.21 d^{-1}

2-28 Plot concentration after shutdown

Given: Data from Problem 2-26, C_o = 100 mg/L, k = 0.3478 d^{-1}

Solution:

a. Using Eqn 2-40 set up parameters for spreadsheet

$$C_{out} = C_o \exp\left[-\left(\frac{1}{\theta} + k\right)t\right]$$

C_o is the effluent concentration at time t = 0 because the lagoon is assumed to be CMFR.

C_o = 20 mg/L

$$\theta = \frac{\forall}{Q} = \frac{(10\text{ha})(10000\,\text{m}^2/\text{ha})(1\text{m})}{8640\,\text{m}^3/\text{d}} = 11.574\text{d}$$

$$\frac{1}{\theta} = 0.0864$$

For spreadsheet

C_{out} = 20exp[- (0.0864 + 0.3478)t]

C_{out} = 20exp[- (0.4342)t]

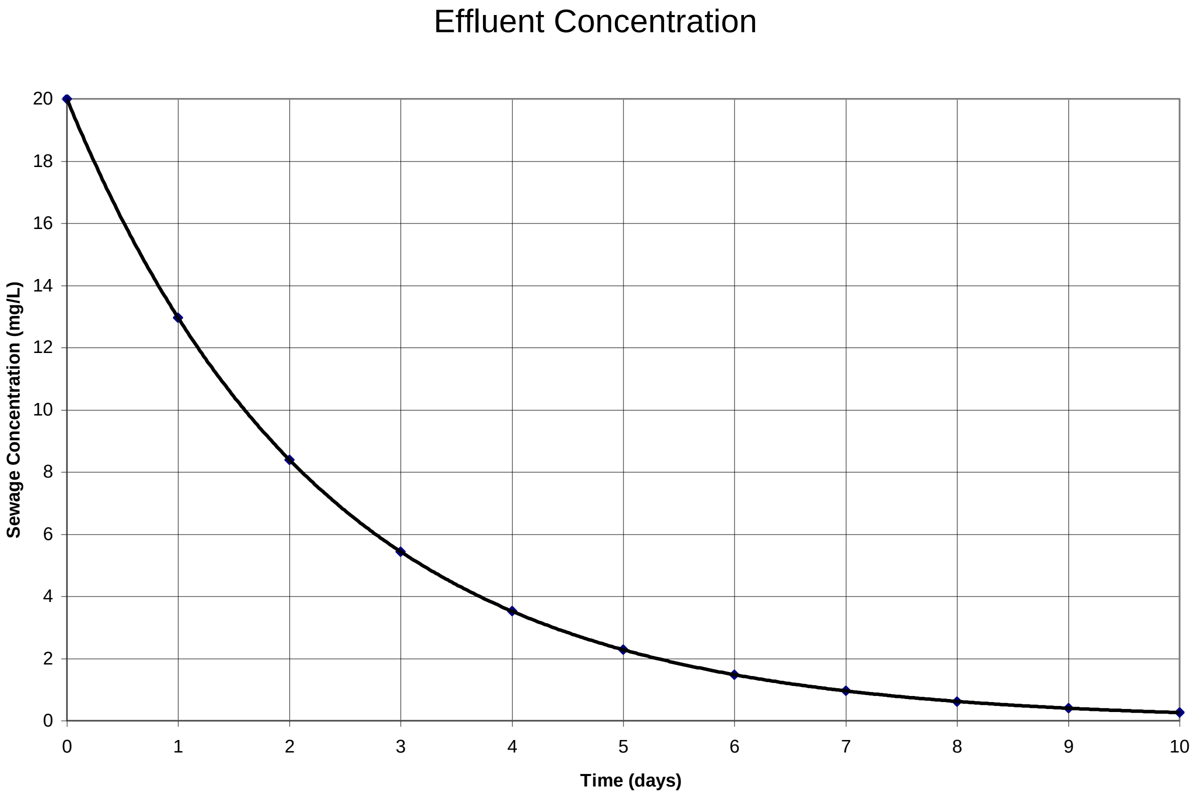

Effluent Concentration
Sewage Concentration (mg/L)
20
18
16
14
12
10
8
6
4
2
0
0
1
2
3
4
5
6
7
8
9
10
Time (days)

Figure S-2-28

2-29 Purging basement of radon

Given: $\forall = 90\ \text{m}^3$, radon = 1.5 Bq/L, radon decay rate constant = $2.09 \times 10^{-6}\ \text{s}^{-1}$, vent at $0.14\ \text{m}^3/\text{s}$, allowable radon = 0.15 Bq/L, assume CMFR.

Solution:

a. Using Eqn 2-40

$$C_{out} = C_o \exp\left[-\left(\frac{1}{\theta}+k\right)t\right]$$

$$\theta = \frac{\cancel{V}}{Q} = \frac{90\text{m}^3}{0.14\,\text{m}^3/\text{s}} = 642.857\text{s}$$

$$\frac{0.15}{1.5} = \exp\left[-\left(\frac{1}{642.857\text{s}}+2.09\times10^{-6}\right)t\right]$$

$0.10 = \exp[-(1.558 \times 10^{-3})t]$

Take the natural log of both sides

$-2.303 = (-1.558 \times 10^{-3})t$

$t = 1.478 \times 10^{3}$ s or 24.64 min or 25 min

2-30 Decay of bacteria from ocean outfall

Given: 5000 m from outfall to beach
10^5 coliforms per mL
Discharge flow rate = $0.3\ \text{m}^3/\text{s}$
$k = 0.3\ \text{h}^{-1}$
Current speed = 0.5 m/s
Assume current behaves as pipe carrying $600\ \text{m}^3/\text{s}$ of seawater

Solution:

a. The concentration resulting from mixing with the seawater pipe

$(10^5\ \text{coliforms/mL})(0.3\ \text{m}^3/\text{s}) = (C_{seawater})(600\ \text{m}^3/\text{s})$

$$C_{seawater} = \frac{(10^5\ \text{coliforms / mL})(0.3\,\text{m}^3/\text{s})}{600\,\text{m}^3/\text{s}} = 50\,\text{coliforms/mL}$$

b. Concentration of coliforms at beach

Travel time to travel to beach

$$t = \frac{5000\text{m}}{0.5\,\text{m/s}} = 10{,}000\text{s} \ \text{ or } 2.78\ \text{h}$$

Decay in plug flow reactor (Eqn 2-22) with $\theta = 2.78$ h

$C_{beach} = (C_{seawater})\exp[-(0.3\ \text{h}^{-1})(2.78\ \text{h})]$

$C_{beach} = (50\ \text{coliforms/mL})(0.43)$

$= 21.73$ or 20 coliforms/mL

2-31 Compare efficiency of CMFR and PRF

Given: $\forall = 280\ \text{m}^3$, $Q = 14\ \text{m}^3/\text{d}$, $k = 0.05\ \text{d}^{-1}$

Solution:

a. CMFR

From Table 2-2

$$C_t = \frac{C_o}{1 + k\theta}$$

$$\theta = \frac{\cancel{V}}{Q} = \frac{280\text{m}^3}{14\,\text{m}^3/\text{d}} = 20\text{d}$$

$$\frac{C_t}{C_o} = \left(\frac{1}{1 + (0.05)(20)}\right) = 0.50$$

Using Eqn 2-8

$$\eta = \frac{C_o - 0.50C_o}{C_o} \times 100\% = 50\%$$

b. PFR

From Table 2-2

$C_t = C_o\exp(-k\theta)$

$$\frac{C_{out}}{C_o} = \exp(-k\theta)$$

$$\frac{C_{out}}{C_o} = \exp(-(0.05)(20))$$

$$\frac{C_{out}}{C_o} = 0.37$$

Using Eqn 2-8

$$\eta = \frac{C_o - 0.37C_o}{C_o} \times 100\% = 63\%$$

2-32 Volume required to achieve 95% efficiency

Given: $Q = 14\ m^3/d$, $k = 0.05$

Solution:

a. Solve Eqn 2-8 for fraction of C_o

$$\eta = 0.95 = \frac{C_o - (X)C_o}{C_o}$$

$1 - X = 0.95$

$X = 0.05$

Therefore

$$\frac{C_t}{C_o} = 0.05$$

b. CMFR

From Table 2-2

$$C_t = \frac{C_o}{1 + k\theta}$$

Solve for θ

$$\frac{1}{1 + k\theta} = \frac{C_t}{C_o}$$

$$\frac{C_o}{C_t} = 1 + k\theta$$

$$k\theta = \frac{C_o}{C_t} - 1$$

$$\theta = \frac{\frac{C_o}{C_t} - 1}{k}$$

Substituting values,

$$\theta = \frac{20 - 1}{0.05} = 380\text{d}$$

Solve for the volume

$$\theta = \frac{\forall}{Q}$$

$\forall = (\theta)(Q) = (380\text{ d})(14\text{ m}^3/\text{d}) = 5{,}320\text{ m}^3$

c. PFR

From Table 2-2

$$\frac{C_t}{C_o} = \exp[-k\theta]$$

As in (a.) above

$0.05 = \exp(-0.05\theta)$

Take the natural log of both sides

$-2.9957 = -0.05\theta$

$\theta = 59.9147$ d

Solve for volume

$$\theta = \frac{\forall}{Q}$$

$\forall = (\theta)(Q) = (59.9147 \text{ d})(14 \text{ m}^3/\text{d}) = 838.8 \text{ m}^3$

2-33 Melting ice

Given: 2 kg of ice, 200 W electric heater

Solution:

a. This is an application of the latent heat of fusion (333 kJ/kg). The energy requires

(2 kg)(333 kJ/kg) = 666 kJ

Since 200 W = 200 J/s

$$t = \frac{666 \times 10^3 \text{ kJ}}{200 \text{ J/s}} = 3330 \text{s} \text{ or } 55.5 \text{ min}$$

2-34 Evaporation cooler

Given: Examples 2-12 and 2-13
40 m^3 of wastewater
Discharge temperature = 100°C
Final Temperature = 20°C

Solution:

a. Required enthalpy change

$\Delta H = (40 \text{ m}^3)(1000 \text{ kg/m}^3)(4.186 \text{ kJ/kg} \cdot \text{K})(373.15 \text{ K} - 293.15 \text{ K}) = 13{,}395{,}200 \text{ kJ}$

b. Noting the enthalpy of vaporization is 2257 kJ/kg from text

$$\text{MassWater} = \frac{13{,}395{,}200\,\text{J}}{2257\,\text{kJ}/\text{kg}} = 5934.96\,\text{kg}$$

c. Volume of water

$$\cancel{V} = \frac{5934.9\,\text{kg}}{1000\,\text{kg}/\text{m}^3} = 5.93 \text{ or } 6\ \text{m}^3$$

d. Note: this is about

$$\frac{6\text{m}^3}{40\text{m}^3} \times 100\% = 15\% \text{ of the total volume}$$

2-35 Heating water in wastewater treatment

Given: Flow rate = 30 m^3/d, current temperature = 15º C, required temperature = 40º C

Solution:

a. Use Eqn 2-45, assume 1 m^3 of water = 1000 kg and that $C_p = C_r$. The specific heat of water from Table 2-3 is 4.186 kJ/kg · K. The temperatures in K are:

273.15 + 15 = 288.15
273.15 + 40 = 313.15

$$\frac{\Delta H}{\Delta T} = (30\ \text{m3/d})(1000\ \text{kg/m3})(4.186\ \text{kJ/kg} \cdot \text{K})(313.15\,\text{K} - 288.15\,\text{K})$$

$$= 3{,}139{,}500 \text{ kJ/d or } 3.14 \text{ GJ/d}$$

2-36 Temperature of river after cooling water discharge

Given: River flow rate = 40 m^3/s, river temperature = 18º C, power plant discharge = 2 m^3/s, cooling water temperature = 80º C

Solution:

This is a simple energy balance as in Example 2-12. Assume the density of water is 1000 kg/m^3. The balance equation would be:

$$Q_{\text{river}}(\rho)(C_p)(\Delta T) = Q_{\text{cooling water}}(\rho)(C_p)(\Delta T)$$

Because the density is assumed constant and the specific heat is the same the equivalence reduces to:

$Q_{river}(T - (273.15 + 18)) = Q_{cooling\ water}((273.15 + 80) - T)$

Or,

$40(T - 291.15) = 2(353.15 - T)$

$40T - 11{,}646 = 706.30 - 2T$

$42T = 12{,}352.30$

$T = 294.10$ K or 20.95ºC or 21º C

2-37 Cooling water temperature

Given: Seine River flow rate = 28 m^3/s
Seine upstream temp = 20°C
Seine downstream temp = 27°C
Cooling water flow rate = 10 m^3/s

Solution:

a. Loss of enthalpy of cooling water

$\Delta H = (1000\ kg/m^3)(10\ m^3/s)(4.186\ kJ/kg \cdot K)(T - 300.15\ K)$

$\Delta H = 41{,}860\ T - 12{,}564{,}279$

b. Gain of enthalpy of Seine

$\Delta H = (1000\ kg/m^3)(28\ m^3/s)(4.186\ kJ/kg \cdot K)(300.15\ K - 293.15\ K)$

$\Delta H = 820{,}456$

c. From Eqn. 2-46, enthalpy loss = enthalpy gain

$41{,}860\ T - 12{,}564{,}279 = 820{,}456$

$41{,}860\ T = 13{,}384{,}734$

$T = 319.75$ K or 46.6 °C or 47°C

2-38 Lagoon temperature in winter

Given: 3,420 m^3 in lagoon

Flow rate of sewage = 300 m^3/d
Lagoon temperature = 0°C (not frozen)
Sewage temperature = 15°C

Solution:

a. Enthalpy loss of sewage

$$\Delta H = (1000\ kg/m^3)(300\ m^3/d)(1\ d)(4.186\ kJ/kg \cdot K)(288.15\ K - T)$$

$$\Delta H = 361{,}858{,}770 - 1{,}255{,}800\ T$$

b. Enthalpy gain of lagoon

$$\Delta H = (1000\ kg/m^3)(3{,}420\ m^3)(4.186\ kJ/kg \cdot K)(T - 273.15\ K)$$

$$\Delta H = 14{,}316{,}120\ T - 3{,}910{,}448{,}178$$

c. Enthalpy loss = enthalpy gain

$$361{,}858{,}770 - 1{,}255{,}800\ T = 14{,}\ 316{,}120\ T - 3{,}910{,}448{,}178$$

$$-1{,}255{,}800\ T = 14{,}316{,}120\ T - 4{,}272{,}306{,}448$$

$$-15{,}571{,}920\ T = -4{,}272{,}306{,}448$$

$$T = 274.36\ K \text{ or } 1.21°C$$

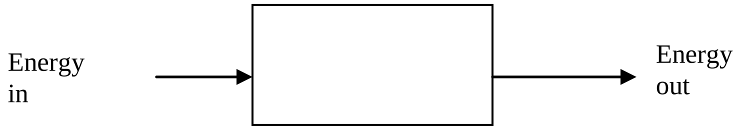
2-39 Lagoon temperature in winter after 7 days

Given: Problem 2-38
Flow out of lagoon = flow into lagoon

Solution:

a. This is an energy balance of the form of Eqn. 2-49. Because of the differential this problem must be solved stepwise rather than in one 7 day stage

b. The energy balance diagram is



c. Note that the lagoon is completely mixed and the temperature of the lagoon is the same as the temperature of the wastewater leaving the lagoon

d. Defining the following:

Sewage constant, $K_S = (1000\ \text{kg/m}^3)(300\ \text{m}^3/\text{d})(4.186\ \text{kJ/kg} \cdot \text{K})(1\ \text{d})$

$= 1{,}255{,}800\ \text{kJ/K}$

Lagoon constant, $K_L = (1000\ \text{kg/m}^3)(3{,}420\ \text{m}^3)(4.186\ \text{kJ/kg} \cdot \text{K})$

$= 14{,}316{,}120\ \text{kJ/K}$

Energy out constant $= K_S$

e. Equation for heat balance

$$K_S(T_{WW} - T) = K_L(T - T_{LAGOON}) + K_S(T - T_{LAGOON})$$

f. Solving for T

$$T = \frac{(K_L + K_S)(T_{LAGOON}) + K_S T_{WW}}{K_L + 2K_S}$$

g. Use this equation to perform spreadsheet iterations

Time (days)	Temperature (K)
initial	273.15
0	274.27
1	275.31
2	276.26
3	277.15
4	277.97
5	278.73
6	279.43
7	280.08

2-40 Cooling water pond

Given: Inlet temp = 45.0°C
Outlet temp = 35.5°C
$h_0 = 0.0412\ \text{kJ/s} \cdot \text{m}^2$
Flow rate = $17.2\ \text{m}^3/\text{s}$

Solution:

a. Enthalpy change required

$$\frac{dH}{dt} = (17.2\,\mathrm{m3/s})(1000\,\mathrm{kg/m3})(4.186\,\mathrm{kJ/kg\cdot K})(318.15\,\mathrm{K}\ \ 308.65\,\mathrm{K})$$

$$= 683{,}992.40\ \mathrm{kJ/s}$$

b. Enthalpy change in cooling pond

$$\frac{dH}{dt} = h_0 A\left(T_2 - T_1\right)$$

$$= (0.0412\ \mathrm{kJ/s \cdot m^2})(A)(318.15\text{-}308.65)$$

$$= 0.39\ A$$

c. Setting “b” = “a” and solving for A

0.39 A = 683,992.40

A = 1,747,553.40 m^2 or 174.76 ha or 175 ha

2-41 Heating a pump house

Given: Dimensions 2m X 3 m X 2.4 m high, wood 1 cm thick (h_c = 0.126 W/m · K), glass-wool 10 cm thick (h_c = 0.0377 W/m · K), indoor temperature = 10º C, outdoor temperature = -18º C, ignore floor heat loss.

Solution:

a. This is an application of Eqn 2-50. Begin by computing the surface area of the pump house.

3 m * 2.4 m * 2 = 14.40
2 m * 2.4 m * 2 = 9.60
3 m * 2 m = 6.00
30.00 m^2

b. For wood walls

$$\frac{\Delta H}{\Delta T} = (0.126\ \mathrm{W/m\cdot K})(30\,\mathrm{m^2})(283.15\,\mathrm{K} - 255.15\,\mathrm{K})\left(\frac{1}{0.10\mathrm{m}}\right)$$

= 1,058.4 W or 1.06 k

c. For glass-wool

$$\frac{\Delta H}{\Delta T} = (0.0377\ \text{W/m}\cdot\text{K})(30\ \text{m}^2)(283.15\ \text{K} - 255.15\ \text{K})\left(\frac{1}{0.10\text{m}}\right)$$

= 316.68 W or 0.32 kW

2-42 Freezing of sewage lagoon

Given: Lagoon wastewater temperature = 15°C
Air temperature = -8°C
$h_0 = 0.5\ \text{kJ/s} \cdot \text{m}^2 \cdot \text{K}$

Solution:

a. Note: Solve in two steps. First calculate the time to lower the temperature to 0°C. Then calculate the time to release all the enthalpy of fusion.

b. Enthalpy of lagoon at start

$$\Delta H = (1000\ \text{kg/m}^3)(3{,}420\ \text{m}^3)(4.186\ \text{kJ/kg}\cdot\text{K})(288.15\ \text{K} - 273.15\ \text{K})$$

$$\Delta H = 214{,}741{,}800\ \text{kJ}$$

c. Area of lagoon

$$A = \frac{3420\text{m}^3}{3\text{m}} = 1140\text{m}^2$$

d. Enthalpy change due to aerator mixing

$$\frac{\Delta H}{\Delta t} = (0.5\ \text{kJ/s}\cdot\text{m}^2\cdot\text{K})(1{,}140\ \text{m2})(288.15\ \text{K}\ \ 265.15\ \text{K})$$

= 13,110 kJ/s

e. Time to lower temp to 0°C

$$\frac{214{,}741{,}800\,\text{kJ}}{13{,}110\,\text{kJ/s}} = 16{,}380\,\text{s} \text{ or } 4.55 \text{ h}$$

f. Enthalpy of fusion

$\Delta H = (333 \text{ kJ/kg})(1000 \text{ kg/m}^3)(3{,}420 \text{ m}^3)$

$= 1{,}138{,}860{,}000 \text{ kJ}$

g. Time to freeze

$$\frac{1{,}138{,}860{,}000\,\text{kJ}}{13{,}110\,\text{kJ/s}} = 86{,}869.57\,\text{s} \text{ or } 24.13 \text{ h}$$

h. Total time

$= 4.55 \text{ h} + 24.13 \text{ h} = 28.68 \text{ h}$ or 29 h

2-43 Overall efficiency of energy production

Given: Heat of combustion = 31.4 MJ/kg, electrical energy = 2.2 kWh/kg coal

Solution:

a. The efficiency is of the form

$$\eta = \frac{W}{Q} = \frac{2.2\,\text{kWh/kg}}{\left(31.4\times10^{6} \text{ J/kg}\right)\left(2.7778\times10^{-7} \text{ kWh/J}\right)}\times100\%$$

$\eta = 25.22\%$

Note on conversion factor:

$\text{kWh} = (1000 \text{ J/s})(3600 \text{ s/h}) = 3.60 \times 10^6 \text{ J/kWh}$ and

$$\frac{1}{3.60\times10^{6} \text{ J/kWh}} = 2.7778\times10^{-7} \text{ kWh/J}$$

DISCUSSION QUESTIONS

2-1 Limestone rock dissolving

Given: limestone rock at bottom of Lake Michigan is dissolving

Solution:

The system is at steady state.

2-2 Benzene in a pond

Given: can of benzene has spilled into a small pond. What data are required to calculate the concentration of benzene in the water leaving the pond.

Solution:

The approach to answer this question is to write a mass-balance equation for the system and use it to identify the data required for solution. In simplified terms, assume steady state and no sorption to the bottom of the pond.

Mass in = Mass that volatilizes + Mass that flows out of the pond downstream

The ‘Mass in’ is a function of the benzene concentration and the volume in the can.

The ‘Mass that volatilizes’ can be estimated from Henry’s law and the rate of vaporization.

The mass that flows downstream may be estimated from the concentration and the flow rate of the stream. Because the concentration is the required unknown, the problem may be solved for concentration.

2-3 Specific heat capacities

Given: Table 2-3, Why are c_p for meat and vegetables higher than for metals?

Solution:

Note that c_p for water is higher then that of metals. Because meat and vegetables are predominately water, their c_p is higher.

2-4 Cold coming into your hand

Given: cold beverage glass

Solution:

Thermodynamically speaking, the cold does not come into your hand. Rather, the heat leaves your hand faster than your body can replace it.

2-5 Bick floor and wood floor

Given: brick floor feels cooler than wood floor

Solution:

A brick floor feels cooler than a wood floor because a brick floor hs a higher thermal conductivity then wood and it removes heat from your bear feet more quickly