INSTRUCTOR'S SOLUTIONS MANUAL

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FUNDAMENTALS OF DIFFERENTIAL EQUATIONS

EIGHTH EDITION

and

FUNDAMENTALS OF DIFFERENTIAL EQUATIONS AND BOUNDARY VALUE PROBLEMS

SIXTH EDITION

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Notes to the Instructor

One goal in our writing has been to create flexible texts that afford the instructor a variety of topics and make available to the student an abundance of practice problems and projects. We recommend that the instructor read the discussion given in the preface in order to gain an overview of the prerequisites, topics of emphasis, and general philosophy of the text.

Supplements

Student's Solutions Manual: By Viktor Maymeskul. Contains complete, worked-out solutions to most odd-numbered exercises, providing students with an excellent study tool. ISBN 13: 978-0-321-74834-8; ISBN 10: 0-321-74834-4.

Companion Web site: Provides additional resources for both instructors and students, including helpful links keyed to sections of the text, access to Interactive Differential Equations, suggestions for incorporating Interactive Differential Equations modules, suggested syllabi, index of applications, and study tips for students. Access: `www.pearsonhighered.com/nagle`

Interactive Differential Equations: By Beverly West (Cornell University), Steven Strogatz (Cornell University), Jean Marie McDill (California Polytechnic State University – San Luis Obispo), John Cantwell (St. Louis University), and Hubert Hohn (Massachusetts College of Arts) is a popular software directly tied to the text that focuses on helping students visualize concepts. Applications are drawn from engineering, physics, chemistry, and biology. Access: `www.pearsonhighered.com/nagle`

Instructor's MAPLE/MATHLAB/MATHEMATICA manuals: By Thomas W. Polaski (Winthrop University), Bruno Welfert (Arizona State University), and Maurino Bautista (Rochester Institute of Technology). A collection of worksheets and projects to aid instructors in integrating computer algebra systems into their courses. Available in the Pearson Instructor Resource Center at `www.pearsonhighered.com/irc`.

MATLAB Manual ISBN 13: 978-0-321-53015-8; ISBN 10: 0-321-53015-2
MAPLE Manual ISBN 13: 978-0-321-38842-1; ISBN 10: 0-321-38842-9
MATHEMATICA Manual ISBN 13: 978-0-321-52178-1; ISBN 10: 0-321-52178-1

Computer Labs

A computer lab in connection with a differential equations course can add a whole new dimension to the teaching and learning of differential equations. As more and more colleges and universities set up computer labs with software such as MAPLE, MATLAB, DERIVE, MATHEMATICA, PHASEPLANE, and MACMATH, there will be more opportunities to include a lab as part of the differential equations course. In our teaching and in our texts, we have tried to provide a variety of exercises, problems, and projects that encourage the student to use the computer to explore. Even one or two hours at a computer generating phase plane diagrams can provide the students with a feeling of how they will use technology together with the theory to investigate real world problems. Furthermore, our experience is that they thoroughly enjoy these activities. Of course, the software, provided free with the texts, is especially convenient for such labs.

Group Projects

Although the projects that appear at the end of the chapters in the text can be worked out by the conscientious student working alone, making them *group* projects adds a social element that encourages discussion and interactions that simulate a professional work place atmosphere. Group sizes of 3 or 4 seem to be optimal. Moreover, requiring that each individual student separately write up the group's solution as a formal technical report for grading by the instructor also contributes to the professional flavor.

Typically, our students each work on 3 or 4 projects per semester. If class time permits, oral presentations by the groups can be scheduled and help to improve the communication skills of the students.

The role of the instructor is, of course, to help the students solve these elaborate problems on their own and to recommend additional reference material when appropriate.

Some additional Group Projects are presented in this guide (see page 10).

Technical Writing Exercises

The technical writing exercises at the end of most chapters invite students to make documented responses to questions dealing with the concepts in the chapter. This not only gives students an opportunity to improve their writing skills, but it helps them organize their thoughts and better understand the new concepts. Moreover, many questions deal with critical thinking skills that will be useful in their careers as engineers, scientists, or mathematicians.

Since most students have little experience with technical writing, it may be necessary to return *ungraded* the first few technical writing assignments with comments and have the students redo the the exercise. This has worked well in our classes and is much appreciated by the students. Handing out a "model" technical writing response is also helpful for the students.

Student Presentations

It is not uncommon for an instructor to have students go to the board and present a solution to a problem. Differential equations is so rich in theory and applications that it is an excellent course to allow (require) a student to give a presentation on a special application (e.g., almost any topic from Chapters 3 and 5), on a new technique not covered in class (e.g., material from Section 2.6, Projects A, B, or C in Chapter 4), or on additional theory (e.g., material from Chapter 6 which generalizes the results in Chapter 4). In addition to improving students' communication skills, these "special" topics are long remembered by the students. Here, too, working in groups of 3 or 4 and sharing the presentation responsibilities can add substantially to the interest and quality of the presentation. Students should also be encouraged to enliven their communication by building physical models, preparing part of their lectures with the aid of video technology, and utilizing appropriate internet web sites.

Homework Assignments

We would like to share with you an obvious, non-original, but effective method to encourage students to do homework problems.

An essential feature is that it requires little extra work on the part of the instructor or grader. We assign homework problems (about 5 of them) after each lecture. At the end of the week (Fridays), students are asked to turn in their homework (typically, 3 sets) for that week. We then choose at random one problem from each assignment (typically, a total of 3) that will be graded. (The point is that the student does not know in advance which problems will be chosen.) Full credit is given for any of the chosen problems for which there is evidence that the student has made an honest attempt at solving. The homework problem sets are returned to the students at the next meeting (Mondays) with grades like 0/3, 1/3, 2/3, or 3/3 indicating the proportion of problems for which the student received credit. The homework grades are tallied at the end of the semester and count as one test grade. Certainly, there are variations on this theme. The point is that students are motivated to do their homework.

Syllabus Suggestions

To serve as a guide in constructing a syllabus for a one-semester or two-semester course, the prefaces to the texts list sample outlines that emphasize methods, applications, theory, partial differential equations, phase plane analysis, computation, or combinations of these. As a further guide in making a choice of subject matter, we provide (starting on the next page) a listing of text material dealing with some common areas of emphasis.

Numerical, Graphical, and Qualitative Methods

The sections and projects dealing with numerical, graphical, and qualitative techniques of solving differential equations include:

Section 1.3: *Direction Fields*

Section 1.4: *The Approximation Method of Euler*

Project A for Chapter 1: *Taylor Series Method*

Project B for Chapter 1: *Picard's Method*

Project C for Chapter 1: *The Phase Line*

Section 3.6: *Improved Euler's Method*, which includes step-by-step outlines of the improved Euler's method subroutine and improved Euler's method with tolerance. These outlines are easy for the student to translate into a computer program (pp. 127–128).

Section 3.7: *Higher-Order Numerical Methods*: *Taylor and Runge-Kutta*, which includes outlines for the Fourth Order Runge-Kutta subroutine and algorithm with tolerance (see pp. 135–136).

Project H for Chapter 3: *Stability of Numerical Methods*

Project I for Chapter 3: *Period Doubling and Chaos*

Section 4.8: *Qualitative Considerations for Variable Coefficient and Non-linear Equations*, which discusses the energy integral lemma, as well as the Airy, Bessel, Duffing, and van der Pol equations.

Section 5.3: *Solving Systems and Higher-Order Equations Numerically*, which describes the vectorized forms of Euler's method and the Fourth Order Runge-Kutta method, and discusses an application to population dynamics.

Section 5.4: *Introduction to the Phase Plane*, which introduces the study of trajectories of autonomous systems, critical points, and stability.

Section 5.8: *Dynamical Systems, Poincaré Maps, and Chaos*, which discusses the use of numerical methods to approximate the Poincarè map and how to interpret the results.

Project A for Chapter 6: *Computer Algebra Systems and Exponential Shift*

Project D for Chapter 6: *Higher-Order Difference Equations*

Project A for Chapter 8: *Alphabetization Algorithms*

Project D for Chapter 10: *Numerical Method for* $\Delta u = f$ *on a Rectangle*

Project D for Chapter 11: *Shooting Method*

Project E for Chapter 11: *Finite-Difference Method for Boundary Value Problems*

Project C for Chapter 12: *Computing Phase Plane Diagrams*

Project D for Chapter 12: *Ecosystem of Planet GLIA-2*

Section 13.1: *Introduction: Successive Approximations*

Appendix B: *Newton's Method*

Appendix C: *Simpson's Rule*

Appendix E: *Method of Least Squares*

Appendix F: *Runge-Kutta Procedure for Equations*

The instructor who wishes to emphasize numerical methods should also note that the text contains an extensive chapter of series solutions of differential equations (Chapter 8).

Engineering/Physics Applications

Since Laplace transforms is a subject vital to engineering, we have included a detailed chapter on this topic – see Chapter 7. Stability is also an important subject for engineers, so we have included an introduction to the subject in Section 5.4 along with an entire chapter addressing this topic – see Chapter 12. Further material dealing with engineering/physics applications include:

Project C for Chapter 2: *Torricelli's Law of Fluid Flow.*

Project I for Chapter 2: *Designing a Solar Collector.*

Section 3.1: *Mathematical Modeling.*

Section 3.2: *Compartmental Analysis*, which contains a discussion of mixing problems and of population models.

Section 3.3: *Heating and Cooling off Buildings*, which discusses temperature variations in the presence of air conditioning or furnace heating.

Section 3.4: *Newtonian Mechanics.*

Section 3.5: *Electrical Circuits.*

Project C for Chapter 3: *Curve of Pursuit.*

Project D for Chapter 3: *Aircraft Guidance in a Crosswind.*

Project E for Chapter 3: *Feedback and the Op Amp.*

Project F for Chapter 3: *Bang-Bang Controls.*

Section 4.1: *Introduction: The Mass-Spring Oscillator.*

Section 4.8: *Qualitative Considerations for Variable-Coefficient and Non-linear Equations.*

Section 4.9: *A Closer Look at Free Mechanical Vibrations.*

Section 4.10: *A Closer Look at Forced Mechanical Vibrations.*

Project B for Chapter 4: *Apollo Re-entry*

Project C for Chapter 4: *Simple Pendulum*

Section 5.1: *Interconnected Fluid Tanks.*

Section 5.4: *Introduction to the Phase PLane.*

Section 5.6: *Coupled Mass-Spring Systems.*

Section 5.7: *Electrical Systems.*

Section 5.8: *Dynamical Systems, Poincaré Maps, and Chaos .*

Project A for Chapter 5: *Designing a Landing System for Interplanetary Travel.*

Project C for Chapter 5: *Things that Bob.*

Project D for Chapter 5: *Hamiltonian Systems.*

Project C for Chapter 6: *Transverse Vibrations of a Beam.*

Chapter 7: *Laplace Transforms*, which in addition to basic material includes discussions of transfer functions, the Dirac delta function, and frequency response modelling.

Project B for Chapter 8, *Spherically Symmetric Solutions to Schrödinger's Equation for the Hydrogen Atom*

Project D for Chapter 8, *Buckling of a Tower*

Project E for Chapter 8, *Aging Spring and Bessel Functions*

Section 9.6: *Complex Eigenvalues*, includes discussion of normal (natural) frequencies.

Project B for Chapter 9: *Matrix Laplace Transform Method.*

Project C for Chapter 9: *Undamped Second-Order Systems.*

Chapter 10: *Partial Differential Equations*, which includes sections on Fourier series, the heat equation, wave equation, and Laplace's equation.

Project A for Chapter 10: *Steady-State Temperature Distribution in a Circular Cylinder*.

Project B for Chapter 10: *A Laplace Transform Solution of the Wave Equation*.

Project A for Chapter 11: *Hermite Polynomials and the Harmonic Oscillator*.

Section 12.4: *Energy Methods*, which addresses both conservative and non-conservative autonomous mechanical systems.

Project A for Chapter 12: *Solitons and Korteweg-de Vries Equation*.

Project B for Chapter 12: *Burger's Equation*.

Students of engineering and physics would also find Chapter 8 on series solutions particularly useful, especially Section 8.8 on special functions.

Biology/Ecology Applications

Project C for Chapter 1: *The Phase Plane*, which discusses the logistic population model and bifurcation diagrams for population control.

Project A for Chapter 2: *Oil Spill in a Canal*.

Project B for Chapter 2: *Differential Equations in Clinical Medicine*.

Section 3.1: *Mathematical Modelling*.

Section 3.2: *Compartmental Analysis*, which contains a discussion of mixing problems and population models.

Project A for Chapter 3: *Dynamics of HIV Infection*.

Project B for Chapter 3: *Aquaculture*, which deals with a model of raising and harvesting catfish.

Section 5.1: *Interconnected Fluid Tanks*, which introduces systems of equations.

Section 5.3: *Solving Systems and Higher-Order Equations Numerically*, which contains an application to population dynamics.

Section 5.5: *Applications to Biomathematics: Epidemic and Tumor Growth Models.*

Project B for Chapter 5: *Spread of Staph Infections in Hospitals – Part I.*

Project E for Chapter 5: *Cleaning Up the Great Lakes*

Project F for Chapter 5: *A Growth Model for Phytoplankton – Part I.*

Problem 19 in Exercises 10.5 , which involves chemical diffusion through a thin layer.

Project D for Chapter 12: *Ecosystem on Planet GLIA-2*

Project E for Chapter 12: *Spread of Staph Infections in Hospitals – Part II.*

Project F for Chapter 12: *A Growth Model for Phytoplankton – Part II.*

The basic content of the remainder of this instructor's manual consists of supplemental group projects, answers to the even-numbered problems, and detailed solutions to the most even-numbered problems in Chapters 1, 2, 3, 4, and 7. These answers are not available any place else since the text and the *Student's Solutions Manual* only provide answers and solutions to odd-numbered problems.

We would appreciate any comments you may have concerning the answers in this manual. These comments can be sent to the authors' email addresses below. We also would encourage sharing with us (the authors and users of the texts) any of your favorite group projects.

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