Gravitation and the Waltz of the Planets 2

Chapter Outline

2-1 Science is both a body of knowledge *and* a process of learning about nature
2-2 The belief in a Sun-centered cosmology formed slowly
2-3 Copernicus devised the first comprehensive heliocentric cosmology
2-4 Tycho Brahe made astronomical observations that disproved ancient ideas about the heavens
2-5 Kepler's laws describe orbital shapes, changing speeds, and the lengths of planetary years
2-6 Galileo's discoveries strongly supported a heliocentric cosmology
2-7 Newton formulated three laws that describe fundamental properties of physical reality
2-8 Newton's description of gravity accounts for Kepler's laws
2-9 Frontiers yet to be discovered

In This Chapter Students Will Discover…

• What makes a theory scientific
• The scientific discoveries that revealed that Earth is not the center of the universe, as previously believed
• Copernicus's argument that the planets orbit the Sun
• Why the direction of motion of each planet on the celestial sphere sometimes appears to change
• That Kepler's determination of the shapes of planetary orbits depended on the careful observations of his mentor Tycho Brahe
• How Isaac Newton formulated an equation to describe the force of gravity and how he thereby explained why the planets and moons remain in orbit

Suggested Learning Objectives

At the end of this chapter, the student should be able to

1. Compare and contrast the Ptolemaic and Copernican cosmologies by explaining a variety of naked-eye observations, using both models.
2. State Kepler's three laws of planetary motion; describe the geometric content and observational consequences of each.
3. List Galileo's telescopic observations and explain the success or failure of Ptolemaic and Copernican models in accounting for them.
4. State and identify examples of Newton's three laws of motion.
5. State Newton's law of universal gravitation; identify the characteristics of this law that explain Kepler's laws in terms of Newton's laws.

Teaching Hints and Strategies

This chapter deals with the development of concepts of motion. It provides an opportunity to enhance student understanding of the role of scientific models by comparing Ptolemaic, Copernican, Keplerian, and Newtonian planetary theory. It is worth emphasizing that this historical sequence involves much more than a change in the inferred architecture of the planetary system. The gradual acceptance of the Copernican, and later the Keplerian, system was one of several circumstances that stimulated intense scientific interest in the problem of motion during the sixteenth and seventeenth centuries. Not until the publication of Newton's *Principia* did dynamical theory regain compatibility with planetary theory. With the consolidation of the Newtonian system, even the standards for what a scientific model should do had changed dramatically.

The constant change in human understanding of the universe is very disturbing for students who just want to know what is "right." An understanding of this constant change should be a central issue in any general education science course. Excellent background material can be found in the first 15 chapters of *Foundations of Modern Physical Science* by G. Holton and D. H. D. Roller (Addison-Wesley, 1958) and in *The Copernican Revolution* by Thomas Kuhn (Harvard University Press, 1957).

The introductory section and Astronomer's Toolbox 2-1 deal with the efforts of ancient astronomers to observe and describe planetary motions. The geocentric theories, which were originated by the ancient Greeks, were based on the assumptions that the Earth is stationary and that the motion of celestial objects is uniform motion on a circular path. A class discussion of the observational basis for these assumptions can lead students to examine the scientific merit of these early cosmologies. Their acquaintance with the celestial sphere/ecliptic model in Chapter 1 will help students to recognize the viability of a geocentric point of view for explaining diurnal motions and secular solar or lunar motions. A brief summary of the Aristotelian theory of motion is necessary if students are to appreciate that the reliance upon deferents, epicycles, and other Ptolemaic structures was neither arbitrary nor bizarre. By carefully contrasting the Aristotelian and Newtonian perspectives, an instructor can enhance student understanding of both and reveal the extent to which "common sense" is substantially Aristotelian, whereas Newtonian concepts are nontrivially abstract.

A discussion of archaeoastronomy and such sites as Stonehenge (Great Britain) or Chimney Rock (Colorado) will develop the students' appreciation for pre-modern astronomy. Discussion of the need for accurate calendars in early civilizations will provide an understanding of the applied nature of ancient astronomy.

Section 2-1 introduces the Copernican heliocentric system in the context of planetary configurations. The description of retrograde motion in the Copernican system is quite different from that in the geocentric system, and this difference should be emphasized. In addition, the differences between sidereal and synodic periods should be discussed. Note that, whereas the synodic period is directly measurable, the sidereal period is not. Indeed, the sidereal period is a Copernican theoretical construct that plays no role in the Ptolemaic model; if one assumes a heliocentric model, the sidereal period assumes a counterfeit degree of "obviousness" absent in a geocentric framework. It is worth emphasizing to students that Copernicus advocated his model primarily on mathematical, aesthetic, and philosophical grounds. The archive of astronomical observations available to him and his contemporaries was explained just as successfully by the Ptolemaic model as by his heliocentric model. Astronomical tables (predictions) based on the Copernican model did provide a better match to later observations, but it is easy to understand why scientifically conservative contemporaries did not rush to embrace the new model.

Tycho's observations mentioned in Section 2-2 were crucial for further progress in understanding the structure of the solar system because they were the most accurate observations ever performed.

Remind students that the telescope had not yet been invented. Despite being a great observer, Tycho was not a great human being. Students are often fascinated with Tycho's lifestyle and therefore a short discussion of it sparks their interest.

Kepler's laws of planetary motion are presented in Section 2-3. The following classroom demonstration can be helpful in showing the properties of ellipses but also in developing a feeling for this difficult concept, which Kepler had solved. Use a 9″ by 15″ Styrofoam block and map tacks as an ellipse maker. Make a loop of string to generate a circle with a radius equal to 4 inches. Mark foci at half-inch intervals along a horizontal line to generate ellipses, using several pens of different colors. The circle and ellipses have eccentricities equal to 0, 0.14, 0.33, and 0.6 for the first four figures. Generate these in front of the class and compute the eccentricities. Most students find it peculiar that the first two ellipses look like circles. Mark one of the foci for each ellipse with the appropriate colored pen. Have the students compare the eccentricities of planetary orbits given in Table A-1 (Appendix at the end of the book) with those of the ellipses generated. The result is that all planetary orbits correspond to low-eccentricity ellipses that are almost indistinguishable from eccentric circles, that is, circles with the Sun located at a position offset from the center by an amount proportional to the eccentricity. Note that the use of eccentric rather than concentric circles allowed Hipparchus and Ptolemy to explain, for instance, the variation in apparent solar orbital speed (indicated by the unequal subdivision of the solar year by equinoxes and solstices) without abandoning the dynamical principal of uniform circular motion. It is instructive to point out that Kepler's laws constitute an empirical description of planetary motion but provide no physical explanation. Their descriptive success, however, stimulated Newton's search for an adequate dynamical theory. For Kepler and many of his contemporaries, that search was for a plausible tangential force that weakened with distance from the Sun. Newton was the first to recognize that a radial force was required to account for planetary orbits.

Galileo's telescopic observations (Section 2-4) provide an excellent example of the use of observations to test a scientific theory. The discovery of Jupiter's Galilean satellites removed the uniqueness of the Earth as a second center of motion in the Copernican system. The sequence of phases and apparent sizes of Venus were shown by Galileo to be consistent with a heliocentric but not with a geocentric cosmology. This critical test should be carefully demonstrated with a light source and sphere. Be careful to investigate an adequate sample of Venus's orbital positions in each system, identifying the configuration, apparent size, and phase of the planet at each position.

Newton's laws of motion (Section 2-5) require careful attention because most students cling to a quasi-Aristotelian conceptual picture even after lengthy (and, from the instructor's perspective, lucid) discussion. Recent research in physics education has provided pedagogically useful insights into the nature and persistence of the conceptual difficulties experienced by most students when they first encounter Newtonian kinematics and dynamics. Instructors who hope to convey substantial understanding of these topics will find an invaluable commentary on this research in *Guide to Introductory Physics Teaching* by Arnold Arons (Wiley, 1990). Lecture demonstrations with rolling balls, moving carts, and so on are helpful in assuring that the students appreciate the difference between velocity and acceleration. The concept of a force is relatively easy for students to grasp; however, the idea of forces acting without contact is somewhat foreign at first. They will accept this idea because of their previous exposure to gravity as the explanation for falling bodies.

In one respect, the law of inertia is easily demonstrated by using a feather and a coin in a tube, which can be evacuated by means of a vacuum pump. (This experiment was performed by Apollo astronauts on the Moon, using a feather and a hammer, and is available on film loops or videodiscs.) Yet, the idea that external forces are always required for the persistence of motion is very difficult to overcome. The robustness of this Aristotelian preconception illustrates how difficult it is for most students to acquire the full Newtonian perspective. It is important in lectures to examine "commonsense" experience, for instance, by identifying the "hidden" forces due to wind resistance and friction that explain the need for constant force to achieve uniform linear motion. But hands-on experience with low-friction demonstration apparatus is the most effective teaching tool. Instructors whose resources include such laboratory equipment would be wise to consider incorporating it in their astronomy syllabi.

Two subtle aspects of Newton's second law are the vector nature of forces and the concept of a net force. All forces do not produce accelerations; only net forces do. Accelerations always occur in the direction of the net force. This fact is an essential element in the recognition of the third law of motion. Newton's third law is very difficult for students to accept. They frequently become confused when applying the second and third laws together. If all forces are equal and opposite, how can nonzero net force ever arise? Remind the students that the equal and opposite forces in an action-reaction pair are always applied to different objects. It is helpful to discuss two general problems in relation to this law. The first has to do with hidden forces. Frequently, the presence of the reaction force is masked by friction. The second has to do with mass differences. The third law says that the forces are equal in magnitude. Two objects having very different masses undergo (by the second law) very different accelerations. Remember to point out that acceptance of this law requires that "no" celestial object is fixed and at the center of the universe. This conclusion is the first implication that the concept of the "center" of the universe may not be as straightforward as it once appeared. Universal gravitation can be presented as a logical extension of Kepler's laws of planetary motion and Galileo's experimental results with falling bodies—the Moon and the apple! The concepts of induction and deduction are neatly presented in Newton's development of the laws of motion and gravitation and his subsequent derivation of Kepler's laws. The inverse square nature of gravitational forces should be emphasized because it appears later in other contexts. The derivation of Kepler's laws is an excellent example of the predictive nature of a successful scientific theory. The derived forms of Kepler's law are much more general and are shown to be just special cases that result from the three basic laws of motion and universal gravitation.

It is also interesting to note here that the Newtonian synthesis, the use of the same laws to explain celestial and terrestrial motion, is a turning point in the development of astronomy. The ancient (Aristotelian) view held that the Earth and the heavens were governed by different rules and made of different materials. If this were the case, then our terrestrial experiences could not be used to understand celestial objects. Newton explained the motion of apples and moons using the same rules. The assumption that celestial bodies are subject to the same laws of nature as is the Earth is an essential ingredient in modern astronomy and removes the Earth forever from the unique position it held in ancient science.

Class Discussion and Projects

1. Using *Starry Night*™ to plot the retrograde motion of Mars and the other superior planets is a rewarding and worthwhile class assignment. If your classroom has a video projection system, project the image and discuss what is happening at each stage of the motion.

2. Compare the Ptolemaic and Copernican explanations for the observed path of a given planet on the sky (perhaps determined as in number 1, above). In the Ptolemaic case, the students can determine the periods, rotational directions, and ratio of deferent and epicycle radii that will yield the observed motion. Note that the relative radii are not uniquely determined by the motion; hence, the Ptolemaic model lacks a natural scale. In the Copernican case, the required parameters are the ratio of orbital radii and the ratio of sidereal periods. Note that one of the Ptolemaic solutions is numerically identical to the Copernican solution, thereby demonstrating that the deferential and epicyclic motions are images of the two Copernican orbital motions.

3. Discuss the role of verification in modern scientific theories in the context of Newton's derivation of Kepler's laws, and the experimental verification of the theory of relativity.

Review Questions

1. Newton proposed and developed the law of gravitation.

2. (b) The Earth orbits the Sun in an elliptical orbit.

3. (b) Uranus takes the longest because it is the farthest from the Sun.

4. A Sun-centered model is a heliocentric model.

5. A sidereal period is measured with respect to the distant stars. The sidereal period of the Earth is 365.26 days.

6. By placing the Sun at the center of the solar system, the retrograde motion of the superior planets can be explained by visualizing the Earth on an inside track overtaking and passing the superior planet on the outer track. The same applies for Mercury and Venus as they pass the Earth in their orbits around the Sun.

7. Inferior planets, Mercury and Venus, can never go through opposition, which is when planets are on the opposite side of the Earth from the Sun. Superior planets can never be seen at inferior conjunction, which is when the planet is between the Earth and the Sun.

8. In either case, the best time to observe a planet is when it appears farthest in angle from the Sun. Mercury or Venus can therefore be seen best at greatest eastern or western elongation. Mars, Jupiter, and Saturn are best seen at opposition.

9. The sidereal periods are the true orbital periods. They are measured with reference to the position of a planet or other body compared to the background stars. The synodic periods express the time taken to cycle through the configurations, such as superior conjunction to superior conjunction.

10. Kepler deduced three laws of planetary motion from Tycho Brahe's observational data. The first law states that the planets orbit around the Sun in elliptical paths with the Sun at one focus. His second law says that a line joining a planet and the Sun sweeps out equal areas in equal times. The consequence of this second law is that planets move faster the closer they are to the Sun, while slowing down the farther they are away from it. The third law states that the square of a planet's sidereal period around the Sun is directly proportional to the cube of the length of the semimajor axis of its orbit. This law implies that for any pair of planets, the one that has the greater average distance from the Sun has the longer year. The three laws are important because they summarize how planets orbit the Sun and how moons orbit planets.

11. The phases of Venus and the brightness of Venus at its various phases are only possible if Venus orbits the Sun. The motion of Jupiter's satellites provided evidence for objects orbiting an object other than the Earth.

12. Whereas his predecessors, like Kepler, approached planetary motion from astronomical observations, Newton approached it from a theoretical, mathematical description of gravity. His equations described the orbits of the planets, as well as the motion of other objects, but he derived them first from principles using gravitational theory. Comparison of the predictions of Newton's equations with the predictions of Kepler's laws (deduced from observed motion), served to test the validity of the equations.

13. Weight is the force of attraction on a mass due to the gravity of the Earth or any other body. Mass is a measure of the number of particles an object has, regardless of its location.

14. The existence and location of Neptune were correctly predicted from Newton's law of universal gravitation in advance of Neptune's discovery. Its presence was inferred by its gravitational effect on the orbit of Uranus.

15. An astronaut has to exert a force on a weightless object to move it because the object has inertia. Inertia is the resistance any object has to changing its direction of motion or speed.

Advanced Questions

16. Use the definitions of kinetic energy and momentum:

$$KE = \frac{1}{2}mv^2 \quad \text{and} \quad p = mv$$
$$p^2 = m^2v^2$$
$$\frac{p^2}{2m} = \frac{1}{2}mv^2$$
$$\frac{p^2}{2m} = KE$$

17. Use the conversion factors from Astronomer's Toolbox 2-1:

(a) 1 pc = 3.26 ly. So, 8.3 pc × 3.26 ly/pc = 27.058 ly = 27.1 ly.

(b) 6.52 ly × (1 pc/3.26 ly) = 2.00 pc.

(c) 1 AU = 1.5×10^8 km. 8450 AU × 1.5×10^8 km/AU = 1.27×10^{12} km ≅ 1.3×10^{12} km.

(d) 1 kpc = 10^3 pc and 1 Mpc = 10^6 pc. 2.7×10^3 Mpc × 10^3 kpc/Mpc = 2.7×10^6 kpc.

18. No, it is not possible for any object other than Earth in the solar system to have a synodic period of exactly one year. Kepler's laws show that all objects at different distances from the Sun have different orbital periods. If an object were farther from the Sun than Earth, that object would have to move faster than its normal orbital speed for it to have a synodic period of one year and thus be aligned with the Earth at the same time each year. One that was closer would have to move more slowly than normal to have the same synodic period as Earth. There is no mechanism for speeding up or slowing down a body in space. If the object were in the same orbit as the Earth, it would never change position with respect to us and so it would not have a synodic period.

19. The time from one opposition to the next is a superior planet's synodic period. The farther a planet is from the Sun, the slower that planet orbits the Sun. Therefore, the farther the planet is from the Sun, the more quickly the Earth can return to being directly between it and the Sun—that is, the shorter the time from opposition to opposition compared to the same intervals for superior planets closer to the Earth.

20. According to Kepler's law of equal areas, if an area of 5.2 AU^2 is swept out in one particular year, then an area of 5.2 AU^2 is swept out every year. So, if 5.2 AU^2 is swept out in 2007 then

5 × 5.2 AU^2 = 26.0 AU^2 is swept out in 5 years.

21. Use Kepler's third law ($P^2 = a^3$) with P = 1000 yr. $P^2 = 1000^2 = 10^6 = a^3$. So,

$a = (10^6)^{1/3} = 10^2$ = 100 AU. The average distance from the Sun is 100 AU. The average equals ½ of the length of the major axis. For a very elliptical orbit, the maximum distance to the focus where the Sun resides will be approximately the same as the length of the major axis. Thus, the maximum distance is 200 AU, which corresponds to the limiting case of a long, skinny elliptical orbit that grazes the Sun.

22. The average distance from the Sun is 1/2(0.5 AU + 3.5 AU) = 2 AU. Using Kepler's third law with a = 2 AU, we find $P = (2\text{ AU})^{3/2} = 2.8$ yr.

23. Jupiter is almost full all the time as seen from Earth. From Saturn, Jupiter's phases would resemble those of Venus or Mercury as seen from Earth. See Figure 2-10.

24. The planets would seem to move eastward normally but westward during retrograde as seen from Australia, just as in the northern hemisphere.

25. Astrology is not a science. If you were to objectively test the predictions of astrology, you would find them either to be invalid or so general that they can neither be proved nor disproved. Astrology fails the scientific method.

Discussion Questions

26. Mars shows a great variation in brightness due to the difference between its closest distance

(~ .5 AU) and greatest distance (~ 2.5 AU). Venus has a larger change in distance but its brightness is low at inferior conjunction as it is around new phase.

27. See Figure 2-8.

What If …

28. At 2 AU, our period would be 2.83 years, and the seasons would be 2.83 times as long. Solar radiation would be about one-fourth of what it is at 1 AU, and the Earth would be much colder.

29 At .5 AU, between Venus and Mercury, our period would be .35 years, seasons would be much shorter, and the solar radiation would be four times greater, so although summer vacation would be very short, you would get sunburned very quickly!

30. Because the force of gravity varies as the square of the distance, the gravitational force between the Earth and the Sun would *decrease* by a factor of 100 if the distance is *increased* by a factor of 10.

31. Remember that $\frac{\mathrm{G}m_{Sun}m_{Earth}}{d^2} = m_{Earth}a$. The mass of the Earth cancels and the acceleration does not depend on the mass of the Earth! However, if the mass of the Sun changed appreciably, we would be in for a big change!

32. The Earth would fly away in a straight line. It would take some time for the Earth to cool down due to heat energy stored in the atmosphere, so humans would have time to contemplate freezing to death while they dealt with the ultimate energy crisis.

33. If skies were perpetually cloudy, we would know nothing about the cosmos. Humans could learn only by voyaging above the clouds in balloons or rockets or by using radio waves, which pass through the cloud layers.

34. If scientists remained believers in the first theory, then new data would have to be ignored if it contradicted the theory. Since this is contrary to the scientific method, science as we know it would not exist.

Web Questions

Answers will vary.

Got It? Questions

Answers provided in the back of the book.

Observing Projects

42. This is a simple observing project with no specific questions.

43.

(a) To the left (eastward); January 23, 2012; April 14, 2012.
(b) To the left (eastward); January 23, 2012; April 14, 2012; Yes, these dates are the same as in part (a).
(c) March 3, 2012; the date is midway between the two dates when Mars appears to be stationary. At this time, the faster-orbiting Earth is overtaking the slower-orbiting Mars and Mars will appear to be moving backward, that is, in a retrograde way, when observed from the moving Earth. As can be seen from this space view of the solar system, Mars is at opposition as seen from Earth at this midway point on the retrograde motion.

44.

(a) The phase of Venus will depend upon the date but can be seen to go through the full cycle of phases from new through crescent to gibbous and on to full phase, and then following this sequence in reverse.
(b) Venus is closer to the Earth than is the Sun when in its crescent phase and farther from the Earth than the Sun when in its gibbous phase. (You will see that there is a relationship between the angular size of Venus and its phase, the crescent Venus appearing much bigger than the gibbous and full phase Venus.)
(c) Venus will be coming toward us when its angular size is increasing.
(d) Venus shows the observed sequence of phases because its smaller orbit around the Sun allows Venus to pass between the Earth and the Sun.
(e) The cycles of phases are different for Mars and Venus because Venus is an inferior planet, having a smaller orbital radius than Earth, whereas Mars is a superior planet, its orbit being bigger than that of the Earth. Thus, Mars will never pass between Earth and the Sun and will never show a crescent phase.

45.

(a) Mercury, Venus, Earth, Mars
(b) Mercury, Venus Earth, Mars
(c) These lists are the same.
(d) This agreement means that the farther the planet is from the Sun, the longer it takes to move around a complete orbit.
(e) This behavior is consistent with Kepler’s third law.

The History of Astronomy Web site can provide information on all aspects of the subject: www.astro.uni-bonn.de/~pbrosche/astoria.html. Web searches using "History of Astronomy" as the search term will yield *millions* of results. Being specific will help narrow the search.

In 1980 the Historical Astronomy Division (HAD) was established as a special subject division of the American Astronomical Society. Its purpose is to advance interest in topics relating to the historical nature of astronomy. History is broadly interpreted to include traditional history of astronomy, archaeoastronomy, and the application of historical records to modern astrophysical problems. Members of HAD include historians and astronomers with an interest in the history of their profession. had.aas.org/index.html

Find information on the tides of history that brought Tycho and Kepler together. It is a fascinating story! When visiting Denmark, visit Tycho's observatory and the IMAX Tycho Brahe Planetarium: www.tycho.dk/in_english/.

Starry Night™ can be very helpful when visualizing retrograde motion. See Favourites-Observing Projects-Planetary Motion.

Consult *Sky & Telescope* or an almanac to determine if an eclipse by a Jovian moon is visible. Even a small telescope can reveal a black dot on the surface of Jupiter that is the eclipse shadow. www.skyandtelescope.com/observing/objects/planets. You can see what Galileo saw!

Center for Archaeoastronomy: www.wam.umd.edu/~tlaloc/archastro/index.html

Chapter 2: Stars and Constellations.

1. Crux, Sagitta, Circinus, (Equuleus)
2. Hydra, Ursa Major, Hercules, (Centaurus)
3. Hydra stretches almost 100° across the sky.
4. Bootes, Virgo, Leo and Canes Venatici.
5. Coma Berenices.
6. Spica, Arcturus, Cor Caroli, Denebola.
7. The Little Dipper, The W, Kids.
8. Big Dipper Stars: Alkaid, Mizar, Alioth, Megrez, Phecda, Merak, Dubhe.
9. Alkaid = HIP 67301 = Zeta Ursa Majoris = 85 Ursa Majoris.
10. (a) Upper-left corner, (b) Lower-right corner.
11. Merak and Dubhe.
12. 34º 04'.
13. Kochab, Pherkad
14. Caph, Schedar, Gamma Cassiopeiae, Ruchbah, Segin.
15. (a) Elnath (b) Taurus.
16. Delta Bootes, Nekkar, Seginus, Rho Bootes, Izar
17. Cor Caroli, Chara.
18. Coma Berenices and Canes Venatici
19. Leo Minor.