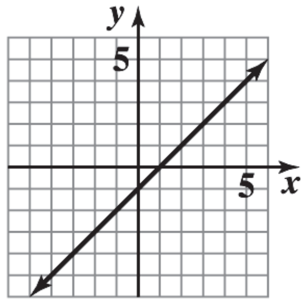
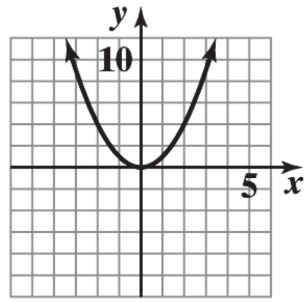
2 FUNCTIONS AND GRAPHS

EXERCISE 2-1

2.



4.



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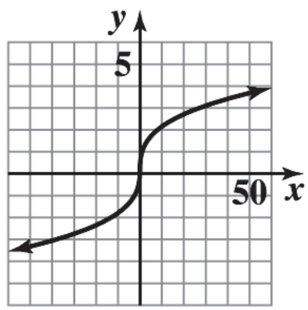

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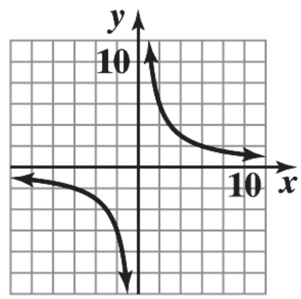


10. The table specifies a function, since for each domain value there corresponds one and only one range value.

12. The table does not specify a function, since more than one range value corresponds to a given domain value.
(Range values 1, 2 correspond to domain value 9.)

14. This is a function.

16. The graph specifies a function; each vertical line in the plane intersects the graph in at most one point.

18. The graph does not specify a function. There are vertical lines which intersect the graph in more than one point. For example, the y-axis intersects the graph in two points.

20. The graph does not specify a function.

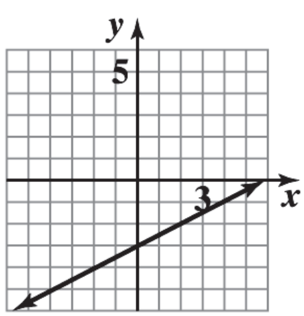
22. $y = 10 - 3x$ is linear.

24. $x^2 - y = 8$ is neither linear nor constant.

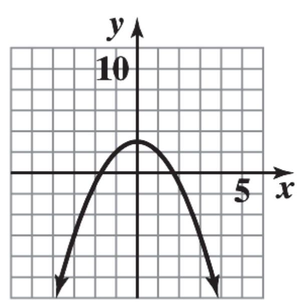
26. $y = \frac{2+x}{3} + \frac{2-x}{3} = \frac{2}{3} + \frac{x}{3} + \frac{2}{3} - \frac{x}{3}$
$= \frac{4}{3}$ which is constant.

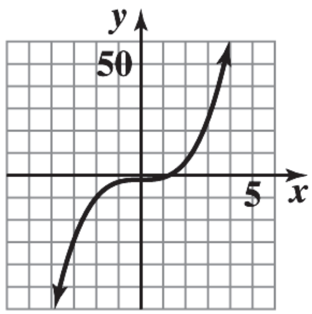
28. $9x - 2y + 6 = 0$ is linear.

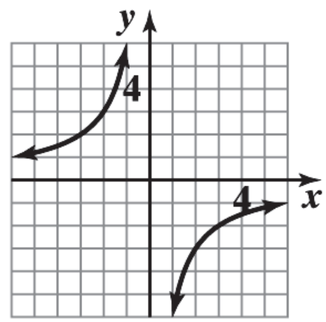
30.



32.



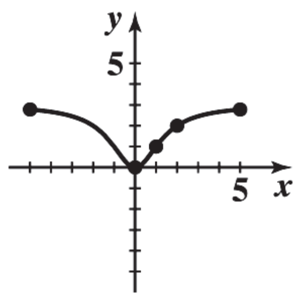
34. 

36. 

38. $f(x) = \dfrac{3x^2}{x^2+2}$. Since the denominator is bigger than 1, we note that the values of f are between 0 and 3.

Furthermore, the function f has the property that $f(-x) = f(x)$. So, adding points $x = 3$, $x = 4$, $x = 5$, we have:

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$F(x)$	2.78	2.67	2.45	2	1	0	1	2	2.45	2.67	2.78

The sketch is: 

40. $y = f(4) = 0$

42. $y = f(-2) = 3$

44. $f(x) = 3, x < 0$ at $x = -4, -2$

46. $f(x) = 4$ at $x = 5$

48. All real numbers

50. All real numbers except $x = 2$

52. $x > -5$

54. Given $6x - 7y = 21$. Solving for y we have: $-7y = 21 - 6x$ and $y = \dfrac{6}{7}x - 3$.

This equation specifies a function. The domain is R, the set of real numbers.

56. Given $x(x+y) = 4$. Solving for y we have: $xy + x^2 = 4$ and $y = \dfrac{4-x^2}{x}$.

This equation specifies a function. The domain is all real numbers except 0.

58. Given $x^2 + y^2 = 9$. Solving for y we have: $y^2 = 9 - x^2$ and $y = \pm\sqrt{9-x^2}$.

This equation does not define y as a function of x. For example, when $x = 0$, $y = \pm 3$.

60. Given $\sqrt{x} - y^3 = 0$.. Solving for y we have: $y^3 = \sqrt{x}$ and $y = x^{1/6}$.

This equation specifies a function. The domain is all nonnegative real numbers, i.e., $x \ge 0$.

62. $f(-5) = (-5)^2 - 4 = 25 - 4 = 21$

64. $f(x-2) = (x-2)^2 - 4 = x^2 - 4x + 4 - 4 = x^2 - 4x$

66. $f(10x) = (10x)^2 - 4 = 100x^2 - 4$

68. $f(\sqrt{x}) = \left(\sqrt{x}\right)^2 - 4 = x - 4$

70. $f(-3) + f(h) = (-3)^2 - 4 + h^2 - 4 = 5 + h^2 - 4 = h^2 + 1$

72. $f(-3+h) = (-3+h)^2 - 4 = 9 - 6h + h^2 - 4 = 5 - 6h + h^2$

74. $f(-3+h) - f(-3) = \left[(-3+h)^2 - 4\right] - \left[(-3)^2 - 4\right] = (9 - 6h + h^2 - 4) - (9 - 4) = -6h + h^2$

76. (A) $f(x+h) = -3(x+h)+9 = -3x-3h+9$

(B) $f(x+h)-f(x) = (-3x-3h+9)-(-3x+9) = -3h$

(C) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-3h}{h} = -3$

78. (A) $f(x+h) = 3(x+h)^2+5(x+h)-8$
$= 3(x^2+2xh+h^2)+5x+5h-8$
$= 3x^2+6xh+3h^2+5x+5h-8$

(B) $f(x+h)-f(x) = (3x^2+6xh+3h^2+5x+5h-8)-(3x^2+5x-8)$
$= 6xh+3h^2+5h$

(C) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{6xh+3h^2+5h}{h} = 6x+3h+5$

80. (A) $f(x+h) = x^2+2xh+h^2+40x+40h$

(B) $f(x+h)-f(x) = 2xh+h^2+40h$

(C) $\dfrac{f(x+h)-f(x)}{h} = 2x+h+40$

82. Given $A = \ell w = 81$.

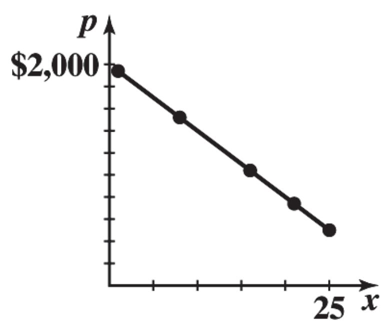
Thus, $w = \dfrac{81}{\ell}$. Now $P = 2\ell + 2w = 2\ell + 2\left(\dfrac{81}{\ell}\right) = 2\ell + \dfrac{162}{\ell}$.

The domain is $\ell > 0$.

84. Given $P = 2\ell + 2w = 160$ or $\ell + w = 80$ and $\ell = 80 - w$.

Now $A = \ell w = (80-w)w$ and $A = 80w - w^2$.

The domain is $0 < w < 80$. [<u>Note</u>: $w < 80$ since $w \geq 80$ implies $\ell \leq 0$.]

86. (A) 

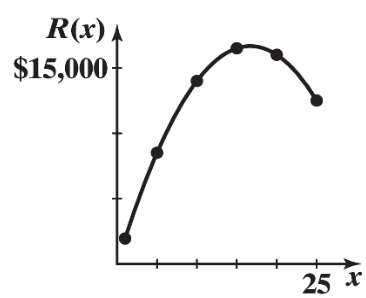
(B) $p(11) = 1{,}340$ dollars per computer
$p(18) = 920$ dollars per computer

88. (A) $R(x) = xp(x)$
$= x(2{,}000 - 60x)$ thousands of dollars

Domain: $1 \leq x \leq 25$

(B) Table 11 Revenue

x(thousands)	$R(x)$(thousands)
1	$1,940
5	8,500
10	14,000
15	16,500
20	16,000
25	12,500

(C) 

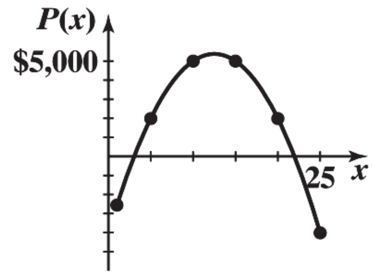
90. (A) $P(x) = R(x) - C(x)$
$= x(2{,}000 - 60x) - (4{,}000 + 500x)$ thousand dollars
$= 1{,}500x - 60x^2 - 4{,}000$

Domain: $1 \le x \le 25$

(B) Table 13 Profit

x (thousands)	$P(x)$ (thousands)
1	–\$2,560
5	2,000
10	5,000
15	5,000
20	2,000
25	–4,000

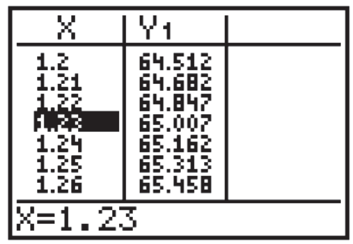
(C)



92. (A) 1.2 inches

(B) Evaluate the volume function for x = 1.21, 1.22, …, and choose the value of x whose volume is closest to 65.

(C) $x = 1.23$ to two decimal places



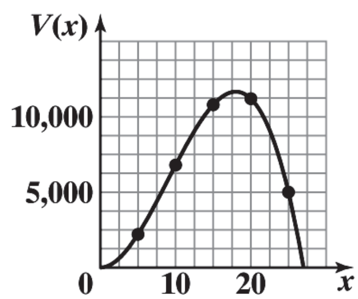
94. (A) $V(x) = x^2(108 - 4x)$

(B) $0 < x < 27$

(C) Table 16 Volume

x	$V(x)$
5	2,200
10	6,800
15	10,800
20	11,200
25	5,000

(D)



96. (A) Given $5v - 2s = 1.4$. Solving for v, we have:
$v = 0.4s + 0.28$.
If $s = 0.51$, then $v = 0.4(0.51) + 0.28 = 0.484$ or 48.4%.

(B) Solving the equation for s, we have:
$s = 2.5v - 0.7$.
If $v = 0.51$, then $s = 2.5(0.51) - 0.7 = 0.575$ or 57.5%.

EXERCISE 2-2

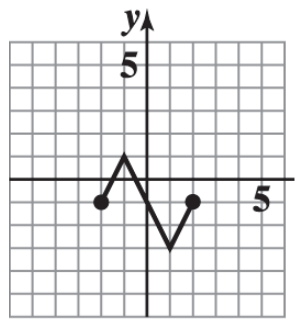
2. $f(x) = -4x + 12$ Domain: all real numbers; range: all real numbers.

4. $f(x) = 3 + \sqrt{x}$ Domain: $[0, \infty)$; range: $[3, \infty)$.

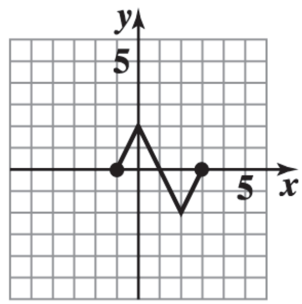
6. $f(x) = -5|x| + 2$ Domain: all real numbers; range: $(-\infty, 2]$.

8. $f(x) = 20 - 10\sqrt[3]{x}$ Domain: all real numbers; range: all real numbers.

10.



12.



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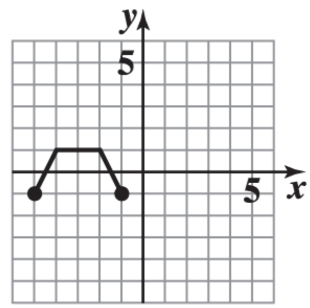

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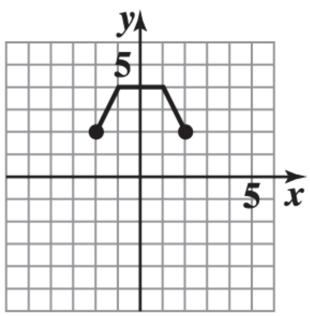

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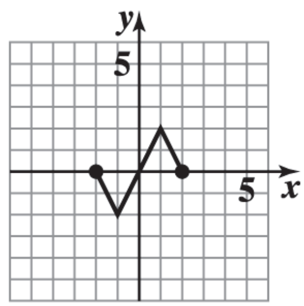
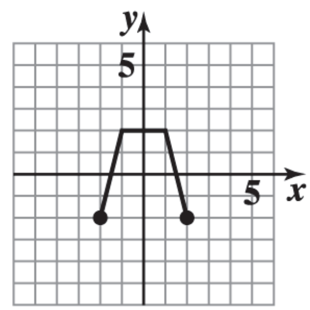
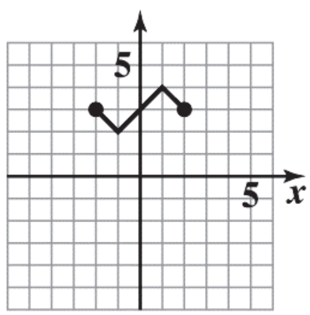


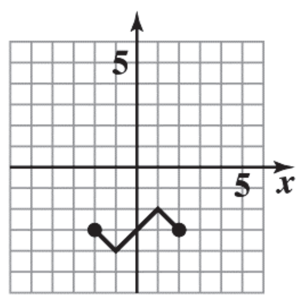
20.



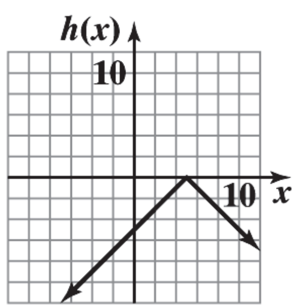
22.



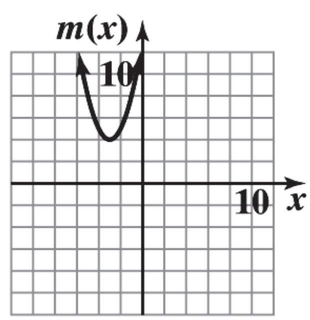
24.



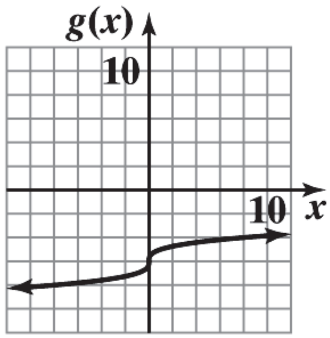
26. The graph of $h(x) = -|x - 5|$ is the graph of $y = |x|$ reflected in the x axis and shifted 5 units to the right.



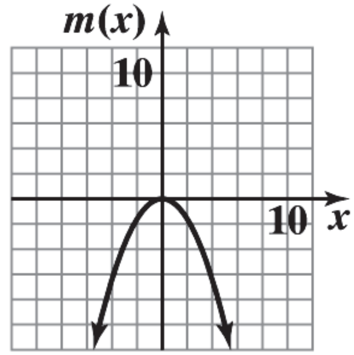
28. The graph of $m(x) = (x + 3)^2 + 4$ is the graph of $y = x^2$ shifted 3 units to the left and 4 units up.



30. The graph of $g(x) = -6 + \sqrt[3]{x}$ is the graph of $y = \sqrt[3]{x}$ shifted 6 units down.



32. The graph of $m(x) = -0.4x^2$ is the graph of $y = x^2$ reflected in the x axis and vertically contracted by a factor of 0.4.



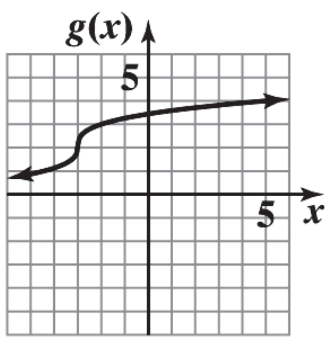
34. The graph of the basic function $y = |x|$ is shifted 3 units to the right and 2 units up. $y = |x - 3| + 2$

36. The graph of the basic function $y = |x|$ is reflected in the x axis, shifted 2 units to the left and 3 units up. Equation: $y = 3 - |x + 2|$

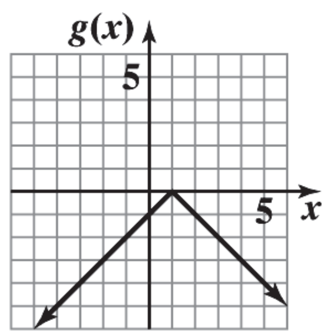
38. The graph of the basic function $\sqrt[3]{x}$ is reflected in the x axis and shifted up 2 units. Equation: $y = 2 - \sqrt[3]{x}$

40. The graph of the basic function $y = x^3$ is reflected in the x axis, shifted to the right 3 units and up 1 unit. Equation: $y = 1 - (x - 3)^3$

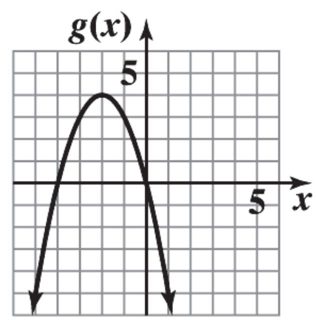
42. $g(x) = \sqrt[3]{x+3} + 2$



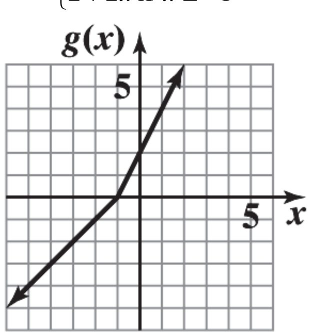
44. $g(x) = -|x - 1|$



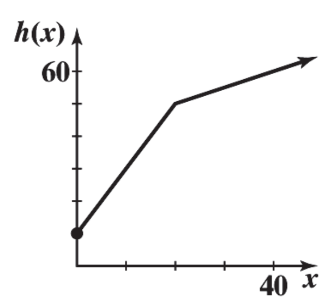
46. $g(x) = 4 - (x + 2)^2$



48. $g(x) = \begin{cases} x+1 & \text{if } x < -1 \\ 2+2x & \text{if } x \geq -1 \end{cases}$

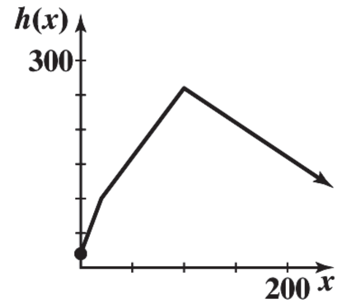


50. $h(x) = \begin{cases} 10+2x & \text{if } 0 \leq x \leq 20 \\ 40+0.5x & \text{if } x > 20 \end{cases}$



52.

$$h(x) = \begin{cases} 4x+20 & \text{if} \quad 0 \le x \le 20 \\ 2x+60 & \text{if} \quad 20 < x \le 100 \\ -x+360 & \text{if} \quad x > 100 \end{cases}$$



54. The graph of the basic function $y = x$ is reflected in the x axis and vertically expanded by a factor of 2. Equation: $y = -2x$

56. The graph of the basic function $y = |x|$ is vertically expanded by a factor of 4. Equation: $y = 4|x|$

58. The graph of the basic function $y = x^3$ is vertically contracted by a factor of 0.25. Equation: $y = 0.25x^3$.

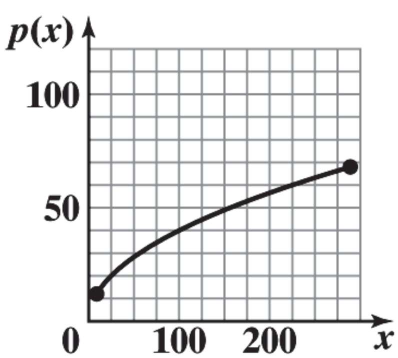
60. Vertical shift, reflection in y axis.
Reversing the order does not change the result. Consider a point (a, b) in the plane. A vertical shift of k units followed by a reflection in y axis moves (a, b) to $(a, b + k)$ and then to $(-a, b + k)$. In the reverse order, a reflection in y axis followed by a vertical shift of k units moves (a, b) to $(-a, b)$ and then to $(-a, b + k)$. The results are the same.

62. Vertical shift, vertical expansion.
Reversing the order can change the result. For example, let (a, b) be a point in the plane. A vertical shift of k units followed by a vertical expansion of h $(h > 1)$ moves (a, b) to $(a, b + k)$ and then to $(a, bh + kh)$. In the reverse order, a vertical expansion of h followed by a vertical shift of k units moves (a, b) to (a, bh) and then to $(a, bh + k)$; $(a, bh + kh) \ne (a, bh + k)$.

64. Horizontal shift, vertical contraction.
Reversing the order does not change the result. Consider a point (a, b) in the plane. A horizontal shift of k units followed by a vertical contraction of h $(0 < h < 1)$ moves (a, b) to $(a + k, b)$ and then to $(a + k, bh)$. In the reverse order, a vertical contraction of h followed by a horizontal shift of k units moves (a, b) to (a, bh) and then to $(a + k, bh)$. The results are the same.

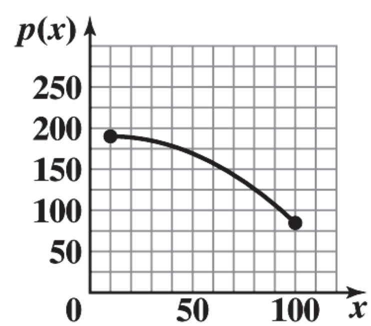
66. (A) The graph of the basic function $y = \sqrt{x}$ is vertically expanded by a factor of 4.

(B)



68. (A) The graph of the basic function $y = x^2$ is reflected in the x axis, vertically contracted by a factor of 0.013, and shifted 10 units to the right and 190 units up.

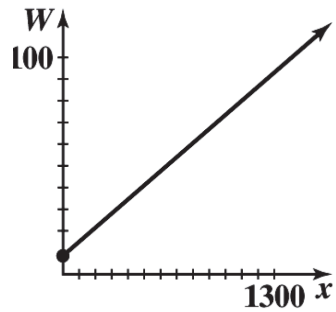
(B)



70. (A) Let x = number of kwh used in a winter month. For $0 \le x \le 700$, the charge is $8.5 + .065x$. At $x = 700$, the charge is \$54. For $x > 700$, the charge is $54 + .053(x - 700) = 16.9 + 0.053x$. Thus,

$$W(x) = \begin{cases} 8.5 + .065x & \text{if } 0 \le x \le 700 \\ 16.9 + 0.053x & \text{if } x > 700 \end{cases}$$

(B)

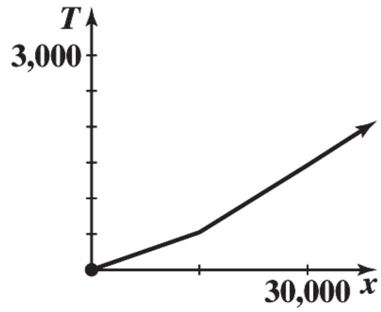


72. (A) Let x = taxable income. If $0 \le x \le 15{,}000$, the tax due is $\$.035x$. At $x = 15{,}000$, the tax due is \$525. For $15{,}000 < x \le 30{,}000$, the tax due is $525 + .0625(x - 15{,}000) = .0625x - 412.5$. For $x > 30{,}000$, the tax due is $1{,}462.5 + .0645(x - 30{,}000) = .0645x - 472.5$.

Thus,

$$T(x) = \begin{cases} 0.035x & \text{if } 0 \le x \le 15{,}000 \\ 0.0625x - 412.5 & \text{if } 15{,}000 < x \le 30{,}000 \\ 0.0645x - 472.5 & \text{if } x > 30{,}000 \end{cases}$$

(B)

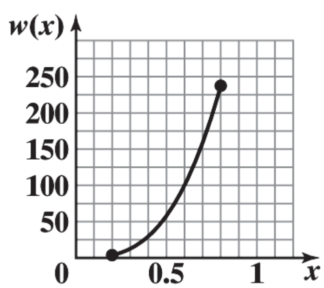


(C) $T(20{,}000) = \$837.50$

$T(35{,}000) = \$1{,}785$

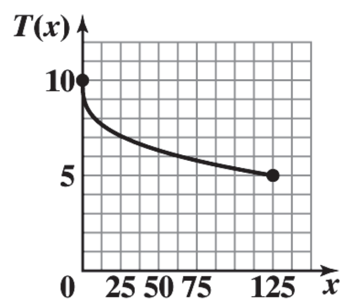
74. (A) The graph of the basic function $y = x^3$ is vertically expanded by a factor of 463.

(B)



76. (A) The graph of the basic function $y = \sqrt[3]{x}$ is reflected in the x axis and shifted up 10 units.

(B)



EXERCISE 2-3

2. $x^2 + 16x$ (standard form)

$x^2 + 16x + 64 - 64$ (completing the square)

$(x + 8)^2 - 64$ (vertex form)

4. $x^2 - 12x - 8$ (standard form)

$(x^2 - 12x) - 8$

$(x^2 - 12x + 36) + 8 - 36$ (completing the square)

$(x - 6)^2 - 44$ (vertex form)

6. $3x^2 + 18x + 21$ (standard form)

$3(x^2 + 6x) + 21$

$3(x^2 + 6x + 9 - 9) + 21$ (completing the square)

$3(x + 3)^2 + 21 - 27$

$3(x + 3)^2 - 6$ (vertex form)

8. $-5x^2 + 15x - 11$ (standard form)

$-5(x^2 - 3x) - 11$

$-5(x^2 - 3x + \frac{9}{4} - \frac{9}{4}) - 11$ (completing the square)

$-5(x - \frac{3}{2})^2 - 11 + \frac{45}{4}$

$-5(x - \frac{3}{2})^2 + \frac{1}{4}$ (vertex form)

10. The graph of $g(x)$ is the graph of $y = x^2$ shifted right 1 unit and down 6 units; $g(x) = (x - 1)^2 - 6$.

12. The graph of $n(x)$ is the graph of $y = x^2$ reflected in the x axis, then shifted right 4 units and up 7 units; $n(x) = -(x - 4)^2 + 7$.

14. (A) g (B) m (C) n (D) f

16. (A) x intercepts: –5, –1; y intercept: –5 (B) Vertex: (–3, 4)
(C) Maximum: 4 (D) Range: $y \le 4$ or $(-\infty, 4]$

18. (A) x intercepts: 1, 5; y intercept: 5 (B) Vertex: (3, –4)
(C) Minimum: –4 (D) Range: $y \ge -4$ or $[-4, \infty)$

20. $g(x) = -(x + 2)^2 + 3$

(A) x intercepts:

$$-(x + 2)^2 + 3 = 0$$
$$(x + 2)^2 = 3$$
$$x + 2 = \pm\sqrt{3}$$
$$x = -2 - \sqrt{3}, -2 + \sqrt{3}$$

y intercept: –1

(B) Vertex: (–2, 3) (C) Maximum: 3 (D) Range: $y \le 3$ or $(-\infty, 3]$

22. $n(x) = (x - 4)^2 - 3$

(A) x intercepts:

$$(x - 4)^2 - 3 = 0$$
$$(x - 4)^2 = 3$$
$$x - 4 = \pm\sqrt{3}$$
$$x = 4 - \sqrt{3}, 4 + \sqrt{3}$$

y intercept: 13

(B) Vertex: (4, –3) (C) Minimum: –3 (D) Range: $y \ge -3$ or $[-3, \infty)$

24. $y = -(x - 4)^2 + 2$

26. $y = [x - (-3)]^2 + 1$ or $y = (x + 3)^2 + 1$

28. $g(x) = x^2 - 6x + 5 = x^2 - 6x + 9 - 4 = (x - 3)^2 - 4$

(A) x intercepts:
$$(x - 3)^2 - 4 = 0$$
$$(x - 3)^2 = 4$$
$$x - 3 = \pm 2$$
$$x = 1, 5$$

y intercept: 5

(B) Vertex: (3, –4) (C) Minimum: –4 (D) Range: $y \ge -4$ or $[-4, \infty)$

30. $s(x) = -4x^2 - 8x - 3 = -4\left[x^2 + 2x + \frac{3}{4}\right] = -4\left[x^2 + 2x + 1 - \frac{1}{4}\right]$
$$= -4\left[(x+1)^2 - \frac{1}{4}\right] = -4(x + 1)^2 + 1$$

(A) x intercepts:
$$-4(x + 1)^2 + 1 = 0$$
$$4(x + 1)^2 = 1$$
$$(x + 1)^2 = \frac{1}{4}$$
$$x + 1 = \pm\frac{1}{2}$$
$$x = -\frac{3}{2}, -\frac{1}{2}$$

y intercept: –3

(B) Vertex: (–1, 1) (C) Maximum: 1 (D) Range: $y \le 1$ or $(-\infty, 1]$

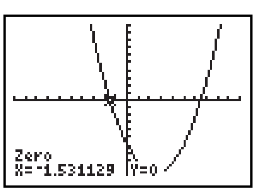
32. $v(x) = 0.5x^2 + 4x + 10 = 0.5[x^2 + 8x + 20] = 0.5[x^2 + 8x + 16 + 4]$
$$= 0.5[(x + 4)^2 + 4]$$
$$= 0.5(x + 4)^2 + 2$$

(A) x intercepts: none
y intercept: 10

(B) Vertex: (–4, 2) (C) Minimum: 2 (D) Range: $y \ge 2$ or $[2, \infty)$

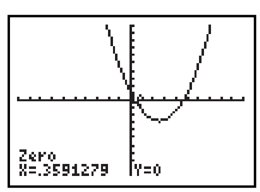
34. $g(x) = -0.6x^2 + 3x + 4$

(A) $g(x) = -2$: $-0.6x^2 + 3x + 4 = -2$
$$0.6x^2 - 3x - 6 = 0$$



$x = -1.53, 6.53$

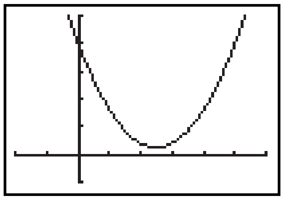
(B) $g(x) = 5$: $-0.6x^2 + 3x + 4 = 5$
$$-0.6x^2 + 3x - 1 = 0$$
$$0.6x^2 - 3x + 1 = 0$$



$x = 0.36, 4.64$

(C) $g(x) = 8$: $-0.6x^2 + 3x + 4 = 8$

$$-0.6x^2 + 3x - 4 = 0$$
$$0.6x^2 - 3x + 4 = 0$$



No solution

36. Using a graphing utility with $y = 100x - 7x^2 - 10$ and the calculus option with maximum command, we obtain 347.1429 as the maximum value.

38. $m(x) = 0.20x^2 - 1.6x - 1 = 0.20(x^2 - 8x - 5)$

$= 0.20[(x-4)^2 - 21] = 0.20(x-4)^2 - 4.2$

(A) x intercepts:

$0.20(x-4)^2 - 4.2 = 0$

$(x-4)^2 = 21$

$x - 4 = \pm\sqrt{21}$

$x = 4 - \sqrt{21} = -0.6,\ 4 + \sqrt{21} = 8.6;$

y intercept: –1

(B) Vertex: (4, –4.2) (C) Minimum: –4.2 (D) Range: $y \geq -4.2$ or $[-4.2, \infty)$

40. $n(x) = -0.15x^2 - 0.90x + 3.3 = -0.15(x^2 + 6x - 22) = -0.15[(x+3)^2 - 31] = -0.15(x+3)^2 + 4.65$

(A) x intercepts:

$-0.15(x+3)^2 + 4.65 = 0$

$(x+3)^2 = 31$

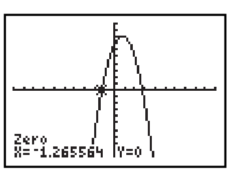
$x + 3 = \pm\sqrt{31}$

$x = -3 - \sqrt{31} = -8.6, -3 + \sqrt{31} = 2.6;$

y intercept: 3.30

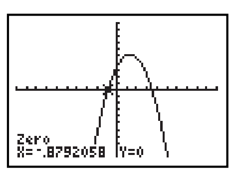
(B) Vertex: (–3, 4.65) (C) Maximum: 4.65 (D) Range: $x \leq 4.65$ or $(-\infty, 4.65]$

42.



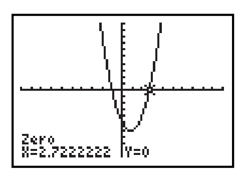
$x = -1.27, 2.77$

44.



$-0.88 \leq x \leq 3.52$

46.



$x < -1$ or $x > 2.72$

48. f is a quadratic function and max $f(x) = f(-3) = -5$

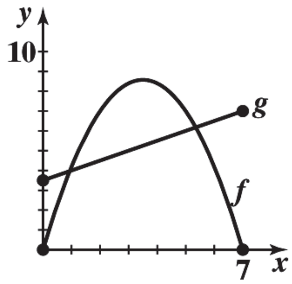
Axis: $x = -3$

Vertex: $(-3, -5)$

Range: $y \le -5$ or $(-\infty, -5]$

x intercepts: None

50. (A)



(B) $f(x) = g(x)$: $-0.7x(x-7) = 0.5x + 3.5$

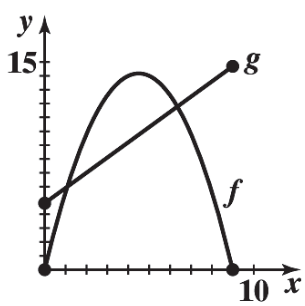
$$-0.7x^2 + 4.4x - 3.5 = 0$$

$$x = \frac{-4.4 \pm \sqrt{(4.4)^2 - 4(0.7)(3.5)}}{-1.4} = 0.93, 5.35$$

(C) $f(x) > g(x)$ for $0.93 < x < 5.35$

(D) $f(x) < g(x)$ for $0 \le x < 0.93$ or $5.35 < x \le 7$

52. (A)



(B) $f(x) = g(x)$: $-0.7x^2 + 6.3x = 1.1x + 4.8$

$$-0.7x^2 + 5.2x - 4.8 = 0$$

$$0.7x^2 - 5.2x + 4.8 = 0$$

$$x = \frac{-(-5.2) \pm \sqrt{(-5.2)^2 - 4(0.7)(4.8)}}{1.4} = 1.08, 6.35$$

(C) $f(x) > g(x)$ for $1.08 < x < 6.35$

(D) $f(x) < g(x)$ for $0 \le x < 1.08$ or $6.35 < x \le 9$

54. A quadratic with no real zeros will not intersect the x–axis.

56. Such an equation will have $b^2 - 4ac = 0$.

58. Such an equation will have $\frac{k}{a} < 0$.

60. $ax^2 + bx + c = a(x-h)^2 + k$

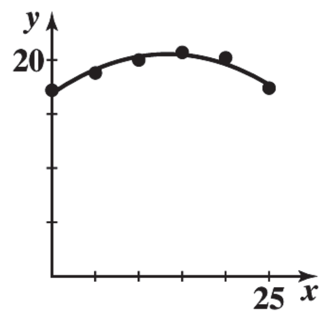
$= a(x^2 - 2hx + h^2) + k$

$= ax^2 - 2ahx + ah^2 + k$

Equating constant terms gives $k = c - ah^2$. Since h is the vertex, we have $h = -\frac{b}{2a}$. Substituting then gives $k = c - ah^2$

$= \frac{4ac - b}{4a}$

(B)



$f(x) = -0.0169x^2 + 0.47x + 17.1$

(A)

x	Mkt Share	$f(x)$
0	17.2	17.1
5	18.8	19.0
10	20.0	20.1
15	20,7	20.3
20	20.2	19.7
25	17.4	18.3
30	16.4	16.0

(C) For 2020, $x = 40$ and $f(40) = -0.0169(30)^2 + 0.47(40) + 17.1 = 8.9\%$

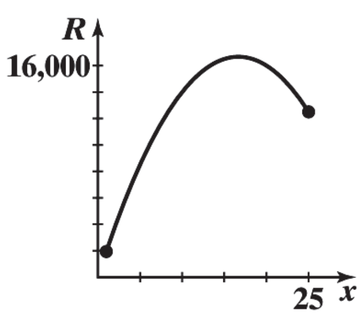
For 2025, $x = 45$ and $f(45) = -0.0169(45)^2 + 0.47(45) + 17.1 = 4.0\%$

(D) Market share rose from 17.2% in 1980 to a maximum of 20.7% in 1995 and then fell to 16.4% in 2010.

64. Verify

66.

(A)



(B) $R(x) = 2{,}000x - 60x^2$

$= -60\left(x^2 - \frac{100}{3}x\right)$

$= -60\left[x^2 - \frac{100}{3}x + \frac{2500}{9} - \frac{2500}{9}\right]$

$= -60\left[\left(x - \frac{50}{3}\right)^2 - \frac{2500}{9}\right]$

$= -60\left(x - \frac{50}{3}\right)^2 + \frac{50{,}000}{3}$

16.667 thousand computers(16,667 computers);
16,666.667 thousand dollars ($16,666,667)

(C) $2000 - 60(50/3) =$ $1,000

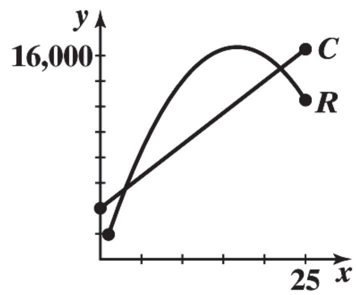
68. (A)

$p\left(\frac{50}{3}\right) =$

$2{,}000 -$

$60\left(\frac{50}{3}\right)$

$= \$1{,}000$



(C) Loss: $1 \le x < 3.035$ or $21.965 < x \le 25$;
Profit: $3.035 < x < 21.965$

(B) $R(x) = C(x)$

$x(2{,}000 - 60x) = 4{,}000 + 500x$

$2{,}000x - 60x^2 = 4{,}000 + 500x$

$60x^2 - 1{,}500x + 4{,}000 = 0$

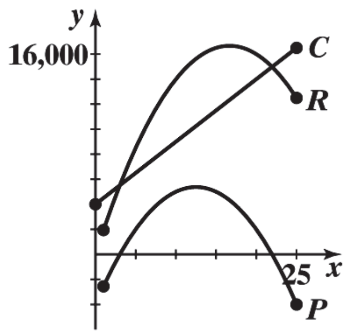
$6x^2 - 150x + 400 = 0$

$x = 3.035, 21.965$

Break-even at 3.035 thousand (3,035) and 21.965 thousand (21,965)

70. (A) $P(x) = R(x) - C(x)$

$= 1{,}500x - 60x^2 - 4{,}000$



(B) and (C) Intercepts and break-even points: 3,035 computers and 21,965 computers

(D) and (E) Maximum profit is \$5,375,000 when 12,500
computers are produced. This is much smaller
than the maximum revenue of \$16,666,667.

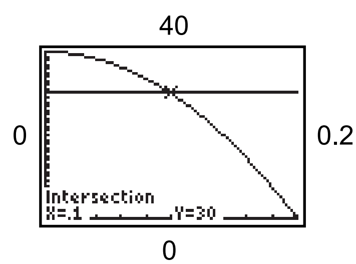
72. Solve: $f(x) = 1{,}000(0.04 - x^2) = 30$

$40 - 1000x^2 = 30$

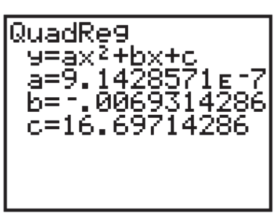
$1000x^2 = 10$

$x^2 = 0.01$

$x = 0.10$ cm



74.



For $x = 2{,}300$, the estimated fuel consumption is
$y = a(2{,}300)^2 + b(2{,}300) + c = 5.6$ mpg.

EXERCISE 2-4

2. $f(x) = 72 + 12x$

(A) Degree: 1

(B) $72 + 12x = 0$

$12x = -72$

$x = -6$

x-intercept: $x = -6$

(C) $f(0) = 72 - 12(0) = 72$

y-intercept: 72

4. $f(x) = x^3(x+5)$

(A) Degree: 4

(B) $x^3(x+5) = 0$

$x = 0, -5$

x-intercepts: $0, -5$

(C) $f(0) = 0(0+5) = 0$

y-intercept: 0

6. $f(x) = x^2 - 4x - 5$

(A) Degree: 2

(B) $(x-5)(x+1) = 0$

$x = -1, 5$

x-intercepts: $-1, 5$

(C) $f(0) = -5$

y-intercept: -5

8. $f(x) = (x^2 - 4)(x^3 + 27)$

(A) Degree: 5

(B) $(x^2 - 4)(x^3 + 27) = 0$

$x = -2, 2, -3$

x-intercepts: $x = -2, 2, -3$

(C) $f(0) = -4(27) = -108$

y-intercept: -108

10. $f(x) = (x+3)^2(8x-4)^6$

(A) Degree: 8

(B) $(x+3)(8x-4) = 0$

$x = -3, \frac{1}{2}$

x-intercepts: $-3, 1/2$

(C) $f(0) = 3^2(-4)^6 = 36,864$

y-intercept: 36,864

12. (A) Minimum degree: 2

(B) Negative – it must have even degree, and positive values in the domain are mapped to negative values in the range.

14. (A) Minimum degree: 3

(B) Negative – it must have odd degree, and positive values in the domain are mapped to negative values in the range.

16. (A) Minimum degree: 4

(B) Positive – it must have even degree, and positive values in the domain are mapped to positive values in the range.

18. (A) Minimum degree: 5

(B) Positive – it must have odd degree, and positive values in the domain are mapped to positive values in the range.

20. A polynomial of degree 7 can have at most 7 x-intercepts.

22. A polynomial of degree 6 may have no x intercepts. For example, the polynomial $f(x) = x^6 + 1$ has no x- intercepts.

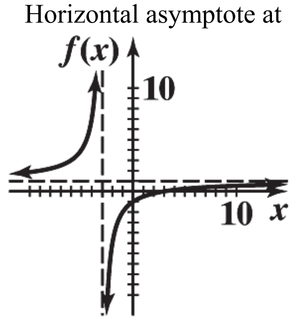
24. (A) Intercepts:

x-intercept(s):	y-intercept:
$x - 3 = 0$ $x = 3$ (3, 0)	$f(0) = \frac{0-3}{0+3} = -1$ (0, –1)

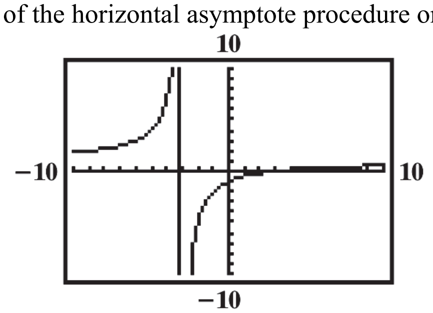
(B) Domain: all real numbers except $x = -3$

(C) Vertical asymptote at $x = -3$ by case 1 of the vertical asymptote procedure on page 90.
Horizontal asymptote at $y = 1$ by case 2 of the horizontal asymptote procedure on page 90.

(D)



(E)



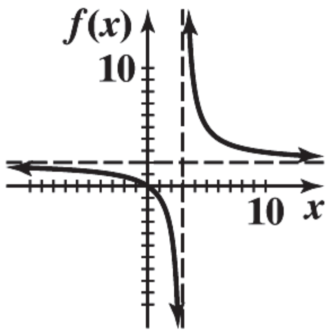
26. (A) Intercepts:

x-intercept(s):	y-intercept:
$2x = 0$ $x = 0$ (0, 0)	$f(0) = \frac{2(0)}{0-3} = 0$ (0, 0)

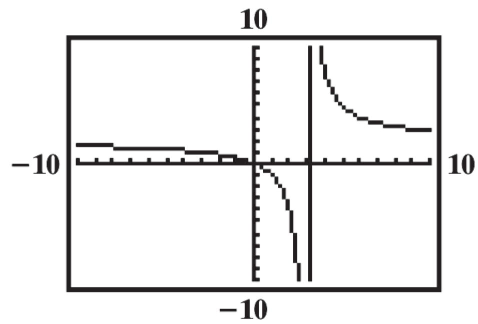
(B) Domain: all real numbers except $x = 3$.

(C) Vertical asymptote at $x = 3$ by case 1 of the vertical asymptote procedure on page 90.
Horizontal asymptote at $y = 2$ by case 2 of the horizontal asymptote procedure on page 90.

(D)



(E)



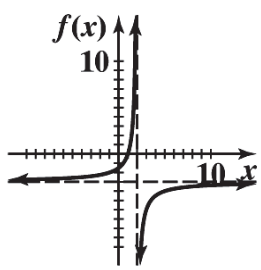
28. (A) Intercepts:

x-intercept:	y-intercept:
$3-3x=0$ $x=1$ (1, 0)	$f(0)=\frac{3-3(0)}{0-2}=-\frac{3}{2}$ $\left(0,-\frac{3}{2}\right)$

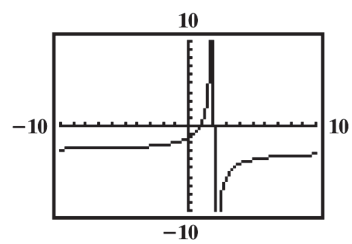
(B) Domain: all real numbers except $x = 2$

(C) Vertical asymptote at $x = 2$ by case 1 of the vertical asymptote procedure on page 90.
Horizontal asymptote at $y = -3$ by case 2 of the horizontal asymptote procedure on page 90.

(D)

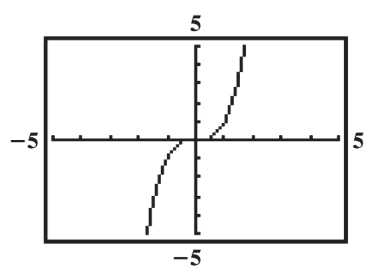


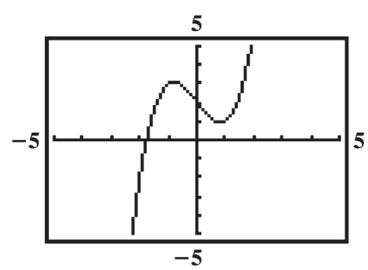
(E)



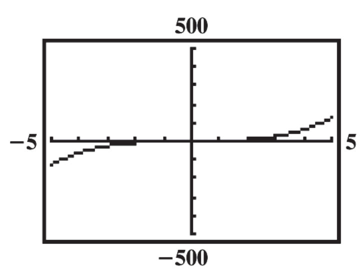
30.

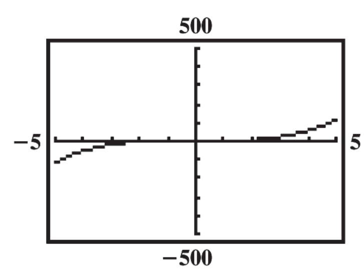
(A)





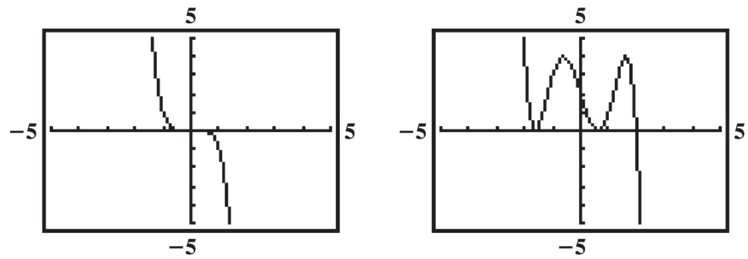
(B)





32.

(A)



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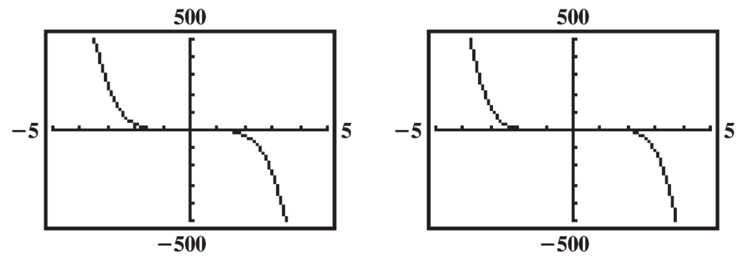
(B)



34. $y = \frac{6}{4}$, by case 2 for horizontal asymptotes on page 90.

36. $y = -\frac{1}{2}$, by case 2 for horizontal asymptotes on page 90.

38. $y = 0$, by case 1 for horizontal asymptotes on page 90.

40. No horizontal asymptote, by case 3 for horizontal asymptotes on page 90.

42. Here we have denominator $(x^2 - 4)(x^2 - 16) = (x-2)(x+2)(x-4)(x+4)$. Since none of these linear terms are factors of the numerator, the function has vertical asymptotes at $x = 2$, $x = -2$, $x = 4$, and $x = -4$.

44. Here we have denominator $x^2 + 7x - 8 = (x-1)(x+8)$. Also, we have numerator $x^2 - 8x + 7 = (x-1)(x-7)$. By case 2 of the vertical asymptote procedure on page 90, we conclude that the function has a vertical asymptote at $x = -8$.

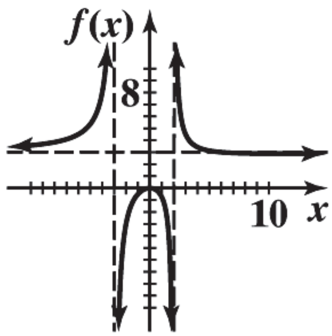
46. Here we have denominator $x^3 - 3x^2 + 2x = x(x^2 - 3x + 2) = x(x-2)(x-1)$. We also have numerator $x^2 + x - 2 = (x+2)(x-1)$. By case 2 of the vertical asymptote procedure on page 90, we conclude that the function has a vertical asymptotes at $x = 0$ and $x = 2$.

48. (A) Intercepts:

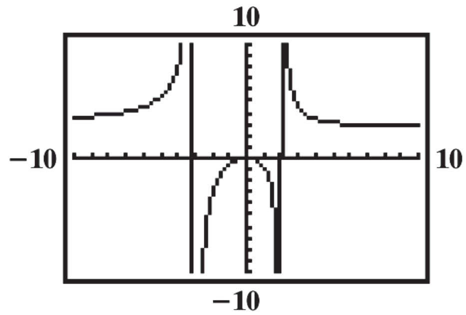
x-intercept(s):	y-intercept:
$3x^2 = 0$	$f(0) = 0$
$x = 0$	
(0, 0)	(0, 0)

(B) Vertical asymptote when $x^2 + x - 6 = (x-2)(x+3) = 0$; so, vertical asymptotes at $x = 2$, $x = -3$. Horizontal asymptote $y = 3$.

(C)



(D)

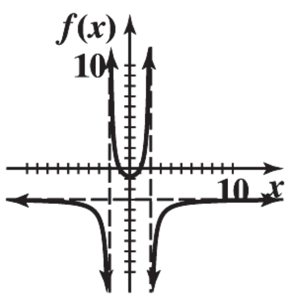


50. (A) Intercepts:

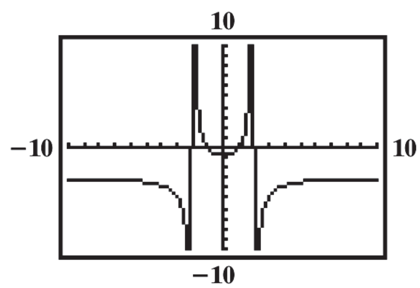
x-intercept(s):	y-intercept:
$3-3x^2=0$ $3x^2=3$ $x=\pm 1$ (1,0), (–1,0)	$f(0)=-\frac{3}{4}$ $\left(0,\frac{-3}{4}\right)$

(B) Vertical asymptotes when $x^2-4=0$; i.e. at $x = 2$ and $x = -2$.
Horizontal asymptote at $y=-3$

(C)



(D)

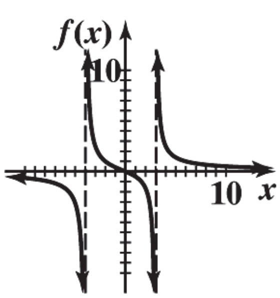


52. (A) Intercepts:

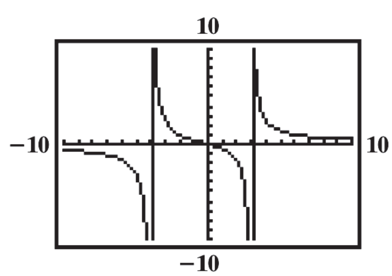
x-intercept(s):	y-intercept:
$5x-10=0$ $x=2$ (2,0)	$f(0)=\frac{-10}{-12}=\frac{5}{6}$ (0,5/6)

(B) Vertical asymptote when $x^2+x-12=(x+4)(x-3)=0$; i.e. when $x = -4$ and when $x = 3$.
Horizontal asymptote at $y = 0$.

(C)



(D)



54. $f(x) = -(x+2)(x-1) = -x^2 - x + 2$

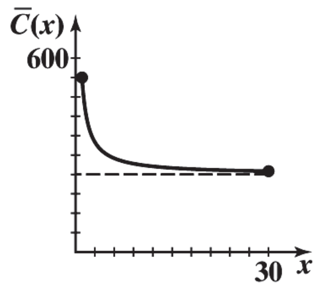
56. $f(x) = x(x+1)(x-1) = x\left(x^2-1\right) = x^3 - x$

58. (A) We want $C(x) = mx + b$. Fixed costs are $b = \$300$ per day. Given $C(20) = 5{,}100$ we have

$$m(20) + 300 = 5{,}100$$
$$20m = 4800$$
$$m = 240$$
$$C(x) = 240x + 300$$

(B) $\bar{C}(x) = \dfrac{C(x)}{x} = \dfrac{240x + 300}{x} = 240 + \dfrac{300}{x}$

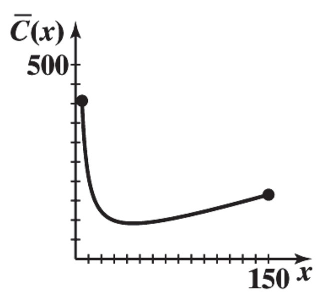
(C)



(D) Average cost tends towards \$240 as production increases.

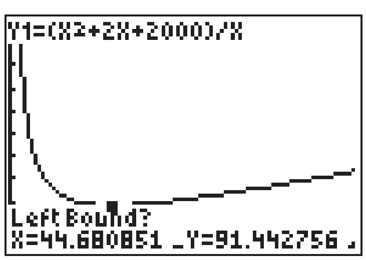
60. (A) $\bar{C}(x) = \dfrac{x^2 + 2x + 2{,}000}{x}$

(B)



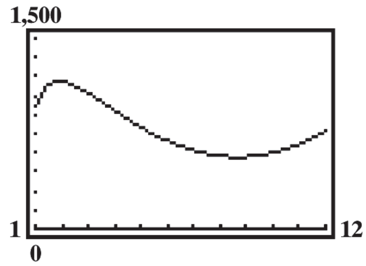
(C) A daily production level of $x = 45$ units per day, results in the lowest average cost of $\bar{C}(45) = \$91.44$ per unit.

(D)



62. (A) $\bar{C}(x) = \dfrac{20x^3 - 360x^2 + 2,300x - 1,000}{x}$

(B)



(C) A minimum average cost of \$566.84 is achieved at a production level of $x = 8.67$ thousand cases per month.

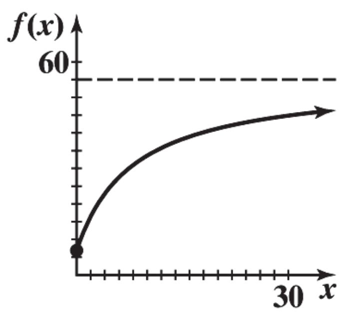
64. (A)

```
CubicReg
 y=ax³+bx²+cx+d
 a=-.0091111111
 b=.5004761905
 c=-7.655555556
 d=269.3571429
```

(B) $y(42) = 156$ eggs

66. (A) The horizontal asymptote is $y = 55$.

(B)



68. (A)

```
CubicReg
 y=ax³+bx²+cx+d
 a=4.4444444E-5
 b=-.0065833333
 c=.2471031746
 d=2.073809524
```

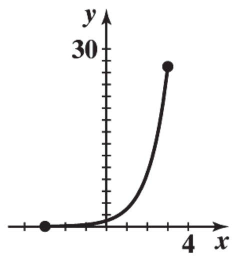
(B) This model gives an estimate of 2.5 divorces per 1,000 marriages.

EXERCISE 2-5

2. A. graph g B. graph f C. graph h D. graph k

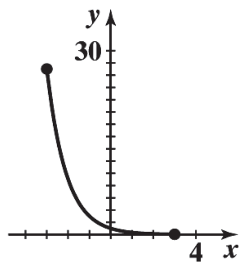
4. $y = 3^x$; [−3, 3]

x	y
–3	$\frac{1}{27}$
–1	$\frac{1}{3}$
0	1
1	3
3	27



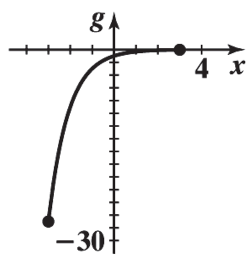
6. $y = 3^{-x}$; [−3, 3]

x	y
–3	27
–1	3
0	1
1	$\frac{1}{3}$
3	$\frac{1}{27}$



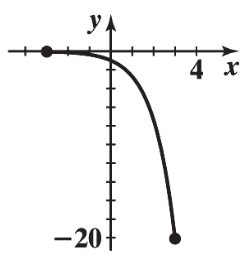
8. $g(x) = -3^{-x}$; [−3, 3]

x	$g(x)$
–3	–27
–1	–3
0	–1
1	$-\frac{1}{3}$
3	$-\frac{1}{27}$



10. $y = -e^x$; [−3, 3]

x	y
–3	≈ –0.05
–1	≈ –0.37
0	–1
1	≈ –2.72
3	≈ –20.09



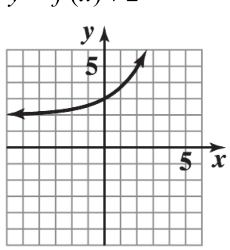
12. The graph of g is the graph of f shifted 2 units to the right.

14. The graph of g is the graph of f reflected in the x axis.

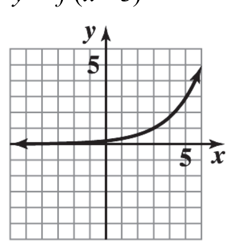
16. The graph of g is the graph of f shifted 2 units down.

18. The graph of g is the graph of f vertically contracted by a factor of 0.5 and shifted 1 unit to the right.

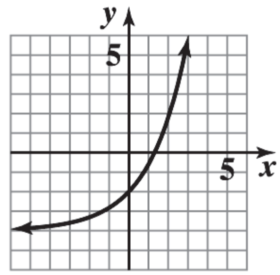
20. A. $y = f(x) + 2$



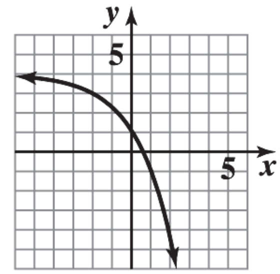
B. $y = f(x - 3)$



C. $y = 2f(x) - 4$

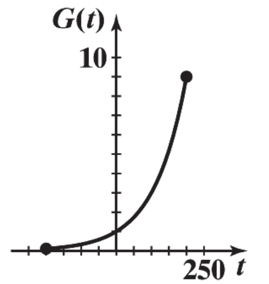


D. $y = 4 - f(x+2)$



22. $G(t) = 3^{t/100}$; [–200, 200]

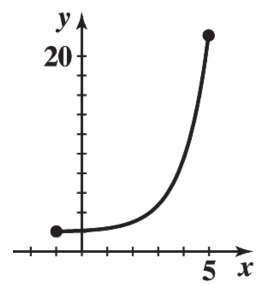
x	$G(t)$
–200	$\frac{1}{9}$
–100	$\frac{1}{3}$
0	1
100	3
200	9



24.

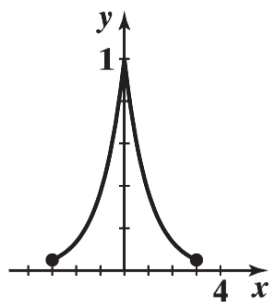
$y = 2 + e^{x-2}$; [–1, 5]

x	y
–1	≈ 2.05
0	≈ 2.14
1	≈ 2.37
3	≈ 4.72
5	≈ 22.09



26. $y = e^{-|x|}$; [–3, 3]

x	y
–3	≈ 0.05
–1	≈ 0.37
0	1
1	≈ 0.37
3	≈ 0.05



28. $a = 2$, $b = -2$ for example. The exponential function property: For $x \neq 0$, $a^x = b^x$ if and only if $a = b$ assumes $a > 0$ and $b > 0$.

30. $5^{3x} = 5^{4x-2}$

$3x = 4x - 2$

$-x = -2$

$x = 2$

32. $7^{x^2} = 7^{2x+3}$

$x^2 = 2x + 3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = -1, 3$

34. $(1-x)^5 = (2x-1)^5$

$1 - x = 2x - 1$

$-3x = -2$

$x = \frac{2}{3}$

36. $10xe^x - 5e^x = 0$

$e^x(10x - 5) = 0$

$10x - 5 = 0$ (since $e^x \neq 0$)

$x = 1/2$

38. $x^2e^{-x} - 9e^{-x} = 0$

$e^{-x}(x^2 - 9) = 0$

$(x^2 - 9) = 0$ (since $e^{-x} \neq 0$)

$x = -3, 3$

40. $e^{4x} + e > 0$ for all x;

$e^{4x} + e = 0$ has no solutions.

42. $e^{3x-1} - e = 0$

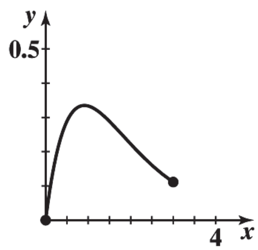
$e^{3x-1} = e^1$

$3x - 1 = 1$

$x = 2/3$

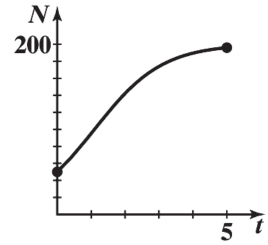
44. $m(x) = x(3^{-x})$; $[0, 3]$

x	$m(x)$
0	0
1	$\frac{1}{3}$
2	$\frac{2}{9}$
3	$\frac{1}{9}$



46. $N = \frac{200}{1+3e^{-t}}$; $[0, 5]$

x	N
0	50
1	≈ 95.07
2	≈ 142.25
3	≈ 174.01
4	≈ 189.58
5	≈ 196.04



48.
$$A = Pe^{rt}$$
$$A = (24{,}000)e^{(0.0435)(7)}$$
$$A = (24{,}000)e^{0.3045}$$
$$A = (24{,}000)(1.35594686)$$
$$A = \$32{,}542.72$$

50. (A)
$$A = P(1+\tfrac{r}{m})^{mt}$$
$$A = 4000(1+\tfrac{0.06}{52})^{(52)(0.5)}$$
$$A = 4000(1.0011538462)^{26}$$
$$A = 4000(1.030436713)$$
$$A = \$4121.75$$

(B)
$$A = P(1+\tfrac{r}{m})^{mt}$$
$$A = 4000(1+\tfrac{0.06}{52})^{(52)(10)}$$
$$A = 4000(1.0011538462)^{520}$$
$$A = 4000(1.821488661)$$
$$A = \$7285.95$$

52.
$$A = P(1+\tfrac{r}{m})^{mt}$$
$$40{,}000 = P(1+\tfrac{0.055}{365})^{(365)(17)}$$
$$40{,}000 = P(1.0001506849)^{6205}$$
$$40{,}000 = P(2.547034043)$$
$$P = \$15{,}705$$

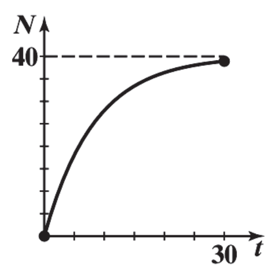
54. (A)
$$A = P(1+\tfrac{r}{m})^{mt}$$
$$A = 10{,}000(1+\tfrac{0.0135}{4})^{(4)(5)}$$
$$A = 10{,}000(1.003375)^{20}$$
$$A = 10{,}000(1.069709)$$
$$A = \$10{,}697.09$$

(B)
$$A = P(1+\tfrac{r}{m})^{mt}$$
$$A = 10{,}000(1+\tfrac{0.0130}{12})^{(12)(5)}$$
$$A = 10{,}000(1.00108333)^{60}$$
$$A = 10{,}000(1.067121479)$$
$$A = \$10{,}671.21$$

(C)
$$A = P(1+\tfrac{r}{m})^{mt}$$
$$A = 10{,}000(1+\tfrac{0.0125}{365})^{(365)(5)}$$
$$A = 10{,}000(1.000034247)^{1825}$$
$$A = 10{,}000(1.06449332)$$
$$A = \$10{,}644.93$$

56. $N = 40(1-e^{-0.12t})$; $[0, 30]$

x	N
0	0
10	≈ 27.95
20	≈ 36.37
30	≈ 38.91



The maximum number of boards an average employee can be expected to produce in 1 day is 40.

58.

```
ExpReg
 y=a*b^x
 a=1008.958664
 b=1.098151058
```

(A) The average salary in 2022: $y(32) \approx$ \$20,186,000.

(B) The model gives an average salary of $y(7) \approx$ \$1,943,000 in 1997.

60. (A) $I(50) = I_o e^{-0.00942(50)} \approx 62\ \%$ (B) $I(100) = I_o e^{-0.00942(100)} \approx 39\ \%$

62. (A) $P = 94e^{0.032t}$.
(B) Population in 2025: $P(13) = 94e^{0.032(13)} \approx 142{,}000{,}000$;
Population in 2035: $P(23) = 94e^{0.032(23)} \approx 196{,}000{,}000$.

64.

```
ExpReg
 y=a*b^x
 a=71.63144793
 b=1.002343596
```

Life expectancy for a person born in 2025: $y(55) \approx 81.5$ years.

EXERCISE 2-6

2. $\log_2 32 = 5 \Rightarrow 32 = 2^5$ **4.** $\log_e 1 = 0 \Rightarrow e^0 = 1$ **6.** $\log_9 27 = \frac{3}{2} \Rightarrow 27 = 9^{3/2}$ **8.** $36 = 6^2 \Rightarrow \log_6 36 = 2$

10. $9 = 27^{2/3} \Rightarrow \log_{27} 9 = \frac{2}{3}$ **12.** $M = b^x \Rightarrow \log_b M = x$ **14.** $\log_{10} 100{,}000 = \log_{10} 10^5 = 5$ **16.** $\log_3 \frac{1}{3} = \log_3 3^{-1} = -1$

18. $\log_4 1 = \log_4 4^0 = 0$ **20.** $\ln e^{-5} = -5$ **22.** $\log_b FG = \log_b F + \log_b G$ **24.** $\log_b w^{15} = 15 \log_b w$

26. $\dfrac{\log_3 P}{\log_3 R} = \log_R P$

28. $\log_2 x = 2$
$2^2 = x$
$4 = x$

30. $\log_3 27 = y$
$3^y = 27$
$3^y = 3^3$
$y = 3$

32. $\log_b e^{-2} = -2$
$e^{-2} = b^{-2}$
$e = b$

34. $\log_{25} x = \frac{1}{2}$
$25^{1/2} = x$
$5 = x$

36. False; an example of a polynomial function of odd degree that is not one–to–one is $f(x) = x^3 - x$.
$f(-1) = f(0) = f(1) = 0$.

38. True; the graph of every function (not necessarily one–to–one) intersects each vertical line exactly once.

40. False; $x = -1$ is in the domain of *f*, but cannot be in the range of *g*.

42. True; since *g* is the inverse of *f*, then (*a*, *b*) is on the graph of *f* if and only if (*b*, *a*) is on the graph of *g*. Therefore, *f* is also the inverse of *g*.

44. $\log_b x = \frac{2}{3}\log_b 27 + 2\log_b 2 - \log_b 3$
$\log_b x = \log_b 27^{2/3} + \log_b 2^2 - \log_b 3$
$\log_b x = \log_b 9 + \log_b 4 - \log_b 3$
$\log_b x = \log_b \frac{(9)(4)}{3}$
$\log_b x = \log_b 12$
$x = 12$

46. $\log_b x = 3\log_b 2 + \frac{1}{2}\log_b 25 - \log_b 20$
$\log_b x = \log_b 2^3 + \log_b 25^{1/2} - \log_b 20$
$\log_b x = \log_b 8 + \log_b 5 - \log_b 20$
$\log_b x = \log_b \frac{(8)(5)}{20}$
$\log_b x = \log_b 2$
$x = 2$

48. $\log_b(x+2)+\log_b x=\log_b 24$

$$\log_b(x+2)x=\log_b 24$$
$$\log_b(x^2+2x)=\log_b 24$$
$$x^2+2x=24$$
$$x^2+2x-24=0$$
$$(x+6)(x-4)=0$$
$$x=-6,4$$

Since the domain of a logarithmic function is $(0,\infty)$, omit the negative solution. Therefore, the solution is $x=4$.

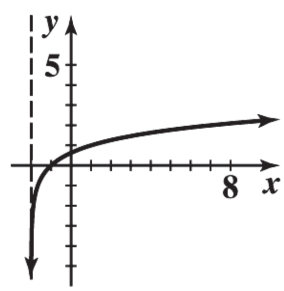
50. $\log_{10}(x+6)-\log_{10}(x-3)=1$

$$\log_{10}\frac{x+6}{x-3}=1$$
$$10^1=\frac{x+6}{x-3}$$
$$10(x-3)=x+6$$
$$10x-30=x+6$$
$$x=4$$

52. $y=\log_3(x+2)$

$$3^y=x+2$$
$$3^y-2=x$$

x	y
$-\frac{53}{27}$	-3
$-\frac{17}{9}$	-2
$-\frac{5}{3}$	-1
-1	0
1	1
7	2
25	3



54. The graph of $y=\log_3(x+2)$ is the graph of $y=\log_3 x$ shifted to the left 2 units.

56. The domain of logarithmic function is defined for positive values only. Therefore, the domain of the function is $x-1>0$ or $x>1$. The range of a logarithmic function is all real numbers. In interval notation the domain is $(1,\infty)$ and the range is $(-\infty,\infty)$.

58. A. $\log 72.604=1.86096$

B. $\log 0.033041=-1.48095$

C. $\ln 40,257=10.60304$

D. $\ln 0.0059263=-5.12836$

60. A. $\log x=2.0832$

$$x=\log^{-1}(2.0832)=10^{2.0832}$$
$$x=121.1156$$

B. $\log x=-1.1577$

$$x=\log^{-1}(-1.1577)=10^{-1.1577}$$
$$x=0.0696$$

C. $\ln x=3.1336$

$$x=\ln^{-1}(3.1336)=e^{3.1336}$$
$$x=22.9565$$

D. $\ln x = -4.3281$

$x = \ln^{-1}(-4.3281) = e^{-4.3281}$

$x = 0.0132$

62. $10^x = 153$

$\log 10^x = \log 153$

$x = 2.1847$

64. $e^x = 0.3059$

$\ln e^x = \ln 0.3059$

$x = -1.1845$

66. $1.02^{4t} = 2$

$\ln 1.02^{4t} = \ln 2$

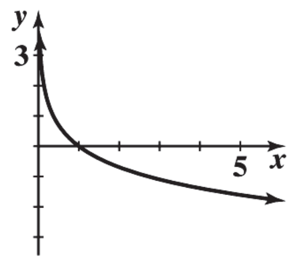
$4t \ln 1.02 = \ln 2$

$t = \dfrac{\ln 2}{4 \ln 1.02}$

$t = 8.7507$

68. $y = -\ln x;\ x > 0$

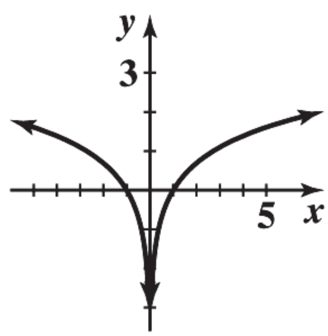
x	y
0.5	≈ 0.69
1	0
2	≈ −0.69
4	≈ −1.39
5	≈ −1.61



Based on the graph above, the function is decreasing on the interval $(0, \infty)$.

70. $y = \ln|x|$

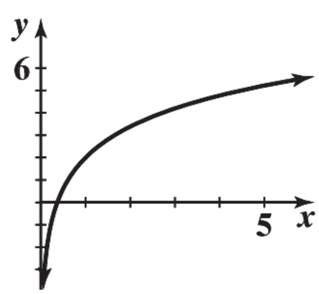
x	y
−5	≈ 1.61
−2	≈ 0.69
1	0
2	≈ 0.69
5	≈ 1.61



Based on the graph above, the function is decreasing on the interval $(-\infty, 0)$ and increasing on the interval $(0, \infty)$.

72. $y = 2\ln x + 2$

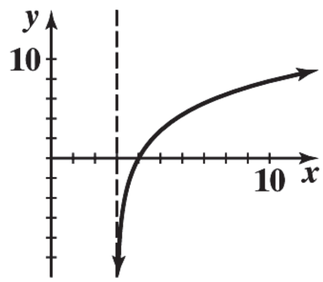
x	y
0.5	≈ 0.61
1	2
2	≈ 3.39
4	≈ 4.77
5	≈ 5.22



Based on the graph above, the function is increasing on the interval $(0, \infty)$.

74. $y = 4\ln(x-3)$

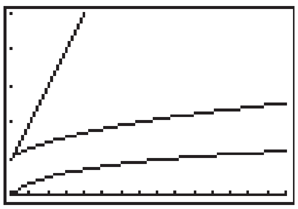
x	y
4	0
6	≈ 4.39
8	≈ 6.44
10	≈ 7.78
12	≈ 8.79



Based on the graph above, the function is increasing on the interval $(3, \infty)$.

76. It is not possible to find a power of 1 that is an arbitrarily selected real number, because 1 raised to any power is 1.

78.



A function f is "smaller than" a function g on an interval $[a, b]$ if $f(x) < g(x)$ for $a \le x \le b$. Based on the graph above, $\log x < \sqrt[3]{x} < x$ for $1 < x \le 16$.

80. Use the compound interest formula: $A = P(1+r)^t$. The problem is asking for the original amount to double, therefore $A = 2P$.

$$2P = P(1+0.0958)^t$$
$$2 = (1.0958)^t$$
$$\ln 2 = \ln(1.0958)^t$$
$$\ln 2 = t\ln(1.0958)$$
$$\frac{\ln 2}{\ln 1.0958} = t$$
$$7.58 \approx t$$

It will take approximately 8 years for the original amount to double.

82. Use the compound interest formula:
$A = P(1+\frac{r}{m})^{mt}$.

(A)
$$7500 = 5000(1+\tfrac{0.08}{2})^{2t}$$
$$1.5 = (1.04)^{2t}$$
$$\ln 1.5 = \ln(1.04)^{2t}$$
$$\ln 1.5 = 2t\ln(1.04)$$
$$\frac{\ln 1.5}{2\ln 1.04} = t$$
$$5.17 \approx t$$

It will take approximately 5.17 years for \$5000 to grow to \$7500 if compounded semiannually.

(B)
$$7500 = 5000(1+\tfrac{0.08}{12})^{12t}$$
$$1.5 = (1.0066667)^{12t}$$
$$\ln 1.5 = \ln(1.0066667)^{12t}$$
$$\ln 1.5 = 12t\ln(1.0066667)$$
$$\frac{\ln 1.5}{12\ln 1.0066667} = t$$
$$5.09 \approx t$$

It will take approximately 5.09 years for \$5000 to grow to \$7500 if compounded monthly.

It will take approximately 5.09 years for \$5000 to grow to \$7500 if compounded monthly.

84. Use the compound interest formula: $A = Pe^{rt}$.

$$41{,}000 = 17{,}000e^{0.0295t}$$
$$\tfrac{41}{17} = e^{0.0295t}$$
$$\ln\tfrac{41}{17} = \ln e^{0.0295t}$$
$$\ln\tfrac{41}{17} = 0.0295t$$
$$\frac{\ln\tfrac{41}{17}}{0.0295} = t$$
$$29.84 \approx t$$

It will take approximately 29.84 years for \$17,000 to grow to \$41,000 if compounded continuously.

86. Equilibrium occurs when supply and demand are equal. The models from Problem 85 have the demand and supply functions defined by $y = 256.4659159 - 24.03812068\ln x$ and $y = -127.8085281 + 20.01315349\ln x$, respectively. Set both equations equal to each other to yield:

$$256.4659159 - 24.03812068\ln x = -127.8085281 + 20.01315349\ln x$$
$$384.274444 = 44.05127417\ln x$$
$$\frac{384.274444}{44.05127417} = \ln x$$
$$e^{384.274444/44.05127417} = e^{\ln x}$$
$$6145 \approx x$$

Substitute the value above into either equation.

$$y = 256.4659159 - 24.03812068\ln x$$
$$y = 256.4659159 - 24.03812068\ln(6145)$$
$$y = 256.4659159 - 24.03812068(8.723394022)$$
$$y = 46.77$$

Therefore, equilibrium occurs when 6145 units are produced and sold at a price of \$46.77.

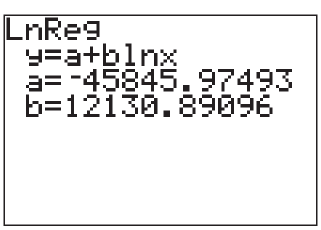
88. (A) $N = 10\log\frac{I}{I_0} = 10\log\frac{10^{-13}}{10^{-16}} = 10\log 10^3 = 30$

(B) $N = 10\log\frac{I}{I_0} = 10\log\frac{3.16\times 10^{-10}}{10^{-16}} = 10\log 3.16\times 10^6 \approx 65$

(C) $N = 10\log\frac{I}{I_0} = 10\log\frac{10^{-8}}{10^{-16}} = 10\log 10^8 = 80$

(D) $N = 10\log\frac{I}{I_0} = 10\log\frac{10^{-1}}{10^{-16}} = 10\log 10^{15} = 150$

90.



2024: $t = 124$; $y(124) \approx 12{,}628$. Therefore, according to the model, the total production in the year 2024 will be approximately 12,628 million bushels.

92.
$$A = A_0 e^{-0.000124t}$$
$$0.1A_0 = A_0 e^{-0.000124t}$$
$$0.1 = e^{-0.000124t}$$
$$\ln 0.1 = \ln e^{-0.000124t}$$
$$\ln 0.1 = -0.000124t$$
$$18,569 \approx t$$

If 10% of the original amount is still remaining, the skull would be approximately 18,569 years old.

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Section 2-1 Functions

Goal: To evaluate function values and to determine the domain of functions

The domain of the following functions will be the set of real numbers unless it meets one of the following conditions:

1. The function contains a fraction whose denominator has a variable.
   The domain of such a function is the set of real numbers EXCEPT the values of the variable that make the denominator zero.
2. The function contains an even root (square root $\sqrt{\ }$, fourth root $\sqrt[4]{\ }$, etc.).
   The domain of such a function is limited to values of the variable that make the radicand (the part under the radical) greater than or equal to 0.

1. Evaluate the following function at the specified values of the independent variable and simplify the results.

$f(x) = 4x - 5$

a) $f(1) = 4(1) - 5$
$f(1) = 4 - 5$
$f(1) = -1$

b) $f(-3) = 4(-3) - 5$
$f(-3) = -12 - 5$
$f(-3) = -17$

c) $f(x-1) = 4(x-1) - 5$
$f(x-1) = 4x - 4 - 5$
$f(x-1) = 4x - 9$

d) $f\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right) - 5$
$f\left(\frac{1}{4}\right) = 1 - 5$
$f\left(\frac{1}{4}\right) = -4$

In problems 2–10 evaluate the given function for $f(x) = x^2 + 1$ and $g(x) = x - 4$.

2. $(f+g)(3) = f(3) + g(3)$
$= 10 + (-1)$
$(f+g)(3) = 9$

$f(3) = (3)^2 + 1$
$f(3) = 9 + 1$
$f(3) = 10$

$g(3) = 3 - 4$
$g(3) = -1$

3. $(f-g)(2c) = f(2c) - g(2c)$ $\quad f(2c) = (2c)^2 + 1$ $\quad g(2c) = 2c - 4$

$= 4c^2 + 1 - (2c - 4)$ $\quad f(2c) = 4c^2 + 1$

$(f-g)(2c) = 4c^2 - 2c + 5$

4. $(fg)(-4) = f(-4)g(-4)$ $\quad f(-4) = (-4)^2 + 1$ $\quad g(-4) = -4 - 4$

$= (17)(-8)$ $\quad f(-4) = 16 + 1$ $\quad g(-4) = -8$

$(fg)(-4) = -136$ $\quad f(-4) = 17$

5. $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)}$ $\quad f(0) = (0)^2 + 1$ $\quad g(0) = 0 - 4$

$= \frac{1}{-4}$ $\quad f(0) = 0 + 1$ $\quad g(0) = -4$

$\left(\frac{f}{g}\right)(0) = -\frac{1}{4}$ $\quad f(0) = 1$

6. $2 \cdot g(-2) = 2(-6)$ $\quad g(-2) = -2 - 4$

$2 \cdot g(-2) = -12$ $\quad g(-2) = -6$

7. $3 \cdot f(4) - 2 \cdot g(-1) = 3(17) - 2(-5)$ $\quad f(4) = (4)^2 + 1$ $\quad g(-1) = -1 - 4$

$= 51 + 10$ $\quad f(4) = 16 + 1$ $\quad g(-1) = -5$

$3 \cdot f(4) - 2 \cdot g(-1) = 61$ $\quad f(4) = 17$

8. $\frac{f(3) - g(2)}{f(1)} = \frac{10 - (-2)}{2}$ $\quad f(3) = (3)^2 + 1$ $\quad f(1) = (1)^2 + 1$ $\quad g(2) = 2 - 4$

$= \frac{12}{2}$ $\quad f(3) = 9 + 1$ $\quad f(1) = 1 + 1$ $\quad g(2) = -2$

$\frac{f(3) - g(2)}{f(1)} = 6$ $\quad f(3) = 10$ $\quad f(1) = 2$

9. $\dfrac{g(-1+h)-g(-1)}{h}=\dfrac{h-5-(-5)}{h}$

$=\dfrac{h}{h}$

$\dfrac{g(-1+h)-g(-1)}{h}=1$

$g(-1+h)=-1+h-4$

$g(-1+h)=h-5$

$g(-1)=-1-4$

$g(-1)=-5$

10. $\dfrac{f(2+h)-f(2)}{h}=\dfrac{h^2+4h+5-(5)}{h}$

$=\dfrac{h^2+4h}{h}$

$\dfrac{f(2+h)-f(2)}{h}=h+4$

$f(2+h)=(2+h)^2+1$

$f(2+h)=h^2+4h+4+1$

$f(2+h)=h^2+4h+5$

$f(2)=(2)^2+1$

$f(2)=4+1$

$f(2)=5$

In problems 11–18 find the domain of each function.

11. $g(x)=\dfrac{5}{x-2}$

The domain is restricted by the denominator. Since it cannot equal zero, the domain is all real numbers except 2.

12. $f(x)=\dfrac{2x}{3x+7}$

The domain is restricted by the denominator. Since it cannot equal zero, the domain is all real numbers except $-\dfrac{7}{3}$.

13. $h(t)=\sqrt[4]{1-2t}$

The domain is restricted by the radicand since it has an even root. Since the radicand must be greater than or equal to zero, the domain is $t\le\dfrac{1}{2}$.

14. $g(x)=1-2x^2$

There are no restrictions on the domain, therefore the domain is all real numbers.

15. $f(x) = \sqrt[3]{x+4}$

There are no restrictions on the domain since it has an odd root, therefore the domain is all real numbers.

16. $h(w) = \sqrt{w-3}$

The domain is restricted by the radicand since it has an even root. Since the radicand must be greater than or equal to zero, the domain is $w \geq 3$.

17. $f(x) = 2x^3 + 5x^2 - x + 17$

There are no restrictions on the domain, therefore the domain is all real numbers.

18. $g(x) = \dfrac{2x^3}{5}$

There are no restrictions on the domain since there is no variable in the denominator, therefore the domain is all real numbers.

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Section 2-2 Elementary Function: Graphs and Transformations

Goal: To describe the shapes of graphs based on vertical and horizontal shifts and reflections, stretches, and shrinks

Basic Elementary Functions:

$f(x) = x$	Identity function
$h(x) = x^2$	Square function
$m(x) = x^3$	Cube function
$n(x) = \sqrt{x}$	Square root function
$p(x) = \sqrt[3]{x}$	Cube root function
$g(x) = \|x\|$	Absolute value function

In problems 1–14 describe how the graph of each function is related to the graph of one of the six basic functions. State the domain of each function. (Do not use a graphing calculator and do not make a chart.)

1. $g(x) = x^2 - 4$

The graph is the square function that is shifted down 4 units. There are no restrictions on the domain, therefore, the domain is all real numbers.

2. $f(x) = \sqrt{x} + 5$

The graph is the square root function that is shifted up 5 units. The domain is restricted by the radicand since it has an even root. Since the radicand must be greater than or equal to zero, the domain is $x \geq 0$.

3. $f(x) = -\sqrt{x}$

The graph is the square root function that is reflected over the x–axis. The domain is restricted by the radicand since it has an even root. Since the radicand must be greater than or equal to zero, the domain is $x \geq 0$.

4. $f(x) = \sqrt[3]{x-2}$

The graph is the cube root function that is shifted 2 units to the right. There are no restrictions on the domain since it has an odd root, therefore the domain is all real number.

5. $g(x) = (x-5)^2 - 3$

The graph is the square function that is shifted to the right 5 units and down 3 units. There are no restrictions on the domain, therefore, the domain is all real numbers.

6. $f(x) = -x^2 + 1$

The graph is the square function that is reflected over the x–axis and shifted up 1 unit. There are no restrictions on the domain, therefore, the domain is all real numbers.

7. $g(x) = 2 - \sqrt{x-4}$

The graph is the square root function that is shifted 4 units to the right, reflected over the x–axis, and shifted 2 units up. The domain is restricted by the radicand since it has an even root. Since the radicand must be greater than or equal to zero, the domain is $x \geq 4$.

8. $h(x) = |x+5|$

The graph is the absolute value function that is shifted 5 units to the left. There are no restrictions on the domain, therefore, the domain is all real numbers.

9. $g(x) = \sqrt[3]{x} - 3$

The graph is the cube root function that is shifted 3 units down. There are no restrictions on the domain since it has an odd root, therefore the domain is all real number.

10. $f(x) = |x+2| - 4$

The graph is the absolute value function that is shifted 2 units to the left and 4 units down. There are no restrictions on the domain, therefore, the domain is all real numbers.

11. $h(x) = -|x-3| + 2$

The graph is the absolute value function that is shifted 3 units to the right, reflected over the x–axis, and shifted 2 units up. There are no restrictions on the domain, therefore, the domain is all real numbers.

12. $f(x) = x^3 + 2$

The graph is the cube function that is shifted 2 units up. There are no restrictions on the domain, therefore, the domain is all real numbers.

13. $f(x) = -(x+2)^3 + 4$

The graph is the cube function that is shifted 2 units to the left, reflected over the *x*–axis, and then shifted 4 units up. There are no restrictions on the domain, therefore, the domain is all real numbers.

14. $h(x) = 3 - \sqrt[3]{x-4}$

The graph is the cube root function that is shifted 4 units to the right, reflected over the *x*–axis, and then shifted 3 units up. There are no restrictions on the domain since it has an odd root, therefore the domain is all real numbers.

In Problems 15–23 write an equation for a function that has a graph with the given characteristics.

15. The shape of $y = x^3$ shifted 6 units right.

$y = (x-6)^3$

16. The shape of $y = \sqrt{x}$ shifted 4 units down.

$y = \sqrt{x} - 4$

17. The shape of $y = |x|$ reflected over the *x*-axis and shifted 2 units up.

$y = -|x| + 2$

18. The shape of $y = x^2$ shifted 2 units right and 4 units up.

$y = (x-2)^2 + 4$

19. The shape of $y = \sqrt[3]{x}$ reflected over the *x*-axis and shifted 1 unit up.

$y = 1 - \sqrt[3]{x}$

20. The shape of $y = x^2$ reflected over the *x*-axis and shifted 3 units down.

$$y = -x^2 - 3$$

21. The shape of $y = \sqrt{x}$ shifted 4 units left.

$$y = \sqrt{x+4}$$

22. The shape of $y = x^3$ shifted 6 units right and 2 units down.

$$y = (x-6)^3 - 2$$

23. The shape of $y = |x|$ shifted 6 units right and 5 units up.

$$y = |x-6| + 5$$

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Section 2-3 Quadratic Functions

Goal: To describe functions that are quadratic in nature

Quadratic Functions:

Standard form of a quadratic: $f(x) = ax^2 + bx + c$ where a,b,c are real and $a \neq 0$.

Vertex form of a quadratic: $f(x) = a(x-h)^2 + k$ where $a \neq 0$ and (h,k) is the vertex.

Axis of symmetry: $x = h$

Minimum/Maximum value:

If $a > 0$, then the turning point (or vertex) is a minimum point on the graph and the minimum value would be *k*.
If $a < 0$, then the turning point (or vertex) is a maximum point on the graph and the maximum value would be *k*.

For 1–8 find:

a. the domain
b. the vertex
c. the axis of symmetry
d. the *x*-intercept(s)
e. the *y*-intercept
f. the maximum or minimum value of the function

then:

g. Graph the function.
h. State the range.
i. State the interval over which the function is decreasing.
j. State the interval over which the function is increasing.

1. $f(x) = (x-1)^2 - 3$

   a. The function is a quadratic, therefore the domain is all real numbers.
   b. The function is in vertex form, therefore the vertex is (1, – 3).
   c. The axis of symmetry is the *x*–value of the vertex, therefore the axis of symmetry is $x = 1$.
   d. The *x*-intercepts are found by setting $f(x) = 0$.

   $$f(x) = (x-1)^2 - 3$$
   $$0 = x^2 - 2x + 1 - 3$$
   $$0 = x^2 - 2x - 2$$

   Solve using the quadratic equation

   $$x = 1 \pm \sqrt{3}$$

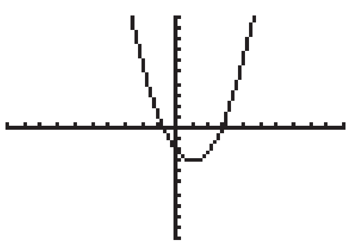
   Therefore, the *x*-intercepts are $(1+\sqrt{3}, 0)$ and $(1-\sqrt{3}, 0)$.

   e. The *y*-intercepts are found by setting $x = 0$.

   $$f(x) = (x-1)^2 - 3$$
   $$f(0) = (0-1)^2 - 3$$
   $$f(0) = (-1)^2 - 3$$
   $$f(0) = -2$$

   Therefore, the *y*-intercept is (0, –2).

   f. The graph opens upward, therefore the graph has a minimum value which is the *y*-coordinate of the vertex or –3.
   g.



   h. The graph has a minimum value of –3, therefore the range is $y \geq -3$.
   i. Based on the graph, the function is decreasing over the interval $(-\infty, 1)$.
   j. Based on the graph, the function is increasing over the interval $(1, \infty)$.

2. $f(x)=(x-2)^2+4$

a. The function is a quadratic, therefore the domain is all real numbers.
b. The function is in vertex form, therefore the vertex is (2, 4).
c. The axis of symmetry is the x–value of the vertex, therefore the axis of symmetry is $x=2$.
d. The x-intercepts are found by setting $f(x)=0$.

$$f(x)=(x-2)^2+4$$
$$0=x^2-4x+4+4$$
$$0=x^2-4x+8$$

Solving the above equation by the quadratic equation will result in complex roots, therefore no x-intercepts are present.

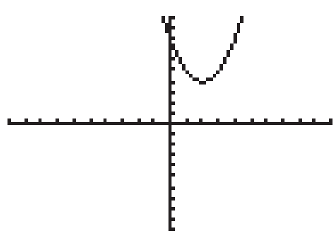
e. The y-intercepts are found by setting $x=0$.

$$f(x)=(x-2)^2+4$$
$$f(0)=(0-2)^2+4$$
$$f(0)=(-2)^2+4$$
$$f(0)=8$$

Therefore, the y-intercept is (0, 8).

f. The graph opens upward, therefore the graph has a minimum value which is the y-coordinate of the vertex or 4.

g.



h. The graph has a minimum value of 4, therefore the range is $y\geq 4$.
i. Based on the graph, the function is decreasing over the interval $(-\infty, 2)$.
j. Based on the graph, the function is increasing over the interval $(2, \infty)$.

3. $f(x) = -x^2 + 7$

a. The function is a quadratic, therefore the domain is all real numbers.
b. The function in vertex form is $f(x) = -(x-0)^2 + 7$, therefore the vertex is (0, 7).
c. The axis of symmetry is the x–value of the vertex, therefore the axis of symmetry is $x = 0$.
d. The x-intercepts are found by setting $f(x) = 0$.

$$f(x) = -x^2 + 7$$
$$0 = -x^2 + 7$$

Solve using the quadratic equation

$$x = \pm\sqrt{7}$$

Therefore, the x-intercepts are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$.

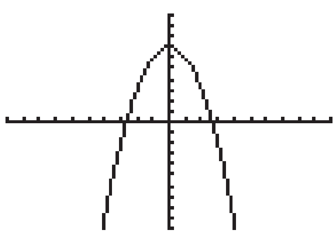
e. The y-intercepts are found by setting $x = 0$.

$$f(x) = -x^2 + 7$$
$$f(0) = -0^2 + 7$$
$$f(0) = 0 + 7$$
$$f(0) = 7$$

Therefore, the y-intercept is (0, 7).

f. The graph opens downward, therefore the graph has a maximum value which is the y-coordinate of the vertex or 7.

g.



h. The graph has a maximum value of 2, therefore the range is $y \le 0$.
i. Based on the graph, the function is decreasing over the interval $(0, \infty)$.
j. Based on the graph, the function is increasing over the interval $(-\infty, 0)$.

4. $f(x) = -(x-1)^2 - 1$

a. The function is a quadratic, therefore the domain is all real numbers.
b. The function is in vertex form, therefore the vertex is (1, –1).
c. The axis of symmetry is the *x*–value of the vertex, therefore the axis of symmetry is $x = 1$.
d. The *x*-intercepts are found by setting $f(x) = 0$.

$$f(x) = -(x-1)^2 - 1$$
$$0 = -x^2 + 2x - 1 - 1$$
$$0 = -x^2 + 2x - 2$$

Solving the above equation by the quadratic equation will result in complex roots, therefore no *x*-intercepts are present.

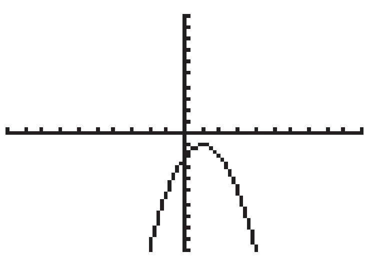
e. The *y*-intercepts are found by setting $x = 0$.

$$f(x) = -(x-1)^2 - 1$$
$$f(0) = -(0-1)^2 - 1$$
$$f(0) = -(-1)^2 - 1$$
$$f(0) = -2$$

Therefore, the *y*-intercept is (0, –2).

f. The graph opens downward, therefore the graph has a maximum value which is the *y*-coordinate of the vertex or –1.

g.



h. The graph has a maximum value of –1, therefore the range is $y \le -1$.
i. Based on the graph, the function is decreasing over the interval $(1, \infty)$.
j. Based on the graph, the function is increasing over the interval $(-\infty, 1)$.

5. $f(x) = x^2 - 4x$

a. The function is a quadratic, therefore the domain is all real numbers.

b. The function in vertex form is $f(x) = (x-2)^2 - 4$, therefore the vertex is (2, – 4).

c. The axis of symmetry is the x–value of the vertex, therefore the axis of symmetry is $x = 2$.

d. The x-intercepts are found by setting $f(x) = 0$.

$$f(x) = x^2 - 4x$$
$$0 = x^2 - 4x$$
$$0 = x(x-4)$$
$$x = 0, 4$$

Therefore, the x-intercepts are $(0,0)$ and $(4,0)$.

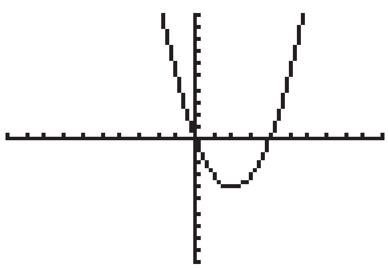
e. The y-intercepts are found by setting $x = 0$.

$$f(x) = x^2 - 4x$$
$$f(0) = 0^2 - 4(0)$$
$$f(0) = 0 + 0$$
$$f(0) = 0$$

Therefore, the y-intercept is (0, 0).

f. The graph opens upward, therefore the graph has a minimum value which is the y-coordinate of the vertex or – 4.

g.



h. The graph has a minimum value of – 4, therefore the range is $y \geq -4$.

i. Based on the graph, the function is decreasing over the interval $(-\infty, 2)$.

j. Based on the graph, the function is increasing over the interval $(2, \infty)$.

6. $f(x) = x^2 + 2x - 4$

a. The function is a quadratic, therefore the domain is all real numbers.

b. The function in vertex form is $f(x) = (x+1)^2 - 5$, therefore the vertex is (–1, –5).

c. The axis of symmetry is the *x*–value of the vertex, therefore the axis of symmetry is $x = -1$.

d. The *x*-intercepts are found by setting $f(x) = 0$.

$$f(x) = (x+1)^2 - 5$$
$$0 = x^2 + 2x + 1 - 5$$
$$0 = x^2 + 2x - 4$$

Solve using the quadratic equation

$$x = -1 \pm \sqrt{5}$$

Therefore, the *x*-intercepts are $(-1+\sqrt{5}, 0)$ and $(-1-\sqrt{5}, 0)$.

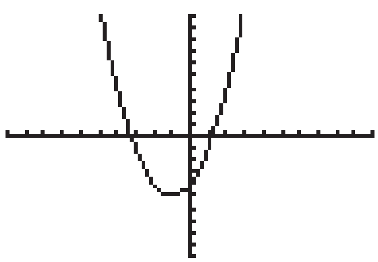
e. The *y*-intercepts are found by setting $x = 0$.

$$f(x) = (x+1)^2 - 5$$
$$f(0) = (0+1)^2 - 5$$
$$f(0) = (1)^2 - 5$$
$$f(0) = -4$$

Therefore, the *y*-intercept is (0, –4).

f. The graph opens upward, therefore the graph has a minimum value which is the *y*-coordinate of the vertex or –5.

g.



h. The graph has a minimum value of –5, therefore the range is $y \geq -5$.

i. Based on the graph, the function is decreasing over the interval $(-\infty, -1)$.

j. Based on the graph, the function is increasing over the interval $(-1, \infty)$.

7. $f(x) = x^2 + 2x + 1$

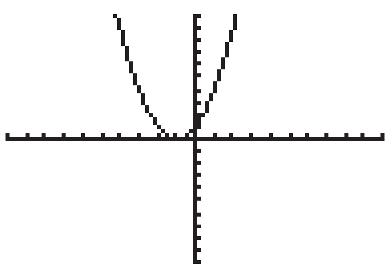
a. The function is a quadratic, therefore the domain is all real numbers.
b. The function in vertex form is $f(x) = (x+1)^2 + 0,$ therefore the vertex is (–1, 0).
c. The axis of symmetry is the *x*–value of the vertex, therefore the axis of symmetry is $x = -1$.
d. The *x*-intercepts are found by setting $f(x) = 0$.

$$f(x) = x^2 + 2x + 1$$
$$0 = x^2 + 2x + 1$$
$$0 = (x+1)(x+1)$$
$$x = -1$$

Therefore, there is only one *x*-intercept which is $(-1, 0)$.
e. The *y*-intercepts are found by setting $x = 0$.

$$f(x) = x^2 + 2x + 1$$
$$f(0) = 0^2 + 2(0) + 1$$
$$f(0) = 0 + 0 + 1$$
$$f(0) = 1$$

Therefore, the *y*-intercept is (0, 1).
f. The graph opens upward, therefore the graph has a minimum value which is the *y*-coordinate of the vertex or 0.
g.



h. The graph has a minimum value of 0, therefore the range is $y \geq 0$.
i. Based on the graph, the function is decreasing over the interval $(-\infty, -1)$.
j. Based on the graph, the function is increasing over the interval $(-1, \infty)$.

8. $f(x) = -x^2 + 10x - 19$

a. The function is a quadratic, therefore the domain is all real numbers.

b. The function in vertex form is $f(x) = -(x-5)^2 + 6$, therefore the vertex is (5, 6).

c. The axis of symmetry is the x–value of the vertex, therefore the axis of symmetry is $x = 5$.

d. The x-intercepts are found by setting $f(x) = 0$.

$$f(x) = -x^2 + 10x - 19$$
$$0 = -x^2 + 10x - 19$$
$$0 = x^2 - 10x + 19$$

Solve using the quadratic equation

$$x = 5 \pm \sqrt{6}$$

Therefore, the x-intercepts are $(5+\sqrt{6}, 0)$ and $(5-\sqrt{6}, 0)$.

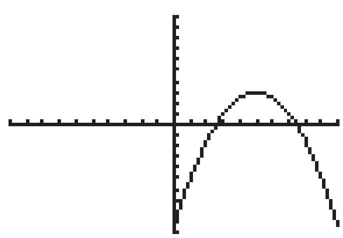
e. The y-intercepts are found by setting $x = 0$.

$$f(x) = -x^2 + 10x - 19$$
$$f(0) = -0^2 + 10(0) - 19$$
$$f(0) = 0 + 0 - 19$$
$$f(0) = -19$$

Therefore, the y-intercept is (0, –19).

f. The graph opens downward, therefore the graph has a maximum value which is the y-coordinate of the vertex or 6.

g.



h. The graph has a maximum value of 4, therefore the range is $y \le 6$.

i. Based on the graph, the function is decreasing over the interval $(5, \infty)$.

j. Based on the graph, the function is increasing over the interval $(-\infty, 5)$.

9. The revenue and cost functions for a company that manufactures components for washing machines were determined to be:

$$R(x) = x(200 - 4x) \qquad \text{and} \qquad C(x) = 160 + 20x$$

where *x* is the number of components in millions and *R(x)* and *C(x)* are in millions of dollars.

a) How many components must be sold in order for the company to break even? (Break-even points are when $R(x) = C(x)$.) (Round answers to nearest million.)

$$R(x) = C(x)$$
$$x(200 - 4x) = 160 + 20x$$
$$200x - 4x^2 = 160 + 20x$$
$$0 = 4x^2 - 180x + 160$$

Solve the equation by the quadratic equation

$$x \approx 0.9, 44.09$$

The company would need to sell approximately 1 million or 44 million to break even.

b) Find the profit equation. ($P(x) = R(x) - C(x)$)

$$P(x) = R(x) - C(x)$$
$$P(x) = x(200 - 4x) - (160 + 20x)$$
$$P(x) = 200x - 4x^2 - 160 - 20x$$
$$P(x) = -4x^2 + 180x - 160$$

c) Determine the maximum profit. How many components must be sold in order to achieve that maximum profit?

The maximum profit occurs at the vertex of the profit function. The *x*-coordinate is $x = -\frac{b}{2a} = -\frac{180}{2(-4)} = \frac{-180}{-8} = 22.5$. To find the *y*-coordinate of the vertex, substitute the value into the function as follows:

$$P(x) = -4x^2 + 180x - 160$$
$$P(22.5) = -4(22.5)^2 + 180(22.5) - 160$$
$$P(22.5) = -2025 + 4050 - 160$$
$$P(22.5) = 1865$$

The maximum profit of \$1,865 million is achieved when 22.5 million components are sold.

10. A company keeps records of the total revenue (money taken in) in thousands of dollars from the sale of x units (in thousands) of a product. It determines that total revenue is a function $R(x)$ given by

$$R(x) = 300x - x^2$$

It also keeps records of the total cost of producing x units of the same product. It determines that the total cost is a function $C(x)$ given by

$$C(x) = 40x + 1600$$

a) Find the break-even points for this company. (Round answer to nearest 1000.)

$$R(x) = C(x)$$
$$300x - x^2 = 40x + 1600$$
$$0 = x^2 - 260x + 1600$$

Solve the equation by the quadratic equation

$$x \approx 6.307, 253.693$$

The company would need to sell approximately 6,000 or 254,000 to break even.

b) Determine at what point profit is at a maximum. What is the maximum profit? How many units must be sold in order to achieve maximum profit?

The profit equation is:

$$P(x) = R(x) - C(x)$$
$$P(x) = 300x - x^2 - (40x + 1600)$$
$$P(x) = -x^2 + 260x - 1600$$

The maximum profit occurs at the vertex of the profit function. The x-coordinate is $x = -\frac{b}{2a} = -\frac{260}{2(-1)} = \frac{-260}{-2} = 130.$ To find the y-coordinate of the vertex, substitute the value into the function as follows:

$$P(x) = -x^2 + 260x - 1600$$
$$P(130) = -(130)^2 + 260(130) - 1600$$
$$P(130) = -16,900 + 33,800 - 1600$$
$$P(130) = 15,300$$

The maximum profit of \$15,300 thousands or \$15,300,000 is achieved when 130,000 units are sold.

11. The cost, *C(x)*, of building a house is a function of the number of square feet, *x*, in the house. If the cost function can be approximated by

$$C(x) = 0.01x^2 - 20x + 25,000 \qquad \text{where } 1000 \le x \le 3500$$

a) What would be the cost of building a 1500 square foot house?

Substitute the value of 1500 into the cost function:

$$C(x) = 0.01x^2 - 20x + 25,000$$
$$C(1500) = 0.01(1500)^2 - 20(1500) + 25,000$$
$$C(1500) = 0.01(2,250,000) - 30,000 + 25,000$$
$$C(1500) = 17,500$$

It will cost \$17,500 to build a 1500 square foot house.

b) Find the minimum cost to build a house. How many square feet would that house have?

The minimum cost occurs at the vertex of the cost function. The x-coordinate is $x = -\frac{b}{2a} = -\frac{-20}{2(0.01)} = \frac{20}{0.02} = 1000.$ To find the y-coordinate of the vertex, substitute the value into the function as follows:

$$C(x) = 0.01x^2 - 20x + 25000$$
$$C(1000) = 0.01(1000)^2 - 20(1000) + 25,000$$
$$C(1000) = 10,000 - 20,000 + 25,000$$
$$C(1000) = 15,000$$

The minimum cost of \$15,000 is achieved when a 1000 square foot home is built.

12. The cost of producing computer software is a function of the number of hours worked by the employees. If the cost function can be approximated by

$$C(x) = 0.04x^2 - 20x + 6000 \qquad \text{where } 200 \le x \le 1000$$

a) What would be the cost if the employees worked 800 hours?

Substitute the value of 800 into the cost function:

$$C(x) = 0.04x^2 - 20x + 6000$$
$$C(800) = 0.04(800)^2 - 20(800) + 6000$$
$$C(800) = 0.04(640,000) - 16,000 + 6000$$
$$C(800) = 15,600$$

If the employees work 800 hours, it will cost \$15,600 to produce the software.

b) Find the number of hours the employees should work in order to minimize the cost. What would the minimum cost be?

The minimum cost occurs at the vertex of the cost function. The *x*-coordinate is $x = -\frac{b}{2a} = -\frac{-20}{2(0.04)} = \frac{20}{0.08} = 250$. To find the *y*-coordinate of the vertex, substitute the value into the function as follows:

$$C(x) = 0.04x^2 - 20x + 6000$$
$$C(250) = 0.04(250)^2 - 20(250) + 6000$$
$$C(250) = 2500 - 5000 + 6000$$
$$C(250) = 3500$$

The minimum cost of \$3500 is achieved when the employees work 250 hours.

Name ________________________________ **Date** ______________ **Class** ____________

Section 2-4 Polynomial and Rational Functions

Goal: To describe and identify functions that are polynomial and rational in nature

Definition: Polynomial function

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ for n a nonnegative integer, called the degree of the polynomial. The coefficients $a_0, a_1, \ldots, a_n$ are real numbers with $a_n \neq 0$. The domain of a polynomial function is the set of all real numbers.

Definition: Rational function

$f(x) = \dfrac{n(x)}{d(x)}$ $d(x) \neq 0$ where *n(x)* and *d(x)* are polynomials. The domain is the set of all real numbers such that $d(x) \neq 0$.

Vertical Asymptotes:

Case 1: Suppose *n(x)* and *d(x)* have no real zero in common. If *c* is a real number such that $d(x) = 0$, then the line $x = c$ is a vertical asymptote of the graph.

Case 2: If *n(x)* and *d(x)* have one or more real zeros in common, cancel common linear factors and apply Case 1 to the reduced fraction.

Horizontal Asymptotes:

Case 1: If degree *n(x)* < degree *d(x)*, then *y* = 0 is the horizontal asymptote.

Case 2: If degree *n(x)* = degree *d(x)*, then *y* = *a/b* is the horizontal asymptote, where *a* is the leading coefficient of *n(x)* and *b* is the leading coefficient of *d(x)*.

Case 3: If degree *n(x)* > degree *d(x)*, there is no horizontal asymptote.

For 1–6 determine each of the following for the polynomial functions:
a. the degree of the polynomial
b. the *x*–intercept(s) of the graph of the polynomial
c. the *y*–intercept of the graph of the polynomial

1. $f(x) = x^3 - 7x - 6 = (x+2)(x-3)(x+1)$

   a. The degree of the polynomial is the highest exponent, which is 3.
   b. The x–intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form, therefore if we set the factors equal to zero, the zeros of the polynomial occur at –1, –2, and 3.
   c. The y-intercept occurs when the x value is zero. If the x value is zero, the only term that will not be zero is the constant term, therefore the y-intercept is (0, –6).

2. $f(x) = x^3 + 4x^2 - 4x - 16 = (x-2)(x+2)(x+4)$

   a. The degree of the polynomial is the highest exponent, which is 3.
   b. The x–intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form, therefore if we set the factors equal to zero, the zeros of the polynomial occur at 2, –2, and –4.
   c. The y-intercept occurs when the x value is zero. If the x value is zero, the only term that will not be zero is the constant term, therefore the y-intercept is (0, –16).

3. $f(x) = x^3 - 3x^2 - 10x + 24 = (x+3)(x-2)(x-4)$

   a. The degree of the polynomial is the highest exponent, which is 3.
   b. The x–intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form, therefore if we set the factors equal to zero, the zeros of the polynomial occur at –3, 2, and 4.
   c. The y-intercept occurs when the x value is zero. If the x value is zero, the only term that will not be zero is the constant term, therefore the y-intercept is (0, 24).

4. $f(x) = x^3 + 4x^2 - x - 4 = (x+4)(x+1)(x-1)$

a. The degree of the polynomial is the highest exponent, which is 3.
b. The *x*–intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form, therefore if we set the factors equal to zero, the zeros of the polynomial occur at –4, –1, and 1.
c. The *y*-intercept occurs when the *x* value is zero. If the *x* value is zero, the only term that will not be zero is the constant term, therefore the *y*-intercept is (0, –4).

5. $f(x) = x^4 - 2x^3 + x^2 + 2x - 2 = (x-1)(x+1)(x^2 - 2x + 2)$

a. The degree of the polynomial is the highest exponent, which is 4.
b. The *x*–intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form. The third factor must be solved using the quadratic equation, therefore, the zeros of the polynomial occur at 1, –1, and $1 \pm \sqrt{3}$.
c. The *y*-intercept occurs when the *x* value is zero. If the *x* value is zero, the only term that will not be zero is the constant term, therefore the *y*-intercept is (0, –2).

6. $f(x) = x^5 + 5x^4 - 20x^2 - x + 15 = (x+3)(x-1)(x+1)(x^2 + 2x - 5)$

a. The degree of the polynomial is the highest exponent, which is 5.
b. The *x*–intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form. The fourth factor must be solved using the quadratic equation, therefore, the zeros of the polynomial occur at –3, –1, 1, and $-1 \pm \sqrt{6}$.
c. The *y*-intercept occurs when the *x* value is zero. If the *x* value is zero, the only term that will not be zero is the constant term, therefore the *y*-intercept is (0, –15).

For the given rational functions in 7–12:
a. Find the domain.
b. Find any x-intercept(s).
c. Find any y-intercept.
d. Find any vertical asymptote.
e. Find any horizontal asymptote.
f. Sketch a graph of $y = f(x)$ for $-10 \le x \le 10$.

7. $f(x) = \dfrac{1}{2x}$

a. The function is defined everywhere except when the denominator is zero. The domain is therefore all real numbers except 0.

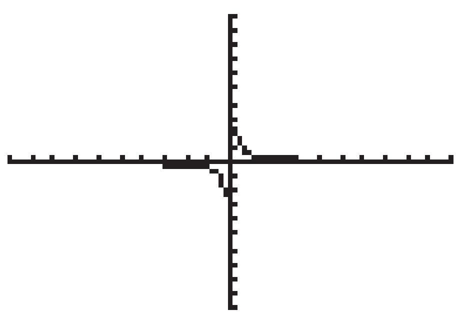
b. The x–intercept(s) are found by setting the function equal to 0. Since the function is a rational expression and the numerator cannot be zero, the function value cannot be zero. Therefore, there is no x–intercept.

c. The y–intercept is found when the value of x is zero. Since $x = 0$ is not in the domain, there is no y–intercept.

d. Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for $x = 0$, the vertical asymptote is the line $x = 0$.

e. Horizontal asymptotes are found by dividing all terms by the highest power of x. Therefore, $f(x) = \dfrac{1}{2x} = \dfrac{\frac{1}{2x}}{\frac{2x}{2x}} = \dfrac{\frac{1}{2x}}{1}$, as x increases or decreases without bound, the denominator is always 1 and the numerator tends to 0; so $f(x)$ tends to 0. The horizontal asymptote is the line $y = 0$.

f.



8. $f(x) = \dfrac{3x}{x-5}$

a. The function is defined everywhere except when the denominator is zero.

$$x - 5 = 0$$
$$x = 5$$

The domain is therefore all real numbers except 5.

b. The x–intercept(s) are found by setting the function equal to 0. Since the function is a rational expression, the function value is zero when the numerator is zero.

$$3x = 0$$
$$x = 0$$

Therefore, the x–intercept is (0, 0).

c. The y–intercept is found when the value of x is zero.

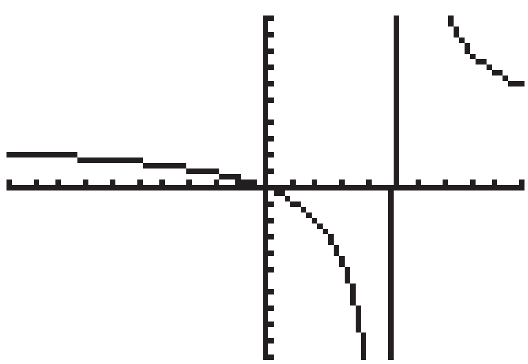
$$f(x) = \frac{3x}{x-5}$$
$$f(0) = \frac{3(0)}{0-5}$$
$$f(0) = \frac{0}{-5} = 0$$

Therefore, the y–intercept is (0, 0).

d. Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for $x = 5$, the vertical asymptote is the line $x = 5$.

e. Horizontal asymptotes are found by dividing all terms by the highest power of x. Therefore, $f(x) = \dfrac{3x}{x-5} = \dfrac{\frac{3x}{x}}{\frac{x}{x} - \frac{5}{x}} = \dfrac{3}{1 - \frac{5}{x}}$, as x increases or decreases without bound, the numerator is always 3 and the denominator tends to 1 – 0, or 1; so $f(x)$ tends to 3. The horizontal asymptote is the line $y = 3$.

f.



9. $f(x) = \dfrac{3x}{x-3}$

a. The function is defined everywhere except when the denominator is zero.

$$x - 3 = 0$$
$$x = 3$$

The domain is therefore all real numbers except 3.

b. The x–intercept(s) are found by setting the function equal to 0. Since the function is a rational expression, the function value is zero when the numerator is zero.

$$3x = 0$$
$$x = 0$$

Therefore, the x–intercept is (0, 0).

c. The y–intercept is found when the value of x is zero.

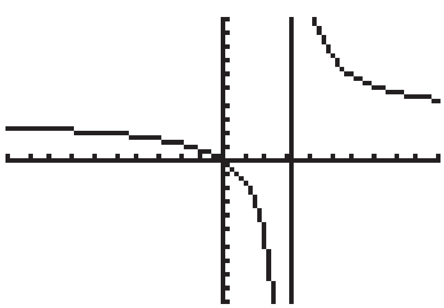
$$f(x) = \frac{3x}{x-3}$$
$$f(0) = \frac{3(0)}{0-3}$$
$$f(0) = \frac{0}{-3} = 0$$

Therefore, the y–intercept is (0, 0).

d. Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for $x = 3$, the vertical asymptote is the line $x = 3$.

e. Horizontal asymptotes are found by dividing all terms by the highest power of x. Therefore, $f(x) = \dfrac{3x}{x-3} = \dfrac{\frac{3x}{x}}{\frac{x}{x} - \frac{3}{x}} = \dfrac{3}{1 - \frac{3}{x}}$, as x increases or decreases without bound, the numerator is always 3 and the denominator is tends to 1 – 0, or 1; so $f(x)$ tends to 3. The horizontal asymptote is the line $y = 3$.

f.



10. $f(x)=\dfrac{2x-4}{x+3}$

a. The function is defined everywhere except when the denominator is zero.

$$x+3=0$$
$$x=-3$$

The domain is therefore all real numbers except –3.

b. The *x*–intercept(s) are found by setting the function equal to 0. Since the function is a rational expression, the function value is zero when the numerator is zero.

$$2x-4=0$$
$$2x=4$$
$$x=2$$

Therefore, the *x*–intercept is (2, 0).

c. The *y*–intercept is found when the value of *x* is zero.

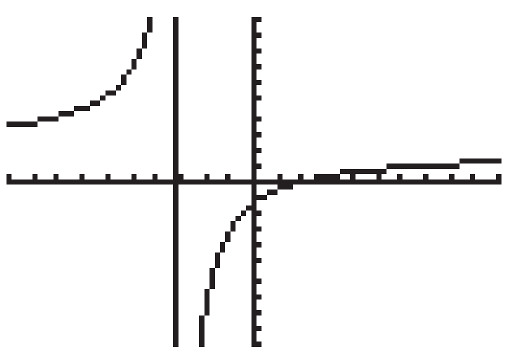
$$f(x)=\frac{2x-4}{x+3}$$
$$f(0)=\frac{2(0)-4}{0+3}$$
$$f(0)=\frac{-4}{3}$$

Therefore, the *y*–intercept is $(0,-\frac{4}{3})$.

d. Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for $x = -3$, the vertical asymptote is the line $x = -3$.

e. Horizontal asymptotes are found by dividing all terms by the highest power of *x*. Therefore, $f(x)=\dfrac{2x-4}{x+3}=\dfrac{\frac{2x}{x}-\frac{4}{x}}{\frac{x}{x}+\frac{3}{x}}=\dfrac{2-\frac{4}{x}}{1+\frac{3}{x}}$, as *x* increases or decreases without bound, the numerator tends to 2 – 0, or 2 and the denominator tends to 1 – 0, or 1; so *f*(*x*) tends to 2. The horizontal asymptote is the line $y = 2$.

f.



11. $f(x) = \dfrac{4+x}{4-x}$

a. The function is defined everywhere except when the denominator is zero.

$$4 - x = 0$$
$$4 = x$$

The domain is therefore all real numbers except 4.

b. The x–intercept(s) are found by setting the function equal to 0. Since the function is a rational expression, the function value is zero when the numerator is zero.

$$4 + x = 0$$
$$x = -4$$

Therefore, the x–intercept is (–4, 0).

c. The y–intercept is found when the value of x is zero.

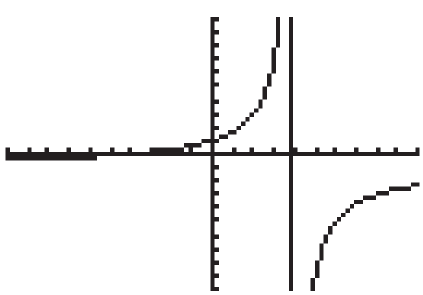
$$f(x) = \frac{4+x}{4-x}$$
$$f(0) = \frac{4+0}{4-0}$$
$$f(0) = \frac{4}{4} = 1$$

Therefore, the y–intercept is $(0,1)$.

d. Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for $x = 4$, the vertical asymptote is the line $x = 4$.

e. Horizontal asymptotes are found by dividing all terms by the highest power of x. Therefore, $f(x) = \dfrac{4+x}{4-x} = \dfrac{\frac{4}{x}+\frac{x}{x}}{\frac{4}{x}-\frac{x}{x}} = \dfrac{\frac{4}{x}+1}{\frac{4}{x}-1}$, as x increases or decreases without bound, the numerator tends to 0 + 1, or 1 and the denominator tends to 0 – 1, or –1; so $f(x)$ tends to –1. The horizontal asymptote is the line $y = -1$.

f.



12. $f(x)=\dfrac{1-5x}{1+2x}$

a. The function is defined everywhere except when the denominator is zero.

$$\begin{aligned}1+2x&=0\\2x&=-1\\x&=-\frac{1}{2}\end{aligned}$$

The domain is therefore all real numbers except $-\frac{1}{2}$.

b. The x–intercept(s) are found by setting the function equal to 0. Since the function is a rational expression, when the numerator is zero, the function value is zero.

$$\begin{aligned}1-5x&=0\\-5x&=-1\\x&=\frac{1}{5}\end{aligned}$$

Therefore, the x–intercept is $(\frac{1}{5},0)$.

c. The y–intercept is found when the value of x is zero.

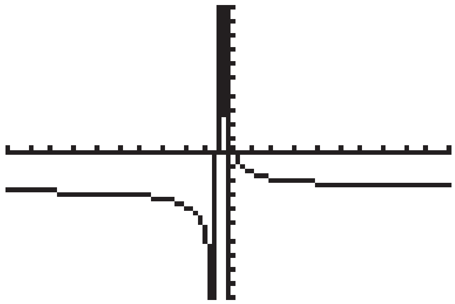
$$\begin{aligned}f(x)&=\frac{1-5x}{1+2x}\\f(0)&=\frac{1-5(0)}{1+2(0)}\\f(0)&=\frac{1}{1}=1\end{aligned}$$

Therefore, the y–intercept is $(0,1)$.

d. Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for $x=-\frac{1}{2}$, the vertical asymptote is the line $x=-\frac{1}{2}$.

e. Horizontal asymptotes are found by dividing all terms by the highest power of x. Therefore, $f(x)=\dfrac{1-5x}{1+2x}=\dfrac{\frac{1}{x}-\frac{5x}{x}}{\frac{1}{x}+\frac{2x}{x}}=\dfrac{\frac{1}{x}-5}{\frac{1}{x}+2}$, as x increases or decreases without bound, the numerator tends to 0 – 5, or –5 and the denominator tends to 0 + 2, or 2; so $f(x)$ tends to $-\frac{5}{2}$. The horizontal asymptote is the line $y=-\frac{5}{2}$.

f.



13. A video production company is planning to produce a documentary. The producer estimates that it will cost \$52,000 to produce the video and \$20 per video to copy and distribute the tape.

a) Assuming that the total cost to market the video, *C(n),* is linearly related to the total number, *n*, of videos produced, write an equation for the cost function.

$$C(n) = 20n + 52,000$$

b) The average cost per video for an output of *n* videos is given by $\bar{C}(n) = \frac{C(n)}{n}$.

Find the average cost function. $\bar{C}(n) = \frac{C(n)}{n} = \frac{20n + 52,000}{n}$

c) Sketch a graph of the average cost function for $500 \le n \le 3000$.

The *x*–axis scale shown is from 500 to 3000. Each tick mark is 500 units. The *y*–axis scale shown is from 0 to 200. Each tick mark is 50 units.

d) What does the average cost per video tend to as production increases?

To find the value that the function tends to go towards, you will find the horizontal asymptote of the function. Horizontal asymptotes are found by dividing all terms by the highest power of *n*. Therefore, $\bar{C}(n) = \frac{\frac{20n}{n} + \frac{52,000}{n}}{\frac{n}{n}} = \frac{20 + \frac{52,000}{n}}{1}$, as *n* increases without bound, the numerator tends to 20 + 0, or 20 and the denominator is always 1; so $\bar{C}(n)$ tends to 20. This means that the average cost per video tends towards \$20 each.

14. A contractor purchases a piece of equipment for \$36,000. The equipment requires an average expenditure of \$8.25 per hour for fuel and maintenance, and the operator is paid \$13.50 per hour to operate the machinery.

a) Assuming that the total cost per day, *C(h)*, is linearly related to the number of hours, *h*, that the machine is operated, write an equation for the cost function.

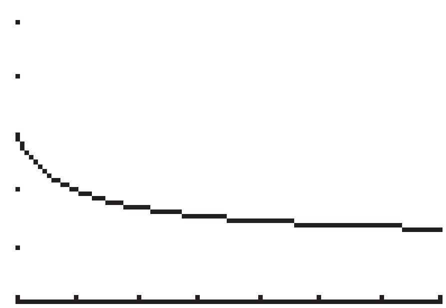
$$C(h) = 21.75h + 36,000$$

b) The average cost per hour of operating the machine is given by $\overline{C}(h) = \frac{C(h)}{h}$.

Find the average cost function. $\overline{C}(h) = \frac{C(h)}{h} = \frac{21.75h + 36,000}{h}$

c) Sketch a graph of the average cost function for $1000 \le h \le 8000$.

The *x*–axis scale shown is from 1000 to 8000. Each tick mark is 1000 units. The *y*–axis scale shown is from 0 to 100. Each tick mark is 20 units.



d) What cost per hour does the average cost per hour tend to as the number of hours of use increases?

To find the value that the function tends to go towards, you will find the horizontal asymptote of the function. Horizontal asymptotes are found by dividing all terms by the highest power of *h*. Therefore, $\overline{C}(h) = \frac{\frac{21.75n}{h} + \frac{36,000}{h}}{\frac{h}{h}} = \frac{21.75 + \frac{36,000}{h}}{1}$, as *h* increases without bound, the numerator tends to 21.75 + 0, or 21.75 and the denominator is always 1; so $\overline{C}(h)$ tends to 21.75. This means that the average cost per hour tends towards \$21.75 as the number of hours of use increases.

15. The daily cost function for producing *x* printers for home computers was determined to be:

$$C(x) = x^2 + 8x + 6000$$

The average cost per printer at a production level of *x* printers per day is $\bar{C}(x) = \frac{C(x)}{x}$.

a) Find the average cost function.

$$\bar{C}(x) = \frac{C(x)}{x} = \frac{x^2 + 8x + 6000}{x}$$

b) Sketch a graph of the average cost function for $25 \le x \le 150$.

The *x*–axis scale shown is from 25 to 150. Each tick mark is 25 units. The *y*–axis scale shown is from 50 to 400. Each tick mark is 50 units.

c) At what production level is the daily average cost at a minimum? What is that minimum value?

Based on the graph above, the minimum value occurs around the third tick mark, which has a value of 75. By substituting values into the average value equation, we would have the following:

$\bar{C}(75) = 163$

$\bar{C}(76) = 162.947$

$\bar{C}(77) = 162.922$

$\bar{C}(78) = 162.923$

$\bar{C}(79) = 162.949$

Therefore, the minimum average cost of 162.92 occurs when 77 printers are produced.

16. The monthly cost function for producing x brake assemblies for a certain type of car is given by:

$$C(x) = 3x^2 + 36x + 9000$$

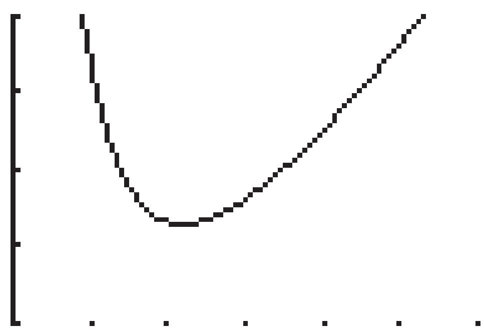
The average cost per brake assembly at a production level of x assemblies per month is $\overline{C}(x) = \frac{C(x)}{x}$.

a) Find the average cost function.

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{3x^2 + 36x + 9000}{x}$$

b) Sketch a graph of the average cost function for $0 \le x \le 150$.

The x–axis scale shown is from 0 to 150. Each tick mark is 25 units. The y–axis scale shown is from 300 to 500. Each tick mark is 50 units.



c) At what production level is the daily average cost at a minimum? What is that minimum value?

Based on the graph above, the minimum value occurs just beyond the third tick mark, which has a value of 50. By substituting values into the average value equation, we would have the following:

$$\overline{C}(53) = 364.811$$
$$\overline{C}(54) = 364.667$$
$$\overline{C}(55) = 364.636$$
$$\overline{C}(56) = 364.714$$

Therefore, the minimum average cost of 364.63 occurs when 55 brake assemblies are produced.

Name ______________________________ **Date** _____________ **Class** ___________

Section 2-5 Exponential Functions

Goal: To describe and solve functions that are exponential in nature

Rules for Exponents:

$a^m \cdot a^n = a^{m+n}$ Product Rule $a^0 = 1,\ \ a \neq 0$ Zero Exponent Rule

$\dfrac{a^m}{a^n} = a^{m-n}$ Quotient Rule $\left(a^m\right)^n = a^{mn}$ Power Rule

In problems 1–8, describe in words the transformations that can be used to obtain the graph of $g(x)$ from the graph of $f(x)$.

1. $g(x) = 4^{x+3} - 4;\ \ f(x) = 4^x$

   The function f is shifted 3 units to the left and 4 units down.

2. $g(x) = -3^x - 5;\ \ f(x) = 3^x$

   The function f is reflected over the x–axis and shifted 5 units down.

3. $g(x) = 2^{x-4} - 6;\ \ f(x) = 2^x$

   The function f is shifted 4 units to the right and 6 units down.

4. $g(x) = -5^{x-3} + 2;\ \ f(x) = 5^x$

   The function f is shifted 3 units to the right, reflected over the x–axis, and shifted 2 units up.

5. $g(x) = 10^{x-2} - 5;\quad f(x) = 10^x$

The function *f* is shifted 2 units to the right and 5 units down.

6. $g(x) = -10^x - 3;\quad f(x) = 10^x$

The function *f* is reflected over the *x*–axis and shifted 3 units down.

7. $g(x) = e^{x+1} + 2;\quad f(x) = e^x$

The function *f* is shifted 1 unit to the left and 2 units up.

8. $g(x) = -e^x + 5;\quad f(x) = e^x$

The function *f* is reflected over the *x*–axis and shifted 5 units up.

In Problems 9–20, solve each equation for *x*.

9. $10^{2x-3} = 10^{5x+4}$

$$2x - 3 = 5x + 4$$
$$-7 = 3x$$
$$-\frac{7}{3} = x$$

10. $10^{x^2} = 10^{2x+8}$

$$x^2 = 2x + 8$$
$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
$$x = -2, 4$$

11. $6^{5x-4} = 6^{x^2}$

$$5x - 4 = x^2$$
$$x^2 - 5x + 4 = 0$$
$$(x-4)(x-1) = 0$$
$$x = 1, 4$$

12. $8^{x^2} = 8^{8x}$

$$x^2 = 8x$$
$$x^2 - 8x = 0$$
$$x(x-8) = 0$$
$$x = 0, 8$$

13. $(x+6)^3 = (3x-8)^3$

$$x + 6 = 3x - 8$$
$$14 = 2x$$
$$7 = x$$

14. $(2x-7)^5 = (x+1)^5$

$$2x - 7 = x + 1$$
$$x = 8$$

15. $(e^3)^4 = e^x$

$$3(4) = x$$
$$12 = x$$

16. $(e^x)^4 = e^{x^2}$

$$4x = x^2$$
$$x^2 - 4x = 0$$
$$x(x-4) = 0$$
$$x = 0, 4$$

17. $(e^{2x})^x = e^{15+x}$

$$2x^2 = 15 + x$$
$$2x^2 - x - 15 = 0$$
$$(2x+5)(x-3) = 0$$
$$x = -\tfrac{5}{2}, 3$$

18. $3^x \cdot 3^4 = 3^{3x^2}$

$$x + 4 = 3x^2$$
$$3x^2 - x - 4 = 0$$
$$(3x-4)(x+1) = 0$$
$$x = \tfrac{4}{3}, -1$$

19. $2^{x^2} = 2^{12x} \cdot 2^{-32}$

$$x^2 = 12x - 32$$
$$x^2 - 12x + 32 = 0$$
$$(x-4)(x-8) = 0$$
$$x = 4, 8$$

20. $9^x \cdot 9 = 9^{2x^2}$

$$x + 1 = 2x^2$$
$$2x^2 - x - 1 = 0$$
$$(2x+1)(x-1) = 0$$
$$x = -\tfrac{1}{2}, 1$$

INTEREST FORMULAS

Simple Interest: $A = P(1+rt)$

Compound Interest: $A = P\left(1+\dfrac{r}{m}\right)^{mt}$

Continuous Compound Interest: $A = Pe^{rt}$

where P is the amount invested (principal), r (expressed as a decimal) is the annual interest rate, t is time invested (in years), m is the number of times a year the interest is compounded, and A is the amount of money in the account after t years (future value).

(Round answers for 21 –28 to the nearest dollar)

21. Fred inherited $35,000 from his uncle. He decides to invest his money for 5 years in order to have the greatest down-payment when he buys a house. He can choose from 3 different banks.

Bank A offers 1% compounded monthly.
Bank B offers .5% compounded continuously.
Bank C offers .75% compounded daily.

Which bank offers the best plan so Fred can earn the most money from his investment?

Bank A:

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 35,000\left(1+\frac{0.01}{12}\right)^{12(5)}$$
$$A = 35000(1.000833)^{60}$$
$$A \approx \$36,794$$

Bank B:

$$A = Pe^{rt}$$
$$A = 35,000e^{(0.005)(5)}$$
$$A = 35,000e^{0.025}$$
$$A \approx \$35,886$$

Bank C:

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 35,000\left(1+\frac{0.0075}{365}\right)^{365(5)}$$
$$A = 35,000(1.000021)^{1825}$$
$$A \approx \$36,337$$

Therefore, Bank A is the best option.

22. The day your first child is born you invest $10,000 in an account that pays 1.2% interest compounded quarterly. How much will be in the account when the child is 18 years old and ready to start to college?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 10,000\left(1+\frac{0.012}{4}\right)^{4(18)}$$
$$A = 10,000(1.003)^{72}$$
$$A \approx \$12,407$$

23. When your second child is born, you are able to invest only \$5000 but the account pays 1% interest compounded daily. How much will be in the account when this child is 18 years old and ready to start to college?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$A = 5000\left(1+\frac{0.01}{365}\right)^{365(18)}$$

$$A = 5000(1.000027)^{6570}$$

$$A \approx \$5986$$

24. When your third child comes along, money is even tighter and you are able to invest only \$1000, but you are able to find a bank that will let you invest the money at 1.75% compounded continuously. How much will be in the account when this third child is 18 years old and ready to start to college?

$$A = Pe^{rt}$$

$$A = 1000e^{(0.0175)(18)}$$

$$A = 1000e^{0.315}$$

$$A \approx \$1370$$

25. Joe Vader plans to start his own business in ten years. How much money would he need to invest today in order to have \$25,000 in ten years if Joe's bank offers a 10-year CD that pays 1.8% interest compounded monthly.

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$25,000 = P\left(1+\frac{0.018}{12}\right)^{12(10)}$$

$$25,000 = P(1.0015)^{120}$$

$$P \approx \$20,885$$

26. Bill and Sue plan to buy a home in 5 years. How much would they need to invest today at 1.2% compounded daily in order to have $30,000 in five years?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$30,000 = P\left(1+\frac{0.012}{365}\right)^{365(5)}$$

$$30,000 = P(1.000088)^{1825}$$

$$P \approx \$28,253$$

27. Suppose you invest $3000 in a four-year certificate of deposit (CD) that pays 1.5%interest compounded monthly the first 3 years and 2.2% compounded daily the last year. What is the value of the CD at the end of the four years?

$$A = P\left(1+\frac{r}{m}\right)^{mt}\left(1+\frac{r}{m}\right)^{mt}$$

$$A = 3000\left(1+\frac{0.015}{12}\right)^{12(3)}\left(1+\frac{0.022}{365}\right)^{365(1)}$$

$$A = 3000(1.00125)^{36}(1.000060)^{365}$$

$$A \approx \$3208$$

28. Suppose you invest $8000 in a 10-year certificate of deposit (CD) that pays 1.25% interest compounded daily the first 6 years and 2% compounded continuously the last four years. What is the value of the CD at the end of the 10 years?

$$A = P\left(1+\frac{r}{m}\right)^{mt} e^{rt}$$

$$A = 8000\left(1+\frac{0.0125}{365}\right)^{365(6)} e^{(0.02)(4)}$$

$$A = 8000(1.000034)^{2190} e^{0.08}$$

$$A \approx \$9341$$

Name ______________________________ **Date** _____________ **Class** ____________

Section 2-6 Logarithmic Functions

Goal: To solve problems that are logarithmic in nature

Properties of Logarithms

$\log_a x + \log_a y = \log_a xy$

$\log_a x - \log_a y = \log_a \frac{x}{y}$

$\log_a x^n = n \log_a x$

Exponential Function: $f(x) = a^x,\ a > 0,\ a \neq 1$

$y = \log_a x$ means $x = a^y$

In Problems 1–20 find the value of *x*. (Evaluate to four decimal places if necessary.)

1. $\log_3 x = 4$

$$x = 3^4$$
$$x = 81$$

2. $\log_3(x+1) = 2$

$$x + 1 = 3^2$$
$$x + 1 = 9$$
$$x = 8$$

3. $\log_3 3^8 = 7 + 3x$

$$3^{7+3x} = 3^8$$
$$7 + 3x = 8$$
$$3x = 1$$
$$x = \frac{1}{3}$$

4. $\log_2 2^6 = 4 - 3x$

$$2^{4-3x} = 2^6$$
$$4 - 3x = 6$$
$$-3x = 2$$
$$x = -\frac{2}{3}$$

5. $\ln(x+6)=2$

$$e^2 = x+6$$
$$e^2 - 6 = x$$
$$1.3891 \approx x$$

6. $\log_x(2x+3)=2$

$$x^2 = 2x+3$$
$$x^2 - 2x - 3 = 0$$
$$(x-3)(x+1)=0$$
$$x = 3$$
$$x \neq -1$$

The log function cannot have a negative base.

7. $\log_2(7x-19)=\log_2(2x+9)$

$$7x-19 = 2x+9$$
$$5x = 28$$
$$x = 5.6$$

8. $\log_3(8-5x)=\log_3(2x-13)$

$$8-5x = 2x-13$$
$$21 = 7x$$
$$3 = x$$

Since the value of 3 creates a negative log, there is no solution.

9. $\ln x + \ln 3 = 3$

$$\ln 3x = 3$$
$$e^3 = 3x$$
$$\frac{e^3}{3} = x$$
$$6.6952 \approx x$$

10. $\ln x - \ln 3 = 1$

$$\ln \tfrac{x}{3} = 1$$
$$e^1 = \tfrac{x}{3}$$
$$3e = x$$
$$8.1548 \approx x$$

11. $\log(x-1) - \log 4 = 3$

$$\log \tfrac{x-1}{4} = 3$$
$$10^3 = \tfrac{x-1}{4}$$
$$(4)10^3 = x-1$$
$$4(1000)+1 = x$$
$$4001 = x$$

12. $\ln(x-1) - \ln 6 = 2$

$$\ln \tfrac{x-1}{6} = 2$$
$$e^2 = \tfrac{x-1}{6}$$
$$6e^2 = x-1$$
$$6e^2 + 1 = x$$
$$45.3343 \approx x$$

13. $2^{3x} = 12$

$$\begin{aligned} \log 2^{3x} &= \log 12 \\ 3x \log 2 &= \log 12 \\ x &= \frac{\log 12}{3\log 2} \\ x &\approx 1.1950 \end{aligned}$$

14. $5^{x-1} = 17$

$$\begin{aligned} \log 5^{x-1} &= \log 17 \\ (x-1)\log 5 &= \log 17 \\ x\log 5 - \log 5 &= \log 17 \\ x\log 5 &= \log 17 + \log 5 \\ x &= \frac{\log 17 + \log 5}{\log 5} \\ x &\approx 2.7604 \end{aligned}$$

15. $7^{x-1} = 8^x$

$$\begin{aligned} \log 7^{x-1} &= \log 8^x \\ (x-1)\log 7 &= x\log 8 \\ x\log 7 - \log 7 &= x\log 8 \\ x\log 7 - x\log 8 &= \log 7 \\ x(\log 7 - \log 8) &= \log 7 \\ x &= \frac{\log 7}{\log 7 - \log 8} \\ x &\approx -14.5727 \end{aligned}$$

16. $4^{2x+3} = 5^{x-2}$

$$\begin{aligned} \log 4^{2x+3} &= \log 5^{x-2} \\ (2x+3)\log 4 &= (x-2)\log 5 \\ 2x\log 4 + 3\log 4 &= x\log 5 - 2\log 5 \\ 2x\log 4 - x\log 5 &= -2\log 5 - 3\log 4 \\ x(2\log 4 - \log 5) &= -2\log 5 - 3\log 4 \\ x &= \frac{-2\log 5 - 3\log 4}{2\log 4 - \log 5} \\ x &\approx -6.3429 \end{aligned}$$

17. $7^{x+1} = 10^{2x}$

$$\begin{aligned} \log 7^{x+1} &= \log 10^{2x} \\ (x+1)\log 7 &= 2x\log 10 \\ x\log 7 + \log 7 &= 2x\log 10 \\ x\log 7 - 2x\log 10 &= -\log 7 \\ x(\log 7 - 2\log 10) &= -\log 7 \\ x &= \frac{-\log 7}{\log 7 - 2\log 10} \\ x &\approx 0.7317 \end{aligned}$$

18. $e^{x+4} = 14.654$

$$\begin{aligned} \ln e^{x+4} &= \ln(14.654) \\ x+4 &= \ln(14.654) \\ x &= \ln(14.654) - 4 \\ x &\approx -1.3153 \end{aligned}$$

19. $x + 5 = e^3$

$$\begin{aligned} x &= e^3 - 5 \\ x &\approx 15.0855 \end{aligned}$$

20. $e^{4x} = e^{20}$

$$\begin{aligned} 4x &= 20 \\ x &= 5 \end{aligned}$$

21. You want to accumulate \$20,000 by your son's eighteenth birthday. How much do you need to invest on the day he is born in an account that will pay 1.4% interest compounded quarterly? (Round your answer to the nearest dollar.)

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$20,000 = P\left(1+\frac{0.014}{4}\right)^{4(18)}$$
$$20,000 = P(1.0035)^{72}$$
$$\$15,552 \approx P$$

22. Using the information in Problem 21, how much would you need to invest if you waited until he is 10 years old to start the fund?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$20,000 = P\left(1+\frac{0.014}{4}\right)^{4(8)}$$
$$20,000 = P(1.0035)^{32}$$
$$\$17,884 \approx P$$

23. A bond that sells for \$1000 today can be redeemed for \$1200 in 10 years. If interest is compounded quarterly, what is the annual interest rate for this investment? (Round your answer to two decimal places when written as a percentage.)

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$1200 = 1000\left(1+\frac{r}{4}\right)^{4(10)}$$
$$1.2 = \left(1+\frac{r}{4}\right)^{40}$$
$$1.2^{1/40} = 1+\frac{r}{4}$$
$$1.2^{1/40} - 1 = \frac{r}{4}$$
$$4(1.2^{1/40} - 1) = r$$
$$1.83\% \approx r$$

24. A bond that sells for $18,000 today can be redeemed for $20,000 in 6 years. If interest is compounded monthly, what is the annual interest rate for this investment? (Round your answer to two decimal places when written as a percentage.)

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$20,000 = 18,000\left(1+\frac{r}{12}\right)^{12(6)}$$

$$1.111 = \left(1+\frac{r}{12}\right)^{72}$$

$$1.111^{1/72} = 1+\frac{r}{12}$$

$$1.111^{1/72} - 1 = \frac{r}{12}$$

$$12(1.111^{1/72} - 1) = r$$

$$1.76\% \approx r$$

25. What is the minimum number of months required for an investment of $10,000 to grow to at least $20,000 (double in value) if the investment earns 1.8% annual interest rate compounded monthly? What would be the actual value of the investment after that many months?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$20,000 = 10,000\left(1+\frac{0.018}{12}\right)^{12t}$$

$$2 = (1.0015)^{12t}$$

$$\ln 2 = \ln(1.0015)^{12t}$$

$$\ln 2 = 12t\ln(1.0015)$$

$$\frac{\ln 2}{12\ln 1.0015} = t$$

$$38.5 \approx t$$

It will take about $(38.5)(12) = 462$ months to double.

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 10,000\left(1+\frac{0.018}{12}\right)^{12(38.5)}$$
$$A = 10,000(1.0015)^{462}$$
$$A = 10,000(1.998668)$$
$$A \approx \$19,987$$

26. What is the minimum number of months required for an investment of $10,000 to grow to at least $30,000 (triple in value) if the investment earns 1.8% annual interest rate compounded monthly? What would be the actual value of the investment after that many months?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$30,000 = 10,000\left(1+\frac{0.018}{12}\right)^{12t}$$
$$3 = (1.0015)^{12t}$$
$$\ln 3 = \ln(1.0015)^{12t}$$
$$\ln 3 = 12t\ln(1.0015)$$
$$\frac{\ln 3}{12\ln 1.0015} = t$$
$$61.1 \approx t$$

It will take about $(61.1)(12) = 733$ months to triple.

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 10,000\left(1+\frac{0.018}{12}\right)^{12(61.1)}$$
$$A = 10,000(1.0015)^{733}$$
$$A = 10,000(3.000192)$$
$$A \approx \$30,002$$

27. Some years ago Ms. Martinez invested \$7000 at 2% compounded quarterly. The account now contains \$10,000. How long ago did she start the account? (Round your answer UP to the next year.)

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$10,000 = 7000\left(1+\frac{0.02}{4}\right)^{4t}$$

$$\tfrac{10}{7} = (1.005)^{4t}$$

$$\ln\tfrac{10}{7} = \ln(1.005)^{4t}$$

$$\ln\tfrac{10}{7} = 4t\ln(1.005)$$

$$\frac{\ln\frac{10}{7}}{4\ln 1.005} = t$$

$$17.88 \approx t$$

It took approximately 18 years to have a balance of \$10,000.

28. Some years ago Mr. Tang invested \$18,000 at 3% compounded monthly. The account now contains \$24,000. How long ago did he start the account? (Round your answer UP to the next year.)

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$24,000 = 18,000\left(1+\frac{0.03}{12}\right)^{12t}$$

$$\tfrac{4}{3} = (1.0025)^{12t}$$

$$\ln\tfrac{4}{3} = \ln(1.0025)^{12t}$$

$$\ln\tfrac{4}{3} = 12t\ln(1.0025)$$

$$\frac{\ln\frac{4}{3}}{12\ln 1.0025} = t$$

$$9.60 \approx t$$

It took approximately 10 years to have a balance of \$24,000.

29. In a certain country the number of people above the poverty level is currently 25 million and growing at a rate of 4% annually. Assuming that the population is growing continuously, the population, P (in millions), t years from now, is determined by the formula:

$$P = 25e^{0.04t}$$

In how many years will there be 30 million people above the poverty level? 40 million? (Round your answers to nearest tenth of a year.)

30 million people

$$P = 25e^{0.04t}$$
$$30 = 25e^{0.04t}$$
$$1.2 = e^{0.04t}$$
$$\ln 1.2 = \ln e^{0.04t}$$
$$\ln 1.2 = 0.04t$$
$$\frac{\ln 1.2}{0.04} = t$$
$$4.6 \approx t$$

40 million people

$$P = 25e^{0.04t}$$
$$40 = 25e^{0.04t}$$
$$1.6 = e^{0.04t}$$
$$\ln 1.6 = \ln e^{0.04t}$$
$$\ln 1.6 = 0.04t$$
$$\frac{\ln 1.6}{0.04} = t$$
$$11.8 \approx t$$

It will take approximately 4.6 years to reach 30 million people and 11.8 years to reach 40 million people.

30. The number of bacteria present in a culture at time t is given by the formula $N = 20e^{0.35t}$, where t is in hours. How many bacteria are present initially (that is when t = 0)? How many are present after 24 hours? How many hours does it take for the bacteria population to double? (Round your answers to nearest whole number.)

Initially there are $N = 20e^{0.35(0)} = 20e^0 = 20$ bacteria present.
After 24 hours there will be $N = 20e^{0.35(24)} = 20e^{8.4} = 20(4447.066748) = 88,941$ bacteria present.

$$N = 20e^{0.35t}$$
$$40 = 20e^{0.35t}$$
$$2 = e^{0.35t}$$
$$\ln 2 = \ln e^{0.35t}$$
$$\ln 2 = 0.35t$$
$$\frac{\ln 2}{0.35} = t$$
$$1.98 \approx t$$

The number of bacteria will double after approximately 2 hours.

Mini Lecture 1.1

Introduction to Algebra: Variables and Mathematical Models

Learning Objectives:

1. Evaluate algebraic expressions.
2. Translate English phrases into algebraic expressions.
3. Determine whether a number is a solution of an equation.
4. Translate English sentences into algebraic equations.
5. Evaluate formulas.

Examples:

1. Evaluate each expression for $x = 5$.

   a. $4(x-3)$ b. $\dfrac{6x-15}{3x}$

2. Evaluate each expression for $x = 3$ and $y = 6$.

   a. $5(x+y)$ b. $\dfrac{2x+3y}{2y}$

3. Write each English phrase as an algebraic expression. Let x represent the number.
   a. the difference of a number and six
   b. eight more than four times a number
   c. four less than the quotient of a number and twelve

4. Determine whether the given number is a solution of the equation.
   a. $x - 8 = 12$; 20 b. $4x - 7 = 9$; 3 c. $3(y - 5) = 6$; 7

5. Write each English sentence as an equation. Let x represent the number.
   a. The product of a number and seven is twenty-one.
   b. The difference of twice a number and three is equal to twenty-seven.
   c. Six less than three times a number is the same as the number increased by twelve.

Teaching Notes:

- It may be helpful to draw students' attention to the word "evaluate." Help them see the letters v – a – l – u . This will help them remember that evaluate means to find the value of an expression.
- Students often make mistakes with the phrase "less than" so they should be cautioned about the order of the subtraction.
- Translating from English to algebra is an important skill that will be used often.

Answers: 1a. 8 b. 1 2a. 45 b. 2 3a. $x-6$ b. $4x+8$ c. $\dfrac{x}{12}-4$ 4a. yes b. not a solution c. yes 5a. $7x = 21$ b. $2x - 3 = 27$ c. $3x - 6 = x + 12$

Mini Lecture 1.2
Fractions in Algebra

Learning Objectives:

1. Convert between mixed numbers and improper fractions.
2. Write the prime factorization of a composite number.
3. Reduce or simplify fractions.
4. Multiply fractions.
5. Divide fractions.
6. Add and subtract fractions with identical denominators.
7. Add and subtract fractions with unlike denominators.
8. Solve problems involving fractions in algebra.

Examples:

1. Convert each mixed number to an improper fraction.

a. $3\frac{7}{10}$ b. $8\frac{3}{7}$ c. $5\frac{2}{3}$ d. $9\frac{1}{4}$

2. Convert each improper fraction to a mixed number.

a. $\frac{13}{8}$ b. $\frac{12}{11}$ c. $\frac{25}{3}$ d. $\frac{37}{7}$

3. Give the prime factorization of each of the following composite numbers.

a. 24 b. 48 c. 90 d. 108

4. What makes a number a prime?

5. Reduce the following fractions to lowest terms by factoring each numerator and denominator and dividing out common factors.

a. $\frac{10}{12}$ b. $\frac{32}{48}$ c. $\frac{24}{50}$ d. $\frac{77}{98}$

6. Perform the indicated operation. Always reduce answer, if possible.

a. $\frac{3}{4}+\frac{1}{6}$ b. $8\frac{1}{8}+3\frac{1}{3}$ c. $\frac{7}{10}-\frac{3}{8}$

d. $10\frac{11}{12}-4\frac{1}{4}$ e. $\left(\frac{7}{9}\right)\left(\frac{18}{19}\right)$ f. $\left(6\frac{2}{3}\right)\left(2\frac{1}{4}\right)$

g. $\frac{7}{8}\div\frac{3}{4}$ h. $5\frac{3}{8}\div 2\frac{1}{4}$

Teaching Notes:

- When teaching factorization, it is often helpful to review divisibility rules.
- To add or subtract fractions, you must have a LCD.
- To divide fractions, multiply by the reciprocal of the divisor.
- To multiply or divide mixed numbers, change to improper fractions first.

Answers: 1. a. 37/10 b. 59/7 c. 17/3 d. 37/4 2. a. 1 5/8 b. 1 1/11 c. 8 1/3 d. 5 2/7
3. a. $2\cdot 2\cdot 2\cdot 3$ b. $2\cdot 2\cdot 2\cdot 2\cdot 3$ c. $2\cdot 3\cdot 3\cdot 5$ d. $2\cdot 2\cdot 3\cdot 3\cdot 3$
4. a number whose only factors are 1 and itself 5. a. 5/6 b. 2/3 c. 12/25 d. 11/14 6. a. 11/12
b. $\frac{275}{24}$ or 11 11/24 c. 13/40 d. $\frac{20}{3}$ or 6 2/3 e. 14/19 f. 15 g. 7/6 or 1 1/6 h. 43/18 or 2 7/18

Mini Lecture 1.3
The Real Numbers

Learning Objectives:

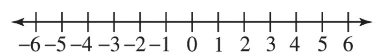
1. Define the sets that make up the set of real numbers.
2. Graph numbers on a number line.
3. Express rational numbers as decimals.
4. Classify numbers as belonging to one or more sets of the real numbers.
5. Understand and use inequality symbols.
6. Find the absolute value of a real number.

Examples:

1. Answer the following questions about each number:
   Is it a natural number? Is it rational?
   Is it a whole number? Is it irrational?
   Is it an integer? Is it a real number?

a. 18 b. –3.5 c. $\sqrt{5}$ d. 0 e. $-\frac{3}{4}$ f. π g. –5 h. 0.45

2. Graph each number on the number line.



a. 5.5 b. $-\frac{16}{4}$ c. $2\frac{1}{4}$ d. –3.2

3. Express each rational number as a decimal.

a. $\frac{7}{8}$ b. $\frac{9}{11}$ c. $\frac{5}{3}$ d. $\frac{1}{4}$

4. Use $>$ or $<$ to compare the numbers.

a. 18 □ –20 b. –16 □ –13 c. –4.3 □ –6.2

d. $\frac{4}{7}$ □ $\frac{8}{11}$ e. $-\frac{3}{5}$ □ $\frac{2}{3}$

5. Give the absolute value.

a. $|8|$ b. $|-5|$ c. $|-3.2|$ d. $|22|$

Teaching Notes:

- Make sure the students have minimal understanding of square roots.
- Absolute value is ALWAYS POSITIVE because it measures distance from zero.
- Remind students that a number cannot be rational and irrational.
- To change a rational number to a decimal, divide the numerator by the denominator.

Answers: 1. a. natural, whole, integer, rational , real b. rational, real c. irrational, real d. whole, integer, rational, real e. rational, real f. irrational, real g., integer, rational, real h. rational, real 2. See below 3. a. 0.875 b. 0.81 c. 0.6 d. 0.25 4. a. $>$ b. $<$ c. $>$ d. $<$ e. $<$ 5. a. 8 b. 5 c. 3.2 d. 22

Mini Lecture 1.4
Basic Rules of Algebra

Learning Objectives:

1. Understand and use the vocabulary of algebraic expressions.
2. Use commutative properties.
3. Use associative properties.
4. Use the distributive property.
5. Combine like terms.
6. Simplify algebraic expressions.

Examples:

1. Fill in the blanks.

Algebraic Expression	# of terms	coefficients	like terms
a. $6y - 3x - 4y + 8$	________	__________	________
b. $5x^2 + 2y - 2x^2 + 9 - 3y$	________	__________	________
c. $6x^2 - 9y + 4x + 8 - y + 5$	________	__________	________

2. Name the property being illustrated and then simplify if possible.
   a. $6(x + 2) = 6x + 12$ ________________________________

   b. $(9 \cdot 12)5 = 9(12 \cdot 5)$ ________________________________

   c. $(x + 4) + 8 = x + (4 + 8)$ ________________________________

   d. $(2)(3.14)(5) = 2(5)(3.14)$ ________________________________

3. Simplify.
   a. $6x - x + 2x =$ ____________ b. $3a - 8 + 2a + 10 =$ _______________
   c. $6(x + 3) - 5 =$ ___________ d. $2\,(x - 4) - (x - 2) =$ ______________
   e. $5\,(y - 2) + 3(4 - y) =$ ____________

Teaching Notes:

- A coefficient is the number factor of a term.
- Like terms have the very same variables raised to the same exponents.
- When applying the commutative property, only the order changes.
- The commutative property holds for addition and multiplication only.
- When applying the associative property the grouping changes.
- The associative property holds for addition and multiplication only.
- When combining like terms, add or subtract the coefficients, the variable part remains the same.
- Always use parentheses when substituting a value for a variable.

Answers: 1 a. 4; 6, –3, –4, 8; $6y$ and $-4y$ b. 5; 5, 2, –2, 9, –3; $5x^2$ and $-2x^2$; $2y$ and $-3y$
c. 6; 6, –9, 4, 8, –1, 5; $9y$ and $-y$; 8 and 5 2. a. distributive b. associative of multiplication
c. associative of addition d. commutative of multiplication 3. a. $7x$ b. 5a +2 c. 6x + 13
d. $x - 6$ e. $2y + 2$

Mini Lecture 1.5
Addition of Real Numbers

Learning Objectives:

1. Add numbers with a number line.
2. Find sums using identity and inverse properties.
3. Add numbers without a number line.
4. Use addition rules to simplify algebraic expressions.
5. Solve applied problems using a series of additions.

Examples:

1. Find each sum using a number line.

   a. $3 + -5$ b. $-4 + -6$ c. $-1 + 2$ d. $5 + 4$

2. Add without using a number line.

   a. $-7 + -11$ b. $-0.4 + -3.2$ c. $-\frac{4}{5} + -\frac{3}{10}$ d. $-15 + 4$

   e. $7.1 + 8.5$ f. $-8 + 25$ g. $-6.4 + 6.1$ h. $\frac{5}{8} + -\frac{3}{4}$

3. Simplify the following.

   a. $-30x + 5x$ b. $-2y + 5x + 8x + 3y$ c. $-2(3x + 5y) + 6(x + 2y)$

4. Write a sum of signed numbers that represents the following situation. Then, add to find the overall change.

   If the stock you purchased last week rose 2 points, then fell 4, rose 1, fell 2, and rose 1, what was the overall change for the week?

Teaching Notes:

- When adding numbers with like signs, add and take the sign.
- When adding numbers with unlike signs, subtract the smaller absolute value from the larger absolute value, and the answer will have the sign of the number with the larger absolute value.

Answers: 1. a. -2 b. -10 c. 1 d. 9 2. a. -18 b. -3.6 c. $-\frac{11}{10}$ or $-1\frac{1}{10}$ d. -11 e. 15.6
f. 17 g. -0.3 h. $-\frac{1}{8}$ 3. a. $-25x$ b. $13x + y$ c. $2y$
4. $2 + (-4) + 1 + (-2) + 1 = -2$; fell 2 points

Mini Lecture 1.6
Subtraction of Real Numbers

Learning Objectives:

1. Subtract real numbers.
2. Simplify a series of additions and subtractions.
3. Use the definition of subtraction to identify terms.
4. Use the subtraction definition to simplify algebraic expressions.
5. Solve problems involving subtraction.

Examples:

1. Subtract by changing each subtraction to addition of the opposite first.

   a. $6 - 12$ b. $-15 - 15$ c. $13 - 21$ d. $\frac{2}{5} - \frac{5}{6}$

   e. $4.2 - 6.8$ f. $25 - (-25)$ g. $-51 - (-13)$ h. $14 - (-13)$

2. Simplify.

   a. $-16 - 14 - (-10)$ b. $-20.3 - (-40.1) - 18$

   c. $15 - (-3) - 10 - 18$ d. $-11 - 21 - 31 - 41$

3. Identify the number of terms in each expression; then name the terms.

   a. $4x - 6y + 12 - 3y$ b. $16 - 2x - 15$

   c. $15a - 2ab + 3b - 6a + 18$ d. $5y - x + 3y - 14xy$

4. Simplify each algebraic expression.

   a. $8x + 7 - x$ b. $-11y - 14 + 2y - 10$

   c. $15a - 10 - 12a + 12$ d. $25 - (-3x) - 15 - (-2x)$

5. Applications.

   a. The temperature at dawn was –7 degrees but fortunately the sun came out and by 4:00 p.m. the temperature had reached 38 degrees. What was the difference in the temperature at dawn and 4:00 p.m.?

   b. Express 214 feet below sea level as a negative integer. Express 10,510 above sea level as a positive integer. What is the difference between the two elevations?

Teaching Notes:

- Say the problem to yourself. When you hear the word "minus", immediately make a "change-change". That means to "change" the subtraction to addition and "change" the sign of the number that follows to its opposite.
- Remember, the sign in front of a term goes with the term.
- The symbol "–"can have different meanings:
  1. subtract or "minus" only when it is between 2 terms
  2. the opposite of
  3. negative

Answers: 1. a. –6 b. –30 c. –8 d. $-\frac{13}{30}$ e. –2.6 f. 50 g. –38 h. 27 2. a. –20 b. 1.8 c. –10 d. –104 3. a. 4 terms; $4x, -6y, 12, -3y$ b. 3 terms; $16, -2x, -15$ c. 5 terms; $15a, -2ab, 3b, -6a, 18$ d. 4 terms; $5y, -x, 3y, -14xy$ 4. a. $7x + 7$ b. $-9y - 24$ c. $3a + 2$ d. $5x + 10$ 5. a. 45 degrees b. –214 feet. 10,500 feet; 10, 724 feet

Mini Lecture 1.7
Multiplication and Division of Real Numbers

Learning Objectives:

1. Multiply real numbers.
2. Multiply more than two real numbers.
3. Find multiplicative inverses.
4. Use the definition of division.
5. Divide real numbers.
6. Simplify algebraic expressions involving multiplication.
7. Determine whether a number is a solution of an equation.
8. Use mathematical models involving multiplication and division.

Examples:

1. Multiply.

   a. $(3)(-4)$ b. $(-6)(-5)$ c. $(-8)(0)$ d. $(-3.2)(-1.1)$ e. $\left(-\frac{3}{4}\right)\left(\frac{2}{9}\right)$

   f. $(-5)(2)(-1)$ g. $(-2)(2)(-3)(-3)$

2. Find the multiplicative inverse of each number.

   a. -8 b. $\frac{2}{5}$ c. -7 d. $\frac{1}{4}$

3. Use the definition of division to find each quotient.

   a. $-49 \div 7$ b. $\frac{-24}{-4}$

4. Divide or state that the expression is undefined.

   a. $\frac{-18}{0}$ b. $-\frac{4}{5} \div \frac{20}{25}$ c. $-32.4 \div 8$ d. $0 \div -8$

5. Simplify.

   a. $-3(2x)$ b. $9x + x$ c. $-12a + 4a$ d. $-(5x - 3)$

   e. $-2(3y + 4)$ f. $2(3x + 4) - (4x - 6)$

Teaching Notes:

- The product of an even number of negative numbers is positive.
- The product of an odd number of negative numbers is negative.
- Any product using zero as a factor will equal zero.
- The quotient of two real numbers with different signs is negative.
- The quotient of two real numbers with same signs is positive.
- Division of a non-zero number by zero is undefined.
- Any non-zero number divided into 0 is 0.

Answers: 1. a. -12 b. 30 c. 0 d. 3.52 e. $-\frac{1}{6}$ f. 10 g. -36 2. a. $-\frac{1}{8}$ b. $\frac{5}{2}$ c. $-\frac{1}{7}$ d. $\frac{4}{1}$
3. a. -7 b. 6 4. a. undefined b. -1 c. -4.05 d. 0 5.a. $-6x$ b. $10x$ c. $-8a$ d. $-5x + 3$
e. $-6y - 8$ f. $2x + 14$

Mini Lecture 1.8
Exponents and Order of Operations

Learning Objectives:

1. Evaluate exponential expressions.
2. Simplify algebraic expressions with exponents.
3. Use order of operations agreement.
4. Evaluate mathematical models.

Examples:

1. Identify the base and the exponent, then evaluate.
   a. 3^4 b. $(-4)^3$ c. -8^2 d. $(-8)^2$

2. Evaluate.
   a. 13^2 b. 2^5 c. $(-3)^3$ d. 5^2

3. Simplify if possible.
   a. $6x^2 - x^2$ b. $5y^3 + 2y - 3y^3$ c. $6a^2 + 2a - 4a^2 - 6a$
   d. $10p^3 - 8p^2$

4. Simplify by using the order of operations.
   a. $30 \div 2 \cdot 3 - 52$
   b. $14 - (33 \div 11) + 4$
   c. $(5 + 2)^2$
   d. $10 - 7(32 \div 8) + 5 \cdot 3$
   e. $\left(\frac{1}{4}\right) + \left(\frac{1}{3}\right)^2$
   f. $15 - 3[8 - (-12 \div 2^2) - 4^2]$
   g. $\dfrac{16 + 4^2 \div 8}{-2 - (-5)}$
   h. $22 + 5(x + 7) - 3x - 10$

5. 5. Evaluate each expression for the given value.
   a. $-a - a^2$ if $a = -3$ b. $-a - a^2$ if $a = 3$ c. $4x^2 - x + 3x$ if $x = -1$

6. Use the formula for perimeter of a rectangle, $P = 2w + 2l$ to find the perimeter of a rectangle if the length is 28 cm and the width is 15 cm.

Teaching Notes:

- If the negative sign is part of the base, it will be inside the parentheses.
- **NEVER** multiply the base and the exponent together.
- The exponent tells how many times to write the base as a factor.
- Always use parentheses when substituting a value for a variable.
- The Order of Operations must be followed on every problem.

Answers: 1. a. 81 b. –64 c. –64 d. 64 2. a. 169 b. 32 c. –27 d. 25 3. a. $5x^2$ b. $2y^3 + 2y$ c. $2a^2 - 4a$ d. $10p^3 - 8p^2$ 4. a. –7 b. 15 c. 49 d. –3 e. $\frac{13}{36}$ f. 30 g. 6 h. 2x + 47 5. a. –6 b. –12 c. 2 6. 86 cm

Mini Lecture 2.1

The Addition Property of Equality

Learning Objectives:

1. Identify linear equations in one variable.
2. Use the addition property of equality to solve equations.
3. Solve applied problems using formulas.

Examples:

1. Identify the linear equations in one variable.
   a. $x+7=10$
   b. $x^2-2=7$
   c. $\frac{3}{x}=5$
   d. $|x+1|=6$
2. Solve the following equations using the addition property of equality. Be sure to check your proposed solution.
   a. $x+2=17$
   b. $-12=x-9$
   c. $x-\frac{1}{2}=4$
   d. $3x-2x=8$
   e. $5x+1=4(x-2)$
   f. $x+3.5=4.8$
   g. $2x+5=x-2$
   h. $3x+5=2x+5$

3. If Sue is 2 years older than John then we will use S to represent Sue's age and J to represent John's age. Use the equation $S=J+2$ to find John's age if Sue is 41.

Teaching Notes:

- Solving an equation is the process of finding the number (or numbers) that make the equation a true statement. These numbers are called the solutions, or roots, or the equation.
- To apply the addition property of equality, one must add the same number or expression to both sides of the equation.
- Equivalent equations are equations that have the same solution.

Answers: 1. a. linear b. not linear c. not linear d. not linear 2. a. 15 b. -3 c. $4\frac{1}{2}$ or $\frac{9}{2}$ d. 8 e. -9 f. 1.3 g. -7 h. 0 3. 39

Mini Lecture 2.2
The Multiplication Property of Equality

Learning Objectives:

1. Use multiplication property of equality to solve equations.
2. Solve equations in the form $-x = c$.
3. Use addition and multiplication properties to solve equations.
4. Solve applied problems using formulas.

Examples:

1. Multiply both sides of the equation by the reciprocal of the coefficient of the variable to solve for the variable.
   a. $\frac{x}{3} = 6$ b. $\frac{x}{-2} = -7$ c. $\frac{y}{15} = -10$ d. $8 = \frac{x}{-3}$

2. Divide both sides of the equation by the coefficient of the variable to solve for the variable.
   a. $6x = 18$ b. $-2x = -14$ c. $15y = -10$ d. $24 = -3x$

Both of the above methods of isolating the variable are effective for solving equations.

3. Solve each equation by multiplying or dividing.
   a. $18y = -108$ b. $\frac{3}{5}x = 12$ c. $124 = \frac{x}{3}$ d. $-7x = -63$

4. Multiply or divide both sides of each equation by –1 to get a positive x.
   a. $-x = -7$ b. $82 = -x$ c. $-a = -\frac{3}{7}$ d. $14 = -x$

5. Solve each equation using both the addition and multiplication properties of equality.
   a. $3x - 5 = 13$ b. $18 - 6x = 14 - 2x$ c. $23 = 2a - 7$
   d. $-6y - 21 = 21$ e. $33 - x = 3x - 11$ f. $\frac{2}{3}x - 6 = 12$

Teaching Notes:

- Remind students that reciprocals always have the same sign.
- When students see $-x$ they must realize the coefficient is –1.

Answers: 1. a. $x = 18$ b. $x = 14$ c. $y = -150$ d. $x = -24$ 2. a. $x = 3$ b. $x = 7$ c. $y = -\frac{2}{3}$ d. $x = -8$
3. a. $y = -6$ b. $x = 20$ c. $x = 372$ d. $x = 9$ 4. a. $x = 7$ b. $x = -82$ c. $a = \frac{3}{7}$ d. $x = -14$
5. a. $x = 6$ b. $x = 1$ c. $a = 15$ d. $y = -7$ e. $x = 11$ f. $x = 27$

Mini Lecture 2.3
Solving Linear Equations

Learning Objectives:

1. Solve linear equations.
2. Solve linear equations containing fractions.
3. Solve linear equations containing decimals.
4. Identify equations with no solution or infinitely many solutions.
5. Solve applied problems using formulas.

Examples:

1. $3x + 2x + 8 = -7 + x + 11$
2. $6x = 3(x + 9)$
3. $5(2x - 1) - 15 = 3(4x + 2) + 4$
4. $\frac{x}{5} = \frac{2x}{3} + \frac{7}{15}$
5. $1.2x + 1.8 = 0.6x$
6. $1.3x + 1.7 = -1 - 1.4x$
7. $2x + 9 = 2(x + 4)$
8. $4(x + 2) + 5 = 5(x + 1) + 8$
9. Use the formula $P = 4s$ to find the length of a side of a square whose perimeter is 32 in.

Teaching Notes:

- Simplify the algebraic expression on each side of the equal sign.
- Collect variable terms on one side of the equal sign and all constant terms on the other side of the equal sign.
- Isolate the variable and solve.
- Check your solution in the original expression.

Answers: 1. –1 2. 9 3. –15 4. –1 5. –3 6. –1 7. inconsistent, no solution 8. 0 9. 8 inches

Mini Lecture 2.4
Formulas and Percents

Learning Objectives:

1. Solve a formula for a variable.
2. Use the percent formula.
3. Solve applied problems involving percent change.

Examples:

1. Solve the formula for the indicated variable by isolating the variable.

   a. $A = \dfrac{B_1 + B_2}{2}$ for B_1 b. $P = a + b + c$ for c

   c. $A = \pi r^2 h$ for h d. $4p + H = M$ for p

   e $Ax + By = C$ for A f. $y = mx + b$ for b

2. Translate each question into an equation using the percent formula, $A = PB$, then solve the equation.

   a. What is 15 percent of 60? b. 62% of what number is 31?

   c. What percent of 132 is 33? d. 60 is what percent of 500?

3. The average, or mean A of the three exam grades, x, y, z, is given by formula $A = \dfrac{x+y+z}{3}$.

   a. Solve the formula for z.

   b. If your first two exams are 75% and 83% ($x = 75$, $y = 83$), what must you get on the third exam to have an average of 80%?

Teaching Notes:

- Many students have trouble solving formulas for a letter and need to be reminded the same steps are used when solving for a letter in a formula as are used when solving any equation for a variable.
- When changing a decimal to a percent, move the decimal point two places to the right and use the % symbol.
- When changing a percent to a decimal, move the decimal point two places to the left and drop the % symbol.
- When translating English into a mathematical equation, the word "is" translates to equals and the word "of" means multiply.

Answers: 1. a. $B_1 = 2A - B_2$ b. $c = P - a - b$ c. $h = \dfrac{A}{\pi r^2}$ d. $p = \dfrac{M-H}{4}$ e. $A = \dfrac{C - By}{x}$
f. $b = y - mx$ 2. a. $x = 0.15(60)$; 9 b. $0.62x = 31$; 50 c. $x \cdot 132 = 33$; 25% d. $60 = x \cdot 500$; 12%
3. a. $z = 3A - x - y$ b. 82%

Mini Lecture 2.5

An Introduction to Problem Solving

Learning Objectives:

1. Translate English phrases into algebraic expressions.
2. Solve algebraic word problems using linear equations.

Examples:

1. Translate each English phrase into an algebraic expression. Let "x" represent the unknown.
   a. Three times a number decreased by 11.
   b. The product of seven and a number increased by 2.
   c. Eight more than a number.

2. Translate each sentence into an algebraic equation and then solve the equation.
   a. Twice a number less five is eleven.
   b. Five times the sum of a number and eight is 30.

3. Identify all unknowns, set up an equation, and then solve.
   a. Bill earns five dollars more per hour than Joe. Together their pay for one hour totals \$21. How much does each man earn per hour?
   b. Two consecutive even integers equal 42. Find the integers.

Teaching Notes for solving algebraic equations:

- Make sure to familiarize all students with basic mathematical terms and the proper way to translate to algebraic terms.
- First, read the problem carefully and assign a variable for one of the unknown quantities.
- Write expressions if necessary for any other unknown quantities in terms of same variable.
- Write an equation for the stated problem.
- Solve the equation and answer the question.
- Check the solution in the original stated problem.

Answers:

1. a. $3x - 11$
   b. $7x + 2$
   c. $x + 8$

2. a. $2x - 5 = 11$
   $x = 8$
   b. $5(x + 8) = 30$
   $x = -2$

3. a. x = Joe
   $x + 5$ = Bill
   $x + (x + 5) = 21$
   x = \$8 (Joe)
   $x + 5$ = \$13 (Bill)

   b. $x = 1^{st}$ even integer
   $x + 2 = 2^{nd}$ even integer
   $x + (x + 2) = 42$
   $x = 20$
   $x + 2 = 22$

Mini Lecture 2.6
Problem Solving in Geometry

Learning Objectives:

1. Solve problems using formulas for perimeter and area.
2. Solve problems using formulas for a circle's area and circumference.
3. Solve problems using formulas for volume.
4. Solve problems involving the angles of a triangle.
5. Solve problems involving complementary and supplementary angles.

Examples:

1. A triangular flower bed has an area of 48 square feet and a height of 12 feet. Find the base of the flower bed.

2. The diameter of a fish pond is 6 feet. Find the area and circumference of the fish pond. First express answer in terms of π, then round both answers to the nearest square foot and foot respectively.

3. Which is the better buy: a 3 liter bottle of soft drink for $2.99 or a 1.2 liter bottle for $1.10?

4. Find the volume of a cylinder with a radius of 2 inches and height of 6 inches. Give answer in π form and then round answer to nearest cubic inch.

5. A volleyball has a radius of 3 inches. Find how much air is needed to fill the ball. Give answer in π form and then round answer to nearest cubic inch.

6. Given a right triangle and knowing that the two acute angles are complementary, find the measure of each if one angle is twice the measure of the other.

Teaching Notes:

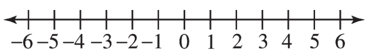
- Make sure to emphasize the formulas outlined in the section.
- Write formula, substitute the given values, and solve for the unknown.

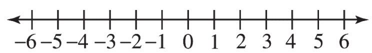
Answers: 1. base = 8 ft. 2. area = 9π ft^2, 28 ft^2; circumference = 6π ft., 19 ft. 3. 1.2 liter bottle
4. 24π in^3, 75 ft^3 5. 36π in^3, 113 in^3 6. 30°, 60°

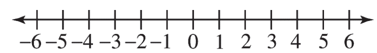
Mini Lecture 2.7
Solving Linear Inequalities

Learning Objectives:
1. Graph the solutions of an inequality on a number line.
2. Use interval notation.
3. Understand properties used to solve linear inequalities.
4. Solve linear inequalities.
5. Identify inequalities with no solution or true for all real numbers.
6. Solve problems using linear inequalities.

Examples:
1. Graph each inequality on the number line.
   a. $x \geq -4$ b. $x < 3$ c. $-1 \leq x < 5$



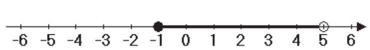


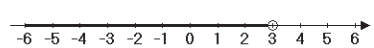


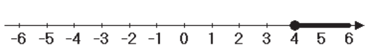
2. Solve each inequality. Write answers in set builder notation and interval notation.
   a. $4x - 3 \leq 5$ b. $6 - x \geq 3$ c. $6x - 12 < 8x - 14$

3. Solve each inequality and give the solution in set builder notation: Graph solution on a number line.
   a. $\frac{1}{5}x > -3$ b. $4(6 - 2x) \geq 12 - 4x$ c. $12x - 3 \geq 4(3x + 2)$
   d. $5(x - 3) \geq 5x - 15$ e. $20 < 3x + 5$ f. $2(x - 5) > 5x + 3$

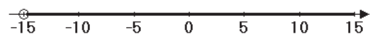
Teaching Notes:
- When graphing the solution of an inequality:
  Use a solid dot when the end point is included in the solution. ($\geq$ or $\leq$)
- When graphing the solution of an inequality:
  Use an open dot when the end point is not included in the solution. ($>$ or $<$)
- When an inequality is multiplied or divided by a negative value, the inequality symbol must be reversed.

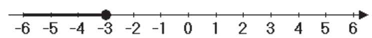
Answers: 1. a. 

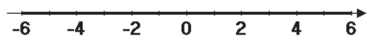
b. 

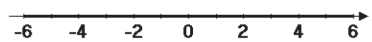
c. 

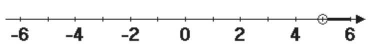
2. a. $\{x \mid x \leq 2\}$ $(-\infty, 2]$ b. $\{x \mid x \leq 3\}$ $(-\infty, 3]$ c. $\{x \mid x > 1\}$ $(1, \infty)$

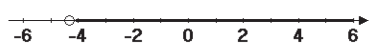
3. a. $x > -15$ $\{x \mid x > -15\}$ 

b. $x \leq 3$ $\{x \mid x \leq 3\}$ 

c. No Solution { } or Ø 

d. All Real Numbers $\{x \mid x \text{ is a real number}\}$ 

e. $x > 5$ $\{x \mid x > 5\}$ 

f. $x > -\frac{13}{3}$ $\left\{x \mid x > -\frac{13}{3}\right\}$ 

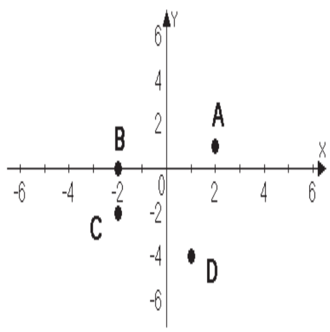
Mini Lecture 3.1
Graphing Linear Equations in Two Variables

Learning Objectives:

1. Plot ordered pairs in the rectangular coordinate system.
2. Find coordinates of points in the rectangular coordinate system.
3. Determine whether an ordered pair is a solution of an equation.
4. Find solutions of an equation in two variables.
5. Use point plotting to graph linear equations.
6. Use graphs of linear equations to solve problems.

Examples:

1. Plot the given points in a rectangular coordinate system. Indicate in which quadrant each point lies.
   a. (–2, –4)
   b. (3, 1)
   c. (–2, 3)
   d. (5, –2)
2. Give the ordered pairs that correspond to the points labeled.



3. Determine if the ordered pair is a solution for the given equation.
   a. $2x + 3y = 10$ (2, 2) b. $3x - y = 5$ (–1, 2)

4. Find five solutions for $y = 2x - 1$ by completing the table of values.
   a.

x	$y = 2x - 1$	(x, y)
–2		
–1		
0		
1		
2		

   b. Plot the ordered pairs to graph the line $y = 2x - 1$.

5. Find five solutions for $y = -x + 1$ by completing the table of values.

a.

x	$y = -x + 1$	(x, y)
–2		
–1		
0		
1		
2		

b. Plot the ordered pairs to graph the quadratic equation $y = -x + 1$.

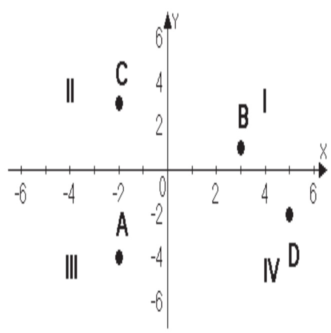
6. a. Your cell phone contract has a base charge of $10 per month and a $.03 per minute charge for nation-wide calling. Create a table of values for $y = .03x + 10$. Use 0, 60, 120, 180, 240 for x.

b. Plot the ordered pairs to graph the above equation.

Teaching Notes:

- When graphing linear equations $(y = mx + b)$ on a coordinate plane, the ordered pairs will form a line when connected.
- It is very important to be able to plot points accurately. Students often have problems with points in the form (0, b).

Answers: 1.

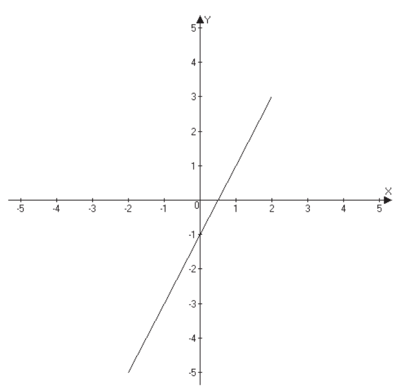


2.a. (2, 1) b. (–2, 0) c. (–1, –2) d. (1, –4)

3.a. yes b. no

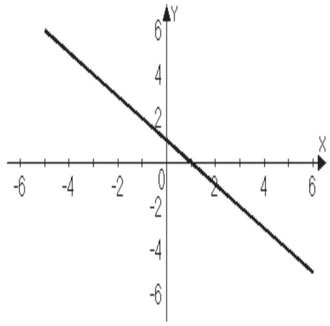
4. a. (–2, –5) (–1, –3) (0, –1) (1, 1) (2, 3)

b.



5. a. (–2, 3) (–1, 2) (0, 1) (1, 0) (2, –1)

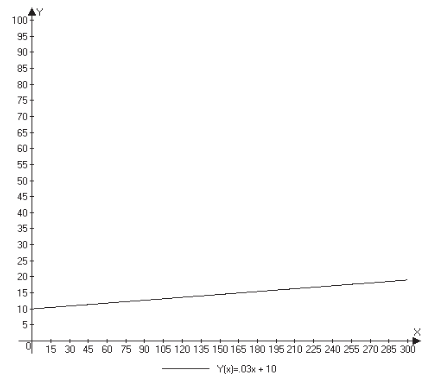
b.



b.

6. a.

x	$y = 0.03x + 10$	(x, y)
0	$y = 0 + 10$	(0, 10)
60	$y = 1.8 + 10$	(60, 11.80)
120	$y = 3.6 + 10$	(120, 13.60)
180	$y = 5.4 + 10$	(180, 15.40)
240	$y = 7.2 + 10$	(240, 17.20)

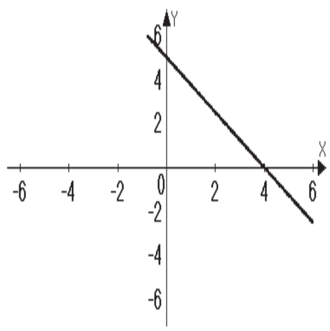


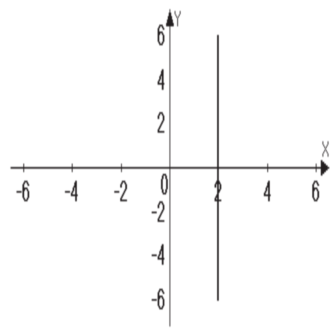
Mini Lecture 3.2
Graphing Linear Equations Using Intercepts

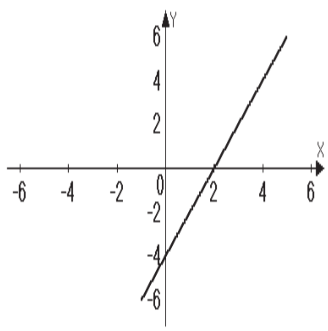
Learning Objectives:

1. Use a graph to identify intercepts.
2. Graph a linear equation in two variables using intercepts.
3. Graph horizontal and vertical lines.

Examples:

1. Identify the x and y-intercepts of each line.
   a.


   b.


   c.


2. Find the x-intercept of the graphs of each of the following equations by substituting 0 in for y and solving for x.
   a. $4x + 7y = 12$ b. $y = 3x - 3$ c. $x - 2y = -8$

3. Find the y-intercept of the graphs of each of the following equations by substituting 0 in for the x and solving for y.
   a. $3x + 2y = -12$ b. $y = 2x + 7$ c. $5x - y = 3$

4. Graph each of the following equations by finding the x and y-intercepts and a check point. Label the intercepts.
   a. $2x - 4y = 12$ b. $5x + 3y = -15$ c. $y = 2x + 6$

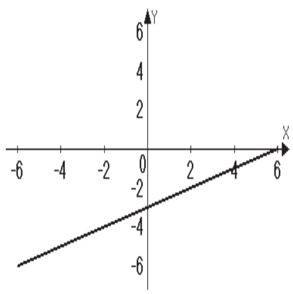
5. Graph each equation on the coordinate plane.
   a. $y + 8 = 12$ b. $x = -3$

Teaching Notes:

- Intercepts are points not just numbers.
- The *x*-intercept is the point where a graph crosses the *x* axis. The value of *y* is always zero at the *x*-intercept.
- The *y*-intercept is the point where a graph crosses the *y* axis. The value of *x* is always zero at the *y*-intercept.
- When an equation is in standard form and *a* and *b* are factors of *c*, then finding intercepts is a good method to choose for graphing.
- A table is often useful to find intercepts.
- A vertical line has no *y*-intercept, unless it is the *y*-axis ($x = 0$).
- A horizontal line has no *x*-intercept, unless it is the *x*-axis ($y = 0$).

Answers: 1. a. *x*-intercept (4,0); *y*-intercept (0,5) b. *x*-intercept (2,0); *y*-intercept (none) c. *x*-intercept (2,0); *y*-intercept (0, –4) 2. a. (3,0) b. (1,0) c. (–8,0) 3. a. (0, –6) b. (0,7) c. (0, –3)

4. a.



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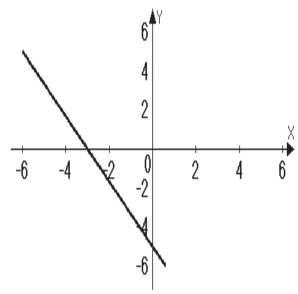
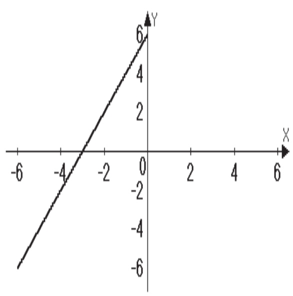
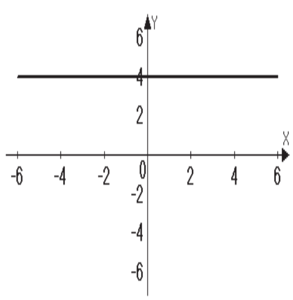


c.



5. a.



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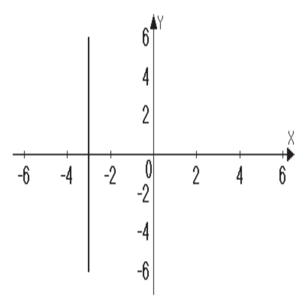


Mini Lecture 3.3
Slope

Learning Objectives:

1. Compute a line's slope.
2. Use slope to show that lines are parallel.
3. Use slope to show that lines are perpendicular.
4. Calculate rate of change in applied situations.

Examples:

1. Using the formula for slope $m = \frac{y_2 - y_1}{x_2 - x_1}$, find the slope of the line passing through each pair of points.
   a. (2, 4) (–3, 1)

   b. (–4, 2) (3, –1)

   c. (1, 5) (2, 5)

   d. (–8, 3) (–8, 1)

2. Determine whether a line passing through points (1, 4) and (5, 6) is parallel or perpendicular to a line passing through points (1, –5) and (0, –3).

3. Determine whether a line passing through points (3, 4) and (5, 2) is parallel or perpendicular to a line passing through points (–3, 5) and (–1, 3).

4. Determine whether a line passing through points (5, –8) and (3, 2) is parallel or perpendicular to a line passing through the points (–6, 3) and (–1, 4).

5 Property taxes have continued to increase year after year. Given that in 1990 a home's taxes were \$1200 and that same home's taxes were \$2600 in 2010. If x represents the year and y the real estate tax, calculate the slope and explain the meaning of your answer.

Teaching Notes:

- Slope is defined as $\frac{rise}{run}\left(\frac{horizontal\ change}{vertical\ change}\right)$.
- $m = \frac{y_2 - y_1}{x_2 - x_1}$, where m represents slope and comes from the French verb "*monter*" meaning to rise or ascend.
- Four slope possibilities:
  1. $m > 0$, positive slope, rises from left to right
  2. $m < 0$, negative slope, falls from left to right
  3. $m = 0$, line is horizontal
  4. m is undefined, line is vertical

Answers: 1.a. $\frac{3}{5}$ b. $-\frac{3}{7}$ c. 0 d. undefined 2. Perpendicular 3. Parallel 4. Perpendicular

5. slope is $\frac{70}{1}$; taxes went up $70 per year.

Mini Lecture 3.4

The Slope-Intercept Form of the Equation of a Line

Learning Objectives:

1. Find a line's slope and y-intercept of a line from its equation.
2. Graph lines in slope-intercept form.
3. Use slope and y-intercept to graph $Ax + By = C$.
4. Use slope and y-intercept to model data.

Examples:

1. Find the slope and y-intercept of each line with the following equations: (Write the y-intercept as a point.)

   a. $y = \frac{2}{3}x - 4$ b. $y = -3x + 2$ c. $y = -1$

   d. $y = \frac{1}{2}x$ e. $y = 4x - 5$ f. $y = -\frac{3}{4}x + 8$

2. Put each equation in slope-intercept form by solving for y. (Isolate y) Then name the slope and y-intercept.

   a. $2x + y = -6$ b. $-4x - 3y = 6$ c. $x - 2y = 8$

   d. $5y = 10x + 4$ e. $x + y = 10$ f. $3x - 4y = 7$

3. Graph each equation using the slope and the y-intercept.

   a. $4x - 2y = 6$ b. $y = -\frac{1}{2}x + 2$ c. $3x - y = -3$

Teaching Notes:

- In an equation in the form $y = mx + b$, m is the slope of line and b is the y-coordinate of the y-intercept.
- To graph: use the y-intercept as the starting point. Then use the slope to plot at least two more points.
- Remember, the slope must be in fraction form, $\frac{rise}{run}$. If the slope is an integer, it can be put over 1 to form a fraction.

Answers: 1. a. slope $\frac{2}{3}$; y-intercept (0, –4) b. slope –3; y-intercept (0, 2) c. slope 0; y-intercept (0, –1)

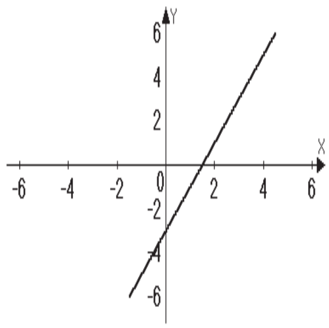
d. slope $\frac{1}{2}$; y-intercept (0, 0) e. slope 4, y-intercept (0, –5) f. slope $-\frac{3}{4}$; y-intercept (0, 8)

2. a. $y = -2x - 6$; slope –2; y-intercept (0, –6) b. $y = -\frac{4}{3}x - 2$; slope $-\frac{4}{3}$; y-intercept (0, –2)

c. $y = \frac{1}{2}x - 4$; slope $\frac{1}{2}$; y-intercept (0, –4) d. $y = 2x + \frac{4}{5}$; slope 2; y-intercept (0, $\frac{4}{5}$)

e. $y = -x + 10$; slope –1, y-intercept (0, 10) f. $y = \frac{3}{4}x - \frac{7}{4}$; slope $\frac{3}{4}$, y-intercept $\left(0, -\frac{7}{4}\right)$

3. a.



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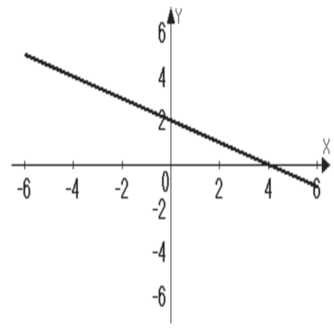

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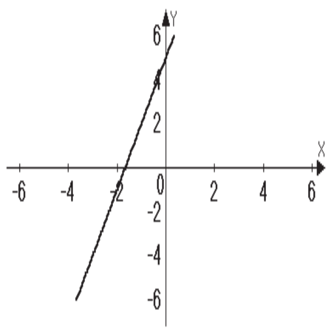


Mini Lecture 3.5
The Point-Slope Form of the Equation of a Line

Learning Objectives:

1. Use the point-slope form to write equations of a line.
2. Write linear equations that model data and make predictions.

Examples:

1. Write the point-slope form and the slope-intercept form of the equation of the line with slope 3 that passes through the point (–1, 4).
2. Write the point-slope form and the slope-intercept form of the equation of the line through the points (1, 4) (–2, 3).
3. The cost of graphing calculators over time has decreased. In 2000, one particular brand sold for $136, in 2011 that same calculator sold for $92. Use the coordinates of the two points (2000, 136) (2011, 92) to write the equation in point-slope and slope-intercept form.

Teaching Notes:

- Distinguish between 2 forms of an equation of a line: point-slope (4.3) and slope-intercept (4.4).
- Point-slope equation with the slope of m and passing through point (x_1, y_1) is $y - y_1 = m(x - x_1)$.
- Use the point-slope equation when given at least one point and the slope.
- If given two points and asked to find the equation of the line, one must first find the slope, section 3.3 and then use the slope and one of the given points in the point-slope equation.

Answers: 1. $y = 4(x + 1)$; $y = 3x + 7$ 2. $y - 4 = \frac{1}{3}(x - 1)$; $y = \frac{1}{3}x + \frac{11}{3}$
3. $y - 136 = -4(x - 2000)$; $y = -4x + 8136$ or $y - 92 = -4(x - 2011)$; $y = -4x + 8136$

Mini Lecture 3.6

Linear Inequalities in Two Variables

Learning Objectives:

1. Determine whether an ordered pair is a solution of an inequality.
2. Graph linear inequalities in two variables.
3. Solve applied problems involving linear inequalities in two variables.

Examples:

Determine which ordered pairs are solutions to the given inequalities.

1. $2x + 3y < 10$
   a. (–1, 4) b. (4, –1) c. (0, 3) d. (3, 2)

2. $y \geq -x + 3$
   a. (4, 7) b. (–3, 0) c. (5, 2) d. (–1, –1)

3. $4x - 2y \leq 8$
   a. (0, 2) b. (2, 0) c. (–2, –2) d. (1, –5)

Put each inequality in slope-intercept form then graph the boundary line and shade the appropriate half plane.

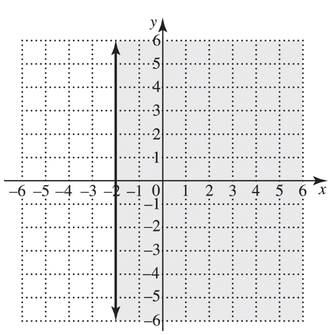
4. a. $2x + 7 \geq 3$ b. $3x - 3y > 6$ c. $y \geq \frac{1}{4}x - 3$
   d. $x < y$ e. $4x + 6y \leq -12$ f. $x \geq 4$

Teaching Notes:

- An inequality has many solutions, therefore shading must be used to show the solutions.
- The graphed line is a boundary and it divides the coordinate plane into two half planes, one of which will be shaded.
- Remind students to use a solid line if the points on the line are included in the solution (≥ or ≤).
- Remind students to use a broken or dashed line if points on the line are not included in the solution (> or <).
- Students tend to forget to reverse the inequality symbol when multiplying or dividing both sides by a negative number.
- Remind students to use a straight edge when graphing lines.

Answers: 1.no; yes; yes; no 2. yes; no; yes; no 3. yes; yes; yes; no

4. a.



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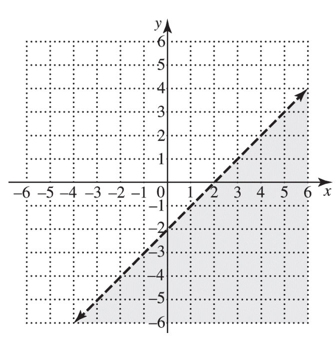
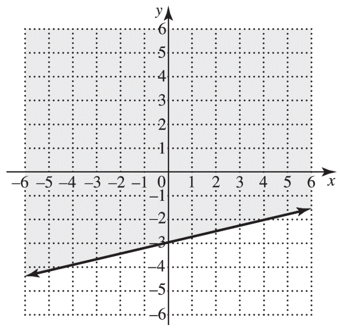
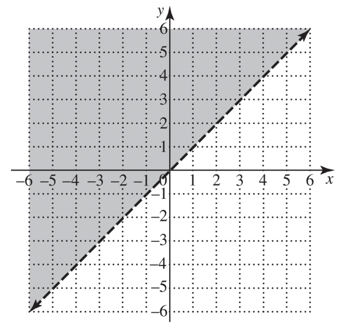


Boundary line dashed

c.

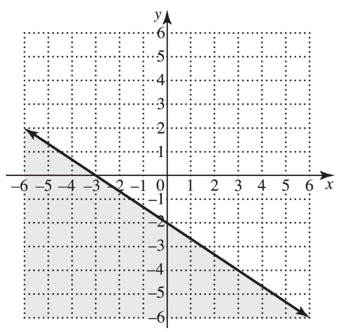


d.

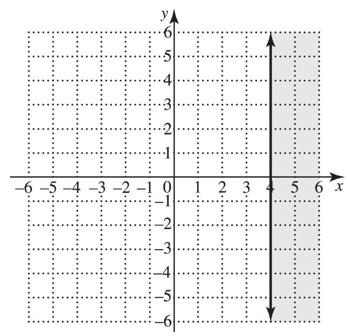


Boundary line dashed

e.



f.



Mini Lecture 4.1

Solving Systems of Linear Equations by Graphing

Learning Objectives:

1. Decide whether an ordered pair is a solution of a linear system.
2. Solve systems of linear equations by graphing.
3. Use graphing to identify systems with no solution or infinitely many solutions.
4. Use graphs of linear systems to solve problems.

Examples:

1. Consider the system.
   $x + y = -3$
   $2x + y = 1$

   Determine if each ordered pair is a solution of the system.
   a. (4, 7) b. (4, –7)

2. Solve the following systems by graphing. State the solution (the intersection point) as an ordered pair (x, y) or state if there is no solution, or state if there are an infinite number of solutions.

   a. $2x + y = -3$
   $y = -2x - 3$

   b. $2x + y = 3$
   $3x - 2y = 8$

   c. $x + 2y = 6$
   $x + 2y = 2$

Teaching Notes:

- When graphing a system of linear equations, there are three possible outcomes:
  1. The two lines can intersect at one point, meaning there is one solution to the system.
  2. The two lines can be parallel to one another, meaning there is no solution to the system.
  3. The two lines are identical or coincide, meaning there are infinitely many solutions to the system.
- When two lines are parallel the system is inconsistent and has no solution.
- When two lines are coinciding, they are called dependent equations and have infinitely many solutions.

Answers:
1. a. not a solution b. yes, a solution 2. a. infinitely many solutions b. (2, –1) c. lines parallel, no solution

Mini Lecture 4.2

Solving Systems of Linear Equations by the Substitution Method

Learning Objectives:

1. Solve linear systems by the substitution method.
2. Use the substitution method to identify systems with no solution or infinitely many solutions.
3. Solve problems using the substitution method.

Examples:

Solve each system using the substitution method. If there is no solution or an infinite number of solutions, so state.

1. a. $x + y = 3$, $y = x + 5$ b. $3x - 2y = 5$, $x = 4y - 5$ c. $7x + 6y = -9$, $y = -2x + 1$ d. $5x - 6y = -4$, $x = y$

2. a. $x + 3y = 4$, $x - 2y = -1$ b. $-2x - y = -3$, $3x + y = 0$ c. $8x - y = 15$, $3x + 4y = 10$ d. $3x - 5y = 12$, $x + 2y = 4$

3. a. $3x + 5y = -3$, $x - 5y = -5$ b. $2x - 4y = -4$, $x + 2y = 8$ c. $7x - 6y = -1$, $x - 2y = -1$ d. $2x - y = 1$, $4x + y = 8$

4. a. $6x + 3y = 1$, $y = -2x - 5$ b. $4x - 4y = 8$, $x - y = 2$ c. $4x - 2y = 8$, $2x - y = 4$ d. $y = -3x + 2$, $6x + 2y = 1$

Teaching Notes:

- Students like to follow specific steps so give them a list of steps to use for solving systems by substitution. Begin with: Isolate a variable with a coefficient of 1 first.
- Many students think they must solve for *y*. Stress that it does not matter whether the variable solved for is *x* or *y*.
- Use colored pens or markers to underline in one equation what will be substituted in the other equation.
- If a graphing calculator is being used in the class, graphing on the calculator is a good way to check solutions.

Answers: 1. a. (–1, 4) b. (3, 2) c. (3, –5) D. (4, 4) 2. a. (1, 1) b. (–3, 9) c. (2, 1) d. (4, 0)
3. a. $(-2, \frac{3}{5})$ b. $(3, \frac{5}{2})$ c. $(\frac{1}{2}, \frac{3}{4})$ d. $(\frac{3}{2}, 2)$ 4. a. No solution b. Infinite solutions
c. Infinite solutions d. No solution

Mini Lecture 4.3

Solving Systems of Linear Equations by the Addition Method

Learning Objectives:

1. Solve linear systems by the addition method.
2. Use the addition method to identify systems with no solution or infinitely many solutions.
3. Determine the most efficient method for solving a linear system.

Examples:

Solve the following systems by the addition method.

1. $x + y = 10$
   $x - y = 8$

2. $4x + 3y = 7$
   $-4x + y = 5$

3. $3x - y = 8$
   $x + 2y = 5$

4. $2w - 3z = -1$
   $3w + 4z = 24$

5. $4x - 5y = 8$
   $-4x + 5y = -8$

6. $2x = 5y + 4$
   $2x - 5y = 6$

Teaching Notes:

- When solving a system of linear equations there are three methods:
  Graphing (4.1)
  Substitution (4.2)
  Addition (4.3)
- Any of the three methods will work when solving a system and produce the correct answer.
- Teach students how to determine which of the three methods is the most efficient when solving a system of equations.

Answers: 1. (9, 1) 2. $\left(-\frac{1}{2},\ 3\right)$ 3. (3, 1) 4. (4, 3) 5. infinitely many solutions 6. no solution

Mini Lecture 4.4
Problem Solving Using Systems of Equations

Learning Objectives:
1. Solve problems using linear systems.

Examples:
Use variables to represent unknown quantities. Write a: Let x = and y = statement for each problem. (Do not solve).
1. The sum of two numbers is 14. One number is six times larger than the other. Find the two numbers.
2. Three pairs of socks and two pairs of mitten cost \$42. One pair of the same kind of socks and four pair of the mittens cost \$24. Find out how much one pair of socks and one pair of mittens cost.
3. John has \$5 bills and \$10 bills in his wallet. He has a total of \$80. He has twice as many \$5 bills as \$10 bills. How many \$5 bills and how many \$10 bills does he have?

Now, for problems 4 – 6, write a system of equations that models the conditions of each problem. (Do not solve).
4.
5.
6.

Solve each of the following using a system of equations.
7. The sum of two numbers is 11. The second number is 1 less than twice the first number. Find the two numbers.
8. Alexis has \$1.65 in nickels and quarters. She has 9 coins altogether. How many coins of each kind does she have?
9. Paul invested \$12,000 in two accounts. One account paid 4% interest and one account paid 5% interest. At the end of the year his money had earned \$560 in interest. How much did he invest in each account?
10. A department store receives 2 shipments of bud vases and picture frames. The first shipment of 5 bud vases and 4 picture frames costs \$62. The second shipment of 10 bud vases and 3 picture frames cost \$84. Find the cost of a vase and a picture frame.

Teaching Notes:
- Stress the importance of reading the problem several times before beginning. Reading aloud really helps.
- Have students write a Let x= and y = statements for each word problem before trying to write the system of equations.
- Help students look at the system they have created and determine which method of solving will work best.
- Remind students to make sure their answers make sense for the given situation.
- Try to build confidence with word problems.

Answers: 1. Let x = one number; let y = the other number. 2. Let x = cost of 1 pair of socks; let y = cost of 1 pair of mittens. 3. Let x = number of \$5 bills; let y = number of \$10 bills

4. $x + y = 14$	5. $3x + 2y = 42$	6. $5x + 10y = 80$
$x = 6y$	$x + 4y = 24$	$x = 2y$

7. The numbers are 4 and 7 8. 3 nickels, 6 quarters 9. \$4000 invested @ 4% and \$8000 invested @ 5%
10. bud vases \$6, picture frames \$8

Mini Lecture 4.5
Systems of Linear Inequalities

Learning Objectives:

1. Use mathematical models involving systems of linear inequalities.
2. Graph the solution sets of systems of linear inequalities.

Examples:

1. Graph the solution set of each system.

   a. $y < -x + 3$
      $y \geq x - 4$

   b. $y > -\frac{1}{2}x + 3$
      $x \geq 3$

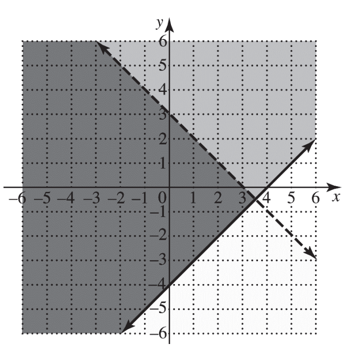
   c. $y \geq x - 1$
      $y < -\frac{1}{3}x + 1$

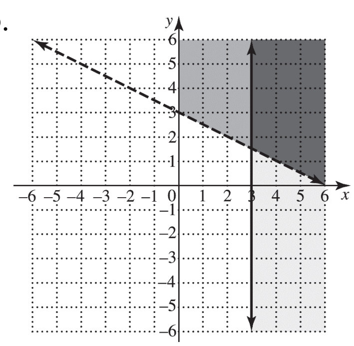
2. Name one point that is a solution for each system of linear inequalities in examples 1a, 1b, and 1c.
   a. b. c.

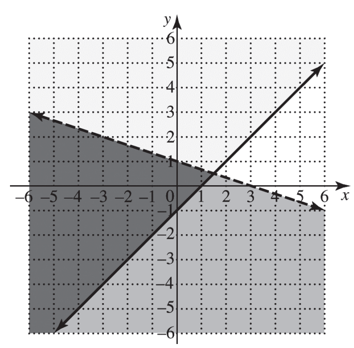
Teaching Notes:

- When the inequality symbol is > or <, the line should be dashed (- - - - -).
- When the inequality symbol is ≥ or ≤, the line should be solid (________).
- When graphing inequalities, it is easy to see the overlap of the graphs if different colored pencils are used to graph each inequality.

Answers:

1. a.


b.


c.


2. a. Answers will vary b. Answers will vary c. Answers will vary