Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–1. Show that Pascal's law applies within a fluid that is accelerating, provided there is no shearing stresses acting within the fluid.

SOLUTION

Consider the free-body diagram of a triangular element of fluid as shown in Fig. 2–2b. If this element has acceleration components of a_x, a_y, a_z, then since $dm = \rho d\forall$ the equations of motion in the y and z directions give

$$\Sigma F_y = dma_y; \qquad p_y(\Delta x)(\Delta s \sin\theta) - \left[p(\Delta x \Delta s)\right]\sin\theta = \rho\left(\frac{1}{2}\Delta x(\Delta s\cos\theta)(\Delta s\sin\theta)\right)a_y$$

$$\Sigma F_z = dma_z; \qquad p_z(\Delta x)(\Delta s\cos\theta) - \left[p(\Delta x\Delta s)\right]\cos\theta - \gamma\left[\frac{1}{2}\Delta x(\Delta s\cos\theta)(\Delta s\sin\theta)\right] = \rho\left(\frac{1}{2}\Delta x(\Delta s\cos\theta)(\Delta s\sin\theta)\right)a_z$$

Dividing by $\Delta x \Delta s$ and letting $\Delta s \rightarrow 0$, so the element reduces in size, we obtain

$$p_y = p$$
$$p_z = p$$

By a similar argument, the element can be rotated 90° about the z axis and $\Sigma F_x = dma_x$ can be applied to show $p_x = p$. Since the angle θ of the inclined face is *arbitrary,* this indeed shows that the pressure at a point is the *same in all directions* for any fluid that has no shearing stress acting within it.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–2. The water in a lake has an average temperature of 15°C. If the barometric pressure of the atmosphere is 720 mm of Hg (mercury), determine the gage pressure and the absolute pressure at a water depth of 14 m.

SOLUTION

From Appendix A, $T = 15°\text{C}$.

$$\rho_w = 999.2\ \text{kg/m}^3$$

$$p_g = \rho_w g h = \left(999.2\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(14\ \text{m})$$

$$= 137.23\left(10^3\right)\ \text{Pa} = 137\ \text{kPa} \qquad \textbf{Ans.}$$

$$p_{\text{atm}} = \rho_{\text{Hg}}\, g h = \left(13\,550\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.720\ \text{m}) = 95.71\ \text{kPa}$$

$$p_{\text{abs}} = p_{\text{atm}} + p_g = 95.71\ \text{kPa} + 137.23\ \text{kPa}$$

$$= 233\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
$p_g = 137\ \text{kPa}$, $p_{\text{abs}} = 233\ \text{kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \mathrm{kg/m^3}$ and its specific weight to be $\gamma_w = 62.4\ \mathrm{lb/ft^3}$. Also, assume all pressures are gage pressures.

2–3. If the absolute pressure in a tank is 140 kPa, determine the pressure head in mm of mercury. The atmospheric pressure is 100 kPa.

SOLUTION

$$p_{\mathrm{abs}} = p_{\mathrm{atm}} + p_g$$

$$140\ \mathrm{kPa} = 100\ \mathrm{kPa} + p_g$$

$$p_g = 40\ \mathrm{kPa}$$

From Appendix A, $\rho_{\mathrm{Hg}} = 13\,550\ \mathrm{kg/m^3}$.

$$p = \gamma_{\mathrm{Hg}} h_{\mathrm{Hg}}$$

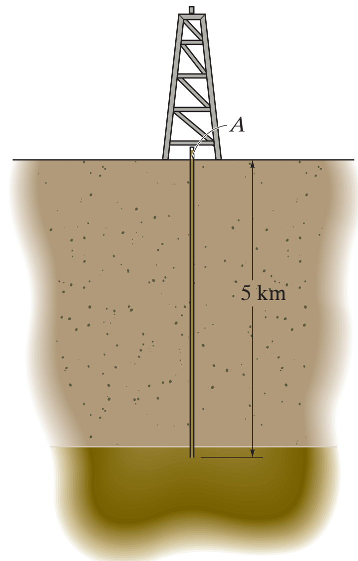
$$40(10^3)\ \mathrm{N/m^2} = (13\,550\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})h_{\mathrm{Hg}}$$

$$h_{\mathrm{Hg}} = 0.3009\ \mathrm{m} = 301\ \mathrm{mm} \qquad \textbf{Ans.}$$

Ans:
$h_{\mathrm{Hg}} = 301\ \mathrm{mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–4.** The oil derrick has drilled 5 km into the ground before it strikes a crude oil reservoir. When this happens, the pressure at the well head A becomes 25 MPa. Drilling "mud" is to be placed into the entire length of pipe to displace the oil and balance this pressure. What should be its density so that the pressure at A becomes zero?



SOLUTION

Consider the case when the crude oil is pushing out at A where $p_A = 25(10^6)$ Pa, Fig. *a*. Here, $\rho_o = 880\ \text{kg/m}^3$ (Appendix A) hence $p_o = \rho_o g h = (880\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5000\ \text{m}) = 43.164(10^6)$ Pa

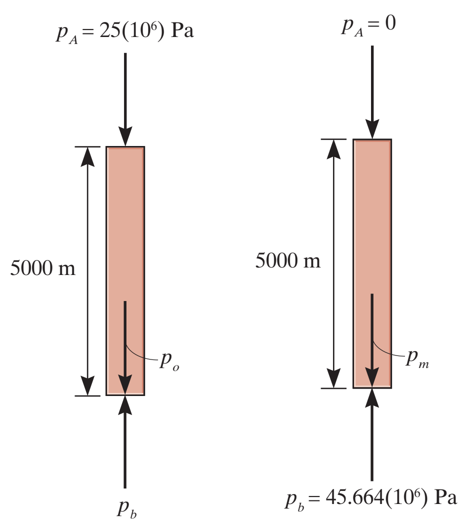
$$p_b = p_A + p_o = 25(10^6)\ \text{Pa} + 43.164(10^6)\ \text{Pa} = 68.164(10^6)\ \text{Pa}$$

It is required that $p_A = 0$, Fig. *b*. Thus

$$p_b = p_m = \rho_m g h$$

$$68.164(10^6)\frac{\text{N}}{\text{m}^2} = \rho_m(9.81\ \text{m/s}^2)(5000\ \text{m})$$

$$\rho_m = 1390\ \text{kg/m}^3 \qquad \textbf{Ans.}$$



(a) (b)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–5. In 1896, S. Riva-Rocci developed the prototype of the current sphygmomanometer, a device used to measure blood pressure. When it was worn as a cuff around the upper arm and inflated, the air pressure within the cuff was connected to a mercury manometer. If the reading for the high (or systolic) pressure is 120 mm and for the low (or diastolic) pressure is 80 mm, determine these pressures in psi and pascals.

SOLUTION

Mercury is considered to be incompressible. From Appendix A, the density of mercury is $\rho_{\text{Hg}} = 13\,550\ \text{kg/m}^3$. Thus, the systolic pressure is

$$p_S = \rho_{\text{Hg}} g h_s = \left(13\,550\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.12\ \text{m}) = 15.95\ \text{kPa}$$

$$= 16.0\left(10^3\right)\ \text{Pa} \qquad \textbf{Ans.}$$

$$p_S = \left[15.95\left(10^3\right)\frac{\text{N}}{\text{m}^2}\right]\left(\frac{1\ \text{lb}}{4.4482\ \text{N}}\right)\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right)^2\left(\frac{1\text{ft}}{12\ \text{in.}}\right)^2 = 2.31\ \text{psi} \qquad \textbf{Ans.}$$

The diastolic pressure is

$$p_d = \rho_{\text{Hg}} g h_d = \left(13\,550\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.08\ \text{m}) = 10.63\left(10^3\right)\ \text{Pa}$$

$$= 10.6\ \text{kPa} \qquad \textbf{Ans.}$$

$$p_d = \left[10.63\left(10^3\right)\frac{\text{N}}{\text{m}^2}\right]\left(\frac{1\ \text{lb}}{4.4482\ \text{N}}\right)\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right)^2\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 1.54\ \text{psi} \qquad \textbf{Ans.}$$

Ans:
$p_s = 16.0\ \text{kPa} = 2.31\ \text{psi}$
$p_d = 10.6\ \text{kPa} = 1.54\ \text{psi}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–6. Show why water would not be a good fluid to use for a barometer by computing the height to which standard atmospheric pressure will elevate it in a glass tube. Compare this result with that of mercury. Take $\gamma_w = 62.4\ \text{lb/ft}^3$, $\gamma_{Hg} = 846\ \text{lb/ft}^3$.

SOLUTION

For water barometer, Fig. *a*,

$$p_w = \gamma_w h_w = p_{atm}$$

$$\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)h_w = \left(14.7\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2$$

$$h_w = 33.92\text{ ft} = 33.9\text{ ft} \qquad \textbf{Ans.}$$

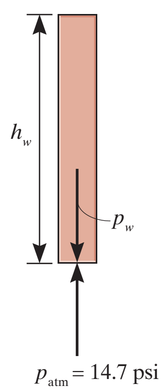
For mercury barometer, Fig. *b*,

$$p_{Hg} = \gamma_{Hg} h_{Hg} = p_{atm}$$

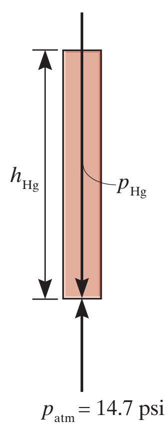
$$847\frac{\text{lb}}{\text{ft}^3}h_{Hg} = \left(14.7\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2$$

$$h_{Hg} = (2.4992\text{ ft})\left(\frac{12\text{ in.}}{1\text{ ft}}\right) = 30.0\text{ in.} \qquad \textbf{Ans.}$$

A water barometer is not suitable since it requires a very long tube.



(a)

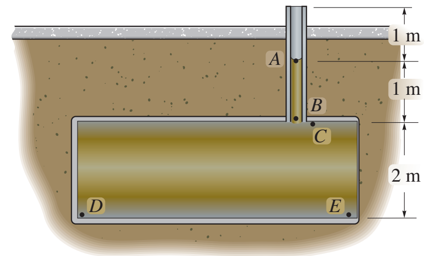


(b)

Ans:
$h_w = 33.9$ ft
$h_{Hg} = 30.0$ in.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–7. The underground storage tank used in a service station contains gasoline filled to the level A. Determine the gage pressure at each of the five identified points. Note that point B is located in the stem, and point C is just below it in the tank. Take $\rho_g = 730\ \text{kg/m}^3$.



SOLUTION

Since the tube is open-ended, point A is subjected to atmospheric pressure, which has zero gauge pressure.

$$p_A = 0 \qquad \textbf{Ans.}$$

The pressures at points B and C are the same since they are at the same horizontal level with $h = 1$ m.

$$p_B = p_C = \left(730\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1\ \text{m}) = 7.16\ \text{kPa} \qquad \textbf{Ans.}$$

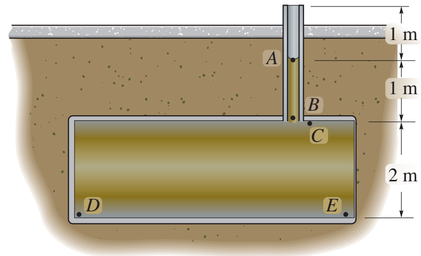
For the same reason, pressure at points D and E is the same. Here, $h = 1$ m $+ 2$ m $= 3$ m.

$$p_D = p_E = \left(730\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(3\ \text{m}) = 21.5\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
$p_A = 0$
$p_B = p_C = 7.16$ kPa
$p_D = p_E = 21.5$ kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–8.** The underground storage tank contains gasoline filled to the level A. If the atmospheric pressure is 101.3 kPa, determine the absolute pressure at each of the five identified points. Note that point B is located in the stem, and point C is just below it in the tank. Take $\rho_g = 730\ \text{kg/m}^3$.



SOLUTION

Since the tube is open-ended, point A is subjected to atmospheric pressure, which has an absolute pressure of 101.3 kPa.

$$p_A = p_{\text{atm}} + p_g$$

$$p_A = 101.3\left(10^3\right)\ \text{N/m}^2 + 0 = 101.3\ \text{kPa} \qquad \textbf{Ans.}$$

The pressures at points B and C are the same since they are at the same horizontal level with $h = 1$ m.

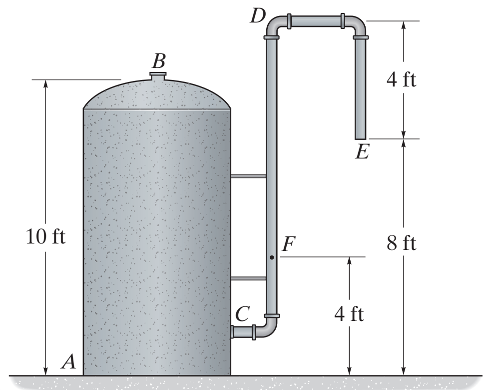
$$p_B = p_C = 101.3\left(10^3\right)\ \text{N/m}^2 + \left(730\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1\ \text{m})$$
$$= 108\ \text{kPa} \qquad \textbf{Ans.}$$

For the same reason, pressure at points D and E is the same. Here, $h = 1\ \text{m} + 2\ \text{m} = 3\ \text{m}$.

$$p_D = p_E = 101.3\left(10^3\right)\ \text{N/m}^2 + \left(730\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(3\ \text{m})$$
$$= 123\ \text{kPa} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–9. The field storage tank is filled with oil. This standpipe is connected to the tank at C, and the system is open to the atmosphere at B and E. Determine the maximum pressure in the tank is psi if the oil reaches a level of F in the pipe. Also, at what level should the oil be in the tank, so that the absolute maximum pressure occurs in the tank? What is this value? Take $\rho_o = 1.78 \text{ slug/ft}^3$.



SOLUTION

Since the top of the tank is open to the atmosphere, the free surface of the oil in the tank will be the same height as that of point F. Thus, the maximum pressure which occurs at the base of the tank (level A) is

$$\begin{aligned}(p_A)_g &= \gamma h \\ &= \left(1.78 \text{ slug/ft}^3\right)\left(32.2 \text{ ft/s}^2\right)(4 \text{ ft}) \\ &= 229.26 \frac{\text{lb}}{\text{ft}^2}\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2 = 1.59 \text{ psi} \qquad \textbf{Ans.}\end{aligned}$$

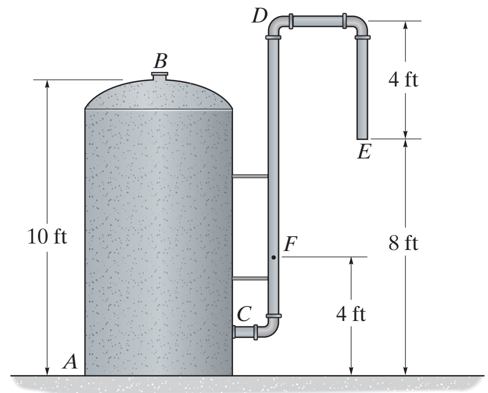
Absolute maximum pressure occurs at the base of the tank (level A) when the **oil reaches level B.**

$$\begin{aligned}(p_A)_{\substack{\text{abs}\\ \text{max}}} &= \gamma h \\ &= \left(1.78 \text{ slug/ft}^3\right)\left(32.2 \text{ ft/s}^2\right)(10 \text{ ft}) \\ &= 573.16 \text{ lb/ft}^2\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2 = 3.98 \text{ psi} \qquad \textbf{Ans.}\end{aligned}$$

Ans:
$(p_A)_g = 1.59$ psi
Absolute maximum pressure occurs when the oil reaches level B.
$(p_A)_{\substack{\text{abs}\\ \text{max}}} = 3.98$ psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–10. The field storage tank is filled with oil. The standpipe is connected to the tank at C and open to the atmosphere at E. Determine the maximum pressure that can be developed in the tank if the oil has a density of 1.78 slug/ft^3. Where does this maximum pressure occur? Assume that there is no air trapped in the tank and that the top of the tank at B is closed.



SOLUTION

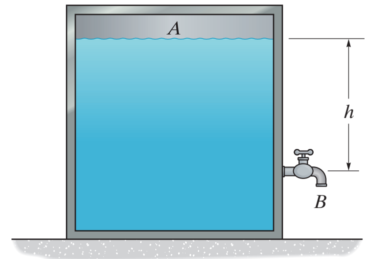
Level D is the highest the oil is allowed to rise in the tube, and the maximum gauge pressure occurs at the base of the tank (level A).

$$\begin{aligned}(p_{\max})_g &= \gamma h \\ &= (1.78 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(8 \text{ ft} + 4 \text{ ft}) \\ &= \left(687.79 \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2 = 4.78 \text{ psi}\end{aligned}$$ **Ans.**

Ans:
$(p_{\max})_g = 4.78 \text{ psi}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–11. The closed tank was completely filled with carbon tetrachloride when the valve at B was opened, slowly letting the carbon tetrachloride level drop as shown. If the valve is then closed and the space within A is a vacuum, determine the pressure in the liquid near valve B when $h = 25$ ft. Also, determine at what level h the carbon tetrachloride will stop flowing out when the valve is opened. The atmospheric pressure is 14.7 psi.



SOLUTION

From the Appendix, $\rho_{ct} = 3.09 \text{ slug/ft}^3$. Since the empty space A is a vacuum, $p_A = 0$. Thus, the absolute pressure at B when $h = 25$ ft is

$$\begin{aligned}(p_B)_{\text{abs}} &= p_A + \gamma h \\ &= 0 + (3.09 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(25 \text{ ft}) \\ &= \left(2487.45 \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2 = 17.274 \text{ psi}\end{aligned}$$

The gauge pressure is given by

$$\begin{aligned}(p_B)_{\text{abs}} &= p_{\text{atm}} + (p_B)_g \\ 17.274 \text{ psi} &= 14.7 \text{ psi} + (p_B)_g \\ (p_B)_g &= 2.57 \text{ psi} \qquad \textbf{Ans.}\end{aligned}$$

When the absolute at B equals the atmospheric pressure, the water will stop flowing. Thus,

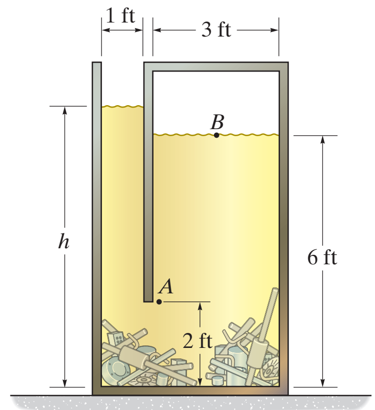
$$\begin{aligned}(p_B)_{\text{abs}} &= p_A + \gamma h \\ \left(14.7 \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)^2 &= 0 + (3.09 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)h \\ h &= 21.3 \text{ ft} \qquad \textbf{Ans.}\end{aligned}$$

Note: When the vacuum is produced, it actually becomes an example of a Rayleigh–Taylor instability. The lower density fluid (air) will migrate up into the valve B and then rise into the space A, increasing the pressure, and pushing some water out the valve. This back-and-forth effect will in time drain the tank.

Ans:
$(p_B)_g = 2.57$ psi
$h = 21.3$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–12.** The soaking bin contains ethyl alcohol used for cleaning automobile parts. If $h = 7$ ft, determine the pressure developed at point A and at the air surface B within the enclosure. Take $\gamma_{ea} = 49.3\ \text{lb/ft}^3$.



SOLUTION

The gauge pressures at points A and B are

$$p_A = \gamma_{ea}\, h_A = \left(49.3\,\frac{\text{lb}}{\text{ft}^3}\right)(7\text{ft} - 2\text{ft})$$

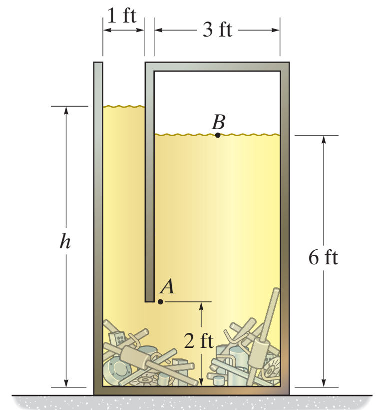
$$= \left(246.5\,\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 1.71\ \text{psi} \qquad \textbf{Ans.}$$

$$p_B = \gamma_{ea}\, h_B = (49.3\ \text{lb/ft}^3)(7\ \text{ft} - 6\ \text{ft})$$

$$= \left(49.3\,\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 0.342\ \text{psi} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–13. The soaking bin contains ethyl alcohol used for cleaning automobile parts. If the pressure in the enclosure is $p_B = 0.5$ psi, determine the pressure developed at point A and the height h of the ethyl alcohol level in the bin. Take $\gamma_{ea} = 49.3\ \text{lb/ft}^3$.



SOLUTION

The gauge pressure at point A is

$$(p_A)_g = p_B + \gamma_{ea}\, h_{BA}$$

$$= 0.5\ \text{psi} + \left(49.3\frac{\text{lb}}{\text{ft}^3}\right)(6\ \text{ft} - 2\ \text{ft})\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 1.869\ \text{psi} = 1.87\ \text{psi} \qquad \textbf{Ans.}$$

The gauge pressure for the atmospheric pressure is $(p_{\text{atm}})_g = 0$. Thus,

$$(p_B)_g = (p_{\text{atm}})_g + \gamma_{ea}\, h_B$$

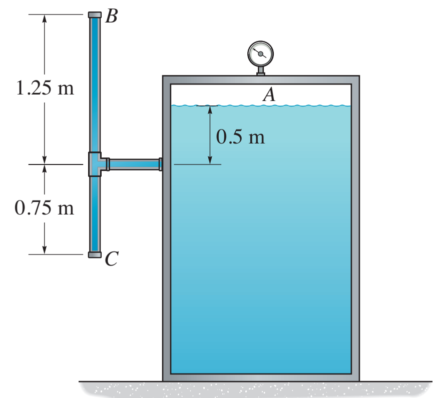
$$\left(0.5\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 = 0 + \left(49.3\frac{\text{lb}}{\text{ft}^3}\right)(h - 6)$$

$$h = 7.46\ \text{ft} \qquad \textbf{Ans.}$$

Ans:
$(p_A)_g = 1.87$ psi
$h = 7.46$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–14. The pipes connected to the closed tank are completely filled with water. If the absolute pressure at A is 300 kPa, determine the force acting on the inside of the end caps at B and C if the pipe has an inner diameter of 60 mm.



SOLUTION

Thus, the force due to pressure acting on the cap at B and C are

$$F_B = p_B A = \left[292.64(10^3)\ \text{N/m}^2\right]\left[\pi(0.03\ \text{m})^2\right]$$

$$= 827.43\ \text{N} = 827\ \text{N} \qquad \textbf{Ans.}$$

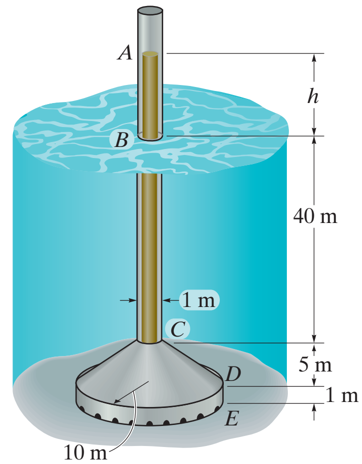
$$F_C = p_C A = \left[312.26(10^3)\ \text{N/m}^2\right]\left[\pi(0.03\ \text{m})^2\right]$$

$$= 882.90\ \text{N} = 883\ \text{N} \qquad \textbf{Ans.}$$

Ans:
$F_B = 827\ \text{N}$
$F_C = 883\ \text{N}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–15. The structure shown is used for the temporary storage of crude oil at sea for later loading into ships. When it is not filled with oil, the water level in the stem is at B (sea level). Why? As the oil is loaded into the stem, the water is displaced through exit ports at E. If the stem is filled with oil, that is, to the depth of C, determine the height h of the oil level above sea level. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1020\ \text{kg/m}^3$.



SOLUTION

The water level remains at B when empty because the gage pressure at B must be zero. It is required that the pressure at C caused by the water and oil be the same. Then

$$(p_C)_w = (p_C)_o$$

$$\rho_w g h_w = \rho_o g h_o$$

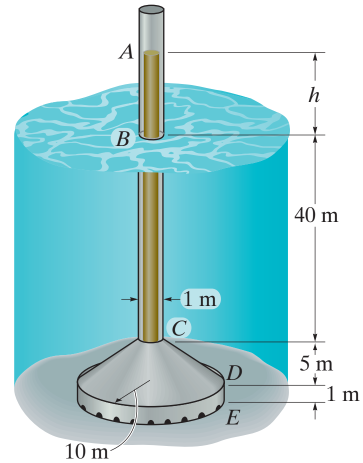
$$(1020\ \text{kg/m}^3)(g)(40\ \text{m}) = (900\ \text{kg/m}^3)g(40\ \text{m} + h)$$

$$h = 5.33\ \text{m}$$ **Ans.**

Ans:
5.33 m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–16.** If the water in the structure in Prob. 2–15 is displaced with crude oil to the level D at the bottom of the cone, then how high h will the oil extend above sea level? Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1020\ \text{kg/m}^3$.



SOLUTION

It is required that the pressure at D caused by the water and oil be the same.

$$(p_D)_w = (p_D)_o$$

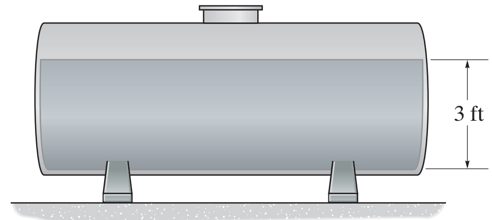
$$\rho_w g h_w = \rho_o g h_o$$

$$(1020\ \text{kg/m}^3)(g)(45\ \text{m}) = (900\ \text{kg/m}^3)(g)(45\ \text{m} + h)$$

$$h = 6.00\ \text{m} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–17. The tank is filled with aqueous ammonia (ammonium hydroxide) to a depth of 3 ft. The remaining volume of the tank contains air under absolute pressure of 20 psi. Determine the gage pressure at the bottom of the tank. Would the results be different if the tank had a square bottom rather than a curved one? Take $\rho_{am} = 1.75\ \text{slug/ft}^3$. The atmospheric pressure is $\rho_{atm} = 14.7\ \text{lb/in}^2$.



SOLUTION

The gage pressure of the air in the tank is

$$(p_{air})_{abs} = p_{atm} + (p_{air})_g$$

$$20\frac{\text{lb}}{\text{in.}^2} = 14.7\frac{\text{lb}}{\text{in.}^2} + (p_{air})_g$$

$$(p_{air})_g = \left(5.3\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2 = 763.2\frac{\text{lb}}{\text{ft}^2}$$

Using this result, the gage pressure at the bottom of tank can be obtained.

$$(p_b)_g = (p_{air})_g + \gamma h$$

$$= 763.2\frac{\text{lb}}{\text{ft}^2} + \left(1.75\frac{\text{slug}}{\text{ft}^3}\right)(32.2\ \text{ft/s}^2)(3\ \text{ft})$$

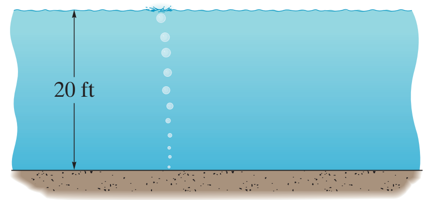
$$= \left(932.25\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2 = 6.47\ \text{psi} \qquad \textbf{Ans.}$$

No, it does not matter what shape the bottom of the tank is.

Ans:
No, it does not matter what shape the bottom of the tank is.
$(p_b)_g = 6.47\ \text{psi}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–18. A 0.5-in.-diameter bubble of methane gas is released from the bottom of a lake. Determine the bubble's diameter when it reaches the surface. The water temperature is 68°F and the atmospheric pressure is 14.7 lb/in².



SOLUTION

Applying the ideal gas law, $p = \rho RT$ of which T is constant in this case. Thus,

$$\frac{p}{\rho} = \text{constant}$$

Since $\rho = \frac{m}{V\!\!\!\!-}$, where m is also constant, then

$$\frac{p}{m/V\!\!\!\!-} = \text{constant}$$

$$pV\!\!\!\!- = \text{constant} \qquad \textbf{(1)}$$

At the bottom of the lake, the absolute pressure is

$$p_b = p_{\text{atm}} + \gamma_W h_W$$

$$= \left(14.7\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)^2 + \left(62.4 \text{ lb/ft}^3\right)(20 \text{ ft}) = 3364.8 \text{ lb/ft}^2$$

At the surface of the lake, the absolute pressure is

$$p_s = p_{\text{atm}} = \left(14.7\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)^2 = 2116.8 \text{ lb/ft}^2$$

Using Eq. (1), we can write

$$p_b V\!\!\!\!-_b = p_s V\!\!\!\!-_s$$

$$\left(3364.8 \text{ lb/ft}^2\right)\left[\frac{4}{3}\pi\left(\frac{0.5 \text{ in.}}{2}\right)^3\right] = \left(2116.8 \text{ lb/ft}^2\right)\left[\frac{4}{3}\pi\left(\frac{d_s}{2}\right)^3\right]$$

$$d_s = 0.5835 \text{ in.} = 0.584 \text{ in.} \qquad \textbf{Ans.}$$

Ans:
$d_s = 0.584$ in.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–19. The Burj Khalifa in Dubai is currently the world's tallest building. If air at 40°C is at an atmospheric pressure of 105 kPa at the ground floor (sea level), determine the absolute pressure at the top of the tower, which has an elevation of 828 m. Assume that the temperature is constant and that air is compressible. Work the problem again assuming that air is incompressible.

SOLUTION

For compressible air, with $R = 286.9\ \text{J/(kg}\cdot\text{K)}$ (Appendix A), $T_0 = 40°\text{C} + 273 = 313\ \text{K}$, $z_0 = 0$, and $z = 828\ \text{m}$,

$$p = p_0 e^{-(g/RT_0)(z-z_0)}$$

$$p = (105\ \text{kPa})e^{-[9.81/286.9(313)](828-0)}$$

$$= 95.92\ \text{kPa} \qquad \textbf{Ans.}$$

For incompressible air, with $\rho = 1.127\ \text{kg/m}^3$ at $T = 40°\text{C}$ (Appendix A),

$$p = p_0 - \rho g h$$

$$= 105(10^3)\ \text{N/m}^2 - (1.127\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(828\ \text{m})$$

$$= 95.85\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
For compressible air, $p = 95.92\ \text{kPa}$
For incompressible air, $p = 95.85\ \text{kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–20.** The Burj Khalifa in Dubai is currently the world's tallest building. If air at 100°F is at an atmospheric pressure of 14.7 psi at the ground floor (sea level), determine the absolute pressure at the top of the building, which has an elevation of 2717 ft. Assume that the temperature is constant and that air is compressible. Work the problem again assuming that air is incompressible.

SOLUTION

For compressible air, with $R = 1716\ \text{ft}\cdot\text{lb/(slug}\cdot\text{R)}$ (Appendix A), $T_0 = (100 + 460)°\text{R} = 560°\text{R}$

$$p = p_0 e^{-(g/RT_o)(z-z_o)}$$

$$p = (14.7\ \text{psi})e^{-[32.2/1716(560)](2717-0)}$$

$$= 13.42\ \text{psi} \qquad \textbf{Ans.}$$

For incompressible air, with $\rho = 0.00220\ \text{slug/ft}^3$ at $T = 100°\text{F}$ (Appendix A),

$$p = p_0 - \gamma h$$

$$= 14.7\ \text{lb/in}^2 - (0.00220\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(2717\ \text{ft})\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 13.36\ \text{psi} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–21. The density ρ of a fluid varies with depth h, although its bulk modulus E_V can be assumed constant. Determine how the pressure varies with depth h. The density at the surface of the fluid is ρ_0.

SOLUTION

The fluid is considered compressible.

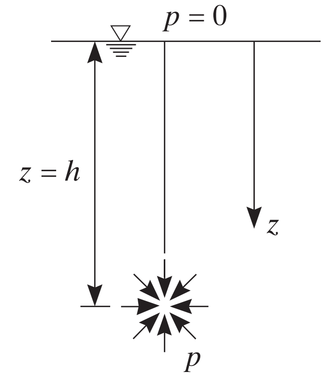
$$E_V = -\frac{dp}{dV/V}$$

However, $V = \dfrac{m}{\rho}$. Then,

$$\frac{dV}{V} = \frac{-(m/\rho^2)d\rho}{m/\rho} = -\frac{d\rho}{\rho}$$

Therefore,

$$E_V = \frac{dp}{d\rho/\rho}$$



(a)

At the surface, where $p = 0$, $\rho = \rho_0$, Fig. a, then

$$E_V \int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = \int_0^p dp$$

$$E_V \ln\left(\frac{\rho}{\rho_0}\right) = p$$

or

$$\rho = \rho_{0}e^{p/E_V}$$

Also,

$$p = p_0 + \rho g z$$

$$dp = \rho g dz$$

$$\frac{dp}{\rho} = g dz$$

Since the pressure $p = 0$ at $z = 0$ and p at $z = h$, Fig. a.

$$\int_0^p \frac{dp}{\rho_0 e^{p/E_V}} = \int_0^h g dz$$

$$\frac{E_V}{\rho_0}\left(1 - e^{-p/E_V}\right) = gh$$

$$1 - e^{-p/E_V} = \frac{\rho_0 g h}{E_V}$$

$$p = -E_V \ln\left(1 - \frac{\rho_0 g h}{E_V}\right) \qquad \textbf{Ans.}$$

Ans:

$$p = -E_V \ln\left(1 - \frac{\rho_o g h}{E_V}\right)$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–22. Due to its slight compressibility, the density of water varies with depth, although its bulk modulus $E_V = 2.20$ GPa (absolute) can be considered constant. Accounting for this compressibility, determine the pressure in the water at a depth of 300 m, if the density at the surface of the water is $\rho = 1000\ \text{kg/m}^3$. Compare this result with assuming water to be incompressible.

SOLUTION

The water is considered compressible. Using the definition of bulk modulus,

$$E_V = \frac{dp}{dV/V}$$

However, $V = \dfrac{m}{\rho}$. Then

$$\frac{dV}{V} = \frac{-(m/\rho^2)d\rho}{m/\rho} = -\frac{dp}{\rho}$$

Therefore,

$$E_V = \frac{dp}{d\rho/\rho}$$

At the surface, $p = 0$ and $\rho = 1000\ kg/m^3$, Also, $E_V = 2.20$ Gpa. Then

$$\left[2.20(10^9)\ \text{N/m}^2\right]\int_{1000\ \text{kg/m}^3}^{\rho}\frac{d\rho}{\rho} = \int_0^p dp$$

$$p = 2.20(10^9)\ln\left(\frac{\rho}{1000}\right)$$

$$\rho = 1000\, e^{\frac{p}{2.20(10^9)}} \qquad \textbf{(1)}$$

Also,

$$dp = \rho g dz$$

$$\frac{dp}{\rho} = 9.81dz \qquad \textbf{(2)}$$

Substitute Eq. (1) into (2).

$$\frac{dp}{1000e^{\frac{p}{2.20(10^9)}}} = 9.81dz$$

Since the pressure $p = 0$ at $z = 0$ and p at $z = 300$ m

$$\int_0^P \frac{dp}{1000e^{\frac{p}{2.20(10^9)}}} = \int_0^{300\ \text{m}} 9.81dz$$

$$-2.2(10^6)e^{-\frac{p}{2.20(10^9)}}\Big|_0^p = 9.81z\Big|_0^{300\ \text{m}}$$

2–22. Continued

$$-2.2\left(10^6\right)\left[e^{-\frac{p}{2.20\left(10^9\right)}} - 1\right] = 2943$$

$$e^{-\frac{p}{2.20\left(10^9\right)}} = 0.9987$$

$$\ln e^{-\frac{p}{2.20\left(10^9\right)}} = \ln 0.9987$$

$$-\frac{p}{2.20\left(10^9\right)} = -1.3386\left(10^{-3}\right)$$

Compressible:

$$p = 2.945\left(10^6\right) \text{ Pa} = 2.945 \text{ MPa}$$ **Ans.**

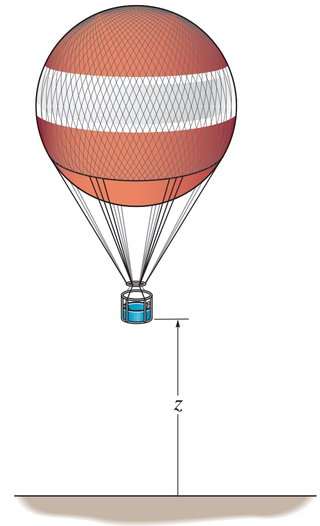
If the water is considered incompressible,

$$p = \rho_o g h = \left(1000 \text{ kg/m}^3\right)\left(9.81 \text{m/s}^2\right)(300 \text{ m})$$
$$= 2.943\left(10^6\right) \text{ Pa} = 2.943 \text{ MPa}$$ **Ans.**

Ans:
Incompressible: $p = 2.943$ MPa
Compressible: $p = 2.945$ MPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–23. As the balloon ascends, measurements indicate that the temperature begins to decrease at a constant rate, from $T = 20°\text{C}$ at $z = 0$ to $T = 16°\text{C}$ at $z = 500$ m. If the absolute pressure and density of the air at $z = 0$ are $p = 101$ kPa and $\rho = 1.202\ \text{kg/m}^3$, determine these values at $z = 500$ m.



SOLUTION

We will first determine the absolute temperature as a function of z.

$$T = 293 - \left(\frac{293 - 289}{500}\right)z = (293 - 0.008z)\ \text{K}$$

Using this result to apply the ideal gas law with $R = 286.9\ \text{J/(kg}\cdot\text{K)}$,

$$p = \rho RT; \qquad \rho = \frac{p}{RT} = \frac{p}{286.9(293 - 0.008z)}$$

$$= \frac{p}{84061.7 - 2.2952z}$$

$$dp = -\gamma dz = -\rho g dz$$

$$dp = -\frac{p(9.81)dz}{84061.7 - 2.2952z}$$

$$\frac{dp}{p} = -\frac{9.81dz}{84061.7 - 2.2952z}$$

When $z = 0$, $p = 101(10^3)$ Pa. Then

$$\int_{101(10^3)}^{p} \frac{dp}{p} = -9.81\int_0^z \frac{dz}{84061.7 - 2.2952z}$$

$$\ln p\Big|_{101(10^3)}^{p} = (-9.81)\left[-\frac{1}{2.2952}\ln(84061.7 - 2.2952z)\right]\Big|_0^z$$

$$\ln\left[\frac{p}{101(10^3)}\right] = 4.2741\ln\left(\frac{84061.7 - 2.2952z}{84061.7}\right)$$

$$\ln\left[\frac{p}{101(10^3)}\right] = \ln\left[\left(\frac{84061.7 - 2.2952z}{84061.7}\right)^{4.2741}\right]$$

$$\frac{p}{101(10^3)} = \left(\frac{84061.7 - 2.2952z}{84061.7}\right)^{4.2741}$$

$$p = 90.3467(10^{-18})(84061.7 - 2.2952z)^{4.2741}$$

2–23. Continued

At $z = 500$ m,

$$p = 90.3467\left(10^{-18}\right)\left[84061.7 - 2.2952(500)\right]^{4.2741}$$
$$= 95.24\left(10^{3}\right)\text{ Pa} = 95.2\text{ kPa} \qquad \textbf{Ans.}$$

From the ideal gas law;

$$p = \rho RT; \qquad \frac{p}{\rho T} = R = \text{constant}$$

Thus,

$$\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2}$$

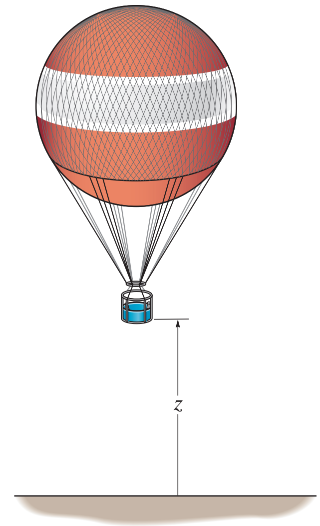
where $p_1 = 101$ kPa, $\rho_1 = 1.202\text{ kg/m}^3$, $T_1 = 293$ K, $p_2 = 95.24$ kPa, $T_2 = 289$ K. Then

$$\frac{101\text{ kPa}}{\left(1.202\text{ kg/m}^3\right)(293\text{ K})} = \frac{95.24\text{ kPa}}{\rho_2(289\text{ K})} \qquad \rho_2 = 1.149\text{ kg/m}^3 = 1.15\text{ kg/m}^3 \qquad \textbf{Ans.}$$

Ans:
$p = 95.2$ kPa
$\rho = 1.15\text{ kg/m}^3$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–24.** As the balloon ascends, measurements indicate that the temperature begins to decrease at a constant rate, from $T = 20°\text{C}$ at $z = 0$ to $T = 16°\text{C}$ at $z = 500$ m. If the absolute pressure of the air at $z = 0$ is $p = 101$ kPa, plot the variation of pressure (vertical axis) verses altitude for $0 \leq z \leq 3000$ m. Give values for increments of $u\ \Delta z = 500$ m.



SOLUTION

We will first determine the absolute temperature as a function of z

$$T = 293 - \left(\frac{293 - 289}{500}\right)z = (293 - 0.008z)\text{k}$$

Using this result to apply this ideal gas law with $R = 286.9\ \text{J/kg}\cdot\text{k}$

$$p = \rho RT; \qquad \rho = \frac{p}{RT} = \frac{p}{286.9(293 - 0.008z)}$$

$$= \frac{p}{84061.7 - 2.2952z}$$

$$dp = -\gamma dz = -\rho g dz$$

$$dp = -\frac{p(9.81)dz}{84061.7 - 2.2952z}$$

$$\frac{dp}{p} = -\frac{9.81dz}{84061.7 - 2.2952z}$$

When $z = 0$, $p = 101(10^3)$ Pa, Then

$$\int_{101(10^3)}^{p} \frac{dp}{p} = -9.81\int_0^z \frac{dz}{84061.7 - 2.2952z}$$

$$\ln p\Big|_{101(10^3)}^{p} = (-9.81)\left[-\frac{1}{2.2952}\ln\left(84061.7 - 2.2952z\right)\right]\Big|_0^z$$

$$\ln\left[\frac{p}{101(10^3)}\right] = 4.2741\ln\left(\frac{84061.7 - 2.2952z}{84061.7}\right)$$

$$\ln\left[\frac{p}{101(10^3)}\right] = \ln\left(\frac{84061.7 - 2.2952z}{84061.7}\right)^{4.2741}$$

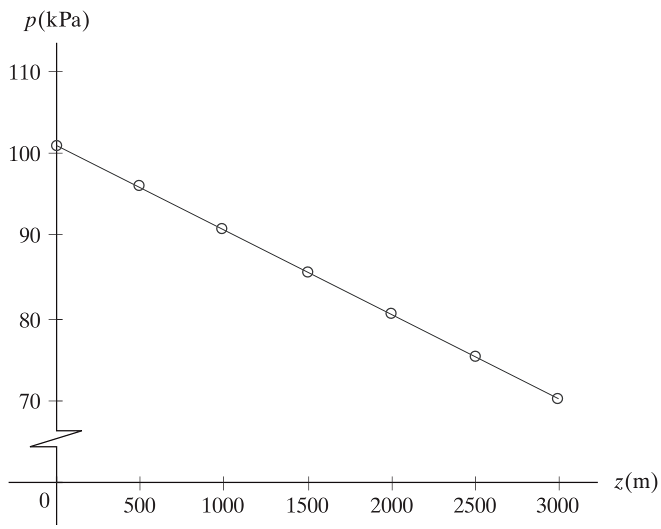
$$\frac{p}{101(10^3)} = \left(\frac{84061.7 - 2.2952z}{84061.7}\right)^{4.2741}$$

$$p = \left[90.3467(10^{-18})(84061.7 - 2.2952z)^{4.2741}\right]\ \text{Pa Where } z \text{ is in } m$$

2–24. Continued

The plot of p vs. z is shown in Fig. a

$z(m)$	0	500	1000	1500	2000	2500	3000
p (kPa)	101	95.2	89.7	84.5	79.4	74.7	70.1



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–25. In the troposphere, the absolute temperature of the air varies with elevation such that $T = T_0 - Cz$, where C is a constant. If $p = p_0$ at $z = 0$, determine the absolute pressure as a function of elevation.

SOLUTION

$$dp = -\gamma dz = -\rho g dz$$

Since the ideal gas law gives $p = \rho RT$ or $\rho = \dfrac{p}{RT}$,

$$\frac{dp}{p} = -\frac{g\,dz}{R\,T}$$

Since $p = p_0$ at $z = 0$, integrating this equation gives

$$\int_{p_0}^{p} \frac{dp}{p} = -\frac{g}{R}\int_0^z \frac{dz}{T_0 - Cz}$$

$$\ln p\Big|_{p_0}^{p} = \frac{g}{R}\left[\frac{1}{C}\ln\left(\mathrm{T}_0 - Cz\right)\right]_0^z$$

$$\ln\frac{p}{p_0} = \frac{g}{RC}\ln\left(\frac{T_0 - Cz}{T_0}\right)$$

Therefore,

$$p = p_0\left(\frac{T_0 - Cz}{T_0}\right)^{g/RC}$$ **Ans.**

Ans:

$$p = p_0\left(\frac{T_0 - Cz}{T_0}\right)^{g/RC}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–26. In the troposphere the absolute temperature of the air varies with elevation such that $T = T_0 - Cz$, where C is a constant. Using Fig. 2–11, determine the constants, T_0 and C. If $p_0 = 101$ kPa at $z_0 = 0$, determine the absolute pressure in the air at an elevation of 5 km.

SOLUTION

From Fig. 2–11, $T_0 = 15°\text{C}$ at $z = 0$. Then

$$15°\text{C} = T_0 - C(0)$$

$$T_0 = 15°\text{C} \qquad \textbf{Ans.}$$

Also, $T_C = -56.5°\text{C}$ at $z = 11.0(10^3)$ m. Then

$$-56.5°\text{C} = 15°\text{C} - C\left[11.0(10^3)\text{m}\right]$$

$$C = 6.50(10^{-3})°\text{C/m} \qquad \textbf{Ans.}$$

Thus,

$$T_C = \left[15 - 6.50(10^{-3})z\right]°\text{C}$$

The absolute temperature is therefore

$$T = (15 - 6.50z) + 273 = \left[288 - 6.50(10^{-3})z\right]\text{K} \qquad \textbf{(1)}$$

Substitute the ideal gas law $p = \rho RT$ or $\rho = \dfrac{p}{RT}$ into $dp = -\gamma dz = -\rho g dz$,

$$dp = -\frac{p}{RT}gdz$$

$$\frac{dp}{p} = \frac{g}{RT}dz$$

From the table in Appendix A, gas constant for air is $R = 286.9\ \text{J/(kg}\cdot\text{K)}$.

Also, $p = 101$ kPa at $z = 0$. Then

$$\int_{101(10^3)\text{ Pa}}^{p}\frac{dp}{p} = -\left(\frac{9.81\ \text{m/s}^2}{286.9\ \text{J/(kg}\cdot\text{K)}}\right)\int_0^z\frac{dz}{288 - 6.50(10^{-3})z}$$

$$\ln p\Big|_{101(10^3)\text{ Pa}}^{p} = 5.2605\ \ln\left[288 - 6.50(10^{-3})z\right]\Big|_0^z$$

$$\ln\left[\frac{p}{101(10^3)}\right] = 5.2605\ \ln\left[\frac{288 - 6.50(10^{-3})z}{288}\right]$$

$$p = 101(10^3)\left[\frac{288 - 6.50(10^{-3})z}{288}\right]^{5.2605}$$

At $z = 5(10^3)$ m,

$$p = 101(10^3)\left\{\frac{288 - \left[6.50(10^{-3})\right]\left[5(10^3)\right]}{288}\right\}^{5.2605}$$

$$= 53.8(10^3)\ \text{Pa} = 53.8\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
$T_0 = 15°\text{C}$
$C = 6.50(10^{-3})°\text{C/m}$
$p = 53.8$ kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–27. The density of a nonhomogeneous liquid varies as a function of depth h, such that $\rho = (850 + 0.2h)\ \text{kg/m}^3$, where h is in meters. Determine the pressure when $h = 20$ m.

SOLUTION

Since $p = \rho g h$, then the liquid is considered compressible.

$$dp = \rho g\, dh$$

Integrating this equation using the gage pressure $p = 0$ at $h = 0$ and p at h. Then,

$$\int_0^P dp = \int_0^h (850 + 0.2\,h)\,(9.81)\,dh$$

$$p = \left(8338.5\,h + 0.981\,h^2\right)\ \text{Pa}$$

At $h = 20$ m, this equation gives

$$p = \left[8338.5(20) + 0.981\left(20^2\right)\right]\ \text{Pa}$$

$$= 167.16\left(10^3\right)\ \text{Pa} = 167\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
167 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \mathrm{kg/m^3}$ and its specific weight to be $\gamma_w = 62.4\ \mathrm{lb/ft^3}$. Also, assume all pressures are gage pressures.

***2–28.** The density of a non-homogeneous liquid varies as a function of depth h, such that $\rho = (635 + 60h)\ \mathrm{kg/m^3}$, where h is in meters. Plot the variation of the pressure (vertical axis) versus depth for $0 \le h < 10$ m. Give values for increments of 2 m.

SOLUTION

$h(m)$	0	2	4	6	8	10
p(kPa)	0	13.6	29.6	48.0	68.7	91.7

The liquid is considered compressible. Use

$$dp = \rho g dh$$

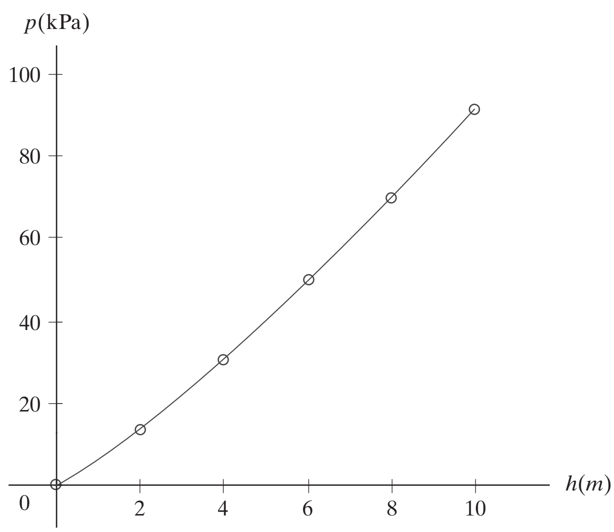
Integrate this equation using the gage pressure $p = 0$ at $h = 0$ and p at h. Then

$$\int_0^p dp = \int_0^h (635 + 60h)(9.81)\, dh$$

$$p = \left[9.81\left(635h + 30h^2\right)\right] \text{ Pa}$$

$$p = \left[0.00981\left(635h + 30h^2\right)\right] \text{ kPa where } h \text{ is in } m.$$

The plot of p *vs* h is shown in Fig. *a*.



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–29. In the troposphere, which extends from sea level to 11 km, it is found that the temperature decreases with altitude such that $dT/dz = -C$, where C is the constant lapse rate. If the temperature and pressure at $z = 0$ are T_0 and p_0, determine the pressure as a function of altitude.

SOLUTION

First, we must establish the relation between T, and z using $T = T_0$ at $z = 0$,

$$\int_{T_0}^{T} dT = -c\int_0^z dz$$

$$T - T_0 = -Cz$$

$$T = T_0 - Cz$$

Applying the ideal gas lan

$$p = \rho RT; \qquad \rho = \frac{p}{RT} = \frac{p}{R(T_0 - Cz)}$$

$$dp = -\gamma dz = -\rho g dz$$

$$dp = -\frac{gp dz}{R(T_0 - Cz)}$$

$$\frac{dp}{p} = \frac{-g}{R}\left(\frac{dz}{T_0 - Cz}\right)$$

Using $p = p_0$ at $z = 0$,

$$\int_{p_0}^{p}\frac{dp}{p} = \frac{-g}{R}\int_0^z \frac{dz}{T_0 - Cz}$$

$$\ln p\bigg|_{p_0}^{p} = -\frac{g}{R}\left[\left(-\frac{1}{C}\right)\ln\,(T_0 - Cz)\right]\bigg|_0^z$$

$$\ln\frac{p}{p_0} = \frac{g}{CR}\ln\left(\frac{T_0 - Cz}{T_0}\right)$$

$$\ln\frac{p}{p_0} = \ln\left[\left(\frac{T_0 - Cz}{T_0}\right)^{g/CR}\right]$$

$$\frac{p}{p_0} = \left(\frac{T_0 - Cz}{T_0}\right)^{g/CR}$$

$$p = p_0\left(\frac{T_0 - Cz}{T_0}\right)^{g/CR} \qquad \textbf{Ans.}$$

Ans:

$$p = p_0\left(\frac{T_0 - Cz}{T_0}\right)^{g/RC}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–30. At the bottom of the stratosphere the temperature is assumed to remain constant at $T = T_0$. If the pressure is $p = p_0$, where the elevation is $z = z_0$, derive an expression for the pressure as a function of elevation.

SOLUTION

$$p = \rho R T_0$$

$$dp = -\rho g dz = \frac{-pg}{RT_0} dz$$

$$\frac{dp}{p} = -\frac{g}{RT_0} dz$$

$$\ln p = -\frac{zg}{RT_0} + C$$

At $z = z_0$, $p = p_0$, so that

$$\frac{p}{p_0} = e^{-(z-z_0)g/RT_0}$$

$$p = p_0\, e^{-(z-z_0)g/RT_0}$$ **Ans.**

Ans:
$p = p_0 e^{-(z-z_0)g/RT_0}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–31. Determine the pressure at an elevation of $z = 20$ km into the stratosphere if the temperature remains constant at $T_0 = -56.5°\text{C}$. Assume the stratosphere beings at $z = 11$ km as shown in Fig. 2–11.

SOLUTION

Within the Troposphere $T_C = T_0 - Cz$, and Fig. 2-7 gives $T_C = 15°\text{C}$ at $z = 0$. Then

$$15°\text{C} = T_0 - C(0)$$

$$T_0 = 15°\text{C}$$

Also, $T_C = -56.5°\text{C}$ at $z = 11.0(10^3)$ m. Then

$$-56.5°\text{C} = 15°\text{C} - C\left[11.0(10^3)\ \text{m}\right]$$

$$C = 6.50(10^{-3})\ °\text{C/m}$$

Thus

$$T_C = \left[15 - 6.50(10^{-3})z\right]\ °\text{C}$$

The absolute temperature is therefore

$$T = 15 - 6.50(10^{-3})z + 273 = \left[288 - 6.50(10^3)z\right]k \qquad \textbf{(1)}$$

Substitute the ideal gas law $p = \rho RT$ or $\rho = \dfrac{p}{RT}$ into Eq. 2–4 $dp = -\gamma dz = -\rho g dz$,

$$dp = -\frac{p}{RT}gdz$$

$$\frac{dp}{p} = -\frac{g}{RT}dz \qquad \textbf{(2)}$$

From table in Appendix A, gas constant for air is $R = 286.9\ \text{J/kg}\cdot\text{K}$. Also, $p = 101$ kPa at $z = 0$. Then

$$\int_{101(10^3)\ \text{Pa}}^{p}\frac{dp}{p} = -\left(\frac{9.81\ \text{m/s}^2}{286.9\ \text{J/kg}\cdot\text{K}}\right)\int_0^z \frac{dz}{288 - 6.50(10^{-3})z}$$

$$\ln p\Big|_{101(10^3)\ \text{Pa}}^{p} = 5.2605\ \ln\left[288 - 6.50(10^{-3})z\right]\Big|_0^z$$

$$\ln\left[\frac{p}{101(10^3)}\right] = 5.2605\ \ln\left[\frac{288 - 6.50(10^{-3})z}{288}\right]$$

$$p = 101(10^3)\left[\frac{288 - 6.50(10^{-3})z}{288}\right]^{5.2605}$$

2–31. Continued

At $z = 11.0(10^3)$ m,

$$p = 101(10^3)\left\{\frac{288 - [6.50(10^{-3})][11.0(10^3)]}{288}\right\}^{5.2605} = 22.51(10^3) \text{ Pa}$$

Integrate Eq. (2) using this result and $T = -56.5°\text{C} + 273 = 216.5 \text{ K}$

$$\int_{22.51(10^3)\text{ Pa}}^{p} \frac{dp}{p} = -\left[\frac{9.81 \text{ m/s}^2}{(286.9 \text{ J/kg}\cdot\text{K})(216.5 \text{ K})}\right]\int_{11(10^3)\text{ m}}^{z} dz$$

$$\ln p\Big|_{22.51(10^3)\text{ Pa}}^{p} = -0.1579(10^{-3})z\Big|_{11(10^3)\text{ m}}^{z}$$

$$\ln\frac{p}{22.51(10^3)} = 0.1579(10^{-3})[11(10^3) - z]$$

$$p = \left[22.51(10^3)e^{0.1579(10^{-3})[11(10^3)-z]}\right] \text{ Pa}$$

At $z = 20(10^3)$ m,

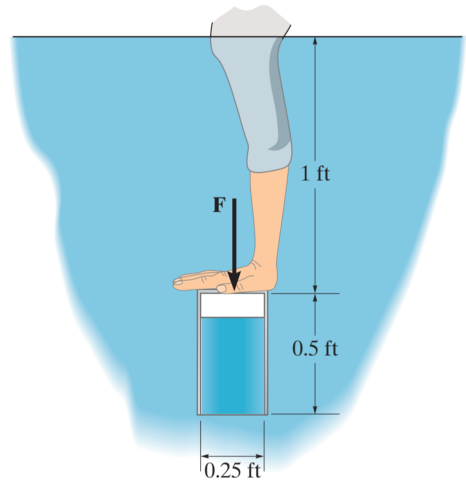
$$p = \left[22.51(10^3)e^{0.1579(10^{-3})[11(10^3)-20(10^3)]}\right] \text{ Pa}$$

$$= 5.43(10^3) \text{ Pa} = 5.43 \text{ kPa}$$ **Ans.**

Ans:
5.43 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–32.** The can, which weighs 0.2 lb, has an open end. If it is inverted and pushed down into the water, determine the force **F** needed to hold it under the surface. Assume the air in the can remains at the same temperature as the atmosphere, and that is 70°F. *Hint*: Account for the change in volume of air in the can due to the pressure change. The atmospheric pressure is $p_{\text{atm}} = 14.7$ psi.



SOLUTION

When submerged, the density of the air in the can changes due to pressure changes. According to the ideal gas law,

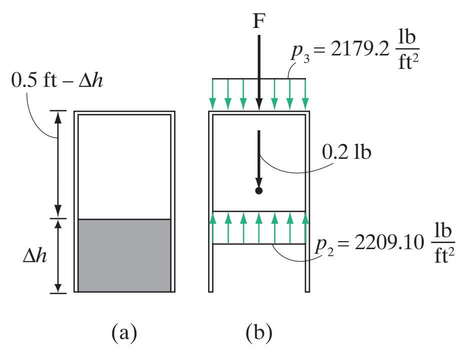
$$p\forall = mRT$$

Since the temperature T is constant, mRT is also constant. Thus,

$$p_1\forall_1 = p_2\forall_2 \qquad \textbf{(1)}$$

When $p_1 = p_{\text{atm}} = \left(14.7\dfrac{\text{lb}}{\text{in.}^2}\right)\left(\dfrac{12\ \text{in.}}{1\ \text{ft}}\right)^2 = 2116.8\dfrac{\text{lb}}{\text{ft}^2}$, $\forall_1 = \pi(0.125\ \text{ft})^2(0.5\ \text{ft})$

$$= 7.8125\left(10^{-3}\right)\pi\ \text{ft}^3.$$



When the can is submerged, the water fills the space shown shaded in Fig. *a*. Thus,

$$p_2 = p_{\text{atm}} + \gamma_w h = \left(2116.8\frac{\text{lb}}{\text{ft}^2}\right) + \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(1.5\ \text{ft} - \Delta h)$$

$$= (2210.4 - 62.4\Delta h)\frac{\text{lb}}{\text{ft}^2}$$

$$\forall_2 = \pi(0.125\ \text{ft})^2(0.5\ \text{ft} - \Delta h) = \left[0.015625\pi(0.5\ \text{ft} - \Delta h)\right]\ \text{ft}^3$$

Substituting these values into Eq. (1),

$$\left(2116.8\,\frac{\text{lb}}{\text{ft}^2}\right)\left[7.8125\left(10^{-3}\right)\pi\ \text{ft}^3\right] = \left[(2210.4 - 62.4\Delta h)\,\frac{\text{lb}}{\text{ft}^2}\right]\left[0.015625\pi(0.5 - \Delta h)\ \text{ft}^3\right]$$

$$62.4\Delta h^2 - 2241.6\Delta h + 46.8 = 0$$

Solving for the root < 0.5 ft, we obtain

$$\Delta h = 0.02089\ \text{ft}$$

Then

$$p_2 = 2210.4 - 62.4(0.02089) = 2209.10\frac{\text{lb}}{\text{ft}^2}$$

The pressure on top of the can is

$$p_3 = p_{\text{atm}} + \gamma_w h = \left(2116.8\,\frac{\text{lb}}{\text{ft}^2}\right) + \left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(1\ \text{ft}) = 2179.2\,\frac{\text{lb}}{\text{ft}^2}$$

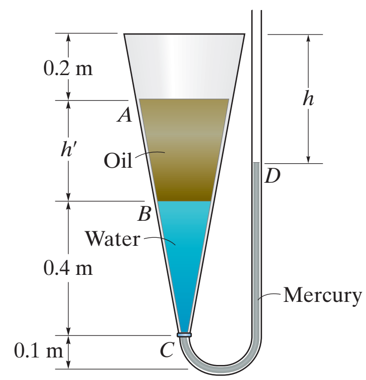
Considering the free-body diagram of the can, Fig. *b*,

$$+\uparrow\Sigma F_y = 0; \qquad \left(2209.10\,\frac{\text{lb}}{\text{ft}^2}\right)\left[\pi(0.125\ \text{ft})^2\right] - 0.2\ \text{lb} - \left(2179.2\,\frac{\text{lb}}{\text{ft}^2}\right)\left[\pi(0.125\ \text{ft})^2\right] - F = 0$$

$$F = 1.27\ \text{lb} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–33. The funnel is filled with oil and water to the levels shown. Determine the depth of oil h' that must be in the funnel so that the water remains at a depth C, and the mercury is at $h = 0.8$ m from the top of the funnel. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1000\ \text{kg/m}^3$, $\rho_{\text{Hg}} = 13\,550\ \text{kg/m}^3$.



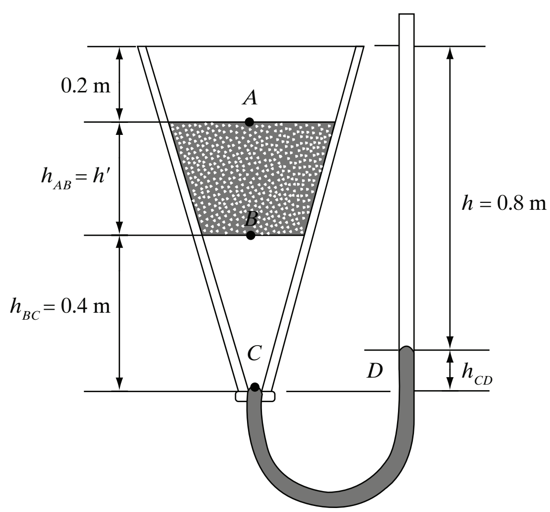
SOLUTION

Referring to Fig. a, $h_{CD} = 0.2\ \text{m} + h' + 0.4\ \text{m} - 0.8\ \text{m} = h' - 0.2\ \text{m}$. Then the manometer rule gives

$$p_A + \rho_o g h_{AB} + \rho_w g h_{BC} - \rho_{\text{Hg}} g h_{CD} = p_D$$

$$0 + \left(900\ \text{kg/m}^3\right)gh' + \left(1000\ \text{kg/m}^3\right)g(0.4) - \left(13\,550\ \text{kg/m}^3\right)g(h' - 0.2\ \text{m}) = 0$$

$$h' = 0.2458\ \text{m} = 246\ \text{mm}$$ **Ans.**

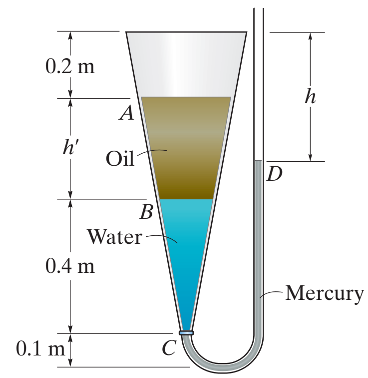


(a)

Ans:
246 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–34. The funnel is filled with oil to a depth of $h' = 0.3$ m and water to a depth of 0.4 m. Determine the distance h the mercury level is from the top of the funnel. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1000\ \text{kg/m}^3$, $\rho_{\text{Hg}} = 13\,550\ \text{kg/m}^3$.



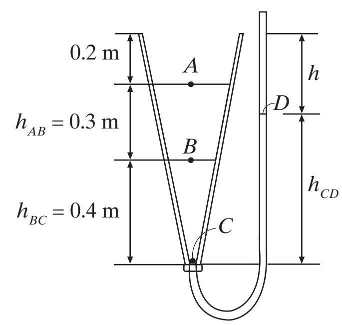
SOLUTION

Referring to Fig. a, $h_{CD} = 0.2\text{ m} + 0.3\text{ m} + 0.4\text{ m} - \text{h} = 0.9\text{ m} - h$. Then the manometer rule gives

$$p_A + \rho_o g h_{AB} + \rho_w g h_{BC} - \rho_{\text{Hg}} g h_{CD} = p_D$$

$$0 + \left(900\ \text{kg/m}^3\right)g(0.3\text{ m}) + \left(1000\ \text{kg/m}^3\right)g(0.4\text{ m}) - \left(13\,550\ \text{kg/m}^3\right)(g)(0.9\text{ m} - h) = 0$$

$$h = 0.8506\text{ m} = 851\text{ mm}$$ **Ans.**

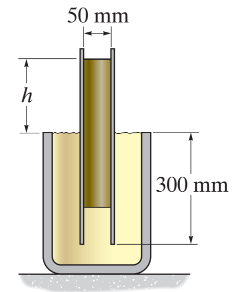


(a)

Ans:
851 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–35. The 150-mm-diameter container is filled to the top with glycerin, and a 50-mm-diameter thin pipe is inserted within it to a depth of 300 mm. If 0.00075 m³ of kerosene is then poured into the pipe, determine the height h to which the kerosene rises from the top of the glycerin.



SOLUTION

The height of the kerosene column in the pipe, Fig. a, is

$$h_{ke} = \frac{V_{ke}}{\pi r^2} = \frac{0.00075\ \text{m}^3}{\pi(0.025\ \text{m})^2} = \left(\frac{1.2}{\pi}\right)\text{m}$$

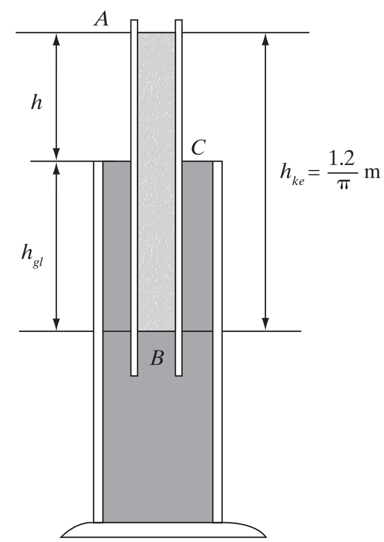
From Appendix A, $\rho_{ke} = 814\ \text{kg/m}^3$ and $\rho_{gl} = 1260\ \text{kg/m}^3$ writing the manometer equation from $A \rightarrow B \rightarrow C$ by referring to Fig. a,

$$p_{\text{atm}} + \rho_{ke} g h_{ke} - \rho_{gl} g h_{gl} = p_{\text{atm}}$$

$$h_{gl} = \left(\frac{\rho_{ke}}{\rho_{gl}}\right) h_{ke} = \left(\frac{814\ \text{kg/m}^3}{1260\ \text{kg/m}^3}\right)\left(\frac{1.2}{\pi}\ \text{m}\right) = 0.2468\ \text{m}$$

Thus,

$$h = h_{ke} - h_{gl} = \frac{1.2}{\pi}\ \text{m} - 0.2468\ \text{m} = 0.1352\ \text{m} = 135\ \text{mm} \qquad \textbf{Ans.}$$

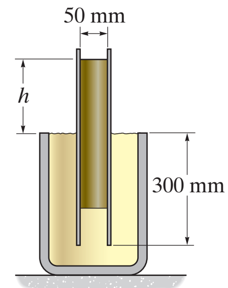


(a)

Ans:
135 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–36.** The 150-mm-diameter container is filled to the top with glycerin, and a 50-mm-diameter thin pipe is inserted within it to a depth of 300 mm. Determine the maximum volume of kerosene that can be poured into the pipe so it does not come out from the bottom end. How high h does the kerosene rise above the glycerin?



SOLUTION

From Appendix A, $\rho_{ke} = 814\ \text{kg/m}^3$ and $\rho_{gl} = 1260\ \text{kg/m}^3$. The kerosene is required to heat the bottom of the tube as shown in Fig. *a*. Write the manometer equation from $A \rightarrow B \rightarrow C$,

$$p_{\text{atm}} + \rho_{ke} g h_{ke} - \rho_{gl} g h_{gl} = p_{\text{atm}}$$

$$h_{ke} = \frac{\rho_{gl}}{\rho_{ke}} h_{gl}$$

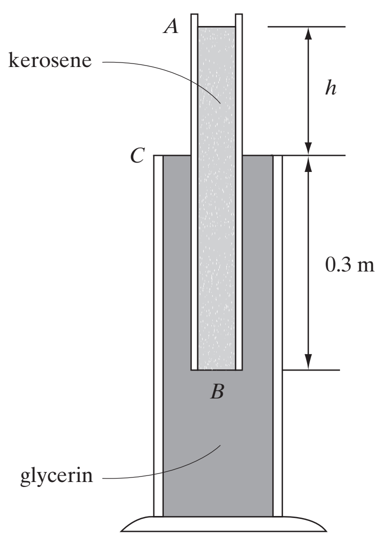
Here, $h_{ke} = (h + 0.3)$ m and $h_{gl} = 0.3$ m. Then

$$(h + 0.3)\ \text{m} = \left(\frac{1260\ \text{kg/m}^3}{814\ \text{kg/m}^3}\right)(0.3\ \text{m})$$

$$h_{ke} = 0.1644\ \text{m} = 164\ \text{mm} \qquad \textbf{Ans.}$$

Thus, the volume of the kerosene in the pipe is

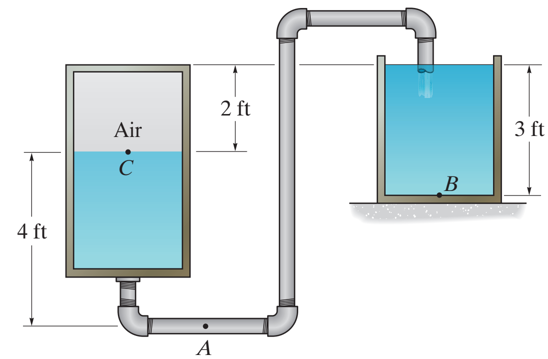
$$\forall_{ke} = \pi r^2 h_{ke} = \pi(0.025\ \text{m})^2(0.1644\ \text{m} + 0.3\ \text{m}) = 0.9118\left(10^{-3}\right)\ \text{m}^3$$
$$= 0.912\left(10^{-3}\right)\ \text{m}^3 \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–37. Determine the pressures at points A and B. The containers are filled with water.



SOLUTION

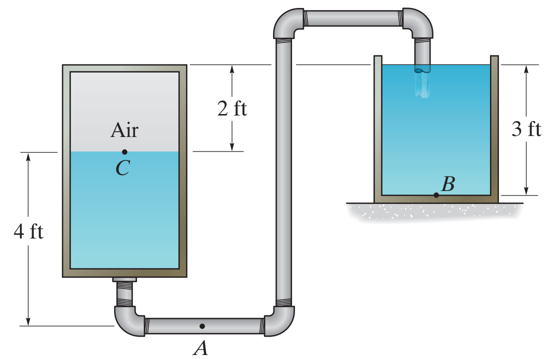
$$p_A = \gamma_A h_A = (62.4\ \text{lb/ft}^3)(2\text{ft} + 4\text{ft}) = \left(374.4\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 2.60\ \text{psi} \quad \textbf{Ans.}$$

$$p_B = \gamma_B h_B = (62.4\ \text{lb/ft}^3)(3\ \text{ft}) = \left(187.2\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 1.30\ \text{psi} \quad \textbf{Ans.}$$

Ans:
$p_A = 2.60$ psi, $p_B = 1.30$ psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–38. Determine the pressure at point C. The containers are filled with water.



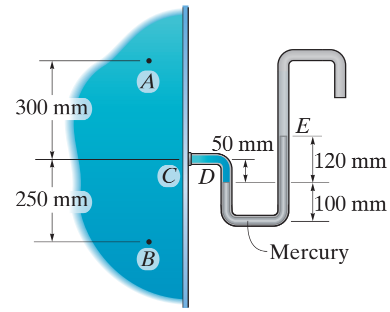
SOLUTION

$$p_C = \gamma h_C = (62.4\ \text{lb/ft}^3)(2\ \text{ft}) = \left(124.8\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 0.870\ \text{psi} \qquad \textbf{Ans.}$$

Ans:
0.870 psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–39. Butyl carbitol, used in the production of plastics, is stored in a tank having the U-tube manometer. If the U-tube is filled with mercury to level E, determine the pressure in the tank at point A. Take $S_{\text{Hg}} = 13.55$, and $S_{bc} = 0.957$.



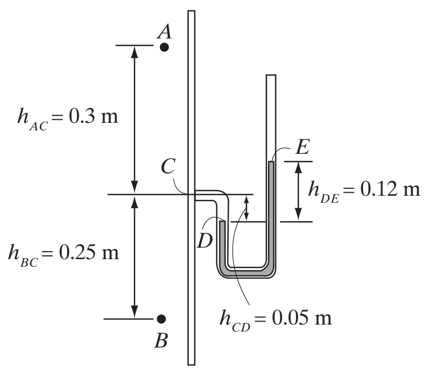
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_E + \rho_{\text{Hg}} g h_{DE} - \rho_{bc} g (h_{CD} + h_{AC}) = p_A$$

$$0 + 13.55(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.120\ \text{m}) - 0.957(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.05\ \text{m} + 0.3\ \text{m}) = p_A$$

$$p_A = 12.67(10^3)\ \text{Pa} = 12.7\ \text{kPa} \qquad \textbf{Ans.}$$

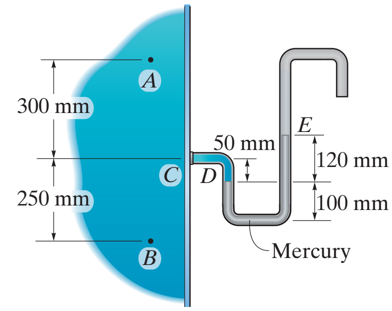


(a)

Ans:
12.7 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–40.** Butyl carbitol, used in the production of plastics, is stored in a tank having the U-tube manometer. If the U-tube is filled with mercury to level E, determine the pressure in the tank at point B. Take $S_{\text{Hg}} = 13.55$, and $S_{bc} = 0.957$.



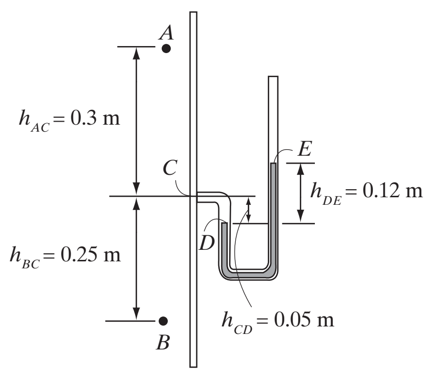
SOLUTION

Referring to Fig. *a*, the manometer rule gives

$$p_E + \rho_{\text{Hg}} g h_{DE} + \rho_{bc} g(-h_{CD} + h_{BC}) = p_B$$

$$0 + 13.55\left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.120\ \text{m}) + 0.957\left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(-0.05\ \text{m} + 0.25\ \text{m}) = p_B$$

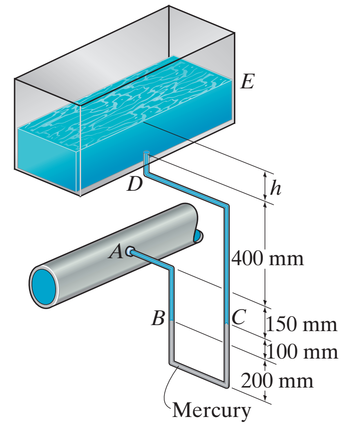
$$p_B = 17.83\left(10^3\right)\ \text{Pa} = 17.8\ \text{kPa}$$ **Ans.**



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–41. Water in the reservoir is used to control the water pressure in the pipe at A. If $h = 200$ mm, determine this pressure when the mercury is at the elevation shown. Take $\rho_{\text{Hg}} = 13\,550\ \text{kg/m}^3$. Neglect the diameter of the pipe.



SOLUTION

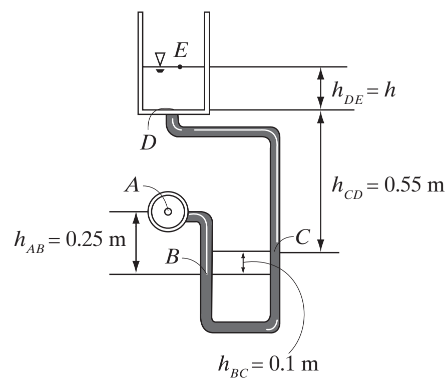
Referring to Fig. *a* with $h = 0.2$ m, the manometer rule gives

$$p_A + \rho_w g h_{AB} - \rho_{\text{Hg}} g h_{BC} - \rho_w g (h_{CD} + h_{DE}) = p_E$$

$$p_A + \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.25\ \text{m}) - \left(13\,550\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.1\ \text{m})$$

$$- \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.55\ \text{m} + 0.2\text{m}) = 0$$

$$p_A = 18.20\left(10^3\right)\ \text{Pa} = 18.2\ \text{kPa} \qquad \textbf{Ans.}$$

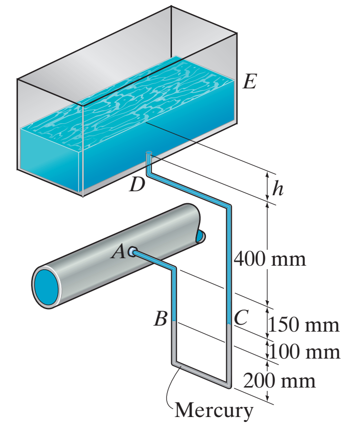


(a)

Ans:
18.2 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–42. If the water pressure in the pipe at A is to be 25 kPa, determine the required height h of water in the reservoir. Mercury in the pipe has the elevation shown. Take $\rho_{\text{Hg}} = 13\,550\ \text{kg/m}^3$. Neglect the diameter of the pipe.



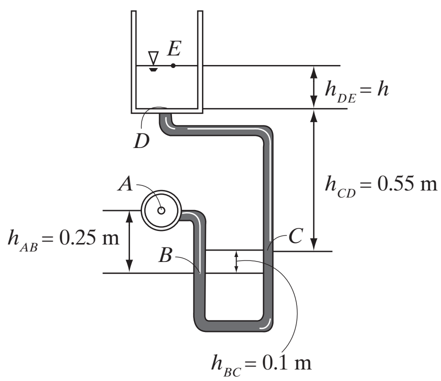
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_A + \rho_w g h_{AB} - \rho_{\text{Hg}} g h_{BC} - \rho_w g (h_{CD} + h_{DE}) = p_E$$

$$25(10^3)\ \text{N/m}^2 + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.25\ \text{m}) - (13\,550\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m})$$
$$- (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.550\ \text{m} + h) = 0$$

$$h = 0.8934\ \text{m} = 893\ \text{mm}$$ **Ans.**

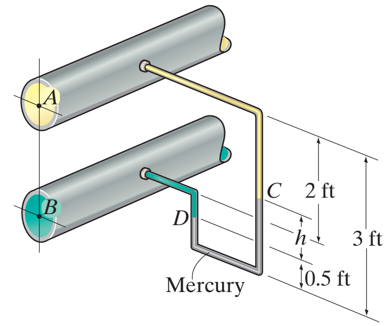


(a)

Ans:
893 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–43. A solvent used for plastics manufacturing consists of cyclohexanol in pipe A and ethyl lactate in pipe B that are being transported to a mixing tank. Determine the pressure in pipe A if the pressure in pipe B is 15 psi. The mercury in the manometer is in the position shown, where $h = 1$ ft. Neglect the diameter of the pipe. Take $S_c = 0.953$, $S_{Hg} = 13.55$, and $S_{el} = 1.03$.



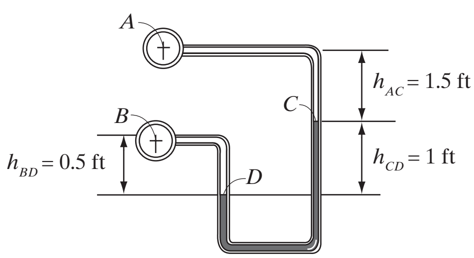
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_A + \gamma_c g h_{AC} + \gamma_{Hg} h_{CD} - \gamma_{el} g h_{BD} = p_B$$

$$p_A + 0.953(62.4\ \text{lb/ft}^3)(1.5\ \text{ft}) + (13.55)(62.4\ \text{lb/ft}^3)(1\ \text{ft}) - (1.03)(62.4\ \text{lb/ft}^3)(0.5\ \text{ft}) = \left(15\frac{\text{lb}}{\text{in.}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2$$

$$p_A = 1257.42\frac{\text{lb}}{\text{ft}^2}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 8.73\ \text{psi}$$ **Ans.**

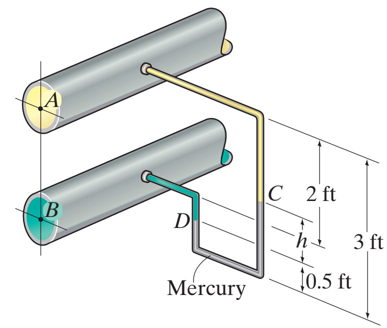


(a)

Ans:
8.73 psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–44.** A solvent used for plastics manufacturing consists of cyclohexanol in pipe A and ethyl lactate in pipe B that are being transported to a mixing tank. If the pressure in pipe A is 18 psi, determine the height h of the mercury in the manometer so that a pressure of 25 psi is developed in pipe B. Neglect the diameter of the pipes. Take $S_c = 0.953$, $S_{\text{Hg}} = 13.55$, and $S_{el} = 1.03$.



SOLUTION

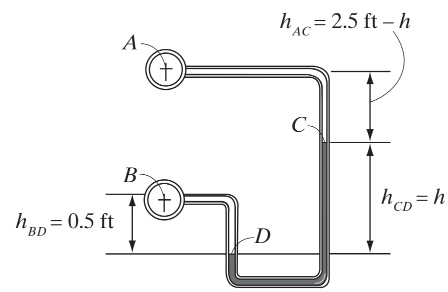
Referring to Fig. a, the manometer rule gives

$$p_A + \gamma_c h_{AC} + \gamma_{\text{Hg}} h_{CD} - \gamma_{el} h_{BD} = p_B$$

$$\frac{18\ \text{lb}}{\text{in.}^2}\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 + 0.953\left(62.4\ \text{lb/ft}^3\right)(2.5\ \text{ft} - h) + 13.55\left(62.4\ \text{lb/ft}^3\right)(h)$$

$$-(1.03)\left(62.4\ \text{lb/ft}^3\right)(0.5\ \text{ft}) = \frac{25\ \text{lb}}{\text{in.}^2}\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2$$

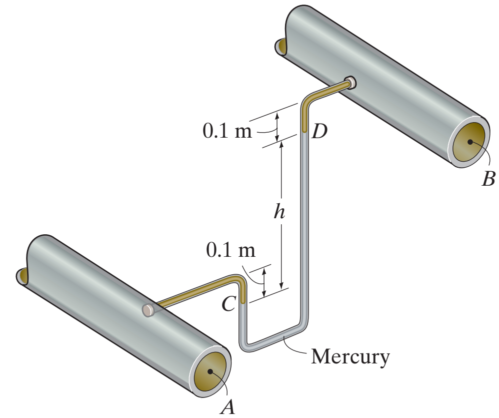
$$h = 1.134\ \text{ft} = 1.13\ \text{ft}$$ **Ans.**



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–45. The two pipes contain hexylene glycol, which causes the level of mercury in the manometer to be at $h = 0.3$ m. Determine the differential pressure in the pipes, $p_A - p_B$. Take $\rho_{hgl} = 923\ \text{kg/m}^3$, $\rho_{\text{Hg}} = 13\,550\ \text{kg/m}^3$. Neglect the diameter of the pipes.



SOLUTION

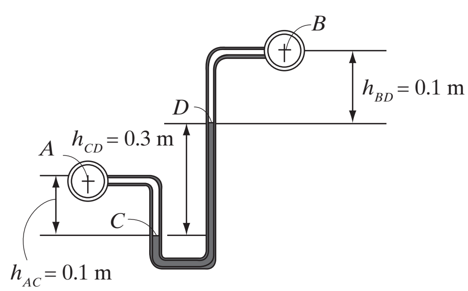
Referring to Fig. *a*, the manometer rule gives

$$p_A + \rho_{hgl} g h_{AC} - \rho_{\text{Hg}} g h_{CD} - \rho_{hgl} g h_{BD} = p_B$$

$$p_A + (923\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m}) - (13\,550\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.3\ \text{m})$$

$$- (923\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m}) = p_B$$

$$p_A - p_B = 39.88(10^3)\ \text{Pa} = 39.9\ \text{kPa} \qquad \textbf{Ans.}$$

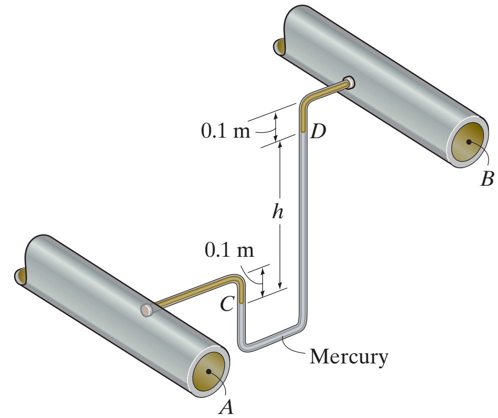


(a)

Ans:
39.9 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–46. The two pipes contain hexylene glycol, which causes the differential pressure reading of the mercury in the manometer to be at $h = 0.3$ m. If the pressure in pipe A increases by 6 kPa, and the pressure in pipe B decreases by 2 kPa, determine the new differential reading h of the manometer. Take $\rho_{hgl} = 923\ \text{kg/m}^3$, $\rho_{\text{Hg}} = 13\,550\ \text{kg/m}^3$. Neglect the diameter of the pipes.



SOLUTION

As shown in Fig. a, the mercury level is at C and D. Applying the manometer rule,

$$p_A + \rho_{hgl} g h_{AC} - \rho_{\text{Hg}} g h_{CD} - \rho_{hgl} g h_{DB} = p_B$$

$$p_A + (923\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m}) - (13\,550\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.3\ \text{m})$$
$$-(923\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m}) = p_B$$

$$p_A - p_B = 39\,877.65\ \text{Pa} \qquad \textbf{(1)}$$

When the pressure at A and B changes, the mercury level will be at C' and D', Fig. a. Then, the manometer rule gives

$$(p_A + \Delta p_A) + \rho_{hgl} g h_{AC'} - \rho_{\text{Hg}} g h_{C'D'} - \rho_{hgl} g h_{D'B} = (p_B - \Delta p_B)$$

$$\left[p_A + 6(10^3)\ \text{N/m}^2\right] + (923\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m} + \Delta h)$$
$$-(13.550\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.3\ \text{m} + 2\Delta h)$$
$$-(923\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m} - \Delta h) = \left[p_B - 2(10^3)\ \text{N/m}^2\right]$$

$$p_A - p_B = 31\,877.65 + 247741.74\ \Delta h \qquad \textbf{(2)}$$

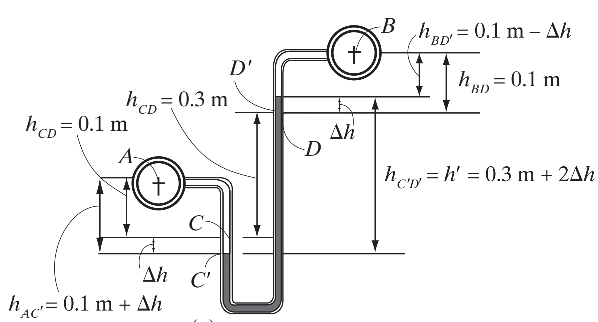
Equating Eqs. (1) and (2), we obtain

$$39\,877.65 = 31\,877.65 + 247741.74\ \Delta h$$

$$\Delta h = 0.03229$$

Thus,

$$h' = 0.3\ \text{m} + 2\Delta h$$
$$= 0.3\ \text{m} + 2(0.03229\ \text{m}) = 0.36458\ \text{m} = 365\ \text{mm} \qquad \textbf{Ans.}$$

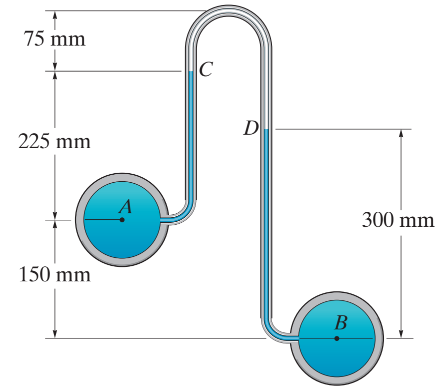


(a)

Ans:
365 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–47. The inverted U-tube manometer is used to measure the difference in pressure between water flowing in the pipes at A and B. If the top segment is filled with air, and the water levels in each segment are as indicated, determine this pressure difference between A and B. $\rho_w = 1000\ \text{kg/m}^3$.



SOLUTION

Notice that the pressure throughout the air in the tube is constant. Referring to Fig. *a*,

$$p_A = (p_w)_1 + p_a = \rho_w g(h_w)_1 + p_a$$

And

$$p_B = (p_w)_2 + p_a = \rho_w g(h_w)_2 + p_a$$

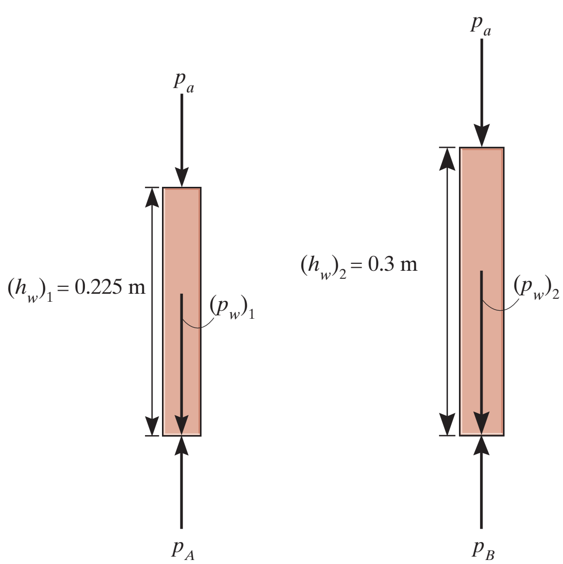
Therefore,

$$\begin{aligned} p_B - p_A &= [\rho_w g(h_w)_2 + p_a] - [\rho_w g(h_w)_1 + p_a] \\ &= \rho_w g[(h_w)_2 - (h_w)_1] \\ &= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.3\ \text{m} - 0.225\ \text{m}) \\ &= 735.75\ \text{Pa} = 736\ \text{Pa} \end{aligned}$$ **Ans.**

Also, using the manometer equation,

$$p_A - \rho_w g h_{AC} + \rho_w g h_{DB} = p_B$$

$$p_B - p_A = \rho_w g[h_{DB} - h_{AC}]$$

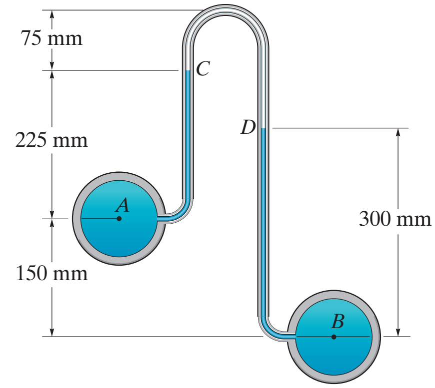


(a)

Ans:
736 Pa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–48.** Solve Prob. 2–47 if the top segment is filled with an oil for which $\rho_o = 800\ \text{kg/m}^3$.



SOLUTION

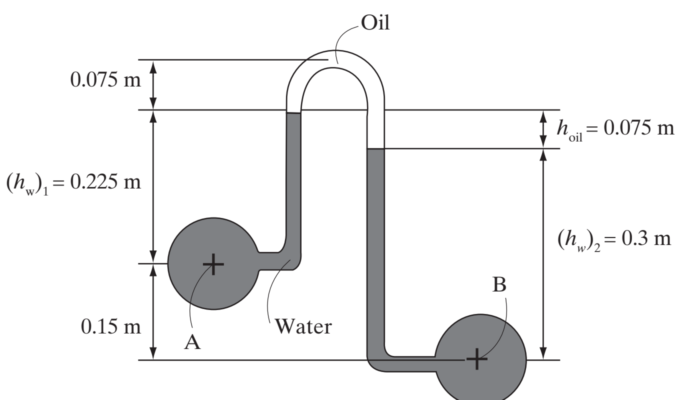
Referring to Fig. *a*, write the manometer equation starting at A and ending at B,

$$p_A - \rho_w g(h_w)_1 + \rho_{oil} g h_{oil} + \rho_w g(h_w)_2 = p_B$$

$$p_B - p_A = \rho_w g\left[(h_w)_2 - (h_w)_1\right] + \rho_{oil} g h_{oil}$$

$$= \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.3\ \text{m} - 0.225\ \text{m}) + \left(800\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.075\ \text{m})$$

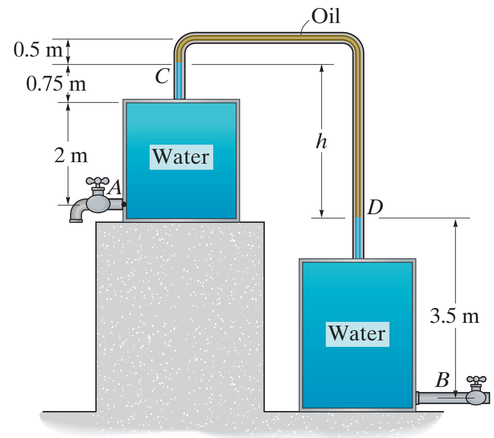
$$= 1.324\left(10^3\right)\ \text{Pa} = 1.32\ \text{kPa} \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–49. The pressure in the tank at the closed valve A is 300 kPa. If the differential elevation in the oil level in $h = 2.5$ m, determine the pressure in the pipe at B. Take $\rho_o = 900\ \text{kg/m}^3$.



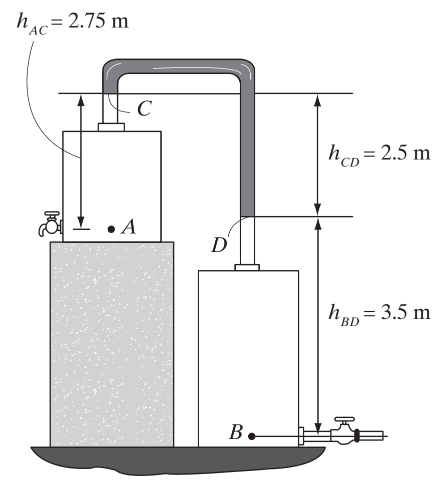
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_A - \rho_w g h_{AC} + \rho_o g h_{CD} + \rho_w g h_{BD} = p_B$$

$$300(10^3)\ \text{N/m}^2 - (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.75\ \text{m}) + (900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.5\ \text{m})$$
$$+ (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3.5\ \text{m}) = p_B$$

$$p_B = 329.43(10^3)\ \text{Pa} = 329\ \text{kPa} \qquad \textbf{Ans.}$$

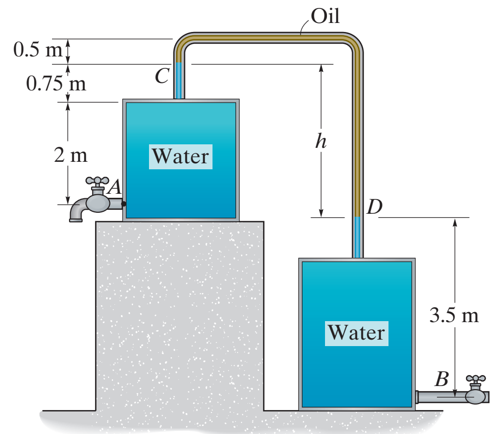


(a)

Ans:
329 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–50. The pressure in the tank at B is 600 kPa. If the differential elevation of the oil is $h = 2.25$ m, determine the pressure at the closed valve A. Take $\rho_o = 900 \text{ kg/m}^3$.



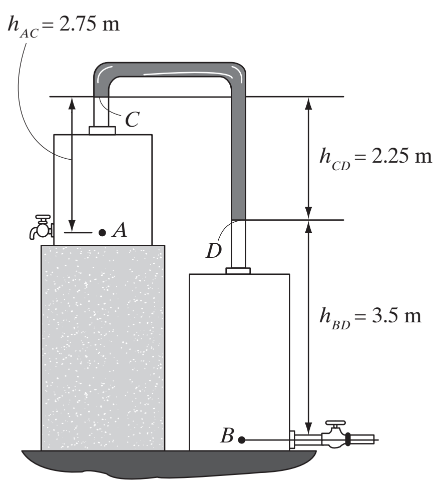
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_A - \rho_w g h_{AC} + \rho_o g h_{CD} + \rho_w g h_{BD} = p_B$$

$$p_A - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.75 \text{ m}) + (900 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.25 \text{ m}) + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3.5 \text{ m}) = 600(10^3) \text{ N/m}^2$$

$$p_A = 572.78(10^3) \text{ Pa} = 573 \text{ kPa}$$ **Ans.**

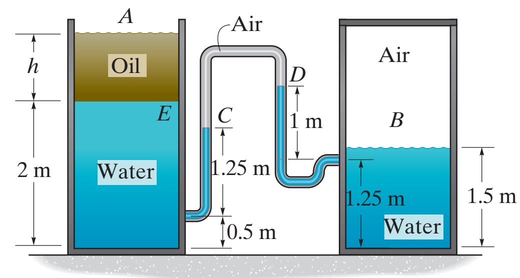


(a)

Ans:
573 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–51. The two tanks A and B are connected using a manometer. If waste oil is poured into tank A to a depth of $h = 0.6$ m, determine the pressure of the entrapped air in tank B. Air is also trapped in line CD as shown. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1000\ \text{kg/m}^3$.



SOLUTION

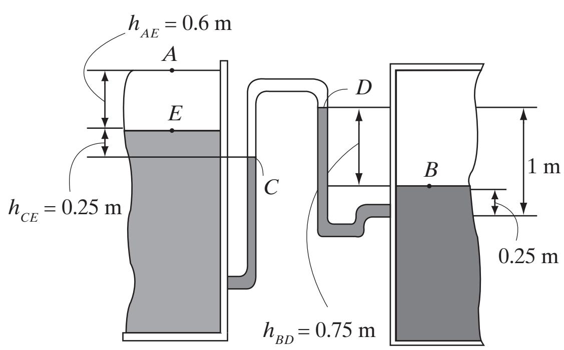
Referring to Fig. a, the manometer rule gives

$$p_A + \rho_o g h_{AE} + \rho_w g h_{CE} + \rho_w g h_{BD} = p_B$$

$$0 + \left(900\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.6\ \text{m}) + \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.25\ \text{m})$$

$$+ \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.75\ \text{m}) = p_B$$

$$p_B = 15.11\left(10^3\right)\ \text{Pa} = 15.1\ \text{kPa} \qquad \textbf{Ans.}$$

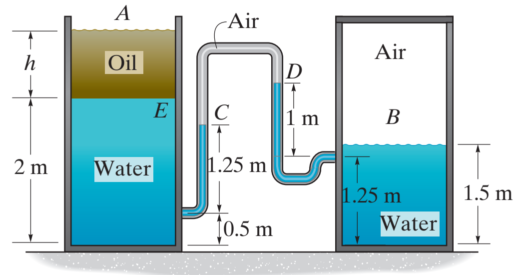


(a)

Ans:
15.1 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–52.** The two tanks A and B are connected using a manometer. If waste oil is poured into tank A to a depth of $h = 1.25$ m, determine the pressure of the trapped air in tank B. Air is also trapped in line CD as shown. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1000\ \text{kg/m}^3$.



SOLUTION

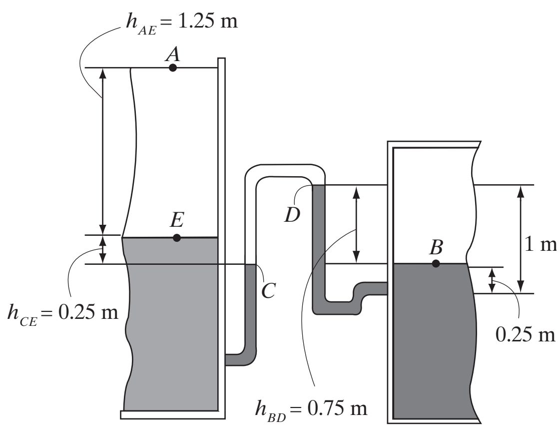
Referring to Fig. a, the manometer rule gives

$$p_A + \rho_o g h_{AE} + \rho_w g h_{CE} + \rho_w g h_{BD} = p_B$$

$$0 + \left(900\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1.25\ \text{m}) + \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.25\ \text{m})$$

$$+ \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.75\ \text{m}) = p_B$$

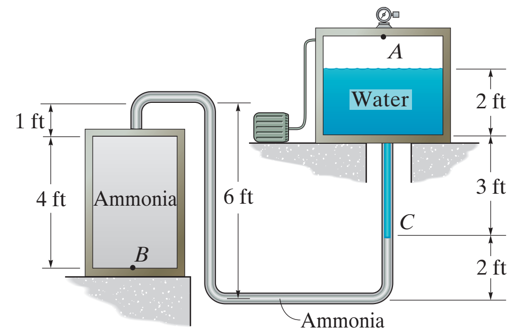
$$p_B = 20.846\left(10^3\right)\ \text{Pa} = 20.8\ \text{kPa}$$ **Ans.**



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–53. Air is pumped into the water tank at A such that the pressure gage reads 20 psi. Determine the pressure at point B at the bottom of the ammonia tank. Take $\rho_{am} = 1.75\ \text{slug/ft}^3$.



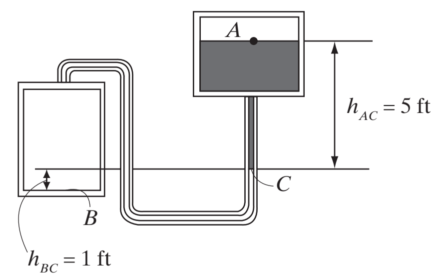
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_A + \gamma_w h_{AC} + \gamma_{Am} h_{BC} = p_B$$

$$\frac{20\ \text{lb}}{\text{in}^2}\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 + \left(62.4\ \text{lb/ft}^3\right)(5\ \text{ft}) + \left(1.75\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)(1\ \text{ft}) = p_B$$

$$p_B = 3248.35\ \frac{\text{lb}}{\text{ft}^2}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 22.6\ \text{psi} \qquad \textbf{Ans.}$$



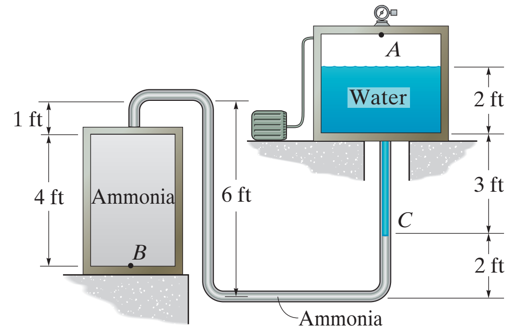
(a)

Note: This is actually an example of a Rayleigh–Taylor instability. The lower density fluid (ammonia) will actually migrate up into the water tank pushing some of the water below the ammonia. This will take place over time.

Ans:
22.6 psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–54. Determine the pressure that must be supplied by the pump so that the air in the tank at A develops a pressure of 50 psi at B in the ammonia tank. Take $\rho_{am} = 1.75\ \text{slug/ft}^3$.



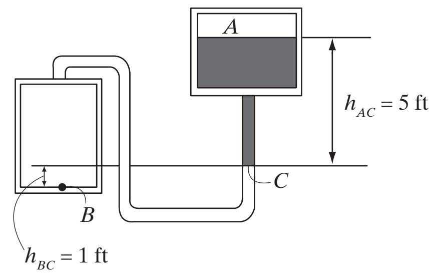
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_A + \gamma_w h_{AC} + \gamma_{Am} h_{BC} = p_B$$

$$p_A + (62.4\ \text{lb/ft}^3)(5\ \text{ft}) + (1.75\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(1\ \text{ft}) = \left(50\ \frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2$$

$$p_A = \left(6831.65\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 47.4\ \text{psi} \qquad \textbf{Ans.}$$



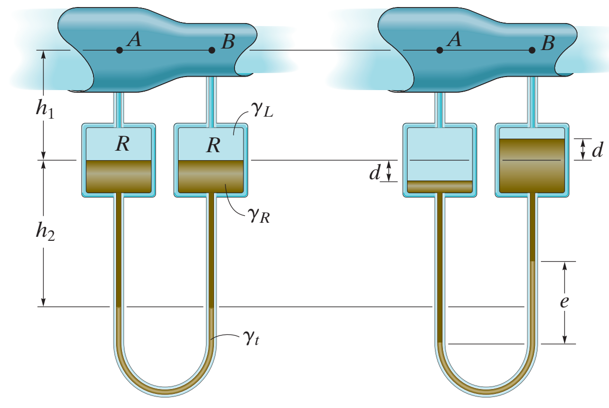
(a)

Note: This is actually an example of a Rayleigh–Taylor instability. The lower density fluid (ammonia) will actually migrate up into the water tank pushing some of the water below the ammonia. This will take place over time.

Ans:
47.4 psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–55. The *micro-manometer* is used to measure small differences in pressure. The reservoirs R and upper portion of the lower tubes are filed with a liquid having a specific weight of γ_R, whereas the lower portion is filled with a liquid having a specific weight of γ_t, Fig. (*a*). When the liquid flows through the venturi meter, the levels of the liquids with respect to the original levels are shown in Fig. (*b*). If the cross-sectional area of each reservoir is A_R and the cross-sectional area of the U-tube is A_t, determine the pressure difference $p_A - p_B$. The liquid in the Venturi meter has a specific weight of γ_L.



(a) (b)

SOLUTION

Write the manometer equation starting at A and ending at B, Fig. *a*

$$p_A + \gamma_L(h_1 + d) + \gamma_R\left(h_2 - d + \frac{e}{2}\right) - \gamma_t e$$

$$-\gamma_R\left(h_2 - \frac{e}{2} + d\right) - \gamma_L(h_1 - d) = p_B$$

$$p_A - p_B = 2\gamma_R d - 2\gamma_L d + \gamma_t e - \gamma_R e \qquad \textbf{(1)}$$

Since the same amount of liquid leaving the left reservoir will enter into the left tube,

$$A_R d = A_t\left(\frac{e}{2}\right)$$

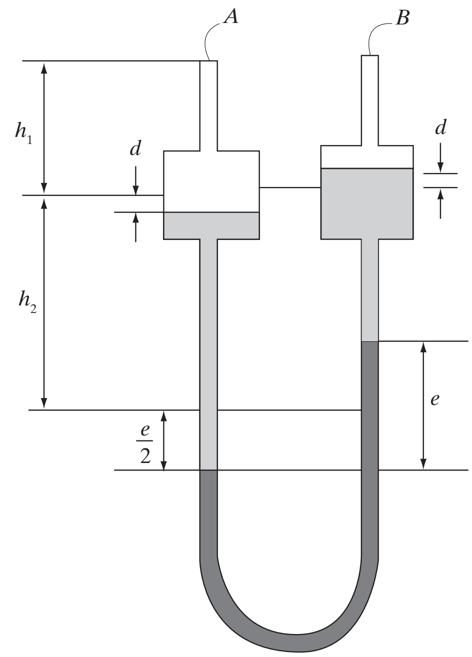
$$d = \left(\frac{A_t}{2A_R}\right)e$$

Substitute this result into Eq. (1),

$$p_A - p_B = 2\gamma_R\left(\frac{A_t}{2A_R}\right)e - 2\gamma_L\left(\frac{A_t}{2A_R}\right)e + \gamma_t e - \gamma_R e$$

$$= e\left[\left(\frac{A_t}{A_R}\right)\gamma_R - \left(\frac{A_t}{A_R}\right)\gamma_L + \gamma_t - \gamma_R\right]$$

$$= e\left[\gamma_t - \left(1 - \frac{A_t}{A_R}\right)\gamma_R - \left(\frac{A_t}{A_R}\right)\gamma_L\right] \qquad \textbf{Ans.}$$



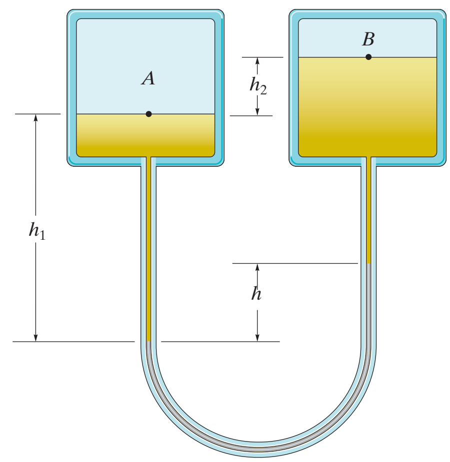
(a)

Ans:

$p_A - p_B = e\left[\gamma_t - \left(1 - \dfrac{A_t}{A_R}\right)\gamma_R - \left(\dfrac{A_t}{A_R}\right)\gamma_L\right]$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–56.** The Morgan Company manufactures a micro-manometer that works on the principles shown. Here there are two reservoirs filled with kerosene, each having a cross-sectional area of 300 mm^2. The connecting tube has a cross-sectional area of 15 mm^2 and contains mercury. Determine h if the pressure difference $p_A - p_B = 40$ Pa. What would h be if water were substituted for mercury? $\rho_{Hg} = 13\,550\ \text{kg/m}^3$, $\rho_{ke} = 814\ \text{kg/m}^3$. *Hint*: Both h_1 and h_2 can be eliminated from the analysis.



SOLUTION

Referring to Fig. *a,* write the manometer equation starting at A and ending at B.

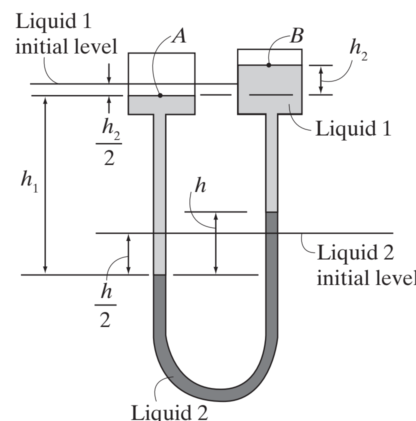
$$p_A + \rho_1 g h_1 - \rho_2 g h - \rho_1 g(h_1 - h + h_2) = p_B$$

$$p_A - p_B = \rho_2 g h - \rho_1 g h + \rho_1 g h_2 \qquad \textbf{(1)}$$

Since the same amount of liquid leaving the left reservoir will enter the left tube

$$A_R = \left(\frac{h_2}{2}\right) = A_t\left(\frac{h}{2}\right)$$

$$h_2 = \left(\frac{A_t}{A_R}\right)h$$



(a)

Substitute this result into Eq. (1)

$$p_A - p_B = \rho_2 g h - \rho_1 g h + \rho_1 g\left(\frac{A_t}{A_R}\right)h$$

$$p_A - p_B = h\left[\rho_2 g - \left(1 - \frac{A_t}{A_R}\right)\rho_1 g\right] \qquad \textbf{(2)}$$

When $\rho_1 = \rho_{ke} = 814\ \text{kg/m}^3$, $\rho_2 = \rho_{Hg} = 13550\ \text{kg/m}^3$ and $p_A - p_B = 40$ Pa,

$$40\ \text{N/m}^2 = h\left[(13\,550\ \text{kg/m}^3)(9.81\ \text{m/s}^2) - \left(1 - \frac{15}{300}\right)(814\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\right]$$

Mercury: $h = 0.3191\left(10^{-3}\right)\ \text{m} = 0.319\ \text{mm}$ **Ans.**

When $\rho_1 = \rho_{ke} = 814\ \text{kg/m}^3$, $\rho_2 = \rho_w = 1000\ \text{kg/m}^3$ and $P_A - P_B = 40$ Pa

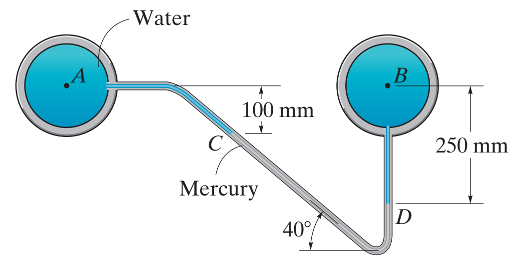
$$40\ \text{N/m}^2 = h\left[(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2) - \left(1 - \frac{15}{300}\right)(814\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\right]$$

Water: $h = 0.01799\ \text{m} = 18.0\ \text{mm}$ **Ans.**

From the results, we notice that if $\rho_2 \gg \rho_1$, h will be too small to be read. Hence, when choosing the liquid to be used, ρ_2 should be slightly larger than ρ_1 so that the sensitivity of the micromanometer is increased.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–57. Determine the difference in pressure $p_B - p_A$ between the centers A and B of the pipes, which are filled with water. The mercury in the inclined-tube manometer has the level shown $S_{Hg} = 13.55$.



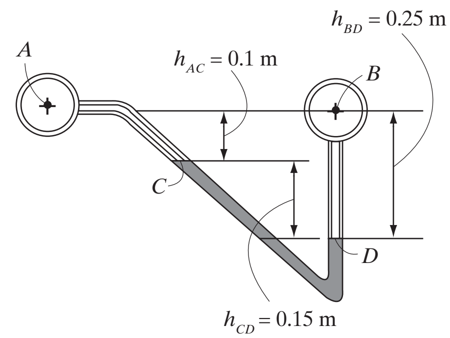
SOLUTION

Referring to Fig. *a*, the manometer rule gives

$$p_A + \rho_w g h_{AC} + \rho_{Hg} g h_{CD} - \rho_w g h_{DB} = p_B$$

$$p_A + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m}) + 13.55(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.15\ \text{m}) - (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.250\ \text{m}) = p_B$$

$$p_B - p_A = 18.47(10^3)\ \text{Pa} = 18.5\ \text{kPa} \qquad \textbf{Ans.}$$

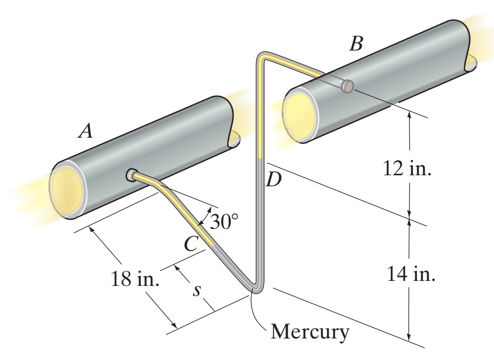


(a)

Ans:
18.5 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–58. Trichlorethylene, flowing through both pipes, is to be added to jet fuel produced in a refinery. A careful monitoring of pressure is required through the use of the inclined-tube manometer. If the pressure at A is 30 psi and the pressure at B is 25 psi, determine the position s that defines the level of mercury in the inclined-tube manometer. Take $S_{Hg} = 13.55$ and $S_t = 1.466$. Neglect the diameter of the pipes.



SOLUTION

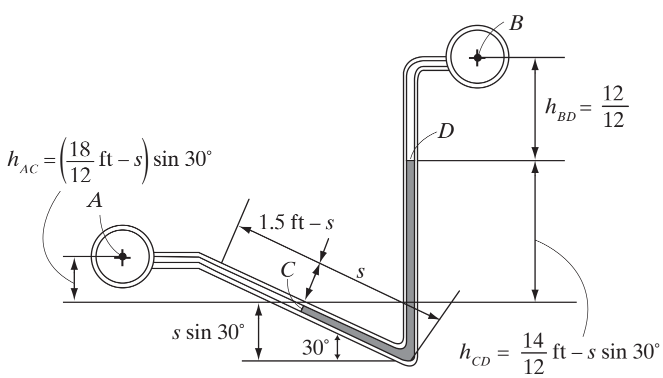
Referring to Fig. a, the manometer rule gives

$$p_A + \gamma_t h_{AC} - \gamma_{Hg} h_{CD} - \gamma_t h_{BD} = p_B$$

$$\left(\frac{30\ \text{lb}}{\text{in}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 + 1.466\left(62.4\ \text{lb/ft}^3\right)\left(\frac{18}{12}\ \text{ft} - s\right)\sin 30° - 13.55\left(62.4\ \text{lb/ft}^3\right)\left(\frac{14}{12}\ \text{ft} - s\sin 30°\right)$$

$$1.466\left(62.4\ \text{lb/ft}^3\right)\left(\frac{12}{12}\ \text{ft}\right) = 25\frac{\text{lb}}{\text{in}^2}\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2$$

$$s = 0.7674\ \text{ft}\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right) = 9.208\ \text{in} = 9.21\ \text{in.} \qquad \textbf{Ans.}$$

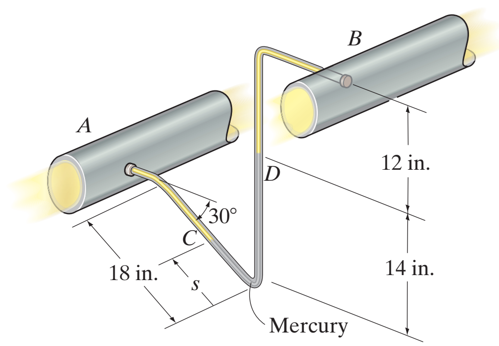


(a)

Ans:
9.21 in.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–59. Trichlorethylene, flowing through both pipes, is to be added to jet fuel produced in a refinery. A careful monitoring of pressure is required through the use of the inclined-tube manometer. If the pressure at A is 30 psi and $s = 7$ in., determine the pressure at B. Take $S_{Hg} = 13.55$ and $S_t = 1.466$. Neglect the diameter of the pipes.



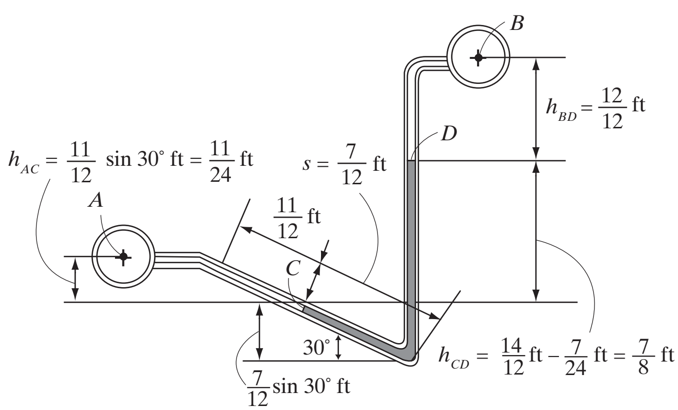
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_A + \gamma_t h_{AC} - \gamma_{Hg} h_{CD} - \gamma_t h_{BD} = p_B$$

$$\frac{30\ \text{lb}}{\text{in.}^2}\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 + 1.466\left(62.4\ \text{lb/ft}^3\right)\left(\frac{11}{24}\ \text{ft}\right) - 13.55\left(62.4\ \text{lb/ft}^3\right)\left(\frac{7}{8}\ \text{ft}\right) - 1.466\left(62.4\ \text{lb/ft}^3\right)\left(\frac{12}{12}\ \text{ft}\right) = p_B$$

$$p_B = 3530.62\ \text{lb/ft}^2\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 24.52\ \text{psi} = 24.5\ \text{psi} \qquad \textbf{Ans.}$$

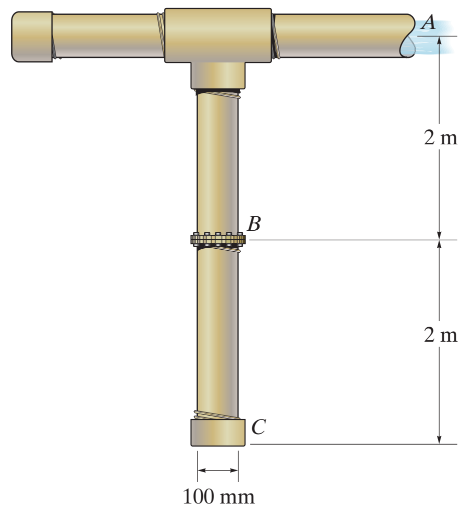


(a)

Ans:
24.5 psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–60.** The vertical pipe segment has an inner diameter of 100 mm and is capped at its end and suspended from the horizontal pipe as shown. If it is filled with water and the pressure at A is 80 kPa, determine the resultant force that must be resisted by the bolts at B in order to hold the flanges together. Neglect the weight of the pipe but not the water within it.



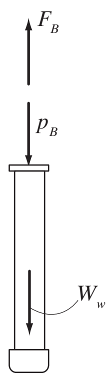
SOLUTION

The forces acting on segment BC of the pipe are indicated on its free-body diagram, Fig. a. Here, $\mathbf{F}_B$ is the force that must be resisted by the bolt, W_w is the weight of the water in segment BC of the pipe, and $\mathbf{P}_B$ is the resultant force of pressure acting on the cross section at B.

$$+\uparrow \Sigma F_y = 0; \qquad F_B - W_w - p_B A_B = 0$$

$$F_B = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2\ \text{m})(\pi)(0.05\ \text{m})^2 + \left[80\left(10^3\right)\text{N/m}^2 + 1000\ \text{kg/m}^3\left(9.81\ \text{m/s}^2\right)(2\ \text{m})\right]\pi(0.05\ \text{m})^2$$

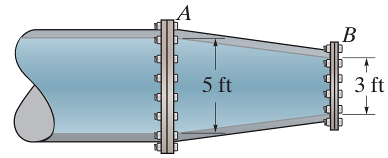
$$= 937\ \text{N} \qquad \textbf{Ans.}$$



(a)

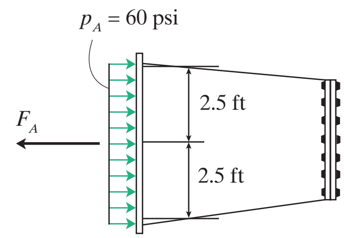
Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–61. Nitrogen in the chamber is at a pressure of 60 psi. Determine the total force the bolts at joints A and B must resist to maintain the pressure. There is a cover plate at B having a diameter of 3 ft.

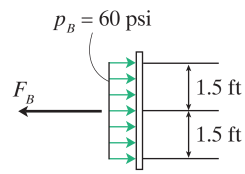


SOLUTION

The force that must be resisted by the bolts at A and B can be obtained by considering the free-body diagrams in Figs. *a* and *b*, respectively. For the bolts at B, Fig. *b*,



(a)



(b)

$$\xrightarrow{+} \Sigma F_x = 0; \qquad p_B A_B - F_B = 0$$

$$F_B = p_B A_B = \left(60\ \text{lb/in}^2\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 \left[(\pi)(1.5\ \text{ft})^2\right]$$

$$= 61073\ \text{lb} = 61.1\ \text{kip} \qquad \textbf{Ans.}$$

For the bolts at A, Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad p_A A_A - F_A = 0$$

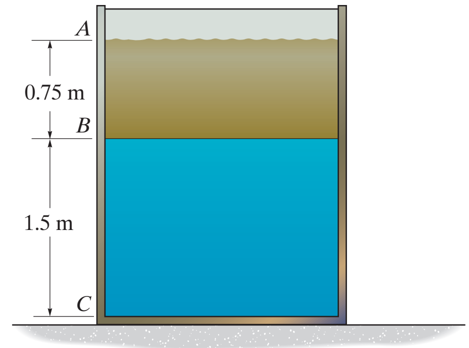
$$F_A = p_A A_A = \left(60\ \text{lb/in}^2\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 \left[(\pi)(2.5\ \text{ft})^2\right]$$

$$= 169646\ \text{lb} = 170\ \text{kip} \qquad \textbf{Ans.}$$

Ans:
$F_B = 61.1$ kip, $F_A = 170$ kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–62. The storage tank contains oil and water acting at the depths shown. Determine the resultant force that both of these liquids exert on the side ABC of the tank if the side has a width of $b = 1.25$ m. Also, determine the location of this resultant, measured from the top of the tank. Take $\rho_o = 900\ \text{kg/m}^3$.

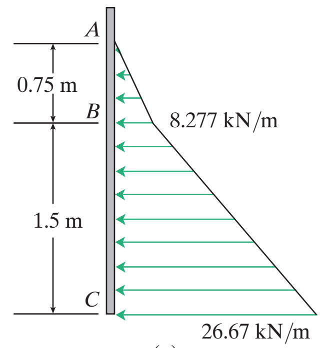


SOLUTION

Loading. Since the side of the tank has a constant width, then the intensities of the distributed loading at B and C, Fig. 2–28b, are

$$w_B = \rho_o g h_{AB} b = \left(900\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.75\ \text{m})(1.25\ \text{m}) = 8.277\ \text{kN/m}$$

$$w_C = w_B + \rho_w g h_{BC} b = 8.277\ \text{kN/m} + \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1.5\ \text{m})(1.25\ \text{m})$$
$$= 26.77\ \text{kN/m}$$



(a)

Resultant Force. The resultant force can be determined by adding the shaded triangular and rectangular areas in Fig. 2–28c. The resultant force is therefore

$$F_R = F_1 + F_2 + F_3$$
$$= \frac{1}{2}(0.75\ \text{m})(8.277\ \text{kN/m}) + (1.5\ \text{m})(8.277\ \text{kN/m}) + \frac{1}{2}(1.5\ \text{m})(18.39\ \text{kN/m})$$
$$= 3.104\ \text{kN} + 12.42\ \text{kN} + 13.80\ \text{kN} = 29.32\ \text{kN} = 29.3\ \text{kN} \qquad \textbf{Ans.}$$

As shown, each of these three parallel resultants acts through the centroid of its respective area.

$$y_1 = \frac{2}{3}(0.75\ \text{m}) = 0.5\ \text{m}$$

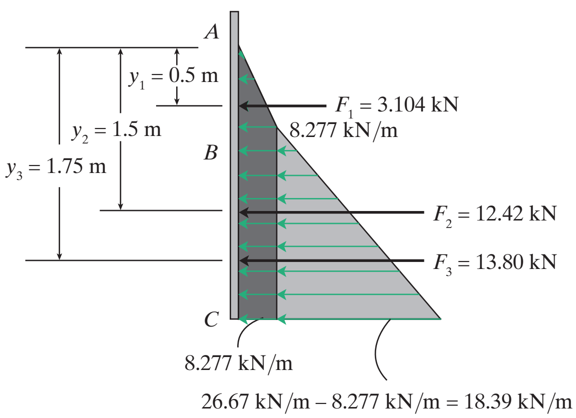
$$y_2 = 0.75\ \text{m} + \frac{1}{2}(1.5\ \text{m}) = 1.5\ \text{m}$$

$$y_3 = 0.75\ \text{m} + \frac{2}{3}(1.5\ \text{m}) = 1.75\ \text{m}$$

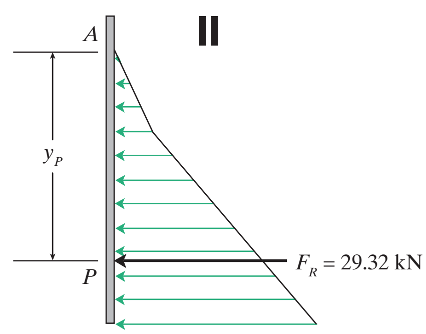
The location of the resultant force is determined by equating the moment of the resultant above A, Fig. 2–28d, to the moments of the component forces about A, Fig. 2–28c. We have,

$$\bar{y}_P F_R = \Sigma y F; \qquad \bar{y}_P(29.32\ \text{kN}) = (0.5\ \text{m})(3.104\ \text{kN}) + (1.5\ \text{m})(12.42\ \text{kN})$$
$$+ (1.75\ \text{m})(13.80\ \text{kN})$$

$$\bar{y}_P = 1.51\ \text{m} \qquad \textbf{Ans.}$$



(b)

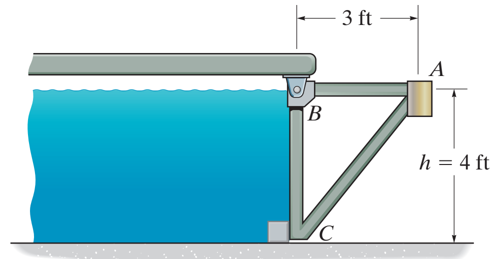


(c)

Ans:
$F_R = 29.3\ \text{kN}, \bar{y}_P = 1.51\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–63. Determine the weight of block A if the rectangular gate begins to open when the water level reaches the top of the channel, $h = 4$ ft. The gate has a width of 2 ft. There is a smooth stop block at C.



SOLUTION

Since the gate has a constant width of $b = 2$ ft, the intensity of the distributed load at C can be computed from

$$w_C = \gamma_w h_C b = \left(62.4\ \text{lb/ft}^3\right)(4\ \text{ft})(2\ \text{ft}) = 499.2\ \text{lb/ft}$$

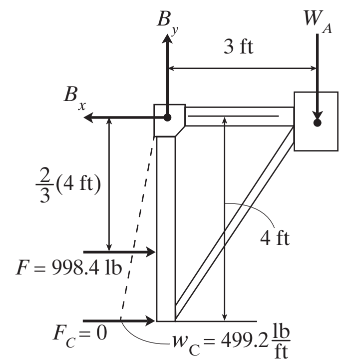
The resultant triangular distributed load is shown on the free-body diagram of the gate, Fig. *a*, and the resultant force of this load is

$$F = \frac{1}{2}w_C h_C = \frac{1}{2}(499.2\ \text{lb/ft})(4\ \text{ft}) = 998.4\ \text{lb}$$

Referring to the free-body diagram of the gate, Fig. *a*,

$$\circlearrowleft + \Sigma M_B = 0; \qquad 998.4\ \text{lb}\left[\frac{2}{3}(4\ \text{ft})\right] - W_A(3\ \text{ft}) = 0$$

$$W_A = 887.47\ \text{lb} = 887\ \text{lb} \qquad \textbf{Ans.}$$

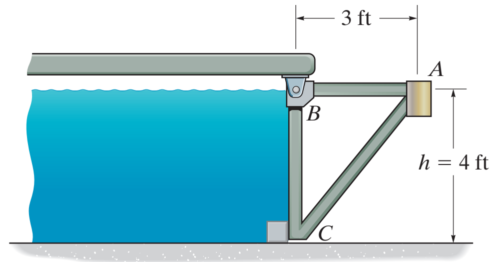


(a)

Ans:
887 lb

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–64.** Determine the weight of block A so that the 2-ft-radius circular gate BC begins to open when the water level reaches the top of the channel, $h = 4$ ft. There is a smooth stop block at C.



SOLUTION

Since the gate is circular in shape, it is convenient to compute the resultant force as follows.

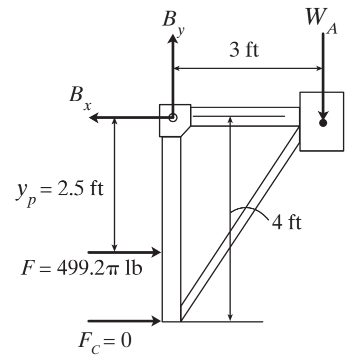
$$F_R = \gamma_w \bar{h} A$$

$$F = \left(62.4\ \text{lb/ft}^3\right)(2\ \text{ft})(\pi)(2\ \text{ft})^2 = 499.2\pi\ \text{lb}$$

The location of the center of pressure can be determined from

$$y_p = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$

$$= \frac{\left(\dfrac{\pi(2\ \text{ft})^4}{4}\right)}{(2\ \text{ft})(\pi)(2\ \text{ft})^2} + 2\ \text{ft} = 2.50\ \text{ft}$$



(a)

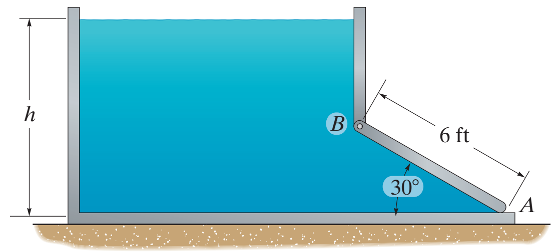
Referring to the free-body diagram of the gate, Fig. *a*,

$$\circlearrowleft + \Sigma M_B = 0; \qquad 499.2\pi\ \text{lb}(2.5\ \text{ft}) - W_A\,(3\ \text{ft}) = 0$$

$$W_A = 1306.90\ \text{lb} = 1.31\ \text{kip} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–65. The uniform rectangular relief gate AB has a weight of 8000 lb and a width of 4 ft. Determine the minimum depth h of water within the canal needed to open it. The gate is pinned at B and rests on a rubber seal at A.



SOLUTION

Here $h_B = h - 6 \sin 30° = (h - 3)$ ft and $h_A = h$. Thus, the intensities of the distributed load at B and A are

$$w_B = \gamma_w h_B b = \left(62.4\ \text{lb/ft}^3\right)(h - 3\ \text{ft})(4\ \text{ft}) = (249.6h - 748.8)\ \text{lb/ft}$$

$$w_A = \gamma_w h_A b = \left(62.4\ \text{lb/ft}^3\right)(h)(4\ \text{ft}) = (249.6h)\ \text{lb/ft}.$$

Thus,

$$\left(F_p\right)_1 = \left[(249.6h - 748.8\ \text{lb/ft})\right](6\ \text{ft}) = (1497.6h - 4492.8)\ \text{lb}$$

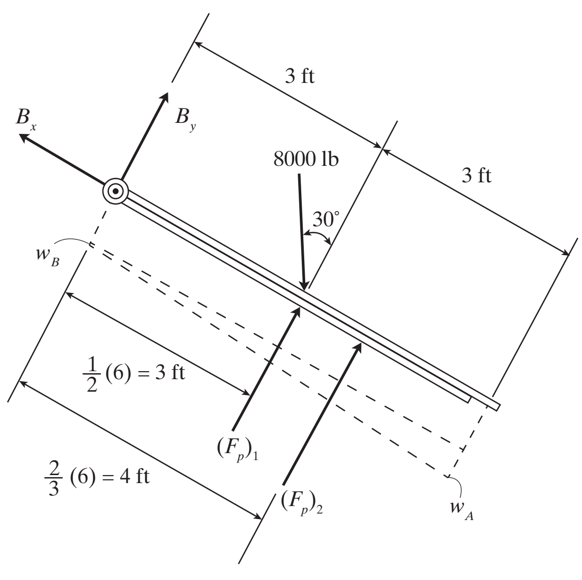
$$\left(F_p\right)_2 = \frac{1}{2}\left[(249.6h\ \text{lb/ft}) - (249.6h - 748.8\ \text{lb/ft})\right](6\ \text{ft}) = 2246.4\ \text{lb}$$

If it is required that the gate is about to open, then the normal reaction at A is equal to zero. Write the moment equation of equilibrium about B, referring to Fig. a,

$$\circlearrowleft + \Sigma M_B = 0;\ \left[(1497.6h - 4492.8\ \text{lb})\right](3\ \text{ft}) + (2246.4\ \text{lb})(4\ \text{ft})$$

$$-(8000\ \text{lb}) \cos 30°(3\ \text{ft}) = 0$$

$$h = 5.626\ \text{ft} = 5.63\ \text{ft}$$ **Ans.**

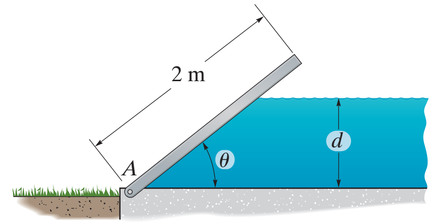


(a)

Ans:
5.63 ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–66. The uniform swamp gate has a mass of 4 Mg and a width of 1.5 m. Determine the angle θ for equilibrium if the water rises to a depth of $d = 1.5$ m.



SOLUTION

Since the gate has a constant width of $b = 1.5$ m, the intensity of the distributed load at A can be computed from

$$w_A = \rho_w g h_A b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1.5\ \text{m})(1.5\ \text{m})$$
$$= 22.07\left(10^3\right)\ \text{N/m}$$

The resulting triangular distributed load is shown on the free-body diagram of the gate, Fig. *a*.

$$F = \frac{1}{2}w_A L = \frac{1}{2}\left[22.07\left(10^3\right)\ \text{N/m}\right]\left(\frac{1.5\ \text{m}}{\sin\theta}\right) = \frac{16.554\left(10^3\right)}{\sin\theta}$$

Referring to the free-body diagram of the gate, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \quad \left[\frac{16.554\left(10^3\right)}{\sin\theta}\right]\left[\frac{1}{3}\left(\frac{1.5\ \text{m}}{\sin\theta}\right)\right] - \left[4000(9.81)\ \text{N}\right]\cos\theta(1\ \text{m}) = 0$$

$$\sin^2\theta\cos\theta = 0.2109$$

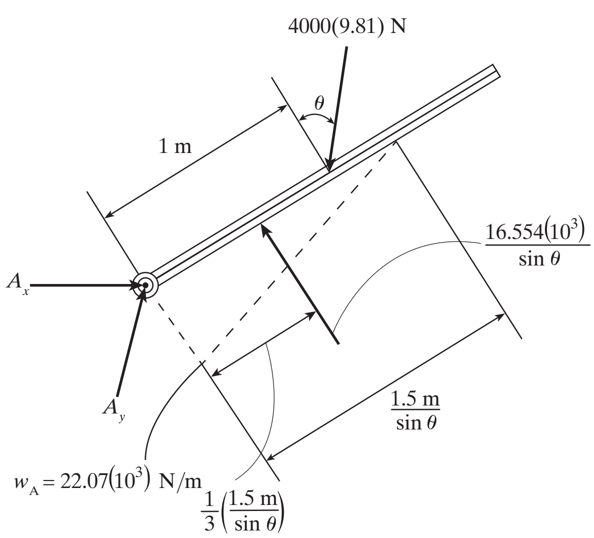
Solving numerically,

$$\theta = 29.49^\circ \text{ or } 77.18^\circ$$

Since $\dfrac{1.5\ \text{m}}{\sin 77.18^\circ} = 1.54\ \text{m} < 2\ \text{m}$ and $\dfrac{1.5\ \text{m}}{\sin 29.49^\circ} = 3.05\ \text{m} > 2\ \text{m}$ only one solution is valid.

$$\theta = 77.2^\circ \qquad \textbf{Ans.}$$

Note: This solution represents an *unstable* equilibrium.

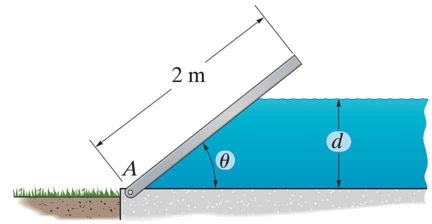


(a)

Ans:
77.2°

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–67. The uniform swamp gate has a mass of 3 Mg and a width of 1.5 m. Determine the depth of the water d if the gate is held in equilibrium at an angle of $\theta = 60°$.



SOLUTION

Since the gate has a constant width of $b = 1.5$ m, the intensity of the distributed load at A can be computed from

$$w_A = \rho_w g h_A b = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(d)(1.5 \text{ m})$$
$$= 14\,715d \text{ N/m}$$

The resulting triangular distributed load is shown on the free-body diagram of the gate, Fig. a.

$$F = \frac{1}{2} w_A L = \frac{1}{2}(14\,715d)\left(\frac{d}{\sin 60°}\right) = \frac{7357.5}{\sin 60°} d^2$$

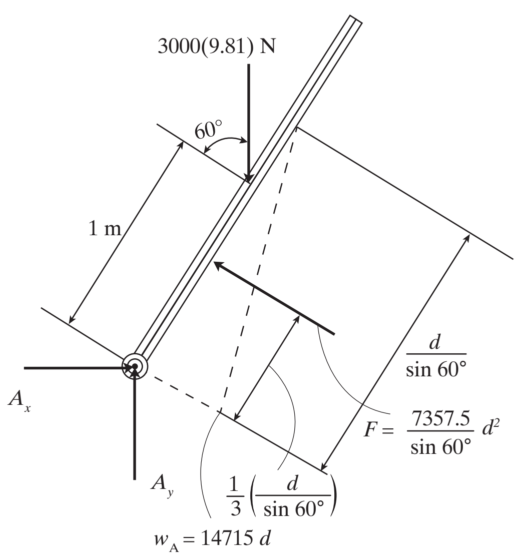
Referring to the free-body diagram of the gate, Fig. a,

$$\zeta + \Sigma M_A = 0; \quad \left(\frac{7357.5}{\sin 60°} d^2\right)\left[\frac{1}{3}\left(\frac{d}{\sin 60°}\right)\right] - \left[3000(9.81) \text{ N}\right] \cos 60°(1 \text{ m}) = 0$$

$$d = 1.6510 \text{ m} = 1.65 \text{ m}$$ **Ans.**

Since $\dfrac{1.6510 \text{ m}}{\sin 60°} = 1.906 \text{ m} < 2 \text{ m}$, this result is valid.

Note: This solution represents an unstable equilibrium. The gate is "held" in place by small external stabilizing forces.

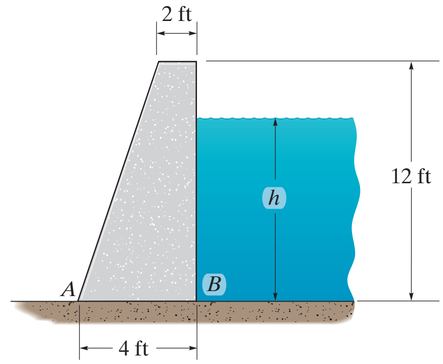


(a)

Ans:
1.65 m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–68.** Determine the critical height h of the water level before the concrete gravity dam starts to tip over due to water pressure acting on its face. The specific weight of concrete is $\gamma_c = 150\ \text{lb/ft}^3$. *Hint:* Work the problem using a 1-ft width of the dam.



SOLUTION

We will consider the dam as having a width of $b = 1$ ft. Then the intensity of the distributed load at the base of the dam is

$$w_B = \gamma_w h b = (62.4\ \text{lb/ft}^3)(h)(1\ \text{ft}) = 62.4h\ \text{lb/ft}$$

The resulting triangular distributed load is shown on the free-body diagram of the dam, Fig. *a*.

$$F = \frac{1}{2}w_B h = \frac{1}{2}(62.4h)h = 31.2h^2$$

It is convenient to subdivide the dam into two parts. The weight of each part is

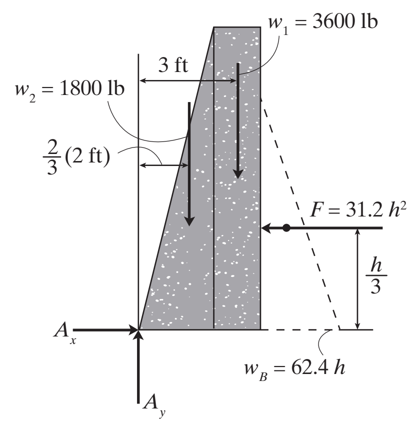
$$W_1 = \gamma_C \mathcal{V}_1 = (150\ \text{lb/ft}^3)\left[2\ \text{ft}(12\ \text{ft})(1\ \text{ft})\right] = 3600\ \text{lb}$$

$$W_2 = \gamma_C \mathcal{V}_2 = (150\ \text{lb/ft}^3)\left[\frac{1}{2}(2\ \text{ft})(12\ \text{ft})(1\ \text{ft})\right] = 1800\ \text{lb}$$

The dam will overturn about point A. Referring to the free-body diagram of the dam, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \quad 31.2h^2\left(\frac{h}{3}\right) - (3600\ \text{lb})(3\ \text{ft}) - (1800\ \text{lb})\left[\frac{2}{3}(2\ \text{ft})\right] = 0$$

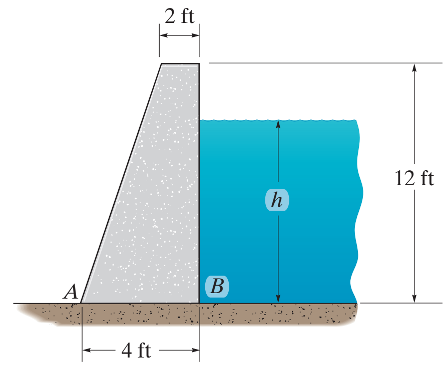
$$h = 10.83\ \text{ft} = 10.8\ \text{ft} \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–69. Determine the critical height h of the water level before the concrete gravity dam starts to tip over due to water pressure acting on its face. Assume water also seeps under the base of the dam. The specific weight of concrete is $\gamma_c = 150\ \text{lb/ft}^3$. *Hint:* Work the problem using a 1-ft width of the dam.



SOLUTION

We will consider the dam having a width of $b = 1$ ft. Then the intensity of the distributed load at the base of the dam is

$$w_B = \gamma_w h_b b = \left(62.4\ \text{lb/ft}^3\right)(h)(1) = 62.4\,h\ \text{lb/ft}$$

The resultant forces of the triangular distributed load and uniform distributed load due the pressure of the seepage water shown on the FBD of the dam, Fig. *a*, are

$$F_1 = \frac{1}{2}w_B h = \frac{1}{2}(62.4\,h)\,h = 31.2\,h^2$$

$$F_2 = w_B L_B = 62.4\,h(4\ \text{ft}) = 249.6\,h$$

It is convenient to subdivide the dam into two parts. The weight of each part is

$$w_1 = \gamma_C \mathcal{V}_1 = \left(150\ \text{lb/ft}^3\right)\left[(2\ \text{ft})(12\ \text{ft})(1\ \text{ft})\right] = 3600\ \text{lb}$$

$$w_2 = \gamma_C \mathcal{V}_2 = \left(150\ \text{lb/ft}^3\right)\left[\frac{1}{2}(2\ \text{ft})(12\ \text{ft})(1\ \text{ft})\right] = 1800\ \text{lb}$$

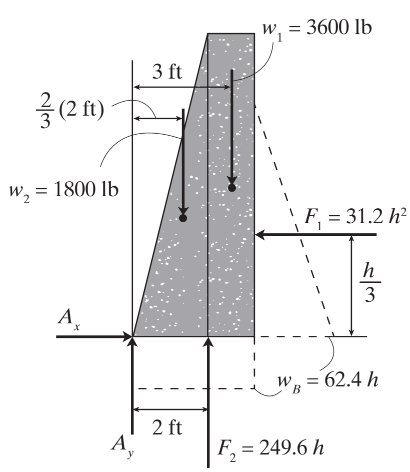
The dam will overturn about point A. Referring to the FBD of the dam, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad 31.2\,h^2\left(\frac{h}{3}\right) + 249.6\,h(2\ \text{ft}) - (3600\ \text{lb})(3\ \text{ft}) - (1800\ \text{lb})\left[\frac{2}{3}(2\ \text{ft})\right] = 0$$

$$10.4\,h^3 + 499.2\,h - 13200 = 0$$

Solve numerically,

$$h = 9.3598\ \text{ft} = 9.36\ \text{ft} \qquad \textbf{Ans.}$$

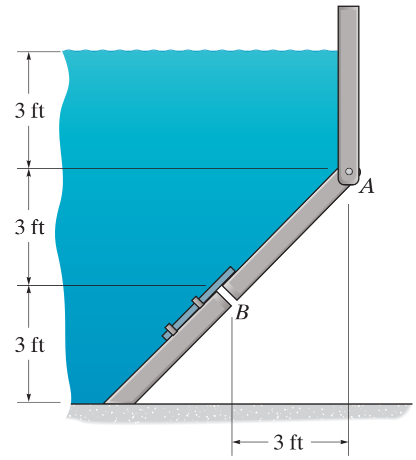


(a)

Ans:
9.36 ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–70. The gate is 2 ft wide and is pinned at A and held in place by a smooth latch bolt at B that exerts a force normal to the gate. Determine this force caused by the water and the resultant force on the pin for equilibrium.



SOLUTION

Since the gate has a width of $b = 2$ ft, the intensities of the distributed loads at A and B can be computed from

$$w_A = \gamma_w h_A b = \left(62.4\ \text{lb/ft}^3\right)(3\ \text{ft})(2\ \text{ft}) = 374.4\ \text{lb/ft}$$

$$w_B = \gamma_w h_B b = \left(62.4\ \text{lb/ft}^3\right)(6\ \text{ft})(2\ \text{ft}) = 748.8\ \text{lb/ft}$$

The resulting trapezoidal distributed load is shown on the free-body diagram of the gate, Fig. *a*. This load can be subdivided into two parts. The resultant force of each part is

$$F_1 = w_A L_{AB} = (374.4\ \text{lb/ft})\left(3\sqrt{2}\ \text{ft}\right) = 1123.2\sqrt{2}\ \text{lb}$$

$$F_2 = \frac{1}{2}(w_B - w_A)L_{AB} = \frac{1}{2}(748.8\ \text{lb/ft} - 374.4\ \text{lb/ft})\left(3\sqrt{2}\ \text{ft}\right) = 561.6\sqrt{2}\ \text{lb}$$

Considering the free-body diagram of the gate, Fig. *a*,

$$\circlearrowleft +\Sigma M_A = 0; \qquad 1123.2\sqrt{2}\ \text{lb}\left(\frac{1}{2}3\sqrt{2}\ \text{ft}\right) + 561.6\sqrt{2}\ \text{lb}\left(\frac{2}{3}3\sqrt{2}\ \text{ft}\right) - N_B\left(3\sqrt{2}\ \text{ft}\right) = 0$$

$$N_B = 1323.7\ \text{lb} = 1.32\ \text{kip} \qquad \textbf{Ans.}$$

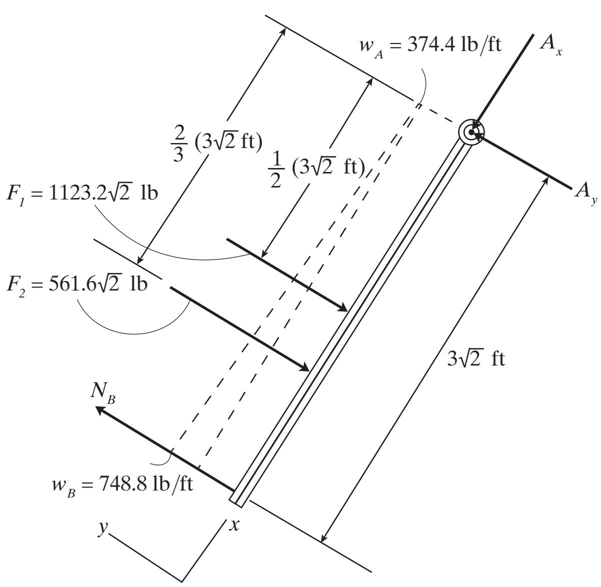
$$\Sigma F_x = 0; \qquad A_x = 0$$

$$\nwarrow +\Sigma F_y = 0; \qquad 1323.7\ \text{lb} - 1123.2\sqrt{2}\ \text{lb} - 561.6\sqrt{2}\ \text{lb} + A_y = 0$$

$$A_y = 1058.96\ \text{lb} = 1.059\ \text{kip}$$

Thus,

$$F_A = \sqrt{(0)^2 + (1.059\ \text{kip})^2} = 1.06\ \text{kip} \qquad \textbf{Ans.}$$

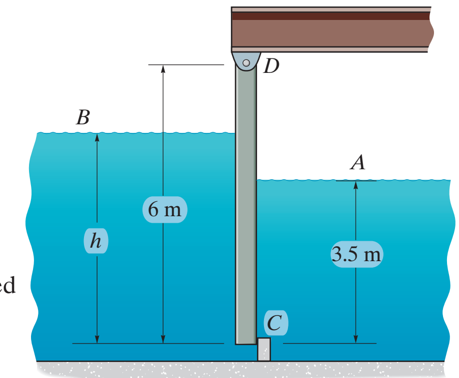


(a)

Ans:
$N_B = 1.32$ kip
$F_A = 1.06$ kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–71. The tide gate opens automatically when the tide water at B subsides, allowing the marsh at A to drain. For the water level $h = 4$ m, determine the horizontal reaction at the smooth stop C. The gate has a width of 2 m. At what height h will the gate be on the verge of opening?



SOLUTION

Since the gate has a constant width of $b = 2$ m, the intensities of the distributed load on the left and right sides of the gate at C are

$$(w_C)_L = \rho_w g h_{BC}(b) = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4\ \text{m})(2\ \text{m})$$
$$= 78.48(10^3)\ \text{N/m}$$
$$(w_C)_R = \rho_w g h_{AC}(b) = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3.5\ \text{m})(2\ \text{m})$$
$$= 68.67(10^3)\ \text{N/m}$$

The resultant triangular distributed load on the left and right sides of the gate is shown on its free-body diagram, Fig. *a*,

$$F_L = \frac{1}{2}(w_C)_L L_{BC} = \frac{1}{2}\left(78.48(10^3)\ \text{N/m}\right)(4\ \text{m}) = 156.96(10^3)\ \text{N}$$
$$F_R = \frac{1}{2}(w_C)_R L_{AC} = \frac{1}{2}\left(68.67(10^3)\ \text{N/m}\right)(3.5\ \text{m}) = 120.17(10^3)\ \text{N}$$

These results can also be obtained as follows

$$F_L = \gamma \bar{h}_L A_L = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2\ \text{m})[(4\ \text{m})(2\ \text{m})] = 156.96(10^3)\ \text{N}$$
$$F_R = \gamma \bar{h}_R A_R = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.75\ \text{m})[3.5\ \text{m}(2\ \text{m})] = 120.17(10^3)\ \text{N}$$

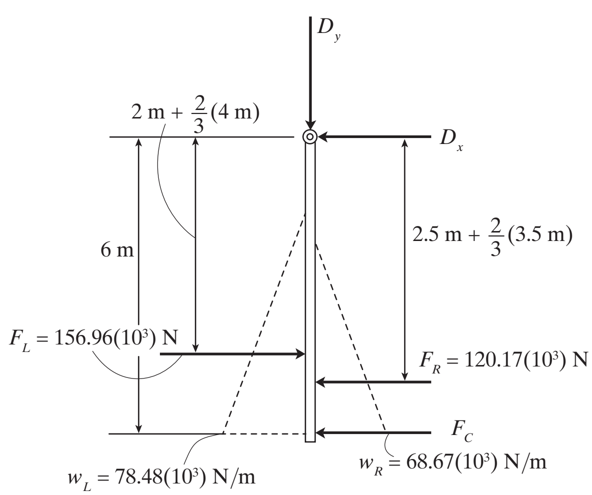
Referring to the free-body diagram of the gate in Fig. *a*,

$$\circlearrowleft + \Sigma M_D = 0; \quad [156.96(10^3)\ \text{N}]\left[2\ \text{m} + \frac{2}{3}(4\ \text{m})\right] - [120.17(10^3)\ \text{N}]\left[2.5\ \text{m} + \frac{2}{3}(3.5\ \text{m})\right] - F_C(6\ \text{m}) = 0$$

$$F_C = 25.27(10^3)\ \text{N} = 25.3\ \text{kN} \qquad \textbf{Ans.}$$

When $h = 3.5$ m, the water levels are equal. Since $F_C = 0$, the gate will open.

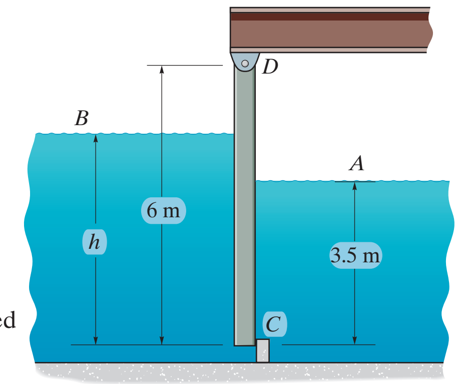
$$h = 3.5\ \text{m} \qquad \textbf{Ans.}$$



Ans:
$F_C = 25.3$ kN
$h = 3.5$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–72.** The tide opens automatically when the tide water at B subsides, allowing the marsh at A to drain. Determine the horizontal reaction at the smooth stop C as a function of the depth h of the water level. Starting at $h = 6$ m, plot values of h for each increment of 0.5 m until the gate begins to open. The gate has a width of 2 m.



SOLUTION

Since the gate has a constant width of $b = 2$ m, the intensities of the distributed loads on the left and right sides of the gate at C are

$$(W_C)_L = \rho_w g h_{BC} b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h)(2\ \text{m}) = 19.62(10^3)h$$

$$(W_C)_R = \rho_w g h_{AC} b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3.5\ \text{m})(2\ \text{m}) = 68.67(10^3)\ \text{N/m}$$

The resultant forces of the triangular distributed loads on the left and right sides of the gate shown on its FBD, Fig. *a*, are

$$F_L = \frac{1}{2}(w_C)_L h_{BC} = \frac{1}{2}\left[19.62(10^3)h\right]h = 9.81(10^3)h^2$$

$$F_R = \frac{1}{2}(w_C)_R h_{AC} = \frac{1}{2}\left[68.67(10^3)\ \text{N/m}\right](3.5\ \text{m}) = 120.17(10^3)\ \text{N}$$

Consider the moment equilibrium about D by referring to the FBD of the gate, Fig. *a*,

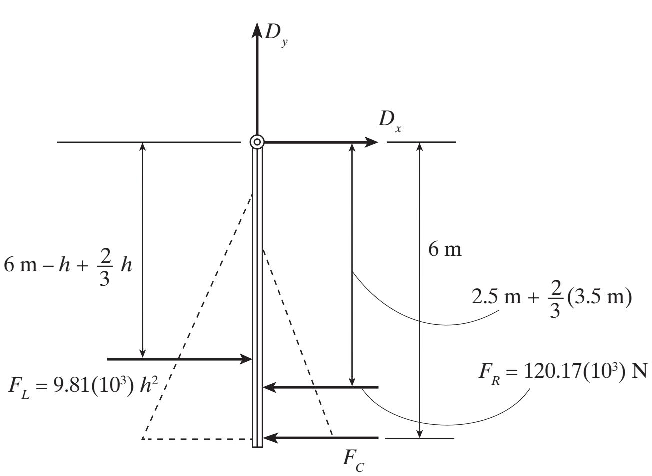
$$\circlearrowleft + \Sigma M_D = 0; \qquad \left[9.81(10^3)h^2\right]\left(6\ \text{m} - h + \frac{2}{3}h\right) - 120.17(10^3)\left[2.5\ \text{m} + \frac{2}{3}(3.5\ \text{m})\right] - F_C(6\ \text{m}) = 0$$

$$58.86(10^3)h^2 - 3.27(10^3)h^3 - 580.83(10^3) - 6F_C = 0$$

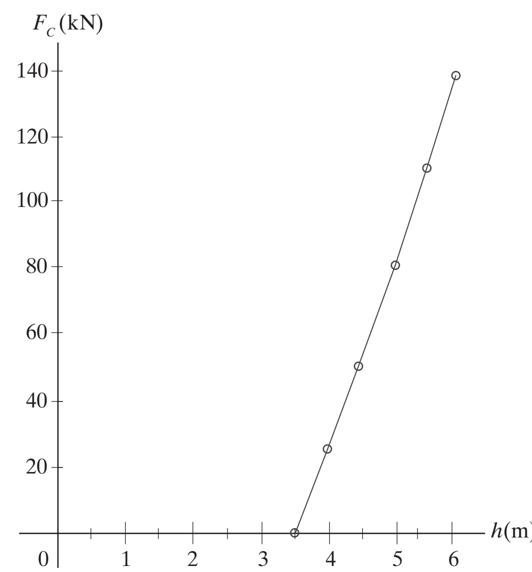
$$F_C = (9.81\ h^2 - 0.545\ h^3 - 96.806)(10^3)\ \text{N}$$

$$F_C = (9.81\ h^2 - 0.545\ h^3 - 96.8)\ \text{kN where } h \text{ is in meters} \qquad \textbf{Ans.}$$

The gate will be on the verge of opening when the water level on both sides of the gate are equal, that is when $h = 3.5$ m. The plot of F_C vs h is shown in Fig. *b*.



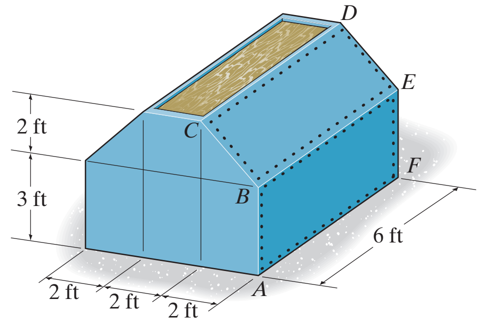
(a)



h(m)	3.5	4	4.5	5.0	5.5	6.0
F_C(kN)	0	25.3	52.2	80.3	109.3	138.6

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–73. The bin is used to store carbon tetrachloride, a cleaning agent for metal parts. If it is filled to the top, determine the magnitude of the resultant force this liquid exerts on each of the two side plates, $AFEB$ and $BEDC$, and the location of the center of pressure on each plate, measured from BE. Take $\rho_{ct} = 3.09\ \text{slug/ft}^3$.



SOLUTION

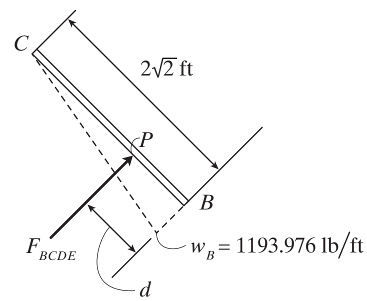
Since the side plate has a width of $b = 6$ ft, the intensities of the distributed load can be computed from

$$w_B = \rho g h_B b = \left(3.09\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)(2\ \text{ft})(6\ \text{ft}) = 1193.976\ \text{lb/ft}$$

$$w_A = \rho g h_A b = \left(3.09\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)(5\ \text{ft})(6\ \text{ft}) = 2984.94\ \text{lb/ft}$$

The resulting distributed load on plates $BCDE$ and $ABEF$ are shown in Figs. *a* and *b*, respectively. For plate $BCDE$,

$$F_{BCDE} = \frac{1}{2}(w_B)L_{BC} = \frac{1}{2}(1193.976\ \text{lb/ft})\left(2\sqrt{2}\ \text{ft}\right) = 1688.54\ \text{lb} = 1.69\ \text{kip} \qquad \textbf{Ans.}$$



(a)

And the center of pressure of this plate from BE is

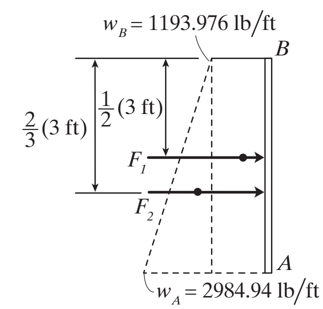
$$d = \frac{1}{3}\left(2\sqrt{2}\ \text{ft}\right) = 0.943\ \text{ft} \qquad \textbf{Ans.}$$

For $ABEF$,

$$F_1 = w_B L_{AB} = (1193.976\ \text{lb/ft})(3\ \text{ft}) = 3581.93\ \text{lb}$$

$$F_2 = \frac{1}{2}(w_A - w_B)L_{AB} = \frac{1}{2}(2984.94\ \text{lb/ft} - 1193.976\ \text{lb/ft})(3\ \text{ft}) = 2686.45\ \text{lb}$$

$$F_{ABEF} = F_1 + F_2 = 3581.93\ \text{lb} + 2686.45\ \text{lb} = 6268.37\ \text{lb} = 6.27\ \text{kip} \qquad \textbf{Ans.}$$

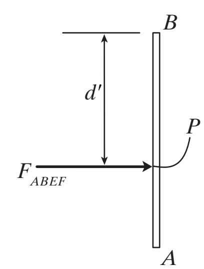


(b)

The location of the center of pressure measured from BE can be obtained by equating the sum of the moments of the forces in Figs. *b* and *c*.

$$\circlearrowleft + M_{R_B} = \Sigma M_B; \quad (6268.37\ \text{lb})d' = (3581.93\ \text{lb})\left[\frac{1}{2}(3\ \text{ft})\right] + (2686.45\ \text{lb})\left[\frac{2}{3}(3\ \text{ft})\right]$$

$$d' = 1.714\ \text{ft} = 1.71\ \text{ft} \qquad \textbf{Ans.}$$

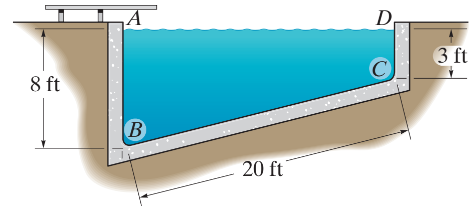


(c)

Ans:
$F_{BCDE} = 1.69$ kip, $d = 0.943$ ft
$F_{ABEF} = 6.27$ kip, $d' = 1.71$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–74. A swimming pool has a width of 12 ft and a side profile as shown. Determine the resultant force the pressure of the water exerts on walls AB and DC, and on the bottom BC.



SOLUTION I

Since the swimming pool has a constant width of $b = 12$ ft, the intensities of the distributed load at B and C can be computed from

$$w_B = \gamma h_{AB} b = \left(62.4\ \text{lb/ft}^3\right)(8\text{ft})(12\text{ft}) = 5990.4\ \text{lb/ft}$$

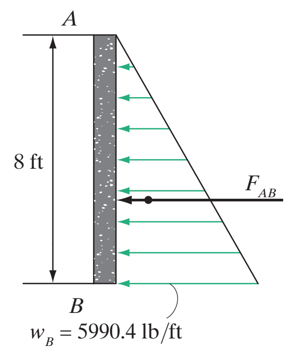
$$w_C = \gamma h_{DC} b = \left(62.4\ \text{lb/ft}^3\right)(3\ \text{ft})(12\ \text{ft}) = 2246.4\ \text{lb/ft}$$

Using these results, the distributed loads acting on walls AB and CD and bottom BC are shown in Figs. *a*, *b*, and *c*.

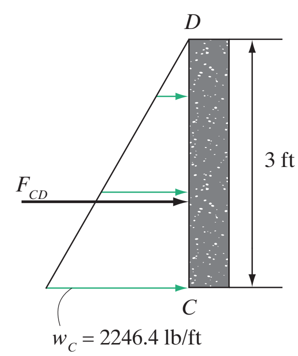
$$F_{AB} = \frac{1}{2} w_B h_{AB} = \frac{1}{2}(5990.4\ \text{lb/ft})(8\ \text{ft}) = 23\,962\ \text{lb} = 24.0\ \text{kip} \qquad \textbf{Ans.}$$

$$F_{DC} = \frac{1}{2} w_C h_{CD} = \frac{1}{2}(2246.4\ \text{lb/ft})(3\ \text{ft}) = 3369.6\ \text{lb} = 3.37\ \text{kip} \qquad \textbf{Ans.}$$

$$F_{BC} = \frac{1}{2}(w_B + w_C)L_{BC} = \frac{1}{2}\left[5990.4\ \text{lb/ft} + 2246.4\text{lb/ft}\right](20\ \text{ft})$$
$$= 82\,368\ \text{lb} = 82.4\ \text{kip} \qquad \textbf{Ans.}$$



(a)



(b)

SOLUTION II

The same result can also be obtained as follows. For wall AB,

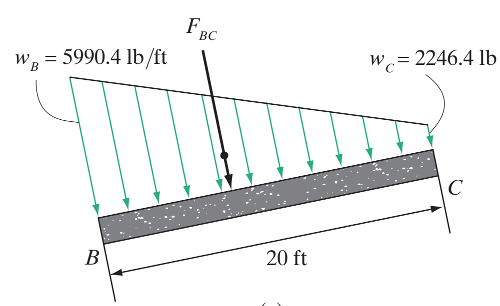
$$F_{AB} = \gamma \bar{h}_{AB} A_{AB} = \left(62.4\ \text{lb/ft}^3\right)(4\ \text{ft})\left[8\ \text{ft}(12\ \text{ft})\right] = 23\,962\ \text{lb} = 24.0\ \text{kip} \qquad \textbf{Ans.}$$

For wall CD,

$$F_{CD} = \gamma \bar{h}_{CD} A_{CD} = \left(62.4\ \text{lb/ft}^3\right)(1.5\ \text{ft})\left[3\ \text{ft}(12\ \text{ft})\right] = 3369.6\ \text{lb} = 3.37\ \text{kip} \qquad \textbf{Ans.}$$

For floor BC,

$$F_{BC} = \gamma \bar{h}_{BC} A_{BC} = \left(62.4\ \text{lb/ft}^3\right)(5.5\ \text{ft})\left[20\ \text{ft}(12\ \text{ft})\right] = 82\,368\ \text{lb} = 82.4\ \text{kip} \qquad \textbf{Ans.}$$

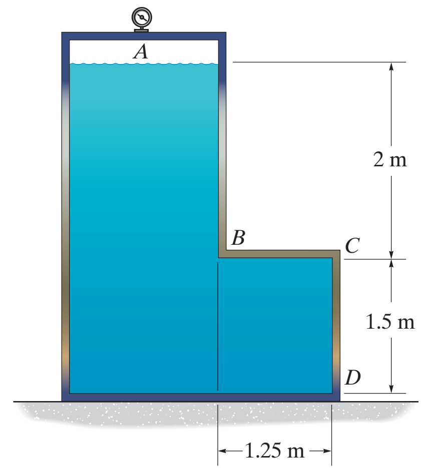


(c)

Ans:
$F_{AB} = 24.0$ kip
$F_{DC} = 3.37$ kip
$F_{BC} = 82.4$ kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–75. The pressure of the air at A within the closed tank is 200 kPa. Determine the resultant force acting on the plates BC and CD caused by the water. The tank has a width of 1.75 m.



SOLUTION

$$p_C = p_B = p_A + \rho g h_{AB}$$
$$= 200(10^3)\ \text{N/m}^2 + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2\ \text{m})$$
$$= 219.62(10^3)\ \text{Pa}$$

$$p_D = p_A + \rho g h_{AD}$$
$$= 200(10^3)\ \text{N/m}^2 + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3.5\ \text{m})$$
$$= 234.335(10^3)\ \text{Pa}$$

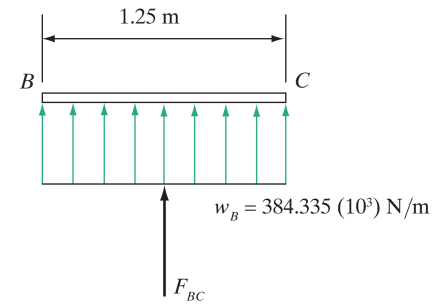
Since plates BC and CD have a constant width of $b = 1.75$ m, the intensities of the distributed load at points B (or C) and D are

$$w_C = w_B = p_B b = (219.62(10^3)\ \text{N/m}^2)(1.75\text{m}) = 384.335(10^3)\ \text{N/m}$$
$$w_D = p_D b = (234.335(10^3)\ \text{N/m}^2)(1.75\ \text{m}) = 410.086(10^3)\ \text{N/m}$$

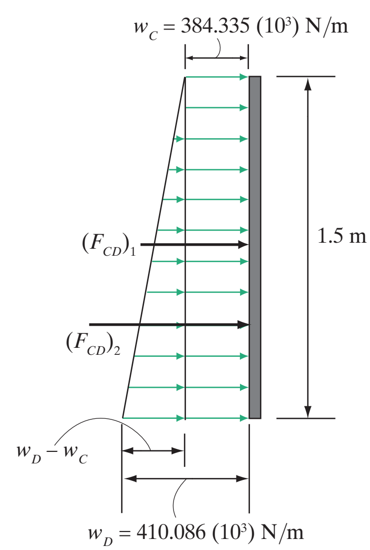
Using these results, the distributed loads acting on plates BC and CD are shown in Figs. a and b, respectively.

$$F_{BC} = w_B L_{BC} = \left[384.335(10^3)\ \frac{\text{N}}{\text{m}}\right](1.25\ \text{m}) = 480.42(10^3)\ \text{N} = 480\ \text{kN} \qquad \textbf{Ans.}$$

$$F_{CD} = (F_{CD})_1 + (F_{CD})_2 = w_C L_{CD} + \frac{1}{2}(w_D - w_C)L_{CD}$$
$$= \left[384.335(10^3)\frac{\text{N}}{\text{m}}\right](1.5\ \text{m}) + \frac{1}{2}\left[410.086(10^3)\frac{\text{N}}{\text{m}} - 384.335(10^3)\frac{\text{N}}{\text{m}}\right](1.5\ \text{m})$$
$$= 595.82(10^3)\ \text{N} = 596\ \text{kN} \qquad \textbf{Ans.}$$



(a)

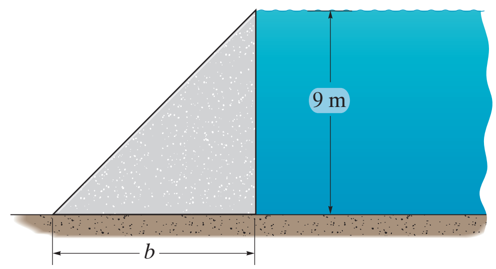


(b)

Ans:
$F_{BC} = 480$ kN, $F_{CD} = 596$ kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–76.** Determine the smallest base length b of the concrete gravity dam that will prevent the dam from overturning due to water pressure acting on the face of the dam. The density of concrete is $\rho_c = 2.4\ \text{Mg/m}^3$. *Hint:* Work the problem using a 1-m width of dam.



SOLUTION

If we consider the dam as having a width of $b = 1$ m, the intensity of the distributed load at the base of the dam is

$$w_b = \rho g h(b) = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(9\ \text{m})(1\ \text{m})$$
$$= 88.29(10^3)\ \text{N/m}$$

The resultant force of the triangular distributed load shown on the free-body diagram of the dam, Fig. *a*. is

$$F = \frac{1}{2}w_b h = \frac{1}{2}\left[88.29(10^3)\ \text{N/m}\right](9\ \text{m}) = 397.305(10^3)\ \text{N}$$

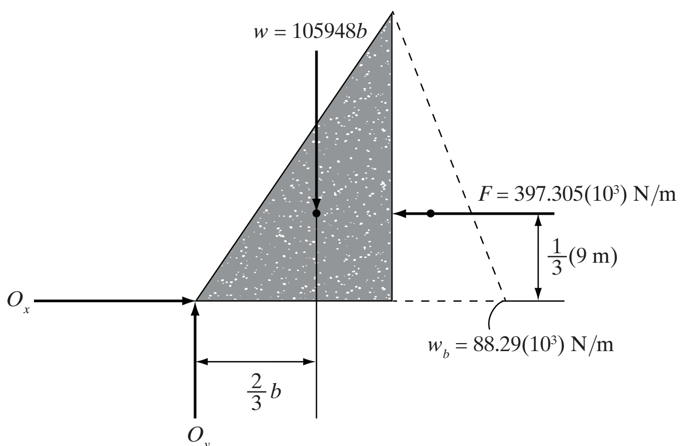
The weight of the dam is given by

$$W = \rho_C g \mathcal{V} = \left[2.4(10^3)\ \text{kg/m}^3\right](9.81\ \text{m/s}^2)\left[\frac{1}{2}(9\ \text{m})(1\ \text{m})b\right]$$
$$= 105\ 948b$$

The dam will overturn about point O. Referring to the free-body diagram of the dam, Fig. *a*,

$$\circlearrowleft + \Sigma M_0 = 0; \qquad \left[397.305(10^3)\ \text{N}\right]\left[\frac{1}{3}(9\ \text{m})\right] - 105\ 948b\left(\frac{2}{3}b\right) = 0$$

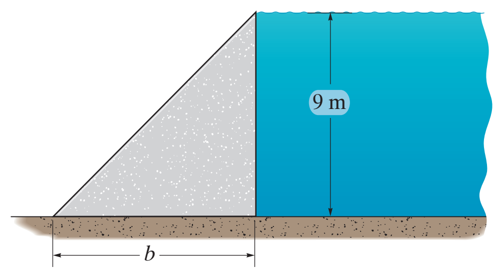
$$b = 4.108\ \text{m} = 4.11\ \text{m}$$ **Ans.**



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–77. Determine the smallest base length b of the concrete gravity dam that will prevent the dam from overturning due to water pressure acting on the face of the dam. Assume water also seeps under the base of the dam. The density of concrete is $\rho_c = 2.4\ \text{Mg/m}^3$. *Hint:* Work the problem using a 1-m width of dam.



SOLUTION

If we consider the dam having a width of $b = 1$ m, the intensity of the distributed load at the base of the dam is

$$w_b = \rho g h b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(9\ \text{m})(1\ \text{m}) = 88.29(10^3)\ \text{N/m}$$

The resultant forces of the triangular distributed load and the uniform distributed load due to the pressure of the seepage water shown on the FBD of the dam, Fig. *a* is

$$F_1 = \frac{1}{2} w_b h = \frac{1}{2}\left[88.29(10^3)\ \text{N/m}\right](9\ \text{m}) = 397.305(10^3)\ \text{N}$$

$$F_2 = w_b b = \left[88.29(10^3)\ \text{N/m}\right] b = 88.29(10^3) b$$

The weight of the dam is given by

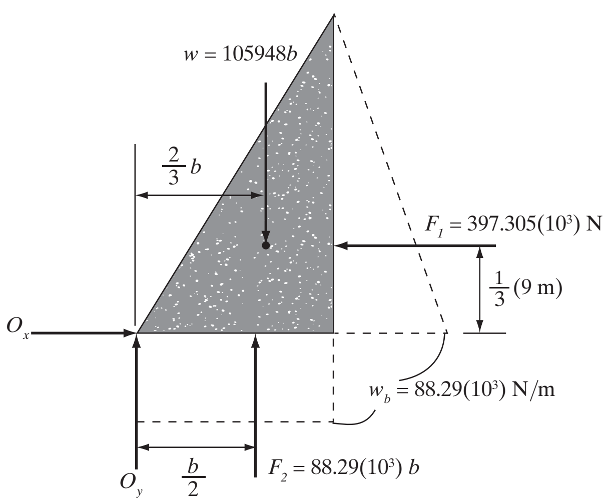
$$W = \rho_C g \forall = \left[2.4(10^3)\ \text{kg/m}^3\right](9.81\ \text{m/s}^2)\left[\frac{1}{2}(\text{b})(9\ \text{m})(1\ \text{m})\right]$$

$$= 105948b$$

The dam will overturn about point O. Referring to the FBD of the dam, Fig. *a*,

$$\circlearrowleft + \Sigma M_0 = 0; \quad \left[397.305(10^3)\ \text{N}\right]\left[\frac{1}{3}(9\ \text{m})\right] + \left[88.29(10^3) b\right]\left(\frac{b}{2}\right) - 105948b\left(\frac{2}{3}b\right) = 0$$

$$b = 6.708\ \text{m} = 6.71\ \text{m}$$ **Ans.**

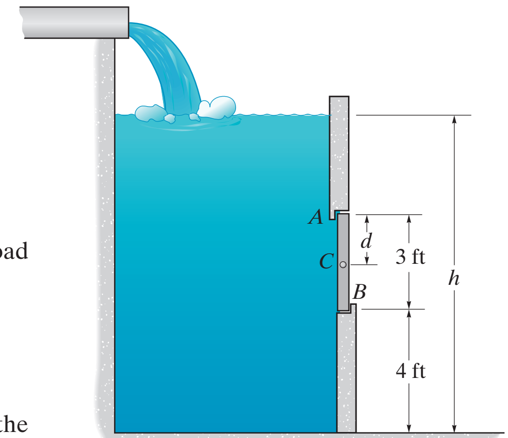


(a)

Ans:
6.71 m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–78. Determine the placement d of the pin on the 2-ft-wide rectangular gate so that it begins to rotate clockwise (open) when waste water reaches a height $h = 10$ ft. What is the resultant force acting on the gate?



SOLUTION

Since the gate has a constant width of $b = 2$ ft, the intensity of the distributed load at A and B can be computed from

$$w_A = \gamma_w h_A b = \left(62.4\ \text{lb/ft}^3\right)(3\ \text{ft})(2\ \text{ft}) = 374.4\ \text{lb/ft}$$

$$w_B = \gamma_w h_B d = \left(62.4\ \text{lb/ft}^3\right)(6\ \text{ft})(2\ \text{ft}) = 748.8\ \text{lb/ft}$$

The resultant trapezoidal distributed load is shown on the free-body diagram of the gate, Fig. a. This load can be subdivided into two parts for which the resultant force of each part is

$$F_1 = w_A L_{AB} = 374.4\ \text{lb/ft}(3\ \text{ft}) = 1123.2\ \text{lb}$$

$$F_2 = \frac{1}{2}\left(w_B - w_A\right)L_{AB} = \frac{1}{2}(748.8\ \text{lb/ft} - 374.4\ \text{lb/ft})(3\ \text{ft}) = 561.6\ \text{lb}$$

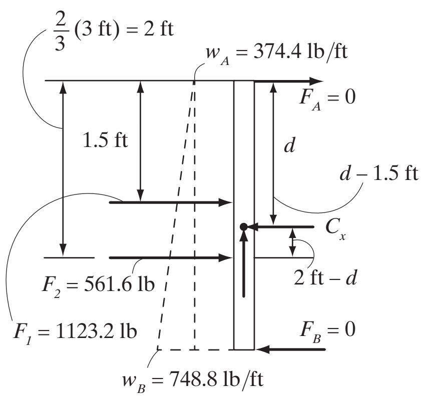
Thus, the resultant force is

$$F_R = F_1 + F_2 = 1123.2\ \text{lb} + 561.6\ \text{lb} = 1684.8\ \text{lb} = 1.68\ \text{kip} \qquad \textbf{Ans.}$$

When the gate is on the verge of opening, the normal force at A and B is zero as shown on the free-body diagram of the gate, Fig. a.

$$\circlearrowleft + \Sigma M_C = 0; \qquad (561.61\text{b})(2\ \text{ft} - d) - (1123.2\ \text{lb})(d - 1.5\ \text{ft}) = 0$$

$$d = 1.67\ \text{ft} \qquad \textbf{Ans.}$$

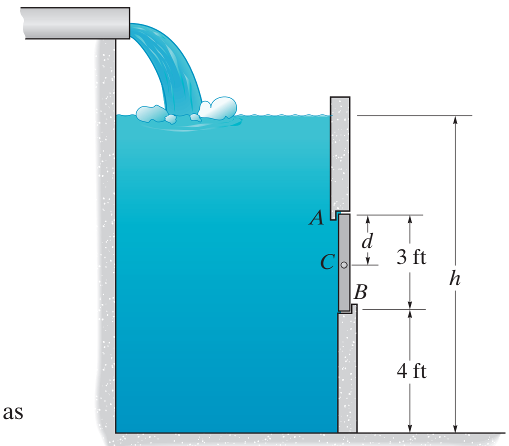


(a)

Ans:
$F_R = 1.68$ kip
$d = 1.67$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–79. Determine the placement d of the pin on the 3-ft-diameter circular gate so that it begins to rotate clockwise (open) when waste water reaches a height $h = 10$ ft. What is the resultant force acting on the gate? Use the formula method.



SOLUTION

Since the gate is circular in shape, it is convenient to compute the resultant force as follows.

$$F_R = \gamma_w \bar{h} A = (62.4\ \text{lb/ft}^3)(10\ \text{ft} - 5.5\ \text{ft})(\pi)(1.5\ \text{ft})^2 = 1984.86\ \text{lb}$$

$$= 1.98\ \text{kip} \qquad \textbf{Ans.}$$

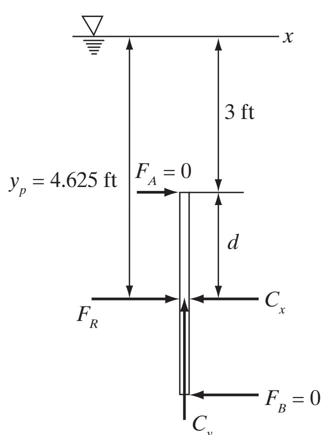
The location of the center of pressure can be determined from

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$

$$= \frac{\left(\dfrac{\pi(1.5\ \text{ft})^4}{4}\right)}{(10\ \text{ft} - 5.5\ \text{ft})(\pi)(1.5\ \text{ft})^2} + (10\ \text{ft} - 5.5\ \text{ft}) = 4.625\ \text{ft}$$

When the gate is on the verge of opening, the normal force at A and B is zero as shown on the free-body diagram of the gate, Fig. *a*. Summing the moments about point C requires that F_R acts through C. Thus,

$$d = y_p - 3\ \text{ft} = 4.625\ \text{ft} - 3\ \text{ft} = 1.625\ \text{ft} = 1.62\ \text{ft} \qquad \textbf{Ans.}$$

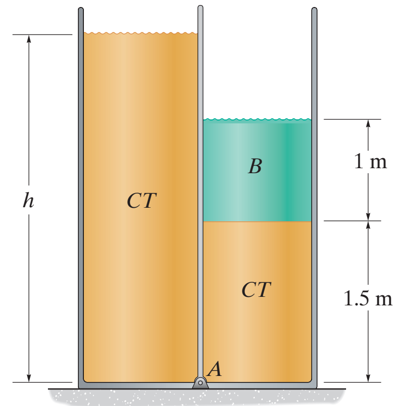


(a)

Ans:
$F_R = 1.98$ kip
$d = 1.62$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–80.** The container in a chemical plant contains carbon tetrachloride, $\rho_{ct} = 1593\ \text{kg/m}^3$, and benzene, $\rho_b = 875\ \text{kg/m}^3$. Determine the height h of the carbon tetrachloride on the left side so that the separation plate, which is pinned at A, will remain vertical.



SOLUTION

Assume 1 m width. The intensities of the distributed load shown in Fig. *a* are,

$$w_2 = \rho_b g h_b b = (875\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1\ \text{m})(1\ \text{m}) = 8.584(10^3)\ \text{N/m}$$

$$w_1 = w_2 + \rho_{CT}g(h_{CT})_R b = \left[8.584(10^3)\ \text{N/m}\right] + (1593\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.5\ \text{m})(1\ \text{m})$$
$$= 32.025(10^3)\ \text{N/m}$$

$$w_3 = \rho_{CT}g(h_{CT})_L b = (1593\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\,\text{h}\ (1\ \text{m})$$
$$= \left[15.63(10^3)h\right]\ \text{N/m}$$

Thus, the resultant forces of these distributed loads are

$$F_1 = \frac{1}{2}\left[32.025(10^3)\ \text{N/m} - 8.584(10^3)\ \text{N/m}\right](1.5\ \text{m}) = 17.58(10^3)\ \text{N}$$

$$F_2 = \left[8.584(10^3)\ \text{N/m}\right](1.5\ \text{m}) = 12.876(10^3)\ \text{N}$$

$$F_3 = \frac{1}{2}\left[8.584(10^3)\ \text{N/m}\right](1\ \text{m}) = 4.292(10^3)\ \text{N}$$

$$F_4 = \frac{1}{2}\left[15.63(10^3)h\ \text{N/m}\right](h) = \left[7.814(10^3)h^2\right]\ \text{N}$$

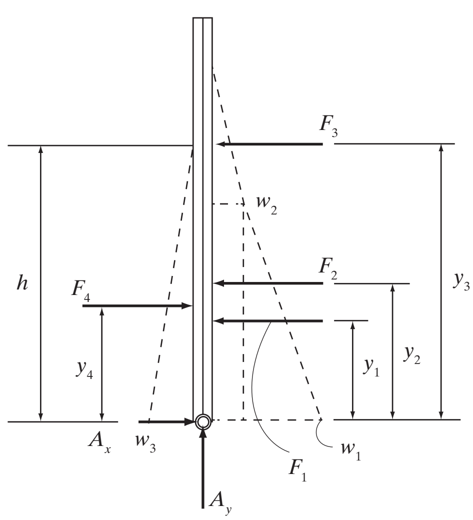
And they act at

$$y_1 = \frac{1.5\ \text{m}}{3} = 0.5\ \text{m} \quad y_2 = \frac{1.5\ \text{m}}{2} = 0.75\ \text{m} \quad y_3 = 1.5\ \text{m} + \frac{1\ \text{m}}{3} = 1.8333\ \text{m}$$
$$y_4 = \frac{h}{3}$$

For the plate to remain vertical,

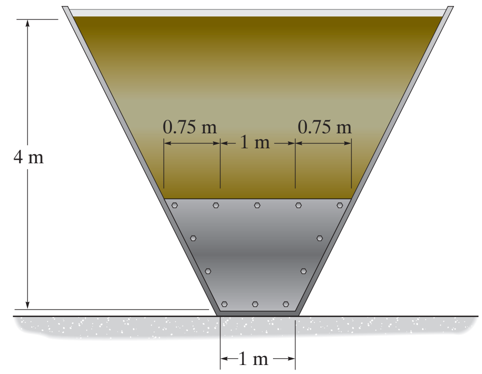
$$\circlearrowleft + \Sigma M_A = 0;\ \left[17.58(10^3)\ \text{N}\right](0.5\ \text{m}) + \left[12.876(10^3)\ \text{N}\right](0.75\ \text{m})$$
$$+ \left[4.292(10^3)\ \text{N}\right](1.8333\ \text{m}) - \left[7.814(10^3)h^2\ \text{N}\right]\left(\frac{h}{3}\right) = 0$$

$$h = 2.167\ \text{m} = 2.16\ \text{m} \qquad \textbf{Ans.}$$



Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–81. The tapered settling tank is filled with oil. Determine the resultant force the oil exerts on the trapezoidal clean-out plate located at its end. How far from the top of the tank does this force act on the plate? Use the formula method. Take $\rho_o = 900\ \text{kg/m}^3$.



SOLUTION

Referring to the geometry of the plate shown in Fig. *a*

$$A = (1\text{ m})(1.5\text{ m}) + \frac{1}{2}(1.5\text{ m})(1.5\text{ m}) = 2.625\text{ m}^2$$

$$\bar{y} = \frac{(3.25\text{ m})[(1\text{ m})(1.5\text{ m})] + (3\text{ m})\left[\frac{1}{2}(1.5\text{ m})(1.5\text{ m})\right]}{2.625\text{ m}^2} = 3.1429\text{ m}$$

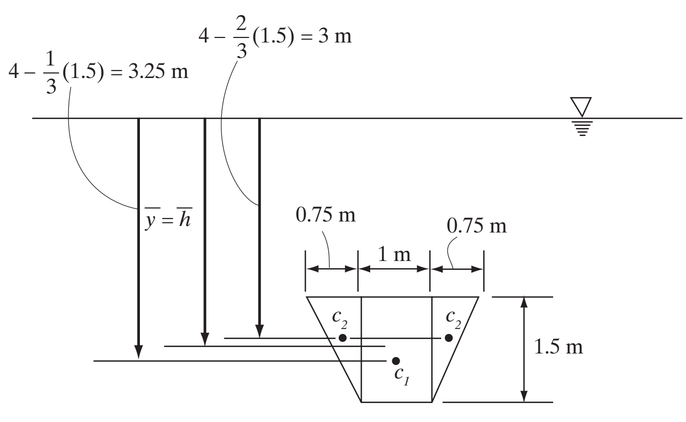
$$\bar{I}_x = \frac{1}{12}(1\text{ m})(1.5\text{ m})^3 + (1\text{ m})(1.5\text{ m})(3.25\text{ m} - 3.1429\text{ m})^2$$
$$+ \frac{1}{36}(1.5\text{ m})(1.5\text{ m})^3 + \frac{1}{2}(1.5\text{ m})(1.5\text{ m})(3.1429\text{ m} - 3\text{ m})^2$$
$$= 0.46205\text{ m}^4$$

The resultant force is

$$F_R = \rho_o g\bar{h}A = (900\text{ kg/m}^3)(9.81\text{ m/s}^2)(3.1429\text{ m})(2.625\text{ m}^2)$$
$$= 72.84(10^3)\text{ N} = 72.8\text{ kN} \qquad \textbf{Ans.}$$

And it acts at

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{0.46205\text{ m}^4}{(3.1429\text{ m})(2.625\text{ m}^2)} + 3.1429\text{ m} = 3.199\text{ m} = 3.20\text{ m} \qquad \textbf{Ans.}$$

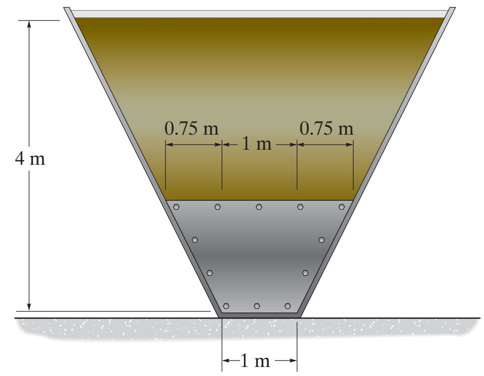


(a)

Ans:
$F_R = 72.8$ kN
$y_P = 3.20$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \mathrm{kg/m^3}$ and its specific weight to be $\gamma_w = 62.4\ \mathrm{lb/ft^3}$. Also, assume all pressures are gage pressures.

2–82. The tapered settling tank is filled with oil. Determine the resultant force the oil exerts on the trapezoidal clean-out plate located at its end. How far from the top of the tank does this force act on the plate? Use the integration method. Take $\rho_o = 900\ \mathrm{kg/m^3}$.



SOLUTION

With respect to x and y axes established, the equation of side AB of the plate, Fig. a is

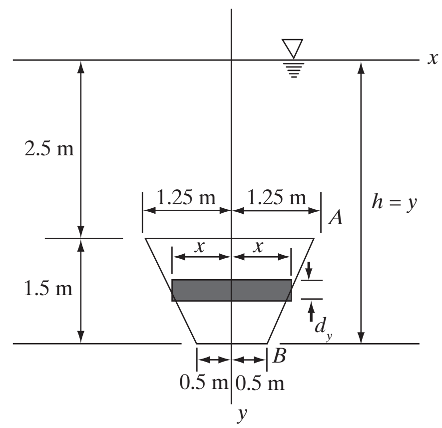
$$\frac{y - 2.5}{x - 1.25} = \frac{4 - 2.5}{0.5 - 1.25}; \qquad 2x = 5 - y$$

Thus, the area of the differential element shown shaded in Fig. a is $dA = 2x dy = 5 - y\, dy$. The pressure acting on this differential element is $p = \rho_0 g h = \left(900\ \mathrm{kg/m^3}\right)\left(9.81\ \mathrm{m/s^2}\right) y = 8829y$. Thus, the resultant force acting on the entire plate is

$$F_R = \int_A p dA = \int_{2.5\ \mathrm{m}}^{4\ \mathrm{m}} 8829y(5 - y)dy$$

$$= 22072.5y^2 - 2943y^3 \Big|_{2.5\ \mathrm{m}}^{4\ \mathrm{m}}$$

$$= 72.84\left(10^3\right)\ \mathrm{N} = 72.8\ \mathrm{kN} \qquad \textbf{Ans.}$$



(a)

And it acts at

$$y_P = \frac{\int_A ypdA}{F_R} = \frac{1}{72.84\left(10^3\right)\ \mathrm{N}} \Big|_{2.5\ \mathrm{m}}^{4\ \mathrm{m}} y(8829y)(5 - y)dy$$

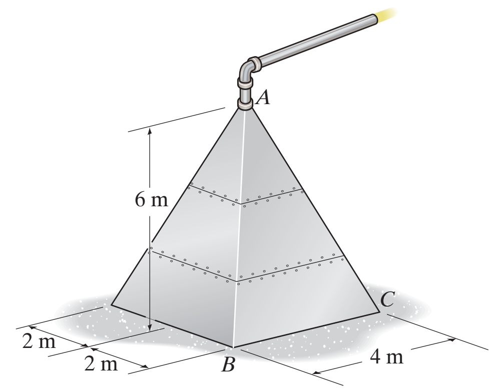
$$= \frac{1}{72.84\left(10^3\right)}\left(14715y^3 - 2207.25y^4\right)\Big|_{2.5\ \mathrm{m}}^{4\ \mathrm{m}}$$

$$= 3.199\ \mathrm{m} = 3.20\ \mathrm{m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 72.8\ \mathrm{kN}$
$y_P = 3.20\ \mathrm{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–83. Ethyl alcohol is pumped into the tank, which has the shape of a four-sided pyramid. When the tank is completely full, determine the resultant force acting on each side, and its location measured from the top A along the side. Use the formula method. $\rho_{ea} = 789\ \text{kg/m}^3$.



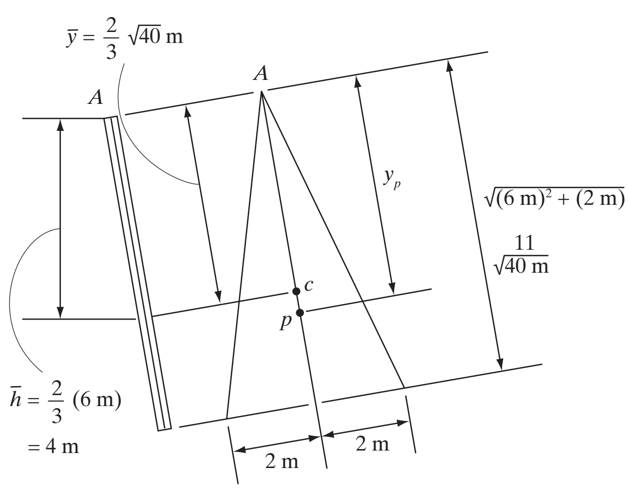
SOLUTION

The geometry of the side wall of the tank is shown in Fig. *a*. In this case, it is convenient to calculate the resultant force as follows.

$$F_R = \gamma_{ea}\bar{h}A = \left(789\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[\frac{2}{3}(6\ \text{m})\right]\left(\frac{1}{2}\right)(4\ \text{m})\left(\sqrt{40}\right)$$
$$= 390.1\left(10^3\right)\ \text{N} = 390\ \text{kN}$$

The location of the center of pressure can be determined from

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$
$$= \frac{\frac{1}{36}(4\ \text{m})\left(\sqrt{40}\ \text{m}\right)^3}{\frac{2}{3}\left(\sqrt{40}\ \text{m}\right)\left(\frac{1}{2}(4\ \text{m})\left(\sqrt{40}\ \text{m}\right)\right)} + \left(\frac{2}{3}\right)\left(\sqrt{40}\ \text{m}\right)$$
$$= 4.74\ \text{m} \qquad \textbf{Ans.}$$

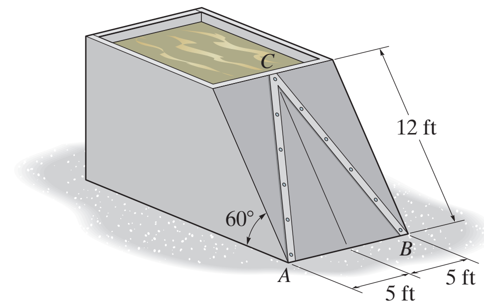


(a)

Ans:
$F_R = 390\ \text{kN}$
$y_P = 4.74\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–84.** The tank is filled to its top with an industrial solvent, ethyl ether. Determine the resultant force acting on the plate ABC, and its location on the plate measured from the base AB of the tank. Use the formula method. Take $\gamma_{ee} = 44.5\ \text{lb/ft}^3$.



SOLUTION

The resultant force is

$$F_R = \gamma_{ee}\bar{h}A = \left(44.5\ \text{lb/ft}^3\right)(8 \sin 60°\ \text{ft})\left[\frac{1}{2}(10\ \text{ft})(12\ \text{ft})\right]$$

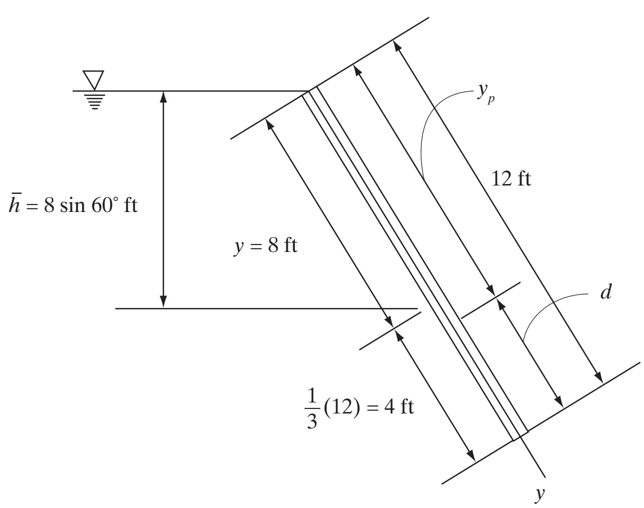
$$= 18.498\left(10^3\right)\ \text{lb} = 18.5\ \text{kip} \qquad \textbf{Ans.}$$

$$\bar{I}_x = \frac{1}{36}bh^3 = \frac{1}{36}(10\ \text{ft})(12\ \text{ft})^3 = 480\ \text{ft. Then}$$

$$y_P = \frac{\bar{I}_x}{\bar{y}_A} + \bar{y} = \frac{480\ \text{ft}}{(8\ \text{ft})\left[\frac{1}{2}(10\ \text{ft})(12\ \text{ft})\right]} + 8\ \text{ft} = 9\ \text{ft}$$

Thus,

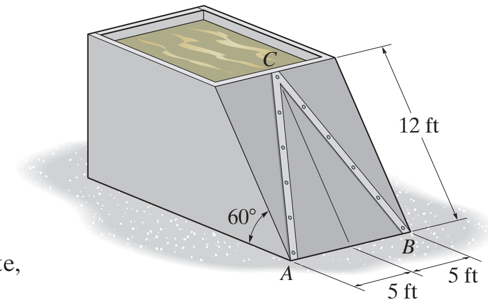
$$d = 12\ \text{ft} - y_P = 12\ \text{ft} - 9\ \text{ft} = 3\ \text{ft} \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–85. Solve Prob. 2–84 using the integration method.



SOLUTION

With respect to x and y axes established, the equation of side AB of the plate, Fig. a, is

$$\frac{y-0}{x-0} = \frac{12-0}{5-0}; \qquad x = \frac{5}{12}y$$

Thus, the area of the differential element shown shaded in Fig. a is $dA = 2x\,dy = 2\left(\frac{5}{12}y\right)dy = \frac{5}{6}y\,dy$. The pressure acting on this differential element is $p = \gamma h = \left(44.5\ \text{lb/ft}^3\right)\left(y \sin 60^\circ\right) = 38.54\,y$. Thus, the resultant force acting on the entire plate is

$$F_R = \int_A p\,dA = \int_0^{12\ \text{ft}} (38.54y)\left(\frac{5}{6}y\,dy\right)$$

$$= 10.71\,y^3\Big|_0^{12\ \text{ft}}$$

$$= 18.50\left(10^3\right)\text{lb} = 18.5\ \text{kip} \qquad \textbf{Ans.}$$

And it acts at

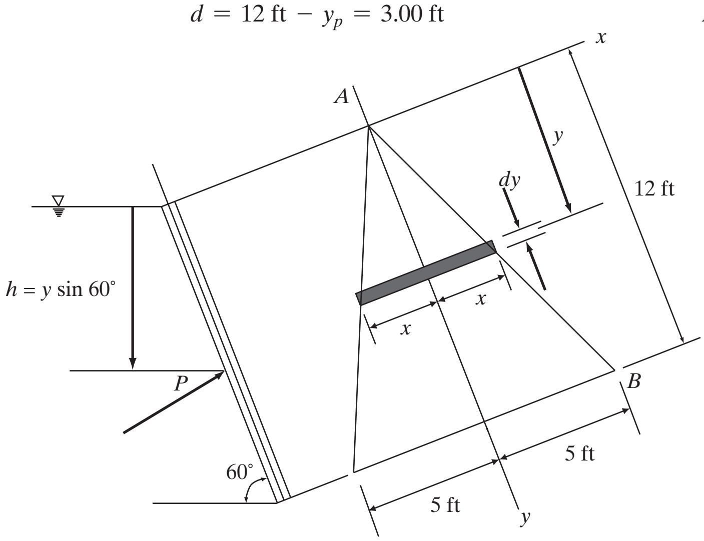
$$y_P = \frac{\int_A y\,p\,dA}{F_R} = \frac{1}{18.50\left(10^3\right)}\int_0^{12\ \text{ft}} y(38.54y)\left(\frac{5}{6}y\,dy\right)$$

$$= \frac{1}{18.50\left(10^3\right)}\left(8.03y^4\right)\Big|_0^{12\ \text{ft}}$$

$$= 9.00\ \text{ft}$$

Thus,

$$d = 12\ \text{ft} - y_p = 3.00\ \text{ft} \qquad \textbf{Ans.}$$

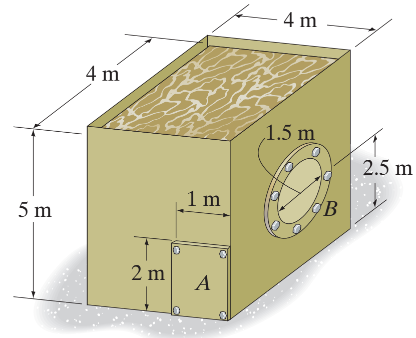


(a)

Ans:
$F_R = 18.5$ kip
$d = 3.00$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–86. Access plates on the industrial holding tank are bolted shut when the tank is filled with vegetable oil as shown. Determine the resultant force that this liquid exerts on plate A, and its location measured from the bottom of the tank. Use the formula method. $\rho_{vo} = 932\ \text{kg/m}^3$.



SOLUTION

Since the plate has a width of $b = 1$ m, the intensities of the distributed load at the top and bottom of the plate can be computed from

$$w_t = \rho_{vo}\, g h_t b = (932\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(1\ \text{m}) = 27.429(10^3)\ \text{N/m}$$
$$w_b = \rho_{vo}\, g h_b b = (932\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})(1\ \text{m}) = 45.715(10^3)\ \text{N/m}$$

The resulting trapezoidal distributed load is shown in Fig. a, and this loading can be subdivided into two parts for which the resultant forces are

$$F_1 = w_t(L) = \left[27.429(10^3)\ \text{N/m}\right](2\ \text{m}) = 54.858(10^3)\ \text{N}$$

$$F_2 = \frac{1}{2}(w_b - w_t)(L) = \frac{1}{2}\left[45.715(10^3)\ \text{N/m} - 27.429(10^3)\ \text{N/m}\right](2\ \text{m}) = 18.286(10^3)\ \text{N}$$

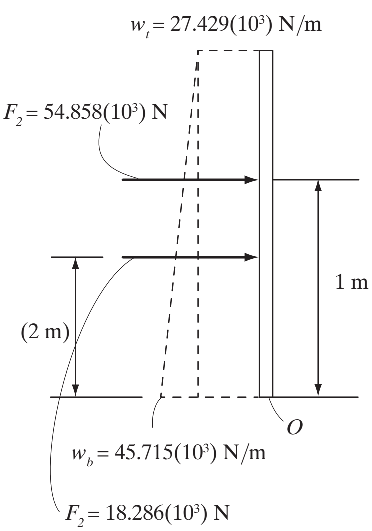
Thus, the resultant force is

$$F_R = F_1 + F_2 = 54.858(10^3)\ \text{N} + 18.286(10^3)\ \text{N} = 73.143(10^3)\ \text{N} = 73.1\ \text{kN} \quad \textbf{Ans.}$$

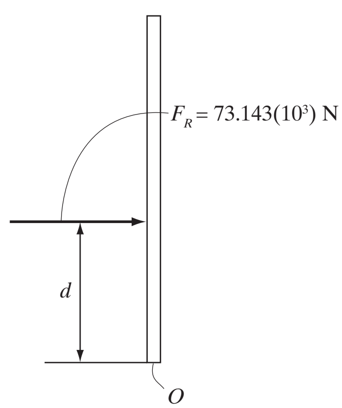
The location of the center of pressure can be determined by equating the sum of the moments of the forces in Figs. a and b about O.

$$\circlearrowleft + (M_R)_O = \Sigma M_O; \qquad \left[73.143(10^3)\ \text{N}\right]d = \left[54.858(10^3)\ \text{N}\right](1\ \text{m}) + \left[18.286(10^3)\ \text{N}\right]\left[\frac{1}{3}(2\ \text{m})\right]$$

$$d = 0.9167\ \text{m} = 917\ \text{mm} \qquad \textbf{Ans.}$$



(a)

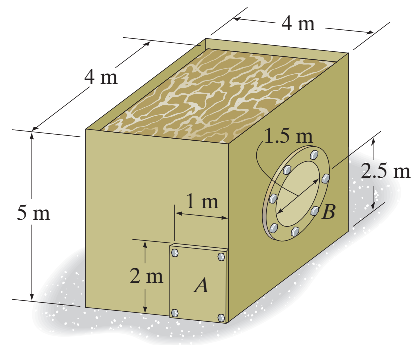


(b)

Ans:
$F_R = 73.1$ kN
$d = 917$ mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–87. Access plates on the industrial holding tank are bolted shut when the tank is filled with vegetable oil as shown. Determine the resultant force that this liquid exerts on plate B, and its location measured from the bottom of the tank. Use the formula method. $\rho_{vo} = 932\ \text{kg/m}^3$.



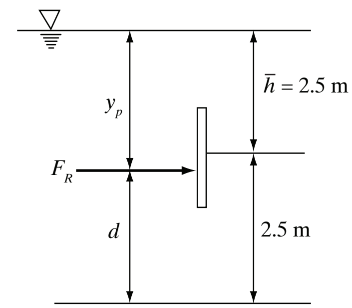
SOLUTION

Since the plate is circular in shape, it is convenient to compute the resultant force as follows

$$F_R = \gamma_{vo}\,\bar{h}A = \left(932\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2.5\ \text{m})\left[\pi(0.75\ \text{m})^2\right]$$
$$= 40.392\left(10^3\right)\text{N} = 40.4\ \text{kN} \qquad \textbf{Ans.}$$

The location of the center of pressure can be determined form

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{\pi\dfrac{(0.75\ \text{m})^4}{4}}{(2.5\ \text{m})(\pi)(0.75\ \text{m})^2} + 2.5\ \text{m}$$
$$= 2.556\ \text{m}$$



(a)

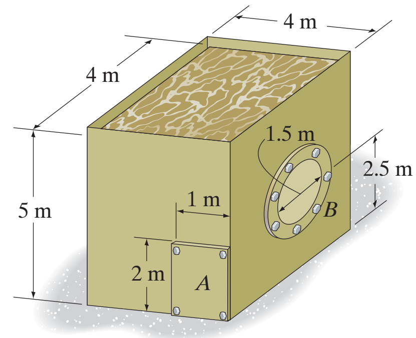
From the bottom of the tank, Fig. *a*,

$$d = 5\ \text{m} - y_P = 5\ \text{m} - 2.556\ \text{m} = 2.44\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 40.4$ kN
$d = 2.44$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–88.** Solve Prob. 2–87 using the integration method.



SOLUTION

With respect to x and y axes established, the equation of the circumference of the circular plate is

$$x^2 + y^2 = 0.75^2; \qquad x = \sqrt{0.75^2 - y^2}$$

Thus, the area of the differential element shown shaded in Fig. a is $dA = 2x\,dy = 2\sqrt{0.75^2 - y^2}\,dy$. The pressure acting on this differential element is $p = \rho_{vo} g h = (932\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.5 - y) = 9142.92(2.5 - y)$. Thus, the resultant force acting on the entire plate is

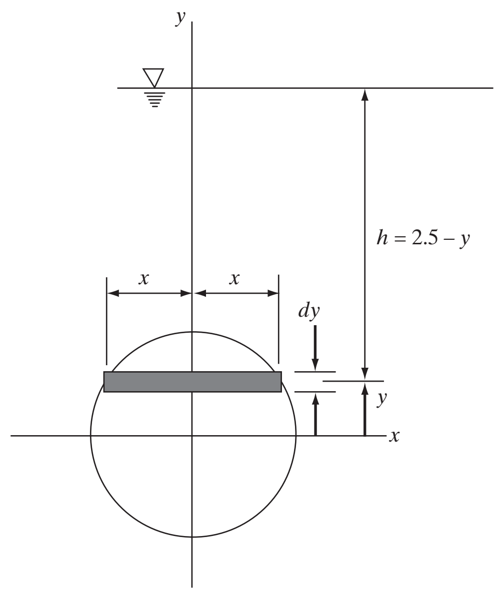
$$F_R = \int_A p\,dA = \int_{-0.75\ \text{m}}^{0.75\ \text{m}} 9142.92(2.5 - y)\left[2\sqrt{0.75^2 - y^2}\,dy\right]$$

$$= 18285.84\int_{-0.75\ \text{m}}^{0.75\ \text{m}} (2.5 - y)\left(\sqrt{0.75^2 - y^2}\right)dy$$

$$= 22857.3\left[y\sqrt{0.75^2 - y^2} + 0.75^2\sin^{-1}\frac{y}{0.75}\right]\Bigg|_{-0.75\ \text{m}}^{0.75\ \text{m}}$$

$$+\ 6095.28\sqrt{(0.75^2 - y^2)^3}\,\Bigg|_{-0.75\ \text{m}}^{0.75\ \text{m}}$$

$$= 40.39(10^3)\ \text{N} = 40.4\ \text{kN} \qquad \textbf{Ans.}$$



(a)

And it acts at

$$y_P = \frac{\displaystyle\int_A (2.5 - y)p\,dA}{F_R}$$

$$= \frac{1}{40.39(10^3)}\int_{-0.75\ \text{m}}^{0.75\ \text{m}} (2.5 - y)\left[9142.92(2.5 - y)\right]\left(2\sqrt{0.75^2 - y^2}\,dy\right)$$

$$= 0.4527\int_{-0.75\ \text{m}}^{0.75\ \text{m}} (6.25 + y^2 - 5y)\left(\sqrt{0.75^2 - y^2}\right)dy$$

$$= 1.4147\left[y\sqrt{0.75^2 - y^2} + 0.75^2\sin^{-1}\frac{y}{a}\right]\Bigg|_{-0.75\ m}^{0.75\ m} + 0.4527\left[-\frac{y}{4}\sqrt{(0.75^2 - y^2)^3}\right.$$

$$\left.+\ \frac{0.75^2}{8}\left(y\sqrt{0.75^2 - y^2} + a^2\sin^{-1}\frac{y}{0.75}\right)\right]\Bigg|_{-0.75\ \text{m}}^{0.75\ \text{m}} + 0.75450\,\Bigg|_{-0.75\ \text{m}}^{0.75\ \text{m}}$$

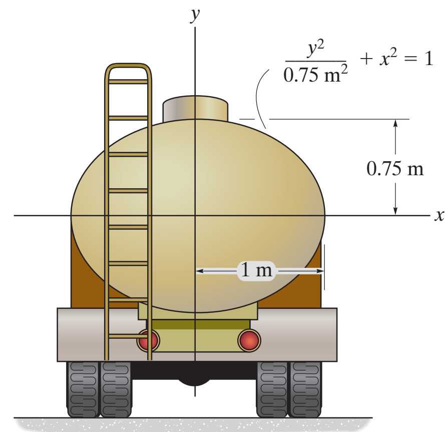
$$= 2.5562\ \text{m}$$

From the bottom of tank is

$$d = 5\ \text{m} - y_p = 5\ \text{m} - 2.5562\ \text{m} = 2.44\ \text{m} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–89. The tank truck is filled to its top with water. Determine the magnitude of the resultant force on the elliptical back plate of the tank, and the location of the center of pressure measured from the top of the tank. Solve the problem using the formula method.



SOLUTION

Using Table 2-1 for the area and moment of inertia about the centroidal $\bar{x}$ axis of the elliptical plate, we get

$$F = \rho_w g \bar{h} A = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.75\ \text{m})(\pi)(0.75\ \text{m})(1\ \text{m})$$
$$= 17.3\ \text{kN} \qquad \textbf{Ans.}$$

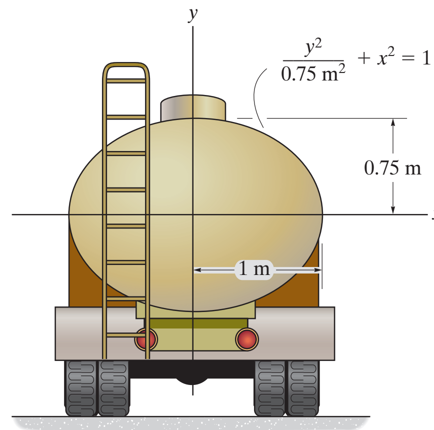
The center of pressure is at

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$
$$= \frac{\left[\frac{1}{4}\pi(1\ \text{m})(0.75\ \text{m})^3\right]}{(0.75\ \text{m})\pi(1\ \text{m})(0.75\ \text{m})} + 0.75\ \text{m}$$
$$= 0.9375\ \text{m} = 0.938\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$F = 17.3\ \text{kN}$
$y_P = 0.938\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–90. Solve Prob. 2–89 using the integration method.



SOLUTION

By integration of a horizontal strip of area

$$dF = p\,dA = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.75\ \text{m} - y)(2x\,dy)$$

$$F = 19\,620\int_{-0.75\ \text{m}}^{0.75\ \text{m}}(0.75 - y)\left(1 - \frac{y^2}{(0.75)^2}\right)^{\frac{1}{2}}dy$$

$$= 19\,620\left[\int_{-0.75\ \text{m}}^{0.75\ \text{m}}\sqrt{(0.75)^2 - y^2}\,dy - \frac{1}{0.75}\int_{-0.75\ \text{m}}^{0.75\ \text{m}}y\sqrt{(0.75)^2 - y^2}\,dy\right]$$

$$= \frac{19\,620}{2}\left[y\sqrt{(0.75)^2 - y^2} + (0.75)^2\sin^{-1}\frac{y}{0.75}\right]_{-0.75}^{0.75} - \frac{19\,620}{0.75}\left[-\frac{1}{3}\sqrt{((0.75)^2 - y^2)^3}\right]_{-0.75}^{0.75}$$

$$= \frac{19\,620\pi(0.75)^2}{2} - 0 = 17\,336\ \text{N} = 17.3\ \text{kN} \qquad \textbf{Ans.}$$

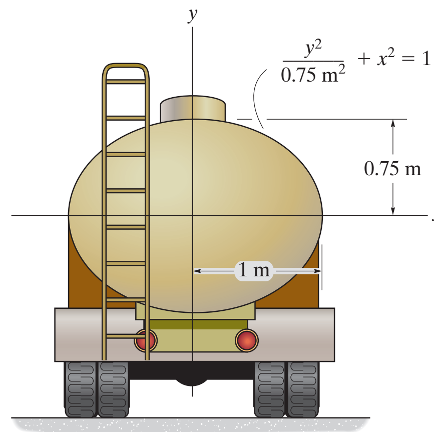
$$y_P = \frac{19\,620\displaystyle\int_{-0.75\ \text{m}}^{0.75\ \text{m}}y(0.75 - y)\left(1 - \frac{y^2}{(0.75)^2}\right)^{\frac{1}{2}}dy}{17\,336\ \text{N}} = -0.1875\ \text{m}$$

$$y_P = 0.75\ \text{m} + 0.1875\ \text{m} = 0.9375\ \text{m} = 0.938\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$F = 17.3\ \text{kN}$
$y_P = 0.938\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \mathrm{kg/m^3}$ and its specific weight to be $\gamma_w = 62.4\ \mathrm{lb/ft^3}$. Also, assume all pressures are gage pressures.

2–91. The tank truck is half-filled with water. Determine the magnitude of the resultant force on the elliptical back plate of the tank, and the location of the center of pressure measured from the x axis. Solve the problem using the formula method. *Hint*: The centroid of a semi-ellipse measured from the x axis is $\bar{y} = \frac{4b}{3\pi}$.



SOLUTION

From Table 2-1, the area and moment of inertia about the x axis of the half-ellipse plate are

$$A = \frac{\pi}{2}ab = \frac{\pi}{2}(1\ \mathrm{m})(0.75\ \mathrm{m}) = 0.375\pi\ \mathrm{m}^2$$

$$I_x = \frac{1}{2}\left(\frac{\pi}{4}ab^3\right) = \frac{1}{2}\left[\frac{\pi}{4}(1\ \mathrm{m})(0.75\ \mathrm{m})^3\right] = 0.05273\pi\ \mathrm{m}^4$$

Thus, the moment of inertia of the half of ellipse about its centroidal $\bar{x}$ axis can be determined by using the parallel-axis theorem.

$$I_x = \bar{I}_x + Ad_y^2$$

$$0.05273\pi\ \mathrm{m}^4 = \bar{I}_x + (0.375\ \pi)\left[\frac{4(0.75\ \mathrm{m})}{3\pi}\right]^2$$

$$\bar{I}_x = 0.046304\ \mathrm{m}^4$$

Since $\bar{h} = \dfrac{4(0.75\ \mathrm{m})}{3\pi} = 0.3183\ \mathrm{m}$, then

$$F_R = \gamma\bar{h}A = \left(1000\ \mathrm{kg/m^3}\right)\left(9.81\ \mathrm{m/s^2}\right)(0.3183\ \mathrm{m})\left(0.375\pi\ \mathrm{m}^2\right)$$
$$= 3.679\left(10^3\right)\ \mathrm{N} = 3.68\ \mathrm{kN}$$ **Ans.**

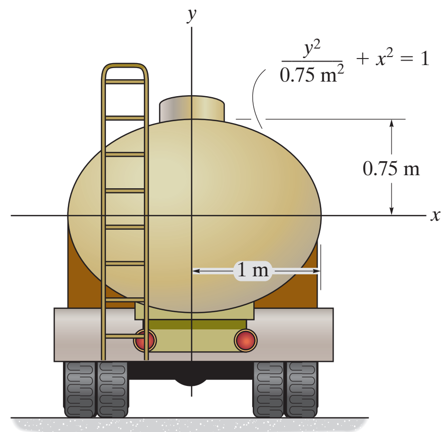
Since $\bar{y} = \bar{h} = 0.3183\ \mathrm{m}$,

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$
$$= \frac{0.046304\ \mathrm{m}^4}{(0.3183\ \mathrm{m})\left(0.375\pi\ \mathrm{m}^2\right)} + 0.3183\ \mathrm{m}$$
$$= 0.4418\ \mathrm{m} = 442\ \mathrm{mm}$$ **Ans.**

Ans:
$F_R = 3.68\ \mathrm{kN}$
$y_P = 442\ \mathrm{mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–92.** Solve Prob. 2–91 using the integration method.



SOLUTION

Using a horizontal strip of area dA,

$$dF = pdA$$

$$dF = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(-y)(2x\ dy)$$

$$F = -19\,620\int_{-0.75\ \text{m}}^{0}(y)\left(1 - \frac{y^2}{0.75^2}\right)^{\frac{1}{2}}dy$$

$$= -\frac{19\,620}{0.75}\int_{-0.75\ \text{m}}^{0} y\sqrt{0.75^2 - y^2}\ dy$$

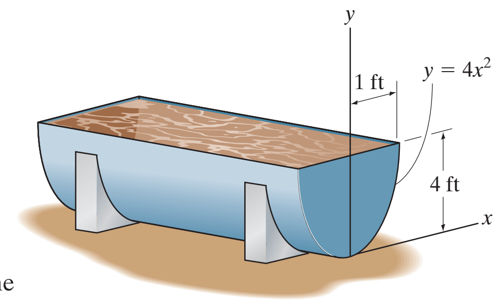
$$= \frac{26\,160}{3}\left[\sqrt{\left(0.75^2 - y^2\right)^3}\right]_{-0.75\ \text{m}}^{0}$$

$$= 3.679\left(10^3\right)\ \text{N} = 3.68\ \text{kN} \qquad \textbf{Ans.}$$

$$y_P = -\frac{-19\,620\displaystyle\int_{-0.75\ \text{m}}^{0} y(y)\left(1 - \frac{y^2}{0.75^2}\right)^{\frac{1}{2}}dy}{3678.75\ \text{N}} = 0.4418\ \text{m} = 442\ \text{mm} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–93. The trough is filled to its top with carbon disulphide. Determine the magnitude of the resultant force acting on the parabolic end plate, and the location of the center of pressure measured from the top. $\rho_{cd} = 2.46\ \text{slug/ft}^3$. Solve the problem using the formula method.



SOLUTION

From Table 2-1, the area and moment of inertia about the centroidal $\bar{x}$ axis of the parabolic plate are

$$A = \frac{2}{3}bh = \frac{2}{3}(2\ \text{ft})(4\ \text{ft}) = 5.3333\ \text{ft}^2$$

$$\bar{I}_x = \frac{8}{175}bh^3 = \frac{8}{175}(2\ \text{ft})(4\ \text{ft})^3 = 5.8514\ \text{ft}^4$$

With $\bar{h} = \frac{2}{5}h = \frac{2}{5}(4\ \text{ft}) = 1.6\ \text{ft}$,

$$F_R = \gamma\bar{h}A = \left(2.46\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)(1.6\ \text{ft})\left(5.3333\ \text{ft}^2\right)$$

$$= 675.94\ \text{lb} = 676\ \text{lb} \qquad \textbf{Ans.}$$

Since $\bar{y} = \bar{h} = 1.6\ \text{ft}$,

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$

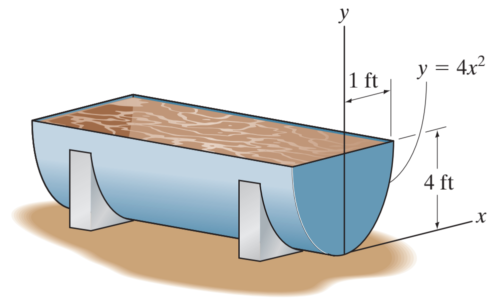
$$= \frac{5.8514\ \text{ft}^4}{(1.6\ \text{ft})\left(5.3333\ \text{ft}^2\right)} + 1.6\ \text{ft}$$

$$= 2.2857\ \text{ft} = 2.29\ \text{ft} \qquad \textbf{Ans.}$$

Ans:
$F_R = 676\ \text{lb}$
$y_P = 2.29\ \text{ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–94. Solve Prob. 2–93 using the integration method.



SOLUTION

Using a horizontal strip of area,

$$F_R = \int_A p\,dA = \int_0^{4\text{ ft}} (2.46\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(4 - y)2x\,dy$$

$$= 158.424\int_0^{4\text{ ft}} (4 - y)\left(\frac{1}{2}\right)\left(y^{\frac{1}{2}}\right)dy$$

$$= 79.212\left(\int_0^{4\text{ ft}} \left(4y^{\frac{1}{2}} - y^{\frac{3}{2}}\right)dy\right)$$

$$= 79.212\left(\frac{8}{3}y^{\frac{3}{2}} - \frac{2}{5}y^{\frac{5}{2}}\right)\Bigg|_0^{4\text{ ft}}$$

$$= 675.94\ \text{lb} = 676\ \text{lb} \qquad \textbf{Ans.}$$

$$F_R(d) = \int_A y(p\,dA) = 158.424\int_0^{4\text{ ft}} y(4 - y)\left(\frac{1}{2}\right)\left(y^{\frac{1}{2}}\right)dy$$

$$(675.94\ \text{lb})(d) = 158.424\int_0^{4\text{ ft}} y(4 - y)\left(\frac{1}{2}\right)y^{\frac{1}{2}}\,dy$$

$$= 79.212\int_0^{4\text{ ft}} \left(4y^{\frac{3}{2}} - y^{\frac{5}{2}}\right)dy$$

$$= 79.212\left(\frac{8}{3}y^{\frac{5}{2}} - \frac{2}{5}y^{\frac{7}{2}}\right)\Bigg|_0^{4\text{ ft}}$$

$$= 1158.76\ \text{lb}\cdot\text{ft}$$

$$d = \frac{1158.76\ \text{lb}\cdot\text{ft}}{675.94\ \text{lb}} = 1.7143\ \text{ft}$$

$$y_P = 4\ \text{ft} - d$$

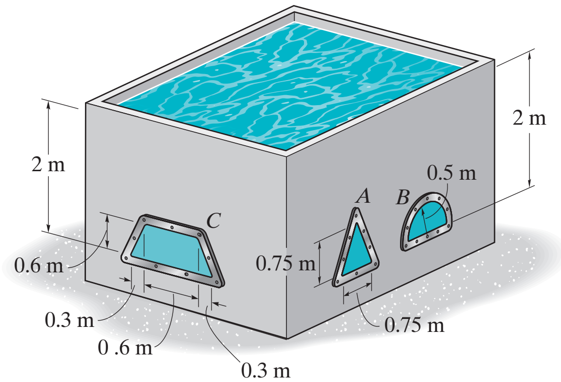
$$= 4\ \text{ft} - 1.7143\ \text{ft}$$

$$= 2.2857\ \text{ft} = 2.29\ \text{ft} \qquad \textbf{Ans.}$$

Ans:
$F_R = 676\ \text{lb}$
$y_P = 2.29\ \text{ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–95. The tank is filled with water. Determine the resultant force acting on the triangular plate A and the location of the center of pressure, measured from the top of the tank. Solve the problem using the formula method.



SOLUTION

The resultant force is

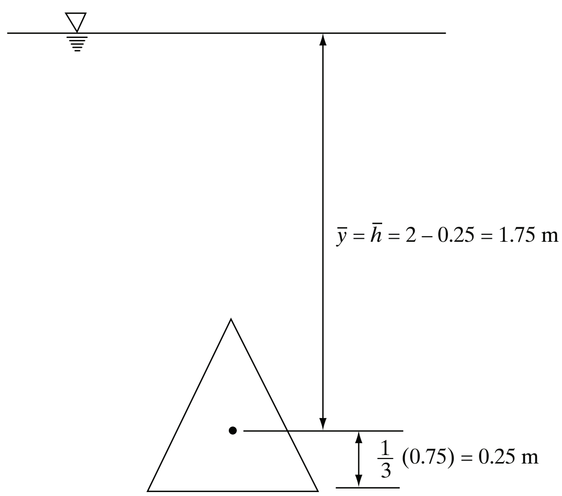
$$F_R = \rho_w g \bar{h} A = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1.75\ \text{m})\left[\frac{1}{2}(0.75\ \text{m})(0.75\ \text{m})\right]$$

$$= 4828.36\ \text{N} = 4.83\ \text{kN} \qquad \textbf{Ans.}$$

$$\bar{I}_x = \frac{1}{36}\left(bh^3\right) = \frac{1}{36}(0.75\ \text{m})(0.75\ \text{m})^3 = 8.7891\left(10^{-3}\right)\ \text{m}^4$$

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{8.7891\left(10^{-3}\right)\ \text{m}^4}{(1.75\ \text{m})\left[\frac{1}{2}(0.75\ \text{m})(0.75\ \text{m})\right]} + 1.75\ \text{m}$$

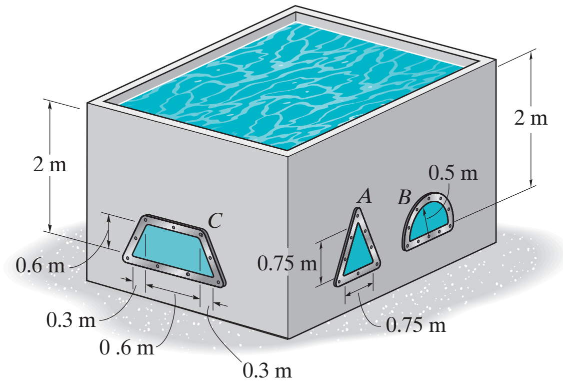
$$= 1.768\ \text{m} = 1.77\ \text{m} \qquad \textbf{Ans.}$$



Ans:
$F_R = 4.83$ kN
$y_P = 1.77$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

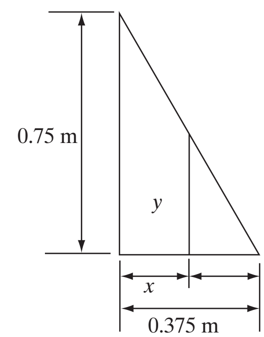
***2–96.** Solve Prob. 2–95 using the integration method.



SOLUTION

From the geometry shown in Fig. *a*

$$\frac{y}{0.75\ \text{m}} = \frac{0.375\ \text{m} - x}{0.375\ \text{m}} \qquad x = (0.375 - 0.5y)\ \text{m}$$



(a)

Referring to Fig. *b*, the pressure as a function of *y* can be written as

$$p = \rho_w g h = \left(1000\ \text{kg/m}^3\right)\left(9.81\text{m/s}^2\right)(2\text{m} - y) = \left[9810(2 - y)\right]\ \text{N/m}^2$$

This pressure acts on the element of area $dA = 2x dy = 2(0.375 - 0.5y)dy$. Thus,

$$dF = p dA = \left[9810(2 - y)\ \text{N/m}^2\right]\left[2(0.375 - 0.5y)dy\right]$$

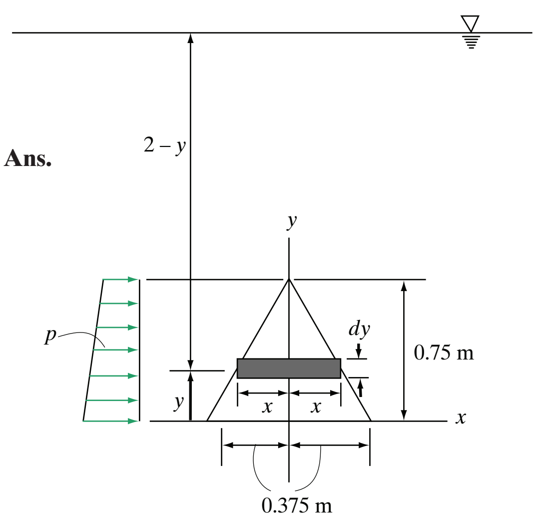
$$= 19\,620\left(0.5y^2 - 1.375y + 0.75\right)dy$$

Then

$$F_R = \int dF = 19\,620 \int_0^{0.75\ \text{m}} \left(0.5y^2 - 1.375y + 0.75\right)dy$$

$$= 19\,620\left[\frac{0.5y^3}{3} - \frac{1.375y^2}{2} + 0.75y\right]\Bigg|_0^{0.75\ \text{m}}$$

$$= 4828.36\ \text{N} = 4.83\ \text{kN}$$ **Ans.**



(b)

And

$$y_p = \frac{\int (2 - y)dF}{F_R}$$

$$= \frac{\int_0^{0.75\ \text{m}} (2 - y)\left[19\,620\left(0.5y^2 - 1.375y + 0.75\right)dy\right]}{4828.36}$$

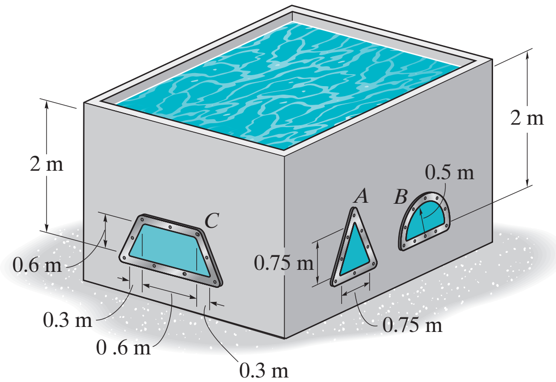
$$= \frac{19\,620 \int_0^{0.75\ \text{m}} \left(-0.5y^3 + 2.375y^2 - 3.5y + 1.5\right)dy}{4828.36}$$

$$= \frac{19\,620\left(-0.125y^4 + 0.79167y^3 - 1.75y^2 + 1.5y\right)\Big|_0^{0.75\ \text{m}}}{4828.36}$$

$$= 1.768 = 1.77\ \text{m}$$ **Ans.**

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–97. The tank is filled with water. Determine the resultant force acting on the semicircular plate B and the location of the center of pressure, measured from the top of the tank. Solve the problem using the formula method.



SOLUTION

The resultant force is

$$F_R = \rho_w g\bar{h}A = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[\left(2 - \frac{2}{3\pi}\right)\text{m}\right]\left[\frac{1}{2}\pi(0.5\ \text{m})^2\right]$$

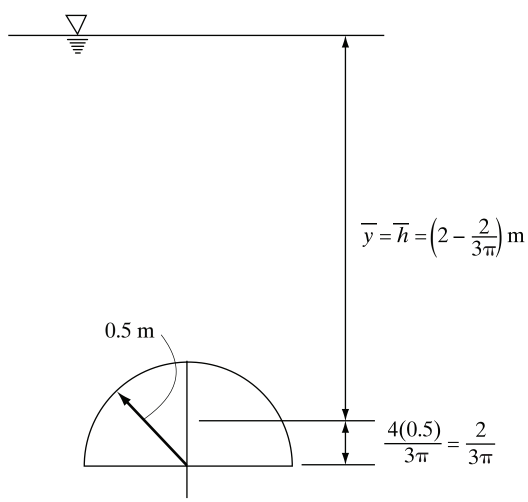
$$= 6887.26\ \text{N} = 6.89\ \text{kN} \qquad \textbf{Ans.}$$

$$\bar{I}_x = 0.1098\ r^4 = 0.1098(0.5\ \text{m})^4 = 6.8625\left(10^{-3}\right)\text{m}^4$$

Then

$$y_p = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{6.8625\left(10^{-3}\right)\text{m}^4}{\left[\left(2 - \frac{2}{3\pi}\right)\text{m}\right]\left[\frac{1}{2}\pi(0.5\ \text{m})^2\right]} + \left(2 - \frac{2}{3\pi}\right)\text{m} = 1.798\ \text{m}$$

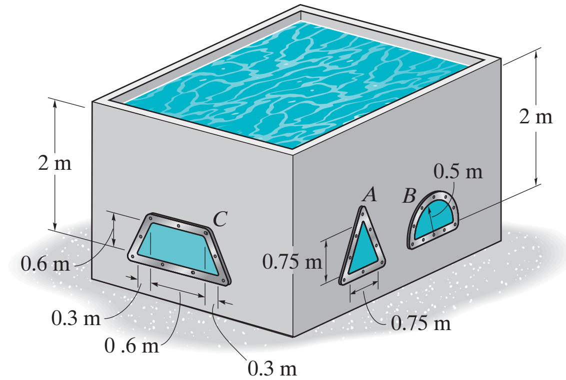
$$= 1.80\ \text{m} \qquad \textbf{Ans.}$$



Ans:
$F_R = 6.89\ \text{kN}$
$y_P = 1.80\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–98. Solve Prob. 2–97 using the integration method.



SOLUTION

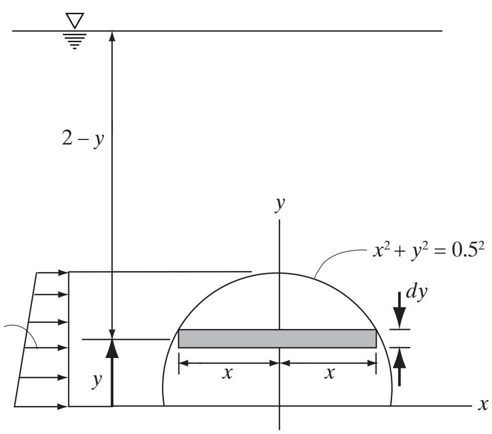
Referring to Fig. *a*, the pressure as a function of y can be written as

$$p = \rho_w g h = (1000\ \text{kg/m}^3)(9.81\text{m/s}^2)(2\text{m} - y) = \left[9810(2 - y)\right]\ \text{N/m}^2$$

This pressure acts on the strip element of area $dA = 2x\,dy$. Here, $x = (0.25 - y^2)^{\frac{1}{2}}$. Thus, $dA = 2(0.25 - y^2)^{\frac{1}{2}}dy$. Then

$$dF = p\,dA = 9810(2 - y)\left\lfloor 2(0.25 - y^2)^{\frac{1}{2}}dy \right\rfloor$$

$$= 19\,620\left[2(0.25 - y^2)^{\frac{1}{2}} - y(0.25 - y^2)^{\frac{1}{2}}\right]dy$$



(a)

Then

$$F_R = \int dF = 19\,620\int_0^{0.5\text{ m}}\left[2(0.25 - y^2)^{\frac{1}{2}} - y\,(0.25 - y^2)^{\frac{1}{2}}\right]dy$$

$$= 19\,620\left[y(0.25 - y^2)^{\frac{1}{2}} + 0.25\sin^{-1}\frac{y}{0.5} + \frac{1}{3}(0.25 - y^2)^{\frac{3}{2}}\right]_0^{0.5\text{ m}}$$

$$= 6887.26\ \text{N}$$

$$= 6.89\ \text{kN} \qquad \textbf{Ans.}$$

And

$$y_P = \frac{\int (2 - y)\,dF}{F_R}$$

$$= \frac{\int_0^{0.5\text{ m}}(2 - y)\left\{19\,620\left[2(0.25 - y^2)^{\frac{1}{2}} - y(0.25 - y^2)^{\frac{1}{2}}\right]dy\right\}}{6887.26}$$

$$= \frac{19\,620\int_0^{0.5\text{ m}}\left[4(0.25 - y^2)^{\frac{1}{2}} - 4y(0.25 - y^2)^{\frac{1}{2}} + y^2(0.25 - y^2)^{\frac{1}{2}}\right]dy}{6887.26}$$

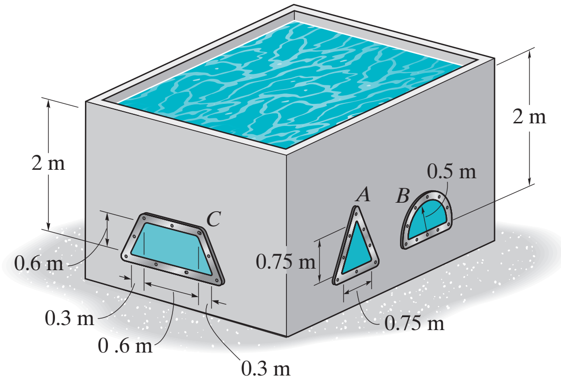
$$= \frac{19\,620\left[2y\left(0.25 - y^2\right)^{\frac{1}{2}} + 0.5\sin^{-1}\frac{y}{0.5} + \frac{4}{3}\left(0.25 - y^2\right)^{\frac{3}{2}} + \frac{y}{4}\left(0.25 - y^2\right)^{\frac{3}{2}} + \frac{y}{32}\left(0.25 - y^2\right)^{\frac{1}{2}} + \frac{1}{128}\sin^{-1}\frac{y}{0.5}\right]\Bigg|_0^{0.5\text{m}}}{6887.26}$$

$$= \frac{12380.29}{6887.26} = 1.798\ \text{m} = 1.80\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 6.89\ \text{kN}$
$y_P = 1.80\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \mathrm{kg/m^3}$ and its specific weight to be $\gamma_w = 62.4\ \mathrm{lb/ft^3}$. Also, assume all pressures are gage pressures.

2–99. The tank is filled with water. Determine the resultant force acting on the trapezoidal plate C and the location of the center of pressure, measured from the top of the tank. Solve the problem using the formula method.



SOLUTION

Referring to the geometry shown in Fig. *a*,

$$A = (0.6\ \mathrm{m})(0.6\ \mathrm{m}) + \frac{1}{2}(0.6\ \mathrm{m})(0.6\ \mathrm{m}) = 0.54\ \mathrm{m^2}$$

$$\bar{y} = \frac{(1.7\ \mathrm{m})(0.6\ \mathrm{m})(0.6\ \mathrm{m}) + (1.8\mathrm{m})\left[\frac{1}{2}(0.6\ \mathrm{m})(0.6\ \mathrm{m})\right]}{0.54\ \mathrm{m^2}} = 1.7333\ \mathrm{m}$$

$$\bar{I}_x = \frac{1}{12}(0.6\ \mathrm{m})(0.6\ \mathrm{m})^3 + (0.6\ \mathrm{m})(0.6\ \mathrm{m})(1.7333\ \mathrm{m} - 1.7\ \mathrm{m})^2$$

$$+ \frac{1}{36}(0.6\ \mathrm{m})(0.6\ \mathrm{m})^3 + \frac{1}{2}(0.6\ \mathrm{m})(0.6\ \mathrm{m})(1.8\ \mathrm{m} - 1.7333\ \mathrm{m})^2$$

$$= 0.0156\ \mathrm{m^4}$$

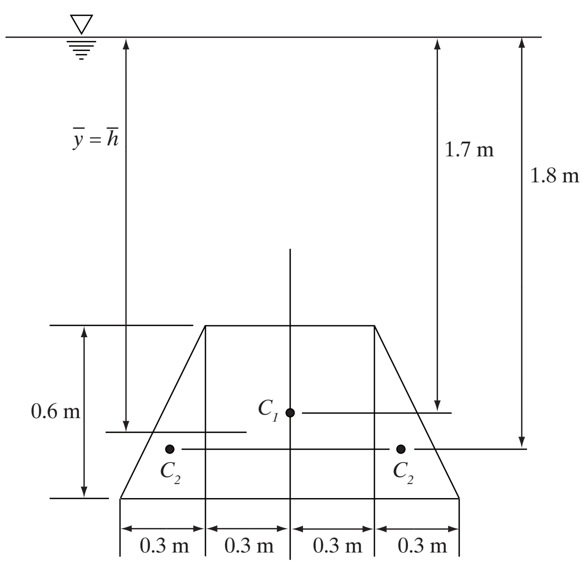
The resultant force is

$$F_R = \rho_w g \bar{h} A = (1000\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})(1.7333\ \mathrm{m})(0.54\ \mathrm{m^2}) = 9182.16\ \mathrm{N}$$

$$= 9.18\ \mathrm{kN} \qquad \textbf{Ans.}$$

And it acts at

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{0.0156\ \mathrm{m^4}}{(1.7333\ \mathrm{m})(0.54\ \mathrm{m})} + 1.7333\ \mathrm{m} = 1.75\ \mathrm{m} \qquad \textbf{Ans.}$$

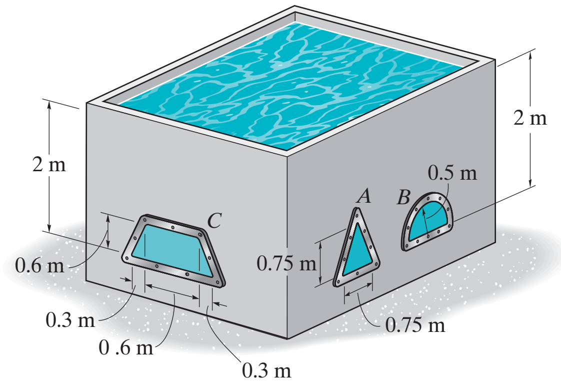


(a)

Ans:
$F_R = 9.18\ \mathrm{kN}$
$y_P = 1.75\ \mathrm{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

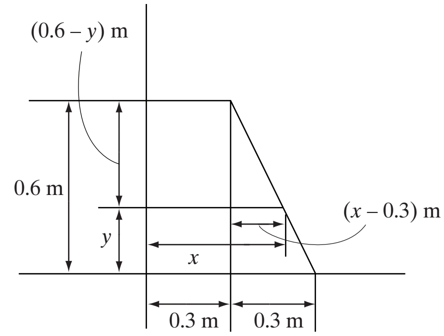
***2–100.** Solve Prob. 2–99 using the integration method.



SOLUTION

Referring to the geometry shown in Fig. a,

$$\frac{0.6\text{ m} - y}{0.6\text{ m}} = \frac{x - 0.3\text{ m}}{0.3\text{ m}}; \qquad x = (0.6 - 0.5y)\text{ m}$$



(a)

Referring to Fig. b, the pressure as a function of y can be written as

$$p = \rho_w g h = \left(1000\ \text{kg/m}^3\right)\left(9.81\text{m/s}^2\right)(2 - y)\text{ m} = \left[9810(2 - y)\right]\text{ N/m}^2$$

This pressure acts on the element of area $dA = 2x\,dy = 2(0.6 - 0.5y)dy = (1.2 - y)dy$. Thus

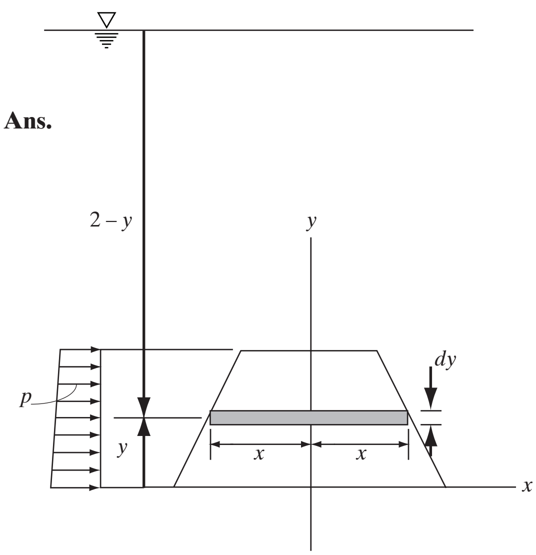
$$dF = p\,dA = 9810(2 - y)(1.2 - y)dy$$
$$= 9810\left(y^2 - 3.2y + 2.4\right)dy$$

Then

$$F_R = \int dF = 9810\int_0^{0.6\text{ m}}\left(y^2 - 3.2y + 2.4\right)dy$$
$$= 9810\left(\frac{y^3}{3} - 1.6y^2 + 2.4y\right)\Bigg|_0^{0.6\text{ m}}$$
$$= 9182.16\text{ N} = 9.18\text{ kN} \qquad \textbf{Ans.}$$

And it acts at

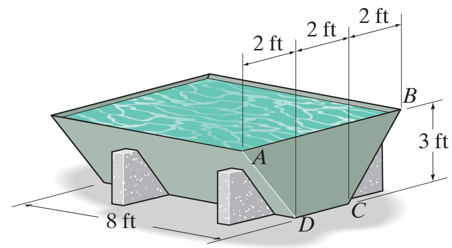
$$y_P = \frac{\int (2 - y)\,dF}{F_R}$$
$$= \frac{\int_0^{0.6\text{ m}}(2 - y)\left[9810\left(y^2 - 3.2y + 2.4\right)dy\right]}{9182.16}$$
$$= \frac{9810\int_0^{0.6\text{ m}}\left(-y^3 + 5.2y^2 - 8.8y + 4.8\right)dy}{9182.16}$$
$$= \frac{9810\left(-\frac{y^4}{4} + 1.7333y^3 - 4.4y^2 + 4.8y\right)\Big|_0^{0.6\text{ m}}}{9182.16}$$
$$= 1.75\text{ m} \qquad \textbf{Ans.}$$



(b)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–101. The open wash tank is filled to its top with butyl alcohol, an industrial solvent. Determine the magnitude of the resultant force on the end plate *ABCD* and the location of the center of pressure, measured from *AB*. Solve the problem using the formula method. Take $\gamma_{ba} = 50.1\text{ lb/ft}^3$.



SOLUTION

First, the location of the centroid of plate *ABCD*, Fig. *a*, measured from edge *AB* must be determined.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{(1.5\text{ ft})\left[3\text{ ft}(2\text{ ft})\right] + (1\text{ ft})\left[\frac{1}{2}(4\text{ ft})(3\text{ ft})\right]}{3\text{ ft}(2\text{ ft}) + \frac{1}{2}(4\text{ ft})(3\text{ ft})} = 1.25\text{ ft}$$

Then, the moment of inertia of plate *ABCD* about its centroid $\bar{x}$ axis is

$$\bar{I}_x = \left[\frac{1}{12}(2\text{ ft})(3\text{ ft})^3 + 2\text{ ft}(3\text{ ft})(1.5\text{ ft} - 1.25\text{ ft})^2\right] + \left[\frac{1}{36}(4\text{ ft})(3\text{ ft})^3 + \frac{1}{2}(4\text{ ft})(3\text{ ft})(1.25\text{ ft} - 1\text{ ft})^2\right] = 8.25\text{ ft}^4$$

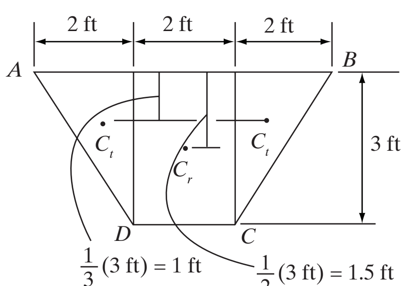
The area of plate *ABCD* is

$$A = 3\text{ ft}(2\text{ ft}) + \frac{1}{2}(4\text{ ft})(3\text{ ft}) = 12\text{ ft}^2$$

Thus,

$$F_R = \gamma \bar{h} A = \left(50.1\text{ lb/ft}^3\right)(1.25\text{ ft})\left(12\text{ ft}^2\right) = 751.5\text{ lb} = 752\text{ lb} \qquad \textbf{Ans.}$$

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{8.25}{1.25(12)} + 1.25 = 1.80\text{ ft} \qquad \textbf{Ans.}$$

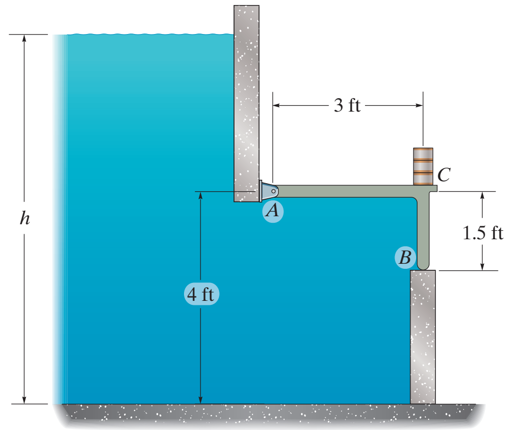


(a)

Ans:
$F_R = 752\text{ lb}$
$y_P = 1.80\text{ ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–102. The control gate ACB is pinned at A and rest on the smooth surface at B. Determine the amount of weight that should be placed at C in order to maintain a reservoir depth of $h = 10$ ft. The gate has a width of 3 ft. Neglect its weight.



SOLUTION

The intensities of the distributed load at C and B shown in Fig. a are

$$w_C = \gamma_w h_C b = \left(62.4\ \text{lb/ft}^3\right)(6\ \text{ft})(3\ \text{ft}) = 1123.2\ \text{lb/ft}$$

$$w_B = \gamma_w h_B b = \left(62.4\ \text{lb/ft}^3\right)(7.5\ \text{ft})(3\ \text{ft}) = 1404\ \text{lb/ft}$$

Thus,

$$F_1 = (1123.2\ \text{lb/ft})(3\ \text{ft}) = 3369.6\ \text{lb}$$

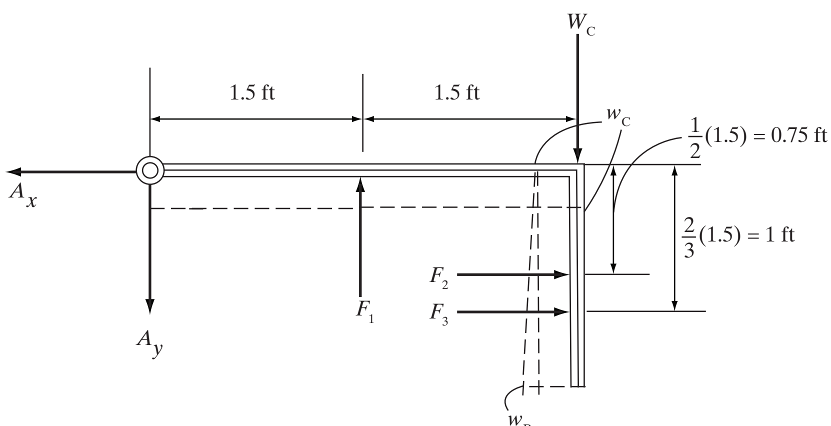
$$F_2 = (1123.2\ \text{lb/ft})(1.5\ \text{ft}) = 1684.8\ \text{lb}$$

$$F_3 = \frac{1}{2}\left[(1404 - 1123.2\ \text{lb/ft})\right](1.5\ \text{ft}) = 210.6\ \text{lb}$$

Since the gate is about to be opened, $N_B = 0$. Write the moment equation of equilibrium about point A by referring to Fig. a,

$$\circlearrowleft + \Sigma M_A = 0; \qquad (3369.6\ \text{lb})(1.5\ \text{ft}) + (1684.8\ \text{lb})(0.75\ \text{ft}) + (210.6\ \text{lb})(1\ \text{ft}) - w_C(3\ \text{ft}) = 0$$

$$W_C = 2176.2\ \text{lb} = 2.18\ \text{kip} \qquad \textbf{Ans.}$$

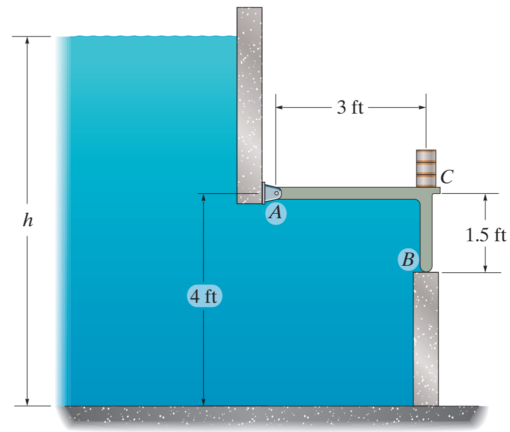


(a)

Ans:
2.18 kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–103. The control gate ACB is pinned at A and rest on the smooth surface at B. If the counterweight C is 2000 lb, determine the maximum depth of water h in the reservoir before the gate begins to open. The gate has a width of 3 ft. Neglect its weight.



SOLUTION

The intensities of the distributed loads at C and B are show in Fig. *a*

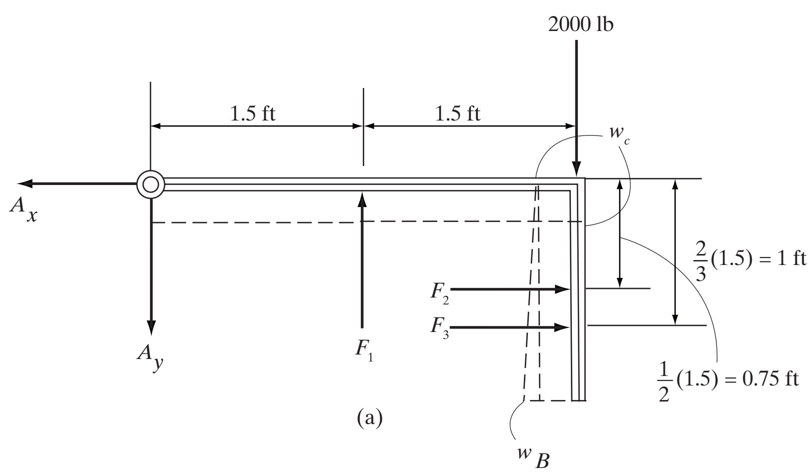
$$w_C = \gamma_w h_C b = \left(62.4\ \text{lb/ft}^3\right)(h - 4\ \text{ft})(3\ \text{ft}) = \left[187.2(h-4)\right]\ \text{lb/ft}$$
$$w_B = \gamma_w h_B b = \left(62.4\ \text{lb/ft}^3\right)(h - 2.5\ \text{ft})(3\ \text{ft}) = \left[187.2(h-2.5)\right]\ \text{lb/ft}$$

Thus,

$$F_1 = (187.2(h-4)\ \text{lb/ft})(3\ \text{ft}) = 561.6(h-4)\ \text{lb}$$
$$F_2 = (187.2(h-4)\ \text{lb/ft})(1.5\ \text{ft}) = 280.8(h-4)\ \text{lb}$$
$$F_3 = \frac{1}{2}\left[187.2(h-2.5)\ \text{lb/ft} - (187.2(h-4)\ \text{lb/ft}\right](1.5\ \text{ft}) = 210.6\ \text{lb}$$

Since the gate is required to be opened $N_B = 0$. Write the moment equation of equilibrium about point A by referring to Fig. *a*

$$\circlearrowleft + \Sigma M_A = 0; \quad \left[561.6(h-4)\ \text{lb}\right](1.5\ \text{ft}) + \left[280.8(h-4)\ \text{lb}\right](0.75\ \text{ft})$$
$$+ (210.6\ \text{lb})(1\ \text{ft}) - (2000\ \text{lb})(3\ \text{ft}) = 0$$
$$1053(h-4) = 5789.4$$
$$h = 9.498\ \text{ft} = 9.50\ \text{ft}$$ **Ans.**

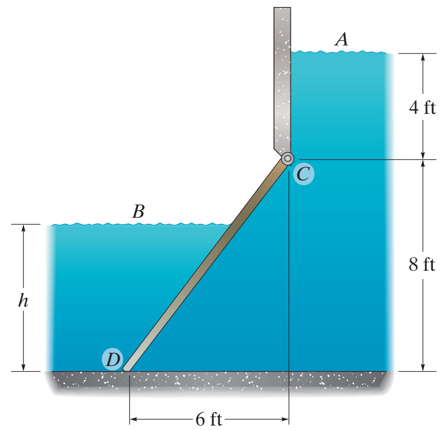


(a)

Ans:
9.50 ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

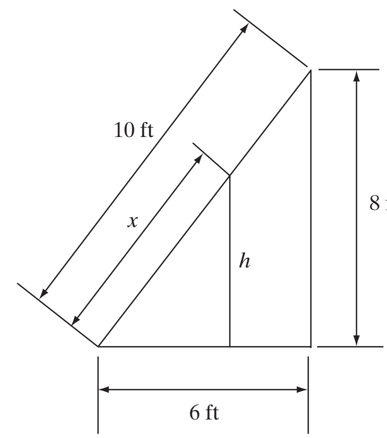
***2–104.** The uniform plate, which is hinged at C, is used to control the level of the water at A to maintain its constant depth of 12 ft. If the plate has a width of 8 ft and a weight of $50(10^3)$ lb, determine the minimum height h of the water at B so that seepage will not occur at D.



SOLUTION

Referring to the geometry in Fig. *a*

$$\frac{x}{10} = \frac{h}{8}; \qquad x = \frac{5}{4}h$$



(a)

The intensities of the distributed load shown in Fig. *b* are

$$w_1 = \gamma_w h_1 b = (62.4\ \text{lb/ft}^3)(4\ \text{ft})(8\ \text{ft}) = 1996.8\ \text{lb/ft}$$
$$w_2 = \gamma_w h_2 b = (62.4\ \text{lb/ft}^3)(12\ \text{ft})(8\ \text{ft}) = 5990.4\ \text{lb/ft}$$
$$w_3 = \gamma_w h_3 b = (62.4\ \text{lb/ft}^3)(h)(8\ \text{ft}) = (499.2h)\ \text{lb/ft}$$

Thus, the resultant forces of these distributed loads are

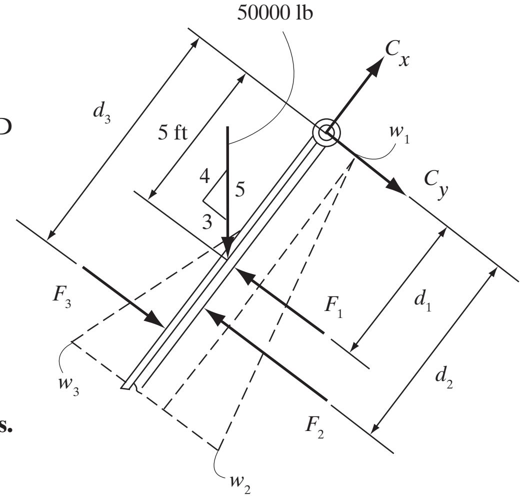
$$F_1 = (1996.8\ \text{lb/ft})(10\ \text{ft}) = 19968\ \text{lb}$$
$$F_2 = \frac{1}{2}(5990.4\ \text{lb/ft} - 1996.8\ \text{lb/ft})(10\ \text{ft}) = 19968\ \text{lb}$$
$$F_3 = \frac{1}{2}(499.2h\ \text{lb/ft})\left(\frac{5}{4}h\right) = (312h^2)\ \text{lb}$$

and act at

$$d_1 = \frac{10\ \text{ft}}{2} = 5\ \text{ft}$$
$$d_2 = \frac{2}{3}(10\ \text{ft}) = 6.667\ \text{ft}$$
$$d_3 = 10\ \text{ft} - \frac{1}{3}\left(\frac{5}{4}h\right) = (10 - 0.4167h)\ \text{ft}$$

For seepage to occur, the reaction at D, must be equal to zero. Referring to the FBD of the gate, Fig. *b*,

$$\circlearrowleft + \Sigma M_C = 0; \qquad (50000\ \text{lb})\left(\frac{3}{5}\right)(5\ \text{ft}) + (312h^2\ \text{lb})(10 - 0.4167h)\ \text{ft}$$
$$- (19968\ \text{lb})(5\ \text{ft}) - (19968\ \text{lb})(6.667\ \text{ft}) = 0$$
$$-130h^3 + 3120h^2 - 82960 = 0$$



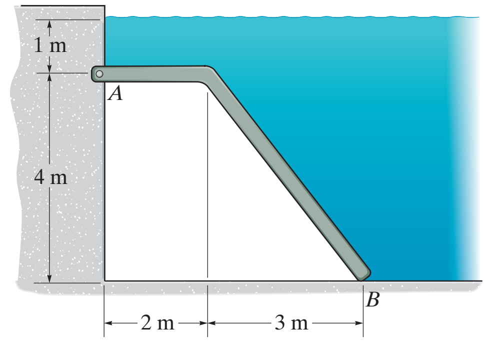
(b)

Solving numerically,

$$h = 5.945\ \text{ft} = 5.95\ \text{ft} < 8\ \text{ft} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–105. The bent plate is 1.5 m wide and is pinned at A and rests on a smooth support at B. Determine the horizontal and vertical components of reaction at A and the vertical reaction at the smooth support B for equilibrium. The fluid is water.

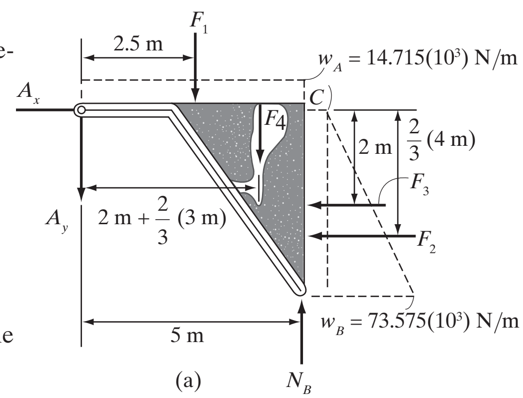


SOLUTION

Since the gate has a width of $b = 1.5$ m, the intensities of the distributed loads at A and B can be computed from

$$w_A = \rho_w g h_A b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1\ \text{m})(1.5\ \text{m}) = 14.715(10^3)\ \text{N/m}$$
$$w_B = \rho_w g h_B b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})(1.5\ \text{m}) = 73.575(10^3)\ \text{N/m}$$

Using these results, the distributed load acting on the plate is shown on the free-body diagram of the gate, Fig. *a*.



(a)

$$F_1 = w_A L_{AB} = (14.715(10^3)\ \text{N/m})(5\ \text{m}) = 73.575(10^3)\ \text{N}$$
$$F_2 = \frac{1}{2}(w_B - w_A)L_{BC} = \frac{1}{2}(73.575(10^3)\ \text{N/m} - 14.715(10^3)\ \text{N/m})(4\ \text{m})$$
$$= 117.72(10^3)\ \text{N}$$
$$F_3 = w_A L_{BC} = (14.715(10^3)\ \text{N/m})(4\ \text{m}) = 58.86(10^3)\ \text{N}$$

$\mathbf{F}_4$ on the free-body diagram is equal to the weight of the water contained in the shaded triangular block, Fig. *a*.

$$F_4 = \rho_w g \forall = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{1}{2}(3\ \text{m})(4\ \text{m})(1.5\ \text{m})\right] = 88.29(10^3)\ \text{N}$$

Considering the free-body diagram of the gate, Fig. *a*.

$$\circlearrowleft + \Sigma M_A = 0; \quad N_B(5\ \text{m}) - 73.575(10^3)\ \text{N}(2.5\ \text{m}) - 58.86(10^3)\ \text{N}(2\ \text{m}) - 117.72(10^3)\ \text{N}\left(\frac{2}{3}(4\ \text{m})\right)$$
$$- 88.29(10^3)\ \text{N}\left(2\ \text{m} + \frac{2}{3}(3\ \text{m})\right) = 0$$

$$N_B = 193.748(10^3)\ \text{N} = 194\ \text{kN} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 58.86(10^3)\ \text{N} - 117.72(10^3)\ \text{N} = 0$$

$$A_x = 176.58(10^3)\ \text{N} = 177\ \text{kN} \qquad \textbf{Ans.}$$

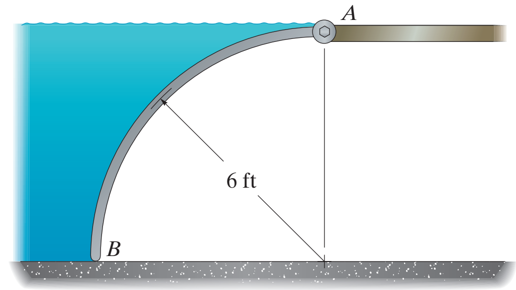
$$+\uparrow \Sigma F_y = 0; \quad -A_y - 73.575(10^3)\ \text{N} - 88.29(10^3)\ \text{N} + 193.748(10^3)\ \text{N} = 0$$

$$A_y = 31.88(10^3)\ \text{N} = 31.9\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$N_B = 194$ kN
$A_x = 177$ kN
$A_y = 31.9$ kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–106. The thin quarter-circular arched gate is 3 ft wide, is pinned at A, and rests on the smooth support at B. Determine the reactions at these supports due to the water pressure.



SOLUTION

Referring to the geometry shown in Fig. *a*,

$$A_{ADB} = (6\text{ ft})(6\text{ ft}) - \frac{\pi}{4}(6\text{ ft})^2 = (36 - 9\pi)\text{ ft}^2$$

$$\bar{x} = \frac{(3\text{ ft})[(6\text{ ft})(6\text{ ft})] - \left(\frac{8}{\pi}\text{ ft}\right)\left[\frac{\pi}{4}(6\text{ ft})^2\right]}{(36 - 9\pi)\text{ ft}^2} = 4.6598\text{ ft}$$

The horizontal component of the resultant force acting on the gate is equal to the pressure force on the vertically projected area of the gate. Referring to Fig. *b*

$$N_B = \gamma_w h_B b = \left(62.4\text{ lb/ft}^3\right)(6\text{ ft})(3\text{ ft}) = 1123.2\text{ lb/ft}$$

Thus,

$$F_h = \frac{1}{2}(1123.2\text{ lb/ft})(6\text{ ft}) = 3369.6\text{ lb}$$

The vertical component of the resultant force acting on the gate is equal to the weight of the column of water above the gate (shown shaded in Fig. *b*).

$$F_v = \gamma_w \forall = \gamma_w A_{ADB} b = \left(62.4\text{ lb/ft}^3\right)\left[(36 - 9\pi)\text{ ft}^2\right](3\text{ ft}) = 1446.24\text{ lb}$$

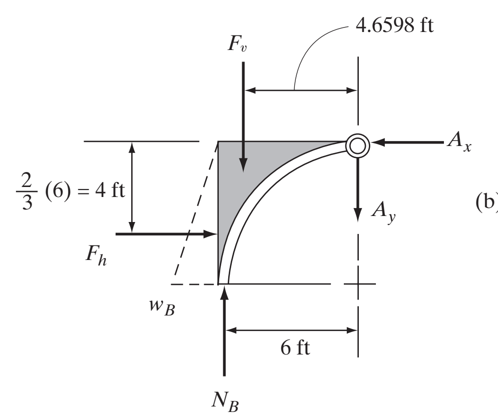
Considering the equilibrium of the FBD of the gate in Fig. *b*,

$\circlearrowleft + \Sigma M_A = 0;\quad (1446.24\text{ lb})(4.6598\text{ ft}) + (3369.6\text{ lb})(4\text{ ft}) - N_B(6\text{ ft}) = 0$

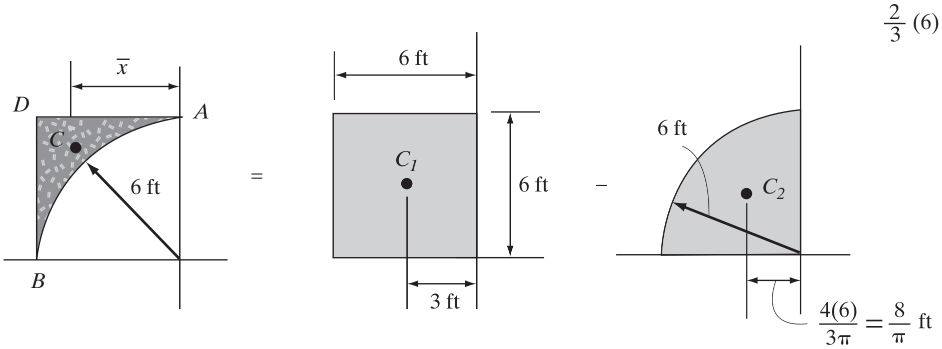
$N_B = 3369.6\text{ lb} = 3.37\text{ kip}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;\quad 3369.6\text{ lb} - A_x = 0 \quad A_x = 3369.6\text{ lb} = 3.37\text{ kip}$ **Ans.**

$\xrightarrow{+} \Sigma F_y = 0;\quad 3369.6\text{ lb} - 1446.24\text{ lb} - A_y = 0 \quad A_y = 1923.36\text{ lb} = 1.92\text{ kip}$ **Ans.**



(b)

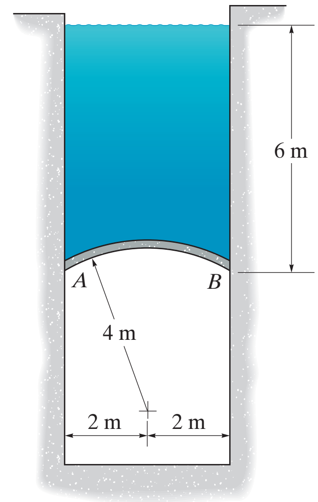


(a)

Ans:
$N_B = 3.37$ kip
$A_x = 3.37$ kip
$A_y = 1.92$ kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–107. Water is confined in the vertical chamber, which is 2 m wide. Determine the resultant force it exerts on the arched roof AB.



SOLUTION

Due to symmetry, the resultant force that the water exerts on arch AB will be vertically downward, and its magnitude is equal to the weight of water of the shaded block in Fig. *a*. This shaded block can be subdivided into two parts as shown in Figs. *b* and *c*. The block in Fig. *c* should be considered a negative part since it is a hole. From the geometry in Fig. *a*,

$$\theta = \sin^{-1}\left(\frac{2 \text{ m}}{4 \text{ m}}\right) = 30^\circ$$

$$h = 4 \cos 30^\circ \text{ m}$$

Then, the area of the parts in Figs. *b* and *c* are

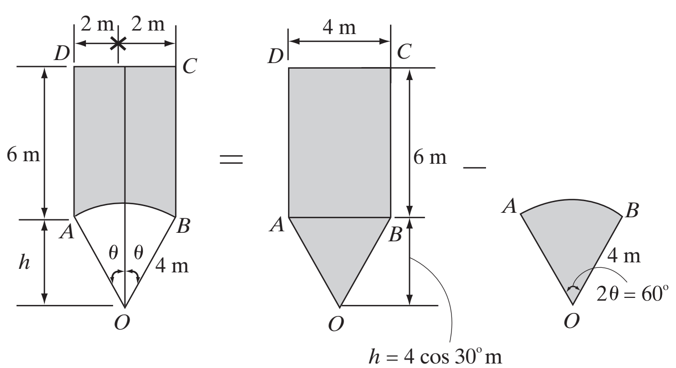
$$A_{OBCDAO} = 6 \text{ m}(4 \text{ m}) + \frac{1}{2}(4 \text{ m})(4 \cos 30^\circ \text{ m}) = 30.928 \text{ m}^2$$

$$A_{OBAO} = \frac{60^\circ}{360^\circ}(\pi r^2) = \frac{60^\circ}{360^\circ}\left[\pi(4 \text{ m})^2\right] = 2.6667\pi \text{ m}^2$$

Therefore,

$$F_R = W = \rho_w g \mathcal{V} = \left(1000 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)\left[\left(30.928 \text{ m}^2 - 2.6667\pi \text{ m}^2\right)(2 \text{ m})\right]$$

$$= 442.44\left(10^3\right) \text{ N} = 442 \text{ kN}$$ **Ans.**

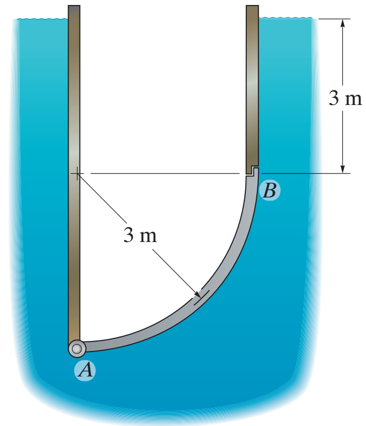


(a) (b) (c)

Ans:
442 kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–108.** Determine the horizontal and vertical components of reaction at the hinge A and the normal reaction at B caused by the water pressure. The gate has a width of 3 m.



SOLUTION

The horizontal component of the resultant force acting on the gate is equal to the pressure force on the vertically projected area of the gate. Referring to Fig. *a*,

$$w_A = \rho_w g h_A b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(6\ \text{m})(3\ \text{m}) = 176.58(10^3)\ \text{N/m}$$

$$w_B = \rho_w g h_B b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(3\ \text{m}) = 88.29(10^3)\ \text{N/m}$$

Thus,

$$(F_h)_1 = \left[88.29(10^3)\ \text{N/m}\right](3\ \text{m}) = 264.87(10^3)\ \text{N} = 264.87\ \text{kN}$$

$$(F_h)_2 = \frac{1}{2}\left[176.58(10^3)\ \text{N/m} - 88.29(10^3)\ \text{N/m}\right](3\ \text{m}) = 132.435(10^3)\ \text{N} = 132.435\ \text{kN}$$

They act at

$$\tilde{y}_1 = \frac{1}{2}(3\ \text{m}) = 1.5\ \text{m} \qquad \tilde{y}_2 = \frac{1}{3}(3\ \text{m}) = 1\ \text{m}$$

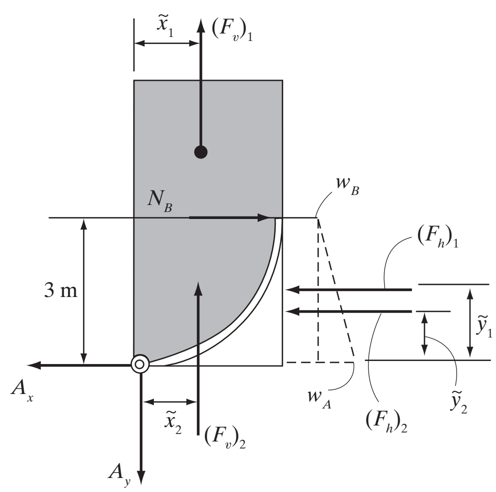
The vertical component of the resultant force acting on the gate is equal to the weight of the imaginary column of water above the gate (shown shaded in Fig. *a*) but acts upward

$$(F_v)_1 = \rho_w g \mathcal{V}_1 = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[(3\ \text{m})(3\ \text{m})(3\ \text{m})\right] = 264.87(10^3)\ \text{N} = 264.87\ \text{kN}$$

$$(F_v)_2 = \rho_w g \mathcal{V}_2 = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{\pi}{4}(3\ \text{m})^2(3\ \text{m})\right] = 66.2175\pi(10^3)\ \text{N} = 66.2175\pi\ \text{kN}$$

They act at

$$\tilde{x}_1 = \frac{1}{2}(3\ \text{m}) = 1.5\ \text{m} \qquad \tilde{x}_2 = \frac{4(3\ \text{m})}{3\pi} = \left(\frac{4}{\pi}\right)\text{m}$$



(a)

2–108. Continued

Considering the equilibrium of the FBD of the gate in Fig. *a*

$$\circlearrowleft + \Sigma M_A = 0; \qquad (264.87\text{ kN})(1.5\text{ m}) + (132.435\text{ kN})(1\text{ m}) + (264.87\text{ kN})(1.5\text{ m})$$
$$+ (66.2175\pi\text{ kN})\left(\frac{4}{\pi}\text{ m}\right) - N_B(3\text{ m}) = 0$$

$$N_B = 397.305\text{ kN} = 397\text{ kN} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 397.305\text{ kN} - 264.87\text{ kN} - 132.435\text{ kN} - A_x = 0$$

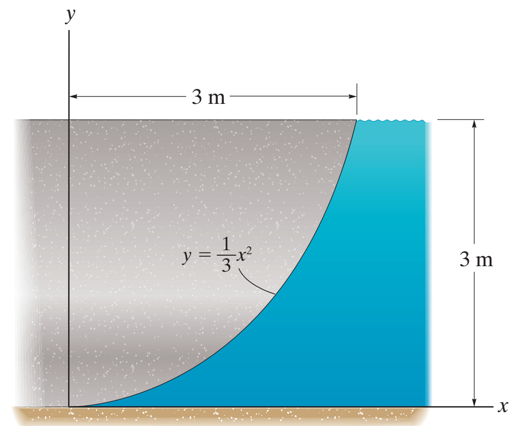
$$A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 264.87\text{ kN} + 66.2175\pi\text{ kN} - A_y = 0$$

$$A_y = 472.90\text{ kN} = 473\text{ kN} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–109. The 5-m-wide overhang is in the form of a parabola, as shown. Determine the magnitude and direction of the resultant force on the overhang.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the wall. Referring to Fig. *a*,

$$w_A = \rho_w g h_A b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(5\ \text{m}) = 147.15(10^3)\ \text{N/m}$$

Thus,

$$F_h = \frac{1}{2}w_A h_A = \frac{1}{2}\left[147.15(10^3)\ \text{N/m}\right](3\ \text{m}) = 220.725(10^3)\ \text{N} = 220.725\ \text{kN}$$

The vertical component of the resultant force is equal to the weight of the imaginary column of water above surface *AB* of the wall (shown shaded in Fig. *a*) but acts upward. The volume of this column of water is

$$\forall = \frac{2}{3}ahb = \frac{2}{3}(3\ \text{m})(3\ \text{m})(5\ \text{m}) = 30\ \text{m}^3$$

Thus,

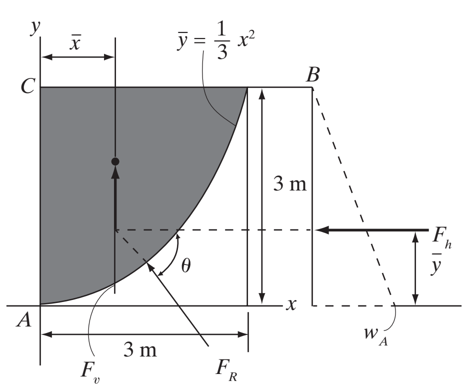
$$F_v = \rho_w g \forall = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(30\ \text{m}^3) = 294.3(10^3)\ \text{N} = 294.3\ \text{kN}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}} = \sqrt{(220.725\ \text{kN})^2 + (294.3\ \text{kN})^2} = 367.875\ \text{kN} = 368\ \text{kN} \quad \textbf{Ans.}$$

Its direction is

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{294.3\ \text{kN}}{220.725\ \text{kN}}\right) = 53.13^\circ = 53.1^\circ \quad \textbf{Ans.}$$

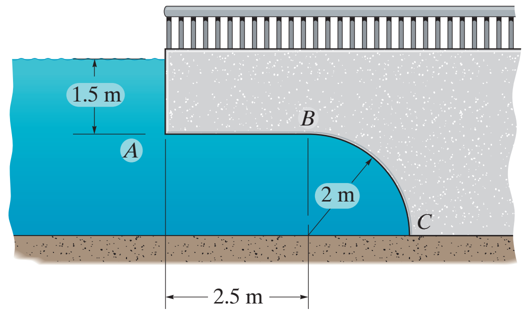


(a)

Ans:
$F_R = 368\ \text{kN}$
$\theta = 53.1^\circ$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

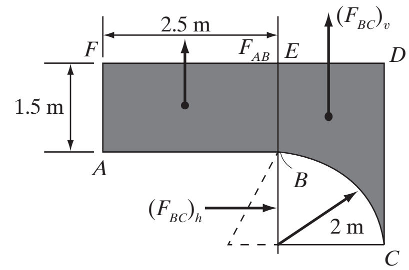
2–110. Determine the resultant force that water exerts on the overhang sea wall along ABC. The wall is 2 m wide.



SOLUTION

Horizontal Component. Since AB is along the horizontal, no horizontal component exists. The horizontal component of the force on BC is

$$(F_{BC})_h = \gamma_w \bar{h} A = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(1.5\ \text{m} + \frac{1}{2}(2\ \text{m})\right)(2\ \text{m}(2\ \text{m})) = 98.1\left(10^3\right)\text{N}$$



(a)

Vertical Component. The force on AB and the vertical component of the force on BC is equal to the weight of the water contained in blocks $ABEFA$ and $BCDEB$ (shown shaded in Fig. a), but it acts upwards. Here, $A_{ABEFA} = 1.5\ \text{m}(2.5\ \text{m}) = 3.75\ \text{m}^2$ and $A_{BCDEB} = (3.5\ \text{m})(2\ \text{m}) - \frac{\pi}{4}(2\ \text{m})^2 = (7 - \pi)\ \text{m}^2$. Then,

$$F_{AB} = \gamma_w \forall_{ABEFA} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[\left(3.75\ \text{m}^2\right)(2\ \text{m})\right]$$
$$= 73.575\left(10^3\right)\text{N} = 73.6\ \text{kN}$$

$$(F_{BC})_v = \gamma_w \forall_{BCDEB} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[(7 - \pi)\ \text{m}^2(2\ \text{m})\right]$$
$$= 75.702\left(10^3\right)\text{N}$$

Therefore,

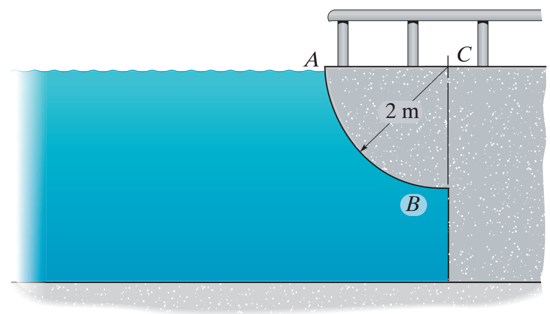
$$F_{BC} = \sqrt{(F_{BC})_h^2 + (F_{BC})_v^2} = \sqrt{\left[98.1\left(10^3\right)\text{N}\right]^2 + \left[75.702\left(10^3\right)\text{N}\right]^2}$$
$$= 123.91\left(10^3\right)\text{N} = 124\ \text{kN}$$

$$F_R = \sqrt{(F_{BC})_h^2 + \left[F_{AB} + (F_{BC})_v\right]^2}$$
$$= \sqrt{\left[98.1\left(10^3\right)\text{N}\right]^2 + \left[73.6\left(10^3\right)\text{N} + 75.702\left(10^3\right)\text{N}\right]^2}$$
$$= 178.6\left(10^3\right)\text{N} = 179\ \text{kN}$$ **Ans.**

Ans:
179 kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \mathrm{kg/m^3}$ and its specific weight to be $\gamma_w = 62.4\ \mathrm{lb/ft^3}$. Also, assume all pressures are gage pressures.

2–111. Determine the magnitude and direction of the resultant hydrostatic force the water exerts on the face AB of the overhang if it is 2 m wide.



SOLUTION

Horizontal Component. The intensity of the distributed load at B is

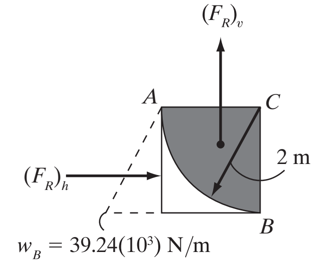
$$w_B = \rho_w g h_B b = (1000\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})(2\ \mathrm{m})(2\ \mathrm{m}) = 39.24(10^3)\ \mathrm{N/m}$$

Then,

$$(F_R)_h = \frac{1}{2}(w_B)h_B = \frac{1}{2}(39.24(10^3)\ \mathrm{N/m})(2\ \mathrm{m}) = 39.24(10^3)\ \mathrm{N}$$

Vertical Component. This component is equal to the weight of the water contained in the block shown shaded in Fig. *a*, but it acts upwards. Then

$$(F_R)_v = \rho_w g \mathcal{V}_{ABCA} = (1000\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})\left[\frac{\pi}{4}(2\ \mathrm{m})^2(2\ \mathrm{m})\right]$$
$$= 61.638(10^3)\ \mathrm{N}\uparrow$$



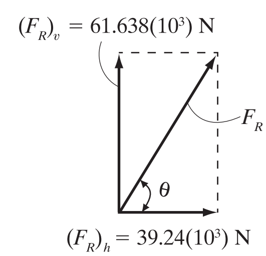
(a)

Thus, the magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_h{}^2 + (F_R)_v{}^2} = \sqrt{[39.24(10^3)\ \mathrm{N}]^2 + [61.638(10^3)\ \mathrm{N}]^2}$$
$$= 73.07(10^3)\ \mathrm{N} = 73.1\ \mathrm{kN} \qquad \textbf{Ans.}$$

And its direction, Fig. *b*, is defined by

$$\theta = \tan^{-1}\left(\frac{(F_R)_v}{(F_R)_h}\right) = \tan^{-1}\left[\frac{61.638(10^3)\ \mathrm{N}}{39.24(10^3)\ \mathrm{N}}\right] = 57.5^\circ \ \measuredangle \qquad \textbf{Ans.}$$

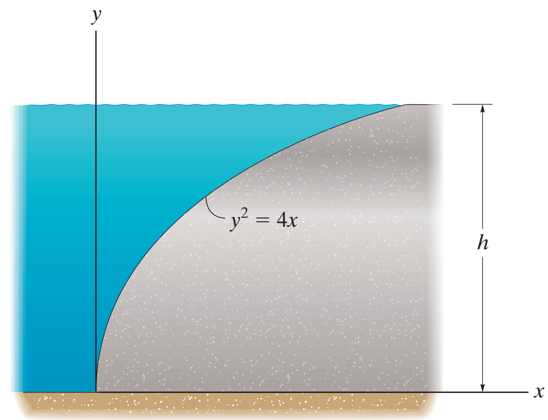


(b)

Ans:
$F_R = 73.1\ \mathrm{kN}$
$\theta = 57.5^\circ \ \measuredangle$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–112.** The 5-m-wide wall is in the form of a parabola. If the depth of the water is $h = 4$ m, determine the magnitude and direction of the resultant force on the wall.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the wall. Referring to Fig. *a*.

$$w_A = \rho_w g h_A b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(4\ \text{m})(5\ \text{m}) = 196.2\left(10^3\right)\ \text{N/m}$$

Thus,

$$F_h = \frac{1}{2} w_A h_A = \frac{1}{2}\left[196.2\left(10^3\right)\ \text{N/m}\right](4\ \text{m}) = 392.4\left(10^3\right)\ \text{N} = 392.4\ \text{kN}$$

It acts at

$$\tilde{y} = \frac{1}{3} h_A = \frac{1}{3}(4\ \text{m}) = \frac{4}{3}\ \text{m}$$

The vertical component of the resultant force is equal to the weight of the column of water above surface AB of the wall (shown shaded in Fig. *a*). The volume of this column of water is

$$\forall = \frac{1}{3} a h b = \frac{1}{3}(4\ \text{m})(4\ \text{m})(5\ \text{m}) = 26.67\ \text{m}^3$$

Thus,

$$F_r = \rho_w g \forall = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(26.67\ \text{m/s}) = 261.6\left(10^3\right)\ \text{N} = 261.6\ \text{kN}$$

It acts at

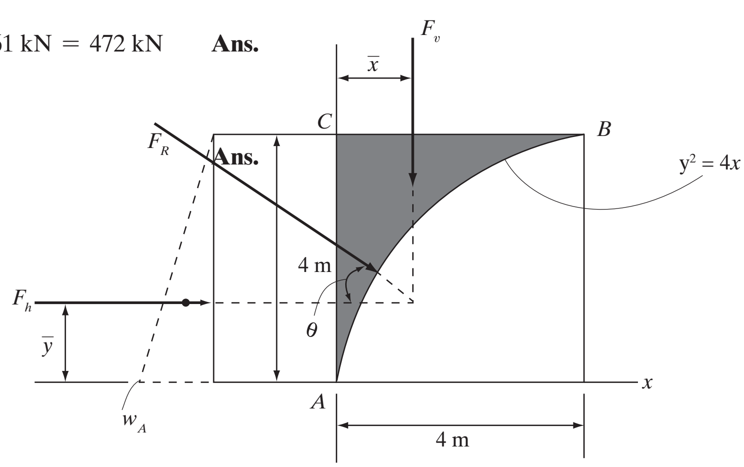
$$\tilde{x} = \frac{3}{10} a = \frac{3}{10}(4\ \text{m}) = \frac{6}{5}\ \text{m}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(392.4\ \text{kN})^2 + (261.6\ \text{kN})^2} = 471.61\ \text{kN} = 472\ \text{kN}$$ **Ans.**

And its direction is

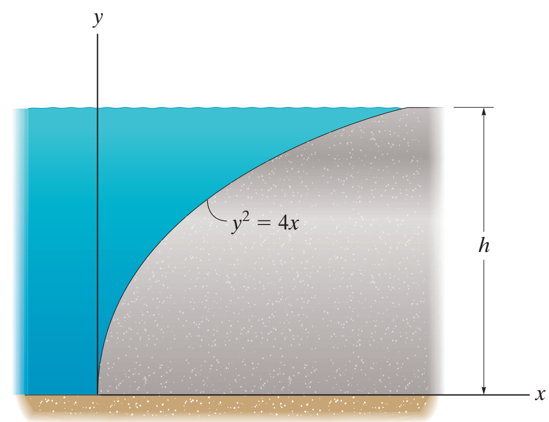
$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{261.6\ \text{kN}}{392.4\ \text{kN}}\right) = 33.69^\circ$$ **Ans.**



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–113. The 5-m wide wall is in the form of a parabola. Determine the magnitude of the resultant force on the wall as a function of depth h of the water. Plot the results of force (vertical axis) versus depth h for $0 \le h \le 4$ m. Give values for increments of $\Delta h = 0.5$ m.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the wall. Referring to Fig. *a*,

$$w_A = \rho_w g h_A b = (1000\ \text{kg/m}^3)(9.81\text{m/s}^2)(h)(5\ \text{m}) = 49.05(10^3)h$$

Thus,

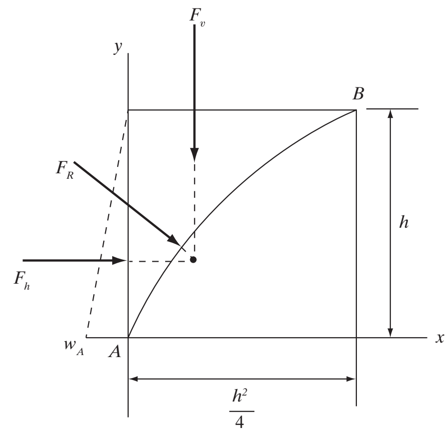
$$F_h = \frac{1}{2} w_A h_A = \frac{1}{2}[49.05(10^3)h]h = 24.525(10^3)h^2$$

The vertical component of the resultant force is equal to the weight of the column of water above surface AB of the wall (shown shaded in Fig. *a*) The volume of this column of water is

$$V\!\!\!/ = \frac{1}{3}ahb = \frac{1}{3}\left(\frac{h^2}{4}\right)(h)(5\ \text{m}) = \frac{5}{12}h^3$$

Thus,

$$F_v = \rho_w g V\!\!\!/ = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left(\frac{5}{12}\text{h}^3\right) = 4087.5\ h^3$$



(a)

Then the magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}}$$

$$F_R = \sqrt{[24.525(10^3)h^2]^2 + [4087.5\ h^3]^2}$$

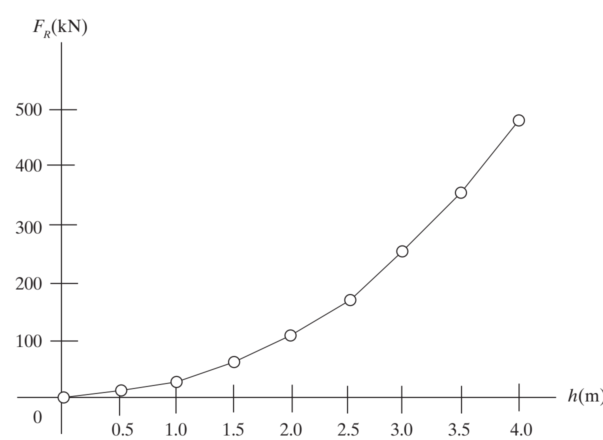
$$F_R = \sqrt{601.48(10^6)h^4 + 16.71(10^6)h^6}$$

The plot of F_R vs h is shown in Fig. *b*

$$F_R = \left[\sqrt{601(10^6)h^4 + 16.7(10^6)h^6}\right]\text{N}$$

where h is in m.

$h(m)$	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
F_R(kN)	0	6.15	24.9	56.9	103.4	166.1	246.8	347.8	471.6



(b)

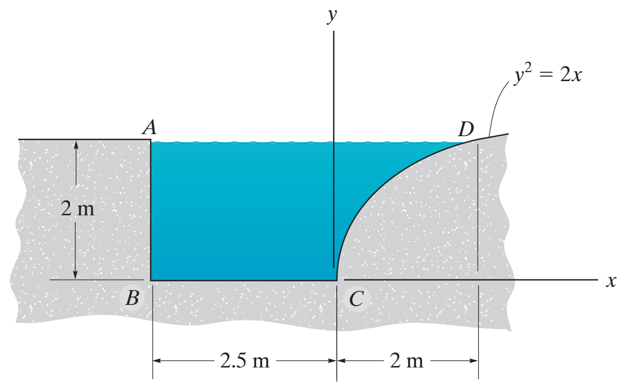
Ans:

$$F_R = \left[\sqrt{601(10^6)h^4 + 16.7(10^6)h^6}\right]\text{N}$$

where h is in m

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–114. Determine the resultant force the water exerts on AB, BC, and CD of the enclosure, which is 3 m wide.



SOLUTION

Horizontal Component. The horizontal component of the force CD is the same as the force on AB. Its magnitude can be determined from

$$F_{AB} = (F_{CD})_h = \gamma_w \bar{h} A = (100 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1 \text{ m})(2 \text{ m}(3 \text{ m}))$$

$$= 58.86(10^3) \text{ N} = 58.9 \text{ kN} \qquad \textbf{Ans.}$$

Vertical Component. The force on BC and the vertical component of the force on CD is equal to the weight of the water contained in blocks $ABCEA$ and $CDEC$ (shown shaded in Fig. *a*). Here, $A_{ABCEA} = 2 \text{ m}(2.5 \text{ m}) = 5 \text{ m}^2$ and $A_{CDEC} = \frac{1}{3} bh = \frac{1}{3}(2 \text{ m})(2 \text{ m}) = 1.3333 \text{ m}^2$ (Table 2-1). Then,

$$F_{BC} = \gamma_w \cancel{V}_{ABCEA} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\left[5 \text{ m}^2(3 \text{ m})\right]$$

$$= 147.15(10^3) \text{ N} = 147 \text{ kN} \qquad \textbf{Ans.}$$

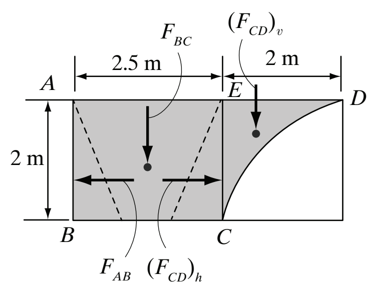
$$(F_{CD})_V = \gamma_w \cancel{V}_{CDEC} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\left[1.3333 \text{ m}^2(3 \text{ m})\right]$$

$$= 39.24(10^3) \text{ N}$$

Therefore,

$$F_{CD} = \sqrt{(F_{CD})_h^2 + (F_{CD})_v^2} = \sqrt{\left[58.86(10^3) \text{ N}\right]^2 + \left[39.24(10^3) \text{ N}\right]^2}$$

$$= 70.74(10^3) \text{ N} = 70.7 \text{ kN} \qquad \textbf{Ans.}$$

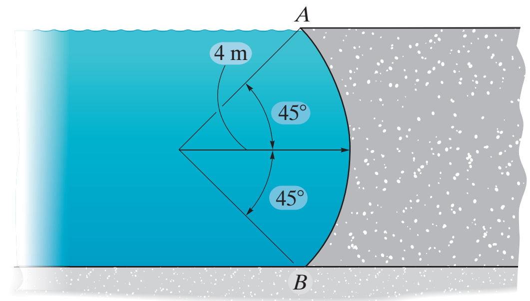


(a)

Ans:
$F_{AB} = 58.9 \text{ kN}$
$F_{BC} = 147 \text{ kN}$
$F_{CD} = 70.7 \text{ kN}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–115. Determine the magnitude of the resultant force the water exerts on the curved vertical wall. The wall is 2 m wide.



SOLUTION

Horizontal Component. This component can be determined by applying

$$(F_{AB})_h = \gamma_w \bar{h} A = (1000\ \text{kg/m}^3)(9.81\text{m/s}^2)(4 \sin 45°)[2(4 \sin 45°\ \text{m})(2\ \text{m})]$$
$$= 313.92(10^3)\ \text{N}$$

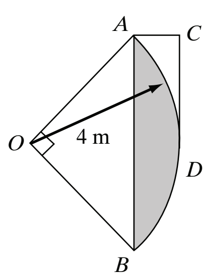
Vertical Component. The downward force on BD and the upward force on AD is equal to the weight of the water contained in blocks $ACDBA$ and $ACDA$, respectively. Thus, the net downward force on ADB is equal to the weight of water contained in block $ADBA$ shown shaded in Fig. *a*. Here, $A_{ADBA} = \frac{\pi}{4}(4\ \text{m})^2 - 2\left[\frac{1}{2}(4 \sin 45°)(4 \cos 45°)\right] = (4\pi - 8)\ \text{m}^2$.

Then,

$$(F_{AB})v = \gamma_w \forall_{ADBA} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[(4\pi - 8)\text{m}^2(2\ \text{m})]$$
$$= 89.59(10^3)\ \text{N}$$

Then,

$$F_{AB} = \sqrt{(F_{AB})_h^2 + (F_{AB})_v^2} = \sqrt{[313.92(10^3)\ \text{N}]^2 + [89.59(10^3)\ \text{N}]^2}$$
$$= 326.45(10^3)\ \text{N} = 326\ \text{kN} \qquad \textbf{Ans.}$$

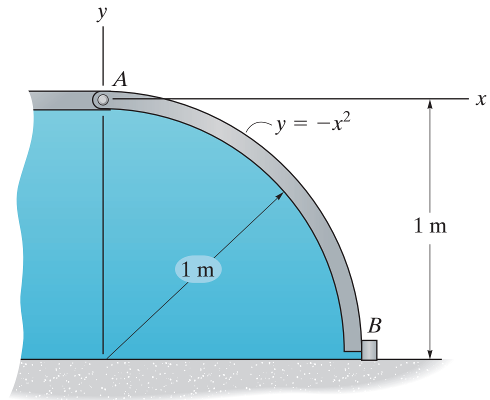


(a)

Ans:
326 kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–116.** Gate AB has a width of 0.5 m and a radius of 1 m. Determine the horizontal and vertical components of reaction at the pin A and the horizontal reaction at the smooth stop B due to the water pressure.



SOLUTION

Vertical Component. This component is equal to the weight of the water contained in the block shown shaded in Fig. *a*, but it acts upward. This block can be subdivided into parts (1) and (2) as shown in Figs. *b* and *c*. Part (2) is a hole and should be considered as a negative part. Thus, the area of the block, Fig. *a*, is $\Sigma A = (1\text{ m})(1\text{ m}) - \frac{\pi}{4}(1\text{ m})^2 = 0.2146\text{ m}^2$ and the horizontal distance measured form its centroid to point A is

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{0.5\text{ m}(1\text{ m})(1\text{ m}) - \dfrac{4(1\text{ m})}{3\pi}\left[\dfrac{\pi}{4}(1\text{ m})^2\right]}{0.2146\text{ m}^2} = 0.7766\text{ m}$$

The magnitude of the vertical component is

$$(F_R)_v = \rho_w g \forall = \left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)\left[0.2146\text{ m}^2(0.5\text{ m})\right]$$
$$= 1052.62\text{ N}$$

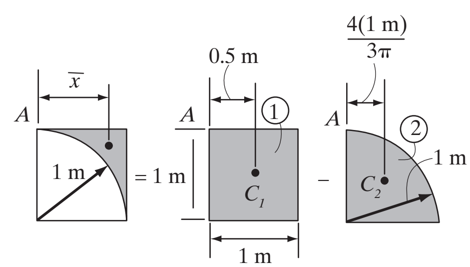
Horizontal Component. The intensity of the distributed load at B is

$$w_B = \rho_w g h_B b = \left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(1\text{ m})(0.5\text{ m}) = 4.905\left(10^3\right)\text{ N/m}$$

Then,

$$(F_R)_h = \frac{1}{2} w_B h_B = \frac{1}{2}\left[4.905\left(10^3\right)\text{ N/m}\right](1\text{ m}) = 2452.5\text{ N}$$

Considering the free-body diagram of the gate in Fig. *d*,



(a) (b) (c)

$$\circlearrowleft + \Sigma M_A = 0;\quad (2452.5\text{ N})\left[\frac{2}{3}(1\text{ m})\right] + (1052.62\text{ N})(0.7766\text{ N}) - F_B(1\text{ m}) = 0$$

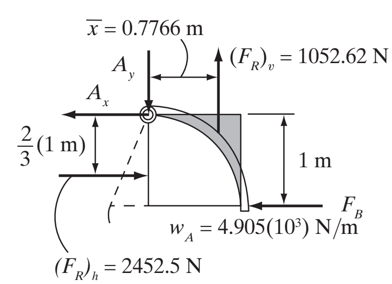
$$F_B = 2452.5\text{ N} = 2.45\text{ kN} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0;\quad 2452.5\text{ N} - 2452.5\text{ N} - A_x = 0$$

$$A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0;\quad 1052.62\text{ N} - A_y = 0$$

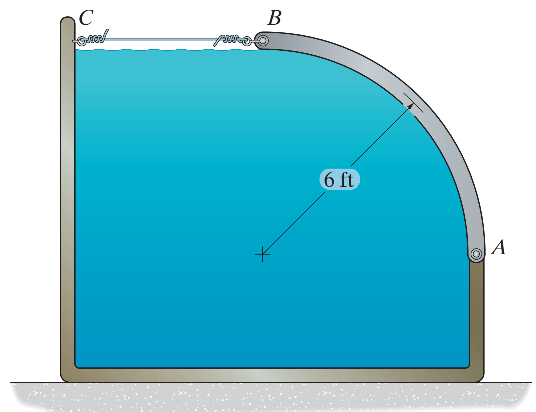
$$A_y = 1052.62\text{ N} = 1.05\text{ kN} \qquad \textbf{Ans.}$$



(d)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–117. A quarter-circular plate is pinned at A and tied to the tank's wall using the cable BC. If the tank and plate are 4 ft wide, determine the horizontal and vertical components of reaction at A, and the tension in the cable due to the water pressure.



SOLUTION

Referring to the geometry shown in Fig. *a*

$$A_{ADB} = (6\ \text{ft})(6\ \text{ft}) - \frac{\pi}{4}(6\ \text{ft})^2 = (36 - 9\pi)\ \text{ft}^2$$

$$\bar{x} = \frac{(3\ \text{ft})\left[(6\ \text{ft})(6\ \text{ft})\right] - \left[\left(6 - \frac{8}{\pi}\right)\text{ft}\right]\left[\frac{\pi}{4}(6\ \text{ft})^2\right]}{(36 - 9\pi)\ \text{ft}^2} = 1.3402\ \text{ft}$$

The horizontal component of the resultant force acting on the shell is equal to the pressure force on the vertically projected area of the shell. Referring to Fig. *b*

$$w_{\bar{A}} = \gamma_w h_A b = \left(62.4\ \text{lb/ft}^3\right)(6\ \text{ft})(4\ \text{ft}) = 1497.6\ \text{lb/ft}$$

Thus,

$$F_h = \frac{1}{2}(1497.6\ \text{lb/ft})(6\ \text{ft}) = 4492.8\ \text{lb}$$

The vertical component of the resultant force acting on the shell is equal to the weight of the imaginary column of water above the shell (shown shaded in Fig. *b*) but acts upwards.

$$F_v = \gamma_w \mathcal{V} = \gamma_w A_{ADB} b = \left(62.4\ \text{lb/ft}^3\right)\left[(36 - 9\pi)\ \text{ft}^2\right](4\ \text{ft}) = 1928.33\ \text{lb}$$

Write the moment equation of equilibrium about A by referring to Fig. *b*,

$$\circlearrowleft + \Sigma M_A = 0;\ T_{BC}(6\ \text{ft}) - (1928.33\ \text{lb})(1.3402\ \text{ft}) - (4492.8\ \text{lb})(2\ \text{ft}) = 0$$

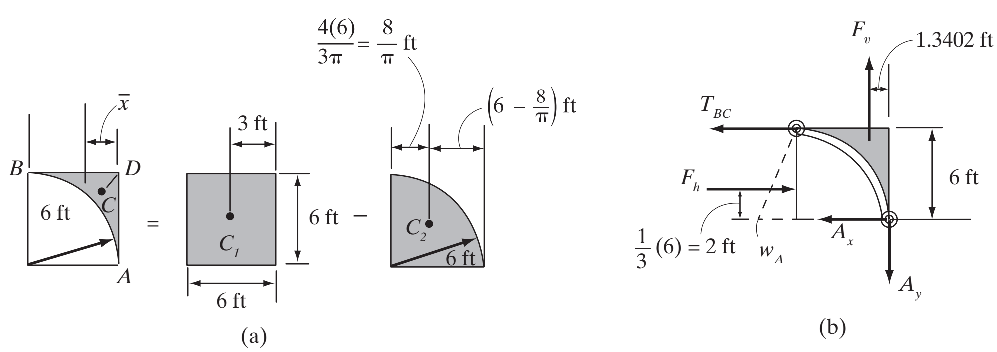
$$T_{BC} = 1928.33\ \text{lb} = 1.93\ \text{kip} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0 \quad -A_x + 4492.8\ \text{lb} - 1928.32\ \text{lb} = 0$$

$$A_x = 2564.5\ \text{lb} = 2.56\ \text{kip} \qquad \textbf{Ans.}$$

$$\uparrow + \Sigma F_y = 0 \quad 1928.33\ \text{lb} - A_y = 0$$

$$A_y = 1928.33\ \text{lb} = 1.93\ \text{kip} \qquad \textbf{Ans.}$$



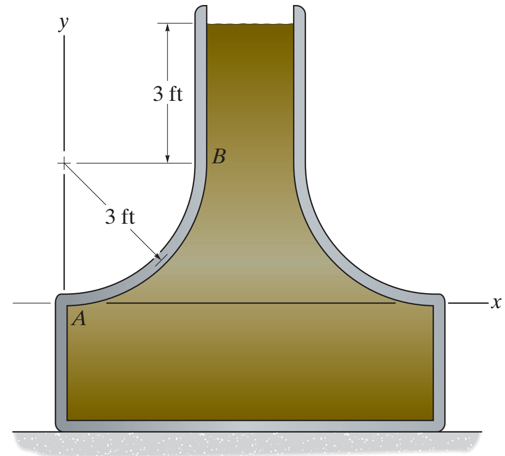
(a)

(b)

Ans:
$T_{BC} = 1.93$ kip
$A_x = 2.56$ kip
$A_y = 1.93$ kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–118. The bin is 4 ft wide and filled with linseed oil. Determine the horizontal and vertical components of the force the oil exerts on the curved segment AB. Also, find the location of the points of application of these components acting on the segment, measured from point A. $\gamma_{lo} = 58.7\ \text{lb/ft}^3$.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of curve AB of the bin. Referring to Fig. a,

$$w_B = \gamma_o h_B b = \left(58.7\ \text{lb/ft}^3\right)(3\ \text{ft})(4\ \text{ft}) = 704.4\ \text{lb/ft}$$

$$w_A = \gamma_o h_A b = \left(58.7\ \text{lb/ft}^3\right)(6\ \text{ft})(4\ \text{ft}) = 1408.8\ \text{lb/ft}$$

Then,

$$(F_h)_1 = (704.4\ \text{lb/ft})(3\ \text{ft}) = 2113.2\ \text{lb}$$

$$(F_h)_2 = \frac{1}{2}(1408.8\ \text{lb/ft} - 704.4\ \text{lb/ft})(3\ \text{ft}) = 1056.6\ \text{lb}$$

Thus,

$$F_h = (F_h)_1 + (F_h)_2 = 2113.2\ \text{lb} + 1056.6\ \text{lb} = 3169.8\ \text{lb} = 3.17\ \text{kip} \qquad \textbf{Ans.}$$

Here, $\tilde{y}_1 = \frac{1}{2}(3\ \text{ft}) = 1.5\ \text{ft}$ and $\tilde{y}_2 = \frac{1}{3}(3\ \text{ft}) = 1\ \text{ft}$. The location of the point of application of F_h can be determined from

$$\bar{y} = \frac{\Sigma \bar{y} F}{\Sigma F} = \frac{(1.5\ \text{ft})(2113.2\ \text{lb}) + (1\ \text{ft})(1056.6\ \text{lb})}{3169.8\ \text{lb}} = 1.3333\ \text{ft}$$

The vertical component of the resultant force is equal to the weight of the imaginary column of water above curve AB of the bin (shown shaded in Fig. a) but acts upwards

$$(F_v)_1 = \gamma_w \forall_1 = \left(58.7\ \text{lb/ft}^3\right)\left[(3\ \text{ft})(3\ \text{ft})(4\ \text{ft})\right] = 2113.2\ \text{lb}$$

$$(F_v)_2 = \gamma_w \forall_2 = \left(58.7\ \text{lb/ft}^3\right)\left[\frac{\pi}{4}(3\ \text{ft})^2(4\ \text{ft})\right] = 1659.70\ \text{lb}$$

Thus,

$$F_v = (F_v)_1 + (F_v)_2 = 2113.2\ \text{lb} + 1659.70\ \text{lb} = 3772.90\ \text{lb} = 3.77\ \text{kip} \qquad \textbf{Ans.}$$

Here, $\tilde{x}_1 = \frac{1}{2}(3\ \text{ft}) = 1.5\ \text{ft}$ and $\tilde{x}_2 = \frac{4(3\ \text{ft})}{3\pi} = \frac{4}{\pi}\ \text{ft}$. The location of the point of application of F_v can be determined from

$$\bar{x} = \frac{\Sigma \bar{x} F}{\Sigma F} = \frac{(1.5\ \text{ft})(2113.2\ \text{lb}) + \left(\frac{4}{\pi}\ \text{ft}\right)(1659.70\ \text{lb})}{3772.90\ \text{lb}} = 1.4002\ \text{ft}$$

The equation of the line of action of F_R is given by

$$y - \bar{y} = -\frac{F_v}{F_h}(x - \bar{x})$$

$$y - 1.3333 = -\frac{3772.9}{3169.8}(x - 1.4002)$$

$$y = -1.1903x + 3$$

2–118. Continued

Use substitution to find the intersection of this line and the circle $x^2 + (y - 3)^2 = 9$:

$$x^2 + \left[(-1.1903x + 3) - 3\right]^2 = 9$$

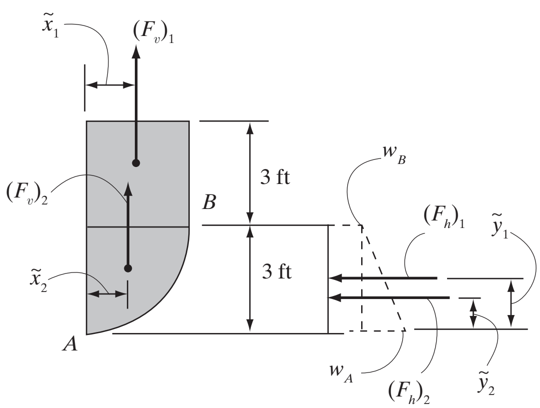
$$2.4167x^2 = 9$$

$$x = 1.9298 \text{ m} = 1.93 \text{ m} \qquad \textbf{Ans.}$$

Back-substituting,

$$y = -1.1903(1.9298) + 3$$

$$= 0.7035 \text{ m} = 0.704 \text{ m} \qquad \textbf{Ans.}$$

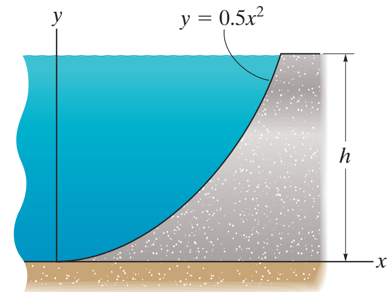


(a)

Ans:
$F_h = 3.17$ kip
$F_v = 3.77$ kip
$x = 1.93$ m
$y = 0.704$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–119. If the water depth is $h = 2$ m, determine the magnitude and direction of the resultant force, due to water pressure acting on the parabolic surface of the dam, which has a width of 5 m.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the dam. Referring to Fig. *a*

$$w_A = \rho_w g h_A b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2\ \text{m})(5\ \text{m}) = 98.1\left(10^3\right)\ \text{N/m}$$

Thus,

$$F_h = \frac{1}{2} w_A h_A = \frac{1}{2}\left[98.1\left(10^3\right)\ \text{N/m}\right](2\ \text{m}) = 98.1\left(10^3\right)\ \text{N} = 98.1\ \text{kN}$$

The vertical component of the resultant force is equal to the weight of the column of water above the dam surface (shown shaded in Fig. *a*). The volume of this column of water is

$$\forall = \frac{2}{3} ahb = \frac{2}{3}(2\ \text{m})(2\ \text{m})(5\ \text{m}) = 13.33\ \text{m}^3$$

Thus,

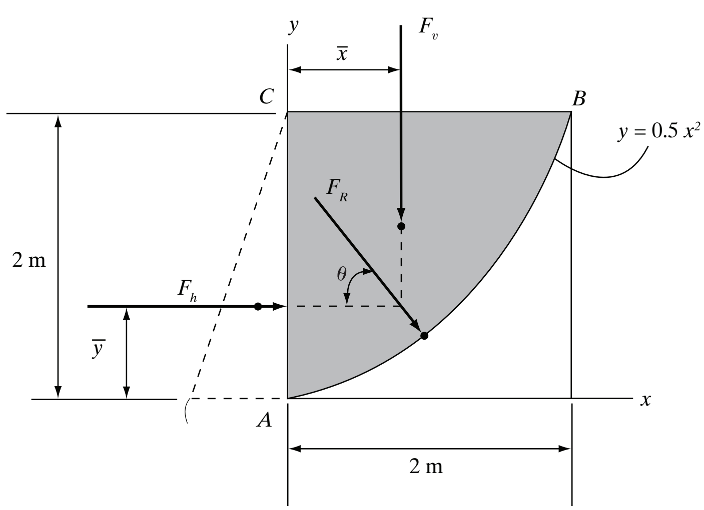
$$F_v = \rho_w g \forall = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(13.33\ \text{m}^3\right) = 130.80\left(10^3\right)\ \text{N} = 130.80\ \text{kN}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}} = \sqrt{(98.1\text{kN})^2 + (130.80\ \text{kN})^2} = 163.5\ \text{kN} \qquad \textbf{Ans.}$$

and its direction is

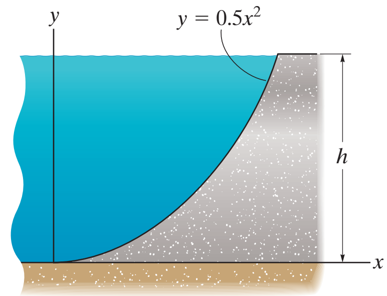
$$\theta = \tan^{-1}\frac{F_v}{F_h} = \tan^{-1}\left(\frac{130.80\ \text{kN}}{98.1\ \text{kN}}\right) = 53.1^\circ \qquad \textbf{Ans.}$$



Ans:
$F_R = 163.5$ kN
$\theta = 53.1^\circ$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–120.** Determine the magnitude of the resultant force due to water pressure acting on the parabolic surface of the dam as a function of the depth h of the water. Plot the results of force (vertical axis) versus depth h for $0 \le h \le 2$ m. Give values for increments of $\Delta h = 0.5$ m. The dam has a width of 5 m.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the dam. Referring to Fig. *a*,

$$w_A = \rho_w g h_A b = \left(1000 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)(h)(5 \text{ m}) = 49.05\left(10^3\right)h$$

Thus,

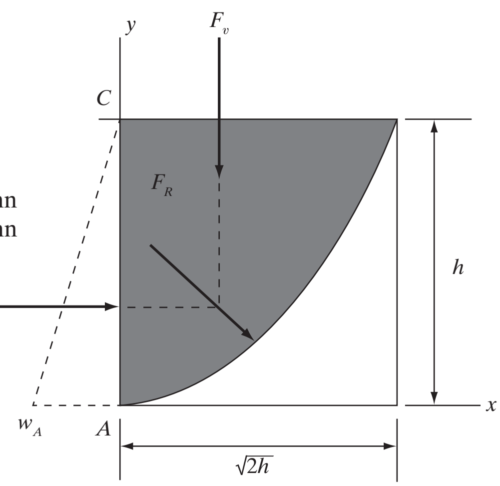
$$F_h = \frac{1}{2} w_A h_A = \frac{1}{2}\left[49.05\left(10^3\right)h\right]h = 24.525\left(10^3\right)h^2$$

The vertical component of the resultant force is equal to the weight of the column of water above the dam surface (shown shaded in Fig. *a*). The volume of this column of water is

$$V\!\!\!/ = \frac{2}{3}ahb = \frac{2}{3}\left(\sqrt{2\,h}\right)(h)(5) = 4.7140\,h^{3/2}$$

Thus,

$$F_v = \rho_w g V\!\!\!/ = \left(1000 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)\left(4.7140\,h^{3/2}\right) = 46.2447\left(10^3\right)h^{3/2}$$



(a)

Then the magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}}$$

$$F_R = \sqrt{\left[24.525\left(10^3\right)h^2\right]^2 + \left[46.2447\left(10^3\right)h^{3/2}\right]^2}$$

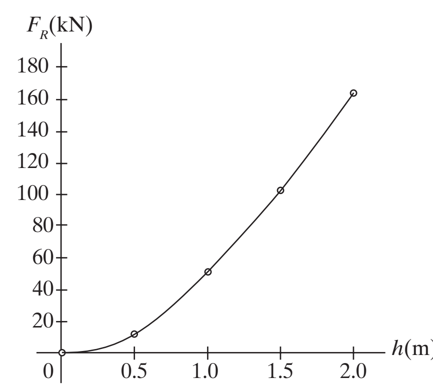
$$F_R = \sqrt{601.476\left(10^6\right)h^4 + 2.13858\left(10^9\right)h^3}$$

The plot of F_R vs h is shown in Fig. *b*.

$$F_R = \left[\sqrt{601\left(10^6\right)h^4 + 2.14\left(10^9\right)h^3}\,\right]\text{N}$$

where h is in m. **Ans.**

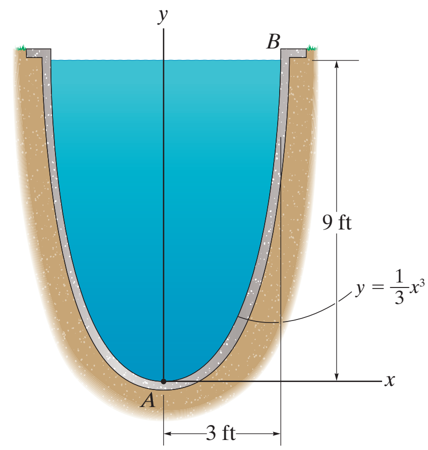
$h(m)$	0	0.5	1.0	1.5	2.0
F_R(kN)	0	17.5	52.3	101.3	163.5



(b)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–121. The canal transports water and has the cross section shown. Determine the magnitude and direction of the resultant force per unit length acting on wall AB, and the location of the center of pressure on the wall, measured with respect to the x and y axes.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of curve AB of the canal. Referring to Fig. a,

$$w_A = \gamma_w h_A b = \left(62.4\ \text{lb/ft}^3\right)(9\ \text{ft})(1\ \text{ft}) = 561.6\ \text{lb/ft}$$

Thus,

$$F_h = \frac{1}{2}(561.6\ \text{lb/ft})(9\ \text{ft}) = 2527.2\ \text{lb}$$

And it acts at

$$\bar{y} = \frac{1}{3}(9\ \text{ft}) = 3\ \text{ft}$$

The vertical component of the resultant force is equal to the weight of the column of water above curve AB of the canal

$$F_v = \int dF = \int \gamma_w\, d\mathcal{V} = \gamma_w \int b\,dA = \gamma_w \int (1)(x\,dy)$$

However, $y = \frac{1}{3}x^3$ or $x = 3^{\frac{1}{3}}y^{\frac{1}{3}}$. Then

$$F_v = 62.4\int_0^{9\ \text{ft}} 3^{\frac{1}{3}}y^{\frac{1}{3}}\,dy = 62.4\left(3^{\frac{1}{3}}\right)\left[\frac{3}{4}y^{\frac{4}{3}}\right]\Bigg|_0^{9\ \text{ft}} = 1263.6\ \text{lb}$$

The location of its point of application can be determined from

$$\bar{x} = \frac{\int \tilde{x}\,dF}{F_v} \quad \text{where } \tilde{x} = \frac{x}{2} \text{ and } dF = \gamma_w x\,dy = 62.4x\,dy$$

Thus,

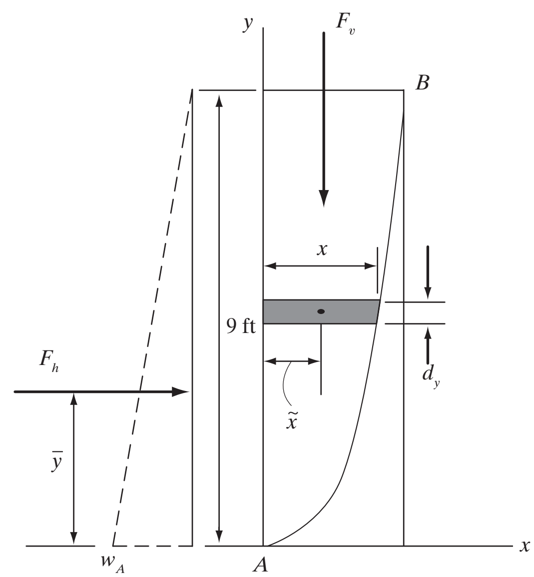
$$\bar{x} = \frac{\int_0^{9\ \text{ft}}\left(\frac{x}{2}\right)(62.4x\,dy)}{1263.6}$$

$$= \frac{31.2\int_0^{9\ \text{ft}} x^2\,dy}{1263.6}$$

$$= \frac{31.2\int_0^{9\ \text{ft}} 3^{\frac{2}{3}}y^{\frac{2}{3}}\,dy}{1263.6}$$

$$= \frac{31.2\left(3^{\frac{2}{3}}\right)\left(\frac{3}{5}y^{\frac{5}{3}}\right)\Big|_0^{9\ \text{ft}}}{1263.6}$$

$$= 1.20\ \text{ft}$$



(a)

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}} = \sqrt{(2527.2\ \text{lb})^2 + (1263.6\ \text{lb})^2} = 2825.50\ \text{lb} = 2.83\ \text{kip} \quad \textbf{Ans.}$$

2–121. Continued

And its direction is defined by

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{1263.6 \text{ lb}}{2527.2 \text{ lb}}\right) = 26.67° \quad \textbf{Ans.}$$

The equation of the line of action of F_R is

$$y - \bar{y} = m(x - \bar{x});\ y - 3 = -\tan\ 26.57°(x - 1.20)$$

$$y = -0.5x + 3.6$$

The intersection point of the line of action of F_R and surface AB can be obtained by solving simultaneously this equation and that of AB.

$$\frac{1}{3}x^3 = -0.5x + 3.6$$

$$\frac{1}{3}x^3 + 0.5x - 3.6 = 0$$

Solving numerically

$$x = 1.9851 \text{ ft} = 1.99 \text{ ft} \quad \textbf{Ans.}$$

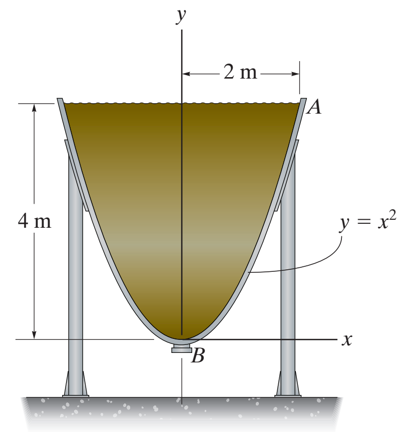
when $x = 1.9851$ ft,

$$y = \frac{1}{3}\left(1.9851^3\right) = 2.61 \text{ ft} \quad \textbf{Ans.}$$

Ans:
$F_R = 2.83$ kip
$\theta = 26.6°$
$x = 1.99$ ft
$y = 2.61$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–122. The settling tank is 3 m wide and contains turpentine having a density of $860\ \text{kg/m}^3$. If the parabolic shape is defined by $y = (x^2)$ m, determine the magnitude and direction of the resultant force the turpentine exerts on the side AB of the tank.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of surface AB. Referring to Fig. a,

$$w_B = \rho_t g h_B b = (860\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4\ \text{m})(3\ \text{m}) = 101.24(10^3)\ \text{N/m}$$

Thus,

$$F_h = \frac{1}{2}\left[101.24(10^3)\ \text{N/m}\right](4\ \text{m}) = 202.48(10^3)\ \text{N} = 202.48\ \text{kN}$$

The vertical component of the resultant forced is equal to the weight of the column of turpentine above surface AB of the wall (shown shaded in Fig. a). The volume of this column of water is

$$\mathcal{V} = \frac{2}{3}ahb = \frac{2}{3}(2\ \text{m})(4\ \text{m})(3\ \text{m}) = 16\ \text{m}^3$$

Thus,

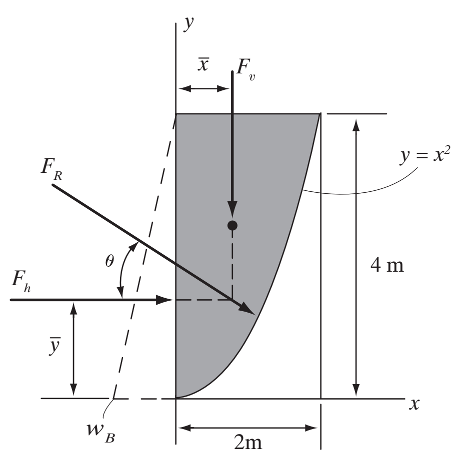
$$F_v = \rho_t g \mathcal{V} = (860\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(16\ \text{m}^3) = 134.99(10^3)\ \text{N} = 134.99\ \text{kN}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}} = \sqrt{(202.48\ \text{kN})^2 + (134.99\ \text{kN})^2} = 243.35\ \text{kN} = 243\ \text{kN} \quad \textbf{Ans.}$$

And its direction as defined by

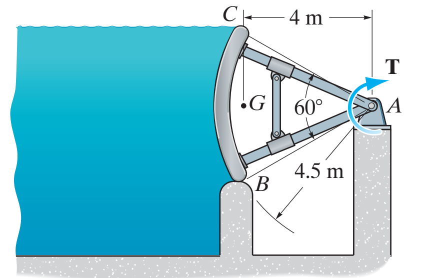
$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{134.99\ \text{kN}}{202.48\ \text{kN}}\right) = 33.79° \quad \textbf{Ans.}$$



Ans:
$F_R = 243\ \text{kN}$
$\theta = 33.7°$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–123. The radial gate is used to control the flow of water over a spillway. If the water is at its highest level as shown, determine the torque **T** that must be applied at the pin A in order to open the gate. The gate has a mass of 5 Mg and a center of mass at G. It is 3 m wide.



SOLUTION

Horizontal Component. This component can be determined by applying

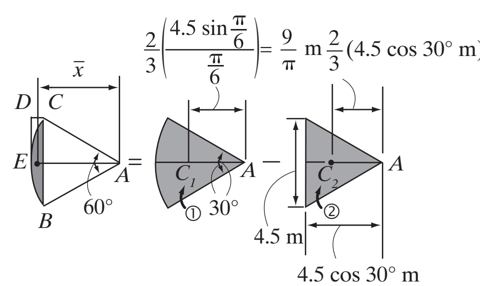
$$(F_{BC})_h = \gamma_w \bar{h} A = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4.5 \sin 30°\ \text{m})\left[2(4.5 \sin 30°\ \text{m})(3\ \text{m})\right]$$
$$= 297.98(10^3)\ \text{N}$$

Vertical Component. The upward force on BE and downward force on CE is equal to the weight of the water contained in blocks $BCDEB$ and $CEDC$, respectively. Thus, the net upward force on BEC is equal to the weight of the water contained in block $BCEB$ shown shaded in Fig. *a*. This block can be subdivided into parts (1) and (2), Figs. *a* and *b*, respectively. However, part (2) is a hole and should be considered as a negative part. The area of block $BCEB$ is $\Sigma A = \left[\frac{\pi}{6}(4.5\ \text{m})^2\right] - \frac{1}{2}(4.5\ \text{m})(4.5 \cos 30°\ \text{m}) = 1.8344\ \text{m}^2$ and the horizontal distance measured from its centroid to point A is

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{\left(\frac{9}{\pi}\ \text{m}\right)\left[\frac{\pi}{6}(4.5\ \text{m})^2\right] - \frac{2}{3}(4.5 \cos 30°\ \text{m})\left[\frac{1}{2}(4.5\ \text{m})(4.5 \cos 30°\ \text{m})\right]}{1.8344\ \text{m}^2}$$
$$= 4.1397\ \text{m}$$

The magnitude of the vertical component is

$$(F_{BC})_v = \gamma_w \forall_{BCEB} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[1.8344\ \text{m}^2(3\ \text{m})\right]$$
$$= 53.985(10^3)\ \text{N}$$



(a) (b) (c)

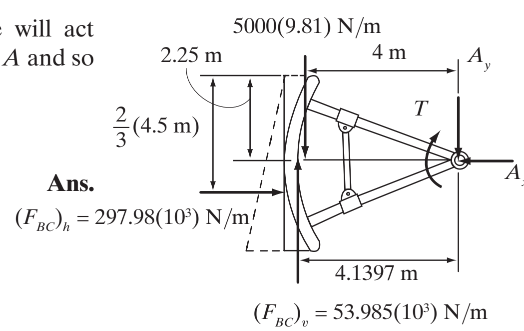
When the gate is on the verge of opening, $N_B = 0$. Referring to the free-body diagram of the gate in Fig. *d*,

$$\circlearrowleft + \Sigma M_A = 0; \quad \left[5000(9.81)\ \text{N}\right](4\ \text{m}) + \left[297.98(10^3)\ \text{N}\right]\left[\frac{2}{3}(4.5\ \text{m}) - 2.25\ \text{m}\right]$$
$$- \left[53.985(10^3)\ \text{N}\right](4.1397\ \text{m}) - T = 0$$

$$T = 196.2(10^3)\ \text{N}\cdot\text{m} = 196\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

This solution can be simplified if one realizes that the resultant force will act perpendicular to the circular surface. Therefore, F_{BC} will act through point A and so produces no moment about this point. Hence,

$$\circlearrowleft + \Sigma M_A = 0; \quad \left[5000(9.81)\ \text{N}\right](4\ \text{m}) - T = 0$$
$$T = 196.2(10^3)\ \text{N}\cdot\text{m} = 196\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

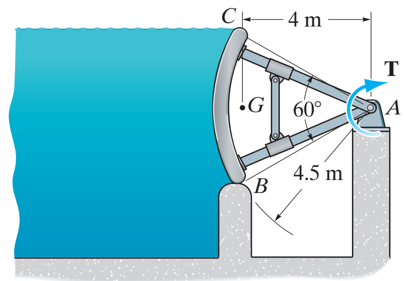


(d)

Ans:
196 kN · m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–124.** The radial gate is used to control the flow of water over a spillway. If the water is at its highest level as shown, determine the horizontal and vertical components of reaction at pin A and the vertical reaction at the spillway crest B. The gate has a weight of 5 Mg and a center of gravity at G. It is 3 m wide. Take $T = 0$.



SOLUTION

Horizontal Component. This component can be determined from

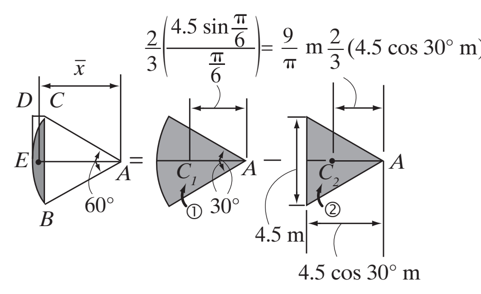
$$(F_{BC})_h = \gamma_w \bar{h} A = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(4.5\ \sin 30°\ \text{m})\left[2(4.5\ \sin 30°\ \text{m})(3\ \text{m})\right]$$

$$= 297.98\left(10^3\right)\ \text{N}$$

Vertical Component. The upward force on BE and downward force on CE is equal to the weight of the water contained in blocks $BCDEB$ and $CEDC$, respectively. Thus, the net upward force on BEC is equal to the weight of the water contained in block $BCEB$ shown shaded in Fig. a. This block can be subdivided into parts (1) and (2), Fig. a and b, respectively. However, part (2) is a hole and should be considered as a negative part. The area of block $BCEB$ is $\Sigma A = \left[\frac{\pi}{6}(4.5\ \text{m})^2\right] - \frac{1}{2}(4.5\ \text{m})(4.5\cos 30°\ \text{m}) = 1.8344\ \text{m}^2$ and the horizontal distance measured from its centroid to point A is

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{\left(\frac{9}{\pi}\ \text{m}\right)\left[\frac{\pi}{6}(4.5\ \text{m})^2\right] - \frac{2}{3}(4.5\cos 30°\ \text{m})\left[\frac{1}{2}(4.5\ \text{m})(4.5\cos 30°\ \text{m})\right]}{1.8344\ \text{m}^2}$$

$$= 4.1397\ \text{m}$$



(a) (b) (c)

Thus, the magnitude of the vertical component is

$$(F_{BC})_v = \gamma_w \mathcal{V}_{BCEB} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[1.8344\ \text{m}^2(3\ \text{m})\right]$$

$$= 53.985\left(10^3\right)\ \text{N}$$

Considering the free-body diagram of the gate in Fig. d,

$$\circlearrowleft + \Sigma M_A = 0; \qquad \left[5000(9.81)\ \text{N}\right](4\ \text{m}) + \left[297.98\left(10^3\right)\ \text{N}\right]\left[\frac{2}{3}(4.5\ \text{m}) - 2.25\ \text{m}\right]$$

$$- \left[53.985\left(10^3\right)\ \text{N}\right](4.1397\ \text{m})$$

$$-N_B(4.5\ \cos\ 30°\ \text{m}) = 0$$

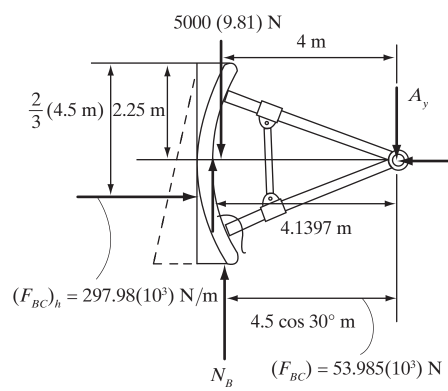
$$N_B = 50.345\left(10^3\right)\ \text{N} = 50.3\ \text{kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 50.345\left(10^3\right)\ \text{N} + 53.985\left(10^3\right)\ \text{N} - 5000(9.81)\ \text{N} - A_y = 0$$

$$A_y = 55.28\left(10^3\right)\ \text{N} = 55.3\ \text{kN} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 297.98\left(10^3\right)\ \text{N} - A_x$$

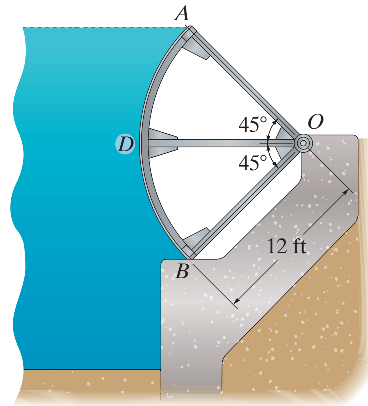
$$A_x = 297.98\left(10^3\right)\ \text{N} = 298\ \text{kN} \qquad \textbf{Ans.}$$



(d)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–125. The 6-ft-wide plate in the form of a quarter-circular arc is used as a sluice gate. Determine the magnitude and direction of the resultant force of the water on the bearing O of the gate. What is the moment of this force about the bearing?



SOLUTION

Referring to the geometry in Fig. *a*,

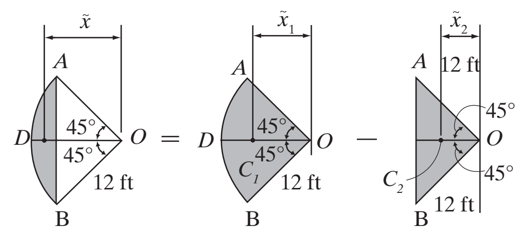
$$A_{ADB} = \frac{\pi}{4}(12\ \text{ft}^2) - \frac{1}{2}\left[(2)(12 \sin 45°\ \text{ft})(12 \cos 45°\ \text{ft})\right] = 41.097\ \text{ft}^2$$

$$\tilde{x}_1 = \frac{2}{3}\left(\frac{12\ \sin 45°\ \text{ft}}{\pi/4}\right) = 7.2025\ \text{ft}$$

$$\tilde{x}_2 = \frac{2}{3}(12 \cos 45°\ \text{ft}) = 5.6569\ \text{ft}$$

$$\bar{x} = \frac{(7.2025\ \text{ft})\left[\frac{\pi}{4}(12\ \text{ft})^2\right] - (5.6569\ \text{ft})\left[\frac{1}{2}(2)(12 \sin 45°\ \text{ft})(12 \cos 45°\ \text{ft})\right]}{41.097\ \text{ft}^2}$$

$$= 9.9105\ \text{ft}$$



(a)

The horizontal component of the resultant force is equal to the pressure force on the vertically projected area of the gate. Referring to Fig. *b*

$$w_B = \gamma_w h_B b = (62.4\ \text{lb/ft}^3)(16.971\ \text{ft})(6\ \text{ft}) = 6353.78\ \text{lb/ft}$$

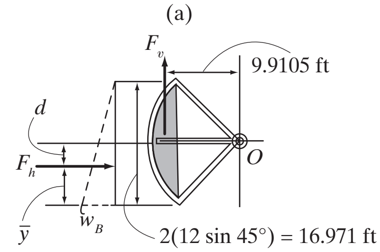
Thus,

$$F_h = \frac{1}{2}(6353.78\ \text{lb/ft})(16.971\ \text{ft}) = 53.9136(10^3)\ \text{lb} = 53.9136\ \text{kip}$$

It acts at

$$\bar{y} = \frac{1}{3}(16.971\ \text{ft}) = 5.657\ \text{ft}$$

$$d = 12\ \sin 45°\ \text{ft} - 5.657\ \text{ft} = 2.8284\ \text{ft}$$



(b)

The vertical component of the resultant force is equal to the weight of the block of water contained in sector ADB shown in Fig. *a* but acts upward.

$$F_v = \gamma_w \mathcal{V}_{ADB} = \gamma_w A_{ADB} b = (62.4\ \text{lb/ft}^3)(41.097\ \text{ft}^2)(6\ \text{ft}) = 15.3868(10^3)\ \text{lb} = 15.3868\ \text{kip}$$

Thus, the magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}} = \sqrt{(53.9136\ \text{kip})^2 + (15.3868\ \text{kip})^2} = 56.07\ \text{kip} = 56.1\ \text{kip} \qquad \textbf{Ans.}$$

Its direction is

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{15.3868\ \text{kip}}{53.9136\ \text{kip}}\right) = 15.93° = 15.9° \qquad \textbf{Ans.}$$

By referring to Fig. *b*, the moment of F_R about O is

$$\circlearrowleft + (M_R)_O = \Sigma M_O;\ (M_R)_O = (53.9136\ \text{kip})(2.8284\ \text{ft}) - (15.3868\ \text{kip})(9.9105\ \text{ft})$$

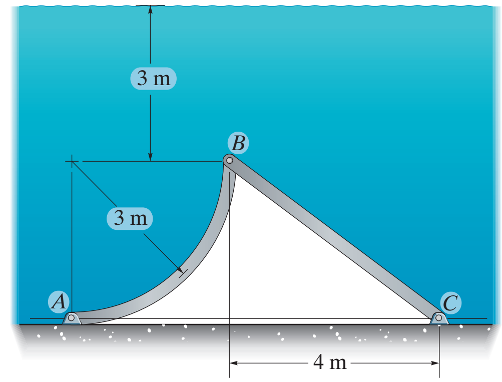
$$= 0 \qquad \textbf{Ans.}$$

This result is expected since the gate is circular in shape. Thus, F_R is always directed toward center O of the circular gate.

Ans:
$F_R = 56.1\ \text{kip}$
$\theta = 15.9°$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–126. The curved and flat plates are pin connected at A, B, and C. They are submerged in water at the depth shown. Determine the horizontal and vertical components of reaction at pin B. The plates have a width of 4 m.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force on the vertically projected area of the plate. Referring to Fig. *a*

$$w_B = \rho_w g h_B b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(4\ \text{m}) = 117.72(10^3)\ \text{N/m}$$

$$w_A = w_C = \rho_w g h_C b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(6\ \text{m})(4\ \text{m}) = 235.44(10^3)\ \text{N/m}$$

Thus,

$$(F_h)_{AB1} = (F_h)_{BC1} = [117.72(10^3)\ \text{N/m}](3\ \text{m}) = 353.16(10^3)\ \text{N} = 353.16\ \text{kN}$$

$$(F_h)_{AB2} = (F_h)_{BC2} = \frac{1}{2}[235.44(10^3)\ \text{N/m} - 117.72(10^3)\ \text{N/m}](3\ \text{m}) = 176.58(10^3)\ \text{N} = 176.58\ \text{kN}$$

They act at

$$\tilde{y}_2 = \tilde{y}_4 = \frac{1}{2}(3) = 1.5\ \text{m} \quad \tilde{y}_1 = \tilde{y}_3 = \frac{1}{3}(3\ \text{m}) = 1\ \text{m}$$

The vertical component of the resultant force is equal to the weight of the column of water above the plates shown shaded in Fig. a

$$(F_v)_{AB_1} = \rho_w g V = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[(3\ \text{m})(3\ \text{m})(4\ \text{m})] = 353.16(10^3)\ \text{N} = 353.16\ \text{kN}$$

$$(F_v)_{AB_2} = \rho_w g V = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{1}{4}\pi(3\ \text{m})^2(4\ \text{m})\right] = 88.29\pi(10^3)\ \text{N} = 88.29\pi\ \text{kN}$$

$$(F_v)_{BC_1} = \rho_w g V = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[(3\ \text{m})(4\ \text{m})(4\ \text{m})] = 470.88(10^3)\ \text{N} = 470.88\ \text{kN}$$

$$(F_v)_{BC_2} = \rho_w g V = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{1}{2}(3\ \text{m})(4\ \text{m})(4\ \text{m})\right] = 235.44(10^3)\ \text{N} = 235.44\ \text{kN}$$

They act at

$$\tilde{x}_1 = \frac{1}{2}(3\ \text{m}) = 1.5\ \text{m} \qquad \tilde{x}_2 = \frac{4(3\ \text{m})}{3\pi} = \frac{4}{\pi}\ \text{m} \qquad \tilde{x}_3 = \frac{1}{2}(4\ \text{m}) = 2\ \text{m} \qquad \tilde{x}_4 = \frac{1}{3}(4\ \text{m}) = \frac{4}{3}\ \text{m}$$

Referring to Fig. *a* and writing the moment equations of equilibrium about A and C

$$\circlearrowleft + \Sigma M_A = 0; \quad B_x(3\ \text{m}) - B_y(3\ \text{m}) - (353.16\ \text{kN})(1.5\ \text{m}) - (88.29\pi\ \text{kN})\left(\frac{4}{\pi}\ \text{m}\right)$$

$$- (353.16\ \text{kN})(1.5\ \text{m}) - (176.58\ \text{kN})(1\ \text{m}) = 0$$

$$B_x - B_y = 529.74 \qquad \textbf{(1)}$$

2–126. Continued

$$\circlearrowleft +\Sigma M_C = 0; \qquad (470.88\text{ kN})(2\text{ m}) + (235.44\text{ kN})\left(\frac{4}{3}\text{m}\right) + (353.16\text{ kN})(1.5\text{ m})$$

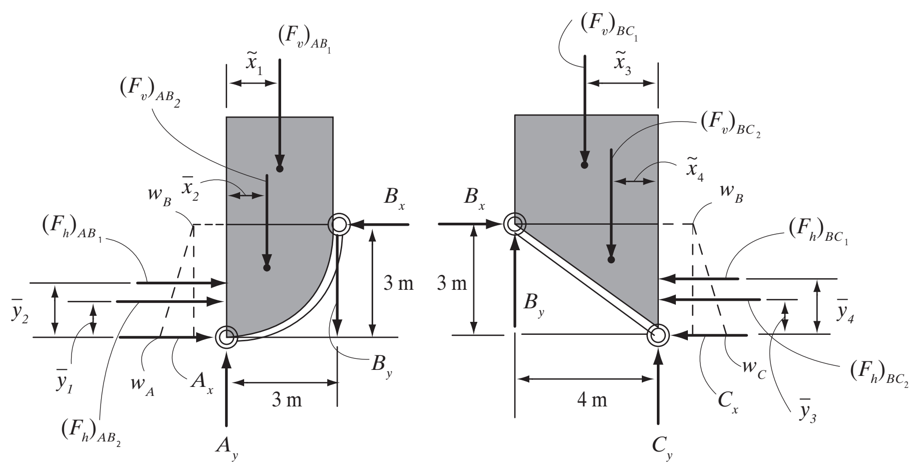
$$+ (176.58\text{ kN})(1\text{ m}) - B_x(3\text{ m}) - B_y(4\text{ m}) = 0$$

$$3B_x - 4B_y = 1962 \qquad \textbf{(2)}$$

Solving Eq. (1) and (2)

$B_x = 582.99\text{ kN} = 583\text{ kN}$ **Ans.**

$B_y = 53.2\text{ kN}$ **Ans.**

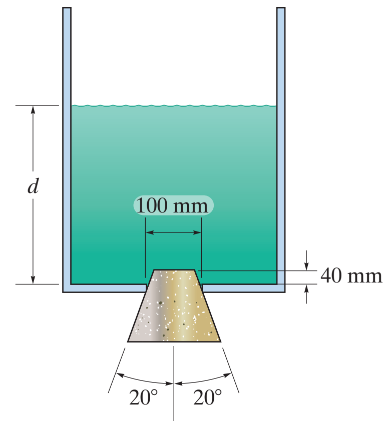


(a)

Ans:
$B_x = 583\text{ kN}$
$B_y = 53.2\text{ kN}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–127. The stopper in the shape of a frustum is used to plug the 100-mm-diameter hole in the tank that contains amyl acetate. If the greatest vertical force the stopper can resist is 100 N, determine the depth d before it becomes unplugged. Take $\rho_{aa} = 863\ \text{kg/m}^3$. *Hint:* The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.



SOLUTION

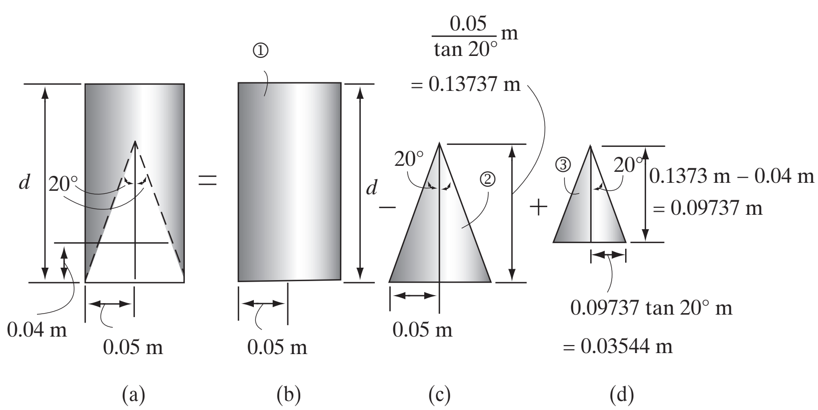
The vertical downward force on the conical stopper is due to the weight of the liquid contained in the block shown shaded in Fig *a*. This block can be subdivided into parts (1), (2), and (3), shown in Figs. *b*, *c*, and *d*, respectively. Part (2) is a hole and should be considered as a negative part. Thus, the volume of the shaded block in Fig. *a* is

$$
\begin{aligned}
V &= V_1 - V_2 + V_3 \\
&= \pi(0.05\ \text{m})^2 d - \frac{1}{3}\pi(0.05\ \text{m})^2(0.13737\ \text{m}) + \frac{1}{3}\pi(0.03544\ \text{m})^2(0.09737\ \text{m}) \\
&= \left[2.5\left(10^{-3}\right)\pi d - 0.2316\left(10^{-3}\right)\right]\ \text{m}^3
\end{aligned}
$$

The vertical force on the stopper is required to be equal to 100 N. Then,

$$
\begin{aligned}
F &= \rho w g V \\
100\ \text{N} &= \left(863\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[2.5\left(10^{-3}\right)\pi d - 0.2316\left(10^{-3}\right)\right] \\
d &= 1.5334\ \text{m} = 1.53\ \text{m}
\end{aligned}
$$

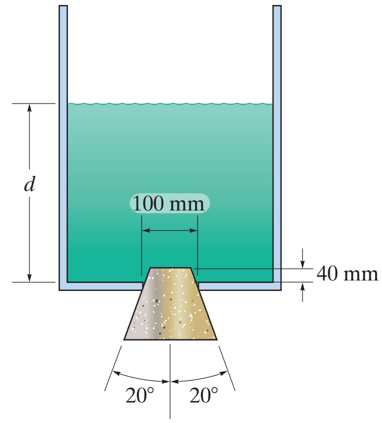
Ans.



Ans:
1.53 m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–128.** The stopper in the shape of a frustum is used to plug the 100-mm-diameter hole in the tank that contains amyl acetate. Determine the vertical force this liquid exerts on the stopper. Take $d = 0.6$ m and $\rho_{aa} = 863\ \text{kg/m}^3$. *Hint:* The volume of a cone is $\mathcal{V} = \frac{1}{3}\pi r^2 h$.



SOLUTION

The vertical downward force on the conical stopper is due to the weight of the liquid contained in the block shown shaded in Fig. *a*. This block can be subdivided into parts (1), (2), and (3), shown in Figs. *b*, *c*, and *d*, respectively. Part (2) is a hole and should be considered as a negative part. Thus, the volume of the shaded block in Fig. *a* is

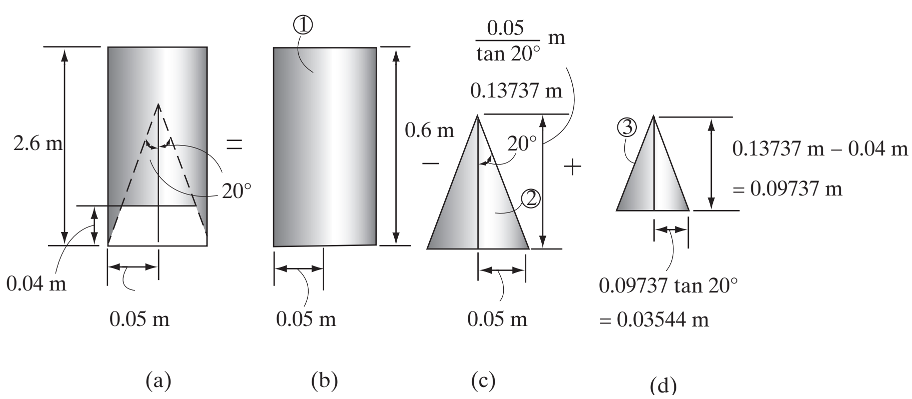
$$\mathcal{V} = \mathcal{V}_1 - \mathcal{V}_2 + \mathcal{V}_3$$

$$= \pi(0.05\text{ m})^2(0.6\text{ m}) - \frac{1}{3}\pi(0.05\text{ m})^2(0.13737\text{ m}) + \frac{1}{3}\pi(0.03544\text{ m})^2(0.09737\text{ m})$$

$$= 4.4808\left(10^{-3}\right)\text{ m}^3$$

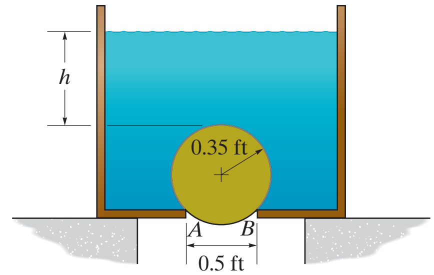
Then,

$$F = \rho_w g \mathcal{V}$$
$$= \left(863\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[4.4808\left(10^{-3}\right)\text{ m}^3\right]$$
$$= 37.93\text{ N} = 37.9\text{ N} \qquad \textbf{Ans.}$$



Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–129. The steel cylinder has a specific weight of $490\ \text{lb/ft}^3$ and acts as a plug for the 1-ft-long slot in the tank. Determine the resultant force the bottom of the tank exerts on the cylinder when the water in the tank is at a depth of $h = 2$ ft.



SOLUTION

The vertical downward force and the vertical upward force are equal to the weight of the water contained in the blocks shown shaded in Figs. *a* and *b*, respectively. The volume of the shaded block in Fig. *a* is

$$\mathcal{V}_1 = \left[2.35\ \text{ft}(0.7\ \text{ft}) - \frac{\pi}{2}(0.35\ \text{ft})^2\right](1\ \text{ft}) = 1.4526\ \text{ft}^3$$

The volume of the shaded block in Fig. *b* is

$$\mathcal{V}_2 = 2\left\{0.1\ \text{ft}(2.35\ \text{ft}) + \left[\frac{44.42^\circ}{360^\circ}(\pi)(0.35\ \text{ft})^2 - \frac{1}{2}(0.25\ \text{ft})(0.2449\ \text{ft})\right]\right\}(1\ \text{ft})$$

$$= 0.5037\ \text{ft}^3$$

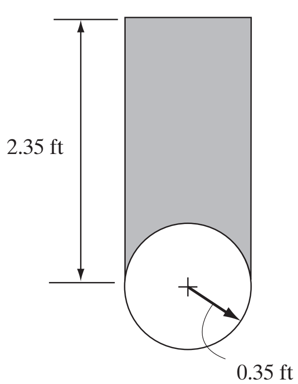
Then,

$$F = \gamma_w(\mathcal{V}_1 - \mathcal{V}_2)$$
$$= \left(62.4\ \text{lb/ft}^3\right)\left(1.4526\ \text{ft}^3 - 0.5037\ \text{ft}^3\right)$$
$$= 59.21\ \text{lb} \downarrow$$

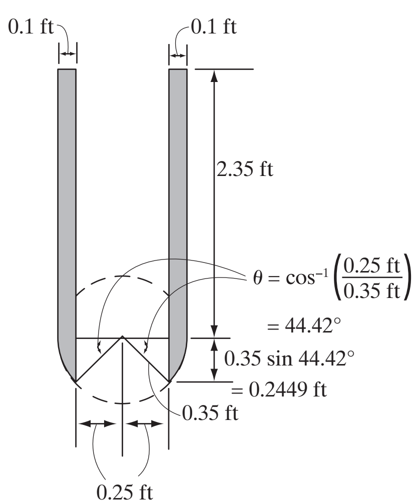
The weight of the cylinder is $W = \gamma_{st}\mathcal{V}_C = \left(490\ \text{lb/ft}^2\right)\left[\pi(0.35\ \text{ft})^2(1\ \text{ft})\right] = 188.57$ lb. Considering the free-body diagram of the cylinder, Fig. *c*, we have

$+\uparrow\Sigma F_y = 0;\qquad N - 59.21\ \text{lb} - 188.57\ \text{lb} = 0$

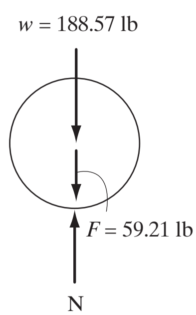
$N = 247.78\ \text{lb} = 248\ \text{lb}$ **Ans.**



(a)



(b)

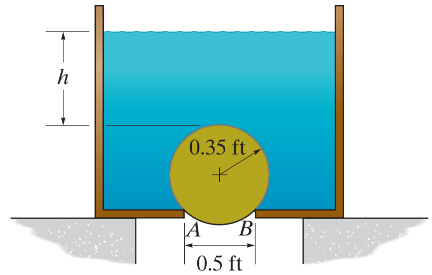


(c)

Ans:
248 lb

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–130. The steel cylinder has a specific weight of $490\ \text{lb/ft}^3$ and acts as a plug for the 1-ft-long slot in the tank. Determine the resultant force the bottom of the tank exerts on the cylinder when the water in the tank just covers the top of the cylinder, $h = 0$.



SOLUTION

The vertical downward force and the vertical upward force are equal to the weight of the water contained in the blocks shown shaded in Figs. *a* and *b*, respectively. The volume of the shaded block in Fig. *a* is

$$\mathcal{V}_1 = \left[0.35\ \text{ft}(0.7\ \text{ft}) - \frac{\pi}{2}(0.35\ \text{ft})^2\right](1\ \text{ft}) = 0.05258\ \text{ft}^3$$

The volume of the shaded block in Fig. *b* is

$$\mathcal{V}_2 = 2\left\{0.35\ \text{ft}(0.1\ \text{ft}) + \left[\frac{44.42°}{360°}(\pi)(0.35\ \text{ft})^2 - \frac{1}{2}(0.25\ \text{ft})(0.2449\ \text{ft})\right]\right\}(1\ \text{ft})$$

$$= 0.10372\ \text{ft}^3$$

Then,

$$F = \gamma_w(\mathcal{V}_1 - \mathcal{V}_2)$$

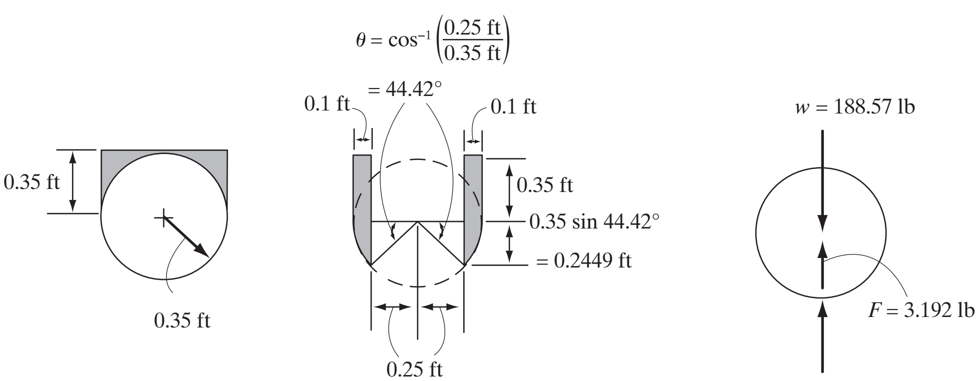
$$= \left(62.4\ \text{lb/ft}^3\right)\left(0.10372\ \text{ft}^3 - 0.05258\ \text{ft}^3\right)$$

$$= 3.192\ \text{lb}\uparrow$$

The weight of the cylinder is $W = \gamma_{st}\mathcal{V}_C = \left(490\ \text{lb/ft}^2\right)\left[\pi(0.35\ \text{ft})^2(1\ \text{ft})\right] = 188.57\ \text{lb}$. Considering the force equilibrium vertically by free-body diagram of the cylinder, Fig. *c*, we have

$+\uparrow\Sigma F_y = 0;\qquad N + 3.192\ \text{lb} - 188.57\ \text{lb} = 0$

$N = 185.38\ \text{lb} = 185\ \text{lb}$ **Ans.**

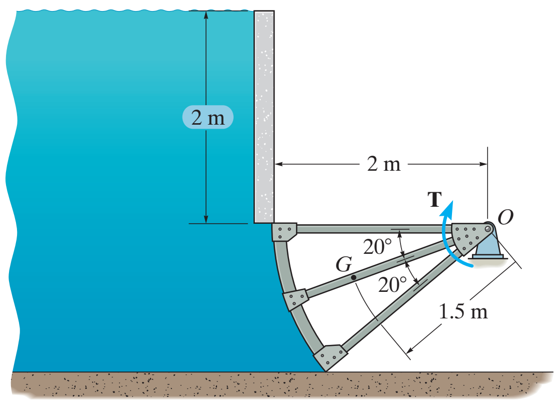


(a) (b) (c)

Ans:
185 lb

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–131. The sluice gate for a water channel is 1.5 m wide and in the closed position, as shown. Determine the magnitude of the resultant force of the water acting on the gate. Solve the problem by considering the fluid acting on the horizontal and vertical projections of the gate. Determine the smallest torque **T** that must be applied to open the gate if its weight is 30 kN and its center of gravity is at G.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the gate. Referring to Fig. *a*

$$w_1 = \rho_w g h_1 b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2\ \text{m})(1.5\ \text{m}) = 29.43\left(10^3\right)\ \text{N/m}$$

$$w_2 = \rho_w g h_2 b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2\ \text{m} + 2\text{m}\sin 40°)(1.5\ \text{m}) = 48.347\left(10^3\right)\ \text{N/m}$$

Then

$$(F_h)_1 = \left[29.43\left(10^3\right)\ \text{N/m}\right](2\sin 40°\text{m}) = 37.834\left(10^3\right)\ \text{N} = 37.834\ \text{kN}$$

$$(F_h)_2 = \frac{1}{2}\left[(48.347 - 29.43)\left(10^3\right)\ \text{N/m}\right](2\ \sin\ 40°\text{m}) = 12.160\ \left(10^3\right)\ \text{N} = 12.160\ \text{kN}$$

$$F_h = (F_h)_1 + (F_h)_2 = 37.834\left(10^3\right)\ \text{N} + 12.160\left(10^3\right)\ \text{N} = 49.994\left(10^3\right)\ \text{N} = 49.994\ \text{kN}$$

Also

$$\tilde{y}_1 = \frac{1}{2}(2\ \text{m}\sin 40°) = 0.6428\ \text{m} \quad \text{and} \quad \tilde{y}_2 = \frac{2}{3}(2\ \text{m}\sin 40°) = 0.8571\ \text{m}$$

The vertical component of the resultant force is equal to the weight of the imaginary column of water above the gate (shown shaded in Fig. *a*) but acts upward. The volume of this column of water is

$$\forall = \left[(2\ \text{m} - 2\ \text{m}\cos 40°)(2\ \text{m}) + \frac{1}{2}(2\ \text{m})^2\left(\frac{40°}{180°}\pi\ \text{rad}\right) - \frac{1}{2}(2\ \text{m}\cos 40°)(2\ \text{m}\sin 40°)\right](1.5\ \text{m})$$

$$= 2.0209\ \text{m}^3$$

$$F_v = \rho_w g \forall = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(2.0209\ \text{m}^3\right) = 19.825\left(10^3\right)\ \text{N} = 19.825\ \text{kN}$$

Referring to Fig *b* and *c*

$$\bar{r} = \frac{2}{3}\left(\frac{2\ \text{m}\sin 20°}{\frac{20}{180}\pi}\right) = 1.3064\ \text{m} \qquad \tilde{x}_2 = 1.3064\ \text{m}\cos 20° = 1.2276\ \text{m}$$

$$\tilde{x}_1 = 2\ \text{m}\cos 40° + \left(\frac{2\ \text{m} - 2\ \text{m}\cos 40°}{2}\right) = 1.7660\ \text{m}$$

$$\tilde{x}_3 = \frac{2}{3}(2\ \text{m}\cos 40°) = 1.0214\ \text{m}$$

Thus, F_v acts at

$$\bar{x} = \frac{(1.7660\ \text{m})(2\ \text{m} - 2\ \text{m}\cos 40°)(2\ \text{m}) + (1.2276\ \text{m})\left[\frac{1}{2}(2\ \text{m})^2\left(\frac{40}{180}\pi\ \text{rad}\right)\right] - (1.0214\ \text{m})\left[\frac{1}{2}(2\ \text{m}\cos 40°)(2\ \text{m}\sin 40°)\right]}{(2\ \text{m} - 2\ \text{m}\cos 40°)(2\ \text{m}) + \frac{1}{2}(2\ \text{m})^2\left(\frac{40}{180}\pi\ \text{rad}\right) - \frac{1}{2}(2\ \text{m}\cos 40°)(2\ \text{m}\sin 40°)}$$

$$= 1.7523\ \text{m}$$

2–131. Continued

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(49.994\text{ kN})^2 + (19.825\text{ kN})^2} = 53.78\text{ kN} = 53.8\text{ kN} \quad \textbf{Ans.}$$

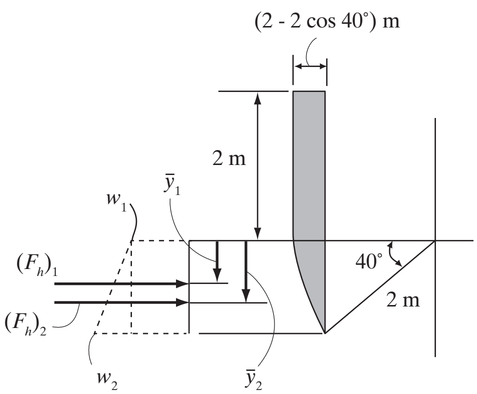
Referring to the FBD of the gate shown in Fig *d*,

$$\circlearrowleft + \Sigma M_0 = 0; \qquad (30\text{ kN})(1.5\cos 20°\text{m}) + (37.834\text{ kN})(0.6428\text{ m}) + (12.160\text{ kN})(0.8571\text{ m})$$

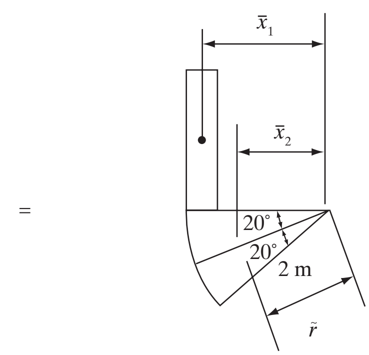
$$-(19.825\text{ kN})(1.7524\text{ m}) - T = 0$$

$$T = 42.29\text{ kN}\cdot\text{m} = 42.3\text{ kN}\cdot\text{m} \quad \textbf{Ans.}$$

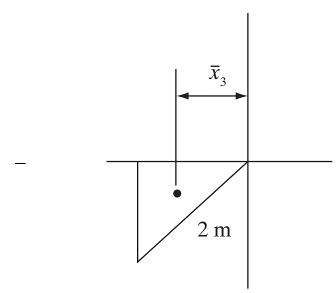
Note that the resultant force of the write acting on the give must act normal to its surface, and therefore it will pass through the pin at O. Therefore it produces moment about the pin.



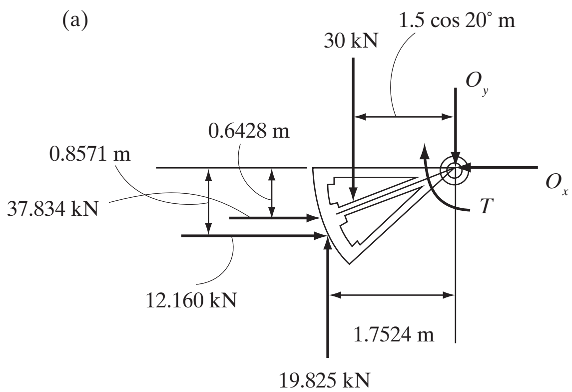
(a)



(b)



(c)

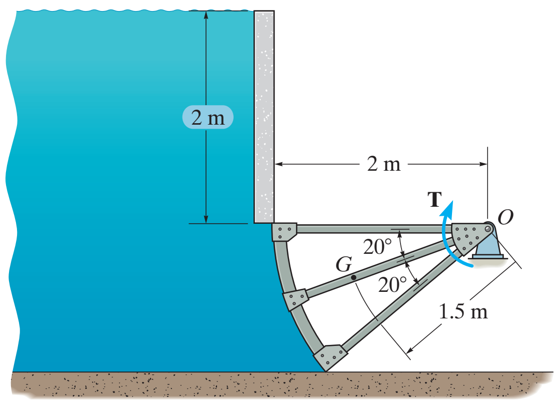


(d)

Ans:
$F_R = 53.8\text{ kN}$
$T = 42.3\text{ kN}\cdot\text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–132.** Solve the first part of Prob. 2-131 by the integration method using polar coordinates.

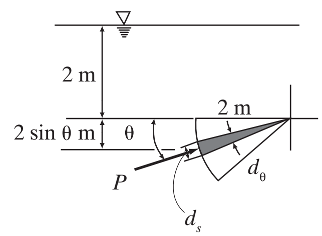


SOLUTION

Referring to Fig a, $h = (2 + 2\sin\theta)$ m. Thus, the pressure acting on the gate as a function of θ is

$$p = \rho_w g h = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2 + 2\sin\theta)\ \text{m} = [19620(1 + \sin\theta)]\ \text{N/m}^2$$

This pressure is acting on the element of area $dA = b\,ds = 1.5\,ds = 1.5\,(2\,d\theta) = 3\,d\theta$.



(a)

Thus,

$$dF = p\,d_A = 19620(1 + \sin\theta)(3\,d\theta). = 58.86(10^3)(1 + \sin\theta)\,d\theta$$

The horizontal and vertical components of $d\mathbf{F}$ are

$$(dF)_h = 58.86(10^3)(1 + \sin\theta)\cos\theta\,d\theta$$

$$= 58.86(10^3)(\cos\theta + \sin\theta\cos\theta)\,d\theta$$

$$(dF)_v = 58.86(10^3)(1 + \sin\theta)\sin\theta\,d\theta$$

$$= 58.86(10^3)(\sin\theta + \sin^2\theta)\,d\theta$$

Since $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 1 - 2\sin^2\theta$, then

$$(dF)_h = 58.86(10^3)\left(\cos\theta + \frac{1}{2}\sin 2\theta\right)d\theta$$

$$(dF)_v = 58.86(10^3)\left(\sin\theta + \frac{1}{2} - \frac{1}{2}\cos 2\theta\right)d\theta$$

The horizontal and vertical components of the resultant force are

$$F_h = \int (dF)_h = 58.86(10^3)\int_0^{\frac{2\pi}{9}}\left(\cos\theta + \frac{1}{2}\sin 2\theta\right)d\theta$$

$$= 58.86(10^3)\left[\sin\theta - \frac{1}{4}\cos 2\theta\right]\Bigg|_0^{\frac{2\pi}{9}}$$

$$= 49.994(10^3)\ \text{N} = 49.994\ \text{kN}$$

$$F_v = \int (dF)_v = 58.86(10^3)\int_0^{\frac{2\pi}{9}}\left(\sin\theta + \frac{1}{2} - \frac{1}{2}\cos 2\theta\right)d\theta$$

$$= 58.86(10^3)\left(-\cos\theta + \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)\Bigg|_0^{\frac{2\pi}{9}}$$

$$= 19.825(10^3)\ \text{N} = 19.825\ \text{kN}$$

Thus, the magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(49.994\ \text{kN})^2 + (19.825\ \text{kN})^2} = 53.78\ \text{kN} = 53.8\ \text{kN} \quad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–133. A flat-bottomed boat has vertical sides and a bottom surface area of $0.75\ \text{m}^2$. It floats in water such that its draft (depth below the surface) is 0.3 m. Determine the mass of the boat. What is the draft when a 50-kg man stands in the center of the boat?

SOLUTION

Equilibrium requires that the weight of the empty boat is equal to the buoyant force.

$$W_b = F_b = \rho_w g \forall Disp = \left(1000\ \text{kg/m}^3\right)\left(9.81\text{m/s}^2\right)\left(0.75\ \text{m}^2\right)(0.3\ \text{m})$$
$$= 2207.25\ \text{N}$$

Thus, the mass of the boat is given by

$$m_b = \frac{W_b}{g} = \frac{2207.25\ \text{N}}{9.81\ \text{m/s}^2} = 225\ \text{kg} \qquad \textbf{Ans.}$$

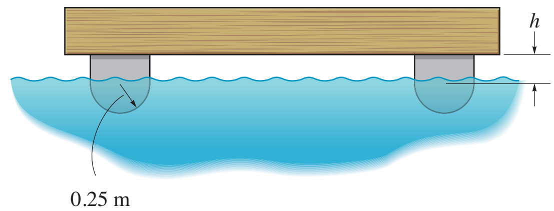
When the man steps into the boat, the total mass is $m_{b+m} = 225\ \text{kg} + 50\ \text{kg} = 275\ \text{kg}$. Then $W_{b+m} = m_{b+m}g = 275\ \text{kg}\left(9.81\ \text{m/s}^2\right) = 2697.75\ \text{N}$. Under this condition, the boat will sink further to create a greater buoyancy force to balance the additional weight. Thus,

$$W_{b+m} = \rho_w g \forall' Disp$$
$$2697.75\ \text{N} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(0.75\ \text{m}^2\right)(h)$$
$$h = 0.367\ \text{m} \qquad \textbf{Ans.}$$

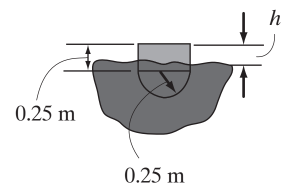
Ans:
$m_b = 225\ \text{kg}$
$h = 0.367\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–134. The raft consists of a uniform platform having a mass of 2 Mg and four floats, each having a mass of 120 kg and a length of 4 m. Determine the height h at which the platform floats from the water surface. Take $\rho_w = 1\text{Mg/m}^3$.



SOLUTION



(a)

Each float must support a weight of

$$W = \left[\frac{1}{4}(2000\ \text{kg}) + 120\ \text{kg}\right]9.81\ \text{m/s}^2 = 6082.2\ \text{N}$$

For equilibrium, the buoyant force on each float is required to be

$$+\uparrow\Sigma F_y = 0; \qquad F_b - 6082.2\ \text{N} = 0 \qquad F_b = 6082.2\ \text{N}$$

Therefore, the volume of water that must be displaced to generate this force is

$$F_b = \gamma \mathcal{V}; \qquad 6082.2\ \text{N} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\mathcal{V}$$

$$\mathcal{V} = 0.620\ \text{m}^3$$

Since the semicircular segment of a float has a volume of $\frac{1}{2}(\pi)(0.25\ \text{m})^2(4\ \text{m}) = 0.3927\ \text{m}^3 < 0.620\ \text{m}^3$, then it must be fully submerged to develop F_b. As shown in Fig. a, we require

$$0.620\ \text{m}^3 = \frac{1}{2}(\pi)(0.25\ \text{m})^2(4\ \text{m}) + (0.25\ \text{m} - h)(0.5\ \text{m})(4\ \text{m})$$

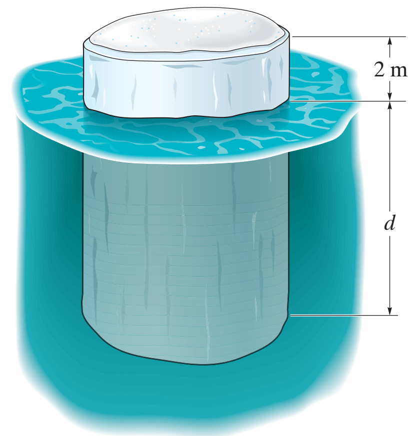
Thus,

$$h = 0.136\ \text{m} = 136\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
136 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–135. Consider an iceberg to be in the form of a cylinder of arbitrary diameter and floating in the ocean as shown. If the cylinder extends 2 m above the ocean's surface, determine the depth of the cylinder below the surface. The density of ocean water is $\rho_w = 1024\ \text{kg/m}^3$, and the density of the ice is $\rho_i = 935\ \text{kg/m}^3$.



SOLUTION

The weight of the iceberg is

$$W = \rho_i g \mathcal{V}_i = \left(935\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[\pi r^2(2 + d)\right]$$

The buoyant force is

$$F_b = \rho_{sw} g \mathcal{V}_{\text{sub}} = \left(1024\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(\pi r^2 d\right)$$

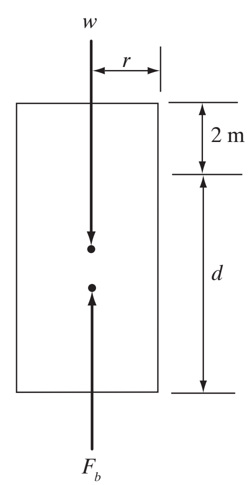
Referring to the FBD of the iceberg, Fig. *a*, equilibrium requires,

$$+\uparrow \Sigma F_y = 0; \qquad F_b - w = 0$$

$$\left(1024\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(\pi r^2 d\right) - \left(935\ \text{kg/m}^3\right)\left(9.81\text{m/s}^2\right)\left[\pi r^2(2 + d)\right] = 0$$

$$1024\, d = 935(2 + d)$$

$$d = 21.01\ \text{m} = 21.0\ \text{m} \qquad \textbf{Ans.}$$

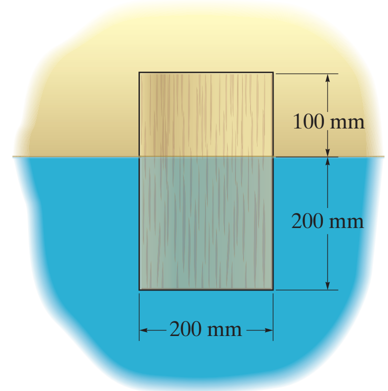


(a)

Ans:
21.0 m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–136.** The cylinder floats in the water and oil to the level shown. Determine the weight of the cylinder. $\rho_o = 910\ \text{kg/m}^3$.



SOLUTION

The buoyant force fuel to the submerging in oil and water are

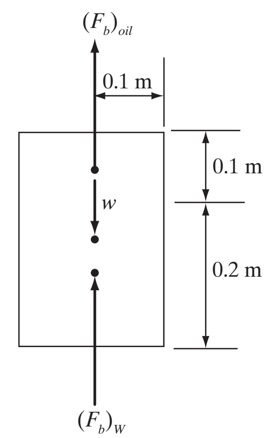
$$(F_b)_{oil} = \rho_{oil} g (V_{\text{sub}})_{oil} = (910\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\pi(0.1\text{m})^2(0.1\ \text{m})\right] = 8.9271\,\pi\ \text{N}$$

$$(F_b)_N = \rho_w g (V_{\text{sub}})_w = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\pi(0.1\ \text{m})^2(0.2\ \text{m})\right] = 19.62\,\pi\ \text{N}$$

Referring to the FBD of the cylinder, Fig. *a*, equilibrium requires,

$$+\uparrow\Sigma F_y = 0; \qquad 8.9271\,\pi\ \text{N} + 19.62\,\pi\ \text{N} - W = 0$$

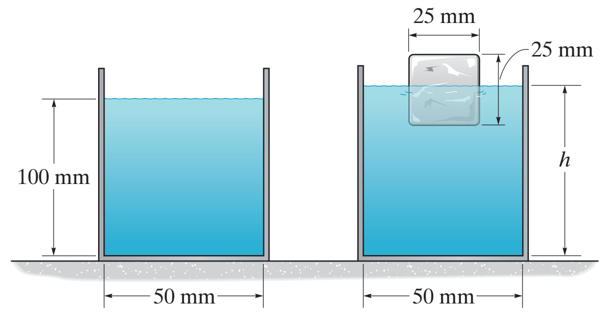
$$W = 89.68\ \text{N} = 89.7\ \text{N} \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–137. A glass having a diameter of 50 mm is filled with water to the level shown. If an ice cube with 25 mm sides is placed into the glass, determine the new height h of the water surface. Take $\rho_w = 1000\ \text{kg/m}^3$ and $\rho_{ice} = 920\ \text{kg/m}^3$. What will the water level h be when the ice cube completely melts?



SOLUTION

Since the ice floats, the buoyant force is equal to the weight of the ice cube which is

$$F_b = W_i = \rho_i \mathcal{V}_i g = (920\ \text{kg/m}^3)(0.025\ \text{m})^3(9.81\ \text{m/s}^2) = 0.1410\ \text{N}$$

This buoyant force is also equal to the weight of the water displaced by the submerged ice cube with at a depth h_s.

$$F_b = \rho_w g \mathcal{V}_s; \qquad 0.1410\ \text{N} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[(0.025\ \text{m})^2 h_s\right]$$

$$h_s = 0.023\ \text{m}$$

Referring to Fig. *a*,

$$\mathcal{V}_1 = \mathcal{V}_2 - \mathcal{V}_3$$

$$\left[\pi(0.025\ \text{m})^2\right](0.1\ \text{m}) = \left[\pi(0.025\text{m})^2\right]h - (0.025\text{m})^2(0.023\ \text{m})$$

$$h = 0.1073\ \text{m} = 107\ \text{mm} \qquad \textbf{Ans.}$$

The mass of ice cube is

$$M_i = \rho_i \mathcal{V}_i = (920\ \text{kg/m}^3)(0.025\ \text{m})^3 = 0.014375\ \text{kg}.$$

Thus, the nice in water level due to the additional water of the melting ice cubs: can be determined from

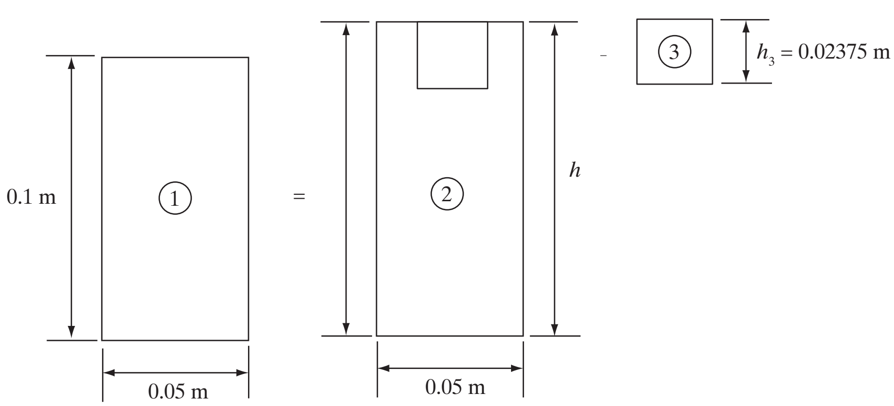
$$M_i = \rho_w \mathcal{V}_w; \qquad 0.014375\ \text{kg} = (1000\ \text{kg/m}^3)\left[\pi(0.025\ \text{m})^2 \Delta h\right]$$

$$\Delta h = 0.007321$$

Thus,

$$h' = 0.1\ \text{m} + 0.007321\ \text{m} = 107\ \text{mm} \qquad \textbf{Ans.}$$

Note The water level h remains unchanged as the cube melts.

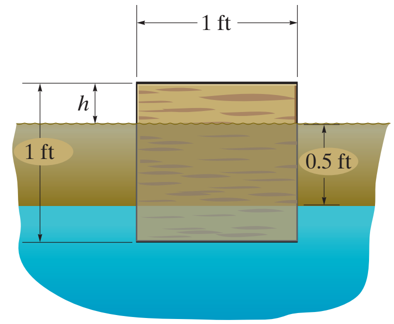


(a)

Ans:
107 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–138. The wood block has a specific weight of $45\ \text{lb/ft}^3$. Determine the depth h at which it floats in the oil–water system. The block is 1 ft wide. Take $\rho_o = 1.75\ \text{slug/ft}^3$.



SOLUTION

The weight of the block is

$$W = \gamma_b \forall_b = \left(45\ \text{lb/ft}^3\right)\left[(1\text{ft})^3\right] = 45\ \text{lb}$$

Assume that block floats in both oil and water, Fig, *a*. Then, the volume of the water and oil being displaced is

$$(\forall_{oil})_{Disp} = 0.5\ \text{ft}(1\ \text{ft})(1\ \text{ft}) = 0.5\ \text{ft}^3$$

$$(\forall_w)_{Disp} = (0.5\ \text{ft} - h)(1\ \text{ft})(1\ \text{ft}) = (0.5 - h)\ \text{ft}^3$$

Thus, the buoyancy forces on the block due to the oil and water are

$$(F_b)_{oil} = \gamma_{oil}(\forall_{oil})_{Disp} = \left(1.75\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)\left(0.5\ \text{ft}^2\right) = 28.175\ \text{lb}$$

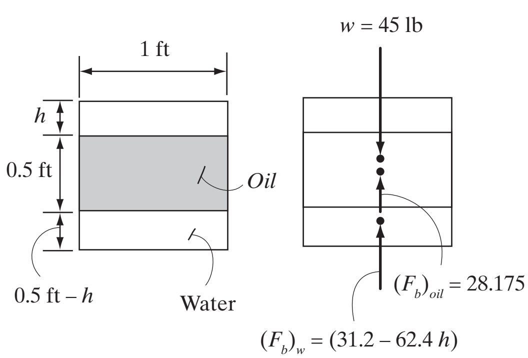
$$(F_b)_w = \gamma_w(\forall_w)_{Disp} = \left(62.4\ \text{slug/ft}^3\right)(0.5 - h)\ \text{ft}^3 = (31.2 - 62.4h)\ \text{lb}$$

Considering the free-body diagram in Fig. *b*,

$+\uparrow \Sigma F_y = 0;\qquad (31.2 - 62.4h)\ \text{lb} + 28.175\ \text{lb} - 45\ \text{lb} = 0$

$h = 0.2304\ \text{ft} = 0.230\ \text{ft}$ **Ans.**

Since $h < 0.5$ ft, the assumption was correct and the result is valid.

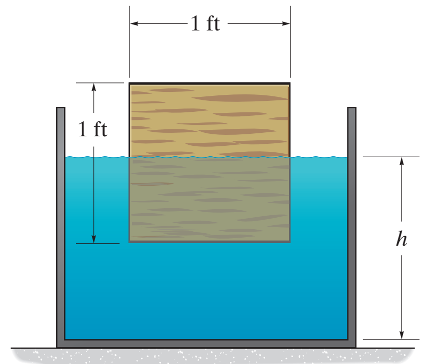


(a) (b)

Ans:
0.230 ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–139. Water in the container is originally at a height of $h = 3$ ft. If a block having a specific weight of 50 lb/ft^3 is placed in the water, determine the new level h of the water. The base of the block is 1 ft square, and the base of the container is 2 ft square.



SOLUTION

The weight of the block is

$$W_b = \gamma_b V_b = (50 \text{ lb/ft}^3)\left[(1\text{ft})^3\right] = 50 \text{ lb}$$

Equilibrium requires that the buoyancy force equal the weight of the block, so that $F_b = 50$ lb. Thus, the displaced volume is

$$F_b = \gamma_w V_{Disp} \qquad 50 \text{ lb} = (62.4 \text{ lb/ft}^3) V_{Disp}$$

$$V_{Disp} = 0.8013 \text{ ft}^3$$

The volume of the water is

$$V_w = 2 \text{ ft}(2 \text{ ft})(3 \text{ ft}) = 12 \text{ ft}^3$$

When the level of the water in the container has a height of h,

$$V_w = V' - V_{Disp}$$

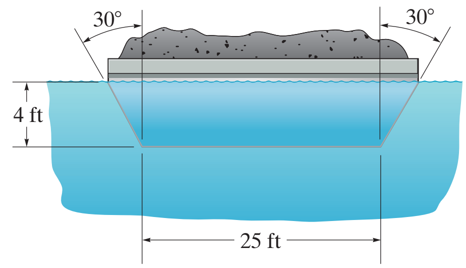
$$12 \text{ ft}^3 = 4\,h \text{ ft}^3 - 0.8013 \text{ ft}^3$$

$$h = 3.20 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
3.20 ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

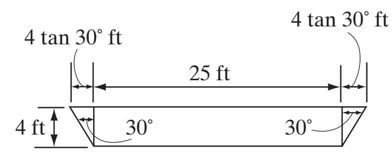
***2–140.** The cross section of the front of a barge is shown. Determine the buoyant force acting per foot length of the hull when the water level is at the indicated depth.



SOLUTION

Referring to the geometry shown in Fig. *a*, the volume of the water displaced per foot length of the hull is

$$\forall_{Disp} = 25\ \text{ft}(4\ \text{ft}) + 2\left[\frac{1}{2}(4\ \tan\ 30°\ \text{ft})(4\ \text{ft})\right] = 109.24\ \text{ft}^3/\text{ft}$$



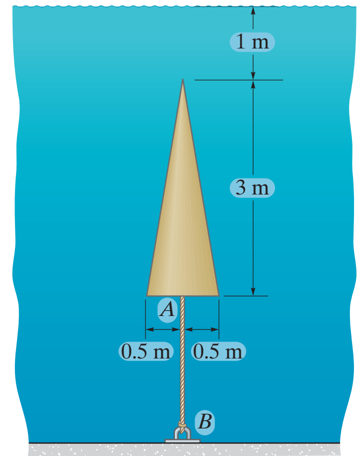
(a)

Thus, the buoyancy force acting per foot length of the hull is

$$F_b = \gamma_w \forall_b = (62.4\ \text{lb/ft}^3)(109.24\ \text{ft}^3/\text{ft})$$
$$= 6816\ \text{lb/ft} = 6.82\ \text{kip/ft} \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–141. The cone is made of wood having a density of $\rho_{wood} = 650\ \text{kg/m}^3$. Determine the tension in rope AB if the cone is submerged in the water at the depth shown. Will this force increase, decrease, or remain the same if the cord is shortened? Why? *Hint*: The volume of a cone is $\forall = \frac{1}{3}\pi r^2 h$.



SOLUTION

The weight of the wooden cone is

$$W = \rho_{wood}\, g \forall_c = \left(650\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[\frac{1}{3}\pi(0.5\ \text{m})^2(3\ \text{m})\right] = 1594.125\pi\ \text{N}$$

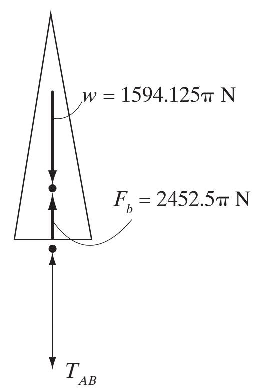
The volume of water that is displaced is the same as the volume of the cone. Thus, the buoyancy force is

$$F_b = \rho_w g \forall_c = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[\frac{1}{3}\pi(0.5\ \text{m})^2(3\ \text{m})\right] = 2452.5\pi\ \text{N}$$

Considering the free-body diagram of the cone in Fig. *a*,

$+\uparrow\Sigma F_y = 0;\qquad 2452.5\pi\ \text{N} - 1594.125\pi\ \text{N} - T_{AB} = 0$

$T_{AB} = 2696.66\ \text{N} = 2.70\ \text{kN}$ **Ans.**



(a)

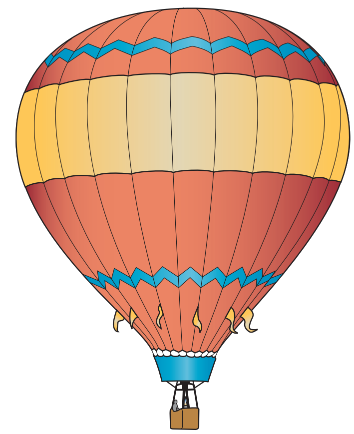
The tension in rope AB *remains the same* since the buoyancy force does not change. For a fully submerged body, the buoyancy force is independent of the depth to which the body is submerged.

Remains the same **Ans.**

Ans:
$T_{AB} = 2.70\ \text{kN}$
Remains the same

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–142. The hot-air balloon contains air having a temperature of 180°F, while the surrounding air having a temperature of 60°F. Determine the maximum weight of the load the balloon can lift if the volume of air it contains is $120(10^3)\ \text{ft}^3$. The empty weight of the balloon is 200 lb.



SOLUTION

From the Appendix, the densities of the air inside the balloon where $T = 180°\ \text{F}$ and outside the balloon where $T = 60°\ \text{F}$, are

$$\rho_a\big|_{T=60°\ \text{F}} = 0.00237\ \text{slug/ft}^3$$

$$\rho_a\big|_{T=180°\ \text{F}} = 0.00193\ \text{slug/ft}^3$$

Thus, the weight of the air inside the balloon is

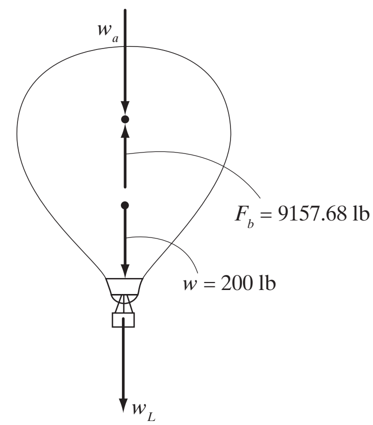
$$W_a\big|_{T=180°\ \text{F}} = \rho_a\big|_{T=180°\text{F}}g\mathcal{V} = \left(0.00193\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)\left[120\left(10^3\right)\text{ft}^3\right]$$

$$= 7457.52\ \text{lb}$$

The buoyancy force is equal to the weight of the displaced air outside of the balloon. This gives

$$F_b = \rho_a\big|_{T=60°\text{F}}g\mathcal{V} = \left(0.00237\ \text{slug/ft}^3\right)\left(32.2\text{ft/s}^2\right)\left[120\left(10^3\right)\text{ft}^3\right]$$

$$= 9157.68\ \text{lb}$$



(a)

Considering the free-body diagram of the balloon in Fig. *a*,

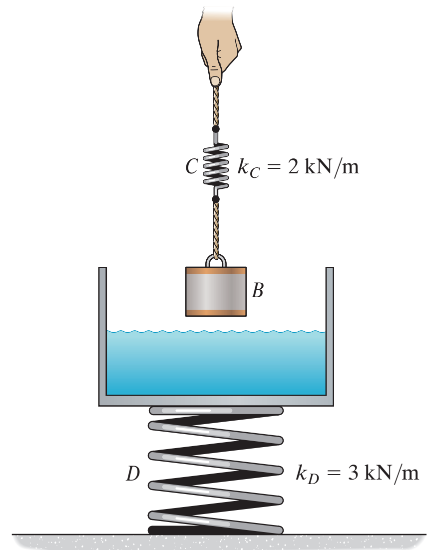
$+\uparrow\Sigma F_y = 0;\qquad 9157.68\ \text{lb} - 7457.52\ \text{lb} - 200\ \text{lb} - W_L = 0$

$W_L = 1500.16\ \text{lb} = 1.50\ \text{kip}$ **Ans.**

Ans:
1.50 kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–143. The container with water in it has a mass of 30 kg. Block B has a density of 8500 kg/m^3 and a mass of 15 kg. If springs C and D have an unstretched length of 200 mm and 300 mm, respectively, determine the length of each spring when the block is submerged in the water.



SOLUTION

The volume of block B is,

$$\forall_B = \frac{m_B}{\rho} = \frac{15\ \text{kg}}{8500\ \text{kg/m}^3} = 1.7647\left(10^{-3}\right)\text{m}^3$$

Thus, the bouyant force is

$$F_b = \rho_w g \forall_{\text{sub}} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(1.7647\left(10^{-3}\right)\text{m}^3\right) = 17.31\ \text{N}$$

Referring to the *FBD* of block B, Fig. *a*,

$$+\uparrow\Sigma F_y = 0; \qquad (F_{sp})_c + 17.31\ \text{N} - \left[15(9.81)\ \text{N}\right] = 0 \qquad (F_{sp})_c = 129.84\ \text{N}$$

Referring to the FBD of the container, Fig. *b*,

$$+\uparrow\Sigma F_y = 0; \qquad (F_{sp})_D - 17.31\ \text{N} - \left[30(9.81)\ \text{N}\right] = 0 \qquad (F_{sp})_D = 311.61\ \text{N}$$

Thus, the deformations of springs C dand D are

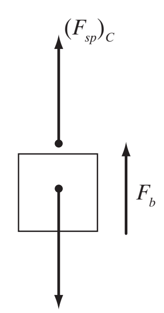
$$\delta_C = \frac{(F_{sp})_C}{k_C} = \frac{129.84\ \text{N}}{2000\ \text{N/m}} = 0.06492\ \text{m} = 64.92\ \text{mm}$$

$$\delta_D = \frac{(F_{sp})_D}{k_D} = \frac{311.61\ \text{N}}{3000\ \text{N/m}} = 0.1039\ \text{m} = 103.87\ \text{mm}$$

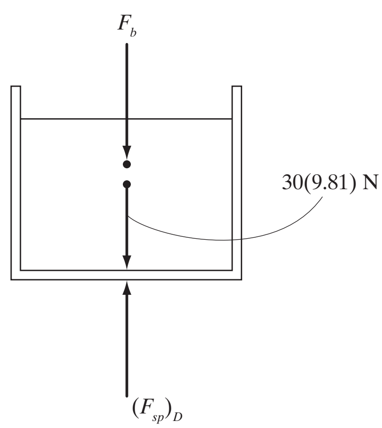
Thus

$$l_C = (l_o)_C + \delta_c = 200\ \text{mm} + 64.92\ \text{mm} = 264.92\ \text{mm} = 265\ \text{mm} \qquad \textbf{Ans.}$$

$$l_D = (l_o)_D + \delta_D = 300\ \text{mm} - 103.87\ \text{mm} = 196.13\ \text{mm} = 196\ \text{mm} \qquad \textbf{Ans.}$$



(a)

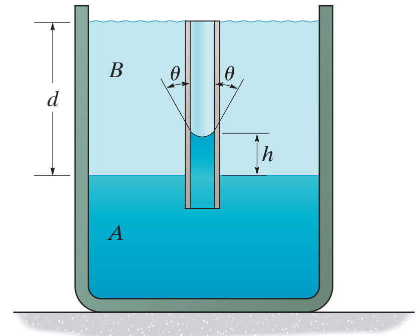


(b)

Ans:
265 mm
196 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–144.** An open-ended tube having an inner radius r is placed in a wetting liquid A having a density ρ_A. The top of the tube is just below the surface of a surrounding liquid B, which has a density ρ_B, where $\rho_A > \rho_B$. If the surface tension σ causes liquid A to make a wetting angle θ with the tube wall as shown, determine the rise h of liquid A within the tube. Show that the result is independent of the depth d of liquid B.



SOLUTION

The volume of the column of liquid A in the rise is $\forall = \pi r^2 h$. Thus, its weight is

$$W = \gamma_A \forall = \rho_A g\left(\pi r^2 h\right) = \pi g \rho_A r^2 h$$

The force on the top and bottom of the column is $p_t A = \rho_B(\overline{d} - h)\left(\pi r^2\right)$ and $\rho_B(d)\left(\pi r^2\right)$. The difference in these forces is the bouyont force.

$$F_B = \gamma_B \forall = \rho_B g\left(\pi r^2 h\right) = \pi g \rho_B r^2 h$$

The force equilibrium along the vertical, Fig. a, requires.

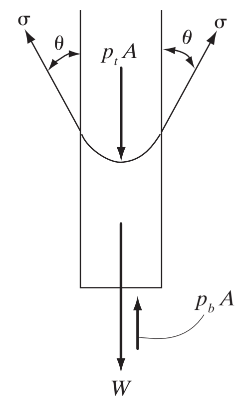
$+\uparrow \Sigma F_y = 0;\qquad \sigma(2\pi r)\cos\theta + \pi g \rho_B r^2 h - \pi g \rho_A r^2 h = 0$

$$2\pi r\sigma\cos\theta + \pi g r^2 h(\rho_B - \rho_A) = 0$$

$$2\pi r\sigma\cos\theta = (\rho_A - \rho_B)\pi g r^2 h$$

$$h = \frac{2\sigma\cos\theta}{gr(\rho_A - \rho_B)}\qquad \textbf{Ans.}$$

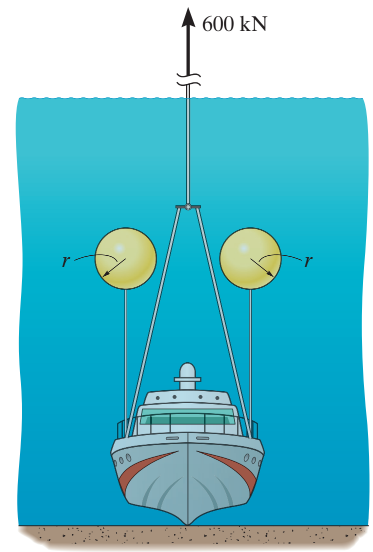
Note that the result is independent of d.



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–145. A boat having a mass of 80 Mg rests on the bottom of the lake and displaces $10.25\ \text{m}^3$ of water. Since the lifting capacity of the crane is only 600 kN, two balloons are attached to the sides of the boat and filled with air. Determine the smallest radius r of each spherical balloon that is needed to lift the boat. What is the mass of air in each balloon if the water temperature is 12°C? The balloons are at an average depth of 20 m. Neglect the mass of air and of the balloon for the calculation required for the lift. The volume of a sphere is $V = \frac{4}{3}\pi r^3$.



SOLUTION

The bouyant force acting on the boat and a balloon are

$$(F_b)_B = \rho_w g (V_B)_{\text{sub}} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(10.25\ \text{m}^3) = 100.55(10^3)\ \text{N}$$
$$= 100.55\ \text{kN}$$

$$(F_b)_b = \rho_w g (V_b)_{\text{sub}} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{4}{3}\pi r^3\right] = 13.08\pi r^3 (10^3)\ \text{N}$$
$$= 13.08\pi r^3\ \text{kN}$$

Referring to the FBD of the boat, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad 2T + 100.55\ \text{kN} + 600\ \text{kN} - [80(9.81)\ \text{kN}] = 0$$
$$T = 42.124\ \text{kN}$$

Referring to the FBD of the balloon Fig. *b*

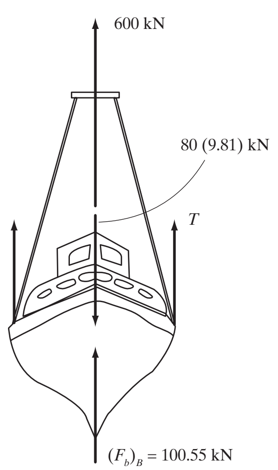
$$+\uparrow \Sigma F_y = 0; \qquad 13.08\pi r^3 - 42.125\ \text{kN} = 0$$
$$r = 1.008\ \text{m} = 1.01\ \text{m} \qquad \textbf{Ans.}$$

Here, $p = p_{\text{atm}} + \rho_w g h = 101(10^3)\ \text{Pa} + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(20\ \text{m}) = 297.2(10^3)\ \text{Pa}$ and $T = 12°\ \text{C} + 273 = 285\ \text{K}$. From Appendix A, $R = 286.9\ \text{J/kg}\cdot\text{K}$. Applying the ideal gas law,

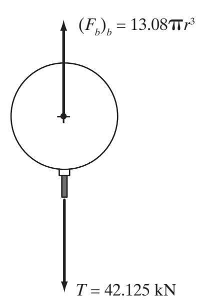
$$p = \rho R T; \qquad \rho = \frac{p}{RT} = \frac{297.2(10^3)\ \text{N/m}^2}{(286.9\ \text{J/kg}\cdot\ \text{K})(285\ \text{K})} = 3.6347\ \text{kg/m}^3$$

Thus,

$$m = \rho V = (3.6347\ \text{kg/m}^3)\left[\frac{4}{3}\pi(1.008\ \text{m})^3\right] = 15.61\ \text{kg} = 15.6\ \text{kg} \qquad \textbf{Ans.}$$



(a)

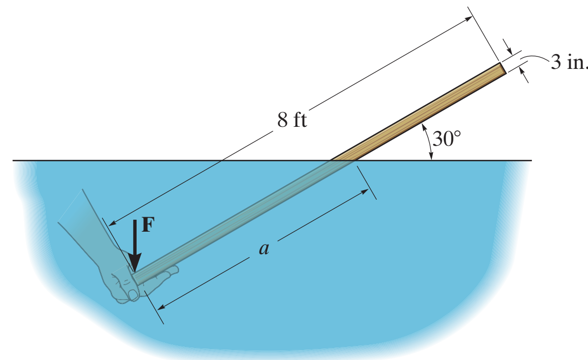


(b)

Ans:
$r = 1.01$ m
$m = 15.6$ kg

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–146. The uniform 8-ft board is pushed down into the water so it makes an angle of $\theta = 30°$ with the water surface. If the cross section of the board measures 3 in. by 9 in., and its specific weight is $\gamma_b = 30\ \text{lb/ft}^3$, determine the length a that will be submerged and the vertical force **F** needed to hold its end in this position.



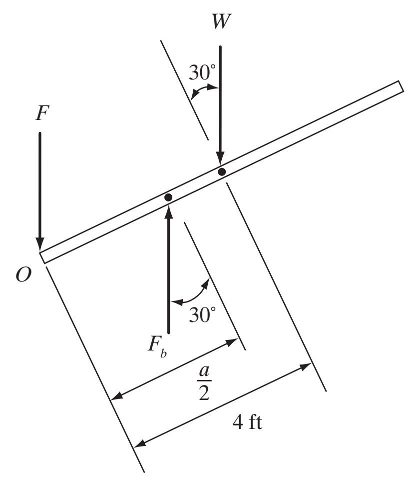
SOLUTION

The weight of the board is

$$W = \gamma_b \forall_b = \left(30\ \text{lb/ft}^3\right)\left[\left(\frac{3}{12}\ \text{ft}\right)\left(\frac{9}{12}\ \text{ft}\right)(8\ \text{ft})\right] = 45\ \text{lb}$$

$$F_b = \gamma_w \forall_{\text{sub}} = \left(62.4\ \text{lb/ft}^3\right)\left[\left(\frac{3}{12}\ \text{ft}\right)\left(\frac{9}{12}\ \text{ft}\right)a\right] = 11.7a\ \text{lb}$$

Referring to the *FBD* of the board, Fig. *a*, equilibrium requires,



$\zeta + \Sigma M_o = 0;\qquad 11.7\,a\,(\cos 30°)\left(\frac{a}{2}\right) - (45\cos 30°\ \text{lb})(4\ \text{ft}) = 0$

$$a = 5.547\ \text{ft} = 5.55\ \text{ft}$$ **Ans.**

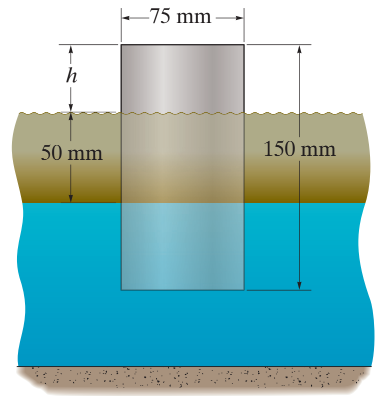
$+\uparrow \Sigma F_y = 0;\qquad 11.7(5.547) - 45\ \text{lb} - F = 0$

$$F = 19.90\ \text{lb} = 19.9\ \text{lb}$$ **Ans.**

Ans:
$a = 5.55$ ft
$F = 19.9$ lb

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–147. The cylinder has a diameter of 75 mm and a mass of 600 g. If it is placed in the tank, which contains oil and water, determine the height h above the surface of the oil at which it will float if maintained in the vertical position. Take $\rho_0 = 980\ \text{kg/m}^3$.



SOLUTION

Since the cylinder floats, the buoyant force is equal to the weight of the cylinder.

$$F_b = (0.6\ \text{kg})(9.81\ \text{m/s}^2) = 5.886\ \text{N}$$

Assuming that the cylinder is submerged below the oil layer, then, the buoyant force produced by the oil layer is

$$(F_b)_{oil} = \rho_{oil}\, g(V_s)_{oil} = (980\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\pi(0.0375\ \text{m})^2(0.05\ \text{m})\right]$$

$$= 2.124\ \text{N} < F_b \qquad \textbf{(O.K!)}$$

The buoyant force produced by the water layer is

$$(F_b)_w = \rho_w g(V_s)_w = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\pi(0.0375\ \text{m})^2(0.15\ \text{m} - 0.05\ \text{m} - h)\right]$$

$$= 43.339\,(0.1 - h)$$

We require

$$F_b = (F_b)_{oil} + (F_b)_w$$

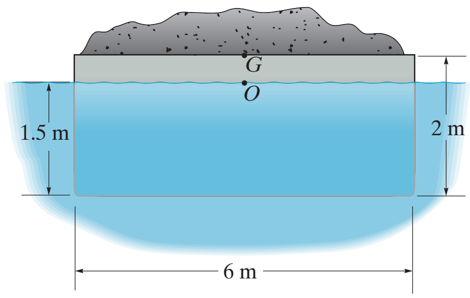
$$5.886\ \text{N} = 2.124\ \text{N} + 43.339(0.1 - h)$$

$$h = 13.2\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$h = 13.2$ mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–148.** When loaded with gravel, the barge floats in water at the depth shown. If its center of gravity is located at G, determine whether the barge will restore itself when a wave causes it to roll slightly at 9°.



SOLUTION

When the barge tips 9°, the submerged portion is trapezoidal in shape, as shown in Fig. *a*. The new center of buoyancy, $C_b{}'$, is located at the centroid of this area. Then

$$x = \frac{(0)(6)(1.0248) + (1)\left[\frac{1}{2}(6)(0.9503)\right]}{(6)(1.0248) + \frac{1}{2}(6)(0.9503)} = 0.3168\ \text{m}$$

$$y = \frac{\frac{1}{2}(1.0248)(6)(1.0248) + \left[1.0248 + \frac{1}{3}(0.9503)\right]\left[\frac{1}{2}(6)(0.9503)\right]}{(6)(1.0248) + \frac{1}{2}(6)(0.9503)} = 0.7751\ \text{m}$$

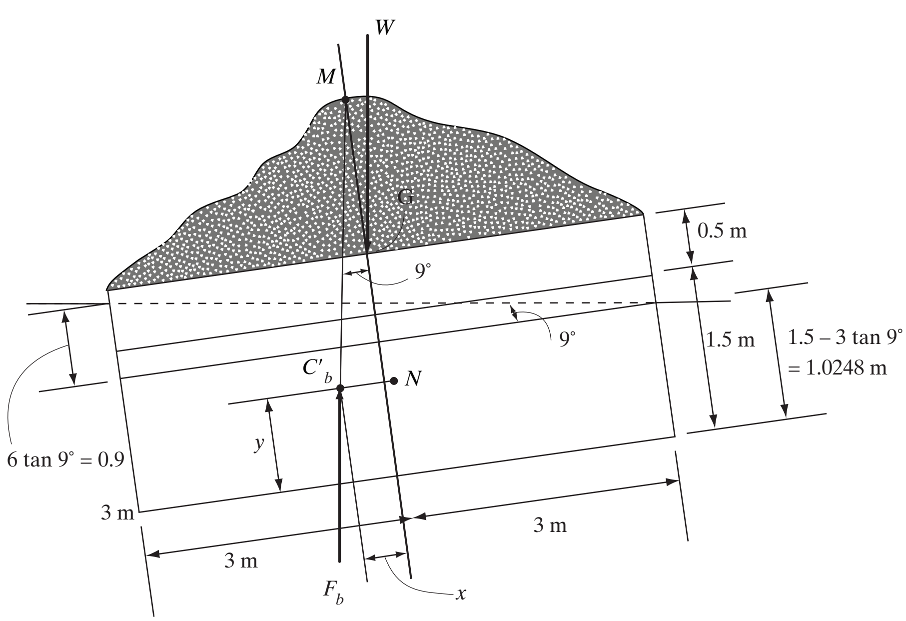
The intersection point M of the line of action of $\mathbf{F}_b$ and the centerline of the barge is the metacenter, Fig. *a*. From the geometry of triangle $MNC_b{}'$ we have

$$MN = \frac{x}{\tan 9°} = \frac{0.3168}{\tan 9°} = 2\ \text{m}$$

Also,

$$GN = 2 - y = 2 - 0.7751 = 1.2249\ \text{m}$$

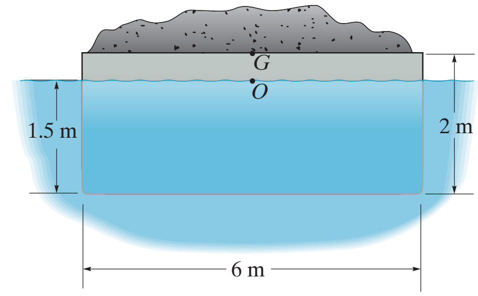
Since $MN > GN$, point M is above G. Therefore, **the barge will restore itself.**



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–149. When loaded with gravel, the barge floats in water at the depth shown. If its center of gravity is located at G determine whether the barge will restore itself when a wave causes it to tip slightly.



SOLUTION

The barge is tilted counterclockwise slightly and the new center of buoyancy $C_b{}'$ is located to the left of the old one. The metacenter M is at the intersection point of the center line of the barge and the line of action of $\mathbf{F}_b$, Fig. *a*. The location of $C_b{}'$ can be obtained by referring to Fig. *b*.

$$\bar{x} = \frac{(1\,\text{m})\left[\frac{1}{2}(6\,\text{m})(6\tan\phi\,\text{m})\right]}{(1.5\,\text{m})(6\,\text{m})} = 2\tan\phi\ \text{m}$$

Then

$$\delta = \bar{x}\cos\phi = 2\,\text{m}\tan\phi\ \cos\phi = (2\,\text{m})\left(\frac{\sin\phi}{\cos\phi}\right)(\cos\phi) = (2\sin\phi)\ \text{m}$$

Since ϕ is very small $\sin\phi = \phi$, hence

$$\delta = 2\phi\ \text{m} \qquad \textbf{(1)}$$

From the geometry shown in Fig. *a*

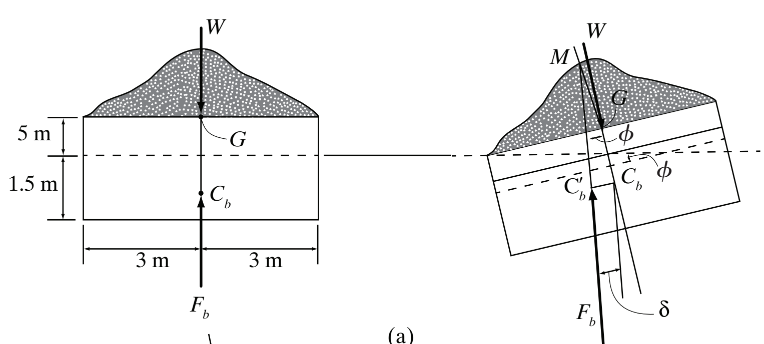
$$\delta = MC_b\ \sin\phi = MC_b\phi \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2)

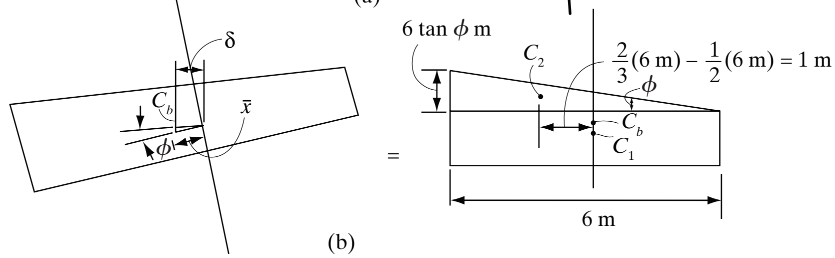
$$2\phi = MC_b\phi$$

$$MC_b = 2\,\text{m}$$

Here, $GC_b = 2\,\text{m} - 0.75\,\text{m} = 1.25\,\text{m}$. Since $MC_b > GC_b$, the barge is in stable equilibrium. Thus, it will restore itself if tilted slightly.



(a)

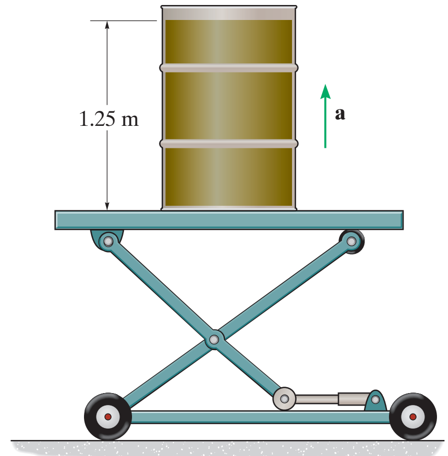


(b)

Ans:
It will restore itself.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–150. The barrel of oil rests on the surface of the scissors lift. Determine the maximum pressure developed in the oil if the lift is moving upward with (a) a constant velocity of 4 m/s, and (b) a constant acceleration of 2 m/s². Take $\rho_o = 900\ \text{kg/m}^3$. The top of the barrel is open to the atmosphere.



SOLUTION

a) Equilibrium

$$p = \rho_o g h = 900\ \text{kg/m}^3(9.81\ \text{m/s}^2)(1.25\ \text{m})$$

$$= 11.0\ \text{kPa} \qquad \textbf{Ans.}$$

b) $p = \rho_o g h\left(1 + \dfrac{a_C}{g}\right)$

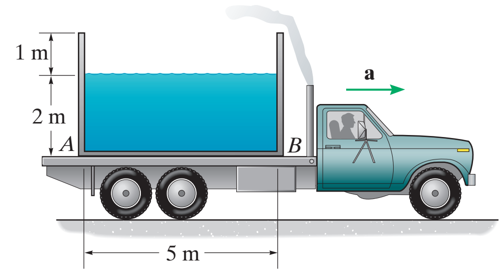
$$p = 900\ \text{kg/m}^3(9.81\ \text{m/s}^2)(1.25\ \text{m})\left(1 + \frac{2\ \text{m/s}^2}{9.81\ \text{m/s}^2}\right)$$

$$p = 13.3\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
a) 11.0 kPa
b) 13.3 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–151. The truck carries an open container of water as shown. If it has a constant acceleration 2 m/s^2, determine the angle of inclination of the surface of the water and the pressure at the bottom corners A and B.



SOLUTION

The free surface of the water in the accelerated tank is shown in Fig. *a*.

$$\tan\theta = \frac{a_c}{g} = \frac{2\ \text{m/s}^2}{9.81\ \text{m/s}^2}$$

$$\theta = 11.52^\circ = 11.5^\circ \qquad \textbf{Ans.}$$

From the geometry in Fig. *a*.

$$\Delta h_A = \Delta h_B = (2.5\ \text{m})\ \tan\ 11.52^\circ$$

$$= 0.5097\ \text{m}$$

Thus,

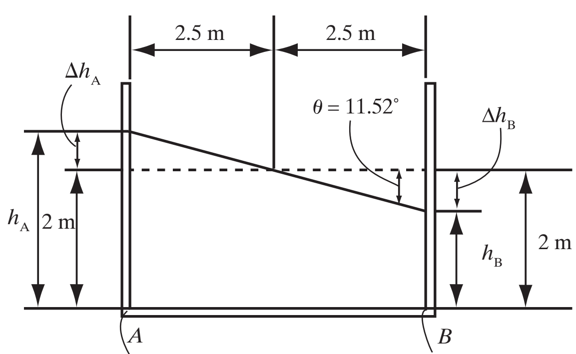
$$p_A = \rho_w g h_A = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2\ \text{m} + 0.5095\ \text{m})$$

$$= 24.6\ \text{kPa}$$

Ans.

$$p_B = \rho_w g h_B = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2\ \text{m} - 0.5095\ \text{m})$$

$$= 14.6\ \text{kPa} \qquad \textbf{Ans.}$$

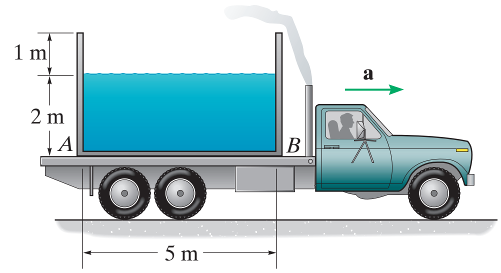


(a)

Ans:
$\theta = 11.5^\circ$
$p_A = 24.6\ \text{kPa}$
$p_B = 14.6\ \text{kPa}$

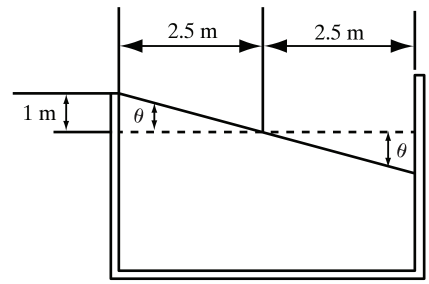
Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–152.** The truck carries an open container of water as shown. Determine the maximum constant acceleration it can have without causing the water to spill out of the container.



SOLUTION

When the tank accelerates, the water spill from the left side wall. The surface of the water under this condition is shown in Fig. *a*.



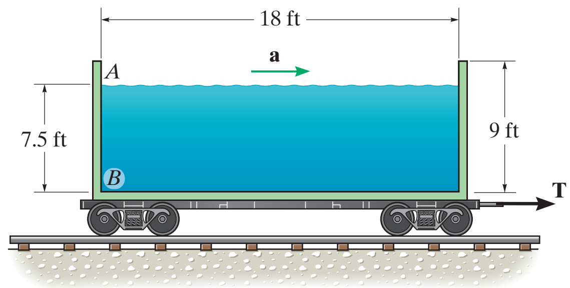
(a)

$$\tan\theta = \frac{1\ \text{m}}{2.5\ \text{m}} = \frac{a_c}{9.81\ \text{m/s}^2}$$

$$a_c = 3.92\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–153. The open rail car is 6 ft wide and filled with water to the level shown. Determine the pressure that acts at point B both when the car is at rest and when the car is moving with a constant acceleration of 10 ft/s². How much water spills out of the car?



SOLUTION

When the car is at rest, the water is at the level shown by the dashed line shown in Fig. *a*

At rest: $p_B = \gamma_w h_B = (62.4\ \text{lb/ft}^3)(7.5\ \text{ft}) = 468\ \text{lb/ft}^2$ **Ans.**

When the car accelerates, the angle θ the water level makes with the horizontal can be determined.

$$\tan\theta = \frac{a_c}{g} = \frac{10\ \text{ft/s}^2}{32.2\ \text{ft/s}^2}; \qquad \theta = 17.25°$$

Assuming that the water will spill out. Then the water level when the car accelerates is indicated by the solid line shown in Fig. *a*. Thus,

$$h = 9\ \text{ft} - 18\ \text{ft}\tan 17.25° = 3.4099\ \text{ft}$$

The original volume of water is

$$\forall = (7.5\ \text{ft})(18\ \text{ft})(6\ \text{ft}) = 810\ \text{ft}^3$$

The volume of water after the car accelerate is

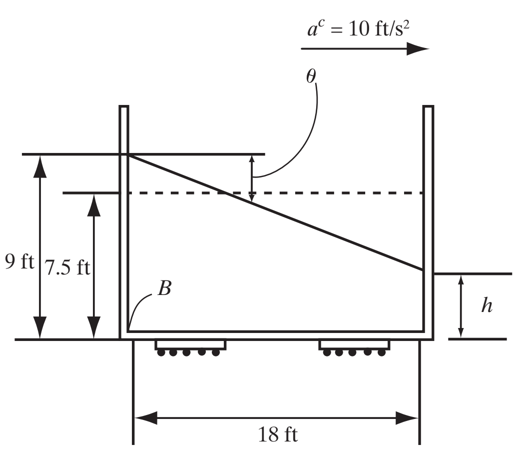
$$\forall' = \frac{1}{2}(9\ \text{ft} + 3.4099\ \text{ft})(18\ \text{ft})(6\ \text{ft}) = 670.14\ \text{ft}^3 < 810\ \text{ft}^3 \qquad \textbf{(OK!)}$$

Thus, the amount of water spilled is

$$\Delta\forall = \forall - \forall' = 810\ \text{ft}^3 - 670.14\ \text{ft}^3 = 139.86\ \text{ft}^3 = 140\ \text{ft}^3 \qquad \textbf{Ans.}$$

The pressure at B when the car accelerates is

With acceleration: $p_B = \gamma_w h_B = (62.4\ \text{lb/ft}^3)(9\ \text{ft}) = 561.6\ \text{lb/ft}^2 = 562\ \text{lb/ft}^2$ **Ans.**

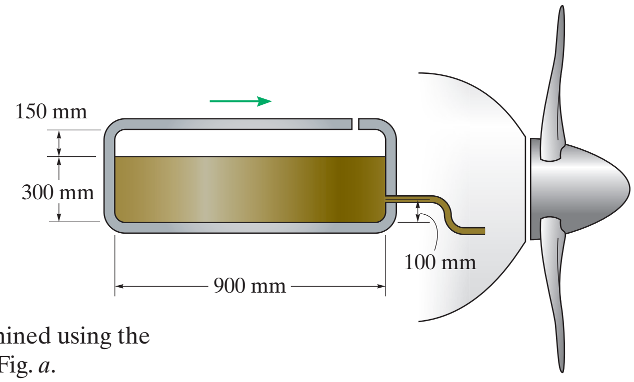


(a)

Ans:
At rest: $p_B = 468\ \text{lb/ft}^2$
With acceleration: $\Delta\forall = 140\ \text{ft}^3$
$p_B = 562\ \text{lb/ft}^2$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–154. The fuel tank, supply line, and engine for an airplane are shown. If the gas tank is filled to the level shown, determine the largest constant acceleration a that the plane can have without causing the engine to be starved of fuel. The plane is accelerating to the right for this to happen. Suggest a safer location for attaching the fuel line.



SOLUTION

If the fuel width of the tank is w, the volume of the fuel can be determined using the fuel level when the airplane is at rest indicated by the dashed line in Fig. a.

$$\forall_f = (0.9\ \text{m})(0.3\ \text{m})w = 0.27w$$

It is required that the fuel level is about to drop lower than the supply line. In this case, the fuel level is indicated by the solid line in Fig. a.

$$\forall_f = (0.9\ \text{m})(0.45\ \text{m})w - \frac{1}{2}(0.45\ \text{m} - 0.1\ \text{m})(0.9\ \text{m} - b)w = 0.27w$$

$$b = 0.1286\ \text{m}$$

Thus,

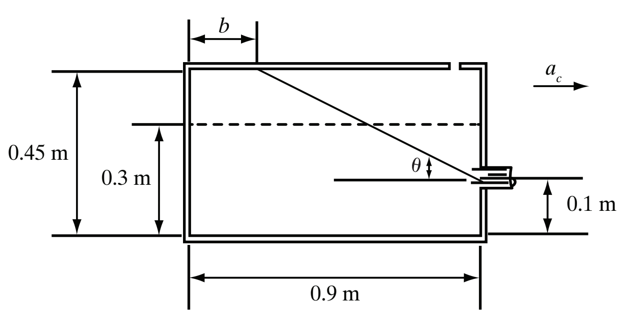
$$\tan\theta = \frac{0.45\ \text{m} - 0.1\ \text{m}}{0.9\ \text{m} - 0.1286\ \text{m}} = 0.4537$$

And so

$$\tan\theta = \frac{a_c}{g}; \qquad 0.4537 = \frac{a_c}{9.81\ \text{m/s}^2}$$

$$a_c = 4.45\ \text{m/s}^2 \qquad \textbf{Ans.}$$

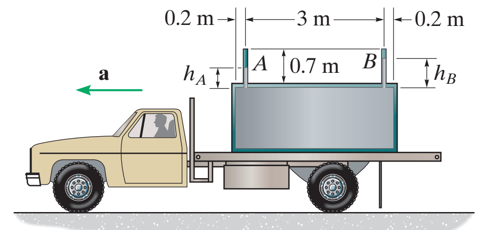
The safer location for attaching the fuel line is at the bottom of the tank.



Ans:
$a_c = 4.45\ \text{m/s}^2$
The safer location is at the bottom of the tank.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–155. A large container of benzene is transported on the truck. Determine the level in each of the vent tubes A and B if the truck accelerates at $a = 1.5\ \text{m/s}^2$. When the truck is at rest, $h_A = h_B = 0.4$ m.



SOLUTION

The imaginary surface of the benzene in the accelerated tank is shown in Fig. *a*.

$$\tan\theta = \frac{a_c}{g} = \frac{1.5\ \text{m/s}^2}{9.81\ \text{m/s}^2}$$

$$\theta = 8.6935°$$

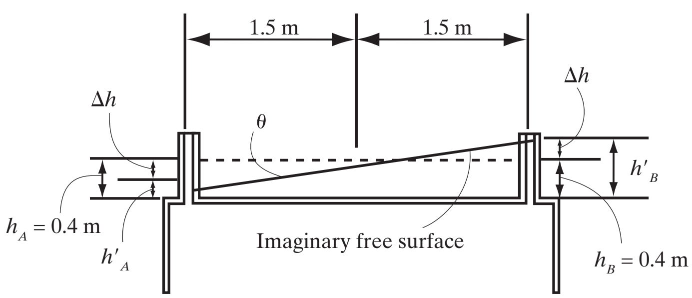
Then,

$$\Delta h = (1.5\ \text{m})\ \tan\ 8.6935° = 0.2294\ \text{m}$$

Thus,

$$h'_A = h_A - \Delta h = 0.4\ \text{m} - 0.2294\ \text{m} = 0.171\ \text{m} \qquad \textbf{Ans.}$$

$$h'_B = h_B + \Delta h = 0.4\ \text{m} + 0.2294\ \text{m} = 0.629\ \text{m} \qquad \textbf{Ans.}$$

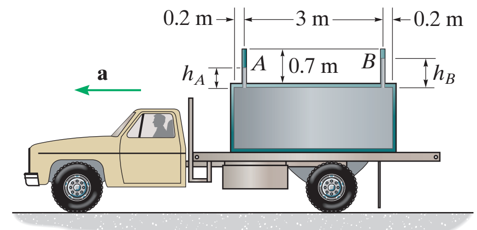


(a)

Ans:
$h'_A = 0.171$ m
$h'_B = 0.629$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–156.** A large container of benzene is being transported by the truck. Determine its maximum constant acceleration so that no benzenes will spill from the vent tubes A or B. When the truck is at rest, $h_A = h_B = 0.4$ m.

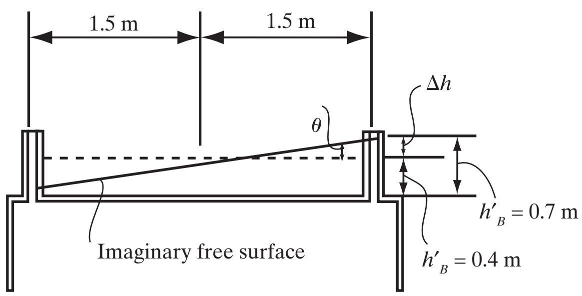


SOLUTION

The imaginary surface of the benzene in the accelerated tank is shown in Fig. a. Under this condition, the water will spill from vent B. Thus, $\Delta h = h'_B - h_B = 0.7\ \text{m} - 0.4\ \text{m} = 0.3\ \text{m}$.

$$\tan\theta = \frac{0.3\ \text{m}}{1.5\ \text{m}} = 0.2 = \frac{a_c}{g}$$

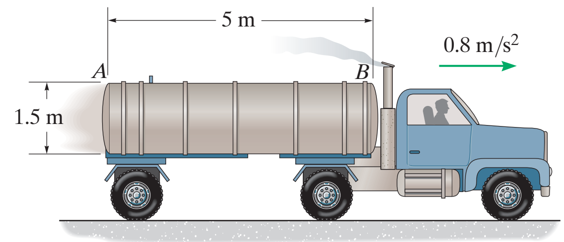
$$a_c = 0.2\left(9.81\ \text{m/s}^2\right) = 1.96\ \text{m/s}^2 \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–157. The closed cylindrical tank is filled with milk, for which $\rho_m = 1030 \text{ kg/m}^3$. If the inner diameter of the tank is 1.5 m, determine the difference in pressure within the tank between corners A and B when the truck accelerates at 0.8 m/s^2.



SOLUTION

The imaginary surface of the milk in the accelerated tank is shown in Fig. *a*.

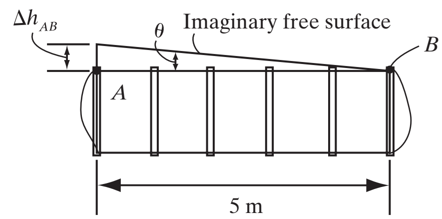
$$\tan\theta = \frac{a_c}{g} = \frac{0.8 \text{ m/s}^2}{9.81 \text{ m/s}^2} = 0.08155$$

Then,

$$\Delta h_{AB} = L_{AB}\tan\theta = (5 \text{ m})(0.08155) = 0.4077 \text{ m}$$

Finally,

$$\Delta p_{AB} = \rho_m g \Delta h_{AB} = \left(1030 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)(0.4077 \text{ m})$$
$$= 4.12\left(10^3\right) \text{ Pa} = 4.12 \text{ kPa} \qquad \textbf{Ans.}$$

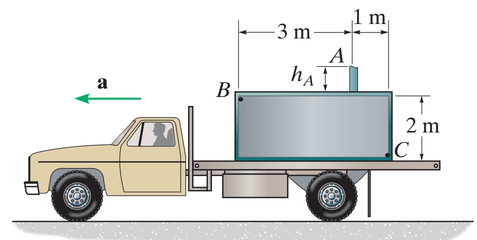


(a)

Ans:
4.12 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–158. Determine the water pressure at points B and C in the tank if the truck has a constant acceleration $a_c = 2 \text{ m/s}^2$. When the truck is at rest, the water level in the vent tube A is at $h_A = 0.3$ m.



SOLUTION

The water level at vent tube A will not change when the tank is accelerated since the water in the tank is confined (no other vent tube). Thus, the imaginary free surface must pass through the free surface at vent tube A.

$$\tan\theta = \frac{a_C}{g}; \qquad \tan\theta = \frac{2 \text{ m/s}^2}{9.81 \text{ m/s}^2} \qquad \theta = 11.52°$$

From the geometry in Fig. *a*,

$$\Delta h_B = (3 \text{ m}) \tan 11.52° = 0.6116 \text{ m}$$

$$\Delta h_C = (1 \text{ m}) \tan 11.52° + 0.3 \text{ m} = 0.5039 \text{ m}$$

Then,

$$h_B = -(\Delta h_B - 0.3 \text{ m}) = -(0.6116 \text{ m} - 0.3 \text{ m}) = -0.3116 \text{ m}$$

$$h_C = \Delta h_C + 2 \text{ m} = 0.5039 \text{ m} + 2 \text{ m} = 2.5039 \text{ m}$$

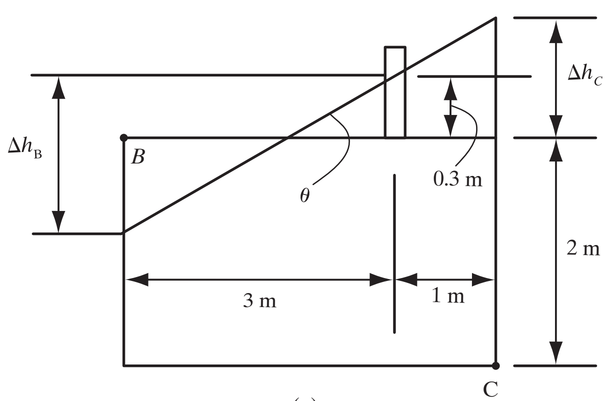
Thus,

$$p_B = \rho_w g h_B = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(-0.3116 \text{ m})$$

$$= -3.057(10^3) \text{ Pa} = -3.06 \text{ kPa} \qquad \textbf{Ans.}$$

$$p_C = \rho_w g h_C = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.5039 \text{ m})$$

$$= 24.563(10^3) \text{ Pa} = 24.6 \text{ kPa} \qquad \textbf{Ans.}$$

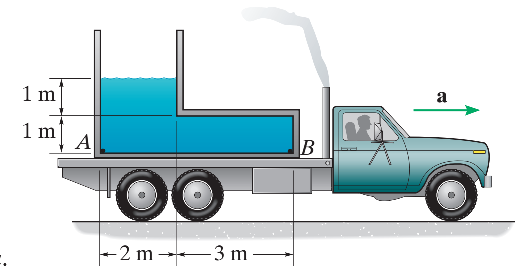


(a)

Ans:
$p_B = -3.06$ kPa
$p_C = 24.6$ kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–159. If the truck has a constant acceleration of $2\ \text{m/s}^2$, determine the water pressure at the bottom corners A and B of the water rank.



SOLUTION

The imaginary free surface of the water in the accelerated tank is shown in Fig. *a*.

$$\tan\theta = \frac{a_C}{g} = \frac{2\ \text{m/s}^2}{9.81\ \text{m/s}^2} = 0.2039$$

From the geometry in Fig. *a*,

$$\Delta h_A = (1\ \text{m})\tan\theta = (1\ \text{m})(0.2039) = 0.2039\ \text{m}$$

$$\Delta h_B = (1\ \text{m} + 3\ \text{m})\tan\theta = (4\ \text{m})(0.2039) = 0.8155\ \text{m}$$

Then

$$h_A = 2\ \text{m} + \Delta h_A = 2\ \text{m} + 0.2039\ \text{m} = 2.2039\ \text{m}$$

$$h_B = 2\ \text{m} - \Delta h_B = 2\ \text{m} - 0.8155\ \text{m} = 1.1845\ \text{m}$$

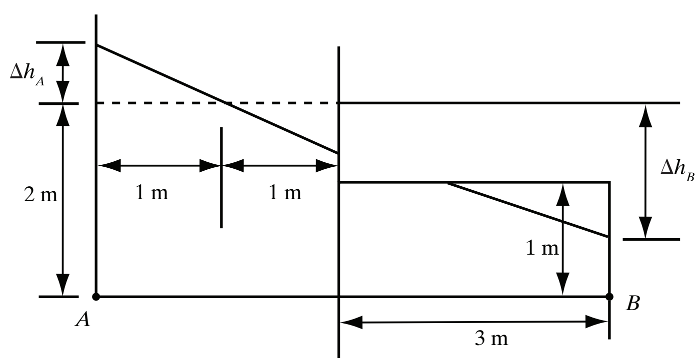
Finally,

$$p_A = \rho_w g h_A = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2.2039\ \text{m})$$

$$= 21.62\left(10^3\right)\ \text{Pa} = 21.6\ \text{kPa} \qquad \textbf{Ans.}$$

$$p_B = \rho_w g h_B = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1.1845\ \text{m})$$

$$= 11.62\left(10^3\right)\ \text{Pa} = 11.6\ \text{kPa} \qquad \textbf{Ans.}$$

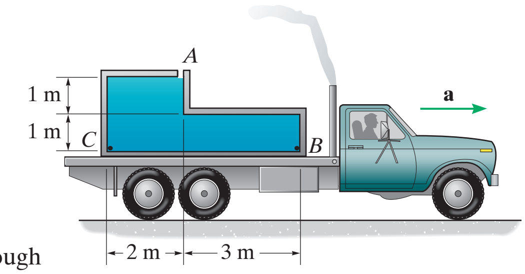


(a)

Ans:
$p_A = 21.6\ \text{kPa}$
$p_B = 11.6\ \text{kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–160.** If the truck has a constant acceleration of $2\ \text{m/s}^2$, determine the water pressure at the bottom corners B and C of the water tank. There is a small opening at A.



SOLUTION

Since the water in the tank is confined, the imaginary free surface must pass through A as shown in Fig. a. We have

$$\tan\theta = \frac{a_C}{g} = \frac{2\ \text{m/s}^2}{9.81\ \text{m/s}^2} = 0.2039$$

From the geometry in Fig. a,

$$\Delta h_C = (2\ \text{m})\tan\theta = (2\ \text{m})(0.2039) = 0.4077\ \text{m}$$

$$\Delta h_B = (3\ \text{m})\tan\theta = (3\ \text{m})(0.2039) = 0.6116\ \text{m}$$

Then

$$h_C = 2\ \text{m} + \Delta h_A = 2\ \text{m} + 0.4077\ \text{m} = 2.4077\ \text{m}$$

$$h_B = 2\ \text{m} - \Delta h_B = 2\ \text{m} - 0.6116\ \text{m} = 1.3884\ \text{m}$$

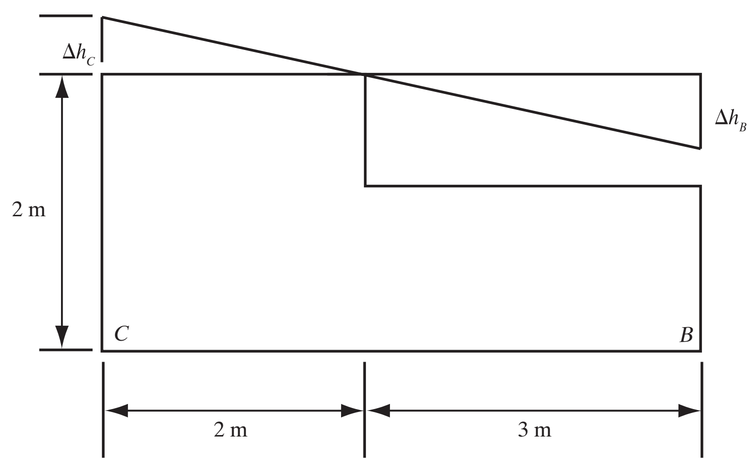
Finally,

$$p_C = \rho_w g h_C = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2.4077\ \text{m})$$

$$= 23.62\left(10^3\right)\ \text{Pa} = 23.6\ \text{kPa} \qquad \textbf{Ans.}$$

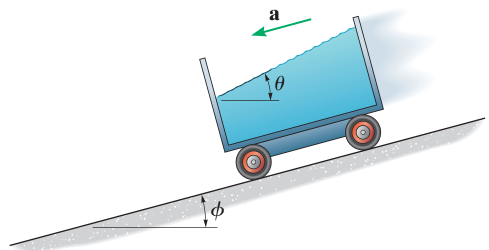
$$p_B = \rho_w g h_B = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1.3884\ \text{m})$$

$$= 13.62\left(10^3\right)\ \text{Pa} = 13.6\ \text{kPa} \qquad \textbf{Ans.}$$



Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–161. The cart is allowed to roll freely down the inclined plane due to its weight. Show that the slope of the surface of the liquid, θ, during the motion is $\theta = \phi$.



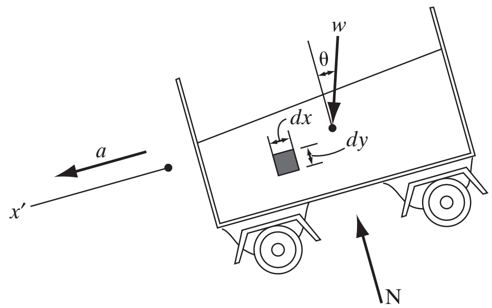
SOLUTION

Referring to the free-body diagram of the container in Fig. *a*,

$\underset{\swarrow}{+}\Sigma F_{x'} = ma_{x'}$

$$w \sin\phi = \frac{w}{g}a$$

$$a = g\sin\phi$$

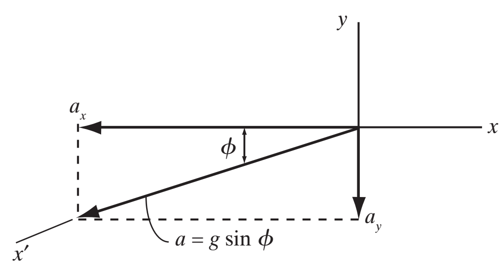


(a)

Referring to Fig. *b*,

$$a_x = -(g\sin\phi)\cos\phi$$

$$a_y = -(g\sin\phi)\sin\phi$$



(b)

We will now apply Newton's equations of notation, Fig. *c*.

$\xrightarrow{+}\Sigma F_x = ma_x;\qquad -\left(p_x + \dfrac{\partial p_x}{\partial x}dx\right)dydz + p_x\,dydz = \dfrac{\gamma(dxdydz)}{g}a_x$

$$dp_x = -\frac{\gamma dx}{g}a_x$$

In y direction,

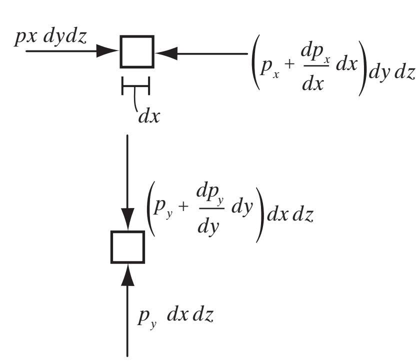
$+\uparrow\Sigma F_y = ma_y;\qquad p_y dxdz - \left(p_y + \dfrac{\partial p_y}{\partial y}dy\right)dxdz - \gamma dxdydz = \dfrac{\gamma dxdydz}{g}a_y$

$$dp_y = -\gamma dy\left(1 + \frac{a_y}{g}\right)$$

At the surface, p is constant, so that $dp_x + dp_y = 0$, or $dp_x = -dp_y$.

$$\frac{\gamma dx}{g}a_x = -\gamma dy\left(1 + \frac{a_y}{g}\right)$$

$$\frac{dy}{dx} = -\frac{a_x}{g + a_y} = \frac{g\sin\phi\cos\phi}{g - g\sin\phi\sin\phi} = \frac{\sin\phi\cos\phi}{\cos^2\phi} = \frac{\sin\phi}{\cos\phi} = \tan\phi$$



(c)

Since at the surface,

$$\frac{dy}{dx} = -\tan\theta$$

then

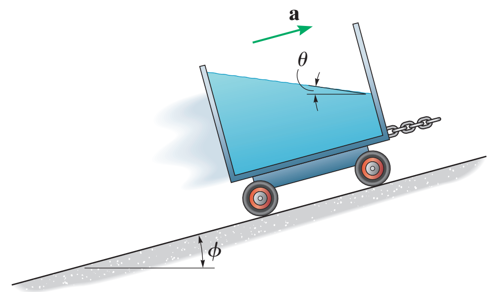
$$\tan\theta = \tan\phi$$

or

$$\theta = \phi \qquad \textbf{Q.E.D.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \mathrm{kg/m^3}$ and its specific weight to be $\gamma_w = 62.4\ \mathrm{lb/ft^3}$. Also, assume all pressures are gage pressures.

2–162. The cart is given a constant acceleration **a** up the plane, as shown. Show that the lines of constant pressure *within* the liquid have a slope of $\tan\theta = (a\cos\phi)/(a\sin\phi + g)$.



SOLUTION

As in the preceding solution, we determine that

$$\frac{dy}{dx} = -\frac{a_x}{g + a_y} \qquad \textbf{(1)}$$

Here, the slope of the surface of the liquid, Fig. *a*, is

$$\frac{dy}{dx} = -\tan\theta \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2), we obtain

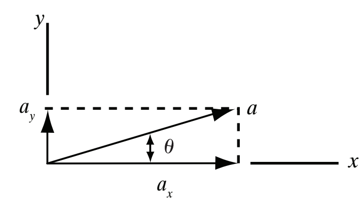
$$\tan\theta = \frac{a_x}{g + a_y} \qquad \textbf{(3)}$$

By establishing the x and y axes shown in Fig. *a*,

$$a_x = a\ \cos\phi \qquad\qquad a_y = a\ \sin\phi$$

Substituting these values into Eq. (3),

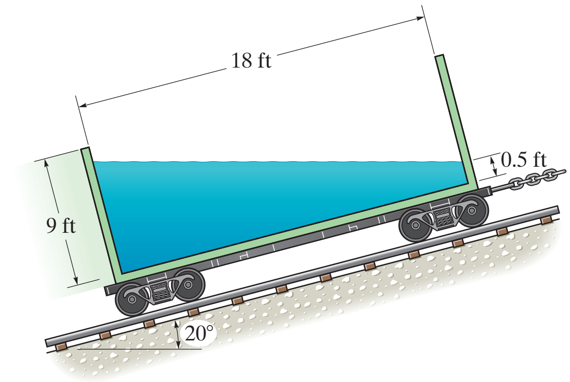
$$\tan\theta = \frac{a\ \cos\phi}{a\ \sin\phi + g} \qquad \textbf{Q.E.D}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–163. The open railcar is used to transport water up the 20° incline. When the car is at rest, the water level is as shown. Determine the maximum acceleration the car can have when it is pulled up the incline so that no water will spill out.



SOLUTION

The volume of the water can be determined by using the water level when the car is at rest, indicated by dashed line in Fig. *a*. Here

$$a = 0.5\ \text{ft} + 18 \tan 20°\ \text{ft} = 7.0515\ \text{ft}$$

If the width of the car is w

$$\forall_w = \frac{1}{2}(0.5\ \text{ft} + 7.0515\ \text{ft})(18\ \text{ft})w = 67.9632\ w$$

It is required that the water is about to spill out. In this case, the water is indicated by the solid line in Fig. *a*,

$$\forall_w = \frac{1}{2}(9\ \text{ft})(\text{b})\ w = 67.9632\ w$$

$$b = 15.1029\ \text{ft} < 18\ \text{ft} \qquad \textbf{(O.K.)}$$

Then,

$$\theta = \tan^{-1}\left(\frac{9\ \text{ft}}{15.1029\ \text{ft}}\right) - 20° = 10.7912°$$

Consider the vertical block of water of weight $\delta_w = \gamma_w h \delta A$ shown shaded in Fig. *a*

$$+\uparrow \Sigma F_y = ma_y; \qquad p\delta_A - \gamma_w h \delta_A = \frac{\gamma_w h \delta_A}{g} a \sin 20°$$

$$p = \frac{\gamma_w h}{g} a \sin 20° + \gamma_w h$$

$$p = \frac{\gamma_w h}{g}(a \sin 20° + g) \qquad \textbf{(1)}$$

Consider the horizontal block of water of weight $\delta_w = \gamma_w x \delta A$ shown shaded in Fig. *a*

$$\underset{\rightarrow}{+} \Sigma F_x = ma_x; \qquad p_2 \delta_A - p_1 \delta_A = \frac{\gamma_w x \delta_A}{g} a \cos 20°$$

$$p_2 - p_1 = \frac{\gamma_w x}{g} a \cos 20° \qquad \textbf{(2)}$$

However, from Eq. (1), p_2 is at h_2 and p_1 is at h_1, so that

$$p_2 - p_1 = \frac{\gamma_w}{g}(h_2 - h_1)(a \sin 20° + g)$$

Substituting this result into Eq. (2), we have

$$\frac{\gamma_w}{g}(h_2 - h_1)(a \sin 20° + g) = \frac{\gamma_w x}{g} a \cos 20°$$

$$\frac{h_2 - h_1}{x} = \frac{a \cos 20°}{a \sin 20° + g}$$

2–163. Continued

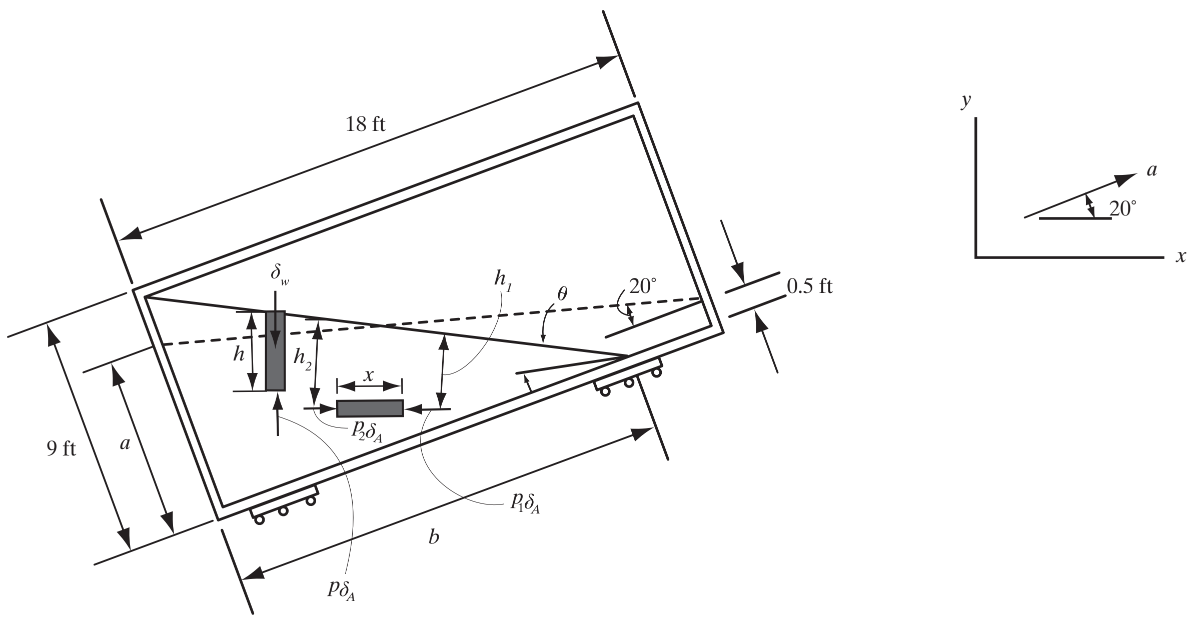
However, $\tan\theta = \dfrac{h_2 - h_1}{x}$. Thus

$$\tan\theta = \frac{a\cos 20^\circ}{a\sin 20^\circ + g}$$

Here, $\theta = 10.7912^\circ$. Then

$$\tan 10.7912^\circ = \frac{a\cos 20^\circ}{a\sin 20^\circ + 32.2\ \text{ft/s}^2}$$

$$a = 7.02\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

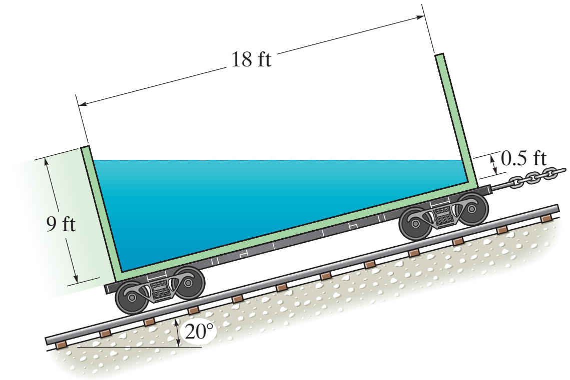


(a)

Ans:
$7.02\ \text{ft/s}^2$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–164.** The open railcar is used to transport water up the 20° incline. When the car is at rest, the water level is as shown. Determine the maximum deceleration the car can have when it is pulled up the incline so that no water will spill out.



SOLUTION

The volume of water can be determined using the water level when the car is at rest indicated by the dashed line in Fig. *a*. Here,

$$a = 0.5\text{ ft} + 18\text{ ft}\ \tan\ 20° = 7.0515\text{ ft}$$

If the width of the car is w,

$$\forall_w = \frac{1}{2}(0.5\text{ ft} + 7.0515\text{ ft})(18\text{ ft})w = 67.9632\ w$$

It is required that the water is about to spill out. In this case, the water level is indicated by the solid line in Fig. *a*

$$\forall_w = \frac{1}{2}(9\text{ ft})(b)(w) = 67.9632\ w$$

$$b = 15.1029\text{ ft} < 18\text{ ft} \qquad \textbf{(O.K)}$$

Then

$$\theta = \tan^{-1}\left(\frac{9\text{ ft}}{15.1029\text{ ft}}\right) + 20° = 50.7912°$$

Consider the vertical block of water of weight $\delta_w = \gamma_w h \delta_A$ shown shaded in Fig. *a*.

$$+\uparrow \Sigma F_y = ma_y; \qquad p\,\delta_A - \gamma_w h \delta_A = \frac{\gamma_w h \delta_A}{g}(-a \sin 20°)$$

$$p = \frac{\gamma_w h}{g}(g - a \sin 20°) \qquad \textbf{(1)}$$

Consider the horizontal block of water of weight $\delta_w = \gamma_w x \delta_A$ shown shaded in Fig. *a*

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad p_1\delta_A - p_2\delta_A = \frac{\gamma_w x \delta_A}{g}(-a \cos 20°)$$

$$p_2 - p_1 = \frac{\gamma_w x}{g}\, a \cos 20° \qquad \textbf{(2)}$$

However, from Eq. (1), since p_1 is at h_1 and p_2 is at h_2.

$$p_2 - p_1 = \frac{\gamma_w}{g}(h_2 - h_1)(g - a \sin 20°)$$

Substituting this result into Eq. (2)

$$\frac{\gamma_w}{g}(h_2 - h_1)(g - a \sin 20°) = \frac{\gamma_w x}{g}\, a \cos 20°$$

$$\frac{h_2 - h_1}{x} = \frac{a \cos 20°}{g - a \sin 20°}$$

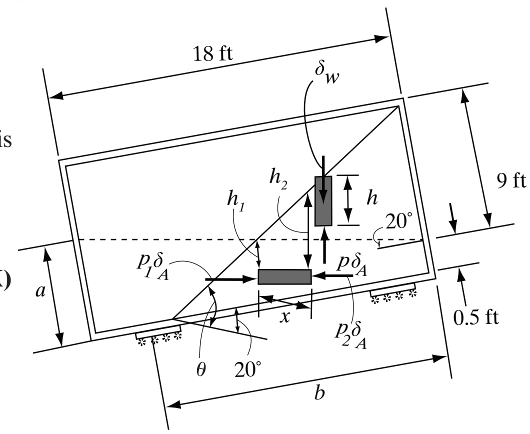
However, $\tan\theta = \dfrac{h_2 - h_1}{x}$. Thus

$$\tan\theta = \frac{a\ \cos 20°}{g - a\ \sin 20°}$$

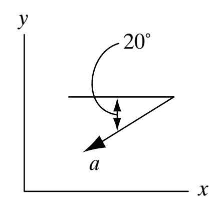
Here, $\theta = 50.7912°$. Then

$$\tan 50.7912° = \frac{a \cos 20°}{32.2\text{ ft/s}^2 - a \sin 20°}$$

$$a = 29.04\text{ ft/s}^2 = 29.0\text{ ft/s}^2 \qquad \textbf{Ans.}$$



(a)



Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–165. A woman stands on a horizontal platform that is rotating at 1.5 rad/s. If she is holding a cup of coffee, and the center of the cup is 4 m from the axis of rotation, determine the slope angle of the coffee's surface. Neglect the size of the cup.

SOLUTION

Since the coffee cup is rotating at a constant velocity about the vertical axis of rotation, then its acceleration is always directed horizontally toward the axis of rotation and its magnitude is given by

$$a_r = \omega^2 r = (1.5\ \text{rad/s})^2(4\ \text{m}) = 9\ \text{m/s}^2$$

Thus, the slope of coffee surface is

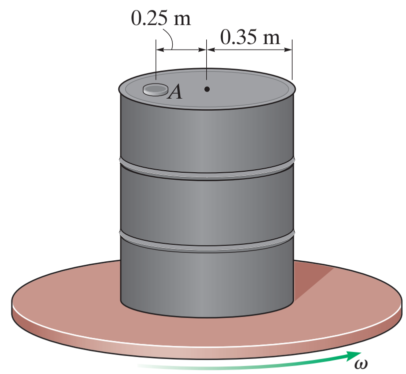
$$m = \tan\theta = \frac{a_r}{g} = \frac{9\ \text{m/s}^2}{9.81\ \text{m/s}^2} = 0.917$$

$$\theta = 42.5^\circ \qquad \textbf{Ans.}$$

Ans:
42.5°

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures.

2–166. The drum is filled to the top with oil and placed on the platform. If the platform is given a rotation of $\omega = 12 \text{ rad/s}$, determine the pressure the oil will exert on the cap at A. Take $\rho_o = 900 \text{ kg/m}^3$.



SOLUTION

We observe from Fig. a that $h = h_A$ at $r = 0.25$ m.

$$h_A = \frac{\omega^2}{2g}r^2$$

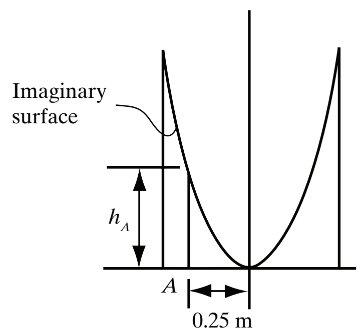
$$h_A = \left[\frac{(12 \text{ rad/s})^2}{2\left(9.81 \text{ m/s}^2\right)}\right](0.25 \text{ m})^2$$

$$= 0.4587 \text{ m}$$

$$p_A = \rho_o g h_A = \left(900 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)(0.4587 \text{ m})$$

$$= 4.05\left(10^3\right) \text{ Pa}$$

$$= 4.05 \text{ kPa}$$ **Ans.**

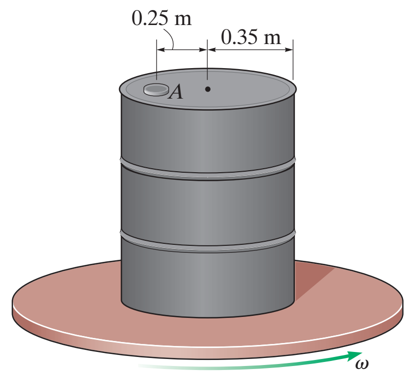


(a)

Ans:
4.05 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–167. The drum is filled to the top with oil and placed on the platform. Determine the maximum rotation of the platform if the maximum pressure the cap at A can sustain before it opens is 40 kPa. Take $\rho_o = 900\ \text{kg/m}^3$.



SOLUTION

It is required that $p_A = 40$ kPa. Thus, the pressure head for the oil is

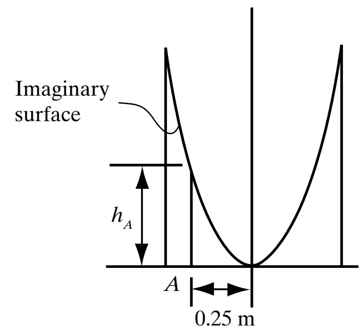
$$h_A = \frac{p_A}{\gamma_O} = \frac{40(10^3)\ \text{N/m}^2}{(900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} = 4.531\ \text{m}$$

We observe from Fig. *a* that $h = h_A$ at $r = 0.25$ m.

$$h_A = \frac{\omega^2}{2g} r^2$$

$$4.531\ \text{m} = \left[\frac{\omega^2}{2(9.81\ \text{m/s}^2)}\right](0.25\ \text{m})^2$$

$$\omega = 37.7\ \text{rad/s}$$ **Ans.**

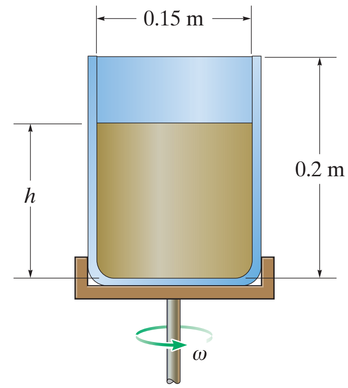


(a)

Ans:
37.7 rad/s

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–168.** The beaker is filled to a height of $h = 0.1$ m with kerosene and place on the platform. What is the maximum angular velocity ω it can have so that no kerosene spills out of the beaker?



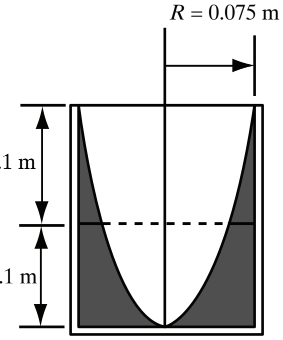
SOLUTION

When the kerosene is about to spill out of the beaker, Fig. *a*, $h/2 = 0.1$ m or $h = 0.2$ m.

$$h = \frac{\omega^2}{2g} r^2$$

$$0.2\ \text{m} = \frac{\omega^2}{2(9.81\ \text{m/s}^2)} \left(\frac{0.15\ \text{m}}{2}\right)^2$$

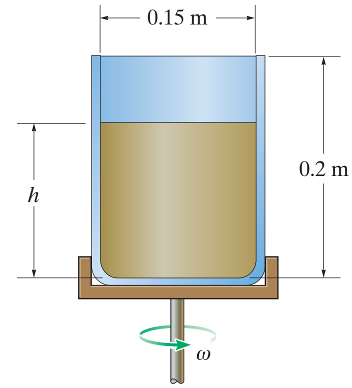
$$\omega = 26.4\ \text{rad/s} \qquad \textbf{Ans.}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–169. The beaker is filled to a height of $h = 0.1$ m with kerosene and placed on the platform. To what height $h = h'$ does the kerosene rise against the wall of the beaker when the platform has an angular velocity of $\omega = 15$ rad/s?



SOLUTION

$$H = \frac{\omega^2}{2g} r^2$$

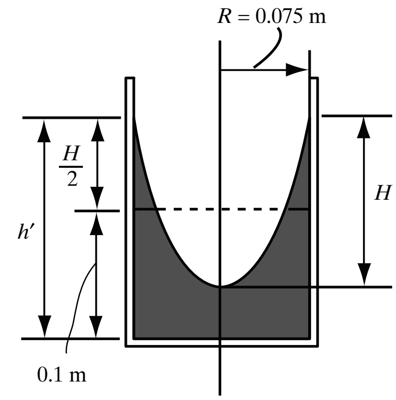
$$H = \frac{(15\ \text{rad/s})^2}{2(9.81\ \text{m/s}^2)}\left(\frac{0.15\ \text{m}}{2}\right)^2$$

$$= 0.0645\ \text{m}$$

From Fig. a, we observe that

$$h' = 0.1\ \text{m} + \frac{0.0645}{2}\ \text{m}$$

$$h' = 0.132\ \text{m} = 132\ \text{mm} \qquad \textbf{Ans.}$$

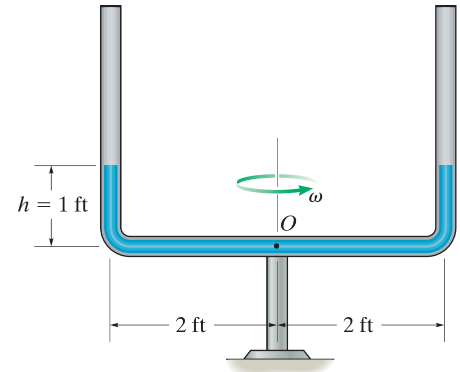


(a)

Ans:
132 mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–170. The tube is filled with water to the level $h = 1$ ft. Determine the pressure at point O when the tube has an angular velocity of $\omega = 8$ rad/s.



SOLUTION

The level of the water in the tube will not change. Therefore, the imaginary surface will be as shown in Fig. *a*.

$$H = \frac{\omega^2 R^2}{2g} = \frac{(8\ \text{rad/s})^2(2\ \text{ft})^2}{2(32.2\ \text{ft/s}^2)} = 3.9752\ \text{ft}$$

We observe from Fig. *a* that

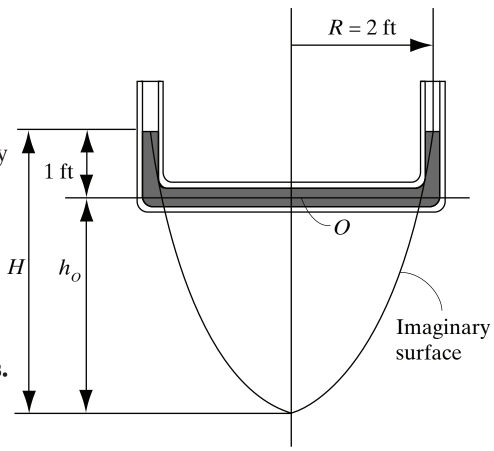
$$h_O = H - 1\ \text{ft} = 3.9752\ \text{ft} - 1\ \text{ft} = 2.9752\ \text{ft}$$

Finally, the pressure at O must be negative since it is 2.9752 ft *above* the imaginary surface of the liquid.

$$p_O = \gamma h_O = (62.4\ \text{lb/ft}^3)(-2.9752\ \text{ft})$$

$$= \left(-185.65\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= -1.29\ \text{psi} \qquad \textbf{Ans.}$$

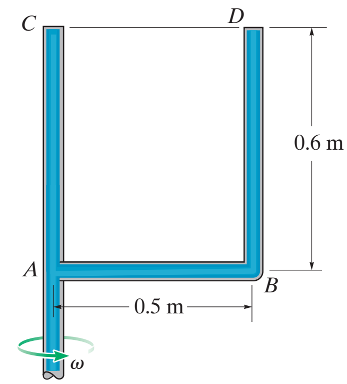


(a)

Ans:
−1.29 psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–171. The sealed assembly is completely filled with water such that the pressures at C and D are zero. If the assembly is given an angular velocity of $\omega = 15\ \text{rad/s}$, determine the difference in pressure between points C and D.



SOLUTION

$$H = \frac{\omega^2 R^2}{2g}$$

$$= \frac{(15\ \text{rad/s})^2(0.5\ \text{m})^2}{2(9.81\ \text{m/s}^2)}$$

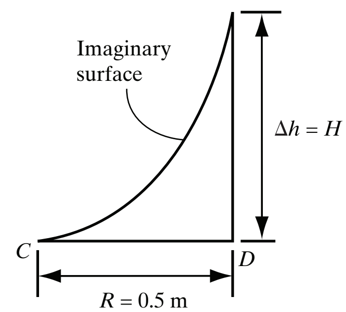
$$= 2.867\ \text{m}$$

From Fig. a, $\Delta h = H = 2.867\ \text{m}$. Then,

$$\Delta p = p_D - p_C = \rho_w g \Delta h$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.867\ \text{m})$$

$$= 28.13(10^3)\ \text{Pa} = 28.1\ \text{kPa}$$ **Ans.**

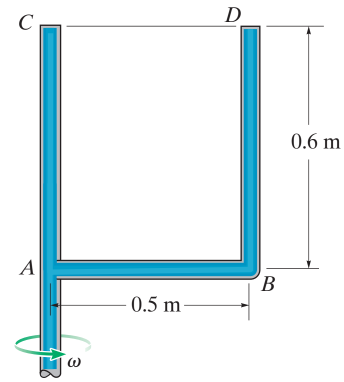


(a)

Ans:
28.1 kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–172.** The sealed assembly is completely filled with water such that the pressures at C and D are zero. If the assembly is given an angular velocity of $\omega = 15\ \text{rad/s}$, determine the difference in pressure between points A and B.



SOLUTION

$$H = \frac{\omega^2 R^2}{2g}$$

$$= \frac{(15\ \text{rad/s})^2(0.5\ \text{m})^2}{2(9.81\ \text{m/s}^2)}$$

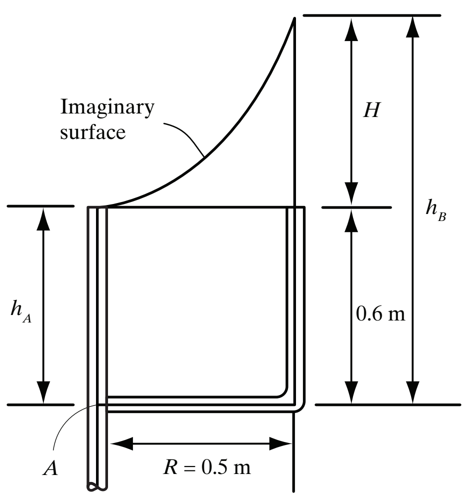
$$= 2.867\ \text{m}$$

From Fig. a, $\Delta h = h_B - h_A = H = 2.867\ \text{m}$. Then,

$$\Delta p = p_B - p_A = \rho_w g \Delta h$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.867\ \text{m})$$

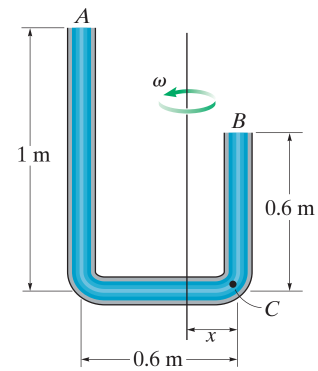
$$= 28.13(10^3)\ \text{Pa} = 28.1\ \text{kPa}$$ **Ans.**



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–173. The U-tube is filled with water and A is open while B is closed. If the axis of rotation is at $x = 0.2$ m, determine the constant rate of rotation so that the pressure at B is zero.



SOLUTION

Since points A and B have zero gauge pressure, the imaginary free surface must pass through them as shown in Fig. *a*.

$$h'_A = \frac{\omega^2 r_A^2}{2g} = \frac{\omega^2(0.4\ \text{m})^2}{2(9.81\ \text{m/s}^2)} = 0.008155\omega^2$$

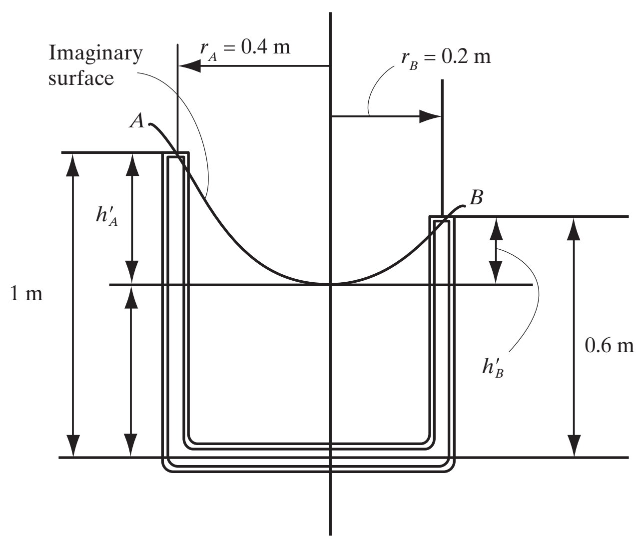
$$h'_B = \frac{\omega^2 r_B^2}{2g} = \frac{\omega^2(0.2\ \text{m})^2}{2(9.81\ \text{m/s}^2)} = 0.002039\omega^2$$

From Fig. *a*,

$$1\ \text{m} - h'_A = 0.6\ \text{m} - h'_B$$

$$1\ \text{m} - 0.008155\omega^2 = 0.6\ \text{m} - 0.002039\omega^2$$

$$\omega = 8.09\ \text{rad/s}$$ **Ans.**

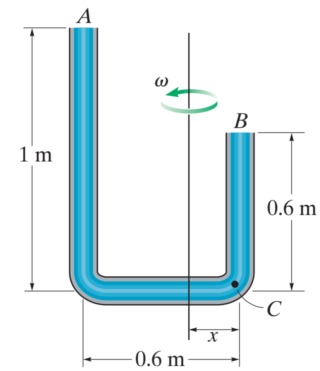


(a)

Ans:
8.09 rad/s

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–174. The U-tube is filled with water and A is open while B is closed. If the axis of rotation is at $x = 0.2$ m and the tube is rotating at a constant rate of $\omega = 10\ \text{rad/s}$, determine the pressure at points B and C.



SOLUTION

Since point A has zero gauge pressure, the imaginary free surface must pass through this point as shown in Fig. *a*.

$$H = \frac{\omega^2 R^2}{2g} = \frac{(10\ \text{rad/s})^2(0.4\ \text{m})^2}{2(9.81\ \text{m/s}^2)} = 0.8155\ \text{m}$$

and

$$h' = \frac{\omega^2 r^2}{2g} = \frac{(10\ \text{rad/s})^2(0.2\ \text{m})^2}{2(9.81\ \text{m/s}^2)} = 0.2039\ \text{m}$$

From Fig. *a*,

$$a = 1\ \text{m} - H = 1\ \text{m} - 0.8155\ \text{m} = 0.1845\ \text{m}$$

Then,

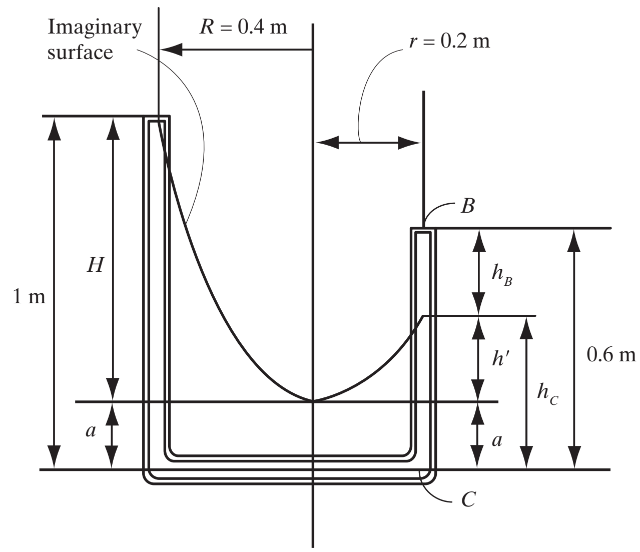
$$h_B = -(0.6\ \text{m} - h' - a) = -(0.6\ \text{m} - 0.2039\ \text{m} - 0.1845\ \text{m}) = -0.2116\ \text{m}$$

$$h_C = h' + a = 0.2039\ \text{m} + 0.1845\ \text{m} = 0.3884\ \text{m}$$

Finally,

$$p_B = \rho_w g h_B = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(-0.2116\ \text{m})$$
$$= -2.076(10^3)\ \text{Pa}$$
$$= -2.08\ \text{kPa} \qquad \textbf{Ans.}$$

$$p_C = \rho_w g h_C = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.3884\ \text{m})$$
$$= 3.81(10^3)\ \text{Pa}$$
$$= 3.81\ \text{kPa} \qquad \textbf{Ans.}$$

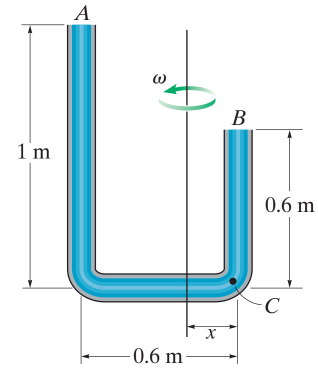


(a)

Ans:
$p_B = -2.08$ kPa
$p_C = 3.81$ kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–175. The U-tube is filled with water and A is open while B is closed. If the axis of rotation is at $x = 0.4$ m and the tube is rotating at a constant rate of $\omega = 10$ rad/s, determine the pressure at points B and C.



SOLUTION

Since point A has zero gauge pressure, the imaginary free surface must pass through this point as shown in Fig. a.

$$H = \frac{\omega^2 R^2}{2g} = \frac{(10\ \text{rad/s})^2(0.4\ \text{m})^2}{2\left(9.81\ \text{m/s}^2\right)} = 0.8155\ \text{m}$$

and

$$h'_A = \frac{\omega^2 r_A{}^2}{2g} = \frac{(10\ \text{rad/s})^2(0.2\ \text{m})^2}{2\left(9.81\ \text{m/s}^2\right)} = 0.2039\ \text{m}$$

From Fig. a,

$$a = 1\ \text{m} - h'_A = 0.7961\ \text{m}$$

Then,

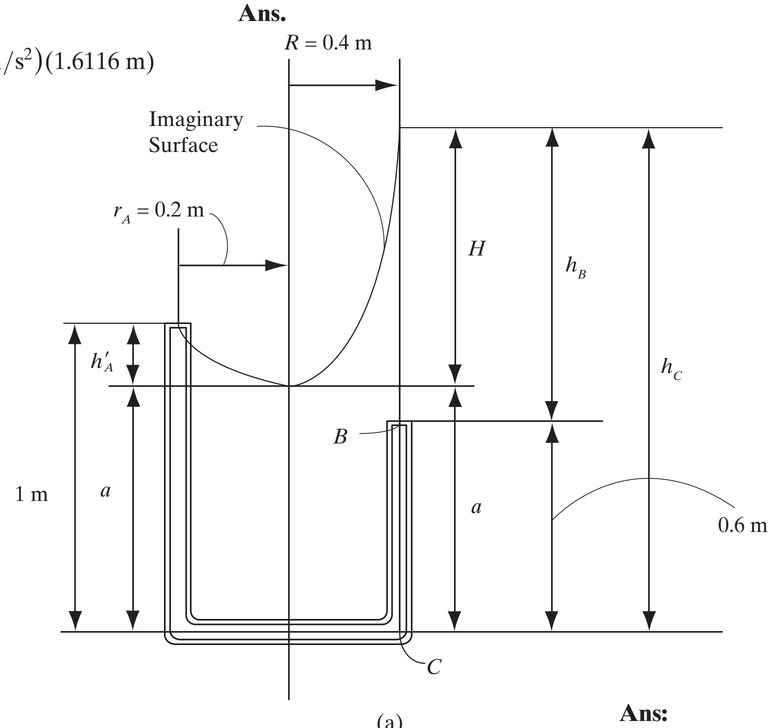
$$h_C = H + a = 0.8155\ \text{m} + 0.7961\ \text{m} = 1.6116\ \text{m}$$

$$h_B = h_C - 0.6\ \text{m} = 1.6116\ \text{m} - 0.6\ \text{m} = 1.0116\ \text{m}$$

Finally,

$$p_B = \rho_w g h_B = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1.0116\ \text{m})$$
$$= 9.924\left(10^3\right)\ \text{Pa}$$
$$= 9.92\ \text{kPa} \qquad \textbf{Ans.}$$

$$p_C = \rho_w g h_C = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1.6116\ \text{m})$$
$$= 15.81\left(10^3\right)\ \text{Pa}$$
$$= 15.8\ \text{kPa} \qquad \textbf{Ans.}$$

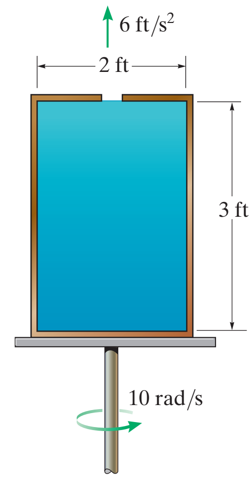


(a)

Ans:
$p_B = 9.92$ kPa
$p_C = 15.8$ kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

***2–176.** The cylindrical container has a height of 3 ft and a diameter of 2 ft. If it is filled with water through the hole in its center, determine the maximum pressure the water exerts on the container when it undergoes the motion shown.



SOLUTION

Pressure due to vertical acceleration.

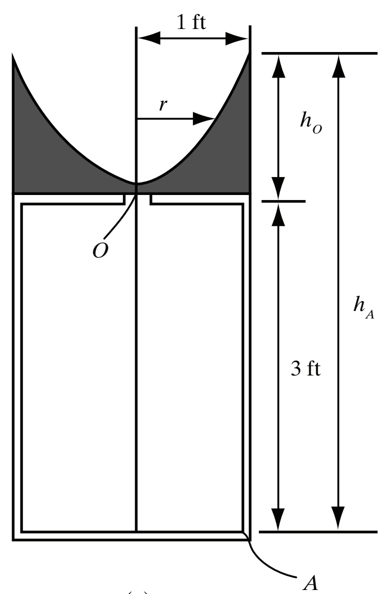
The container undergoes an upward acceleration of $a_C = 6\ \text{ft/s}^2$, and the resulting Maximum pressure occurs at the bottom of the container.

$$(p_{a_c})_{max} = \gamma h\left(1 + \frac{a_C}{g}\right) = (62.4\ \text{lb/ft}^3)(3\ \text{ft})\left(1 + \frac{6\ \text{ft/s}^2}{32.2\ \text{ft/s}}\right) = 222.08\ \text{lb/ft}^2$$

Pressure due to rotation.

Since the container is fully filled and the pressure at the center, O, of the lid is atmospheric pressure, the imaginary parabolic surface above the lid will be formed as if there were no lid, Fig. *a*,

$$h = \frac{\omega^2}{2g}r^2; \qquad h_o = \left[\frac{(10\ \text{rad/s})^2}{2(32.2\ \text{ft/s}^2)}\right](1\ \text{ft})^2 = 1.5528\ \text{ft}$$



(a)

Total pressure.

The maximum pressure is

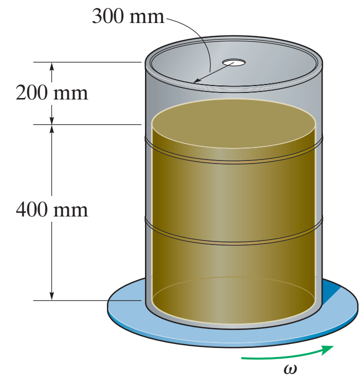
$$\begin{aligned} p_{max} &= (p_{a_c})_{max} + (p_w)_{max} \\ &= 222.08\ \text{lb/ft}^2 + (62.4\ \text{lb/ft}^3)(1.5528\ \text{ft}) \\ &= (318.98\ \text{lb/ft}^2)\left(\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right) = 2.22\ \text{psi} \end{aligned}$$ **Ans.**

Note: The same answer is obtained if the vertical acceleration is treated as "additional gravity," leading to $g = 32.2 + 6 = 38.2\ \text{ft/s}^2$ and $\gamma_w = 62.4\,(38.2/32.2) = 74.03\ \text{lb/ft}^3$, and from there the problem is treated as a rotation-only scenario.

Ans:
2.22 psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–177. The drum has a hole in the center of its lid and contains kerosene to a level of 400 mm when $\omega = 0$. If the drum is placed on the platform and it attains an angular velocity of 12 rad/s, determine the resultant force the kerosene exerts on the lid.



SOLUTION

The volume of the air contained in the paraboloid must be the same as the volume of air in the drum when it is not rotating. Since the volume of the paraboloid is equal to one half the volume of the cylinder of the same radius and height, then

$$\forall_{Air} = \forall_{pb}$$

$$\pi(0.3\text{ m})^2(0.2) = \frac{1}{2}(\pi r_i^2 h)$$

$$r_i^2 h = 0.036 \qquad \textbf{(1)}$$

Then

$$h = \frac{w^2}{2g}r^2; \qquad h = \left[\frac{12^2}{2(9.81)}\right]r_i^2 = 7.3394 r_i^2 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$h = 0.5140\text{ m} \qquad r_i = 0.2646\text{ m}$$

From Section 2–14, beneath the lid,

$$p = \left(\frac{\gamma\omega^2}{2g}\right)r^2 + C$$

Since $\gamma = \rho g$, this equation becomes

$$p = \left(\frac{\rho\omega^2}{2}\right)r^2 + C$$

At $r = r_i$, $p = 0$. Then

$$0 = \frac{\rho\omega^2}{2}r_i^2 + C$$

$$C = -\frac{\rho\omega^2}{2}r_i^2$$

Thus,

$$p = \frac{\rho\omega^2}{2}(r^2 - r_i^2)$$

2–177. Continued

Then the differential force dF acting on the differential annular element of area $dA = 2\pi r dr$ shown shaded in Fig. *a* is

$$dF = pdA = \frac{\rho\omega^2}{2}\left(r^2 - r_i^2\right)(2\pi r dr)$$

$$= \pi\rho\omega^2\left(r^3 - r_i^2 r\right)dr$$

$$F = \int dF = \pi\rho\omega^2 \int_{r_i}^{r_o} \left(r^3 - r_i^2 r\right)dr$$

$$= \pi\rho\omega^2\left(\frac{r^4}{4} - \frac{r_i^2}{2}r^2\right)\Bigg|_{r_i}^{r_o}$$

$$= \pi\rho\omega^2\left(\frac{r_o^4}{4} - \frac{r_i^2 r_o^2}{2} + \frac{r_i^4}{4}\right)$$

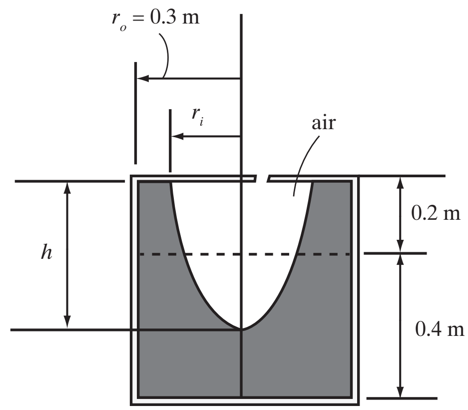
$$= \frac{\pi}{4}\rho\omega^2\left(r_o^4 - 2r_i^2 r_o^2 + r_i^4\right)$$

$$= \frac{\pi}{4}\rho\omega^2\left(r_o^2 - r_i^2\right)^2$$

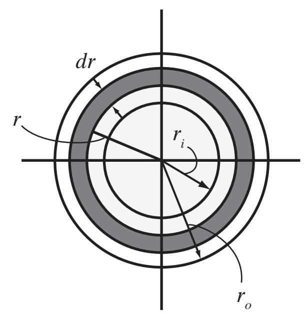
Here $\rho = \rho_{ke} = 814\ \text{kg/m}^3$, $\omega = 12\ \text{rad/s}$, $r_o = 0.3\ \text{m}$ and $r_i = 0.2646\ \text{m}$

$$F = \frac{\pi}{4}\left(814\ \text{kg/m}^3\right)(12\ \text{rad/s})^2\left[(0.3\ \text{m})^2 - (0.2646\ \text{m})^2\right]^2$$

$$= 36.69\ \text{N} = 36.7\ \text{N}$$ **Ans.**



(a)



(b)

Ans:
36.7 N