Chapter 2

Fluid Statics

2.1. From the given data: $p_{\text{atm}} = 101\,\text{kPa}$, $h_{\text{k}} = 0.62\,\text{m}$, and $h_{\text{w}} = 2.05\,\text{m}$. For water, $\gamma_{\text{w}} = 9.79\,\text{kN·m}^3$, and for kerosene, $\rho_{\text{k}} = 808\,\text{kg/m}^3$, which gives $\gamma_{\text{k}} = 7.92\,\text{kN·m}^3$ (from Appendix B.4). The absolute pressure on the bottom of the tank, p_{b}, is calculated as follows:

$$p_{\text{b}} = p_{\text{atm}} + \gamma_{\text{k}} h_{\text{k}} + \gamma_{\text{w}} h_{\text{w}} = 101 + 7.92(0.62) + 9.79(2.05) = \boxed{125.98\,\text{kPa}}$$

As a gauge pressure, the pressure on the bottom of the tank is $125.98\,\text{kPa} - 101\,\text{kPa} = \boxed{24.98\,\text{kPa}}$.

2.2. (a) Taking $\gamma_{\text{w}} = 9.79$ kN/m^3 and $p = 101.3$ kPa, the depth, h, below the water surface corresponding to a gauge pressure of 101.3 kPa is given by

$$h = \frac{p}{\gamma_{\text{w}}} = \frac{101.3}{9.79} = \boxed{10.3\,\text{m}}$$

(b) From the given data: $\Delta h = 1.65\,\text{m}$. Therefore, the pressure difference, Δp, is given by

$$\Delta p = \gamma \Delta h = (9.79)(1.65) = \boxed{16.2\,\text{kPa}}$$

2.3. From the given data: $\text{SG}_1 = 0.98$, $z_1 = 0\,\text{m}$, $\text{SG}_2 = 1.07$, and $z_2 = -12\,\text{m}$. Since SG varies linearly,

$$\text{SG} = \text{SG}_1 + \frac{\text{SG}_2 - \text{SG}_1}{z_2 - z_1}(z - z_1) = 0.98 + \frac{1.07 - 0.98}{-12 - 0}(z - 0) = 0.98 - 0.0075z \qquad (1)$$

The relationship between specific gravity (SG) and specific weight (γ) is given by

$$\gamma\ (\text{kN/m}^3) = g \cdot \text{SG} = 9.807 \cdot \text{SG} \qquad (2)$$

Combining Equations 1 and 2 gives the following equation for the variation of specific weight with depth,

$$\gamma = 0.611 - 0.0736\,z\,\text{kN/m}^3 \qquad (3)$$

Using the hydrostatic pressure distribution (Equation 2.10) the difference in pressure between $z = -12\,\text{m}$ and $z = 0\,\text{m}$ is given by Equation 2.11 as

$$p_2 - p_1 = -\int_{z_1}^{z_2} \gamma\,\text{d}z = -\int_0^{-12} (0.611 - 0.0736\,z)\,\text{d}z = -\left[9.611\,z - 0.03678\,z^2\right]_0^{-12} = \boxed{121\,\text{kPa}}$$

This is a $\boxed{\text{gauge pressure}}$, relative to atmospheric pressure on the surface of the liquid.

2.4. (a) From the given data: $h = 12\,\text{m}$ and $p_\text{A} = 200\,\text{kPa}$. For water at 20°C, $\gamma = 9.789\,\text{kN/m}^3$. By definition of absolute pressure,

$$p_\text{A} = p_\text{atm} + \gamma h \quad \rightarrow \quad 200 = p_\text{atm} + (9.789)(12) \quad \rightarrow \quad \boxed{p_\text{atm} = 91.5\,\text{kPa}}$$

(b) From the given data: SG = 0.85, $\rho = 850\,\text{kg/m}^3$, $\gamma = 8.336\,\text{kN/m}^3$, and $h = 6\,\text{m}$. Since the pressure distribution is hydrostatic,

$$p_\text{g} = \gamma h = (8.336)(6) = \boxed{50.0\,\text{kPa}}, \qquad p_\text{A} = p_\text{atm} + p_\text{g} = 91.5 + 50.0 = \boxed{141.5\,\text{kPa}}$$

2.5. From the given data: $h = 10\,\text{m}$. For water at 20°C, $\gamma = 9.79\,\text{kg/m}^3$. For standard atmospheric pressure, $p_\text{atm} = 101.3\,\text{kPa}$. The gauge pressure, p, at the valve is given by

$$p = \gamma h = (9.79)(10) = \boxed{97.9\,\text{kPa}}$$

The absolute pressure, p_abs, at the valve location is given by

$$p_\text{abs} = p + p_\text{atm} = 97.9 + 101.3 = \boxed{199.2\,\text{kPa}}$$

2.6. From the given data: $p_0 = 14$ kPa, and $\gamma_\text{oil} = 0.80$, $\gamma_\text{w} = 0.80(9.79) = 7.83$ kN/m^3. Hence, at a depth $h = 1.5$ m below the surface of the oil, the pressure, p, is given by

$$p = p_0 + \gamma_\text{oil} h = 14 + (7.83)(1.5) = \boxed{25.7\,\text{kPa}}$$

2.7. From the given data: $p_\text{B} = 5\,\text{kPa}$, $\Delta z_1 = 0.30\,\text{m}$, and $\Delta z_2 = 0.62\,\text{m}$. For water at 20°C, $\gamma = 9.789\,\text{kN/m}^3$. The air pressures in tanks A and C are as follows:

$$p_\text{A} = p_\text{A} + \gamma \Delta z_1 = 5 + (9.789)(0.30) = \boxed{7.94\,\text{kPa}}$$

$$p_\text{C} = p_\text{A} - \gamma \Delta z_2 = 7.94 - (9.789)(0.62) = \boxed{1.87\,\text{kPa}}$$

2.8. From the given data: $D_1 = 5\,\text{mm}$, $h = 30\,\text{m}$, and $T = 15$°C. For water at 15°C, $\rho = 999.1\,\text{kg/m}^3$ and $\gamma = 9798\,\text{N/m}^3$. Under standard conditions, $p_\text{atm} = 101.3\,\text{kPa}$. The initial volume of the bubble, V_1, the initial pressure, p_1, and the final pressure, p_2, are given by

$$V_1 = \frac{\pi D_1^3}{6} = \frac{\pi (0.005)^3}{6} = 6.545 \times 10^{-8}\,\text{m}^3$$

$$p_1 = p_\text{atm} + \gamma h = 101.3 \times 10^3 + (9798)(30) = 3.953 \times 10^5\,\text{Pa}$$

$$p_2 = p_\text{atm} = 1.013 \times 10^5\,\text{Pa}$$

Applying the ideal gas law to the air in the bubble and assuming isothermal conditions gives

$$p_1 V_1 = p_2 V_2 \quad \rightarrow \quad V_2 = \left(\frac{p_1}{p_2}\right) V_1 = \left(\frac{3.953 \times 10^5}{1.013 \times 10^5}\right)(6.545 \times 10^{-8}) = 2.554 \times 10^{-7}\,\text{m}^3$$

Therefore, the diameter, D_2, at the surface is given by

$$D_2 = \left[\frac{6V_2}{\pi}\right]^{\frac{1}{3}} = \left[\frac{6(2.554 \times 10^{-7})}{\pi}\right]^{\frac{1}{3}} = 0.00787\,\text{m} \approx \boxed{7.9\,\text{mm}}$$

2.9. From the given data: $\Delta z = 20\,\text{m}$, $p_{\text{atm}} = 101.3\,\text{kPa}$, and $T = 20°\text{C}$. At 20°C, the density of seawater is given by Appendix B.4 as $\rho = 1023\,\text{kg/m}^3$, which corresponds to $\gamma = 10.03\,\text{kN/m}^3$. Since the pressure distribution in the ocean is hydrostatic and the pressure of the air inside the bubble is equal to the pressure of the water outside the bubble, and the temperature is constant, the ratio of densities is given by

$$\frac{\rho_1}{\rho_2} = \frac{p_1}{p_2} = \frac{p_{\text{atm}} + \gamma\Delta z}{p_{\text{atm}}} = \frac{101.3 + (10.03)(20)}{101.3} = \boxed{2.98}$$

2.10. From the given data: $h_1 = 7$ m, $\gamma_1 = 9$ kN/m^3, $h_2 = 2.3$ m, and $p_{\text{bot}} = 92$ kPa. For water at 4°C, $\gamma_{\text{w}} = 9.81$ kN/m^3. The specific gravity, SG, can be derived from the following hydrostatic pressure relationship,

$$p_{\text{bot}} = \gamma_1 h_1 + [\text{SG} \cdot \gamma_{\text{w}}]h_2 \rightarrow 92 = (9)(7) + [\text{SG} \cdot (9.81)](2.3) \quad \rightarrow \quad \text{SG} = \boxed{1.3}$$

$\boxed{\text{Yes}}$ the liquid on the bottom must necessarily be denser than the liquid on the top.

2.11. For water at 20°C, Table 1.9 gives $\gamma = 9.79$ kN/m^3. The pressure head, h, corresponding to $p = 450$ kPa is therefore

$$h = \frac{p}{\gamma} = \frac{450}{9.79} = \boxed{46.0\,\text{m}}$$

2.12. For $p = 800$ kPa, the pressure head, h, is given by

$$h = \frac{p}{\gamma_{\text{w}}} = \frac{800}{9.79} = \boxed{81.7\,\text{m (of water)}}$$

For crude oil at 20°C, $\rho_{\text{oil}} = 856$ kg/m^3 (from Appendix B.4), which gives $\gamma_{\text{oil}} = 8.40$ kN/m^3. For $p = 800$ kPa,

$$h = \frac{p}{\gamma_{\text{oil}}} = \frac{800}{8.40} = \boxed{95.2\,\text{m (of crude oil)}}$$

2.13. Pressure, p_1, corresponding to $h_{\text{w}} = 80$ mm of water is

$$p_1 = \gamma_{\text{w}} h_{\text{w}} = (9.79)(0.080) = 0.783\,\text{kPa}$$

and the pressure, p_2, corresponding to $h_{\text{f}} = 60$ mm of a fluid whose specific weight is $\gamma_{\text{f}} = 2.90\gamma_{\text{w}} = 2.90(9.79) = 28.4$ kN/m^3 is

$$p_2 = \gamma_{\text{f}} h_{\text{f}} = (28.4)(0.060) = 1.70\,\text{kPa}$$

The total pressure, p, is therefore given by

$$p = p_1 + p_2 = 0.783 + 1.70 = 2.48\,\text{kPa}$$

and the pressure head, h_{Hg}, in mm of mercury (taking $\gamma_{\text{Hg}} = 133$ kN/m^3) is

$$h_{\text{Hg}} = \frac{p}{\gamma_{\text{Hg}}} = \frac{2.48}{133} \times 1000 = \boxed{18.6\text{ mm Hg}}$$

2.14. For $p_{\text{atm}} = 101.3$ kPa, the pressure head, h_{Hg}, in mm of mercury ($\gamma_{\text{Hg}} = 133$ kN/m^3) is given by

$$h_{\text{Hg}} = \frac{p_{\text{atm}}}{\gamma_{\text{Hg}}} = \frac{101.3}{133} \times 1000 = \boxed{762 \text{ mm Hg}}$$

2.15. From the given data: $D = 7$ mm, and $h' = 80$ mm. For water at 20°C, $\sigma = 72.8$ mN/m = 0.0728 N/m, and $\gamma = 9789$ N/m^3 (from Appendix B.1). For water and clean glass, $\theta = 0°$.

(a) The rise height, Δh, due to surface tension is calculated as

$$\Delta h = \frac{4\sigma\cos\theta}{\gamma D} = \frac{4(0.0728)\cos 0°}{(9789)(0.007)} = 4.45 \times 10^{-3}\,\text{m} = \boxed{4.45\,\text{mm}}$$

(b) In accordance with Equation 2.20, the pressure head, h, at the attachment point is given by

$$h = h' - \Delta h = 80 - 4.45 = 75.75\,\text{mm} \approx \boxed{75.8\,\text{mm}}$$

2.16. When the reservoir is half-full, the pipeline pressure is 350 kPa, and the height, h_0, of the mid-point of the reservoir above the pipeline is

$$h_0 = \frac{350}{\gamma_{\text{w}}} = \frac{350}{9.79} = \boxed{35.8\,\text{m}}$$

Note that the pressures of liquids in pipes are generally given as gauge pressures unless stated otherwise. When the pressure in the pipeline is 500 kPa, the height, h_1, of the water in the reservoir above the pipeline is

$$h_1 = \frac{500}{\gamma_{\text{w}}} = \frac{500}{9.79} = 51.1\,\text{m}$$

Hence the minimum space between the mid-point and top of the reservoir is 51.1 m − 35.8 m = $\boxed{15.3 \text{ m}}$.

2.17. From the given data: $x = 120$ mm Hg, $y = 70$ mm Hg, $\Delta z_{\text{head}} = 0.5$ m, $\Delta z_{\text{toe}} = 1.5$ m, and $\rho = 1060$ kg/m^3. From the given density, $\gamma = 10.40$ kN/m^3.

(a) The following pressure differences can be calculated:

$$\text{heart-head} = \gamma \cdot \Delta z_{\text{head}} = (10.40)(0.5) = 5.20\,\text{kPa} = 39 \text{ mm Hg}$$
$$\text{heart-toe} = \gamma \cdot \Delta z_{\text{toe}} = (10.40)(1.5) = 15.6\,\text{kPa} = 117 \text{ mm Hg}$$

The blood pressures in the head and toes are:

$$\text{head} = \frac{120 - 39}{70 - 39} = \boxed{81/31}$$

$$\text{toes} = \frac{120 + 117}{70 + 117} = \boxed{237/187}$$

(b) The maximum pressure is $p = 120$ mm Hg = 16.0 kPa. Therefore, the height, h, that blood would rise in the tube is given by

$$h = \frac{p}{\gamma} = \frac{16.0}{10.40} = \boxed{1.54\,\text{m}}$$

2.18. From the given data: p = 150 mm Hg = 20.00 kPa, and ρ = 1025 kg/m^3. Taking g = 9.807 m/s^2, the height h between arm level and fluid level is given by

$$h = \frac{p}{\rho g} = \frac{20.00 \times 10^3}{(1025)(9.807)} = \boxed{1.99\,\text{m}}$$

2.19. From the given data: Δz = 6 m, and ρ = 1060 kg/m^3. The specific weight of the blood is γ = 10.4 kN/m^3.

(a) When the giraffe drinks, the change in pressure in the head, Δp, is given by

$$\Delta p = \gamma \cdot \Delta z = (10.4)(6) = 67.6\,\text{kPa} = \boxed{507 \text{ mm Hg}}$$

(b) The difference in pressure between the head and the heart is 507 mm/2 = 254 mm. Since the maximum pressure at the heart level is given as 280 mm, then the maximum pressure in the head is 280 mm + 254 mm = $\boxed{534 \text{ mm Hg}}$.

2.20. From the given data: p_{air} = 300 kPa, A_1 = 7 cm^2 = 0.0007 m^2, W_1 = 50 N = 0.05 kN, A_2 = 500 cm^2 = 0.05 m^2, W_2 = 800 N = 0.8 kN, Δz = 1 m, Δs_1 = 10 cm, ρ = 900 kg/m^3, and $\gamma = \rho g$ = 8.83 kN/m^3.

(a) The force, F, exerted by the compressed air on the piston is given by

$$F = p_{\text{air}} A_1 = (300)(0.0007) = \boxed{0.21\,\text{kN}}$$

(b) Let W be the weight mounted on the platform, then

$$\frac{F + W_1}{A_1} - \gamma \Delta z = \frac{W + W_2}{A_2} \rightarrow \frac{0.21 + 0.05}{0.0007} - (8.83)(1) = \frac{W + 0.8}{0.05} \rightarrow W = \boxed{17.3\,\text{kN}}$$

(c) If Δs_2 is the displacement of the platform, then

$$A_1 \Delta s_1 = A_2 \Delta s_2 \quad \rightarrow \quad (7)(10) = (500)\Delta s_2 \quad \rightarrow \quad \Delta s_2 = \boxed{0.14\,\text{cm}}$$

2.21. From the given data: F_1 = 500 N, D_1 = 25 mm, and D_2 = 100 mm. If the force on the 100-mm piston is F_2, and noting that performance of the hydraulic system will not be compromised if both pistons exert the same pressure, then

$$\frac{F_1}{D_1^2} = \frac{F_2}{D_2^2} \quad \rightarrow \quad \frac{500}{25^2} = \frac{F_2}{100^2} \quad \rightarrow \quad F_2 = \boxed{8000\,\text{N}}$$

2.22. From the given data: z = 4342 m = 4.342 km. For the standard atmosphere, T_0 = 15°C = 288.2 K, b = 6.5°C/km, p_0 = 101.3 kPa, and g/Rb = 5.26. The standard-atmosphere temperature, T, at the summit is calculated using Equation 2.25 as

$$T = T_0 - bz = 15 - (6.5)(4.342) = \boxed{-13.2^\circ\text{C}} = 259.9\,\text{K}$$

The standard-atmosphere pressure, p, at the summit is calculated using Equation 2.26 as

$$p = p_0 \left(\frac{T}{T_0}\right)^{\frac{g}{Rb}} = 101.3 \left(\frac{259.9}{288.2}\right)^{5.26} = \boxed{58.8\,\text{kPa}}$$

The calculated standard-atmosphere temperature and pressure are fairly close to the measured values of −11°C and 58 kPa.

2.23. From the given data: $z_1 = 11$ km, $z_2 = 20$ km, $T_0 = -56.5$°C = 216.7 K, and $p_1 = 22.63$ kPa. The average value of g is $\bar{g} = 9.769$ m/s^2. For air, $R = 287.1$ J/kg·K. Using Equation 2.29, the theoretical pressure, p_2, at the top of the stratosphere is given by

$$p_2 = p_1 \exp\left[-\frac{\bar{g}(z_2 - z_1)}{RT_0}\right] = (22.63)\exp\left[-\frac{(9.769)(20000 - 11000)}{(287.1)(216.7)}\right] = \boxed{5.51\,\text{kPa}}$$

The standard-atmosphere pressure at $z = 20$ km (from Appendix B.3) is $\boxed{5.529 \text{ kPa}}$, so the theoretical and standard values are very close.

2.24. From the given data: $b = 6.5$°C/km, $p_0 = 101.325$ kPa, and $T_0 = 15$°C = 288.15 K. For air, $R = 287.1$ J/kg·K, which gives $g/Rb = 5.255$. Assuming a uniform lapse rate and a hydrostatic pressure distribution, the temperature, T, and pressure, p, at any elevation are given by

$$T = T_0 - bz, \qquad p = p_0\left(\frac{T}{T_0}\right)^{\frac{g}{Rb}}$$

The results of applying these equations and comparing the predictions to the standard atmosphere is given in the following table.

z (km)	T (°C)	T (K)	p (kPa)	T_{std} (°C)	p_{std} (kPa)	ΔT (°C)	Δp (kPa)
0	15.0	288.15	101.325	15.00	101.325	0.00	0.000
1	8.5	281.65	89.876	8.50	89.876	0.00	0.000
2	2.0	275.15	79.498	2.00	79.501	0.00	−0.003
3	−4.5	268.65	70.112	−4.49	70.121	−0.01	−0.009
4	−11.0	262.15	61.644	−10.98	61.660	−0.02	−0.016
5	−17.5	255.65	54.024	−17.47	54.048	−0.03	−0.024
6	−24.0	249.15	47.186	−23.96	47.217	−0.04	−0.031
7	−30.5	242.65	41.065	−30.45	41.110	−0.05	−0.045
8	−37.0	236.15	35.605	−36.94	35.651	−0.06	−0.046
9	−43.5	229.65	30.747	−43.42	30.800	−0.08	−0.053
10	−50.0	223.15	26.441	−49.90	26.499	−0.10	−0.058
11	−56.5	216.65	22.636	−56.50	22.632	0.00	0.004

Bases on the results presented in the the above table, the maximum temperature difference is $\boxed{-0.10°\text{C}}$, and the maximum pressure difference is $\boxed{-0.058\,\text{kPa}}$.

2.25. Taking the pressure distribution in the atmosphere as hydrostatic,

$$\frac{\text{d}p}{\text{d}z} = -\rho g; \quad \rho = \frac{p}{RT}; \quad T = a + bz$$

Using these equations:

$$\frac{\text{d}p}{p} = -\frac{g}{RT}dz = -\frac{g}{R(a+bz)}\text{d}z$$

$$\int_1^2 \frac{\mathrm{d}p}{p} = -\frac{g}{R}\int_1^2 \frac{\mathrm{d}z}{a+bz}$$

$$\frac{p_2}{p_1} = \left(\frac{a+bz_2}{a+bz_1}\right)^{-g/Rb} \tag{1}$$

From the given data: p_1 = 101 kPa, p_2 = 1 Pa, z_1 = 0 m, a = 273 + 20 = 293 K, $b = -6.3$ K/km = -0.0063 K/m, R = 287 J/kg·K (for air), which yields

$$-\frac{g}{Rb} = \frac{9.81}{(287)(0.0063)} = 5.426$$

Substituting into Equation 2.23 gives

$$\frac{0.001}{101} = \left(\frac{293 - 0.0063z_2}{293}\right)^{5.426}$$

which yields z_2 = 52,070 m = $\boxed{\text{52.1 km}}$.

2.26. From the given data: z = 2256 m, T = 5°C = 278 K, T_0 = 27°C = 300 K, and p_0 = 101 kPa. The lapse rate, b, can be estimated as

$$b = \frac{T_0 - T}{z} = \frac{300 - 278}{2256} = 0.00975 \text{ K/m} = 9.75 \text{ K/km}$$

For the standard atmosphere, b = 6.50 K/km, g/Rb = 5.26, and so for b = 9.75 K/km it is estimated that

$$\frac{g}{Rb} = 5.26 \times \frac{6.50}{9.75} = 3.51$$

(a) The pressure, p, at the Peak can be calculated using Equation 2.26 which gives

$$p = p_0\left(\frac{T}{T_0}\right)^{\frac{g}{Rb}} = (101)\left(\frac{278}{300}\right)^{3.51} = \boxed{77.3\,\text{kPa}}$$

(b) The vapor pressure of water is equal to 77.3 kPa when the temperature of the water is 92°C (from Appendix B.1). Therefore, water boils at $\boxed{92°\text{C}}$ at the Peak.

2.27. For the standard atmosphere, b = 6.50 K/km = 0.00650 K/m. For air, M = 28.96 g/mol = 0.02896 kg/mol. Constants are R = 8.314 J/mol·K and g = 9.81 m/s^2. Therefore,

$$\frac{gM}{Rb} = \frac{(9.81)(0.02896)}{(8.314)(0.00650)} = \boxed{5.26}$$

Under standard atmospheric conditions, p_0 = 101.3 kPa and T_0 = 15°C = 288.15 K. In La Paz, z = 3640 m and estimated atmospheric conditions are as follows:

$$T = T_0 - bz = 288.15 - (0.00650)(3640) = 264.5\ K (= -8.66°\text{C})$$

$$p = p_0\left(\frac{T}{T_0}\right)^{\frac{gM}{Rb}} = (101.3)\left(\frac{264.5}{288.15}\right)^{5.26} = 64.56\,\text{kPa}$$

The temperature of water at which the saturation vapor pressure is 64.56 kPa is the temperature at which water boils and is equal to approximately $\boxed{87.6°\text{C}}$.

2.28. From the given data: $\Delta z = 3000\,\text{m}$. For standard air, $R = 287.1\,\text{J/kg·K}$, and for a standard atmosphere at sea level, $p_1 = 101.325\,\text{kPa}$ and $T_1 = 15°\text{C} = 288.15\,\text{K}$. Assume that the temperature remains constant at 15°C over the depth of the shaft. Using Equation 2.29 gives

$$p_2 = p_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right] \quad \rightarrow \quad p_2 = (101.325)\exp\left[-\frac{(9.807)(-3000)}{(287.1)(288.15)}\right] = \boxed{145\,\text{kPa}}$$

2.29. From the given data: $p_0 = 755$ mm, $z = 829.8$ m $= 0.8298$ km, and $T_0 = 35.5°\text{C} = 308.7$ K. Assuming standard atmospheric conditions, $b = 6.5°\text{C/km}$, and $g/Rb = 5.26$. The estimated temperature, T, at the top of the building is calculated using Equation 2.25 as

$$T = T_0 - bz = 35.5 - (6.5)(0.8298) = 30.10°\text{C} = 303.3\,\text{K}$$

The barometric pressure, p, at the top of the building can be estimated using Equation 2.26 as

$$p = p_0\left(\frac{T}{T_0}\right)^{\frac{g}{Rb}} = 755\left(\frac{303.3}{308.7}\right)^{5.26} = \boxed{688\text{ mm Hg}}$$

2.30. From the given data: $p_1 = 750$ mm, and $p_2 = 690$ mm. For a standard atmosphere: $p_0 = 760$ mm, $T_0 = 15°\text{C} = 288.15$ K, $b = 6.5$ K/km, and $g/Rb = 5.26$. Using Equation 2.26,

$$p_1 = p_0\left[1 - \frac{bz_1}{T_0}\right]^{\frac{g}{Rb}} \quad \rightarrow \quad 750 = (760)\left[1 - \frac{(6.5)z_1}{288.15}\right]^{5.26} \rightarrow z_1 = 0.111\,\text{km}$$

$$p_2 = p_0\left[1 - \frac{bz_2}{T_0}\right]^{\frac{g}{Rb}} \quad \rightarrow \quad 690 = (760)\left[1 - \frac{(6.5)z_1}{288.15}\right]^{5.26} \rightarrow z_2 = 0.807\,\text{km}$$

Therefore the change in elevation is estimated as $0.807\,\text{km} - 0.111\,\text{km} = 0.696\,\text{km} = \boxed{696\text{ m}}$

2.31. From the given data: $h_{\text{air}} = 0.3$ m, $h_{\text{g1}} = 1.2$ m, $h_{\text{g2}} = 0.8$ m., $h_{\text{g3}} = 1.9$ m, and $p_{\text{atm}} = 101$ kPa. For gasoline at 20°C, $\rho_{\text{g}} = 680\text{ kg/m}^3$, which gives $\gamma_{\text{g}} = 6.67\text{ kN/m}^3$. If p_0 is the pressure at the Bourdon gauge, then

$$p_0 + \gamma_{\text{g}}h_{\text{g1}} - \gamma_{\text{g}}h_{\text{g3}} = 0 \quad \rightarrow \quad p_0 + 6.67(1.2 - 1.9) = 0 \quad \rightarrow \quad p_0 = \boxed{4.67\,\text{kPa}}$$

Note that the Bourdon gauge reads gauge pressure, and the variation of hydrostatic pressure in the air is negligible.

2.32. From the given data: $\text{SG}_1 = 0.9$, $\Delta z_1 = 0.25\,\text{m}$, $\text{SG}_2 = 2.5$, and $\Delta z_2 = 0.25\,\text{m}$. The specific weights corresponding to the given specific gravities are determined by the relation

$$\gamma = \text{SG}\cdot\rho_0 g = \text{SG}\cdot(1000)(9.807) = 9807\cdot\text{SG}\,\text{N/m}^3 = 9.807\cdot\text{SG}\,\text{kN/m}^3$$

Using this relation, the specific weights of the light and dense fluids are

$$\gamma_1 = 9.807(0.9) = 8.826\,\text{kN/m}^3, \qquad \gamma_2 = 9.807(2.5) = 24.52\,\text{kN/m}^3$$

(a) Assuming that both the top of the light fluid and the air above the liquid are at the same atmospheric pressure, then

$$\cancel{p_{\text{atm}}} + \gamma_1\Delta z_1 - \gamma_2\Delta z = \cancel{p_{\text{atm}}} \quad \rightarrow \quad \Delta z = \frac{\gamma_1}{\gamma_2}\Delta z_1 = \frac{8.826}{24.52}(0.25) = \boxed{0.090\,\text{m}}$$

(b) Since the pressure distribution is hydrostatic, the gauge pressure on the bottom of the tank, p_0, is given by

$$p_0 = \gamma_1 \Delta z_1 + \gamma_2 \Delta z_2 = (8.826)(0.25) + (24.52)(0.25) = \boxed{8.34\,\text{kPa}}$$

2.33. From Figure 2.50,

$$p_\text{A} = p_\text{B} - \gamma_\text{f}(0.10) - \gamma_\text{w}(0.15)$$

where $p_\text{B} = 0$ kPa (gauge pressure), $\gamma_\text{f} = 40$ kN/m^3, and $\gamma_\text{w} = 9.79$ kN/m^3. Hence,

$$p_\text{A} = 0 - 40(0.10) - 9.79(0.15) = \boxed{-5.47\,\text{kPa}}$$

Alternative solution:
In terms of absolute pressure, $p_\text{B} = 101.33$ kPa, $\gamma_\text{f} = 40$ kN/m^3, and $\gamma_\text{w} = 9.79$ kN/m^3. Hence,

$$p_\text{A} = 101.33 - 40(0.10) - 9.79(0.15) = \boxed{95.86\,\text{kPa}}$$

It should be noted that the pressure of liquids in pipes is seldom given in terms of absolute pressure, so $p_\text{A} = -5.47$ kPa is the preferred answer.

2.34. For SAE 30 oil and mercury at 20°C: $\rho_\text{oil} = 918$ kg/m^3, and $\rho_\text{Hg} = 13550$ kg/m^3 (from Appendix B.4). These values correspond to: $\gamma_\text{oil} = 9.00$ kN/m^3, and $\gamma_\text{Hg} = 133$ kN/m^3. Applying the hydrostatic pressure equation gives

$$p_\text{air} + \gamma_\text{oil} h_\text{oil} - \gamma_\text{Hg} h_\text{Hg} = p_\text{atm}$$

$$p_\text{air} + (9.00)(1) - (133)(0.25) = p_\text{atm} \quad \rightarrow \quad p_\text{air} - p_\text{atm} = 24.3\,\text{kPa}$$

2.35. From Figure 2.52,

$$p_\text{A} = p_\text{B} + \gamma_\text{w} h_3 - \gamma_\text{f} h_2 - \gamma_\text{w} h_1$$

which simplifies to

$$\boxed{p_\text{A} - p_\text{B} = \gamma_\text{w}(h_3 - h_1) - \gamma_\text{f} h_2}$$

2.36. From the given data: $\gamma_\text{w} = 9.79$ kN/m^3, $\gamma_\text{g} = 18.3$ kN/m^3, $h_1 = 0.5$ m, and $h_2 = 0.3$ m. Applying the hydrostatic pressure relation between points 1 and 2 gives

$$p_1 - \gamma_\text{w} h_1 - \gamma_\text{g} h_2 + \gamma_\text{w}(h_1 + h_2) = p_2$$

$$p_1 - 9.79(0.5) - 18.3(0.3) + 9.79(0.5 + 0.3) = p_2 \quad \rightarrow \quad p_1 - p_2 = \boxed{2.55\,\text{kPa}}$$

2.37. For equilibrium,

$$p_\text{w} + \gamma_\text{w}(0.15) - \gamma_1(0.10) - \gamma_2(0.20) + \gamma_3(0.15) = p_0$$

Taking $\gamma_\text{w} = 9.79$ kN/m^3:

$$\begin{aligned} p_0 - p_\text{w} &= \gamma_\text{w}[0.15 - \text{SG}_1(0.10) - \text{SG}_2(0.20) + \text{SG}_3(0.15)] \\ &= (9.79)[0.15 - (13.6)(0.10) - (0.68)(0.20) + (0.86)(0.15)] = -11.9\,\text{kPa} \end{aligned}$$

So the pressure difference is $\boxed{11.9\text{ kPa}}$.

2.38. From the given data: $D_1 = 1\,\text{m}$, $D_2 = 10\,\text{mm}$, $\Delta p = 200\,\text{Pa}$, and $\Delta s = 200\,\text{mm}$. For SAE 30 oil at 20°C, $\rho = 918\,\text{kg/m}^3$ and $\gamma = \rho g = 9003\,\text{N/m}^3$ (from Appendix B.4). The following preliminary calculations of the cross-sectional area, A_1, of the tank and the cross-sectional area, A_2, of the manometer are useful,

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi 1^2}{4} = 0.7854\,\text{m}^2, \qquad A_2 = \frac{\pi D_2^2}{4} = \frac{\pi (0.2)^2}{4} = 7.854\times10^{-5}\,\text{m}^2$$

Let Δh be the change in oil level in the reservoir corresponding to Δp, and let p_0 be atmospheric pressure, then the continuity and hydrostatic-pressure relationships require that

$$\Delta h A_1 = \Delta s A_2 \quad\rightarrow\quad \Delta h = \frac{A_2}{A_1}\Delta s \tag{1}$$

$$\cancel{p_0} + \Delta p - \gamma\Delta h - \gamma\Delta s\sin\theta = \cancel{p_0} \tag{2}$$

Combining Equations 1 and 2 to eliminate Δh gives

$$\sin\theta = \left[\frac{\Delta p}{\gamma\Delta s} - \frac{A_2}{A_1}\right] = \left[\frac{200}{(9003)(0.200)} - \frac{7.854\times10^{-5}}{0.7854}\right] = 0.1110 \quad\rightarrow\quad \theta = \boxed{6.37^\circ}$$

2.39. For a water temperature of 15°C, $\gamma_\text{w} = 9.80\ \text{kN/m}^3$. For the given manometer setup,

$$p_\text{w} = p_0 + \gamma_\text{w} h_2 - \text{SG}\gamma_\text{w} L_1 \sin\theta - \gamma_\text{w} L_2 \sin\theta$$

Noting that sin $\theta = 8/12 = 0.667$, the above equation gives

$$p_\text{w} = 30 + (9.80)(0.50) - (2.4)(9.80)(0.06)(0.667) - (9.80)(0.06)(0.667) = 33.6\,\text{kPa}$$

Therefore the water pressure in the pipe is $\boxed{33.6\ \text{kPa}}$.

NEW From the given data: $I_{xx} = 8.553\,\text{m}^4$, and $\theta = 70^\circ$. The given dimensions are shown in Figure 2.1, where the inclined distance from the water surface to the top of the plane surface is $1.5\sin 70^\circ = 1.596\,\text{m}$.

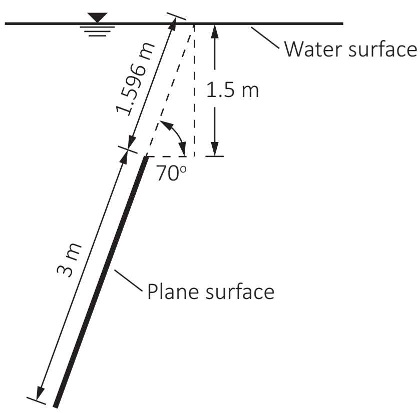


Figure 2.1: Side view of inclined surface

For water at 20°C, $\gamma = 9.789\,\text{kN/m}^3$. From the given dimensions of the plane surface, the following geometric properties can be calculated:

$$A_1 = (3)(2) = 6\,\text{m}^2, \qquad A_2 = (5)(1) = 5\,\text{m}^2$$

$$\bar{y}_1 = 1.596 + \frac{2}{2} = 2.596\,\text{m}, \qquad \bar{y}_2 = 1.596 + 2 + \frac{1}{2} = 4.096\,\text{m}$$

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2} = 3.278\,\text{m}, \qquad A = 11\,\text{m}^2$$

Using the calculated data, the resultant force, F, and location, y_cp, are given by

$$F = \gamma A\bar{y}\sin\theta = (9.789)(11)(3.278)\sin 70^\circ = \boxed{332\,\text{kN}}$$

$$y_\text{cp} = \bar{y} + \frac{I_{xx}}{A\bar{y}} = 3.278 + \frac{8.553}{(11)(3.278)} = \boxed{3.52\,\text{m}}$$

2.40. From the given data: b = 3 m, d = 4 m, W = 20 kN, h = 2 m, and μ = 0.05. For water, γ = 9.79 kN/m^3. The geometric properties of the gate are:

$$\bar{y} = h + \frac{d}{2} = 2 + \frac{4}{2} = 4\,\text{m}, \qquad A = bd = (3)(4) = 12\,\text{m}^2$$

The hydrostatic force, F, on the gate is given by

$$F = \gamma A\bar{y} = (9.79)(12)(4) = 469.9\,\text{kN}$$

The frictional force, F_f, and the total force, F_lift, required to lift the gate are given by

$$F_\text{f} = \mu F = (0.05)(469.9) = 23.50\,\text{kN}$$

$$F_\text{lift} = F_\text{f} + W = 23.50 + 20 = \boxed{43.5\,\text{kN}}$$

2.41. From the given data: $h = 4$ m, $L = 3.5$ m, $w = 0.3$ m, and SG = 2.5. The following preliminary calculations are useful:

$$A = Lh = (3.5)(4) = 14\,\text{m}^2, \qquad \gamma = \text{SG}\cdot(9.807) = 24.52\,\text{kN/m}^3$$

$$\bar{y} = \frac{h}{2} = \frac{4}{2} = 2\,\text{m}, \qquad I_\text{cc} = \frac{Lh^3}{12} = \frac{(4)(3.5)^3}{12} = 18.67\,\text{m}^4$$

Using the given and derived data, the support force, F, and location, y_cp, are calculated as follows,

$$F = \gamma A\bar{y} = (24.52)(14)(2) = 687\,\text{kN}, \qquad y_\text{cp} = \bar{y} + \frac{I_\text{cc}}{A\bar{y}} = 2 + \frac{18.67}{(14)(2)} = 2.67\,\text{m}$$

Therefore, the magnitude of the required support force on each side of the form is $\boxed{687\text{ kN}}$. This support should be located $4\,\text{m} - 2.67\,\text{m} = \boxed{1.33\text{ m}}$ from the bottom of the wall section. The lateral location is $L/2 = 3.5/2 = \boxed{1.75\text{ m}}$ from the edge of the wall section.

2.42. From the given data: $b = 2$ m, $d = 3$ m, $\theta = 60°$, $h_{\rm top} = 2.5$ m. For water, $\gamma = 9.79$ kN/m^3. The geometric properties of the gate are calculated as follows:

$$A = bd = (2)(3) = 6\,\text{m}^2, \qquad y_{\rm top} = \frac{h_{\rm top}}{\sin\theta} = \frac{2.5}{\sin 60°} = 2.887\,\text{m}$$

$$\bar{y} = y_{\rm top} + \frac{d}{2} = 2.887 + \frac{3}{2} = 4.387\,\text{m}, \qquad I_{xx} = \frac{bd^3}{12} = \frac{(2)(3)^3}{12} = 4.500\ \text{m}^4$$

The resultant force, F, and the center of pressure, $y_{\rm cp}$, are given by

$$F = \gamma A\bar{y}\sin\theta = (9.79)(6)(4.387)\sin 60° = \boxed{223\,\text{kN}}$$

$$y_{\rm cp} = \bar{y} + \frac{I_{xx}}{A\bar{y}} = 4.387 + \frac{4.500}{(6)(4.387)} = 4.558\,\text{m}$$

The center of pressure is $4.558\sin 60° = \boxed{3.95\ \text{m}}$ below the water surface.

2.43. Force, F, on gate given by

$$F = \gamma A\bar{y}$$

where $\gamma = 9.79$ kN/m^3, $A = \pi D^2/4 = \pi(2)^2/4 = 3.142$ m^2, and $\bar{y} = 4$ m. Therefore

$$F = (9.79)(3.142)(4) = \boxed{123\,\text{kN}}$$

The location of F is given by $y_{\rm cp}$, where

$$y_{\rm cp} = \bar{y} + \frac{I_{\rm cc}}{A\bar{y}}$$

For a circle

$$I_{\rm cc} = \frac{\pi D^4}{64} = \frac{\pi(2)^4}{64} = 0.785\ \text{m}^4$$

therefore,

$$y_{\rm cp} = 4 + \frac{0.785}{(3.142)(4)} = \boxed{4.06\,\text{m}}$$

Moment of hydrostatic force about A, $M_{\rm A}$, is the minimum moment needed to open the gate,

$$M_{\rm A} = F(y_{\rm cp} - 3) = 123(4.06 - 3) = \boxed{130\ \text{kN·m}}$$

2.45. From the given data: $H = 3\,\text{m}$, $T = 1\,\text{m}$, $\rho_{\rm c} = 2800\,\text{kg/m}^3$, $\rho_{\rm s} = 1500\,\text{kg/m}^3$, and $\mu = 0.35$. Considering a unit length of slurry wall (perpendicular to the page), the following preliminary calculations are useful,

$$\gamma_{\rm c} = \rho_{\rm c} g = 27.46\,\text{kN/m}^3, \qquad \gamma_{\rm s} = \rho_{\rm s} g = 14.71\,\text{kN/m}^3$$

$$W = \gamma_{\rm c} V_{\rm c} = (27.46)(3\times 1) = 82.38\,\text{kN}, \qquad F_{\rm f} = \mu W = (0.35)(82.38) = 28.83\,\text{kN}$$

$$F_{\rm h} = \gamma_{\rm s} A\bar{y} = (14.71)(h)\left(\frac{h}{2}\right) = 7.355h^2, \qquad y_{\rm cp} = \bar{y} + \frac{I_{\rm cc}}{A\bar{y}} = \frac{h}{2} + \frac{h^3/12}{h\cdot h/2} = \frac{2}{3}h$$

where W = weight of retaining wall, $F_{\rm f}$ = friction force, and $F_{\rm h}$ = horizontal hydrostatic force.

(a) For shear failure, the horizontal hydrostatic force is equal to the friction force, which requires that

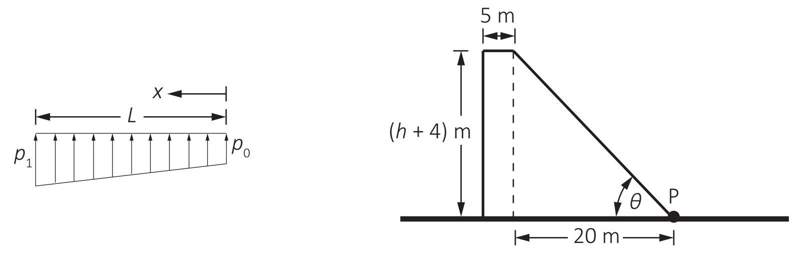
$$F_\text{h} = F_\text{f} \quad \rightarrow \quad 7.355h^2 = 28.83\,\text{kN} \quad \rightarrow \quad \boxed{h = 1.98\,\text{m}}$$

(b) For overturning about the point P, the ground reaction is equal to zero and

$$W \cdot \frac{T}{2} = F_\text{h} \cdot (h - y_\text{cp}) \quad \rightarrow \quad (82.38)(0.5) = (7.355h^2)\left(h - \tfrac{2}{3}h\right) \quad \rightarrow \quad \boxed{h = 2.56\,\text{m}}$$

The more likely failure mode is by $\boxed{\text{shear failure}}$, since this failure will occur at with a lower slurry depth (1.98 m vs. 2.56 m).

2.46. From the given data: $L = 25\,\text{m}$, $T = 5\,\text{m}$, $s = 4\,\text{m}$, $\text{SG}_\text{c} = 2.4$, and $y_2 = 3\,\text{m}$. From the given specific gravity of concrete, the specific weight of concrete is $\gamma_\text{c} = \text{SG} \cdot g = 23.53\,\text{kN/m}^3$. For water at 20°C, $\gamma_\text{w} = 9.789\,\text{kN/m}^3$. The illustrations given in Figure 2.2 are useful in the calculations.



(a) Pressure distribution (b) Geometry of dam

Figure 2.2: Definition diagrams for calculations

The slope of the downstream side of the dam is measured by θ, which can be expressed in terms of the upstream height, h using the relation

$$\sin\theta = \frac{h+4}{\sqrt{(h+4)^2 + 20^2}} \tag{1}$$

Using the subscript "1" to designate upstream and the subscript "2" to designate downstream, and taking a unit length if the dam (perpendicular to the page), the following preliminary calculations are useful:

$$A_1 = (1)h = h, \qquad A_2 = (1)\left(\frac{3}{\sin\theta}\right) = \frac{3}{\sin\theta}$$

$$\bar{y}_1 = \frac{h}{2}, \qquad \bar{y}_2 = \frac{1.5}{\sin\theta}$$

$$I_{100} = \frac{(1)(h)^3}{12} = \frac{h^3}{12}, \qquad I_{200} = \frac{(1)(3/\sin\theta)^3}{12} = \frac{2.25}{\sin^3\theta}$$

The centers of pressure are calculated as follows:

$$y_{1\text{cp}} = \bar{y}_1 + \frac{I_{100}}{A_1\bar{y}_1} = \frac{h}{2} + \frac{h^3/12}{(h)(h/2)} = \frac{2h}{3}$$

$$y_{2\text{cp}} = \bar{y}_2 + \frac{I_{200}}{A_2\bar{y}_2} = \frac{1.5}{\sin\theta} + \frac{2.25/\sin^3\theta}{(3/\sin\theta)(1.5/\sin\theta)} = \frac{2}{\sin\theta}$$

The horizontal hydrostatic forces on the upstream and downstream faces are:

$$F_1 = \gamma_\text{w} A_1\bar{y}_1 = (9.789)(h)(h/2) = 4.895h^2$$

$$F_2 = \gamma_\text{w} A_2\bar{y}_2 \sin\theta = (9.789)\left[\frac{3}{\sin\theta}\right]\left[\frac{1.5}{\sin\theta}\right]\sin\theta = \frac{44.05}{\sin\theta}$$

The moment, M_1, about the toe of the dam (Point P in Figure 2.2) caused by the uplift pressure is obtained with the following calculations:

$$p = p_0 + \frac{p_1 - p_0}{L}x \rightarrow p = 3(9.789) + \frac{9.789h - 9.789(3)}{25}x \rightarrow p = 29.37 + [0.3916h - 1.175]x$$

$$M_1 = \int_0^{25} x\cdot p\,\text{d}x = 14.69x^2\Big|_0^{25} + [0.1305h - 0.3917]x^3\Big|_0^{25} = 3060.9 + 2039.1h$$

The moment, M_2, about the toe of the dam caused by the weight of the concrete is obtained with the following calculations:

$$M_2 = 5(h+4)\gamma_\text{c}(20+2.5) + \tfrac{1}{2}(20)(h+4)\gamma_\text{c}\left(20 - \frac{20}{3}\right)$$

$$= 5(h+4)(23.53)(20+2.5) + \tfrac{1}{2}(20)(h+4)(23.53)\left(20 - \frac{20}{3}\right) \quad\rightarrow\quad M_2 = 5784(h+4)$$

At the instant of overturning, the ground reaction is equal to zero and the sum of the moments about P is equal to zero, which requires that

$$-F_1(h - y_{1\text{cp}}) + F_2\left(\frac{3}{\sin\theta} - y_{2\text{cp}}\right) - M_1 + M_2 = 0$$

$$\rightarrow\quad -4.895h^2(h - \tfrac{2}{3}h) + \frac{44.05}{\sin\theta}\left(\frac{3}{\sin\theta} - \frac{2}{\sin\theta}\right) - (3060.9 + 2039.1h) + [5784(h+4)] = 0$$

$$\rightarrow\quad -1.632h^3 + \frac{44.05}{\sin^2\theta} + 7823h + 20075 = 0$$

$$\rightarrow\quad -1.632h^3 + (44.05)\frac{(h+4)^2 + 20^2}{h+4} + 3744.9h + 20075 = 0 \quad\rightarrow\quad h = \boxed{50.7\,\text{m}}$$

2.48. From the given data: $D = 2.5$ m, $R = D/2 = 1.25$ m, $\theta = 35°$, $h_\text{c} = 1.5$ m, and $W = 500$ kN. For water, $\gamma = 9.79$ kN/m^3.

(a) The hydrostatic force that would exist on the top surface of the gate is the same as that which exists on the bottom surface of the gate. Work with a top-of-gate perspective. The relevant geometric properties of the gate are as follows:

$$\bar{y} = \frac{1.5}{\sin 35^\circ} = 2.615\,\text{m}, \qquad y_{\text{top}} = \bar{y} - R = 2.615 - 1.25 = 1.365\,\text{m}$$

$$A = \pi R^2 = \pi(1.25)^2 = 4.909\,\text{m}^2, \qquad I = \frac{\pi R^4}{4} = \frac{\pi(1.25)^4}{4} = 1.918\ \text{m}^4$$

Therefore the resultant force, F, and its location, y_{cp}, are given by

$$F = \gamma A h_{\text{c}} = (9.79)(4.909)(1.5) = \boxed{72.1\,\text{kN}}$$

$$y_{\text{cp}} = \bar{y} + \frac{I}{A\bar{y}} = 2.615 + \frac{1.918}{(4.909)(2.615)} = 2.765\,\text{m}$$

The location of the resultant relative to the top of the gate is $y_{\text{F}} = 2.765\,\text{m} - 1.365\,\text{m} = \boxed{1.400\,\text{m}}$.

(b) When the gate is about to open, and F_{b} is the applied (vertical) force at the bottom of the gate, taking moments about the top of the gate gives,

$$F \cdot y_{\text{F}} + F_{\text{b}} \cdot (D\cos\theta) = W \cdot (R\cos\theta)$$

$$(72.1)(1.400) + F_{\text{b}}(2.5\cos 35^\circ) = (500)(1.25\cos 35^\circ) \quad \rightarrow \quad F_{\text{b}} = \boxed{201\,\text{kN}}$$

2.49. From the given data: $\theta = 50^\circ$, $d = 15$ m, and $R = 3$ m. For water, $\gamma = 9.79$ kN/m^3. The useful geometric properties of a semicircle (from Appendix C) are

$$y_{\text{c}} = \frac{4R}{3\pi}, \qquad I_{xc} = 0.1098R^4$$

where, in this case, y_{c} is the distance from the shaft to the centroid, and I_{xc} is the moment of inertia about an axis parallel to the shaft and passing through the centroid. Using these properties the following derived geometric properties can be calculated:

$$\bar{y} = \frac{d}{\sin\theta} - \left[R - \frac{4R}{3\pi}\right] = \frac{15}{\sin 50^\circ} - \left[3 - \frac{4(3)}{3\pi}\right] = 17.85\,\text{m}$$

$$h_{\text{c}} = \bar{y}\sin\theta = (17.85)(\sin 50^\circ) = 13.68\,\text{m}$$

$$A = \frac{1}{2}\pi R^2 = \frac{1}{2}\pi(3)^2 = 14.14\,\text{m}^2$$

$$I_{xc} = 0.1098R^4 = 0.1098(3)^4 = 8.894\ \text{m}^4$$

$$y_{\text{cp}} = \bar{y} + \frac{I_{xc}}{A\bar{y}} = 17.85 + \frac{8.894}{(14.14)(17.85)} = 17.89\,\text{m}$$

Using these results, the hydrostatic force, F, calculated as follows

$$F = \gamma A h_{\text{c}} = (9.79)(14.14)(13.68) = 1893\,\text{kN}$$

The distance from the shaft to the center of pressure, y_{cp1}, is given by

$$y_{\text{cp1}} = y_{\text{cp}} - \left[\frac{d}{\sin\theta} - R\right] = 17.89 - \left[\frac{15}{\sin 50^\circ} - 3\right] = 1.309\,\text{m}$$

The support force, F_P, is derived by considering the gate as a free body and taking moments about the shaft, which yields

$$F_\text{P} R = F y_{\text{cp1}} \quad \rightarrow \quad F_\text{P} = F\frac{y_{\text{cp1}}}{R} = 1893\frac{1.309}{3} = \boxed{826\,\text{kN}}$$

2.50. From the given data: $\theta = 35^\circ$, $R = 420$ mm, and $\bar{y} = 3$ m. The force, F, on the hatch is

$$F = \gamma A \bar{y} \sin\theta$$

where $A = \pi R^2 = \pi(0.42)^2 = 0.554$ m^2, and therefore

$$F = (9.79)(0.554)(3)\sin 35^\circ = \boxed{9.33\,\text{kN}}$$

This force is located at a distance y_{cp} from the surface, where

$$y_{\text{cp}} = \bar{y} + \frac{I_{\text{cc}}}{A\bar{y}}$$

For the circular hatch,

$$I_{\text{cc}} = \frac{\pi D^4}{64} = \frac{\pi(0.84)^4}{64} = 0.0244 \text{ m}^4$$

hence

$$y_{\text{cp}} = 3 + \frac{0.0244}{(0.554)(3)} = 3.01\,\text{m}$$

The resultant hydrostatic force is therefore $\boxed{3.01 \text{ m}}$ below the water surface, measured along the sloping wall.

2.51. Calculate the force on the gate:

$$F = \gamma A \bar{y} \sin\theta \tag{1}$$

where $\theta = 90^\circ - \sin^{-1}(3/5) = 53.1^\circ$, $\bar{y} = 4/\sin(53.1^\circ) + 2.5 = 7.502$ m, $A = (5)(4) = 20$ m^2, and $\gamma = 9.79$ kN/m^3. Substituting into Equation 1 and also calculating the center of pressure gives:

$$F = (9.79)(20)(7.502)\sin(53.1^\circ) = 1175\,\text{kN}$$

$$y_{\text{cp}} = \bar{y} + \frac{I}{A\bar{y}} = 7.502 + \frac{\frac{(4)(5)^3}{12}}{(20)(7.502)} = 7.780\,\text{m}$$

Taking moments about the hinge and taking into consideration that the reaction force at P acts normal to the surface gives

$$[P\cos(53.1^\circ)](5) = (7.502 + 2.5 - 7.780)(1175)$$

which gives $\boxed{P = 869 \text{ kN}}$.

2.52. From the given data: $W = 500$ kg, $w = 5$ m, and $\theta = 45°$. Taking $\gamma = 9.79$ kN/m^3, the hydrostatic force on the gate, F_h, is given by

$$F_\mathrm{h} = \gamma A\bar{y}\sin\theta = \gamma A\bar{h} = (9.79)\left(\frac{3}{\sin 45°}\times 5\right)(0.5+1.5) = 415\,\mathrm{kN}$$

The center of pressure, y_cp, is given by

$$y_\mathrm{cp} = \bar{y} + \frac{I}{A\bar{y}} = \left(\frac{2}{\sin 45°}\right) + \frac{5\left(\frac{3}{\sin 45°}\right)^3\frac{1}{12}}{\left(\frac{3}{\sin 45°}\times 5\right)\left(\frac{2}{\sin 45°}\right)} = 3.36\,\mathrm{m}$$

(a) If the force is applied at the center of the gate, taking moments about B gives

$$F\left(\frac{1.5}{\sin 45°}\right) = F_\mathrm{h}(y_\mathrm{cp} - \bar{y}) + W(1.5)$$

$$F\left(\frac{1.5}{\sin 45°}\right) = (415)\left(3.36 - \frac{0.5}{\sin 45°}\right) + \frac{500(9.81)(1.5)}{1000}$$

which gives $F = \boxed{522\text{ kN}}$.

(b) The minimum force would be required if it were applied at the $\boxed{\text{bottom of the gate}}$. In this case, taking moments about B gives

$$F\left(\frac{3}{\sin 45°}\right) = 415\left(3.36 - \frac{0.5}{\sin 45°}\right) + \frac{500(9.81)(1.5)}{1000}$$

which gives $F = \boxed{261\text{ kN}}$.

2.53. From the given data: $\rho_\mathrm{f} = 998$ kg/m^3, $\rho_\mathrm{s} = 1025$ kg/m^3, and $w = 100$ lb/m $= 0.4448$ kN/m. A sketch of the dimensions used in solving this problem is shown in Figure 2.3.

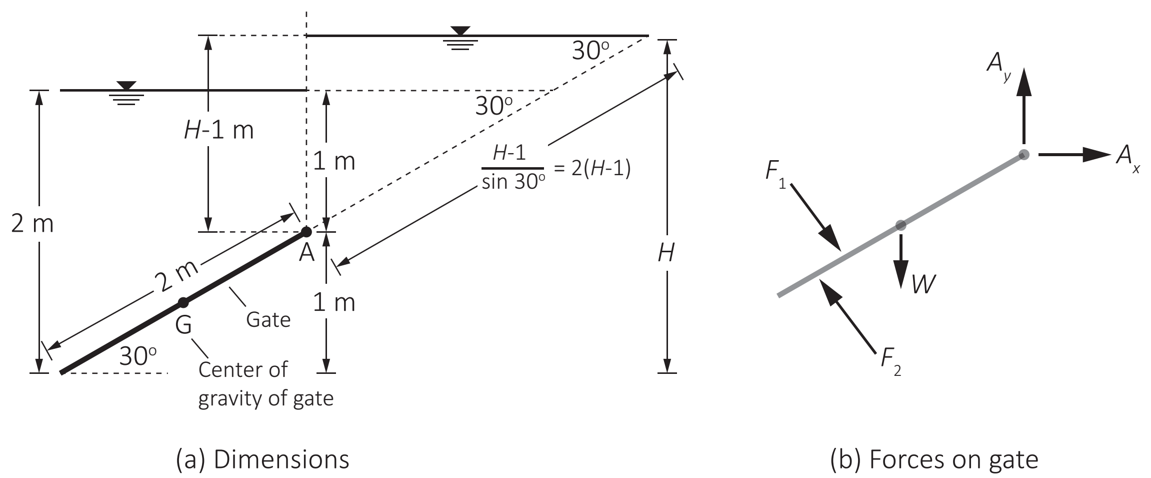


Figure 2.3: Gate dimensions and forces

Using the given data and referring to Figure 2.3,

$$\gamma_\mathrm{w} = (998)(9.81) = 9.79\text{ kN/m}^3$$

$$\gamma_s = (1025)(9.81) = 10.06 \text{ kN/m}^3$$
$$F_1 = \gamma_w A\bar{y}_1 \sin\theta = (9.79)(2\times 1)(2+1)\sin 30^\circ = 29.37\,\text{kN}$$
$$y_{1cp} = \bar{y}_1 + \frac{I}{A\bar{y}_1}$$
$$I = \frac{bd^3}{12}$$
$$A = bd$$
$$y_{1cp} = \bar{y}_1 + \frac{d^2}{12\bar{y}_1} = 3 + \frac{2^2}{12(3)} = 3.11\,\text{m}$$
$$F_2 = \gamma_s A\bar{y}_2 \sin\theta = (10.06)(2\times 1)[2(H-1)+1]\sin 30^\circ = 10.06(2H-1)\,\text{kN}$$
$$y_{2cp} = \bar{y}_2 + \frac{d^2}{12\bar{y}_2} = (2H-1) + \frac{2^2}{12(2H-1)} = (2H-1) + \frac{0.3333}{2H-1}$$

Taking moments about A ($\sum M = 0$) yields

$$29.37(1.11) + 0.4448\cos 30^\circ(1) = 10.06(2H-1)\left[1 + \frac{0.3333}{(2H-1)}\right]$$
$$32.99 = 10.06(2H-1) + 3.353$$

which yields $H = \boxed{1.97 \text{ m}}$.

2.54. The hydrostatic force, F, on the gate is given by

$$F = \gamma A\bar{y}\sin\theta$$

For an elliptical surface, Table C.1 in Appendix C gives

$$A = \frac{\pi bh}{4}$$

where

$$b = D = 1.2\,\text{m}, \qquad h = \frac{D}{\sin\theta} = \frac{1.2}{\sin 30^\circ} = 2.4\,\text{m}$$

and therefore

$$A = \frac{\pi bh}{4} = \frac{\pi(1.2)(2.4)}{4} = 2.26\,\text{m}^2$$

The location of the centroid, $\bar{y}$, is given by

$$\bar{y} = \frac{9}{\sin\theta} = \frac{9}{\sin 30^\circ} = 18\,\text{m}$$

and the net hydrostatic force on the gate is

$$F = \gamma A\bar{y}\sin\theta = (9.79)(2.26)(18)\sin 30^\circ = \boxed{199\,\text{kN}}$$

The location of the center of pressure, y_{cp}, is given by

$$y_{cp} = \bar{y} + \frac{I_{cc}}{A\bar{y}}$$

where Table C.1 in Appendix C gives

$$I_{\rm cc} = \frac{\pi b h^3}{64} = \frac{\pi (1.2)(2.4)^3}{64} = 0.814 \text{ m}^4$$

hence

$$y_{\rm cp} = 18 + \frac{0.814}{(2.26)(18)} = \boxed{18.0\,\text{m}}$$

The moment of the hydrostatic force about P, M_P, is given by

$$M_{\rm P} = 199\left[y_{\rm cp} - \left(9 - \frac{1.2}{2}\right)\frac{1}{\sin 30^\circ}\right] = 199\left[18.0 - \left(9 - \frac{1.2}{2}\right)\frac{1}{\sin 30^\circ}\right] = 239 \text{ kN·m}$$

The moment required to keep the gate closed is $\boxed{239 \text{ kN·m}}$.

2.55. Consider the flap gate as a free body as shown in Figure 2.4

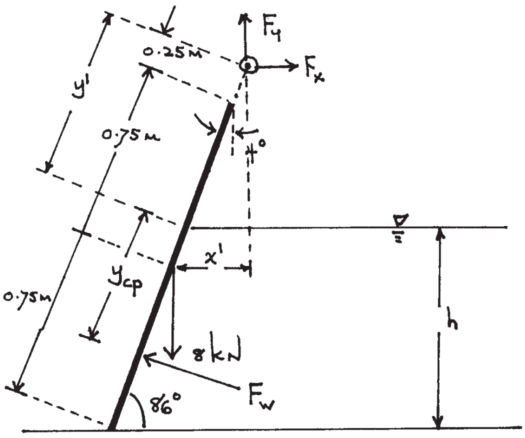


Figure 2.4: Flap gate free body

The area, A, of the gate under water is given by

$$A = 1.5\frac{h}{\sin 86^\circ} = 1.504h$$

and the distance to the centroid of the gate from the water surface, $\bar{y}$, measured along the gate, is given by

$$\bar{y} = \frac{1}{2}\frac{h}{\sin 86^\circ} = 0.5012h$$

The hydrostatic force, $F_{\rm w}$, exerted by the water is

$$F_{\rm w} = \gamma A \bar{y} \sin\theta \tag{1}$$

where γ=9.79 kN/m^3 (at 20°C) and $\theta = 86^\circ$. Substituting known and derived data into Equation 1 gives

$$F_{\rm w} = (9.79)(1.504h)(0.5012h)\sin 86^\circ = 7.362h^2\,\text{kN}$$

The distance, $y_{\rm cp}$, below the water surface to the center of pressure is given by

$$y_{\rm cp} = \frac{I_{\rm oo}}{A\bar{y}} + \bar{y} \tag{2}$$

where

$$I_{\rm oo} = \frac{bd^3}{12} = \frac{(1.5)\left(\frac{h}{\sin 86^\circ}\right)^3}{12} = 0.1259h^3 \tag{3}$$

Combining Equations 2 and 3 and taking $A = 1.504h$ and $\bar{y} = 0.5012h$ gives

$$y_{\rm cp} = \frac{0.1259h^3}{(1.504h)(0.5012h)} + 0.5012h = 0.6682h$$

The distance from the hinge to the water surface, measured along the gate (y' in Figure 2.4) is given by

$$y' = 1.75 - \frac{h}{\cos 4^\circ} = 1.75 - 1.002h$$

and the horizontal distance from the hinge to the center of gravity of the gate (x' in Figure 2.4) is given by

$$x' = (1\,\text{m})\cos 86^\circ = 0.06976\,\text{m}$$

Taking moments about the hinge, with the weight of the gate (W) equal to 8 kN, yields

$$F_{\rm w} \cdot (y' + y_{\rm cp}) = W \cdot x'$$
$$(7.362h^2)(1.75 - 1.002h + 0.6682h) = (8)(0.06976)$$

which simplifies to

$$7.362h^2(1.75 - 0.3338h) = 0.5581$$

This cubic equation has the following three solutions

$$h = 5.23\,\text{m}, \quad 0.212\,\text{m}, \quad \text{and} -0.204\,\text{m}$$

The only realistic solution is $\boxed{h = 0.212\text{ m}}$.

2.56. From the given data: $b = 2\,\text{m}$ and the other dimensions are given in the problem diagram. Take $\gamma = 9.79\,\text{kN·m}^3$. For reference, the sketch shown in Figure 2.5 is useful.

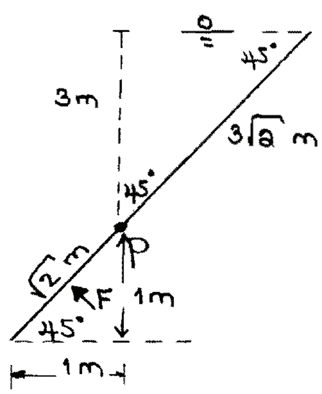


Figure 2.5: Force on gate

Using the given data:

$$\sin 45^\circ = \sqrt{2}, \qquad F = \gamma A\bar{y}\sin\theta$$

$$\bar{y} = 3\sqrt{2} + \frac{\sqrt{2}}{2} = 4.950\,\text{m}, \qquad F = (9.79)(\sqrt{2}\times 2)(4.950)\left(\frac{1}{\sqrt{2}}\right) = 96.9\,\text{kN}$$

$$y_\text{cp} = \bar{y} + \frac{I}{A\bar{y}}, \qquad I = \frac{bd^3}{12} = \frac{(2)(\sqrt{2})^3}{12} = 0.4714\ \text{m}^4$$

$$A = (\sqrt{2})(2) = 2.828\,\text{m}^2, \qquad y_\text{cp} = 4.950 + \frac{0.4714}{(2.828)(4.950)} = 4.984\,\text{m}$$

Taking moments about P gives

$$F(y_\text{cp} - 3\sqrt{2}) = F_\text{s}(1) \quad \rightarrow \quad F_\text{s} = 96.9\frac{(4.984 - 3\sqrt{2})}{1} = \boxed{71.8\,\text{kN}}$$

2.57. From the given data: $b = 1.52\,\text{m}$. Assume $T = 20^\circ\text{C}$, the fluid properties are: $\gamma_\text{w} = 9.789\,\text{kg/m}^3$, and $\gamma_\text{sw} = 1.025(9.789) = 10.03\,\text{kg/m}^3$. A schematic diagram of the important variables is shown in Figure 2.6. The centroidal depth on the freshwater side is given by $\bar{y} = 0.5(3.05\,\text{m}) = 1.525\,\text{m}$.

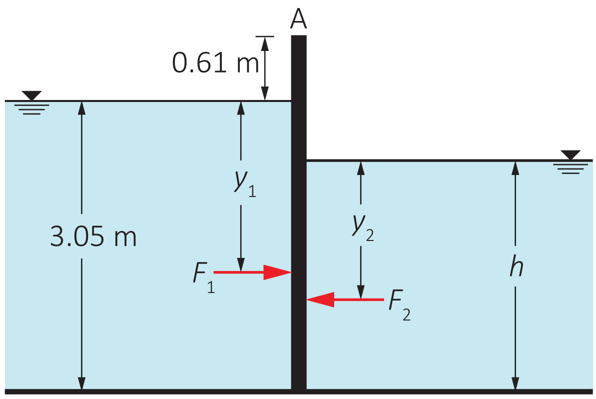


Figure 2.6: Schematic diagram of vertical gate

$$F_1 = \gamma_\text{w} A\bar{y} = (9.789)(3.05\times 1.525)(1.525) = 69.21\,\text{kN}$$

$$y_1 = \bar{y} + \frac{I_0}{A\bar{y}} = 1.525 + \frac{\frac{(1.525)(3.05)^3}{12}}{(1.525\times 3.05)(1.525)} = 2.033\,\text{m}$$

$$F_2 = \gamma_\text{sw} A\bar{y} = (10.03)(1.525h)\left(\frac{h}{2}\right) = 7.623h^2\,\text{kN}$$

$$y_2 = \bar{y} + \frac{I_0}{A\bar{y}} = \frac{h}{2} + \frac{\frac{(1.525)(h)^3}{12}}{(1.525\times h)(h/2)} = \frac{2}{3}h$$

Taking moments about A,

$$F_1(y_1{+}0.61) = F_2\left(3.05 + 0.61 - h + \frac{2}{3}h\right) \quad \rightarrow \quad (69.21)(2.033{+}0.61) = 7.632h^2\left(3.66 - \frac{h}{3}\right)$$

which yields $h = 3.002\,\text{m}$. Therefore, the gate will open when the depth of seawater is less than $\boxed{3.00\,\text{m}}$.

2.58. From the given data: $h_1 = 0.5\,\text{m}$, $h_2 = 0.7\,\text{m}$, and $w = 3\,\text{m}$. Use the subscript "b" to indicate the portion of the gate below the hinge, and the subscript "t" to indicate the portion of the gate above the hinge. The following preliminary calculations are useful:

$$A_\text{b} = wh_1 = (3)(0.5) = 1.5\,\text{m}^2, \qquad I_\text{bc} = \frac{wh_1^3}{12} = \frac{(3)(0.5)^3}{12} = 0.03125\,\text{m}^4$$

$$A_\text{t} = wh_2 = (3)(0.7) = 2.1\,\text{m}^2, \qquad I_\text{tc} = \frac{wh_2^3}{12} = \frac{(3)(0.7)^3}{12} = 0.08575\,\text{m}^4$$

Calculate the resultant hydrostatic forces and their locations on the portions of the gate below and above the hinge:

$$F_\text{b} = \gamma A_\text{b} h_\text{bc} = \gamma A_\text{b}(h - 0.25) \tag{1}$$

$$y_\text{bcp} = \bar{y}_\text{b} + \frac{I_\text{bc}}{A_\text{b}\bar{y}_\text{b}} = (h - 0.25) + \frac{I_\text{bc}}{A_\text{b}(h - 0.25)} \tag{2}$$

$$F_\text{t} = \gamma A_\text{t} h_\text{tc} = \gamma A_\text{t}(h - 0.85) \tag{3}$$

$$y_\text{tcp} = \bar{y}_\text{t} + \frac{I_\text{tc}}{A_\text{t}\bar{y}_\text{t}} = (h - 0.85) + \frac{I_\text{tc}}{A_\text{t}(h - 0.85)} \tag{4}$$

When the gate is just about to open, the reaction of the stopper is equal to zero and the sum of the moments about the hinge is equal to zero. Therefore,

$$F_\text{b} \cdot [y_\text{bcp} - (h - 0.5)] = F_\text{t} \cdot [(h - 0.5) - y_\text{tcp}] \tag{5}$$

Substituting the expressions from Equations 1 to 4 into Equation 5 and making h the subject of the formula yields

$$h = \frac{I_\text{bc} - 0.25^2 A_\text{b} + (0.35)(0.85)A_\text{t} + I_\text{tc}}{0.35A_\text{t} - 0.25A_\text{b}} \tag{6}$$

(a) Substituting the values of the given and derived parameters into Equation 6 yields

$$h = \frac{(0.03125) - 0.25^2(1.5) + (0.35)(0.85)(2.1) + (0.08575)}{0.35(2.1) - 0.25(1.5)} = \boxed{1.80\,\text{m}}$$

(b) Since the specific weight of the liquid, γ, $\boxed{\text{does not appear in the expression for } h}$ given by Equation 6, the calculated depth of liquid, h does not depend on the specific weight or the density of the liquid.

2.59. Consider the side of the trough shown as a free body in Figure 2.7.

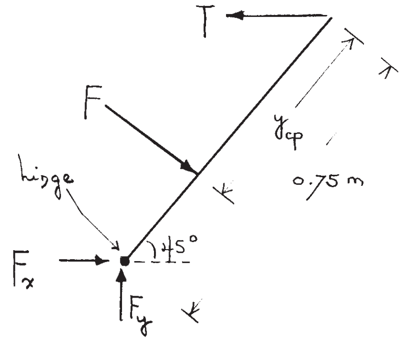


Figure 2.7: Free-Body Diagram

The hydrostatic force, F, is given by

$$F = \gamma A \bar{y} \sin\theta = (9.79)(0.75 \times 6)\left(\frac{0.75}{2}\right) \sin 45^\circ = 11.68\,\text{kN}$$

The center of pressure, y_{cp}, is given by

$$y_{\text{cp}} = \bar{y} + \frac{I_{\text{cc}}}{A\bar{y}} = 0.375 + \frac{\frac{6\times 0.75^3}{12}}{(0.75\times 6)(0.375)} = 0.50\,\text{m}$$

Taking moments about the hinge gives

$$F(0.75 - y_{\text{cp}}) = T(0.75 \sin 45^\circ)$$

which yields

$$T = \frac{F(0.75 - y_{\text{cp}})}{0.75\sin 45^\circ} = \frac{11.68(0.75 - 0.50)}{0.75 \sin 45^\circ} = \boxed{5.51\,\text{kN}}$$

2.60. From the given data: $W = 3\,\text{m}$, $L = 2\,\text{m}$, $\theta = 30^\circ$, $p_0 = 300\,\text{kPa}$, $d_1 = 2\,\text{m}$, and $d_2 = 1\,\text{m}$. For water at 20°C, $\gamma = 9.789\,\text{m/s}^2$. The following preliminary calculations are useful,

$$h_{\text{c}} = d_2 + [d_1 + \tfrac{1}{2}L]\sin\theta = 1 + [2 + \tfrac{1}{2}(2)]\sin 30^\circ = 2.5\,\text{m}$$

$$A = WL = (3)(2) = 6\,\text{m}^2, \qquad \bar{y} = \frac{h_{\text{c}}}{\sin\theta} = \frac{2.5}{\sin 30^\circ} = 5\,\text{m}, \qquad I_{\text{cc}} = \frac{WL^3}{12} = \frac{(3)(2)^3}{12} = 2\,\text{m}^4$$

Substituting these data into Equations 2.41 and 2.48 gives,

$$F = [p_0 + \gamma h_{\text{c}}]A = [300 + (9.789)(2.5)](6) = 1947\,\text{kN} = \boxed{1.947\,\text{MN}}$$

$$y_{\text{cp}} = \bar{y} + \frac{\gamma \sin\theta I_{\text{cc}}}{[p_0 + \gamma\bar{y}\sin\theta]A} = 5 + \frac{(9.789)\sin 30^\circ(2)}{[300 + (9.789)(5)\sin 30^\circ](6)} = 5.005\,\text{m}$$

The depth, h_{cp}, if the resultant force below the water surface is given by

$$h_{\text{cp}} = y_{\text{cp}} \sin\theta = 5.005 \sin 30^\circ = \boxed{2.50\,\text{m}}$$

2.61. From the given data: $R = 2$ m, and $d = 3$ m. For water at 20°C, $\gamma = 9.79$ kN/m^3. Using these data:

$$I_{\rm xc} = I_{\rm yc} = 0.05488R^4 = 0.05488(2)^4 = 0.8781 \text{ m}^4$$
$$I_{\rm xyc} = -0.01647R^4 = -0.01647(2)^4 = -0.2635 \text{ m}^4$$
$$A = \frac{1}{4}\pi R^2 == \frac{1}{4}\pi(2)^2 = 3.142\,\text{m}^2$$
$$\bar{y} = d + \frac{4R}{3\pi} = 3 + \frac{4(2)}{3\pi} = 3.849\,\text{m}$$
$$\bar{x} = \frac{4R}{3\pi} = \frac{4(2)}{3\pi} = 0.849\,\text{m}$$
$$F = \gamma A\bar{y} = (9.79)\left[\frac{1}{4} \times \pi(2)^2\right](3.849) = \boxed{118\,\text{kN}}$$
$$y_{\rm cp} = \bar{y} + \frac{I_{\rm xc}}{A\bar{y}} = 3.849 + \frac{0.8781}{(3.142)(3.849)} = \boxed{3.921\,\text{m}}$$
$$x_{\rm cp} = \bar{x} + \frac{I_{\rm xyc}}{A\bar{y}} = 0.849 + \frac{-0.2635}{(3.142)(3.849)} = \boxed{0.827\,\text{m}}$$
$$M_{\rm XX} = F \cdot x_{\rm cp} = (118) \cdot (0.827) = \boxed{97.6 \text{ kN·m}}$$

2.62. The ellipse parameters as referenced to the geometric properties in the Appendix are: $a = 1$ m, $b/2 = 1\,\text{m} \rightarrow b = 2\,\text{m}$. From the other given data: $d = 2$ m. For water at 20°C, $\gamma = 9.789$ kN/m^3. Using these data with the same axis references as in the Appendix:

$$I_{\rm yc} = \tfrac{1}{128}\pi ba^3 = \tfrac{1}{128}\pi(2)(1)^3 = 0.04909 \text{ m}^4$$
$$I_{\rm xyc} = 0 \text{ m}^4$$
$$A = \tfrac{1}{8}\pi ab = \tfrac{1}{8}\pi(1)(2) = 0.7854\,\text{m}^2$$
$$\bar{y} = d + \frac{a}{2} = 2 + \frac{1}{2} = 2.500\,\text{m}$$
$$\bar{x} = \frac{2b}{3\pi} = \frac{2(2)}{3\pi} = 0.4244\,\text{m}$$
$$F = \gamma A\bar{y} = (9.789)(0.7854)(2.5) = \boxed{19.22\,\text{kN}}$$
$$y_{\rm cp} = \bar{y} + \frac{I_{\rm yc}}{A\bar{y}} = 2.500 + \frac{0.04909}{(0.7854)(2.500)} = \boxed{2.525\,\text{m}}$$
$$x_{\rm cp} = \bar{x} + \frac{I_{\rm xyc}}{A\bar{y}} = 0.4244 + 0 = \boxed{0.4244\,\text{m}}$$
$$M_{\rm XX} = F \cdot x_{\rm cp} = (19.22) \cdot (0.4244) = \boxed{8.157 \text{ kN·m}}$$

2.63. For the upper portion of the gate: $A_{\rm U} = 1 \text{ m}\times 1 \text{ m} = 1 \text{ m}^2$, $\gamma_{\rm U} = 9.50$ kN/m^3, $\bar{y}_{\rm U} = 2 + 1/2 = 2.5$ m, $I_{\rm cU} = bd^3/12 = (1)(1)^3/12 = 0.08333$ m^4, hence

$$F_{\rm U} = \gamma_{\rm U} A_{\rm U}\bar{y}_{\rm U} = (9.50)(1)(2.5) = 23.75\,\text{kN}$$
$$y_{\rm cpU} = \bar{y}_{\rm U} + \frac{I_{\rm cU}}{A_{\rm U}\bar{y}_{\rm U}} = 2.5 + \frac{0.0833}{(1)(2.5)} = 2.533\,\text{m}$$

For the lower portion of the gate, with depths taken relative to the interface: $p_0 = \gamma_U h_U = (9.50)(3) = 28.5$ kPa, $A_L = 2$ m$\times$1 m $= 2$ m^2, $\gamma_L = 9.90$ kN/m^3, $\bar{y}_L = 2/2 = 1$ m, $I_{cL} = bd^3/12 = (1)(2)^3/12 = 0.6667$ m^4, hence

$$F_L = [p_0 + \gamma_L \bar{y}_L]A_L = [28.5 + (9.90)(1)](2) = 76.80\,\text{kN}$$

$$y_{cpL} = \bar{y}_L + \frac{\gamma_L I_{cL}}{[p_0 + \gamma_L \bar{y}_L]A_L} = \bar{y}_L + \frac{\gamma_L I_{cL}}{F_L} = 1 + \frac{(9.90)(0.6667)}{76.8} = 1.086\,\text{m}$$

Therefore, the total force, F, and location, y_{cp}, are given by

$$F = F_U + F_L = 23.75 + 76.80 = \boxed{100.6\,\text{kN}}$$

$$y_{cp} = \frac{F_U y_{cpU} + F_L(3 + y_{cpL})}{F} = \frac{(23.75)(2.533) + (76.80)(3 + 1.086)}{100.6} = \boxed{3.72\,\text{m}}$$

2.64. The parameters of the ellipse, as described in the Appendix, are $a = 1$ m, and $b = 2$ m. From the given data: $d = 3$ m. For the upper portion of the gate:

$$A_U = \tfrac{1}{8}\pi ab = \tfrac{1}{8}\pi(1)(2) = 0.7854\,\text{m}^2, \qquad \gamma_U = 9.40\,\text{kN/m}^3$$

$$\bar{y}_U = d - \frac{2b}{3\pi} = 3 - \frac{2(2)}{3\pi} = 2.576\,\text{m}, \qquad I_{cU} = \tfrac{1}{128}\pi ab^3 = \tfrac{1}{128}\pi(1)(2)^3 = 0.1963\,\text{m}^4$$

$$F_U = \gamma_U A_U \bar{y}_U = (9.40)(0.7854)(2.576) = 19.02\,\text{kN}$$

$$y_{cpU} = \bar{y}_U + \frac{I_{cU}}{A_U \bar{y}_U} = 2.576 + \frac{0.1963}{(0.7854)(2.576)} = 2.673\,\text{m}$$

For the lower portion of the gate, with depths taken relative to the interface: $p_0 = \gamma_U d = (9.40)(3) = 28.2$ kPa, and

$$A_L = A_U = 0.7854\,\text{m}^2, \qquad \gamma_L = 9.80\,\text{kN/m}^3$$

$$\bar{y}_L = \frac{2b}{3\pi} = \frac{2(2)}{3\pi} = 0.4244\,\text{m}, \qquad I_{cL} = I_{cU} = 0.1963\,\text{m}^4$$

$$F_L = [p_0 + \gamma_L \bar{y}_L]A_L = [28.2 + (9.80)(0.4244)](0.7854) = 25.41\,\text{kN}$$

$$y_{cpL} = \bar{y}_L + \frac{\gamma_L I_{cL}}{[p_0 + \gamma_L \bar{y}_L]A_L} = \bar{y}_L + \frac{\gamma_L I_{cL}}{F_L} = 0.4244 + \frac{(9.80)(0.1963)}{25.41} = 0.5001\,\text{m}$$

Therefore, the total force, F, and location, y_{cp}, are given by

$$F = F_U + F_L = 19.02 + 25.41 = \boxed{44.43\,\text{kN}}$$

$$y_{cp} = \frac{F_U y_{cpU} + F_L(3 + y_{cpL})}{F} = \frac{(19.02)(2.673) + (25.41)(3 + 0.5001)}{44.43} = \boxed{3.146\,\text{m}}$$

NEW For water at 20°C, $\gamma = 9.789\,\text{kN/m}^3$. For the bent part of the surface the perimeter of the quarter circle is 1 m, and the radius, R, of the quarter circle is calculated as follows:

$$P = \tfrac{1}{4}\cdot 2\pi R \quad\rightarrow\quad 1 = \tfrac{1}{4}\cdot 2\pi R \quad\rightarrow\quad R = 0.6366\,\text{m}$$

Using the calculated value of R, the horizontal and vertical forces on the surface are given by

$$F_x = \gamma A_{\text{v1}}\bar{y}_{\text{v1}} + \gamma A_{\text{v1}}\bar{y}_{\text{v1}} = 9.789[(2\times 3)(1.5+1) + (0.6366\times 5)(1.5+2+0.6366)]$$
$$= \boxed{265.8\,\text{kN}}$$

$$F_z = \gamma V = \gamma\left[\tfrac{1}{4}\pi R^2 + 3.5R\right]W = 9.789\left[\tfrac{1}{4}\pi(0.6366)^2 + 3.5(0.6366)\right](5) = \boxed{124.6\,\text{kN}}$$

2.65. Because of symmetry, the net horizontal hydrostatic force is zero. The pressure at the top of the cone, p_0, is given by

$$p_0 = 150 - \gamma_{\text{w}}(7\,\text{m}) = 150 - (9.79)(7) = 81.47\,\text{kPa}$$

This gives an equivalent height of water, H, of

$$H = \frac{p_0}{\gamma} = \frac{81.47}{9.79} = 8.32\,\text{m}$$

Therefore, the vertical force on the cone, F, is given by

$$F = \left[\pi R^2 H + \frac{1}{3}\pi R^2 h\right]\gamma = \left[\pi(1)^2(8.32) + \frac{1}{3}\pi(1)^2(4)\right](9.79) = \boxed{297\,\text{kN}}$$

2.66. From the given data: $F_0 = 2500\,\text{kN}$, $L = 10\,\text{m}$, and $h = 2.4\,\text{m}$. For water at 20°C, $\gamma = 9.789\,\text{kN/m}^3$. For any given step height (= width), x, the horizontal force, F_x, is a function of x as follows:

$$A_{\text{v}} = w(h+4x) = 10(2.4+4x), \qquad \bar{y}_{\text{v}} = \frac{h+4x}{2} = 1.2 + 2x$$

$$F_x = \gamma A_{\text{v}}\bar{y}_{\text{v}} = 9.789[10(2.4+4x)](1.2+2x)$$

Setting $F_x(x) = 2500\,\text{kN}$ yields $x = \boxed{1.187\text{ m}}$. Using this value of x, the vertical force on the dam, F_y, is given by

$$F_y = \gamma V_0 = \gamma w(x^2 + 2x^2 + 3x^2 + 4x^2) = (9.789)(10)(1.187)^2(1+2+3+4) = \boxed{1379\,\text{kN}}$$

2.67. From the given data: $L = 5\,\text{m}$, $h_{\text{f}} = 4\,\text{m}$, $h_{\text{s}} = 2\,\text{m}$, and $R = 2\,\text{m}$. For fresh water at 20°C, $\gamma_{\text{f}} = 9.789\,\text{kN/m}^3$, and for salt water at 20°C, $\gamma_{\text{s}} = 10.03\,\text{kN/m}^3$ (from Appendix B.4). The following preliminary calculations are useful:

$$A_{\text{f}} = Lh_{\text{f}} = (5)(4) = 20\,\text{m}^2, \qquad \bar{y}_{\text{f}} = \tfrac{1}{2}h_{\text{f}} = \tfrac{1}{2}(4) = 2\,\text{m}$$

$$A_{\text{s}} = Lh_{\text{s}} = (5)(2) = 10\,\text{m}^2, \qquad \bar{y}_{\text{s}} = \tfrac{1}{2}h_{\text{s}} = \tfrac{1}{2}(1) = 1\,\text{m}$$

$$V_{0\text{f}} = L\left[R^2 - \tfrac{1}{4}\pi R^2\right] = (5)\left[(2)^2 - \tfrac{1}{4}\pi(2)^2\right] = 4.292\,\text{m}^3$$

where $V_{0\text{f}}$ is the volume of the space between the top of the wall and the freshwater surface. The horizontal and vertical components of the net hydrostatic force on the wall are given by:

$$F_x = \gamma_\text{f} A_\text{f} \bar{y}_\text{f} - \gamma_\text{s} A_\text{s} \bar{y}_\text{s} = (9.789)(20)(2) - (10.03)(10)(1) = \boxed{291\,\text{kN}}$$

$$F_y = \gamma_\text{f} V_{0\text{f}} = (9.789)(4.292) = \boxed{42.0\,\text{kN}}$$

2.68. From the given data: $R = h_\text{f} = 3.5\,\text{m}$, and $w = 4.8\,\text{m}$. For fresh water at 20°C, $\gamma_\text{f} = 9.789\,\text{kN/m}^3$, and for salt water at 20°C, $\gamma_\text{s} = 10.03\,\text{kN/m}^3$ (from Appendix B.4). The following preliminary calculations are useful:

$$A_\text{vf} = h_\text{f} w = (3)(4.8) = 16.8\,\text{m}^2, \qquad A_\text{vs} = \tfrac{1}{2} h w = 2.4 h\,\text{m}^2$$

$$\bar{y}_\text{vf} = \tfrac{1}{2} h_\text{f} = \tfrac{1}{2}(3.5) = 1.75\,\text{m}, \qquad \bar{y}_\text{vs} = \tfrac{1}{2} h$$

$$V_{0\text{f}} = R h_\text{f} w - \tfrac{1}{4}\pi R^2 w = (3.5)(3.5)(4.8) - \tfrac{1}{4}\pi(3.5)^2(4.8) = 12.62\,\text{m}^2$$

where $V_{0\text{f}}$ is the volume of the space between the top of the gate and the freshwater surface.

(a) For the horizontal hydrostatic forces to be equal,

$$\gamma_\text{f} A_\text{vf} \bar{y}_\text{vf} = \gamma_\text{s} A_\text{vs} \bar{y}_\text{vs}$$

Substituting the given and derived relationship into this equation and solving for h gives

$$h = \sqrt{\left(\frac{\gamma_\text{f}}{\gamma_\text{s}}\right) \frac{2 A_\text{vf} \bar{y}_\text{vf}}{w}} = \sqrt{\left(\frac{9.789}{10.03}\right) \frac{2(16.8)(1.75)}{4.8}} = \boxed{3.458\,\text{m}}$$

(b) For the vertical hydrostatic forces to be equal,

$$\gamma_\text{f} V_{0\text{f}} = \gamma_\text{s} V_{0\text{s}} \quad \rightarrow \quad V_{0\text{s}} = \left(\frac{\gamma_\text{f}}{\gamma_\text{s}}\right) V_{0\text{f}} = \left(\frac{9.789}{10.03}\right)(12.62) = 12.32\,\text{m}^3$$

Consider the geometry of the gate shown in Figure 2.8, and recall that the area of a segment of a circle with central angle θ is equal to $\frac{1}{2}R^2\theta$.

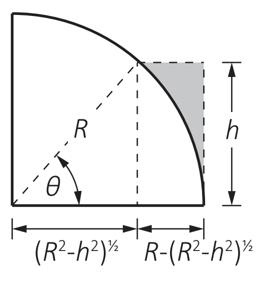


Figure 2.8: Segment of a circle

For any height h, the volume above the gate, $V_{0\text{s}}$, is equal to the shaded area and, using the geometric relations in Figure 2.8 yields

$$V_{0\text{s}} = w[h(R - \sqrt{R^2 - h^2})] - \left[\frac{1}{2}R^2 \sin^{-1}\left(\frac{h}{R}\right) - \frac{1}{2}\sqrt{R^2 - h^2}h\right]$$

Since $V_{0s} = 12.32\,\text{m}^3$ when the vertical hydrostatic forces are equal, then

$$(4.8)[h(3.5 - \sqrt{3.5^2 - h^2})] - \left[\frac{1}{2}(3.5)^2 \sin^{-1}\left(\frac{h}{3.5}\right) - \frac{1}{2}\sqrt{3.5^2 - h^2}h\right] = 12.32$$

which yields $h = \boxed{2.598\,\text{m}}$

2.69. From the given data: $R = 1\,\text{m}$, and $W = 40\,\text{kN/m}$. For water at 20°C, $\gamma = 9.789\,\text{kN/m}^3$. If F_h and F_v are the horizontal and vertical hydrostatic forces on the gate, then the magnitude, F, and direction, θ, of the hydrostatic force are given by

$$F = \sqrt{F_\text{h}^2 + F_\text{v}^2}, \qquad \sin\theta = \frac{F_\text{v}}{\sqrt{F_\text{h}^2 + F_\text{v}^2}} \tag{1}$$

The vertical force on the gate, F_v, is given by

$$F_\text{v} = \gamma[(h-R)R + \tfrac{1}{4}\pi R^2] = (9.789)[(h-1)(1) + \tfrac{1}{4}\pi(1)^2] \quad \rightarrow \quad F_\text{v} = 9.789h - 2.101\,\text{kN} \tag{2}$$

The resultant hydrostatic force acts through the center of the circular quadrant, the weight of the gate acts vertically through the centroid of the gate, and the centroid of the gate is located at a distance $4R/3\pi$ from the center of the quadrant. Taking moments about the pin when the gate is just about to open (i.e., the reaction is equal to zero) and using Equation 1 gives

$$FR\sin\theta = W\left[R - \frac{4R}{3\pi}\right] \rightarrow \cancel{\sqrt{F_\text{h}^2 + F_\text{v}^2}}\,(1)\frac{F_\text{v}}{\cancel{\sqrt{F_\text{h}^2 + F_\text{v}^2}}} = 40\left[1 - \frac{4(1)}{3\pi}\right] \quad \rightarrow \quad F_\text{v} = 23.02\,\text{kN}$$

Combining this result with Equation 2 gives $\boxed{h = 2.57\,\text{m}}$.

NEW (a) From the given data: $h = 2.7\,\text{m}$, $r_1 = 1\,\text{m}$, and $r_2 = 0.95\,\text{m}$. For water at 20°C, $\gamma = 9.789\,\text{kN/m}^3$. For a unit length of gate, $L = 1\,\text{m}$ and x- and y-components of the hydrostatic force on the gate, and the weight of the gate, are given by:

$$F_x = \gamma\bar{y}A = \gamma[h - 0.5r_1][r_1 L] = 9.879[2.7 - 0.5(1)][1(1)] = 21.54\,\text{kN}$$

$$F_y = \gamma V = \gamma[(h - r_1)r_1 + 0.25\pi r_1^2]L = 9.789[(2.7-1)(1) + 0.25\pi(1)^2](1) = 24.33\,\text{kN}$$

$$W = \rho_\text{g} g V_\text{g} = \rho_\text{g} g \pi L[r_1^2 - r_2^2] = \rho_\text{g}(9.807)\pi(1)[1^2 - 0.95^2] = 3.004\rho_\text{g}\,\text{N} = 3.004 \times 10^{-3}\rho_\text{g}\,\text{kN}$$

The calculated values of F_x and F_y can be used to determine the magnitude and direction of the resultant force as follows:

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{21.54^2 + 24.33^2} = 32.49\,\text{kN}, \qquad \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{24.33}{32.49}\right) = 36.83^\circ$$

Taking moments about the hinge when the gate is about to open, yields:

$$F\cdot r_1\sin\theta - W\cdot r_1 = 0 \quad \rightarrow \quad (32.49)\cdot(1)\sin 36.83^\circ - (3.004\rho_\text{g}\times 10^{-3})\cdot(1) = 0 \quad \rightarrow \quad \rho_\text{g} = \boxed{8099\,\text{kg/m}^3}$$

(b) Consider now a gate material with density $\rho_g = 8100\,\text{kg/m}^3$. When the gate is filled with water:

$$W = 3.004\rho_g + \gamma\pi r_2^2 L = 3.004(8100) + (9.789)\pi(0.95)^2(1) = 52.08\,\text{kN}$$

$$F_y = \gamma\left[(h - r_1)r_1 + 0.25\pi r_1^2\right] L = (9.789)\left[(h-1)(1) + 0.25\pi(1)^2\right](1) = 9.789(h - 0.2146)\,\text{kN}$$

$$\sin\theta = \frac{F_y}{F} = \frac{9.789(h - 0.2146)}{F}$$

Taking moments about the hinge when the gate is about to open, yields:

$$F \cdot r_1 \sin\theta - W \cdot r_1 = 0 \quad \rightarrow \quad F \cdot (1)\frac{9.789(h-0.2146)}{F} - 52.08 \cdot (1) = 0 \quad \rightarrow \quad \boxed{h = 5.54\,\text{m}}$$

2.70. From the given data: $\gamma = 9.79\,\text{kN/m}^3$, $A_\text{v} = (3)(20) = 60\,\text{m}^2$, and $\bar{y}_\text{v} = 6.5\,\text{m}$. The horizontal force, F_x, on viewing glass is given by

$$F_x = \gamma A_\text{v}\bar{y}_\text{v} = (9.79)(60)(6.5) = 3820\,\text{kN}$$

The volume of water, V_0, above the viewing glass is given by

$$V_0 = \left[(8)(3) - \frac{1}{4}\cdot\frac{\pi}{4}(6)^2\right](20) = 338.6\,\text{m}^3$$

Vertical force, F_z, on the viewing glass isgiven by

$$F_z = \gamma V_0 = (9.79)(338.6) = 3310\,\text{kN}$$

The net force on the viewing glass is given by

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(3820)^2 + (3310)^2} = \boxed{5050\,\text{kN}}$$

2.71. Determine the hydrostatic forces on the plane and curved surfaces separately and then add them up. Assume that $\gamma = 9.79$ kN/m^3. The geometric relationships and relevant dimensions are shown in Figure 2.9.

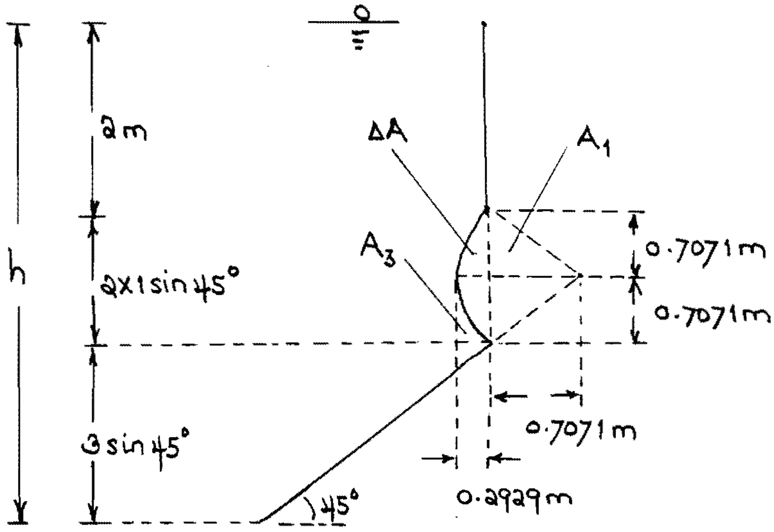


Figure 2.9: Geometric Relationships

`Plane Surface:` The force normal to the surface, F, is given by

$$F = \gamma A \bar{y} \sin\theta \tag{1}$$

Using the given data and the geometric relationships shown in Figure 2.9:

$$A = (3)(2) = 6\,\text{m}^2$$
$$\bar{y}\sin\theta = 2 + 2\sin 45^\circ + \frac{3}{2}\sin 45^\circ = 4.475\,\text{m}$$

Substituting these parameters into Equation 1 gives

$$F = (9.79)(6)(4.475) = 262.9\,\text{kN}$$

This force has x and y components as follows:

$$F_x = \quad 262.9\cos 45^\circ = 185.9\,\text{kN}$$
$$F_y = -262.9\sin 45^\circ = -185.9\,\text{kN}$$

`Curved Surface:` The x-component of the force on the curved surface is given by

$$F_x = \gamma A y_\text{v} \tag{2}$$

Using the given data and the geometric relationships shown in Figure 2.9:

$$A = (2 \times 0.7071)(2) = 2.828\,\text{m}^2$$
$$y_v = 2 + 0.7071 = 2.7071\,\text{m}$$

Substituting these parameters into Equation 2 gives

$$F_x = (9.79)(2.828)(2.7071) = 74.95\,\text{kN}$$

The y component of the hydrostatic force on the upper curved surface is given by

$$F_{y1} = -\gamma V_1 \tag{3}$$

Using the given data and the geometric relationships shown in Figure 2.9:

$$\begin{aligned}
A_1 &= \frac{1}{2}(0.7071)(0.7071) = 0.2500\,\mathrm{m}^2 \\
A_0 &= \frac{\pi}{8}r^2 = \frac{\pi}{8}(1)^2 = 0.3927\,\mathrm{m}^2 \\
\Delta A &= A_0 - A_1 = 0.3927 - 0.2500 = 0.1427\,\mathrm{m}^2 \\
V_1 &= [(2 + 0.7071)(0.2929) - 0.1427](2) = 1.300\,\mathrm{m}^3
\end{aligned}$$

Substituting into Equation 3 gives

$$F_{y1} = -(9.79)(1.300) = -12.73\,\mathrm{kN}$$

The y component of the hydrostatic force on the lower curved surface is given by

$$F_{y2} = \gamma V_2 \tag{4}$$

Using the given data and the geometric relationships shown in Figure 2.9:

$$\begin{aligned}
A_3 &= (0.2929)(0.7071) - \Delta A = (0.2929)(0.7071) - 0.1427 = 0.0644\,\mathrm{m}^2 \\
V_2 &= [(2 + 2 \times 0.7071)(0.2929) - 0.0644](2) = 1.871\,\mathrm{m}^3
\end{aligned}$$

Substituting into Equation 4 gives

$$F_{y2} = (9.79)(1.871) = 18.32\,\mathrm{kN}$$

Therefore, the net vertical hydrostatic force on the curved portion of the gate is given by

$$F_y = F_{y1} + F_{y2} = -12.73 + 18.32 = 5.59\,\mathrm{kN}$$

This force could also be determined by simply calculating the buoyant force on the gate.

`Total Hydrostatic Force:` The x and y components of the total hydrostatic force are equal to the sum of the hydrostatic forces on the plane and curved portions of the gate, so

$$\begin{aligned}
F_x &= 185.9 + 74.95 = \boxed{260.9\,\mathrm{kN}} \\
F_y &= -185.9 + 5.59 = \boxed{-180.3\,\mathrm{kN}}
\end{aligned}$$

2.72. From the given data: width of the gate = 5 m, weight of gate = 10 kN. Let R_x and R_y be the reaction of the gate to the hydrostatic force. Hence,

$$R_x = \gamma A_\mathrm{v} \bar{y}_\mathrm{v}$$

where $\gamma = 9.79$ kN/m^3, $A_\mathrm{v} = 3 \times 5 = 15$ m^2, and $\bar{y}_\mathrm{v} = 3/2 = 1.5$ m. Substituting gives

$$R_x = (9.79)(15)(1.5) = 220\,\mathrm{kN}$$

This force is located at y_{cp} below the water surface, where

$$y_{\text{cp}} = \bar{y}_{\text{v}} + \frac{I_{\text{cc}}}{A_{\text{v}}\bar{y}_{\text{v}}}$$

where

$$I_{\text{cc}} = \frac{bd^3}{12} = \frac{5(3)^3}{12} = 11.25 \text{ m}^4$$

and hence

$$y_{\text{cp}} = 1.5 + \frac{11.25}{(15)(1.5)} = 2.0\,\text{m}$$

The vertical reaction, R_y, is equal to the weight of water above the gate,

$$R_y = \gamma V = (9.79)\left[\frac{\pi(3)^2}{4}\right](5) = 346\,\text{kN}$$

This force acts through the centroid of the circle quadrant occupied by the gate, which is $4r/3\pi$ from P, where

$$\frac{4r}{3\pi} = \frac{4(3)}{3\pi} = 1.27\,\text{m}$$

The net hydrostatic force, R, on the gate is therefore given by

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{220^2 + 346^2} = \boxed{410\,\text{kN}}$$

The moment, M, tending to open the gate is

$$M = 346(1.27) - 220(2.0) - 10(1) = -10.6 \text{ kN·m}$$

Hence the moment required to open the gate is $\boxed{10.6 \text{ kN·m}}$.

Note: This moment is numerically equal to the moment due only to the weight of the gate. This is an expected result since the resultant hydrostatic force on a circular gate will necessarily act through the center of the gate.

2.73. From the given data: $H = 15.25$ m and assume $\gamma = 9.79$ kN/m^3. The horizontal component of the force is given by

$$F_{\text{h}} = \gamma A\bar{y}_{\text{v}} = (9.79)(15.25 \times 1)\left(\frac{15.25}{2}\right) = 1138\,\text{kN}$$

The vertical force is the weight of the fluid above the dam. The x coordinate at the water level, x_0, is given by

$$y = \frac{x^2}{2.4} \quad \rightarrow \quad 15.25 = \frac{x_0^2}{2.4} \quad \rightarrow \quad x_0 = 6.050\,\text{m}$$

and the vertical force on the dam is given by

$$F_{\text{v}} = \gamma V = 9.79\int_0^{6.050}(1)\left(15.25 - \frac{x^2}{2.4}\right)\,\text{d}x = 9.79\left[15.25x - \frac{x^3}{7.2}\right]_0^{6.050} = 602.1\,\text{kN}$$

Therefore the resultant force is $\sqrt{1138^2 + 602.1^2} = \boxed{1287\text{ kN}}$ and this force makes an angle of $\tan^{-1}(1138/602.1) = 62.1°$ with the vertical. The horizontal force acts at the center of pressure given by

$$y_{\rm cp} = \bar{y} + \frac{I}{A\bar{y}} = \frac{15.25}{2} + \frac{\dfrac{(1)(15.25)^3}{12}}{(15.25\times 1)\left(\dfrac{15.25}{2}\right)} = 10.17\,\text{m}$$

The vertical force acts through the center of gravity given by

$$x_{\rm cg} = \frac{1}{\left(\dfrac{602.1}{9.79}\right)}\int_0^{6.050} x\left(15.25 - \frac{x^2}{2.4}\right)\,{\rm d}x = 0.01626\left[\frac{15.25x^2}{2} - \frac{x^4}{9.6}\right]_0^{6.050} = 2.269\,\text{m}$$

Using these data give

$$\text{AB} = 2.269\,\text{m} + (15.25\,\text{m} - 10.17\,\text{m})\tan 62.1° = \boxed{11.87\,\text{m}}$$

2.74. Considering the normal force on the viewing glass, $F_{\rm N}$, and the limit of 100 N that can be supported by each rivet,

$$F_{\rm N} = \gamma A\bar{y} = (9.79)\left(\frac{\pi(1)^2}{4}\right)(5) = 38.45\,\text{kN}$$

$$\text{required rivets} = \frac{38450}{100} = 385 \text{ rivets}$$

Considering the shear force on the viewing glass, $F_{\rm S}$, and the limit of 5 N that can be supported by each rivet,

$$F_{\rm S} = \gamma\frac{1}{2}\left[\frac{4}{3}\pi R^3\right] = (9.79)\frac{1}{2}\left[\frac{4}{3}\pi(0.5)^3\right] = 2.56\,\text{kN}$$

$$\text{required rivets} = \frac{2560}{5} = 512 \text{ rivets}$$

Therefore, at least $\boxed{\text{512 rivets}}$ are needed to support the weight of the water in the viewing glass, plus additional rivets to support the weight of the glass itself. If a flat viewing glass is used instead, at least $\boxed{\text{385 rivets}}$ would be required.

The force on the top half of the viewing glass, $F_{\rm T}$, is given by

$$F_{\rm T} = \gamma A\bar{y} = \gamma\left[\frac{1}{2}\pi R^2\right]\left[5 - \frac{4R}{3\pi}\right] = (9.79)\left[\frac{1}{2}\pi(0.5)^2\right]\left[5 - \frac{4(0.5)}{3\pi}\right] = 18.41\,\text{kN}$$

The force on the bottom half of the viewing glass, $F_{\rm B}$, is given by

$$F_{\rm B} = \gamma A\bar{y} = \gamma\left[\frac{1}{2}\pi R^2\right]\left[5 + \frac{4R}{3\pi}\right] = (9.79)\left[\frac{1}{2}\pi(0.5)^2\right]\left[5 + \frac{4(0.5)}{3\pi}\right] = 20.04\,\text{kN}$$

Therefore the force ratio is $18.41/20.04 = \boxed{0.92}$. More rivets will be required on the $\boxed{\text{bottom}}$.

2.75. Take one-half of the trough as a free body. The horizontal component of the hydrostatic force, F_{h}, is given by

$$F_{\text{h}} = \gamma A\bar{y} = (9.79)(3 \times 0.5)\left(\frac{0.5}{2}\right) = 3.67\,\text{kN}$$

The vertical component of the hydrostatic force, F_{v}, is given by

$$F_{\text{v}} = \gamma V = (9.79)\left[\frac{1}{4}\pi(0.5)^2 \times 3\right] = 5.77\,\text{kN}$$

The line of action of F_{h} is y_{h} from the water surface, where

$$y_{\text{h}} = \bar{y} + \frac{I}{A\bar{y}} = \frac{0.5}{2} + \frac{\frac{3(0.5)^2}{12}}{(3)(0.5)\left(\frac{0.5}{2}\right)} = 0.333\,\text{m}$$

The line of action of F_{v} is x_{v} from the centerline of the trough, where

$$x_{\text{v}} = \frac{4R}{3\pi} = \frac{(4)(0.5)}{3\pi} = 0.2122\,\text{m}$$

Taking moments about the hinge gives

$$(R - y_{\text{h}})F_{\text{h}} + x_{\text{v}}F_{\text{v}} = RT$$
$$(0.5 - 0.333)(3.67) + (0.2122)(5.77) = (0.5)T$$

which yields $T = \boxed{3.67\text{ kN}}$.

2.76. Look at the gate as a free body as shown in Figure 2.10, where $2 - 2\sin 45° = 0.586\,\text{m}$. The net horizontal force, F_{H}, is given by

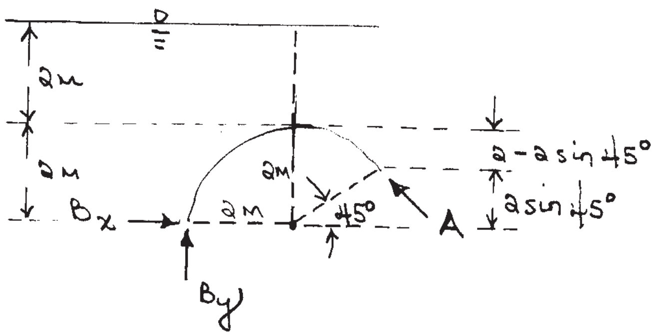


Figure 2.10: Gate as a Free Body

$$F_{\text{H}} = \gamma A_{1\text{v}}\bar{y}_1 - \gamma A_{2\text{v}}\bar{y}_2 = (10.05)\left[(2 \times 3)(3) - (0.586 \times 3)\left(2 + \frac{0.586}{2}\right)\right] = 140.4\,\text{kN}$$

The net vertical force, F_V, is given by

$$
\begin{aligned}
F_\mathrm{h} &= \gamma V \\
&= (10.05)\left[4\times 2 - \frac{1}{4}\pi(2)^2 + 4\times 2\cos 45^\circ - \left(\frac{1}{2}\times 2\cos 45^\circ\times 2\sin 45^\circ\right) - \frac{1}{8}\pi(2)^2\right](3) \\
&= 239.5\,\mathrm{kN}
\end{aligned}
$$

The net hydrostatic force on the gate acts through the center of the circle. Taking moments about the center of the circle gives

$$2B_y = 2A \rightarrow B_y = A \tag{1}$$

For equilibrium in the x-direction,

$$B_x + 140.4 = A\cos 45^\circ \tag{2}$$

and for equilibrium in the y direction

$$B_y + A\sin 45^\circ = 239.5\,\mathrm{kN} \tag{3}$$

Combining Equations 1 to 3 gives

$$\boxed{A = 140.3\,\mathrm{kN},\quad B_x = -41.2\,\mathrm{kN},\quad B_y = 140.3\,\mathrm{kN}}$$

2.77. From the given data: D = 400 mm, t = 4 mm, and p = 800 kPa. For equilibrium:

$$pA = 2\sigma Lt \quad\rightarrow\quad p(LD) = 2\sigma Lt \quad\rightarrow\quad \sigma = \frac{pD}{2t}$$

where σ is the circumferential stress, and L is any arbitrary length of pipe. Substituting given values:

$$\sigma = \frac{(800)(400)}{2(4)} = 40000\,\mathrm{kPa} = \boxed{40\,\mathrm{MPa}}$$

2.78. From the given data: W_air = 40 N, and W_water = 25 N. For water at 20°C, γ_w = 9.79 kN/m^3. Let γ_s be the density of the solid object and let V_s be its volume, then

$$\gamma_\mathrm{s}V_\mathrm{s} = 40\,\mathrm{N} \quad\rightarrow\quad V_\mathrm{s} = \frac{40}{\gamma_\mathrm{s}} \tag{1}$$

$$\gamma_\mathrm{s}V_\mathrm{s} - \gamma_\mathrm{w}V_\mathrm{s} = 25\,\mathrm{N} \quad\rightarrow\quad V_\mathrm{s} = \frac{25}{\gamma_\mathrm{s} - \gamma_\mathrm{w}} \tag{2}$$

Combining Equations 1 and 2 yields

$$\frac{40}{\gamma_\mathrm{s}} = \frac{25}{\gamma_\mathrm{s} - \gamma_\mathrm{w}} \quad\rightarrow\quad \frac{40}{\gamma_\mathrm{s}} = \frac{25}{\gamma_\mathrm{s} - 9.79} \quad\rightarrow\quad \gamma_\mathrm{s} = \boxed{26.1\,\mathrm{kN/m^3}}$$

The volume of the object, V_s, can therefore be estimated as

$$V_\mathrm{s} = \frac{40}{\gamma_\mathrm{s}} = \frac{40}{26.1\times 10^3} = \boxed{1.53\times 10^{-3}\,\mathrm{m^3}}$$

2.79. (a) Use the subscript "o" to denote the object, "w" to denote water, and "a" to denote air. The basic equations to be used are as follows:

$$W_\text{a} = \gamma_\text{o} V_\text{o}, \qquad W_\text{w} = \gamma_\text{o} V_\text{o} - \gamma_\text{w} V_\text{o}$$

where W represents weight, and V represents volume. Dividing the second equation by the first equation gives

$$\frac{W_\text{w}}{W_\text{a}} = 1 - \frac{\gamma_\text{w}}{\gamma_\text{o}} = 1 - \frac{1}{\text{SG}} \quad \rightarrow \quad \boxed{\text{SG} = \frac{1}{1 - W_\text{w}/W_\text{a}}}$$

(b) From the give data: $W_\text{a} = 40\,\text{N}$, and $W_\text{o} = 25\,\text{N}$. Substituting these data into the derived equation gives

$$\text{SG} = \frac{1}{1 - W_\text{w}/W_\text{a}} = \frac{1}{1 - 25/40} = \boxed{2.67}$$

2.80. From the given data: $D_\text{b} = 15\,\text{m}$, $R_\text{b} = D_\text{b}/2 = 7.5\,\text{m}$, and $W = 2\,\text{kN}$. For standard atmospheric conditions at sea level, $p_0 = 101.3\,\text{kPa}$ and $T_0 = 15°\text{C} = 288.15\,\text{K}$. For air, $R = 287.1\,\text{J/kg·K}$. The volume of the balloon, V_b, is given by

$$V_\text{b} = \tfrac{4}{3}\pi R_\text{b}^3 = \tfrac{4}{3}\pi(7.5)^3 = 1767\,\text{m}^3$$

At liftoff, the weight of the air in the balloon plus the attached weight to be lifted is equal to the volume of air displaced by the balloon. If T is the temperature of the air in the balloon under this condition, then using the ideal gas law to calculate the density of air gives

$$\frac{p_0 g}{RT} V_\text{b} + W = \frac{p_0 g}{RT_0} V_\text{b} \quad \rightarrow \quad T = \frac{p_0 g V_\text{b}}{R} \left[\frac{p_0 g V_\text{b}}{RT_0} - W \right]^{-1}$$

$$\rightarrow \quad T = \frac{(101.1 \times 10^3)(9.807)(1767)}{287.1} \left[\frac{(101.3 \times 10^3)(9.807)(1767)}{(287.1)(288.15)} - 2000 \right]^{-1} = 318.1\,\text{K}$$

Therefore, the temperature of the air in the balloon must be raised to $318.1\,\text{K} - 273.15\,\text{K} = \boxed{45.0°\text{C}}$.

2.81. From the given data: $D = 3\,\text{m}$, and $M = 8\,\text{kg}$. The volume of the balloon is given by

$$V = \frac{\pi D^3}{6} = \frac{\pi(3)^3}{6} = 14.14\,\text{m}^3$$

The balloon stabilizes when the weight of the balloon is equal to the weight of the air displaced by the balloon, which requires that

$$M\cancel{g} = \rho_\text{air}\cancel{g}V \quad \rightarrow \quad \rho_\text{air} = \frac{M}{V} = \frac{8}{14.14} \quad \rightarrow \quad \rho_\text{air} = 0.5658\,\text{kg/m}^3$$

Referring to the standard atmosphere in Appendix B.3, the density in the atmosphere is equal to 0.5658 kg/m^3 at an elevation of 7.38 km. Therefore, the balloon stabilizes at an elevation of $\boxed{7.38\text{ km}}$.

2.82. From the given data: $W = 1.5\,\text{kN}$, $p_\text{atm} = 101\,\text{kPa}$, $T_\text{a} = 20°\text{C} = 293.15\,\text{K}$, $w = 80\ \text{g/m}^2$, and $T_\text{b} = 80°\text{C} = 353.15\,\text{K}$. For air, $R = 287.1\,\text{J/kg·K}$. The density of the atmospheric air, ρ_a, and the density of the air in the balloon, ρ_b, can be derived from the ideal gas law as follows:

$$\rho_\text{a} = \frac{p}{RT_\text{a}} = \frac{101\times10^3}{(287.1)(293.15)} = 1.200\,\text{kg/m}^3, \qquad \rho_\text{b} = \frac{p}{RT_\text{b}} = \frac{101\times10^3}{(287.1)(353.15)} = 0.9962\,\text{kg/m}^3$$

Under stable conditions, the weight of the balloon plus the air in the balloon plus the supported weight is equal to the weight of the air displaced by the balloon, which requires that

$$W + wg\cdot\pi D^2 + \rho_\text{b} g\cdot\frac{\pi D^3}{6} = \rho_\text{a} g\cdot\frac{\pi D^3}{6}$$

$$1500 + (0.080)(9.807)\pi D^2 + (0.9962)(9.807)\frac{\pi D^3}{6} = (1.200)(9.807)\frac{\pi D^3}{6} \quad\rightarrow\quad \boxed{D = 12.1\,\text{m}}$$

2.83. When the sum of the forces equal zero,

$$F_\text{D} + F_\text{B} - W = 0 \quad\rightarrow\quad 3\pi\mu v D + \gamma_\text{w}\frac{\pi D^3}{6} - \gamma_\text{p}\frac{\pi D^3}{6} = 0$$

which simplifies to

$$v = \frac{1}{3\pi\mu D}(\gamma_\text{p} - \gamma_\text{w})\frac{\pi D^3}{6} \quad\rightarrow\quad v = \frac{(\gamma_\text{p} - \gamma_\text{w})D^2}{18\mu} \tag{1}$$

In this case, $\gamma_\text{p} = 2.65\gamma_\text{w} = 2.65(9.79) = 25.9\ \text{kN/m}^3 = 25900\ \text{N/m}^3$, $\gamma_\text{w} = 9.79\ \text{kN/m}^3 = 9790\ \text{N/m}^3$, $D = 2\ \text{mm} = 0.002\ \text{m}$, and $\mu = 1.00\times10^{-3}\ \text{N·s/m}^2$ at 20°C. Substituting into Equation 1 gives

$$v = \frac{(25900 - 9790)(0.002)^2}{18(1.00\times10^{-3})} = \boxed{3.58\ \text{m/s}}$$

2.84. From the given data: $L = 10$ m, $L_1 = 7$ m, $L_2 = 3$ m, $W = 15$ m, $H = 4$ m, $\text{SG}_1 = 1.5$, and $\text{SG}_2 = 3.0$. For fresh water at 20°C, $\gamma_\text{fw} = 9.79\ \text{kN/m}^3$. For fresh water at 4°C, $\gamma_\text{w} = 9.807\ \text{kN/m}^3$. The specific weights of the two parts of the body are calculated as follows,

$$\text{specific weight of light section, } \gamma_1 = \text{SG}_1\cdot\gamma_\text{w} = 1.5(9.807) = 14.71\,\text{kN/m}^3$$
$$\text{specific weight of heavy section, } \gamma_2 = \text{SG}_2\cdot\gamma_\text{w} = 3.0(9.807) = 29.42\,\text{kN/m}^3$$

The volumes of the sections of cabin are calculated as follows,

$$\text{volume light section, } V_1 = L_1\times W\times H = (7)(15)(4) = 420\,\text{m}^3$$
$$\text{volume of heavy section, } V_2 = L_2\times W\times H = (3)(15)(4) = 180\,\text{m}^3$$
$$\text{volume of entire cabin, } V_\text{c} = V_1 + V_2 = 420 + 180 = 600\,\text{m}^3$$

The forces on various parts of the body are as follows,

$$\text{buoyant force on the entire body, } F_\text{c} = \gamma_\text{fw}V_\text{c} = (9.79)(600) = 5874\,\text{kN}$$
$$\text{weight of light section, } W_1 = \gamma_1 V_1 = (14.71)(420) = 6178\,\text{kN}$$

$$\text{weight of heavy section, } W_2 = \gamma_2 V_2 = (29.42)(180) = 5296\,\text{kN}$$

If the support force is F, then equilibrium of forces in the vertical direction requires that

$$F = W_1 + W_2 - F_\text{c} = 6178 + 5296 - 5874 = \boxed{5600\,\text{kN}}$$

Considering the cabin as a free body and taking moments about the centroid, accounting for the fact that the buoyant force acts through the centroid of the body, the moment equation gives

$$W_1 \cdot \left[\frac{L - L_1}{2}\right] + F \cdot x = W_2 \cdot \left[\frac{L - L_2}{2}\right]$$

$$(6178) \cdot \left[\frac{10 - 7}{2}\right] + (5600) \cdot x = (5296) \cdot \left[\frac{10 - 3}{2}\right] \quad \rightarrow \quad x = \boxed{1.66\,\text{m}}$$

2.85. From the given data: $L_\text{c} = 0.15\,\text{m}$, and $f_\text{o} = 0.15$. For water at 20°C, $\rho_\text{w} = 998.2\,\text{kg/m}^3$ (Appendix B.1), and for SAE 30 oil at 20°C, $\rho_\text{o} = 918\,\text{kg/m}^3$ (Appendix B.4). If ρ_c is the density of the cube, noting that the buoyancy force is equal to the weight of fluid displaced, then for equilibrium,

$$\rho_\text{c} L^3 g = L^3 g[f_\text{o}\rho_\text{o} + (1 - f_\text{o})\rho_\text{w}] \quad \rightarrow \quad \rho_\text{c} = f_\text{o}\rho_\text{o} + (1 - f_\text{o})\rho_\text{w}$$

$$\rightarrow \quad \rho_\text{c} = (0.15)(918) + (1 - 0.15)(998.2) = \boxed{986\,\text{kg/m}^3}$$

2.86. The weight, W, of a floating object in a fluid of specific weight γ_f is related to the displacement volume, V, by the relation: $W = \gamma_\text{f} V$. Therefore,

$$\boxed{\frac{W_2 - W_1}{W_1} \times 100 = \frac{\gamma_\text{f} V_2 - \gamma_\text{f} V_1}{\gamma_\text{f} V_1} \times 100 = \frac{V_2 - V_1}{V_1} \times 100}$$

This shows that the percentage change in V is the same as the percentage change in W.

2.87. From the given data: $f = 0.90$, $t = 25\,\text{mm}$, $W_\text{i} = 500\,\text{N}$, and $\rho_\text{s} = 8000\,\text{kg/m}^3$. For seawater, $\rho_\text{s} = 1023\,\text{kg/m}^3$ (from Appendix B.4). The specific weights corresponding to the given densities are $\gamma_\text{s} = 78.56\,\text{kN/m}^3$ and $\gamma_\text{w} = 10.03\,\text{kN/m}^3$. If D is the (outer) diameter of the sphere, then for 90% (= f) of the sphere below water, putting the buoyant force equal to the weight of the sphere plus the instrumentation gives

$$f \cdot \gamma_\text{w} \frac{\pi D^3}{6} = \gamma_\text{s} \left[\frac{\pi D^3}{6} - \frac{\pi (D - 2t)^3}{6}\right] + W_\text{i}$$

$$(0.9)(10.03)\frac{\pi D^3}{6} = (78.56) \left[\frac{\pi D^3}{6} - \frac{\pi (D - 50 \times 10^{-3})^3}{6}\right] + 500 \times 10^{-3}$$

which yields $\boxed{D = 1.12\,\text{m}}$.

2.88. From the given data: $d_1 = 1\,\text{m}$, $d_2 = 0.1\,\text{m}$, and SG $= 0.85$. Assume that the specific gravity specification applies to this object in this water. Use the subscript "o" to denote the total object, the subscript "s" to denote the portion of the object that is submerged, and the subscript "w" to denote water. Taking L to be the length of the object, for vertical equilibrium,

$$\gamma_\text{o} V_\text{o} = \gamma_\text{w} V_\text{s} \quad \rightarrow \quad \gamma_\text{o} A_\text{o} \cancel{L} = \gamma_\text{w} A_\text{s} \cancel{L} \quad \rightarrow \quad A_\text{s} = \left(\frac{\gamma_\text{o}}{\gamma_\text{w}}\right) A_\text{o} \quad \rightarrow \quad A_\text{s} = \text{SG} \cdot \pi R^2 \qquad (1)$$

The geometry of the partially submerged object is shown in Figure 2.11.

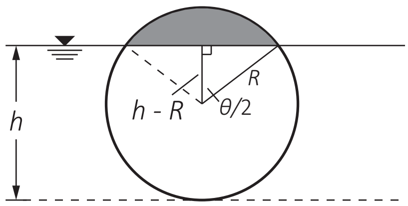


Figure 2.11: Geometry of partially submerged object

In this case, $h = d_1 - d_2 = 1\,\text{m} - 0.1\,\text{m} = 0.9\,\text{m}$ and the submerged area, A_s, is given by

$$A_\text{s} = \pi R^2 - \left[R^2\left(\frac{\theta}{2}\right) - R\sin\left(\frac{\theta}{2}\right)(h - R)\right], \quad \text{where} \quad \frac{\theta}{2} = \cos^{-1}\left(\frac{h-R}{R}\right) \qquad (2)$$

Combining Equations 1 and 2 and solving for R (with $h = 0.9\,\text{m}$ and SG $= 0.85$) yields $R = 0.568\,\text{m}$ and hence $D = 2R = 1.14\,\text{m}$. Therefore, the maximum diameter of the object that will satisfy the given constraints is $\boxed{1.14\text{ m}}$.

2.89. From the given data: $W_1 = 800$ N and $W_2 = 200$ N. For water at 20°C, $\gamma_\text{w} = 9790$ N/m^3. Let L be any given load carried by the canoe, and let V_1 and V_2 be the displacement volumes corresponding to W_1 and W_2, respectively, then

$$L + \gamma_\text{w} V_1 = W_1 \quad \rightarrow \quad L + 9790\, V_1 = 800 \qquad (1)$$

$$L + \gamma_\text{w} V_2 = W_2 \quad \rightarrow \quad L + 9790\, V_2 = 200 \qquad (2)$$

Subtracting Equations 1 and 2 to eliminate L gives

$$V_1 - V_2 = \frac{800 - 200}{9790} = \boxed{0.0613\,\text{m}^3}$$

2.90. Let V_t be the total volume of the body, V_a be the volume of the body above the surface of the liquid, γ_1 be the specific weight of the body, and γ_2 be the specific weight of the liquid. For equilibrium,

$$\gamma_1 V_\text{t} = \gamma_2 (V_\text{t} - V_\text{a}) \quad \rightarrow \quad \text{SG}_1 \cdot V_\text{t} = \text{SG}_2 \cdot (V_\text{t} - V_\text{a}) \quad \rightarrow \quad \frac{V_\text{a}}{V_\text{t}} = \frac{\text{SG}_2 - \text{SG}_1}{\text{SG}_2}$$

The fraction, f_a, of the body that is above the water surface is given by

$$\boxed{f_\text{a} = \frac{V_\text{a}}{V_\text{t}} = \frac{\text{SG}_2 - \text{SG}_1}{\text{SG}_2}} \qquad (1)$$

In the case of the iceberg in seawater, $SG_1 = 0.92$ and $SG_2 = 1.03$, and Equation 1 gives

$$f_a = \frac{1.03 - 0.92}{1.03} = \boxed{0.11}$$

2.91. From the given data: SG = 0.8, and $D = 10\,\text{mm}$. Let N = number of bubbles per m^3, V_b = volume of each bubble, ρ = density of water, and ρ_b = density of bubbly water. Using these definitions and neglecting the mass of air in the bubbles,

$$\text{volume of air in 1 m}^3 = NV_b$$

$$\text{volume of water in 1 m}^3 = 1 - NV_b$$

$$\text{mass of water in 1 m}^3 = \rho(1 - NV_b)$$

$$\text{density of bubbly water, } \rho_b = \rho(1 - NV_b)$$

The ship sinks when the density of the bubbly water is equal to the density of the ship, in which case

$$\text{SG} \cdot \rho = \rho_b \quad \rightarrow \quad \text{SG} \cdot \rho = \rho(1 - NV_b) \quad \rightarrow \quad N = \frac{1 - \text{SG}}{V_b} \tag{1}$$

From the given bubble diameter, $V_b = \frac{1}{6}\pi D^3 = 5.236 \times 10^{-7}\,\text{m}^3$, and Equation 1 gives

$$N = \frac{1 - 0.8}{5.236 \times 10^{-7}} = \boxed{3.82 \times 10^5 \text{ bubbles/m}^3}$$

2.92. For equilibrium, the weight of the pool must at least equal the buoyant force. Let the depth of water in the pool be x, then for equilibrium (see Figure 2.12)

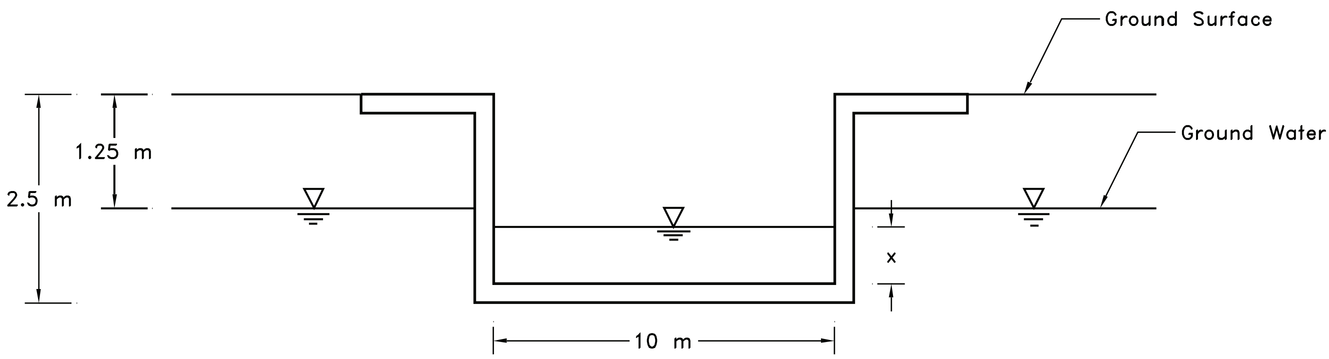


Figure 2.12: Swimming pool

$$\begin{aligned}
\text{Wt. of pool} + \text{Wt. of water in pool} &= \text{Buoyant force} \\
500 + (10 \times 5 \times x)(9.79) &= [10 \times 5 \times (2.5 - 1.25)](9.79 \\
500 + 490x &= 612 \\
x &= 0.23\,\text{m}
\end{aligned}$$

Therefore at least $\boxed{23 \text{ cm}}$ of water must be maintained in the pool.

2.93. When the barge is fully loaded, the draft, V, is given by

$$\gamma_{\rm sw}V = (20 + 250)\,\text{kN}$$

For $S = 1.03$,

$$V = \frac{20 + 250}{(1.03)(9.79)} = 26.78\,\text{m}^3 \tag{1}$$

and from geometry

$$V = \left(6y + 2\frac{y^2}{4}\right)(3) = 18y + \frac{3}{2}y^2 \tag{2}$$

Combining Equations 1 and 2 gives

$$18y + \frac{3}{2}y^2 = 26.8$$

and solving gives the draft as $y = \boxed{1.34\text{ m}}$.

2.94. From the given data: $f_1 = 0.75$, and $f_2 = 0.90$. For water, $\rho_{\rm w} = 998$ kg/m^3. The average density of the body, $\bar{\rho}_{\rm b}$, can be derived from Equation 2.75 as follows

$$\frac{V_{\rm f}}{V_{\rm b}} = \frac{\bar{\rho}_{\rm b}}{\rho_{\rm f}} \quad \rightarrow \quad f_1 = \frac{\bar{\rho}_{\rm b}}{\rho_{\rm w}} \quad \rightarrow \quad \bar{\rho}_{\rm b} = f_1\rho_{\rm w} = (0.75)(998) = \boxed{749\,\text{kg/m}^3}$$

The average density of the solid material is $\bar{\rho}_{\rm s}$ and the fraction of the body that is open space is f_2. Representing the mass of the solid by $M_{\rm s}$ and the volume of the solid by $V_{\rm s}$, then

$$\bar{\rho}_{\rm b} = \frac{M_{\rm s}}{V_{\rm b}} = \frac{M_{\rm s}}{V_{\rm s}/(1-f_2)} = (1-f_2)\bar{\rho}_{\rm s} \quad \rightarrow \quad \bar{\rho}_{\rm s} = \frac{1}{1-f_2}\bar{\rho}_{\rm b}$$

Substituting the known values of f_2 and $\bar{\rho}_{\rm b}$ yields

$$\bar{\rho}_{\rm s} = \frac{1}{1-0.9}(749) = \boxed{7490\,\text{kg/m}^3}$$

2.95. From the given data: $H = 2\,\text{m}$, $A = LW$, $\text{SG}_1 = 1.2$, $\Delta z_1 = 1.2\,\text{m}$, and $\text{SG}_2 = 1.6$. Denote the specific weights of the body, top layer, and bottom layer by $\gamma_{\rm b}$, γ_1, and γ_2, respectively. For vertical equilibrium, where h is the depth of penetration into the bottom layer,:

$$\gamma_{\rm b}AH = \gamma_1 A\Delta z_1 + \gamma_2 Ah \quad \rightarrow \quad h = \frac{\gamma_{\rm b}}{\gamma_2}H - \frac{\gamma_1}{\gamma_2}\Delta z_1 \quad \rightarrow \quad h = \frac{\text{SG}_{\rm b}}{\text{SG}_2}H - \frac{\text{SG}_1}{\text{SG}_2}\Delta z_1 \tag{1}$$

(a) The minimum specific gravity of the body for full penetration of the top layer can be derived by setting $h = 0$ in Equation 1 which gives

$$\text{SG}_{\rm b}\cdot H = \text{SG}_1\Delta z_1 \quad \rightarrow \quad \text{SG}_{\rm b} = \frac{\Delta z_1}{H}\text{SG}_1 = \frac{1.2}{2}(1.2) = \boxed{0.72}$$

(b) The depth of penetration, h, into the bottom layer when $\text{SG}_{\rm b} = 1.0$ can be obtained by substituting directly into Equation 1, which yields

$$h = \frac{1.0}{1.6}(2) - \frac{1.2}{1.6}(1.2) = \boxed{0.35\,\text{m}}$$

2.96. From the given data: $L = 3$ m, $D = 200$ mm $= 0.2$ m, and SG $= 0.6$. For seawater, $\gamma_{\text{sw}} = 10.03$ kN/m^3 (from Appendix B.4). The the relevant forces and dimensions in the problem are shown in Figure 2.13, where $\mathbf{W}$ is the weight of the buoy, $\mathbf{T}$ is the tension in the support cable, and $\mathbf{F}_\text{b}$ is the buoyant force.

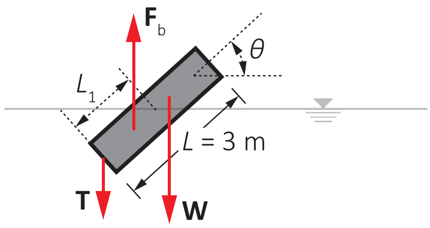


Figure 2.13: Buoy at low tide

Let A be the cross-sectional area of the buoy and V_b be the volume of the buoy, then

$$A_\text{buoy} = \frac{\pi D^2}{4} = \frac{\pi 0.2^2}{4} = 0.03142\,\text{m}^2$$

$$V_\text{buoy} = A_\text{buoy} L = (0.03142)(3) = 0.09426\,\text{m}^3$$

The magnitudes of $\mathbf{W}$ and $\mathbf{F}_\text{b}$ can be expressed in terms of other variables as follows:

$$W = \gamma_\text{buoy} V_\text{buoy}$$
$$F_\text{b} = \gamma_\text{sw} V_\text{sub}$$

(a) This is the case of the partially submerged buoy. Taking moments about the point where the support cable is attached to the buoy,

$$\gamma_\text{buoy} \cancel{A_\text{buoy}} L \frac{L}{\cancel{2}} \cancel{\cos\theta} = \gamma_\text{sw} \cancel{A_\text{buoy}} L_1 \frac{L_1}{\cancel{2}} \cancel{\cos\theta} \rightarrow \frac{L_1}{L} = \sqrt{\frac{\gamma_\text{buoy}}{\gamma_\text{sw}}} = \sqrt{\text{SG}} \qquad (1)$$

For vertical equilibrium,

$$T + \gamma_\text{buoy} V_\text{buoy} = \gamma_\text{sw} V_\text{sub} \quad \rightarrow \quad T + \gamma_\text{buoy} A_\text{buoy} L = \gamma_\text{sw} A_\text{buoy} L_1$$
$$\rightarrow \quad T + \text{SG} \cdot \gamma_\text{sw} A_\text{buoy} L = \gamma_\text{sw} A_\text{buoy} L_1$$

which simplifies to

$$T = \gamma_\text{sw} A_\text{buoy} L \left[\frac{L_1}{L} - \text{SG}\right] = \gamma_\text{sw} V_\text{buoy} \left[\frac{L_1}{L} - \text{SG}\right] \qquad (2)$$

Combining Equations 1 and 2 and evaluating gives

$$T = \gamma_\text{sw} V_\text{buoy} \left[\sqrt{\text{SG}} - \text{SG}\right] = (10.03)(0.09426)\left[\sqrt{0.6} - 0.6\right] = \boxed{0.165\,\text{kN}}$$

(b) This is the case of the fully submerged buoy. For vertical equilibrium,

$$T = \gamma_\text{sw} V_\text{buoy} - \gamma_\text{buoy} V_\text{buoy} = \gamma_\text{sw} V_\text{buoy}(1 - \text{SG}) = (10.03)(0.09426)(1 - 0.6) = \boxed{0.378\,\text{kN}}$$

2.97. From the given data: $W = 0.246$ N and $D_0 = 10$ mm $= 0.01$ m. For pure water at 20°C, $\gamma_\mathrm{w} = 9790$ N/m^3. Calculate the cross-sectional area, A_0, of the hydrometer stem and the volume, V_0, of the hydrometer below the water mark in pure water:

$$A_0 = \frac{\pi}{4}D_0^2 = \frac{\pi}{4}(0.01)^2 = 7.854 \times 10^{-5}\,\mathrm{m}^2$$
$$\gamma_\mathrm{w} V_0 = 0.246\,\mathrm{N} \rightarrow 9790V_0 = 0.246 \rightarrow V_0 = 2.513 \times 10^{-5}\,\mathrm{m}^3$$

For $\Delta h = 2$ cm $= 0.02$ m, the specific gravity of the fluid, $\mathrm{SG_f}$, is given by

$$\mathrm{SG_f} = \frac{V_0}{V_0 + A_0\Delta h} = \frac{2.513}{2.513 + (7.854)(0.02)} = \boxed{0.94}$$

2.98. From the given data: $D = 9\,\mathrm{mm}$, $V_0 = 20\,\mathrm{cm}^3 = 2 \times 10^4\,\mathrm{mm}^3$, and $\mathrm{SG} = 1.2$. The cross-sectional area, A_0, of the hydrometer stem is given by

$$A_0 = \frac{\pi}{4}D^2 = \frac{\pi}{4}(9)^2 = 63.62\,\mathrm{mm}^2$$

The relationship between the hydrometer displacement, Δh, and the specific gravity, SG, is given by Equation 2.78 as

$$\mathrm{SG} = \frac{V_0}{V_0 + A_0\Delta h} \quad \rightarrow \quad \Delta h = \left[\frac{V_0}{\mathrm{SG}} - V_0\right]\frac{1}{A_0} = \left[\frac{2 \times 10^4}{1.2} - 2 \times 10^4\right]\frac{1}{63.62} = -52.4\,\mathrm{mm}$$

Therefore the distilled-water mark will be $\boxed{52.4\ \mathrm{mm}}$ above the liquid surface.

2.99. From the given data: $D = 0.70\,\mathrm{m}$, $R = D/2 = 0.35\,\mathrm{m}$, $L = 0.60\,\mathrm{m}$, $\mathrm{SG_b} = 0.65$, and $\mathrm{SG}_\ell = 0.90$. The centroidal moment of inertia, I_{00}, of the circular area that intersects the liquid surface is given by (Appendix C.1):

$$I_{00} = \frac{\pi R^4}{4} = \frac{\pi(0.35)^4}{4} = 1.179 \times 10^{-2}\,\mathrm{m}^4 \tag{1}$$

When the cylindrical body is placed in the liquid, the weight of the body is equal to the weight of the liquid displaced, which requires that

$$\gamma_\ell V_\mathrm{sub} = \gamma_\mathrm{b}\pi R^2 L \quad \rightarrow \quad V_\mathrm{sub} = \frac{\gamma_\mathrm{b}}{\gamma_\ell}\pi R^2 L = \frac{0.65}{0.90}\pi(0.35)^2(0.60) \quad \rightarrow \quad V_\mathrm{sub} = 0.1668\,\mathrm{m}^3 \tag{2}$$

where the relation $\gamma_\mathrm{b}/\gamma_\ell = \mathrm{SG_b}/\mathrm{SG}_\ell$ has been used. The submerged height, h, can be derived from the buoyancy relationship given in Equation 2 as follows,

$$V_\mathrm{sub} = \frac{\gamma_\mathrm{b}}{\gamma_\ell}\pi R^2 L \quad \rightarrow \quad \cancel{\pi R^2} h = \frac{\gamma_\mathrm{b}}{\gamma_\ell}\cancel{\pi R^2} L \quad \rightarrow \quad h = \frac{\mathrm{SG_b}}{\mathrm{SG}_\ell}L = \frac{0.65}{0.90}(0.60) = 0.4333\,\mathrm{m}$$

The distance between the center of gravity and the center of buoyancy, GB, is therefore given by

$$\mathrm{GB} = \frac{L}{2} - \frac{h}{2} = \frac{0.60}{2} - \frac{0.4333}{2} = 0.0833\,\mathrm{m} \tag{3}$$

The metacentric height, GM, can be calculated from Equation 2.79 using the results from Equations 1 to 3 to yield

$$\mathrm{GM} = \frac{I_{00}}{V_\mathrm{sub}} - \mathrm{GB} = \frac{1.179\times10^{-2}}{0.1668} - 0.0833 = \boxed{-0.0127\,\mathrm{m}}$$

Since the metacentric height is negative, the cylindrical body is $\boxed{\text{unstable}}$ at the orientation at which it is placed in the liquid.

NEW From the given data: $D = 300\,\mathrm{mm}$, $R = D/2 = 150\,\mathrm{mm} = 0.15\,\mathrm{m}$, and $\rho_\mathrm{log} = 512\,\mathrm{kg/m^3}$. For water at 20°C, $\rho_\mathrm{wat} = 998\,\mathrm{kg/m^3}$. Let x be the depth of the log below water when the axis is vertical, then for equilibrium

$$\rho_\mathrm{log}\cancel{g\pi R^2}L = \rho_\mathrm{wat}\cancel{g\pi R^2}x \quad\rightarrow\quad x = \frac{\rho_\mathrm{log}}{\rho_\mathrm{wat}}L = \frac{512}{998}L \quad\rightarrow\quad x = 0.5130L$$

At the limit of stability,

$$\mathrm{GM} = 0 = \frac{I_{00}}{V_\mathrm{sub}} - \mathrm{GB} \quad\rightarrow\quad 0 = \frac{\dfrac{\pi R^4}{4}}{\pi R^2 x} - \left(\frac{L}{2} - \frac{x}{2}\right)$$

substituting known quantities gives

$$0 = \frac{\dfrac{\pi\,0.15^4}{4}}{\pi\,0.15^2(0.5130L)} - \left(\frac{L}{2} - \frac{0.5130L}{2}\right) \quad\rightarrow\quad \boxed{L = 0.212\,\mathrm{m}}$$

NEW From the given data: $r_\mathrm{b} = \frac{1}{2}(4\,\mathrm{m}) = 2\,\mathrm{m}$, $h_\mathrm{b} = 2\,\mathrm{m}$, $r_\mathrm{p} = \frac{1}{2}(2\,\mathrm{m}) = 1\,\mathrm{m}$, $\rho_\mathrm{w} = 998\,\mathrm{kg/m^3}$, and $\rho_\mathrm{b} = 170\,\mathrm{kg/m^3}$. Let h be the height of the pole. The following equations must be satisfied:

$$V = \pi r_\mathrm{p}^2 h_\mathrm{p} + \pi r_\mathrm{b}^2 h_\mathrm{b}, \qquad y_\mathrm{g} = \frac{\pi r_\mathrm{p}^2 h_\mathrm{p}(h_\mathrm{b} + \frac{1}{2}h_\mathrm{p}) + \pi r_\mathrm{b}^2 h_\mathrm{b}(\frac{1}{2}h_\mathrm{b})}{V}, \qquad V_\mathrm{sub} = \pi r_\mathrm{b}^2 h_\mathrm{sub} = \frac{\gamma_\mathrm{b}}{\gamma_\mathrm{w}}V$$

$$h_\mathrm{sub} = \frac{\gamma_\mathrm{w} V}{\gamma_\mathrm{b}\pi r_\mathrm{b}^2}, \qquad I_{00} = \tfrac{1}{4}\pi r_\mathrm{b}^2, \qquad \mathrm{GB} = y_\mathrm{g} - \tfrac{1}{2}h_\mathrm{sub}$$

$$\mathrm{GM} = \frac{I_{00}}{V_\mathrm{sub}} - \mathrm{GB}, \qquad \mathrm{GM} = 0, \qquad \mathrm{GB} = \frac{I_{00}}{V_\mathrm{sub}}$$

Noting that $\gamma_\mathrm{w}/\gamma_\mathrm{b} = 170/998 = 0.1703$, substituting the given data and combining the above equations yield the following equation for h_p:

$$h_\mathrm{p}\left(2 + \tfrac{1}{2}h_\mathrm{p}\right) - 0.02129(h_\mathrm{p} + 8)^2 - 0.1549 = 0 \quad\rightarrow\quad h_\mathrm{p} = \boxed{4.45\,\mathrm{m}}$$

A final check is necessary to ensure that the base is not submerged in water. Calculating h_sub gives

$$h_\mathrm{sub} = \left(\frac{\gamma_\mathrm{b}}{\gamma_\mathrm{w}}\right)\frac{V}{\pi r_\mathrm{b}^2} = 0.53\,\mathrm{m}$$

Since the base is not submerged ($h_\mathrm{sub} < h_\mathrm{b}$) the calculations are validated.

2.100. At the limit of stability, the metacentric height is equal to zero, so

$$\text{GM} = \frac{I_{00}}{V_{\text{sub}}} - \text{GB} \quad \rightarrow \quad \frac{I_{00}}{V_{\text{sub}}} - \text{GB} = 0 \quad \rightarrow \quad \text{GB} = \frac{I_{00}}{V_{\text{sub}}}$$

$$I_{00} = \frac{bd^3}{12} = \frac{4(0.70)^3}{12} = 0.114\,\text{m}^4$$

$$V_{\text{sub}} = \left[\frac{1}{2}(0.70)(0.15) + (0.3)(0.7)\right](4) = 1.05\,\text{m}^3$$

$$\text{GB} = \frac{I_{00}}{V_{\text{sub}}} = \frac{0.114}{1.05} = 0.109\,\text{m}$$

Find the distance of the center of buoyancy, z_B above the bottom:

$$z_\text{B} = \frac{A_1 z_1 + A_2 z_2}{A_1 + A_2} = \frac{(0.7 \times 0.3)(0.15 + 0.30/2) + (\frac{1}{2} \times 0.7 \times 0.15)(\frac{2\times 0.15}{3})}{0.21 + 0.0525} = 0.26\,\text{m}$$

If the height of the center of gravity above the bottom is z_G, then

$$z_\text{G} - z_\text{B} = 0.109\,\text{m} \rightarrow z_\text{G} = z_\text{B} + 0.109 = 0.26 + 0.109 = 0.37\,\text{m}$$

Hence, the limit of stability occurs when the center of gravity is $\boxed{0.37\text{ m}}$ above the bottom of the canoe.

2.101. From the given data: D = 0.3 m, Δz = 0.5 m, a_x = 0, and a_z = 1 m/s^2. For kerosene at 20°C, ρ = 808 kg/m^3 (from Appendix B.4). Applying Equation 2.95 gives the pressure increase from bottom to top, Δp, as follows

$$\Delta p = -\rho a_x \Delta x - \rho(g + a_z)\Delta z = -808(0)(0) - 808(9.81 + 1)(0.5) = -4367\,\text{Pa} = -4.37\,\text{kPa}$$

Therefore, the gauge pressure on the bottom of the cylinder is $\boxed{4.37\text{ kPa}}$. The area, A, of the bottom of the container is given by

$$A = \frac{1}{4}\pi D^2 = \frac{1}{4}\pi(0.3)^2 = 0.07069\,\text{m}^2$$

Hence the force, F, exerted by the fluid in the cylinder on the elevator is given by

$$F = p_{\text{bottom}} A = (4.37)(0.07069) = 0.309\,\text{kN} = \boxed{309\,\text{N}}$$

2.102. From the given data: Δx = 10 m, D = 2 m, a_x = 2 m/s^2, and a_z = 0. For water at 20°C, ρ = 998 kg/m^3. The pressure difference, Δp, between opposing locations at the front and back of the tank is given by Equation 2.95 as

$$\Delta p = -\rho a_x \Delta x - \rho(g + a_z)\Delta z \quad \rightarrow \quad \Delta p = -(998)(2)(10) - 0 = 19\ 960\,\text{Pa} = 19.96\,\text{kPa}$$

The area of the front and back of the tank is $A = \pi D^2/4$ = 3.142 m^2, so the force difference, ΔF, is given by

$$\Delta F = \Delta p \cdot A = (19.96)(3.142) = \boxed{62.7\,\text{kN}}$$

2.103. From the given data: $\rho = 1040\,\text{kg/m}^3$. For spillage to occur: $\Delta z = -0.8$ m for $\Delta x = 2$ m, and $a_z = 0$.

(a) Taking $g = 9.81$ m/s^2, the limiting acceleration, a_x, is obtained from Equation 2.96 as follows

$$\frac{\Delta z}{\Delta x} = -\frac{a_x}{g + a_z} \quad \rightarrow \quad \frac{-0.8}{2} = -\frac{a_x}{9.81 + 0} \quad \rightarrow \quad a_x = \boxed{3.92 \text{ m/s}^2}$$

(b) Under the limiting (spill) condition, the depth of liquid at the front of the tank is $1.2\,\text{m} - 0.8\,\text{m} = 0.4\,\text{m}$. In accordance with Equation 2.23, the gauge pressure, p_bf, at the bottom front of the tank is therefore given by

$$p_\text{bf} = \rho g (0.4\,\text{m}) = (1040)(9.807)(0.4) = 4.08 \times 10^3\,\text{Pa} = \boxed{4.08\,\text{kPa}}$$

Under the limiting (spill) condition, the depth of liquid at the back of the tank is 2 m. In accordance with Equation 2.23, the gauge pressure, p_bb, at the bottom back of the tank is therefore given by

$$p_\text{bb} = \rho g (2\,\text{m}) = (1040)(9.807)(2) = 2.04 \times 10^4\,\text{Pa} = \boxed{20.4\,\text{kPa}}$$

2.104. From the given data: $L = 3$ m, $W = 0.8$ m, $H = 1.6$ m, and $d = 1.2$ m. Consider both alignments of the tank separately.

(a) Long side aligned with the direction of truck motion. For spillage to occur: $\Delta z = 0.4$ m for $\Delta x = 1.5$ m. The limiting acceleration, a_x, is obtained from Equation 2.96 as follows

$$\frac{\Delta z}{\Delta x} = \frac{a_x}{g} \quad \rightarrow \quad \frac{0.4}{1.5} = \frac{a_x}{9.81} \quad \rightarrow \quad a_x = 2.62 \text{ m/s}^2$$

(b) Short side aligned with the direction of truck motion. For spillage to occur: $\Delta z = 0.4$ m for $\Delta x = 0.4$ m. The limiting acceleration, a_x, is obtained from Equation 2.96 as follows

$$\frac{\Delta z}{\Delta x} = \frac{a_x}{g} \quad \rightarrow \quad \frac{0.4}{0.4} = \frac{a_x}{9.81} \quad \rightarrow \quad a_x = 9.81 \text{ m/s}^2$$

Therefore, the maximum allowable acceleration of $\boxed{9.81 \text{ m/s}^2}$ occurs with the $\boxed{\text{side orientation}}$.

2.105. From the given data: $\theta = 10°$. Applying Equation 2.96 gives

$$\tan\theta = -\frac{a_x}{g} \quad \rightarrow \quad \tan 10° = -\frac{a_x}{9.81} \quad \rightarrow \quad a_x = \boxed{-1.73 \text{ m/s}^2}$$

2.106. From the given data: $\Delta V = 90$ km/h $= 25$ m/s, and $\Delta t = 10$ s. Since the truck decelerates at a constant rate,

$$a_x = \frac{\Delta V}{\Delta t} = \frac{25}{10} = 2.5 \text{ m/s}^2$$

Let θ be the slope of the liquid surface, then Equation 2.96 gives

$$\tan\theta = -\frac{a_x}{g} \quad \rightarrow \quad \tan\theta = -\frac{2.5}{9.81} \quad \rightarrow \quad \theta = \boxed{-14.3°}$$

2.107. From the given data: $a = 5\,\text{m/s}^2$ and $\theta = 25°$. The components of the acceleration are:

$$a_x = 5\cos 25° = 4.532\,\text{m/s}^2$$

$$a_z = 5\sin 25° = 2.113\,\text{m/s}^2$$

Taking $g = 9.807\,\text{m/s}^2$ and substituting into Equation 2.96 gives

$$\frac{\Delta z}{\Delta x} = -\frac{a_x}{g + a_z} = -\frac{4.532}{9.807 + 2.113} = -0.3802$$

The slope of the surface in the tanker is therefore equal to $\tan^{-1}(0.3802) = \boxed{20.1°}$. Therefore the slope of the liquid in the tank is $\boxed{\text{less than}}$ the slope of the incline.

2.108. Under the given conditions, the truck is free-falling down the incline with $a_x = 0$ and $a_z = -g$. Substituting these values into Equation 2.96 gives

$$\frac{\Delta z}{\Delta x} = -\frac{a_x}{g + a_z} = -\frac{0}{g - g} = \frac{0}{0}$$

Therefore, the slope of the water surface is $\boxed{\text{indeterminate}}$. This result is a consequence of there being no effective gravity force to keep the liquid contained in the tank.

2.109. From the given data: W = 300 mm = 0.3 m, H = 280 mm = 0.28 m, and p_atm = 101 kPa. For water at 20°C, ρ = 998 kg/m^3 and p_svp = 2.34 kPa (from Appendix B.4).

(a) In this case Δz = 40 mm and Δx = 300 mm, and Equation 2.96 gives the required acceleration, a_x, as

$$\frac{\Delta z}{\Delta x} = -\frac{a_x}{g + a_z} \rightarrow \frac{-40}{300} = -\frac{a_x}{9.81 + 0} \quad \rightarrow \quad a_x = \boxed{1.308\,\text{m/s}^2}$$

(b) In this case $r_1 = 0$, $r_2 = 0.15$ m, $z_1 = 0$, $z_2 = 0.28$ m, $p_1 = p_\text{svp} = 2.34$ kPa, and $p_2 = p_\text{atm} = 101$ kPa. The required rate of rotation, ω, for these conditions to occur is given by Equation 2.104 as follows:

$$p_2 - p_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g[z_2 - z_1]$$

$$101 - 2.34 = \frac{(998)\omega^2}{2}(0.28^2 - 0^2) - (998)(9.81)[0.28 - 0] \rightarrow \omega = 15.90\,\text{rad/s} = \boxed{152\,\text{rpm}}$$

2.110. From the given data: $z_1 - z_2$ = 40 mm = 0.04 m, r_1 = 0.15 m + 0.05 m = 0.20 m, and r_2 = 0.15 m − 0.05 m = 0.10 m. The corresponding rate of rotation can be derived from Equation 2.103 as follows

$$z_1 - z_2 = \frac{\omega^2}{2g}(r_1^2 - r_2^2) \rightarrow 0.04 = \frac{\omega^2}{2(9.807)}(0.2^2 - 0.1^2) \rightarrow \omega = 5.114\,\text{rad/s} = \boxed{48.8\,\text{rpm}}$$

2.111. From the given data: $R = 0.2\,\mathrm{m}$, $\omega = 450\,\mathrm{rpm} = 47.12\,\mathrm{rad/s}$, and $\Delta = 0.1\,\mathrm{m}$. Identify locations in the tube using the axes shown in Figure 2.83.

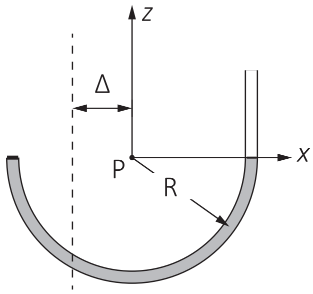


Figure 2.14: Reference axes

Since the U-tube has a circular shape of radius R, then

$$x^2 + z^2 = R^2 \tag{1}$$

The pressure distribution in the U-tube is given by Equation 2.104, from which the gauge pressure can be expressed as

$$p_\mathrm{g} = \frac{\rho\omega^2}{2}[x_*^2 - x_{*0}^2] - \rho g(z - z_0) \tag{2}$$

where x_* = x-coordinate measured from the axis of rotation, x_{*0} = x-coordinate liquid surface that is open to the atmosphere, and z_0 (= 0) = z coordinate of the open surface.

(a) Combining Equations 1 and 2 and noting that the axis of rotation is Δ from the origin (P) gives

$$p_\mathrm{g} = \frac{\rho\omega^2}{2}[(x+\Delta)^2 - (R+\Delta)^2] - \rho g(-\sqrt{R^2 - x^2}) \tag{3}$$

The pressure is a minimum where $\mathrm{d}p_\mathrm{g}/\mathrm{d}x = 0$. Differentiating Equation 3 and setting the result equal to zero gives

$$\frac{\mathrm{d}p_\mathrm{g}}{\mathrm{d}x} = \frac{\rho\omega^2}{2}[2(x+\Delta)] + \frac{\rho g}{2}[(R^2 - x^2)^{-\frac{1}{2}}(-2x)] = 0$$

which simplifies to

$$\frac{\omega^4}{g^2}(x+\Delta)^2(R^2 - x^2) - x^2 = 0 \rightarrow \frac{47.12^4}{9.807^2}(x+0.1)^2(0.2^2 - x^2) - x^2 = 0 \rightarrow x = -0.1026\,\mathrm{m}$$

Therefore, the minimum gauge pressure occurs at a location that is $0.1026 - 0.1 \approx =$ $\boxed{0.003\ \mathrm{m}}$ to the left of the rotation axis.

(b) The gauge pressure as a function of x is given by Equation 3 and is plotted in Figure 2.15.

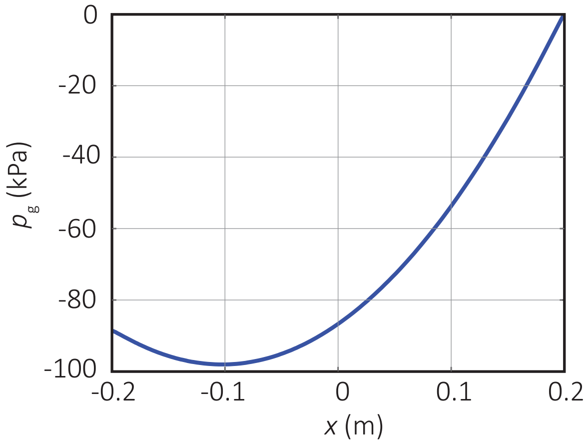


Figure 2.15: Gage pressure in U-tube

(c) Setting $x = -0.1026\,\text{m}$ in Equation 3 gives the minimum gauge pressure as $-98.06\,\text{kPa}$, which corresponds to an absolute pressure of $101.3\,\text{kPa} - 98.06\,\text{kPa} = \boxed{3.24\text{ kPa}}$. The vapor pressure of water at 20°C is 2.337 kPa, so cavitation will $\boxed{\text{not occur}}$ under the given conditions.

2.112. From the given data: $D = 0.5$ m, $R = D/2 = 0.25$ m, and $\omega = 30$ rad/s. For crude oil at 20°C, $\rho = 856\text{ kg/m}^3$ (from Appendix B.4). The pressure difference, Δp, between the center and perimeter is given by Equation 2.104 as

$$\Delta p = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) = \frac{(856)(30)^2}{2}(0.25^2 - 0^2) = 24080\,\text{Pa} = \boxed{24.1\,\text{kPa}}$$

2.113. From the given data: $R = 0.5/2 = 0.25\,\text{m}$, $\omega = 400\,\text{rpm} = 41.89\,\text{rad/s}$, and $p_0 = 200\,\text{kPa}$. For water at 20°C, $\rho = 998.2\,\text{kg/m}^3$. The pressure distribution on the top of the cylinder is given by Equation 2.104, which can be expressed as

$$p_0 - p = \frac{\rho\omega^2}{2}(R^2 - r^2) \quad \rightarrow \quad p = a + br^2 \tag{1}$$

where

$$a = p_0 - \frac{\rho\omega^2 R^2}{2} = 200 \times 10^3 - \frac{(998.2)(41.89)^2(0.25)^2}{2} = 1.453 \times 10^5\text{ Pa}$$

$$b = \frac{\rho\omega^2}{2} = \frac{(998.2)(1.89)^2}{2} = 8.757 \times 10^5\text{ Pa/m}^2$$

Using Equation 1, the force, F, on the top surface of the cylinder is given by

$$F = \int_0^R (a + br^2)2\pi r\,\text{d}r = 2\pi\left[\int_0^R ar\,\text{d}r + \int_0^R br^3\,\text{d}r\right] \quad \rightarrow \quad F = 2\pi\left[\frac{aR^2}{2} + \frac{bR^4}{4}\right]$$

Substituting the values of the given and derived parameters into the above equation yields

$$F = 2\pi\left[\frac{(1.453\times10^5)(0.25)^2}{2} + \frac{(8.757\times10^5)(0.25)^4}{4}\right][\times10^{-3}\text{ kN/N}] = \boxed{33.9\,\text{kN}}$$

2.114. From the given data: $D = 3$ cm, $R = D/2 = 1.5$ cm $= 0.015$ m, and $\Delta z = 1$ cm $= 0.01$ m. In accordance with Equation 2.108,

$$\Delta z = 2\left[\frac{\omega^2 R^2}{4g}\right] \quad \rightarrow \quad 0.01 = 2\left[\frac{\omega^2 (0.015)^2}{4(9.81)}\right] \rightarrow \omega = 29.5\,\text{rad/s} = \boxed{282\,\text{rpm}}$$

2.115. From the given data: $D = 1.5$ m, $R = D/2 = 0.75$ m, and $d = 1$ m.

(a) In accordance with Equation 2.108, the liquid surface intersects the bottom of the cylinder when

$$d = \frac{\omega^2 R^2}{4g} \quad \rightarrow \quad 1 = \frac{\omega^2 (0.75)^2}{4(9.81)} \quad \rightarrow \quad \omega = 8.35\,\text{rad/s} = \boxed{79.8\,\text{rpm}}$$

(b) From the given data: $\omega = 40$ rpm $= 4.189$ rad/s. In accordance with Equation 2.108, the required height, Δz, above the static water level is given by

$$\Delta z = \frac{\omega^2 R^2}{4g} = \frac{(4.189)^2 (0.75)^2}{4(9.81)} = 0.25\,\text{m}$$

The cylinder must be at least 1 m + 0.25 m = $\boxed{1.25\text{ m}}$ high to avoid spillage.