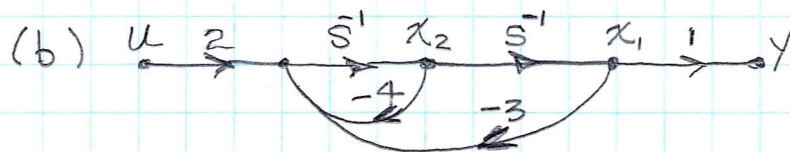


CHAPTER 3 PROBLEM SOLUTIONS

3.1 (a) $\ddot{y} + 4\dot{y} + 3y = 2u$

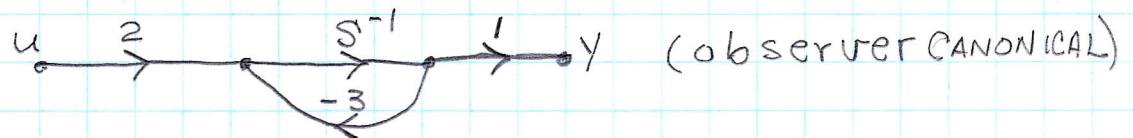
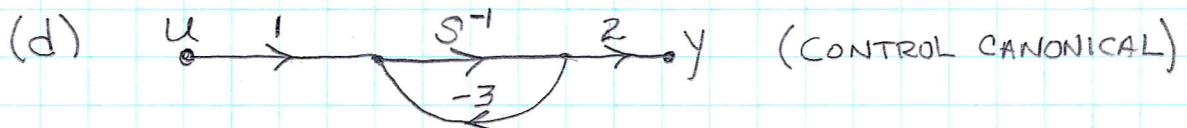
LET: $x_1 = y$, $\dot{x}_1 = \dot{y} = x_2$, $\dot{x}_2 = -3x_1 - 4x_2 + 2u$

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \end{bmatrix} u$$



(c) $\dot{y} + 3y = 2u$; LET $x = y$

$$\dot{x} = -3x + 2u, \quad y = x$$

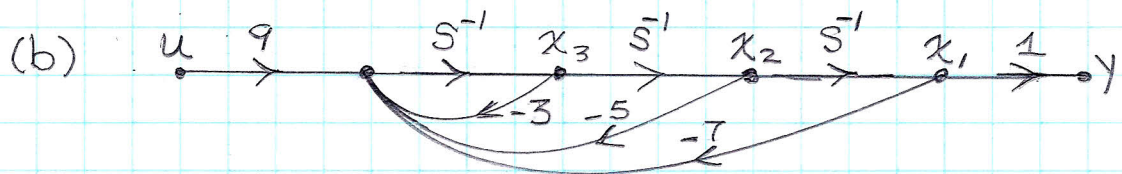


3.2 (a) $\dot{x}_3 + 3x_3 + 5x_2 + 7x_1 = 9u$

$\dot{x}_1 = x_2$, $\dot{x}_2 = x_3$, $\dot{x}_3 = -7x_1 - 5x_2 - 3x_3 + 9u$

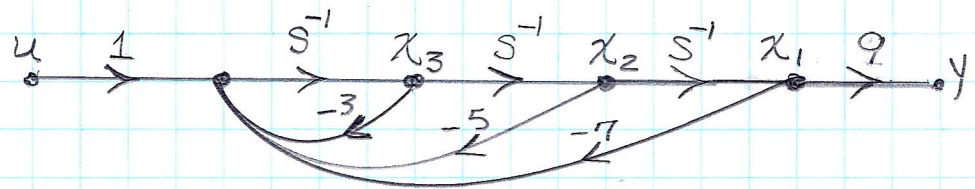
$$\underline{\dot{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -5 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ 9 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \end{bmatrix} u$$



(c) OBSERVER CANONICAL FORM (SEE (b))

CONTROL CANONICAL FORM

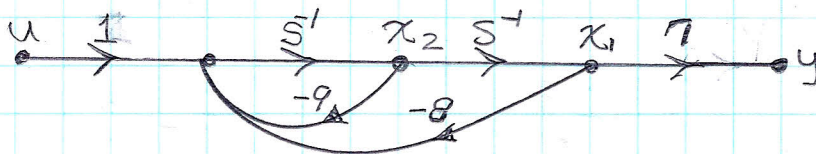


$$3.3 \text{ (a)} \quad \frac{Y(s)}{U(s)} = \frac{7s^{-2}}{1 - (-9s^{-1} - 8s^{-2})}$$

(SEE MASON'S GAIN FORMULA)

$$M_1 \Delta_1 = 7s^{-2}$$

$$\Delta = 1 - (-9s^{-1} - 8s^{-2})$$



$$(b) \quad \dot{x}_1 = x_2, \quad \dot{x}_2 = -8x_1 - 9x_2 + u$$

$$y = 7x_1$$

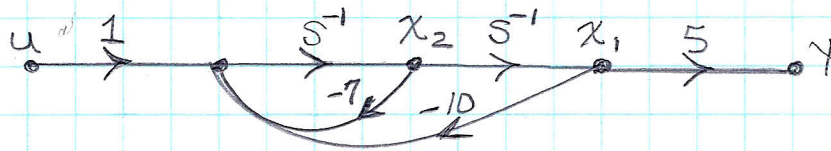
$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -8 & -9 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 7 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \end{bmatrix} u$$

(CONTROL
CANONICAL
FORM)

3.4 (a)

$$\frac{Y(s)}{U(s)} = \frac{5s^{-2}}{1 - (-7s^{-1} - 10s^{-2})}$$



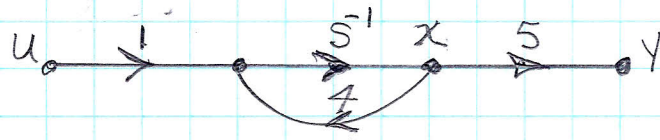
$$(b) \quad \dot{x}_1 = x_2, \quad \dot{x}_2 = -10x_1 - 7x_2 + u$$

$$y = 5x_1$$

$$\underline{\dot{x}} = \begin{bmatrix} 0 & 1 \\ -10 & -7 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [5 \quad 0] \underline{x} + [0] u$$

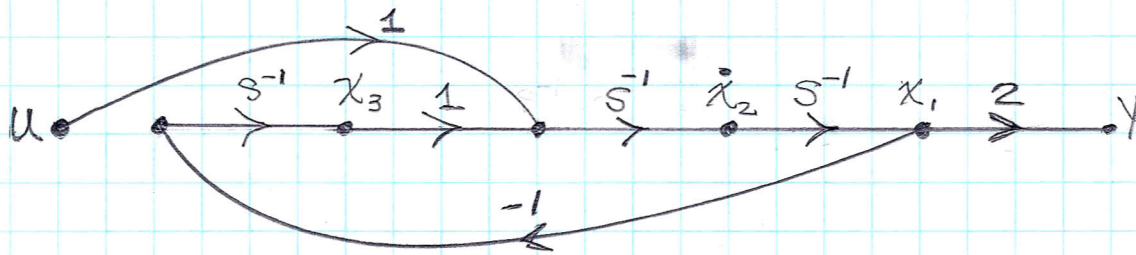
$$(c) \quad \frac{Y(s)}{U(s)} = \frac{5s^{-1}}{1 - (4s^{-1})}$$



$$\dot{x} = 4x + u; \quad y = 5x$$

$$y = 5x$$

$$3.4(d) \quad \frac{y(s)}{u(s)} = \frac{2s^{-2}}{1 - (-s^{-3})}$$



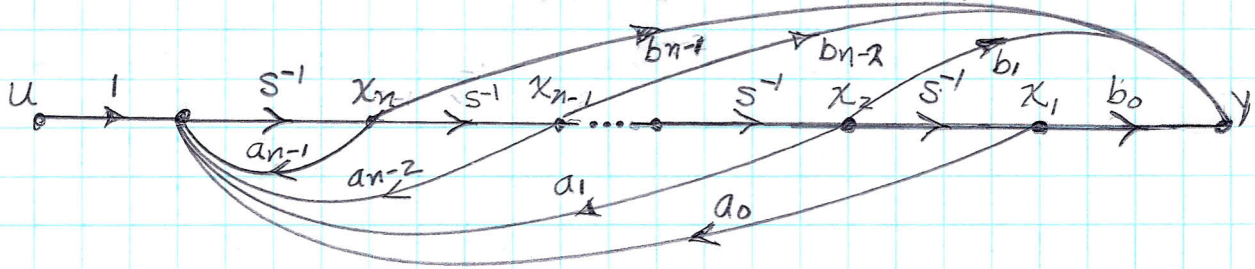
$$\dot{x}_1 = x_2 ; \quad \dot{x}_2 = x_3 + u ; \quad \dot{x}_3 = -x_1$$

$$y = 2x_1$$

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

$$y = [2 \ 0 \ 0] \underline{x}$$

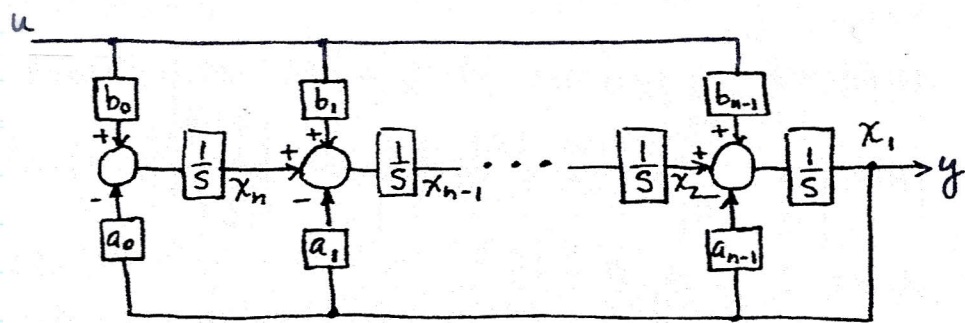
$$3.5 (a) \quad G(s) = \frac{b_{n-1}s^{-1} + \dots + b_1s^{-1} + b_0s^{-n}}{1 - (-a_{n-1}s^{-1} - \dots - a_1s^{-1} - a_0s^{-n})}$$



$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 1 \\ -a_0 & -a_1 & \dots & -a_{n-2} & -a_{n-1} \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [b_0 \quad b_1 \quad \dots \quad b_{n-2} \quad b_{n-1}] \underline{x} + [0] u$$

(b)



$$\dot{\underline{x}} = \begin{pmatrix} -a_{n-1} & 1 & 0 & \dots & 0 \\ -a_{n-2} & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -a_1 & 0 & 0 & \dots & 1 \\ -a_0 & 0 & 0 & \dots & 0 \end{pmatrix} \underline{x} + \begin{pmatrix} b_{n-1} \\ b_{n-2} \\ \vdots \\ b_1 \\ b_0 \end{pmatrix} u$$

$$y = (1 \ 0 \ 0 \ \dots \ 0) \underline{x}$$

$$3.6 (a) \quad \dot{\underline{x}} = \begin{bmatrix} -5 & -4 \\ -1 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 4 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} s+9 & 4 \\ 1 & s+3 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} s+3 & -4 \\ -1 & s+9 \end{bmatrix}}{s^2 + 12s + 23}$$

$$(b) \quad [sI - A]^{-1} = \begin{bmatrix} \frac{s+3}{s^2+12s+23} & \frac{-4}{s^2+12s+23} \\ \frac{-1}{s^2+12s+23} & \frac{s+9}{s^2+12s+23} \end{bmatrix} = \underline{\Phi}(s)$$

$$|sI - A| = s^2 + 12s + 23 \simeq (s + 2.4)(s + 9.6)$$

$$(c) \quad \phi(t) = \mathcal{F}^{-1}\{[sI - A]^{-1}\} = \begin{bmatrix} 0.084e^{-2.4t} + 0.916e^{-9.6t} & -0.555(e^{-2.4t} - e^{-9.6t}) \\ -0.139(e^{-2.4t} - e^{-9.6t}) & 0.916e^{-2.4t} + 0.083e^{-9.6t} \end{bmatrix}$$

$$3.7(a) \quad \dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ -4 & -5 & 15 \\ -1 & -2 & -2 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \underline{x}$$

$$(b) \quad [sI - A] = \begin{bmatrix} s & -1 & 0 \\ 4 & s+5 & -15 \\ 1 & 2 & s+2 \end{bmatrix}$$

$$\Phi(s) = [sI - A]^{-1}$$

$$\Phi(s) = \frac{\begin{bmatrix} s^2 + 17s + 60 & s+2 & 15 \\ -(4s+33) & s(s+2) & 15s \\ -(s-3) & -(2s+1) & s^2+15s+9 \end{bmatrix}}{s^3 + 17s^2 + 69s + 3}$$

$$(c) \quad \phi(t) = \mathcal{L}^{-1}\{\Phi(s)\}$$

$$\phi_{11}(t) = 0.995e^{-0.55t} + 1.083e^{-5.45t} - 1.034e^{-11t}$$

$$\phi_{12}(t) = 0.0283e^{-0.55t} + 0.1269e^{-5.45t} - 0.1552e^{-11t}$$

$$\phi_{13}(t) = 0.293e^{-0.55t} - 0.5516e^{-5.45t} + 0.2586e^{-11t}$$

$$\phi_{21}(t) = -0.5478e^{-0.55t} - 0.5901e^{-5.45t} + 1.1379e^{-11t}$$

$$\phi_{22}(t) = -0.0156e^{-0.55t} - 0.6913e^{-5.45t} + 1.7069e^{-11t}$$

$$\phi_{23}(t) = -0.1613e^{-0.55t} + 3.006e^{-5.45t} - 2.8448e^{-11t}$$

$$\phi_{31}(t) = 0.0694e^{-0.55t} - 0.3107e^{-5.45t} + 2.414e^{-11t}$$

$$\phi_{32}(t) = 0.0020e^{-0.55t} - 0.3640e^{-5.45t} + 0.3621e^{-11t}$$

$$\phi_{33}(t) = 0.0202e^{-0.55t} + 1.5830e^{-5.45t} - 0.6034e^{-11t}$$

$$3.8 \quad (a) \quad [sI - A] = s + 3, \quad B = 4$$

$$\Phi(s) = [sI - A]^{-1} = \frac{1}{s+3}$$

$$(b) \quad \phi(t) = \mathcal{L}^{-1}\{\Phi(s)\} = e^{-3t}$$

$$(c) \quad (3-24): x(t) = \phi(t)x(0) + \int_0^t \phi(\tau)Bu(t-\tau)d\tau$$

$$x(t) = \int_0^t 4e^{-3\tau}d\tau = \left. -\frac{4}{3}e^{-3\tau} \right|_0^t$$

$$x(t) = \frac{4}{3}(1 - e^{-3t})u(t)$$

$$(d) \quad x(0) = -1 \Rightarrow x(t) = -e^{-3t} + \frac{4}{3}(1 - e^{-3t})$$

$$x(t) = \left(\frac{4}{3} - \frac{7}{3}e^{-3t}\right)u(t)$$

$$(e) \quad sx(s) = -3x(s) + 4u(s) \Rightarrow u(s) = \frac{(s+3)x(s)}{4}$$

$$y(s) = x(s)$$

$$\frac{y(s)}{u(s)} = \frac{4}{s+3} \Rightarrow y(s) = \frac{4u(s)}{s+3}$$

$$\mathcal{L}\{u(t)\} = \frac{1}{s} \therefore y(s) = \frac{4}{s(s+3)}$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{4}{s(s+3)}\right\} = \mathcal{L}^{-1}\left\{\frac{4/3}{s} - \frac{4/3}{s+3}\right\}$$

$$y(t) = \frac{4}{3}(1 - e^{-3t})u(t)$$

$$(f) \quad (3-20): x(s) = (sI - A)^{-1}x(0) + (sI - A)^{-1}B u(s)$$

$$x(0) = -1$$

$$x(s) = \frac{-1}{s+3} + \frac{1}{s+3}(4)\left(\frac{1}{s}\right)$$

$$x(t) = (-e^{-3t} + \frac{4}{3}(1 - e^{-3t}))u(t)$$

$$x(t) = \left(\frac{4}{3} - \frac{7}{3}e^{-3t}\right)u(t) \quad \checkmark$$

3.9 $A = -1, B = 1, C = 1, D = 1$

$$sI - A = s + 1$$

(a) $\Phi(s) = [sI - A]^{-1} = \frac{1}{s+1}$

(b) $\phi(t) = \mathcal{L}^{-1}\{\Phi(s)\} = e^{-t}$

(c) $x(t) = \int_0^t \phi(\tau) B u(t-\tau) d\tau = \int_0^t e^{-\tau} d\tau$

$$x(t) = (1 - e^{-t}), t \geq 0$$

$$y(t) = x(t) + u(t) = 2 - e^{-t}, t \geq 0$$

(d) $x(0) = -1$

$$x(t) = \phi(t)x(0) + \int_0^t \phi(\tau) B u(\tau) d\tau$$

$$x(t) = -1e^{-t} + 1 - e^{-t} = 1 - 2e^{-t}, t \geq 0$$

$$y(t) = x(t) + u(t) = 2 - 2e^{-t}, t \geq 0$$

(e) $sX(s) = -X(s) + U(s) \Rightarrow X(s) = \frac{U(s)}{s+1}$

$$Y(s) = X(s) + U(s)$$

$$Y(s) = \frac{U(s)}{s+1} + U(s) = \left(\frac{1}{s+1} + 1\right)U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{s+2}{s+1} \Rightarrow Y(s) = \frac{s+2}{s(s+1)}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{2}{s} + \frac{-1}{s+1}\right\} = 2 - e^{-t}, t \geq 0$$

(f) $X(s) = (sI - A)^{-1}x(0) + (sI - A)^{-1}B U(s)$

$$= \frac{1}{s+1}(-1) + \frac{1}{s(s+1)} = \frac{-s+1}{s(s+1)}$$

$$Y(s) = X(s) + U(s) = \frac{-s+1}{s(s+1)} + \frac{1}{s} = \frac{-s+1+s+1}{s(s+1)}$$

$$Y(s) = \frac{2}{s(s+1)} \Rightarrow y(t) = 2 - 2e^{-t}, t \geq 0$$

$$3.10 \quad [sI - A] = \begin{bmatrix} s & -2 \\ 2 & s+5 \end{bmatrix}$$

$$(a) \quad \Phi(s) = [sI - A]^{-1} = \frac{\begin{bmatrix} s+5 & 2 \\ -2 & s \end{bmatrix}}{s^2 + 5s + 4}$$

$$\Phi(s) = \begin{bmatrix} \frac{s+5}{(s+1)(s+4)} & \frac{2}{(s+1)(s+4)} \\ \frac{-2}{(s+1)(s+4)} & \frac{s}{(s+1)(s+4)} \end{bmatrix}$$

$$\Phi(s) = \begin{bmatrix} \frac{4/3}{s+1} + \frac{-1/3}{s+4} & \frac{2/3}{s+1} + \frac{-2/3}{s+4} \\ \frac{-2/3}{s+1} + \frac{2/3}{s+4} & \frac{-1/3}{s+1} + \frac{4/3}{s+4} \end{bmatrix}$$

$$(b) \quad \phi(t) = \mathcal{L}^{-1}\{\Phi(s)\} = \begin{bmatrix} \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t} & \frac{2}{3}e^{-t} - \frac{2}{3}e^{-4t} \\ -\frac{2}{3}e^{-t} + \frac{2}{3}e^{-4t} & -\frac{1}{3}e^{-t} + \frac{4}{3}e^{-4t} \end{bmatrix}$$

$$(c) \quad \underline{x}(t) = \phi(t) \underline{x}(0) + \int_0^t \phi(t-\tau) B u(t-\tau) d\tau$$

$$\underline{x}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\underline{x}(t) = \int_0^t \phi(\tau) \begin{bmatrix} 0 \\ 1 \end{bmatrix} 1 d\tau$$

$$= \int_0^t \begin{bmatrix} \frac{2}{3}e^{-\tau} - \frac{2}{3}e^{-4\tau} \\ -\frac{1}{3}e^{-\tau} + \frac{4}{3}e^{-4\tau} \end{bmatrix} d\tau$$

$$= \begin{bmatrix} \int_0^t \frac{2}{3}(e^{-\tau} - e^{-4\tau}) d\tau \\ \int_0^t \frac{1}{3}(-e^{-\tau} + 4e^{-4\tau}) d\tau \end{bmatrix}$$

$$\underline{x}(t) = \begin{bmatrix} \frac{1}{2} - \frac{2}{3}e^{-t} + \frac{1}{6}e^{-4t} \\ \frac{1}{3}e^{-t} - \frac{1}{3}e^{-4t} \end{bmatrix}$$

$$y(t) = [1 \ 0] \underline{x}(t) = \left(\frac{1}{2} - \frac{2}{3}e^{-t} + \frac{1}{6}e^{-4t} \right), t \geq 0$$

$$3.10 \text{ (d)} \quad \underline{x}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{aligned} \phi(t) \underline{x}(0) &= \phi(t) \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 8/3 e^{-t} - 5/3 e^{-4t} \\ -4/3 e^{-t} + 10/3 e^{-4t} \end{bmatrix} \end{aligned}$$

$$\underline{x}(t) = \phi(t) \underline{x}(0) + \int_0^t \phi(t-\tau) B u(t-\tau) d\tau$$

$$\begin{aligned} \text{From (c)} \\ &= \begin{bmatrix} 8/3 e^{-t} - 5/3 e^{-4t} \\ -4/3 e^{-t} + 10/3 e^{-4t} \end{bmatrix} + \begin{bmatrix} 1/2 - 2/3 e^{-t} + 1/6 e^{-4t} \\ 1/3 e^{-t} - 1/3 e^{-4t} \end{bmatrix} \end{aligned}$$

$$\underline{x}(t) = \begin{bmatrix} 0.5 + 2e^{-t} - 1.5e^{-4t} \\ -1e^{-t} + 3e^{-4t} \end{bmatrix}$$

$$y(t) = [1 \quad 0] \underline{x}(t) = 0.5 + 2e^{-t} - 1.5e^{-4t}, \quad t \geq 0$$

$$(e) \quad \frac{Y(s)}{U(s)} = C[sI - A]^{-1} B + D$$

$$= [1 \quad 0] \Phi(s) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = [1 \quad 0] \begin{bmatrix} \frac{2}{(s+1)(s+4)} \\ \frac{-1/3 s}{(s+1)(s+4)} \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = \frac{2}{(s+1)(s+4)}, \quad U(s) = 1/s$$

$$Y(s) = \frac{2}{s(s+1)(s+4)} = \frac{0.5}{s} - \frac{2/3}{s+1} + \frac{1/6}{s+4}$$

$$y(t) = 0.5 - 2/3 e^{-t} + 1/6 e^{-4t}, \quad t \geq 0$$

$$3.11(a) [sI - A] = \begin{bmatrix} s & -1 \\ 3 & s+6 \end{bmatrix} \Rightarrow [sI - A]^{-1} = \frac{\begin{bmatrix} s+6 & 1 \\ -3 & s \end{bmatrix}}{s^2 + 6s + 3}$$

$$\Phi(s) = \begin{bmatrix} \frac{s+6}{s^2+6s+3} & \frac{1}{s^2+6s+3} \\ \frac{-3}{s^2+6s+3} & \frac{s}{s^2+6s+3} \end{bmatrix}$$

$$(b) \phi(t) = \mathcal{L}^{-1}\{\Phi(s)\} = \mathcal{L}^{-1}\left\{\begin{bmatrix} \frac{s+6}{(s+0.55)(s+5.45)} & \frac{1}{(s+0.55)(s+5.45)} \\ \frac{-3}{(s+0.55)(s+5.45)} & \frac{s}{(s+0.55)(s+5.45)} \end{bmatrix}\right\}$$

$$\phi(t) = \begin{bmatrix} 1.1124e^{-0.55t} - 0.1124e^{-5.45t} & 0.2041(e^{-0.55t} - e^{-5.45t}) \\ 0.6124(e^{-0.55t} - e^{-5.45t}) & -0.1124e^{-0.55t} + 1.1124e^{-5.45t} \end{bmatrix}$$

$$(c) \underline{x}(t) = \phi(t) \underline{x}(0) + \int_0^t \phi(\tau) B u(t-\tau) d\tau$$

$$\therefore \underline{x}(t) = \int_0^t \phi(\tau) \begin{bmatrix} 0 \\ 1 \end{bmatrix} 1 d\tau$$

$$= \begin{bmatrix} \int_0^t 0.2041(e^{-0.55\tau} - e^{-5.45\tau}) d\tau \\ \int_0^t (-0.1124e^{-0.55\tau} + 1.1124e^{-5.45\tau}) d\tau \end{bmatrix}$$

$$\underline{x}(t) = \begin{bmatrix} -0.371(e^{-0.55t} - 1) - 0.375(e^{-5.45t} - 1) \\ 0.2042(e^{-0.55t} - 1) - 0.2042(e^{-5.45t} - 1) \end{bmatrix}$$

$$\underline{x}(t) = \begin{bmatrix} 0.3335 - 0.371e^{-0.55t} + 0.0375e^{-5.45t} \\ 0.2042(e^{-0.55t} - e^{-5.45t}) \end{bmatrix}$$

$$3.11(d) \quad \underline{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\underline{x}(t) = \phi(t) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \int_0^t \phi(t-\tau) \begin{bmatrix} 0 \\ 1 \end{bmatrix} (1) d\tau$$

$$= \begin{bmatrix} 1.1124e^{-0.55t} - 0.1124e^{-5.45t} \\ 0.6124(e^{-0.55t} - e^{-5.45t}) \end{bmatrix}$$

$$+ \begin{bmatrix} 0.2041 \int_0^t (e^{-0.55\tau} - e^{-5.45\tau}) d\tau \\ \int_0^t (-0.5505e^{-0.55\tau} + 8.7866e^{-5.45\tau}) d\tau \end{bmatrix}$$

$$= \begin{bmatrix} 0.7414e^{-0.55t} - 0.0749e^{-5.45t} \\ 1.6124e^{-0.55t} - 2.2244e^{-5.45t} \end{bmatrix}$$

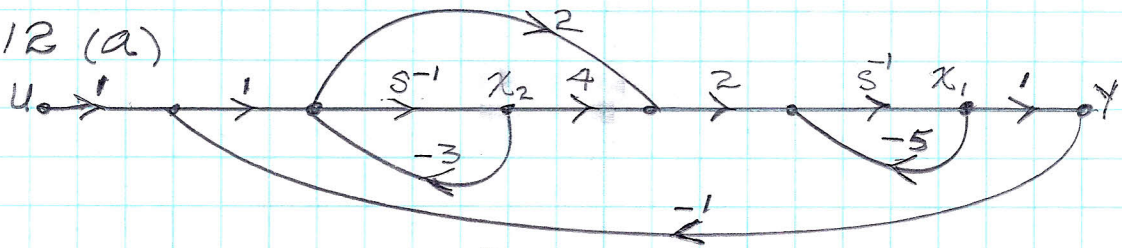
$$(e) \quad \underline{x}(s) = [sI - A]^{-1} \underline{x}(0) + [sI - A]^{-1} B U(s) \quad [(3-20)]$$

$$= \frac{\begin{bmatrix} s+6 & 1 \\ -3 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left(\frac{1}{s}\right)}{(s+0.55)(s+5.45)}$$

$$= \begin{bmatrix} \frac{1}{s(s+0.55)(s+5.45)} \\ \frac{1}{s(s+0.55)(s+5.45)} \end{bmatrix}$$

$$\underline{x}(t) = \begin{bmatrix} 0.3335 - 0.371e^{-0.55t} + 0.0375e^{-5.45t} \\ 0.2042e^{-0.55t} - 0.2042e^{-5.45t} \end{bmatrix}$$

3.12 (a)



$$\dot{\underline{x}} = \begin{bmatrix} -5 & -4 & 8 & -12 \\ -1 & & -3 & \end{bmatrix} \underline{x} + \begin{bmatrix} 4 \\ 1 \end{bmatrix} u = \begin{bmatrix} -9 & -4 \\ -1 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 4 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \end{bmatrix} u$$

$$(b) \quad G_c(s) = \frac{2+4s^{-1}}{1+3s^{-1}} = \frac{2s+4}{s+3}$$

$$G_p(s) = \frac{2s^{-1}}{1+5s^{-1}} = \frac{2}{s+5}$$

$$(c) \quad \frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} = \frac{\frac{4s+8}{(s+3)(s+5)}}{1 + \frac{4s+8}{(s+3)(s+5)}} = \frac{4s+8}{s^2+12s+23}$$

$$(d) \quad |sI-A| = \begin{vmatrix} s & 0 \\ 0 & s \end{vmatrix} - \begin{bmatrix} -9 & -4 \\ -1 & -3 \end{bmatrix} = \begin{vmatrix} s+9 & 4 \\ 1 & s+3 \end{vmatrix} = (s+9)(s+3) - (4)(1)$$

$$|sI-A| = s^2 + 12s + 23$$

$$(e) \quad \Delta = 1 - (-3s^{-1} - 5s^{-1} - 8s^{-2} - 4s^{-1}) + (-3s^{-1})(-5s^{-1}) = 1 - (-12s^{-1} - 8s^{-2}) + 15s^{-2} = s^{-2}(s^2 + 12s + 23) = |sI-A|$$