# Solutions Manual

## Discrete-Event System Simulation

*Fifth Edition*

Jerry Banks
John S. Carson II
Barry L. Nelson
David M. Nicol

August 10, 2009

# Contents

# Foreword

There are over three hundred exercises for solution in the text. These exercises emphasize principles of discrete-event simulation and provide practice in utilizing concepts found in the text.

Answers provided here are selective, in that not every problem in every chapter is solved. Answers in some instances are suggestive rather than complete. These two caveats hold particularly in chapters where building of computer simulation models is required. The solutions manual will give the instructor a basis for assisting the student and judging the student's progress. Some instructors may interpret an exercise differently than we do, or utilize an alternate solution method; they are at liberty to do so. We have provided solutions that our students have found to be understandable.

When computer solutions are provided they will be found on the text web site, `www.bcnn.net`, rather than here. Solutions in addition to those noted below may be developed and added to the book's web site.

Jerry Banks
John S. Carson II
Barry L. Nelson
David M. Nicol

# Chapter 1

# Introduction to Simulation

1.1

| | SYSTEM | ENTITIES | ATTRIBUTES | ACTIVITIES | EVENTS | STATE VARIABLES |
|---|---|---|---|---|---|---|
| | Small appliance repair shop | Appliances | Type of appliance<br><br>Age of appliance<br><br>Nature of problem | Repairing the appliance | Arrival of a job<br><br>Completion of a job | Number of appliances waiting to be repaired<br><br>Status of repair person busy or idle |
| a. | Cafeteria | Diners | Size of appetite<br><br>Entree preference | Selecting food<br><br>Paying for food | Arrival at service line<br><br>Departures from service line | Number of diners in waiting line<br><br>Number of servers working |
| b. | Grocery store | Shoppers | Length of grocery list | Checking out | Arrival at checkout counters<br><br>Departure from checkout counter | Number of shoppers in line<br>Number of checkout lanes in operation |
| c. | Laundromat | Washing machine | Breakdown rate | Repairing a machine | Occurrence of breakdowns<br><br>Completion of service | Number of machines running<br>Number of machines in repair<br>Number of Machines waiting for repair |

| | SYSTEM | ENTITIES | ATTRIBUTES | ACTIVITIES | EVENTS | STATE VARIABLES |
|---|---|---|---|---|---|---|
| d. | Fast food restaurant | Customers | Size of order desired | Placing the order<br><br>Paying for the order | Arrival at the counter<br><br>Completion of purchase | Number of customers waiting<br><br>Number of positions operating |
| e. | Hospital emergency room | Patients | Attention level required | Providing service required | Arrival of the patient<br><br>Departure of the patient | Number of patients waiting<br><br>Number of physicians working |
| f. | Taxicab company | Fares | Origination<br><br>Destination | Traveling | Pick-up of fare<br><br>Drop-off of fare | Number of busy taxi cabs<br><br>Number of fares waiting to be picked up |
| g. | Automobile assembly line | Robot welders | Speed<br><br>Breakdown rate | Spot welding | Breaking down | Availability of machines |

1.3 Abbreviated solution:

| Iteration | Problem Formulation | Setting of Objectives and Overall Project Plan |
|---|---|---|
| 1 | Cars arriving at the intersection are controlled by a traffic light. The cars may go straight, turn left, or turn right. | How should the traffic light be sequenced? Criterion for evaluating effectiveness: average delay time of cars. Resources required: 2 people for 5 days for data collection, 1 person for 2 days for data analysis, 1 person for 3 days for model building, 1 person for 2 days for running the model, 1 person for 3 days for implementation. |
| 2 | Same as 1 above plus the following: Right on red is allowed after full stop provided no pedestrians are crossing and no vehicle is approaching the intersection. | How should the traffic light be sequenced? Criterion for evaluating effectiveness: average delay time of cars. Resources required: 2 people for 8 days for data collection, 1 person for 3 days for data analysis, 1 person for 4 days for model building, 1 person for 2 days for running the model, 1 person for 3 days for implementation. |
| 3 | Same as 2 above plus the following: Trucks arrive at the intersection. Vehicles break down in the intersection making one lane impassable. Accidents occur blocking traffic for varying amounts of time. | How should the traffic light be sequenced? Should the road be widened to 4 lanes? Method of evaluating effectiveness: average delay time of all vehicles. Resources required: 2 people for 10 days for data collection, 1 person for 5 days for data analysis, 1 person for 5 days for model building, 1 person for 3 days for running the model, 1 person for 4 days for implementation. |

1.4 Data Needed

Number of guests attending

Time required for boiling water

Time required to cook pasta

Time required to dice onions, bell peppers, mushrooms

Time required to saute onions, bell peppers, mushrooms, ground beef

Time required to add necessary condiments and spices

Time required to add tomato sauce, tomatoes, tomato paste

Time required to simmer sauce

Time required to set the table

Time required to drain pasta

Time required to dish out the pasta and sauce

Events

Begin cooking

Complete pasta cooking<br>Complete sauce cooking } Simultaneous

Arrival of dinner guests

Begin eating

Activities

Boiling the water

Cooking the pasta

Cooking sauce

Serving the guests

State variables

Number of dinner guests

Status of the water (boiling or not boiling)

Status of the pasta (done or not done)

Status of the sauce (done or not done)

1.5 Event

Deposit

Withdrawal

Activities

Writing a check

Cashing a check

Making a deposit

Verifying the account balance

Reconciling the checkbook with the bank statement

1.12 (a) 1971 with 1200 attendees

(b) 1972

(c) From Dec. 8, 1971 to Jan. 17, 1973, 1.11 years

(d) DC, Southeast, West

1.15 The pupose of the WSC Foundation is to develop and manage a fund to help insure the continuance and high quality of the WSC.

# Chapter 2

# Simulation Examples in a Spreadsheet

*For additional solutions check the course web site at* `www.bcnn.net`. *The numbers resulting from a student's spreadsheet simulation may differ from the results here, depending on the random numbers used.*

*In the spreadsheet solutions, the columns labeled "RD Assignment" are for manual solutions using the random digits in Table A. 1. You can ignore these columns when solving the problem in Excel, and instead use the methods in the textbook.*

2.1

| Customer | Interarrival Time (Minutes) | Clock Arrival Time | Service Time (Minutes) | Clock Time Service Begins | Waiting Time in Queue (Minutes) | Clock Time Service Ends | Time Customer Spends in System (Minutes) | Idle Time of Server (Minutes) |
|---|---|---|---|---|---|---|---|---|
| 1 | | 0 | 25 | 0 | 0 | 25 | 25 | |
| 2 | 0 | 0 | 50 | 25 | 25 | 75 | 75 | 0 |
| 3 | 60 | 60 | 37 | 75 | 15 | 112 | 52 | 0 |
| 4 | 60 | 120 | 45 | 120 | 0 | 165 | 45 | 8 |
| 5 | 120 | 240 | 50 | 240 | 0 | 290 | 50 | 75 |
| 6 | 0 | 240 | 62 | 290 | 50 | 352 | 112 | 0 |
| 7 | 60 | 300 | 43 | 352 | 52 | 395 | 95 | 0 |
| 8 | 120 | 420 | 48 | 420 | 0 | 468 | 48 | 25 |
| 9 | 0 | 420 | 52 | 468 | 48 | 519 | 99 | 0 |
| 10 | 120 | 540 | 38 | 540 | 0 | 578 | 38 | 21 |
| Average | | | 45 | | 19 | | 112 | |

(a) The average time in the queue for the 10 new jobs is 19 minutes.

(b) The average processing time of the 10 new jobs is 45 minutes.

(c) The maximum time in the system for the 10 new jobs is 112 minutes.

2.2 Profit = Revenue from retail sales - Cost of bagels made + Revenue from grocery store sales - Lost profit.

Let $Q$ = number of dozens baked/day

$S = \sum_i 0_i$, where $0_i$ = Order quantity in dozens for the $i$th customer

$Q - S$ = grocery store sales in dozens, $Q > S$

$S - Q$ = dozens of excess demand, $S > Q$

$$\text{Profit} = \$5.40 \min(S, Q) - \$3.80Q + \$2.70(Q - S) - \$1.60(S - Q)$$

| Number of Customers | Probability | Cumulative Probability | RD Assignment |
|---|---|---|---|
| 8 | .35 | .35 | 01-35 |
| 10 | .30 | .65 | 36-65 |
| 12 | .25 | .90 | 66-90 |
| 14 | .10 | 1.00 | 91-100 |

| Dozens Ordered | Probability | Cumulative Probability | RD Assignment |
|---|---|---|---|
| 1 | .4 | .4 | 1-4 |
| 2 | .3 | .7 | 5-7 |
| 3 | .2 | .9 | 8-9 |
| 4 | .1 | 1.0 | 0 |

Pre-analysis

$$\begin{aligned} E(\text{Number of Customers}) &= .35(8) + .30(10) + .25(12) + .10(14) \\ &= 10.20 \end{aligned}$$

$$E(\text{Dozens ordered}) = .4(1) + .3(2) + .2(3) + .1(4) = 2$$

$$E(\text{Dozens sold}) = \bar{S} = (10.20)(2) = 20.4$$

$$\begin{aligned} E(\text{Profit}) &= \$5.40\text{Min}(\bar{S}, Q) - \$3.80Q + \$2.70(Q - \bar{S}) - \$1.60(\bar{S} - Q) \\ &= \$5.40\text{Min}(20.4, Q) - \$3.80Q + \$2.70(Q - 20.4) \\ &\quad -\$0.67(20.4 - Q) \end{aligned}$$

$$E(\text{Profit}|Q = 0) = 0 - 0 + \$1.60(20.4) = -\$32.64$$

$$\begin{aligned} E(\text{Profit}|Q = 10) &= \$5.40(10) - \$3.80(10) + 0 - \$1.60(20.4 - 10) \\ &= -\$0.64 \end{aligned}$$

$$\begin{aligned} E(\text{Profit}|Q = 20) &= \$5.40(20) - \$3.80(20) + 0 - \$1.60(20.4 - 20) \\ &= \$15.36 \end{aligned}$$

$$\begin{aligned} E(\text{Profit}|Q = 30) &= \$5.40(20.4) - \$3.80(30) + \$2.70(30 - 20.4) - 0 \\ &= \$22.08 \end{aligned}$$

$$\begin{aligned} E(\text{Profit}|Q = 40) &= \$5.40(20.4) - \$3.80(40) + \$2.70(40 - 20.4) - 0 \\ &= \$11.08 \end{aligned}$$

The pre-analysis, based on expectation only, indicates that simulation of the policies $Q = 20, 30$, and 40 should be sufficient to determine the policy. The simulation should begin with $Q = 30$, then proceed to $Q = 40$, then, most likely to $Q = 20$.

Initially, conduct a simulation for $Q = 20, 30$ and 40. If the profit is maximized when $Q = 30$, it will become the policy recommendation.

The problem requests that the simulation for each policy should run for 5 days. This is a very short run length to make a policy decision.

$Q = 30$

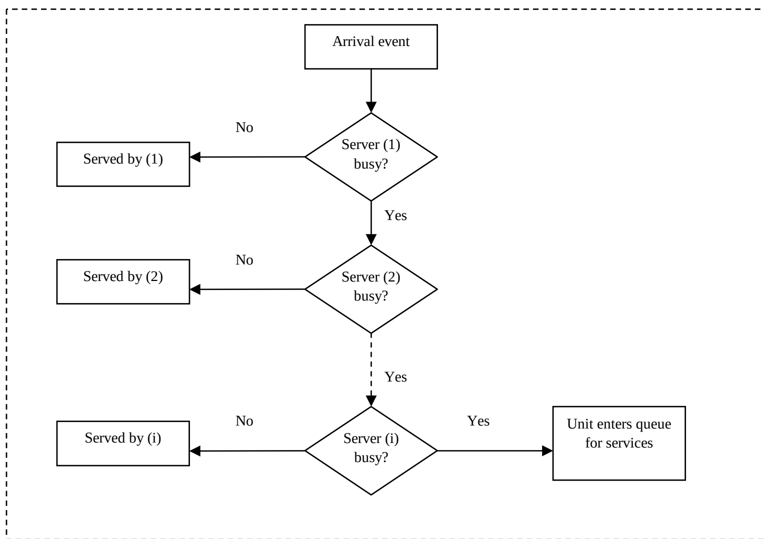
| Day | RD for Customer | Number of Customers | RD for Demand | Dozens Ordered | Revenue from Retail $ | Lost Profit $ |
|---|---|---|---|---|---|---|
| 1 | 44 | 10 | 8 | 3 | 16.20 | 0 |
| | | | 2 | 1 | 5.40 | 0 |
| | | | 4 | 1 | 5.40 | 0 |
| | | | 8 | 3 | 16.20 | 0 |
| | | | 1 | 1 | 5.40 | 0 |
| | | | 6 | 2 | 10.80 | 0 |
| | | | 3 | 1 | 5.40 | 0 |
| | | | 0 | 4 | 21.60 | 0 |
| | | | 2 | 1 | 5.40 | 0 |
| | | | 0 | 4 | 21.60 | 0 |
| | | | | 21 | 113.40 | 0 |

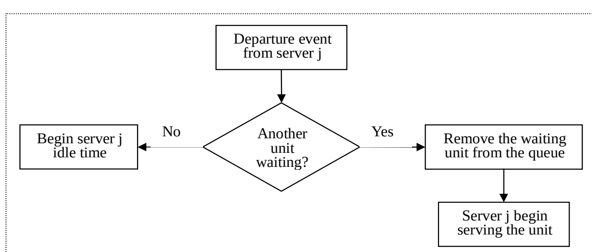
For Day 1,

Profit $= \$113.40 - \$152.00 + \$24.30 - 0 = \$14.30$

Days 2, 3, 4 and 5 are now analyzed and the five day total profit is determined.

2.3 For a queueing system with i channels, first rank all the servers by their processing rate. Let (1) denote the fastest server, (2) the second fastest server, and so on.




2.4

| Time Between Calls | Probability | Cumulative Probability | RD Assignment |
|---|---|---|---|
| 15 | .14 | .14 | 01-14 |
| 20 | .22 | .36 | 15-36 |
| 25 | .43 | .79 | 37-79 |
| 30 | .17 | .96 | 80-96 |
| 35 | .04 | 1.00 | 97-00 |

| Service Time | Probability | Cumulative Probability | RD Assignment |
|---|---|---|---|
| 5 | .12 | .12 | 01-12 |
| 15 | .35 | .47 | 13-47 |
| 25 | .43 | .90 | 48-90 |
| 35 | .06 | .96 | 91-96 |
| 45 | .04 | 1.00 | 97-00 |

First, simulate for one taxi for 5 days.<br>Then, simulate for two taxis for 5 days. } Shown on simulation tables

Comparison

Smalltown Taxi would have to decide which is more important—paying for about 43 hours of idle time in a five day period with no customers having to wait, or paying for around 4 hours of idle time in a five day period, but having a probability of waiting equal to 0.59 with an average waiting time for those who wait of around 20 minutes.

One Taxi

| Day | Call | RD for Time between Calls | Time between Calls | Call Time | RD for Service Time | Service Time | Time Service Begins | Time Customer Waits | Time Service Ends | Time Customer in System | Idle Time of Taxi |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 15 | - | 0 | 01 | 5 | 0 | 0 | 5 | 5 | 0 |
| | 2 | 01 | 20 | 20 | 53 | 25 | 20 | 0 | 55 | 25 | 0 |
| | 3 | 14 | 15 | 35 | 62 | 25 | 55 | 20 | 80 | 45 | 0 |
| | 4 | 65 | 25 | 60 | 55 | 25 | 80 | 20 | 105 | 45 | 0 |
| | 5 | 73 | 25 | 85 | 95 | 35 | 105 | 20 | 140 | 55 | 0 |
| | 6 | 48 | 25 | 110 | 22 | 15 | 140 | 30 | 155 | 45 | 0 |
| | ⋮ | | | | | | | | | | |
| | 20 | 77 | 25 | 444 | 63 | 25 | 470 | 25 | 495 | 50 | 0 |
| 2 | | | | | | | | | | | |
| ⋮ | | | | | | | | | | | |

Typical results for a 5 day simulation:

Total idle time = 265 minutes = 4.4 hours

Average idle time per call = 2.7 minutes

Proportion of idle time = .11

Total time customers wait = 1230 minutes

Average waiting time per customer = 11.9 minutes

Number of customers that wait = 61 (of 103 customers)

Probability that a customer has to wait = .59

Average waiting time of customers that wait = 20.2 minutes

Two taxis (using common RDs for time between calls and service time)

| | | | | | Taxi 1 | | | Taxi 2 | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Day | Call | Time between Calls | Call Time | Service Time | Time Service Begins | Service Time | Time Service Ends | Time Service Begins | Service Time | Time Service Ends | Time Customer Waits | Time Customer in System | Idle Time Taxi 1 | Idle Time Taxi 2 |
| 1 | 1 | - | 0 | 5 | 0 | 5 | 5 | | | | 0 | 5 | | |
| | 2 | 20 | 20 | 25 | 20 | 25 | 45 | | | | 0 | 25 | | |
| | 3 | 15 | 35 | 25 | | | | 35 | 25 | 60 | 0 | 25 | | 35 |
| | 4 | 25 | 60 | 25 | 60 | 25 | 85 | | | | 0 | 25 | 15 | |
| | 5 | 25 | 85 | 35 | 80 | 35 | 120 | | | | 0 | 35 | | |
| | 6 | 25 | 110 | 15 | | | | 110 | 15 | 125 | 0 | 15 | | 50 |
| | ⋮ | | | | | | | | | | | | | |
| | 20 | 20 | 480 | 25 | 480 | 25 | 505 | | | | 0 | 25 | 10 | |
| 2 | | | | | | | | | | | | | | |
| ⋮ | | | | | | | | | | | | | | |

Typical results for a 5 day simulation:

Idle time of Taxi 1 = 685 minutes

Idle time of Taxi 2 = 1915 minutes

Total idle time = 2600 minutes = 43 hours

Average idle time per call = 25.7 minutes

Proportion of idle time = .54

Total time customers wait = 0 minutes

Number of customers that wait = 0

2.5 For manual simulations, $RNN_x$ is a random normal number from Table A. 2. For spreadsheet simulations, it is generated from the appropriate VBA function as described in Chapter 2. Similarly for $RNN_y$ and $RNN_z$.

$$
\begin{aligned}
X &= 100 + 10RNN_x \\
Y &= 300 + 15RNN_y \\
Z &= 40 + 8RNN_z
\end{aligned}
$$

Typical results...

| | $RNN_x$ | $X$ | $RNN_y$ | $Y$ | $RNN_z$ | $Z$ | $W$ |
|---|---|---|---|---|---|---|---|
| 1 | -.137 | 98.63 | .577 | 308.7 | -.568 | 35.46 | 11.49 |
| 2 | .918 | 109.18 | .303 | 304.55 | -.384 | 36.93 | 11.20 |
| 3 | 1.692 | 116.92 | -.383 | 294.26 | -.198 | 38.42 | 10.70 |
| 4 | -.199 | 98.01 | 1.033 | 315.50 | .031 | 40.25 | 10.27 |
| 5 | -.411 | 95.89 | .633 | 309.50 | .397 | 43.18 | 9.39 |
| ⋮ | | | | | | | |

After generating the 50 values of $W$, you can use a bar chart in Excel to develop the histogram.

2.6

| Value of B | Probability | Cumulative Probability | RD Assignment |
|---|---|---|---|
| 0 | 0.2 | 0.2 | 1-2 |
| 1 | 0.2 | 0.4 | 3-4 |
| 2 | 0.2 | 0.6 | 5-6 |
| 3 | 0.2 | 0.8 | 7-8 |
| 4 | 0.2 | 1 | 9-0 |

| Value of C | Probability | Cumulative Probability | RD Assignment |
|---|---|---|---|
| 10 | 0.1 | 0.1 | 1-10 |
| 20 | 0.25 | 0.35 | 11-35 |
| 30 | 0.5 | 0.85 | 36-85 |
| 40 | 0.15 | 1 | 86-1 |

| Customer | A | B | C | D |
|---|---|---|---|---|
| 1 | 79.23 | 2 | 30 | 2 |
| 2 | 113.04 | 3 | 30 | 32 |
| 3 | 58.53 | 0 | 20 | 1.46 |
| 4 | 99.68 | 0 | 20 | 2.49 |
| 5 | 87.15 | 0 | 10 | 4.36 |
| 6 | 91.05 | 1 | 40 | 0.83 |
| 7 | 66.97 | 1 | 30 | 0.7 |
| 8 | 104.88 | 3 | 30 | 0.5 |
| 9 | 61.6 | 1 | 30 | 0.61 |
| 10 | 98.92 | 3 | 30 | 0.4 |
| Average | 86.1 | 1.4 | 27 | 4.53 |

2.7

| Lead Time (Days) | Probability | Cumulative Probability | RD Assignment |
|---|---|---|---|
| 0 | .166 | .166 | 001-166 |
| 1 | .166 | .332 | 167-332 |
| 2 | .166 | .498 | 333-498 |
| 3 | .166 | .664 | 499-664 |
| 4 | .166 | .830 | 665-830 |
| 5 | .166 | .996 | 831-996 |
| | | | 996-000 (discard) |

Assume 5-day work weeks.

$$D = \text{Demand}$$
$$D = 5 + 1.5(RNN)(\text{ Rounded to nearest integer})$$

| Week | Day | Beginning Inventory | $RNN$ for Demands | Demand | Ending Inventory | Order Quantity | RD for Lead Time | Lead Time | Lost Sales |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 18 | -1.40 | 3 | 15 | | | | 0 |
| | 2 | 15 | -.35 | 4 | 11 | | | | 0 |
| | 3 | 11 | -.38 | 4 | 7 | 13 | 691 | 4 | 0 |
| | 4 | 7 | .05 | 5 | 2 | | | | 0 |
| | 5 | 2 | .36 | 6 | 0 | | | | 4 |
| 2 | 6 | 0 | .00 | 5 | 0 | | | | 5 |
| | 7 | 0 | -.83 | 4 | 0 | | | | 4 |
| | 8 | 13 | -1.83 | 2 | 11 | | | | 0 |
| | 9 | 11 | -.73 | 4 | 7 | 13 | 273 | 1 | 0 |
| | 10 | 7 | -.89 | 4 | 3 | | | | 0 |
| ⋮ | | | | | | | | | |

Typical results
Average number of lost sales/week = 24/5 = 4.8 units/weeks

2.8 Material *A* (200kg/box)

| Interarrival Time | Probability | Cumulative Probability | RD Assignment |
|---|---|---|---|
| 3 | .2 | .2 | 1-2 |
| 4 | .2 | .4 | 3-4 |
| 5 | .2 | .6 | 5-6 |
| 6 | .2 | .8 | 7-8 |
| 7 | .2 | 1.0 | 9-0 |

| Box | RD for Interarrival Time | Interarrival Time | Clock Time |
|---|---|---|---|
| 1 | 1 | 3 | 3 |
| 2 | 4 | 4 | 7 |
| 3 | 8 | 6 | 13 |
| 4 | 3 | 4 | 17 |
| ⋮ | | | |
| 14 | 4 | 4 | 60 |

Material *B* (100kg/box)

| Box | 1 | 2 | 3 | ⋯ | 10 |
|---|---|---|---|---|---|
| Clock Time | 6 | 12 | 18 | ⋯ | 60 |

Material *C* (50kg/box)

| Interarrival Time | Probability | Cumulative Probability | RD Assignment |
|---|---|---|---|
| 2 | .33 | .33 | 01-33 |
| 3 | .67 | 1.00 | 34-00 |

| Box | RD for Interarrival Time | Interarrival Time | Clock Time |
|---|---|---|---|
| 1 | 58 | 5 | 3 |
| 2 | 92 | 3 | 6 |
| 3 | 87 | 3 | 9 |
| 4 | 31 | 2 | 11 |
| ⋮ | ⋮ | ⋮ | ⋮ |
| 22 | 62 | 3 | 60 |

| Clock Time | *A* Arrival | *B* Arrival | *C* Arrival |
|---|---|---|---|
| 3 | 1 | | 1 |
| 6 | | 1 | 2 |
| 7 | 2 | | |
| 9 | | | 3 |
| 11 | | | 4 |
| 12 | | 2 | |
| ⋮ | | | |