Finite Mathematics and Its Applications 12th Edition Goldstein Test Bank

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Perform the indicated elementary row operation.

1)
$$\begin{cases} x - 5y = 4 \\ 2x + 2y = 5 \end{cases} \xrightarrow{R_2 + (-2)R_1}$$

Answer:
$$\begin{cases} x - 5y = 4 \\ 12y = -3 \end{cases}$$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

2) Find the result of performing the elementary row operation $R_3 + (5) R_2$ on the system $\begin{bmatrix} 1 & 0 & 3 & | & 9 \\ 0 & 1 & -3 & | & 2 \\ 0 & -5 & 4 & | & 1 \end{bmatrix}$.

A)
$$\begin{bmatrix} 1 & 0 & 3 & | & 9 \\ 0 & 0 & -3 & | & 2 \\ 0 & -5 & 4 & | & 1 \end{bmatrix}$$

B)
$$\begin{bmatrix} 1 & 0 & 3 & | & 9 \\ 0 & 1 & -3 & | & 2 \\ 0 & 0 & -11 & | & 11 \end{bmatrix}$$

C)
$$\begin{bmatrix} 1 & 0 & 3 & | & 9 \\ 0 & -5 & -11 & | & 11 \\ 0 & -5 & 4 & | & 1 \end{bmatrix}$$

D)
$$\begin{bmatrix} 1 & 0 & 3 & | & 9 \\ 0 & 1 & -3 & | & 2 \\ 10 & 5 & 14 & | & 11 \end{bmatrix}$$

Answer: B

3) Find the result of performing the elementary row operation $R_2 + (-1) R_3$ on the system $\begin{bmatrix} 1 & 0 & 3 & | & 9 \\ 0 & 1 & -3 & | & 2 \\ 0 & -5 & 4 & | & 1 \end{bmatrix}$.

A)
$$\begin{bmatrix} 1 & 0 & 3 & | & 9 \\ 0 & 1 & -3 & | & 2 \\ 0 & -7 & 2 & | & 0 \end{bmatrix}$$

B)
$$\begin{bmatrix} 1 & 0 & 3 & | & 9 \\ 0 & 6 & -7 & | & 1 \\ 0 & -5 & 4 & | & 1 \end{bmatrix}$$

C)
$$\begin{bmatrix} 1 & 0 & 3 & | & 9 \\ 0 & -7 & 2 & | & 0 \\ 0 & -5 & 4 & | & 1 \end{bmatrix}$$

D)
$$\begin{bmatrix} 1 & 0 & 3 & | & 9 \\ 0 & -1 & -3 & | & 2 \\ 0 & -4 & -7 & | & 1 \end{bmatrix}$$

Answer: B

4) The system $\begin{bmatrix} 1 & 0 & 0 & | & -2 \\ 0 & 1 & 3 & | & 5 \\ 1 & 1 & -3 & | & 4 \end{bmatrix}$ is equivalent to the system

A)
$$\begin{bmatrix} 1 & 0 & 0 & | & -2 \\ 0 & 1 & 3 & | & 5 \\ 0 & 1 & -3 & | & 6 \end{bmatrix}.$$

B)
$$\begin{bmatrix} 1 & 0 & 0 & | & -2 \\ 0 & 1 & -3 & | & 5 \\ 0 & 0 & 0 & | & 4 \end{bmatrix}.$$

C)
$$\begin{bmatrix} 1 & 0 & 0 & | & -2 \\ 0 & 3 & -9 & | & 5 \\ 1 & 1 & -3 & | & 4 \end{bmatrix}.$$

D)
$$\begin{bmatrix} 1 & 0 & 0 & | & -2 \\ 0 & 1 & -3 & | & 5 \\ 0 & 1 & -3 & | & 4 \end{bmatrix}.$$

Answer: A

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5) The system $\begin{bmatrix} 1 & 0 & 1 & -1 \\ 3 & 1 & 3 & 7 \\ 2 & 1 & 4 & 4 \end{bmatrix}$ is equivalent to the system

A)
$$\begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 7 \\ 2 & 1 & 4 & 4 \end{bmatrix}$$

B)
$$\begin{bmatrix} 1 & 0 & 1 & -1 \\ 3 & 1 & 3 & 7 \\ 0 & 1 & 3 & 2 \end{bmatrix}$$

C)
$$\begin{bmatrix} 1 & 0 & 1 & -1 \\ 3 & 1 & 3 & 7 \\ 0 & -1 & 2 & 2 \end{bmatrix}$$

D)
$$\begin{bmatrix} 1 & 0 & 1 & -1 \\ 3 & 1 & 3 & 7 \\ 0 & 1 & 2 & 2 \end{bmatrix}$$

Answer: C

Use the indicated row operation to change the matrix.

6) Replace R_2 by $R_1 + (-1)R_2$.

$$\begin{bmatrix} 1 & -3 & 4 \\ 2 & 3 & 1 \end{bmatrix}$$

A) $\begin{bmatrix} 1 & -3 & 3 \\ 3 & 0 & 4 \end{bmatrix}$

B) $\begin{bmatrix} 1 & -3 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

C) $\begin{bmatrix} 1 & -3 & 3 \\ 1 & 6 & -2 \end{bmatrix}$

D) $\begin{bmatrix} 1 & -3 & 4 \\ -1 & -6 & 3 \end{bmatrix}$

Answer: D

7) Replace R_2 by $\frac{1}{2}R_1 + \frac{1}{2}R_2$.

$$\begin{bmatrix} 2 & 0 & | & 2 \\ -2 & 2 & | & 8 \end{bmatrix}$$

A) $\begin{bmatrix} 2 & 0 & | & 2 \\ 0 & 0 & | & 5 \end{bmatrix}$

B) $\begin{bmatrix} 2 & 0 & | & 2 \\ 0 & 1 & | & 5 \end{bmatrix}$

C) $\begin{bmatrix} 2 & 0 & | & 2 \\ 0 & 2 & | & 10 \end{bmatrix}$

D) $\begin{bmatrix} 2 & 0 & | & 2 \\ -1 & 1 & | & 4 \end{bmatrix}$

Answer: B

8) Replace R_2 by $\frac{1}{3}R_1 + \frac{1}{2}R_2$.

$$\begin{bmatrix} 3 & 0 & | & 12 \\ -2 & 4 & | & 6 \end{bmatrix}$$

A) $\begin{bmatrix} 3 & 0 & | & 12 \\ 1 & 4 & | & 18 \end{bmatrix}$

B) $\begin{bmatrix} 3 & 0 & | & 12 \\ 0 & 0 & | & 7 \end{bmatrix}$

C) $\begin{bmatrix} 3 & 0 & | & 12 \\ -1 & 2 & | & 3 \end{bmatrix}$

D) $\begin{bmatrix} 3 & 0 & | & 12 \\ 0 & 2 & | & 7 \end{bmatrix}$

Answer: D

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

State the next elementary row operation that should be performed to put the matrix into diagonal form.

9)
$$\begin{bmatrix} 1 & 2 & -4 & | & 5 \\ 0 & 0 & 3 & | & 6 \\ 0 & 1 & 4 & | & 1 \end{bmatrix}$$
Answer: $R_2 \leftrightarrow R_3$

State and perform the next elementary row operation that should be performed to put the matrix in diagonal form.

10)
$$\begin{bmatrix} 1 & 0 & 3 & | & 9 \\ 0 & 1 & -3 & | & 2 \\ 0 & -5 & 4 & | & 1 \end{bmatrix}$$
Answer:

$$\xrightarrow{R_3 + 5R_2} \begin{bmatrix} 1 & 0 & 3 & | & 9 \\ 0 & 1 & -3 & | & 2 \\ 0 & 0 & -11 & | & 11 \end{bmatrix}$$

11)
$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Answer:

$$\xrightarrow{R_2 + 2R_3} \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

12)
$$\begin{bmatrix} 1 & 2 & -3 & 3 \\ 0 & -4 & 6 & 7 \\ 2 & 6 & 5 & -1 \end{bmatrix}$$

Answer:

$$\xrightarrow{R_3 + (-2)R_1} \begin{bmatrix} 1 & 2 & -3 & 3 \\ 0 & -4 & 6 & 7 \\ 0 & 2 & 11 & -7 \end{bmatrix}$$

13) Is $x = 2$, $y = -1$, $z = 4$ a solution of the system of equations shown below? Explain your answer.

$$\begin{cases} x - y + z = 7 \\ 2x \quad - z = 0 \\ 2y + 3z = 9 \end{cases}$$

Answer: $x = 2$, $y = -1$, $z = 4$ is not a solution because these values do not satisfy the third equation.

14) Is $x = 2$, $y = -1$, $z = 4$ a solution of the system of equations shown below? Explain your answer.

$$\begin{cases} x - y + z = 7 \\ 2x \quad - z = 0 \\ 2y + 3z = 10 \end{cases}$$

Answer: $x = 2$, $y = -1$, $z = 4$ is a solution because these values satisfy every equation.

15) When you solve a system of equations, what set of values are you looking for?

Answer: The solution to a system of equations is the set of all values such that each value satisfies every equation in the system.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the Gauss–Jordan method to solve the system of equations.

16)
$$\begin{cases} 2x + 4y = 14 \\ 2x + 3y = 9 \end{cases}$$
   A) $x = 5$, $y = -3$
   B) $x = -3$, $y = -5$
   C) $x = -3$, $y = 5$
   D) No solution

Answer: C

17)
$$\begin{cases} 3x + 5y = 16 \\ 3x = -9 \end{cases}$$
   A) $x = 5, y = -3$
   B) $x = -3, y = 5$
   C) $x = -3, y = -5$
   D) No solution

Answer: B

18)
$$\begin{cases} 5x + y = 12 \\ 6x + 3y = 9 \end{cases}$$
   A) $x = -3, y = -3$
   B) $x = -3, y = 3$
   C) $x = 3, y = -3$
   D) No solution

Answer: C

19)
$$\begin{cases} x + y + z = 7 \\ x - y + 2z = 6 \\ 5x + y + z = 3 \end{cases}$$
   A) $x = 5, y = -1, z = 3$
   B) $x = -1, y = 3, z = 5$
   C) $x = 5, y = 3, z = -1$
   D) No solution

Answer: B

20)
$$\begin{cases} x - y + 4z = -6 \\ 5x + z = -1 \\ x + 3y + z = 5 \end{cases}$$
   A) $x = -1, y = 0, z = 2$
   B) $x = -1, y = 2, z = 0$
   C) $x = 0, y = 2, z = -1$
   D) No solution

Answer: C

21)
$$\begin{cases} x - y + z = 3 \\ x + y + z = -1 \\ x + y - z = 5 \end{cases}$$
   A) $x = 4, y = -2, z = -3$
   B) $x = 4, y = -3, z = -2$
   C) $x = -3, y = 4, z = -2$
   D) No solution

Answer: A

22)

$$\begin{cases} 2x - y - 9z = -71 \\ 6x + 6z = 66 \\ 9y + z = 53 \end{cases}$$

A) $x = 3$, $y = 8$, $z = 5$
B) $x = 3$, $y = 5$, $z = 8$
C) $x = -3$, $y = 5$, $z = 6$
D) No solution

Answer: B

Fill in the missing numbers in the next step of the Gauss–Jordan elimination method.

23)

$$\begin{bmatrix} 1 & 7 & 4 & | & -5 \\ 4 & 1 & -9 & | & 9 \\ -4 & -3 & 5 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 7 & 4 & | & -5 \\ 0 & -27 & \square & | & \square \\ 0 & 25 & \square & | & \square \end{bmatrix}$$

A)
$$\begin{bmatrix} 1 & 7 & 4 & | & -5 \\ 0 & -27 & -25 & | & 29 \\ 0 & 25 & 21 & | & -20 \end{bmatrix}$$

B)
$$\begin{bmatrix} 1 & 7 & 4 & | & -5 \\ 0 & -27 & -13 & | & 14 \\ 0 & 25 & 1 & | & -21 \end{bmatrix}$$

C)
$$\begin{bmatrix} 1 & 7 & 4 & | & -5 \\ 0 & -27 & -24 & | & 31 \\ 0 & 25 & 20 & | & -19 \end{bmatrix}$$

D)
$$\begin{bmatrix} 1 & 7 & 4 & | & -5 \\ 0 & -27 & -27 & | & 28 \\ 0 & 25 & 24 & | & -20 \end{bmatrix}$$

Answer: A

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Solve the problem.

24) The sales division of a company purchased type A cell phones for $95 each and type B cell phones for $110 each. It purchased a total of 18 cell phones at a total cost of $1815. How many type A cell phones did the division purchase?

Answer: 11 type A cell phones

25) A marketing company wishes to conduct a $15,000,000 advertising campaign in three media–radio, television, and newspaper. The company wants to spend twice as much money in television advertising as in radio and newspaper advertising combined. Also, the company wants to spend a total of $13,000,000 in radio and television combined. Let x, y and z represent the amount in millions of dollars spent in radio, television, and newspaper respectively. How much should the company spend in each medium? (Express the problem in terms of a system of linear equations and solve it using Gaussian elimination.)

Answer:
$$\begin{cases} x + y + z = 15 \\ -2x + y - 2z = 0 \\ x + y = 13 \end{cases}$$

x = 3, y = 10, z = 2; The company should spend $3,000,000 in radio advertising, $10,000,000 in television advertising, and $2,000,000 in newspaper advertising.

26) At a certain party, 72 people are invited. Only half of those invited come. Of those who come, twice as many are women as are men. How many men and how many women come? Let x be the number of women and y be the number of men.

Answer: x = 24 = number of women; y = 12 = number of men

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

27) A waiter made a deposit of $194. If his deposit consisted of 82 bills, some of them one–dollar bills and the rest five–dollar bills, how many one–dollar bills did he deposit?
   A) 28 one–dollar bills
   B) 44 one–dollar bills
   C) 54 one–dollar bills
   D) 49 one–dollar bills

Answer: C

28) At a local zoo, adult visitors must pay $3 and children pay $1. On one day, the zoo collected a total of $770. If the zoo had 410 visitors that day, how many adults and how many children visited?
   A) 230 adults and 180 children
   B) 192 adults and 218 children
   C) 115 adults and 295 children
   D) 180 adults and 230 children

Answer: D

29) The annual salaries of a software engineer and a project supervisor total $192,400. If the project supervisor makes $15,200 more than the software engineer, find each of their salaries.
   A) software engineer: $103,800
      project supervisor: $119,000
   B) software engineer: $88,600
      project supervisor: $103,800
   C) software engineer: $73,400
      project supervisor: $119,000
   D) software engineer: $73,400
      project supervisor: $88,600

Answer: B

30) A particular computer takes 74 nanoseconds (ns) to carry out 5 sums and 9 products, and 64 nanoseconds to perform 4 sums and 8 products. How long does the computer take to carry out one sum and one product?

   A) one sum: 4 ns
      one product: 6 ns

   B) one sum: 6 ns
      one product: 8 ns

   C) one sum: 6 ns
      one product: 4 ns

   D) one sum: 4 ns
      one product: 8 ns

Answer: A

31) Barges from ports X and Y went to cities A and B. X sent 30 barges and Y sent 8. City A received 21 barges and B received 17. Shipping costs $220 from X to A, $300 from X to B, $400 from Y to A, and $180 from Y to B. $8760 was spent. How many barges went where?

   A) 15 from X to A, 15 from X to B, 6 from Y to A, and 2 from Y to B
   B) 17 from X to A, 17 from X to B, 4 from Y to A, and 4 from Y to B
   C) 19 from X to A, 11 from X to B, 2 from Y to A, and 6 from Y to B
   D) 21 from X to A, 9 from X to B, 0 from Y to A, and 8 from Y to B

Answer: D

32) Factories A and B sent rice to stores 1 and 2. A sent 14 loads and B sent 22. Store 1 received 20 loads and store 2 received 16. It cost $200 to ship from A to 1, $350 from A to 2, $300 from B to 1, and $250 from B to 2. $8600 was spent. How many loads went where?

   A) 12 from A to 1, 2 from A to 2, 8 from B to 1, and 14 from B to 2
   B) 0 from A to 1, 14 from A to 2, 16 from B to 1, and 6 from B to 2
   C) 13 from A to 1, 1 from A to 2, 7 from B to 1, and 4 from B to 2
   D) 14 from A to 1, 0 from A to 2, 6 from B to 1, and 16 from B to 2

Answer: D

33) Pivot the matrix $\begin{bmatrix} 1 & 3 & 5 \\ 5 & 6 & 2 \end{bmatrix}$ about the element 6.

A)
$$\begin{bmatrix} 1 & 3 & 5 \\ 3 & 0 & -8 \end{bmatrix}$$

B)
$$\begin{bmatrix} 1 & 3 & 5 \\ \dfrac{5}{6} & 1 & \dfrac{1}{3} \end{bmatrix}$$

C)
$$\begin{bmatrix} 1 & 2 & -\dfrac{8}{3} \\ 0 & 1 & \dfrac{23}{9} \end{bmatrix}$$

D)
$$\begin{bmatrix} -\dfrac{3}{2} & 0 & 4 \\ \dfrac{5}{6} & 1 & \dfrac{1}{3} \end{bmatrix}$$

Answer: D

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

34) Pivot the matrix $\begin{bmatrix} 1 & 3 \\ 4 & -2 \end{bmatrix}$ about the element 3.

Answer:
$$\begin{bmatrix} \dfrac{1}{3} & 1 \\ \dfrac{14}{3} & 0 \end{bmatrix}$$

35) Pivot the matrix $\begin{bmatrix} 1 & 2 & 4 \\ 0 & \underline{1} & 2 \\ 0 & -5 & 3 \end{bmatrix}$ about the underlined element 1.

Answer:
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 13 \end{bmatrix}$$

36) Pivot the matrix $\begin{bmatrix} 3 & 2 & -2 \\ 1 & \frac{1}{2} & -\frac{3}{2} \\ 0 & -1 & 9 \end{bmatrix}$ about the element $\frac{1}{2}$.

Answer:
$$\begin{bmatrix} -1 & 0 & 4 \\ 2 & 1 & -3 \\ 2 & 0 & 6 \end{bmatrix}$$

Solve the system of linear equations using the Gaussian elimination method. If there is no solution, state so; if there are infinitely many solutions, find two of them.

37)
$$\begin{cases} x - y + 2z = 2 \\ y - 2z = 1 \\ -3x + 5y - 10z = -4 \end{cases}$$

Answer: z = any value, x = 3, y = 2z + 1;
two possible solutions: x = 3, y = 1, z = 0 and x = 3, y = 3, z = 1

38)
$$\begin{cases} x - 3y - 5z = 1 \\ -3x + 7y + 9z = 1 \\ x + 4z = -5 \end{cases}$$

Answer: z = any value, x = -4z - 5, y = -3z -2;
two possible solutions: x = -5, y = -2, z = 0 and x = -9, y = -5, z = 1

39)
$$\begin{cases} x - y - 2z = 2 \\ y - 2z = 1 \\ -3x + 5y - 10z = -4 \end{cases}$$

Answer: unique solution; x = 3, y = 1, z = 0

Solve the linear system by using the Gauss–Jordan elimination method.

40)
$$\begin{cases} 3x + 6y = -6 \\ -2x - 7y = 1 \end{cases}$$

Answer: x = -4, y = 1

41)
$$\begin{cases} x - 3y - 5z = 1 \\ -3x + 7y + 9z = 1 \\ x + 4z = -5 \end{cases}$$

Answer: z = any value, x = -4z - 5, y = -3z - 2

For the system of equations, state whether there is one, none, or infinitely many solutions. If there are one or more solutions, give all values of x, y, and z that satisfy the system.

42)
$$\begin{cases} x - y + 3z = 0 \\ z = 7 \end{cases}$$

Answer: infinitely many solutions: y = any value, x = y - 21, z = 7

43)
$$\begin{cases} x + 3y + 2z = 4 \\ y - 3z = -1 \\ z = 17 \end{cases}$$
Answer: one solution: $x = -180$, $y = 50$, $z = 17$

44)
$$\begin{cases} x + y - z = 1 \\ y - 2z = 1 \\ x + y - z = 2 \end{cases}$$
Answer: no solution

45) When solving a system of linear equations with the unknowns x, y, and z using the Gauss–Jordan elimination method, the following matrix was obtained. What can be concluded about the solution of the system?
$$\begin{bmatrix} 1 & 0 & 0 & | & 4 \\ 0 & 1 & 1 & | & 3 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$
Answer: The system has infinitely many solutions; the general solution is:
    z = any value, $x = 4$, $y = 3 - z$.

46) When solving a system of linear equations with the unknowns x, y, and z using the Gauss–Jordan elimination method, the following matrix was obtained. What can be concluded about the general solution of the system?
$$\begin{bmatrix} 1 & 0 & 2 & | & 4 \\ 0 & 1 & 1 & | & 3 \end{bmatrix}$$
Answer: The system has infinitely many solutions; the general solution is:
    z = any value, $x = 4 - 2z$, $y = 3 - z$.

47) When solving a system of linear equations with the unknowns x, y, and z using the Gauss–Jordan elimination method, the following matrix was obtained. What can be concluded about the solution of the system?
$$\begin{bmatrix} 1 & 0 & 0 & | & 4 \\ 0 & 1 & -1 & | & 3 \\ 0 & 0 & 0 & | & 2 \end{bmatrix}$$
Answer: The system has no solution.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

48) Consider the system: $\begin{cases} x - y = 7 \\ 2x - 2y = k \end{cases}$. Which of the following statements is true?
    A) If $k = 14$, the system has no solution.
    B) If $k \neq 14$, the system has exactly one solution.
    C) If $k = 14$, the system has infinitely many solutions.
    D) none of these
Answer: C

49) Consider the system: $\begin{cases} x - y = 7 \\ 2x - 2y = k \end{cases}$. Which of the following statements is true?
    A) If $k = 10$, the system has no solution.
    B) If $k \neq 10$, the system has exactly one solution.
    C) If $k = 10$, the system has infinitely many solutions.
    D) none of these
Answer: A

50) Which of the following systems have infinitely many solutions?

I. $\begin{cases} x - 2y + 5z = 7 \\ \quad\quad y - 2z = 4 \end{cases}$
  II. $\begin{cases} x - y + 4z + 3w = 2 \\ \quad\quad\quad z + 2w = 8 \\ \quad\quad\quad\quad\quad w = 3 \end{cases}$

III. $\begin{cases} x + y - \;\; z = 1 \\ \quad\quad y - 2z = 1 \\ \quad\quad 3y - 6z = 4 \end{cases}$
  IV. $\begin{cases} x + 2y - \;\; z = 2 \\ x \quad\quad - \;\; z = 3 \\ 2x \quad\quad - 2z = 6 \end{cases}$

   A) III and IV only
   B) I, II, and III only
   C) I and II only
   D) I, II, and IV only
   E) all of these

Answer: D

51) Which of the following systems has no solution?
   A)
   $\begin{cases} 2x + 4y = 10 \\ 3x + 6y = 20 \end{cases}$

   B)
   $\begin{cases} 2x + 4y = 10 \\ 3x + 6y = 15 \end{cases}$

   C)
   $\begin{cases} 2x + 4y = 10 \\ 2x + 6y = 10 \end{cases}$

   D)
   $\begin{cases} 2x + 4y = 10 \\ 4x + 8y = 20 \end{cases}$

Answer: A

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

52) Suppose the line L has equation $2x + 3y = 5$ and the line M has equation $x + 4y = -5$. Use Gauss–Jordan elimination method to determine whether L and M intersect in one point, are distinct but parallel, or coincide.

Answer: L and M intersect in one point: (7, –3).

53) Suppose line L has equation $3x + 2y = 7$ and line H has equation $9x + Ky = 14$. Find the value for K so that L and M are parallel.

Answer: K = 6

54) If possible, find the general solution of the following system.
$\begin{cases} x + \;\; y + \;\; z = 1 \\ x + \;\; y + 2z = -1 \\ 2x + 2y + 3z = 0 \end{cases}$

Answer: z = –2, x + y = 3; the general solution is: x = any value, y = 3 – x, z = –2.

55) Find three solutions to $\begin{cases} 8x + 2y - z = 7 \\ \quad\quad\; y \quad\;\; = 2 \end{cases}$.

   Answer: Answers will vary. Solutions are of the form: x = any value, y = 2, z = 8x – 3.
   Three possible solutions are:
   x = 0, y = 2, z = –3
   x = 1, y = 2, z = 5
   x = 2, y = 2, z = 13

56) Find a value for K so that the following system of equations
   $\begin{cases} 6x + 3y = 15 \\ Kx + \;y = 5 \end{cases}$ has infinitely many solutions.
   Answer: K = 2

57) Find a value for K so that the following system of equations
   $\begin{cases} 6x + 3y = 15 \\ Kx + \;y = 5 \end{cases}$ has exactly one solution.
   Answer: K ≠ 2

58) Find a specific solution to a system of linear equations whose general solution is
   y = any value
   w = any value
   x = 3 – 6y
   z = 2
   Answer: One possible solution is x = 3, y =0, z = 2, w = 1.

59) Find a specific solution to a system of linear equations whose general solution is
   y = any value
   w = 2x – z
   x = 3 – 6y
   z = 2
   Answer: One possible solution is x = 3, y =0, z = 2, w = 4.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

60) The two lines –2x + y = 3 and –3x + y = 2
   A) are parallel.
   B) are perpendicular.
   C) coincide.
   D) intersect in exactly one point.
   E) none of these
   Answer: D

61) The two lines x + 5y = 3 and 5x – y = 6
   A) coincide.
   B) are perpendicular.
   C) intersect in exactly one point.
   D) are parallel.
   E) none of these
   Answer: B

Use the Gauss–Jordan method to solve the system of equations.

62) $-3x - 8y = -9$
$-6x - 16y = -9$
   A) $(2, 2)$
   B) $(-9, -9)$
   C) $\left(3 - \dfrac{8}{3}y, y\right)$
   D) No solution

Answer: D

63) $-3x - 7y = 59$
$-9x - 21y = 177$
   A) $(-8, -5)$
   B) $\left(-\dfrac{59}{3} + \dfrac{7}{3}y, y\right)$
   C) $\left(-\dfrac{59}{3} - \dfrac{7}{3}y, y\right)$
   D) No solution

Answer: C

64) $-4x - 2y = 6$
$-16x - 8y = -1$
   A) $(6, -1)$
   B) $(4, 4)$
   C) $\left(-\dfrac{1}{3}x - \dfrac{2}{3}y, y\right)$
   D) No solution

Answer: D

A matrix, A, corresponding to a system of linear equations and the matrix rref(A) obtained after the Gauss–Jordan elimination method is applied to A, are given. Write the system of linear equations corresponding to A, and use rref(A) to give all solutions of the system of linear equations.

65) $A = \begin{bmatrix} 1 & 5 & -5 & | & 12 \\ 2 & 11 & -12 & | & 25 \end{bmatrix}$; $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 5 & | & 7 \\ 0 & 1 & -2 & | & 1 \end{bmatrix}$

A)
$\begin{cases} x + 5y - 5z = 12 \\ 2x + 11y - 12z = 25 \end{cases}$
$z = $ any value, $y = 1 + 2z$, $x = 7 - 5z$

B)
$\begin{cases} x + 5y - 5z = 7 \\ 2x + 11y - 12z = 1 \end{cases}$
$z = $ any value, $y = 1 - 2z$, $x = 7 + 5z$

C)
$\begin{cases} x + 5y - 5z = 7 \\ 2x + 11y - 12z = 1 \end{cases}$
$z = $ any value, $y = -2$, $x = 5$

D)
$\begin{cases} x + 5z = 7 \\ y - 2z = 1 \end{cases}$
$z = $ any value, $y = 1 + 2z$, $x = 7 - 5z$

Answer: A

15

66) $A = \begin{bmatrix} 1 & 2 & 6 & | & 7 \\ 2 & 5 & 16 & | & 19 \\ -2 & -5 & -16 & | & -19 \end{bmatrix}$; $\text{rref}(A) = \begin{bmatrix} 1 & 0 & -2 & | & -3 \\ 0 & 1 & 4 & | & 5 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$

A)
$$\begin{cases} x + 2y + 6z = 7 \\ 2x + 5y + 16z = 19 \\ -2x - 5y - 16z = -19 \end{cases}$$
$z = 0, y = 5, x = -3$

B)
$$\begin{cases} x \quad\ - 2z = -3 \\ \quad\ y + 4z = 5 \end{cases}$$
$z = $ any value, $y = 5 - 4z$, $x = -3 + 2z$

C)
$$\begin{cases} x + 2y + 6z = -3 \\ 2x + 5y + 16z = 5 \\ -2x - 5y - 16z = 0 \end{cases}$$
$z = $ any value, $y = 5 + 4z$, $x = 3 - 2z$

D)
$$\begin{cases} x + 2y + 6z = 7 \\ 2x + 5y + 16z = 19 \\ -2x - 5y - 16z = -19 \end{cases}$$
$z = $ any value, $y = 5 - 4z$, $x = -3 + 2z$

Answer: D

Solve the problem.

67) Janet is planning to visit Arizona, New Mexico, and California on a 14–day vacation. If she plans to spend as much time in New Mexico as she does in the other two states combined, how can she allot her time in the three states? (Let x denote the number of days in Arizona, y the number of days in New Mexico, and z the number of days in California. Let z be the parameter.)
   A) $x = 7 - z, y = 7, 0 \le z \le 7$
   B) $x = 7, y = z - 7, 0 \le z \le 7$
   C) $x = z - 7, y = 7, 0 \le z \le 7$
   D) $x = 7, y = 7 - z, 0 \le z \le 7$

Answer: A

68) A company is introducing a new soft drink and is planning to have 48 advertisements distributed among TV ads, radio ads, and newspaper ads. If the cost of TV ads is $500 each, the cost of radio ads is $200 each, and the cost of newspaper ads is $200 each, how can the ads be distributed among the three types if the company has $13,800 to spend for advertising? (Let x denote the number of TV ads, y the number of radio ads, and z the number of newspaper ads. Let z be the parameter.)
   A) $x = 14, y = z - 34, 0 \le z \le 34$
   B) $x = 14, y = 34 - z, 0 \le z \le 34$
   C) $x = z - 34, y = 14, 0 \le z \le 34$
   D) $x = 34 - z, y = 14, 0 \le z \le 34$

Answer: B

69) An investor has $400,000 to invest in stocks, bonds, and commodities. If he plans to put three times as much into stocks as in bonds, how can he distribute his money among the three types of investments? (Let x denote the amount put into stocks, y the amount put into bonds, and z the amount put into commodities. Let all amounts be in dollars, and let z be the parameter.)
   A) x = 100,000 – z/2, y = 300,000 – z/2, 0 ≤ z ≤ 400,000
   B) x = 300,000 – 3z/4, y = 100,000 – z/4, 0 ≤ z ≤ 400,000
   C) x = 300,000 – z/2, y = 100,000 – z/2, 0 ≤ z ≤ 400,000
   D) x = 100,000 – 3z/4, y = 300,000 – z/4, 0 ≤ z ≤ 400,000

Answer: B

70) A company has 120 sales representatives, each to be assigned to one of four marketing teams. If the first team is to have three times as many members as the second team and the third team is to have twice as many members as the fourth team, how can the members be distributed among the teams? (Let x denote the number of members assigned to the first team, y the number assigned to the second team, z the number assigned to the third team, and w the number assigned to the fourth team. Let w be the parameter.)
   A) x = 120 – 3w, y = 40 – w, z = 3w – 40, 0 ≤ w ≤ 40
   B) x = 90 – 9w/4, y = 30 – 3w/4, z = 2w, 0 ≤ w ≤ 40
   C) x = 30 – 3w/4, y = 10 – w/4, z = 2w, 0 ≤ w ≤ 20
   D) x = 60 – 3w, y = 20 – w, z = 40 + 3w, 0 ≤ w ≤ 20

Answer: B

71) A school library has $27,000 to spend on new books among the four categories of biology, chemistry, physics, and mathematics. If the amount spent on biology books is to be the same as the amount spent on chemistry books and if the amount spent on mathematics books is to be the same as the total spent on chemistry and physics books, how can the money be distributed among the four types of books? (Let x denote the amount spent on biology books, y the amount spent on chemistry books, z the amount spent on physics books, and w the amount spent on mathematics books. Let all amounts be in dollars, and let w be the parameter.)
   A) x = 27,000 – 3w, y = 27,000 – 3w, z = 4w – 27,000, 9000 ≤ w ≤ 18,000
   B) x = 27,000 – w, y = 27,000 – w, z = 2w – 27,000, 4500 ≤ w ≤ 9000
   C) x = 27,000 – 2w, y = 27,000 – 2w, z = 3w – 27,000, 9000 ≤ w ≤ 13,500
   D) x = 27,000 + w, y = 27,000 + w, z = w – 27,000, 4500 ≤ w ≤ 13,500

Answer: C

72) A recording company is to release 210 new CDs in the categories of rock, country, jazz, and classical. If twice the number of rock CDs is to equal three times the number of country CDs and if the number of jazz CDs is to equal the number of classical CDs, how can the CDs be distributed among the four types? (Let x be the number of rock CDs, y the number of country CDs, z the number of jazz CDs, and w the number of classical CDs. Let w be the parameter.)
   A) x = 126 – w, y = 56 – w, z = w, 0 ≤ w ≤ 70
   B) x = 105 – 2w, y = 105 – 2w/5, z = w, 0 ≤ w ≤ 105
   C) x = 105 – 3w/5, y = 70 – 2w/5, z = w, 0 ≤ w ≤ 91
   D) x = 126 – 6w/5, y = 84 – 4w/5, z = w, 0 ≤ w ≤ 105

Answer: D

73) A politician is planning to spend a total of 40 hours on a campaign swing through the southern states of Arkansas, Louisiana, Mississippi, Alabama, and Georgia. Assume that he spends the same amount of time in Mississippi as in Alabama; half the amount of time in Georgia as in Arkansas; and the same amount of time in Mississippi, Alabama, and Georgia (combined) as in Arkansas and Louisiana (combined). How can he distribute his time among the five states? (Let a be the hours spent in Arkansas, b the hours spent in Louisiana, c the hours spent in Mississippi, d the hours spent in Alabama, and e the hours spent in Georgia. Let e be the parameter.)

   A) $a = e$, $b = 20 - e/2$, $c = 10 - e/4$, $d = 10 - e/4$, $0 \le e \le 20$
   B) $a = 2e$, $b = 20 - 2e$, $c = 10 - e/2$, $d = 10 - e/2$, $0 \le e \le 10$
   C) $a = 2e$, $b = 20 - e$, $c = 10 - e$, $d = 10 - e$, $0 \le e \le 20$
   D) $a = 2e$, $b = 20 - e$, $c = 10 - e/2$, $d = 10 - e/2$, $0 \le e \le 10$

Answer: B

74) Which of the following calculations can be performed?

I. $[4 \ 0 \ 2] + \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$ II. $[6] + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ III. $[5]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ IV. $\begin{bmatrix} 2 & 1 & 2 \\ 4 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

   A) III only
   B) IV only
   C) III and IV only
   D) I and II only
   E) all of these

Answer: B

75) What is the identity matrix of size 3?

   A)
$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

   B)
$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

   C)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

   D)
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Answer: C

76) If B is a 1 × 6 matrix and A is a 6 × 1 matrix, determine the size of AB.
   A) 6 × 6
   B) 6 × 1
   C) 1 × 1
   D) 1 × 6
   E) none of these
   Answer: A

77) If B is a 4 × 2 matrix and A is a 3 × 4 matrix, determine the size of AB.
   A) 3 × 2
   B) 4 × 4
   C) 3 × 4
   D) 2 × 3
   E) none of these
   Answer: A

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Determine the values of x and y, if any, that make A = B.

78) $A = \begin{bmatrix} 5 & 3 & -2 \\ x & 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & y & -2 \\ 8 & 4 & 0 \end{bmatrix}$

Answer: $x = 8, y = 3$

79) $A = \begin{bmatrix} x & 1 \\ 2 & 3 \\ 4 & y \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2 & 4 \\ 1 & 3 & -2 \end{bmatrix}$

Answer: There are no values of x and y that make A = B.

80) In each case below, give an example of a matrix that meets the specified condition.
   (a) A and B so that AB is not defined
   (b) a 2 × 2 matrix that has no inverse
   (c) C and D so that CD is defined but DC is not defined

Answer: Answers will vary.

   (a) Possible solutions: any two matrices such that the number of columns in A is not the same as the number of rows in B.

   One possible example is $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 1 \\ 7 \end{bmatrix}$.

   (b) Possible solution: any 2 × 2 matrix such that $\Delta = 0$, or having a row of zeros.

   Possible examples: $\begin{bmatrix} 6 & 4 \\ 3 & 2 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

   (c) Possible solutions: any two matrices such that the number of columns in C is the same as the number of rows in D, but the number of rows in C is not equal to the number of columns in D.

   One possible example is $C = \begin{bmatrix} 3 & 2 & 1 \\ 1 & -2 & 0 \end{bmatrix}$ and $D = \begin{bmatrix} 4 \\ 8 \\ -2 \end{bmatrix}$.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Provide an appropriate response.

81) Which choice best describes the following matrix?

$$\begin{bmatrix} 24 \\ 26 \\ 20 \\ 17 \\ 18 \end{bmatrix}$$

A) Row matrix
B) Square matrix
C) 1 × 5 matrix
D) Column matrix

Answer: D

Answer the question about the 2 × 3 matrix A.

82) Find a_{12} and a_{23}.

$$A = \begin{bmatrix} -1 & 7 & 2 \\ -1 & 0 & 9 \end{bmatrix}$$

A) 7, 9
B) 7, −1
C) −1, 9
D) −1, 2

Answer: A

83) For what values of i and j does $a_{ij} = 9$?

$$A = \begin{bmatrix} -6 & 9 & 2 \\ -2 & 0 & 4 \end{bmatrix}$$

A) i = 2, j = 1
B) i = 2, j = 2
C) i = 1, j = 2
D) i = 1, j = 1

Answer: C

84)

$$\begin{bmatrix} 9 & -2 \\ -7 & 2 \\ 8 & -7 \end{bmatrix} + \begin{bmatrix} -5 & -8 \\ 2 & -7 \\ 4 & -2 \end{bmatrix} =$$

A)
$$\begin{bmatrix} 14 & 6 \\ -9 & 9 \\ 4 & -6 \end{bmatrix}$$

B)
$$\begin{bmatrix} 4 & -10 \\ 5 & 2 \\ 12 & 9 \end{bmatrix}$$

C)
$$\begin{bmatrix} 4 & 2 \\ -5 & -5 \\ 12 & -9 \end{bmatrix}$$

D)
$$\begin{bmatrix} 4 & -10 \\ -5 & -5 \\ 12 & -9 \end{bmatrix}$$

Answer: D

85)

$$\begin{bmatrix} -1 & 0 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 3 \\ 3 & 1 \end{bmatrix} =$$

A)
$$\begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}$$

B)
$$[-3]$$

C)
$$\begin{bmatrix} -2 & 3 \\ 6 & 2 \end{bmatrix}$$

D)
$$\begin{bmatrix} 0 & -3 \\ 0 & 0 \end{bmatrix}$$

Answer: D

Perform the indicated operation, where possible.

86) $\begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 6 & 2 \\ 4 & 2 \end{bmatrix}$

A) $\begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix}$

B) $\begin{bmatrix} 3 & 3 \\ 6 & 7 \end{bmatrix}$

C) $\begin{bmatrix} 3 & 4 \\ 3 & 7 \end{bmatrix}$

D) Not possible

Answer: B

87) $\begin{bmatrix} 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 \\ 8 \end{bmatrix}$

    A) $\begin{bmatrix} 4 & 12 \end{bmatrix}$

    B) $\begin{bmatrix} 4 \\ 12 \end{bmatrix}$

    C) $\begin{bmatrix} 3 & 1 \\ 4 & 8 \end{bmatrix}$

    D) Not possible

Answer: D

88) $\begin{bmatrix} -4 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 3 \end{bmatrix}$

    A) $\begin{bmatrix} -8 & 5 & 1 \end{bmatrix}$

    B) $\begin{bmatrix} -8 & 8 & 1 \end{bmatrix}$

    C) $\begin{bmatrix} -8 & 5 & 4 \end{bmatrix}$

    D) Not possible

Answer: D

89) $\begin{bmatrix} 3 \\ -3 \\ -4 \end{bmatrix} + \begin{bmatrix} -5 \\ 4 \\ 9 \end{bmatrix}$

    A) $\begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$

    B) $\begin{bmatrix} -2 & 1 & 5 \end{bmatrix}$

    C) $\begin{bmatrix} 3 & -5 \\ -3 & 4 \\ -4 & 9 \end{bmatrix}$

    D) $\begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$

Answer: D

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Let A, B, and C be the following matrices:

$$A = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 2 & 3 \end{bmatrix} \quad C = \begin{bmatrix} -2 & 3 & 1 \\ 5 & 6 & -3 \end{bmatrix}$$

State whether or not each of the following calculations is possible. If possible, perform the calculation.

90) A + B

    Answer: The calculation is not possible.

91) B – C

    Answer:

$$\begin{bmatrix} 4 & -3 & -3 \\ -5 & -4 & 6 \end{bmatrix}$$

92) 2A – B

    Answer: The calculation is not possible.

93) 2C + B

Answer:

$$\begin{bmatrix} -2 & 6 & 0 \\ 10 & 14 & -3 \end{bmatrix}$$

State whether the calculation is possible. If possible, perform the calculation.

94)

$$\begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ \frac{1}{2} & 0 \end{bmatrix} - \begin{bmatrix} 6 & 6 \\ 1 & 2 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} -1 & -5 \\ -\frac{7}{2} & -1 \end{bmatrix}$$

95)

$$\begin{bmatrix} 3 \\ 30 \\ 17 \end{bmatrix} + \begin{bmatrix} 14 \\ 0 \\ 8 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} 17 \\ 30 \\ 25 \end{bmatrix}$$

96)

$$[6] + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Answer: The calculation is not possible.

97)

$$\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} + [4 \quad 0 \quad 6]$$

Answer: The calculation is not possible.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

98) $[4 \ 0 \ 6] \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} =$

A)
$[4 \quad 0 \ -12]$

B)
$\begin{bmatrix} 4 & 12 & -8 \\ 0 & 0 & 0 \\ 6 & 18 & -12 \end{bmatrix}$

C)
$\begin{bmatrix} 4 \\ 0 \\ -12 \end{bmatrix}$

D)
$\begin{bmatrix} 4 & 0 & 6 \\ 12 & 0 & 18 \\ -8 & 0 & -12 \end{bmatrix}$

E)
$[-8]$

Answer: E

99) $\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} [4 \ 0 \ 6] =$

A)
$\begin{bmatrix} 4 & 0 & 6 \\ 12 & 0 & 18 \\ -8 & 0 & -12 \end{bmatrix}$

B)
$\begin{bmatrix} 4 & 12 & -8 \\ 0 & 0 & 0 \\ 6 & 18 & -12 \end{bmatrix}$

C)
$[-8]$

D)
$\begin{bmatrix} 4 \\ 0 \\ -12 \end{bmatrix}$

E)
$[4 \quad 0 \ -12]$

Answer: A

100) Let $A = \begin{bmatrix} -2 & 1 & 3 \\ 1 & 0 & 5 \\ 0 & 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & -3 \\ 1 & 3 & -4 \\ 3 & 0 & 1 \end{bmatrix}$. Find the second row of AB.

A) [15 –1 2]
B) [18 1 2]
C) [18 –1 2]
D) [15 1 2]
E) none of these

Answer: D

101) Let $A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 0 & 3 \\ 1 & 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 5 & 2 \\ 7 & 1 & 3 \end{bmatrix}$. Find the third row of AB.

A) [31 11 16]
B) [7 4 6]
C) [25 18 58]
D) [18 23 16]
E) none of these

Answer: D

102) Let $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}$. Find the entry in the second row, first column of AB.

A) 6
B) 34
C) 18
D) 22
E) 8

Answer: C

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

State whether the calculation is possible. If possible, perform the calculation.

103)

$\begin{bmatrix} 2 & 1 & 0 \\ 3 & 0 & 2 \end{bmatrix} \begin{bmatrix} 7 & -1 \\ 1 & 0 \\ 5 & 2 \end{bmatrix}$

Answer:

$\begin{bmatrix} 15 & -2 \\ 31 & 1 \end{bmatrix}$

104)

$\begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix} [4 \ 0 \ 3]$

Answer:

$\begin{bmatrix} 4 & 0 & 3 \\ 8 & 0 & 6 \\ -20 & 0 & -15 \end{bmatrix}$

105)

$$\begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ 6 & 4 \\ 1 & 6 \end{bmatrix}$$

Answer: The calculation is not possible.

106)

$$\begin{bmatrix} 1 & 0 & -2 \\ 2 & 4 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}$$

Answer: The calculation is not possible.

107)

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

108) If $A = \begin{bmatrix} 6 & -2 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} \frac{1}{2} & 1 \\ 1 & 3 \end{bmatrix}$, find AB and BA.

Answer: $AB = BA = I_2$

109) If $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$, does AB = BA? Explain.

Answer: No.

$$AB = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 17 & 14 \\ 14 & 13 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 10 & 25 \end{bmatrix}$$

110) Find a and b so that $\begin{bmatrix} 1 & 6 & -3 \\ 7 & 7 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & 0 \\ 2 & 6 & 1 \\ 4 & 2 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 28 & 0 \\ 2 & 36 & 15 \\ 4 & 2 & 2 \end{bmatrix}$

Answer: a = –4, b = –2

111) If $A^4 = \begin{bmatrix} -3 & 4 \\ -4 & 5 \end{bmatrix}$ and $A^5 = \begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix}$, what is A?

Answer: $A = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

The sizes of two matrices A and B are given. Find the sizes of the product AB and the product BA, whenever these products exist.

    112) A is 4 × 4, and B is 4 × 4.

        A) 4 × 8; 4 × 8

        B) 8 × 4; 8 × 4

        C) 4 × 4; 4 × 4

        D) 1 × 1; 1 × 1

    Answer: C

    113) A is 2 × 1, and B is 1 × 2.

        A) 2 × 2; 1 × 1

        B) AB does not exist; 1 × 1

        C) 1 × 1; 2 × 2

        D) 2 × 2; BA does not exist.

    Answer: A

    114) A is 4 × 3, and B is 4 × 3.

        A) 4 × 3; 3 × 4

        B) 3 × 4; 4 × 3

        C) 4 × 4; 3 × 3

        D) AB does not exist; BA does not exist.

    Answer: D

Find the matrix product, if possible.

115) $\begin{bmatrix} -2 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -2 & 0 \\ -1 & 1 \end{bmatrix}$

    A) $\begin{bmatrix} 3 & 1 \\ 2 & -8 \end{bmatrix}$

    B) $\begin{bmatrix} 4 & -6 \\ -4 & -1 \end{bmatrix}$

    C) $\begin{bmatrix} 1 & 3 \\ -8 & 2 \end{bmatrix}$

    D) $\begin{bmatrix} 4 & 0 \\ -3 & 2 \end{bmatrix}$

    Answer: C

116) $\begin{bmatrix} -1 & 3 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 0 & -2 & 6 \\ 1 & -3 & 2 \end{bmatrix}$

    A) $\begin{bmatrix} 0 & -6 \\ 18 & 3 \\ -18 & 12 \end{bmatrix}$

    B) $\begin{bmatrix} 3 & -7 & 0 \\ 6 & -24 & 30 \end{bmatrix}$

    C) $\begin{bmatrix} 3 & 6 & -7 \\ -24 & 0 & 30 \end{bmatrix}$

    D) Does not exist

    Answer: B

117) $\begin{bmatrix} 3 & -2 & 1 \\ 0 & 4 & -2 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ -2 & 2 \end{bmatrix}$

A) $\begin{bmatrix} 15 & 0 \\ 0 & 8 \end{bmatrix}$

B) $\begin{bmatrix} 15 & -6 \\ -10 & 12 \\ 5 & -6 \end{bmatrix}$

C) $\begin{bmatrix} 15 & -10 & 5 \\ -6 & 12 & -6 \end{bmatrix}$

D) Does not exist

Answer: D

118) $\begin{bmatrix} 1 & 3 & -2 \\ 3 & 0 & 5 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -2 & 1 \\ 0 & 5 \end{bmatrix}$

A) $\begin{bmatrix} -3 & -7 \\ 9 & 25 \end{bmatrix}$

B) $\begin{bmatrix} 3 & -6 & 0 \\ 0 & 0 & 25 \end{bmatrix}$

C) $\begin{bmatrix} -7 & -3 \\ 25 & 9 \end{bmatrix}$

D) Does not exist

Answer: A

119) Find the system of equations that is equivalent to the matrix equation, $\begin{bmatrix} -2 & 5 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$.

A)
$\begin{cases} -2x + 5y = 3 \\ x + 4y = 6 \end{cases}$

B)
$\begin{cases} -2x + 5x = 3 \\ y + 4y = 6 \end{cases}$

C)
$\begin{cases} -2x + y = 3 \\ 5x + 4y = 6 \end{cases}$

D)
$\begin{cases} -2x + y = 3 \\ 5y + 4y = 6 \end{cases}$

Answer: A

120) Find the matrix equation that is equivalent to the system of equations, $\begin{cases} x - 2y + z = 4 \\ 3x + y + z = 1 \\ x + y + 2z = 0 \end{cases}$.

A)
$$\begin{bmatrix} 1 & 3 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}.$$

B)
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & -2 & 1 \\ 3 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}.$$

C)
$$\begin{bmatrix} 1 & -2 & 1 \\ 3 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}.$$

D)
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 3 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}.$$

Answer: C

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

121) Write the system of linear equations as a matrix equation.
$\begin{cases} x + 3y = 6 \\ 2x - y = 3 \end{cases}$

Answer:
$$\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

122) Write the system of linear equations as a matrix equation.
$\begin{cases} x + 2y + 3z = 4 \\ 6y + 7z = 8 \\ x = 5 \end{cases}$

Answer:
$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 6 & 7 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 5 \end{bmatrix}$$

123) Write the system of linear equations as a matrix equation.
$\begin{cases} x + y + 4z = 3 \\ 4x + y - 2z = -6 \\ -3x + 2z = 1 \end{cases}$

Answer:
$$\begin{bmatrix} 1 & 1 & 4 \\ 4 & 1 & -2 \\ -3 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \\ 1 \end{bmatrix}$$

124) Give the system of linear equations that is equivalent to the matrix equation:

$$\begin{bmatrix} 3 & 1 & 2 \\ -1 & 0 & 2 \\ 0 & 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

Answer:

$$\begin{cases} 3x + y + 2z = 1 \\ -x \quad\; + 2z = 2 \\ \quad\; 4y + z = -1 \end{cases}$$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Write the matrix equation as a system of linear equations without matrices.

125)

$$\begin{bmatrix} -7 & 0 \\ 1 & 1 \\ -8 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -8 \\ 8 \\ -7 \end{bmatrix}$$

A) $-7x_1 + x_2 = -8$
$x_1 + 8x_2 = 8$
$-8x_1 - 5x_2 = -7$

B) $-7x_1 + x_2 + x_3 = -8$
$x_1 - 5x_2 = 8$

C) $-7x_1 = -8$
$x_1 + x_2 = 8$
$-8x_1 - 5x_2 = -7$

Answer: C

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Solve the problem.

126) In a certain state legislature the percentage of legislators voting for or against lowering the legal drinking age by various party affiliations is summarized by this matrix.

$$\begin{array}{c} \\ \text{Democrat} \\ \text{Republican} \\ \text{Independent} \end{array} \overset{\begin{array}{cc} \text{For} & \text{Against} \end{array}}{\begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \\ 1 & 0 \end{bmatrix}} = A$$

There are 60 Democratic, 30 Republicans, and 10 Independents in the legislature. Use a matrix calculation to determine how many legislators voted for lowering the drinking age and how many voted against lowering the drinking age.

Answer: Let $B = [\, 60 \quad 30 \quad 10 \,]$

$$BA = [\, 60 \quad 30 \quad 10 \,] \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \\ 1 & 0 \end{bmatrix} = [\, 55 \quad 45 \,]$$

The vote is 55 in favor, 45 against.

127) A company has three factories I, II, and III. Each factory employs both skilled and unskilled worker. The break down of the number of workers by type in the three factories is summarized by matrix A below. The hourly wage of the skilled worker is $15 and that of the unskilled worker is $10. Multiply appropriate matrices and compute the total hourly pay at each factory.

$$A = \begin{matrix} \text{skilled} \\ \text{unskilled} \end{matrix} \begin{matrix} \text{I} & \text{II} & \text{III} \\ \begin{bmatrix} 25 & 34 & 24 \\ 42 & 53 & 62 \end{bmatrix} \end{matrix}$$

Answer: Let B = [15 10]. Then BA = [795 1040 980].
    Therefore, the total hourly pay for factory I = $795, for II = $1,040, and for III = $980.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

128) Suppose one hour's output (measured in bottles) in a brewery is described by the following matrix:

	Production line 1	Production line 2	
Regular Beer	300	200	
Light Beer	400	100	= A
Malt Beer	100	50	

Let $H = \begin{bmatrix} x \\ y \end{bmatrix}$ where x represents the number of hours Production line 1 is in operation in a day and y represents the number of hours Production line 2 is in operation in a day. Then the entry in the second row, first column of AH, represents
   A) the total number of hours in a day spent producing light beer.
   B) the total number of hours in a day spent producing regular beer.
   C) the total number of bottles of regular beer produced in a day.
   D) the total number of bottles of light beer produced in a day.
   E) none of these

Answer: D

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

129) Two hundred students are registered in a certain mathematics course. Experience suggests that if x and y represent the number of students who earn a grade of C or better on Exam I and the number of students who earn a grade of D or F on Exam I, respectively, and if p and n represent the number of students who earn a grade of C or better on Exam II and the number of students who earn a grade of D or F on Exam II, respectively, then

$$0.9x + 0.3y = p$$
$$0.1x + 0.7y = n$$

(a) Write the system of equations in matrix form.

(b) Solve the resulting matrix equation for $X = \begin{bmatrix} x \\ y \end{bmatrix}$.

(c) Suppose 120 students earn a grade of C or better on Exam II. How many students earned a grade of D or F on Exam I?

Answer: (a)

$$\begin{bmatrix} 0.9 & 0.3 \\ 0.1 & 0.7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} p \\ n \end{bmatrix}$$

(b)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \dfrac{7}{6} & -\dfrac{1}{2} \\ -\dfrac{1}{6} & \dfrac{3}{2} \end{bmatrix} \begin{bmatrix} p \\ n \end{bmatrix}$$

(c) $y = -\dfrac{1}{6}(120) + \dfrac{3}{2}(80) = 100$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

130) Barnes and Able sell life, health, and auto insurance. Their sales, in dollars, for May and June are given in the following matrices.

	Life	Health	Auto	
May:	20,000	15,000	8000	Able
	30,000	0	17,000	Barnes

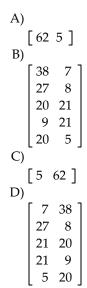|				
June:	70,000	0	30,000	Able
	20,000	25,000	32,000	Barnes

Find a matrix that gives total sales, in dollars, of each type of insurance by each salesman for the two–month period.

A) $\begin{bmatrix} 90,000 & 15,000 & 38,000 \\ 50,000 & 25,000 & 49,000 \end{bmatrix}$

B) $\begin{bmatrix} 90,000 & 0 & 38,000 \\ 50,000 & 0 & 49,000 \end{bmatrix}$

C) $\begin{bmatrix} 90,000 & 15,000 & 38,000 \\ 50,000 & 25,000 & 32,000 \end{bmatrix}$

D) $\begin{bmatrix} 140,000 & 25,000 & 49,000 \end{bmatrix}$

Answer: A

131) The matrices give points and rebounds for five starting players in two games. Find the matrix that gives the totals.

$$F = \begin{array}{cc} \text{Points} & \text{Rebounds} \\ \left[\begin{array}{cc} 20 & 3 \\ 16 & 5 \\ 8 & 12 \\ 5 & 11 \\ 10 & 2 \end{array}\right] \end{array}$$

$$S = \begin{array}{cc} \text{Points} & \text{Rebounds} \\ \left[\begin{array}{cc} 18 & 4 \\ 11 & 3 \\ 12 & 9 \\ 4 & 10 \\ 10 & 3 \end{array}\right] \end{array}$$

A)
$$\left[\begin{array}{cc} 62 & 5 \end{array}\right]$$

B)
$$\left[\begin{array}{cc} 38 & 7 \\ 27 & 8 \\ 20 & 21 \\ 9 & 21 \\ 20 & 5 \end{array}\right]$$

C)
$$\left[\begin{array}{cc} 5 & 62 \end{array}\right]$$

D)
$$\left[\begin{array}{cc} 7 & 38 \\ 27 & 8 \\ 21 & 20 \\ 21 & 9 \\ 5 & 20 \end{array}\right]$$

Answer: B

132) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

A)
$$\begin{bmatrix} \dfrac{1}{2} & -\dfrac{3}{2} \\ -1 & 2 \end{bmatrix}$$

B)
$$\begin{bmatrix} -2 & 1 \\ \dfrac{3}{2} & -\dfrac{1}{2} \end{bmatrix}$$

C) not defined

D)
$$\begin{bmatrix} 1 & \dfrac{1}{2} \\ \dfrac{1}{3} & \dfrac{1}{4} \end{bmatrix}$$

E) none of these

Answer: B

133) Find the inverse of the matrix $\begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$.

A)
$$\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$$

B) not defined

C)
$$\begin{bmatrix} 4 & 1 \\ -3 & 1 \end{bmatrix}$$

D)
$$\begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$$

E) none of these

Answer: D

134) The matrix $\begin{bmatrix} 3 & x \\ 4 & 36 \end{bmatrix}$ has no inverse if x is

A) 0.
B) 3.
C) 48.
D) $\dfrac{1}{3}$.
E) 27.

Answer: E

135) The matrix $\begin{bmatrix} 3 & 12 \\ x & 36 \end{bmatrix}$ has no inverse if x is

    A) 48.
    B) 9.
    C) 27.
    D) 12.
    E) 3.

    Answer: B

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find the inverse of the matrix, if it exists.

136) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

    Answer: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

137) $\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$

    Answer: Inverse does not exist.

138) $\begin{bmatrix} 2 & -1 \\ -6 & 8 \end{bmatrix}$

    Answer: $\begin{bmatrix} 0.8 & 0.1 \\ 0.6 & 0.2 \end{bmatrix}$

139) $\begin{bmatrix} 0.6 & 0.1 \\ 0.4 & 0.9 \end{bmatrix}$

    Answer: $\begin{bmatrix} 1.8 & -0.2 \\ -0.8 & 1.2 \end{bmatrix}$

140) $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$

    Answer: $\begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ 0 & 1 \end{bmatrix}$

141) $\begin{bmatrix} 3 & 1 \\ 0 & 0 \end{bmatrix}$

    Answer: Inverse does not exist.

142) [6]

    Answer: $\begin{bmatrix} \frac{1}{6} \end{bmatrix}$

143) [0.4]

    Answer: [2.5]

144) Two n × n matrices A and B are called inverses of each other if both products AB and BA equal I_n. Are the following matrices inverses of each other?

$A = \begin{bmatrix} 7 & 4 \\ 5 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}$

Answer: Yes

145) Are the following matrices inverses of each other?

$A = \begin{bmatrix} 1 & 4 \\ 8 & 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 8 \\ 3 & -2 \end{bmatrix}$

Answer: No.

146) Given that $\begin{bmatrix} 2 & -2 & -1 \\ -5 & 3 & 4 \\ -6 & 5 & 4 \end{bmatrix}$ and $\begin{bmatrix} 8 & -3 & 5 \\ 4 & -2 & 3 \\ 7 & -2 & 4 \end{bmatrix}$ are inverses of each other,

find A so that $(A - I)^{-1} = \begin{bmatrix} 8 & -3 & 5 \\ 4 & -2 & 3 \\ 7 & -2 & 4 \end{bmatrix}$.

Answer:

$A = \begin{bmatrix} 3 & -2 & -1 \\ -5 & 4 & 4 \\ -6 & 5 & 5 \end{bmatrix}$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

147)

Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} x & y & 0 \\ \dfrac{3}{5} & -\dfrac{1}{5} & 0 \\ 0 & 0 & \dfrac{1}{2} \end{bmatrix}$

In order for A and B to be inverses, x and y must be

A) $x = -5, y = \dfrac{5}{3}$.

B) $x = 1, y = 0$.

C) $x = 1, y = \dfrac{1}{2}$.

D) $x = -\dfrac{1}{5}, y = \dfrac{2}{5}$.

E) none of these

Answer: D

148)

$$\text{Let A} = \begin{bmatrix} -4 & 1 & 2 \\ 7 & -1 & -4 \\ -\dfrac{1}{2} & 0 & \dfrac{1}{2} \end{bmatrix} \text{ and B} = \begin{bmatrix} x & y & 4 \\ 3 & 2 & 4 \\ 1 & 1 & 6 \end{bmatrix}$$

In order for A and B to be inverses, x and y must be

A) $x = 1, y = 1$.
B) $x = -1, y = 1$.
C) $x = 1, y = -1$.
D) $x = -1, y = -1$.
E) none of these

Answer: A

Find the inverse, if it exists, for the matrix.

149) $\begin{bmatrix} 2 & -6 \\ -3 & 1 \end{bmatrix}$

A)
$$\begin{bmatrix} -\dfrac{3}{16} & -\dfrac{1}{8} \\ -\dfrac{1}{16} & -\dfrac{3}{8} \end{bmatrix}$$

B)
$$\begin{bmatrix} -\dfrac{1}{16} & -\dfrac{3}{8} \\ -\dfrac{3}{16} & -\dfrac{1}{8} \end{bmatrix}$$

C)
$$\begin{bmatrix} -\dfrac{1}{8} & -\dfrac{3}{8} \\ -\dfrac{3}{16} & -\dfrac{1}{16} \end{bmatrix}$$

D)
$$\begin{bmatrix} -\dfrac{1}{16} & \dfrac{3}{8} \\ \dfrac{3}{16} & -\dfrac{1}{8} \end{bmatrix}$$

Answer: B

150) $\begin{bmatrix} 0 & -2 \\ 5 & 4 \end{bmatrix}$

A)
$$\begin{bmatrix} \dfrac{2}{5} & -\dfrac{1}{5} \\ \dfrac{1}{2} & 0 \end{bmatrix}$$

B)
$$\begin{bmatrix} -\dfrac{1}{2} & 0 \\ \dfrac{2}{5} & \dfrac{1}{5} \end{bmatrix}$$

C)
$$\begin{bmatrix} \dfrac{2}{5} & \dfrac{1}{5} \\ -\dfrac{1}{2} & 0 \end{bmatrix}$$

D)
$$\begin{bmatrix} 0 & \dfrac{1}{5} \\ -\dfrac{1}{2} & \dfrac{2}{5} \end{bmatrix}$$

Answer: C

151) $\begin{bmatrix} -6 & -2 \\ 4 & 6 \end{bmatrix}$

A) $\begin{bmatrix} -\dfrac{1}{2} & -\dfrac{1}{6} \\ \dfrac{1}{3} & \dfrac{1}{2} \end{bmatrix}$

B) $\begin{bmatrix} 6 & 2 \\ -4 & -6 \end{bmatrix}$

C) $\begin{bmatrix} -\dfrac{1}{2} & \dfrac{1}{6} \\ -\dfrac{1}{3} & \dfrac{1}{2} \end{bmatrix}$

D) No inverse

Answer: D

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152) $\begin{bmatrix} -1 & 4 \\ 0 & 6 \end{bmatrix}$

A)

$$\begin{bmatrix} 0 & \dfrac{1}{6} \\ -1 & \dfrac{2}{3} \end{bmatrix}$$

B)

$$\begin{bmatrix} -1 & -\dfrac{2}{3} \\ 0 & \dfrac{1}{6} \end{bmatrix}$$

C)

$$\begin{bmatrix} \dfrac{1}{6} & \dfrac{2}{3} \\ 0 & -1 \end{bmatrix}$$

D)

$$\begin{bmatrix} -1 & \dfrac{2}{3} \\ 0 & \dfrac{1}{6} \end{bmatrix}$$

Answer: D

153) Given that the matrices $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -2 & -2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ are inverses of each other, find the solution $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ of the

system $\begin{cases} -5x - 2y - 2z = 1 \\ -x + y \quad\quad = 2 \\ -x \quad\quad + z = -3 \end{cases}$.

A)

$$\begin{bmatrix} 5 & -2 & -2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}.$$

B)

$$\begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}.$$

C)

$$\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}.$$

D)

$$\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \begin{bmatrix} 5 & -2 & -2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}.$$

Answer: B

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SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

154) Solve the system $\begin{cases} x + y = 2 \\ 2x + y = -1 \end{cases}$ by using the inverse of an appropriate matrix.

Answer: $x = -3, y = 5$

155) The matrices $\begin{bmatrix} 5 & -2 & -2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ are inverses of each other.

Use this fact to solve the system $\begin{cases} x + 2y + 2z = 1 \\ x + 3y + 2z = -1 \\ x + 2y + 3z = -1 \end{cases}$.

Answer: $x = 9, y = -2, z = -2$

156) The matrices $\begin{bmatrix} 5 & 2 & -2 \\ 2 & 1 & -1 \\ -3 & -1 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & -2 & 0 \\ -1 & 4 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ are inverses of each other.

Use this fact to solve the system $\begin{cases} x - 2y = 5 \\ -x + 4y + z = 1 \\ x - y + z = 2 \end{cases}$.

Answer: $x = 23, y = 9, z = -12$

157) Use the fact that

$$\begin{bmatrix} 3 & 0 & -6 & 0 \\ 1 & 1 & 1 & 4 \\ 0 & 2 & 1 & 6 \\ 1 & 0 & 0 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} \dfrac{1}{9} & \dfrac{4}{3} & -\dfrac{2}{3} & -\dfrac{2}{3} \\[2mm] \dfrac{2}{9} & \dfrac{5}{3} & -\dfrac{1}{3} & -\dfrac{7}{3} \\[2mm] -\dfrac{1}{9} & \dfrac{2}{3} & -\dfrac{1}{3} & -\dfrac{1}{3} \\[2mm] -\dfrac{1}{18} & -\dfrac{2}{3} & \dfrac{1}{3} & \dfrac{5}{6} \end{bmatrix}$$

to solve the system

$\begin{cases} 3x - 6z = 0 \\ x + y + z + 4w = 2 \\ 2y + z + 6w = 1 \\ x + 2w = 0 \end{cases}$.

Answer: $x = 2, y = 3, z = 1, w = -1$

158) Consider the system $\begin{cases} 3x + 2y = 7 \\ y = 8 \end{cases}$

   (a) Rewrite it in the form $AX = B$, where A, B, and X are appropriate matrices.

   (b) Find the inverse of A.

   (c) Solve the system by computing $A^{-1}B$.

   Answer: (a)

$$\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

   (b)

$$\begin{bmatrix} \dfrac{1}{3} & -\dfrac{2}{3} \\ 0 & 1 \end{bmatrix}$$

   (c) $x = -3, y = 8$

159) Consider the system $\begin{cases} 2x + 3y = 4 \\ -2x - y = 8 \end{cases}$

   (a) Rewrite it in the form $AX = B$, where A, B, and X are appropriate matrices.

   (b) Find the inverse of A.

   (c) Solve the system by computing $A^{-1}B$.

   Answer: (a)

$$\begin{bmatrix} 2 & 3 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

   (b)

$$\begin{bmatrix} -\dfrac{1}{4} & -\dfrac{3}{4} \\ \dfrac{1}{2} & \dfrac{1}{2} \end{bmatrix}$$

   (c) $x = -7, y = 6$

160) Consider the system $\begin{cases} 0.7x + 0.2y = 3 \\ 0.3x + 0.8y = 2 \end{cases}$

   (a) Rewrite it in the form $AX = B$ where, A, B, and X are appropriate matrices.

   (b) Find the inverse of A.

   (c) Solve the system by computing $A^{-1}B$.

   Answer: (a)

$$\begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

   (b)

$$\begin{bmatrix} 1.6 & -0.4 \\ -0.6 & 1.4 \end{bmatrix}$$

   (c) $x = 4, y = 1$

161) (a) Find the inverse of the matrix $\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$.

    (b) Use the result from (a) to solve the system $\begin{cases} 2x + y = 2 \\ 3x + y = -1 \end{cases}$.

    Answer: (a) $\begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$ (b) $x = -3, y = 8$

162) Find the 2 × 2 matrix C such that AC = B, where $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 \\ 2 & -3 \end{bmatrix}$.

    Answer:
$$C = \begin{bmatrix} 16 & 11 \\ -22 & -16 \end{bmatrix}$$

163) Find the 3 × 3 matrix C such that AC = B, where $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

    Answer:
$$C = \begin{bmatrix} 5 & 7 & 9 \\ -4 & -5 & -6 \\ -8 & -13 & -18 \end{bmatrix}$$

164) Solve the equation AX = B, where $A = \begin{bmatrix} 13 & -5 \\ -5 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

    Answer:
$$X = \begin{bmatrix} 12 \\ 31 \end{bmatrix}$$

165) Let A and B be n × n invertible matrices and C be an n × 1 column vector.
    Solve AX + BY = C for X.

    Answer: $X = A^{-1} (C - BY)$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the system of equations by using the inverse of the coefficient matrix if it exists and by the echelon method if the inverse doesn't exist.

166) $-3x + 9y = 9$
    $3x + 2y = 13$
      A) $x = -2, y = -3$
      B) $x = 2, y = 3$
      C) $x = -3, y = -2$
      D) $x = 3, y = 2$

    Answer: D

167) $3x + 6y = -3$
    $2x - 2y = -14$
      A) $x = -5, y = 2$
      B) No inverse, no solution for system
      C) $x = 2, y = -5$
      D) $x = -5, y = -2$

    Answer: A

168) 2x + y = 2
     5x + 3y = 4
       A) x = –2, y = –2
       B) x = 2, y = –2
       C) x = –2, y = 2
       D) No inverse, no solution for system

   Answer: B

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Solve the problem.

169) A charitable organization estimates that 60% of the people in a certain area who make a contribution in one
     year will also contribute the next year, and that 10% of those who do not contribute one year will contribute the
     next. Let x and y denote the number of people who contribute in one year respectively, and let u and v be the
     corresponding numbers for the following year.

   (a) Write a matrix equation relating $\begin{bmatrix} x \\ y \end{bmatrix}$ to $\begin{bmatrix} u \\ v \end{bmatrix}$.

   (b) Solve the equation for $\begin{bmatrix} x \\ y \end{bmatrix}$.

   (c) Suppose that out of 10,000 people, 1500 made a contribution this year. How many made a contribution last
     year?

   Answer: (a)
$$\begin{bmatrix} 0.6 & 0.1 \\ 0.4 & 0.9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix}.$$

   (b)
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1.8 & -0.2 \\ -0.8 & 1.2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

   (c) 1000

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

170) A bakery sells three types of cakes, each requiring the amount of ingredients shown.

	Cake I	Cake II	Cake III
flour (cups)	2	4	2
sugar (cups)	2	1	2
eggs	2	1	3

   To fill its orders for these cakes, the bakery used 72 cups of flour, 48 cups of sugar, and 62 eggs. How many cakes
   of each type were made?
       A) 6 cake I, 8 cake II, 14 cake III
       B) 8 cake I, 8 cake II, 14 cake III
       C) 14 cake I, 8 cake II, 6 cake III
       D) 30 cake I, 8 cake II, 12 cake III

   Answer: A

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

171) Use the Gauss–Jordan method to compute the inverse of the matrix $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$.

Answer: $\left[\begin{array}{cc|cc} 3 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right]$, $\left[\begin{array}{cc|cc} 1 & \frac{1}{3} & \frac{1}{3} & 0 \\ 2 & 1 & 0 & 1 \end{array}\right]$, $\left[\begin{array}{cc|cc} 1 & \frac{1}{3} & \frac{1}{3} & 1 \\ 0 & \frac{1}{3} & -\frac{2}{3} & 1 \end{array}\right]$, $\left[\begin{array}{cc|cc} 1 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 1 & -2 & 3 \end{array}\right]$, $\left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & -2 & 3 \end{array}\right]$

$\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$

172) Use the Gauss–Jordan method to find the inverse of the matrix A, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 4 & 5 & 5 \end{bmatrix}$.

Answer:

$A^{-1} = \begin{bmatrix} 5 & 0 & -1 \\ -3 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}$

173) Use the Gauss–Jordan method to compute the inverse of the matrix, if it exists. $\begin{bmatrix} -1 & 2 \\ -3 & 7 \end{bmatrix}$

Answer:
$\begin{bmatrix} -7 & 2 \\ -3 & 1 \end{bmatrix}$

174) Use the Gauss–Jordan method to explain why the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ has no inverse.

Answer: Row reduction of the original matrix results in a matrix with a row of zeros.

175) Use the Gauss–Jordan method to compute $\begin{bmatrix} -1 & 2 & -4 \\ 1 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$.

Answer:
$\begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

176) Use the Gauss–Jordan method to compute the inverse of the matrix, if it exists.
$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$
Answer: Inverse does not exist.

177) Use the Gauss–Jordan method to compute the inverse of the matrix, if it exists.
$\begin{bmatrix} 2 & 6 & 0 \\ -1 & -3 & 0 \\ 7 & 7 & 1 \end{bmatrix}$
Answer: Inverse does not exist.

178) If $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 0 & 1 \\ 3 & 1 & 5 \end{bmatrix}$, find A.

Answer:

$$\begin{bmatrix} 1 & 0 & 0 \\ -13 & -5 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the inverse, if it exists, of the given matrix.

179) $A = \begin{bmatrix} 5 & -6 & 0 & 5 \\ 15 & -12 & 0 & 10 \\ 6 & -24 & 0 & 15 \\ 0 & 0 & -6 & 0 \end{bmatrix}$

A)
$$\begin{bmatrix} -\dfrac{2}{5} & \dfrac{1}{5} & 0 & 0 \\ \dfrac{11}{10} & -\dfrac{3}{10} & -\dfrac{1}{6} & 0 \\ 0 & 0 & 0 & -\dfrac{1}{6} \\ \dfrac{48}{25} & -\dfrac{14}{25} & -\dfrac{1}{5} & 0 \end{bmatrix}$$

B)
$$\begin{bmatrix} \dfrac{1}{5} & -\dfrac{1}{6} & 0 & \dfrac{1}{5} \\ \dfrac{1}{15} & -\dfrac{1}{12} & 0 & \dfrac{1}{10} \\ \dfrac{1}{6} & -\dfrac{1}{24} & 0 & \dfrac{1}{15} \\ 0 & 0 & -\dfrac{1}{6} & 0 \end{bmatrix}$$

C)
$$\begin{bmatrix} -\dfrac{2}{5} & \dfrac{11}{10} & 0 & \dfrac{48}{25} \\ \dfrac{1}{5} & -\dfrac{3}{10} & 0 & -\dfrac{14}{25} \\ 0 & -\dfrac{1}{6} & 0 & -\dfrac{1}{5} \\ 0 & 0 & -\dfrac{1}{6} & 0 \end{bmatrix}$$

D) No inverse

Answer: A

45

180) $A = \begin{bmatrix} 0 & 3 & 3 \\ -3 & 0 & 6 \\ 0 & 8 & 0 \end{bmatrix}$

A)
$$\begin{bmatrix} \dfrac{2}{3} & 0 & \dfrac{1}{3} \\[2mm] -\dfrac{1}{3} & 0 & 0 \\[2mm] -\dfrac{1}{4} & \dfrac{1}{8} & -\dfrac{1}{8} \end{bmatrix}$$

B) Does not exist

C)
$$\begin{bmatrix} \dfrac{2}{3} & -\dfrac{1}{3} & -\dfrac{1}{4} \\[2mm] 0 & 0 & \dfrac{1}{8} \\[2mm] \dfrac{1}{3} & 0 & -\dfrac{1}{8} \end{bmatrix}$$

D)
$$\begin{bmatrix} -\dfrac{2}{3} & -\dfrac{1}{3} & -\dfrac{1}{4} \\[2mm] -\dfrac{1}{8} & 0 & \dfrac{1}{8} \\[2mm] \dfrac{1}{3} & 0 & 0 \end{bmatrix}$$

Answer: C

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

181) Use matrix inversion to solve the system of linear equations.
$$\begin{cases} 5x + 2y - 2z = 5 \\ 2x + y - z = 1 \\ -3x - y + 2z = 2 \end{cases}$$
Answer: $x = 3$, $y = 1$, $z = 6$

182) Use matrix inversion to solve the system of linear equations.
$$\begin{cases} x + y + 3z = 10 \\ 2x + y - z = 0 \\ -x - y + 2z = 5 \end{cases}$$
Answer: $x = 2$, $y = -1$, $z = 3$

183) Use matrix inversion to solve the system of linear equations.
$$\begin{cases} x + y + z = 6 \\ x - y + z = 3 \\ x - y - z = 0 \end{cases}$$
Answer: $x = 3$, $y = \dfrac{3}{2}$, $z = \dfrac{3}{2}$

184) (a) Using the Gauss–Jordan method, find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -4 \\ 0 & 0 & 2 \end{bmatrix}$.

(b) Based on your answer to (a), solve the system $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -4 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

Answer: (a)
$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

(b) $x = -2,\ y = 8,\ z = \dfrac{3}{2}$

185) (a) Using the Gauss–Jordan method, find the inverse of the matrix $\begin{bmatrix} -2 & 2 & -1 \\ 3 & -5 & 4 \\ 5 & -6 & 4 \end{bmatrix}$.

(b) Based on your answer to (a), solve the system $\begin{bmatrix} -2 & 2 & -1 \\ 3 & -5 & 4 \\ 5 & -6 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$.

Answer: (a)
$$\begin{bmatrix} 4 & -2 & 3 \\ 8 & -3 & 5 \\ 7 & -2 & 4 \end{bmatrix}$$

(b) $x = 13,\ y = 23,\ z = 19$

186) (a) Using the Gauss–Jordan method, find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$.

(b) Based on your answer to (a), solve the following system.
$$\begin{cases} x & + z = 2 \\ 2x + y & = 3 \\ y - z = 4 \end{cases}$$

Answer: (a)
$$\begin{bmatrix} -1 & 1 & -1 \\ 2 & -1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$$

(b) $x = -3,\ y = 9,\ z = 5$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the system of equations by using the inverse of the coefficient matrix.

187) $x + y + z = 3$
$x - y + 5z = 17$
$5x + y + z = 7$
   A) $x = 1,\ y = -1,\ z = 3$
   B) $x = 3,\ y = 1,\ z = -1$
   C) $x = 3,\ y = -1,\ z = 1$
   D) No inverse, no solution for system

Answer: A

188) $x - y + 4z = 17$
$5x + z = 5$
$x + 2y + z = 11$
   A) $x = 5, y = 0, z = 3$
   B) $x = 0, y = 3, z = 5$
   C) $x = 5, y = 3, z = 0$
   D) No inverse, no solution for system

Answer: B

189) $x - y + z = 6$
$x + y + z = 8$
$x + y - z = 4$
   A) $x = 5, y = 1, z = 2$
   B) $x = 5, y = 2, z = 1$
   C) $x = 2, y = 5, z = 1$
   D) No inverse, no solution for system

Answer: A

190) $x - y + 4z = 1$
$5x + z = 0$
$x + 2y + z = -2$
   A) $x = 0, y = 0, z = -1$
   B) $x = 0, y = -1, z = 0$
   C) $x = 0, y = -1, z = 1$
   D) No inverse, no solution for system

Answer: B

Solve the problem.

191) A company makes 3 types of cable. Cable A requires 3 black wires, 3 white wires, and 2 red wires. Cable B requires 1 black, 2 white, and 1 red. Cable C requires 2 black, 1 white, and 2 red. If 100 black wires, 110 white wires, and 90 red wires were used, then how many of each cable were made?
   A) 10 cable A, 103 cable B, 20 cable C
   B) 10 cable A, 30 cable B, 20 cable C
   C) 20 cable A, 30 cable B, 10 cable C
   D) 10 cable A, 30 cable B, 93 cable C

Answer: B

192) A company makes 3 types of cable. Cable A requires 3 black wires, 3 white wires, and 2 red wires. Cable B requires 1 black, 2 white, and 1 red. Cable C requires 2 black, 1 white, and 2 red. If 95 black wires, 100 white wires, and 90 red wires were used, then how many of each cable were made?
   A) 5 cable A, 18 cable B, 25 cable C
   B) 30 cable A, 5 cable B, 25 cable C
   C) 5 cable A, 30 cable B, 25 cable C
   D) 58 cable A, 30 cable B, 22 cable C

Answer: C

193) A basketball fieldhouse seats 15,000. Courtside seats cost $8, endzone seats cost $6, and balcony seats cost $5. The total revenue for a sellout is $86,000. If half the courtside seats, half the balcony seats, and all the endzone seats are sold; then the total revenue is $49,000. How many of each type of seat are there?
    A) 4000 courtside, 3000 endzone, 8000 balcony
    B) 3200 courtside, 1800 endzone, 10,000 balcony
    C) 3000 courtside, 2000 endzone, 10,000 balcony
    D) 3000 courtside, 4000 endzone, 8000 balcony

Answer: C

194) The economy of a small country can be regarded as consisting of three industries, I, II, and III, whose input–output matrix is

$$A = \begin{array}{cc} & \begin{array}{ccc} I & II & III \end{array} \\ \begin{array}{c} I \\ II \\ III \end{array} & \begin{bmatrix} 0.20 & 0.01 & 0.30 \\ 0.30 & 0.10 & 0.02 \\ 0.05 & 0.40 & 0.10 \end{bmatrix} \end{array}$$

The entry in the third row, second column of matrix A means that
    A) industry II uses 40% of industry III's output.
    B) industry III uses 40% of industry II's output.
    C) to make $1 worth of output, industry III needs $0.40 worth of input from industry II.
    D) to make $1 worth of output, industry II needs $0.40 worth of input from industry III.
    E) none of these

Answer: D

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

195) Solve the following matrix equation for X: $AX = BX + C$.

Answer: $X = (A - B)^{-1}C$

196) Solve the matrix equation for $AX = BX + C$ for $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

where $A = \begin{bmatrix} 6 & 0 \\ 15 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 \\ 4 & -2 \end{bmatrix}$, and $C = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

Answer: $X = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

197) If A is given by $A = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.3 \end{bmatrix}$, find $I - A$.

Answer:
$$\begin{bmatrix} 0.4 & -0.4 \\ -0.2 & 0.7 \end{bmatrix}$$

198) Let $A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$. Find $(I - A)^{-1}$.

Answer:
$$\begin{bmatrix} \frac{1}{2} & -1 \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

199) If A is given by $A = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.3 \end{bmatrix}$, find $(I - A)^{-1}$

Answer:
$$\begin{bmatrix} 3.5 & 2 \\ 1 & 2 \end{bmatrix}$$

200) Solve the matrix equation $X = AX + D$ for the unknown matrix $X = \begin{bmatrix} x \\ y \end{bmatrix}$, where $A = \begin{bmatrix} 9 & 3 \\ 2 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 30 \\ 40 \end{bmatrix}$.

Answer: $x = 6$, $y = -26$

201) Let $A = \begin{bmatrix} 0.3 & 0.5 & 0 \\ 0 & 0.2 & 0.4 \\ 0.1 & 0.1 & 0.2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, and $D = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$. Solve the matrix equation $(I - A)X = D$.

Answer:
$$I - A = \begin{bmatrix} 0.7 & -0.5 & 0 \\ 0 & 0.8 & -0.4 \\ -0.1 & -0.1 & 0.8 \end{bmatrix}, (I - A)^{-1} = \begin{bmatrix} 1.5 & 1 & 0.5 \\ 0.1 & 1.4 & 0.7 \\ 0.2 & 0.3 & 1.4 \end{bmatrix}, X = \begin{bmatrix} 50 \\ 50 \\ 50 \end{bmatrix}$$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

202) Find the solution X for the matrix equation $X - AX = D$.
  A) $X = D(A - I)^{-1}$
  B) $X = (A - I)^{-1} D$
  C) $X = D(I - A)^{-1}$
  D) $X = (I - A)^{-1} D$
  E) none of these
Answer: D

203) Find the solution X for the matrix equation $XA + XB = C$.
  A) $X = (B + A)^{-1} C$
  B) $X = (A + B)^{-1} C$
  C) $X = C(B + A)^{-1}$
  D) $X = C(A + B)^{-1}$
  E) none of these
Answer: D

Solve the problem.

204) The economy of a small country can be regarded as consisting of three industries, I, II, and III, whose input–output matrix is

$$
\begin{array}{cccc}
 & \text{I} & \text{II} & \text{III} \\
A = \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \end{array} & \begin{bmatrix} 0.20 & 0.01 & 0.30 \\ 0.30 & 0.10 & 0.02 \\ 0.05 & 0.40 & 0.10 \end{bmatrix}
\end{array}
$$

Suppose x, y, and z represent the output of industries I, II, and III, respectively. An algebraic expression for the amount of output from industry III that can be exported is

A) z + (0.30x + 0.02y + 0.10z).
B) z – (0.30x + 0.02y + 0.10z).
C) z + (0.05x + 0.40y + 0.10z).
D) z – (0.05x + 0.40y + 0.10z).
E) none of these

Answer: D

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

205) An economy consists of steel and coal. The steel industry consumes $ 0.25 of coal and $ 0.02 of steel to produce $1 of steel. The coal industry requires $0.04 of coal and $0.01 of steel to produce $1 of coal.

(a) If the total production in millions of dollars of steel and coal is given by the matrix $X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, find the amounts of steel and coal used in production.

(b) Compute the amounts of steel and coal available which can be consumed or exported.

Answer: (a) $\begin{bmatrix} 0.02 & 0.01 \\ 0.25 & 0.04 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.07 \\ 0.62 \end{bmatrix}$, that is $70,000 steel, $620,000 coal.

(b) $1.93 million steel, $2.38 million coal

206) A large insurance company has a data processing division and an actuarial division. For each $1 worth of output the data processing division needs $0.20 worth of data processing and $0.40 worth of actuarial science. For each $1 worth of output, the actuarial division needs $0.30 worth of data processing and $0.10 of actuarial science.

(a) Write the input–output matrix for the insurance company.
(b) The company estimates a demand of 5 million dollars for the data processing division and 2 million for the actuarial division. Let x be the production level of the data processing division and let y be the production level of the actuarial division. Write a matrix equation to fit this problem.
(c) At what levels should each division produce to meet the demand?

Answer: (a)
$$
\begin{array}{ccc}
 & \text{DPD} & \text{AD} \\
\begin{array}{c} \text{DPD} \\ \text{AD} \end{array} & \begin{bmatrix} 0.20 & 0.30 \\ 0.40 & 0.10 \end{bmatrix}
\end{array}
$$

(b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.20 & 0.30 \\ 0.40 & 0.10 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

(c) data processing division should produce $8.5 million, actuarial division should produce $6 million

207) The economy of Mongovia is composed of three industries: I, II, and III. Suppose that in order to produce $1 worth of industry I, it takes $0.02 worth of I, $0.20 worth of II and $0.30 worth of III. To produce $1 worth of industry II, its takes $0.05 worth of I, $0.02 worth of I, and $0.20 worth of III. To produce $1 worth of industry III, it takes $0.10 worth of I, $0.15 worth of II, and $0.02 worth of III.
(a) Write the input–output matrix for this economy.
(b) How much should each industry produce to allow for consumption at these levels: $3 million for I, $2 million for II, and $3 million for III?

Answer: (a)

$$\begin{array}{c} \\ I \\ II \\ III \end{array} \begin{array}{ccc} I & II & III \end{array} \\ \begin{bmatrix} 0.02 & 0.05 & 0.10 \\ 0.20 & 0.02 & 0.15 \\ 0.30 & 0.20 & 0.02 \end{bmatrix}$$

(b) $3.75 million worth of industry I, $3.56 million worth of industry II, $4.93 million worth of industry III.

208) An economy consists of steel and coal. The steel industry consumes $ 0.25 of coal and $ 0.02 of steel to produce $1 of steel. The coal industry requires $0.04 coal and $0.01 steel to produce $1 coal.
(a) What is the input–output matrix for this economy?
(b) What is the amount of coal needed for a steel production level of $30 million?

Answer: (a)

$$\begin{array}{c} \\ steel \\ coal \end{array} \begin{array}{cc} steel & coal \end{array} \\ \begin{bmatrix} 0.02 & 0.01 \\ 0.25 & 0.04 \end{bmatrix}$$

(b) $7.5 million

209) The economy of a small country can be regarded as consisting of three industries, I, II, III, whose input–output matrix is

$$A = \begin{array}{c} \\ I \\ II \\ III \end{array} \begin{array}{ccc} I & II & III \end{array} \\ \begin{bmatrix} 0.2 & 0.3 & 0.05 \\ 0.01 & 0.1 & 0.4 \\ 0.3 & 0.02 & 0.1 \end{bmatrix}$$

If x, y, and z represent the outputs of industries I, II, and III respectively, write an algebraic expression for the amount of output from industry II that can be exported.

Answer: $y - (0.01x + 0.1y + 0.4z)$

210) An economy consisting of agriculture (I) and manufacturing (II) has the following input–output matrix.

$$A = \begin{array}{c} \\ I \\ II \end{array} \begin{array}{cc} I & II \end{array} \\ \begin{bmatrix} 0.1 & 0.3 \\ 0.3 & 0.4 \end{bmatrix}$$

How many units of agriculture and manufacturing should be produced in order to meet a demand for 15 units from I and 9 units from II?

Answer: 26 units of agriculture and 28 of manufacturing

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

211) Suppose the following matrix represents the input–output matrix of a primitive economy that depends on two basic goods, yams and pigs. How much of each commodity should be produced to get 12 bushels of yams and 14 pigs? (It is not necessary to round to a whole–number quantity of pigs.)

$$\begin{array}{c} \text{yams pigs} \\ \begin{array}{c} \text{yams} \\ \text{pigs} \end{array} \begin{bmatrix} 1/4 & 1/6 \\ 1/2 & 0 \end{bmatrix} \end{array}$$

A) 33.75 bushels of yams and 18.75 pigs
B) 21.5 bushels of yams and 29.25 pigs
C) 21.5 bushels of yams and 24.75 pigs
D) 28.5 bushels of yams and 18.75 pigs

Answer: C

212) Suppose the following matrix represents the input–output matrix of a simplified economy that involves just three commodity categories: manufacturing, agriculture, and transportation. How many units of each commodity should be produced to satisfy a demand of 1300 units for each commodity?

$$\begin{array}{c} \text{Mfg Agri Trans} \\ \begin{array}{c} \text{Mfg} \\ \text{Agri} \\ \text{Trans} \end{array} \begin{bmatrix} 0 & 1/4 & 1/3 \\ 1/2 & 0 & 1/4 \\ 1/4 & 1/4 & 0 \end{bmatrix} \end{array}$$

A) 3263 units of manufacturing, 3640 units of agriculture, and 3029 units of transportation
B) 3640 units of manufacturing, 3029 units of agriculture, and 3263 units of transportation
C) 3237 units of manufacturing, 3666 units of agriculture, and 3029 units of transportation
D) 3263 units of manufacturing, 3029 units of agriculture, and 3237 units of transportation

Answer: C

213) Suppose the following matrix represents the input–output matrix of a primitive economy that depends on two basic goods, yams and pigs. How much of each commodity should be produced to satisfy a demand for 63 bushels of yams and 52 pigs? (It is not necessary to round to a whole–number quantity of pigs.)

$$\begin{array}{c} \text{yams pigs} \\ \begin{array}{c} \text{yams} \\ \text{pigs} \end{array} \begin{bmatrix} 1/4 & 1/6 \\ 1/2 & 0 \end{bmatrix} \end{array}$$

A) 107.5 bushels of yams and 105.75 pigs
B) 153 bushels of yams and 74.25 pigs
C) 107.5 bushels of yams and 129.38 pigs
D) 133.5 bushels of yams and 74.25 pigs

Answer: A

214) Suppose the following matrix represents the input–output matrix of a simplified economy with just three sectors: manufacturing, agriculture, and transportation.

$$
\begin{array}{c}
\quad\quad \text{Mfg} \quad \text{Agri} \quad \text{Trans} \\
\begin{array}{c} \text{Mfg} \\ \text{Agri} \\ \text{Trans} \end{array}
\begin{bmatrix}
0 & 0.25 & 0.33 \\
0.50 & 0 & 0.25 \\
0.25 & 0.25 & 0
\end{bmatrix}
\end{array}
$$

Suppose also that the demand matrix is as follows:

$$
D = \begin{bmatrix} 504 \\ 267 \\ 157 \end{bmatrix}
$$

Find the amount of each commodity that should be produced.
   A) 1114 units of manufacturing, 1061 units of agriculture, and 732 units of transportation
   B) 928 units of manufacturing, 884 units of agriculture, and 610 units of transportation
   C) 928 units of manufacturing, 1061 units of agriculture, and 610 units of transportation
   D) 670 units of manufacturing, 635 units of agriculture, and 93 units of transportation

Answer: B

215) Suppose the following matrix represents the input–output matrix, T, of a simplified economy.

$$
\begin{array}{c}
\quad\quad\quad \text{Manufacturing} \quad \text{Agriculture} \quad \text{Transportation} \\
\begin{array}{c} \text{Manufacturing} \\ \text{Agriculture} \\ \text{Transportation} \end{array}
\begin{bmatrix}
0 & 1/4 & 1/3 \\
1/2 & 0 & 1/4 \\
1/4 & 1/4 & 0
\end{bmatrix}
\end{array}
$$

The demand matrix is

$$
D = \begin{bmatrix} 1300 \\ 1300 \\ 1300 \end{bmatrix}
$$

Find the amount of each commodity that should be produced.
   A) 3224 units of manufacturing, 3668.6 units of agriculture, and 3023.8 units of transportation.
   B) 3263 units of manufacturing, 3029 units of agriculture, and 3224 units of transportation.
   C) 3640 units of manufacturing, 3029 units of agriculture, and 3263 units of transportation.
   D) 3263 units of manufacturing, 3640 units of agriculture, and 3023.8 units of transportation.

Answer: A

216) A simplified economy is based on agriculture, manufacturing, and transportation. Each unit of agricultural output requires 0.3 unit of its own output, 0.5 of manufacturing, and 0.2 unit of transportation output. Each unit of manufacturing output requires 0.3 unit of its own output, 0.1 of agricultural, and 0.4 unit of transportation output. Each unit of transportation output requires 0.4 unit of its own output, 0.2 of agricultural, and 0.1 of manufacturing output. There is demand for 80 units of agricultural, 65 units of manufacturing, and 15 units of transportation output. How many units should each segment of the economy produce?
   A) Agriculture: 179; manufacturing: 251; transportation: 339
   B) Agriculture: 339; manufacturing: 179; transportation: 251
   C) Agriculture: 339; manufacturing: 251; transportation: 179
   D) Agriculture: 251; manufacturing: 339; transportation: 179

Answer: C