Heat and Mass Transfer, SI Edition
Solutions Manual
Second Edition

This solutions manual sets down the answers and solutions for the Discussion Questions, Class Quiz Questions, and Practice Problems. There will likely be variations of answers to the discussion questions as well as the class quiz questions. For the practice problems there will likely be some divergence of solutions, depending on the interpretation of the processes, material behaviors, and rigor in the mathematics. It is the author's responsibility to provide accurate and clear answers. If you find errors please let the author know of them at <rolle@uwplatt.edu>.

Chapter 2

Discussion Questions

Section 2-1

1. Describe the physical significance of thermal conductivity.

   Thermal conductivity is a parameter or coefficient used to quantitatively describe the amount of conduction heat transfer occurring across a unit area of a bounding surface, driven by a temperature gradient.

2. Why is thermal conductivity affected by temperature?

   Conduction heat transfer seems to be the mechanism of energy transfer between adjacent molecules or atoms and the effectiveness of these transfers is strongly dependent on the temperatures. Thus, to quantify conduction heat transfer with thermal conductivity means that thermal conductivity is strongly affected by temperature.

3. Why is thermal conductivity not affected to a significant extent by material density?

   Thermal conductivity seems to not be strongly dependent on the material density since thermal conductivity is an index of heat or energy transfer between adjacent molecules and while the distance separating these molecules is dependent on density, it is not strongly so.

Section 2-2

4. Why is heat of vaporization, heat of fusion, and heat of sublimation accounted as energy generation in the usual derivation of energy balance equations?

   Heats of vaporization, fusion, and sublimation are energy measures accounting for phase changes and not directly to temperature or pressure changes. It is convenient, therefore, to account these phase change energies as lumped terms, or energy generation.

Section 2-3

5. Why are heat transfers and electrical conduction similar?

Heat transfer and electrical conduction both are viewed as exchanges of energy between adjacent moles or atoms, so that they are similar.

6. Describe the difference among thermal resistance, thermal conductivity, thermal resistivity, and R-Values.

Thermal Resistance is the distance over which conduction heat transfer occurs times the inverse of the area across which conduction occurs and the thermal conductivity, and thermal resistivity is the distance over which conduction occurs times the inverse of the thermal conductivity. The R-Value is the same as thermal resistivity, with the stipulation that in countries using the English unit system, 1 R-Value is 1 $hr \cdot ft^2 \cdot {}^0F$ per Btu.

Section 2-4

7. Why do solutions for temperature distributions in heat conduction problems need to converge?

Converge is a mathematical term used to describe the situation where an answer approaches a unique, particular value.

8. Why is the heat conduction in a fin not able to be determined for the case where the base temperature is constant, as in Figure 2-9?

The fin is an extension of a surface and at the edges where the fin surface coincides with the base, it is possible that two different temperatures can be ascribed at the intersection, which means there is no way to determine precisely what that temperature is. Conduction heat transfer can then not be completely determined at the base.

9. What is meant by an isotherm?

An isotherm is a line or surface of constant or the same temperature.

10. What is meant by a heat flow line?

A heat flow line is a path of conduction heat transfer. Conduction cannot cross a heat flow line.

Section 2-5

11. What is a shape factor?

The shape factor is an approximate, or exact, incorporating the area, heat flow paths, isotherms, and any geometric shapes that can be used to quantify conduction heat flow between two isothermal surfaces through a heat conducting media. The product of the shape factor, thermal conductivity, and temperature difference of the two surfaces predicts the heat flow.

12. Why should isotherms and heat flow lines be orthogonal or perpendicular to each other?

Heat flow occurs because of a temperature difference and isotherms have no temperature difference. Thus heat cannot flow along isotherms, but must be perpendicular or orthogonal to isotherms.

Section 2-6

13. Can you identify a physical situation when the partial derivatives from the left and right (see Equations 2-85 and 2-86) are not the same?

Often at a boundary between two different conduction materials the left and the right gradients could be different. Another situation could be if radiation or convection heat transfer occurs at a boundary and then again the left and right gradients or derivatives could be different.

Section 2-7

14. Can you explain when fins may not be advantageous in increasing the heat transfer at a surface?

Fins may not be a good solution to situations where a highly corrosive, extremely turbulent, or fluid having many suspended particles is in contact with the surface.

15. Why should thermal contact resistance be of concern to an engineer?

Thermal contact resistance inhibits good heat transfer, can mean a significant change in temperature at a surface of conduction heat transfer, and can provide a surface for potential corrosion.

Class Quiz Questions

1. What is the purpose of the negative sign in Fourier's law of conduction heat transfer?

The negative sign provides for assigning a positive heat transfer for negative temperature gradients.

2. If a particular 20-cm thick material has a thermal conductivity of 17 W/ m·K, what is its R-value?

The R-value is the thickness times the inverse thermal conductivity;

$$R - Value = \frac{thick}{\kappa} = (20 \times 10^{-2} m)/(17\ \mathrm{W}/m \cdot K) = 0.012\ \mathrm{m \cdot K/W}$$

3. What is the thermal resistance of a 10-m^2 insulation board, 30 cm thick, and having thermal conductivity of 0.03 W/m·K?

The thermal resistance is

$$\Delta x / A \cdot \kappa = (0.3m)/(10m^2)(0.03W / m \cdot K) = 1.0K / W$$

4. What is the difference between heat conduction in series and in parallel between two materials?

The thermal resistance, or thermal resistivity are additive for series. In parallel the thermal resistance needs to be determined with the relationship

$R_{eq} = (R_1)(R_2)/(R_1 + R_2)$

5. Write the conduction equation for radial heat flow of heat through a tube that has inside diameter of D_i and outside diameter of D_0.

$$\dot{Q} = 2\pi\kappa L \frac{\Delta T}{\ln(D_0/D_i)}$$

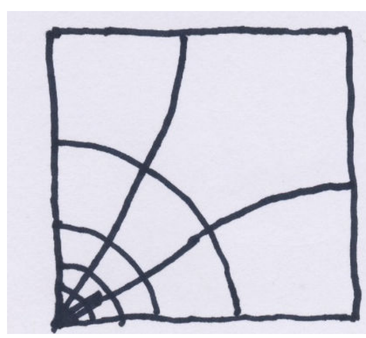
6. Write the Laplace equation for two-dimensional conduction heat transfer through a homogeneous, isotropic material that has constant thermal conductivity.

$$\frac{\partial^2 T(\mathrm{x,y})}{\partial x^2} + \frac{\partial^2 T(\mathrm{x,y})}{\partial y^2} = 0$$

7. Estimate the heat transfer from an object at 40^0C to a surface at 5^0C through a heat conducting media having thermal conductivity of 8.5 W/m·K if the shape factor is 30 cm.

$$\dot{Q} = S\kappa\,\Delta T = (0.3\,m)(8.5\,W/m\cdot K)(35°\mathrm{C}) = 89.25\,W$$

8. Sketch five isotherms and appropriate heat flow lines for heat transfer per unit depth through a 5 cm x 5 cm square where the heat flow is from a high temperature corner to the four sides at a uniform lower temperature. Use one isothermal as the corner and another isothermal as the side of the square.



9. If the thermal contact resistance between a clutch surface and a driving surface is 0.0023 $m^2 \cdot {}^0C/W$, estimate the temperature drop across the contacting surfaces, per unit area when 200 W/m^2 of heat is desired to be dissipated.

The temperature drop is

$$\Delta T = \dot{Q} R_{TCR} = (200 W/m^2)(0.0023 m^2 \cdot {}^0 C/W) = 0.46^0 C$$

10. Would you expect the wire temperature to be greater or less for a 0.14 cm dia. copper wire as compared to a 0.18 cm dia. copper wire, both conducting the same electrical current?

A 0.14 cm dia. copper wire has a smaller diameter and a greater electrical resistance per unit length. Therefore it would be expected to have a higher temperature than the 0.18 cm dia. copper wire.

Practice Problems

Section 2-1

1. Compare the value for thermal conductivity of Helium at 20^{0}C using Equation 2-3 and the value from Appendix Table B-4.

Solution

Using Equation 2-3 for helium

$\kappa = 0.8762x10^{-4}\sqrt{T} = 0.0015W/cm\cdot K = 0.15W/m\cdot K$ From Appendix Table B-4

$\kappa = 0.152W/m\cdot K$

2. Predict the thermal conductivity for neon gas at 95^{0}C. Use a value of 3.9 Å for the collision diameter for neon.

Solution

Assuming neon behaves as an ideal gas, with MW of 20, and 367K, and using Equation 2-1

$$\kappa = 8.328x10^{-4}\sqrt{\frac{T}{MW\cdot\Gamma}} = 8.328x10^{-4}\sqrt{\frac{367K}{(20)(3.9)}} = 18.05x10^{-4}W/cm\cdot K$$

3. Show that thermal conductivity is proportional to temperature to the 1/6th power for a liquid according to Bridgeman's equation (2-6).

Solution

From Bridgeman's equation $\kappa = 3.865x10^{-23}\left(V_s/x_m^2\right)$ Also, V_s (sonic velocity) ~$\sqrt{E_b/\rho}$ ~$\rho^{-1/2}$ the mean separation distance between molecules $x_m^2 = (mm/\rho)^{2/3}$ ~$\rho^{-2/3}$ so that $\kappa \sim \rho^{-2/3+1/2} = \rho^{-1/6} \sim T^{1/6}$

4. Predict a value for thermal conductivity of liquid ethyl alcohol at 300 K. Use the equation suggested by Bridgman's equation (2-6).

Solution

Bridgeman's equation (2-6) uses the sonic velocity in the liquid, $\sqrt{E_b/\rho}$, which for ammonia at 300 K is nearly 1.14 x 10^5 cm/s. The equation also uses the mean distance between molecules, assuming a uniform cubic arrangement of the molecules, which is $\sqrt[3]{mm/\rho}$, mm being the mass of one molecule in grams, the molecular mass divided by Avogadro's number. Using data from a chemistry handbook the value of x_m is nearly 0.459 x 10^{-7} cm. Using Equation 2-6,

$$\kappa = 3.865x10^{-23}\left(V_s/x_m^2\right) = 20.9x10^{-4}W / \mathrm{c}\,m\cdot K = 0.209W / m\cdot K$$

5. Plot the value for thermal conductivity of copper as a function of temperature as given by Equation 2-10. Plot the values over a range of temperatures from -40^{0}C to 70^{0}C.

Solution

Using Equation 2-10 and coefficients from Appendix Table B-8E

$$\kappa = \kappa_{TO} + \alpha(T - T_0) = 393\,\frac{\mathrm{W}}{\mathrm{m}\cdot\mathrm{k}} - 0.019\,\frac{\mathrm{W}}{\mathrm{m}\cdot\mathrm{K}^2}\;(T - 273\mathrm{K})$$

This can be plotted on a spreadsheet or other modes.

6. Estimate the thermal conductivity of platinum at -100^{0}C if its electrical conductivity is 6 x 10^7 mhos/m, based on the Wiedemann-Franz law. Note: 1 mho = 1 amp/volt = 1 coulomb/s, 1 W = 1 J/s = 1 volt-coulomb.

Solution

Using the Wiedemann-Franz law, Equation 2-9 gives

$$\kappa = Lz\cdot T = \left(2.43x10^{-8}\,V^2/K^2\right)\left(6x10^7\,amp/V\cdot m\right)\left(173K\right) = 252.2W / m\cdot K$$

7. Calculate the thermal conductivity of carbon bisulfide using Equation 2-6 and compare this result to the listed value given in Table 2-2.

Solution

Equation 2-6 uses the sonic velocity in the material. This is $V_s = \sqrt{E_b/\rho} = 1.18\ x\ 10^5 cm/s$, where E_b is the bulk modulus. The mean distance between adjacent molecules, assuming a uniform cubic arrangement, is also used. This is $x_m = \sqrt{mm/\rho}$ where mm is the mass of one molecule; MW/Avogadro's number. This gives $x_m = 0.466\ x\ 10^{-7} cm$. then

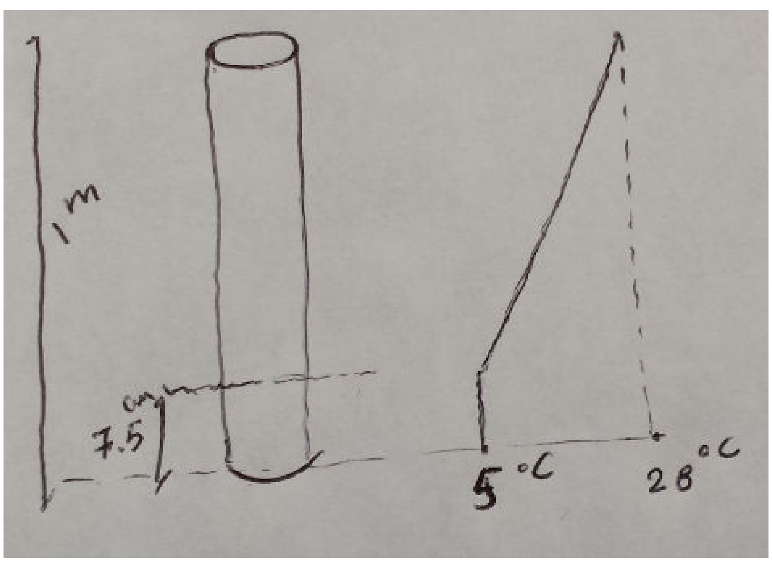
$$\kappa = 3.865x10^{-23}\frac{V_s}{x_m^2} = 0.0021W/cm\cdot{}^0C$$

Section 2-2

8. Estimate the temperature distribution in a stainless steel rod, 2.5 cm in diameter, that is 1 meter long with 7.5 cm of one end submerged in water at 5⁰C and the other end held by a person. Assume the person's skin temperature is 28⁰C, the temperature in the rod is uniform at any point in the rod, and steady state conditions are present.

Solution

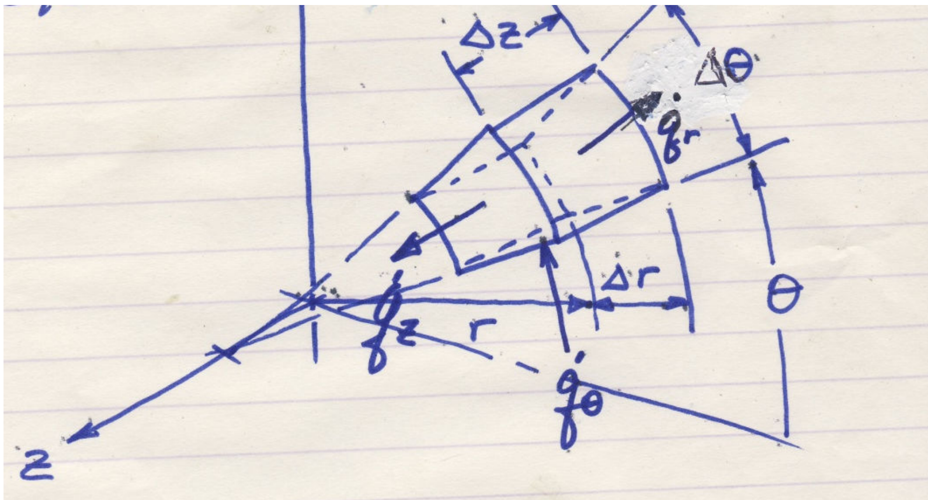
Assuming the heat flow to be axial and not radial and also 5⁰C for the first 7.5 cm of the rod, the temperature distribution between x = 7.5 cm and out to x = 1 m we can use Fourier's law of conduction and then for 7.5 ≤ x ≤ 100 cm, identifying the slope and x-intercept $T(x) = 0.249\ x + 3.135$ The sketched graph is here included. One could now predict the heat flow axially through the rod, using Fourier's law and using a thermal conductivity for stainless steel.



9. Derive the general energy equation for conduction heat transfer through a homogeneous, isotropic media in cylindrical coordinates, Equation 2-19.

Solution

Referring to the cylindrical element sketch, you can apply an energy balance, Energy in – Energy Out = Energy Accumulated in the Element. Then, accounting the energies in and out as conduction heat transfer we can write



$$\left[\dot{q}_r\right]_r = \left[-\kappa r \Delta z \Delta\theta \frac{\partial T}{\partial r}\right]_r$$

an **in** energy

$$\left[\dot{q}_\theta\right]_\theta = \left[-\kappa \Delta r \Delta z \frac{1}{r}\frac{\partial T}{\partial \theta}\right]_\theta$$

an **in** energy

$$\left[\dot{q}_z\right]_z = \left[-\kappa\left(r+\frac{\Delta r}{2}\right)\Delta\theta\Delta r\frac{\partial T}{\partial z}\right]_z$$

an **in** energy

$$\left[\dot{q}_r\right]_{r+\Delta r} = \left[-\kappa\left(r+\Delta r\right)\Delta z\Delta\theta\frac{\partial T}{\partial r}\right]_{r+\Delta r}$$

an **out** energy

$$\left[\dot{q}_{\theta+\Delta\theta}\right]_{\theta+\Delta\theta} = \left[-\kappa \Delta r \Delta z \frac{1}{r}\frac{\partial T}{\partial \theta}\right]_{\theta+\Delta\theta}$$

an **out** energy

$$\left[\dot{q}_{z+\Delta z}\right]_{z+\Delta z} = \left[-\kappa\left(r+\frac{\Delta r}{2}\right)\Delta\theta\Delta r\frac{\partial T}{\partial x}\right]_{z+\Delta z}$$

an **out** energy

$$\rho\left(r+\frac{\Delta r}{2}\right)\left(\Delta\theta\cdot\Delta z\cdot\Delta r\right)c_p\frac{\partial T}{\partial t}$$

The rate of energy accumulated in the element. If you put the three energy in terms and the three out terms on the left side of the energy balance and the accumulated energy on the right, divide all terms by $(r + r/2)(\Delta\theta \cdot \Delta z \cdot \Delta r)$, and take the limits as $\Delta r \rightarrow 0$, *Δz → 0,* and *Δϑ→ 0* gives, using calculus, Equation 2-19

$$\frac{1}{r}\frac{\partial}{\partial r}\kappa r\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial}{\partial \theta}\kappa\frac{\partial T}{\partial \theta}+\frac{\partial}{\partial z}\kappa\frac{\partial T}{\partial z}=\rho c_p\frac{\partial T}{\partial t}$$

10. Derive the general energy equation for conduction heat transfer through a homogeneous, isotropic media in spherical coordinates, Equation 2-20.

Solution

Referring to the sketch of an element for conduction heat transfer in spherical coordinates, you can balance the energy in – the energy out equal to the energy accumulated in the element. Using Fourier's law of conduction

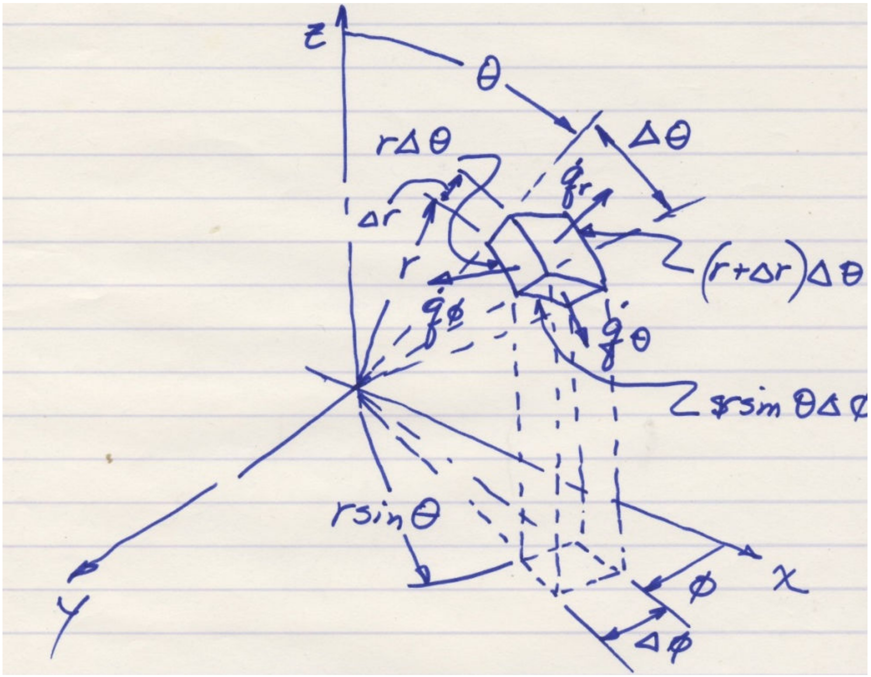
$$\left[\dot{Q}_r\right]_r=\left[-\kappa r\Delta\theta r\sin\theta\Delta\phi\frac{\partial T}{\partial r}\right]_r$$ an **in** term

$$\left[\dot{Q}_\theta\right]_\theta=\left[-\kappa\frac{1}{r}\Delta r\cdot r\sin\theta\Delta\phi\frac{\partial T}{\partial \theta}\right]_\theta$$ an **in** term

$$\left[\dot{Q}_\phi\right]_\phi=\left[-\kappa r\Delta\theta\Delta r\frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\right]_\phi$$ an **in** term

$$\left[\dot{Q}_{r+\Delta r}\right]_{r+\Delta r}=\left[-\kappa\left(r+\Delta r\right)\Delta\theta\left(r+\Delta r\right)sin\theta\Delta\phi\frac{\partial T}{\partial r}\right]_{r+\Delta r}$$ an **out** term

$$\left[\dot{Q}_{\theta+\Delta\theta}\right]_{\theta+\Delta\theta}=\left[-\kappa\frac{1}{r}r\sin\theta\Delta\phi r\Delta\theta\frac{\partial T}{\partial \theta}\right]_{\theta+\Delta\theta}$$ an **out** term



$$\left[\dot{Q}_{\phi+\Delta\phi}\right]_{\phi+\Delta\phi} = \left[-\kappa r\Delta\theta \frac{\Delta r}{r\sin\theta}\frac{\partial T}{\partial\phi}\right]_{\phi+\Delta\phi}$$

an **out** term

$$\rho\Delta V c_p \frac{\partial T}{\partial t} = \rho\left(r\sin\theta\Delta\phi\right)\cdot\Delta r\cdot r\Delta\theta\frac{\partial T}{\partial t}$$

Which is the accumulated energy. Inserting the three in terms as positive on the left side of the energy balance, inserting the three out terms as negative on the left side of the balance, inserting the accumulated term on the right side, and dividing all terms by the quantity $(rsin\theta\Delta\varphi)\cdot\Delta r\cdot\Delta\theta$ gives the following

$$\frac{\kappa\left(r+\Delta r\right)^2\sin\theta\Delta\theta\Delta\phi\left(\frac{\partial T}{\partial r}\right)_{r+\Delta r} - \kappa\left(r\right)^2\sin\theta\Delta\theta\Delta\phi\left(\frac{\partial T}{\partial r}\right)_r}{r^r\sin\theta\Delta r\Delta\theta\Delta\phi} +$$

$$\frac{\kappa r\sin\theta\Delta\phi\Delta\theta\left(\frac{\partial T}{\partial\theta}\right)_{\theta+\Delta\theta} - \kappa r\sin\theta\Delta\phi\Delta\theta\left(\frac{\partial T}{\partial\theta}\right)_\theta}{r^2\sin\theta\Delta r\Delta\theta\Delta\phi} +$$

$$\frac{\frac{\kappa r\Delta\theta\Delta r}{r\sin\theta}\left(\frac{\partial T}{\partial\phi}\right)_{\phi+\Delta\phi} - \frac{\kappa\Delta\theta\Delta r}{r\sin\theta}\left(\frac{\partial T}{\partial\phi}\right)_\phi}{r^2\sin\theta\Delta r\Delta\theta\Delta\phi} = \rho c_p\frac{\partial T}{\partial t}$$

Taking the limits as *$\Delta r \to 0$, $\Delta\vartheta \to 0$, $\Delta\varphi \to 0$* and reducing

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\kappa r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\kappa\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left(\kappa\frac{\partial T}{\partial\phi}\right) = \rho c_p\frac{\partial T}{\partial t}$$

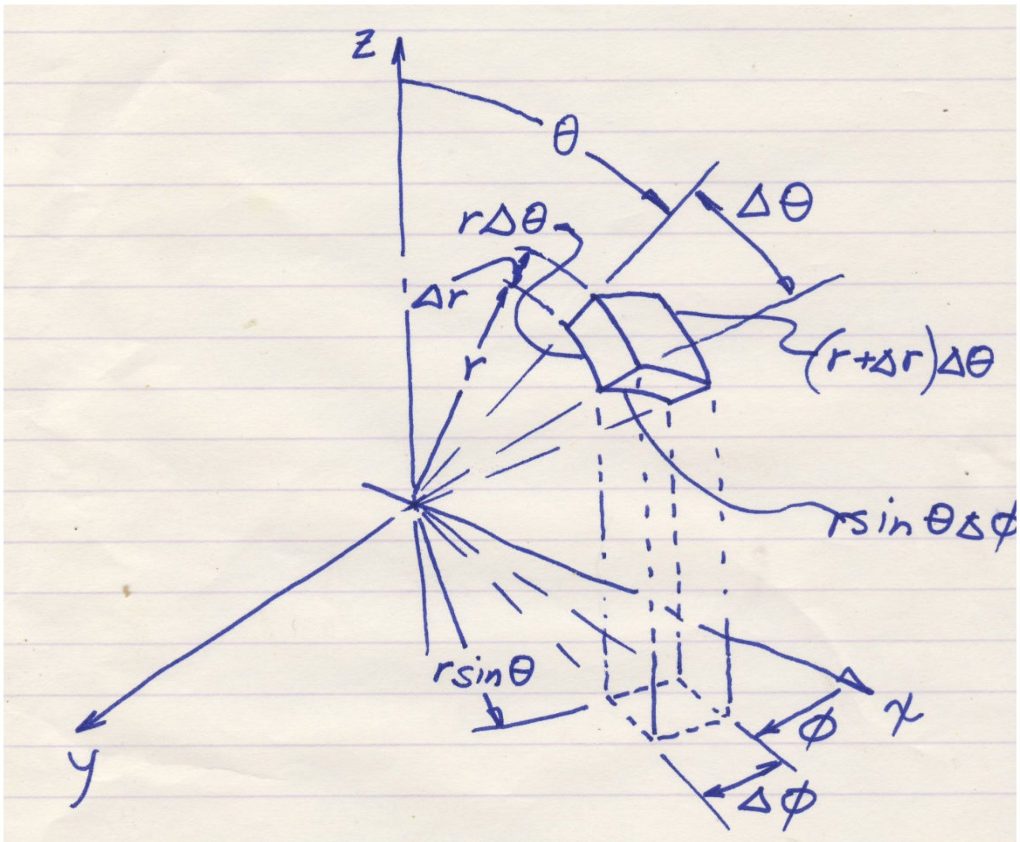
which is Equation 2-20, conservation of energy for conduction heat transfer in spherical coordinates.

11. Determine a relationship for the volume element in spherical coordinates.

Solution

Referring to the sketch for an element in spherical coordinates, and guided by the concept of a volume element gives,

$$\Delta V = \left(r\sin\theta\Delta\phi\right)\cdot\left(\Delta r\right)\cdot\left(r\Delta\theta\right)$$



Section 2-3

12. An ice-storage facility uses sawdust as an insulator. If the outside walls are 60 cm thick sawdust and the sideboard thermal conductivity is neglected, determine the R-value of the walls. If the inside temperature is -5^{0}C and the outside is 30^{0}C, estimate the heat gain of the storage facility per square foot of outside wall.

Solution

Assuming steady state conditions and that the thermal conductivity is the value listed in Appendix Table B-2,

$$R - Value = \frac{\Delta x}{\kappa} = \frac{0.6\ \mathrm{m}}{0.059\ \mathrm{W/m \cdot K}} = 10.17 \frac{\mathrm{m^2 \cdot K}}{\mathrm{W}}$$

$$\dot{q}_A = \kappa \frac{\Delta T}{\Delta x} = \frac{\Delta T}{R - Value} = \frac{30°\mathrm{C} - (-5°\mathrm{C})}{10.17} = 3.44\ \frac{W}{m^2} = 1.09 \frac{Btu}{hr \cdot ft^2}$$

13. The combustion chamber of an internal combustion engine is at 800^0C when fuel is burned in the chamber. If the engine is made of cast iron with an average thickness of 6.5 cm between the combustion chamber and the outside surface, estimate the heat transfer per unit area if the outside surface temperature is 50^0C and the outside air temperature is 30^0C.

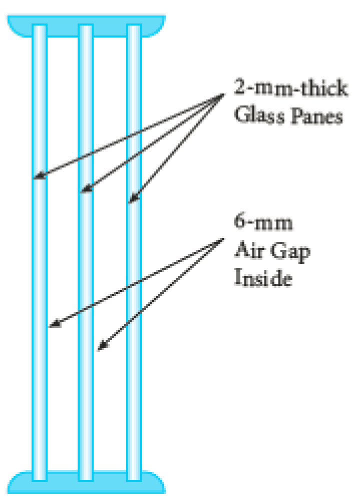
Solution

Assuming steady state one-dimensional conduction and using a thermal conductivity that is assumed constant and has a value from Table B-2,

$$\dot{q}_A = \kappa \frac{\Delta T}{\Delta x} = \left(39 \frac{W}{m \cdot K}\right) \frac{800^0 - 50^0 K}{0.065 m} = 450 kW / m^2$$

14. Triple-pane window glass has been used in some building construction. Triple pane glass is a set of three glass panels, each separated by a sealed air gap as shown in Figure 2-49. Estimate the R-Value for triple pane windows and compare this to the R-Value for single pane glass. Note that the air within the gap is sealed and cannot move so that it acts as a conducting medium only.

FIG 2-49 Triple pane window.



Solution

Assume the air in the gaps do not move so that they are essentially conducting media. Then the R-Value is

$$R - Value = 3(\frac{\Delta x}{\kappa})_{glass} + 2(\frac{\Delta x}{\kappa})_{air} = 3(\frac{0.002}{1.4}) + 2(\frac{0.006}{0.026}) = 0.4658\ \mathrm{m}^2 \cdot \frac{\mathrm{K}}{\mathrm{W}}$$

The R-Value for a single pane window is

$$R - Value = \left(\frac{\Delta x}{\kappa}\right)_{glass} = \frac{0.002\ \mathrm{m}}{1.4\,\frac{\mathrm{W}}{\mathrm{m \cdot K}}} = 0.1429\ \mathrm{m}^2 \cdot \frac{\mathrm{K}}{\mathrm{W}}$$

The ratio of the R-Value for the triple pane to the R-Value for a single pane is roughly 324

15. For the outside wall shown in Figure 2-50, determine the R-Value, the heat transfer through the wall per unit area and the temperature distribution through the wall if the outside surface temperature is 36^0C and the inside surface temperature is 15^0C.

Solution

The R-Value is the sum of the three materials; pine, plywood, and limestone, with thermal conductivity

$$R - Value = R_V = \left(\frac{\Delta x}{\kappa}\right)_{pine} + \left(\frac{\Delta x}{\kappa}\right)_{plywood} + \left(\frac{\Delta x}{k}\right)_{\lim estone} = \frac{0.04}{0.15} + \frac{0.02}{0.12} + \frac{0.06}{2.15} = 0.462 m^2 \cdot K / W$$

values obtained from Appendix Table B-2.The conversion to English units is 0.176 $\mathrm{m^2K/W}$ = 1 R-Value so that $R - Value = 2.62$. The heat transfer per unit area is

$$\dot{q}_A = \frac{\Delta T}{R_V} = \frac{36-15}{0.462} = 45.45 W / m^2$$

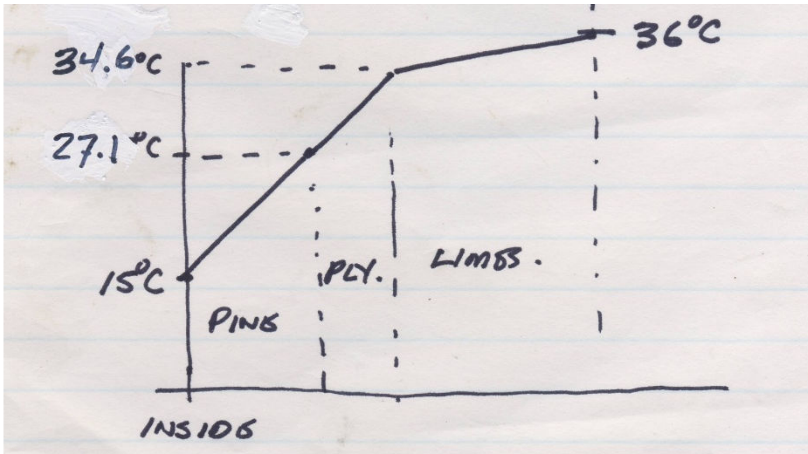
The temperature distribution is determined by noting that the heat flow is the same through each material. For the pine,

$$\dot{q}_{A,pine} = 45.45 W / m^2 = \frac{T_1 - 15^0 C}{R_{V.\mathrm{pine}}} = \frac{T_1 - 15}{0.04 / 0.15}$$

so that T_1 at the surface between the pine and the plywood, is 27.1^0C. Similarly, to determine the temperature between the plywood and the limestone, again noting that the heat flow is the same as before

$$\dot{q}_A = 45.45 = \frac{T_2 - T_1}{R_{V,plywood}} = \frac{T_2 - 27.1}{0.02 / 0.12}$$

so that T_2 is 34.6^{0}C. This is sketched in the figure.



16. Determine the heat transfer per foot of length through a copper tube having an outside diameter of 5 cm and an inside diameter of 3.8 cm. The pipe contains 80^{0}C ammonia and is surrounded by 25^{0}C air.

__Solution__

Assuming steady state and only conduction heat transfer, for a tube cylindrical coordinates is the appropriate means of analysis. Then

$$\dot{q}_l = \frac{2\pi\kappa\Delta T}{\ln\left({}^{D_o}/_{D_i}\right)} = \frac{2\pi\left(400\ {}^{\mathrm{W}}/_{\mathrm{m}\cdot\mathrm{K}}\right)(80^\circ\mathrm{C} - 25^\circ\mathrm{C})}{\ln(\frac{5\ \mathrm{cm}}{3.8\ \mathrm{cm}})} = 503{,}690\ {}^{\mathrm{W}}/_{\mathrm{m}}$$

$$= 523{,}980\ {}^{\mathrm{Btu}}/_{\mathrm{hr}\cdot\mathrm{ft}}$$

17. A steam line is insulated with 15 cm of rock wool. The steam line is a 5 cm OD iron pipe with a 5 mm thick wall. Estimate the heat loss through the pipe per meter length if steam at 120^{0}C is in the line and the surrounding temperature is 20^{0}C. Also determine the temperature distribution through the pipe and insulation.

__Solution__

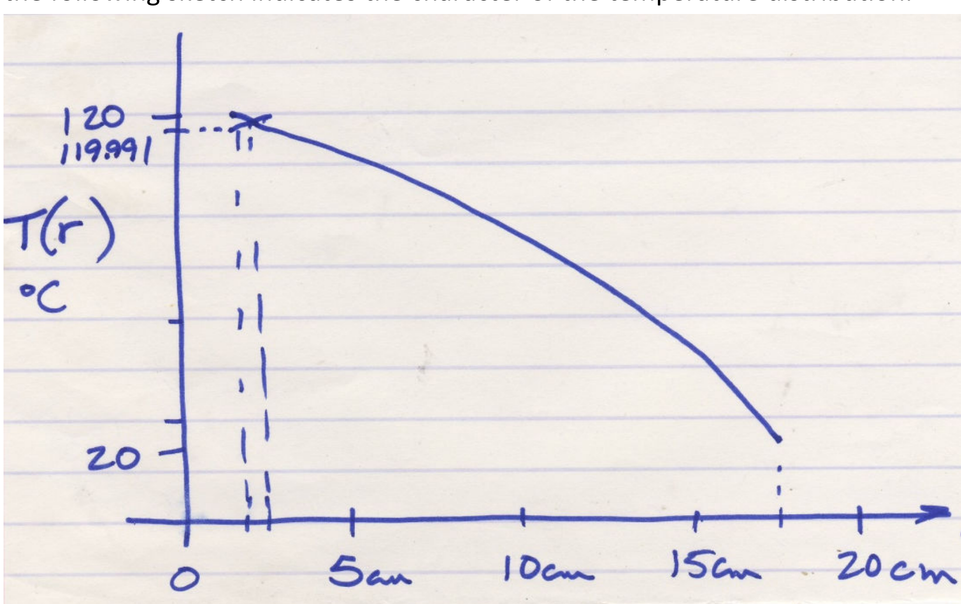
Assume heat flow is one-dimensional radial and steady state. The heat flow is then the overall temperature difference divided by the sum of the radial thermal resistances. We have

$$\dot{q}_l = \frac{(\Delta T)_{overall}}{\left(\frac{\ln(D_0/D_i)}{2\pi\kappa}\right)_{pipe} + \left(\frac{\ln(D_0/D_i)}{2\pi\kappa}\right)_{wool}} = \frac{(120-20)}{\left(\frac{\ln(5/4)}{2\pi\cdot 51}\right) + \left(\frac{\ln(35/5)}{2\pi\cdot 0.04}\right)} = 12.91 W/m^2$$

To determine the temperature distribution through the pipe and wool insulation the radial heat flow will be same through the iron pipe and the wool insulation. The temperature at the interface between the iron pipe and the insulation is determined by

$$\dot{q}_l = 1(20 - T_{pipeOD})\frac{2\pi(51)}{\ln(5/4)}$$

From this the interface temperature, $T_{pipeOd} = 119.991^0C = T_{woolID}$ The temperature in a homogeneous radial section is $(r) = T_0 + Clnr$. For the iron pipe, the two boundary conditions 1.) $T = 120^0C$ @ $r = 2$ *cm* and 2.) $T = 119.991^0C$ @ $r = 2.5$ *cm* can be used to solve for $T(r)$ and resulting in two separate equations. Solving these two simultaneously gives that $T_0 = 120.028^0C$ and $C = -0.040$. For the iron pipe then $T(r) = 120.928 - 0.040lnr$. For the wool insulation the two boundary conditions 1.) $T = 119.991^0C$ @ $r = 2.5$ *cm* and 2.) $T = 20^0C$ @ $r = 17.5$ *cm* can be substituted into the equation to solve for $T(r)$. Solving these two equations simultaneously for T_0 and C gives that $T_0 = 167.07$ and $C = -51.385$. For the wool insulation $T(r) = 157.07 - 51.385lnr$. the following sketch indicates the character of the temperature distribution.



18. Evaporator tubes in a refrigerator are constructed of 2.5 cm OD aluminum tubing with 3 mm thick walls. The air surrounding the tubing is at -5^{0}C and the refrigerant in the evaporator is at -10^{0}C. Estimate the heat transfer to the refrigerant over 30 cm of length.

Solution

Assume steady state one-dimensional radial conduction heat transfer and using a thermal conductivity value from Appendix Table B-2

$$\dot{Q} = \frac{2\pi\kappa L}{\ln\left({r_o}/{r_i}\right)}(T_o - T_i) = \frac{2\pi(236\ {W}/{m \cdot K})(0.3\ m)}{\ln\left({1.25\ cm}/{(0.95\ cm)}\right)}\left(-5°C - (-10°C)\right)$$

$$= 8{,}105\ W$$

19. Teflon tubing of 4 cm OD and 2.7 cm ID conducts 1.9 W/m when the outside temperature is 80^{0}C. Estimate the inside temperature of the tubing. Also predict the thermal resistance per unit length.

Solution

Assume steady state one-dimensional radial conduction heat transfer. Reading the thermal conductivity from Appendix Table B-2, applying the Fourier's Law of conduction for radial heat flow

$$\dot{q}_l = \frac{2\pi\kappa}{\ln\left(r_0/r_i\right)}\left(T_i - T_0\right) = 1.9\ W/m$$

and solving for T_i

$$T_i = T_0 + \frac{(1.9W/m)\ln\left(2cm/1.35cm\right)}{2\pi\left(0.35W/m\cdot^0 C\right)} = 80.34^0 C$$

and the thermal resistance per unit of length is

$$R_{TL} = \frac{\ln\left(r_0/r_i\right)}{2\pi\kappa} = \frac{\ln\left(4/2.7\right)}{2\pi\left(0.35\right)} = 0.1787 m\cdot K/W$$

20. A spherical flask, 4 m in diameter with a 5 mm thick wall, is used to heat grape juice. During the heating process the outside surface of the flask is 100^{0}C and the inside surface is 80^{0}C. Estimate the thermal resistance of the flask; the heat transfer through the flask, if it is assumed that only the bottom half is heated; and the temperature distribution through the flask wall.

Solution

Assume steady state one-dimensional, radial conduction heat transfer with constant properties. Since only the bottom half is heated you need to recall that a surface area of a hemisphere is 2πr² rather than 4πr². Then

$$\dot{Q} = \frac{2\pi\kappa(T_0 - T_i)}{\frac{1}{r_i} - \frac{1}{r_0}} = \frac{2\pi(1.4W/m\cdot K)(100-80^0C)}{\frac{1}{1.905m} - \frac{1}{2m}} = 7056W$$

The thermal resistance for the full flask would be

$$R_T = \left(\frac{1}{r_i} - \frac{1}{r_0}\right)\left(\frac{1}{4\pi\kappa}\right) = \left(\frac{1}{1.905m} - \frac{1}{2m}\right)\left(\frac{1}{4\pi(1.4W/m\cdot K)}\right) = 0.001417 m\cdot K/W$$

For such a small thermal resistance, the temperature distribution will be nearly constant through the wall. Yet for the bottom half of the flask we can write

$$T(r) = T_i + \frac{\dot{Q}}{2\pi\kappa}\left(\frac{1}{r_i} - \frac{1}{r}\right) = 80^0C + \frac{7056W}{2\pi(1.4\,\mathrm{W/m\cdot^0}C)}\left(\frac{1}{1.905m} - \frac{1}{r}\right)$$

or

$$T(\mathrm{r}) = 80^0C + 401m\cdot^0C\left(0.525\,\mathrm{m}^{-1} - \frac{1}{r}\right)$$

21. A Styrofoam spherical container having a 2.5 cm thick wall and 60 cm diameter holds dry ice (solid carbon dioxide) at -65⁰C. If the outside temperature is 15⁰C, estimate the heat gain in the container and establish the temperature distribution through the 2.5-cm wall.

Solution

Assuming steady state one-dimensional radial conduction heat transfer and using the thermal conductivity value for Styrofoam from Appendix Table B-2

$$\dot{Q} = \frac{4\pi\kappa}{\frac{1}{r_i} - \frac{1}{r_o}}(T_o - T_i) = \frac{4\pi(0.029\ \mathrm{W/m\cdot K})}{\frac{1}{0.275\ \mathrm{m}} - \frac{1}{0.3\ \mathrm{m}}}(15°\mathrm{C} - (-65°\mathrm{C})) = 96.2\ \mathrm{W}$$

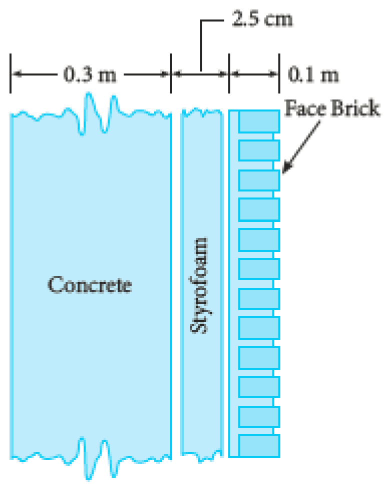
The temperature distribution for T(r) is

$$T(r) = -65°\mathrm{C} + \frac{96.2\,W}{4\pi(0.029\ W/m\cdot K)}\left(\frac{1}{0.275\ \mathrm{m}} - \frac{1}{r}\right)$$
$$= -65°\mathrm{C} + 264.11\left(\frac{1}{0.275\ \mathrm{m}} - \frac{1}{r}\right)$$

where r is in meters.

22. Determine the overall thermal resistance per unit area for the wall shown in Figure 2-51. Exclude the thermal resistance due to convection heat transfer in the analysis. Then, if the heat transfer is expected to be 19.0 W/m^2 and the exposed brick surface is 10^{0}C, estimate the temperature distribution through the wall.

FIG 2-51 Structural wall.



Solution

The overall thermal resistance will be the sum of the thermal resistances of the three components,

$$R_V = \frac{0.3m}{1.6W/m\cdot K} + \frac{0.025m}{0.029W/m\cdot K} + \frac{0.1m}{0.7W/m\cdot K} = 1.192m^2\cdot K/W$$

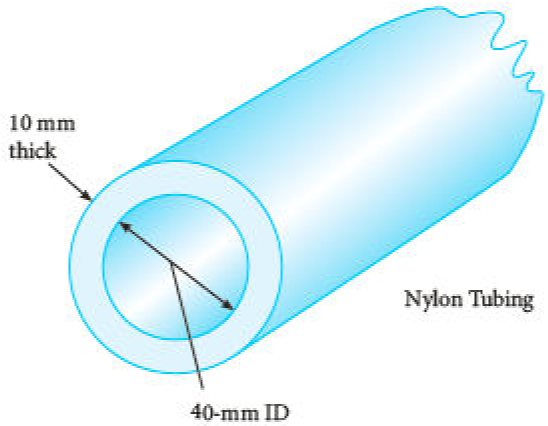
Since there is expected to be 19.0 W/m^2 of conduction heat transfer through each of the three components, the temperatures at the inside surface and the two interface surfaces are $T_{inside} = \left(19.0W/m^2\right)\left(1.192m^2\cdot^0 C/W\right) + 10^0C = 32.6^0C$ which is the inside surface temperature. The temperature between the concrete and the Styrofoam is $T_{c-styr} = T_{inside} - \dot{Q}\cdot R_{concrete} = 32.6^0C - \left(19W/m^2\right)\left(0.1875m^2\cdot^0 C/W\right) = 29.0^0C$

and the temperature between the Styrofoam and the brick facing is

$$T_{styr-brick} = T_0 + \dot{Q}\cdot R_{brick} = 10^0C + \left(19W/m^2\right)\left(0.143m^2\cdot^0 C/W\right) = 12.7^0C$$

23. Determine the thermal resistance per unit length of the tubing (nylon) shown in Figure 2-52. Then predict the heat transfer through the tubing if the inside ambient temperature is -10^{0}C and the outside is 20^{0}C.

FIG 2-52 Tubing.



Solution

The nylon tubing has properties of Teflon, the inside diameter is 40mm, and the outside diameter is 60mm. Then

$$R_{TL} = \frac{\ln(D_0/D_i)}{2\pi\kappa} = \frac{\ln(r_0/r_i)}{2\pi\kappa} = \frac{\ln(60mm/40mm)}{2\pi(0.35W/m\cdot K)} = 0.184 m\cdot K/W$$

Assuming steady state one-dimensional radial conduction heat transfer,

$$\dot{q}_l = \frac{\Delta T}{R_{TL}} = \frac{30^0 C}{0.184 m\cdot{}^0 C/W} = 162.7 W/m$$

24. Determine the temperature distribution through the wall of Example Problem 2-5 if the thermal conductivity is affected by temperature through the relationship

$$\kappa = 0.638 + 0.00270\,T\ \frac{\text{W}\cdot\text{cm}}{\text{m}^2\cdot\text{K}}$$

where T is in Kelvin.

Solution

In Example 2-5 the wall is 40 cm thick, has a temperature of 12^0C on one side and 40^0F on the other. Assuming steady state one-dimensional conduction heat transfer

$$\dot{q}_A = -\kappa\frac{dT}{dx} = -(0.638 + 0.00270\,T)\frac{dT}{dx}$$

separating variables and integrating

$$\dot{q}_A \int dx = \dot{q}_A(40\ cm) = -\int (0.638 + 0.00270\ T)dT$$

$$= -9.2(12 - 40) - \frac{1}{2}(0.00270)(12^2 - 40^2)$$

and then solving for the heat transfer per unit area gives

$$\dot{q}_A = \frac{1}{40}\left[257.6\ \frac{W \cdot cm}{m^2} + 1.96 \frac{W \cdot cm}{m^2}\right] = 6.49\ \frac{W}{m^2}$$

25. Determine the temperature distribution through a slab if κ = $aT^{0.001}$, T is in Kelvin, and *a* is a constant. Then compare this to the case where κ = *a.*

Solution

$\dot{q}_A = -\kappa \frac{dT}{dx} = -aT^{0.001}\frac{dT}{dx}$ If the variables are now separated and integrating

$\dot{q}_A \int dx = \dot{q}_A x = -a\int T^{0.001} dT = -\frac{a}{1.001}T^{1.001} - C$ defining a boundary condition of $T = T_0$ @ $x = 0$ allows the constant C to be defined as $C = -\frac{a}{1.001}T_0^{1.001}$ the temperature distribution is then $T(\text{x}) = \sqrt[1.001]{T_0^{1.001} - \dot{q}_A x\left(\frac{1.001}{a}\right)}$

For $\kappa = a$ and $T = T_0$ @ $x = 0$ $T = T_0 - \dot{q}_A \frac{x}{a}$

Section 2-4

26. Show that $T(x, y) = (a\ sin\ px + b\ cos\ px)(ce^{-py} + de^{py})$ satisfies Laplace's equation $\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = 0.$

Solution

Taking first and second derivatives $\frac{\partial T}{\partial x} = \left(ap\cos px - bp\sin px\right)\left(ce^{-py} + de^{py}\right)$

and $\frac{\partial^2 T}{\partial x^2} = -\left(ap^2 \sin px + bp^2 \cos px\right)\left(ce^{-py} + de^{py}\right)$ taking the first and second partial derivative with respect to y give, for the second derivative that

$$\frac{\partial^2 T}{\partial y^2} = \left(a \sin px + b \cos px\right) p^2 \left(ce^{-py} + de^{py}\right)$$

summing these last two equations gives Laplace's equation.

27. For the wall of Example Problem 2-11, determine the heat transfer in the y-direction at 3 feet above the base. Then plot the temperature distribution at this level.

Solution

The solution to the wall temperature of example 2-11 is $T(x, y) = \left(50^0 F\right) e^{-\pi y/L} \sin \frac{\pi x}{L}$

The heat transfer in the y-direction can be determined,

$$\dot{Q}_y = -\kappa A_y \frac{\partial T}{\partial y} = -\kappa W \int_0^L \frac{\partial T}{\partial y} dx = -\kappa W \int \left(-50^0 F\right)\left(\frac{\pi}{L}\right) e^{-\pi y/L} \sin \frac{\pi x}{L}$$

For *W = 1 ft, L = 3 ft,* and *y = 3 ft* this equation can then be finalized

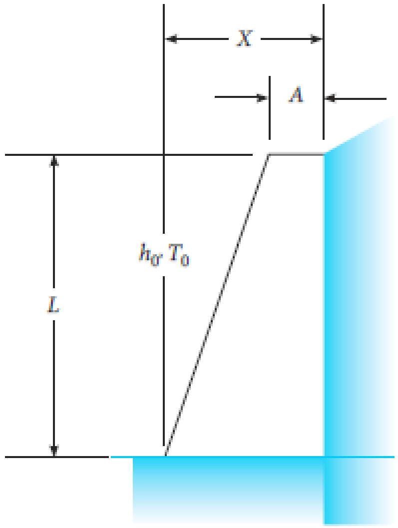
$$\dot{Q}_y = -\left(50^0 F\right)\left(\frac{\kappa \pi}{3 ft}\right) \int_0^{3 ft} e^{-\pi} \sin \frac{\pi x}{3 ft} dx = \left(50^0 F\right)\left(\frac{\kappa \pi}{3 ft}\right) e^{-\pi} \left[\frac{3 ft}{\pi} \cos \frac{\pi x}{3 ft}\right]_0^{3 ft} = \left(50^0 F\right)\left(\kappa\right) e^{-\pi} \left(2\right)$$

For a thermal conductivity of *0.925Btu/hr·ft°F* from Appendix Table B-2E, the heat transfer is about *4.00 Btu/hr.* The temperature distribution at y = 3 ft for *0 ≤ x ≤ 3ft* is

$$T(x, y = 3 ft) = 2.15 \sin \frac{\pi x}{3 ft}$$

28. Write the governing equation and the necessary boundary conditions for the problem of a tapered wall as shown in Figure 2-53.

FIG 2-53 Tapered wall with heat transfer.



Solution

For steady state conduction in two-dimensions the governing equation will be $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$. Calling T_g the ground temperature the following four (4) boundary conditions may be used:

B.C. 1 $T(x,0) = T_g \quad for\ 0 < x \le X$

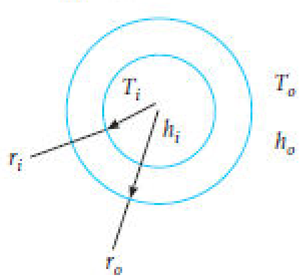
B.C. 2 $T(X,y) = T_g \quad for\ 0 \le y \le L$

B.C. 3 $T(x,L) = T_0 \quad for\ X - A \le x \le X$

B.C. 4 $T(x,y) = T_0 \quad for\ 0 \le y \le L \quad and \quad y = (L/X - A)\,x$

29. Write the governing equation and the necessary boundary conditions for the problem of a heat exchanger tube as shown in Figure 2-54.

FIG 2-54 Heat exchanger tube.



Solution

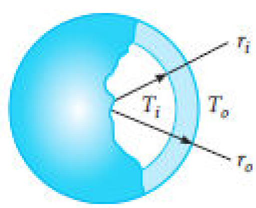
A heat exchanger tube with convection heat transfer at the inside and the outside surfaces can be analyzed for steady state one-dimensional radial heat transfer with the equation $\frac{1}{r}\frac{d}{dr}\kappa r\frac{dT}{dr}=0$ and with, as a possibility, the following two boundary conditions

B.C. 1 $\dot{q}_r = 2\pi r_i h_i (T_i - T)$ @ $r = r_i$

B.C. 2 $\dot{q}_r = 2\pi r_0 h_0 (T - T_0)$ @ $r = r_0$

30. Write the governing equation and the necessary boundary conditions for the problem of a spherical concrete shell as sketched in Figure 2-55.

FIG 2-55 Spherical thick walled shell.



Solution

For steady state one-dimensional radial conduction heat transfer in spherical coordinates the governing equation for analyzing this and two suggested boundary conditions are $\frac{1}{r^2}\frac{d}{dr}\kappa r^2\frac{dT}{dr}=0$

B.C. 1 $T = T_0$ @ $r = r_0$

B.C. 2 $T = T_i$ @ $r = r_i$

31. Determine the Fourier coefficient A_n for the problem resulting in a temperature distribution of $T(x,y) = \sum_{n=0}^{\infty} A_n e^{-n\pi y/L} sin(n\pi x/L)$ involving a boundary temperature distribution given by $T(x,0) = cos(\pi x/L)$ for $0 \le x \le L$.

Solution

The Fourier coefficient is defined as

$$A_n = \frac{2}{L}\int_0^L T(x,0)\sin\left(\frac{n\pi x}{L}\right)dx = \frac{2}{L}\int_0^L \cos\left(\frac{\pi x}{L}\right)\sin\left(\frac{n\pi x}{L}\right)dx$$

and using an identity

$$A_n = \frac{2}{L}\int_0^L \frac{1}{2}\left(\sin\left(\frac{n\pi x}{L}+\frac{\pi x}{L}\right)+\sin\left(\frac{n\pi x}{L}-\frac{\pi x}{L}\right)\right)dx = \frac{1}{L}\int_0^L\left[\sin\left(\frac{n\pi x}{L}+\frac{\pi x}{L}\right)+\sin\left(\frac{n\pi x}{L}-\frac{\pi x}{L}\right)\right]dx$$

For n = 0 the Fourier coefficient, A_0 becomes

$$A_0 = \frac{1}{L}\int_0^L\left[\sin\left(\frac{\pi x}{L}\right)+\sin\left(\frac{-\pi x}{L}\right)\right] = 0$$

For n = 1 the Fourier coefficient becomes

$$A_1 = \frac{1}{L}\int_0^L \sin\left(\frac{2\pi x}{L}\right)dx = -\frac{L}{2\pi L}\cos\left(\frac{2\pi x}{L}\right)\Bigg|_0^L = 0$$

For n = 2, the Fourier coefficient is

$$A_2 = \frac{1}{L}\int_0^L\left(\sin\left(\frac{3\pi x}{L}\right)+\sin\left(\frac{\pi x}{L}\right)\right)dx = -\frac{1}{3\pi}\cos\left(\frac{3\pi x}{L}\right)-\frac{1}{\pi}\cos\left(\frac{\pi x}{L}\right) = \frac{2}{3\pi}+\frac{2}{\pi}$$

For n =4

$$A_4 = \frac{2}{5\pi}+\frac{2}{3\pi}$$

For any even integer of n, such as 6, 8, 10, etc. the Fourier coefficient is

$$A_n = \frac{2}{(n+1)\pi}+\frac{2}{(n-1)\pi}$$

By reviewing the first coefficient, A_1 it turns out that for all odd integers of n, such as 3, 5, 7, 9, 11, etc, the Fourier coefficient is zero, 0.

32. Determine the Fourier coefficient A_n for the problem involving a boundary temperature distribution given by $T(x,0) = T_0\left(1-\frac{x}{L}\right)$ and where the solution to the temperature field is $T(x,y) = \sum_{n=0}^{\infty} A_n e^{-n\pi y/L} sin(n\pi x/L)$.

Solution

$$A_n = \frac{2}{L}\int_0^L T(\mathrm{x},0)\sin\frac{n\pi x}{L}dx = \frac{2}{L}\int_0^L T_0\left(1-\frac{x}{L}\right)\sin\frac{n\pi x}{L}dx$$

By inspection $A_0 = 0$ for $n = 0$.

For n = 1

$$A_1 = \frac{2T_0}{L}\int_0^L \sin\frac{\pi x}{L}dx - \frac{2T_0}{L^2}\int_0^L x\sin\frac{\pi x}{L}dx$$ Using integral tables in Appendix Table A-4

$$A_1 = \frac{2T_0}{L}\left(-\frac{L}{\pi}\cos\frac{\pi x}{L}\right)_0^L - \frac{2T_0}{L^2}\left(\frac{L^2}{\pi^2}\sin\frac{\pi x}{L} - \frac{L}{\pi}\cos\frac{\pi x}{L}\right)_0^L = \frac{4T_0}{\pi} - \frac{2T_0}{\pi} = \frac{2T_0}{\pi}$$ For n even, such as 2, 4, 6, 8, ...

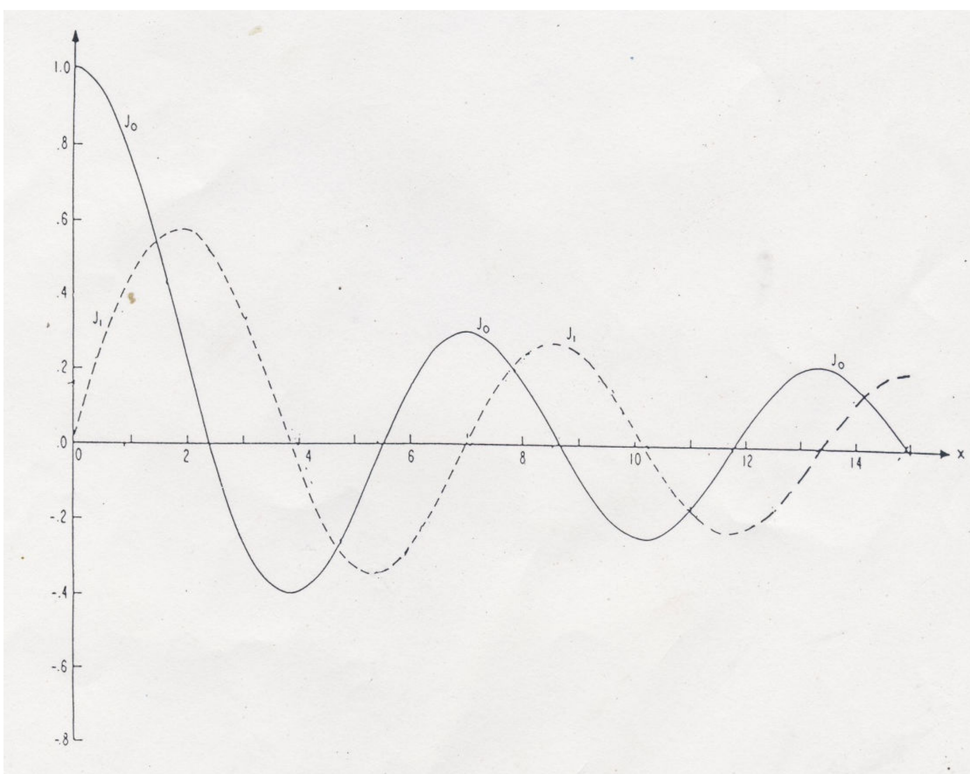
$$A_n = \frac{2T_0}{n\pi}$$ and for n odd, such as 3, 5, 7, 9, ...

$$A_n = \frac{2T_0}{n\pi}$$ which is the same as for n even

33. Plot the Bessel's function of the first kind of zero and first order, J_0 and J_1, for arguments from 0 to 10.

Solution

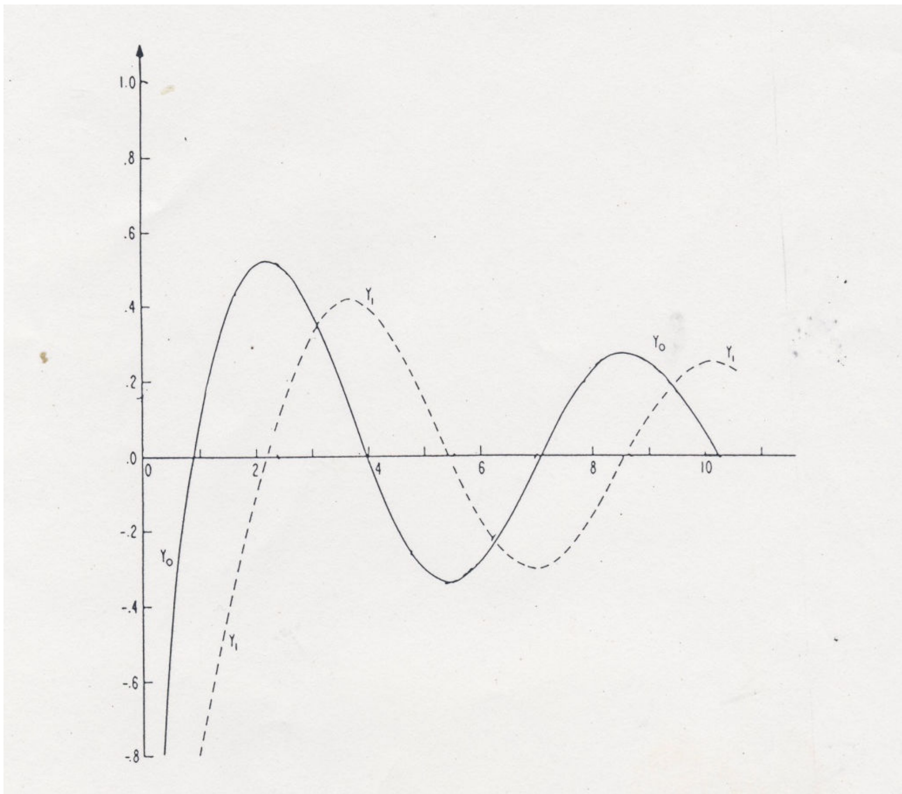
Appendix Table A-10-1 tabulates the Bessel's Function of arguments from 0 to 10. The plot is shown.



34. Plot the Bessel's Function of the second kind of zero and first order, Y_0 and Y_1 for arguments from 0 to 10.

Solution

The Bessel's Functions of the second kind of zeroth and first order are tabulated in Appendix Table A-10-1, plotted in Appendix Figure A-10-2, and here shown.



35. A silicon rod 20 cm in diameter and 30 cm long is exposed to a high temperature at one end so that the end is at 400^0C whereas the remaining surfaces are at 60^0C. Estimate the centerline temperature distribution through the rod.

Solution

The ratio of the length to radius, L/R is 3.0 so, using Figure 2-22 the following values can be read:

x/L	$(T - T_0)/(T_f - T_0)$	T(x) ^{0}Celsius
0.0	0.000	60.00
0.2	0.002	60.68
0.4	0.020	67.20
0.6	0.086	90.96
0.8	0.360	189.6
1.0	1.000	400.0

The values for T(x) are computed from the equation

$$T(\mathrm{x},0)=\left(\frac{T-T_0}{T_f-T_0}\right)\left(400-60^0C\right)+60^0C$$

36. A Teflon rod 15 cm in diameter and 60 cm long is at 110^{0}C. It is then exposed at one end to cool air so that that end reaches 25^{0}C whereas the cylindrical surface cools to 65^{0}C. The other end remains at 110^{0}C at steady state. Determine the expected temperature distribution.

Solution

To determine the centerline temperature distribution you can use Figure 2-22b. Since the L/R value is 60/7.5 = 8 we need to extrapolate on the graph for approximate values. Also, the centerline temperature will not change significantly for values of z/L less than about 0.6. In addition, a principle of superposition will provide the rigorous solution. Yet, since the axial lengths are such that the distance from the 110^{0}C end will be the total length minus the length from the 27^{o}F end, $z_{110} = L - z_{27}$. Since the temperature of the center of the rod, axially, does not change significantly from the 65^{0}C (T_0) for z/L ≤ 0.6, we can just consider each end separately. For the model of a rod at 65^{o}C with one end at 25^{0}C we have, say at z/L of 0.8, from Figure 2-22b that *$(T - T_0)/(25^0F - T_0) = (T - 65)/(25 - 65) = (T - 27)/(-40)$* has a value of about 0.15. Therefore, at z = 12 cm (corresponding to z = 48 cm from the 65^{0}C end) from the 25^{o}C end the centerline temperature is $T(12\ \mathrm{cm},0) = (0.15)(25-65)+65 = 595^0\mathrm{C}$*)*. At say z = 21 cm (0.4 m from the 65^{o}C end), z/L = 0.65, and from Figure 2-22b, *$(T - T_0)/(25^0F - T_0)$ ≈*

0.03 and then the centerline temperature at 21 cm from the 25^{0}C end is $T(21\text{ cm}, 0) = (0.03)(25 - 65) + 65 = 64.4^0\text{C}$. Similarly, for the end at 110^{0}C with the rod at 65^0 C, at z/l = 0.8, corresponding to 12 cm from the 110^{0}C end, the centerline temperature is $T(12\text{ cm}, 0) = (0.15)(110 - 65) + 65 = 72°\text{C}$). At z/L = 0.65 (corresponding to 21 cm from the 110°C end) the centerline temperature is $T(21\text{ cm}, 0) = (0.03)(110 - 65) + 65 = 66.4°\text{C}$

Section 2-5

37. A water line of 5-cm diameter is buried horizontally 1.2 m deep in earth. Estimate the heat loss per foot from the water line if water at 10^{0}C flows through the line and the outside temperature of the line is assumed to be 10^{0}C. The surface temperature of the earth is -30^{0}C.

Solution

Using the shape factor from Table 2-3, item 8, where L » r

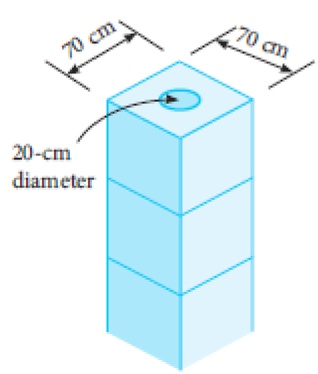
$$S = \frac{2\pi L}{\cosh^{-1}\frac{Y}{r}} = \frac{2\pi L}{\cosh^{-1}\frac{1.2\text{ m}}{2.5 \times 10^{-2}\text{m}}} = \frac{2\pi L}{4.564}$$

The thermal conductivity of earth is about 1.5 W/m·K from Appendix Table B-2 so that the heat transfer per unit length is

$$\dot{q}_l = S\kappa\Delta T(1/L) = \left(\frac{2\pi}{4.654}\right)\left(1.5\ \frac{\text{W}}{\text{m·K}}\right)(40°\text{C}) \approx 81\frac{\text{W}}{\text{m}}$$

38. A chimney is constructed of square concrete blocks with a round flue as shown in Figure 2-56. Estimate the heat loss through the cement blocks per meter of chimney if the outer surface temperature is -10^{0}C and the inner surface temperature is 150^{0}C.

FIG 2-56 Chimney and flue.



Solution

Assuming steady state conduction and using the shape factor from Table 2-3, item 4, the heat loss can be estimated. From Appendix Table B-2 the thermal conductivity of

$$\frac{S}{L} = \frac{2\pi}{\ln\left(0.54\frac{W}{r}\right)} = \frac{2\pi}{\ln\left(0.54(7)\right)} = 4.725$$

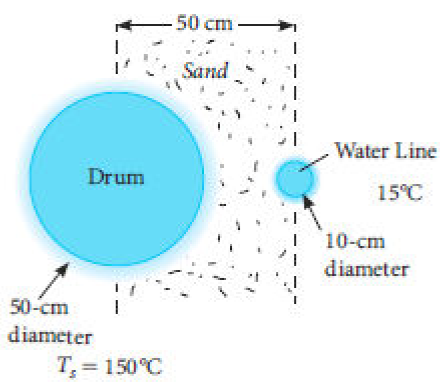
concrete may be taken as 1.4 W/m·K so that

The heat loss can then be calculated from

$$\dot{q}_l = \frac{S}{L}\kappa\Delta T = (4.725)\left(1.4\frac{W}{m\cdot K}\right)\left(150-(-10)^0 C\right) = 1.058\frac{kW}{m}$$

39. Nuclear waste is placed in drums 50 cm in diameter by 100 cm long and buried in sand. Water lines are buried adjacent to the drums to keep them cool. The suggested typical arrangement is shown in Figure 2-57. Estimate the heat transfer between a drum and the water line.

FIG 2-57 Nuclear waste drums.



Solution

Assume steady state, infinite media, and all heat transfer occurs between the 100 cm long drum and an adjacent 10 cm long water line. Using, item 11 from Table 2-3 with r = r_1/r_2 = 25/5 = 5, and L = 50 cm/5 cm = 10, gives

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{L^2-1-r^2}{2r}\right)} = \frac{2\pi}{\cosh^{-1}(7.4)} = 2.336$$

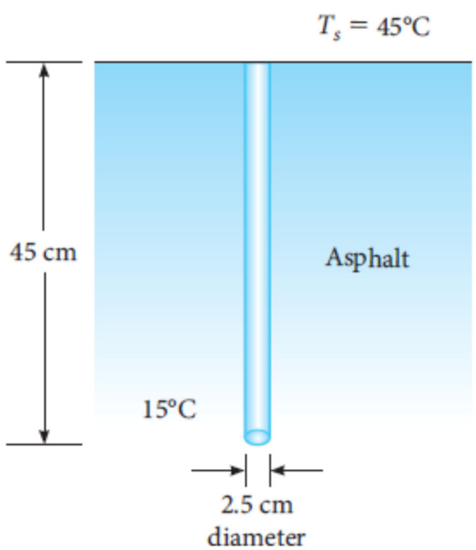
Assuming dry sand with a thermal

conductivity from Appendix Table B-2 of 0.3 W/m·K, the heat transfer is

$$\dot{Q} = S\kappa L\Delta T = (2.336)\left(0.3\frac{W}{m\cdot K}\right)(1m)(135K) = 94.6W$$

40. Steel pins are driven into asphalt pavement as shown in Figure 2-58. Estimate the heat transfer between a pin when it is at 15^{0}C and the surface when it is at 45^{0}C.

FIG 2-58 Steel pins in asphalt.



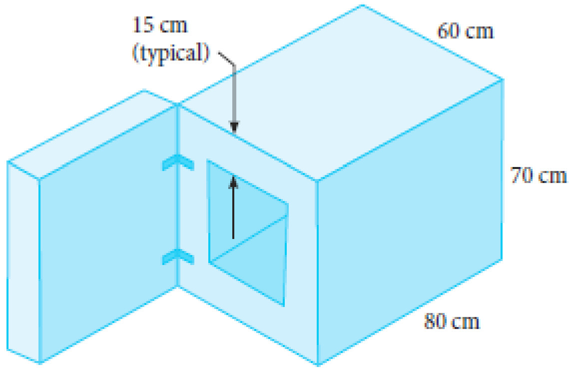
Solution

Assume steady state conduction. Using a value of 0.062 W/m·K for the thermal conductivity of asphalt from Appendix Table B-2, a uniform pin temperature of 15°C and the asphalt surface is 45°C,

$$\dot{Q} = S\kappa\,\Delta T = \frac{2\pi L}{\ln\frac{2L}{R}}\kappa\,\Delta T = \frac{2\pi(0.45\text{ m})}{\ln\frac{2(0.45\text{ m})}{0.025\text{ m}}}\left(0.062\frac{\text{W}}{\text{m}\cdot\text{K}}\right)(45°\text{C} - 15°\text{C})$$
$$= 1.47\text{ W}$$

41. A heat treat furnace sketched in Figure 2-59 has an inside surface temperature of 1200^{0}C and an outside surface temperature of 60^{0}C. If the walls are assumed to be homogeneous with thermal properties the same as asbestos, estimate the heat transfer from the walls, excluding the door.

FIG 2-59 Heat treat furnace.



Solution

The heat transfer between the inside and the outside is $\dot{Q} = \dot{Q}_{top} + \dot{Q}_{bottom} + \dot{Q}_{back} + 2\dot{Q}_{side} + 4\dot{Q}_{sideedge} + 2\dot{Q}_{backedge} + 2\dot{Q}_{uprightedges} + 4\dot{Q}_{corners}$. All of these can be modeled with shape factors from Table 2-3. The first four terms are just one-dimensional conduction through a sheet, or plate. The next three are edges and the last is a corner. Combining all this $\dot{Q} = \frac{\kappa \Delta T}{\Delta x}\left[30x65cm^2 + 30x65 + 30x40 + 2x40x65\right] +$

$\kappa \Delta T\left[4(0.559)(65) + 2(0.559)(30) + 2(0.559)(40) + 4(0.15)(15)\right]$

substituting the thermal conductivity, the thickness Δx , and the temperature difference

$\dot{Q} = 1634.8W$

42. A small refrigerator freezer, 40 cm x 40 cm x 45 cm outer dimensions, has an inside surface temperature of -10^{0}C and an outside surface temperature of 25^{0}C. If the walls are uniformly 7.5 cm thick, homogeneous, and with thermal properties the same as Styrofoam, estimate the heat transfer through the walls and door of the refrigerator.

Solution

Using shape factor methods we can list

$$\dot{Q} = \dot{Q}_{door} + \dot{Q}_{back} + \dot{Q}_{top} + \dot{Q}_{bottom} + 2\dot{Q}_{side} + 4\dot{Q}_{edge} + 4\dot{Q}_{upedge} + 4\dot{Q}_{back/frontedge} + 8\dot{Q}_{corner}$$

The thermal conductivity of Styrofoam is 0.029 W/m·K, the temperature difference is 35^{o}C. The first five terms are just heat transfer through a flat plate, the next three are edges, and the last is a corner. Using items 1, 17, and 18 from Table 2-3 we get

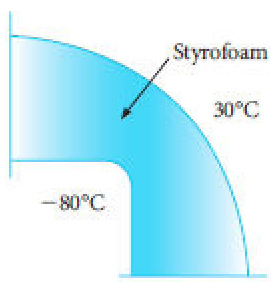
$$\dot{Q} = \frac{\kappa\,\Delta T}{\Delta x}[0.25 \times 0.25\ \text{cm}^2 + 0.25 \times 0.25\ \text{cm}^2 + 0.25 \times 0.3\ \text{cm}^2 + 0.25 \times 0.3\ \text{cm}^2 + 2 \times 0.25 \times 0.3\ \text{cm}^2\ ] + \kappa\Delta T(0.559)(4 \times 0.3 + 4 \times 0.25 + 4 \times 0.25) + \kappa\Delta T(8 \times 0.0125 * 0.075)$$

The total heat transfer is then

$$\dot{Q} = 10.3\ W$$

43. Using graphical methods, estimate the temperatue distribution and the heat transfer per meter depth between the two surfaces at the corner shown in Figure 2-60. Notice that the scale is 1 to 8, or that the figure is only one-eighth the actual size.

FIG 2-60 Heat transfer at a corner.

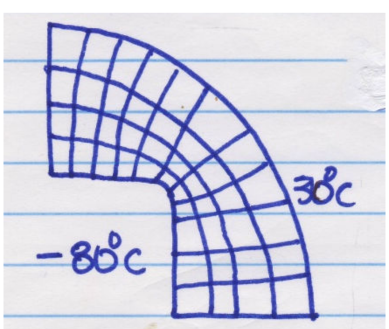


Scale: 1 to 8

Solution

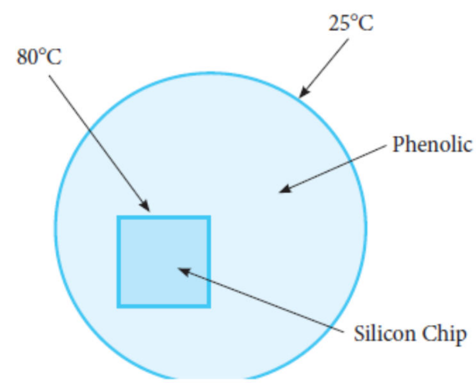
The sketch shown shows that there are 11 heat flow paths, M = 11, and 4 temperature steps, N = 4. Thus, the shape factor is roughly M/N = 2.75 and the heat transfer is

$$\dot{Q} = S\kappa\Delta T = (2.75)(0.029 W/m\cdot K)(110 K) = 8.7725 W/m$$



44. Using graphical methods, estimate the temperature distribution through the phenolic disk surrounding the silicon chip sketched in Figure 2-61. Then estimate the heat transfer per millimeter of depth.

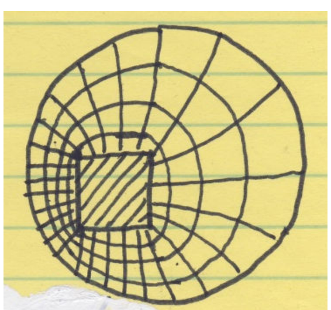
FIG 2-61 Silicon chip embedded in phenolic.



Solution

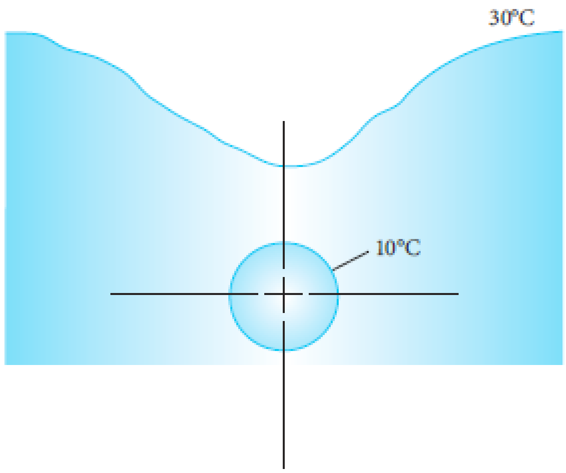
Here we have that the shape factor is the heat flow paths, M, divided by the temperature steps, N, so that S = M/N. From the sketch shown there are about 25 heat flow paths and 4 temperature steps. Using a thermal conductivity of 0.35 W/m·K for nylon as an approximation for phenolic from Appendix Table B-2, we have

$$\dot{q}_l = S\kappa\,\Delta T = \frac{25}{4}\left(0.35\,\frac{\mathrm{W}}{\mathrm{m\cdot K}}\right)(80°\mathrm{C} - 25°\mathrm{C}) = 120.3\,\frac{\mathrm{W}}{\mathrm{m}}$$



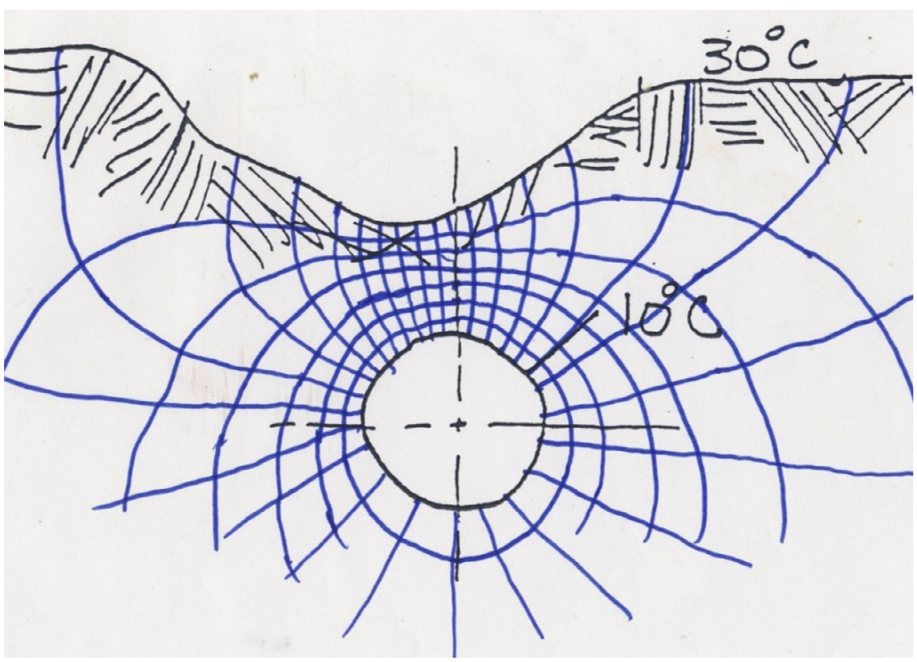
45. Using graphical techniques estimate the temperature distribution through the earth around the electrical power line shown in Figure 2-62. Then estimate the heat transfer necessary between the line and the ground surface for steady state conditions. Express your answer in W/m.

FIG 2-62 Buried high-power line.



Solution

The temperature distribution and the heat transfer can be approximated with a sketch of the heat flow lines and isotherms. These two sets of lines need to be orthogonal or perpendicular at all times and the spacing between adjacent isotherms and heat flow



lines need to approximate a square. The Shape factor, S, will be the ratio of the heat flow paths, M, to the isotherms, N. The sketch shows a possible approximate solution where the temperature steps or isotherms is seven (7) and the number of heat flow paths is twenty seven (27). Then

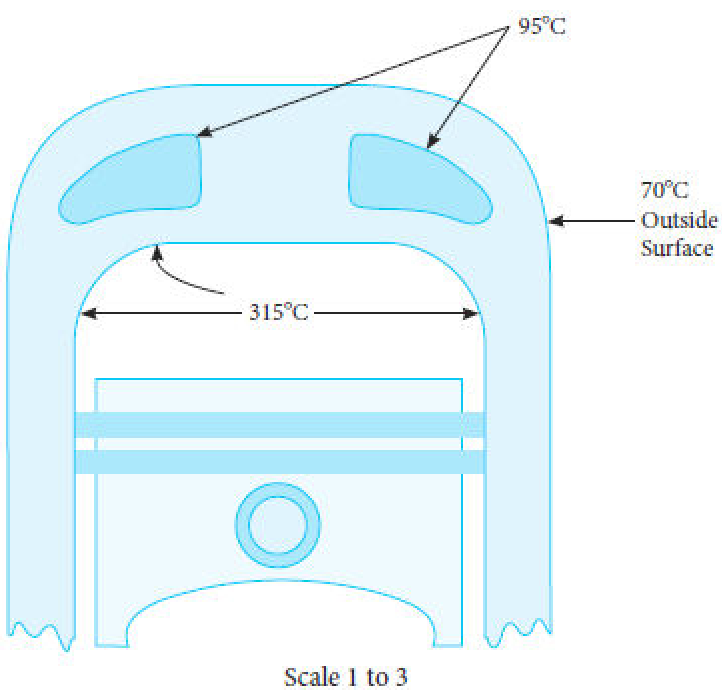
$$\dot{q}_l = S\kappa\Delta T = \frac{M}{N}\kappa\Delta T = \frac{27}{7}\left(0.52\frac{W}{m\cdot K}\right)(20K) = 40.1\frac{W}{m}$$

Notice that the shape factor is 27/7 = 3.86, which is a value in close agreement with item 8 of Table B-3 for a buried line,

$$S = \frac{2\pi}{\cosh^{-1}\frac{Y}{R}} = \frac{2\pi}{\cosh^{-1}\frac{4}{2}} = 4.77$$

46. Using graphical techniques, estimate the temperature distribution through the cast iron engine block and head shown in Figure 2-63.

FIG 2-63 Sketch of a gasoline engine.



Solution

Referring to the sketch of the piston-cylinder and assuming symmetry, there are five (5) isothermal steps so N = 5. Also there are estimated to be twenty-two (22) heat flow paths for one half the cylinder for heat exchange between the cylinder at 315°C and the surroundings at 70°C. From Appendix Table B-2 the thermal conductivity for cast iron may be taken as 39 W/m·K. Then the heat transfer is

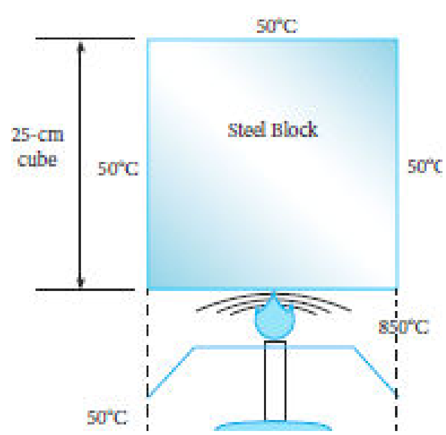
$$\dot{q}_r = \kappa \frac{M}{N}\,\Delta T = \left(39\ \frac{\mathrm{W}}{\mathrm{m\cdot K}}\right)\left(\frac{22}{5}\right)(315\text{°C} - 70\text{°C}) = 42042\ \frac{\mathrm{W}}{\mathrm{m\cdot radious}}$$

and if we assume an effective radius of 0.09 m and rotate the 22 heat flow paths through one revolution, 2πr, then the heat transfer will be

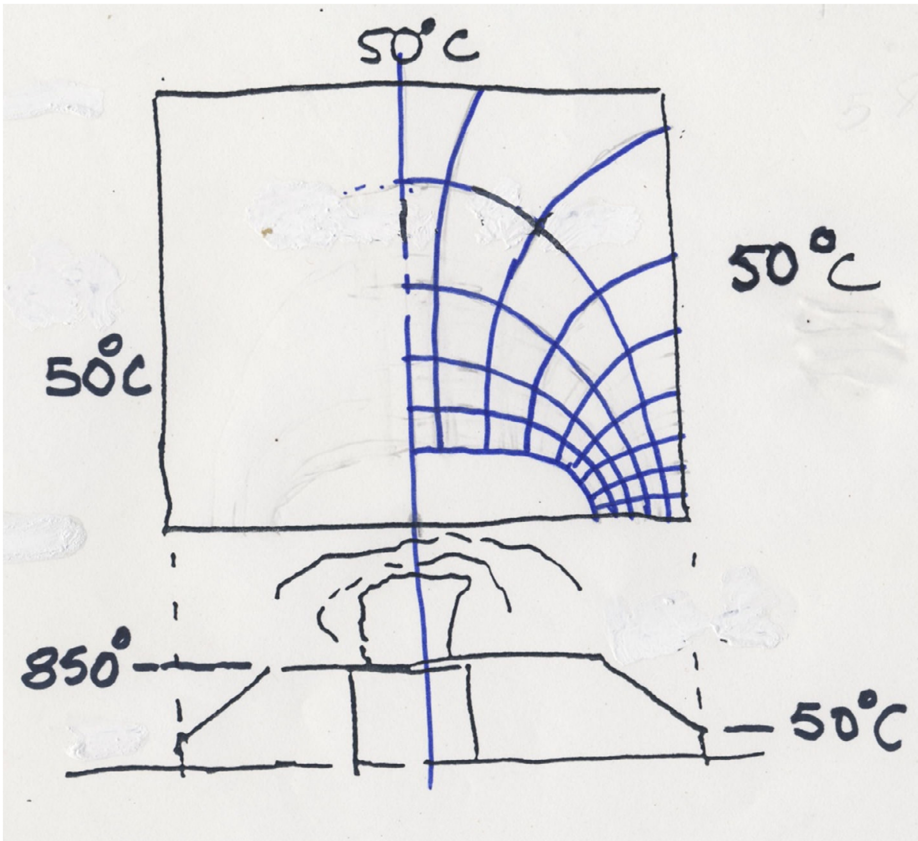
$$\dot{Q} = 2\pi r_{effective}\dot{q}_r = 2\pi(0.09\ \mathrm{m})\left(42042\ \frac{\mathrm{W}}{\mathrm{m}}\right) = 23{,}774\ \mathrm{W}$$

47. A Bunsen burner is used to heat a block of steel. The surfaces of the steel may be taken as 50°C except on the bottom, where the burner is heating the block. Figure 2-64 shows the temperature profile at the bottom surface and the overall configuration of the heating process. Using graphical techniques, estimate the temperature profile through the block and the heat transfer through the block.

FIG 2-64 Bunsen burner and steel block.



Solution



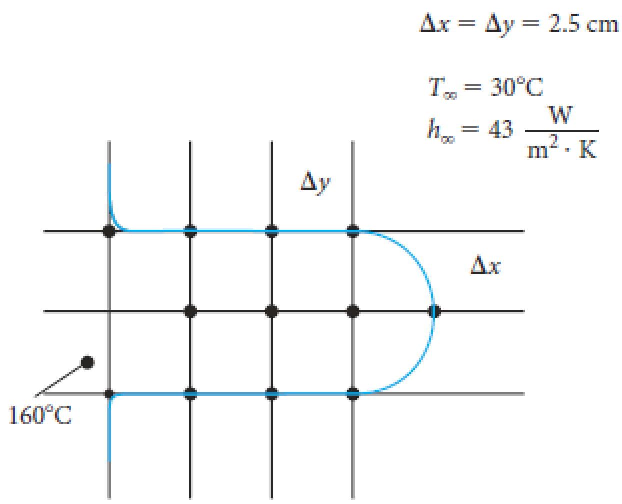
Using graphical techniques requires that a web of approximately square elements are formed between adjacent heat flow lines and isotherms. An approximate solution is shown, noting that the 850^{0}C is assumed to be in the block. The number of heat flow paths for one-half the block is nine (9) and the number of isotherms is five (5). Assuming a carbon steel the thermal conductivity is taken as 60.5 W/m·K from Appendix Table B-2. Since the block is 25 cm square

$$\dot{Q} = \kappa L \frac{M}{N} \Delta T = \left(60.5 \frac{W}{m \cdot K}\right)(0.25m)\left(\frac{9}{5}\right)(850 - 50K) = 21.78kW$$

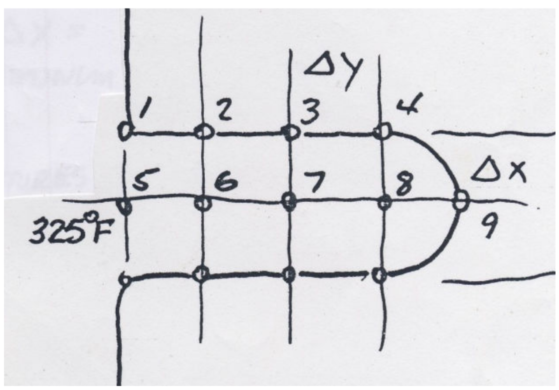
Section 2-6

48. Estimate the heat transfer from the fin shown in Figure 2-65. Write the necessary node equations and then solve for the temperatures. Assume the fin is aluminum.

FIG 2-65 Fin heat transfer.



Solution



Referring to the sketch, assuming symmetry so that only 9 nodes need to be identified and using node neighborhoods of 2.5 cm squares (Δx = Δy = 2.5cm), and assuming the temperature of node 5 is 160^{0}C the node equations can be written for steady state conduction two-dimensional heat transfer. The thermal conductivity of aluminum is 236 W/m·K from Appendix Table B-2. For node 1:

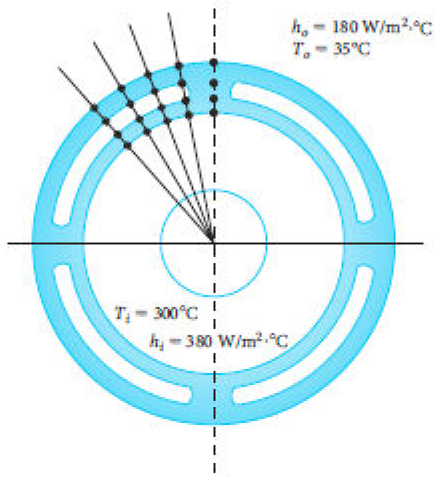
$$\kappa\left(\frac{\Delta y}{2}\right)\left(\frac{T_5 - T_1}{\Delta x}\right) + \kappa\left(\frac{\Delta x}{2}\right)\left(\frac{T_2 - T_1}{\Delta y}\right) + h_\infty \frac{\Delta y}{2}\left(T_\infty - T_1\right) = 0$$

substituting into this node equation, for the Temperature matrix

$$T = \begin{bmatrix} 289.5 \\ 293.3 \\ 289.29 \\ 289.09 \\ 296.15 \\ 290.78 \\ 288.38 \\ 284.56 \end{bmatrix}$$

49. Write the node equations for the model of heat transfer through the compressor housing section shown in Figure 2-66. Then solve for the node temperatures by using EES, Mathcad, or MATLAB.

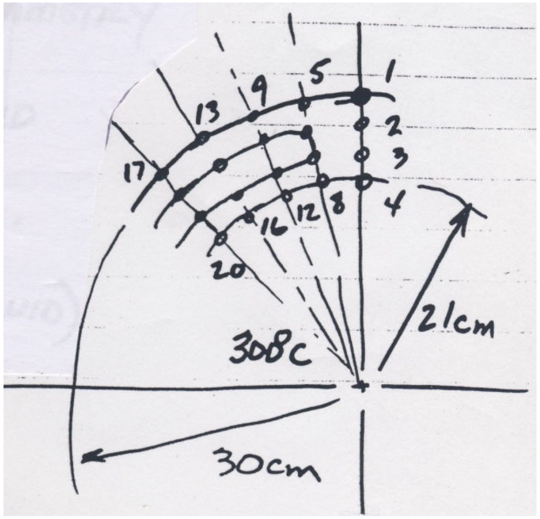
FIG 2-66 Compressor housing section.

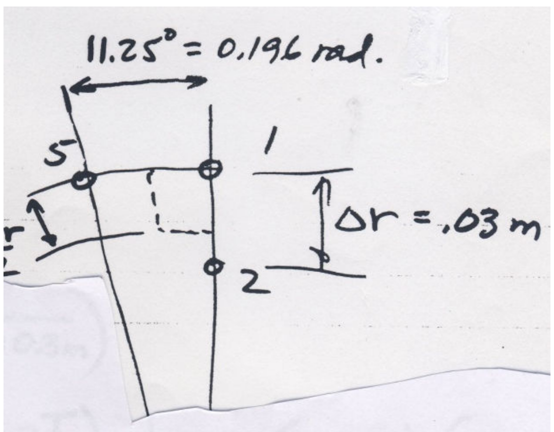


Cast iron, Scale: 1" = 6"

Solution

Referring to Figure 2-66, which is a scale 1 to 6, the inner radius is assumed to be 21 cm and the outer radius is then 30 cm. The housing is cast iron so that the thermal conductivity is 39 W/m·K from Appendix Table B-2 and assuming that the slots have quiescent fluid at with a thermal conductivity of 1 W/m·K, the node equations may be written out. Referring to the following sketches some of the nodes are identified, others need to be to be inferred, and node 1 is shown in some detail.





Node 1 neighborhood. The angular displacement between nodes is 11.25^0 or 0.196 radians. For node 1

$$\kappa\frac{\Delta r}{2}\frac{1}{(0.196)(0.3m)}(T_5-T_1)+\kappa\frac{(0.196rad)(0.285m)}{2}\left(\frac{T_2-T_1}{\Delta r}\right)+h_0\left(\frac{(0.196)(0.3m)}{2}\right)(35^0C-T_1)=0$$

Substituting the thermal conductivity, convective heat transfer coefficient, and radius change $9.95(T_5-T_1)+36.3(T_2-T_1)+5.292(35-T_1)=0$ which is the equation for node 1

An energy balance for node 2 gives

$$\kappa\Delta r\left(\frac{T_6-T_2}{0.196(0.27m)}\right)+\kappa\left(\frac{0.196(0.285m)}{2}\right)\left(\frac{T_1-T_2}{\Delta r}\right)+\kappa\left(\frac{0.196(0.255m)}{2}\right)\left(\frac{T_3-T_2}{\Delta r}\right)=0$$

or $22.1(T_6-T_2)+36.309(T_1-T_2)+32.487(T_3-T_2)=0$ which is the equation for node 2. An energy balance of the heat flows to each of the nodes can be made and the following equations result

$24.872\left(T_7-T_3\right)+32.487\left(T_2-T_3\right)+28.665\left(T_4-T_3\right)=0$ t which is the equation for node 3, $14.21\left(T_8-T_4\right)+28.665\left(T_3-T_4\right)+7.82\left(300-T_4\right)=0$ which is the equation for node 4. After applying energy balances to all of the 20 nodes the following set of equations result

$$51.542T_1 - 36.3T_2 - 9.95T_5 = 185.22$$

$$90.896T_2 - 36.3T_1 - 22.1T_6 - 32.487T_3 = 0$$

$$86.024T_3 - 32.487T_2 - 28.665T_4 - 24.872T_7 = 0$$

$$50.695T_4 - 28.665T_3 - 14.21T_8 = 2346$$

$$103.084T_5 - 9.95T_1 - 72.6T_6 - 9.95T_9 = 370.44$$

$$142.077T_6 - 22.1T_2 - 72.6T_5 - 36.32T_7 - 11.057T_{10} = 0$$

$$130.96T_7 - 24.87T_3 - 36.32T_6 - 57.33T_8 - 12.44T_{11} = 0$$

$$101.39T_8 - 14.21T_4 - 57.33T_7 - 14.21T_{12} = 4692$$

$$103.084T_9 - 9.95T_5 - 72.6T_{10} - 9.95T_{13} = 370.44$$

$$94.757T_{10} - 11.057T_6 - 72.6T_9 - 0.043T_{11} - 11.057T_{14} = 0$$

$$82.253T_{11} - 12.44T_7 - 0.043T_{10} - 57.33T_{12} - 12.44T_{15} = 0$$

$$101.39T_{12} - 14.21T_8 - 57.33T_{11} - 14.21T_{16} = 4692$$

$$103.084T_{13} - 9.95T_9 - 72.6T_{14} - 9.95T_{17} = 370.44$$

$$22.114T_{14} - 11.057T_{10} - 72.6T_{13} - 0.043T_{15} - 11.057T_{18} = 0$$

$$82.253T_{15} - 12.44T_{11} - 0.043T_{14} - 57.33T_{16} - 12.44T_{19} = 0$$

$$101.39T_{16} - 14.21T_{12} - 57.33T_{15} - 14.21T_{20} = 4692$$

$$51.542T_{17} - 9.95T_{13} - 36.3T_{18} = 185.22$$

$$47.3785T_{18} - 11.057T_{14} - 36.3T_{17} - 0.0213T_{19} = 0$$

$$41.126T_{19} - 12.44T_{15} - 0.0213T_{18} - 28.665T_{20} = 0$$

$$50.695T_{20} - 14.21T_{16} - 28.665T_{19} = 2346$$

With this set of equations the temperatures can be determined. Using Mathcad, noting that the results are tabulated in the final column with node 1 being listed as 0, node 2 as 1, and so on.

Solving for the temperature field in an air compressor using Mathcad:

Guess Values

$T1 := 40$
$T2 := 80$
$T3 := 150$
$T4 := 250$
$T5 := 40$
$T6 := 80$
$T7 := 150$
$T8 := 250$
$T9 := 40$
$T10 := 70$
$T11 := 180$
$T12 := 260$
$T13 := 40$
$T14 := 70$
$T15 := 200$
$T16 := 270$
$T17 := 40$
$T18 := 80$
$T19 := 160$
$T20 := 280$

THESE VALUES WERE ESTIMATED FROM THE BOUNDARY CONDITIONS

Given

$$51.542 \cdot T1 - 36.3 \cdot T2 - 9.95 \cdot T5 = 185.22$$

$$90.896 \cdot T2 - 36.3 \cdot T1 - 22.1 \cdot T6 - 32.487 \cdot T3 = 0$$

$$86.024 \cdot T3 - 32.487 \cdot T2 - 28.665 \cdot T4 - 24.872 \cdot T7 = 0$$

$$50.695 \cdot T4 - 28.665 \cdot T3 - 14.21 \cdot T8 = 2346$$

$$103.084 \cdot T5 - 9.95 \cdot T1 - 72.6 \cdot T6 - 9.95 \cdot T9 = 370.44$$

$$143.167 \cdot T6 - 22.1 \cdot T2 - 72.6 \cdot T5 - 37.133 \cdot T7 - 11.333 \cdot T10 = 0$$

$$132.093 \cdot T7 - 24.87 \cdot T3 - 37.133 \cdot T6 - 57.33 \cdot T8 - 12.76 \cdot T11 = 0$$

$$101.39 \cdot T8 - 14.21 \cdot T4 - 57.33 \cdot T7 - 14.21 \cdot T12 = 4692$$

$$103.084 \cdot T9 - 9.95 \cdot T5 - 72.6 \cdot T10 - 9.95 \cdot T13 = 370.44$$

;

$96.9 \cdot T10 - 11.333 \cdot T6 - 72.6 \cdot T9 - 1.65 \cdot T11 - 11.333 \cdot T14 = 0$

$84.5 \cdot T11 - 12.76 \cdot T7 - 1.65 \cdot T10 - 57.33 \cdot T12 - 12.76 \cdot T15 = 0$

$101.39 \cdot T12 - 14.21 \cdot T8 - 57.33 \cdot T11 - 14.21 \cdot T16 = 4692$

$103.084 \cdot T13 - 9.95 \cdot T9 - 72.6 \cdot T14 - 9.95 \cdot T17 = 370.44$

$96.9 \cdot T14 - 11.333 \cdot T10 - 72.6 \cdot T13 - 1.65 \cdot T15 - 11.333 \cdot T18 = 0$

$84.5 \cdot T15 - 12.76 \cdot T11 - 1.65 \cdot T14 - 57.33 \cdot T16 - 12.76 \cdot T19 = 0$

$101.39 \cdot T16 \quad 14.21 \cdot T12 \quad 57.33 \cdot T15 \quad 14.21 \cdot T20 - 4692$

$51.542 \cdot T17 \quad 9.95 \cdot T13 \quad 36.3 \cdot T18 \quad 185.22$

$48.458 \cdot T18 \quad 11.333 \cdot T14 \quad 36.3 \cdot T17 \quad 0.825 \cdot T19 \quad 0$

$42.25 \cdot T19 \quad 12.76 \cdot T15 \quad 0.825 \cdot T18 \quad 28.665 \cdot T20 \quad 0$

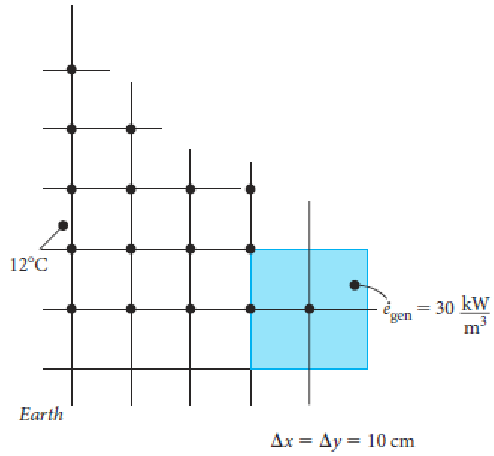
$50.695 \cdot T20 \quad 14.21 \cdot T16 \quad 28.665 \cdot T19 \quad 2346$

Find(T1, T2, T3, T4, T5, T6, T7, T8, T9, T10, T11, T12, T13, T14, T15, T16, T17, T18, T19, T20) =

	0
0	149.086
1	168.279
2	196.333
3	221.284
4	139.743
5	158.633
6	204.22
7	228.297
8	103.989
9	111.481
10	244.077
11	253.53
12	86.96
13	92.856
14	258.605
15	265.758
16	82.177
17	87.743
18	262.428
19	269.157

50. Write the node equations for describing heat transfer through the buried waste shown schematically in Figure 2-67. Notice that there is energy generation that occurs due to a pyrolytic reaction of the waste (slow chemical reaction) and that there are boundaries that require reference to Table 2-6.

FIG 2-67 Buried waste mass.



Solution

For doing a finite difference analysis the following grid may be used. Then the heat from the waste mass per unit depth (1 m) is $\dot{E}_g = 30\ kW/m^2\,(0.2m)^2 = 1.2\ kW$. Estimate that the power or heat to node 1 is 0.6 kW and 0.3 kW to node 2. Using a thermal conductivity of 0.52 W/m·K for earth or soil from Appendix Table B-2, and utilizing symmetry in the x-direction, one-half of the neighborhood for node 1 will be 0.05 m wide and

$$\kappa\left(\frac{\Delta x}{2}\right)\left(\frac{T_3 - T_1}{\Delta y}\right) + \kappa\left(\frac{\Delta y}{2}\right)\left(\frac{T_2 - T_1}{\Delta x}\right) + 600W = 0$$

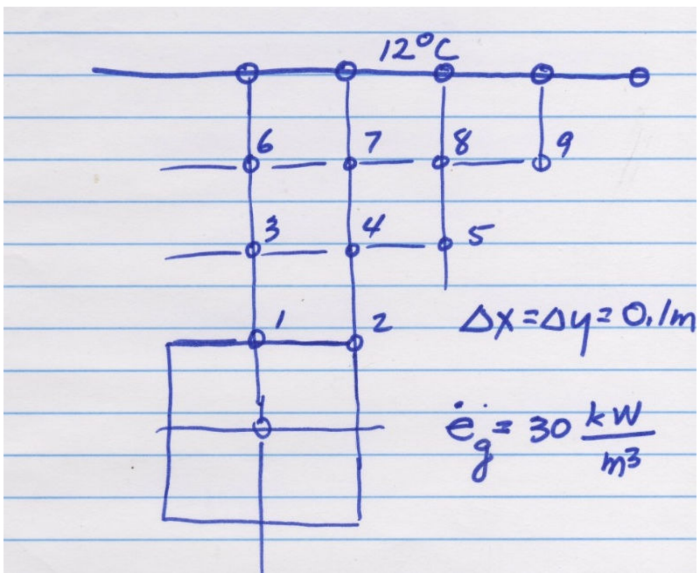
or, for node 1

$$(0.52)\left(\frac{T_3 - T_1}{2}\right) + (0.26)(T_2 - T_1) + 600W = 0$$

Similarly, for the remaining nodes,

$$(0.26)(T_1 - T_2) + (0.52)(T_5 - T_2) + 300W = 0$$

which is for node 2,



$$(0.26)(T_1 - T_3) + (0.26)(T_6 - T_3) + (0.52)(T_4 - T_3) = 0$$ which is the node equation for node 3 $4T_4 - T_3 - T_2 - T_5 - T_7 = 0$

$$3T_6 - \frac{1}{2}T_3 - T_7 = 6^0 C$$

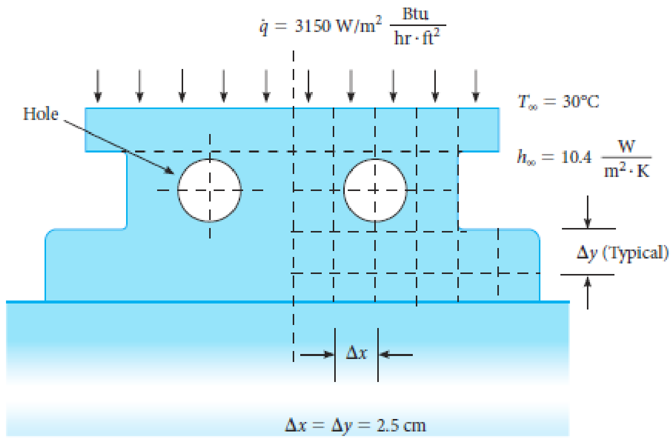
$$4T_7 - T_6 - T_4 - T_8 = 12^0 C$$

$$4T_8 - T_7 - T_5 - T_9 = 12^0 C$$

$$2T_9 - T_8 = 12^0 C$$

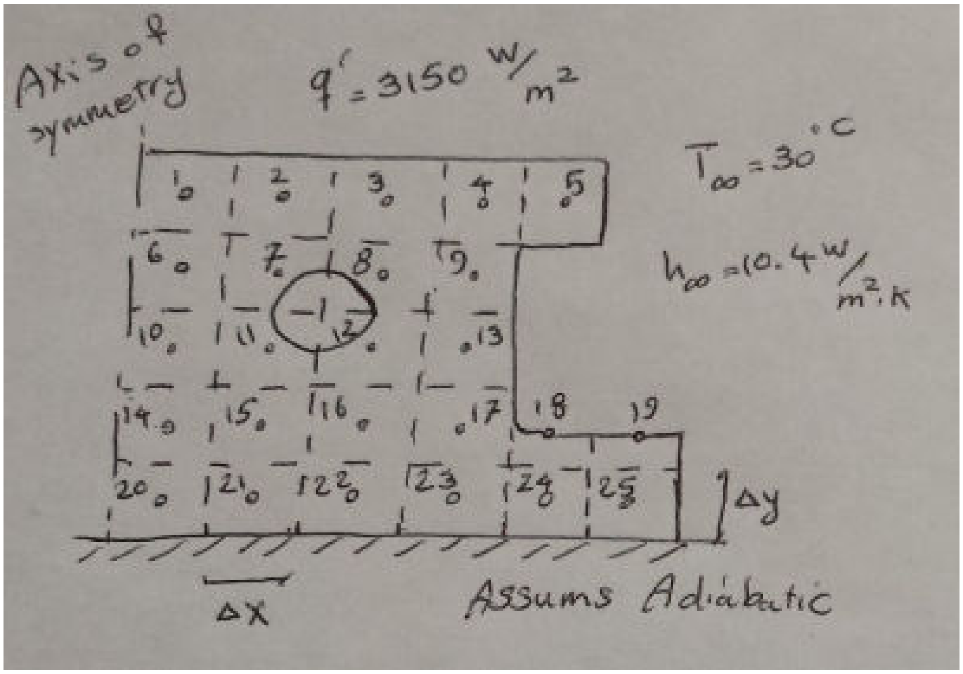
51. Write the node equations for determining the temperature distribution through the cast iron lathe slide shown in Figure 2-68. Notice that the sliding surface is assumed to be adiabatic and that there are irregular boundary profiles.

FIG 2-68 Lathe slide.



Solution

A proposed node layout is shown



The node neighborhoods are $\Delta x = \Delta y = 2.5\ cm$, assume the hole has air at 30°C with a convective heat transfer coefficient of 10.4 W/m^2·^{0}C, and the thermal conductivity for cast iron may be taken as 39 W/m·^{0}C from Appendix Table B-2. Applying an energy balance to node neighborhood 1, the following equation results

$$\kappa(T_2 - T_1) + \kappa(T_6 - T_1) + \Delta x \left(3154.6 \frac{W}{m^2}\right) = 0$$

Substituting for thermal conductivity and node neighborhood size,

$$2T_1 - T_2 - T_6 = 2.0^0F$$

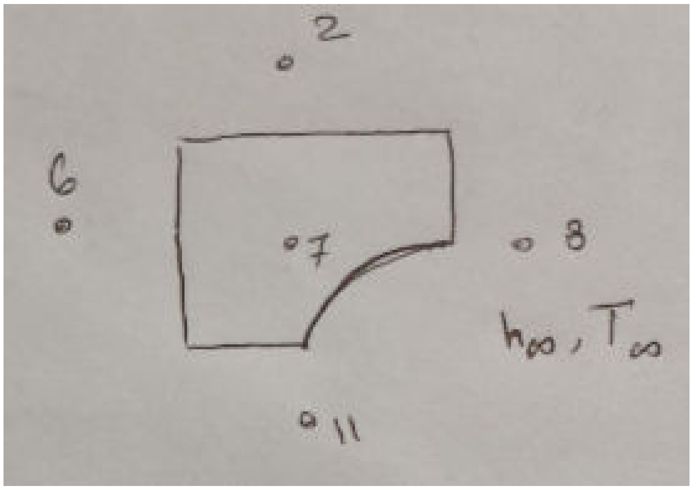
For nodes 2 through 6

$$3T_2 - T_3 - T_7 = -15.7^0F$$

$$1.044T_5 - T_4 = 3.5^0F$$

$$2T_6 - T_1 - T_7 - T_{10} = 17.8$$

Nodes 7, 8, 11, and 12 require some adjusting. Referring to the sketch for node 7



The energy balance can be approximated by

$$\kappa\frac{\Delta y}{\Delta x}\left(T_6-T_7\right)+\kappa\frac{\Delta y}{\Delta x}\left(T_2-T_7\right)+\kappa\frac{\Delta y}{2\Delta x}\left(T_8-T_7\right)+\kappa\frac{\Delta x}{2\Delta y}\left(T_{11}-T_7\right)+h_\infty\frac{\pi}{2}\left(\frac{\Delta x}{2}\right)\left(90^0F-T_7\right)=0$$

$$\kappa\frac{\Delta y}{\Delta x}(T_6-T_7)+\kappa\frac{\Delta y}{\Delta x}(T_2-T_7)+\kappa\frac{\Delta y}{2\,\Delta x}(T_8-T_7)+\kappa\frac{\Delta x}{2\,\Delta y}(T_{11}-T_7)$$
$$+h_\infty\frac{\pi}{2}\left(\frac{\Delta x}{2}\right)(30°\text{C}-T_7)=0$$

which becomes $3.017T_7-T_6-T_2-0.5T_8-0.5T_{11}=0.57$

Similarly, for node 8 $3.017T_8-T_2-T_9-0.5T_7-0.5T_{12}=0.57$

And for nodes 11 and 12 $3.017T_{11}-T_{10}-T_{15}-0.5T_{12}-0.5T_7=0.57$

$$3.017T_{12}-T_{16}-T_{13}-0.5T_{11}-0.5T_8=0.57$$

The remaining node equations are straightforward energy balances and are,

For node 9 $33.022T_9-T_4-T_8-T_{13}=-536.2^0\text{C}$

For node 10 $3T_{10}-T_6-T_{11}-T_{14}=0$

For node 13 $33.022T_{13}-T_9-T_{12}-T_{17}=-536.2^0\text{C}$

For node 14 $3T_{14}-T_{10}-T_{15}-T_{20}=0$

For node 15 $4T_{15}-T_{11}-T_{14}-T_{16}-T_{21}=0$

For node 16 $4T_{16}-T_{12}-T_{15}-T_{17}-T_{22}=0$

For node 17 $3.511T_{17}-T_{13}-T_{16}-T_{23}-0.5T_{18}=0.34°\text{C}$

For node 18 $2.022T_{18}-T_{17}-T_{19}-T_{24}=18.5°\text{C}$

For node 19 $1.533T_{19} - 0.5T_{18} - T_{25} = 0.49°C$

For node 20 $2T_{20} - T_{14} - T_{21} = 0$

For node 21 $3T_{21} - T_{15} - T_{20} - T_{22} = 0$

For node 22 $3T_{22} - T_{16} - T_{21} - T_{23} = 0$

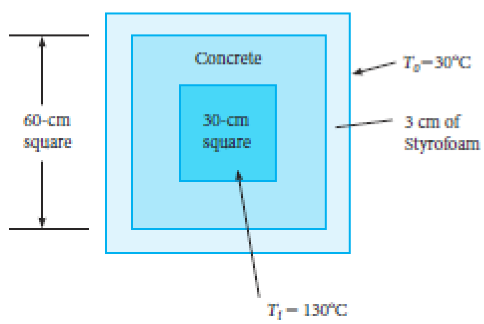
For node 23 $3T_{23} - T_{17} - T_{22} - T_{24} = 0$

For node 24 $3T_{24} - T_{18} - T_{23} - T_{25} = 0$

For node 25 $2.022T_{25} - T_{19} - T_{24} = 0.72°C$

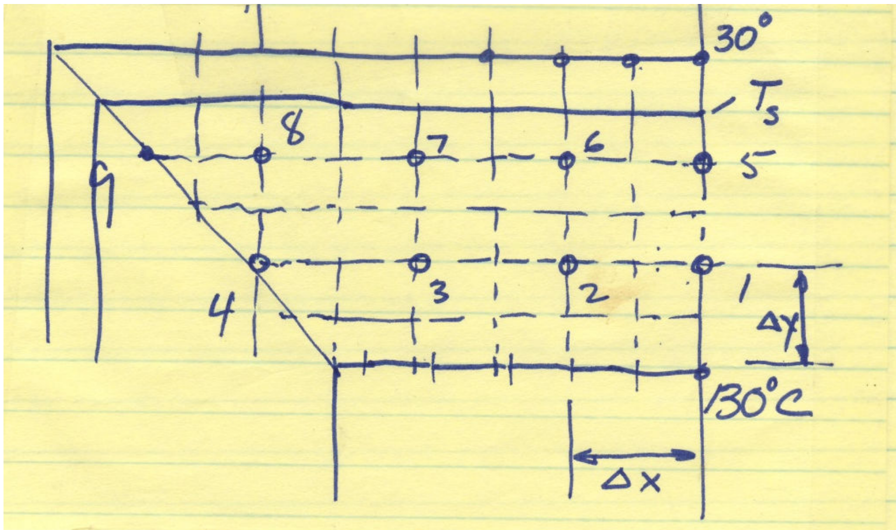
52. A concrete chimney flue is surrounded by a Styrofoam insulator as shown in Figure 2-69. Construct an appropriate grid model and then write the node equations needed to determine the temperature distribution.

FIG 2-69 Chimney flue.



Solution

Assume symmetry for the chimney so that only one quarter of the section needs to be considered, as shown in the sketch



Writing the energy balance for node 1

$$\kappa_{con}\frac{\Delta x}{2}\left(\frac{130-T_1}{\Delta y}\right)+\kappa_{con}\Delta y\left(\frac{T_2-T_1}{\Delta x}\right)+\kappa_{con}\frac{\Delta x}{2}\left(\frac{T_5-T_1}{\Delta y}\right)=0$$

Which can be reduced to $2T_1 - 0.5T_5 - T_2 = 65^0C$

Fore node 2 $4T_2 - T_1 - T_6 - T_3 = 130^0C$

For node 3 $4T_3 - T_2 - T_4 - T_7 = 130^0C$

For node 4 $2.25T_4 - T_3 - T_8 = 32.5^0C$

For nodes 5 through 9 the Styrofoam impacts the energy balance so

$$\kappa_{con}\frac{\Delta x}{2}\left(\frac{T_1-T_5}{\Delta y}\right)+\kappa_{con}\Delta y\left(\frac{T_6-T_5}{\Delta x}\right)+\kappa_{sty}\frac{\Delta x}{2}\left(\frac{30-T}{\Delta y/2}\right)=0$$

Also, since the boundary temperature between the Styrofoam and the concrete is not yet known we write

$$\dot{Q}_{30C-node5}=\kappa_{con}\frac{\Delta x}{2}\left(\frac{T_s-T_5}{\Delta y/2}\right)=\kappa_{sty}\frac{\Delta x}{2}\left(\frac{30-T_s}{\Delta y/2}\right)$$

Solving this equation for T_s and substituting back into the node equation gives

$$\left(1.5+\frac{\kappa_{sty}}{\kappa_{con}+\kappa_{sty}}\right)T_5 - 0.5T_1 - T_6 = \frac{\kappa_{sty}}{\kappa_{con}+\kappa_{sty}}(30^0)$$

For node 6 $$\left(3+\frac{\kappa_{sty}}{\kappa_{con}+\kappa_{sty}}\right)T_6 - T_5 - T_2 - T_7 = \frac{\kappa_{sty}}{\kappa_{con}+\kappa_{sty}}(30^0)$$

For node 7 $$\left(3+\frac{\kappa_{sty}}{\kappa_{con}+\kappa_{sty}}\right)T_7 - T_8 - T_6 - T_3 = \frac{\kappa_{sty}}{\kappa_{con}+\kappa_{sty}}(30^0)$$

For node 8 $\left(3+\frac{\kappa_{sty}}{\kappa_{con}+\kappa_{sty}}\right)T_8 - T_7 - T_4 - T_9 = \frac{\kappa_{sty}}{\kappa_{con}+\kappa_{sty}}(30^0)$

For node 9 $\left(1+\frac{\kappa_{sty}}{\kappa_{con}+\kappa_{sty}}\right)T_9 - T_8 = \frac{\kappa_{sty}}{\kappa_{con}+\kappa_{sty}}(30^0)$

53. Consider the chimney flue of Figure 2-69. If the Styrofoam is removed and the outer boundary condition is the same, write the necessary node equations and solve for the node temperatures. What is the heat transfer through the chimney flue?

Solution

Using the same node arrangement as for Problem 2-52 and referring to the sketch, the node equation for node 1 is $2T_1 - 0.5T_5 - T_2 = 65^0C$

For node 2 $4T_2 - T_1 - T_3 - T_6 = 130^0C$

For node 3 $4T_3 - T_2 - T_4 - T_7 = 130^0C$

For node 4 $2.25T_4 - T_3 - T_8 = 32.5^0C$

For node 5 $2.5T_5 - 0.5T_1 - T_6 = 15^0C$

For node 6 $5T_6 - T_2 - T_5 - T_7 = 60^0C$

For node 7 $5T_7 - T_3 - T_6 - T_8 = 60^0C$

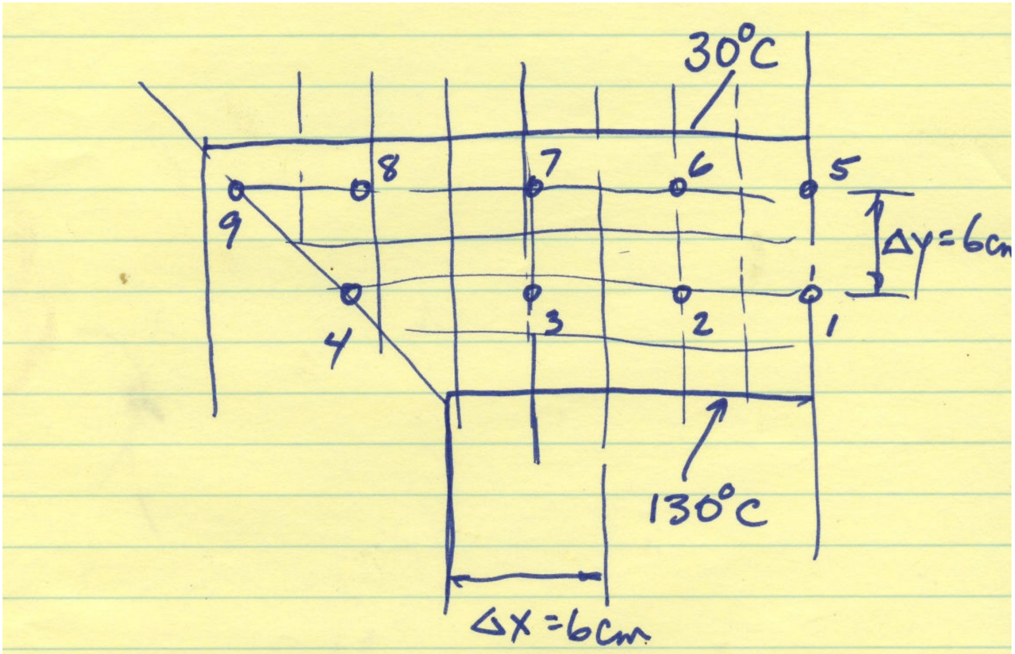
For node 8 $5T_8 - T_4 - T_7 - T_9 = 60^0C$

For node 9 $3T_9 - T_8 = 60^0C$

The heat transfer can be approximated by the equation

$8(\dot{Q}_{1-5} + \dot{Q}_{2-6} + \dot{Q}_{3-7} + \dot{Q}_{4-8})$ which can be written

$$\dot{Q}_{total} = 8\kappa\,[0.5T_1 + T_2 + T_3 + T_4 - 0.5T_5 - T_6 - T_7 - T_8]$$


30°C
8
7
6
5
9
Δy=6cm
4
3
2
1
130°C
Δx=6cm

The temerature field is determined by solving for the nine node equations. Using Mathcad:

$$M := \begin{bmatrix} 2 & -1 & 0 & 0 & -0.5 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2.25 & 0 & 0 & 0 & -1 & 0 \\ -0.5 & 0 & 0 & 0 & 2.5 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 5 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 5 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -1 & 5 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 3 \end{bmatrix}$$

$$v := \begin{bmatrix} 65 \\ 130 \\ 130 \\ 32.5 \\ 15 \\ 60 \\ 60 \\ 60 \\ 60 \end{bmatrix}$$

$$soln := lsolve(M, v)$$

$$soln = \begin{bmatrix} 86.375 \\ 86.7 \\ 83.363 \\ 70.233 \\ 42.101 \\ 47.064 \\ 46.517 \\ 42.161 \\ 34.054 \end{bmatrix}$$

$k := 1.6$
$T1 := 86.375$

$T2 := 86.7$

$T3 := 83.363$

$T4 := 70.233$

$T5 := 42.101$

$T6 := 47.064$

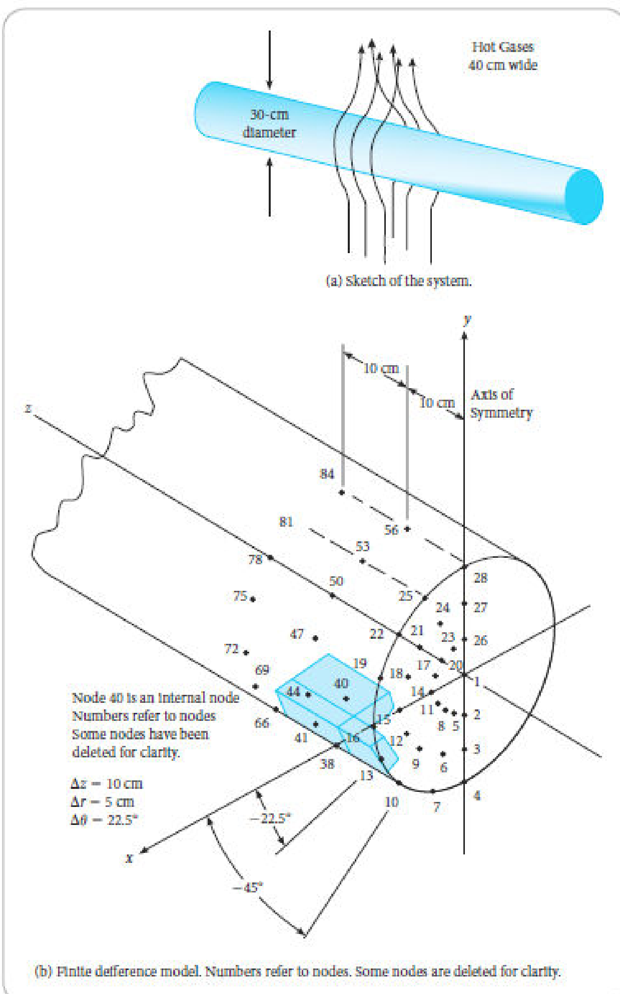
$T7 := 46.517$

$T8 := 42.161$

$Q := 8 \cdot k \cdot (0.5 \cdot T1 + T2 + T3 + T4 - 0.5 \cdot T5 - T6 - T7 - T8)$

$Q = 1.622 \cdot 10^3 \ \frac{W}{m}$

54. Write the node equations for the nodes 1 and 2 of the model of the oak beam sketched in Figure 2-32.

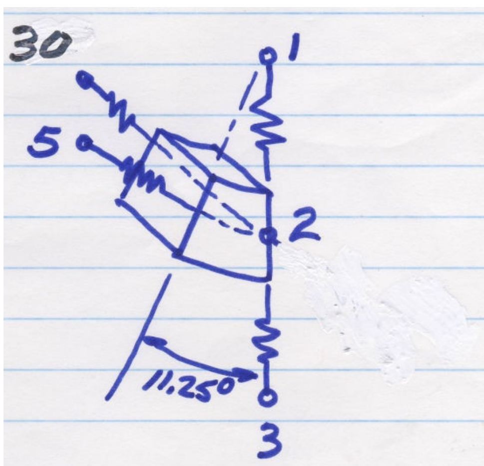


(a) Sketch of the system.

(b) Finite defference model. Numbers refer to nodes. Some nodes are deleted for clarity.

Solution

The model of the round beam is such that axial symmetry is assumed so that a hemispherical section will suffice for nodes. In Figure 2-32 the node numbering scheme follows the pattern of number 1 is in the center, 2, 3, and 4 are radially outward to the outside surface. Then on a 22.5^0 rotation numbers 5, 6, and 7 occur. On the next 22.5^0 rotation numbers 8, 9, and 10 occur. Continuing in this pattern there are three nodes at every 22.5^0 rotation for the first 28 nodes. On the next hemisphere axially parallel to the first hemisphere node number 29 will be on the center position with numbers 30. 31. And 32 outward. Again at a 22.5^0 rotation numbers 33, 34, and 35 occur. Continuing, the pattern is such that nodes on the hemisphere parallel to the succeeding hemisphere will have a number of the previous node plus 28. Thus, node 2 will be adjacent to nodes 29, axially, and also to nodes 3, 5, and 1. This model is sketched. Node 1 has nine (9) adjacent radial nodes; 2, 5, 8, 11, 14, 17, 20, 23, and 26. Node 1 also has an adjacent node on the axis, number 29. This model is sketched.



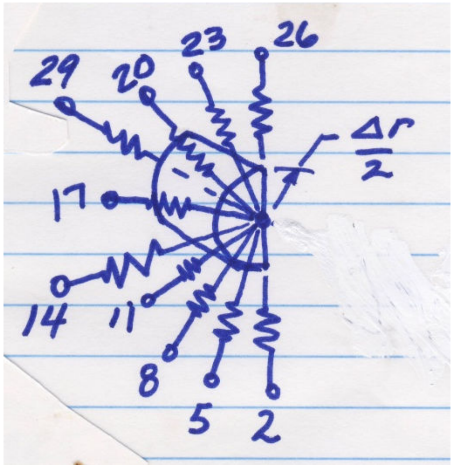
For node 2 there four adjacent nodes with the thermal resistances of

$R_{Tr,1-2} = \frac{64}{\kappa\pi\Delta z}$, $R_{Tr,3-2} = \frac{32}{\kappa\pi\Delta z}$, $R_{Tz,30-2} = \frac{\Delta z}{\kappa\pi\left(\frac{1}{16}\right)\left(r^2-\frac{\Delta r^2}{4}\right)} = \frac{64}{\kappa\pi\Delta z}$, and

$R_{T\theta,5-2} = \frac{\left(\frac{3}{4}\Delta r\right)(\pi/8)}{(\Delta z/2)\Delta r} = \frac{3\pi}{16\Delta z}$ so that the node equation for node 2 can be formed.

Noting that $\dot{Q} = \Delta T / R_T$ the node equation becomes

$$\frac{\kappa\pi}{64}\left(T_1 - T_2\right) + \frac{\kappa\pi\Delta z}{32}\left(T_3 - T_2\right) + \frac{3\kappa\pi\Delta r}{64}\left(T_{30} - T_2\right) + \frac{16\Delta z}{3\pi}\left(T_5 - T_2\right) = 0$$



The thermal resistances for conduction between node 1 and 3, 5, 8, 11, 14, 17, 20, 23, and 26 are

$$R_{Tr,2-1} = R_{Tr,26-1} = \frac{64}{\kappa \pi \Delta z}$$ and

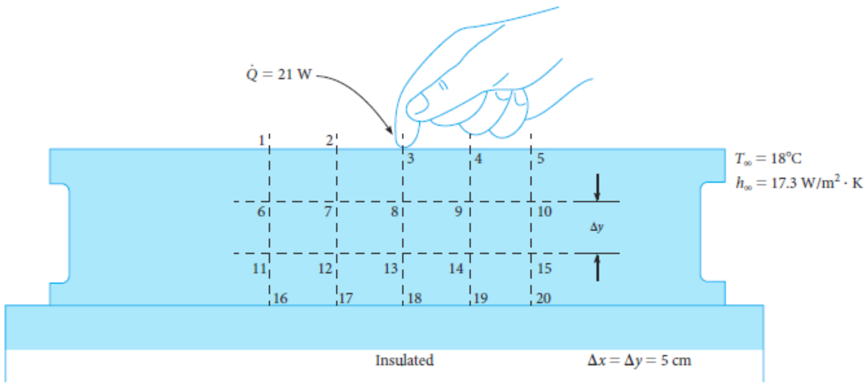
$$R_{Tr,5-1} = R_{Tr,8-1} = R_{Tr,11-1} = R_{Tr,14-1} = R_{Tr,17-1} = R_{Tr,20-1} = R_{Tr,23=1} = \frac{32}{\kappa \pi \Delta z}$$ and for node 29

to 1 $$R_{Tz,29-1} = \frac{8\Delta z}{\kappa \pi \Delta r^2}$$ and the node equation or energy balance for node 1 is

$$\frac{\kappa \pi \Delta z}{64}\left(T_2 + T_{26} - 2T_1\right) + \frac{\kappa \pi \Delta z}{32}\left(T_5 + T_8 + T_{11} + T_{14} + T_{17} + T_{20} + T_{23} - 7T_1\right) + \frac{\kappa \pi \Delta r^2}{8\Delta z}\left(T_{29} - T_1\right) = 0$$

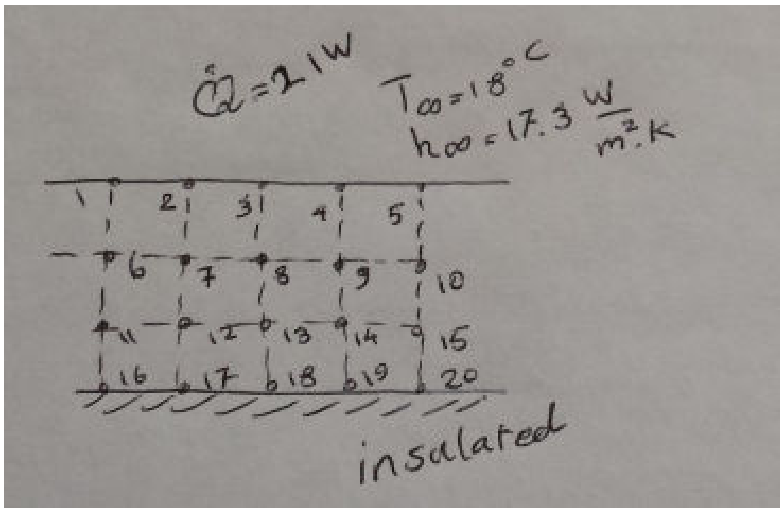
55. Figure 2-70 shows a section of a large surface plate used for precision measurements. A person touches the surface and thereby induces heat transfer through the plate. Neglecting radiation involved, write the node equations for nodes 1, 5, and 12 of the node model of the plate shown in the figure. Assume steady state conditions and that the plate is 65^{0}F beyond the nodes indicated in the figure.

FIG 2-70 Surface plate.



Solution

The sketch of the granite surface plate is shown.



For node 1, the energy balance becomes

$$\kappa\frac{\Delta y}{2}\left(\frac{18°\text{C}-T_1}{\Delta x}\right)+\kappa\Delta x\left(\frac{T_6-T_1}{\Delta y}\right)+\kappa\frac{\Delta y}{2}\left(\frac{T_2-T_1}{\Delta x}\right)+h_\infty\Delta x(T_\infty-T_1)=0$$

which reduces to

$$3.035\,T_1-0.05\,T_2-\,T_6=28.15°\text{C}$$

In a similar fashion, the node equations are

$$3.035\,T_5-0.05\,T_4-\,T_{10}=28.15°\text{C}$$

for node 5, and f or node 12

$$4T_{12}-T_7-T_{13}-T_{17}-T_{11}=0$$

56. Write the complete set of node equations for the surface plate shown in Figure 2-70 and estimate the temperatures and the heat transfer through the plate.

Solution

Referring to the sketch for the nodes of the surface plate, shown in the solution to Problem 2-55, the twenty node equations become

$$3.035T_1 - 0.5T_2 - T_6 = 28.15°\mathrm{C}$$

$$3.035T_2 - 0.5T_1 - 0.5T_3 - T_7 = 37.04°\mathrm{C}$$

$$3.035T_3 - 0.5T_2 - 0.5T_4 - T_8 = 37.04°\mathrm{C}$$

$$3.035T_4 - 0.5T_3 - 0.5T_5 - T_9 = 19°\mathrm{C}$$

$$3.035T_5 - 0.5T_4 - T_{10} = 28.15°\mathrm{C}$$

$$4T_6 - T_1 - T_7 - T_{11} = 18.33°\mathrm{C}$$

$$4T_7 - T_6 - T_2 - T_8 - T_{12} = 0$$

$$4T_8 - T_7 - T_3 - T_9 - T_{13} = 0$$

$$4T_9 - T_8 - T_4 - T_{10} - T_{14} = 0$$

$$4T_{10} - T_9 - T_5 - T_{15} = 18.33°\mathrm{C}$$

$$4T_{11} - T_{12} - T_{16} - T_6 = 18.33°\mathrm{C}$$

$$4T_{12} - T_{11} - T_7 - T_{13} - T_{17} = 0$$

$$4T_{13} - T_{12} - T_8 - T_{14} - T_{18} = 0$$

$$4T_{14} - T_{13} - T_9 - T_{15} - T_{19} = 0$$

$$4T_{15} - T_{14} - T_{10} - T_{20} = 18.33°\mathrm{C}$$

$$2T_{16} - 0.5T_{11} - T_{17} = 9.17°\mathrm{C}$$

$$2T_{17} - T_{12} - 0.5T_{16} - 0.5T_{18} = 0$$

$$2T_{18} - T_{13} - 0.5T_{17} - 0.5T_{19} = 0$$

$$2T_{19} - T_{14} - 0.5T_{18} - 0.5T_{20} = 0$$

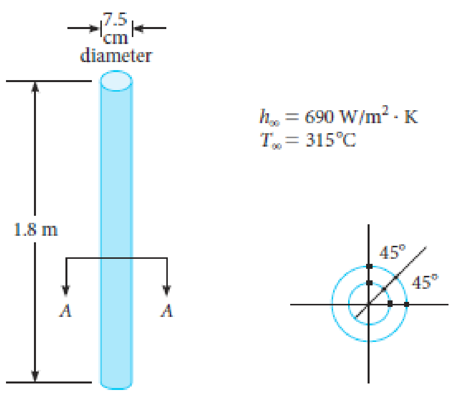
$$2T_{20} - 0.5T_{19} - T_{15} = 9.17°\mathrm{C}$$

This set of 20 x 20 matrix ca be solved with a computer

$$T = \begin{bmatrix} 19.2 \\ 21.1 \\ 28.7 \\ 21.1 \\ 19.2 \\ 19.5 \\ 21 \\ 22.9 \\ 20.9 \\ 19.4 \\ 19.5 \\ 20.5 \\ 21.2 \\ 20.4 \\ 19.4 \\ 19.6 \\ 20.3 \\ 20.7 \\ 20.2 \\ 19.3 \end{bmatrix}$$

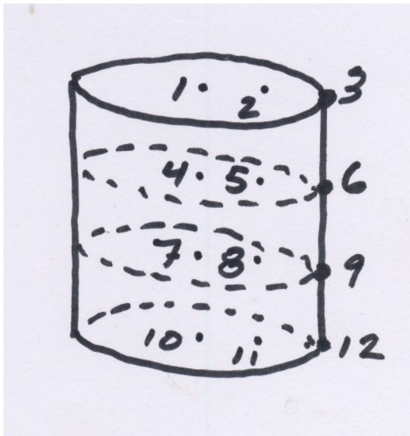
57. A plutonium nuclear fuel rod shown in Figure 2-71 has energy generation in the amount of 112 MW/m^3. For the grid model shown, write the node equations and solve for the temperatures. Assume κ = 10 W/m·K.

FIG 2-71 Plutonium fuel rod.

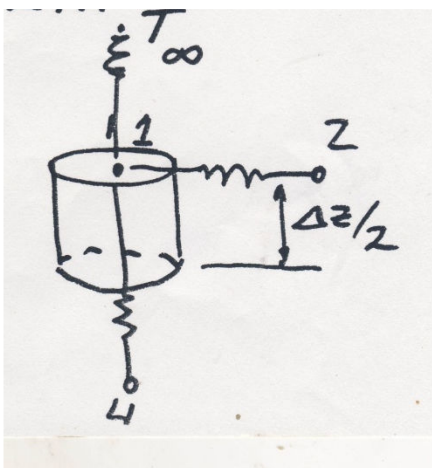


Solution

From Figure 2-71, it can be assumed that the heat flow is radially outward and axially and angularly and axially symmetrical. The node model is sketched



Then for node 1 the adjacent nodes are 4 and 2 plus a convective heat transfer. Referring to the sketch for node 1



The node equation is

$$\kappa\frac{\Delta r}{2}\pi(2)\left(\frac{\Delta z}{2}\right)\left(\frac{T_2-T_1}{\Delta r}\right)+\kappa\pi\left(\frac{\Delta r^2}{4}\right)\left(\frac{T_4-T_1}{\Delta z}\right)+h_\infty\pi\left(\frac{\Delta r^2}{4}\right)(T_\infty-T_1)$$
$$+\left(112\times10^6\,\frac{\mathrm{MW}}{\mathrm{m}^3}\right)\left(\frac{\Delta z}{2}\right)\left(\frac{\pi\Delta r^2}{4}\right)=0$$

Using the following values,

$h_\infty = 690\frac{\mathrm{W}}{\mathrm{m^2\cdot K}}$, $T_\infty = 315°\mathrm{C}$, $\kappa = 10\frac{W}{mK}$, $\Delta r = 0.0375$ m, *and* $\Delta z = 0.3$ m

the following node equation results

$$0.56586T_1 - 0.5T_2 - 0.0009766T_4 = 528.3$$

A similar analysis for node 2, noting that it has three adjacent nodes, 1, 3, and 5, plus a convective heat transfer and energy generation, yielding

$$1.79417T_2 - 0.5T_1 - 9.75T_3 - 0.0347T_5 = 4386.7$$

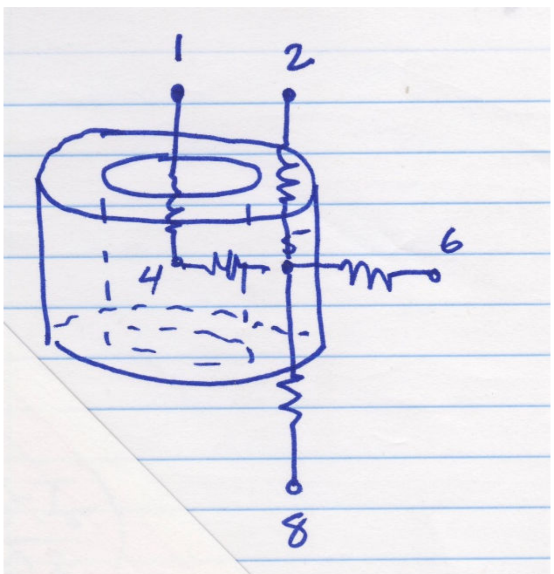
For node 3, the energy balance reduces to

$$17.727T_3 - 91.5T_2 - 0.00684T_6 = 10267$$

For node 4,

$$1.001954T_4 - 0.000977T_1 - 0.000977T_7 - T_5 = 1013.9$$

Node 5 is a bit more complicated. Referring to the sketch the node equation becomes



$$4.015625T_5 - T_4 - 3T_6 - 0.0078125T_2 - 0.0078125T_8 = 4055.6$$

Node 6 has three adjacent nodes plus convection and energy generation so its node equation is

$$20.3T_6 - 3T_5 - 0.00684T_3 - 0.00684T_9 = 9754.2$$

Node 7 energy balance similar to node 4, becomes

$$1.001954T_7 - 0.000977T_4 - 0.000977T_{10} - T_8 = 1013.9$$

For the node 8 node equation, similar to node 5

$$4.015625T_8 - T_7 - 3T_9 - 0.0078125T_5 - 0.0078125T_{11} = 4055.6$$

For node 9, similar to node 6

$$20.3T_9 - 3T_8 - 0.00684T_6 - 0.00684T_{12} = 9754.2$$

The energy balance for node 10 is similar to node 7 except it is only one-half as long as node 7 and there is no lower surface heat transfer.

$$0.500977T_{10} - 0.000977T_7 - 0.5T_{11} = 506.94$$

Node 11 equation is

$$1.328T_{11} - 0.5T_{10} - 0.75T_{12} - 0.0078125T_8 = 4054.3$$

And Node 12 is

$$9.409T_{12} - 0.75T_{11} - 0.00684T_9 = 6278.9$$

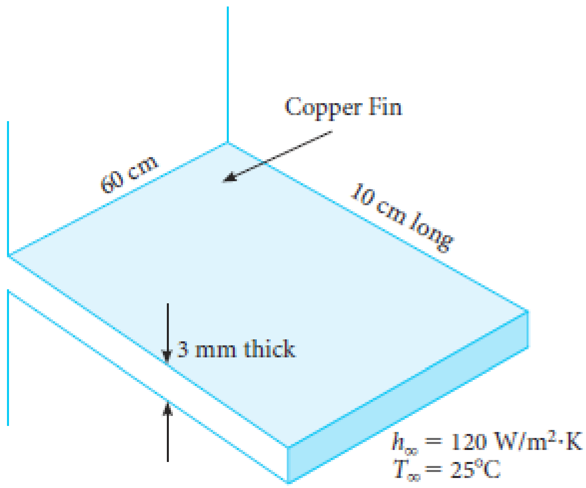
Using Mathcad for the prediction of the 12 node temperatures, the results are

$$T = \begin{bmatrix} 4042.8 \\ 3988.9 \\ 826.7 \\ 3566.7 \\ 2552.2 \\ 858.3 \\ 3579.4 \\ 2561.1 \\ 859.4 \\ 745.6 \\ 645 \\ 1195 \end{bmatrix} \text{°C}$$

Section 2-7

58. Determine the heat transfer and fin efficiency for a copper fin shown in Figure 2-72. The fin can be assumed to be very long and its base temperature taken as 93^0C.

FIG 2-72 Problem 2-58.



Solution

For very long fins the fin efficiency is

$$\eta_{fin} = \frac{1}{L}\sqrt{\frac{\kappa A}{hP}}$$ where $L = 10$ cm $= 0.1$ m

$\kappa = 400$ W/m·k From Appendix Table B-2

$A = 0.6$ m x $(0.003$ mm$) = 0.0018$ m^2

$h = 120$ W/m^2·K

$P = perimeter = 1.2$ m

Then $0.30 = 30\%$

The heat transfer of the fin is

$$\dot{Q} = \eta_{fin}\dot{Q}_o = (0.3)(hA_s)(93 - 25°\text{C}) = 0.293 \text{ W/fin}$$

59. A square bronze fin, 30 cm wide, 1 cm thick, and 5 cm long is surrounded by air at 27^{0}C and h = 300 W/m^2·K. The base temperature of the fin is 170^{0}C. Determine the fin tip temperature, the fin heat transfer, and the fin efficiency.

Solution

For a finite length fin the temperature distribution is given by the equation

$$\Theta(x) = T(x) - T_0 = \Theta_0 \left\{ \frac{\cosh\left[m(L-x)\right] + \dfrac{h}{m\kappa}\sinh\left[m(L-x)\right]}{\cosh mL + \dfrac{h}{m\kappa}\sinh mL} \right\}$$

For this fin $h = 300\ W/m^2K$

$\kappa = 114\ W/m{\cdot}K$

fin thickness, $Y = 0.01\ m$, fin width $W = 0.3\ m$

fin length $L = 0.05\ m$

perimeter, $P = 2W + 2Y = 0.62\ m$, Area, $A = WY = 0.003\ m^2$

60. A square aluminum fin having base temperature of 100^0C, 5 mm width, and 5 cm length is surrounded by water at 40^0C. Using h of 400 $W/m^2{\cdot}K$, compare the heat transfer of the fin predicted by the three conditions: a) very long fin, b) adiabatic tip, and c) uniform convection heat transfer over the fin, including the tip. Assume a width of 1 m.

Solution

From the Appendix Table B.2, $\kappa_{alum} = 236\ W/m{\cdot}K$ Also,

$\Theta_0 = 100 - 40 = 60\ K$, $T_\infty = 40^0\ C$, $h = 400\ W/m^2{\cdot}K$, $t = 0.005\ m$, $L = 0.05\ m$, $W = 1\ m$

$P = 2t + 2W + 2W = 2.01\ m$, $A = LW = 0.05\ m^2$, and

$$m = \sqrt{\frac{Ph}{\kappa A}} = 26.1\ m^{-1}$$

For the very long fin, a) $\dot{Q}_{fin} = \theta_0\sqrt{hP\kappa A} = 1848\ W$

For a fin with an adiabatic tip, $\dot{Q}_{fin} = \theta_0\sqrt{hP\kappa A}\tanh(mL) = 1595\ W$

For a finite length fin,

$$\dot{Q}_{fin} = \theta_0\sqrt{hP\kappa A}\left\{\frac{\sinh mL + \frac{h}{mL}\cosh mL}{\cosh mL + \frac{h}{mL}\sinh mL}\right\} = 1624W$$

61. Show that the fin heat transfer for a square fin having an adiabatic tip is

$$\dot{Q}_{fin} = \theta_0\sqrt{hP\kappa A}\tanh mL$$

Solution

For a square fin with an adiabatic tip the temperature distribution is

$$\theta(x) = T(x) - T_\infty = \theta_0\frac{\cosh\left[m(L-x)\right]}{\cosh mL}$$

The heat transfer is

$$\dot{Q}_{fin} = -\kappa A\left(\frac{\partial T}{\partial x}\right)_{x=0} = -\kappa A\left(\frac{\partial \theta}{\partial x}\right)_{x=0}$$

and

$$\frac{\partial \theta}{\partial x} = -\frac{m\theta_0\sinh\left[m(L-x)\right]}{\cosh mL}$$

At x = 0 this is

$$\left(\frac{\partial \theta}{\partial x}\right)_{x=0} = -\frac{m\theta_0\sinh mL}{\cosh mL} = -m\theta_0\tanh mL$$

The fin hat transfer is then

$$\dot{Q}_{fin} = -\kappa A\left(-m\theta_0\tanh mL\right)$$

but $m = \sqrt{\frac{Ph}{\kappa A}}$

so that

$$\dot{Q}_{fin} = \theta_0 \sqrt{Ph\kappa A} \tanh mL$$

62. Show that the heat transfer for a fin that is square and has fin tip convective heat transfer coefficient h_L can be written

$$\dot{Q}_{fin} = \theta_0 \sqrt{hP\kappa A} \left\{ \frac{\sinh mL + \frac{h_L}{\kappa m} \cosh mL}{\cosh mL + \frac{h_L}{\kappa m} \sinh mL} \right\}$$

Solution

For square fin with convective heat transfer coefficient h_L at the tip, the temperature distribution is

$$\theta(x) = T(x) - T_\infty = \theta_0 \left\{ \frac{\cosh\left[m(L-x)\right] + \frac{h_L}{m\kappa} \sinh\left[m(L-x)\right]}{\cosh mL + \frac{h_L}{m\kappa} \sinh mL} \right\}$$

The fin heat transfer is

$$\dot{Q}_{fin} = -\kappa A \left(\frac{\partial T}{\partial x} \right)_{x=0} = -\kappa A \left(\frac{\partial \theta}{\partial x} \right)_{x=0}$$

Also

$$\frac{\partial \theta}{\partial x} = \frac{-m \sinh\left[m(L-x)\right] - \frac{mh_L}{m\kappa} \cosh\left[m(L-x)\right]}{\cosh mL + \frac{h_L}{m\kappa} \sinh mL}$$

Since

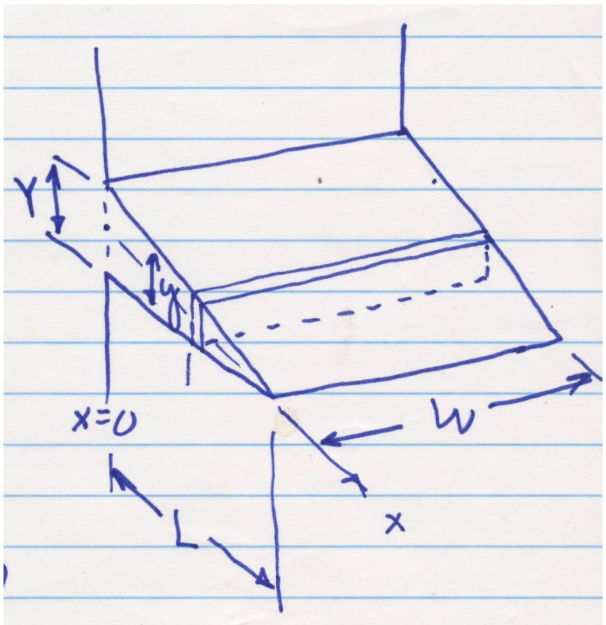
$$m = \sqrt{\frac{Ph}{\kappa A}}$$

$$\dot{Q}_{fin} = \theta_0 \sqrt{hP\kappa A} \left\{ \frac{\sinh mL + \frac{h_L}{m\kappa} \cosh mL}{\cosh mL + \frac{h_L}{m\kappa} \sinh mL} \right\}$$

63. Derive an expression for the heat transfer from a tapered fin having base of Y thickness, L length, κ thermal conductivity, h_0 convective coefficient, and T_0 base temperature. The surrounding fluid temperature is T_∞.

Solution

Referring to the sketch,



$$y = Y\left(1 - \frac{x}{L}\right)$$

From a heat balance through the fin

$$\kappa A \frac{d^2\theta}{dx^2} = h_0 P\theta$$

where $\theta = T - T_\infty$ $\qquad \theta_0 = T_0 - T_\infty$

$P = 2y + 2W \approx 2W$ for y « W.

Then

$$\frac{1}{\theta}\frac{d^2\theta}{dx^2} = \frac{h_0 2WL}{\kappa WY(L-x)} = \frac{2h_0 L}{\kappa Y(L-x)}$$

Using $X = L - x$ and $C = 2h_9L/\kappa Y$

$$-\frac{1}{\theta}\frac{d^2\theta}{dx^2} = \frac{C}{X}$$

with two boundary conditions: B.C. 1, $\vartheta = \vartheta_0$ @ $X = L$

`` B.C. 2, $\vartheta = 0$ @ $= 0$

Now, assuming a series solution so that

$$\theta = c_0 + c_1 X + c_2 X^2 + c_3 X^3 + c_4 X^4 + c_5 X^5 + \ldots + c_n X^n + \ldots$$

From B.C. 2, $c_0 = 0$ and then

$$\theta = c_1 X + c_2 X^2 + c_3 X^3 + c_4 X^4 + c_5 X^5 + \ldots + c_n X^n + \ldots$$ for the second derivative

$$\frac{d^2\theta}{dx^2} = 2c_2 + 6c_3 X + 12c_4 X^2 + 20c_5 X^3 + 30c_6 X^4 + 42c_7 X^5 + 56c_8 X^6 + 72c_9 X^7 + 90c_{10} X^8 + \ldots$$

Using the differential equation $-\frac{d^2\theta}{dx^2} = \frac{C}{X}\theta$ we get

$$2c_2 + 6c_3 X + 12c_4 X^2 + 20c_5 X^3 + 30c_6 X^4 + 42c_7 X^5 + \ldots\ldots = -\frac{C}{X}\left(c_1 X + c_2 X^2 + c_3 X^3 + c_4 X^4 + c_5 X^5 + \ldots\right)$$

Comparing coefficients, $2c_2 = -Cc_1$ *or* $c_2 = -\frac{C}{2}c_1$

, $6c_3 = -Cc_2$ *or* $c_3 = \frac{C^2}{12}c_1$

, $12c_4 = -Cc_3$ *or* $c_4 = -\frac{C^3}{144}c_1$

$20c_5 = -Cc_4$ *or* $c_5 = \frac{C^4}{2880}c_1$ and so on…

For C less than or equal to 1.0, using the first four terms is suitable as higher terms will be significantly smaller. Then,

$$\theta = c_1 X - \frac{C}{2}c_1 X^2 + \frac{C^2}{12}c_1 X^3 - \frac{C^3}{144}c_1 X^4 + ..$$ and using B.C.1

$$\theta = \theta_0 = c_1 L - \frac{C}{2}c_1 L^2 + \frac{C^2}{12}c_1 L^3 - \frac{C^3}{144}c_1 L^4$$ Solving this for c_1 and substituting

$$\theta = \frac{\theta_0\left(X - \frac{C}{2}X^2 + \frac{C^2}{12}X^3 - \frac{C^3}{144}X^4 + \ldots\right)}{\left(L - \frac{C}{2}L^2 + \frac{C^2}{12}L^3 - \frac{C^3}{144}L^4 + \ldots\right)}$$

64. Show that the fin effectiveness is related to the fin efficiency by the equation

$$\varepsilon_{fin} = 1 - \left(\frac{A_{fin}}{A_T} - \eta_{fin} \frac{A_{fin}}{A_T} \right)$$

Solution

For a fin and a base area between succeeding fins, the fin effectiveness is

$$\varepsilon_{fin} = \frac{\dot{Q}_{fin} + \dot{Q}_{base}}{\dot{Q}_0}$$ where

$$\dot{Q}_0 = hA_{fin}\theta_0 + hA_{base}\theta_0 = hA_T\theta_0$$

Where $A_T = A_{fin} + A_{base}$

Also,

$$\dot{Q}_{fin} = \eta_{fin} hA_{fin}\theta_0$$

And

$$\dot{Q}_{base} = hA_{base}\theta_0$$

Substituting into the effectiveness equation

$$\varepsilon_{fin} = \frac{\eta_{fin} hA_{fin}\theta_0 + hA_{base}\theta_0}{hA_T\theta_0} = \frac{\eta_{fin} hA_{fin}\theta_0 + h\left(A_T - A_{fin}\right)\theta_0}{hA_T\theta_0}$$

Cancelling the *h's*, ϑ_0*'s*, and rearranging,

$$\varepsilon_{fin} = 1 + \eta_{fin} \frac{A_{fin}}{A_T} - \frac{A_{fin}}{A_T} = 1 - \left[\frac{A_{fin}}{A_T} - \eta_{fin} \frac{A_{fin}}{A_T} \right]$$

65. A circumferential steel fin is 8 cm long, 3 mm thick, and is on a 20 cm diameter rod. The surrounding air temperature is 20^0C and h = 35 W/m^{2}K, while the surface temperature of the rod is 300^0C. Determine a) Fin Efficiency, and b) Heat transfer from the fin.

Solution

Referring to Figure 2-41

L = 8 cm = 0.08 m, $r_1 = 0.1$ *m,* *y = 3 mm = 0.003 m,* $r_2 = L + r_1$, $L_C = L + y/2 = 0.0815$ *m,* $r_{2C} = r_1 + L_C = 0.1815$ *m, and* $A_m = y(r_{2C} - r_1) = 0.0002445\ m^2$ Using a thermal conductivity of 43 W/mK for steel from Appendix Table B-2

$$L_C^{3/2} = \sqrt{\frac{h}{\kappa A_m}} = 1.342$$ and $$\frac{r_{2C}}{r_1} = 1.815$$. Then, from Figure 2-41,

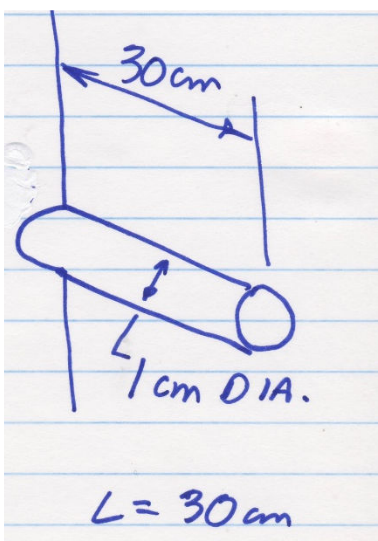
a) $\eta_{fin} \approx 44\ \%$

b)

$$\dot{Q}_{fin} = \eta_{fin} h A_{fin} \theta_0 = (0.44)\left(35\frac{W}{m^2K}\right)\left[(\pi)\left(r_2^2 - r_1^2\right) + 2\pi r_2 y\right](300 - 20K) = 318\frac{W}{fin}$$

66. A bronze rod 1 cm in diameter and 30 cm long protrudes from a bronze surface at 150⁰C. The rod is surrounded by air at 10⁰C with a convective heat transfer coefficient of 10 W/m² K. Determine the heat transfer through the rod.

Solution



Assume the bronze has the same thermal conductivity as brass, 114 W/mK from Appendix Table B-2. Some of the other parameters are: $h = 10\ W/m^2\ K$, $T_\infty = 10^0$ C, $T_0 = 150^0$C,

$\Theta_0 = T_0 - T_\infty = 140^0$ *C,* $P = \pi D = 0.0314159$ *m,* $A = \pi r^2 = 0.00007854\ m^2$*, and*

$$m = \sqrt{\frac{hP}{\kappa A}} = 5.923 m^{-1}$$

and using the case III fin equation, the finite length fin,

$$\dot{Q}_{fin} = \theta_0 \sqrt{hP\kappa A} \left\{ \frac{\sinh mL + \frac{h}{m\kappa}\cosh mL}{\cosh mL + \frac{h}{m\kappa}\sinh mL} \right\} = 7.024 W / rod$$

67. A circumferential cast iron fin attached to a compressor housing is 2.5 cm thick, 7.5 cm long, 7.5 cm diameter, and the convective heat transfer coefficient is 28 W/m^2·K. If the base temperature is 70^{0}C and the surrounding air is 25^{0}C, determine the fin efficiency and the heat transfer through the fin.

Solution

Referring to Figure 2-41, the following parameters are:

r_1 = 3.75 cm = 0.0375 m, r_2 = 0.112 m, L = 0.035 m, y = 0.025m,

L_C = L + y/2 = 0.05 m, r_{2C} = r_1 + L_C = 0.09 m, A_m = y(r_{2C} - r_1) = 0.00125 m^2 ,

r_{2C}/r_1 = 2.333, and $L_C^{3/2}\sqrt{\frac{h}{\kappa A_m}} = 0.486$.

From Figure 2-41

η_{fin} ≈ 82%.

The heat transfer is

$$\dot{Q}_{fin} = \eta_{fin} h A_{fin} (T_0 - T_\infty)$$

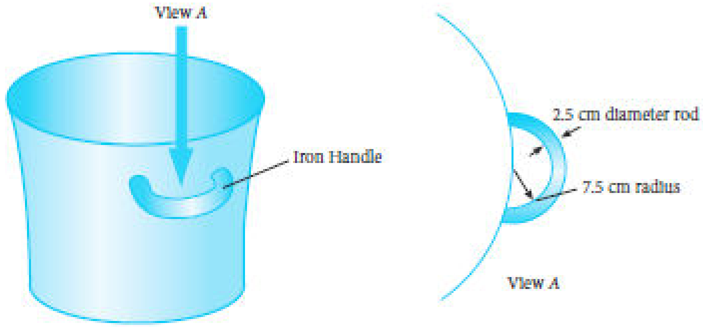
$$= 0.82 \left(28 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}\right) (\pi)(r_2^2 - r_1^2 + 2r_2 y)(70 - 25°\text{C})$$

$$= 181.2 \text{ W}$$

68. A handle on a cooking pot can be modeled as a rod fin with an adiabatic tip at the farthest section from the attachment points. For the handle shown in the sketch, determine the temperature distribution and the heat transfer through the handle if the pot surface is 88^{0}C, the surrounding air temperature is 30^{0}C, and the convective heat transfer coefficient is 277 W/m^2·K.

Solution

FIG 2-73 Cooking pot handle.



Treating this handle as a fin with an adiabatic tip, the important parameters are: Thermal conductivity of 39 W/m^2·K from Appendix Table B-2, L = πr/2 = π(0.075/2) m = 0.12 m, P = π(0.025) m = 0.078 m,   A = π(0.025)^2 (1/4) = 4.9 × 10^{-4} m^2, and

$$m = \sqrt{\frac{hP}{\kappa A}} = \sqrt{\frac{277 \times 0.078}{39 \times 4.9 \times 10^{-4}}} = 33.6\, m^{-1}$$

For an adiabatic tipped fin,

$$\theta = \theta_0 \frac{\cosh[m(L-x)]}{\cosh mL} = (58°\mathrm{C}) \frac{\cosh[33.6(L-x)]}{\cosh 7.255} = (0.082) \cosh[33.6(L-x)]$$

At the extreme outer point of the handle,

$\theta = 0.079°\mathrm{C}$ or

T = 30.079^{0}C

The heat transfer through the fin is

$$\dot{Q}_{fin} = \theta_0 \sqrt{hP\kappa A}\ \tan mL = 39.8\ \mathrm{W}$$

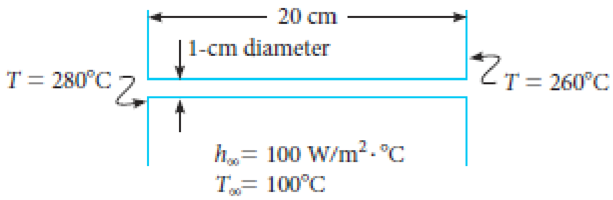
Since the handle has two fins, so to speak,

$\dot{Q}_{handle} = 79.69\ \mathrm{W}$

69. An aluminum fin is attached at both ends in a compact heat exchanger as shown. For the situation shown, determine the temperature distribution and the heat transfer through the fin. Notice that the analysis requires using the governing equation $d^2\,\theta(x)/dx^2 = m^2\,\theta(x)$ with appropriate boundary conditions to determine the temperature distribution.

Solution

FIG 2-74 Compact heat exchanger fin.



For the fin

$$\frac{d^2\theta}{dx^2} = m^2\theta$$

with boundary conditions, B.C. 1 $\theta = \theta_1 = T_1 - T_\infty = 180°\mathrm{C}$ @ $x = 0$

B.C. 2 $\theta = \theta_2 = T_2 - T_\infty = 160°\mathrm{C}$ @ $x = L$

From this equation and the boundary conditions Equation 2-114 is

$$\theta(x) = \frac{1}{e^{2mL} - 1}\left[\left\{\theta_1 e^{2mL} - \theta_2 e^{mL}\right\} e^{-mx} + \left\{\theta_2 e^{mL} - \theta_1\right\} e^{mx}\right]$$

$$m = \sqrt{\frac{hP}{\kappa A}} = \sqrt{\frac{(100)\pi(0.01m)}{(236)\pi(0.005)^2}} = 13m^{-1}$$

where $L = 0.2\ m$,

And then $mL = 2.6$ so that

$$\theta(x) = T(x) - 100 = \frac{1}{e^{5.2} - 1}\left[\left\{180e^{5.2} - 160e^{2.6}\right\} e^{-13x} + \left\{160e^{2.6} - 180\right\} e^{13x}\right]$$

The maximum or minimum temperature occurs at the location predicted by Equation 2-115,

$$x_m = \frac{1}{2m}\ln\left(\frac{\theta_1 e^{2mL} - \theta_2 e^{mL}}{\theta_2 e^{mL} - \theta_1}\right) = 0.1079m$$

Using x = 0.1079 m in the above equation for the temperature distribution,

$T_{\min imum} = 186.08^0 C$ The fin heat transfer is the sum of the two adiabatic stems

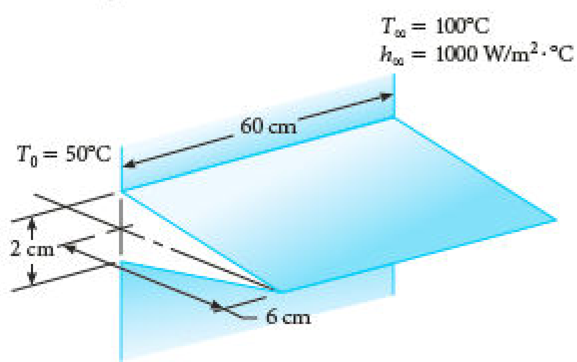
$$\dot{Q}_{fin} = \dot{Q}_{fin1} + \dot{Q}_{fin2} = 180\sqrt{hP\kappa A}\tanh m(0.1079\,\text{m}) + 160\sqrt{hP\kappa A}\tanh m\left(0.2 - 0.1079m\right) = 70.63W$$

70. For the tapered fin shown, determine the temperature distribution, the fin efficiency, and the heat transfer through the fin.

Solution

Referring to the figure,

FIG 2-75 Tapered fin.



The following parameters are known: $L = L_C = 0.06$ m, $Y = 0.02$ m, $A_m = LY/2 = 0.0006$ m^2,

$K = 236$ W/mK, $h = 1000$ W/m^2·K, and

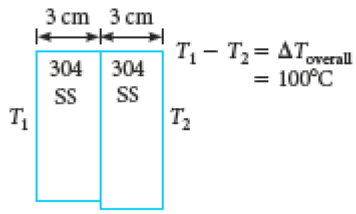
$$L_C^{3/2} = \sqrt{\frac{h}{\kappa A_m}} = 1.235$$

From Figure 2-40, $\eta_{fin} \approx 62\%$ and the heat transfer is

$$\dot{Q}_{fin} = \eta_{fin}\dot{Q}_0 = \eta_{fin} hA\theta_0 = (0.62)(1000)(0.073)(50) = 2263W$$

71. Determine the expected temperature drop at the contact between two 304 stainless steel parts if the overall temperature drop across the two parts is 100^{0}C.

FIG 2-76 Heat transfer at contact surfaces.



Solution

From Table 2-12, using a value for thermal contact

$$\dot{q}_A = \frac{\Delta T_{TL}}{R_{TC} \cdot A} = \frac{T_1 - T_2}{\sum R_V} = \frac{T_1 - T_2}{2\left(\frac{\Delta x}{\kappa}\right)_{304ss} + R_{TC} \cdot A}$$

resistance of 304 stainless at 20°C, assuming it will be unchanged at 100°C, 0.000528 $m^2 \cdot {}^0C/W$, then

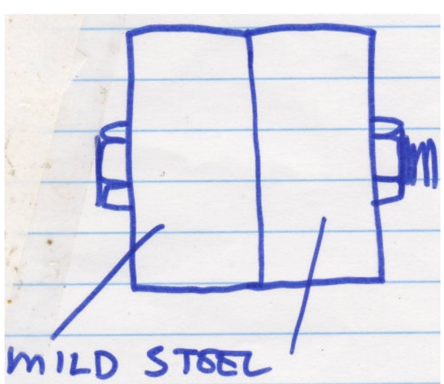
$$\dot{q}_A = \frac{T_1 - T_2}{2\left(\frac{0.03}{14}\right) + 0.000528} = \frac{100}{0.0048137} \frac{W}{m^2}$$

then

$$\Delta T_{TC} = \frac{0.000528}{0.0048137}\left(100^0 C\right) = 10.97^0 C \approx 11^0 C$$

72. A mild steel weldment is bolted to another mild steel surface. The contact pressure is estimated at 2 MPa and the expected heat transfer between the two parts is 136 W/m^2. Estimate the temperature drop at the contact due to thermal contact resistance.

Solution



The temperature drop across the contact surface is

$$\Delta T_{TC} = \dot{q}_A \cdot (R_{TL} \cdot A) = \left(136 \frac{W}{m^2}\right)(R_{TL} \cdot A)$$

The thermal contact resistance, from Table 2-12, is

$$R_{TC} \cdot A = 0.000394 \ \frac{\mathrm{m^2 °C}}{\mathrm{W}}$$

so that

$$\Delta T_{TC} = 52.8°\mathrm{C}$$

73. For Example Problem 2-26, estimate the temperature drop at the contact surface if the heat transfer is reduced to 9.5 W/m^2.

Solution

The thermal contact resistance of the concrete block/styrofoam for example 2-26 is 1.1m^2 ·^{0}C/W. If the heat transfer is reduced to 9.5 W/m^{2,} the temperature drop will be,

$$\Delta T_{TC} = \dot{q}_A \cdot (R_{TC} \cdot A) = 1.74°\mathrm{C}$$

74. A guarded hot plate test results in the following data:

FIG 2-77 Thermal Conductivity Data for Problem 2-74.

Test No.	Heater Data: *A*, amps	Heater Data: *V*, volts	Thermocouple Data (millivolts, mV): 1	Thermocouple Data (millivolts, mV): 2
1	0.05	8.6	2.669	2.775
2	0.055	8.4	2.672	2.780
3	0.049	8.8	2.662	2.771

Thermocouple conversion: 22°/mV

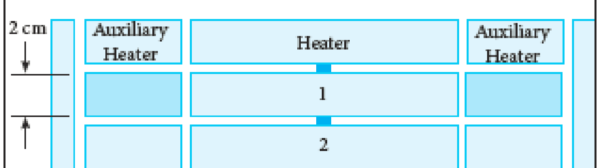


Diagram of testing device

Estimate the thermal conductivity of the test material.

Solution

The arithmetic averages are

Amps = 0.05133,   volts = 8.6,   thermocouple 1 = 2.6677 mv,   thermocouple 2 = 2.7753 mv.

The average power is = amps·volts = 0.44147 W. the average millivolt difference between 1 and 2 is 0.10756 mv. For a 22^0 C /mv setting, the average temperature difference will be 2.366^0C. From Fourier's law

$$\dot{Q} = \kappa A \frac{\Delta T}{\Delta x} = 0.44147W$$

For a sample thickness of 2 cm (0.02 m) and a test area of 0.01 m^2

$$\kappa = \frac{\dot{Q}\Delta x}{A\Delta T} = 0.373 \frac{W}{m \cdot K}$$

75. A steam line has an outer surface diameter of 3 cm and temperature of 160^0C. If the line is surrounded by air at 25^0C and the convective heat transfer coefficient is 3.0 W/m^2·K, determine the heat transfer per meter of line. Then determine the thickness of asbestos insulation needed to provide insulating qualities to the steam line.

Solution

The heat transfer is by convection so

$$\dot{q}_l = h\pi D\left(T_s - T_\infty\right) = \left(3\frac{W}{m^2 \cdot K}\right)\pi\left(0.03m\right)\left(160 - 25K\right) = 38.17W / m$$

The critical radius of insulation needed to make the convection equal to the conduction through the line is

$$r_{oc} = \frac{\kappa}{h_0} = \frac{0.156W/m \cdot K}{3W/m^2 \cdot K} = 5.2cm$$

76. Electric power lines require convective cooling from the surrounding air to prevent excessive temperatures in the wire. If a 2.5 cm diameter line is wrapped with nylon to increase heat transfer with the surroundings, how much nylon can be wrapped around the wire before it begins to act as an insulator? The convective heat transfer coefficient is 8.7 W/m^2 ·^{0}C.

Solution

The critical thickness determines how much insulation wrapped around a cylinder decrease heat transfer. Using properties of Teflon from Appendix Table B-2E,

$$r_{oc} = \frac{\kappa}{h_0} = \frac{0.2023\,Btu/hr \cdot ft \cdot {}^0F}{5\,Btu/hr \cdot ft^2 \cdot {}^0F} = 0.04\,ft = 0.48in$$

$$r_{oc} = \frac{\kappa}{h} = \frac{0.35\ \mathrm{W/(m \cdot \kappa)}}{8.7\ \mathrm{W/m^2 °C}} = 0.04\ \mathrm{m} = 40\ \mathrm{cm}$$

77. Estimate the temperature distribution through a bare 16 gauge copper wire conducting 1.5 amperes of electric current if the surrounding air is at 10^{0}C and the convective heat transfer coefficient is 65 W/m^2· K.

Solution

Equation 2-123 will predict the temperature distribution through the wire.

$$T(r) = T_\infty + \dot{e}_{gen}\left[\frac{r_0}{2h_0} + \frac{1}{4\kappa}\left(r_0^2 - r^2\right)\right]$$

Here $T_\infty = 10^0C$ $\quad h_0 = 65\,W/m^2K$, $\quad \kappa = 400\ W/mK$ from Appendix Table B-2. Then, from Appendix Table B-7, $\quad r_0 = 25.41\,mils = 0.0006454\,m$

$$A_0 = 2{,}583\,cir.mils = 16.664\,x\,10^{-7}\,m^2$$

$$R_e = 4.016\ ohms/1000ft = 13.1756\,x\,10^{-3}ohms/m$$

The energy generation is

$$\dot{e}_{gen} = \frac{I^2 R_e}{A_0} = \frac{(1.5amps)^2\left(13.1756x10^{-3}\,\Omega/m\right)}{16.664x10^{-7}\,m^2} = 1.779x10^4\,W/m^3$$

The temperature distribution is

$$T(r) = 10^0C + 17,790\frac{W}{m^3}\left[\frac{0.0006454m}{2(65W/m^2\cdot K)} + \frac{1}{4(400W/m\cdot K)}\left(0.0006454^2 m^2 - r^2\right)\right]$$

and

$$T(r) = 10^0C + 0.0883^0C + 11.11875\left(r_0^2 - r^2\right)$$ where $r_0 = 0.0006454$ *m*

At the center, where r = 0 *T(r) = 10.088305⁰C*

And at the outer surface, here r = r_0 *T(r) = 10.0883⁰C*

78. Aluminum wire has resistivity of 0.286 x 10^{-7} ohm·m where resistivity is defined as (ohm)·area/length. Determine the temperature distribution through an aluminum wire of 6 mm diameter carrying 200 amperes of current if it is surrounded by air at 25^0C and with a convective heat transfer coefficient of 345 W/m^2·K.

Solution

Equation 2-123 predicts the wire temperature distribution

$$T(r) = T_\infty + \dot{e}_{gen}\left[\frac{r_0}{2h_0} + \frac{1}{4\kappa}\left(r_0^2 - r^2\right)\right]$$

Here, $T_\infty = 25°\text{C}$, $h_0 = 345\ \text{W/m}^2\cdot\text{K}$

r_0 = *3* mm = *0.003* m, *κ = 236* W/m·K, *I = 200 amps,* $A_0 = 2.83\times10^{-5}$ m²

$$R_e = \frac{R}{A_0} = \frac{0.286\times10^{-7}\ \Omega\cdot\text{m}}{2.83\times10^{-5}\ \text{m}^2} = 0.0010\ \Omega/\text{m}$$

and

$$\dot{e}_{gen} = \frac{I^2R_e}{A_0} = \frac{(200\ amps)^2(0.0010\ \Omega/\text{m})}{2.83\times10^{-5}\ \text{m}^2} = 1.41\times10^6\frac{W}{m^3}$$

then

$$T(r) = 25°\text{C} + \left(1.41\times10^6\frac{W}{m^3}\right)\left[\frac{0.003\ \text{m}}{2(345\ \text{W/m}^2\cdot\text{K})} + \frac{1}{4(236\ \text{W/m}\cdot\text{K})}(0.003^2\ \text{m}^2 - r^2)\right]$$

or

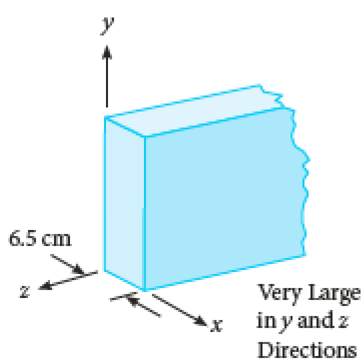
$$T(r) = 31.14°\text{C} - 1493.6\, r^2$$

T(r) = 31.14^{0}C *at the center, r=0*

T(r) = 31.153^{0}C *at the surface, r =* r_0

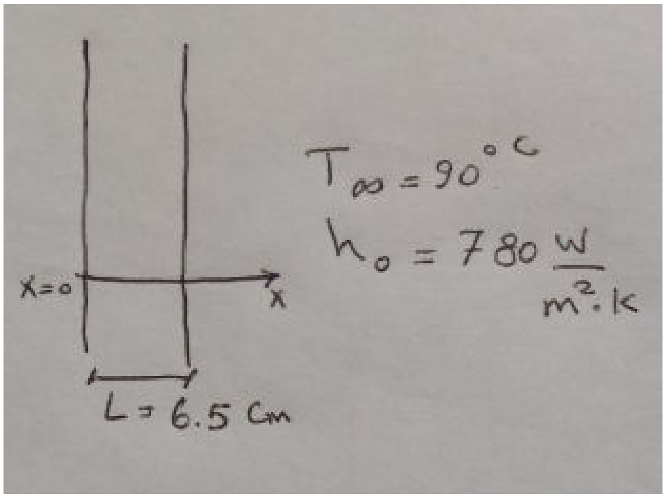
79. Determine the temperature distribution through a uranium slab shown. Assume energy generation of 2.8 MW/m^3 and the slab is surrounded by water at 90^{0}C with a convective heat transfer coefficient of 780 W/m^2 ·K. Use the equation $\frac{d^2T}{dx^2} + \frac{\dot{e}_{gen}}{\kappa} = 0$ with appropriate boundary condition must be made to obtain the temperature distribution function.

FIG 2-78 Uranium slab.



Solution

Using the figure shown and the governing equation for one-dimensional conduction heat transfer with energy generation



$$\frac{d^2T}{dx^2} + \frac{\dot{e}_{gen}}{\kappa} = 0$$

with two boundary conditions: B.C.1

$$\left(\frac{2.8 \times 10^6}{2}\right)(0.065\text{ m}) = 91000\frac{W}{m^2} = h_0(T - 90°\text{C})$$

at x = 0

And $\frac{dT}{dx} = 0$ @ $x = L/2$

Separating variable once gives,

$$\frac{dT}{dx} = -\frac{\dot{e}_{gen}}{\kappa}x + C_1$$

and then again

$$T(x) = -\frac{\dot{e}_{gen}}{2\kappa}x^2 + C_1x + C_2$$

From B.C. 1

$$T = \frac{91000\ W/m^2}{780\ W/m^2 \cdot K} + 90°\text{C} = 206.7\ °\text{C}$$

at x = 0. This means that $C_2 = 206.7\ °\text{C}$

From B.C. 2

$$C_1 = \frac{\dot{e}_{gen}}{2\kappa}L$$

so that the temperature distribution becomes

$$T(x) = -\frac{\dot{e}_{gen}}{2\kappa}x^2 + \frac{\dot{e}_{gen}}{2\kappa}L + 206.7°\text{C}$$

At the center of the slab, where x = 3.25 cm = 0.0325, *T = 159.5*^{0}C

80. Plutonium plates of 6 cm thickness generate 60 kW/m^3 of energy. It is exposed on one side to pressurized water which cannot be more than 280^{0}C. The other surface is well insulated. What must the convective heat transfer coefficient be at the exposed surface?

Solution

Using the governing energy balance equation

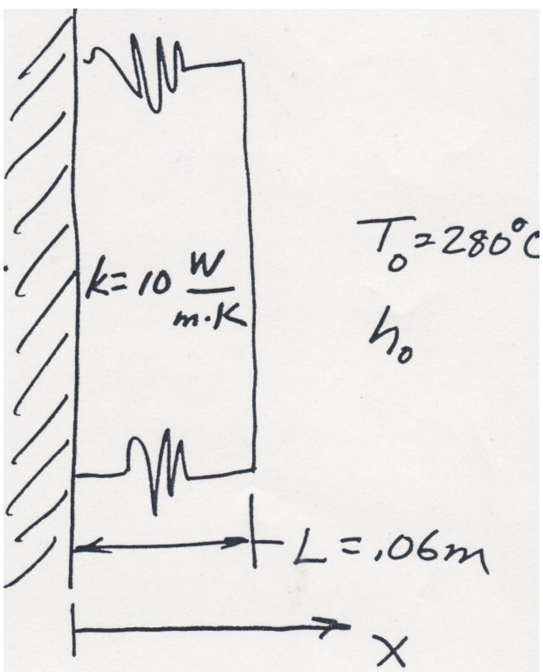
$$\frac{d^2T}{dx^2}+\frac{\dot{e}_{gen}}{\kappa}=0$$

With B. C. 1, $\frac{dT}{dx}=0$ @ $x=0$

B.C. 2 $\dot{e}_{gen}L=h_0(T-T_\infty)$ @ x = L

Separating variables and integrating

$$\frac{dT}{dx}=-\frac{\dot{e}_{gen}}{\kappa}x+C_1$$



And separating variable once more, integrating gives,

$$T(x)=-\frac{\dot{e}_{gen}}{2\kappa}x^2+C_1x+C_2$$

From B.C. 1 $C_1 = 0$