Chapter 2 Exercises

2.1

The electric field outside a charged sphere is the same as for a point source,

$$E(r) = \frac{Q}{4\pi\epsilon_0 r^2},$$

where Q is the charge on the inner surface of radius a. The potential drop is the integral

$$\Delta V = -\int_b^a \frac{Q}{4\pi\epsilon_0 r^2} dr = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right).$$

The capacitance is therefore

$$C = \frac{Q}{\Delta V} = 4\pi\epsilon_0\left(\frac{ab}{b-a}\right)$$

This is inversely proportional to the resistance found in Exercise 1.4.

2.2

The planar capacitor formula is

$$C = \frac{A\kappa\epsilon_0}{d}.$$

Solving for A,

$$A = \frac{Cd}{\kappa\epsilon_0} = \frac{(1\text{ F})(10^{-9}\text{ m})}{10(8.85 \times 10^{-12}\text{ C}^2/\text{N}^2 - \text{m}^2)} = 1.1\text{ m}^2.$$

2.3

The solenoid inductor formula is

$$L = \frac{A\mu_0 N^2}{l}.$$

The loop area is $A \approx \pi(5\text{ cm}^2) = 76\text{ cm}^2 = 7.6 \times 10^{-3}\text{ m}^2$. The density of coils is $N/l \approx (6^2 - 4^2)/(.05^2)(2\pi(.05\text{ m})) = 2.5 \times 10^4/\text{m}$.

Solving for l,

$$l = \frac{L}{A\mu_0(N/l)^2} = \frac{1\text{ H}}{(7.6 \times 10^{-3}\text{ m}^2)(4\pi \times 10^{-7}\text{ N/A}^2)(2.5 \times 10^4/\text{m})^2} = 17\text{ cm}.$$

2.4

Integrate the flux over a rectangular section between the wires of length l. The magnetic field from a wire is

$$B = \frac{\mu_0 I}{2\pi r}.$$

The flux from one wire is

$$\Phi = l \int_b^a \frac{\mu_0 I}{2\pi r} dr = \frac{\mu_0 I l}{2\pi} \ln(a/b).$$

The second wire contributes flux in the same direction, so the total flux is

$$\Phi_{tot} = \frac{\mu_0 I I}{\pi} \ln(b/a).$$

The voltage drop is

$$\Delta V = \frac{\partial \Phi_{tot}}{\partial t} = \frac{\mu_0 l}{\pi} \ln(a/b) \frac{\partial I}{\partial t}.$$

Therefore

$$L = \frac{\mu_0 l}{\pi} \ln(a/b).$$

[Note: Technically we should account for the magnetic field inside each wire. The current inside the radius r is Ir^2/b^2. $B(2\pi r) = \mu_0 Ir^2/b^2$, so

$$B = \frac{\mu_0 r I}{2\pi b^2}$$

inside the wire. The integral of the flux is inside the wire is

$$\Phi = l \int_0^b \frac{\mu_0 r I}{2\pi b^2} = l \frac{\mu_0 I}{4\pi}.$$

The contribution to the inductance L from both wires is then

$$L' = \frac{\mu_0 l}{2\pi}.$$

which implies $L/L' = 2\ln(a/b)$, which means the field inside the wire is negligible if $a \gg b$.]

2.5

The Biot-Savart law is

$$d\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{R}}{R^3}.$$

For a loop of radius L centered at (0,0), the unit vector along $d\vec{l}$ is

$$\hat{\theta} = -\hat{x} \sin\theta + \hat{y} \cos\theta.$$

The distance from a point of the circle to a point r on the x-axis is

$$R = |L(\cos\theta, \sin\theta) - (r, 0)| = \sqrt{(L\cos\theta - r)^2 + L^2 \sin^2\theta}$$

The direction of $\vec{R}$ is

$$\hat{R} = -\frac{L(\cos\theta, \sin\theta) - (r, 0)}{R} = -\frac{1}{R}(\hat{x}(L\cos\theta - r) + \hat{y} L \sin\theta).$$

15

The cross product is

$$\hat{\theta} \times \hat{R} = -\frac{\hat{z}}{R}(r\cos\theta - L).$$

The integral of the Biot-Savart element is then

$$\vec{B} = -\frac{\mu_0 I \hat{z}}{4\pi} \int_0^{2\pi} L d\theta \frac{r\cos\theta - L}{[(L\cos\theta - r)^2 + L^2 \sin^2\theta]^{3/2}}$$

This function involves elliptic integrals. Doing it numerically for $r = 0$ (the center of the loop), gives

$$\vec{B} = \frac{\mu_0 I \hat{z}}{4L},$$

while at $r = L/2$, it is

$$\vec{B} = (3.91/\pi)\frac{\mu_0 I \hat{z}}{4L}.$$

The magnitude of B rises sharply near $r = L$.

2.6

$$V_o = IR = RC\frac{\partial}{\partial t}(V - V_o) = -RC\frac{\partial V_o}{\partial t}.$$

This has the solution

$$V_o(t) = Ve^{-t/RC}.$$

2.7

Kirchhoff:

$$0 = RC\frac{\partial V_o}{\partial t} + V_o.$$

This has the general solution

$$V_o(t) = V_1 e^{-t/RC} + V_2.$$

Setting this to V at $t = t_0$ gives

$$V = V_1 e^{-t_0/RC} + V_2.$$

The current through the capacitor is

$$I(t_0) = -\frac{V}{R} = C\frac{\partial}{\partial t}(V_1 e^{-t/RC} + V_2 R)\Big|_{t=t_0} = C\left(-\frac{V_1}{RC}e^{-t_0/RC}\right)$$

which implies $V_1 = Ve^{t_0/RC}$. Plugging this into the above gives $V_2 = 0$, so the solution is

$$V_o(t) = Ve^{-(t-t_0)/RC}.$$

This is decay to zero with the same time constant RC.

2.8

The integrator circuit of Figure 2.16(a) is governed by equation (2.4.13),

$$V_o + RC\dot{V_o} = V_i.$$

For $V_o = \sin\omega t$, this implies

$$\sin\omega t + \omega RC \cos\omega t = V_i.$$

When $\omega RC \gg 1$, the $\sin\omega t$ term is negligible, and V_i is proportional to $\dot{V_o}$, i.e., V_o is proportional to the antiderivative of V_i.

The differentiator circuit of Figure 2.16(b) is governed by equation (2.4.17),

$$\dot{V_o} + \frac{V_o}{RC} = \dot{V_i}.$$

For $V_o = \sin\omega t$, this implies

$$\omega \cos\omega t + \frac{1}{RC}\sin\omega t = \dot{V_i}.$$

When $\omega RC \ll 1$, the $\cos\omega t$ term is negligible, and $\dot{V_i}$ is proportional to V_o, i.e., V_o is proportional to the derivative of V_i.

2.9

This is the circuit shown in Fig. 2.18(b). We have

$$\frac{V_o}{V_i} = \frac{R}{R + Z_C} = \frac{R}{R - i/\omega C}$$

$$\left|\frac{V_o}{V_i}\right|^2 = \frac{R}{R - i/\omega C}\frac{R}{R + i/\omega C} = \frac{R^2}{R^2 + 1/\omega^2 C^2} = \frac{1}{1 + 1/\omega^2 R^2 C^2}.$$

2.10

$$\frac{V_o}{V_i} = \frac{Z_L}{R + Z_L} = \frac{i\omega L}{R + i\omega L}$$

This is a high-pass filter.

2.11 From Exercise 2.9, we have

$$\left|\frac{V_o}{V_i}\right|^2 = \frac{1}{1 + 1/\omega^2 R^2 C^2}$$

Solve for ω:

$$\omega = \frac{1}{RC}\left(\frac{1}{|V_o/V_i|^2} - 1\right)^{-1/2}$$

a)
$$-3 \text{ dB} = 10 \log_{10} |V_o/V_i|^2$$
$$|V_o/V_i|^2 = 10^{-.3} = 0.5 \rightarrow \omega = 1/RC$$

b)
$$|V_o/V_i|^2 = 10^{-1} = 0.1 \rightarrow \omega = 1/3RC$$

c)
$$|V_o/V_i|^2 = 10^{-2} = 0.01 \rightarrow \omega = 1/10RC$$

2.12
  a)
$$10 \text{ dBm} = 20 \log_{10}(V/V_0)$$
$$\frac{V}{V_0} = 10^{1/2}$$
$$V = 3.1V_0 = 1 \text{ V}$$
$$\bar{P} = \frac{V^2}{2R} = 10 \text{ mW}.$$

We could also have done this just by noting that 10 dBm is 10 times greater than 0 dBm.
  b)
$$35 \text{ dB} = 20 \log_{10} V_2/V_1$$
$$\frac{V_2}{V_1} = 10^{35/20} = 56$$

2.13

$$\sin \omega t = \frac{e^{i\omega t} - e^{-i\omega t}}{2i}$$

From (2.7.11),
$$F(\omega') = 2\pi \frac{\delta(\omega' - \omega) - \delta(\omega' + \omega)}{2i}$$

2.14
  $f(t)$ is given by (note typo in book)

$$f(t) = \begin{cases} 1, & (n-1)T < t < (n-1/2)T \\ 0, & (n-1/2)T < t < nT. \end{cases}$$

$$
\begin{aligned}
c_n &= \frac{1}{T} \int_{T/2}^{T} e^{-i2\pi nt/T} dt \\
&= \frac{e^{-i2\pi n} - e^{-i\pi n}}{-i2\pi n} \\
&= \frac{e^{-3i\pi n/2}(e^{-i\pi n/2} - e^{i\pi n/2})}{-i2\pi n} = \frac{1}{\pi n} e^{-3i\pi n/2} \sin(\pi n/2)
\end{aligned}
$$

n	c_n
0	$\frac{1}{2}$
± 1	$i\pi^{-1}$
± 2	0
± 3	$\frac{i}{3}\pi^{-1}$
± 4	0
± 5	$\frac{i}{5}\pi^{-1}$
± 6	0
± 7	$\frac{i}{7}\pi^{-1}$

$$
f(t) = \sum_{n=-\infty}^{\infty} c_n e^{i2\pi nt/T} = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{i}{\pi n}(e^{i2\pi nt/T} - e^{-i2\pi nt/T}) = \frac{1}{2} - \sum_{n=1}^{\infty} \frac{2}{\pi n} \sin(2\pi nt/T)
$$

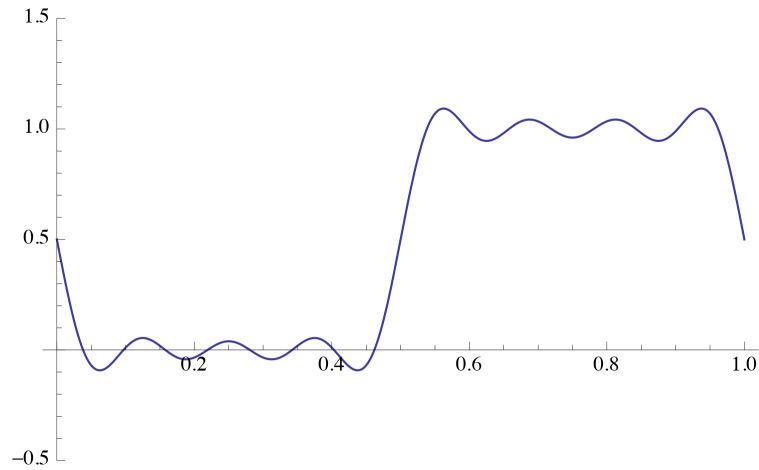The sum of the terms up through $n = 7$ (first five nonzero terms) is shown in Figure 6.

Figure 6: Fourier sum for Exercise 2.14.

2.15

The Fourier transform is

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt.$$

$f(t)$ is given by

$$f(t) = \begin{cases} 1, & (n-1)T < t < (n-1/2)T \\ 0, & (n-1/2)T < t < nT. \end{cases}$$

The transform is

$$F(\omega) = \sum_{n=-\infty}^{\infty} \int_{(n-1)T}^{(n-1/2)T} e^{-i\omega t}dt = \sum_{n=-\infty}^{\infty} \frac{i}{\omega}(e^{-i\omega(n-1/2)T)} - e^{-i\omega(n-1)T)})$$

$$= \frac{i}{\omega}(e^{i\omega T/2} - e^{i\omega T}) \sum_{n=-\infty}^{\infty} e^{-i\omega nT}.$$

The sum will be equal to infinity for $\omega = 2\pi n'/T$ and zero otherwise (this is equivalent to a δ-function). Thus we have

$$F(\omega) = \begin{cases} \dfrac{-2i}{2\pi n'/T} = \dfrac{-i}{\pi n'}, & \omega = 2\pi n'/T \\[2mm] 0, & \text{else.} \end{cases}$$

This is the same as the result of Exercise 2.14.

2.16

The Fourier transform is

$$F(\omega) = \int_{-\infty}^{\infty} \Theta(t)e^{-st}e^{-i\omega t}dt = \int_{0}^{\infty} e^{-st}e^{-i\omega t}dt = -\frac{1}{-s-i\omega} = \frac{-i}{\omega - is}$$

The response function (2.5.7) is

$$\frac{V_o}{V_i} = \frac{-i/\omega C}{R - i/\omega C} = \frac{-i/RC}{\omega - i/RC}$$

and the product is

$$F_o(\omega) = \left(\frac{-i/RC}{\omega - i/RC}\right)\left(\frac{-i}{\omega - is}\right)$$

The reverse Fourier transform is

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\frac{-i/RC}{\omega - i/RC}\right)\left(\frac{-i}{\omega - is}\right) e^{i\omega t}dt.$$

For $t > 0$, $\omega \to +i\infty$ converges, so this becomes

$$f(t) = i(-i/RC)\frac{-i}{i/RC - is}e^{i(i/RC)t} + i\frac{(-i/RC)}{is - i/RC}(-i)e^{i(is)t}.$$

20

Setting $s = 0$ gives

$$f(t) = 1 - e^{-t/RC}.$$

2.17

Loops:

$V_i = IR + I_c Z_C$
$V_i = IR + I_L Z_L$

Node:

$I = I_C + I_L$

Solution:

$$I_C = \frac{Z_L}{RZ_C + RZ_L + Z_C Z_L} V_i = \frac{i\omega L}{-iR/\omega C + i\omega RL + L/C} V_i$$

$$V_C = I_C Z_C = \frac{L/C}{-iR/\omega C + i\omega RL + L/C} V_i$$

2.18

(2.8.2) is

$$\frac{V_o}{V_i} = \frac{R}{R - i/\omega C + i\omega L}.$$

(2.8.12) is

$$\dot{V}_i = \dot{I}R + \frac{I}{C} + L\ddot{I}.$$

For $V_i = V_i(0)e^{i\omega t}$ and $I = I_0 e^{i\omega t + \phi}$, this becomes

$$i\omega V_i(0)e^{i\omega t} = i\omega I_0 R e^{i(\omega t + \phi)} + \frac{I_0}{C} e^{i(\omega t + \phi)} - L\omega^2 I_0 e^{i(\omega t + \phi)}.$$

The output V_o is across R, so $V_o = I_0 R e^{i(\omega t + \phi)}$. The above equation therefore becomes

$$i\omega V_i = i\omega V_o + \frac{V_o}{RC} - \frac{L}{R}\omega^2 V_o.$$

Solving for V_o/V_i gives

$$\frac{V_o}{V_i} = \frac{i\omega}{i\omega + 1/RC - \omega^2 L/R}$$

which is the same as (2.8.2).

2.19

We want a high-pass filter like that shown in Fig. 2.18(b), which has response (see Exercise 2.9)

$$\left|\frac{V_o}{V_i}\right|^2 = \frac{1}{1 + 1/\omega^2 R^2 C^2}$$

We solve for C:

$$C = \frac{\omega}{R}\sqrt{\frac{1}{|V_o/V_i|^2} - 1} = \frac{2\pi(100 \text{ Hz})}{50 \times 10^6 \ \Omega}\sqrt{1/(.95)^2 - 1} = 0.6 \ \mu\text{F}.$$

2.20

Solve for 10% value:

$$e^{-t_{10}^2/2\tau^2} = .1$$

$$t_{10} = \sqrt{-2\tau^2 \ln(0.1)}.$$

For 90% value,

$$t_{90} = \sqrt{-2\tau^2 \ln(0.9)}.$$

On the positive side,

$$t_{10} - t_{90} = \sqrt{-2\tau^2 \ln(0.9)} - \sqrt{-2\tau^2 \ln(0.1)} = \tau\sqrt{2}(\sqrt{-\ln(0.1)} - \sqrt{-\ln(0.9)}) = 1.2\sqrt{2}\tau.$$

2.21

The electric field is purely radial. For charge $+Q$ on the inner sphere and $-Q$ on the outer sphere, the electric field is

$$E(r) = \frac{Q}{4\pi\epsilon_0 r^2}$$

and the potential drop is

$$\Delta V(r) = -\int_{r_1}^{r_2} E(r)dr = \frac{Q}{4\pi\epsilon_0 r_1} - \frac{Q}{4\pi\epsilon_0 r_2}.$$

Comparing to the definition $Q = C\Delta V$, this gives

$$C = 4\pi\epsilon_0 \left(\frac{1}{r_1} - \frac{1}{r_2}\right)^{-1}.$$

2.22

The electric field from a line charge is found from Gauss's law,

$$E = \frac{\sigma}{2\pi r \epsilon_0},$$

where $\sigma = Q/l$ is the charge density. The voltage drop from wire of radius b to a distance a is

$$\Delta V = -\int_b^a E(r)dr = \frac{Q/l}{2\pi\epsilon_0}\ln(a/b).$$

By superposition, the other wire contributes the same, so we multiply by 2. This implies

$$C = \frac{\pi\epsilon_0 l}{\ln(a/b)}.$$

2.23

$$C = \frac{A\epsilon_0}{d} = \frac{lw\epsilon_0}{d}$$

$$L = \frac{\mu_0 l d}{w}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{\mu_0 l d}{w}\frac{lw\epsilon_0}{d}}} = \frac{1}{\sqrt{\mu_0\epsilon_0 l^2}} = \frac{c}{l}.$$

2.24

The high-pass filter of Fig. 2.18(b) has response (see Exercise 2.9)

$$\frac{V_o}{V_i} = \frac{R}{R - i/\omega C} = \frac{R(R + i/\omega C)}{R^2 + 1/\omega^2 C^2}.$$

From (2.5.9),

$$\tan\phi = \frac{\mathsf{Im}\ V_o/V_i}{\mathsf{Re}\ V_o/V_i}$$
$$= \frac{1/\omega C}{R}$$

At high frequency, $\phi = 0$. At low frequency, $\phi = \pi/2$.

2.25

$$\frac{V_o}{V_i} = \frac{i\omega L}{i\omega L + R} = \frac{i\omega L/R}{i\omega L/R + 1}.$$
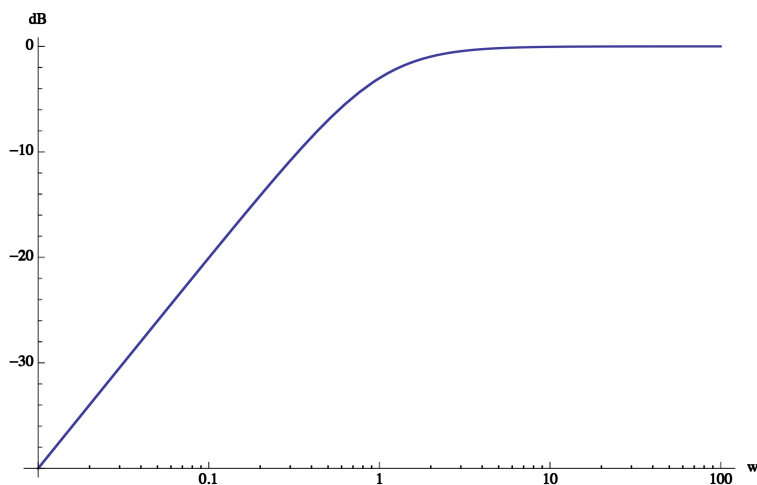
Figure **??** shows the plot with frequency in units of R/L.

Figure 7: Response function for Exercise 2.25.

2.26

$|V_o|^2$ is increased by $(3/2)^2 = 2.25$.

$10(\log_{10} 2.25) = 3.52$ dB

2.27

From Exercises 2.9 and 2.18:

$$\left|\frac{V_o}{V_i}\right|^2 = \frac{1}{1 + 1/\omega^2 R^2 C^2}$$

Solving for ω,

$$\omega = \frac{1}{RC}\left(\frac{1}{|V_o/V_i|^2} - 1\right)^{-1/2} = \frac{0.1}{RC}$$

2.28

The low-pass filter response (2.5.7) is

$$\frac{V_o}{V_i} = \frac{-i/\omega C}{R - i/\omega C}.$$

24

Power response is

$$\left|\frac{V_o}{V_i}\right|^2 = \frac{-i/\omega C}{R - i/\omega C}\frac{i/\omega C}{R + i/\omega C} = \frac{1/\omega^2 C^2}{R^2 + 1/\omega^2 C^2} = \frac{1}{\omega^2 (RC)^2 + 1}.$$

Solve for ω:

$$\omega = \frac{1}{RC}\sqrt{\frac{1}{|V_o/V_i|^2} - 1}$$

10%:
$$\omega = \frac{3}{RC}.$$

90%:
$$\omega = \frac{1}{3RC}.$$

10-90 range:

$$\Delta\omega = \frac{2.667}{RC}.$$

2.29

$$\Delta\text{dB} = 20\log_{10} V_2/V_1$$

$$\frac{V_2}{V_1} = 10^{\Delta\text{dB}/20} = 10^{13/20} = 4.5$$

The amplitude signal-to-noise ratio increases by a factor of 4.5, i.e. from 2 to 9.
It is also common to talk in terms of the signal-to-noise power ratio.

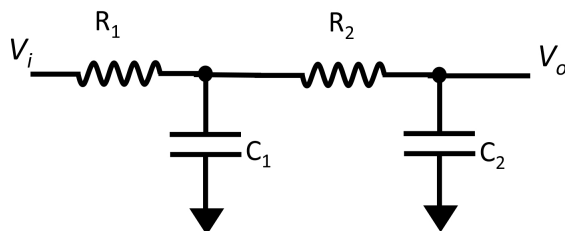**2.30**
The circuit is shown in Figure 8.

Figure 8: Circuit for Exercise 2.30.

Loops:

$$V_i = I_1 R_1 + I_{C1} Z_{C1}$$
$$V_i = I_1 R_1 + I_2 R_2 + V_o$$
$$V_o = I_2 Z_{C2}$$

Node:
$$I_1 = I_{C1} + I_2$$

Set $R_1 = R_2 = R$, $C_1 = C_2 = C$. Solution:

$$\frac{V_o}{V_i} = \frac{Z_C^2}{R^2 + 3RZ_C + Z_C^2}.$$

$$\left| \frac{V_o}{V_i} \right|^2 = \frac{1}{1 + 7C^2 R^2 \omega^2 + C^4 R^4 \omega^4}$$

In the Figure 9, the lower curve is this response function, while the upper curve is the single low-pass response, from Exercise 2.27.
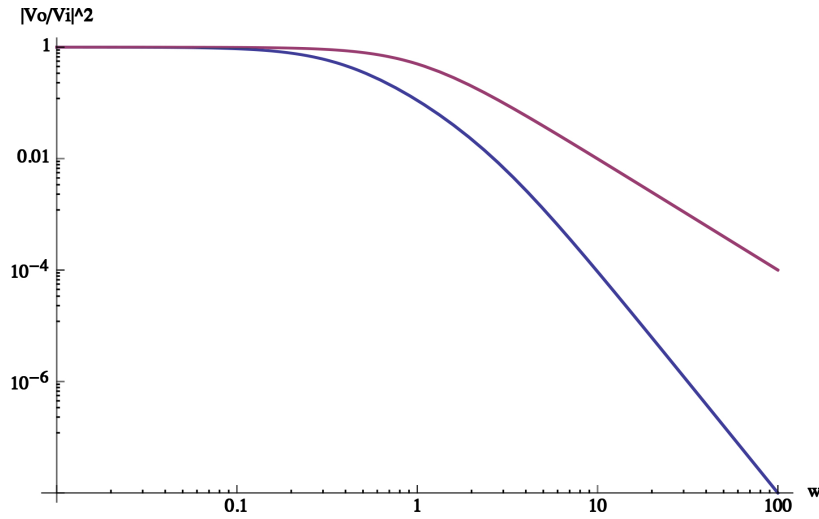

Figure 9: Response functions for Exercise 2.30.

2.31

Assuming perfect detector; no current flows into output; I_o is the current flowing from top to bottom in the right side of the circuit.

Loops:

$$V_i = I_{C1} Z_C + I_{R3} \frac{R}{2}$$

26

$$V_i = I_{R1}R + I_{C3}\frac{Z_C}{2}$$
$$V_i = I_{R1}R + I_oR + V_o$$
$$V_o = I_oZ_C + I_{R3}\frac{R}{2}$$

Nodes:

$$I_{R1} = I_{C3} + I_o$$
$$I_{C1} + I_o = I_{R3}$$

Solution:

$$\frac{V_o}{V_i} = \frac{(-1 + C^2R^2\omega^2)}{-1 - 4iCR\omega + C^2R^2\omega^2}$$

This is plotted in Figure 10. When $\omega = 1/RC$, this equals 0. When $\omega \to 0$ or $\omega \to \infty$, it approaches unity.
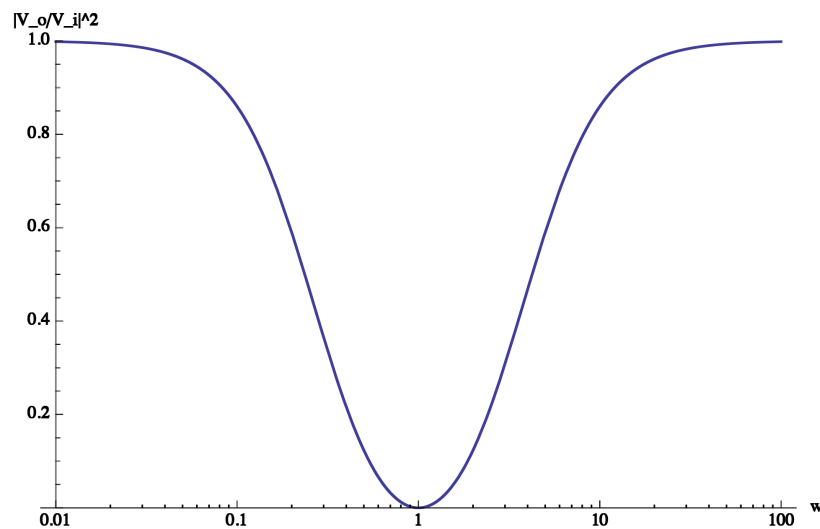

Figure 10: Response function for Exercise 2.31.

2.32

The circuit in the book is missing a 50-Ω resistor between V_i and the rest of the circuit—as drawn, the circuit will give $V_o = V_i$ for all inputs. Putting a resistor there gives

Loops:
$$V_i = IR + V_o$$
$$V_o = I_1(Z_{C1} + Z_{L1})$$

$$V_o = I_2(Z_{C2} + Z_{L2})$$

Node:
$$I = I_1 + I_2$$

Solution:

$$\frac{V_o}{V_i} = \frac{(-1 + C_1 L_1 \omega^2)(-1 + C_2 L_2 \omega^2)}{1 + iC_1 R\omega + iC_2 R\omega - C_1 L_1 \omega^2 - C_2 L_2 \omega^2 - iC_1 C_2 L_1 R\omega^3 - iC_1 C_2 L_2 R\omega^3 + C_1 C_2 L_1 L_2 \omega^4}$$

This is a double notch filter, with zeroes where the two terms in the numerator vanish. Figure 11 shows a plot for the values given:
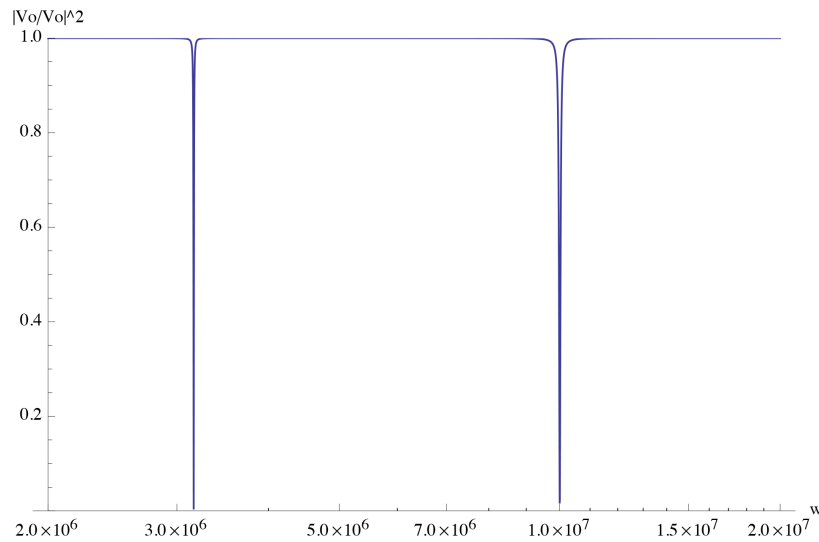

Figure 11: Double notch response function, for Exercise 2.32.

2.33

a) Capacitor relation:
$$I = C\frac{\partial \Delta V_C}{\partial t}$$

Loops:

$$V_s = I_1 R_1 + I_2 R_2$$
$$V_i = \Delta V_C + V_o \rightarrow \dot{V}_i = \frac{I_C}{C} + \dot{V}_o$$
$$V_o = I_2 R_2$$

These become

$$V_s = I_1 R_1 + I_2 R_2$$
$$i\omega V_1 e^{i\omega t} = \frac{I_C}{C} + i\omega V_{AC} e^{i(\omega t + \phi)}$$
$$V_{DC} + V_{AC} e^{i(\omega t + \phi)} = I_2 R_2$$

Node:
$$I_C + I_1 = I_2.$$

Write $I_1 = I_{1DC} + I_{1AC}$ and $I_2 = I_{2DC} + I_{2AC}$ (note that there is no DC component through the capacitor), and set DC terms equal and AC terms equal:

$$V_s = I_{1DC} R_1 + I_{2DC} R_2$$
$$0 = I_{1AC} R_1 + I_{2AC} R_2$$
$$i\omega V_1 e^{i\omega t} = \frac{I_C}{C} + i\omega V_{AC} e^{i(\omega t + \phi)}$$
$$V_{DC} = I_{2DC} R_2$$
$$V_{AC} e^{i(\omega t + \phi)} = I_{2AC} R_2$$
$$I_C + I_{1AC} = I_{2AC}$$
$$I_{1DC} = I_{2DC}$$

We solve these for $I_C, I_{1DC}, I_{1AC}, I_{2DC}, I_{2AC}, V_{DC}$, and $V_{AC} e^{i\phi}$, which gives

$$V_{DC} = \frac{R_2 V_s}{R_1 + R_2},$$

$$
\begin{aligned}
V_{AC} e^{i\phi} &= \frac{C R_1 R_2 \omega}{-iR_1 - iR_2 + C R_1 R_2 \omega} V_1 \\
&= \frac{C R_{\text{eff}} \omega}{-i + C R_{\text{eff}} \omega} V_1
\end{aligned}
$$

where
$$R_{\text{eff}} = \frac{R_1 R_2}{R_1 + R_2}.$$

2.34

We want a low-pass filter that eliminates AC frequency of 60 Hz. We use the circuit shown in Fig. 2.18(a) with a polarized capacitor with the negative side grounded. Formula (2.5.11) gives us
$$|V_o/V_i|^2 = \frac{1}{1 + \omega^2 R^2 C^2}.$$
We would like low series R to prevent DC droop of the voltage supply. Pick $R = 1\ \Omega$, which is small compared to a typical 50 Ω load impedance. Solve for C:

$$C = \frac{1}{\omega R} \sqrt{1/|V_o/V_i|^2 - 1}$$

29

To eliminate 99% of the ripple, pick $|V_o/V_i|^2 = .01$. Setting $\omega = 2\pi(60 \text{ Hz}) = 377 \text{ s}^{-1}$, we then have

$$C = \frac{1}{(377 \text{ s}^{-1})(50 \ \Omega)}\sqrt{99} = 0.00053 \text{ F} = 530 \ \mu\text{F}.$$