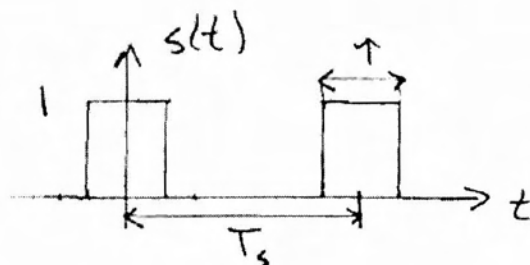


Chapter 3

3-1



$$d = \frac{\tau}{T_s}$$

$$s(t) = \sum_{n=-\infty}^{\infty} a_n \cos(n\omega_s t)$$

$$a_0 = \frac{1}{T_s} \int_0^{T_s} s(t) dt = \frac{\tau}{T_s} = d$$

$$a_n = \frac{2}{T_s} \int_{-T_s/2}^{T_s/2} s(t) \cos(n\omega_s t) dt$$

$$= \frac{2}{T_s} \int_{-\tau/2}^{\tau/2} \cos\left(\frac{n2\pi}{T_s} t\right) dt$$

$$= \frac{2}{T_s} \frac{\sin\left(\frac{n2\pi}{T_s} t\right)}{n2\pi/T_s} \bigg|_{-\tau/2}^{\tau/2}$$

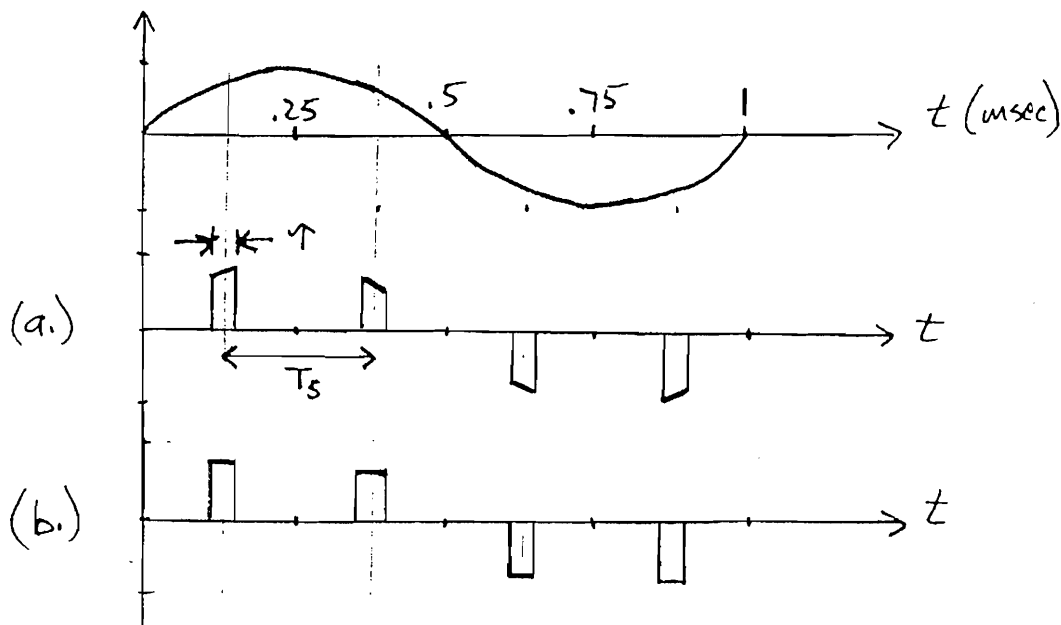
$$\sin(-x) = -\sin(x)$$

$$= \frac{1}{n\pi} \left[\sin\left(n\pi\tau/T_s\right) - \sin\left(-n\pi\tau/T_s\right) \right]$$

$$= \frac{1}{n\pi} \left[2 \sin\left(n\pi\tau/T_s\right) \right]$$

$$= \frac{2d \sin(n\pi d)}{(n\pi d)}$$

3-2



3-3

```

A := 1      f1 := 2500    f2 := 4000
f := 0,200 ..5000

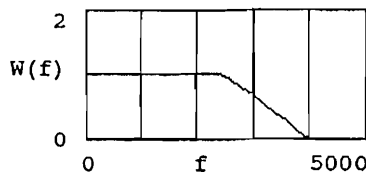
W1(x) := if(|x| < f1,A,0)

W2(x) := [ -A / (f2 - f1) ] * (|x| - f2) * (phi(|x| - f1) - phi(|x| - f2))

W(x) := W1(x) + W2(x)

fs := 10000
tau := 50 10^-6
Ts := 1 / fs
d := tau / Ts

```



Naturally-sampled PAM

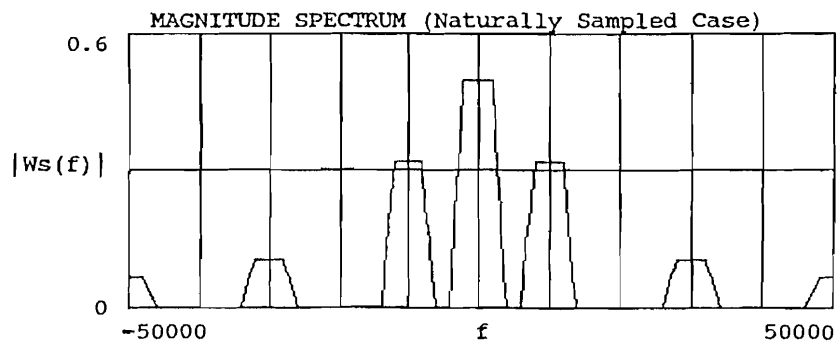
```

n := -5,-4 ..5
Sa(x) := if [ x != 0, sin(x)/x, 1 ]
f := -50000,-48000 ..50000

Ws(f) := d * sum_n (Sa(pi * n * d)) * W(f - n * fs)

```

3-3 Cont'd.



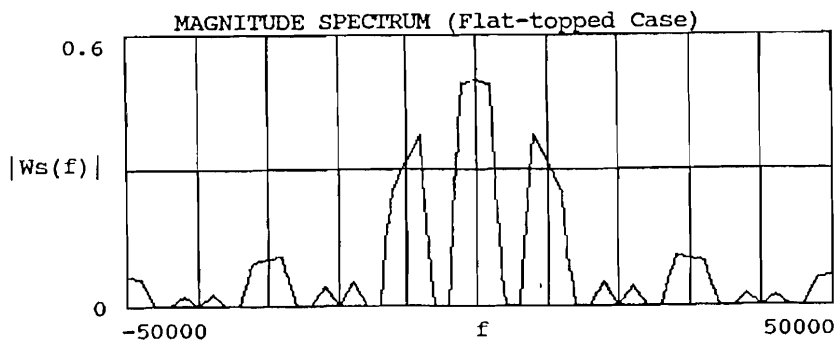
3-4

Flat-topped PAM

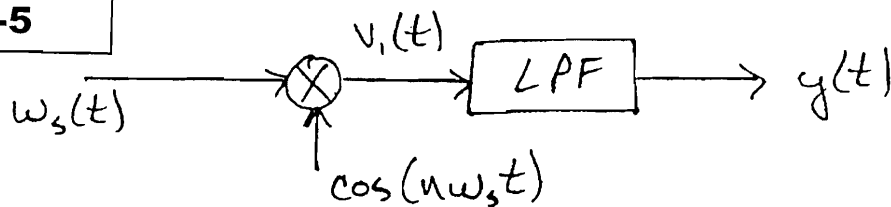
$$H(f) := \tau \text{Sa}(\pi \tau f)$$

$$W_s(f) := \left[\frac{1}{T_s} \right] \cdot H(f) \cdot \sum_n W(f - n f_s)$$

See next screen for plot



3-5



3-5 Cont'd (a.)

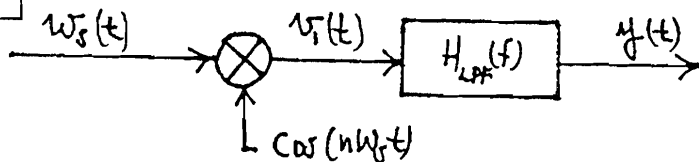
$$w_s(t) = w(t) \left[d + 2d \sum_{k=1}^{\infty} \frac{\sin(k\pi d)}{k\pi d} \cos(k\omega_s t) \right]$$

$$\begin{aligned} v_1(t) &= w_s(t) \cos(n\omega_s t) \quad \begin{array}{l} \text{Deleted by LPF} \\ \text{Deleted by LPF} \end{array} \\ &= w(t) \left\{ d \cos(n\omega_s t) + 2d \sum_{\substack{k=1 \\ k \neq n}}^{\infty} \frac{\sin(k\pi d)}{k\pi d} \cos(k\omega_s t) \cos(n\omega_s t) \right. \\ &\quad \left. + 2d \frac{\sin(n\pi d)}{n\pi d} \underbrace{\cos^2(n\omega_s t)}_{\frac{1}{2} + \frac{1}{2} \cos(2n\omega_s t)} \right\} \quad \begin{array}{l} \text{Deleted by LPF} \\ \text{Deleted by LPF} \end{array} \end{aligned}$$

$$\Rightarrow y(t) = \frac{d \sin(n\pi d)}{n\pi d} w(t) = C w(t)$$

$$(b) \quad C = \frac{d \sin(n\pi d)}{n\pi d}$$

3-6



$$v_1(t) = w_s(t) \cos(n\omega_s t) \longleftrightarrow V_1(f) = W_s(f) * \frac{1}{2} [\delta(f - n f_s) + \delta(f + n f_s)]$$

Using (3-10), we get

$$V_1(f) = \frac{1}{f_s} H(f) \sum_{k=-\infty}^{\infty} W(f - k f_s) * \frac{1}{2} [\delta(f - n f_s) + \delta(f + n f_s)]$$

$$= \frac{1}{f_s} H(f) \sum_{k=-\infty}^{\infty} [W(f - n f_s - k f_s) + W(f + n f_s - k f_s)]$$

$$= \frac{1}{f_s} H(f) \sum_{k=-\infty}^{\infty} [W(f - (n+k) f_s) + W(f - (k-n) f_s)]$$

$$\Rightarrow V_1(f) = \frac{2}{f_s} H(f) W(f) + \frac{1}{f_s} H(f) \left[\sum_{\substack{k=-\infty \\ k \neq -n}}^{\infty} W(f - (n+k) f_s) + \sum_{\substack{k=-\infty \\ k \neq n}}^{\infty} W(f - (k-n) f_s) \right]$$

(Use (3-11)) Does not pass through $H_{LPF}(f)$

$$\Rightarrow Y(f) = \frac{2}{f_s} H(f) W(f) H_{LPF}(f) \stackrel{\text{Set}}{=} 2d \frac{\sin(\pi z f)}{\pi z f} W(f) H_{LPF}(f) = C W(f)$$

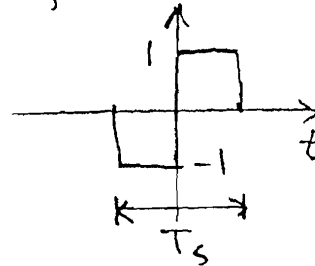
$$\Rightarrow H_{LPF}(f) = \begin{cases} \frac{C}{2d} \frac{\pi z f}{\sin \pi z f}, & |f| < f_{co} \\ 0, & f \text{ elsewhere} \end{cases} \quad \text{where } B < f_{co} < f_s - B$$

3-7 using (3-10)

$$W_s(f) = \frac{H(f)}{T_s} \sum_{k=-\infty}^{\infty} W(f - kf_s)$$

where $H(f)$ is the spectrum of the Manchester encoded pulse, $h(t)$.

Thus



$$\begin{aligned} H(f) &= \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt \\ &= \int_{-T_s/2}^0 (-1) e^{-j\omega t} dt + \int_0^{T_s/2} (1) e^{j\omega t} dt \\ &= \frac{j}{\omega} \left[-e^{-j\omega t} \Big|_{-T_s/2}^0 + e^{j\omega t} \Big|_0^{T_s/2} \right] \\ &= \frac{-j}{\omega} \left[2 - 2 \left(\frac{e^{j\omega T_s/2} + e^{-j\omega T_s/2}}{2} \right) \right] \end{aligned}$$

$\cos \frac{\omega T_s}{2}$

$$H(f) = -j T_s \frac{(1 - \cos \omega T_s/2)}{\omega T_s/2}$$

3-7 Cont'd.

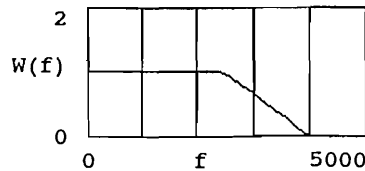
$$A := 1 \quad f_1 := 2500 \quad f_2 := 4000$$

$$f := 0, 200 \dots 5000$$

$$W_1(x) := \text{if}(|x| < f_1, A, 0)$$

$$W_2(x) := \left[\frac{-A}{f_2 - f_1} \right] (|x| - f_2) (\Phi(|x| - f_1) - \Phi(|x| - f_2))$$

$$W(x) := W_1(x) + W_2(x)$$



$$f_s := 10000$$

$$T_s := \frac{1}{f_s}$$

$$j := \sqrt{-1}$$

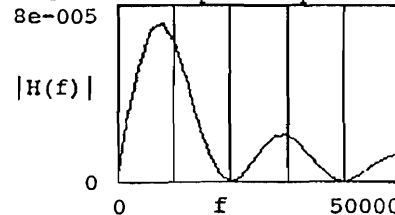
$$n := -5, -4 \dots 5$$

$$S_a(x) := \text{if} \left[x \neq 0, \frac{\sin(x)}{x}, 1 \right]$$

$$f := -50000, -48000 \dots 50000$$

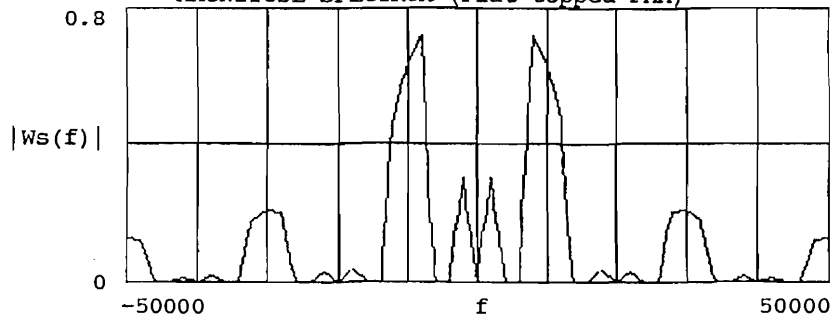
$$H(f) := j \cdot T_s \cdot S_a \left[\pi \cdot f \cdot \frac{T_s}{2} \right] \cdot \sin \left[2 \pi \cdot f \cdot \frac{T_s}{4} \right]$$

Manchester-pulse Spectrum



$$W_s(f) := \left[\frac{1}{T_s} \right] \cdot H(f) \cdot \sum_n W(f - n f_s)$$

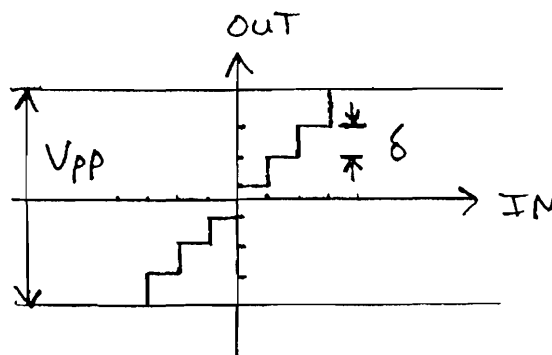
MAGNITUDE SPECTRUM (Flat-topped PAM)



3-8

Binary PCM

$$M = 2^n \text{ levels}$$



3-8. Cont'd for $|u_q| \leq \frac{P}{100} V_{PP}$

we need

$$\text{step size} = \delta = \frac{V_{PP}}{M} \leq \frac{2P}{100} V_{PP}$$

$$\frac{1}{M} \leq \frac{P}{50}$$

$$2^n = M \geq \frac{50}{P}$$

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

$$= \log_a(b) \log_b(x)$$

$$n \geq \log_2 \left(\frac{50}{P} \right)$$

$$\geq \log_2(10) \log_{10} \left(\frac{50}{P} \right)$$

3-9

(a.) $f_s = 2B = 2(100) = \underline{\underline{200 \text{ samples/sec}}}$

(b.) Using the results given in prob. 3-8.

$$n \geq 3.32 \log_{10} \left(\frac{50}{P} \right) = 3.32 \log_{10} \left(\frac{50}{0.1} \right) \\ = 8.96$$

$$\underline{\underline{n = 9 \text{ bits/word}}}$$

$$(c.) R = \left(\frac{n \text{ bits}}{\text{word}} \right) \left(\frac{f_s \text{ words}}{\text{sec}} \right) = 200(9) = \underline{\underline{1.8 \text{ kbits/sec}}}$$

(d.) For binary PCM $D = R$

eq. (3-74) $D = \frac{2B}{1+r}$, for $B_{\min}$, $r=0$

$$\Rightarrow B = \frac{D}{2} = \underline{\underline{900 \text{ Hz}}}$$

3-10

$$\left(\frac{S}{N}\right) = M^2 = 10^{\frac{30}{10}} = 1000 \Rightarrow M = \sqrt{1000} = 31.6$$

$\uparrow$ (30dB)

$$\Rightarrow \text{Use } M=32=2^5 \Rightarrow \text{Need } \underline{5 \text{ bits/sample} = n}$$

$$R = f_s n = \left(8 \frac{\text{ksamples}}{\text{sec}}\right) \left(\frac{5 \text{ bits}}{\text{sample}}\right) = 40 \frac{\text{kbits}}{\text{sec}}$$

$$\Rightarrow R = \left(40 \frac{\text{kbits}}{\text{sec}}\right) \left(\frac{\text{byte}}{8 \text{ bits}}\right) = 5 \frac{\text{kbytes}}{\text{sec}}$$

$$T = \frac{700 \text{ Mbytes}}{5 \frac{\text{kbytes}}{\text{sec}}} = \frac{700 \times 10^6}{5 \times 10^3} \text{ sec} = 140 \times 10^3 \text{ sec}$$

$$\Rightarrow \underline{T = 140 \times 10^3 \text{ sec} = \frac{140 \times 10^3 \text{ sec}}{60 \text{ sec/min}} = 2,333 \text{ min} = \underline{38 \text{ hr}, 53 \text{ min}}}$$

3-11

Using (3-1B),

$$(a) \left(\frac{S}{N}\right)_{\text{dB, peak}} = 6.02n + 4.77 = 55 \Rightarrow n = 8.34 \Rightarrow \text{Use } \underline{n=9 \text{ bits/word length}}$$

$\uparrow$ (5dB)

$$M = 2^n = 2^9 = \underline{512 \text{ quantizing steps}}$$

$$(b) f_s = 2f_{\text{analog}} = 2(4.2 \text{ MHz}) = 8.4 \text{ Msamples/sec}$$

$$R = f_s n = \left(8.4 \frac{\text{Msamples}}{\text{sec}}\right) \left(\frac{9 \text{ bits}}{\text{sample}}\right) = \underline{75.6 \frac{\text{Mbits}}{\text{sec}}}$$

For rectangular pulse shape,

$$\underline{B_{\text{null}} = R = 75.6 \text{ MHz}}$$

3-12

$$(a) f_s \geq 2B_{\text{analog}} = 2(20 \text{ kHz}) = 40 \frac{\text{ksamples}}{\text{sec}}$$

For 8X oversampling of the recovered PCM signal
(used to increase f_s 8X and simplify LPF requirements)

$$\Rightarrow f_{8x} = 8f_s = 320 \frac{\text{ksamples}}{\text{sec}}$$

$$B_{\text{null}} = R = n f_{8x} = \left(\frac{16 \text{ bits}}{\text{sample}}\right) \left(320 \frac{\text{ksamples}}{\text{sec}}\right) = \underline{5.12 \text{ MHz}}$$

(b) Using (3-1B)

$$\left(\frac{S}{N}\right)_{\text{peak}} = 6.02n + 4.77 \text{ dB} = 6.02(5) + 4.77 = \underline{94.77 \text{ dB}}$$

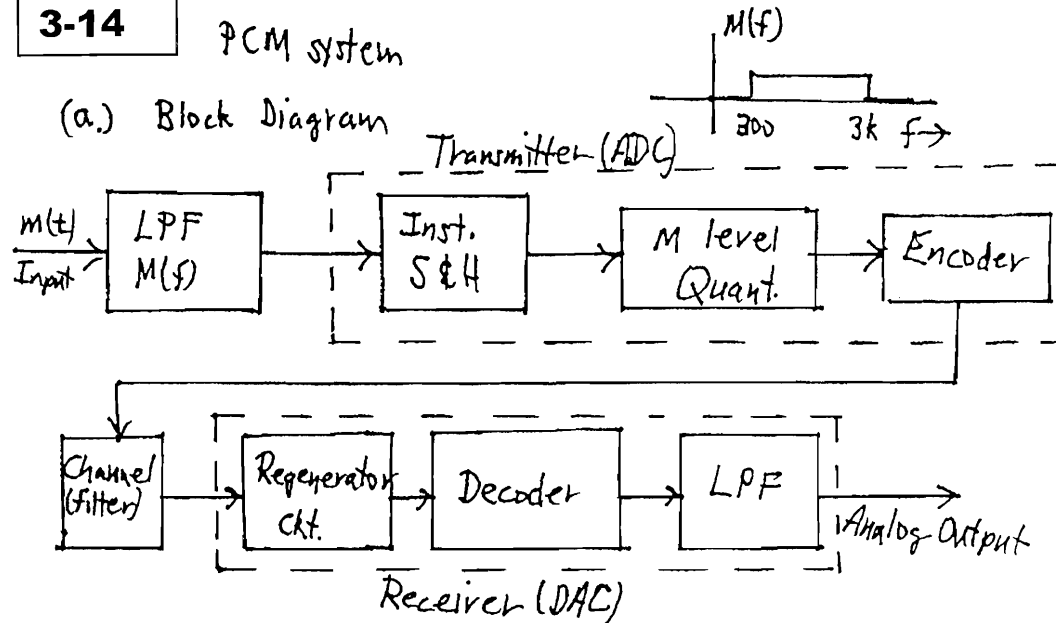
3-13

$$(44.1 \text{ k samples/sec})(16 \text{ bits/sample}) = 705.6 \text{ kb/s for Mono} \\ \Rightarrow \underline{\underline{1,411 \text{ kb/s for Stereo}}}$$

3-14

PCM system

(a.) Block Diagram



(b.) Assume $P_e = 0 \Rightarrow$ Use (3-18).

$$\left(\frac{S}{N}\right)_{\text{dB peak}} = 6.02n + 4.77 = 30 \text{ dB} \Rightarrow n = 4.19$$

$$\Rightarrow \text{Use } n = 5 + \text{parity} = \underline{\underline{6}} \quad (\text{If no parity, } n = 5 \text{ bits})$$

$$\Rightarrow B_{\text{null}} = R = n f_s = (6) \left(\frac{7.4 \text{ samples}}{\text{sec}} \right) = \underline{\underline{42 \text{ kHz (with parity)}}}$$

$$B_{\text{null}} = (5)(7) = \underline{\underline{35 \text{ kHz (No parity)}}}$$

(c) If the audio signal is a voice signal it is likely to have a large peak-to-average voltage ratio. Therefore, a

non-uniform quantizer with smaller step sizes near the zero level will reduce the quantizing noise. A $\mu = 255$

non-uniform characteristic is typically used in the U.S. See the textbook for further discussion.

3-15

For the case of (3-17b) with $P_e \leq 0$,

$$\left(\frac{S}{N}\right) = M^2 \Rightarrow \text{For no more than 0.1\% error } \left(\frac{S}{N}\right) = 0.999M^2$$

Using (3-16b) we get

$$0.999M^2 = \frac{M^2}{1 + 4(M^2 - 1)P_e}$$

$$\Rightarrow P_e = \frac{1.001 \times 10^{-3}}{4(M^2 - 1)} = \frac{2.5025 \times 10^{-4}}{M - 1}$$

For: $M=4$, $P_e \leq 1.668 \times 10^{-5}$

$M=8$, $P_e \leq 3.972 \times 10^{-6}$

$M=16$, $P_e \leq 9.814 \times 10^{-9}$

As the number of levels is increased,
the approximation becomes more constrained.

3-16

(a.) $P_e = 10^{-4}$ $\frac{S}{N} \geq 30 \text{ dB}$

$$\text{Eg. (3-16)} \quad \left(\frac{S}{N}\right)_{\text{dB}} = 10 \log_{10} \left[\frac{3M^2}{1 + 4(M^2 - 1)P_e} \right]$$

$M = 2^n$ levels

for $n=4$: $\left(\frac{S}{N}\right)_{\text{out}} = 28.4 \text{ dB}$

for $n=5$: $\left(\frac{S}{N}\right)_{\text{out}} = 33.4 \text{ dB}$

$\rightarrow M = 2^5 = \underline{\underline{32 \text{ levels}}}$

(b.) $f_s = 2(2.7 \text{ KHz}) = 5.4 \text{ k} \frac{\text{samples}}{\text{sec}}$

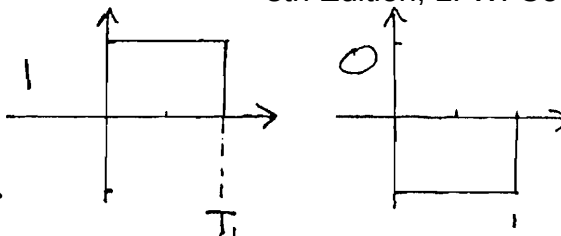
The first zero-crossing of the $\frac{\sin x}{x}$
type spectrum is:

$$B = \frac{n}{T_s} = n f_s = 5(5.4 \text{ k}) = \underline{\underline{27 \text{ KHz} = B}}$$

3-17

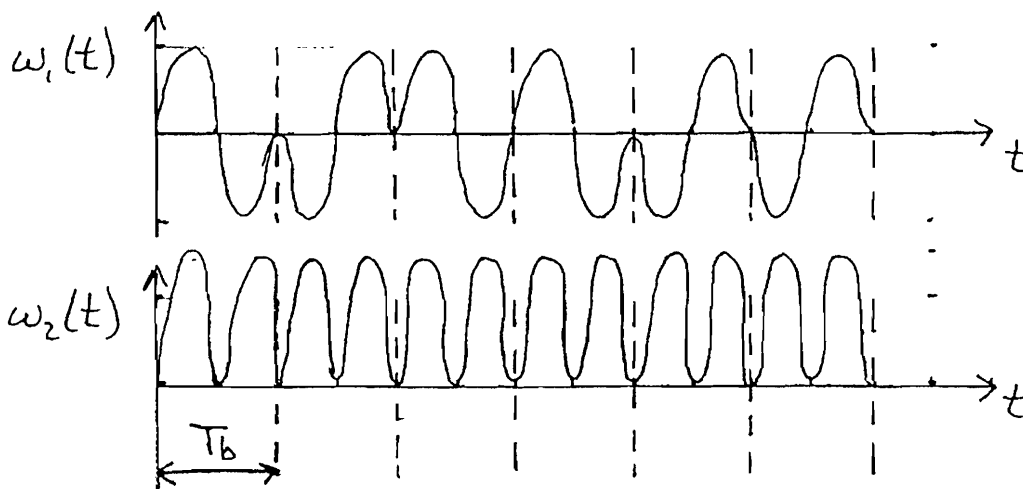
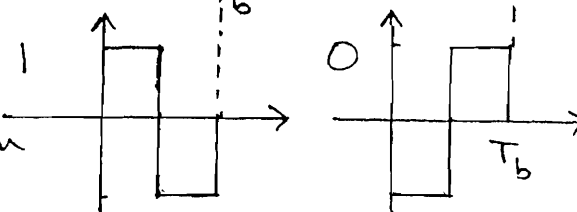
polar -

See Fig. 3-20 for
waveforms



Manchester -

Waveforms shown
below :



fund. freq. of $w_2(t) = \frac{2}{T_b} \neq f_0$

$\therefore$ Synch. shown in Fig. 3-17 will not
work for Manchester format unless
the N.B.F. ($H(f)$) is tuned to $2f_0$.

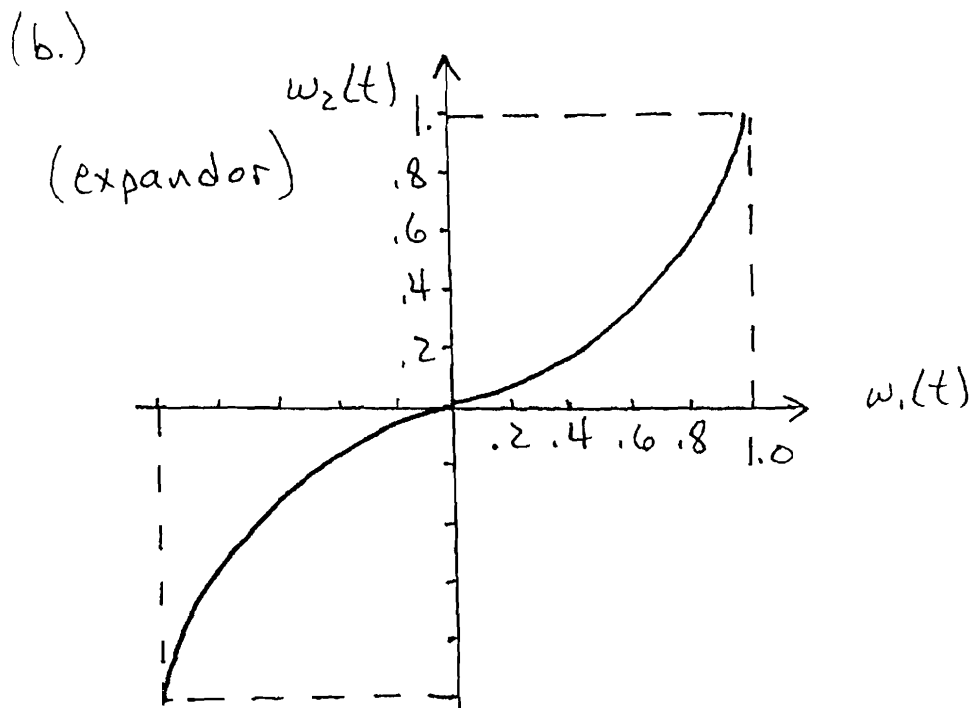
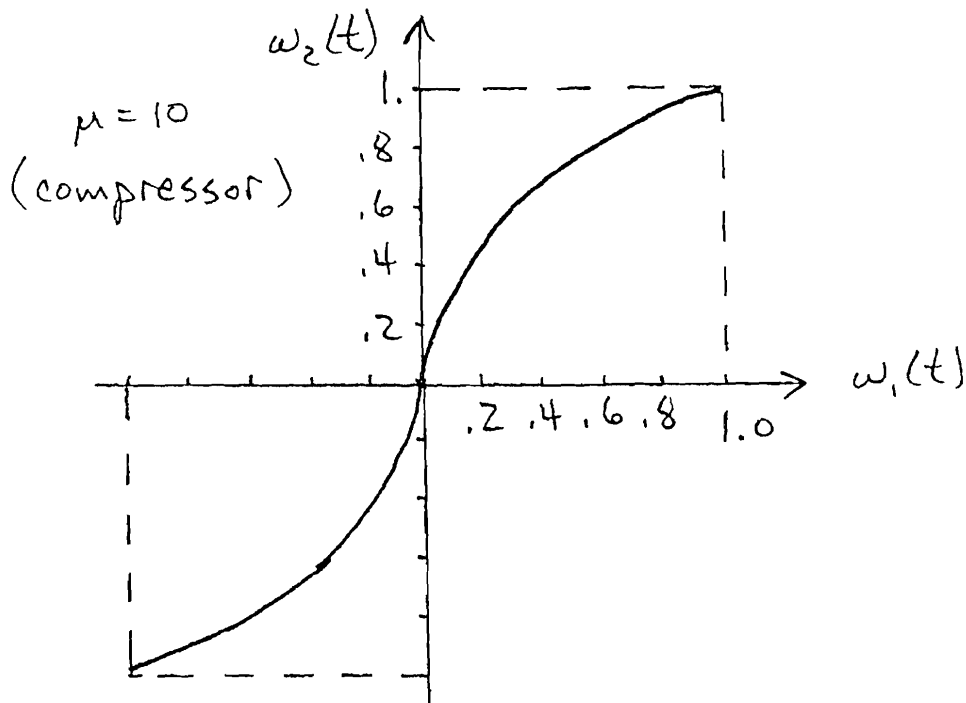
If this change is made :

$w_4(t)$ will consist of pulses $\frac{T_b}{2}$ sec.
apart, and must be passed through
a frequency divider ckt. to produce
pulses at the proper spacing - T_b .

3-18

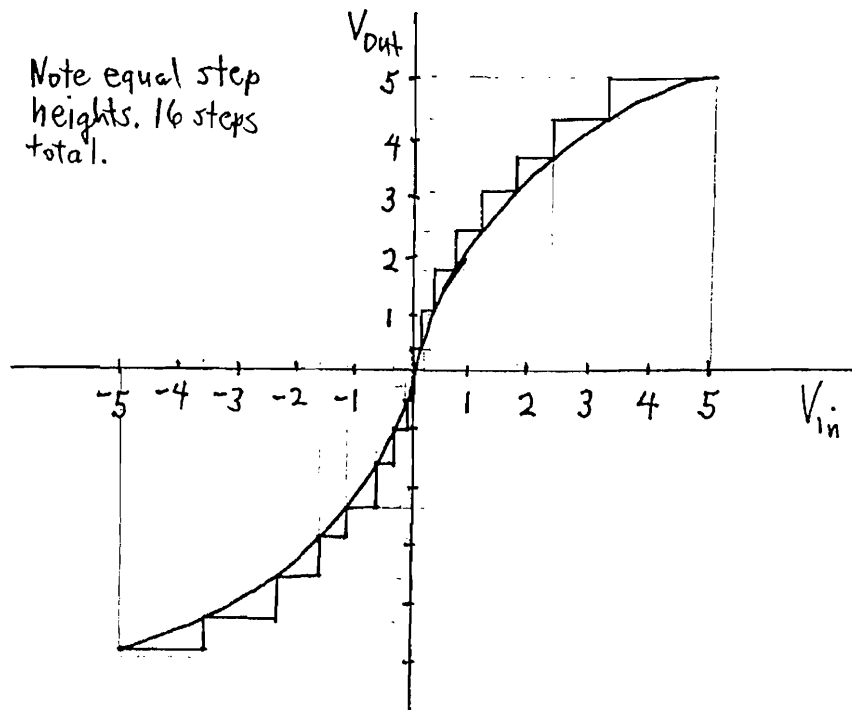
$$(a.) \quad |\omega_2(t)| = \frac{\ln(1 + \mu |\omega_1(t)|)}{\ln(1 + \mu)}$$

$$\text{let } \omega_1(t) = \frac{v_{in}(t)}{5} ; \omega_2(t) = \frac{v_{out}(t)}{5}$$



3-18. Continued

(c.) 16-level non-uniform Quantization



3-19

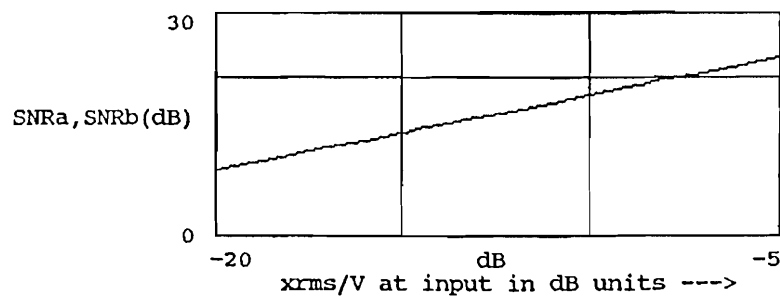
dB := -20, -19 .. -5 n := 4

(a) Using (3-25) and (3-26b), $\mu := 10$

$$\text{SNRa} := 6.02 \cdot n + 4.77 - 20 \cdot \log(\ln(1 + \mu))$$

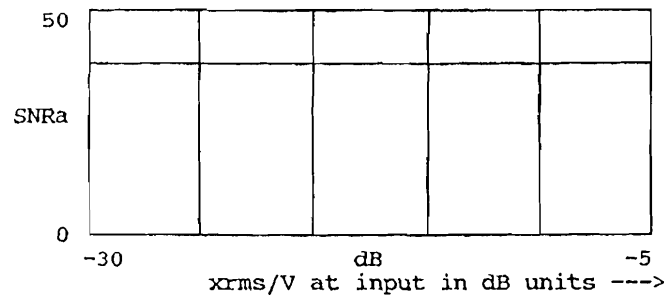
(b) Using (3-25) and (3-26a),

$$\text{SNRb(dB)} := 6.02 \cdot n + 4.77 + \text{dB}$$



3-20

$$\begin{aligned} \text{dB} &:= -30, -29 \dots -5 & M &:= 256 & n &:= \frac{\log(M)}{\log(2)} \\ \mu &:= 255 & n &= 8 \\ \text{SNRa} &:= 6.02 n + 4.77 - 20 \log(\ln(1 + \mu)) \end{aligned}$$



3-21

(a) $L = 2^l = 16 \Rightarrow \underline{\underline{l = 4 \text{ bits/level}}}$

(b) $D = \frac{N}{T} = \frac{1 \text{ symbol}}{0.8 \times 10^{-3} \text{ sec}} = \underline{\underline{1,250 \text{ baud}}}$

(c) $R = lD = 4(1,250) = \underline{\underline{5 \text{ kbits/sec}}}$

3-22

$R = lD = \underbrace{(3 \text{ bits/symbol})}_{\substack{B=2^3 \Rightarrow l=3}} (10.76 \frac{\text{Msymbols}}{\text{sec}}) = \underline{\underline{32.28 \text{ Mbits/sec}}}$

3-23

$$(a) \quad D = \frac{R}{2} = \frac{1,500k}{4} = 375 \text{ kbaud}$$

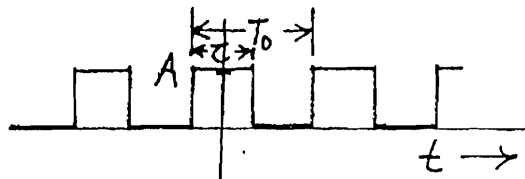
$$B \geq \frac{1}{2} \frac{N}{T} = \frac{1}{2} D = \frac{1}{2} (375k) = \underline{\underline{187.5 \text{ kHz}}} \text{ minimum BW}$$

$$(b) \quad D = \frac{1,500}{8} = 187.5 \text{ kbaud}$$

$$B \geq \frac{1}{2} D = \frac{1}{2} (187k) = \underline{\underline{93.75 \text{ kHz}}} \text{ minimum BW}$$

3-24

For alternating data the waveform is periodic where $T_0 = 2T_b$.



From (2-109) the spectrum is

$$W(f) = \sum_{-\infty}^{\infty} C_n \delta(f - n f_0)$$

$$\begin{aligned} \text{where } C_n &= \frac{1}{T_0} \int_a^{a+T_0} w(t) e^{-jn\omega_0 t} dt = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} A e^{jn\omega_0 t} dt \\ &= \frac{A}{T_0} \left. \frac{e^{jn\omega_0 t}}{jn\omega_0} \right|_{-\frac{T_0}{2}}^{\frac{T_0}{2}} = \frac{2A}{T_0} \frac{e^{jn\omega_0 T_0/2} - e^{-jn\omega_0 T_0/2}}{2jn\omega_0} \\ &= \frac{2A}{T_0} \frac{\sin(n\omega_0 T_0/2)}{n\omega_0} \end{aligned}$$

$$\begin{aligned} \Rightarrow C_n &= \frac{2A}{T_0} \frac{\sin(n\omega_0 T_0/2)}{n\omega_0} = \frac{A}{2} \left(\frac{T_0}{T_b} \right) \frac{\sin(n\pi \frac{T_0}{T_b})}{n\pi \frac{T_0}{T_b}} \\ &= \sum_{n=-\infty}^{\infty} \frac{A}{2} \left(\frac{T_0}{T_b} \right) \left(\frac{\sin(n\pi \frac{T_0}{T_b})}{n\pi \frac{T_0}{T_b}} \right) \delta(f - \frac{n}{T_b}) \quad (A) \end{aligned}$$

where $R = \frac{1}{T_b} = \text{bit rate}$

3-24. Cont'd

Using (A) for NRZ signaling with $\tau = T_b$

$$|W(f)| = \sum_{-\infty}^{\infty} \frac{A}{2} \left| \frac{\sin(\frac{n\pi}{2})}{(\frac{n\pi}{2})} \right| \delta(f - \frac{n}{2}R) \quad \begin{array}{l} \text{Unipolar} \\ \text{NRZ} \\ \text{(alternating} \\ \text{data)} \end{array}$$

If the data are a sequence of four "1"s followed by four "0"s, the waveform would have the same shape except T_0 would be 4 times as large.
i.e. $T_0 = 8T_b$.

$$\Rightarrow |W(f)| = \sum_{-\infty}^{\infty} \frac{A}{2} \left| \frac{\sin(\frac{n\pi}{2})}{(\frac{n\pi}{2})} \right| \delta(f - \frac{n}{8}R) \quad \begin{array}{l} \text{Unipolar NRZ} \\ \text{4 "1"s and 4 "0"s} \end{array}$$

3-25

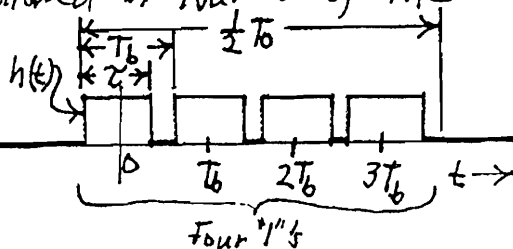
Using (A) for RZ signaling with $\tau = \frac{3}{4}T_b$,

$$|W(f)| = \sum_{-\infty}^{\infty} \frac{3}{8}A \left| \frac{\sin(\frac{3}{8}n\pi)}{(\frac{3}{8}n\pi)} \right| \delta(f - \frac{n}{2}R) \quad \begin{array}{l} \text{Unipolar RZ} \\ \text{(alternating data)} \end{array}$$

For RZ with four "1"s followed by four "0"s, the periodic waveform would appear as shown where $T_0 = 8T_b$. The mathematical calculations are simplified if (2-112) is used

$$C_n = f_0 H(nf_0)$$

where $h(t)$ is the basic waveform that is repeated to create the periodic waveform (as shown in the figure). $h(t)$ consists of the superposition of four rectangular pulses. Using the time delay theorem of Table 2-1



3-25 Cont'd.

and the rectangular pulse spectrum of Table 2-2

$$H(f) = Az \frac{\sin(\pi f z)}{\pi f z} [1 + e^{-j\omega T_b} + e^{-j\omega 2T_b} + e^{-j\omega 3T_b}]$$

$$\text{Or } c_n = \frac{Az}{8T_b} \frac{\sin\left(\frac{n\pi}{8} \frac{z}{T_b}\right)}{\left(\frac{n\pi}{8} \frac{z}{T_b}\right)} [1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3n\pi}{4}}]$$

$$\boxed{f = nf_0 = \frac{n}{T_b} = \frac{n}{8T_b}}$$

For RZ with $z = \frac{3}{4}T_b$, this becomes

$$c_n = \frac{3}{32} A \left(\frac{\sin\left(\frac{3}{32} n\pi\right)}{\left(\frac{3}{32} n\pi\right)} \right) [1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3n\pi}{4}}]$$

Thus, the spectrum for Unipolar RZ with four alternate "1" and "0"s is

$$|W(f)| = \sum_{n=-\infty}^{\infty} \frac{3}{32} A \left| \frac{\sin\left(\frac{3}{32} n\pi\right)}{\left(\frac{3}{32} n\pi\right)} \right| |1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3n\pi}{4}}| \delta\left(f - \frac{n}{8T_b}\right)$$

3-26

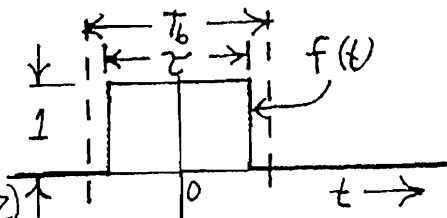
In general the PSD is given by (3-36).
Now follow the text discussion that leads up to (3-37a).
It is seen that for unipolar signaling $R(0) = \frac{A^2}{2}$
and $R(k) = \frac{A^2}{4}$, for $k \neq 0$. Thus,

$$\begin{aligned} P(f)_{\text{unipolar}} &= \frac{|F(f)|^2}{T_b} \left[\frac{A^2}{2} + 2 \frac{A^2}{4} \sum_{k=1}^{\infty} \cos(2\pi k f T_b) \right] \\ &= \frac{A^2}{4T_b} |F(f)|^2 \left[2 + 2 \sum_{k=1}^{\infty} \cos(2\pi k f T_b) \right] \\ &= \frac{A^2}{4T_b} |F(f)|^2 \left[1 + \sum_{k=-\infty}^{\infty} e^{j k \omega T_b} \right] \end{aligned}$$

3-26. Cont'd.

$$\Rightarrow P_{\text{unipolar}}(f) = \frac{A^2}{4T_b} |F(f)|^2 \left[1 + \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta(f - \frac{n}{T_b}) \right]$$

Now evaluate $F(f)$ for the unipolar pulse shape shown.



$$F(f) = \mathcal{F}[f(t)] = \frac{\tau \sin(\pi f \tau)}{(\pi f \tau)}$$

$$\Rightarrow P_{\text{unipolar}}(f) = \frac{A^2}{4} \left(\frac{\tau^2}{T_b} \right) \left[\frac{\sin(\pi f \tau)}{(\pi f \tau)} \right]^2 \left[1 + R \sum_{n=-\infty}^{\infty} \delta(f - nR) \right] \quad (A)$$

where the bit rate is $R = 1/T_b$ and τ is the pulse width.

(a) For Unipolar NRZ signaling let $\tau = T_b$, (A) becomes

$$\Rightarrow P_{\text{unipolar NRZ}}(f) = \frac{A^2}{4} T_b \left(\frac{\sin(\pi f T_b)}{(\pi f T_b)} \right)^2 + \frac{A^2}{4} \delta(f) \quad \text{Unipolar NRZ}$$

because $\frac{\sin(\pi f T_b)}{(\pi f T_b)} = 0$ at $f = nR = \frac{n}{T_b}$ except for $n=0$.

Note that this result is the same as (3-39b)

(b) For the RZ case of $\tau = \frac{3}{4} T_b$, (A) becomes

$$P_{\text{unipolar RZ}}(f) = \frac{9A^2}{64} T_b \left[\frac{\sin(\frac{3}{4} \pi f T_b)}{(\frac{3}{4} \pi f T_b)} \right]^2 \left[1 + R \sum_{n=-\infty}^{\infty} \delta(f - nR) \right] \quad \text{Unipolar RZ}$$

where $R = 1/T_b$.

The magnitude spectra for the periodic case, Prob. 3-24, consists only of line spectra. On the other

3-26. Cont'd.

hand, the random data case gives a continuous spectral component in the PSD as well as some discrete lines. The null bandwidth is the same for both cases. That is, for Unipolar NRZ the null bandwidth is $B_{null} = \frac{1}{T_b} = R$ and the spectral efficiency is $\eta = 1$ (bits/sec)/Hz. For Unipolar RZ with $\tau = \frac{3}{4}T_b$, $B_{null} = \frac{4}{3} \frac{1}{T_b} = \frac{4}{3}R$ and $\eta = \frac{3}{4}$ (bits/sec)/Hz.

3-27

The solution is identical to that of Prob. 3-22(a) where the DC level is set to zero and the peak-to-peak level is now $2A$ instead of A . Using the solution to Prob 3-24, we get

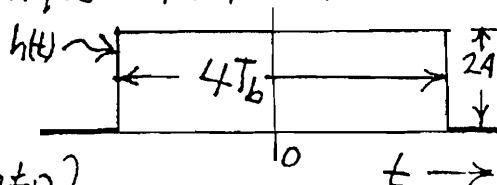
$$|W(f)| = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} A \left| \frac{\sin\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)} \right| \delta\left(f - \frac{n}{2}R\right)$$

For the case of a sequence consisting of four "1"s followed by two "0"s, the calculation is simplified if the technique shown in the

solution to prob. 3-25 is used. Using (2-112)

$$C_n = \begin{cases} f_0 H(nf_0), & n \neq 0 \\ C_0, & n = 0 \end{cases}$$

where $f_0 = \frac{1}{T_b} = \frac{1}{6T_b}$ and $H(f) = \mathcal{F}[h(t)]$ for the $h(t)$ shown.



3-27 Cont'd

The DC level is

$$C_0 = \langle w(t) \rangle = \frac{1}{T_0} \int_{-2T_b}^{4T_b} w(t) dt = \frac{1}{6T_b} [4AT_b - 2AT_b]$$

$$\neq C_0 = \frac{A}{3}$$

Using the rectangular pulse spectrum shown in Table 2-2,

$$H(f) = 2A \left[4T_b \frac{\sin(\pi f 4T_b)}{(\pi f 4T_b)} \right] = 8AT_b \frac{\sin(4\pi f T_b)}{(4\pi f T_b)}$$

Thus,

$$C_n = \left\{ \begin{array}{ll} \frac{4A}{3} \frac{\sin(4\pi f T_b)}{4\pi f T_b}, & n \neq 0 \\ \frac{A}{3}, & n = 0 \end{array} \right\}$$

Using this in (2-109) we get (for polar signaling with a data pattern of four "1"s followed by two "0"s)

$$|W(f)| = \frac{A}{3} \delta(f) + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{4}{3} A \left| \frac{\sin(4\pi f T_b)}{(4\pi f T_b)} \right| \delta(f - \frac{1}{6} n R)$$

3-28

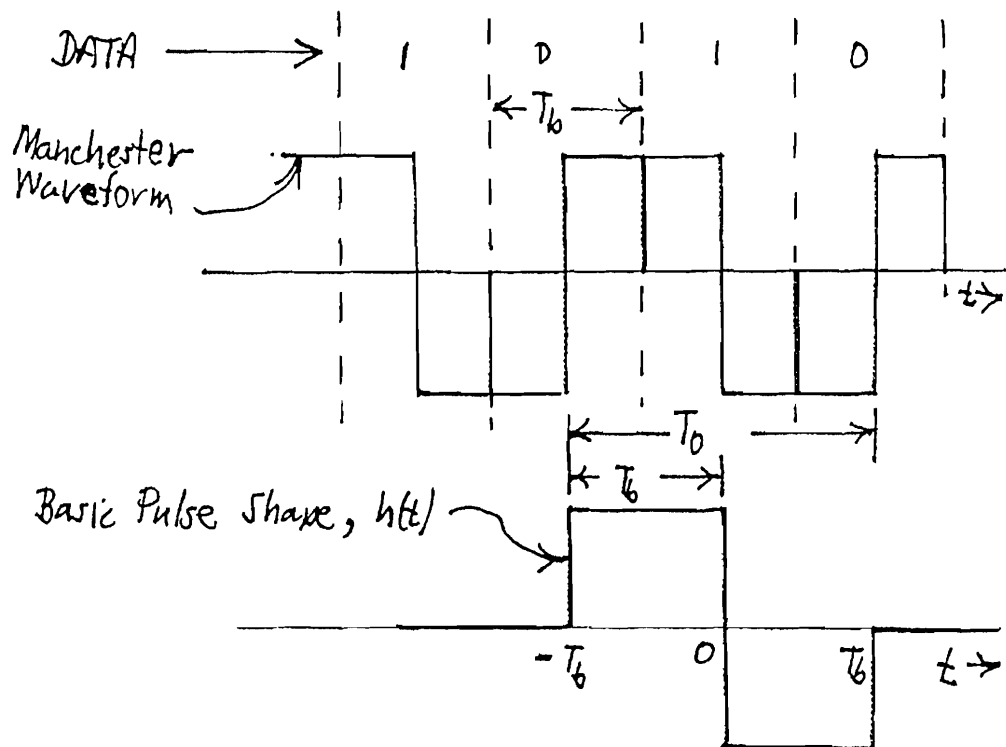
For Manchester signaling, use the pulse shape that is shown on the following page.

$$C_n = f_0 H(nf_0) \text{ and } f_0 = \frac{1}{T_0} = \frac{1}{2T_b}$$

Using Tables 2-1 and 2-2, we get

$$H(f) = \mathcal{F}[h(t)] = A \left[\frac{T_b \sin(\pi f T_b)}{(\pi f T_b)} \right] e^{j\frac{\omega T_b}{2}} - A \left[\frac{T_b \sin(\pi f T_b)}{\pi f T_b} \right] e^{-j\frac{\omega T_b}{2}}$$

3-28 (cont'd.)



$$\Rightarrow H(f) = j 2 A T_b \left(\frac{\sin(\pi f T_b)}{(\pi f T_b)} \right) \left(\frac{e^{j \omega T_b/2} - e^{-j \omega T_b/2}}{j 2} \right)$$

$$= j 2 A T_b \left(\frac{\sin(\pi f T_b)}{(\pi f T_b)} \right) \sin(\pi f T_b)$$

Using $f_0 = \frac{1}{T_b} = \frac{1}{2T_b}$, this reduces to

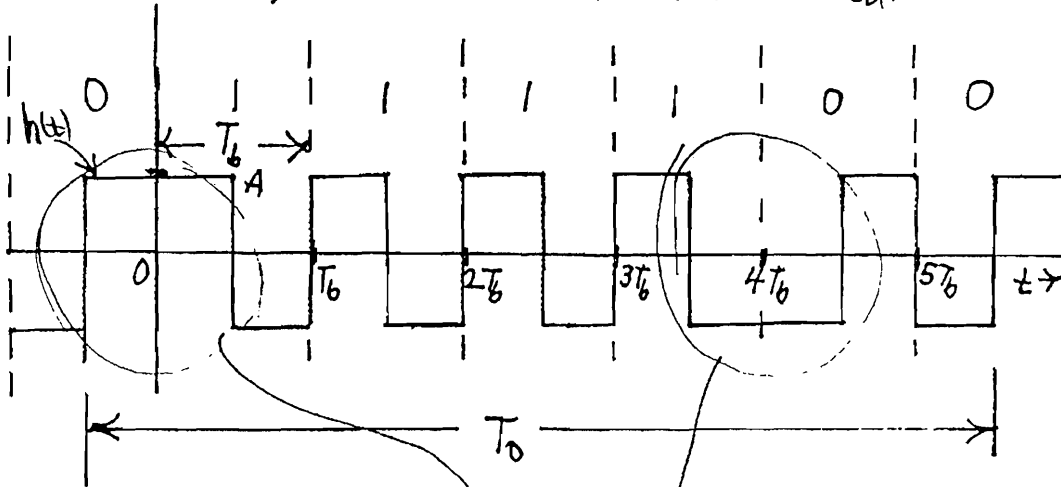
$$C_n = j A \left(\frac{\sin(\frac{\pi}{2} n)}{(\frac{\pi}{2} n)} \right) \sin(\frac{\pi}{2} n)$$

Using (2-109), we get

$$\underline{\underline{|W(f)| = \sum_{-\infty}^{\infty} \frac{A}{2} \left| \frac{\sin(\frac{\pi}{2} n)}{(\frac{\pi}{2} n)} \right| |\sin(\frac{\pi}{2} n)| \delta(f - \frac{n}{2} R)}}$$

3-28 Cont'd

If the test pattern consists of four "1"s followed by two "0"s, the $h(t)$ shown below is used.



$$\begin{aligned} \Rightarrow H(f) = & AT_b \left(\frac{\sin(\pi f T_b)}{\pi f T_b} \right) \left[1 - e^{-j\omega 4T_b} \right] \\ & + \frac{AT_b}{2} \left(\frac{\sin(\pi f \frac{T_b}{2})}{(\pi f \frac{T_b}{2})} \right) \left[-e^{-j\omega \frac{3}{4}T_b} + e^{-j\omega \frac{5}{4}T_b} - e^{-j\omega \frac{7}{4}T_b} \right. \\ & \quad \left. + e^{-j\omega \frac{9}{4}T_b} - e^{-j\omega \frac{11}{4}T_b} + e^{-j\omega \frac{13}{4}T_b} \right. \\ & \quad \left. + e^{-j\omega \frac{15}{4}T_b} - e^{-j\omega \frac{17}{4}T_b} \right] \end{aligned}$$

Using (2-109), the magnitude spectrum for this Manchester data waveform is

$$\underline{\underline{|W(f)| = \sum_{-\infty}^{\infty} f_0 |H(nf_0)| \delta(f - nf_0)}}$$

where $H(f)$ is given above

$$\underline{\underline{\text{and } f_0 = \frac{1}{T_0} = \frac{1}{6T_b} = \frac{1}{6}R}}$$

3-29

(a) Substituting (3-40) into (3-36a)
the PSD for Polar RZ signaling
is

$$P(f) = \frac{A^2}{T_b} |F(f)|^2$$

where the pulse shape, $f(t)$, is shown in the figure. Thus,

$$F(f) = \mathcal{F}[f(t)] = \tau \frac{\sin(\pi f \tau)}{\pi f \tau}$$

$$\text{and } P(f) = \frac{A^2 \tau^2}{T_b} \left[\frac{\sin(\pi f \tau)}{\pi f \tau} \right]^2$$

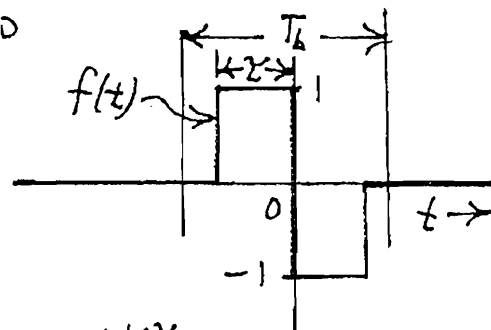
For the case of $\tau = \frac{1}{2} T_b$, this becomes

$$P(f) = \frac{A^2 T_b}{4} \left[\frac{\sin(\frac{\pi}{2} f T_b)}{(\frac{\pi}{2} f T_b)} \right]^2$$

The first-null bandwidth is $B_{null} = \frac{2}{T_b} = 2R$.

and the bandwidth efficiency is $\eta = \frac{1}{2}$ (bit/sec)/Hz.

(b) Equation (3-36) can also
be used to evaluate the PSD
for RZ Manchester signaling
where the pulse shape is
shown in the figure.



$$F(f) = \tau \left(\frac{\sin(\pi f \tau)}{\pi f \tau} \right) \left[e^{j\omega \frac{\tau}{2}} - e^{-j\omega \frac{\tau}{2}} \right]$$

3-29(b.) Cont'd

$$\Rightarrow F(f) = j 2 \tau \left(\frac{\sin(\pi f \tau)}{(\pi f \tau)} \right) \sin\left(\frac{\omega \tau}{2}\right)$$

Using (3-40) and (3-36), the PSD for Manchester signaling is

$$P(f) = \frac{4 A^2 \tau^2}{T_b} \left[\frac{\sin(\pi f \tau)}{(\pi f \tau)} \right]^2 [\sin(\pi f \tau)]^2$$

If $\tau = \frac{1}{4} T_b$, this becomes

$$P(f) = \frac{1}{4} A^2 T_b \left[\frac{\sin(\frac{\pi}{4} f T_b)}{(\frac{\pi}{4} f T_b)} \right]^2 [\sin(\frac{\pi}{4} f T_b)]^2$$

The first-null bandwidth is $B_{null} = \frac{4}{T_b} = 4R$
and the spectral efficiency is $\eta = \frac{1}{4}$ (bits/sec)/Hz.

3-30

From (3-44)

for Bipolar signaling

$$R(k) = \begin{cases} A^2/2, & k=0 \\ -A^2/4, & k=1 \\ 0, & k \text{ otherwise} \end{cases}$$

Substituting this into (3-36a) the PSD for Bipolar signaling

is

$$P(f) = \frac{|F(f)|^2}{T_b} \left[\frac{A^2}{2} - \frac{A^2}{2} \cos(2\pi f T_b) \right] = \frac{A^2}{2T_b} |F(f)|^2 [1 - \cos(2\pi f T_b)]$$

$$\neq P(f) = \frac{A^2}{T_b} |F(f)|^2 \sin^2(\pi f T_b)$$

For Bipolar NRZ (i.e. pulse width is T_b), $F(f) = T_b \frac{\sin(\pi f T_b)}{\pi f T_b}$

so that the PSD becomes

$$P(f) = A^2 T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b} \right]^2 \sin^2(\pi f T_b) \quad \text{Bipolar NRZ}$$

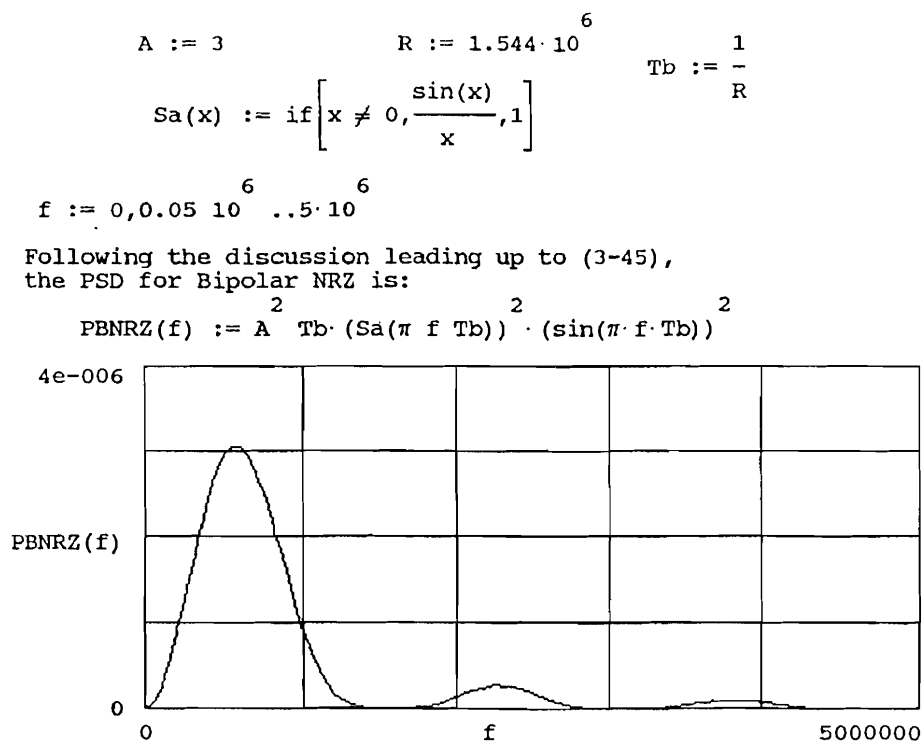
3-30. Cont'd.

For Bipolar RZ (i.e. pulse width is $T_b/2$), $F(f) = \frac{T_b}{2} \frac{\sin(\pi f T_b/2)}{\pi f T_b/2}$

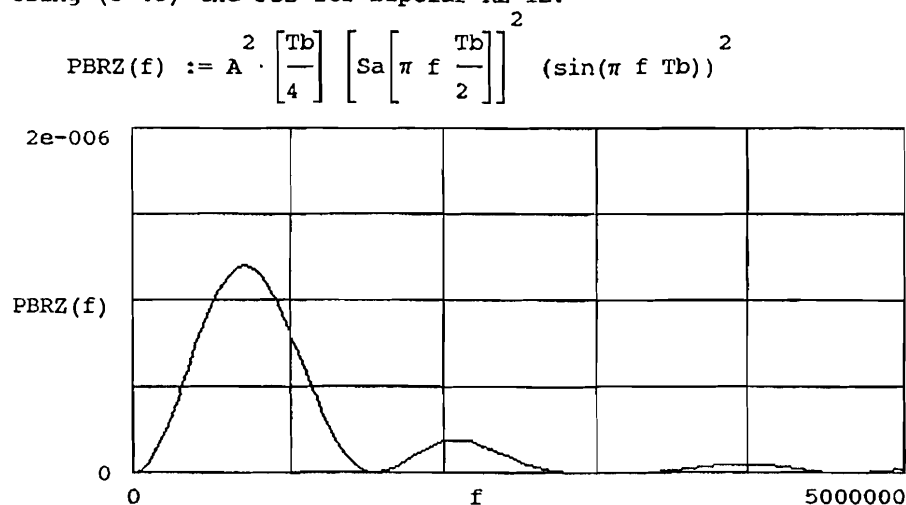
so that the PSD becomes

$$P(f) = \frac{A^2 T_b}{4} \left[\frac{\sin(\pi f T_b/2)}{\pi f T_b/2} \right]^2 \sin^2(\pi f T_b) \quad \text{Bipolar RZ}$$

The PSD's are plotted using MathCAD.



Using (3-45) the PSD for Bipolar RZ is:



3-31

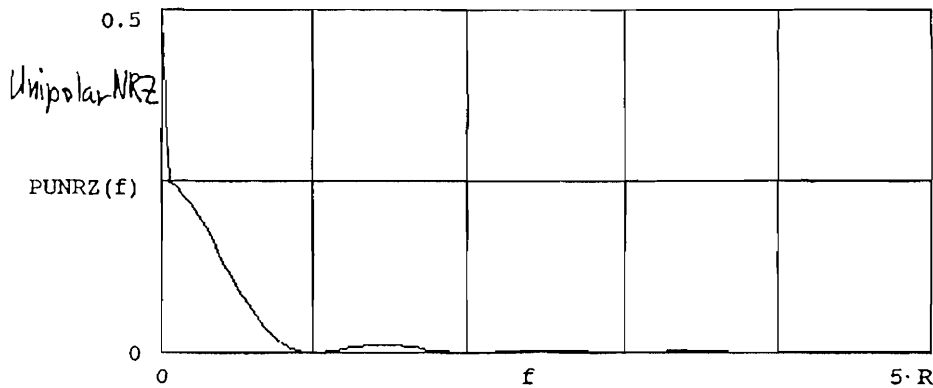
$$A := 1 \quad R := 1 \quad f := 0, 0.05 \dots 5 \quad T_b := \frac{1}{R}$$

$$Sa(x) := \text{if} \left[x \neq 0, \frac{\sin(x)}{x}, 1 \right]$$

The PSD for Unipolar NRZ is given by (3-39b) and consists of both a continuous spectrum and a discrete spectrum. The computer cannot plot infinite values for the delta functions, so plot the weights of the delta functions instead. Thus (3-39b) will be broken into two functions, one for the continuous spectral plot and one for the discrete spectral plot.

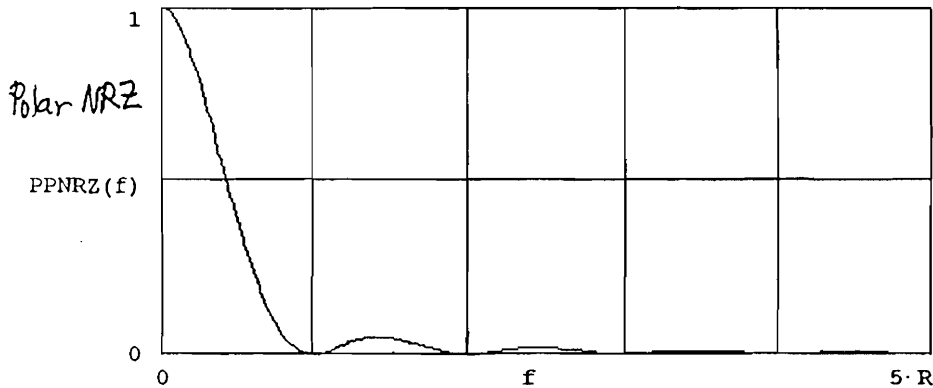
$$PUNRZc(f) := \left[\frac{2 T_b}{A} \right] \cdot (Sa(\pi f T_b))^2 \quad PUNRZd(f) := \text{if} \left[f \neq 0, 0, \frac{A^2}{4} \right]$$

$$PUNRZ(f) := PUNRZc(f) + PUNRZd(f)$$



Use (3-41) for Polar NRZ spectrum:

$$PPNRZ(f) := \left[\frac{2}{A T_b} \right] (Sa(\pi f T_b))^2$$



The PSD for Unipolar RZ is given by (3-43) and consists of both a continuous spectrum and a discrete spectrum. The computer cannot plot infinite values for the delta functions, so plot the weights of the delta functions instead. Thus (3-43) will be broken into two functions, one for the continuous spectral plot and one for the discrete spectral plot.

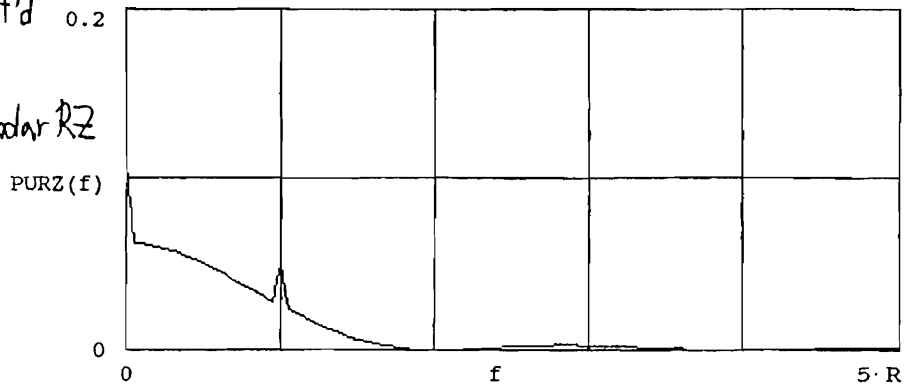
$$PURZc(f) := \left[\frac{2 T_b}{A} \right] \left[\frac{Sa \left[\pi f \frac{T_b}{2} \right]}{2} \right]^2$$

$$PURZd(f) := \text{if} \left[\text{mod}(f, R) \neq 0, 0, \frac{A^2}{16} \left[\frac{Sa \left[\pi f \frac{T_b}{2} \right]}{2} \right]^2 \right]$$

3-31 $PURZ(f) := PURZc(f) + PURZd(f)$

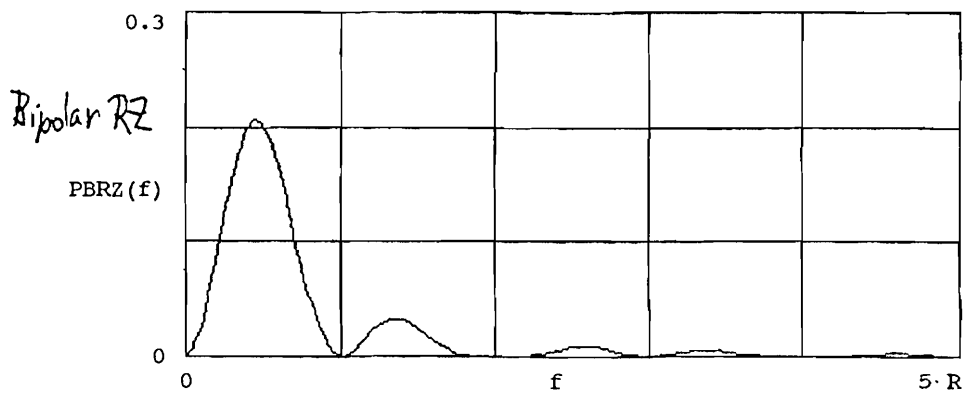
Cont'd

Unipolar RZ



Using (3-45) the PSD for Bipolar RZ is:

$$PBRZ(f) := A^2 \left[\frac{Tb}{4} \right] \cdot \left[Sa \left[\pi f \cdot \frac{Tb}{2} \right] \right]^2 (\sin(\pi f Tb))^2$$

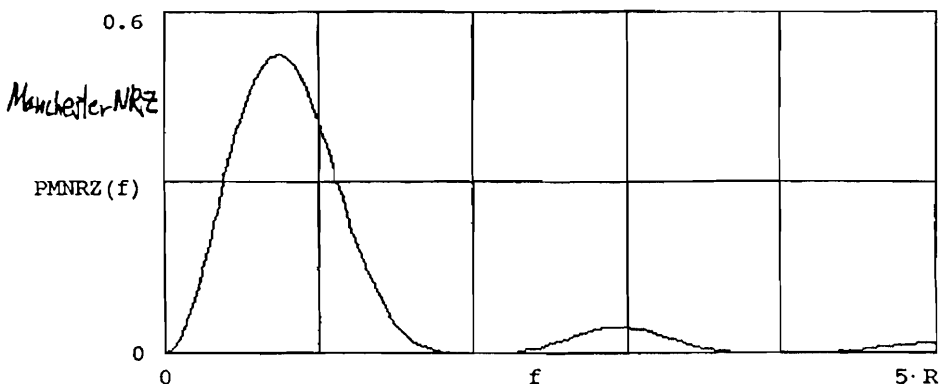


$$A := 1 \quad R := 1 \quad f := 0, 0.05 \dots 5 \quad Tb := \frac{1}{R}$$

$$Sa(x) := \text{if} \left[x \neq 0, \frac{\sin(x)}{x}, 1 \right]$$

Use (3-46c) for the Manchester NRZ spectrum:

$$PMNRZ(f) := A^2 \cdot Tb \left[Sa \left[\pi f \cdot \frac{Tb}{2} \right] \right]^2 \cdot \left[\sin \left[\pi f \cdot \frac{Tb}{2} \right] \right]^2$$



3-32

Referring to (3-36), it is seen that this equation is derived in Chapter 6 and that (3-36) is identical to (6-70b). Furthermore, the solution to this problem is given in the text following (6-70b). Equation (6-70e) shows that there will be delta functions in the spectrum at frequencies $f=nR$ if and only if the line code has a non-zero dc value and $F(nR)$ is not zero.

If the line code is designed to have a delta function at $f=R$ or some harmonic of R , then a phase-locked loop can be used to extract the bit sync signal as discussed in Section 3-5 (Bit Synchronizers).

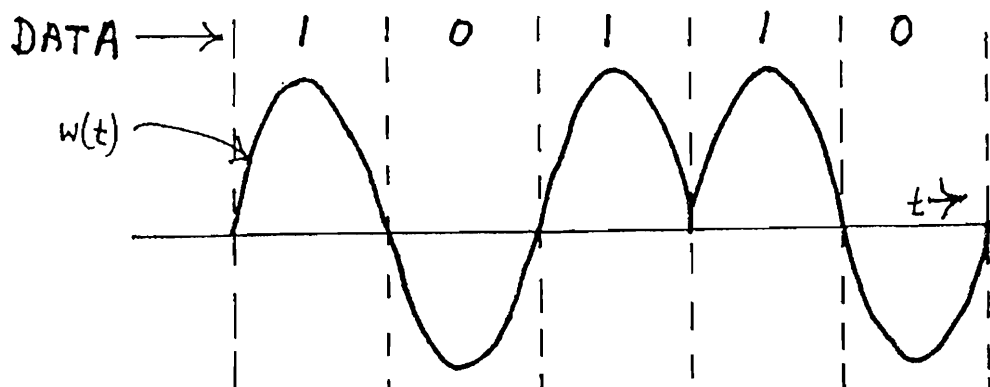
3-33

$$(a.) \quad w(t) = A \sum_{n=-\infty}^{\infty} a_n f(t - nT_b)$$

where $a_n = \pm 1$ random data

$$\Rightarrow w(t) = A \sum_{n=-\infty}^{\infty} a_n \begin{cases} \cos\left(\frac{\pi}{T_b}(t - nT_b)\right), & |t - nT_b| < \frac{T_b}{2} \\ 0, & \text{elsewhere} \end{cases}$$

This is plotted in the figure below for a random data pattern.



3-33 Cont'd

(b) Using (3-40) in (3-36a) the PSD is given by

$$P(f) = \frac{A^2}{T_b} |F(f)|^2 \quad \text{where } F(f) = \mathcal{F}[f(t)]. \quad (1)$$

Now, evaluate $F(f)$:

$$\begin{aligned} F(f) &= \int_{-\frac{T_b}{2}}^{\frac{T_b}{2}} \cos\left(\frac{\pi t}{T_b}\right) e^{-j2\pi f t} dt \\ &= \int_{-\frac{T_b}{2}}^{\frac{T_b}{2}} \cos\left(\frac{\pi t}{T_b}\right) \cos(2\pi f t) dt - j \int_{-\frac{T_b}{2}}^{\frac{T_b}{2}} \underbrace{\cos\left(\frac{\pi t}{T_b}\right)}_{\text{even}} \underbrace{\sin(2\pi f t)}_{\text{odd}} dt \\ &= \int_{-\frac{T_b}{2}}^{\frac{T_b}{2}} \frac{1}{2} \cos\left[2\pi\left(\frac{1}{2T_b} + f\right)t\right] dt + \int_{-\frac{T_b}{2}}^{\frac{T_b}{2}} \frac{1}{2} \cos\left[2\pi\left(\frac{1}{2T_b} - f\right)t\right] dt \\ &= \left[\frac{\sin\left[2\pi\left(\frac{1}{2T_b} + f\right)t\right]}{2\pi\left(\frac{1}{2T_b} + f\right)} \right]_0^{\frac{T_b}{2}} + \left[\frac{\sin\left[2\pi\left(\frac{1}{2T_b} - f\right)t\right]}{2\pi\left(\frac{1}{2T_b} - f\right)} \right]_0^{\frac{T_b}{2}} \end{aligned}$$

Integrals are even $\Rightarrow \int_{-\frac{T_b}{2}}^{\frac{T_b}{2}} \cos(\pi f t) dt = 2 \int_0^{\frac{T_b}{2}} \cos(\pi f t) dt$

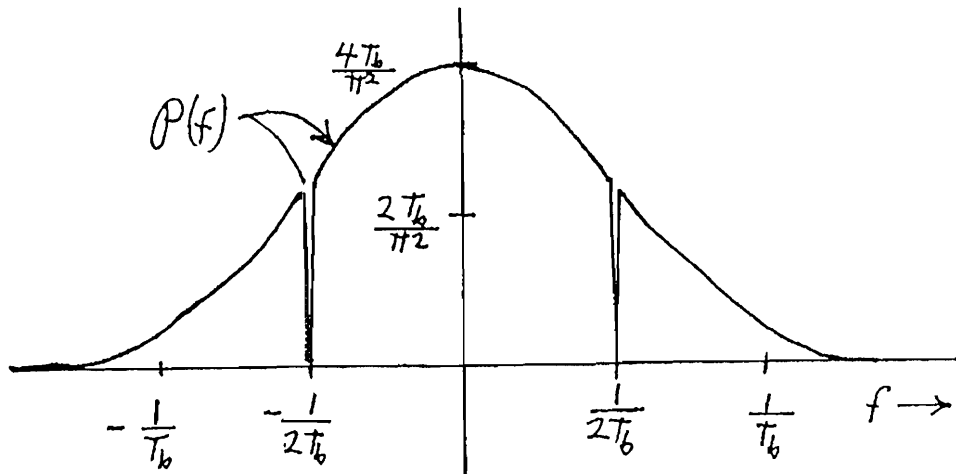
$$\begin{aligned} \Rightarrow F(f) &= \left[\frac{\sin\left[2\pi\left(\frac{1}{2T_b} + f\right)\frac{T_b}{2}\right]}{2\pi\left(\frac{1}{2T_b} + f\right)} + \frac{\sin\left[2\pi\left(\frac{1}{2T_b} - f\right)\frac{T_b}{2}\right]}{2\pi\left(\frac{1}{2T_b} - f\right)} \right] \\ &= \frac{\left(\frac{\pi}{T_b} - 2\pi f\right) \cos(\pi f T_b) + \left(\frac{\pi}{T_b} + 2\pi f\right) \cos(\pi f T_b)}{(2\pi)^2 \left(\frac{1}{2T_b} + f\right) \left(\frac{1}{2T_b} - f\right)} \end{aligned}$$

$$\Rightarrow F(f) = \frac{2T_b}{\pi} \frac{\cos(\pi f T_b)}{1 - (2T_b f)^2}$$

Substituting this into (1), the PSD is

$$P(f) = \frac{4T_b A^2}{\pi^2} \left(\frac{\cos(\pi f T_b)}{1 - (2T_b f)^2} \right)^2$$

3-33.(b.) Cont'd. This result is plotted below.

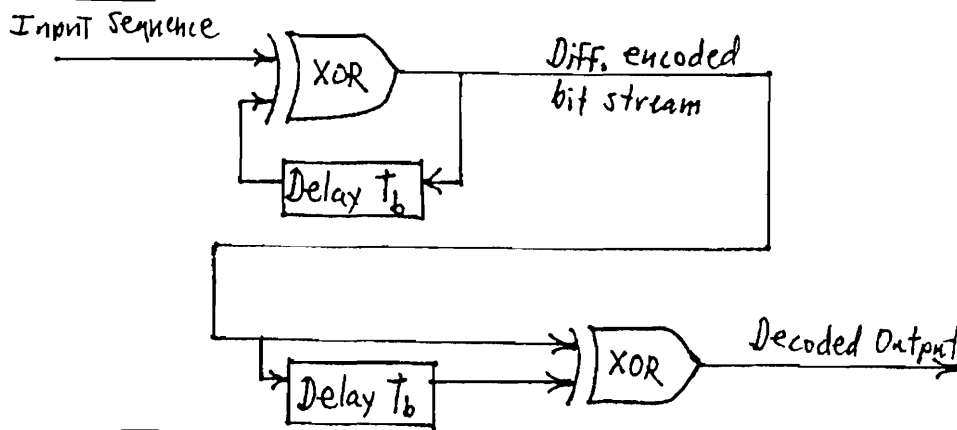


(c) The first-null bandwidth is $B_{null} = \frac{1}{2T_b} = \frac{1}{2}R$ and the spectral efficiency is $\eta = 2 \text{ (bits/sec)/Hz}$.

3-34

Reference digit	
Input Sequence	0 1 1 0 1 0 0 0 1 0 1
Encoded Sequence ①	1 1 0 1 1 0 0 0 0 1 1 0
Encoded Sequence ②	0 0 1 0 0 1 1 1 1 0 0 1

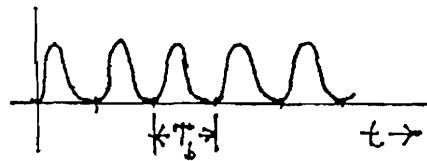
3-35



Reference digit	
Input Sequence	0 0 1 1 1 1 0 1 0 0 0 1
Encoded sequence	1 1 1 0 1 0 1 1 0 0 0 0 1
Decoded sequence	0 0 1 1 1 1 0 1 0 0 0 1

3-36

A possible regenerative repeater for a Polar RZ line code would be almost identical to the Unipolar NRZ repeater shown in Fig. 3-19 except that a monostable output circuit would be required to produce the permitted binary levels of $+V_0$ and $-V_0$ instead of $+V_0$ and 0. A bit synchronizer similar to the square-law synchronizer shown in Fig. 3-20 could be used. In this case the output of the square-law device would be of the Unipolar RZ type and this waveform would be exactly periodic (except for noise) as shown. consequently, we would expect the PSD of this waveform to contain only delta functions. To show that this is the case, use (3-36) where the a_n can take on only one value, call it $+A$, with the probability of 1. Thus,



$$P(f) = \frac{A^2 |F(f)|^2}{T_b} \left[1 + 2 \sum_{k=1}^{\infty} \cos(2\pi k f T_b) \right]^2 = \frac{A^2 |F(f)|^2}{T_b} \left[\sum_{k=-\infty}^{\infty} e^{j k \omega T_b} \right]^2$$

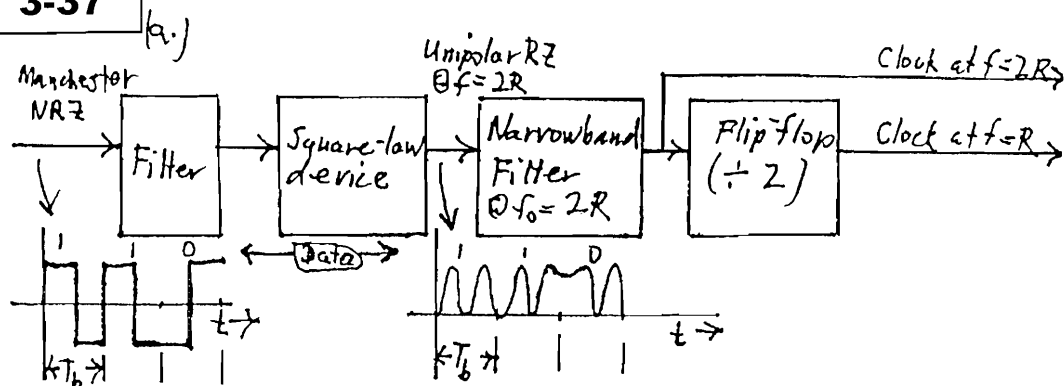
$$R(k) = \sum a_n a_{n+k} p_i = A^2$$

or, using (3-38)

$$P(f) = A^2 R^2 |F(f)|^2 \sum_{n=-\infty}^{\infty} \delta(f - nR)$$

where $R = 1/T_b$. The weight of the delta function at $f=R$ is $A^2 R^2 |F(R)|^2$. Therefore we have a periodic component in the waveform at the frequency $f=R$. Consequently, a low-pass filter (or PLL) centered on $f_0=R$ can be used to extract the bit sync signal as shown in Fig. 3-20.

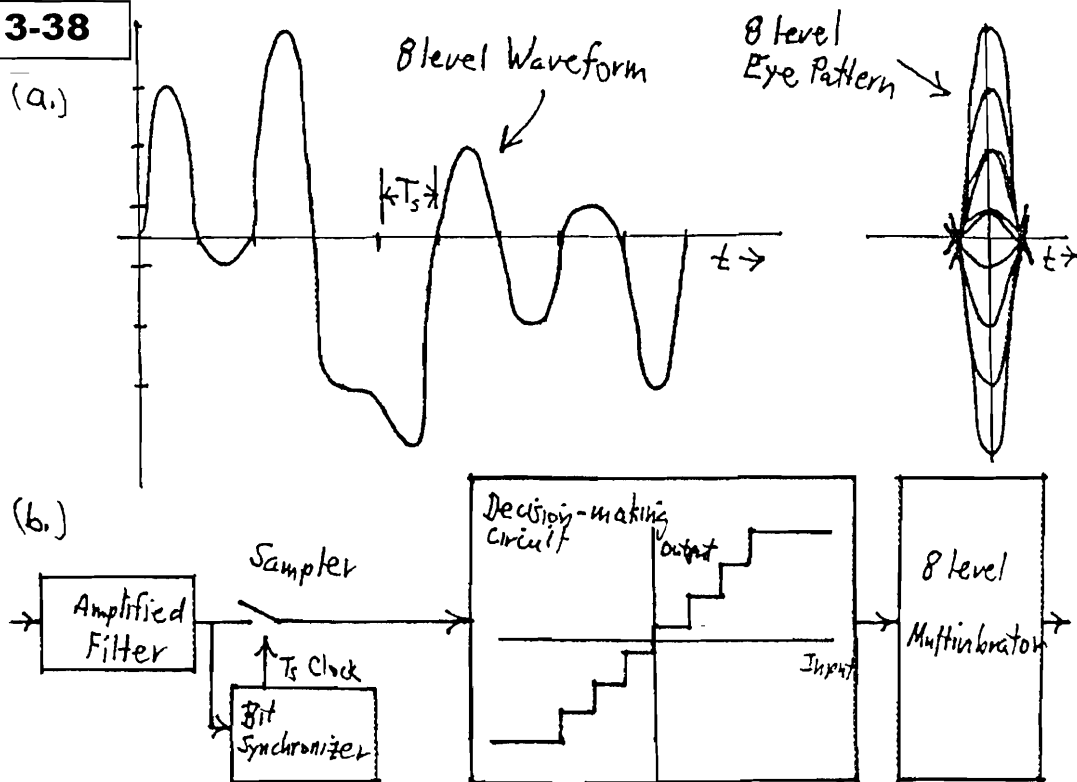
3-37



(b.) and (c.) The Manchester NRZ waveform is filtered to produce a waveform that has RZ characteristics. Consequently, the output of the square-law device will be a Unipolar RZ waveform except that it would have a bit rate of $2R$ instead of R where $R = 1/T_b$ is the bit rate of the input Manchester line code. Thus, using mathematics similar to the solution shown in P3-31 above, there would be delta functions at frequencies $f = n2R$. Consequently, a narrowband filter centered on $f = 2R$ could be used to recover bit sync at twice the clock frequency R and a flip-flop (divide by 2 frequency divider) could be used to obtain the clock signal at $f = R$.

(d.) The Manchester NRZ bit synchronizer would function even when all 1s or all 0s data were received, whereas the corresponding Polar NRZ synchronizer would fail for this type of data because no pulses would appear at the square-law output for the all 1s or all 0s type of data.

3-38



Use a bit synchronizer similar to that shown in Fig. 3-20.

3-38 Cont'd.

(c) The receiver functions as described in the text associated with Fig 3-19 and Fig 3-20 except that multilevel signaling is present so that an 8 level decision circuit and 8 level multivibrator are required.

3-39 Use the result from Prob 3-8.

$$(a) n \geq 3.32 \log_{10} \left(\frac{50}{p} \right) = 3.32 \log_{10} (50) = 5.64 \Rightarrow \text{Use } n = 6 \text{ bits/word.}$$

$$f_s = 2B = 5.4 \text{ kHz} \Rightarrow R_{\min} = n f_s = 6(5.4 \text{ kHz}) = \underline{32.4 \text{ kbits/sec}}$$

$$(b) L = B = 2^l \Rightarrow l = 3 \text{ bits/D/K} \quad D = \frac{R}{l} = \frac{32.4 \text{ kbit/sec}}{3 \text{ bits/symbol}} = \underline{10.8 \text{ ksym/sec}}$$

$$(c) D = \frac{2B}{1+r} \text{ where } r=0 \text{ for min BW} \Rightarrow B = \frac{D}{2} = \underline{5.4 \text{ kHz}}$$

3-40 $L = B = 2^l \Rightarrow l = 3$

$$(a) D = \frac{R}{l} = \frac{9600 \text{ bits/sec}}{3 \text{ bits/symbol}} = \underline{3.2 \text{ ksymbol/sec}}$$

$$(b) D = \frac{2R}{1+r} = \frac{2(3.2 \text{ k})}{1+r} = 3.2 \text{ k} \Rightarrow \underline{r = 0.5}$$

3-41

(a) This is similar to the problem illustrated by Fig. 3-22 except $L = 64$ and the peak limit levels are ± 10 .

$$\begin{aligned} \Rightarrow R(0) &= \sum_{i=1}^{64} (a_n)_i^2 P_i \\ &= \frac{1}{64} \left\{ 2[(10)^2 + (9.68)^2 + (9.37)^2 + \dots + (0.16)^2] \right\} \end{aligned}$$

$$\uparrow P_i = \frac{1}{64}$$

$$\Rightarrow R(0) = 34.39155$$

Using a programmable calculator or a personal computer

Also $R(k) = 0$ for $k \neq 0$

3-41 (cont'd).

Using (3-36a), the PSD for $w(t)$ is

$$P(f) = \frac{|F(f)|^2}{T_s} [34.39 + 0] = \frac{1}{T_s} \left(\frac{T_s}{2} \right)^2 \left(\frac{\sin(\pi f \frac{T_s}{2})}{\pi f \frac{T_s}{2}} \right)^2 34.39$$

$F(f) = \mathcal{F}[f(t)] = \mathcal{F}\left[\pi \left(\frac{2t}{T_s}\right)\right]$

Using $T_s = 6T_b$ since $L = 2^6 = 64$, the PSD becomes

$$P(f) = 17.2 \left(\frac{\sin 3\pi f T_b}{3\pi f T_b} \right)^2$$

(b.) The first-null bandwidth is $B_{null} = \frac{1}{3T_b} = \frac{1}{3}R$ Hz

(c.) The spectral efficiency is

$$\eta = \frac{R}{B_{null}} = 3 \text{ (bits/sec)/Hz}$$

3-42

(a.) $B = \frac{1}{2}(1+r)D = \frac{1}{2}R = \frac{1}{2}(300) = 150 \text{ Hz}$

$r=0, D=R$

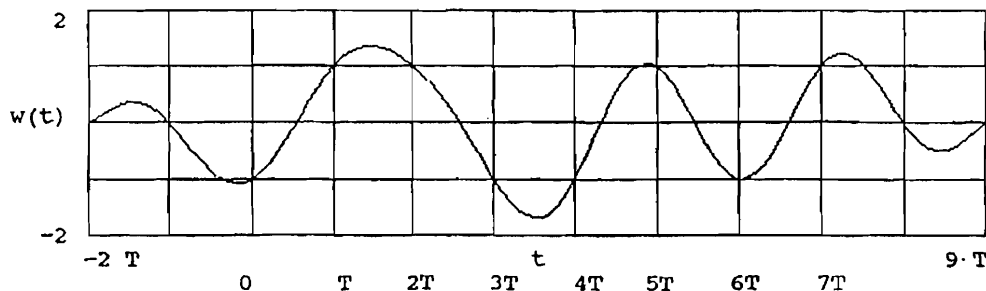
(b.) $f_s := 300$ $T := \frac{1}{f_s}$ $T = 0.003$

$t := -2 \cdot T, -1.9 \cdot T \dots 9 \cdot T$ $k := 0, 1 \dots 7$

$u_k(t) := \pi \cdot \frac{1}{T} \cdot (t - kT)$ $\phi_k(t) := \frac{\sin[u_k(t)]}{u_k(t)}$

$a_0 := -1$ $a_1 := 1$ $a_2 := 1$ $a_3 := -1$ $a_4 := -1$ $a_5 := 1$

$a_6 := -1$ $a_7 := 1$ $w(t) := \sum_k a_k \phi_k(t)$



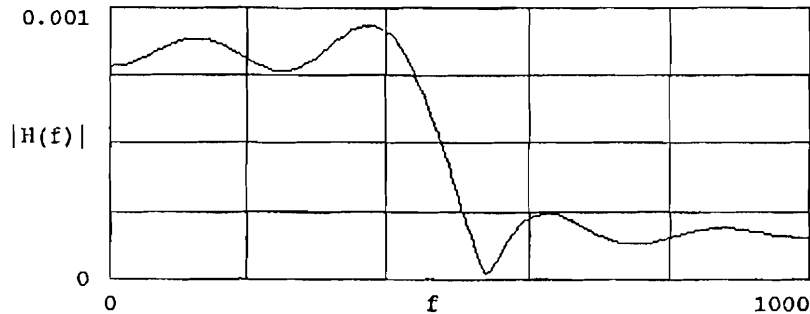
3-43

(a.)

$$f := 0, 10 \dots 1000$$

$$fs := 1000$$

$$H(f) := \int_0^{0.008} \frac{\sin(\pi \cdot fs \cdot (t - 0.004))}{\pi \cdot fs \cdot (t - 0.004)} e^{-2j \cdot \pi \cdot f \cdot t} dt$$



(b.) From the figure above, the bandwidth for the causal approximation is $B = 540 \text{ Hz}$

The bandwidth for the noncausal filter is $B = \frac{1}{2} fs = \underline{\underline{500 \text{ Hz}}}$

3-44

$$h_e(t) = \underbrace{\int_{-f_1}^{f_1} e^{j2\pi ft} df}_{(1)} + \underbrace{\int_{-B}^{-f_1} \frac{1}{2} \left\{ 1 + \cos \left[\frac{\pi}{2f_d} (-f - f_1) \right] \right\} e^{j2\pi ft} df}_{(2)} + \underbrace{\int_{f_1}^B \frac{1}{2} \left\{ 1 + \cos \left[\frac{\pi}{2f_d} (f - f_1) \right] \right\} e^{j2\pi ft} df}_{(3)}$$

$$(1) = \frac{e^{j2\pi f_1 t} - e^{-j2\pi f_1 t}}{j2\pi t} = \frac{\sin(2\pi f_1 t)}{\pi t}$$

$$(2) = \int_{-B}^{-f_1} \frac{1}{2} \left\{ 1 + \cos \left[\frac{\pi}{2f_d} (-f - f_1) \right] \right\} e^{j2\pi ft} df$$

$$\stackrel{\text{Let } y = -f}{=} \int_{f_1}^B \frac{1}{2} \left\{ 1 + \cos \left[\frac{\pi}{2f_d} (y - f_1) \right] \right\} e^{-j2\pi yt} dy = (3)^*$$

$$h_e(t) = (1) + (2) + (3) \text{ and using } c + c^* = 2\text{Re}\{c\}$$

$$\Rightarrow h_e(t) = \frac{\sin(2\pi f_1 t)}{\pi t} + \text{Re} \left\{ \int_{f_1}^B \left(1 + \cos \left[\frac{\pi}{2f_d} (f - f_1) \right] \right) e^{j2\pi ft} df \right\}$$

3-44. Cont'd ; $H_1(f) = \cos\left[\frac{\pi}{2f_\Delta}(f-f_1)\right]$

$$\begin{aligned} h_e(t) &= \frac{\sin(2\pi f_1 t)}{\pi t} + \int_{f_1}^B \cos[2\pi f t] df \\ &\quad + \int_{f_1}^B H_1(f) \cos[2\pi f t] df \\ &= \textcircled{1} + \textcircled{2} + \textcircled{3} \end{aligned}$$

$$\begin{aligned} \textcircled{1} + \textcircled{2} &= \frac{\sin(2\pi f_1 t)}{\pi t} + \frac{\sin(2\pi B t) - \sin(2\pi f_1 t)}{2\pi t} \\ &= \frac{\sin(2\pi f_1 t) + \sin(2\pi B t)}{2\pi t} ; \begin{array}{l} f_1 = f_0 - f_\Delta \\ B = f_0 + f_\Delta \end{array} \\ &= \frac{\sin(2\pi f_0 t) \cos(2\pi f_\Delta t)}{\pi t} \quad (A-13) \end{aligned}$$

$\sin(x-y) + \sin(x+y) = 2 \sin x \cos y$

$$\begin{aligned} \textcircled{3} &= \int_{f_1}^B \cos\left[\frac{\pi}{2f_\Delta}(f-f_1)\right] \cos[2\pi f t] df \\ &= \frac{1}{2} \int_{f_1}^B \cos\left[\frac{\pi}{2f_\Delta}(f-f_1) - 2\pi f t\right] df \\ &\quad + \frac{1}{2} \int_{f_1}^B \cos\left[\frac{\pi}{2f_\Delta}(f-f_1) + 2\pi f t\right] df \end{aligned}$$

Using:

$$B - f_1 = 2f_\Delta ; \sin(\pi - x) = \sin(x) = -\sin(-x);$$

3-44 Cont'd.

$$\textcircled{3} = \frac{1}{2} \frac{\sin(2\pi Bt) + \sin(2\pi f_1 t)}{\pi/2f_\Delta - 2\pi t} + \frac{1}{2} \left(\frac{-\sin(2\pi Bt) - \sin(2\pi f_1 t)}{\pi/2f_\Delta + 2\pi t} \right)$$

Using Sec. A-1 with $x = f_0$ and $y = f_\Delta$

$$\begin{aligned} \textcircled{3} &= [\sin(2\pi f_0 t) \cos(2\pi f_\Delta t)] \\ &\quad \cdot \left[\frac{1}{\pi/2f_\Delta - 2\pi t} - \frac{1}{\pi/2f_\Delta + 2\pi t} \right] \\ &= [\sin(2\pi f_0 t) \cos(2\pi f_\Delta t)] \left[\frac{4f_\Delta t}{\pi(1 - (4f_\Delta t)^2)} \right] \end{aligned}$$

$$\therefore h_e(t) = \frac{2f_0 \sin(2\pi f_0 t)}{2\pi f_0 t} \left[\cos(2\pi f_\Delta t) + \frac{\cos(2\pi f_\Delta t)}{1 - (4f_\Delta t)^2} \cdot (4f_\Delta t)^2 \right]$$

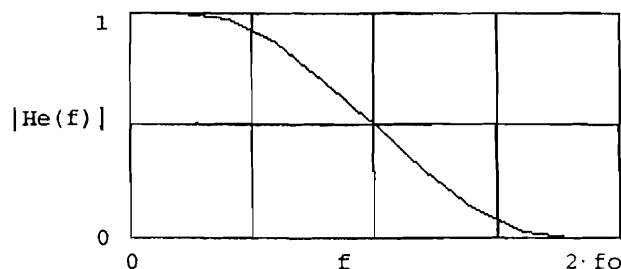
$$\Rightarrow h_e(t) = 2f_0 \frac{\sin(2\pi f_0 t)}{2\pi f_0 t} \left[\frac{\cos[2\pi f_\Delta t]}{1 - (4f_\Delta t)^2} \right]$$

3-45

Select r
(a.) $r := 0.75$ $f_0 := 1$ $f_1 := (1 - r) f_0$ $f_d := r f_0$
 $f := 0, 0.2 \dots 2 f_0$ $B := (1 + r) f_0$

$$H_e(f) := (1 - \Phi(f - f_1)) + 0.5 \left[1 + \cos \left[\pi \frac{f - f_1}{2 f_d} \right] \right] (\Phi(f - f_1) - \Phi(f - B))$$

$r = 0.75$
 $f_1 = 0.25$
 $f_0 = 1$
 $B = 1.75$



3-45 Cont'd.

(b.) $T := \frac{1}{2 f_o}$ $t := 0 + 0.001, 0.05 \dots 4 \cdot T$

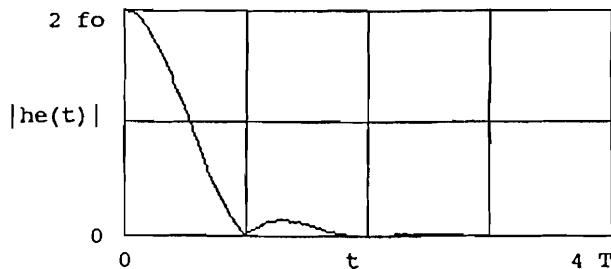
$$h_e(t) := 2 f_o \cdot \left[\frac{\sin(2 \pi f_o t)}{2 \cdot \pi f_o t} \right] \left[\frac{\cos(2 \pi f_d \cdot t)}{1 - (4 f_d \cdot t)^2} \right]$$

$r = 0.75$

$T = 0.5$

$f_o = 1$

$f_d = 0.75$



3-46

The pulse shape (impulse response) for the raised cosine-rolloff channel is given by (3-73) and the PSD for the output polar signal is given by (3-40) in (3-36a). Thus, for $A=1$,

$$P_{out}(f) = \frac{1}{T_b} |H_e(f)|^2$$

where, using (3-69)

$$H_e(f) = \begin{cases} 1 & , |f| < f_1 \\ \frac{1}{2} \left\{ 1 + \cos \left[\frac{\pi(|f| - f_1)}{2 f_\Delta} \right] \right\} & , f_1 < |f| < B \\ 0 & , |f| > B \end{cases}$$

For a maximum bit rate without ISI, from (3-74),

$$R = \frac{2B}{1+r} = \frac{2B}{\frac{3}{2}} = \frac{4}{3} B \Rightarrow \underline{B = \frac{3}{4} R}$$

$\uparrow$
 $r = 0.5$

$$f_\Delta = B - f_0 = B - \frac{f_\Delta}{r} \Rightarrow r f_\Delta = r B - f_\Delta$$

$\uparrow$
 $r = \frac{f_\Delta}{f_0}$

$$\Rightarrow f_\Delta = \frac{rB}{r+1} = \frac{\frac{1}{2}B}{\frac{1}{2}+1} = \frac{\frac{1}{2}B}{\frac{3}{2}} = \frac{1}{3}B$$

$$\stackrel{\uparrow}{=} \frac{1}{3} \left(\frac{3}{4} R \right) = \frac{1}{4} R \Rightarrow \underline{f_\Delta = \frac{1}{4} R}$$

$\uparrow$
 $B = \frac{3}{4} R$

3-46, Cont'd

From Fig. 3-25

$$f_1 = B - 2f_A = \frac{3}{4}R - 2(\frac{1}{4}R) = \frac{1}{4}R$$

$$\Rightarrow \underline{f_1 = \frac{1}{4}R}$$

Thus,

$$H_e(f) = \begin{cases} 1 & , |f| < \frac{1}{4}R \\ \frac{1}{2} \left\{ 1 + \cos \left[\frac{\pi(|f| - \frac{1}{4}R)}{2(\frac{1}{4}R)} \right] \right\} & , \frac{1}{4}R < |f| < \frac{3}{4}R \\ 0 & , f > \frac{3}{4}R \end{cases}$$

or

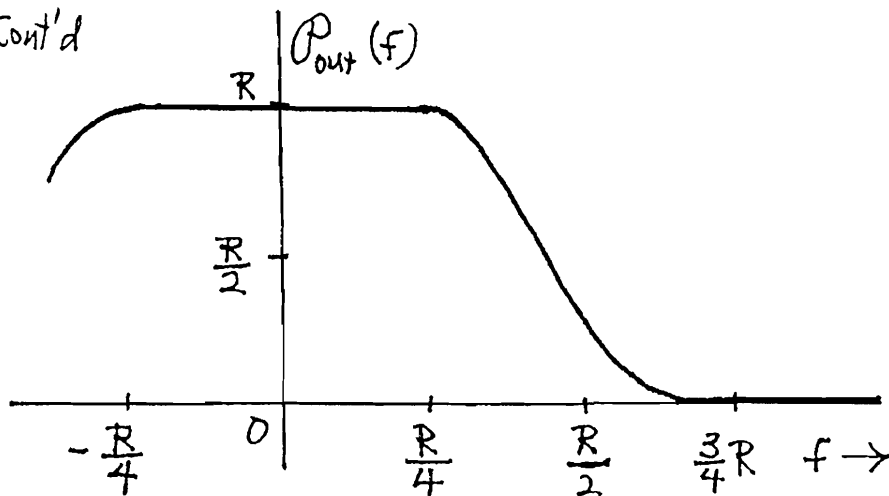
$$H_e(f) = \begin{cases} 1 & , |f| < \frac{1}{4}R \\ \frac{1}{2} \left\{ 1 + \cos \left[2\pi \left(\left| \frac{f}{R} \right| - \frac{1}{4} \right) \right] \right\} & , \frac{1}{4}R < |f| < \frac{3}{4}R \\ 0 & , |f| > \frac{3}{4}R \end{cases}$$

Then, the PSD at the output is

$$\underline{\underline{P_{out}(f) = \begin{cases} R & , |f| < \frac{1}{4}R \\ \frac{R}{4} \left\{ 1 + \cos \left[2\pi \left(\left| \frac{f}{R} \right| - \frac{1}{4} \right) \right] \right\}^2 & , \frac{1}{4}R < |f| < \frac{3}{4}R \\ 0 & , |f| > \frac{3}{4}R \end{cases}}}$$

A sketch of this result is shown at the top of the next page.

3-46. Cont'd



3-47

$$h_e(t) = \int_{-\infty}^{\infty} H_e(f) e^{j2\pi ft} df$$

$$\text{For } t = nT_s \Rightarrow h_e(nT_s) = \int_{-\infty}^{\infty} H_e(f) e^{j2\pi nT_s f} df$$

Break into multiple integrals, each with a $\frac{1}{T_s}$ wide interval.

$$\Rightarrow h_e(nT_s) = \sum_{k=-\infty}^{\infty} \int_{-\frac{1}{2T_s} + \frac{k}{T_s}}^{\frac{1}{2T_s} + \frac{k}{T_s}} H_e(f) e^{j2\pi nT_s f} df$$

$$\text{Let } f_1 = f - \frac{k}{T_s}$$

$$\text{Then } h_e(nT_s) = \sum_{k=-\infty}^{\infty} \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} H_e(f_1 + \frac{k}{T_s}) e^{j2\pi nT_s (f_1 + \frac{k}{T_s})} df_1$$

or

$$h_e(nT_s) = \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} \sum_{k=-\infty}^{\infty} H_e(f_1 + \frac{k}{T_s}) e^{j2\pi nT_s f_1} df_1$$

$$\text{Assume } \sum_{k=-\infty}^{\infty} H_e(f_1 + \frac{k}{T_s}) = T_s, \quad |f_1| < \frac{1}{2T_s}$$

$$\text{Then } h(nT_s) = \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} T_s e^{j2\pi nT_s f_1} df_1 = \left. \frac{T_s e^{j2\pi nT_s f_1}}{j2\pi nT_s} \right|_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}}$$

$$\text{or } h(nT_s) = \frac{e^{j2\pi nT_s \frac{1}{2T_s}} - e^{-j2\pi nT_s \frac{1}{2T_s}}}{j2\pi n} = \frac{\sin(n\pi)}{n\pi} = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$$

Q.E.D.

3-48

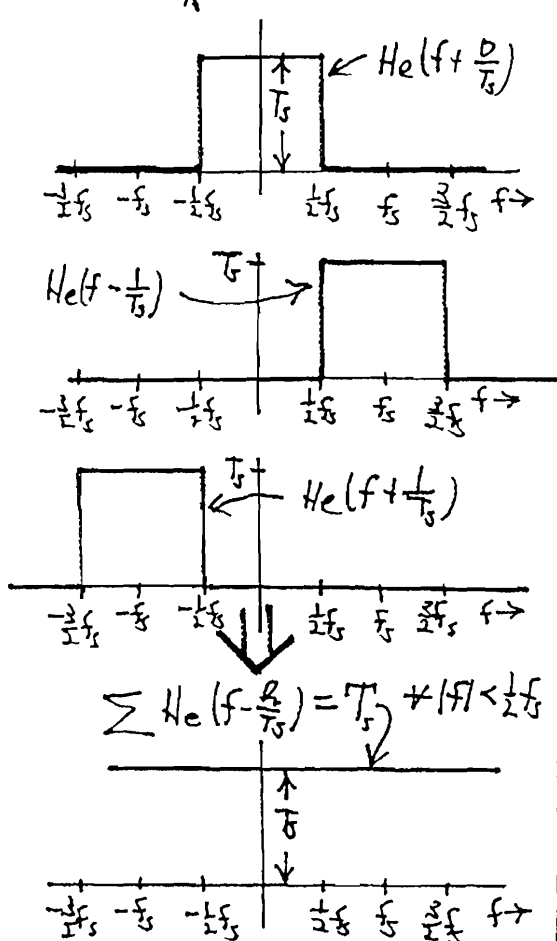
See if $\sum_{k=-\infty}^{\infty} H_e(f + \frac{k}{T_s}) = T_s$

(1)

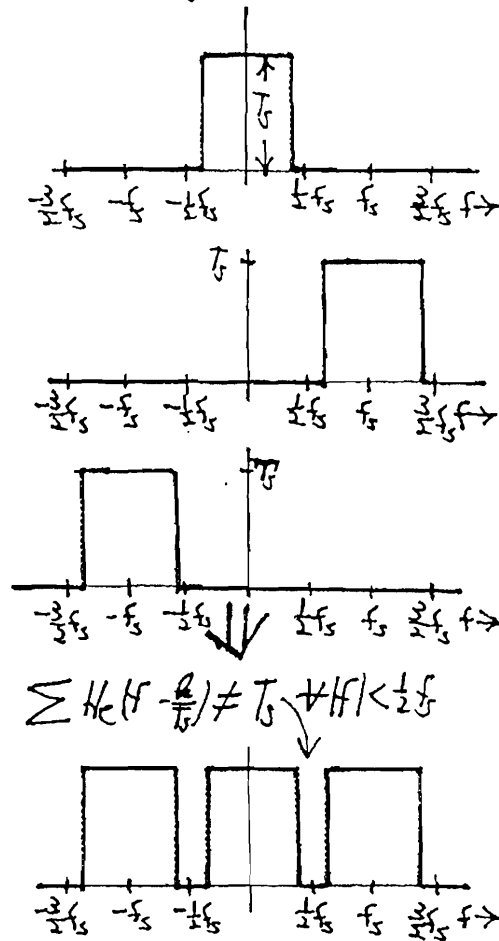
is satisfied for $|f| < \frac{1}{2}f_s$ where $\frac{1}{T_s} = \frac{2}{T_b}$

(a) $H_e(f) = \frac{T_b}{2} \Pi\left(\frac{f}{\frac{2}{T_b}}\right) = T_s \Pi\left(\frac{f}{\frac{2}{T_s}}\right) = T_s \Pi\left(\frac{f}{\frac{1}{f_s}}\right)$

(b) $H_e(f) = \frac{T_b}{2} \Pi\left(\frac{f}{\frac{2}{T_b}}\right) = T_s \Pi\left(\frac{f}{\frac{1}{f_s}}\right) = T_s \Pi\left(\frac{f}{\frac{1}{f_s}}\right)$



Yes, Nyquist criterion is satisfied



No, Nyquist criterion is not satisfied.

3-49

(a.) Substitute (3-69) into

$$H_e(f) = \begin{cases} 1 + Y(f), & |f| < f_0 \\ Y(f), & f_0 < |f| < 2f_0 \end{cases}$$

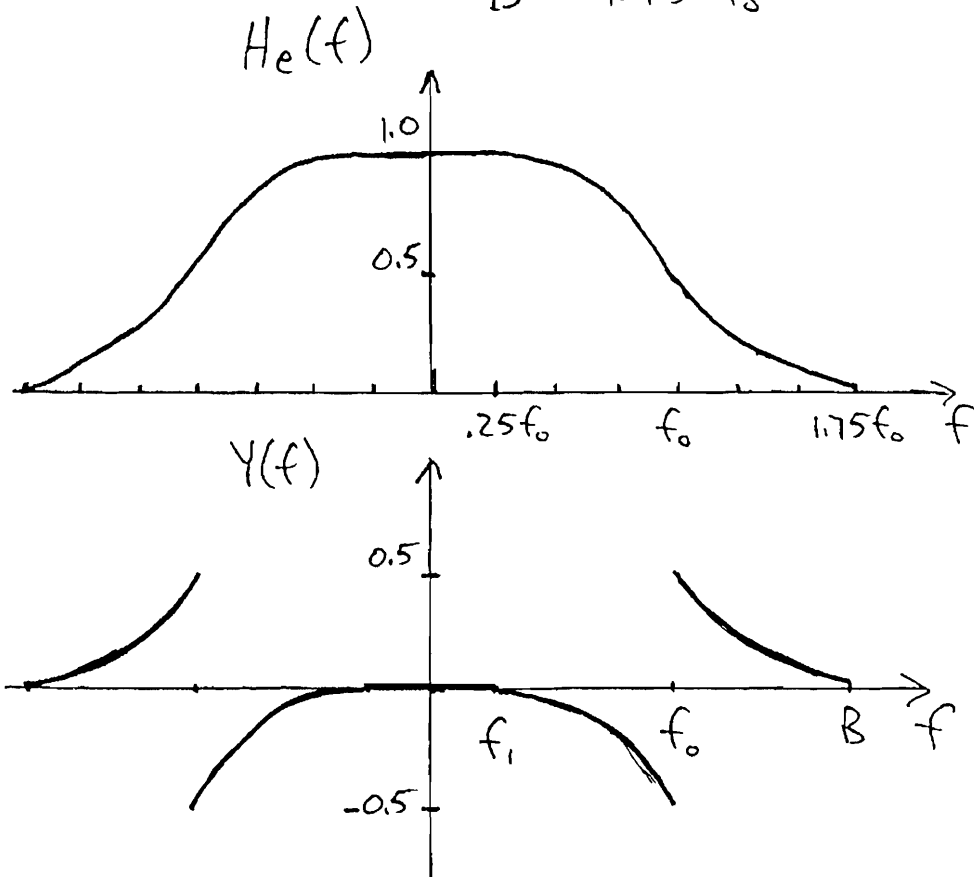
$$\Rightarrow Y(f) = \begin{cases} 0 & |f| < f_1 \\ \frac{1}{2} \left\{ -1 + \cos \left[\frac{\pi(|f| - f_1)}{2f_0} \right] \right\} & f_1 < |f| < f_0 \\ \frac{1}{2} \left\{ 1 + \cos \left[\frac{\pi(|f| - f_1)}{2f_0} \right] \right\} & f_0 < |f| < B \\ 0 & |f| > B \end{cases}$$

3-49 Cont'd (b.)

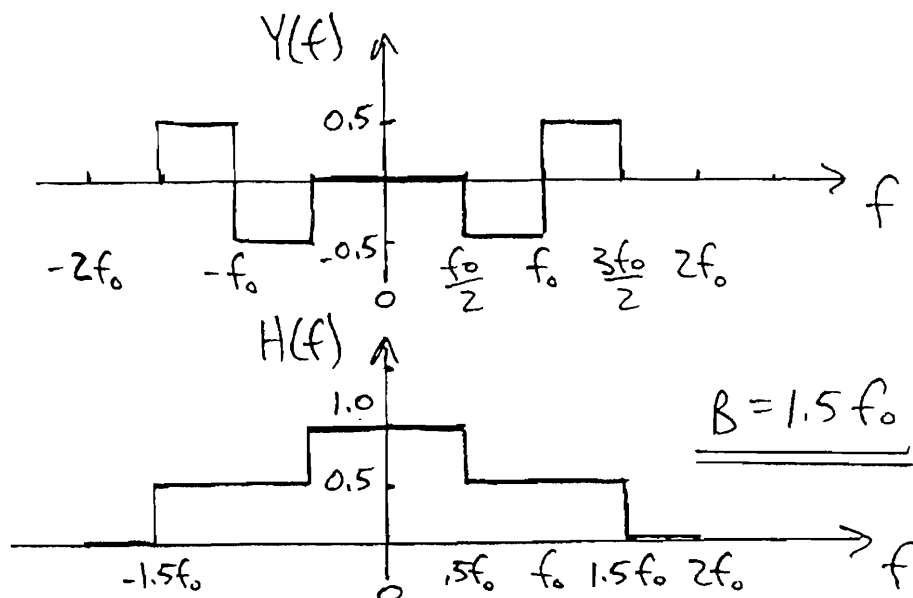
$$r = 0.75 = \frac{f_\Delta}{f_0} \Rightarrow f_1 = f_0 (1 - 0.75)$$

$$f_1 = 0.25 f_0$$

$$B = 1.75 f_0$$



(c.)



3-50 $M=16=2^4 \Rightarrow n=4$

(a) Binary PCM $\Rightarrow l=1$, $R=nf_s=4f_s=D$

$$D = \frac{2B}{1+r} = \frac{2(4\text{ kHz})}{1+0.5} = \underline{\underline{5.33\text{ kbits/sec}}}$$

(b) From (a) $f_s = \frac{D}{4} = \frac{5.33\text{ k}}{4} = 1.33\text{ kHz}$

$$B_{\text{analog max}} = \frac{f_s}{2} = \frac{1.33\text{ k}}{2} = \underline{\underline{667\text{ Hz}}}$$

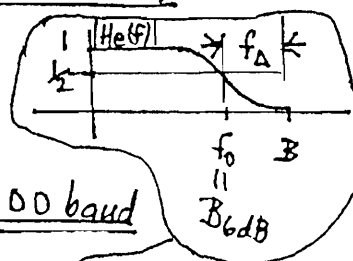
3-51 $L=2^l=4 \Rightarrow l=2$

(a) $D = \frac{R}{l} = \frac{R}{2}$
 $\Rightarrow R=2D=2\left(\frac{2B}{1+r}\right) = \underline{\underline{10.67\text{ kbits/sec}}}$

(b) $M=2^n=L^N=(2^l)^{N/l}=16 \Rightarrow n=4$

$$R=nf_s \Rightarrow f_s = \frac{R}{n} = \frac{10.67\text{ kbits}}{4\text{ bits/sample}} = \underline{\underline{2.66\text{ ksamples/sec}}}$$

$$B_{\text{analog max}} = \frac{f_s}{2} = \frac{2.66\text{ k}}{2} = \underline{\underline{1.33\text{ kHz}}}$$



3-52 (a) $L=2^l=4 \Rightarrow l=2$

$$D = R/l = \frac{2400}{2} = \underline{\underline{1200\text{ baud}}}$$

(b) $B = \frac{1}{2}(1+r)D$ where $r = \frac{f_{\Delta}}{f_0} = 0 \Rightarrow B=f_0=B_{6dB}$

$$\Rightarrow B_{6dB} = \frac{1}{2}(1+0)D = \frac{1}{2}(1200) = \underline{\underline{600\text{ Hz}}}$$

(c)

$$B_{\text{absolute}} = \frac{1}{2}(1+r)D = \frac{1}{2}(1+0.5)(1200) = \frac{3}{4}(1200)$$

$\uparrow$
 $r=0.5$

$$\Rightarrow \underline{\underline{B_{\text{absolute}} = 900\text{ Hz}}}$$

3-53

PCM with $M=255$ companding.

$\Rightarrow$ Use (3-25) with (3-26b)

$$(S/N)_dB = 6.02n + \alpha = 6.02n + 4.77 - 20 \log[\ln(M)] \stackrel{M=255}{=} 6.02n - 10.1$$

$$\Rightarrow n = \frac{40 + 10.1}{6.02} = \frac{50.108}{6.02} = 8.32 \text{ bits}$$

$n = 8.32 \text{ bits} \Rightarrow$ Use 9 bits to get 40dB (S/N)

$$R = f_s n = 2Bn = 2(3400)(9) = \underline{\underline{61.2 \text{ kbits/sec}}}$$

3-54

DPCM Use (3-80a) where, depending on the properties of the analog input signal, $-3 < \alpha < 15$.

$\alpha = -3$ gives the worst case result and $\alpha = 15$ gives the best result.

$$\Rightarrow n = \begin{cases} 7.1, \text{ worst case} \\ 4.1, \text{ best case} \end{cases} \stackrel{\text{use next largest integer value}}{\uparrow} \begin{cases} 8, \text{ worst case} \\ 5, \text{ best case} \end{cases}$$

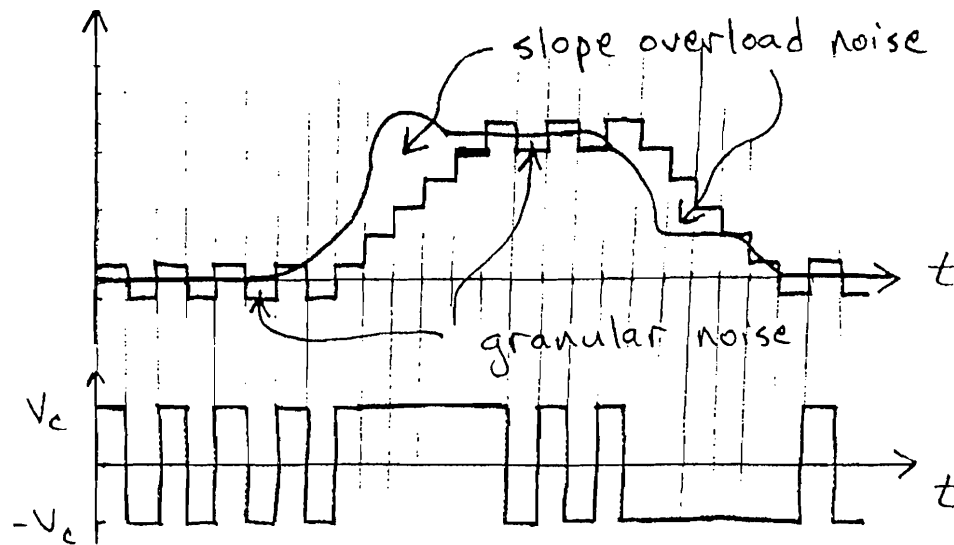
Thus

$$R = 2Bn = 2(3400)n = \begin{cases} 54.4 \text{ kbits/sec, worst case} \\ 34 \text{ kbits/sec, best case} \end{cases}$$

Even for the worst case, the bit rate required for DPCM would be slightly less than that required for $M=255$ PCM. Consequently, DPCM with a bit rate of about 48 kbits/sec might be selected. as the best solution where it is realized that for certain types of inputs, the 40dB S/N goal would not be achieved.

3-55

DM



3-56

(a) To prevent slope overload,

$$\delta > \frac{2\pi f_a A}{f_s} = \frac{2\pi (10 \text{ kHz}) (0.5 \text{ V})}{10(2) (10 \text{ kHz})} = \underline{\underline{0.157 \text{ V}}}$$

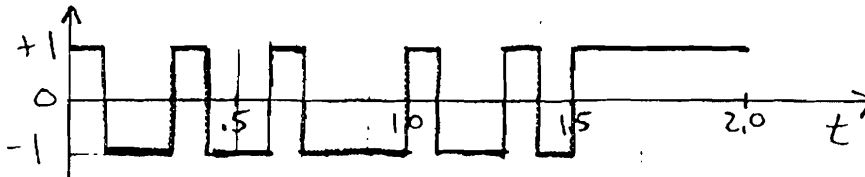
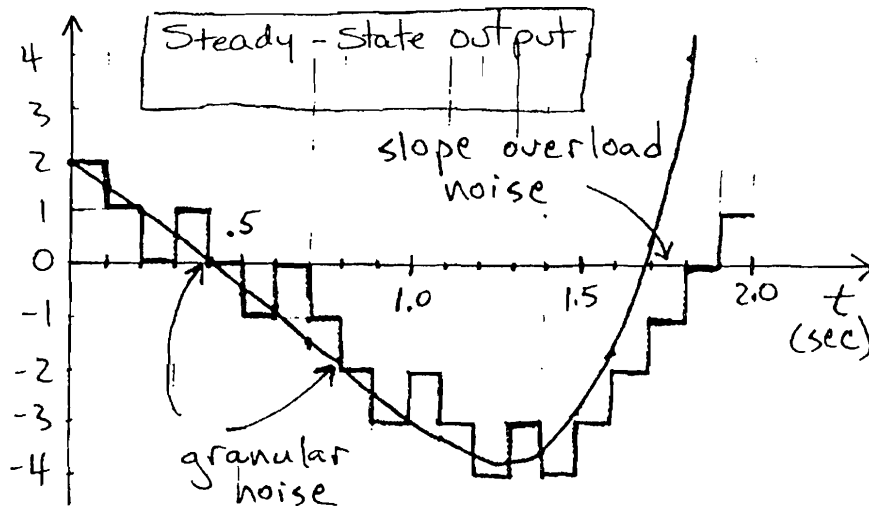
$$(b) \quad P_n(f) = \frac{\delta^2}{6f_s} = \frac{(0.157)^2}{6(200 \text{ kHz})} = \underline{\underline{2.06 \times 10^{-5} \frac{\text{V}^2}{\text{kHz}} = \frac{N_b}{2}}}$$

$$(c) \quad \left(\frac{S}{N}\right)_{\text{recvr out}} = \frac{\frac{A^2}{2}}{2B\left(\frac{N_b}{2}\right)} = \frac{(0.5 \text{ V})^2/2}{2(200 \text{ kHz})(2.06 \times 10^{-5} \frac{\text{V}^2}{\text{kHz}})} = 15.2$$

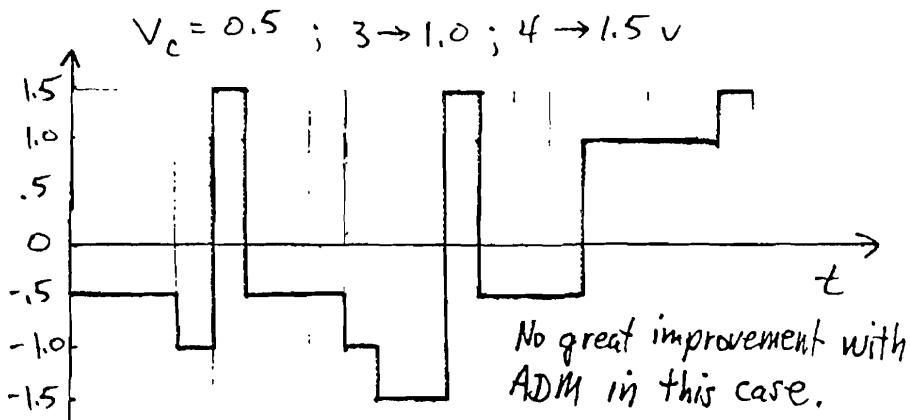
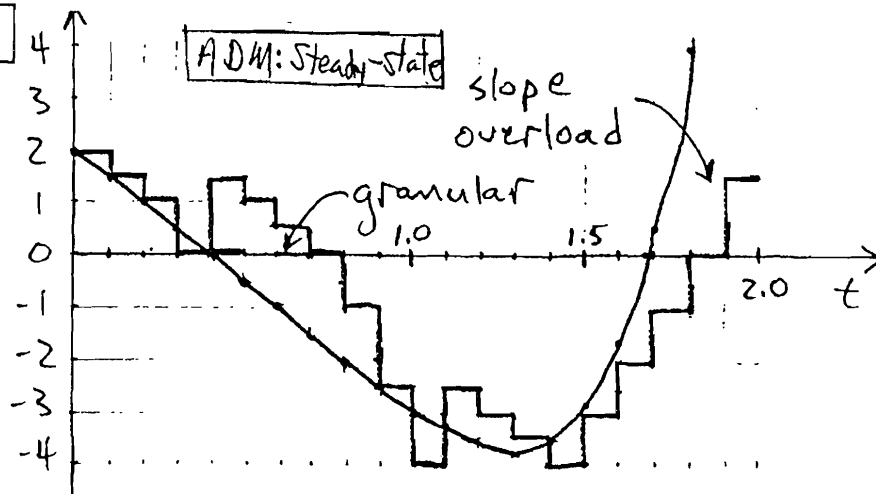
$$\Rightarrow \left(\frac{S}{N}\right)_{\text{dB}} = 10 \log(15.2) = \underline{\underline{11.82 \text{ dB}}}$$

3-57

$$v_{in}(t) = 0.1t^2 - 5t + 2, \quad V_c = 1 \text{ volt}, \quad f_s = 10 \text{ Hz}$$



3-58



3-59

(a.) From (3-84)

$$\delta = \frac{2\pi f_a A}{f_s} ; \quad f_a = 3.4 \text{ kHz} \quad \& \quad A = \frac{1}{2}$$

We need to determine the f_s which the channel can support. Assuming that a $r=0$ roll-off factor is used, then

$$f_s = B = 2B = 2(1 \text{ MHz}) = 2 \times 10^6 \frac{\text{Samples}}{\text{sec}}$$

$$\Rightarrow \delta = \frac{2\pi (3.4 \text{ k}) (\frac{1}{2})}{2 \times 10^6} = \underline{\underline{0.00534}}$$

(Note: The channel has to be equalized with a Nyquist filter.)

(b.)

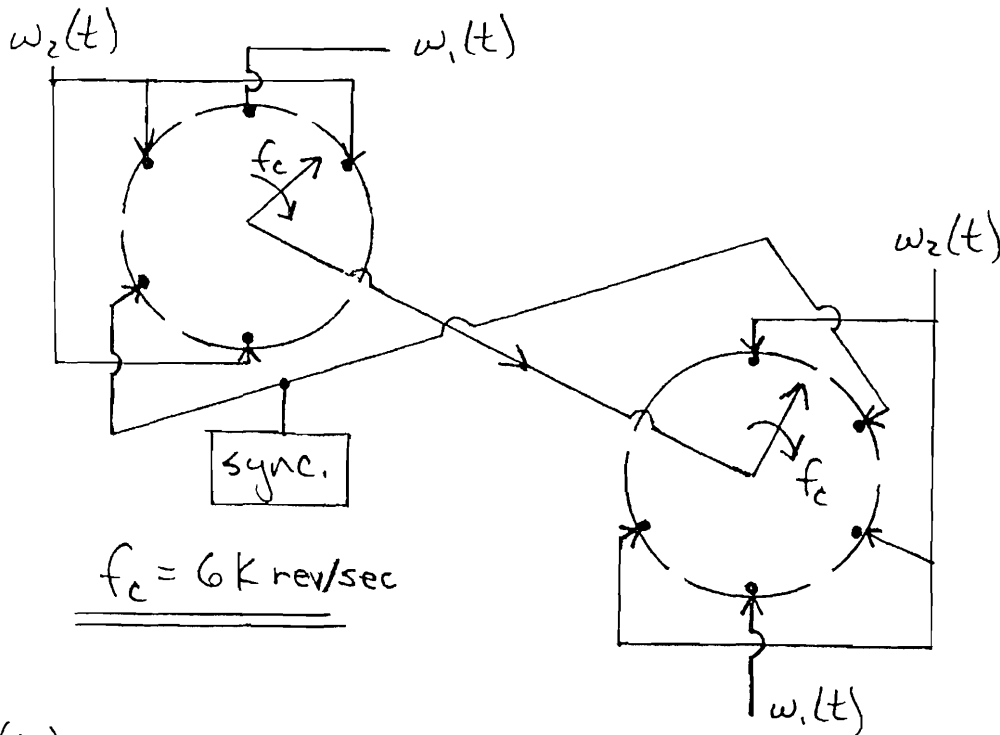
$$\delta = \frac{2\pi (3.4 \text{ k}) (\frac{1}{2})}{25 \times 10^3} = \underline{\underline{0.427}}$$

(Note: No Channel equalization required.)

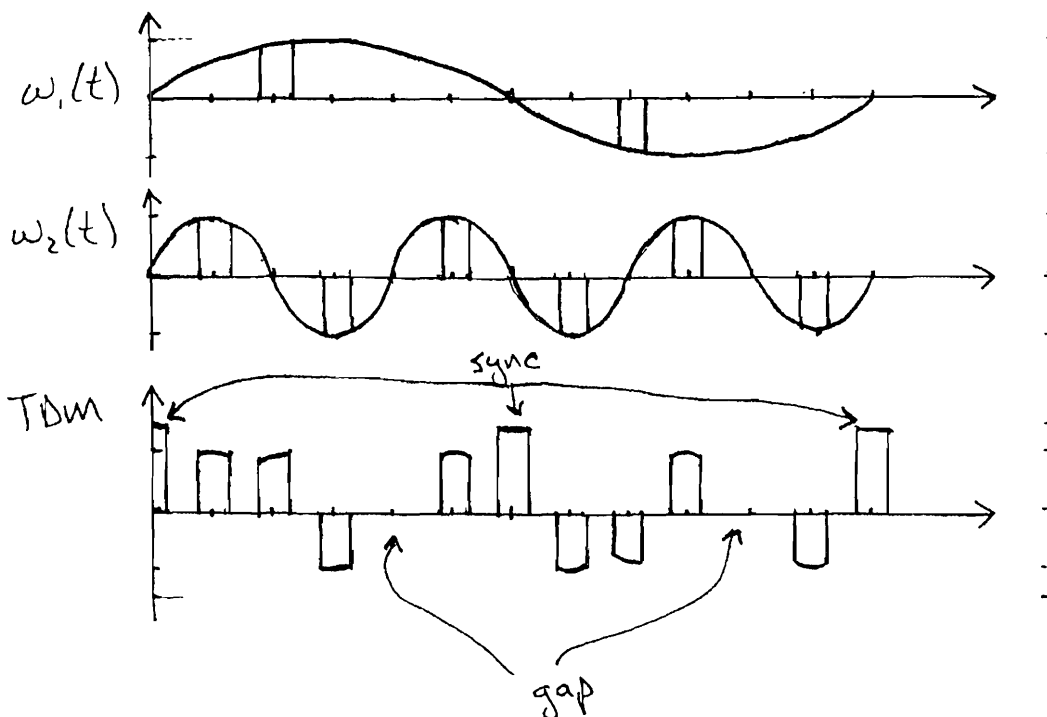
3-60 TDM

(a.) $\omega_1(t)$; $B = 3 \text{ KHz} \Rightarrow f_{s1} = 6 \text{ K samples/sec}$

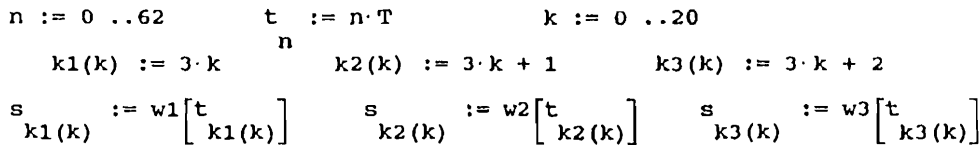
$\omega_2(t)$; $B = 9 \text{ KHz} \Rightarrow f_{s2} = 18 \text{ K samples/sec}$



(b.)

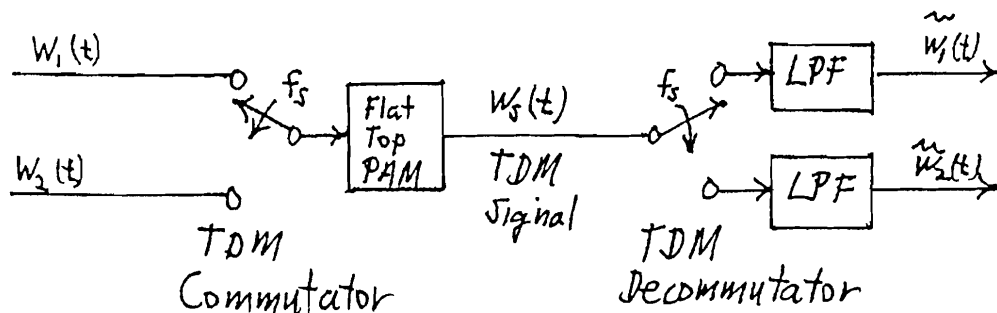


```
T := 0.05          t := 0,0.05 .. 3
```

$$w_3(t) := \text{if}(|t - 1| \leq 1, |t - 1| - 1, 0)$$


⇒ The minimum sampling frequency for each analog signal is $f_s = 2B = 6 \text{ kHz}$

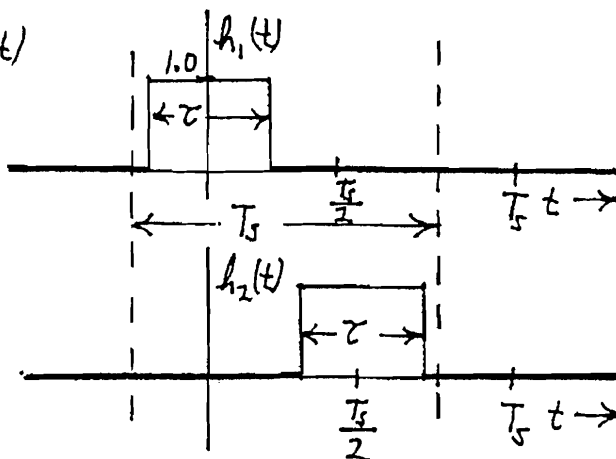
3-63.(a.) Cont'd



(b) Referring to (3-8), the sampled TDM signal is

$$w_s(t) = \sum_{k=-\infty}^{\infty} w_1(kT_s) h_1(t - kT_s) + \sum_{k=-\infty}^{\infty} w_2(kT_s) h_2(t - kT_s)$$

where $h_1(t)$ and $h_2(t)$ are shown in the figure and $\tau \leq \frac{T_s}{2}$ and $f_s \geq 2B$.



Following the same procedure as described in (3-8) thru (3-13), the spectrum of the TDM instantaneously sampled (flat-topped) PAM signal is

$$W_s(f) = \frac{1}{T_s} H_1(f) \sum_{k=-\infty}^{\infty} W_1(f - k f_s) + \frac{1}{T_s} H_2(f) \sum_{k=-\infty}^{\infty} W_2(f - k f_s)$$

where $H_1(f) = \tau \left(\frac{\sin(\pi f \tau)}{\pi f \tau} \right)$ and $H_2(f) = \tau \left(\frac{\sin(\pi f \tau)}{\pi f \tau} \right) e^{-j2\pi f \frac{T_s}{2}}$

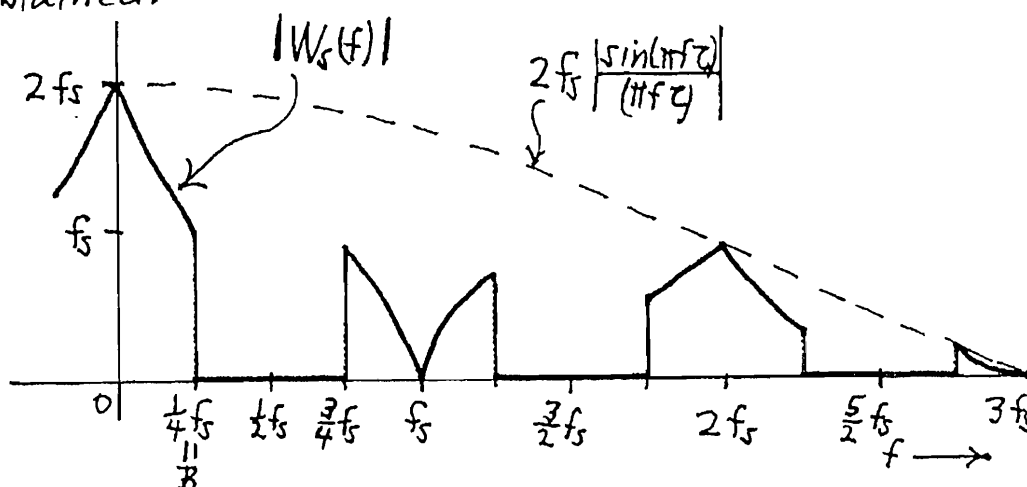
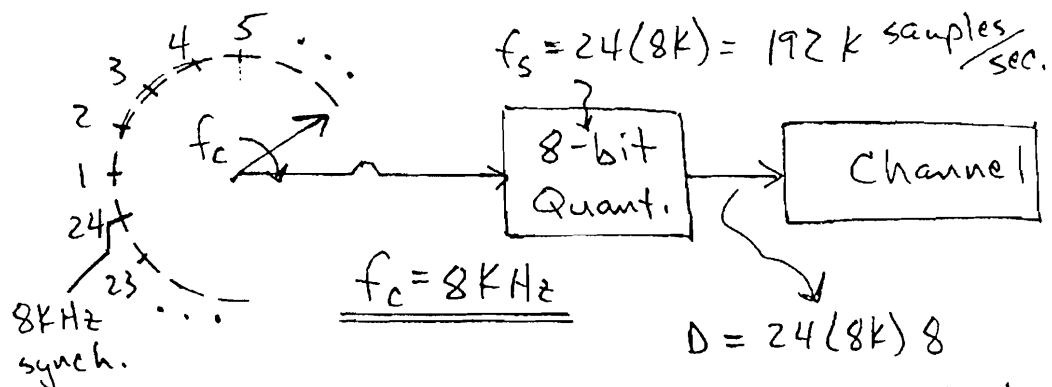
$$\Rightarrow W_s(f) = f_s \tau \frac{\sin(\pi f \tau)}{\pi f \tau} \sum_{k=-\infty}^{\infty} \Pi\left(\frac{f - k f_s}{2B}\right) + 2B\tau \left(\frac{\sin(\pi f \tau)}{\pi f \tau} \right) + 2B\tau \left(\frac{\sin(\pi f \tau)}{\pi f \tau} \right) e^{-j\pi T_s f} \sum_{k=-\infty}^{\infty} \Lambda\left(\frac{f - k f_s}{B}\right)$$

3-63. (b.) Cont'd

Thus,

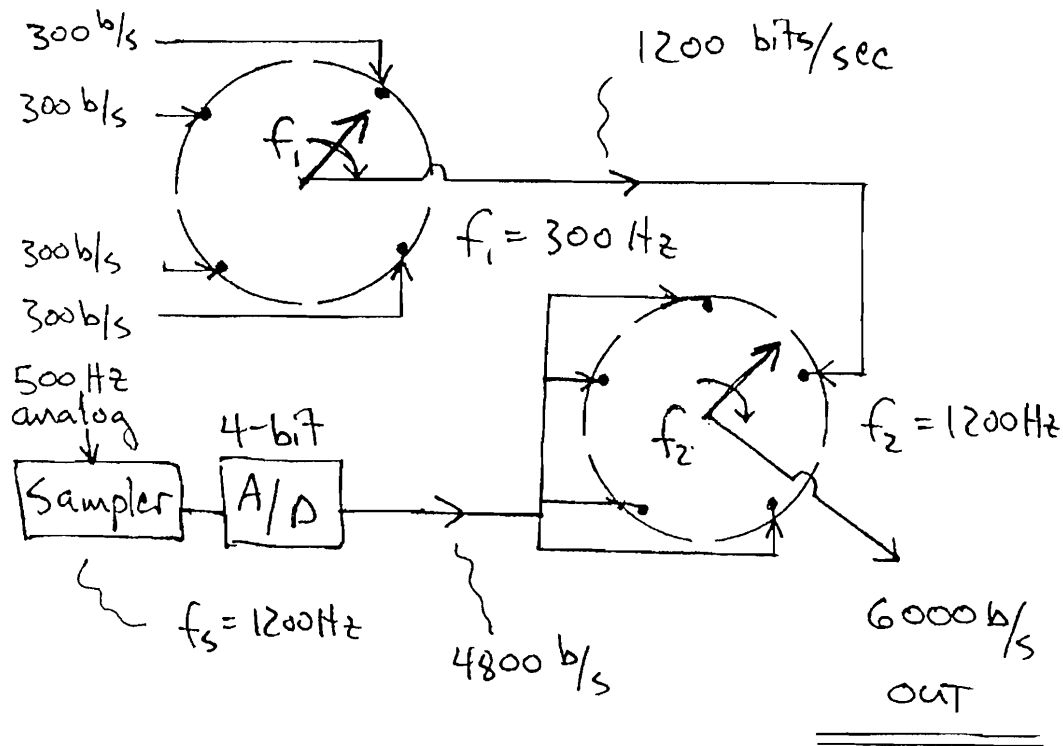
$$|W_s(f)| = f_s \left| \frac{\sin(\pi f \tau)}{(\pi f \tau)} \right| \sum_{k=-\infty}^{\infty} \left| \Pi\left(\frac{f - k f_s}{2B}\right) + e^{j\pi f_s \tau} \Pi\left(\frac{f - k f_s}{B}\right) \right|$$

For the sketch, let the parameters be the same as those shown in Fig. 3-6. Let $\tau/T_s = 1/3$, $f_s = 4B$. Using a programmable calculator, the following sketch is obtained.

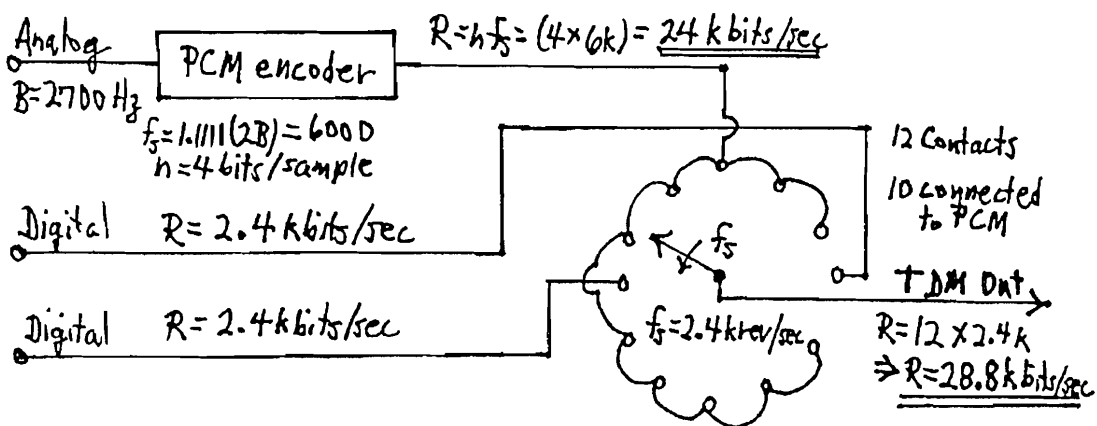

3-64 (a.) TDM - PCM

(b.) $r = 0.75$

$$\beta = \frac{D(1+r)}{2} = \frac{1.536(10^6)(1.75)}{2} = 1.344 \text{ MHz}$$

3-65



3-66



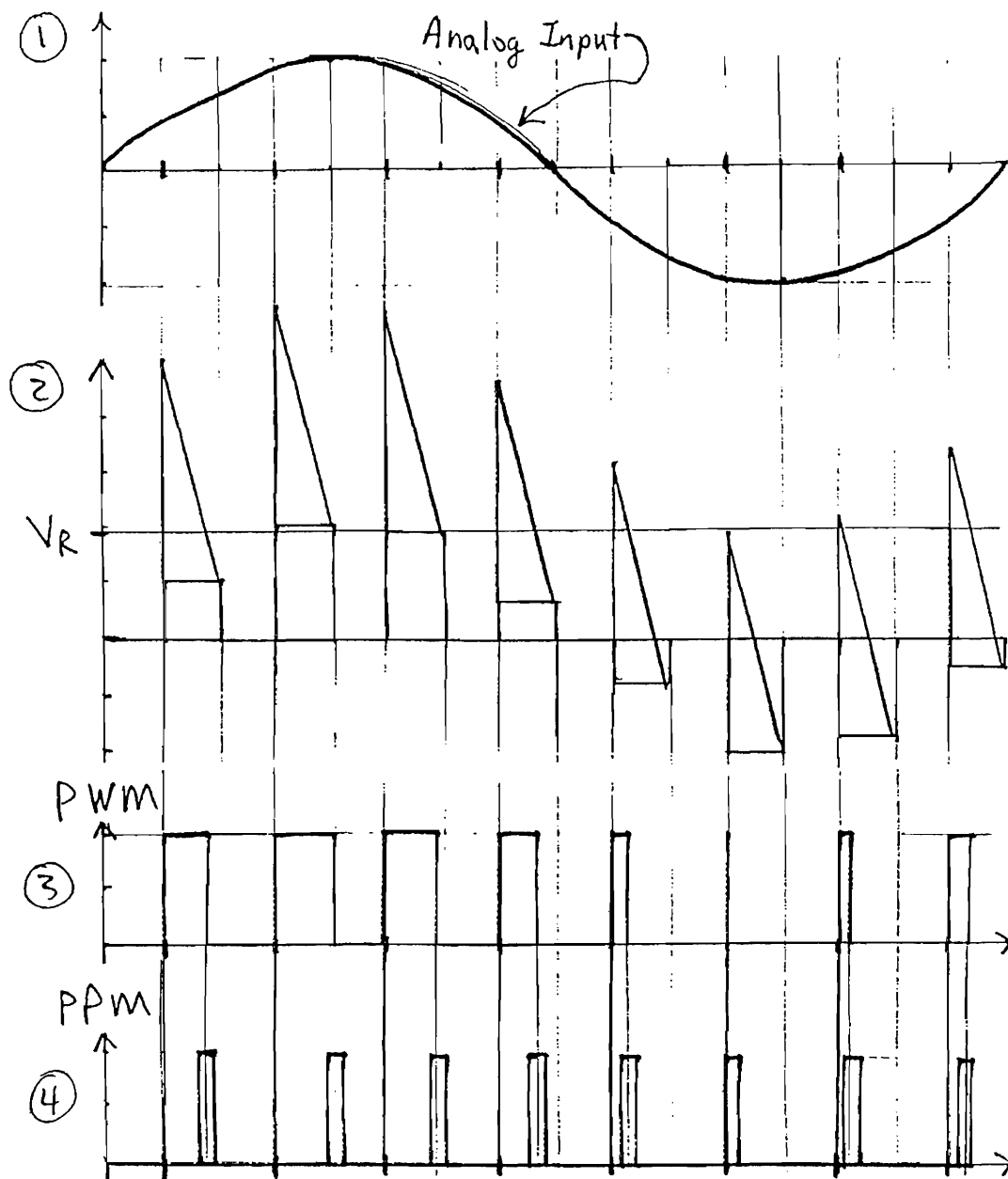
3-67

Available capacity $\triangleq AC = (1.544 \text{ Mbits/sec})(0.99) = 1.53 \text{ Mbits/sec}$

(a) $\frac{AC}{110} = 13,896$, (b) $\frac{AC}{8000} = 191$, (c) $\frac{AC}{9600} = 159$, (d) $\frac{AC}{64 \times 10^3} = 23$

(e) $\frac{AC}{144 \times 10^3} = 10$. If sources are active only 10% of the time, a statistical multiplexer could support 10 times the number of terminals.

3-68



3-69

The information in a PAM signal is contained in the amplitude of each pulse. The information in a PWM signal is contained in the width of each pulse. However, the information in a PPM signal is contained in the position of each pulse relative to each clocking time. A clocking signal must be available at the receiver in order to determine the relative position of each pulse and the clocking signal must be synchronized with the clocking signal used at the transmitter.

3-70

(a) For PCM a $N=8$ dimensional system is used since any of the 256 messages can be represented by

$$s_i(t) = \sum_{j=1}^8 s_{ij} \phi_j(t)$$

where $s_{ij} = \pm 1$ for binary PCM.

$$T_0 = \frac{1}{10 \text{ mess/sec.}} = \underline{\underline{0.1 \text{ sec/message}}}$$

$$B = \frac{1}{2} \left(\frac{N}{T_0} \right) = \frac{1}{2} \left(\frac{8}{0.1} \right) = \underline{\underline{40 \text{ Hz}}}$$

(b) For PPM a $N=256$ dimensional system is used:

$$s_i(t) = \sum_{j=1}^{256} s_{ij} \phi_j(t) \quad \text{where } s_{ij} = \pm 1$$

$$B = \frac{1}{2} \left(\frac{N}{T_0} \right) = \frac{1}{2} \left(\frac{256}{0.1} \right) = \underline{\underline{1,280 \text{ Hz}}}$$