Contents

Before Calculus

Exercise Set 0.1

1. **(a)** $-2.9, -2.0, 2.35, 2.9$ **(b)** None **(c)** $y = 0$ **(d)** $-1.75 \le x \le 2.15$, $x = -3$, $x = 3$

(e) $y_{\max} = 2.8$ at $x = -2.6$; $y_{\min} = -2.2$ at $x = 1.2$

2. **(a)** $x = -1, 4$ **(b)** None **(c)** $y = -1$ **(d)** $x = 0, 3, 5$

(e) $y_{\max} = 9$ at $x = 6$; $y_{\min} = -2$ at $x = 0$

3. **(a)** Yes **(b)** Yes **(c)** No (vertical line test fails) **(d)** No (vertical line test fails)

4. **(a)** The natural domain of f is $x \neq -1$, and for g it is the set of all x. $f(x) = g(x)$ on the intersection of their domains.

(b) The domain of f is the set of all $x \ge 0$; the domain of g is the same, and $f(x) = g(x)$.

5. **(a)** 1999, \$47,700 **(b)** 1993, \$41,600

(c) The slope between 2000 and 2001 is steeper than the slope between 2001 and 2002, so the median income was declining more rapidly during the first year of the 2-year period.

6. **(a)** In thousands, approximately $\dfrac{47.7 - 41.6}{6} = \dfrac{6.1}{6}$ per yr, or \$1017/yr.

(b) From 1993 to 1996 the median income increased from \$41.6K to \$44K (K for 'kilodollars'; all figures approximate); the average rate of increase during this time was $(44 - 41.6)/3$ K/yr $= 2.4/3$ K/yr $=$ \$800/year. From 1996 to 1999 the average rate of increase was $(47.7 - 44)/3$ K/yr $= 3.7/3$ K/yr $\approx$ \$1233/year. The increase was larger during the last 3 years of the period.

(c) 1994 and 2005.

7. **(a)** $f(0) = 3(0)^2 - 2 = -2$; $f(2) = 3(2)^2 - 2 = 10$; $f(-2) = 3(-2)^2 - 2 = 10$; $f(3) = 3(3)^2 - 2 = 25$; $f(\sqrt{2}) = 3(\sqrt{2})^2 - 2 = 4$; $f(3t) = 3(3t)^2 - 2 = 27t^2 - 2$.

(b) $f(0) = 2(0) = 0$; $f(2) = 2(2) = 4$; $f(-2) = 2(-2) = -4$; $f(3) = 2(3) = 6$; $f(\sqrt{2}) = 2\sqrt{2}$; $f(3t) = 1/(3t)$ for $t > 1$ and $f(3t) = 6t$ for $t \le 1$.

8. **(a)** $g(3) = \dfrac{3+1}{3-1} = 2$; $g(-1) = \dfrac{-1+1}{-1-1} = 0$; $g(\pi) = \dfrac{\pi+1}{\pi-1}$; $g(-1.1) = \dfrac{-1.1+1}{-1.1-1} = \dfrac{-0.1}{-2.1} = \dfrac{1}{21}$; $g(t^2-1) = \dfrac{t^2-1+1}{t^2-1-1} = \dfrac{t^2}{t^2-2}$.

(b) $g(3) = \sqrt{3+1} = 2$; $g(-1) = 3$; $g(\pi) = \sqrt{\pi+1}$; $g(-1.1) = 3$; $g(t^2-1) = 3$ if $t^2 < 2$ and $g(t^2-1) = \sqrt{t^2-1+1} = |t|$ if $t^2 \ge 2$.

9. **(a)** Natural domain: $x \neq 3$. Range: $y \neq 0$. **(b)** Natural domain: $x \neq 0$. Range: $\{1, -1\}$.

(c) Natural domain: $x \leq -\sqrt{3}$ or $x \geq \sqrt{3}$. Range: $y \geq 0$.

(d) $x^2 - 2x + 5 = (x-1)^2 + 4 \geq 4$. So $G(x)$ is defined for all x, and is $\geq \sqrt{4} = 2$. Natural domain: all x. Range: $y \geq 2$.

(e) Natural domain: $\sin x \neq 1$, so $x \neq (2n+\frac{1}{2})\pi$, $n = 0, \pm 1, \pm 2, \ldots$. For such x, $-1 \leq \sin x < 1$, so $0 < 1 - \sin x \leq 2$, and $\frac{1}{1-\sin x} \geq \frac{1}{2}$. Range: $y \geq \frac{1}{2}$.

(f) Division by 0 occurs for $x = 2$. For all other x, $\frac{x^2-4}{x-2} = x + 2$, which is nonnegative for $x \geq -2$. Natural domain: $[-2, 2) \cup (2, +\infty)$. The range of $\sqrt{x+2}$ is $[0, +\infty)$. But we must exclude $x = 2$, for which $\sqrt{x+2} = 2$. Range: $[0, 2) \cup (2, +\infty)$.

10. **(a)** Natural domain: $x \leq 3$. Range: $y \geq 0$. **(b)** Natural domain: $-2 \leq x \leq 2$. Range: $0 \leq y \leq 2$.

(c) Natural domain: $x \geq 0$. Range: $y \geq 3$. **(d)** Natural domain: all x. Range: all y.

(e) Natural domain: all x. Range: $-3 \leq y \leq 3$.

(f) For $\sqrt{x}$ to exist, we must have $x \geq 0$. For $H(x)$ to exist, we must also have $\sin\sqrt{x} \neq 0$, which is equivalent to $\sqrt{x} \neq \pi n$ for $n = 0, 1, 2, \ldots$. Natural domain: $x > 0$, $x \neq (\pi n)^2$ for $n = 1, 2, \ldots$. For such x, $0 < |\sin\sqrt{x}| \leq 1$, so $0 < (\sin\sqrt{x})^2 \leq 1$ and $H(x) \geq 1$. Range: $y \geq 1$.

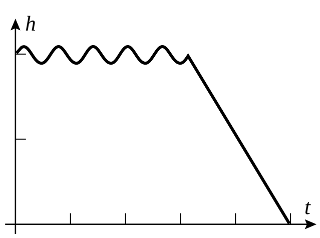
11. **(a)** The curve is broken whenever someone is born or someone dies.

(b) C decreases for eight hours, increases rapidly (but continuously), and then repeats.

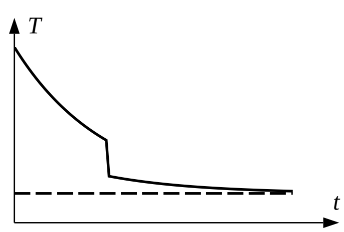
12. **(a)** Yes. The temperature may change quickly under some conditions, but not instantaneously.

(b) No; the number is always an integer, so the changes are in movements (jumps) of at least one unit.

13.



14.



15. Yes. $y = \sqrt{25 - x^2}$.

16. Yes. $y = -\sqrt{25 - x^2}$.

17. Yes. $y = \begin{cases} \sqrt{25 - x^2}, & -5 \leq x \leq 0 \\ -\sqrt{25 - x^2}, & 0 < x \leq 5 \end{cases}$

18. No; the vertical line $x = 0$ meets the graph twice.

19. False. E.g. the graph of $x^2 - 1$ crosses the x-axis at $x = 1$ and $x = -1$.

20. True. This is Definition 0.1.5.

21. False. The range also includes 0.

22. False. The domain of g only includes those x for which $f(x) > 0$.

23. **(a)** $x = 2, 4$ **(b)** None **(c)** $x \leq 2$; $4 \leq x$ **(d)** $y_{\min} = -1$; no maximum value.

24. **(a)** $x = 9$ **(b)** None **(c)** $x \geq 25$ **(d)** $y_{\min} = 1$; no maximum value.

25. The cosine of θ is $(L - h)/L$ (side adjacent over hypotenuse), so $h = L(1 - \cos\theta)$.

26. The sine of $\theta/2$ is $(L/2)/10$ (side opposite over hypotenuse), so $L = 20\sin(\theta/2)$.

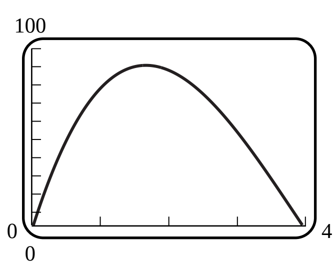
27. **(a)** If $x < 0$, then $|x| = -x$ so $f(x) = -x + 3x + 1 = 2x + 1$. If $x \geq 0$, then $|x| = x$ so $f(x) = x + 3x + 1 = 4x + 1$;
$$f(x) = \begin{cases} 2x + 1, & x < 0 \\ 4x + 1, & x \geq 0 \end{cases}$$

(b) If $x < 0$, then $|x| = -x$ and $|x - 1| = 1 - x$ so $g(x) = -x + (1 - x) = 1 - 2x$. If $0 \leq x < 1$, then $|x| = x$ and $|x - 1| = 1 - x$ so $g(x) = x + (1 - x) = 1$. If $x \geq 1$, then $|x| = x$ and $|x - 1| = x - 1$ so $g(x) = x + (x - 1) = 2x - 1$;
$$g(x) = \begin{cases} 1 - 2x, & x < 0 \\ 1, & 0 \leq x < 1 \\ 2x - 1, & x \geq 1 \end{cases}$$

28. **(a)** If $x < 5/2$, then $|2x - 5| = 5 - 2x$ so $f(x) = 3 + (5 - 2x) = 8 - 2x$. If $x \geq 5/2$, then $|2x - 5| = 2x - 5$ so $f(x) = 3 + (2x - 5) = 2x - 2$;
$$f(x) = \begin{cases} 8 - 2x, & x < 5/2 \\ 2x - 2, & x \geq 5/2 \end{cases}$$

(b) If $x < -1$, then $|x - 2| = 2 - x$ and $|x + 1| = -x - 1$ so $g(x) = 3(2 - x) - (-x - 1) = 7 - 2x$. If $-1 \leq x < 2$, then $|x - 2| = 2 - x$ and $|x + 1| = x + 1$ so $g(x) = 3(2 - x) - (x + 1) = 5 - 4x$. If $x \geq 2$, then $|x - 2| = x - 2$ and $|x + 1| = x + 1$ so $g(x) = 3(x - 2) - (x + 1) = 2x - 7$;
$$g(x) = \begin{cases} 7 - 2x, & x < -1 \\ 5 - 4x, & -1 \leq x < 2 \\ 2x - 7, & x \geq 2 \end{cases}$$

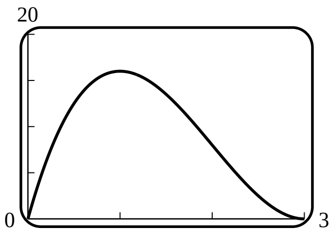
29. **(a)** $V = (8 - 2x)(15 - 2x)x$ **(b)** $0 < x < 4$

(c)


$0 < V \leq 91$, approximately

(d) As x increases, V increases and then decreases; the maximum value occurs when x is about 1.7.

30. **(a)** $V = (6 - 2x)^2 x$ **(b)** $0 < x < 3$

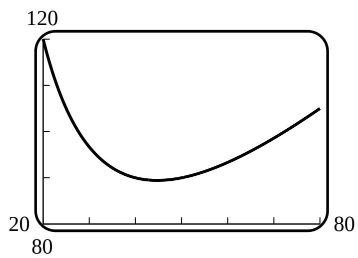


(c) $0 < V \leq 16$, approximately

(d) As x increases, V increases and then decreases; the maximum value occurs when x is about 1.

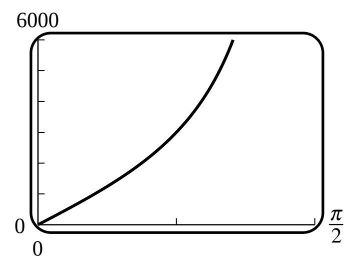
31. (a) The side adjacent to the building has length x, so $L = x + 2y$. **(b)** $A = xy = 1000$, so $L = x + 2000/x$.

(c) $0 < x \leq 100$ **(d)**



$x \approx 44.72$ ft, $y \approx 22.36$ ft

32. (a) $x = 3000 \tan\theta$ **(b)** $0 \leq \theta < \pi/2$ **(c)** 3000 ft

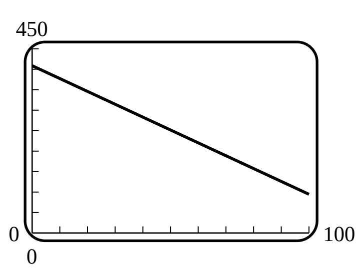


33. (a) $V = 500 = \pi r^2 h$, so $h = \dfrac{500}{\pi r^2}$. Then $C = (0.02)(2)\pi r^2 + (0.01)2\pi r h = 0.04\pi r^2 + 0.02\pi r \dfrac{500}{\pi r^2} = 0.04\pi r^2 + \dfrac{10}{r}$; $C_{\min} \approx 4.39$ cents at $r \approx 3.4$ cm, $h \approx 13.7$ cm.

(b) $C = (0.02)(2)(2r)^2 + (0.01)2\pi r h = 0.16r^2 + \dfrac{10}{r}$. Since $0.04\pi < 0.16$, the top and bottom now get more weight. Since they cost more, we diminish their sizes in the solution, and the cans become taller.

(c) $r \approx 3.1$ cm, $h \approx 16.0$ cm, $C \approx 4.76$ cents.

34. (a) The length of a track with straightaways of length L and semicircles of radius r is $P = (2)L + (2)(\pi r)$ ft. Let $L = 360$ and $r = 80$ to get $P = 720 + 160\pi \approx 1222.65$ ft. Since this is less than 1320 ft (a quarter-mile), a solution is possible.



(b) $P = 2L + 2\pi r = 1320$ and $2r = 2x + 160$, so $L = (1320 - 2\pi r)/2 = (1320 - 2\pi(80 + x))/2 = 660 - 80\pi - \pi x$.

(c) The shortest straightaway is $L = 360$, so we solve the equation $360 = 660 - 80\pi - \pi x$ to obtain $x = \frac{300}{\pi} - 80 \approx 15.49$ ft.

(d) The longest straightaway occurs when $x = 0$, so $L = 660 - 80\pi \approx 408.67$ ft.

35. **(i)** $x = 1, -2$ causes division by zero. **(ii)** $g(x) = x + 1$, all x.

36. **(i)** $x = 0$ causes division by zero. **(ii)** $g(x) = |x| + 1$, all x.

37. **(a)** 25°F **(b)** 13°F **(c)** 5°F

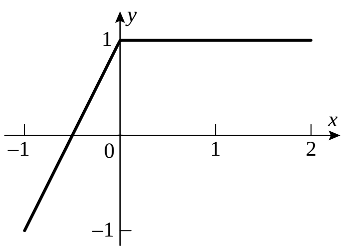
38. If $v = 48$ then $-60 = \text{WCT} \approx 1.4157T - 30.6763$; thus $T \approx -21$°F when WCT $= -60$.

39. If $v = 48$ then $-60 = \text{WCT} \approx 1.4157T - 30.6763$; thus $T \approx 15$°F when WCT $= -10$.

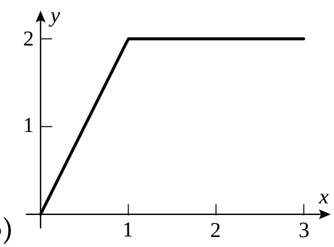
40. The WCT is given by two formulae, but the first doesn't work with the data. Hence $5 = \text{WCT} = -27.2v^{0.16} + 48.17$ and $v \approx 18$mi/h.

Exercise Set 0.2

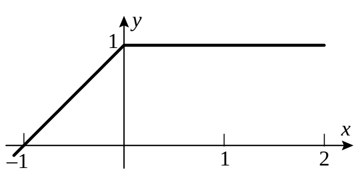
1. **(a)**



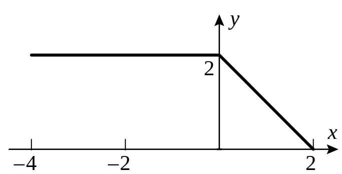
(b)



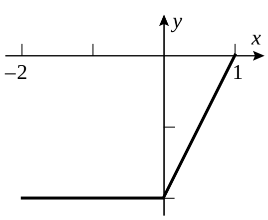
(c)



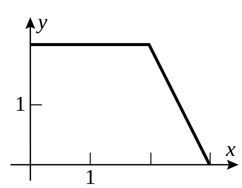
(d)



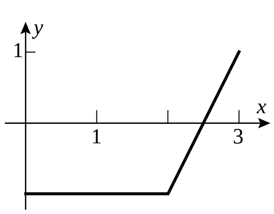
2. **(a)**



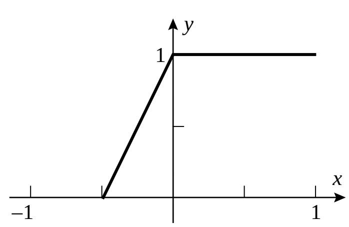
(b)



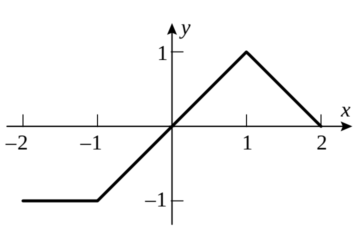
(c)



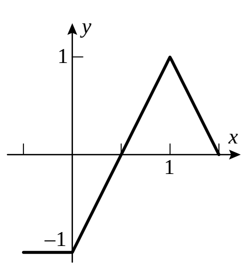
(d)

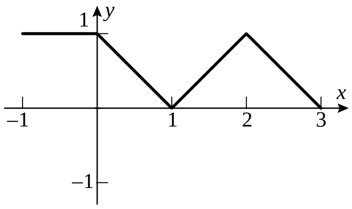


3. **(a)**

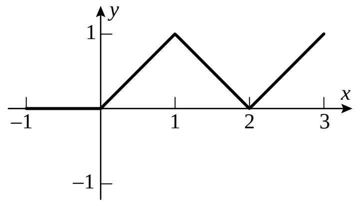


(b)

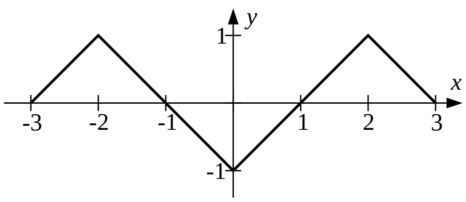




(c)

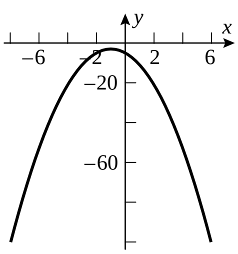


(d)

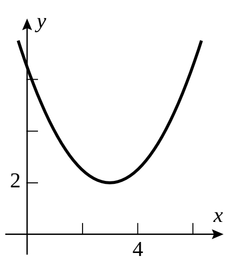


4.

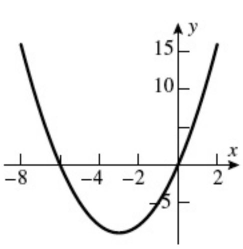
5. Translate left 1 unit, stretch vertically by a factor of 2, reflect over x-axis, translate down 3 units.



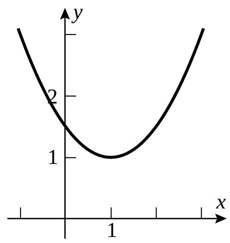
6. Translate right 3 units, compress vertically by a factor of $\frac{1}{2}$, and translate up 2 units.



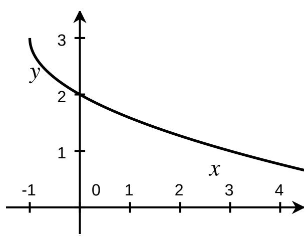
7. $y = (x+3)^2 - 9$; translate left 3 units and down 9 units.



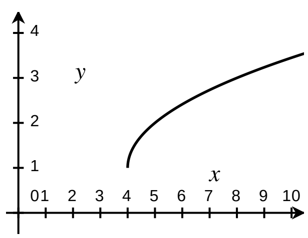
8. $y = \frac{1}{2}[(x-1)^2 + 2]$; translate right 1 unit and up 2 units, compress vertically by a factor of $\frac{1}{2}$



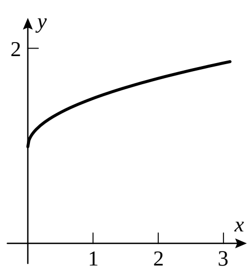
9. Translate left 1 unit, reflect over x-axis, translate up 3 units.



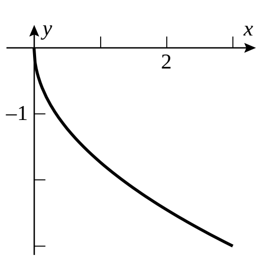
10. Translate right 4 units and up 1 unit.



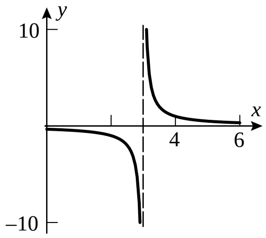
11. Compress vertically by a factor of $\frac{1}{2}$, translate up 1 unit.



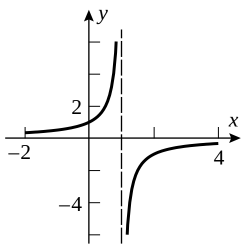
12. Stretch vertically by a factor of $\sqrt{3}$ and reflect over x-axis.



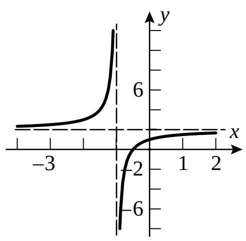
13. Translate right 3 units.



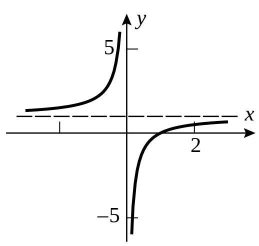
14. Translate right 1 unit and reflect over x-axis.



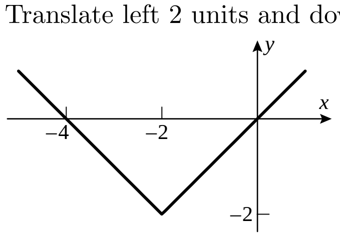
15. Translate left 1 unit, reflect over x-axis, translate up 2 units.



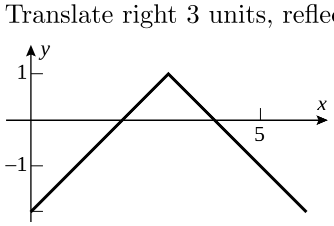
16. $y = 1 - 1/x$; reflect over x-axis, translate up 1 unit.



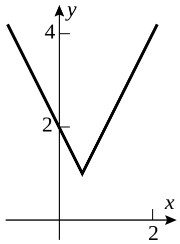
17. Translate left 2 units and down 2 units.



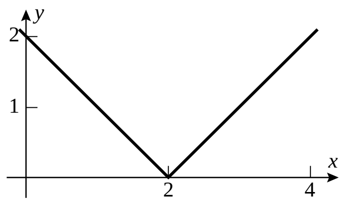
18. Translate right 3 units, reflect over x-axis, translate up 1 unit.



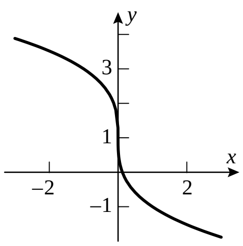
19. Stretch vertically by a factor of 2, translate right 1/2 unit and up 1 unit.



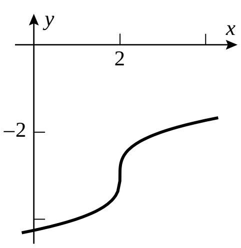
20. $y = |x - 2|$; translate right 2 units.



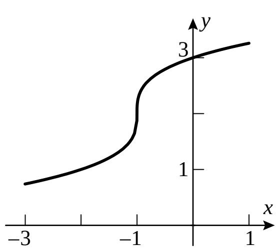
21. Stretch vertically by a factor of 2, reflect over x-axis, translate up 1 unit.



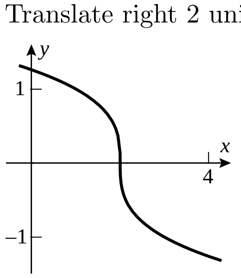
22. Translate right 2 units and down 3 units.

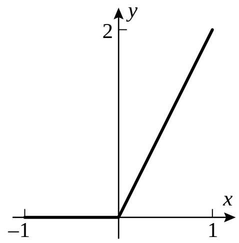


23. Translate left 1 unit and up 2 units.

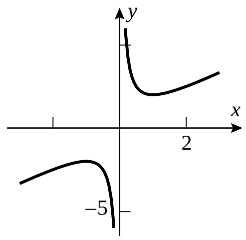


24. Translate right 2 units, reflect over x-axis.



25. (a) 

(b) $y = \begin{cases} 0 \text{ if } x \le 0 \\ 2x \text{ if } 0 < x \end{cases}$

26. 

27. $(f+g)(x) = 3\sqrt{x-1}$, $x \ge 1$; $(f-g)(x) = \sqrt{x-1}$, $x \ge 1$; $(fg)(x) = 2x-2$, $x \ge 1$; $(f/g)(x) = 2, x > 1$

28. $(f+g)(x) = (2x^2+1)/[x(x^2+1)]$, all $x \neq 0$; $(f-g)(x) = -1/[x(x^2+1)]$, all $x \neq 0$; $(fg)(x) = 1/(x^2+1)$, all $x \neq 0$; $(f/g)(x) = x^2/(x^2+1)$, all $x \neq 0$

29. **(a)** 3 **(b)** 9 **(c)** 2 **(d)** 2 **(e)** $\sqrt{2+h}$ **(f)** $(3+h)^3+1$

30. **(a)** $\sqrt{5s+2}$ **(b)** $\sqrt{\sqrt{x}+2}$ **(c)** $3\sqrt{5x}$ **(d)** $1/\sqrt{x}$ **(e)** $\sqrt[4]{x}$ **(f)** $0, x \ge 0$

(g) $1/\sqrt[4]{x}$ **(h)** $|x-1|$ **(i)** $\sqrt{x+h}$

31. $(f \circ g)(x) = 1-x$, $x \le 1$; $(g \circ f)(x) = \sqrt{1-x^2}$, $|x| \le 1$.

32. $(f \circ g)(x) = \sqrt{\sqrt{x^2+3}-3}$, $|x| \ge \sqrt{6}$; $(g \circ f)(x) = \sqrt{x}$, $x \ge 3$.

33. $(f \circ g)(x) = \dfrac{1}{1-2x}$, $x \neq \dfrac{1}{2}, 1$; $(g \circ f)(x) = -\dfrac{1}{2x} - \dfrac{1}{2}$, $x \neq 0, 1$.

34. $(f \circ g)(x) = \dfrac{x}{x^2+1}$, $x \neq 0$; $(g \circ f)(x) = \dfrac{1}{x} + x$, $x \neq 0$.

35. $(f \circ g \circ h)(x) = x^{-6}+1$.

36. $(f \circ g \circ h)(x) = \dfrac{x}{1+x}$.

37. **(a)** $g(x) = \sqrt{x}$, $h(x) = x+2$ **(b)** $g(x) = |x|$, $h(x) = x^2-3x+5$

38. **(a)** $g(x) = x+1$, $h(x) = x^2$ **(b)** $g(x) = 1/x$, $h(x) = x-3$

39. **(a)** $g(x) = x^2$, $h(x) = \sin x$ **(b)** $g(x) = 3/x$, $h(x) = 5+\cos x$

40. **(a)** $g(x) = 3\sin x$, $h(x) = x^2$ **(b)** $g(x) = 3x^2+4x$, $h(x) = \sin x$

41. **(a)** $g(x) = (1+x)^3$, $h(x) = \sin(x^2)$ **(b)** $g(x) = \sqrt{1-x}$, $h(x) = \sqrt[3]{x}$

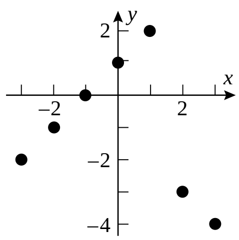
42. **(a)** $g(x) = \dfrac{1}{1-x}$, $h(x) = x^2$ **(b)** $g(x) = |5+x|$, $h(x) = 2x$

43. True, by Definition 0.2.1.

44. False. The domain consists of all x in the domain of g such that $g(x)$ is in the domain of f.

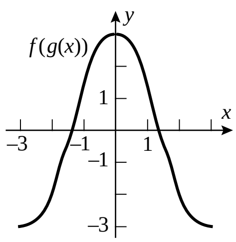
45. True, by Theorem 0.2.3(a).

46. False. The graph of $y = f(x + 2) + 3$ is obtained by translating the graph of $y = f(x)$ left 2 units and up 3 units.

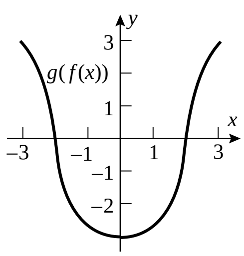
47. 

48. $\{-2, -1, 0, 1, 2, 3\}$

49. Note that $f(g(-x)) = f(-g(x)) = f(g(x))$, so $f(g(x))$ is even.



50. Note that $g(f(-x)) = g(f(x))$, so $g(f(x))$ is even.



51. $f(g(x)) = 0$ when $g(x) = \pm 2$, so $x \approx \pm 1.5$; $g(f(x)) = 0$ when $f(x) = 0$, so $x = \pm 2$.

52. $f(g(x)) = 0$ at $x = -1$ and $g(f(x)) = 0$ at $x = -1$.

53. $\dfrac{3(x+h)^2 - 5 - (3x^2 - 5)}{h} = \dfrac{6xh + 3h^2}{h} = 6x + 3h$; $\dfrac{3w^2 - 5 - (3x^2 - 5)}{w - x} = \dfrac{3(w - x)(w + x)}{w - x} = 3w + 3x$.

54. $\dfrac{(x+h)^2 + 6(x+h) - (x^2 + 6x)}{h} = \dfrac{2xh + h^2 + 6h}{h} = 2x + h + 6$; $\dfrac{w^2 + 6w - (x^2 + 6x)}{w - x} = w + x + 6$.

55. $\dfrac{1/(x+h) - 1/x}{h} = \dfrac{x - (x+h)}{xh(x+h)} = \dfrac{-1}{x(x+h)}$; $\dfrac{1/w - 1/x}{w - x} = \dfrac{x - w}{wx(w - x)} = -\dfrac{1}{xw}$.

56. $\dfrac{1/(x+h)^2 - 1/x^2}{h} = \dfrac{x^2 - (x+h)^2}{x^2h(x+h)^2} = -\dfrac{2x + h}{x^2(x+h)^2}$; $\dfrac{1/w^2 - 1/x^2}{w - x} = \dfrac{x^2 - w^2}{x^2w^2(w - x)} = -\dfrac{x + w}{x^2w^2}$.

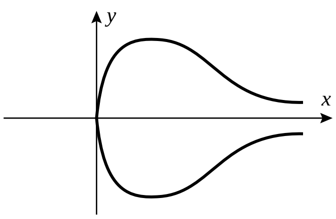
57. Neither; odd; even.

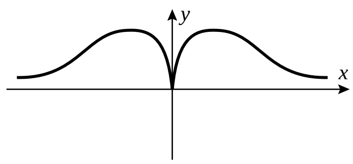
58. **(a)**

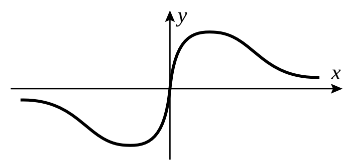
x	-3	-2	-1	0	1	2	3
$f(x)$	1	-5	-1	0	-1	-5	1

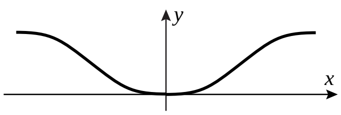
(b)

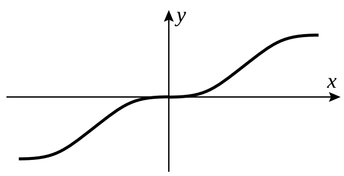
x	-3	-2	-1	0	1	2	3
$f(x)$	1	5	-1	0	1	-5	-1

59. **(a)**


(b)


(c)


60. **(a)**


(b)


61. **(a)** Even. **(b)** Odd.

62. **(a)** Odd. **(b)** Neither.

63. **(a)** $f(-x) = (-x)^2 = x^2 = f(x)$, even. **(b)** $f(-x) = (-x)^3 = -x^3 = -f(x)$, odd.

(c) $f(-x) = |-x| = |x| = f(x)$, even. **(d)** $f(-x) = -x + 1$, neither.

(e) $f(-x) = \dfrac{(-x)^5 - (-x)}{1 + (-x)^2} = -\dfrac{x^5 - x}{1 + x^2} = -f(x)$, odd. **(f)** $f(-x) = 2 = f(x)$, even.

64. **(a)** $g(-x) = \dfrac{f(-x) + f(x)}{2} = \dfrac{f(x) + f(-x)}{2} = g(x)$, so g is even.

(b) $h(-x) = \dfrac{f(-x) - f(x)}{2} = -\dfrac{f(x) - f(-x)}{2} = -h(x)$, so h is odd.

65. In Exercise 64 it was shown that g is an even function, and h is odd. Moreover by inspection $f(x) = g(x) + h(x)$ for all x, so f is the sum of an even function and an odd function.

66. **(a)** x-axis, because $x = 5(-y)^2 + 9$ gives $x = 5y^2 + 9$.

(b) x-axis, y-axis, and origin, because $x^2 - 2(-y)^2 = 3$, $(-x)^2 - 2y^2 = 3$, and $(-x)^2 - 2(-y)^2 = 3$ all give $x^2 - 2y^2 = 3$.

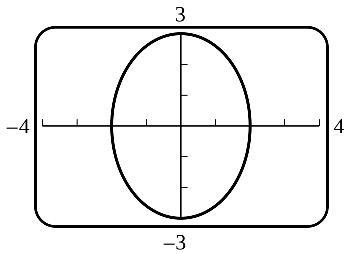
(c) Origin, because $(-x)(-y) = 5$ gives $xy = 5$.

67. **(a)** y-axis, because $(-x)^4 = 2y^3 + y$ gives $x^4 = 2y^3 + y$.

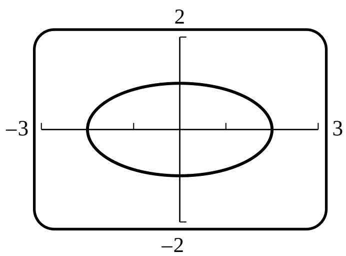
(b) Origin, because $(-y) = \dfrac{(-x)}{3 + (-x)^2}$ gives $y = \dfrac{x}{3 + x^2}$.

(c) x-axis, y-axis, and origin because $(-y)^2 = |x| - 5$, $y^2 = |-x| - 5$, and $(-y)^2 = |-x| - 5$ all give $y^2 = |x| - 5$.

68.



69.

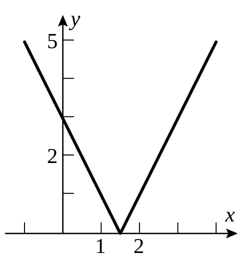


70. (a) Whether we replace x with $-x$, y with $-y$, or both, we obtain the same equation, so by Theorem 0.2.3 the graph is symmetric about the x-axis, the y-axis and the origin.

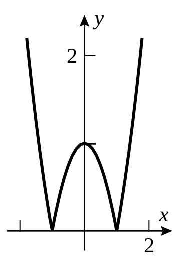
(b) $y = (1 - x^{2/3})^{3/2}$.

(c) For quadrant II, the same; for III and IV use $y = -(1 - x^{2/3})^{3/2}$.

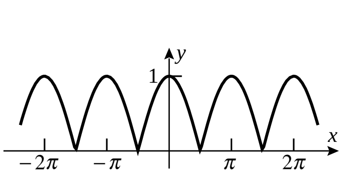
71.



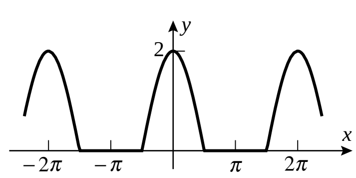
72.



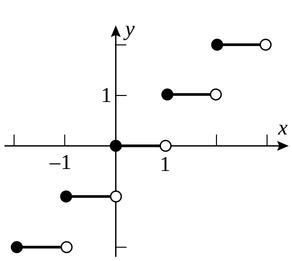
73. (a)



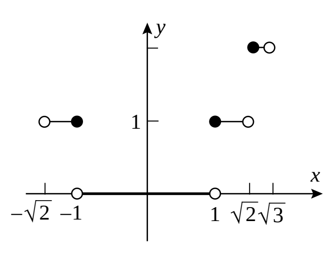
(b)

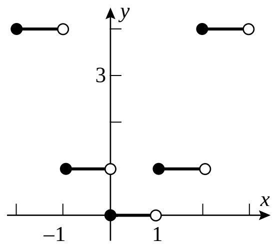


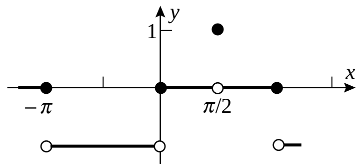
74. (a)



(b)



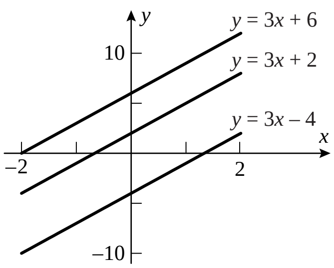
(c) 

(d) 

75. Yes, e.g. $f(x) = x^k$ and $g(x) = x^n$ where k and n are integers.

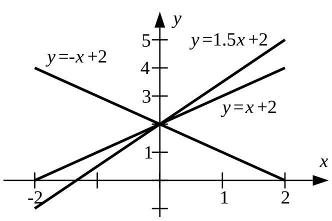
Exercise Set 0.3

1. **(a)** $y = 3x + b$ **(b)** $y = 3x + 6$

(c) 

2. Since the slopes are negative reciprocals, $y = -\frac{1}{3}x + b$.

3. **(a)** $y = mx + 2$ **(b)** $m = \tan\phi = \tan 135° = -1$, so $y = -x + 2$

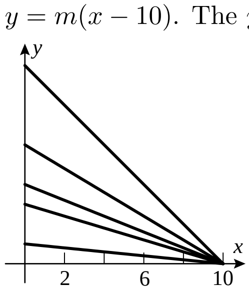
(c) 

4. **(a)** $y = mx$ **(b)** $y = m(x - 1)$ **(c)** $y = -2 + m(x - 1)$ **(d)** $2x + 4y = C$

5. Let the line be tangent to the circle at the point (x_0, y_0) where $x_0^2 + y_0^2 = 9$. The slope of the tangent line is the negative reciprocal of y_0/x_0 (why?), so $m = -x_0/y_0$ and $y = -(x_0/y_0)x + b$. Substituting the point (x_0, y_0) as well as $y_0 = \pm\sqrt{9 - x_0^2}$ we get $y = \pm\dfrac{9 - x_0 x}{\sqrt{9 - x_0^2}}$.

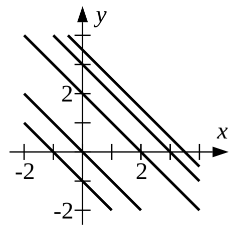
6. Solve the simultaneous equations to get the point $(-2, 1/3)$ of intersection. Then $y = \frac{1}{3} + m(x + 2)$.

7. The x-intercept is $x = 10$ so that with depreciation at 10% per year the final value is always zero, and hence $y = m(x - 10)$. The y-intercept is the original value.

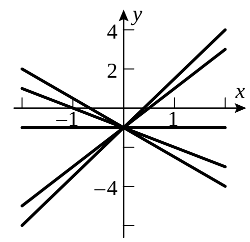


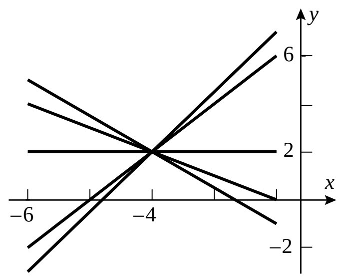
8. A line through $(6,-1)$ has the form $y+1=m(x-6)$. The intercepts are $x=6+1/m$ and $y=-6m-1$. Set $-(6+1/m)(6m+1)=3$, or $36m^2+15m+1=(12m+1)(3m+1)=0$ with roots $m=-1/12,-1/3$; thus $y+1=-(1/3)(x-6)$ and $y+1=-(1/12)(x-6)$.

9. **(a)** The slope is -1.

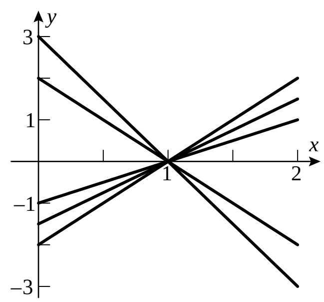


(b) The y-intercept is $y=-1$.



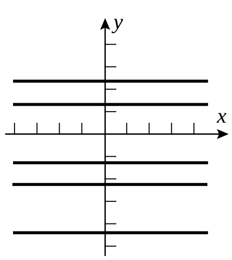


(c) They pass through the point $(-4,2)$.

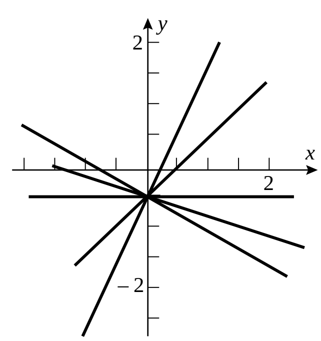


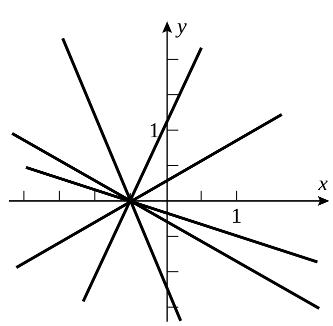
(d) The x-intercept is $x=1$.

10. **(a)** Horizontal lines.

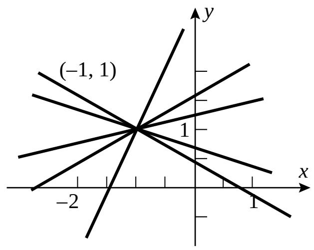


(b) The y-intercept is $y=-1/2$.





(c) The x-intercept is $x=-1/2$.

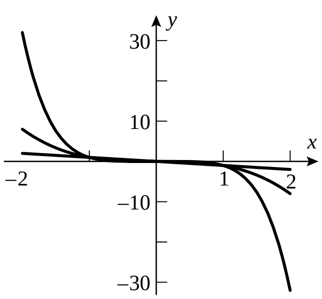


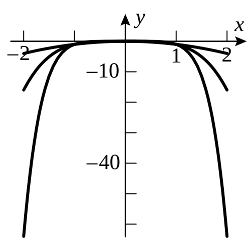
(d) They pass through $(-1, 1)$.

11. **(a)** VI **(b)** IV **(c)** III **(d)** V **(e)** I **(f)** II

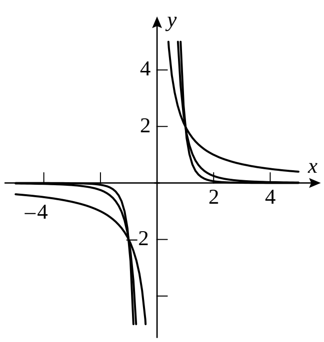
12. In all cases k must be positive, or negative values would appear in the chart. Only kx^{-3} decreases, so that must be $f(x)$. Next, kx^2 grows faster than $kx^{3/2}$, so that would be $g(x)$, which grows faster than $h(x)$ (to see this, consider ratios of successive values of the functions). Finally, experimentation (a spreadsheet is handy) for values of k yields (approximately) $f(x) = 10x^{-3}$, $g(x) = x^2/2$, $h(x) = 2x^{1.5}$.

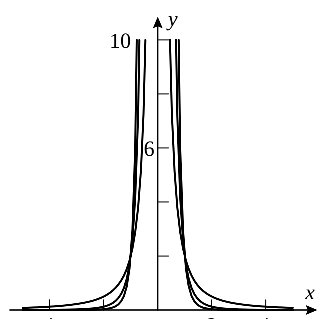
13. **(a)**



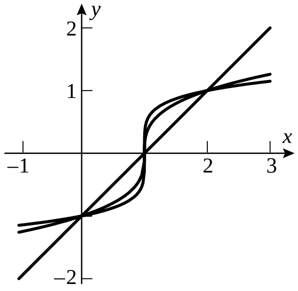


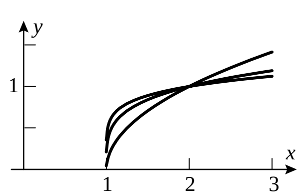
(b)



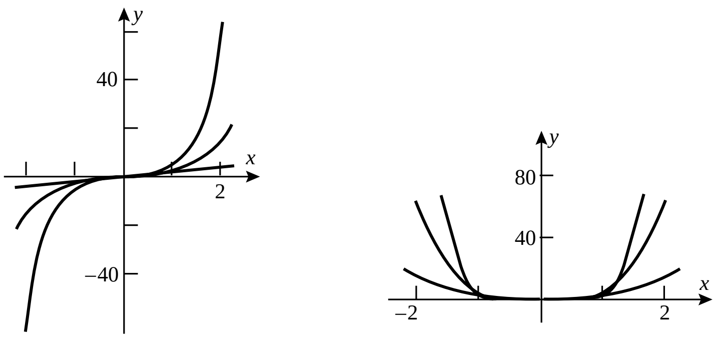


(c)

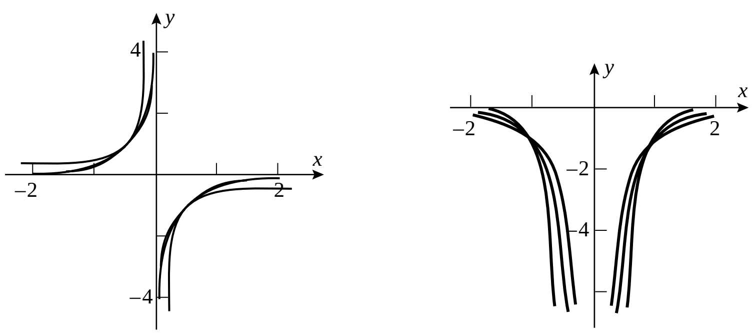




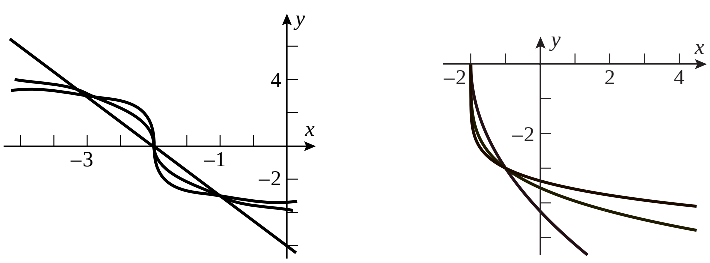
14. (a)



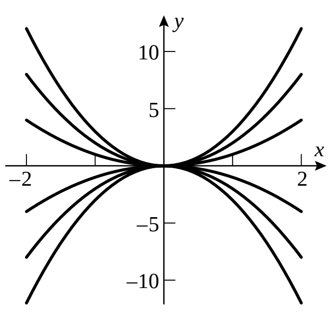
(b)



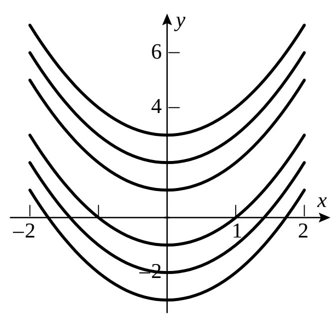
(c)



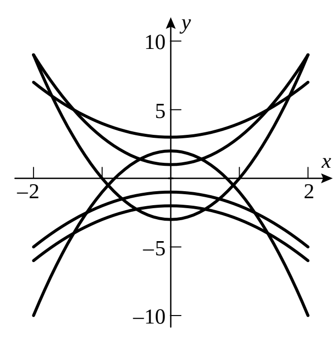
15. (a)



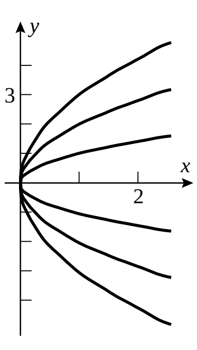
(b)



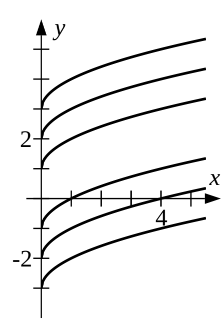
(c)



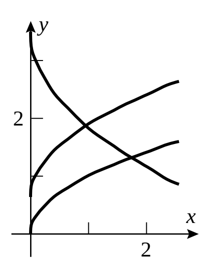
16. (a)



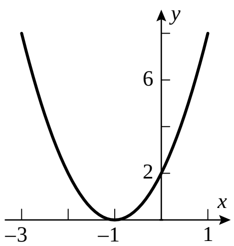
(b)



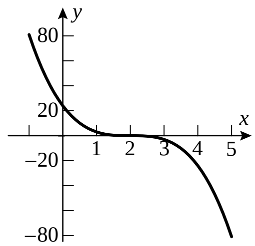
(c)



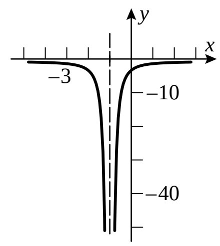
17. **(a)**



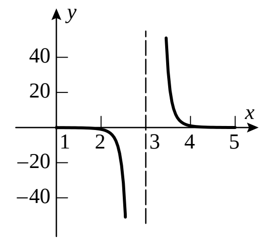
(b)



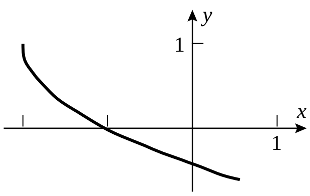
(c)



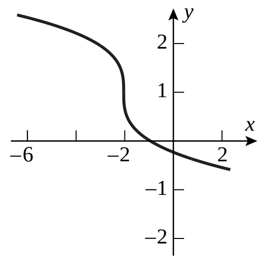
(d)



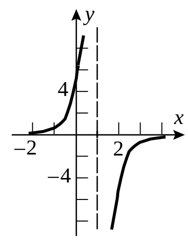
18. **(a)**



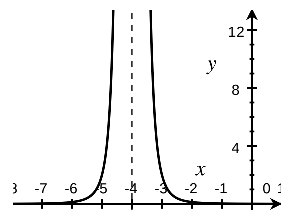
(b)



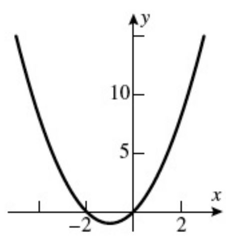
(c)



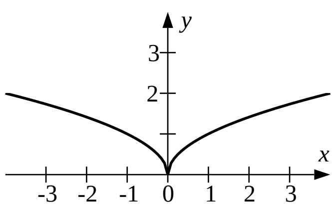
(d)



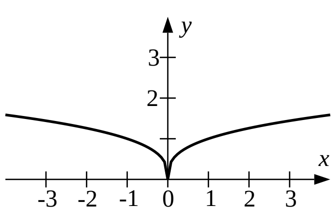
19. $y = x^2 + 2x = (x+1)^2 - 1$.



20. **(a)** The part of the graph of $y = \sqrt{|x|}$ with $x \geq 0$ is the same as the graph of $y = \sqrt{x}$. The part with $x \leq 0$ is the reflection of the graph of $y = \sqrt{x}$ across the y-axis.



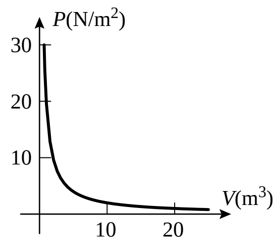
(b) The part of the graph of $y = \sqrt[3]{|x|}$ with $x \geq 0$ is the same as the part of the graph of $y = \sqrt[3]{x}$ with $x \geq 0$. The part with $x \leq 0$ is the reflection of the graph of $y = \sqrt[3]{x}$ with $x \geq 0$ across the y-axis.



21. **(a)** N·m **(b)** k = 20 N·m

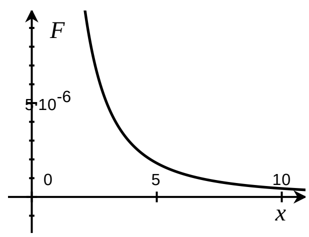
(c)

V(L)	0.25	0.5	1.0	1.5	2.0
P (N/m^2)	80×10^3	40×10^3	20×10^3	13.3×10^3	10×10^3

(d) 

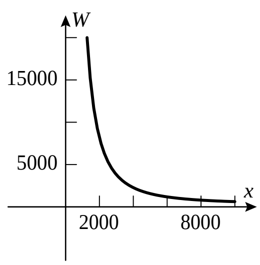
22. If the side of the square base is x and the height of the container is y then $V = x^2y = 100$; minimize $A = 2x^2 + 4xy = 2x^2 + 400/x$. A graphing utility with a zoom feature suggests that the solution is a cube of side $100^{\frac{1}{3}}$ cm.

23. **(a)** $F = k/x^2$ so $0.0005 = k/(0.3)^2$ and $k = 0.000045$ N·m^2. **(b)** $F = 0.000005$ N.

(c) 

(d) When they approach one another, the force increases without bound; when they get far apart it tends to zero.

24. **(a)** $2000 = C/(4000)^2$, so $C = 3.2 \times 10^{10}$ lb·mi^2. **(b)** $W = C/5000^2 = (3.2 \times 10^{10})/(25 \times 10^6) = 1280$ lb.

(c) 

(d) No, but W is very small when x is large.

25. True. The graph of $y = 2x + b$ is obtained by translating the graph of $y = 2x$ up b units (or down $-b$ units if $b < 0$).

26. True. $x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 + \left(c - \frac{b^2}{4}\right)$, so the graph of $y = x^2 + bx + c$ is obtained by translating the graph of $y = x^2$ left $\frac{b}{2}$ units (or right $-\frac{b}{2}$ units if $b < 0$) and up $c - \frac{b^2}{4}$ units (or down $-(c - \frac{b^2}{4})$ units if $c - \frac{b^2}{4} < 0$).

27. False. The curve's equation is $y = 12/x$, so the constant of proportionality is 12.

28. True. As discussed before Example 2, the amplitude is $|-5| = 5$ and the period is $\frac{2\pi}{|A\pi|} = \frac{2}{|A|}$.

29. **(a)** II; $y = 1$, $x = -1, 2$ **(b)** I; $y = 0$, $x = -2, 3$ **(c)** IV; $y = 2$ **(d)** III; $y = 0$, $x = -2$

30. The denominator has roots $x = \pm 1$, so $x^2 - 1$ is the denominator. To determine k use the point $(0, -1)$ to get $k = 1$, $y = 1/(x^2 - 1)$.

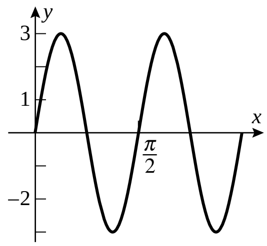
31. **(a)** $y = 3\sin(x/2)$ **(b)** $y = 4\cos 2x$ **(c)** $y = -5\sin 4x$

32. **(a)** $y = 1 + \cos \pi x$ **(b)** $y = 1 + 2\sin x$ **(c)** $y = -5\cos 4x$

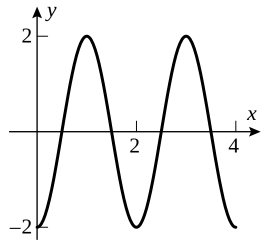
33. **(a)** $y = \sin(x + \pi/2)$ **(b)** $y = 3 + 3\sin(2x/9)$ **(c)** $y = 1 + 2\sin(2x - \pi/2)$

34. $V = 120\sqrt{2}\sin(120\pi t)$.

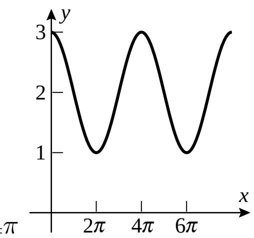
35. **(a)** $3, \pi/2$



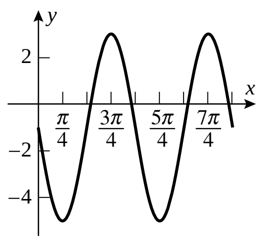
(b) $2, 2$



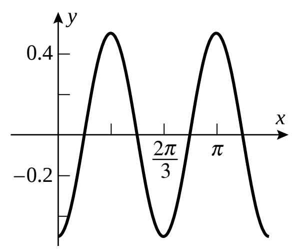
(c) $1, 4\pi$



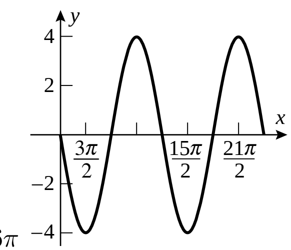
36. **(a)** $4, \pi$



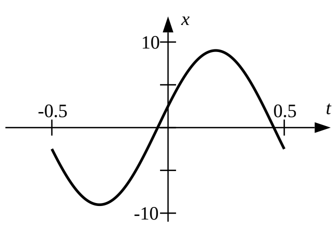
(b) $1/2, 2\pi/3$



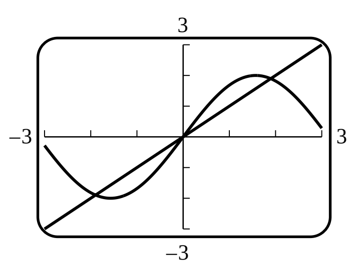
(c) $4, 6\pi$



37. Let $\omega = 2\pi$. Then $A\sin(\omega t + \theta) = A(\cos\theta \sin 2\pi t + \sin\theta\cos 2\pi t) = (A\cos\theta)\sin 2\pi t + (A\sin\theta)\cos 2\pi t$, so for the two equations for x to be equivalent, we need $A\cos\theta = 5\sqrt{3}$ and $A\sin\theta = 5/2$. These imply that $A^2 = (A\cos\theta)^2 + (A\sin\theta)^2 = 325/4$ and $\tan\theta = \dfrac{A\sin\theta}{A\cos\theta} = \dfrac{1}{2\sqrt{3}}$. So let $A = \sqrt{\dfrac{325}{4}} = \dfrac{5\sqrt{13}}{2}$ and $\theta = \tan^{-1}\dfrac{1}{2\sqrt{3}}$. Then (verify) $\cos\theta = \dfrac{2\sqrt{3}}{\sqrt{13}}$ and $\sin\theta = \dfrac{1}{\sqrt{13}}$, so $A\cos\theta = 5\sqrt{3}$ and $A\sin\theta = 5/2$, as required. Hence $x = \dfrac{5\sqrt{13}}{2}\sin\left(2\pi t + \tan^{-1}\dfrac{1}{2\sqrt{3}}\right)$.



38. Three; $x = 0$, $x \approx \pm 1.8955$.



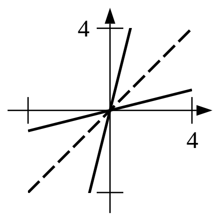
Exercise Set 0.4

1. **(a)** $f(g(x)) = 4(x/4) = x$, $g(f(x)) = (4x)/4 = x$, f and g are inverse functions.

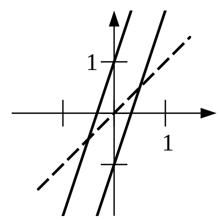
(b) $f(g(x)) = 3(3x - 1) + 1 = 9x - 2 \neq x$ so f and g are not inverse functions.

(c) $f(g(x)) = \sqrt[3]{(x^3 + 2) - 2} = x$, $g(f(x)) = (x - 2) + 2 = x$, f and g are inverse functions.

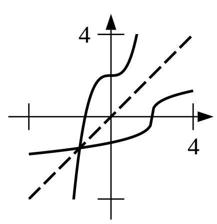
(d) $f(g(x)) = (x^{1/4})^4 = x$, $g(f(x)) = (x^4)^{1/4} = |x| \neq x$, f and g are not inverse functions.



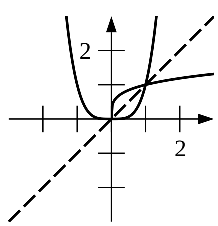
2. **(a)** They are inverse functions.



(b) The graphs are not reflections of each other about the line $y = x$.

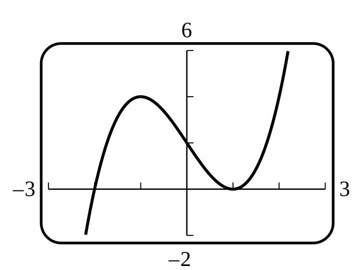


(c) They are inverse functions.

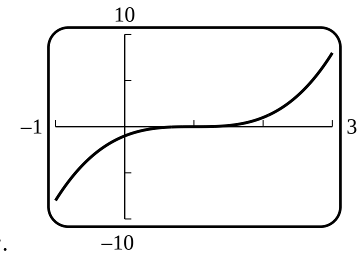


(d) They are not inverse functions.

3. **(a)** yes **(b)** yes **(c)** no **(d)** yes **(e)** no **(f)** no



4. **(a)** The horizontal line test shows the function is not one-to-one.



(b) Yes: $f(x) = (x-1)^3$ so if $f(x) = f(y)$ then $x = \sqrt[3]{f(x)} + 1 = \sqrt[3]{f(y)} + 1 = y$.

5. **(a)** Yes; all outputs (the elements of row two) are distinct.

(b) No; $f(1) = f(6)$.

6. **(a)** Since the point $(0,0)$ lies on the graph, no other point on the line $x = 0$ can lie on the graph, by the vertical line test. Thus the hour hand cannot point straight up or straight down, so noon, midnight, 6AM and 6PM are impossible. To show that other times are possible, suppose the tip of the hour hand stopped at (a,b) with $a \neq 0$. Then the function $y = bx/a$ passes through $(0,0)$ and (a,b).

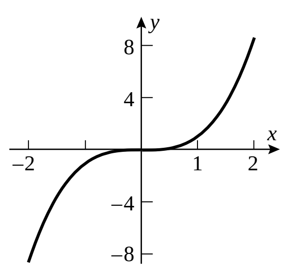
(b) If f is invertible then, since $(0,0)$ lies on the graph, no other point on the line $y = 0$ can lie on the graph, by the horizontal line test. So, in addition to the times mentioned in **(a)**, 3AM, 3PM, 9AM, and 9PM are also impossible.

(c) In the generic case, the minute hand cannot point to 6 or 12, so times of the form 1:00, 1:30, 2:00, 2:30, …, 12:30 are impossible. In case f is invertible, the minute hand cannot point to 3 or 9, so all hours :15 and :45 are also impossible.

7. **(a)** f has an inverse because the graph passes the horizontal line test. To compute $f^{-1}(2)$ start at 2 on the y-axis and go to the curve and then down, so $f^{-1}(2) = 8$; similarly, $f^{-1}(-1) = -1$ and $f^{-1}(0) = 0$.

(b) Domain of f^{-1} is $[-2,2]$, range is $[-8,8]$.

(c)



8. **(a)** The horizontal line test shows this. **(b)** $-3 \leq x \leq -1$; $-1 \leq x \leq 2$; and $2 \leq x \leq 4$.

9. $y = f^{-1}(x)$, $x = f(y) = 7y - 6$, $y = \dfrac{1}{7}(x+6) = f^{-1}(x)$.

10. $y = f^{-1}(x)$, $x = f(y) = \dfrac{y+1}{y-1}$, $xy - x = y + 1$, $(x-1)y = x + 1$, $y = \dfrac{x+1}{x-1} = f^{-1}(x)$.

11. $y = f^{-1}(x)$, $x = f(y) = 3y^3 - 5$, $y = \sqrt[3]{(x+5)/3} = f^{-1}(x)$.

12. $y = f^{-1}(x)$, $x = f(y) = \sqrt[5]{4y+2}$, $y = \dfrac{1}{4}(x^5 - 2) = f^{-1}(x)$.

13. $y = f^{-1}(x)$, $x = f(y) = 3/y^2$, $y = -\sqrt{3/x} = f^{-1}(x)$.

14. $y = f^{-1}(x)$, $x = f(y) = \dfrac{5}{y^2+1}$, $y = \sqrt{\dfrac{5-x}{x}} = f^{-1}(x)$.

15. $y = f^{-1}(x), x = f(y) = \begin{cases} 5/2 - y, & y < 2 \\ 1/y, & y \geq 2 \end{cases}$, $y = f^{-1}(x) = \begin{cases} 5/2 - x, & x > 1/2 \\ 1/x, & 0 < x \leq 1/2 \end{cases}$.

16. $y = f^{-1}(x)$, $x = f(y) = \begin{cases} 2y, & y \leq 0 \\ y^2, & y > 0 \end{cases}$, $y = f^{-1}(x) = \begin{cases} x/2, & x \leq 0 \\ \sqrt{x}, & x > 0 \end{cases}$.

17. $y = f^{-1}(x)$, $x = f(y) = (y+2)^4$ for $y \geq 0$, $y = f^{-1}(x) = x^{1/4} - 2$ for $x \geq 16$.

18. $y = f^{-1}(x)$, $x = f(y) = \sqrt{y+3}$ for $y \geq -3$, $y = f^{-1}(x) = x^2 - 3$ for $x \geq 0$.

19. $y = f^{-1}(x)$, $x = f(y) = -\sqrt{3-2y}$ for $y \leq 3/2$, $y = f^{-1}(x) = (3 - x^2)/2$ for $x \leq 0$.

20. $y = f^{-1}(x)$, $x = f(y) = y - 5y^2$ for $y \geq 1$, $y = f^{-1}(x) = (1 + \sqrt{1 - 20x})/10$ for $x \leq -4$.

21. $y = f^{-1}(x)$, $x = f(y) = ay^2 + by + c$, $ay^2 + by + c - x = 0$, use the quadratic formula to get $y = \dfrac{-b \pm \sqrt{b^2 - 4a(c-x)}}{2a}$;

(a) $f^{-1}(x) = \dfrac{-b + \sqrt{b^2 - 4a(c-x)}}{2a}$ **(b)** $f^{-1}(x) = \dfrac{-b - \sqrt{b^2 - 4a(c-x)}}{2a}$

22. **(a)** $C = \dfrac{5}{9}(F - 32)$.

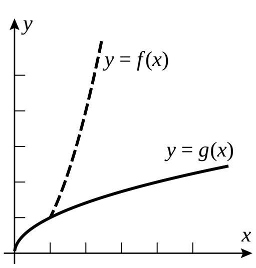
(b) How many degrees Celsius given the Fahrenheit temperature.

(c) $C = -273.15°$ C is equivalent to $F = -459.67°$ F, so the domain is $F \geq -459.67$, the range is $C \geq -273.15$.

23. **(a)** $y = f(x) = \dfrac{10^4}{6.214}x$. **(b)** $x = f^{-1}(y) = (6.214 \times 10^{-4})y$.

(c) How many miles in y meters.

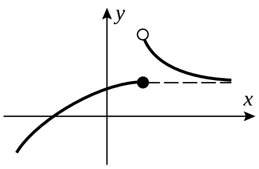
24. **(a)** $f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x, x > 1$; $g(f(x)) = g(x^2) = \sqrt{x^2} = x, x > 1$.

(b)


(c) No, because it is not true that $f(g(x)) = x$ for every x in the domain of g (the domain of g is $x \geq 0$).

25. **(a)** $f(f(x)) = \dfrac{3 - \dfrac{3-x}{1-x}}{1 - \dfrac{3-x}{1-x}} = \dfrac{3 - 3x - 3 + x}{1 - x - 3 + x} = x$ so $f = f^{-1}$.

(b) It is symmetric about the line $y = x$.

26. 

27. If $f^{-1}(x) = 1$, then $x = f(1) = 2(1)^3 + 5(1) + 3 = 10$.

28. If $f^{-1}(x) = 2$, then $x = f(2) = (2)^3/[(2)^2 + 1] = 8/5$.

29. $f(f(x)) = x$ thus $f = f^{-1}$ so the graph is symmetric about $y = x$.

30. **(a)** Suppose $x_1 \neq x_2$ where x_1 and x_2 are in the domain of g and $g(x_1)$, $g(x_2)$ are in the domain of f then $g(x_1) \neq g(x_2)$ because g is one-to-one so $f(g(x_1)) \neq f(g(x_2))$ because f is one-to-one thus $f \circ g$ is one-to-one because $(f \circ g)(x_1) \neq (f \circ g)(x_2)$ if $x_1 \neq x_2$.

(b) f, g, and $f \circ g$ all have inverses because they are all one-to-one. Let $h = (f \circ g)^{-1}$ then $(f \circ g)(h(x)) = f[g(h(x))] = x$, apply f^{-1} to both sides to get $g(h(x)) = f^{-1}(x)$, then apply g^{-1} to get $h(x) = g^{-1}(f^{-1}(x)) = (g^{-1} \circ f^{-1})(x)$, so $h = g^{-1} \circ f^{-1}$.

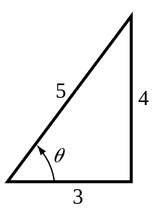
31. False. $f^{-1}(2) = f^{-1}(f(2)) = 2$.

32. False. For example, the inverse of $f(x) = 1 + 1/x$ is $g(x) = 1/(x - 1)$. The domain of f consists of all x except $x = 0$; the domain of g consists of all x except $x = 1$.

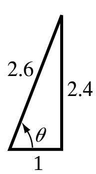
33. True. Both terms have the same definition; see the paragraph before Theorem 0.4.3.

34. False. $\pi/2$ and $-\pi/2$ are not in the range of $\tan^{-1}$.

35. $\tan\theta = 4/3$, $0 < \theta < \pi/2$; use the triangle shown to get $\sin\theta = 4/5$, $\cos\theta = 3/5$, $\cot\theta = 3/4$, $\sec\theta = 5/3$, $\csc\theta = 5/4$.



36. $\sec\theta = 2.6, 0 < \theta < \pi/2$; use the triangle shown to get $\sin\theta = 2.4/2.6 = 12/13, \cos\theta = 1/2.6 = 5/13, \tan\theta = 2.4 = 12/5, \cot\theta = 5/12, \csc\theta = 13/12$.



37. **(a)** $0 \leq x \leq \pi$ **(b)** $-1 \leq x \leq 1$ **(c)** $-\pi/2 < x < \pi/2$ **(d)** $-\infty < x < +\infty$

38. Let $\theta = \sin^{-1}(-3/4)$; then $\sin\theta = -3/4$, $-\pi/2 < \theta < 0$ and (see figure) $\sec\theta = 4/\sqrt{7}$.