CHAPTER 2

Solutions to Chapter-End Problems

A. Key Concepts

Simple Interest:

2.1 $i = 0.12$ per year
P = 500
N = 2 years

I = PiN = 500(0.12)(2) = 120

At the end of two years, the amount of interest owed will be $120.

2.2   P = 3000
N = 6 months
$i = 0.09$ per year
  $= 0.09/12$ per month, or $0.09/2$ per six months

P + I   = P + PiN = P(1 + iN)
        = 3000[1 + (0.09/12)(6)] = 3135
or
        = 3000[1 + (0.09/2)(1)] = 3135

The total amount due is $3135, which is $3000 for the principal amount and $135 in interest.

2.3   I = 150
N = 3 months
$i = 0.01$ per month

P = I/(iN) = 150/[(0.01)(3)] = 5000

A principal amount of $5000 will yield $150 in interest at the end of 3 months when the interest rate is 1% per month.

2.4   I = 2400
N = 2 years
P = 12 000

$i = I/(PN) = 2400/[(12\ 000)(2)] = 0.1$

Copyright © 2013 Pearson Canada Inc.

The simple interest rate is 10% per year.
Nominal and Effective (Discrete) Compound Interest:

2.5 $P = 2000$
   $N = 5$ years
   $i = 0.12$ per year

   $F = P(1+i)^N = 2000(1+0.12)^5 = 3524.68$

   The bank account will have a balance of $3525 at the end of 5 years.

2.6   **(a)** $P = 1000$
      $i = 0.10$ per year
      $N = 2$ years

   $F = P(1+i)^N = 1000(1+0.10)^2 = 1210$

   The balance at the end of 2 years will be €1210.

   (b) $P = 900$
      $i = 0.12$ per year $= 0.01$ per month
      $N = 2$ years $= 24$ months

   $F = P(1+i)^N = 900(1+0.01)^{24} = 1142.76$

   The balance at the end of 24 months (2 years) will be €1143.

2.7   From: $F = P(1 + i)^N$

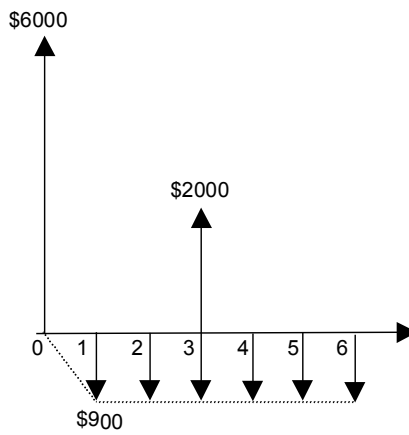 $P = F/(1 + i)^N = 50\,000/(1 + 0.02)^{20} = 33\,648.57$

   Greg should invest about $33 649.

2.8 $F = P(1 + i)^N$
   $50\,000 = 20\,000(1 + i)^{20}$
   $(1+i)^{20} = 5/2$
   $i = (5/2)^{1/20} - 1 = 0.04688 = 4.688\%$ per quarter $= 18.75\%$ per year
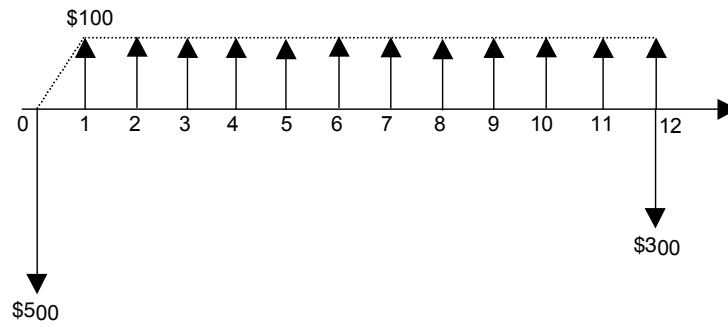
   The investment certificate would have to pay at least 18.75% nominal interest, compounded quarterly.
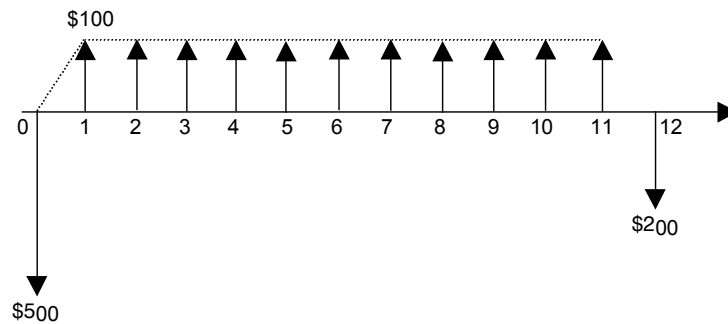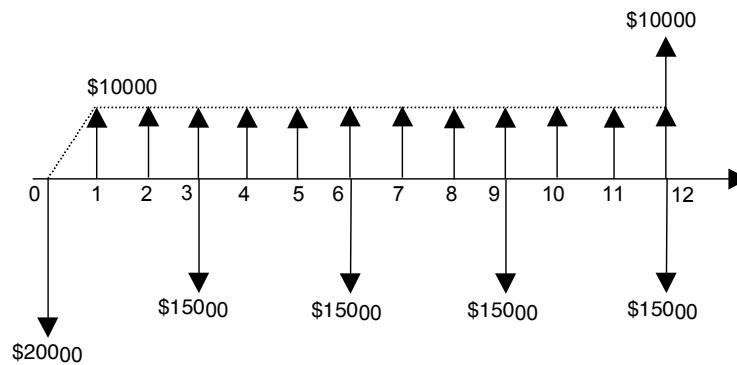
Cash Flow Diagrams:

2.9 Cash flow diagram:

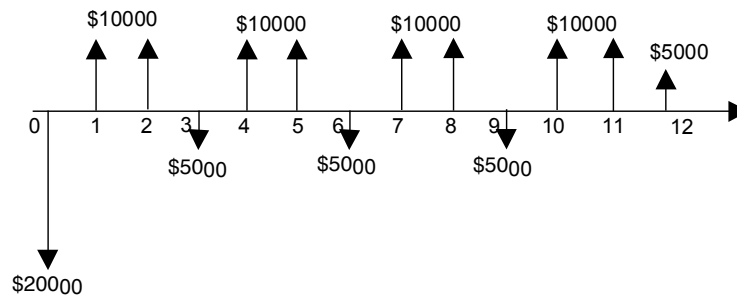**2.10** Showing cash flow elements separately:

Showing net cash flow:

2.11   Showing cash flow elements separately:

Showing net cash flow:

2.12   The calculation of the net cash flow is summarized in the table below.

Time	Payment	Receipt	Net
0	20		−20
1		30	30
2		33	33
3	20	36.3	16.3
4		39.9	39.9
5		43.9	43.9
6	20	48.3	28.3
7		53.1	53.1
8		58.5	58.5
9	20	64.3	44.3
10		70.7	70.7
11		77.8	77.8
12	20	85.6	65.6

Cash flow diagram:
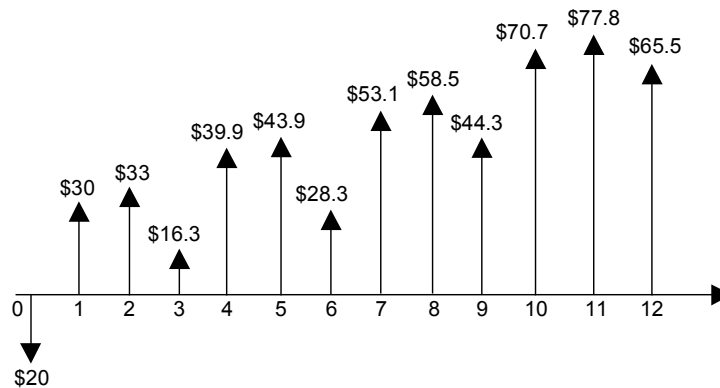
B. Applications

2.13 I = 190.67
P = 550
N = 4 1/3 = 13/3 years

i = I/(PN) = 190.67/[550(13/3)] = 0.08

The simple interest rate is 8% per year.

2.14 $F = P(1 + i)^N$
$50\ 000 = 20\ 000(1 + 0.02)^N$
$(1.02)^N = 5/2$
$N = \ln(5/2)/\ln(1.02) = 46.27$ quarters = 11.57 years

Greg would have to invest his money for about 11.57 years to reach his target.

2.15 $F = P(1 + i)^N$
$= 20\ 000(1 + 0.02)^{20} = 29\ 718.95$

Greg would have accumulated about $29 719.

2.16 First, we shift the time reference point from now to three years ago (i.e., P = 500 000 and F = 650 000). The computations relating the two amounts remain unchanged. Then the formula $F = P(1 + i)^N$ must be solved in terms of i to answer the question: $i = (F/P)^{1/N} - 1$.

(a) With a compounding period of one year, the number of periods is N = 3. This gives:

$i = (F/P)^{1/N} - 1 = (650\ 000/500\ 000)^{1/3} - 1 = 0.09139$

The annual interest rate earned was approximately 9.14%.

(b) With a compounding period of one month, the number of periods is $N = 36$. This gives:

$i = (F/P)^{1/N} - 1 = (650\ 000/500\ 000)^{1/36} - 1 = 0.007315$

The monthly interest rate earned was approximately 0.73%. On an annual basis this would be stated as $(0.73)(12) = 8.76\%$ (nominal) interest.

2.17 **(a)** $P = 5000$
$i = 0.05$ per six months
$F = 8000$

From: $F = P(1 + i)^N$

$N = \ln(F/P)/\ln(1 + i) = \ln(8000/5000)/\ln(1 + 0.05) = 9.633$

The answer that we get is 9.633 (six-month) periods. But what does this mean? It means that after 9 compounding periods, the account will not yet have reached $8000. (You can verify yourself that the account will contain $7757). Since compounding is done only every six months, we must, in fact, wait 10 compounding periods, or 5 years, for the deposit to be worth *more* than $8000. At that time, the account will hold $8144.

(b) $P = 5000$
$r = 0.05$ (for the full year)
$F = 8000$
$i = r/m = 0.05/2 = 0.025$ per six months

From: $F = P(1 + i)^N$

$N = \ln(F/P)/\ln(1 + i) = \ln(8000/5000)/\ln(1 + 0.025) = 19.03$

We must wait 20 compounding periods, or 10 years, for the deposit to be worth more than $8000.

2.18 $P = 500$
$F = 708.31$
$i = 0.01$ per month

From: $F = P(1 + i)^N$

$N = \ln(F/P)/\ln(1 + i) = \ln(708.31/500)/\ln(1 + 0.01) = 35.001$

The deposit was made 35 months ago.

2.19 **(a)** P = 1000
   i = 0.1
   N = 20

$F = P(1 + i)^N = 1000(1+0.1)^{20} = 6727.50$

About $6728 could be withdrawn 20 years from now.

(b) $F = PiN = 1000(0.1)(20) = 2000$

Without compounding, the account would only accumulate $2000 over 20 years.

2.20 Let P = X and F = 2X.

(a) By substituting F = 2X and P = X into the formula, $F = P + I = P + PiN$, we get

$2X = X + XiN = X(1 + iN)$
$2 = 1 + iN$
$iN = 1$
$N = 1/i = 1/0.11 = 9.0909$

It will take 9.1 years.

(b) From $F = P(1 + i)^N$, we get $N = \ln(F/P)/\ln(1 + i)$. By substituting F = 2X and P = X into this expression of N,

$N = \ln(2X/X)/\ln(1 + 0.11) = \ln(2)/\ln(1.11) = 6.642$

Since compounding is done every year, the amount will not double until the 7th year.

(c) Given r = 0.11 per year, the effective interest rate is $i = e^r - 1 = 0.1163$.

From $F = P(1 + i)^N$, we get $N = \ln(F/P)/\ln(1 + i)$. By substituting F = 2X and P = X into this expression of N,

$N = \ln(2X/X)/\ln(1 + 0.1163) = \ln(2)/\ln(1.1163) = 6.3013$

Since interest is compounded continuously, the amount will double after 6.3 years.

2.21   **(a)** $r = 0.25$ and $m = 2$

$i_e = (1 + r/m)^m - 1 = (1 + 0.25/2)^2 - 1 = 0.26563$

The effective rate is approximately 26.6%.

(b) $r = 0.25$ and $m = 4$

$i_e = (1 + r/m)^m - 1 = (1 + 0.25/4)^2 - 1 = 0.27443$

The effective rate is approximately 27.4%.

(c) $i_e = e^r - 1 = e^{0.25} - 1 = 0.28403$

The effective rate is approximately 28.4%.

2.22   **(a)** $i_e = 0.15$ and $m = 12$

From: $i_e = (1 + r/m)^m - 1$

$r = m[(1 + i_e)^{1/m} - 1] = 12[(1 + 0.15)^{1/12} - 1] = 0.1406$

The nominal rate is 14.06%.

(b) $i_e = 0.15$ and $m = 365$

From: $i_e = (1 + r/m)^m - 1$

$r = m[(1 + i_e)^{1/m} - 1] = 365[(1 + 0.15)^{1/365} - 1] = 0.13979$

The nominal rate is 13.98%.

(c) For continuous compounding, we must solve for r in $i_e = e^r - 1$:

$r = \ln(1 + i_e) = \ln(1 + 0.15) = 0.13976$

The nominal rate is 13.98%.

2.23 $F = P(1 + i)^N$
14 800 = 665(1 + i)^{64}
i = 0.04967

The rate of return on this investment was 5%.

2.24 The present value of X is calculated as follows:

$F = P(1 + i)^N$
$3500 = X(1 + 0.075)^5$
X = 2437.96

The value of X in 10 years is then:

$F = 2437.96(1 + 0.075/365)^{3650} = 4909.12$

The present value of X is $2438. In 10 years, it will be $4909.

2.25 r = 0.06 and m = 365

$i_e = (1 + r/m)^m - 1 = (1 + 0.06/365)^{365} - 1 = 0.0618$

The effective interest rate is about 6.18%.

2.26 Effective interest for continuous interest account:
$i_e = e^r - 1 = e^{0.0799} - 1 = 0.08318 = 8.318\%$

Effective interest for daily interest account:
$i_e = (1 + r/m)^m - 1 = (1 + 0.08/365)^{365} - 1 = 0.08328 = 8.328\%$

No, your money will earn slightly less with continuous compounding.

2.27 i_e(weekly) = $(1 + r/m)^m - 1 = (1 + 0.055/52)^{52} - 1 = 0.0565 = 5.65\%$
i_e(monthly) = $(1 + r/m)^m - 1 = (1 + 0.07/12)^{12} - 1 = 0.0723 = 7.23\%$

2.28 i_e(Victory Visa) = $(1 + r/m)^m - 1 = (1 + 0.26/365)^{365} - 1 = 0.297 = 29.7\%$
i_e(Magnificent Master Card) = $(1 + 0.28/52)^{52} - 1 = 0.322 = 32.2\%$
i_e(Amazing Express) = $(1 + 0.3/12)^{12} - 1 = 0.345 = 34.5\%$

Victory Visa has the lowest effective interest rate, so based on interest rate, Victory Visa seems to offer the best deal.

2.29 $F = P(1 + i_e)^N$
$796.25 = 750(1 + i_e)^1$

$i_e = 0.06167$

$i_e = (1 + i_s)^m - 1$
$0.06167 = (1 + i_s)^{12} - 1$
$i_s = 0.004999$

The nominal monthly interest rate was 0.5%.

2.30 First, determine the effective interest rate that May used to get $2140.73 from $2000. Then, determine the nominal interest rate associated with the effective interest:

$F = P(1 + i_e)^N$
$2140.73 = 2000(1 + i_e)^1$
$i_e = 0.070365$

$i_e = e^r - 1$
$0.070365 = e^r - 1$
$r = 0.068$

The *correct* effective interest rate is then:

$i_e = (1 + r/m)^m - 1 = (1 + 0.068/12)^{12} - 1 = 0.07016$

The correct value of $2000 a year from now is:

$F = P(1 + i_e)^N = 2000(1 + 0.07016)^1 = \$2140.32$

2.31 The calculation of the net cash flow is summarized in the table below.

| Time	Investment A			Investment B		
	Payment	**Receipt**	**Net**	**Payment**	**Receipt**	**Net**
0	2400		−2400			0
1		250	250		50	50
2		250	250	500	100	−400
3		250	250		150	150
4		250	250	500	200	−300
5		250	250		250	250
6		250	250	500	300	−200
7		250	250		350	350
8		250	250	500	400	−100
9		250	250		450	450
10		250	250	500	500	0
11		250	250		550	550
12	200	250	50	500	600	100

Cash flow diagram for investment A:
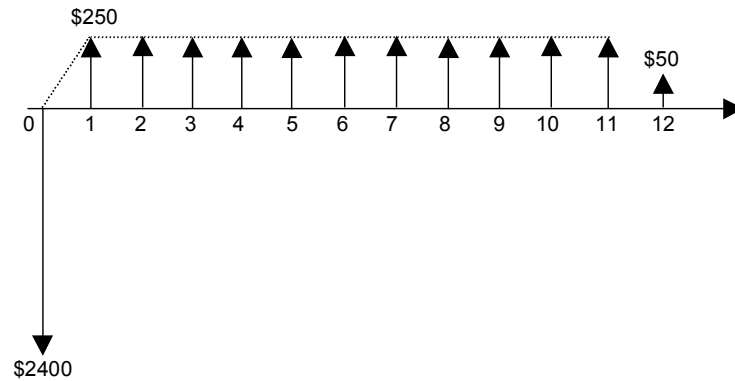
Cash flow diagram for investment B:
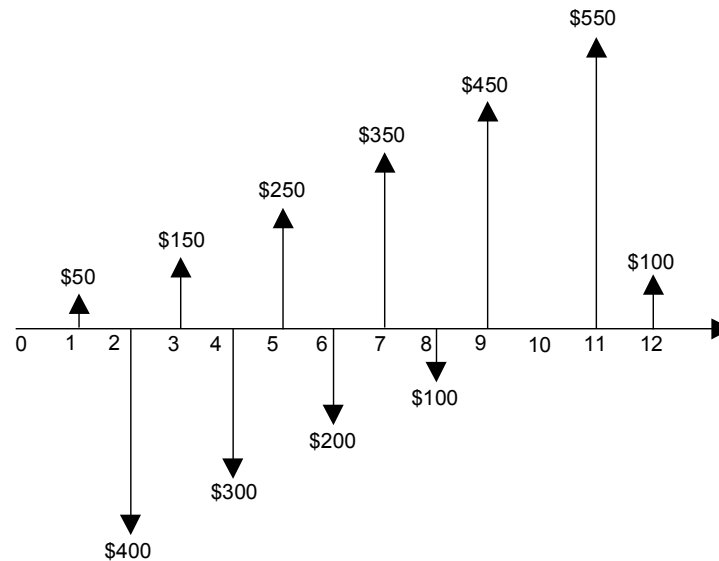
Since the cash flow diagrams do not include the time factor (i.e., interest), it is difficult to say which investment may be better by just looking at the diagrams. However, one can observe that investment A offers uniform cash inflows whereas B alternates between positive and negative cash flows for the first 10 months. On the other hand, investment A requires $2400 up front, so it may not be a preferred choice for someone who does not have a lump sum of money now.

2.32 **(a)** The amount owed at the end of each year on a loan of $100 using 6% interest rate:

Year	Simple Interest	Compound Interest
0	100	100.00
1	106	106.00
2	112	112.36
3	118	119.10
4	124	126.25
5	130	133.82
6	136	141.85
7	142	150.36
8	148	159.38
9	154	168.95
10	160	179.08
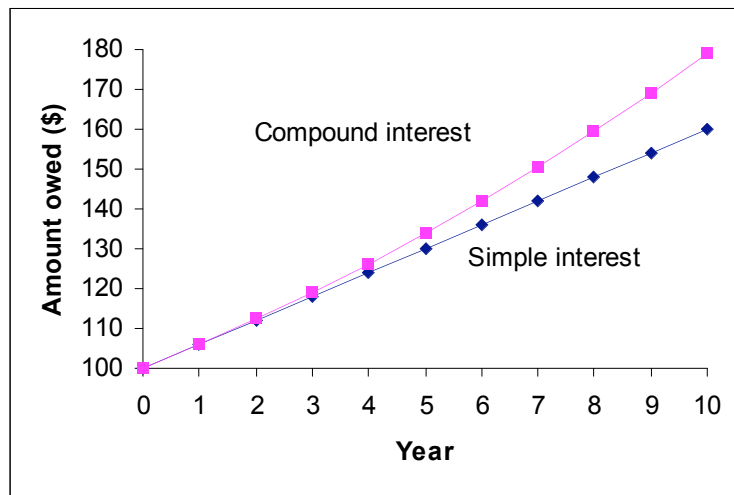
(b) The amount owed at the end of each year on a loan of $100 using 18% interest rate:

Year	Simple Interest	Compound Interest
0	100	100.00
1	118	118.00
2	136	139.24
3	154	164.30
4	172	193.88
5	190	228.78
6	208	269.96
7	226	318.55
8	244	375.89
9	262	443.55
10	280	523.38
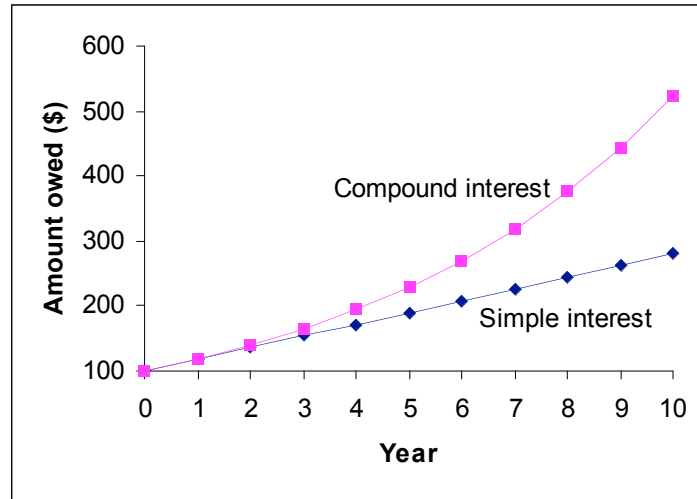
2.33 **(a)** From $F = P(1 + i)^N$, we get $N = \ln(F/P)/\ln(1 + i)$.

At $i = 12\%$:
$N = \ln(1\,000\,000/0.01)/\ln(1 + 0.12) = 162.54$ years

At $i = 18\%$:
$N = \ln(F/P)/\ln(1 + i) = \ln(1\,000\,000/0.01)/\ln(1 + 0.18) = 111.29$ years

(b) The growth in values of a penny as it becomes a million dollars:

| Year	At 12%	At 18%
0	0.01	0.01
10	0.03	0.05
20	0.10	0.27
30	0.30	1.43
40	0.93	7.50
50	2.89	39.27
60	8.98	205.55
70	27.88	1 075.82
80	86.58	5 630.68
90	268.92	29 470.04
100	835.22	154 241.32
110	2 594.07	807 273.70
120	8 056.80	4 225 137.79
130	25 023.21	22 113 676.39
140	77 718.28	115 739 345.70
150	241 381.18	605 760 702.48
160	749 693.30	3 170 451 901.72
170	2 328 433.58	16 593 623 884.84
180	7 231 761.26	86 848 298 654.83

2.34 From the table and the charts below, we can see that £100 will double in
(a) 105 months (or 8.75 years) if interest is 8% compounded monthly

(b) 13 six-month periods (6.5 years) if interest is 11% per year, compounded semi-annually

**(c)** 5.8 years if interest is 12% per year compounded continuously

| Month	8%	11%	12%
0	100.00	100.00	100.00
12	108.30	111.30	112.75
24	117.29	123.88	127.12
36	127.02	137.88	143.33
48	137.57	153.47	161.61
60	148.98	170.81	182.21
72	161.35	190.12	205.44
84	174.74	211.61	231.64
96	189.25	235.53	261.17
108	204.95	262.15	294.47
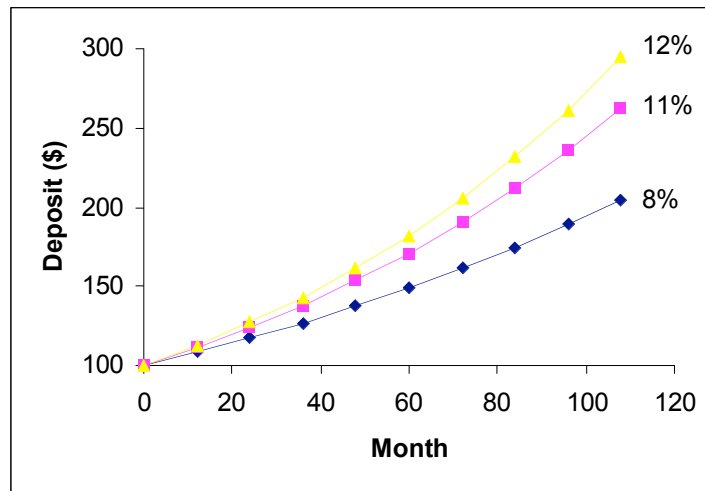
2.35 Effective interest rate for various nominal interest rates as the compounding period becomes shorter and shorter:

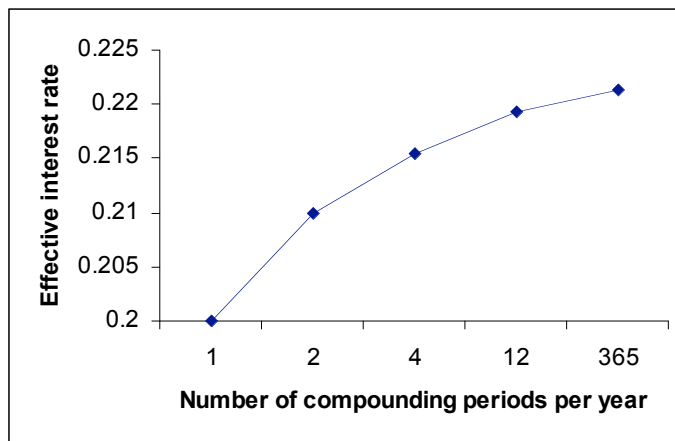| Number of Compounding Periods per Year	Effective Interest Rate for		
	6%	10%	20%
1	0.0600	0.1000	0.2000
2	0.0609	0.1025	0.2100
4	0.0614	0.1038	0.2155
12	0.0617	0.1047	0.2194
365	0.0618	0.1052	0.2213

(a) 6% interest:
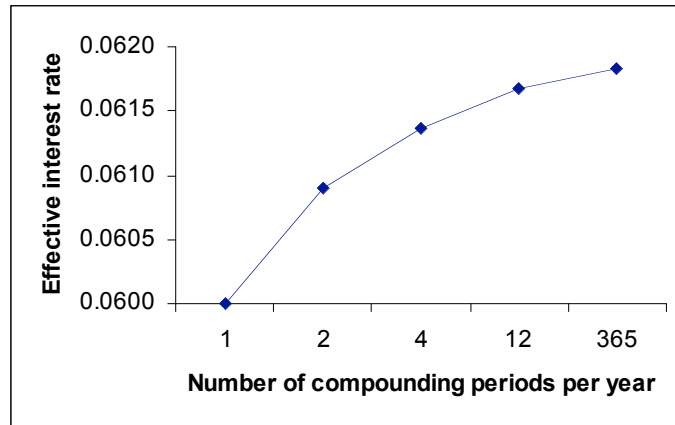
(b) 10% interest:
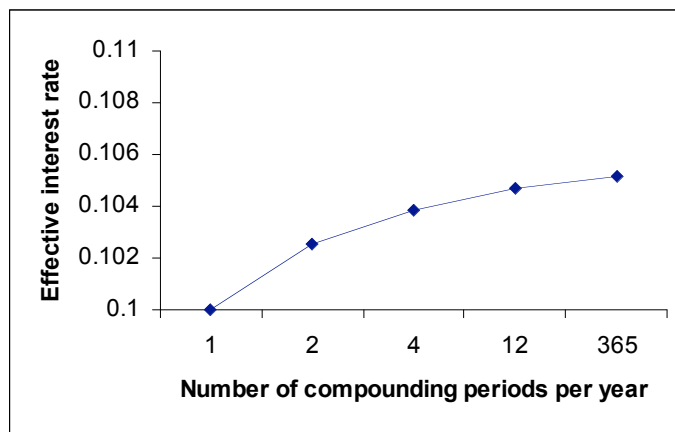
(c) 20% interest

C. More Challenging Problems

2.36 The present worth of each instalment:

Instalment	F	P
1	100000	100000
2	100000	90521
3	100000	81941
4	100000	74174
5	100000	67143
6	100000	60779
7	100000	55018
8	100000	49803
9	100000	45082
10	100000	40809
	Total	665270

Sample calculation for the third instalment, which is received at the end of the second year:

$P = F/(1 + r/m)^N = 100\ 000/(1 + 0.10/12)^{24} = 81\ 941$

The total present worth of the prize is $665 270, not $1 000 000.

2.37 The present worth of the lottery is $665 270. If you take $300 000 today, that leaves a present worth of $365 270. The future worth of $365 270 in 5 years (60 months) is:

$F = P(1 + r/m)^N = 365\ 270(1 + 0.10/12)^{60} = 600\ 982$

The payment in 5 years will be $600 982.

2.38 Brand 1: cost is $4000

Brand 2: cost is $2500 now and $1700 in two years. The present worth of the $1700 is:

$P = F/(1 + r/m)^m = 1700/(1 + 0.08/12)^{24} = 1449.41$

The total present cost of Brand 2 is 2500 + 1449 = $3949.

Brand 2 is about $51 less expensive than Brand 1.

2.39 The first investment has an interest rate of 1% per month (compounded monthly), the second 6% per 6 month period (compounding semi-annually).

(a) Effective semi-annual interest rate for the first investment:
$i_e = (1 + i_s)^N - 1 = (1 + 0.01)^6 - 1 = 0.06152 = 6.152\%$

Effective semi-annual interest rate for the second investment is 6% as interest is already stated on that time period.

(b) Effective annual interest rate for the first investment:
$i_e = (1 + i_s)^N - 1 = (1 + 0.01)^{12} - 1 = 0.1268 = 12.68\%$

Effective annual interest rate for the second investment:
$i_e = (1 + i_s)^N - 1 = (1 + 0.06)^2 - 1 = 0.1236 = 12.36\%$

(c) The first investment is the preferred choice because it has the higher effective interest rate, regardless of on what period the effective rate is computed.

2.40 **(a)** $i = 0.15/12 = 0.0125$, or 1.25% per month

The effective annual rate is:

$i_e = (1 + i)^m - 1 = (1 + 0.0125)^{12} - 1 = 0.1608$ or 16.08%

(b) P = 50 000
   N = 12
   $i = 0.15/12 = 0.0125$, or 1.25% per month

$F = P(1 + i)^N = 50\ 000(1 + 0.0125)^{12} = 58\ 037.73$

You will have $58 038 at the end of one year.

(c) Adam's Fee = 2% of F = 0.02(58037.73) = 1160.75

Realized F = 58 037.73 − 1160.75 = 56 876.97

The effective annual interest rate is:

$F = P(1 + i)^1$
$56\ 876.97 = 50\ 000(1 + i)$
$i = 56\ 876.97/50\ 000 - 1 = 0.1375$ or 13.75%

The effective interest rate of this investment is 13.75%.

2.41 Market equivalence does not apply as the cost of borrowing and lending is not the same. Mathematical equivalence does not hold as neither 6% nor 8% is the rate of exchange between the $100 and the $110 one year from now. Decisional equivalence holds as you are indifferent between the $100 today and the $110 one year from now.

2.42 Decisional equivalence holds since June is indifferent between the two options. Mathematical equivalence does not hold since neither 8% compounded monthly (lending) or 8% compounded daily (borrowing) is the rate of exchange representing the change in the house price ($110 000 now and $120 000 a year later is equivalent to the effective interest rate of 9.09%). Market equivalence also does not hold since the cost of borrowing and lending is not the same.

2.43 **(a)** The amount of the initial deposit, P, can be found from $F = P(1 + i)^N$ with F = $3000, N = 36, and i = 0.10/12.

(b) Having determined P = $2225, then we can figure out the size of the deposit at the end of years 1, 2 and 3.

If you had not invested in the fixed interest rate investment, you would have obtained interest rates of 8%, 10%, and 14% for each of the three years. The table below shows how much the initial deposit would have been worth at the end of each of the three years if you had been able to reinvest each year at the new rate. Because of the surge in interest rates in the third year, with 20/20 hindsight, you would have been better off (by about $60) not to have locked in at 10% for three years.

Year	Fixed Interest Rate	Varying Interest Rate
0	2225	2225
1	2458	2410
2	2715	2662
3	3000	3060
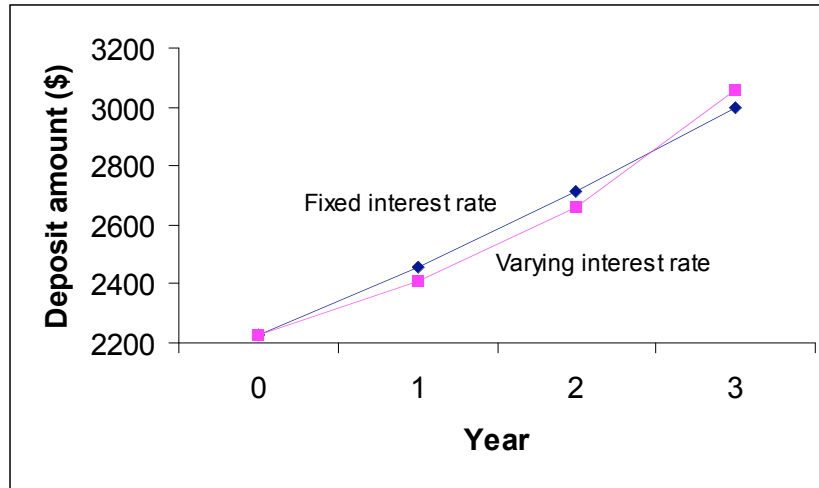
2.44 Interest rate *i* likely has its origins in commonly available interest rates present in Marlee's financial activities such as investing or borrowing money. Interest rate *j* can only be determined by having Marlee choose between X and Y to determine at which interest rate Marlee is indifferent between the choice. Interest rate *k* probably does not exist for Martlee, since it is unlikely that she can borrow and lend money at the same interest rate. If for some reason she could, then *k=j*.

Also, *i* could be either greater or less than *j*.

Notes for Mini-Case 2.1

3) Money will always be lost over the year. If money could be gained, everybody would borrow as much money as possible to invest.

Solutions to All Additional Problems

Note: Solutions to odd-numbered problems are provided on the Student CD-ROM.

2S.1

You can assume that one month is the shortest interval of time for which the quoted rental rates and salaries apply. Assembling the batteries will require 24 person-months, and the associated rental space. To maximize the interest you receive from your savings, and minimize the interest you pay on your line of credit, you should defer this expenditure till as late in the year as possible. So you leave your money in the bank till December 1, then purchase the necessary materials and rent the industrial space. Assume that salaries will be paid at the end of the month.

As of December 1, you have $100\ 000(1.005)^{11}$ = $105 640 in the bank.

You need to spend $360 000 on materials and $240 000 to rent space. After spending all you have in the bank, you therefore need to borrow an additional $494 360 against your line of credit.

As of December 31, you owe $494\ 360(1.01)$ = $499 304 to the bank, and you owe $240 000 in salaries. So after depositing the government cheque and paying these debts, you have

   1 200 000 – 499 304 – 240 000 = $460 696 in the bank.
This example illustrates one of the reasons why Just-in-Time ("JIT") manufacture has become popular in recent years: You want to minimize the time that capital is tied up. An additional motivation for JIT would become evident if you were to consider the cost of storing the finished batteries before delivery.

Be aware, however, that the JIT approach also carries risks. December is typically a time when labour, space, and credit are in high demand so there is a possibility that the resources you need will be unavailable or more expensive than expected, and there will then be no time to recover. We will look at methods for managing risk in Chapter 12.

2S.2

We want to solve the equation

Future worth = Present worth $(1+i)^N$, where the future worth is twice the present worth.

So we have

$2 = (1+i)^N$

Taking logarithms on both sides, we get

$N = \log(2) / \log(1+i)$

For small values of *i*, log (1+*i*) is approximately *i* (this can be deduced from the Taylor series).

And log(2) = 0.69315. So, expressing *i* as a percentage rather than a fraction, we have:

$N = 69.3 / i$

Since this is only an approximation, we will adjust 69.3 to an easily factored integer, 72, thus obtaining

$N = 72 / i$

2S.3

Gita is paying 15% on her loan over a two-week period, so the effective annual rate is

$(1.15^{26} - 1) \times 100\% = 3686\%$

The Grameen Bank was awarded the Nobel Peace Prize in 2006 for making loans available to poor investors in Bangladesh at more reasonable rates.

2S.4

Five hundred years takes us beyond the scope of the tables in Appendix A, so we employ the formula

$P = F / (1+i)^N$

to find the present value of the potential loss.

In this case, we have P = $1 000 000 000 / $(1.05)^{500}$ = $0.025, or two-and-a-half cents. This implies that it is not worth going to any trouble to make the waste repository safe for that length of time.

This is a rather troubling conclusion, because the example is not imaginary; the U.S., for example, is currently trying to design a nuclear waste repository under

Yucca Mountain in Nevada that will be secure for ten thousand years—twenty times as long as in our example. It is not clear how the engineers involved in the project can rationally plan how to allocate their funds, since the tool we usually use for that purpose—engineering economics—gives answers that seem irrational.

2S.5

There is no "right" answer to this question, which is intended for discussion in class or in a seminar. Some of the arguments that might be advanced are as follows:

One option is to say, "You cannot play the numbers game with human lives. Each life is unique and of inestimable value. Attempting to treat lives on the same basis as dollars is both cold-blooded and ridiculous."

But this really won't do. Medical administrators, for example, do have a responsibility to save lives, and they have limited resources to meet this responsibility. If they are to apportion their resources rationally, they must be prepared to compare the results of different strategies.

To support the point of view that future lives saved should be discounted by some percentage in comparison with present lives, the following arguments might be offered:

1.   Suppose we make the comparison fifty years in the future. If we spend our resources on traffic police, we will have saved the lives of those who would have died in accidents, and, because we spent the money that way, the world of fifty years hence will also contain the descendants of those who would have died. So the total number of live humans will be increased by more than fifty.

(This argument assumes that creating a new life is of the same value as preserving an existing life. We do not usually accept that assumption; for example, many governments may promote population control by limiting the number of children born, but it would be unacceptable to control the population by killing off the old and infirm. To give another example, if I am accused of murder, it would not be an acceptable defence to argue that I have fathered two children, and have thus made a greater contribution to society than a law-abiding bachelor.)

2.   Just as I charge you interest on a loan because of the uncertainty of what might happen between now and the due date—you could go bankrupt, I could die, etc.—so we should discount future lives saved because we cannot anticipate how the world will change before the saving is realized. For example, a cure for cancer could be discovered ten years from now, and then all the money spent on the anti-smoking campaign will have been wasted, when it could have been used to save lives lost in highway accidents.

3.   We should not be counting numbers of lives, but years of human life. Thus it is better to spend the money on preventing accidents, because these kill

people of all ages, while cancer and heart disease are mostly diseases of later life; so more years of human life are saved by the first strategy.

4.   The world population is growing. Thus, for humanity as a whole, losing a fixed number of lives can more easily be born in the future than it can now. (This is an extension of the argument that it is worse to kill a member of an endangered species than of a species that is plentiful.)