

CHAPTER 2

Exercises 2.1, page 47

$$1. \xrightarrow{2R_1} \begin{cases} x - 6y = 4 \\ 5x + 4y = 1 \end{cases} \quad 2. \xrightarrow{(-1)R_2} \begin{cases} x + 4y = 6 \\ y = -2 \end{cases} \quad 3. \xrightarrow{R_2 + 5R_1} \begin{cases} x + 2y = 3 \\ 14y = 16 \end{cases} \quad 4. \xrightarrow{R_2 + (-\frac{1}{2})R_1} \begin{cases} x - 6y = 4 \\ 5y = -1 \end{cases}$$

$$5. \xrightarrow{R_3 + (-4)R_1} \begin{cases} x - 2y + z = 0 \\ y - 2z = 4 \\ 9y - z = 5 \end{cases} \quad 6. \xrightarrow{R_3 + 3R_2} \begin{cases} x + 6y - 4z = 1 \\ y + 3z = 1 \\ 16z = 5 \end{cases} \quad 7. \xrightarrow{R_1 + \frac{1}{2}R_2} \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 4 \end{array} \right]$$

$$8. \xrightarrow{R_3 + (-4)R_2} \left[\begin{array}{ccc|c} 1 & 0 & 7 & 9 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 16 & -7 \end{array} \right] \quad 9. \left[\begin{array}{cc|c} -3 & 4 & -2 \\ 1 & -7 & 8 \end{array} \right] \quad 10. \left[\begin{array}{cc|c} \frac{2}{3} & -3 & 4 \\ 0 & 1 & -5 \end{array} \right] \quad 11. \left[\begin{array}{ccc|c} 1 & 13 & -2 & 0 \\ 2 & 0 & -1 & 3 \\ 0 & 1 & 0 & 5 \end{array} \right]$$

$$12. \left[\begin{array}{ccc|c} 0 & 1 & -1 & 22 \\ 2 & 0 & 0 & 17 \\ 1 & -3 & 0 & 12 \end{array} \right] \quad 13. \begin{cases} -2y = 3 \\ x + 7y = -4 \end{cases} \quad 14. \begin{cases} -5x + \frac{2}{3}y = 3 \\ x + 7y = -\frac{5}{8} \end{cases} \quad 15. \begin{cases} 3x + 2y = -3 \\ y - 6z = 4 \\ -5x - y + 7z = 0 \end{cases} \quad 16. \begin{cases} \frac{6}{5}x - y + 12z = -\frac{2}{3} \\ -x = 5 \\ 2y - z = 6 \end{cases}$$

17. Multiply the second row of the matrix by $\frac{1}{3}$. 18. Change the second row of the matrix by adding to it -4 times the first row. 19. Change the first row of the matrix by adding to it 3 times the second row. 20. Multiply the first row of the matrix by -1 . 21. Interchange rows 2 and 3. 22. Interchange rows 1 and 2.

$$23. \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 10 & 5 \end{array} \right] \quad 24. \left[\begin{array}{cc|c} 1 & -4 & -\frac{3}{2} \\ -3 & 4 & 9 \end{array} \right] \quad 25. \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 3 & -2 & 0 \end{array} \right] \quad 26. \left[\begin{array}{cc|c} 1 & 3 & -2 \\ 0 & -8 & 13 \end{array} \right] \quad 27. \left[\begin{array}{cc|c} 1 & 3 & -5 \\ 0 & 1 & 7 \end{array} \right] \quad 28. \left[\begin{array}{cc|c} 1 & 7 & 6 \\ -3 & 2 & 0 \end{array} \right]$$

$$29. R_2 + 2R_1 \quad 30. \frac{1}{2}R_2 \quad 31. R_1 + (-2)R_2 \quad 32. R_3 + (-4)R_1 \quad 33. R_1 \leftrightarrow R_2 \text{ or } R_1 \leftrightarrow R_3 \quad 34. (-\frac{1}{3})R_2$$

$$35. R_1 + (-3)R_3 \text{ or } R_2 + (-2)R_3 \quad 36. R_2 \leftrightarrow R_3 \quad 37. \left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & 10 & 2 & 18 \\ 0 & -6 & 5 & -13 \end{array} \right] \quad 38. \left[\begin{array}{ccc|c} 1 & 2 & 7 & -3 \\ 0 & -7 & -11 & 5 \\ 0 & 14 & 37 & -9 \end{array} \right]$$

$$39. \begin{cases} x + y = 7 \\ x - y = 1 \end{cases}; x = 4, y = 3 \quad 40. \begin{cases} 2x + 3y = 23 \\ 6x - 4y = 4 \end{cases}; x = 4, y = 5 \quad 41. \begin{cases} 3x - 4y = -27 \\ x + 2y = 11 \end{cases}; x = -1, y = 6$$

$$42. \begin{cases} 4x - 3y = 18 \\ 2x - y = 8 \end{cases}; x = 3, y = -2 \quad 43. \begin{cases} 2x + y + 3z = 31 \\ x + y - 2z = 3 \\ 4x - 2y + 5z = 17 \end{cases}; x = 3, y = 10, z = 5 \quad 44. \begin{cases} 2x + 7y + 4z = 1 \\ 3x - 8y + 9z = 20 \\ 4x + 5z = 16 \end{cases}; x = 4, y = -1, z = 0$$

$$45. \begin{cases} 3x + 7y + 2z = 5 \\ 7x - 6y - 3z = 4 \\ 10x + 9y - 7z = 3 \end{cases}; x = 1, y = 0, z = 1 \quad 46. \begin{cases} 3x + 2y + z = 10 \\ 8x - y + 6z = 16 \\ 5x + 3y - z = 9 \end{cases}; x = .5, y = 3, z = 2.5 \quad 47. x = -1, y = 1$$

$$48. x = -6, y = \frac{3}{2} \quad 49. x = -\frac{8}{7}, y = -\frac{9}{7}, z = -\frac{3}{7} \quad 50. x = 6, y = 1, z = 0 \quad 51. x = -1, y = 1 \quad 52. x = 2, y = 0$$

$$53. x = 1, y = 2, z = -1 \quad 54. x = 1, y = 2, z = 3 \quad 55. x = -2.5, y = 15 \quad 56. x = 18, y = -3 \quad 57. x = 1, y = -6, z = 2$$

$$58. x = 1, y = 2, z = 3 \quad 59. x = -1, y = -2, z = 5 \quad 60. x = 1, y = -3, z = 4 \quad 61. 30 \quad 62. 8 \quad 63. d \quad 64. c$$

$$65. 150 \text{ short sleeve, } 200 \text{ long sleeve} \quad 66. 47 \text{ bottles of national brand, } 35 \text{ bottles of store brand} \quad 67. 190 \text{ adults, } 85 \text{ children}$$

$$68. 15 \text{ at-bats, } 3 \text{ hits, } 200 \text{ batting average} \quad 69. x = 3.7, y = 3.9, z = 1.9 \quad 70. x = 13, y = 19, z = 68 \quad 71. 3 \text{ ounces of Brazilian, } 6 \text{ ounces of Columbian, } 7 \text{ ounces of Peruvian}$$

$$72. 5 \text{ ounces of cashews, } 6 \text{ ounces of almonds, } 5 \text{ ounces of walnuts} \quad 73. \$25,000 \text{ in the bond fund, } \$50,000 \text{ in the health sciences fund, } \$25,000 \text{ in the real estate fund} \quad 74. 6 \text{ ounces of food I, } 3 \text{ ounces of food II, } 1 \text{ ounce of food III} \quad 75. 23\frac{1}{3} \text{ pounds of first type, } 85 \text{ pounds of second type, } 201\frac{2}{3} \text{ pounds of third type} \quad 76. \$1250 \text{ in the savings account, } \$1250 \text{ in the certificate of deposit, and } \$2500 \text{ in the prepaid college fund}$$

$$77. \left[\begin{array}{cc|c} 1 & 0 & -5 \\ 0 & 1 & 4 \end{array} \right] \quad 78. \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{58}{13} \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -\frac{9}{13} \end{array} \right] \quad 79. \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{175}{54} \\ 0 & 1 & 0 & \frac{16}{9} \\ 0 & 0 & 1 & \frac{26}{27} \end{array} \right] \quad 80. \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{109}{11} \\ 0 & 1 & 0 & -\frac{7}{11} \\ 0 & 0 & 1 & \frac{13}{11} \end{array} \right] \quad 81. x = -2.5, y = 15$$

$$82. x = 18, y = -3 \quad 83. x = 1, y = -6, z = 2 \quad 84. x = 1, y = 2, z = 3$$

Exercises 2.2, page 56

$$1. \left[\begin{array}{ccc} 1 & -2 & 3 \\ 0 & 13 & -8 \end{array} \right] \quad 2. \left[\begin{array}{cc|c} 0 & 0 & 6 \\ \frac{1}{2} & 1 & -\frac{3}{2} \end{array} \right] \quad 3. \left[\begin{array}{ccc|c} 9 & -1 & 0 & -7 \\ -\frac{1}{2} & \frac{1}{2} & 1 & 3 \\ 5 & -1 & 0 & -3 \end{array} \right] \quad 4. \left[\begin{array}{ccc|c} 15 & 0 & 0 & -28 \\ 7 & 0 & 9 & 0 \\ -1 & 1 & -1 & 4 \end{array} \right] \quad 5. \left[\begin{array}{c|c} 1 & \frac{3}{2} \\ 0 & -9 \\ 0 & \frac{7}{2} \end{array} \right] \quad 6. \left[\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right]$$

$$7. \left[\begin{array}{ccc} 4 & 3 & 0 \\ 1 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & 1 \end{array} \right] \quad 8. \left[\begin{array}{ccc} 0 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 1 \\ -2 & 5 & 0 \end{array} \right] \quad 9. \begin{cases} x + y + 4z = 6 \\ 2x + y + z = 10 \end{cases}; z = \text{any value, } y = 2 - 7z, x = 4 + 3z$$

2-2 Instructor Answers

10. $\begin{cases} 2x - 2y + z = 2 \\ -6x + 6y - 3z = 5 \end{cases}$; no solution 11. $\begin{cases} -5x + 15y - 10z = 5 \\ x - 3y + 2z = 0 \end{cases}$; no solution
12. $\begin{cases} 2x - 6y - 4z = 0 \\ -3x + 9y + 6z = 0 \end{cases}$; $y = \text{any value}, z = \text{any value}, x = 3y + 2z$
13. $\begin{cases} 2x - y + 5z = 12 \\ -x - 4y + 2z = 3 \\ 8x + 5y + 11z = 30 \end{cases}$; $z = \text{any value}, y = z - 2, x = 5 - 2z$ 14. $\begin{cases} 2x - y + 2z = 4 \\ 3x + y + z = -2 \\ x + 2y - z = 5 \end{cases}$; no solution
15. $\begin{cases} x + 2y + 3z - w = 4 \\ 2x + 3y + w = -3 \\ 4x + 7y + 6z - w = 5 \end{cases}$; $z = \text{any value}, w = \text{any value}, y = 11 - 6z + 3w, x = 9z - 5w - 18$
16. $\begin{cases} x + y + z = -1 \\ x + 2y - z = -6 \\ 2x + y + 4z = 3 \end{cases}$; $z = \text{any value}, y = 2z - 5, x = 4 - 3z$ 17. $y = \text{any value}, x = 3 + 2y$ 18. No solution
19. No solution 20. $y = \text{any value}, x = 4 + 3y$ 21. $x = 1, y = 2$ 22. $y = \text{any value}, x = 6y + 12$ 23. No solution
24. $x = 3, y = 2$ 25. No solution 26. $z = \text{any value}, x = 11z + 8, y = 4z + 1$ 27. $z = \text{any value}, x = -6 - z, y = 5$
28. $y = \text{any value}, x = 3y + 2, z = 4$ 29. No solution 30. No solution 31. No solution 32. $x = 3, y = -1, z = 0$
33. $z = \text{any value}, w = \text{any value}, x = 2z + w, y = 5 - 3w$ 34. $w = \text{any value}, x = -\frac{1}{2}w + \frac{11}{2}, y = \frac{1}{2}w - \frac{5}{2}, z = 6$
35. No solution 36. $w = \text{any value}, x = 1 - 4w, y = 2w + 3, z = 0$
37. Possible answers: $z = 0, x = -13, y = 9$; $z = 1, x = -8, y = 6$; $z = 2, x = -3, y = 3$
38. Possible answers: $z = 0, x = -56, y = 13$; $z = 1, x = -64, y = 14$; $z = 2, x = -72, y = 15$
39. Possible answers: $y = 0, x = 23, z = 5$; $y = 1, x = 16, z = 5$; $y = 2, x = 9, z = 5$
40. Possible answers: $z = 0, x = 4, y = 7$; $z = 1, x = 4, y = 10$; $z = 2, x = 4, y = 13$
41. Food 3: $z = \text{any value between } 0 \text{ and } 100$, food 2: $y = 100 - z$, food 1: $x = 300 - z$ 42. No solution
44. 4 grams of food A, 3 grams of food B, 2 grams of food C; 1.5 grams of food A, 3.9 grams of food B, 1.9 grams of food C
45. 50 ottomans, 30 sofas, 40 chairs; 5 ottomans, 55 sofas, 35 chairs; 95 ottomans, 5 sofas, 45 chairs
46. 9 computers, 4 printers, 2 scanners; 8 computers, 2 printers, 5 scanners
47. 6 floral squares, the other 90 any mix of solid green and solid blue
48. The same number of \$7 and \$13 plants, up to 7 of each type, the rest \$10 plants
49. No solution if $k \neq -12$; infinitely many if $k = -12$ 50. 3 51. None
52. No, there still could be a unique solution or infinitely many solutions depending on the other rows of the matrix.
53. One; $x = 7, y = 3$ 54. None 55. None 56. One; $x = 5, y = 6$
57. There has been a pivot about the bottom right element. 58. Does not differ

Exercises 2.3, page 68

1. 2×3 2. 2×1 , column matrix 3. 1×3 , row matrix 4. 2×2 square, identity matrix 5. 2×2 , square matrix
6. 1×1 , square, column, and row matrix 7. $-4; 0$ 8. $-1; 2$ 9. $i = 1, j = 3$ 10. $i = 2, j = 2$
11. $\begin{bmatrix} 9 & 3 \\ 7 & -1 \end{bmatrix}$ 12. $\begin{bmatrix} 13 \\ 3 \end{bmatrix}$ 13. $\begin{bmatrix} 2 & 4 & 2.5 \\ -5.5 & 1 & 1.2 \end{bmatrix}$ 14. $\begin{bmatrix} 3 & 3 & 2 \end{bmatrix}$ 15. $\begin{bmatrix} 1 & 3 \\ 1 & 2 \\ 4 & -2 \end{bmatrix}$ 16. $\begin{bmatrix} .2 & -.5 \\ -.2 & .5 \end{bmatrix}$ 17. $\begin{bmatrix} -7 \\ \frac{1}{6} \end{bmatrix}$
18. $\begin{bmatrix} .8 & 1 & 0 \\ .4 & -1.6 & 1.5 \end{bmatrix}$ 19. $[11]$ 20. $[\frac{1}{2}]$ 21. $[10]$ 22. $[0]$ 23. $\begin{bmatrix} 4 & 0 & -\frac{2}{3} \\ -6 & \frac{1}{2} & \frac{1}{3} \end{bmatrix}$ 24. $\begin{bmatrix} 6 & .75 \\ 0 & 1.8 \end{bmatrix}$ 25. $\begin{bmatrix} 3 & 19 \\ 23 & 3 \end{bmatrix}$
26. $[0 \quad 18]$ 27. Yes; 3×5 28. Yes; 3×4 29. No 30. Yes; 1×1 31. Yes; 3×1 32. No 33. $\begin{bmatrix} 6 & 17 \\ 6 & 10 \end{bmatrix}$ 34. $\begin{bmatrix} 10 \\ 7 \end{bmatrix}$
35. $\begin{bmatrix} 21 \\ -4 \\ 8 \end{bmatrix}$ 36. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ 37. $\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ 38. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 39. $\begin{bmatrix} .48 & .39 \\ .52 & .61 \end{bmatrix}$ 40. $\begin{bmatrix} 8 & -10 & 2 \\ -1 & 6 & 3 \\ 3 & 5 & 11 \end{bmatrix}$ 41. $\begin{bmatrix} 25 & 17 & 2 \\ 3 & -1 & 2 \\ 1 & 1 & 4 \end{bmatrix}$
42. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ 43. $\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$ 44. $\begin{bmatrix} .4 & .4 & .4 \\ .4 & .4 & .4 \\ .2 & .2 & .2 \end{bmatrix}$ 45. $[30 \quad 41]$ 46. $[8 \quad 18]$ 47. $\begin{bmatrix} 10 & 0 \\ 0 & 15 \end{bmatrix}$ 48. $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ 49. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 50. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
51. $\begin{bmatrix} 23 & 24 \\ 25 & 26 \end{bmatrix}$ 52. $\begin{bmatrix} 2.4 & 5.6 \\ 7.8 & 9.9 \end{bmatrix}$ 53. $\begin{cases} 2x + 3y = 6 \\ 4x + 5y = 7 \end{cases}$ 54. $\begin{cases} -3x + 4y = 1 \\ y = 1 \end{cases}$ 55. $\begin{cases} x + 2y + 3z = 10 \\ 4x + 5y + 6z = 11 \\ 7x + 8y + 9z = 12 \end{cases}$ 56. $\begin{cases} x = 1 \\ y = 2 \\ z = 3 \end{cases}$

$$57. \begin{bmatrix} 3 & 2 \\ 7 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad 58. \begin{bmatrix} 5 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix} \quad 59. \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 2 \end{bmatrix} \quad 60. \begin{bmatrix} -2 & 4 & -1 \\ 1 & 6 & 3 \\ 7 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \\ 8 \end{bmatrix}$$

65. (a) $\begin{bmatrix} 340 \\ 265 \end{bmatrix}$ (b) Mike's clothes cost \$340; Don's clothes cost \$265. (c) $\begin{bmatrix} 25 \\ 18.75 \\ 62.50 \end{bmatrix}$ (d) The costs of the three items of clothing after

a 25% increase 66. (a) [15,400 16,050] (b) The monthly sales for Store 1 were \$15,400 and for Store 2 were \$16,050. (c) [275 88 66] (d) The retail prices after a 10% increase 67. (a) [2282.50 2322.50 3550.50], total retail value for the white chocolate-covered,

milk chocolate-covered, and dark chocolate-covered items (b) $\begin{bmatrix} 3138.00 \\ 3337.50 \\ 6772.50 \end{bmatrix}$, total revenue from peanuts, raisins, and espresso beans

(c) $\begin{bmatrix} 94.50 \\ 351.50 \\ 256.50 \end{bmatrix}$, 10% reduction in the number of pounds sold 68. (a) [18,500 21,750 24,250], November wholesale costs for each of the

three stores (b) [18,000 26,500 27,500], December wholesale costs for each of the three stores (c) [31,500 37,250 40,750].

November revenue for each of the three stores (d) [31,000 44,500 46,500]. December revenue for each of the three stores

(e) [200 200 300], profits for each of the three appliances (f) [13,000 15,500 16,500], November profits for each of the three

stores (g) [13,000 18,000 19,000], December profits for each of the three stores (h) $\begin{bmatrix} 50 & 90 & 50 \\ 50 & 40 & 30 \\ 20 & 25 & 65 \end{bmatrix}$, quantities of each of the

appliances sold during November and December (i) [26,000 33,500 35,500], combined November and December profits for each of the three stores (j) [475 427.50 712.50], 5% discount off retail prices 69. (a) I: 2.75, II: 2, III: 1.3 (b) A: 74, B: 112, C: 128, D: 64, F: 22 70. Scheme III 71. 10,100 voting Democratic, 7900 voting Republican 72. (a) Democrats; 56.1% (b) Republicans;

50.6% 73. Carpenters: \$2000, bricklayers: \$2100, plumbers: \$1200 74. (a) $\begin{bmatrix} 232,000 & 260,500 \\ 86,000 & 97,500 \\ 42,000 & 47,000 \end{bmatrix}$ (b) 86,000 (c) 47,000

75. (a) [162 150 143], number of units of each nutrient consumed at breakfast (b) [186 200 239], number of units of each nutrient consumed at lunch (c) [288 300 344], number of units of each nutrient consumed at dinner (d) [5 8], total number of ounces of each food that Mikey eats during a day (e) [636 650 726], number of units of each nutrient consumed per day

76. (a) [100 115 85 75], units of each ingredient needed to fill to order (b) $\begin{bmatrix} 108 \\ 102 \\ 182 \end{bmatrix}$, cost to make each type of cookie (c) [5850], total cost to fill order (d) $\begin{bmatrix} 67 \\ 48 \\ 43 \end{bmatrix}$, profit for each type of cookie (e) [2275], total profit for the order

(f) [8125], total price of the order 77. (a) $\begin{bmatrix} 720 \\ 646 \end{bmatrix}$ (b) \$720

78. (a) $T = \begin{bmatrix} 3 & 5 \\ \frac{1}{2} & 1 \end{bmatrix}$ Assembly Packaging (b) $S = \begin{bmatrix} 30 \\ 20 \end{bmatrix}$ DVDs TVs; 190 hours of assembly, 35 hours of packaging

79. (a) $T = \begin{bmatrix} 30 & 45 \\ 30 & 50 \\ 15 & 10 \end{bmatrix}$ Preparation Baking Finishing (b) $S = \begin{bmatrix} 20 \\ 8 \end{bmatrix}$ Boston cream pie Carrot cake; $TS = \begin{bmatrix} 960 \\ 1000 \\ 380 \end{bmatrix}$ Preparation Baking Finishing

(c) Total baking time: 1000 minutes, or $16\frac{2}{3}$ hours; total finishing time: 380 minutes, or $6\frac{1}{3}$ hours

80. (a) $T = \begin{bmatrix} 20 & 5 & 15 \\ 30 & 5 & 20 \end{bmatrix}$ Preparation Lacquering Drying Manicure Pedicure (b) $S = \begin{bmatrix} 15 & 9 \end{bmatrix}$; $ST = \begin{bmatrix} 570 & 120 & 405 \end{bmatrix}$ (c) Total drying time is 405 minutes, or $6\frac{3}{4}$ hours.

81. (a) $T = \begin{bmatrix} 2 & 3 & 2 \\ 1.5 & 2 & 1 \end{bmatrix}$ Cutting Sewing Finishing Huge One Regular Joe (b) $S = \begin{bmatrix} 32 \\ 24 \end{bmatrix}$ Huge One Regular Joe

(c) $A = \begin{bmatrix} 27 & 56 \end{bmatrix}$; $AT = \begin{bmatrix} 138 & 193 & 110 \end{bmatrix}$; $AS = [2208]$ (d) 193 hours (e) \$2208

2-4 Instructor Answers

82. (a) $BC = [7000]$; The total revenue is \$7000. (b) $AC = \begin{bmatrix} 740 \\ 2065 \\ 90.6 \end{bmatrix}$; The total size is 740 GB, the total battery life is 2065 hours,

and the total weight is 90.6 ounces. (c) The total battery life for all MP3 players sold is 2065 hours. 84. $a = 1, b = -2$

85. $\begin{bmatrix} 3 & -2 & 1 \\ -5 & 6 & 7 \end{bmatrix}$ 86. $\begin{bmatrix} 5 & 4 & -3 \\ 0 & -1 & 2 \end{bmatrix}$ 87. 4×4 88. 3×3 89. $\begin{bmatrix} 9257 & 57,718 & 89,389 \\ 13.9 \\ 14.9 \\ 14.2 \end{bmatrix}$

90. $\begin{bmatrix} 155,959 & 95,997 & 66,554 \\ 250.0 \\ 42.0 \\ 107.8 \end{bmatrix}$ 91. $\begin{bmatrix} 6.4 & -2 & -2.7 \\ 20.5 & 22.5 & -2.4 \\ -14 & 17.6 & 16 \end{bmatrix}$ 92. $\begin{bmatrix} 5.6 & -16 & 3.3 \\ -17.5 & 21.5 & -5.6 \\ 4 & -4.4 & 12 \end{bmatrix}$

93. $\begin{bmatrix} -171.3 & 40.8 & -31.8 \\ 454.6 & -22.5 & 22.7 \\ -2.6 & 122.3 & 53.56 \end{bmatrix}$ 94. $\begin{bmatrix} 27.9 & 130.6 & -69.88 \\ 106.75 & -149.44 & 26.1 \\ -47.5 & 336.2 & -18.7 \end{bmatrix}$ 95. $\begin{bmatrix} 1.2 & 21 & -9 \\ 57 & 1.5 & 4.8 \\ -27 & 33 & 6 \end{bmatrix}$ 96. $\begin{bmatrix} 1.8 & 14 & -6 \\ 38 & 2 & 3.2 \\ -18 & 22 & 5 \end{bmatrix}$

Exercises 2.4, page 78

1. $x = 2, y = 0$ 2. $x = 6, y = 1$ 3. $\begin{bmatrix} 1 & -2 \\ -3 & 7 \end{bmatrix}$ 4. $\begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$ 5. $\begin{bmatrix} 1 & -1 \\ -\frac{5}{2} & 3 \end{bmatrix}$ 6. $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ 7. $\begin{bmatrix} 1.6 & -4 \\ -6 & 1.4 \end{bmatrix}$ 8. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

9. $\begin{bmatrix} \frac{1}{3} \end{bmatrix}$ 10. $[5]$ 11. $x = 4, y = -\frac{1}{2}$ 12. $x = 2, y = -3$ 13. $x = 32, y = -6$ 14. $x = 1, y = 2$ 15. (a) $\begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} m \\ s \end{bmatrix}$

(b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1.4 & -.6 \\ -.4 & 1.6 \end{bmatrix} \begin{bmatrix} m \\ s \end{bmatrix}$ (c) 110,000 married; 40,000 single (d) 130,000 married; 20,000 single 16. (a) $\begin{bmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{2}{3} & \frac{3}{4} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s \\ w \end{bmatrix}$

(b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ -8 & 4 \end{bmatrix} \begin{bmatrix} s \\ w \end{bmatrix}$ (c) 12,000 (d) 24,000 17. (a) $\begin{bmatrix} .7 & .1 \\ .3 & .9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix}$ (b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{6} \\ -\frac{1}{2} & \frac{7}{6} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$ (c) 8500; 4500

18. (a) $\begin{bmatrix} .8 & .5 \\ .2 & .5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix}$ (b) 24; 28 19. $x = 9, y = -2, z = -2$ 20. $x = 5, y = -1, z = -1$ 21. $x = 21, y = 25, z = 26$

22. $x = 6, y = 7, z = 8$ 23. $x = 1, y = 5, z = -4, w = 9$ 24. $x = -4, y = 1, z = 19, w = 5$ 25. $x = 4, y = -19, z = 2, w = -4$ 26. $x = -9, y = 25, z = -4, w = 5$ 28. True

29. (a) $\begin{bmatrix} 1 & 2 \\ .9 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$ (b) After 1 year: 1,170,000 in group I and 405,000 in group II. After 2 years: 1,980,000 in group I and

1,053,000 in group II. (c) 700,000 in group I and 55,000 in group II. 30. $\begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}$

33. One possible answer is $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. 34. One possible answer is $A = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$. 35. $\begin{bmatrix} -\frac{10}{73} & \frac{75}{292} \\ \frac{25}{73} & -\frac{5}{292} \end{bmatrix}$

36. $\begin{bmatrix} -\frac{2}{123} & \frac{11}{82} \\ \frac{10}{41} & \frac{20}{41} \end{bmatrix}$ 37. $\begin{bmatrix} \frac{1020}{8887} & \frac{2910}{8887} & -\frac{500}{8887} \\ \frac{3050}{8887} & \frac{860}{8887} & \frac{1990}{8887} \\ \frac{125}{8887} & \frac{618}{8887} & \frac{810}{8887} \end{bmatrix}$ 38. $\begin{bmatrix} \frac{75}{1901} & -.3414\dots & -\frac{2305}{5703} \\ \frac{220}{1901} & \frac{2525}{5703} & \frac{2110}{5703} \\ \frac{170}{1901} & \frac{655}{5703} & \frac{1112}{5703} \end{bmatrix}$ 39. $x = -\frac{4}{5}, y = \frac{28}{5}, z = 5$

40. $x = \frac{23}{2}, y = \frac{257}{2}, z = \frac{209}{2}$ 41. $x = 0, y = 2, z = 0, w = 2$ 42. $x = -\frac{8}{181}, y = \frac{413}{181}, z = \frac{749}{181}, w = \frac{367}{181}$

Exercises 2.5, page 82

1. $\begin{bmatrix} -2 & 3 \\ 5 & -7 \end{bmatrix}$ 2. $\begin{bmatrix} \frac{1}{11} & \frac{1}{11} \\ -\frac{3}{11} & \frac{5}{22} \end{bmatrix}$ 3. $\begin{bmatrix} \frac{7}{2} & \frac{3}{2} \\ -2 & -1 \end{bmatrix}$ 4. $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ 5. No inverse 6. No inverse 7. $\begin{bmatrix} -1 & 2 & -4 \\ 1 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

8. $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 0 \\ -\frac{3}{2} & -\frac{3}{2} & 1 \end{bmatrix}$ 9. No inverse 10. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{5} & \frac{2}{5} \\ 1 & -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$ 11. $\begin{bmatrix} -5 & 6 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -\frac{1}{46} & \frac{1}{46} \\ 0 & 0 & \frac{25}{46} & -\frac{1}{23} \end{bmatrix}$ 12. $\begin{bmatrix} \frac{1}{4} & 0 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 1 & 0 & -1 \\ -\frac{1}{4} & 0 & \frac{3}{2} & 0 \\ 2 & 0 & -3 & 1 \end{bmatrix}$

13. $x = 2, y = -3, z = 2$ 14. $x = -9, y = 5, z = 1$ 15. $x = 2, y = 1, z = 3$ 16. $x = 2, y = 7, z = -3$

17. $x = 4, y = -4, z = 3, w = -1$ 18. $x = 6, y = 3, z = 0, w = 3$ 19. $\begin{bmatrix} -3 & 5 \\ 10 & -16 \end{bmatrix}$ 20. $\begin{bmatrix} -23 & -10 \\ 9 & 4 \end{bmatrix}$ 21. $x = 42, y = 21, z = 37$

22. $x = 58, y = 27, z = 15$ 23. $x = 82, y = 17, z = 1$ 24. $x = 14, y = 46, z = 22$

Exercises 2.6, page 88

1. 20 cents 2. 15 cents 3. Energy sector 4. Energy sector 5. \$6 million 6. \$4.5 million 7. Manufacturing 8. Services

$$9. AX = \begin{bmatrix} 11.00 \\ 11.50 \\ 9.50 \end{bmatrix} \quad 11. X = \begin{bmatrix} 12.89 \\ 14.06 \\ 13.08 \end{bmatrix} \quad 12. X = \begin{bmatrix} 26.05 \\ 19.90 \\ 31.35 \end{bmatrix} \quad 13. \text{ Coal: \$8.84 billion; steel: \$3.725 billion; electricity: \$9.895 billion}$$

14. Computers: \$482 million; semiconductors: \$298.5 million; business forms: \$155.5 million 15. Computers: \$354 million; semiconductors: \$172 million 16. Coal: \$4.124 billion; steel: \$1.788 billion; electricity: \$3.354 billion 17. \$1.55 billion worth of coal, \$0.86 billion worth of steel, and \$4.55 billion worth of electricity 18. \$358,000,000 worth of computers, \$118,000,000 worth of semiconductors, and \$253,000,000 worth of business forms

$$19. \begin{matrix} T & E \\ \text{(a)} & \begin{bmatrix} .25 & .30 \\ .20 & .15 \end{bmatrix} \end{matrix} \quad \begin{matrix} \text{(b)} & \begin{bmatrix} 1.47 & .52 \\ .35 & 1.30 \end{bmatrix} \end{matrix} \quad \begin{matrix} \text{(c)} & \text{Transportation: \$8.91 billion; energy: \$5.65 billion} \\ \text{(d)} & \text{Transportation: \$3.92 billion; energy: \$2.63 billion} \end{matrix}$$

$$20. \begin{matrix} T & E \\ \text{(a)} & \begin{bmatrix} .25 & .30 \\ .20 & .15 \end{bmatrix} \end{matrix} \quad \begin{matrix} \text{(b)} & \begin{bmatrix} 1.47 & .52 \\ .35 & 1.30 \end{bmatrix} \end{matrix} \quad \begin{matrix} \text{(c)} & \text{Transportation: \$9.52 billion; energy: \$10.50 billion} \\ \text{(d)} & \text{Transportation: \$5.53 billion, energy: \$3.48 billion} \end{matrix}$$

$$21. \text{Plastics: \$955,000; industrial equipment: \$590,000} \quad 22. \text{Plastics: \$1.93 million; industrial equipment: \$3.14 million}$$

$$23. \begin{matrix} W & S & C \\ \text{(a)} & \begin{bmatrix} .30 & 0 & .10 \\ .20 & .30 & .20 \\ .10 & .20 & .05 \end{bmatrix} \end{matrix} \quad \begin{matrix} \text{(b)} & \begin{bmatrix} 1.47 & .05 & .16 \\ .49 & 1.54 & .38 \\ .26 & .33 & 1.15 \end{bmatrix} \end{matrix} \quad \begin{matrix} \text{(c)} & \text{Wood: \$1.99; steel: \$7.41; coal: \$3.88} \\ \text{(d)} & \text{Wood: \$0.98; steel: \$3.39; coal: \$1.87} \end{matrix}$$

$$24. \begin{matrix} W & S & C \\ \text{(a)} & \begin{bmatrix} .30 & 0 & .10 \\ .20 & .30 & .20 \\ .10 & .20 & .05 \end{bmatrix} \end{matrix} \quad \begin{matrix} \text{(b)} & \begin{bmatrix} 1.47 & .05 & .16 \\ .49 & 1.54 & .38 \\ .26 & .33 & 1.15 \end{bmatrix} \end{matrix} \quad \begin{matrix} \text{(c)} & \text{Wood: \$437; steel: \$864; coal: \$1071} \\ \text{(d)} & \text{Wood: \$237; steel: \$564; coal: \$271} \end{matrix}$$

25. Manufacturing: \$398 million; transportation: \$313 million; agriculture: \$452 million

$$26. \begin{matrix} A & E & M \\ \text{(a)} & \begin{bmatrix} .08 & .15 & .25 \\ .10 & .14 & .12 \\ .20 & .10 & .05 \end{bmatrix} \end{matrix} \quad \begin{matrix} \text{(b)} & \begin{bmatrix} 1.19 & .25 & .34 \\ .18 & 1.22 & .20 \\ .27 & .18 & 1.15 \end{bmatrix} \end{matrix} \quad \begin{matrix} \text{(c)} & \text{Agriculture: \$6.18 billion; energy: \$4.75 billion; manufacturing: \$3.91 billion} \\ \text{(d)} & \text{Agriculture: \$2.18 billion; energy: \$1.75 billion; manufacturing: \$1.91 billion} \end{matrix}$$

27. Merchant: \$85,000; baker: \$68,000; farmer: \$103,000 28. U.S.: \$846 million; Canada: \$333 million; England: \$1440 million 30. The second and third columns of $(I - A)^{-1}$ represent the increased production levels required by \$1 billion increases in the final demand for steel and electricity, respectively.

$$31. \begin{bmatrix} 11.91 \\ 15.83 \\ 9.57 \\ 7.26 \end{bmatrix} \quad 32. \begin{bmatrix} 11.61 \\ 8.17 \\ 5.09 \\ 13.32 \end{bmatrix}$$

Chapter 2: Answers to Fundamental Concept Check Exercises, page 93

1. Values of $x, y, z, \dots$ that satisfy each equation in the system 2. Rectangular array of numbers 3. (a) Interchange any two equations (or rows). (b) Multiply an equation (or row) by a nonzero number. (c) Change an equation (or row) by adding to it a multiple of another equation (or row). 4. System of equations: $x = c_1; y = c_2; \dots$; Matrix: all entries on the main diagonal are 1; all entries off the main diagonal are zero 5. Use elementary row operations to make the entry have value 1, and make the other entries in its column have value 0. 6. (a) Create a matrix corresponding to the system of linear equations. (b) Attempt to put the matrix into diagonal form as described in the box following Example 1 of Section 2.2. (c) If the matrix cannot be put into diagonal form, follow the first step in the box following Example 3 of Section 2.2. (d) Write the system of linear equations corresponding to the matrix, and read off the solution(s). 7. Row matrix: a matrix consisting of a single row (that is, a $1 \times n$ matrix); Column matrix: a matrix consisting of a single column (that is, an $m \times 1$ matrix); Square matrix: a matrix having the same number of columns as rows (that is, an $n \times n$ matrix); Identity matrix: a square matrix having 1s on the main diagonal and 0s elsewhere 8. The entry in the i^{th} row and j^{th} column 9. For two matrices of the same size, the sum (difference) is the matrix obtained by adding (subtracting) the corresponding entries of the two matrices. 10. For two matrices A and B , where the number of columns of A is the same as the number of rows of B , the matrix AB is the matrix having the same number of rows as A and the same number of columns as B whose ij^{th} entry is obtained by adding the products of the corresponding entries of the i^{th} row of A with the j^{th} column of B . 11. The scalar product of the number c and the matrix A is the matrix obtained by multiplying each element of A by c . 12. The inverse of the square matrix A is the matrix

whose product with A is an identity matrix. **13.** The inverse of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\frac{1}{D} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $D = ad - bc$ and $D \neq 0$. **14.** Write the matrix form ($AX = B$) of the system of linear equations. If the matrix A has an inverse, then the solution of the system of linear equations is given by the entries of the matrix $A^{-1}B$. **15.** Adjoin an identity matrix to the right of the matrix A and then apply the Gauss–Jordan elimination method to the entire matrix until its left side is an identity matrix if possible. The new right side of the matrix will be the inverse of A . **16.** A square matrix whose ij^{th} entry is the amount of input from the i^{th} industry required to produce one unit of the j^{th} industry; A column matrix whose i^{th} element is the amount of units demanded from the i^{th} industry **17.** If A is an input–output matrix and D is a consumer-demand matrix, then the i^{th} entry of the matrix $(I - A)^{-1}D$ gives the amount of input required from the i^{th} industry to meet the final demand.

Chapter 2: Review Exercises, page 93

1. $\begin{bmatrix} 1 & -2 & \frac{1}{3} \\ 0 & 8 & \frac{16}{3} \end{bmatrix}$ **2.** $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ -12 & 0 & 7 \end{bmatrix}$ **3.** $x = 4, y = 5$ **4.** $x = 50, y = 2, z = -12$ **5.** $x = -1, y = \frac{2}{3}, z = \frac{1}{3}$ **6.** No solution

7. $z = \text{any value}, x = 1 - 3z, y = 4z, w = 5$ **8.** $x = 7, y = 3$ **9.** $\begin{bmatrix} 5 \\ 3 \\ 7 \end{bmatrix}$ **10.** $\begin{bmatrix} 6 & 17 \\ 12 & 26 \end{bmatrix}$ **11.** $\begin{bmatrix} 6 & -\frac{9}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$ **12.** $\begin{bmatrix} .6 & -10 \\ 6.6 & 2 \\ 4 & 11 \end{bmatrix}$

13. 5 **14.** 4 **15.** $x = -2, y = 3$ **16. (a)** $x = 13, y = 23, z = 19$ **(b)** $x = -4, y = 13, z = 14$ **17.** $\begin{bmatrix} -1 & 3 \\ \frac{1}{2} & -1 \end{bmatrix}$

18. $\begin{bmatrix} 5 & -1 & -1 \\ -3 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ **19.** Corn: 500 acres; wheat: 0 acres; soybeans: 500 acres **20. (a)** $\begin{bmatrix} 5455 \\ 5275 \end{bmatrix}$; total month's costs for each store

(b) $\begin{bmatrix} 6600 \\ 6360 \end{bmatrix}$; total month's revenue for each store **(c)** $\begin{bmatrix} 35 \\ 15 \\ 40 \end{bmatrix}$; profit for each piece of equipment **(d)** $\begin{bmatrix} 1145 \\ 1085 \end{bmatrix}$; total month's profit for each store

21. (a) [10,100 8230 4670]; total amount invested in bonds, stocks, and the conservative fixed income fund, respectively
(b) [522.40 1807.30]; total returns on the investments for one year and five years, respectively **(c)** [10,000 16,000 20,000]; the result of doubling the amounts invested **(d)** The total amount invested in stocks is \$8230. **(e)** The total return after one year is \$522.40.

22. (a) $AB = \begin{bmatrix} 328 \\ 336 \\ 323 \\ 326 \end{bmatrix}$; Sara earned \$328, Quinn earned \$336, Tamia earned \$323, and Zack earned \$326. **(b)** Most: Quinn; least: Tamia

(c) Quinn and Zack both earned \$329. **(d)** 30 hours **23.** 4 apples, 9 bananas, 5 oranges **24. (a)** A : 9400, 8980; B : 7300, 7510
(b) A : 10,857, 12,082; B : 6571, 5959 **25.** Industry I: 20; industry II: 20 **26.** 4 **27. (a)** True **(b)** False **(c)** True
28. (a) True **(b)** False

CHAPTER 11

Exercises 11.1, page 495

1. Statement 2. Statement 3. Statement 4. Statement 5. Not a statement—not a declarative sentence. 6. Statement
7. Not a statement—not a declarative sentence. 8. Statement 9. Statement 10. Statement 11. Not a statement— x is not specified. 12. Not a statement—“she” is not specified. 13. Not a statement—not a declarative sentence. 14. Statement
15. Statement 16. “Lagos is the largest city in Nigeria,” “Alpha Centauri is the nearest star to Earth.” 17. $2 + 2 = 5$, $4 < 7$
18. p : China is in Asia. 19. p : The Phelps Library is in New York. 20. p : A U.S. citizen travels overseas.
 q : Chicago is in North America. q : The Phelps Library is in Dallas. q : A U.S. citizen needs a valid passport.
Then we have $p \wedge q$. Then we have $p \vee q$. Then we have $p \rightarrow q$.
21. p : The Smithsonian Museum of Natural History has displays of rocks. 22. p : The number 7 is odd.
 q : The Smithsonian Museum of Natural History has displays of bugs. q : The number 14 is even.
Then we have $p \wedge q$. Then we have $p \wedge q$.
23. p : Amtrak trains go to Chicago. 24. (a) Paris is not called the City of Lights. (b) Paris is not called the City of Lights
 q : Amtrak trains go to Cincinnati. or the Eiffel Tower is located in Paris. (c) If Paris is called the City of Lights, then the
Then we have $\sim p \wedge \sim q$ or $\sim(p \vee q)$. Eiffel Tower is located in Paris. (d) Paris is called the City of Lights or the Eiffel Tower
is located in Paris. (e) Paris is not called the City of Lights and the Eiffel Tower is not
located in Paris. (f) It is not the case that Paris is called the City of Lights or the Eiffel
Tower is located in Paris.
25. (a) Ozone is opaque to ultraviolet light, and life on Earth requires ozone. (b) Ozone is not opaque to ultraviolet light, or life on Earth requires ozone. (c) Ozone is not opaque to ultraviolet light, or life on Earth does not require ozone. (d) If life on Earth does not require ozone, then ozone is opaque to ultraviolet light. 26. (a) $\sim p$ (b) $\sim q \vee p$ (c) $p \rightarrow \sim q$ 27. (a) $a \vee f$ (b) $a \wedge \sim f$
(c) $f \wedge \sim a$ (d) $\sim a \wedge \sim f$

Exercises 11.2, page 502

1. Since r is a statement form, so is $\sim r$. Then $p \wedge \sim r$ is a statement form, and so is $\sim(p \wedge \sim r)$. Since q is a statement form, $\sim(p \wedge \sim r) \vee q$ is a statement form. 2. Since q is a statement form, so is $\sim q$. Since p is a statement form, so is $p \rightarrow \sim q$. Since r is a statement form, so is $(p \rightarrow \sim q) \wedge r$. 3. Since p is a statement form, so is $\sim p$. Since q and r are statement forms, so are $\sim p \vee r$ and $q \wedge r$, and hence also $(\sim p \vee r) \rightarrow (q \wedge r)$. 4. Since p and r are statement forms so is $p \wedge r$. Since q is a statement form so is $(p \wedge r) \rightarrow q$ and hence also $\sim[(p \wedge r) \rightarrow q]$.

5.	p	q	$p \wedge \sim q$	6.	p	q	$\sim(p \vee \sim q) \wedge (p \wedge \sim q)$	7.	p	q	$(p \vee \sim q) \wedge q$
	T	T	T F F		T	T	F T F F F F		T	T	T F T T
	T	F	T T T		T	F	F T T F T T		T	F	T T F F
	F	T	F F F		F	T	T F F F F F		F	T	F F F T
	F	F	F F T		F	F	F T T F F T		F	F	T T F F
(1)	(2)		(4) (3)	(1)	(2)	(7)	(5) (3) (8)	(6) (4)	(1)	(2)	(5) (3) (6) (4)

8.	p	q	$(p \wedge q) \vee (p \wedge \sim q)$	9.	p	q	$\sim[(p \vee q) \wedge (p \wedge q)]$	10.	p	q	$(p \vee \sim q) \oplus \sim p$
	T	T	T T F F		T	T	F T T T		T	T	T F T F
	T	F	F T T T		T	F	T T F F		T	F	T T T F
	F	T	F F F F		F	T	T T F F		F	T	F F T T
	F	F	F F F T		F	F	T F F F		F	F	T T F T
(1)	(2)		(4) (6) (5) (3)	(1)	(2)	(6)	(3) (5) (4)	(1)	(2)		(5) (3) (6) (4)

11.	p	q	$p \oplus (\sim p \vee q)$	12.	p	q	r	$(p \wedge q) \wedge r$	13.	p	q	r	$(p \wedge \sim r) \oplus q$
	T	T	T F F T T		T	T	T	T T T T T		T	T	T	T F F T T
	T	F	T T F F F		T	T	F	T T T F F		T	T	F	T T T F T
	F	T	F T T T T		T	F	T	T F F F T		T	F	T	T F F F F
	F	F	F T T T F		T	F	F	T F F F F		T	F	F	T T T T F
(1)	(2)		(5) (3) (4)		F	T	T	F F T F T		F	T	T	F F F T T
					F	T	F	F F T F F		F	T	F	F F T T T
					F	F	T	F F F F T		F	F	T	F F F F F
					F	F	F	F F F F F		F	F	F	F F T F F
				(1)	(2)	(3)		(4) (5)	(1)	(2)	(3)		(5) (4) (6)

14.

p	q	r	$\sim (p \wedge r) \vee q$
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

(1) (2) (3) (5) (4) (6)

15.

p	q	r	$\sim [(p \wedge r) \vee q]$
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	T

(1) (2) (3) (6) (4) (5)

16.

p	$p \wedge \sim p$
T	F
F	F

(1) (3) (2)

Always false

17.

p	$p \vee \sim p$
T	T
F	T

(1) (3) (2)

Always true

18.

p	q	r	$(p \vee q) \oplus \sim r$
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	T

(1) (2) (3) (4) (6) (5)

19.

p	q	r	$p \oplus (q \oplus r)$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	F

(1) (2) (3) (5) (4)

20.

p	q	r	$p \vee (q \wedge r)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	F

(1) (2) (3) (5) (4)

21.

p	q	r	$(p \vee q) \wedge (p \vee r)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

(1) (2) (3) (4) (6) (5)

22.

p	q	r	$(p \vee q) \wedge (p \vee \sim r)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

(1) (2) (3) (5) (7) (6) (4)

23.

p	q	$(p \vee q) \wedge \sim (p \vee q)$
T	T	F
T	F	F
F	T	F
F	F	F

(1) (2) (3) (6) (5) (4)

24.

p	q	r	$(\sim p \vee q) \wedge r$
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	F

(1) (2) (3) (4) (5) (6)

25.

p	q	r	$\sim (p \vee q) \wedge r$
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

(1) (2) (3) (5) (4) (6)

26.

p	q	r	$\sim$	$[(p \vee q) \wedge r]$
T	T	T	F	T
T	T	F	T	F
T	F	T	F	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	F
F	F	T	T	F
F	F	F	T	F

(1) (2) (3) (6) (4) (5)

27. Not logically equivalent 28. Logically equivalent
 29. Not logically equivalent 30. Not logically equivalent
 31. 16 32. 256

33. (a)

p	p	p
T	T	F
F	F	T

(1) (2)

(b)

p	q	$(p \mid p)$	$(q \mid q)$
T	T	T	T
T	F	T	F
F	T	F	T
F	F	F	F

(1) (2) (3) (5) (4)

(c)

p	q	$(p \mid q)$	$(p \mid q)$
T	T	T	T
T	F	T	F
F	T	F	T
F	F	F	F

(1) (2) (3) (5) (4)

(d)

p	q	p	$[(p \mid q) \mid q]$
T	T	T	T
T	F	T	F
F	T	F	T
F	F	F	F

(1) (2) (5) (3) (4)

34. (a)

p	q	$p \ominus \sim q$
T	T	F
T	F	T
F	T	F
F	F	T

(1) (2) (4) (3)

(b)

p	q	r	$(p \ominus q) \ominus r$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	F

(1) (2) (3) (4) (5)

(c)

p	q	r	$p \ominus (q \ominus r)$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	F

(1) (2) (3) (5) (4)

(d)

p	q	$\sim$	$(p \ominus q) \wedge (p \oplus q)$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	F	F

(1) (2) (5) (3) (6) (4)

35. (a) T (b) F (c) T (d) F (e) F (f) F 36. (a) F (b) T (c) T (d) F 37. T 38. F 39. F
 40. T 41. 1 42. 0 43. (a) F (b) F 44. (a) T (b) T 45. (a) F (b) F 46. (a) F (b) F

47.

p	q	$p \wedge \sim q$
T	T	F
T	F	T
F	T	F
F	F	F

48.

p	q	$\sim(p \vee q)$
T	T	F
T	F	F
F	T	F
F	F	T

49.

p	q	r	$p \wedge (\sim q \vee r)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

50.

p	q	r	$(p \wedge \sim r) \vee (\sim q \wedge r)$
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

51. (Not p) And q, or $\sim p \wedge q$

52. p Or (Not q), or $p \vee \sim q$

Exercises 11.3, page 508

1.

p	q	$\sim p \rightarrow \sim q$
T	T	F
T	F	T
F	T	F
F	F	T

(1) (2) (3) (5) (4)

2.

p	q	r	$p \vee (q \rightarrow \sim r)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

(1) (2) (3) (6) (5) (4)

3.

p	q	$(p \oplus q) \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	F

(1) (2) (3) (4)

4.

p	q	$\sim q$	$\rightarrow$	$\sim p$
T	T	F	T	F
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

(1) (2) (3) (5) (4)

5.

p	q	r	$(\sim p \wedge q) \rightarrow r$
T	T	T	F T F T T T
T	T	F	F T F T T F
T	F	T	F T F F T T
T	F	F	F T F F T F
F	T	T	T F T T T T
F	T	F	T F T T F F
F	F	T	T F F F T T
F	F	F	T F F F T F

(1) (2) (3) (4) (5) (6)

6.

p	q	$\sim (p \rightarrow q)$
T	T	F T T T
T	F	T T F F
F	T	F F T T
F	F	F F T T

(1) (2) (4) (3)

7.

p	q	$(p \rightarrow q) \leftrightarrow (\sim p \vee q)$
T	T	T T T F T T T
T	F	T F F T F T F
F	T	F T T T T F T
F	F	F T F T T F F

(1) (2) (4) (6) (3) (5)

8.

p	q	r	$p \oplus (q \rightarrow r)$
T	T	T	T F T T T
T	T	F	T T T F F
T	F	T	T F F T T
T	F	F	T F F T F
F	T	T	F T T T T
F	T	F	F F T F F
F	F	T	F T F T T
F	F	F	F T F T F

(1) (2) (3) (5) (4)

9.

p	q	r	$(p \rightarrow q) \rightarrow r$
T	T	T	T T T T T
T	T	F	T T T F F
T	F	T	T F F T T
T	F	F	T F F T F
F	T	T	F T T T T
F	T	F	F T T F F
F	F	T	F T F T T
F	F	F	F T F F F

(1) (2) (3) (4) (5)

10.

p	q	r	$p \rightarrow (q \rightarrow r)$
T	T	T	T T T T T
T	T	F	T F T F F
T	F	T	T T F T T
T	F	F	T T F T F
F	T	T	F T T T T
F	T	F	F T T F F
F	F	T	F T F T T
F	F	F	F T F T F

(1) (2) (3) (5) (4)

11.

p	q	$(p \vee q) \leftrightarrow (p \wedge q)$
T	T	T T T T T T T
T	F	T T F F T F F
F	T	F T T F F F T
F	F	F F F T F F F

(1) (2) (3) (5) (4)

12.

p	q	$[(\sim p \wedge q) \oplus q] \rightarrow p$
T	T	F T F T T T T
T	F	F T F F F F T
F	T	T F T T F T F
F	F	T F F F F F F

(1) (2) (3) (4) (5) (6)

13. $[(\sim p) \wedge (\sim q)] \rightarrow [(\sim p) \wedge q]$ 14. $(p \wedge q) \rightarrow [p \vee (\sim q)]$ 15. $[(\sim p) \wedge (\sim q)] \vee r \rightarrow [(\sim q) \wedge r]$ 16. $(p \vee [(\sim q) \wedge r]) \rightarrow (p \vee r)$
 17. T 18. F 19. T 20. T 21. T 22. F 23. F 24. F 25. T 26. F 27. $p \leftrightarrow q$ 28. $p \rightarrow q$; hypothesis: p ; conclusion: q
 29. $q \rightarrow p$; hypothesis: q ; conclusion: p 30. $q \rightarrow p$; hypothesis: q ; conclusion: p 31. $q \rightarrow p$; hypothesis: q ; conclusion: p
 32. $p \rightarrow q$; hypothesis: p ; conclusion: q 33. $\sim p \rightarrow \sim q$; hypothesis: $\sim p$; conclusion: $\sim q$ 34. $q \rightarrow p$; hypothesis: q ; conclusion: p
 35. $\sim p \rightarrow \sim q$; hypothesis: $\sim p$; conclusion: $\sim q$; TRUE 36. $p \rightarrow q$; hypothesis: p ; conclusion: q ; TRUE 37. $\sim q \rightarrow \sim p$; hypothesis: $\sim q$; conclusion: $\sim p$; TRUE 38. $q \rightarrow p$; hypothesis: q ; conclusion: p ; TRUE 39. (a) hyp: I will run a marathon.; con: Amy watches my dogs.
 (b) hyp: A student earns a B.; con: A student passes the course. (c) hyp: The football team is my favorite.; con: The football team wears green uniforms. (d) hyp: Isaac has wrinkled fingers.; con: Isaac was in the bathtub. 40. (a) hyp: I drive too fast.; con: I receive a traffic ticket. (b) hyp: I drive too fast.; con: I receive a traffic ticket. (c) hyp: A person is a delivery driver.; con: A person has a valid license. (d) hyp: A cow is healthy.; con: A cow has clear eyes and a wet nose. 41. (a) If City Sanitation collects the garbage, then the mayor calls. (b) The price of beans goes down if there is no drought. (c) Goldfish swim in Lake Erie if Lake Erie is fresh water. (d) Tap water is not salted if it boils slowly. 42. (a) If Jane is tired, then Jane runs 20 miles. (b) Cindy loves Fred if Fred

loves Cindy. (c) If Jon cashes a check, then the bank is open. (d) Errors are clear if the documentation is complete. (e) Sally's eating the vegetables is sufficient for Sally's getting dessert. (f) Sally's eating the vegetables is a necessary condition for Sally's getting dessert. 43. (a) 4 (b) 4 (c) 4 (d) 4 (e) 6 (f) 6 44. (a) 100 (b) -100 (c) -100 (d) -100 (e) -100 (f) 100 45. (a) 7 (b) 7 (c) 0 (d) 7 (e) 0 (f) 0 46. (a) 3 (b) 2 (c) $\frac{1}{2}$ (d) 0 (e) -1,000,000 (f) -1,000,000 47. (a) 6 (b) -12 (c) 0 (d) 0 (e) 28 (f) 0 48. (a) 0 (b) 4 (c) -30 (d) -3 (e) -30 (f) -30

Exercises 11.4, page 516

1. $[(p \rightarrow q) \wedge q] \rightarrow p$
 F T T T T F F

When p is false and q is true, the statement is FALSE.

2. Show that the corresponding biconditional is a tautology.

(a)

p	q	r	$[p \vee (q \wedge r)]$			$\leftrightarrow$	$[(p \vee q) \wedge (p \vee r)]$		
T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	T	T	F
T	F	T	T	F	F	T	T	F	T
T	F	F	T	F	F	T	T	F	F
F	T	T	F	T	T	T	F	T	T
F	T	F	F	F	F	T	F	T	F
F	F	T	F	F	F	T	F	F	T
F	F	F	F	F	F	T	F	F	F

(1) (2) (3) (7) (4) (9) (5) (8) (6)

(b)

p	q	r	$[p \wedge (q \vee r)]$			$\leftrightarrow$	$[(p \wedge q) \vee (p \wedge r)]$		
T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T	T	F
T	F	T	T	F	T	T	F	F	T
T	F	F	T	F	F	T	F	F	F
F	T	T	F	F	T	T	F	F	T
F	T	F	F	F	T	T	F	F	F
F	F	T	F	F	F	T	F	F	T
F	F	F	F	F	F	T	F	F	F

(1) (2) (3) (7) (4) (9) (5) (8) (6)

3. Show that the corresponding biconditional is a tautology.

p	q	$(p \rightarrow q) \leftrightarrow [\sim(p \wedge \sim q)]$		
T	T	T	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

(1) (2) (3) (7) (6) (5) (4)

4. $[p \vee \sim q] \wedge r \rightarrow p$

$$\begin{aligned}
 &\Leftrightarrow \sim[(p \vee \sim q) \wedge r] \vee p \\
 &\Leftrightarrow \sim(p \vee \sim q) \vee \sim r \vee p \\
 &\Leftrightarrow [\sim(p \wedge \sim q) \vee \sim r] \vee p \\
 &\Leftrightarrow [\sim p \wedge q] \vee \sim r \vee p \\
 &\Leftrightarrow p \vee [\sim p \wedge q] \vee \sim r \\
 &\Leftrightarrow [p \vee (\sim p \wedge q)] \vee \sim r \\
 &\Leftrightarrow [(p \vee \sim p) \wedge (p \vee q)] \vee \sim r \\
 &\Leftrightarrow [t \wedge (p \vee q)] \vee \sim r \\
 &\Leftrightarrow [(p \vee q) \wedge t] \vee \sim r \\
 &\Leftrightarrow (p \vee q) \vee \sim r
 \end{aligned}$$

Implication (10a)
 De Morgan's law (8b)
 De Morgan's law (8a)
 Double negation (1)
 Commutative law (2a)
 Associative law (3a)
 Distributive law (4a)
 Law 7a, Table 1
 Commutative law (2b)
 Identity law (6d)

5.

p	q	c	$(p \rightarrow q) \leftrightarrow [(p \wedge \sim q) \rightarrow c]$		
T	T	F	T	T	F
T	F	F	T	F	F
F	T	F	F	T	F
F	F	F	F	T	F

(1) (2) (3) (6) (8) (5) (4) (7)

6. (a) t Always true
 $\Leftrightarrow \sim(p \wedge q) \vee (p \wedge q)$ Law 7a, Table 1
 $\Leftrightarrow (\sim p \vee \sim q) \vee (p \wedge q)$ De Morgan's law (8b)
 $\Leftrightarrow \sim p \vee [\sim q \vee (p \wedge q)]$ Associative law (3a)
 $\Leftrightarrow \sim p \vee [q \rightarrow (p \wedge q)]$ Implication (10a)
 $\Leftrightarrow p \rightarrow [q \rightarrow (p \wedge q)]$ Implication (10a)
 $p \rightarrow [q \rightarrow (p \wedge q)]$ is a tautology.
- (b) False
7. False 8. True 9. $\sim(\sim p \rightarrow q)$
 10. $p \wedge \sim(q \wedge \sim r)$ 11. $\sim[\sim(p \vee q) \vee \sim(\sim p \vee \sim q)]$
 12. (a) $(\sim p \vee q) \vee p$ (b) $r \vee [\sim(p \vee \sim q)]$ (c) $\sim p \vee r$
 (d) $(\sim p \vee \sim q) \vee r$ 13. $\sim(p \vee q) \vee (q \wedge \sim r)$
 14. $q \wedge (p \vee r)$ 15. $(p \wedge \sim q) \vee (p \vee \sim r)$
 16. $(p \wedge \sim q) \wedge \sim r$ 17. (a) Arizona does not border California, or Arizona does not border Nevada. (b) There are no tickets available, and the agency cannot get tickets. (c) The killer's hat was neither white nor gray.
18. (a) Montreal is not a province in Canada, or Ottawa is not a province in Canada. (b) The salesman does not go to the customer, and the customer does not call the salesman. (c) The hospital admits psychiatric patients and orthopedic patients.
19. $\sim p \wedge q \wedge \sim r$ 20. $\sim p \wedge q \wedge r$ 21. (a) Jeremy does not take 12 credits and Jeremy does not take 15 credits.
 (b) Sandra did not receive a gift from Sally, or Sandra did not receive a gift from Sacha. 22. (a) The plane taken from California to Maine was not on time or not every seat was taken. (b) Kenneth was on time or he was dressed properly. 23. (a) I have a ticket to the theater, and I did not spend a lot of money. (b) Basketball is played on an indoor court, and the players do not wear sneakers.
 (c) The stock market is going up, and interest rates are not going down. (d) Humans have enough water, and humans are not staying healthy. 24. (a) Isaac Newton did not invent the calculus nor did Henry Ford invent the calculus. Or: Neither Isaac Newton nor Henry Ford invented the calculus. (b) Neither James Galway nor Paul Simon plays the piano. 25. (a) If x is an even number, then the sum $2 + x$ is even.; TRUE (b) If "S" is next to "K," then the computer keyboard does not have the QWERTY layout; TRUE
26. (a) If a book is a thesaurus, then it has a list of synonyms; TRUE (b) If the water is not salty, then it does not come from the Atlantic Ocean; TRUE 27. (a) Contrapositive: If a dog is not a Chihuahua, then it is not small; FALSE Inverse: If a dog is not small, then it is not a Chihuahua; TRUE Converse: If a dog is a Chihuahua, then it is small; TRUE (b) Contrapositive: If you stop, then the traffic light is not green; TRUE Inverse: If the traffic light is not green, then you stop; FALSE Converse: If you do not stop, then the traffic light is green; FALSE (c) Contrapositive: If we are not French citizens, then we do not live in France; FALSE Inverse: If we do not live in France, then we are not French citizens; FALSE Converse: If we are French citizens, then we live in France; FALSE
28. (a) Contrapositive: If a rectangle is a square, then it has four equal sides; TRUE Inverse: If a rectangle has four equal sides, then it is square; TRUE Converse: If a rectangle is not a square, then it does not have four equal sides; TRUE Negation: A rectangle does not have four equal sides and it is a square; FALSE (b) Contrapositive: If the probability of a head is not 1/2, then the coin is not fair; TRUE Inverse: If the coin is not fair, then the probability of a head is not 1/2; TRUE Converse: If the probability of a head is 1/2, then the coin is fair; TRUE Negation: A coin is fair and the probability of a head is not 1/2; FALSE (c) Contrapositive: If a nation is not Poland, then it does not have a red and white flag; FALSE Inverse: If a nation does not have a red and white flag, then it is not Poland; TRUE Converse: If a nation is Poland, then has a red and white flag; TRUE Negation: A nation has a red and white flag and it is not Poland; TRUE
29. (a) $(q \vee r) \rightarrow \sim p$ (b) $(q \vee r) \wedge p$; We wait on some other person or some other time, and change will come. 30. (a) $(p \wedge q) \rightarrow r$ (b) $(p \wedge q) \wedge \sim r$; We remove your content due to a copyright violation and you believe this removal was in error, and you do not have 30 days to file an appeal. 31. (a) Inverse: If you wish to claim the premium tax credit for 2015, then you need the information in Part II. Converse: If you do not need the information in Part II, then you do not wish to claim the premium tax credit for 2015. Contrapositive: If you need the information in Part II, then you wish to claim the premium tax credit for 2015. Negation: You do not wish to claim the premium tax credit for 2015 and you do not need the information in Part II.
 (b) Inverse: If your spouse does not itemize deductions, then you can take the standard deduction. Converse: If you can't take the standard deduction, then your spouse itemizes deductions. Contrapositive: If you can take the standard deduction, then your spouse does not itemize deductions. Negation: Your spouse itemizes deductions and you can take the standard deduction. (c) Inverse: If you do not check a box, your tax will change and your refund will change. Converse: If your tax or refund don't change, then you check a box. Contrapositive: If your tax or refund change, then you don't check a box. Negation: You check a box and your tax will change and your refund will change. 32. (a) Inverse: If the patient can assent, then physicians should not explain the plan of care or not tell her what to expect. Converse: If physicians should explain the plan of care and tell the patient what to expect, then she cannot assent. Contrapositive: If physicians should not explain the plan of care or not tell the patient what to expect, then she can assent. Negation: The patient cannot assent and physicians should not explain the plan of care or not tell her what to expect. (b) Inverse: If physicians agree with laws, they should not seek to change them. Converse: If physicians should seek to change laws, then they do not agree with them. Contrapositive: If physicians should not seek to change laws, then they agree with them. Negation: Physicians do not agree with laws and should not seek to change them. (c) Inverse: If a patient does not refuse care from a resident and does not refuse care from a fellow, the attending physician should not be notified. Converse: If the attending physician should be notified, then the patient refuses care from a resident or fellow. Contrapositive: If the attending physician should not be notified, then the patient does not refuse care from a resident and does not refuse care from a fellow. Negation: A patient refuses care from a resident or a fellow and the attending physician should not be notified.
33. We must show that $[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology.

p	q	$[p \wedge (p \rightarrow q)] \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T
(1)	(2)	(4) (3) (5)

34. We must show that $[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$ is a tautology.

p	q	$[(p \rightarrow q) \wedge \sim q]$	$\rightarrow$	$\sim p$
T	T	F	T	F
T	F	F	T	F
F	T	F	T	T
F	F	T	T	T
(1)	(2)	(3)	(4)	(6)

Exercises 11.5, page 523

1. e : "The auto manufacturer follows environmental regulations."
 l : "The auto manufacturer faces legal action."
 1. $e \vee l$ hyp.
 2. $\sim e$ hyp.
 3. l disj. syll. (1, 2)
2. v : "The class votes for an oral final."
 g : "The teacher is glad."
 m : "The exam is scheduled for a Monday."
 1. $v \rightarrow g$ hyp.
 2. $\sim m \rightarrow \sim g$ hyp.
 3. $\sim m$ hyp.
 4. $\sim g$ mod. ponens (2, 3)
 5. $\sim v$ mod. tollens (1, 4)
3. a : "My allowance comes this week."
 p : "I pay the rent."
 b : "My bank account will be in the black."
 e : "I will be evicted."
 1. $(a \wedge p) \rightarrow b$ hyp.
 2. $\sim p \rightarrow e$ hyp.
 3. $\sim e \wedge a$ hyp.
 4. $\sim e$ subtr. (3)
 5. p mod. tollens (2, 4)
 6. a subtr. (3)
 7. b mod. ponens (5, 6, 1)
4. s : "Jane is in the sixth grade."
 u : "Jane understands fractions."
 r : "Jane is in remedial math class."
 1. $(s \rightarrow u) \wedge (s \rightarrow r)$ hyp.
 2. s hyp.
 3. $s \rightarrow (u \vee r)$ constr. dilemma
 4. $u \vee r$ mod. ponens (2, 3)
5. p "The price of oil increases."
 a "The OPEC countries are in agreement."
 d "There is a U.N. debate."
 1. $p \rightarrow a$ hyp.
 2. $\sim d \rightarrow p$ hyp.
 3. $\sim a$ hyp.
 4. $\sim p$ mod. tollens (1, 3)
 5. d mod. tollens (2, 4)
6. j : "Jill wins."
 i : "Jack loses."
 p : "Peter wins."
 s : "Paul loses."
 1. $j \rightarrow i$ hyp.
 2. $p \rightarrow s$ hyp.
 3. $j \vee p$ hyp.
 4. $(j \vee p) \rightarrow (i \vee s)$ const. dilemma (1, 2)
 5. $i \vee s$ mod. ponens (3, 4)
7. g : "The germ is present."
 r : "The rash is present."
 f : "The fever is present."
 1. $g \rightarrow (r \wedge f)$ hyp.
 2. f hyp.
 3. $\sim r$ hyp.
 4. $\sim r \vee \sim f$ addition (3)
 5. $\sim(r \wedge f)$ De Morgan (4)
 6. $\sim g$ mod. tollens (1, 5)
8. i : "Sophia pays her car insurance."
 s : "Sophia stops driving her car."
 f : "Sophia pays a fine."
 1. $\sim i \rightarrow (s \vee f)$ hyp.
 2. $\sim s$ hyp.
 3. $\sim f$ hyp.
 4. $\sim(s \vee f)$ De Morgan (2, 3)
 5. i mod. tollens (1, 4)
9. c : "The material is cotton."
 r : "The material is rayon."
 d : "The material can be made into a dress."
 1. $(c \vee r) \rightarrow d$ hyp.
 2. $\sim d$ hyp.
 3. $\sim(c \vee r)$ mod. tollens (1, 2)
 4. $\sim c \wedge \sim r$ De Morgan (3)
 5. $\sim r$ subtraction (4)

11-8 Instructor Answers

10. m : "There is money in my account."
 c : "I have a check."
 p : "I will pay the rent."
 e : "I am evicted."
- | | |
|---|---------------------------------------|
| 1. $(m \wedge c) \rightarrow p$ | hyp. |
| 2. $\sim c \rightarrow e$ | hyp. |
| 3. $\sim p \rightarrow \sim(m \wedge c)$ | contrapositive (1) |
| 4. $\sim e \rightarrow c$ | contrapositive (2) |
| 5. $\sim p \rightarrow (\sim m \vee \sim c)$ | De Morgan (3) |
| 6. $(\sim e \wedge \sim p) \rightarrow [c \wedge (\sim m \vee \sim c)]$ | contr. dilemma (4, 5) |
| 7. $[c \wedge (\sim m \vee \sim c)] \rightarrow \sim m$ | disj. syllogism (logical implication) |
| 8. $(\sim e \wedge \sim p) \rightarrow \sim m$ | hypoth. syllogism (6, 7) |

11. Invalid 12. Valid 13. Valid 14. Valid 15. Valid 16. Invalid 17. Invalid 18. Valid 19. Invalid 20. Valid

21. s : "Sam goes to the store."
 m : "Sam needs milk."
 $H_1: s \rightarrow m$
 $H_2: \sim m$
 $C: \sim s$
- | | |
|----------------------|---------------------------------|
| 1. s | $\sim C$ |
| 2. $s \rightarrow m$ | H_1 |
| 3. m | $\sim H_2$; mod. ponens (1, 2) |

22. r : "It rains hard."
 p : "There is a picnic."
 f : "Dave brings the frisbee."
 h : "The kids are happy."
 $H_1: r \rightarrow \sim p$
 $H_2: f \rightarrow h$
 $H_3: \sim h \wedge p$
 $C: \sim r \wedge \sim f$
- | | |
|---|----------------------------|
| 1. $\sim(\sim r \wedge \sim f)$ | $\sim C$ |
| 2. $r \vee f$ | De Morgan (1) |
| 3. $r \rightarrow \sim p$ | H_1 |
| 4. $f \rightarrow h$ | H_2 |
| 5. $(r \vee f) \rightarrow (\sim p \vee h)$ | contr. dilemma (3, 4) |
| 6. $\sim p \vee h$ | mod. ponens (2, 5) |
| 7. $h \vee \sim p$ | commutative law (6) |
| 8. $\sim(\sim h \wedge p)$ | $\sim H_3$; De Morgan (7) |

23. n : "The newspaper reports the crime."
 t : "Television reports the crime."
 s : "The crime is serious."
 k : "A person is killed."
 $H_1: (n \wedge t) \rightarrow s$
 $H_2: k \rightarrow n$
 $H_3: k$
 $H_4: t$
 $C: s$
- | | |
|---------------------------------|----------------------------------|
| 1. $\sim s$ | $\sim C$ |
| 2. $(n \wedge t) \rightarrow s$ | H_1 |
| 3. $\sim(n \wedge t)$ | mod. tollens (1, 2) |
| 4. $\sim n \vee \sim t$ | De Morgan (3) |
| 5. t | H_4 |
| 6. $\sim n$ | disj. syllogism (4, 5) |
| 7. $k \rightarrow n$ | H_2 |
| 8. $\sim k$ | $\sim H_3$; mod. tollens (6, 7) |

24. l : "Linda feels ill."
 a : "Linda takes aspirin."
 f : "Linda runs a fever."
 b : "Linda takes a bath."
 $H_1: l \rightarrow a$
 $H_2: f \rightarrow \sim b$
 $H_3: \sim l \rightarrow b$
 $H_4: f$
 $C: a$
- | | |
|---------------------------|----------------------------------|
| 1. $\sim a$ | $\sim C$ |
| 2. $l \rightarrow a$ | H_1 |
| 3. $\sim l$ | mod. tollens (1, 2) |
| 4. $\sim l \rightarrow b$ | H_3 |
| 5. b | mod. ponens (3, 4) |
| 6. $f \rightarrow \sim b$ | H_2 |
| 7. $\sim f$ | $\sim H_4$; mod. tollens (5, 6) |

25. j : "Jimmy finds his keys."
 h : "He does his homework."

Direct proof	Indirect proof
1. $\sim j \rightarrow h$ H_1	1. $\sim j$ $\sim C$
2. $\sim h$ H_2	2. $\sim j \rightarrow h$ H_1
3. j mod. tollens (1, 2)	3. h mod. ponens (1, 2)
	4. $\sim h$ H_2
	} contradiction

26. p : "Spring vacation includes Easter."
 q : "Simone goes to France."
 r : "George goes to France."

Direct proof	Indirect proof
1. $p \rightarrow q$ hyp.	1. r $\sim C$
2. $r \rightarrow \sim q$ hyp.	2. $r \rightarrow \sim q$ hyp.
3. p hyp.	3. $\sim q$ mod. ponens (1, 2)
4. q mod. ponens (1, 3)	4. $p \rightarrow q$ hyp.
5. $\sim r$ mod. tollens (2, 4)	5. $\sim p$ $\sim$ hyp.; mod. tollens (3, 4)

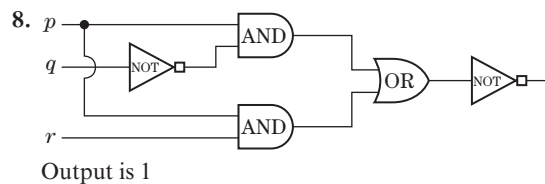
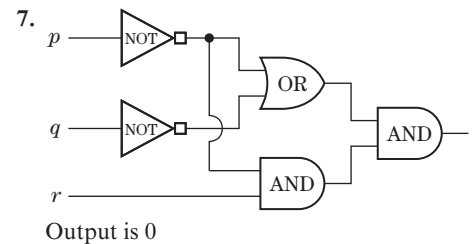
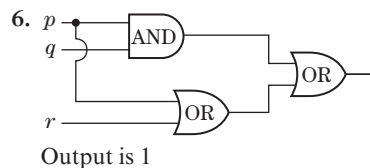
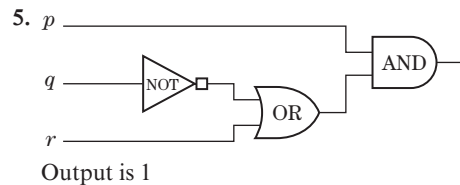
27. m : "Marissa goes to the movies." 1. $\sim m \rightarrow \sim i$ H_1
 k : "Marissa is in a knitting class." 2. $\sim k \rightarrow i$ H_2
 i : "Marissa is idle." 3. $\sim k$ H_3
 4. i mod. ponens (2, 3)
 5. m mod. tollens (1, 4)
28. S : "Sydney collects a penalty payment." 1. $N \rightarrow S$ hyp.
 N : "Norman leaves a mess." 2. $R \vee N$ hyp.
 R : "Rachel cleans the apartment." 3. $\sim R$ hyp.
 4. N disj. syll. (2, 3)
 5. S mod. ponens (1, 4)

Exercises 11.6, page 532

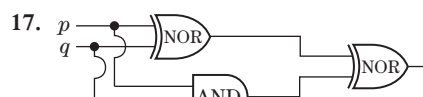
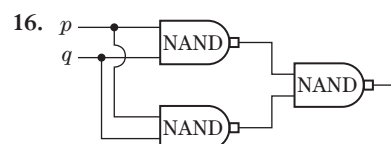
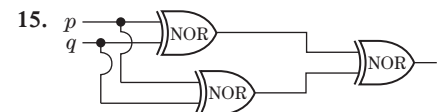
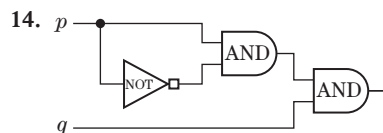
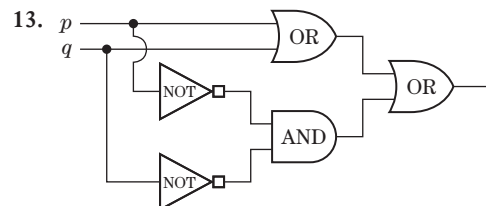
1. (a) F (b) T (c) T (d) T (e) F 2. (a) T (b) F (c) F (d) F 3. $\forall x \sim p(x)$, or $\sim[\exists x p(x)]$. This is FALSE. Abby meant to say, "Not all men cheat on their wives." 4. Not all doors open. 5. (a) $\forall x p(x)$ (b) $\exists x \sim p(x)$ (c) $\exists x p(x)$ (d) $\sim[\forall x p(x)]$ (e) $\forall x \sim p(x)$ (f) $\sim[\exists x p(x)]$ (g) (b) and (d); (e) and (f) 6. (a) $\forall x \sim p(x)$ (b) $\exists x p(x)$ (c) $\sim[\forall x p(x)]$ (d) $\exists x \sim p(x)$ (e) $\sim[\exists x p(x)]$ (f) (a) and (e); (c) and (d) 7. (a) T (b) F 8. (a) T (b) F 9. (a) T (b) F (c) T (d) F (e) F (f) T (g) F (h) T 10. (a) F (b) F (c) T (d) T (e) T (f) T (g) F (h) F (i) T (j) T
11. (a) Not every dog has his day. (b) No men fight wars. (c) Some women are unmarried. (d) There exists a pot without a cover. (e) All children have pets. (f) Every month has 30 days. 12. (a) Some stitch does not save time. (b) Some book has no hard cover. (c) All children are unafraid of snakes. (d) No computer is without a hard disk. (e) All chairs have arms.
13. (a) "The sum of any two nonnegative integers is greater than 12." FALSE: consider $x = 1, y = 2$. "There exist two nonnegative integers whose sum is not greater than 12." (b) "For any nonnegative integer, there is a nonnegative integer that, added to the first, makes a sum greater than 12." TRUE (c) "There is a nonnegative integer that, added to any other nonnegative integer, makes a sum greater than 12." TRUE (Try $x = 13$.) (d) "There are two nonnegative integers, the sum of which is greater than 12." TRUE (Try $x = 6, y = 7$.)
14. (a) FALSE; consider $x = \{a, b\}$ and $y = \{c\}$. (b) FALSE; consider $x = \{a\}$ and $y = \{b\}$ (c) TRUE; consider $x = \emptyset$. (d) TRUE; consider $y = A$. 15. (a) FALSE; let $x = 2, y = 3$. (b) TRUE: For any x , let $y = x$. (c) TRUE: Let $x = 1$. (d) FALSE: No y is divisible by every x . (e) TRUE: for any y , let $x = y$. (f) TRUE: any x divides itself. 16. (a) $\sim p(x) \vee q(x)$ (b) $p(x) \oplus \sim q(x)$ (c) $\sim p(x) \wedge \sim q(x)$ (d) $\sim[p(x) \vee q(x)]$ 17. (a) $S \subseteq T$ translates as $\forall x [x \geq 8 \rightarrow x \leq 10]$. (b) No; consider $x = 11$. 18. No; $3 \in S$ but $3 \notin T$. 19. $S = \{2, 4, 6, 8\}, T = \{1, 2, 3, 4, 6, 8\}$. So $\forall x [x \in S \rightarrow x \in T]$. 20. $S = \{a, b, d\}, T = \{a, b, d, e, g\}$. So $\forall x [x \in S \rightarrow x \in T]$.

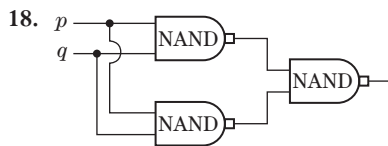
Exercises 11.7, page 536

1. $(p \vee q) \wedge (p \vee \sim q)$ 2. $(\sim p \wedge \sim q) \vee q$ 3. $[(p \wedge q) \wedge \sim r] \wedge (\sim q \vee r)$ 4. $[(p \wedge \sim q) \vee (p \wedge r)] \vee (\sim r \wedge q)$



9. p 10. $(p \vee r) \vee \sim q$ 11. $r \wedge [(p \wedge q) \vee \sim(p \vee q)]$
 12. $\sim(p \vee q) \wedge r$





p	q	$(p \text{ NAND } q)$	$(p \text{ NAND } q)$	$(p \text{ NAND } q)$
0	0	0	1	0
0	1	0	1	1
1	0	1	1	0
1	1	1	0	1
(1)	(2)	(3)	(5)	(4)



20. Answers will vary 21. $(p \text{ And (Not } q)) \text{ Or } r$ 22. $p \text{ Or (Not } q) \text{ Or } r$

Chapter 11: Answers to Fundamental Concept Check Exercises, page 540

1. A declarative statement that is either true or false

2.

p	q	$p \wedge q$	$p \vee q$	$\sim p$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	T	T	F	T	T
T	F	F	T	F	F	F
F	T	F	T	T	T	F
F	F	F	F	T	T	T

3. when p is TRUE and q is FALSE

4. have the same truth tables; compare their truth values

5. $\sim(p \vee q) \Leftrightarrow (\sim p \wedge \sim q)$, $\sim(p \wedge q) \Leftrightarrow (\sim p \vee \sim q)$; to negate statements having the disjunctive or conjunctive connectives.

6. $p; q$ 7. $\sim q \rightarrow \sim p; q \rightarrow p$, TRUE, either TRUE or FALSE 8. Statement: If a person is a resident of Maryland, then they are a resident of the United States. Contrapositive: If a person is not a resident of the United States, then they are not a resident of Maryland. Converse: If a person is a resident of the United States, then they are a resident of Maryland. 9. $p \wedge \sim q$ 10. AND, $\wedge$; OR, $\vee$; NOT, $\sim$; IF... THEN, $\rightarrow$; XOR, $\oplus$ 11. a statement form that has truth value TRUE regardless of the truth values of its component statements; see whether the final column of its truth table has only Ts 12. $\sim[\exists x p(x)] \Leftrightarrow [\forall x \sim p(x)]$; $\sim[\forall x p(x)] \Leftrightarrow [\exists x \sim p(x)]$

Chapter 11: Review Exercises, page 540

1. (a) Statement (b) Not a statement—not a declarative sentence (c) Statement (d) Not a statement—"he" is not specified (e) Statement 2. (a) If two lines are perpendicular, then their slopes are negative reciprocals of each other. (b) If goldfish can live in a fishbowl, then the water is aerated. (c) If it rains, then Jane uses her umbrella. (d) If Sally gives Morris a treat, then he ate all of his food. 3. (a) Contrapositive: If the Yankees are not playing in Yankee Stadium, then they are not in New York City.; Converse: If the Yankees are playing in Yankee Stadium, then they are in New York City. (b) Contrapositive: If the quake is not considered major, then the Richter scale does not indicate the earthquake is a 7.; Converse: If the quake is considered major, then the Richter scale indicates the earthquake is a 7. (c) Contrapositive: If a coat is not warm, then it is not made of fur.; Converse: If a coat is warm, then it is made of fur. (d) Contrapositive: If Jane is not in Moscow, then she is not in Russia.; Converse: If Jane is in Moscow, then she is in Russia.

4. (a) p : "Two triangles are similar."

q : "Their sides are equal."

$p \rightarrow q$ negated becomes $\sim(p \rightarrow q)$ or $p \wedge \sim q$, or "Two triangles are similar but their sides are unequal."

(b) $U = \{\text{real numbers}\}$ and $p(x) = (x^2 = 5)$.

$\exists x p(x)$ negated becomes $\forall x \sim p(x)$ or "For every real number x , $x^2 \neq 5$."

(c) $U = \{\text{positive integers}\}$,

$p(n) = (n \text{ is even})$, and $q(n) = (n^2 \text{ is even})$.

$\forall n [p(n) \rightarrow q(n)]$ negated becomes $\exists n \sim [p(n) \rightarrow q(n)]$ or $\exists n [p(n) \wedge \sim q(n)]$, or "There exists a positive integer n such that n is even and n^2 is not even."

(d) $U = \{\text{real numbers}\}$ and $p(x) = (x^2 + 4 = 0)$.

$\exists x p(x)$ negated becomes $\forall x \sim p(x)$ or "For every real number x , $x^2 + 4 \neq 0$."

5. (a) Tautology (b) Tautology (c) Not a tautology (d) Not a tautology

6. (a)

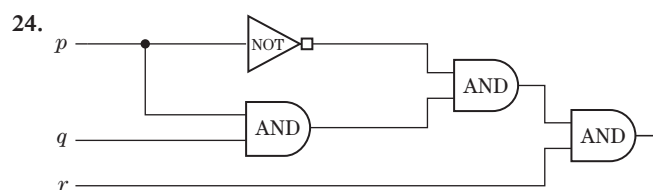
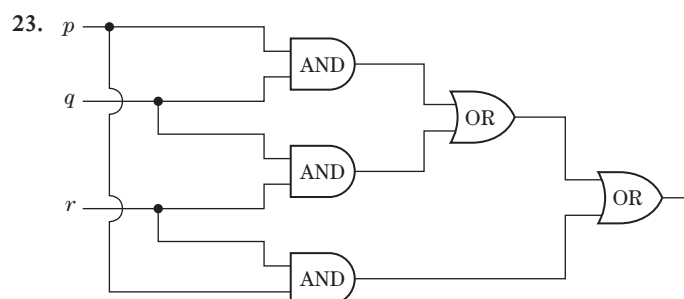
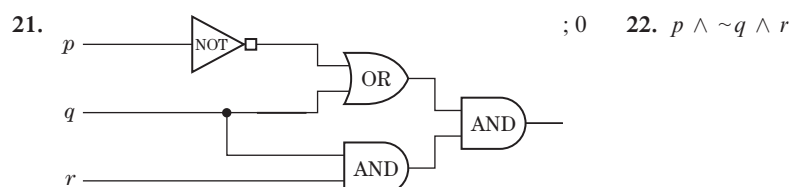
p	q	r	$p \rightarrow (\sim q \vee r)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T
(1)	(2)	(3)	(6)

(b)

p	q	r	$p \wedge [q \leftrightarrow (r \wedge p)]$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F
(1)	(2)	(3)	(6)

7. (a) TRUE (b) FALSE 8. (a) TRUE (b) FALSE 9. (a) FALSE (b) TRUE 10. (a) 17 (b) 100 (c) 100 (d) 100
 11. (a) 50 (b) -25 (c) -15 (d) 10 12. (a) Cannot be determined (b) TRUE (c) TRUE (d) Cannot be determined
 (e) Cannot be determined 13. (a) Cannot be determined (b) Cannot be determined (c) TRUE 14. (a) TRUE (b) FALSE
 (c) TRUE 15. (a) Cannot be determined (b) Cannot be determined (c) TRUE

16. t : "Taxes go up."
 s : "I sell the house."
 m : "I move to India."
 1. $t \rightarrow (s \wedge m)$ hyp.
 2. $\sim m$ hyp.
 3. $\sim s \vee \sim m$ addition (2)
 4. $\sim (s \wedge m)$ De Morgan (3)
 5. $\sim t$ mod. tollens (1, 4)
17. m : "I study mathematics."
 b : "I study business."
 p : "I can write poetry."
 1. $m \wedge b$ hyp.
 2. $b \rightarrow (\sim p \vee \sim m)$ hyp.
 3. b subtraction (1)
 4. $\sim p \vee \sim m$ mod. ponens (2, 3)
 5. m subtraction (1)
 6. $\sim p$ disj. syllogism (4, 5)
18. d : "I shop for a dress."
 h : "I wear high heels."
 s : "I have a sore foot."
 1. $d \rightarrow h$ hyp.
 2. $s \rightarrow \sim h$ hyp.
 3. d hyp.
 4. h mod. ponens (1, 3)
 5. $\sim s$ mod. tollens (2, 4)
19. a : "Asters grow in the garden."
 d : "Dahlias grow in the garden."
 s : "It is spring."
 1. $a \vee d$ hyp.
 2. $s \rightarrow \sim a$ hyp.
 3. s hyp.
 4. $\sim a$ mod. ponens (2, 3)
 5. d disj. syllogism (1, 4)
20. t : "The professor gives a test."
 h : "Nancy studies hard."
 d : "Nancy has a date."
 s : "Nancy takes a shower."
 H_1 : $t \rightarrow h$
 H_2 : $d \rightarrow s$
 H_3 : $\sim t \rightarrow \sim s$
 H_4 : d
 C : h
 1. $\sim h$ $\sim C$
 2. $t \rightarrow h$ H_1
 3. $\sim t$ mod. tollens (1, 2)
 4. $\sim t \rightarrow \sim s$ H_3
 5. $\sim s$ mod. ponens (3, 4)
 6. $d \rightarrow s$ H_2
 7. $\sim d$ $\sim H_4$; mod. tollens (5, 6)



25. $(p \text{ NOR } q) \text{ NOR } (p \text{ AND } q)$