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Chapter 3: Interest Rate and Economic Equivalence

Types of Interest

3.1

$$\$20,000 = \$10,000(1+0.075N)$$

- Simple interest: $(1+0.075N) = 2$

$$N = \frac{1}{0.075} = 13.33 \approx 14 \text{ years}$$

- Compound interest:

$$\$20,000 = \$10,000(1+0.07)^N$$
$$(1+0.07)^N = 2$$
$$N = 10.24 \approx 11 \text{years}$$

3.2

- Simple interest:

$$I = iPN = (0.06)(\$5,000)(5) = \$1,500$$

- Compound interest:

$$I = P[(1+i)^N - 1] = \$5,000(1.3382 - 1) = \$1,691$$

3.3

- Option 1: Compound interest with 8%:

$$F = \$3,000(1+0.08)^5 = \$3,000(1.4693) = \$4,408$$

- Option 2: Simple interest with 9%:

$$\$3,000(1+0.09\times 5) = \$3,000(1.45) = \$4,350$$

$\therefore$ Option 1 is better.

3.4

End of Year	Principal Repayment	Interest payment	Remaining Balance
0			\$10,000
1	\$1,638	\$1,000	\$8,362
2	\$1,802	\$836	\$6,560
3	\$1,982	\$656	\$4,578
4	\$2,180	\$458	\$2,398
5	\$2,398	\$240	\$0

Equivalence Concept

3.5

$$P = \$18,000(P/F, 5\%, 5) = \$18,000(0.7835) = \$14,103$$

3.6

$$F = \$25,000(F/P, 8\%, 3) = \$25,000(1.2597) = \$31,493$$

3.7

$$F = \$100(F/P, 10\%, 10) + \$200(F/P, 10\%, 8) = \$688$$

3.8

$$\begin{aligned} \$1,000(F/P, i, 2) &= \$1,200 \\ \$1,000(1+i)^2 &= \$1,200 \\ i &= \sqrt{1.2} - 1 \\ i &= 9.54\% \end{aligned}$$

Single Payments (Use of *F*/*P* or *P*/*F* Factors)

3.9

$$F = \$180,000(F/P, 6\%, 10) = \$322,353$$

3.10

(a) $F = \$7,000(F/P, 6\%, 5) = \$9,368$

(b) $F = \$3,250(F/P, 5\%, 15) = \$6,757$

(c) $F = \$18,000(F/P, 8\%, 33) = \$228,169$

(d) $F = \$20,000(F/P, 9\%, 8) = \$39,851$

3.11

$$P = \$300,000(P/F, 8\%, 10) = \$138,958$$

3.12

(a) $P = \$15,500(P/F, 14\%, 8) = \$5,434$

(b) $P = \$18,000(P/F, 4\%, 12) = \$11,243$

(c) $P = \$20,000(P/F, 8\%, 9) = \$10,005$

(d) $P = \$55,000(P/F, 11\%, 4) = \$36,230$

3.13

(a) $P = \$12,000(P/F, 13\%, 4) = \$7,360$

(b) $F = \$30,000(F/P, 13\%, 5) = \$55,273$

3.14

$$F = 3P = P(1+0.06)^N$$
$$\log 3 = N \log(1.06)$$
$$N = 18.85 \approx 19 \text{ years}$$

3.15

$$F = 2P = P(1+0.08)^N$$

- $\log 2 = N \log(1.08)$
  $N = 9$ years

- Rule of 72: $72/8 = 9$ years

3.16

(a) Single-payment compound amount $(F/P, i, N)$ factors for

N	9%	10%
35	20.4140	28.1024
40	31.4094	45.2593

To find $(F/P, 9.5\%, 38)$, first, interpolate for $n = 38$:

N	9%	10%
38	27.0112	38.3965

Then, interpolate for $i = 9.5\%$:

$$(F/P, 9.5\%, 38) = 32.7039$$

As compared to formula determination

$$(F/P, 9.5\%, 38) = 31.4584$$

(b) Single-payment compound amount $(P/F, 8\%, N)$ factors for

N	45	50
	0.0313	0.0213

Then, interpolate for $N = 47$

$$(P/F, 8\%, 47) = 0.0273$$

As compared to the value from the interest formula:

$$(P/F, 8\%, 47) = 0.0269$$

3.17

(a)
$$\$18(1+i)^{44} = \$92,400$$
$$i = 21.43\%$$

(b) $F = \$97.8(F/P, 21.43\%, 22) = \$7,007\text{billion}$

Uneven Payment Series

3.18

$$\$1,000 + \frac{\$1,000}{1.1} + \frac{\$1,500}{1.1^3} = \frac{\$1,210}{1.1^2} + \frac{X}{1.1^4}$$
$$X = \$2,981$$

3.19

$$P = \frac{\$25,000}{1.07^2} + \frac{\$33,000}{1.07^3} + \frac{\$46,000}{1.07^4} + \frac{\$38,000}{1.07^5} = \$110,961$$

3.20

$$F = \$2,000(F/P, 6\%, 10) + \$2,500(F/P, 6\%, 8) + \$3,000(F/P, 6\%, 6) = \$11,822$$

3.21

$$\begin{aligned} P &= \$3{,}000{,}000 + \$2{,}400{,}000(P/F, 8\%, 1) + \cdots \\ &\quad + \$3{,}000{,}000(P/F, 8\%, 10) \\ &= \$20{,}734{,}618 \end{aligned}$$

Or,

$$\begin{aligned} P &= \$3{,}000{,}000 + \$2{,}400{,}000(P/A, 8\%, 5) \\ &\quad + \$3{,}000{,}000(P/A, 8\%, 5)(P/F, 8\%, 5) \\ &= \$20{,}734{,}618 \end{aligned}$$

3.22

$$P = \$8{,}000(P/F, 6\%, 2) + \$6{,}000(P/F, 6\%, 5) + \$4{,}000(P/F, 6\%, 7) = \$14{,}264$$

Equal Payment Series

3.23

(a) With deposits made at the end of each year

$$F = \$2{,}000(F/A, 8\%, 15) = \$54{,}304$$

(b) With deposits made at the beginning of each year

$$F = \$2{,}000(F/A, 8\%, 15)(1.08) = \$58{,}649$$

3.24

$$F = \$10{,}000(F/A, 6\%, 20) = \$367{,}856$$

3.25

(a) $F = \$6{,}000(F/A, 8\%, 5) = \$35{,}200$

(b) $F = \$4{,}000(F/A, 6.25\%, 12) = \$68{,}473$

(c) $F = \$9{,}000(F/A, 9.45\%, 20) = \$484{,}359$

(d) $F = \$3{,}000(F/A, 11.75\%, 12) = \$71{,}308$

3.26

(a) $A = \$32{,}000(A/F, 8\%, 15) = \$1{,}179$

(b) $A = \$55{,}000(A/F, 6\%, 10) = \$4{,}173$

(c) $A = \$35,000(A/F, 7\%, 20) = \$853.8$

(d) $A = \$8,000(A/F, 11\%, 4) = \$1,699$

3.27

$$\$50,000(A/F, 6\%, 10) = \$3,793.40$$

3.28

$$\$35,000 = \$2,000(F/A, 6\%, N)$$
$$(F/A, 6\%, N) = 17.5$$
$$N = 12.32 \approx 13\text{years}$$

3.29

$$\$15,000 = A(F/A, 11\%, 5)$$
$$A = \$2,408.57$$

3.30

$$\$5,000 = \$500(F/P, 7\%, 5) + A(F/A, 7\%, 5)$$
$$A = \$747.51$$

3.31

(a) $A = \$12,000(A/P, 4\%, 6) = \$2,289.14$

(b) $A = \$3,500(A/P, 6.7\%, 7) = \$642.66$

(c) $A = \$6,500(A/P, 3.5\%, 5) = \$1,439.63$

(d) $A = \$32,000(A/P, 8.5\%, 15) = \$3,853.47$

3.32

(a) The capital recovery factor $(A/P, i, N)$ for

N	6%	7%
35	0.0690	0.0772
40	0.0665	0.0750

To find $(A/P, 6.25\%, 38)$, first, interpolate for $N = 38$:

N	6%	7%
38	0.0675	0.0759

Then, interpolate for $i = 6.25\%$;

$$(A/P, 6.25\%, 38) = 0.0696:$$

As compared to the value from the interest formula:

$$(A/P, 6.25\%, 38) = 0.0694$$

(b) The equal payment series present-worth factor $(P/A, i, 85)$ for

i	9%	10%
	11.1038	9.9970

Then, interpolate for $i = 9.25\%$:

$$(P/A, 9.25\%, 85) = 10.8271$$

As compared to the value from the interest formula:

$$(P/A, 9.25\%, 85) = 10.8049$$

3.33

- Equal annual payment:

$$A = \$50,000(A/P, 12\%, 3) = \$20,817.45$$

- Interest payment for the second year:

End of Year	Principal Repayment	Interest payment	Remaining Balance
0			\$50,000
1	\$14,817.45	\$6,000	\$35,182.55
2	\$16,595.54	$\boxed{\$4,221.91}$	\$18,587.01
3	\$18,587.01	\$2,230.44	0

3.34

$$A = \$10,000(A/P, 9\%, 10) = \$1,558.2$$

3.35

(a) $P = \$1,000(P/A, 6.8\%, 8) = \$6,017.86$

(b) $P = \$3,500(P/A, 9.5\%, 12) = \$24,443.44$

(c) $P = \$1,900(P/A, 8.25\%, 9) = \$11,746.68$

(d) $P = \$9,300(P/A, 7.75\%, 5) = \$37,378.16$

3.36

$P = \$35,000(P/A, 12\%, 10) = \$197,758$

Since \$200,000 > \$197,758, You should not purchase the equipment.

3.37

(a)

$$P = \$3,875,000 + \$3,125,000(P/F, 6\%, 1) + ... + \$8,875,000(P/F, 6\%, 7)$$
$$= \$39,547,241.99$$

(b)

$$P = \$1,375,000 + \$1,375,000(P/A, 6\%, 7) = \$9,050,774.48$$

Since \$9,050,774.48 > \$8,000,000, the prorated payment option is better choice.

Linear Gradient Series

3.38

$$F = \$10,000(F/A, 8\%, 5) + \$3,000(F/G, 8\%, 5)$$
$$= \$10,000(F/A, 8\%, 5) + \$3,000(A/G, 8\%, 5)(F/A, 8\%, 5)$$
$$= \$91,163.55$$

3.39

$$F = \$7,500(F/A, 8\%, 5) - \$1,500(F/G, 8\%, 5)$$
$$= \$7,500(F/A, 8\%, 5) - \$1,500(P/G, 8\%, 5)(F/P, 8\%, 5)$$
$$= \$27,750.74$$

3.40

$$P = \$100 + [\$100(F/A, 9\%, 7) + \$50(F/A, 9\%, 6) + \$50(F/A, 9\%, 4)$$
$$+ \$50(F/A, 9\%, 2)](P/F, 9\%, 7)$$
$$= \$991.32$$

3.41

$$A = \$15,000 - \$1,000(A/G, 8\%, 12)$$
$$= \$10,404.25$$

3.42

$$P = \$1,000(P/A,6\%,5) + \$250(P/G,6\%,5)$$
$$= \$6,196$$

3.43

Using the geometric gradient series present worth factor, we can establish the equivalence between the loan amount $120,000 and the balloon payment series as

$$\$120,000 = A_1(P/A_1,10\%,9\%,5) = 4.6721A_1$$
$$A_1 = \$25,684.38$$

Payment series

N	Payment
1	$25,684.38
2	$28,252.82
3	$31,078.10
4	$34,185.91
5	$37,604.51

3.44

$$F = \$6,000(P/A_1,5\%,7\%,30)(F/P,7\%,30)$$
$$= \$987,093.8$$

3.45

(a) $P = \$6,000,000(P/A_1,-10\%,12\%,7) = \$21,372,076$

(b) Note that the oil price increases at the annual rate of 5% while the oil production decreases at the annual rate of 10%. Therefore, the annual revenue can be expressed as follows:

$$A_n = \$60(1+0.05)^{n-1}100,000(1-0.1)^{n-1}$$
$$= \$6,000,000(0.945)^{n-1}$$
$$= \$6,000,000(1-0.055)^{n-1}$$

This revenue series is equivalent to a decreasing geometric gradient series with g = -5.5%. So,

$$P = \$6,000,000(P/A_1,-5.5\%,12\%,7) = \$23,847,897$$

(c) Computing the present worth of the remaining series (A_4, A_5, A_6, A_7) at the end of period 3 gives

$$A_4 = 6{,}000{,}000(1-0.055)^3 = 5{,}063{,}451.75$$
$$P = \$5{,}063{,}451.75(P/A_4, -5.5\%, 12\%, 4) = \$14{,}269{,}627.82$$

3.46

$$P = \sum_{n=1}^{20} A_n (1+i)^{-n}$$
$$= \sum_{n=1}^{20} (2{,}000{,}000) n (1.06)^{n-1} (1.06)^{-n}$$
$$= (2{,}000{,}000/1.06) \sum_{n=1}^{20} n (\frac{1.06}{1.06})^n$$
$$= \$396{,}226{,}415$$

3.47

(a) The withdrawal series would be

Period	Withdrawal
11	\$12,000
12	\$12,000(1.08)
13	\$12,000(1.08)(1.08)
14	\$12,000(1.08)(1.08)(1.08)
15	\$12,000(1.08)(1.08)(1.08)(1.08)

$$P_{10} = \$12{,}000(P/A_1, 8\%, 12\%, 5) = \$49{,}879.14$$

Assuming that each deposit is made at the end of each year, then:

$$\$49{,}879.14 = A(F/A, 12\%, 10)$$
$$A = \$2{,}842.32$$

(b) $P_{10} = \$12{,}000(P/A_1, 8\%, 9\%, 5) = \$54{,}045.08$

$$\$54{,}045.08 = A(F/A, 9\%, 10)$$
$$A = \$3{,}557.25$$

Various Interest Factor Relationships

3.48

(a) $(P/F,8\%,67)=(P/F,8\%,50)(P/F,8\%,17)=0.0058$

$$(P/F,8\%,67)=(1+0.08)^{-67}=0.0058$$

(b) $(A/P,i,N)=\dfrac{i}{1-(P/F,i,N)}$

$$(P/F,8\%,42)=(P/F,8\%,40)(P/F,8\%,2)=0.0394$$

$$(A/P,8\%,42)=\frac{0.08}{1-0.0394}=0.0833$$

$$(A/P,8\%,42)=\frac{0.08(1.08)^{42}}{(1.08)^{42}-1}=0.0833$$

(c) $(P/A,i,N)=\dfrac{1-(P/F,i,N)}{i}=\dfrac{1-(P/F,8\%,100)(P/F,8\%,35)}{0.08}=12.4996$

$$(P/A,8\%,135)=\frac{(1.08)^{135}-1}{0.08(1.08)^{135}}=12.4996$$

3.49

(a)

$$(F/P,i,N)=i(F/A,i,N)+1$$

$$\begin{aligned}(1+i)^N &= i\frac{(1+i)^N-1}{i}+1\\ &=(1+i)^N-1+1\\ &=(1+i)^N\end{aligned}$$

(b)

$$(P/F,i,N)=1-(P/A,i,N)i$$

$$\begin{aligned}(1+i)^{-N} &= 1-i\frac{(1+i)^N-1}{i(1+i)^N}\\ &=\frac{(1+i)^N}{(1+i)^N}-\frac{(1+i)^N-1}{(1+i)^N}\\ &=(1+i)^{-N}\end{aligned}$$

(c)

$$(A/F,i,N) = (A/P,i,N) - i$$

$$\frac{i}{(1+i)^N - 1} = \frac{i(1+i)^N}{(1+i)^N - 1} - i = \frac{i(1+i)^N}{(1+i)^N - 1} - \frac{i[(1+i)^N - 1]}{(1+i)^N - 1}$$

$$= \frac{i}{(1+i)^N - 1}$$

(d)

$$(A/P,i,N) = \frac{i}{[1-(P/F,i,N)]}$$

$$\frac{i(1+i)^N}{(1+i)^N - 1} = \frac{i}{\frac{(1+i)^N}{(1+i)^N} - \frac{1}{(1+i)^N}}$$

$$= \frac{i(1+i)^N}{(1+i)^N - 1}$$

(e) , (f) , (g) Divide the numerator and denominator by $(1+i)^N$ and take the limit $N \to \infty$.

Equivalence Calculations

3.50

$$P = [\$100(F/A,12\%,9) + \$50(F/A,12\%,7) + \$50(F/A,12\%,5)](P/F,12\%,10)$$
$$= \$740.49$$

3.51

$$P(1.08) + \$200 = \$200(P/F,8\%,1) + \$120(P/F,8\%,2) + \$120(P/F,8\%,3)$$
$$+ \$300(P/F,8\%,4)$$
$$P = \$373.92$$

3.52

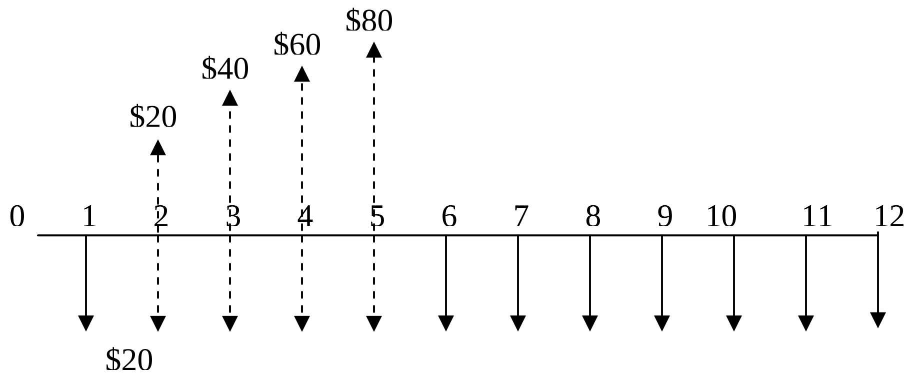
$$A(P/A,13\%,5) = \$100(P/A,13\%,5) + \$20(P/A,13\%,3)(P/F,13\%,2) =$$
$$\$351.72 + \$36.98 = (3.5172)A$$
$$A = \$110.51$$

3.53

$$\begin{aligned} P_1 &= \$200 + \$100(P/A,6\%,5) + \$50(P/F,6\%,1) + \$50(P/F,6\%,4) + \$100(P/F,6\%,5) \\ &= \$782.75 \\ P_2 &= X(P/A,6\%,5) = \$782.75 \\ X &= \$185.82 \end{aligned}$$

3.54

$$\begin{aligned} P &= \$20(P/G,10\%,5) - \$20(P/A,10\%,12) \\ &= \$0.96 \end{aligned}$$



3.55

Establish economic equivalent at $N = 8$:

$$\begin{aligned} C(F/A,8\%,8) - C(F/A,8\%,2)(F/P,8\%,3) &= \$6,000(P/A,8\%,2) \\ 10.6366C - (2.08)(1.2597)C &= \$6,000(1.7833) \\ 8.0164C &= \$10,699.80 \\ C &= \$1,334.73 \end{aligned}$$

3.56

The original cash flow series is

N	A_N	N	A_N
0	0	6	$900
1	$800	7	$920
2	$820	8	$300
3	$840	9	$300
4	$860	10	$300 – $500
5	$880		

3.57

$$\begin{aligned}\$300(F/A,10\%,8)+\$200(F/A,10\%,3) &= 2C(F/P,10\%,8)+C(F/A,10\%,7)\\ \$4,092.77 &= 2C(2.1436)+C(9.4872)\\ C &= \$297.13\end{aligned}$$

3.58

Establishing equivalence at $N = 5$

$$\begin{aligned}&\$200(F/A,8\%,5)-\$50(F/P,8\%,1)\\ &\quad = X(F/A,8\%,5)-(\$200+X)[(F/P,8\%,2)+(F/P,8\%,1)]\\ &\$1,119.32 = X(5.8666)-(\$200+X)(2.2464)\\ &X = \$433.29\end{aligned}$$

3.59

Computing equivalence at $N = 5$

$$X = \$3,000(F/A,9\%,5)+\$3,000(P/A,9\%,5) = \$29,623.08$$

3.60 (b), (d), and (f)

3.61 (b), (d), and (e)

3.62

$$A_1 = (\$50+\$50(A/G,10\%,5)-[\$50+\$50(P/F,10\%,1)](A/P,10\%,5) = \$115.32$$
$$A_2 = A+A(A/P,10\%,5) = 1.2638A$$
$$A = \$91.25$$

3.63(a)

3.64(b)

3.65(b)

$$\begin{aligned}&\$25,000+\$30,000(P/F,10\%,6)\\ &\quad = C(P/A,10\%,12)+\$1,000(P/A,10\%,6)(P/F,10\%,6)\\ &\$41,935 = 6.8137C+\$2,458.43\\ &C = \$5,794\end{aligned}$$

Solving for an Unknown Interest Rate of Unknown Interest Periods

3.66

$$2P = P(1+i)^5$$
$$2^{1/5} = 1+i$$
$$i = 14.87\%$$

3.67

Establishing equivalence at $n = 0$

$$\$2{,}000(P/A,i,6) = \$2{,}500(P/A_1,-25\%,i,6)$$

By Excel software, $i = 92.36\%$

3.68

$$\$35{,}000 = \$10{,}000(F/P,i,5) = \$10{,}000(1+i)^5$$
$$i = 28.47\%$$

3.69

$$\$1{,}000{,}000 = \$2{,}000(F/A,6\%,N)$$
$$500 = \frac{(1+0.06)^N - 1}{0.06}$$
$$31 = (1+0.06)^N$$
$$\log 31 = N \log 1.06$$
$$N = 58.93 \approx 59 \text{years}$$

3.70

Option 1: $\$100{,}000(F/A,7\%,7)(F/P,7\%,13) = 2{,}085{,}484.95$

Option 2: $\$100{,}000(F/A,7\%,13) = 2{,}014{,}064.29$

$$\$100{,}000(F/A,i,7)(F/P,i,13) = \$100{,}000(F/A,i,13)$$
$$i = 6.6\%$$

3.71

Assuming that annual renewal fees are paid at the beginning of each year,

(a)

$$\$15.96 + \$15.96(P/A,6\%,3) = \$58.62$$

It is better to take the offer because of lower cost to renew.

(b)

$$\begin{aligned}\$57.12 &= \$15.96 + \$15.96(P/A, i, 3)\\ i &= 7.96\%\end{aligned}$$

Short Case Studies

ST 3.1

(a)

$$P = 280,000(P/A, 8\%, 19) = 2,689,007.78$$

(b)

$$\begin{aligned}280,000(P/A, i, 19) &= 5,600,000 - 283,770\\ i &= 0.00709\%\end{aligned}$$

ST 3.2

(a)

$$\begin{aligned}P_{\text{Contract}} &= \$5,600,000 + \$7,178,000(P/F, 6\%, 1)\\ &+\$11,778,000(P/F, 6\%, 2) + \dots\\ &+\$17,778,000(P/F, 6\%, 9)\\ &= \$97,102,826.86\end{aligned}$$

(b)

$$\begin{aligned}P_{\text{Bonus}} &= \$5,000,000 + \$5,000,000(P/A, 6\%, 5) + \$778,000(P/A, 6\%, 9)\\ &= \$31,353,535.52 > \$23,000,000\end{aligned}$$

It is better stay with the original plan.

ST 3.3

(a) Compute the equivalent present worth (in 2006) for each option at $i = 6\%$.

$$\begin{aligned}P_{Deferred} &= \$2,000,000 + \$566,000(P/F, 6\%, 1) + \$920,000(P/F, 6\%, 2) + \cdots\\ &+ \$1,260,000(P/F, 6\%, 11) = \$8,574,491\end{aligned}$$

$$P_{Non-Deferred} = \$2,000,000 + \$900,000(P/F,6\%,1) + \$1,000,000(P/F,6\%,2) + \cdots$$
$$+ \$1,975,000(P/F,6\%,5) = \$7,431,562$$

$\therefore$ At $i = 6\%$, the deferred plan is a better choice.

(b) Using either Excel or Cash Flow Analyzer, both plans would be economically equivalent at $i = 15.72\%$.

ST 3.4 Assuming that premiums paid at the end of each year, the maximum amount to invest in the prevention program is

$$P = \$14,000(P/A,12\%,5) = \$50,467.$$

If the premiums paid at the beginning of each year, the solution changes to

$$P = \$14,000 + \$14,000(P/A,12\%,4) = \$56,523.$$

ST 3.5

- Compute the required annual net cash profit to pay off the investment and interest.

$$\$70,000,000 = A(P/A,10\%,5) = 3.7908A$$
$$A = \$18,465,824$$

- Decide the number of shoes, X

$$\$18,465,824 = X(\$100)$$
$$X = 184,658.24$$

ST 3.6

Establish the following equivalence equation:

$$\$140,000 = \$32,639(P/A,i,9).$$

The interest rate makes two options equivalent is $i = 18.10\%$ by Excel.
So, if her rate of return is over 18.10%, it is a good decision.