2-1

a. $\begin{bmatrix} 3 & 2 & 0 \\ 2 & 4 & 5 \\ 0 & 5 & 6 \end{bmatrix}$ 3 x 3, square, symmetric

b. $\begin{Bmatrix} x \\ x^2 \\ x^3 \\ x^4 \end{Bmatrix}$ 4 x 1 column

c. $\begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}$ 2 x 2, square, diagonal

d. $\begin{bmatrix} 1 & y & y^2 & y^3 \end{bmatrix}$ 1 x 4, row

e. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 3 x 3, square, diagonal, identity

f. $\begin{bmatrix} 3 & -1 & 0 & 0 & 0 \\ 2 & 0 & 6 & 0 & 0 \\ 0 & 4 & 1 & 4 & 0 \\ 0 & 0 & 5 & 4 & 2 \\ 0 & 0 & 0 & 7 & 8 \end{bmatrix}$ 5 x 5, square, banded

g. $\begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ 4 x 4, square, upper triangular

h. $\begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & c_3 & 0 \\ 0 & 0 & 0 & c_4 \end{bmatrix}$ 4 x 4, square, diagonal

2.2

a. $$[A]+[B]=\begin{bmatrix}4 & 2 & 1\\ 7 & 0 & -7\\ 1 & -5 & 3\end{bmatrix}+\begin{bmatrix}1 & 2 & -1\\ 5 & 3 & 3\\ 4 & 5 & -7\end{bmatrix}=\begin{bmatrix}5 & 4 & 0\\ 12 & 3 & -4\\ 5 & 0 & -4\end{bmatrix}$$

b. $$[A]-[B]=\begin{bmatrix}4 & 2 & 1\\ 7 & 0 & -7\\ 1 & -5 & 3\end{bmatrix}-\begin{bmatrix}1 & 2 & -1\\ 5 & 3 & 3\\ 4 & 5 & -7\end{bmatrix}=\begin{bmatrix}3 & 0 & 2\\ 2 & -3 & -10\\ -3 & -10 & 10\end{bmatrix}$$

c. $$3[A]=3\begin{bmatrix}4 & 2 & 1\\ 7 & 0 & -7\\ 1 & -5 & 3\end{bmatrix}=\begin{bmatrix}12 & 6 & 3\\ 21 & 0 & -21\\ 3 & -15 & 9\end{bmatrix}$$

d. $$[A][B]=\begin{bmatrix}4 & 2 & 1\\ 7 & 0 & -7\\ 1 & -5 & 3\end{bmatrix}\begin{bmatrix}1 & 2 & -1\\ 5 & 3 & 3\\ 4 & 5 & -7\end{bmatrix}=\begin{bmatrix}18 & 19 & -5\\ -21 & -21 & 42\\ -12 & 2 & -37\end{bmatrix}$$

e. $$[A]\{C\}=\begin{bmatrix}4 & 2 & 1\\ 7 & 0 & -7\\ 1 & -5 & 3\end{bmatrix}\begin{Bmatrix}1\\ -2\\ 4\end{Bmatrix}=\begin{Bmatrix}4\\ -21\\ 23\end{Bmatrix}$$

f. $$[A]^2=\begin{bmatrix}4 & 2 & 1\\ 7 & 0 & -7\\ 1 & -5 & 3\end{bmatrix}\begin{bmatrix}4 & 2 & 1\\ 7 & 0 & -7\\ 1 & -5 & 3\end{bmatrix}=\begin{bmatrix}31 & 3 & -7\\ 21 & 49 & -14\\ -28 & -13 & 45\end{bmatrix}$$

g. $$[I][A]=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}4 & 2 & 1\\ 7 & 0 & -7\\ 1 & -5 & 3\end{bmatrix}=\begin{bmatrix}4 & 2 & 1\\ 7 & 0 & -7\\ 1 & -5 & 3\end{bmatrix}$$

$$[A][I]=\begin{bmatrix}4 & 2 & 1\\ 7 & 0 & -7\\ 1 & -5 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}4 & 2 & 1\\ 7 & 0 & -7\\ 1 & -5 & 3\end{bmatrix}$$

2.3

$$[A_{11}]=\begin{bmatrix}5 & 7 & 2\\ 3 & 8 & -3\end{bmatrix} \qquad [A_{12}]=\begin{bmatrix}0 & 3 & 5\\ -5 & 0 & 8\end{bmatrix}$$

$$[A_{21}]=\begin{bmatrix}1 & 4 & 0\\ 0 & 10 & 5\\ 2 & -5 & 9\end{bmatrix} \qquad [A_{22}]=\begin{bmatrix}7 & 15 & 9\\ 12 & 3 & -1\\ 2 & 18 & -10\end{bmatrix}$$

$$\{B_{11}\}=\begin{Bmatrix}2\\ 8\\ -5\end{Bmatrix} \qquad [B_{12}]=\begin{bmatrix}10 & 0\\ 7 & 5\\ 2 & -4\end{bmatrix}$$

$$\{B_{21}\}=\begin{Bmatrix}4\\ 3\\ 1\end{Bmatrix} \qquad [B_{22}]=\begin{bmatrix}8 & 13\\ 12 & 0\\ 5 & 7\end{bmatrix}$$

$$[A_{11}]\{B_{11}\}+[A_{12}]\{B_{21}\}=\begin{Bmatrix}70\\ 73\end{Bmatrix}$$

$$[A_{21}]\{B_{11}\}+[A_{22}]\{B_{21}\}=\begin{Bmatrix}116\\ 111\\ -29\end{Bmatrix}$$

$$[A_{11}][B_{12}]+[A_{12}][B_{22}]=\begin{bmatrix}164 & 62\\ 80 & 43\end{bmatrix}$$

$$[A_{21}][B_{12}]+[A_{22}][B_{22}]=\begin{bmatrix}319 & 174\\ 207 & 179\\ 185 & -105\end{bmatrix}$$

$$[A][B]=\left[\begin{array}{c|cc}70 & 164 & 62\\ 73 & 80 & 43\\ \hline 116 & 319 & 174\\ 111 & 207 & 179\\ -29 & 185 & -105\end{array}\right]$$

2.4

a.

$$[A]=\begin{bmatrix}1 & 4 & 2\\ 8 & 3 & 6\\ 7 & 1 & -2\end{bmatrix} \qquad [A]^T=\begin{bmatrix}1 & 8 & 7\\ 4 & 3 & 1\\ 2 & 6 & -2\end{bmatrix}$$

$$[B]=\begin{bmatrix}0 & 5 & -1\\ -3 & 1 & 7\\ 2 & 4 & -4\end{bmatrix} \qquad [B]^T=\begin{bmatrix}0 & -3 & 2\\ 5 & 1 & 4\\ -1 & 7 & -4\end{bmatrix}$$

b.

$$\left[\begin{bmatrix}1 & 4 & 2\\ 8 & 3 & 6\\ 7 & 1 & -2\end{bmatrix}+\begin{bmatrix}0 & 5 & -1\\ -3 & 1 & 7\\ 2 & 4 & -4\end{bmatrix}\right]^T=\begin{bmatrix}1 & 9 & 1\\ 5 & 4 & 13\\ 9 & 5 & -6\end{bmatrix}^T=\begin{bmatrix}1 & 5 & 9\\ 9 & 4 & 5\\ 1 & 13 & -6\end{bmatrix}$$

$$[A]^T+[B]^T=\begin{bmatrix}1 & 8 & 7\\ 4 & 3 & 1\\ 2 & 6 & -2\end{bmatrix}+\begin{bmatrix}0 & -3 & 2\\ 5 & 1 & 4\\ -1 & 7 & -4\end{bmatrix}=\begin{bmatrix}1 & 5 & 9\\ 9 & 4 & 5\\ 1 & 13 & -6\end{bmatrix}$$

c.

$$([A][B])^T=\left(\begin{bmatrix}1 & 4 & 2\\ 8 & 3 & 6\\ 7 & 1 & -2\end{bmatrix}\begin{bmatrix}0 & 5 & -1\\ -3 & 1 & 7\\ 2 & 4 & -4\end{bmatrix}\right)^T=\begin{bmatrix}-8 & 17 & 19\\ 3 & 67 & -11\\ -7 & 28 & 8\end{bmatrix}^T=\begin{bmatrix}-8 & 3 & -7\\ 17 & 67 & 28\\ 19 & -11 & 8\end{bmatrix}$$

$$[B]^T[A]^T=\begin{bmatrix}0 & -3 & 2\\ 5 & 1 & 4\\ -1 & 7 & -4\end{bmatrix}\begin{bmatrix}1 & 8 & 7\\ 4 & 3 & 1\\ 2 & 6 & -2\end{bmatrix}=\begin{bmatrix}-8 & 3 & -7\\ 17 & 67 & 28\\ 19 & -11 & 8\end{bmatrix}$$

2.5

a.

$$\begin{vmatrix} 2 & 10 & 0 \\ 16 & 6 & 14 \\ 12 & -4 & 18 \end{vmatrix} = (2)(6)(18)+(10)(14)(12)+(0)(16)(-4) - (10)(16)(18)-(2)(14)(-4)-(0)(6)(12)$$

$$\underline{\underline{\det(A) = -872}} \Leftarrow$$

$$\begin{vmatrix} 2 & 10 & 0 \\ 16 & 6 & 14 \\ 12 & -4 & 18 \end{vmatrix} = 2\begin{vmatrix} 6 & 14 \\ -4 & 18 \end{vmatrix} - 10\begin{vmatrix} 16 & 14 \\ 12 & 18 \end{vmatrix} + 0\begin{vmatrix} 16 & 6 \\ 12 & -4 \end{vmatrix}$$

$$= 2[(6)(18)-(14)(-4)] - 10[(16)(18)-(14)(12)] + 0$$

$$\underline{\underline{\det(A) = -872}} \Leftarrow$$

matrix [B] is singular because elements of second row and first row are linearly dependent.

$$\underline{\underline{\det(B) = 0}} \Leftarrow$$

This result can be shown by direct expansion as well.

b.

$$\underline{\underline{\det([A]^T) = \det(A) = -872}} \Leftarrow$$

2.5 Cont.

c.

$$\det(5[A]) = \begin{vmatrix} 10 & 50 & 0 \\ 80 & 30 & 70 \\ 60 & -20 & 90 \end{vmatrix} = (10)(30)(90) + (50)(70)(60) + 0 - (50)(80)(90) - (10)(70)(-20) - 0$$

$$\underline{\underline{\det(5[A]) = -109000}}$$

Since matrix [A] is 3×3, alternatively,

$$\det(5[A]) = 5^3 \det(A) = (125)(-872) \underline{\underline{= -109000}}$$

2.6

$$\det(A) = \begin{vmatrix} 0 & 5 & 0 \\ 8 & 3 & 7 \\ 9 & -2 & 9 \end{vmatrix} = (0)(3)(9) + (5)(7)(9) + (0)(8)(-2) - (5)(8)(9) - (0)(7)(-2) - (0)(3)(9)$$

$$\det(A) = -45 \Leftarrow$$

$$\det([A]^T) = \det(A) = -45 \Leftarrow$$

2.7

$$\begin{bmatrix} 10875000 & -1812500 & 0 \\ -1812500 & 6343750 & -4531250 \\ 0 & -4531250 & 4531250 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

Following the steps discussed in Section 2.7, we get

$$\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = 10^{-3} \begin{Bmatrix} 0.0883 \\ 0.5297 \\ 0.7062 \end{Bmatrix}$$

2.8

$$\begin{bmatrix} 0 & 5 & 0 \\ 8 & 3 & 7 \\ 9 & -2 & 9 \end{bmatrix}$$

Because of the zero elements in Row 1, the lower triangular matrix will not have a triangular form, instead it becomes

$$[L] = \begin{bmatrix} 0 & 1.0000 & 0 \\ 0.8889 & 0.9556 & 1.0000 \\ 1.0000 & 0 & 0 \end{bmatrix}$$

$$[U] = \begin{bmatrix} 9 & -2 & 9 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Check:

$$[L][U] = \begin{bmatrix} 0 & 1.0000 & 0 \\ 0.8889 & 0.9556 & 1.0000 \\ 1.0000 & 0 & 0 \end{bmatrix} \begin{bmatrix} 9 & -2 & 9 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 0 \\ 8 & 3 & 7 \\ 9 & -2 & 9 \end{bmatrix}$$

2.9

$$[A]=\begin{bmatrix} 10875000 & -1812500 & 0 \\ -1812500 & 6343750 & -4531250 \\ 0 & -4531250 & 4531250 \end{bmatrix}$$

$$\{b\}=\begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

$$[L]=\begin{bmatrix} 1.0000 & 0 & 0 \\ -0.1667 & 1.0000 & 0 \\ 0 & -0.7500 & 1.0000 \end{bmatrix}$$

$$[u]=10^7\begin{bmatrix} 1.0875 & -0.1812 & 0 \\ 0 & 0.6042 & -0.4531 \\ 0 & 0 & 0.1133 \end{bmatrix}$$

$$\{z\}=[L]^{-1}\{b\}=\begin{bmatrix} 1.0000 & 0 & 0 \\ -0.1667 & 1.0000 & 0 \\ 0 & -0.7500 & 1.0000 \end{bmatrix}^{-1}\begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

$$\{U\}=[u]^{-1}\{z\}=10^7\begin{bmatrix} 1.0875 & -0.1812 & 0 \\ 0 & 0.6042 & -0.4531 \\ 0 & 0 & 0.1133 \end{bmatrix}^{-1}\begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

$$\{U\}=10^{-3}\begin{Bmatrix} 0.0883 \\ 0.5297 \\ 0.7062 \end{Bmatrix}$$

Note the difference between **u** denoting upper triangular matrix and **U** denoting the displacement results

2.10

$$[A]=\begin{bmatrix} 10875000 & -1812500 & 0 \\ -1812500 & 6343750 & -4531250 \\ 0 & -4531250 & 4531250 \end{bmatrix}$$

$$\{b\}=\begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

$$[A]^{-1}=10^{-6}\begin{bmatrix} 0.1103 & 0.1103 & 0.1103 \\ 0.1103 & 0.6621 & 0.6621 \\ 0.1103 & 0.6621 & 0.8828 \end{bmatrix}$$

$$\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix}=[A]^{-1}\{b\}=10^{-6}\begin{bmatrix} 0.1103 & 0.1103 & 0.1103 \\ 0.1103 & 0.6621 & 0.6621 \\ 0.1103 & 0.6621 & 0.8828 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}=10^{-3}\begin{Bmatrix} 0.0883 \\ 0.5297 \\ 0.7062 \end{Bmatrix}$$

2.11

(a) Using Gaussian method

$$x_1 + x_2 + x_3 = 6$$
$$2x_1 + 5x_2 + x_3 = 15$$
$$-3x_1 + x_2 + 5x_3 = 14$$

$$\begin{cases} 2x_1 + 5x_2 + x_3 = 15 \\ -2x_1 - 2x_2 - 2x_3 = -12 \end{cases}$$
$$3x_2 - x_3 = 3$$

$$\begin{cases} -3x_1 + x_2 + 5x_3 = 14 \\ 3x_1 + 3x_2 + 3x_3 = 18 \end{cases}$$
$$4x_2 + 8x_3 = 32$$

$$\begin{cases} 3x_2 - x_3 = 3 \\ 4x_2 + 8x_3 = 32 \end{cases} \rightarrow \begin{matrix} x_2 - \frac{1}{3}x_3 = 1 \\ 4x_2 + 8x_3 = 32 \end{matrix}$$

$$\begin{cases} 4x_2 + 8x_3 = 32 \\ -4x_2 + \frac{4}{3}x_3 = -4 \end{cases}$$
$$\frac{28}{3}x_3 = 28 \rightarrow \underline{\underline{x_3 = 3}} \Leftarrow$$

$$x_2 = 1 + \frac{1}{3}x_3 = 1 + \frac{1}{3}(3) = 2 \qquad \underline{\underline{x_2 = 2}} \Leftarrow$$

$$x_1 = 6 - x_2 - x_3 = 6 - 2 - 3 = 1 \qquad \underline{\underline{x_1 = 1}} \Leftarrow$$

2.11 Cont.

(b) Using the LU decomposition method

$$[A] = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix} \qquad \{b\} = \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$u_{11} = a_{11} = 1 \qquad u_{12} = a_{12} = 1 \qquad u_{13} = a_{13} = 1$$

$$l_{21} = \frac{a_{21}}{u_{11}} = \frac{2}{1} = 2 \qquad l_{31} = \frac{a_{31}}{u_{11}} = \frac{-3}{1} = -3$$

$$u_{22} = a_{22} - l_{21}u_{12} = 5 - (2)(1) = 3$$

$$u_{23} = a_{23} - l_{21}u_{13} = 1 - (2)(1) = -1$$

$$l_{32} = \frac{a_{32} - l_{31}u_{12}}{u_{22}} = \frac{1-(-3)(1)}{3} = \frac{4}{3}$$

$$u_{33} = a_{33} - (l_{31}u_{13} + l_{32}u_{23}) = 5 - \left[(-3)(1) + (\tfrac{4}{3})(-1)\right] = \frac{28}{3}$$

$$\therefore \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & \frac{4}{3} & 1 \end{bmatrix}}_{[L]} \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{28}{3} \end{bmatrix}}_{[U]}$$

$$[L]\{z\} = \{b\} \longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & \frac{4}{3} & 1 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$\{z\} = \begin{Bmatrix} 6 \\ 3 \\ 28 \end{Bmatrix}$$

$$[U]\{x\} = \{z\} \longrightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{28}{3} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 3 \\ 28 \end{Bmatrix}$$

$$\{x\} = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix} \Leftarrow$$

2.11 Cont.

(c) by finding the inverse of the Coefficient matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix}^{-1} \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{bmatrix} 0.8571 & -0.1429 & -0.1429 \\ -0.4643 & 0.2857 & 0.0357 \\ 0.6071 & -0.1429 & 0.1071 \end{bmatrix} \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix} \Leftarrow$$

2.12

$$[A]^{-1} = \begin{bmatrix} \frac{1}{5} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \Leftarrow$$

$$[B]^{-1} = \begin{bmatrix} 0.8571 & -0.1429 & -0.1429 \\ -0.4643 & 0.2857 & 0.0357 \\ 0.6071 & -0.1429 & 0.1071 \end{bmatrix} \Leftarrow$$

$$[C] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \underbrace{\begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}}_{[C]^{-1}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} X_{11} \\ X_{21} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \longrightarrow X_{11} = \frac{K_{22}}{K_{11}K_{22} - K_{12}K_{21}}$$

$$X_{21} = \frac{-K_{21}}{K_{11}K_{22} - K_{12}K_{21}}$$

$$\Leftarrow$$

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} X_{12} \\ X_{22} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \longrightarrow X_{12} = \frac{-K_{12}}{K_{11}K_{22} - K_{12}K_{21}}$$

$$X_{22} = \frac{K_{11}}{K_{11}K_{22} - K_{12}K_{21}}$$

2.13

For a 2X2 matrix:

$$\det\left(\alpha\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\right) \stackrel{?}{=} \alpha^2\begin{vmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{vmatrix}$$

$$\alpha\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix} = \begin{bmatrix}\alpha a_{11} & \alpha a_{12}\\ \alpha a_{21} & \alpha a_{22}\end{bmatrix}$$

$$\det\begin{bmatrix}\alpha a_{11} & \alpha a_{12}\\ \alpha a_{21} & \alpha a_{22}\end{bmatrix} = (\alpha a_{11})(\alpha a_{22}) - (\alpha a_{12})(\alpha a_{21})$$

$$\det\left(\alpha\begin{bmatrix}a_{11} & a_{12}\\ a_{12} & a_{22}\end{bmatrix}\right) = \alpha^2\left(a_{11}a_{22} - a_{12}a_{21}\right) = \alpha^2\begin{vmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{vmatrix}$$

Q.E.D.

For a 3X3 matrix:

$$\det\left(\alpha\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\right) \stackrel{?}{=} \alpha^3\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}$$

$$\det\begin{vmatrix}\alpha a_{11} & \alpha a_{12} & \alpha a_{13}\\ \alpha a_{21} & \alpha a_{22} & \alpha a_{23}\\ \alpha a_{31} & \alpha a_{32} & \alpha a_{33}\end{vmatrix} = (\alpha a_{11})(\alpha a_{22})(\alpha a_{33}) + (\alpha a_{12})(\alpha a_{23})(\alpha a_{31}) + \cdots \cdots - (\alpha)(a_{13})(\alpha a_{22})(\alpha a_{31})$$

$$\det\left(\alpha\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\right) = \alpha^3\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}$$

Q.E.D.

In general for a nxn matrix, we have

$$\det\left(\alpha\begin{bmatrix}a_{11} & a_{12} & \cdot & a_{1n}\\ a_{21} & a_{22} & : & a_{2n}\\ \cdot & \cdot & : & \cdot\\ a_{n1} & a_{n2} & \cdot & a_{nn}\end{bmatrix}\right) = \alpha^n\begin{vmatrix}a_{11} & a_{12} & \cdot & a_{1n}\\ a_{21} & a_{21} & \cdot & a_{2n}\\ \cdot & \cdot & \cdot & \cdot\\ a_{n1} & a_{n2} & \cdot & a_{nn}\end{vmatrix}$$

2.14

Using Equation (2.83), we have

$$\begin{bmatrix} -\omega^2 + \frac{2K}{m_1} & -\frac{K}{m_1} \\ -\frac{K}{m_2} & -\omega^2 + \frac{2K}{m_2} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = 0$$

$$\begin{vmatrix} -\omega^2 + \frac{2(100)}{0.1} & -\frac{100}{0.1} \\ -\frac{100}{0.2} & -\omega^2 + \frac{2(100)}{0.2} \end{vmatrix} = 0$$

$$\omega^4 - 3000\,\omega^2 + 1.5 \times 10^6 = 0$$

$$\omega_1^2 = 2366 \ (\text{rad}/s)^2 \qquad \omega_1 = 48.6 \ \text{rad}/s$$

$$\omega_2^2 = 634 \ (\text{rad}/s)^2 \qquad \omega_2 = 25.2 \ \text{rad}/s$$

$$(-2366 + 2000) X_1 - 1000\, X_2 = 0 \ \longrightarrow \ \frac{X_2}{X_1} = -0.366$$

$$(-634 + 2000) X_1 - 1000\, X_2 = 0 \ \longrightarrow \ \frac{X_2}{X_1} = 1.366$$

2.15

```
>> a=[4 2 1;7 0 -7;1 -5 3]

a =

     4     2     1
     7     0    -7
     1    -5     3

>> b=[1 2 -1;5 3 3;4 5 -7]

b =

     1     2    -1
     5     3     3
     4     5    -7

>> c=[1;-2;4]

c =

     1
    -2
     4

>> a+b

ans =

     5     4     0
    12     3    -4
     5     0    -4

>> a-b

ans =

     3     0     2
     2    -3   -10
    -3   -10    10
```

2.15 Cont.

```
>> 3*a

ans =

   12    6    3
   21    0  -21
    3  -15    9

>> a*b

ans =

   18   19   -5
  -21  -21   42
  -12    2  -37

>> a*c

ans =

    4
  -21
   23

>> a*a

ans =

   31    3   -7
   21   49  -14
  -28  -13   45

>> i=[1 0 0;0 1 0;0 0 1]

i =

     1     0     0
     0     1     0
     0     0     1
```

2.15 Cont.

```
>> i*a

ans =

     4     2     1
     7     0    -7
     1    -5     3

>> a*i

ans =

     4     2     1
     7     0    -7
     1    -5     3

>>
```

2.16

```
>> A=[1 4 2;8 3 6;7 1 -2]

A =

     1     4     2
     8     3     6
     7     1    -2

>> B=[0 5 -1;-3 1 7;2 4 -4]

B =

     0     5    -1
    -3     1     7
     2     4    -4

>> A'

ans =

     1     8     7
     4     3     1
     2     6    -2

>> B'

ans =

     0    -3     2
     5     1     4
    -1     7    -4

>> (A+B)'

ans =

     1     5     9
     9     4     5
     1    13    -6
```

2.16 Cont.

```
>> A'+B'

ans =

     1     5     9
     9     4     5
     1    13    -6

>> (A*B)'

ans =

    -8     3    -7
    17    67    28
    19   -11     8

>> B'*A'

ans =

    -8     3    -7
    17    67    28
    19   -11     8

>>
```

2.17

```
>> A=[2 10 0;16 6 14;12 -4 18]

A =

     2    10     0
    16     6    14
    12    -4    18

>> B=[2 10 0;4 20 0;12 -4 18]

B =

     2    10     0
     4    20     0
    12    -4    18

>> det(A)

ans =

  -872

>> det(B)

ans =

     0

>> det((A)')

ans =

  -872

>> det(5*(A))

ans =

   -109000

>>
```

2.18

```
>> A=[0 5 0;8 3 7;9 -2 9]

A =

     0     5     0
     8     3     7
     9    -2     9

>> det(A)

ans =

   -45

>> det((A)')

ans =

   -45

>>
```

2.19

```
>> A=[10875000 -1812500 0;-1812500 6343750 -4531250;0 -4531250 4531250]

A =

   10875000   -1812500          0
   -1812500    6343750   -4531250
          0   -4531250    4531250

>> b=[0;0;800]

b =

     0
     0
   800

>> x=A\b

x =

  1.0e-003 *

    0.0883
    0.5297
    0.7062

>>
```

2.20

```
>> A=[0 5 0;8 3 7;9 -2 9]

A =

     0     5     0
     8     3     7
     9    -2     9

>> [l,u]=lu(A)

l =

         0    1.0000         0
    0.8889    0.9556    1.0000
    1.0000         0         0


u =

     9    -2     9
     0     5     0
     0     0    -1

>> l*u

ans =

     0     5     0
     8     3     7
     9    -2     9
```

2-21

```
>> A=[10875000 -1812500 0;-1812500 6343750 -4531250;0 -4531250 4531250]

A =

  10875000   -1812500          0
  -1812500    6343750   -4531250
         0   -4531250    4531250

>> b=[0;0;800]

b =

     0
     0
   800

>> [l,u]=lu(A)

l =

    1.0000         0         0
   -0.1667    1.0000         0
         0   -0.7500    1.0000


u =

  1.0e+007 *

    1.0875   -0.1812         0
         0    0.6042   -0.4531
         0         0    0.1133

>> z=inv(l)*b

z =

     0
     0
   800

>> U=inv(u)*z

U =

  1.0e-003 *

    0.0883
    0.5297
    0.7062
```

Note the difference between **u** denoting upper triangular matrix and **U** denoting the displacement results

2.22

```
>> A=[10875000 -1812500 0;-1812500 6343750 -4531250;0 -4531250 4531250]

A =

   10875000   -1812500          0
   -1812500    6343750   -4531250
          0   -4531250    4531250

>> b=[0;0;800]

b =

     0
     0
   800

>> Ainverse=inv(A)

Ainverse =

  1.0e-006 *

    0.1103    0.1103    0.1103
    0.1103    0.6621    0.6621
    0.1103    0.6621    0.8828

>> u=Ainverse*b

u =

  1.0e-003 *

    0.0883
    0.5297
    0.7062

>>
```

2.23

```
>> A=[1 1 1;2 5 1;-3 1 5]

A =

     1     1     1
     2     5     1
    -3     1     5

>> b=[6 15 14]

b =

     6    15    14

>> b=[6;15;14]

b =

     6
    15
    14
```

(a) using the Gaussian method

```
>> x=A\b

x =

    1.0000
    2.0000
    3.0000
```

(b) using the LU decomposition method

```
>> [l,u]=lu(A)

l =

   -0.3333    0.2353    1.0000
   -0.6667    1.0000         0
    1.0000         0         0


u =

   -3.0000    1.0000    5.0000
         0    5.6667    4.3333
         0         0    1.6471
```

2.23 Cont.

```
>> z=inv(l)*b

z =

   14.0000
   24.3333
    4.9412

>> x=inv(u)*z

x =

    1.0000
    2.0000
    3.0000
```

(c) by finding the inverse of the coefficient matrix

```
>> x=inv(A)*b

x =

    1.0000
    2.0000
    3.0000

>>
```

2.24

For example, consider the following 4 x 4 matrix, and α =2 and α =3.

```
>> A=[1 2 1 3;2 1 4 1;5 3 0 1;4 1 5 7]

A =

     1     2     1     3
     2     1     4     1
     5     3     0     1
     4     1     5     7

>> det(A)

ans =

   219

>> det(2*A)

ans =

        3504
```

Since matrix A is 4 x 4 then let us examine to see if det(2*A) = 2^4 * det(A)?

```
>> 2^4*det(A)

ans =

        3504

>> det(3*A)

ans =

       17739
```

Or is det(3*A) = 3^4 * det(A)?

```
>> 3^4*det(A)

ans =

       17739
```

2.24 Cont.

Let us now consider the following 3 x 3 matrix, and $\alpha = 2$ and $\alpha = 3$.

```
>> B=[1 2 1;2 1 4;5 3 0]

B =

    1    2    1
    2    1    4
    5    3    0

>> det(B)

ans =

   29

>> det(2*B)

ans =

  232
```

Is $\det(2*B) = 2^3 *\det(B)$?

```
>> 2^3*det(B)

ans =

  232

>> det(3*B)

ans =

  783
```

Or is $\det(3*B) = 3^3 *\det(B)$?

```
>> 3^3*det(B)

ans =

  783

>>
```

2.25

```
>> A=[7.11 -1.23 0 0 0;-1.23 1.99 -0.76 0 0;0 -0.76 0.851 -0.091 0;0 0 -0.091 2.311 -2.22;0 0 0 -2.22 3.69]

A =

    7.1100   -1.2300         0         0         0
   -1.2300    1.9900   -0.7600         0         0
         0   -0.7600    0.8510   -0.0910         0
         0         0   -0.0910    2.3110   -2.2200
         0         0         0   -2.2200    3.6900

>> b=[5.88*20; 0; 0; 0; 1.47*70]

b =

  117.6000
         0
         0
         0
  102.9000

>> T=A\b

T =

   20.5898
   23.4091
   27.9719
   66.0789
   67.6410

>>
```

2.26

```
>> A=[7.11 -1.23 0 0 0;-1.23 1.99 -0.76 0 0;0 -0.76 0.851 -0.091 0;0 0 -0.091 2.311 -2.22;0 0 0 -2.22 3.69]

A =

   7.1100  -1.2300        0        0        0
  -1.2300   1.9900  -0.7600        0        0
        0  -0.7600   0.8510  -0.0910        0
        0        0  -0.0910   2.3110  -2.2200
        0        0        0  -2.2200   3.6900

>> b=[5.88*20; 0; 0; 0; 1.47*70]

b =

  117.6000
         0
         0
         0
  102.9000

>> Ainverse=inv(A)

Ainverse =

   0.1681   0.1585   0.1430   0.0133   0.0080
   0.1585   0.9160   0.8263   0.0771   0.0464
   0.1430   0.8263   1.9323   0.1803   0.1085
   0.0133   0.0771   0.1803   1.0421   0.6269
   0.0080   0.0464   0.1085   0.6269   0.6482

>> T=Ainverse*b

T =

   20.5898
   23.4091
   27.9719
   66.0789
   67.6410
```

2.27

```
>> A=[7.11 -1.23 0 0 0;-1.23 1.99 -0.76 0 0;0 -0.76 0.851 -0.091 0;0 0 -0.091 2.311 -2.22;0 0 0 -2.22 3.69]

A =

    7.1100   -1.2300         0         0         0
   -1.2300    1.9900   -0.7600         0         0
         0   -0.7600    0.8510   -0.0910         0
         0         0   -0.0910    2.3110   -2.2200
         0         0         0   -2.2200    3.6900


>> b=[5.88*20; 0; 0; 0; 1.47*70]

b =

  117.6000
         0
         0
         0
  102.9000

>> [l,u]=lu(A)

l =

    1.0000         0         0         0         0
   -0.1730    1.0000         0         0         0
         0   -0.4276    1.0000         0         0
         0         0   -0.1730    1.0000         0
         0         0         0   -0.9672    1.0000


u =

    7.1100   -1.2300         0         0         0
         0    1.7772   -0.7600         0         0
         0         0    0.5260   -0.0910         0
         0         0         0    2.2953   -2.2200
         0         0         0         0    1.5428

>> z=inv(l)*b

z =

  117.6000
   20.3443
    8.6999
    1.5051
  104.3558

>> T=inv(u)*z

T =

   20.5898
   23.4091
   27.9719
   66.0789
   67.6410

>>
```

2-28

```
>> A=[1 0 0 0 0;-0.0408 0.0888 -0.0408 0 0;0 -0.0408 0.0888 -0.0408 0;0 0 -0.0408 0.0888 -0.0408;
0 0 0 -0.0408 0.04455]

A =

   1.0000        0        0        0        0
  -0.0408   0.0888  -0.0408        0        0
        0  -0.0408   0.0888  -0.0408        0
        0        0  -0.0408   0.0888  -0.0408
        0        0        0  -0.0408   0.0445

>> b=[100;0.144;0.144;0.144;0.075]

b =

  100.0000
    0.1440
    0.1440
    0.1440
    0.0750

>> T=A\b

T =

  100.0000
   75.0387
   59.7901
   51.5633
   48.9064

>>
```

2-29

```
>> A=10^5*[7.2 0 0 0 -1.49 -1.49;0 7.2 0 -4.22 -1.49 -1.49;0 0 8.44 0 -4.22 0;0 -4.22 0 4.22 0 0;-1.49 -1.49
-4.22 0 5.71 1.49;-1.49 -1.49 0 0 1.49 1.49]

A =

    720000         0         0         0   -149000   -149000
         0    720000         0   -422000   -149000   -149000
         0         0    844000         0   -422000         0
         0   -422000         0    422000         0         0
   -149000   -149000   -422000         0    571000    149000
   -149000   -149000         0         0    149000    149000

>> b=[0;0;0;-500;0;-500]

b =

     0
     0
     0
  -500
     0
  -500

>> U=A\b

U =

   -0.0036
   -0.0103
    0.0012
   -0.0115
    0.0024
   -0.0195

>>
```

2.30

```
>> A=[2000 -1000;-500 1000]

A =

    2000    -1000
    -500     1000
```

The eigenvalues are:

```
>> eig(A)

ans =

  1.0e+003 *

    2.3660
    0.6340
```

Note the natural frequencies of the system are equal to the square root of the eigenvalues.

```
>> sqrt(eig(A))

ans =

   48.6418
   25.1789
```

The eigenvector and eigenvlaues are given by:

```
>> [v,e]=eig(A)

v =

    0.9391    0.5907
   -0.3437    0.8069

e =

  1.0e+003 *

    2.3660         0
         0    0.6340
```

2.30 Cont.

Normalizing the eigenvector with respect to X_1, we get:

```
>> -0.3437/0.9391

ans =

  -0.3660
```

Therefore, the first mode is given by $X_2/X_1 = -0.3660$.

```
>> .8069/0.5907

ans =

   1.3660
```

The second mode is then given by $X_2/X_1 = 1.3660$.

Problem 2-31

[K] =

7.11	-1.23	0	0	0
-1.23	1.99	-0.76	0	0
0	-0.76	0.851	-0.091	0
0	0	-0.091	2.31	-2.22
0	0	0	-2.22	3.69

{T} =

T_1
T_2
T_3
T_4
T_5

[F] =

117.6
0
0
0
102.9

$[K]^{-1}$ =

0.1681	0.1585	0.143	0.0134	0.008
0.1585	0.9161	0.8264	0.0772	0.0464
0.143	0.8264	1.9324	0.1805	0.1086
0.0134	0.0772	0.1805	1.0431	0.6276
0.008	0.0464	0.1086	0.6276	0.6486

{T} = $[K]^{-1}$ [F] =

T	Value
T_1	20.59
T_2	23.41
T_3	27.98
T_4	66.15
T_5	67.68

Problem 2-32

[K] =

1	0	0	0	0
-0.041	0.0888	-0.0408	0	0
0	-0.0408	0.0888	-0.041	0
0	0	-0.0408	0.0888	-0.0408
0	0	0	-0.041	0.04455

{T} =

T_1
T_2
T_3
T_4
T_5

[F] =

100
0.144
0.144
0.144
0.075

$[K]^{-1}$ =

1	0	0	0	0
0.688	16.8623	12.191	9.6701	8.85613
0.4974	12.1906	26.532	21.047	19.2751
0.3945	9.67011	21.047	36.137	33.0956
0.3613	8.85613	19.275	33.096	52.7564

{T} = $[K]^{-1}$ [F] =

T	$[K]^{-1}[F]$
T_1	100.00
T_2	75.04
T_3	59.79
T_4	51.56
T_5	48.91

Problem 2-33

[K] =

7.2	0	0	0	-1.49	-1.49
0	7.2	0	-4.22	-1.49	-1.49
0	0	8.44	0	-4.22	0
0	-4.22	0	4.22	0	0
-1.49	-1.49	-4.22	0	5.71	1.49
-1.49	-1.49	0	0	1.49	1.49

{T} =

U2x
U2y
U4x
U4y
U5x
U5y

[F] =

0
0
0
-0.005
0
-0.005

$[K]^{-1}$ =

0.23697	0.23697	0	0.237	0	0.473934
0.23697	0.90811	0	0.9081	0	1.145075
0	-1E-17	0.237	-1E-17	0.23697	-0.23697
0.23697	0.90811	0	1.1451	0	1.145075
0	-2E-17	0.237	-2E-17	0.47393	-0.47393
0.47393	1.14507	-0.237	1.1451	-0.4739	2.764083

{U} =

U2x
U2y
U4x
U4y
U5x
U5y

= **$[K]^{-1}$ [F]** =

-0.0036
-0.0103
0.0012
-0.0115
0.0024
-0.0195