

C H A P T E R 2
Differentiation

Section 2.1 The Derivative and the Slope of a Graph **72**

Section 2.2 Some Rules for Differentiation **89**

Section 2.3 Rates of Change: Velocity and Marginals **96**

Section 2.4 The Product and Quotient Rules **102**

Quiz Yourself **113**

Section 2.5 The Chain Rule **115**

Section 2.6 Higher-Order Derivatives **123**

Section 2.7 Implicit Differentiation **129**

Section 2.8 Related Rates **137**

Review Exercises **141**

Test Yourself **154**

CHAPTER 2

Differentiation

Section 2.1 The Derivative and the Slope of a Graph

Skills Warm Up

1. $P(3, 1), Q(3, 6)$

$$m = \frac{6 - 1}{3 - 3}; m \text{ is undefined.}$$

$$x = 3$$

2. $P(2, 2), Q = (-5, 2)$

$$m = \frac{2 - 2}{-5 - 2} = 0$$

$$y - 2 = 0(x - 2)$$

$$y = 2$$

3. $P(1, 5), Q(4, -1)$

$$m = \frac{-1 - 5}{4 - 1} = \frac{-6}{3} = -2$$

$$y - 5 = -2(x - 1)$$

$$y - 5 = -2x + 2$$

$$y = -2x + 7$$

4. $P(3, 5), Q(-1, -7)$

$$m = \frac{-7 - 5}{-1 - 3} = \frac{-12}{-4} = 3$$

$$y - 5 = 3(x - 3)$$

$$y = 3x - 4$$

$$\begin{aligned} 5. \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} 2x + \Delta x \\ &= 2x \end{aligned}$$

$$\begin{aligned} 6. \lim_{\Delta x \rightarrow 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x[3x^2 + 3x\Delta x + (\Delta x)^2]}{\Delta x} \end{aligned}$$

$$\begin{aligned} &= \lim_{\Delta x \rightarrow 0} 3x^2 + 3x\Delta x + (\Delta x)^2 \\ &= 3x^2 \end{aligned}$$

$$7. \lim_{\Delta x \rightarrow 0} \frac{1}{x(x + \Delta x)} = \frac{1}{x^2}$$

$$\begin{aligned} 8. \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x} \end{aligned}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$

$$\begin{aligned} &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} \\ &= 2x \end{aligned}$$

9. $f(x) = 3x$

$$\text{Domain: } (-\infty, \infty)$$

10. $f(x) = \frac{1}{x - 1}$

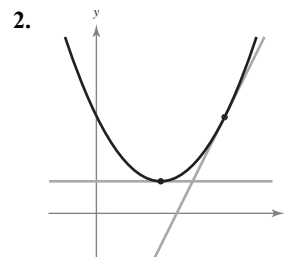
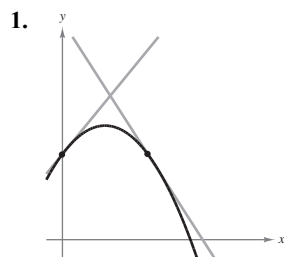
$$\text{Domain: } (-\infty, 1) \cup (1, \infty)$$

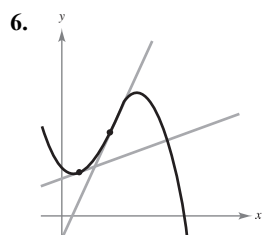
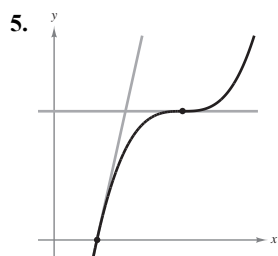
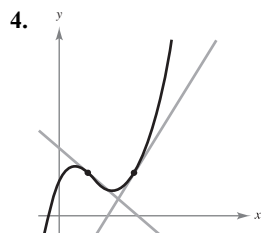
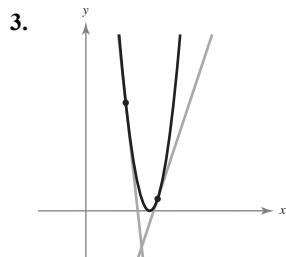
11. $f(x) = \frac{1}{5}x^3 - 2x^2 + \frac{1}{3}x - 1$

$$\text{Domain: } (-\infty, \infty)$$

12. $f(x) = \frac{6x}{x^3 + x}$

$$\text{Domain: } (-\infty, 0) \cup (0, \infty)$$





7. The slope is $m = 1$.

8. The slope is $m = \frac{4}{3}$.

9. The slope is $m = 0$.

10. The slope is $m = \frac{1}{4}$.

11. The slope is $m = -\frac{1}{3}$.

12. The slope is $m = -3$.

13. 2009: $m \approx 118$

2011: $m \approx 375$

The slope is the rate of change in millions of dollars per year of revenue for the years 2009 and 2011 for Under Armour.

14. 2010: $m \approx 500$

2012: $m \approx 500$

The slope is the rate of change in millions of dollars per year of sales for the years 2010 and 2012 for Fossil.

15. $t = 3$: $m \approx 8$

$t = 7$: $m \approx 1$

$t = 10$: $m \approx -10$

The slope is the rate of change of the average temperature in degrees Fahrenheit per month in Bland, Virginia, for March, July, and October.

16. (a) At t_1 , $f'(t_1) > g'(t_1)$, so the runner given by f is running faster.

(b) At t_2 , $g'(t_2) > f'(t_2)$, so the runner given by g is running faster. The runner given by f has traveled farther.

(c) At t_3 , the runners are at the same location, but the runner given by g is running faster.

(d) The runner given by g will finish first because that runner finishes the distance at a lesser value of t .

17. $f(x) = -1$ at $(0, -1)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(0 + \Delta x) - f(0)}{\Delta x} \\ &= \frac{-1 - (-1)}{\Delta x} \\ &= \frac{0}{\Delta x} \\ &= 0 \\ m &= \lim_{\Delta x \rightarrow 0} m_{\text{sec}} = \lim_{\Delta x \rightarrow 0} 0 = 0 \end{aligned}$$

18. $f(x) = 6$ at $(-2, 6)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(-2 + \Delta x) - f(-2)}{\Delta x} \\ &= \frac{6 - 6}{\Delta x} \\ &= \frac{0}{\Delta x} \\ &= 0 \\ m &= \lim_{\Delta x \rightarrow 0} m_{\text{sec}} = \lim_{\Delta x \rightarrow 0} 0 = 0 \end{aligned}$$

19. $f(x) = 13 - 4x$ at $(3, 1)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(3 + \Delta x) - f(3)}{\Delta x} \\ &= \frac{[13 - 4(3 + \Delta x)] - 1}{\Delta x} \\ &= \frac{13 - 12 - 4\Delta x - 1}{\Delta x} \\ &= \frac{-4\Delta x}{\Delta x} \\ &= -4 \\ m &= \lim_{\Delta x \rightarrow 0} m_{\text{sec}} = \lim_{\Delta x \rightarrow 0} (-4) = -4 \end{aligned}$$

20. $f(x) = 6x + 3$ at $(1, 9)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(1 + \Delta x) - f(1)}{\Delta x} \\ &= \frac{[6(1 + \Delta x) + 3] - 9}{\Delta x} \\ &= \frac{6 + 6\Delta x + 3 - 9}{\Delta x} \\ &= \frac{6\Delta x}{\Delta x} \\ &= 6 \\ m &= \lim_{\Delta x \rightarrow 0} m_{\text{sec}} = \lim_{\Delta x \rightarrow 0} 6 = 6 \end{aligned}$$

21. $f(x) = 2x^2 - 3$ at $(2, 5)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(2 + \Delta x) - f(2)}{\Delta x} \\ &= \frac{[2(2 + \Delta x)^2 - 3] - 5}{\Delta x} \\ &= \frac{[2(4 + 4\Delta x + (\Delta x)^2) - 3] - 5}{\Delta x} \\ &= \frac{[8 + 8\Delta x + 2(\Delta x)^2 - 3] - 5}{\Delta x} \\ &= \frac{2\Delta x(4 + \Delta x)}{\Delta x} \\ &= 2(4 + \Delta x) \\ m &= \lim_{\Delta x \rightarrow 0} m_{\text{sec}} = \lim_{\Delta x \rightarrow 0} (2(4 + \Delta x)) = 8 \end{aligned}$$

22. $f(x) = 11 - x^2$ at $(3, 2)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(3 + \Delta x) - f(3)}{\Delta x} \\ &= \frac{11 - (3 + \Delta x)^2 - [11 - (3)^2]}{\Delta x} \\ &= \frac{11 - (9 + 6\Delta x + \Delta x^2) - 2}{\Delta x} \\ &= \frac{11 - 9 - 6\Delta x + (\Delta x)^2 - 2}{\Delta x} \\ &= \frac{-6\Delta x - (\Delta x)^2}{\Delta x} \\ &= \frac{\Delta x(-6 - \Delta x)}{\Delta x} \\ &= -6 - \Delta x \\ m &= \lim_{\Delta x \rightarrow 0} m_{\text{sec}} = \lim_{\Delta x \rightarrow 0} (-6 - \Delta x) = -6 \end{aligned}$$

23. $f(x) = x^3 - 4x$ at $(-1, 3)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(-1 + \Delta x) - f(-1)}{\Delta x} \\ &= \frac{(-1 + \Delta x)^3 - 4(-1 + \Delta x) - [(-1)^3 - 4(-1)]}{\Delta x} \\ &= \frac{-1 + 3\Delta x - 3(\Delta x)^2 + (\Delta x)^3 + 4 - 4\Delta x - 3}{\Delta x} \\ &= \frac{-\Delta x - 3(\Delta x)^2 + (\Delta x)^3}{\Delta x} \\ &= \frac{\Delta x(-1 - 3\Delta x + (\Delta x)^2)}{\Delta x} \\ &= -1 - 3\Delta x + (\Delta x)^2 \\ m &= \lim_{\Delta x \rightarrow 0} m_{\text{sec}} = \lim_{\Delta x \rightarrow 0} (-1 - 3\Delta x + (\Delta x)^2) = -1 \end{aligned}$$

24. $f(x) = 7x - x^3$ at $(-3, 6)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(-3 + \Delta x) - f(-3)}{\Delta x} \\ &= \frac{7(-3 + \Delta x) - (-3 + \Delta x)^3 - [7(-3) - (-3)^3]}{\Delta x} \\ &= \frac{-21 + 7\Delta x - (-27 + 27\Delta x - 9(\Delta x)^2 + (\Delta x)^3) - 6}{\Delta x} \\ &= \frac{-20\Delta x + 9(\Delta x)^2 - (\Delta x)^3}{\Delta x} \\ &= \frac{\Delta x(-20 + 9\Delta x - (\Delta x)^2)}{\Delta x} \\ &= -20 + 9\Delta x - (\Delta x)^2 \\ m &= \lim_{\Delta x \rightarrow 0} m_{\text{sec}} = \lim_{\Delta x \rightarrow 0} (-20 + 9\Delta x - (\Delta x)^2) = -20 \end{aligned}$$

25. $f(x) = 2\sqrt{x}$ at $(4, 4)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(4 + \Delta x) - f(4)}{\Delta x} \\ &= \frac{2\sqrt{4 + \Delta x} - 2\sqrt{4}}{\Delta x} \\ &= \frac{2\sqrt{4 + \Delta x} - 4}{\Delta x} \cdot \frac{2\sqrt{4 + \Delta x} + 4}{2\sqrt{4 + \Delta x} + 4} \\ &= \frac{4(4 + \Delta x) - 16}{\Delta x(2\sqrt{4 + \Delta x} + 4)} \\ &= \frac{16 + 4\Delta x - 16}{\Delta x(2\sqrt{4 + \Delta x} + 4)} \\ &= \frac{4\Delta x}{\Delta x(2\sqrt{4 + \Delta x} + 4)} \\ &= \frac{4}{2\sqrt{4 + \Delta x} + 4} \end{aligned}$$

$$\begin{aligned} m &= \lim_{\Delta x \rightarrow 0} m_{\text{sec}} = \lim_{\Delta x \rightarrow 0} \frac{4}{2\sqrt{4 + \Delta x} + 4} \\ &= \frac{4}{2\sqrt{4} + 4} = \frac{1}{2} \end{aligned}$$

26. $f(x) = \sqrt{x+1}$ at $(8, 3)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(8 + \Delta x) - f(8)}{\Delta x} \\ &= \frac{\sqrt{8 + \Delta x + 1} - \sqrt{8 + 1}}{\Delta x} \\ &= \frac{\sqrt{9 + \Delta x} - 3}{\Delta x} \\ &= \frac{\sqrt{9 + \Delta x} - 3}{\Delta x} \cdot \frac{\sqrt{9 + \Delta x} + 3}{\sqrt{9 + \Delta x} + 3} \\ &= \frac{9 + \Delta x - 9}{\Delta x(\sqrt{9 + \Delta x} + 3)} \\ &= \frac{\Delta x}{\Delta x(\sqrt{9 + \Delta x} + 3)} \\ &= \frac{1}{\sqrt{9 + \Delta x} + 3} \end{aligned}$$

$$\begin{aligned} m &= \lim_{\Delta x \rightarrow 0} m_{\text{sec}} \\ &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{9 + \Delta x} + 3} \\ &= \frac{1}{\sqrt{9} + 3} \\ &= \frac{1}{6} \end{aligned}$$

27. $f(x) = 3$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{3 - 3}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{0}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} 0 \\ &= 0 \end{aligned}$$

28. $f(x) = -2$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-2 - (-2)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{0}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} 0 \\ &= 0 \end{aligned}$$

29. $f(x) = -5x$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-5(x + \Delta x) - (-5x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-5x - 5\Delta x + 5x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-5\Delta x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} -5 \\ &= -5 \end{aligned}$$

30. $f(x) = 4x + 1$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{4(x + \Delta x) + 1 - (4x + 1)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{4x + 4\Delta x + 1 - 4x - 1}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{4\Delta x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} 4 \\ &= 4 \end{aligned}$$

$$31. g(s) = \frac{1}{3}s + 2$$

$$\begin{aligned} g'(s) &= \lim_{\Delta s \rightarrow 0} \frac{g(s + \Delta s) - g(s)}{\Delta s} \\ &= \lim_{\Delta s \rightarrow 0} \frac{\frac{1}{3}(s + \Delta s) + 2 - \left(\frac{1}{3}s + 2\right)}{\Delta s} \\ &= \lim_{\Delta s \rightarrow 0} \frac{\frac{1}{3}s + \frac{1}{3}\Delta s + 2 - \frac{1}{3}s - 2}{\Delta s} \\ &= \lim_{\Delta s \rightarrow 0} \frac{\frac{1}{3}\Delta s}{\Delta s} \\ &= \lim_{\Delta s \rightarrow 0} \frac{1}{3} \\ &= \frac{1}{3} \end{aligned}$$

$$32. h(t) = 6 - \frac{1}{2}t$$

$$\begin{aligned} h'(t) &= \lim_{\Delta t \rightarrow 0} \frac{h(t + \Delta t) - h(t)}{\Delta t} \\ &= \lim_{\Delta t \rightarrow 0} \frac{6 - \frac{1}{2}(t + \Delta t) - \left(6 - \frac{1}{2}t\right)}{\Delta t} \\ &= \lim_{\Delta t \rightarrow 0} \frac{6 - \frac{1}{2}t - \frac{1}{2}\Delta t - 6 + \frac{1}{2}t}{\Delta t} \\ &= \lim_{\Delta t \rightarrow 0} \frac{-\frac{1}{2}\Delta t}{\Delta t} \\ &= \lim_{\Delta t \rightarrow 0} -\frac{1}{2} \\ &= -\frac{1}{2} \end{aligned}$$

$$33. f(x) = 4x^2 - 5x$$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{[4(x + \Delta x)^2 - 5(x + \Delta x)] - (4x^2 - 5x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{[4(x^2 + 2x\Delta x + (\Delta x)^2) - 5x - 5\Delta x] - 4x^2 + 5x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{4x^2 + 8x\Delta x + 4(\Delta x)^2 - 5x - 5\Delta x - 4x^2 + 5x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{4\Delta x\left(2x + \Delta x - \frac{5}{4}\right)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} 4\left(2x + \Delta x - \frac{5}{4}\right) = 8x - 5 \end{aligned}$$

$$34. f(x) = 2x^2 + 7x$$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{[2(x + \Delta x)^2 + 7(x + \Delta x)] - (2x^2 + 7x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{[2(x^2 + 2x\Delta x + (\Delta x)^2) + 7x + 7\Delta x] - 2x^2 - 7x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{2x^2 + 4x\Delta x + 2(\Delta x)^2 + 7x + 7\Delta x - 2x^2 - 7x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{2\Delta x\left(2x + \Delta x + \frac{7}{2}\right)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} 2\left(2x + \Delta x + \frac{7}{2}\right) = 4x + 7 \end{aligned}$$

$$35. h(t) = \sqrt{t-3}$$

$$\begin{aligned} h'(t) &= \lim_{\Delta t \rightarrow 0} \frac{h(t + \Delta t) - h(t)}{\Delta t} \\ &= \lim_{\Delta t \rightarrow 0} \frac{\sqrt{t + \Delta t - 3} - \sqrt{t - 3}}{\Delta t} \\ &= \lim_{\Delta t \rightarrow 0} \frac{\sqrt{t + \Delta t - 3} - \sqrt{t - 3}}{\Delta t} \cdot \frac{\sqrt{t + \Delta t - 3} + \sqrt{t - 3}}{\sqrt{t + \Delta t - 3} + \sqrt{t - 3}} \\ &= \lim_{\Delta t \rightarrow 0} \frac{t + \Delta t - 3 - (t - 3)}{\Delta t(\sqrt{t + \Delta t - 3} + \sqrt{t - 3})} \\ &= \lim_{\Delta t \rightarrow 0} \frac{\Delta t}{\Delta t(\sqrt{t + \Delta t - 3} + \sqrt{t - 3})} \\ &= \lim_{\Delta t \rightarrow 0} \frac{1}{\sqrt{t + \Delta t - 3} + \sqrt{t - 3}} \\ &= \frac{1}{2\sqrt{t - 3}} \\ &= \frac{\sqrt{t - 3}}{2(t - 3)} \end{aligned}$$

$$36. f(x) = \sqrt{x+2}$$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x + 2} - \sqrt{x + 2}}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x + 2} - \sqrt{x + 2}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x + 2} + \sqrt{x + 2}}{\sqrt{x + \Delta x + 2} + \sqrt{x + 2}} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x + \Delta x + 2 - (x + 2)}{\Delta x(\sqrt{x + \Delta x + 2} + \sqrt{x + 2})} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x}{\Delta x(\sqrt{x + \Delta x + 2} + \sqrt{x + 2})} \\ &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x + 2} + \sqrt{x + 2}} \\ &= \frac{1}{2\sqrt{x + 2}} \end{aligned}$$

37. $f(t) = t^3 - 12t$

$$\begin{aligned}
 f'(t) &= \lim_{\Delta t \rightarrow 0} \frac{f(t + \Delta t) - f(t)}{\Delta t} \\
 &= \lim_{\Delta t \rightarrow 0} \frac{(t + \Delta t)^3 - 12(t + \Delta t) - (t^3 - 12t)}{\Delta t} \\
 &= \lim_{\Delta t \rightarrow 0} \frac{t^3 + 3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 - 12t - 12\Delta t - t^3 + 12t}{\Delta t} \\
 &= \lim_{\Delta t \rightarrow 0} \frac{3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 - 12\Delta t}{\Delta t} \\
 &= \lim_{\Delta t \rightarrow 0} \frac{\Delta t(3t^2 + 3t\Delta t + (\Delta t)^2 - 12)}{\Delta t} \\
 &= \lim_{\Delta t \rightarrow 0} (3t^2 + 3t\Delta t + (\Delta t)^2 - 12) \\
 &= 3t^2 - 12
 \end{aligned}$$

38. $f(t) = t^3 + t^2$

$$\begin{aligned}
 f'(t) &= \lim_{\Delta t \rightarrow 0} \frac{f(t + \Delta t) - f(t)}{\Delta t} \\
 &= \lim_{\Delta t \rightarrow 0} \frac{(t + \Delta t)^3 + (t + \Delta t)^2 - (t^3 + t^2)}{\Delta t} \\
 &= \lim_{\Delta t \rightarrow 0} \frac{t^3 + 3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 + t^2 + 2t\Delta t + (\Delta t)^2 - t^3 - t^2}{\Delta t} \\
 &= \lim_{\Delta t \rightarrow 0} \frac{3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 + 2t\Delta t + (\Delta t)^2}{\Delta t} \\
 &= \lim_{\Delta t \rightarrow 0} \frac{\Delta t(3t^2 + 3t\Delta t + (\Delta t)^2 + 2t + \Delta t)}{\Delta t} \\
 &= \lim_{\Delta t \rightarrow 0} (3t^2 + 3t\Delta t + (\Delta t)^2 + 2t + \Delta t) \\
 &= 3t^2 + 2t
 \end{aligned}$$

39. $f(x) = \frac{1}{x+2}$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{x + \Delta x + 2} - \frac{1}{x + 2}}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{x + \Delta x + 2} \cdot \frac{x + 2}{x + 2} - \frac{1}{x + 2} \cdot \frac{x + \Delta x + 2}{x + \Delta x + 2}}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\frac{x + 2 - (x + \Delta x + 2)}{(x + \Delta x + 2)(x + 2)}}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{-\Delta x}{\Delta x(x + \Delta x + 2)(x + 2)} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{-1}{(x + \Delta x + 2)(x + 2)} \\
 &= -\frac{1}{(x + 2)^2}
 \end{aligned}$$

$$40. g(s) = \frac{1}{s-4}$$

$$\begin{aligned} g'(s) &= \lim_{\Delta s \rightarrow 0} \frac{g(s + \Delta s) - g(s)}{\Delta s} \\ &= \lim_{\Delta s \rightarrow 0} \frac{\frac{1}{s + \Delta s - 4} - \frac{1}{s - 4}}{\Delta s} \\ &= \lim_{\Delta s \rightarrow 0} \frac{s - 4 - (s + \Delta s - 4)}{(s + \Delta s - 4)(s - 4)} \cdot \frac{1}{\Delta s} \\ &= \lim_{\Delta s \rightarrow 0} \frac{-\Delta s}{\Delta s(s + \Delta s - 4)(s - 4)} \\ &= \lim_{\Delta s \rightarrow 0} \frac{-1}{(s + \Delta s - 4)(s - 4)} \\ &= -\frac{1}{(s - 4)^2} \end{aligned}$$

$$42. f(x) = -\frac{1}{8}x^2 \text{ at } (-4, -2)$$

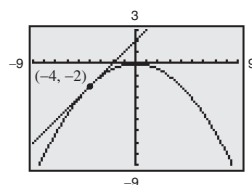
$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{8}(x + \Delta x)^2 - \left(-\frac{1}{8}x^2\right)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{8}(x^2 + 2x\Delta x + (\Delta x)^2) + \frac{1}{8}x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{8}x^2 - \frac{1}{4}x\Delta x - \frac{1}{8}(\Delta x)^2 + \frac{1}{8}x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{4}x\Delta x - \frac{1}{8}(\Delta x)^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x\left(-\frac{1}{4}x - \frac{1}{8}\Delta x\right)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \left(-\frac{1}{4}x - \frac{1}{8}\Delta x\right) \\ &= -\frac{1}{4}x \end{aligned}$$

$$m = f'(-4) = -\frac{1}{4}(-4) = 1$$

$$y - (-2) = 1[x - (-4)]$$

$$y + 2 = x + 4$$

$$y = x + 2$$



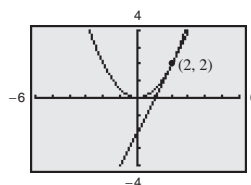
$$41. f(x) = \frac{1}{2}x^2 \text{ at } (2, 2)$$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{2}(x + \Delta x)^2 - \frac{1}{2}x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{2}(x^2 + 2x\Delta x + (\Delta x)^2) - \frac{1}{2}x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x\Delta x + (\Delta x)^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(x + \Delta x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} (x + \Delta x) \\ &= x \end{aligned}$$

$$m = f'(2) = 2$$

$$y - 2 = 2(x - 2)$$

$$y = 2x - 2$$



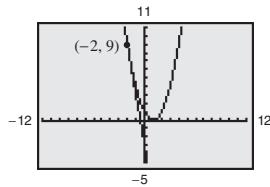
43. $f(x) = (x - 1)^2$ at $(-2, 9)$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x - 1)^2 - (x - 1)^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x - 2x + (\Delta x)^2 - 2\Delta x + 1 - x^2 + 2x - 1}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2 - 2\Delta x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \Delta x - 2)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} (2x + \Delta x - 2) \\ &= 2x - 2 \end{aligned}$$

$$m = f'(-2) = 2(-2) - 2 = -6$$

$$y - 9 = -6[x - (-2)]$$

$$y = -6x - 3$$



44. $f(x) = 2x^2 - 5$ at $(-1, -3)$

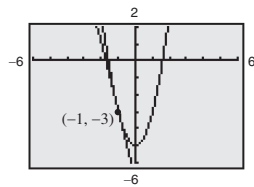
$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{2(x + \Delta x)^2 - 5 - (2x^2 - 5)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{2x^2 + 4x\Delta x + 2(\Delta x)^2 - 5 - 2x^2 + 5}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{4x\Delta x + 2(\Delta x)^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(4x + 2\Delta x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} (4x + 2\Delta x) \\ &= 4x \end{aligned}$$

$$m = f'(-1) = 4(-1) = -4$$

$$y - (-3) = -4(x - (-1))$$

$$y + 3 = -4x - 4$$

$$y = -4x - 7$$



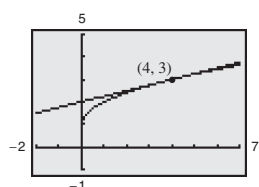
45. $f(x) = \sqrt{x} + 1$ at $(4, 3)$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} + 1 - (\sqrt{x} + 1)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x + \Delta x - x}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})} \\ &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} \\ &= \frac{1}{2\sqrt{x}} \end{aligned}$$

$$m = f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$$y - 3 = \frac{1}{4}(x - 4)$$

$$y = \frac{1}{4}x + 2$$



46. $f(x) = \sqrt{x + 3}$ at $(6, 3)$

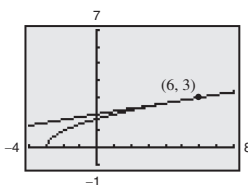
$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x + 3} - \sqrt{x + 3}}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x + 3} - \sqrt{x + 3}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x + 3} + \sqrt{x + 3}}{\sqrt{x + \Delta x + 3} + \sqrt{x + 3}} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x + \Delta x + 3 - (x + 3)}{\Delta x(\sqrt{x + \Delta x + 3} + \sqrt{x + 3})} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x}{\Delta x(\sqrt{x + \Delta x + 3} + \sqrt{x + 3})} \\ &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x + 3} + \sqrt{x + 3}} \\ &= \frac{1}{2\sqrt{x + 3}} \\ &= \frac{\sqrt{x + 3}}{2(x + 3)} \end{aligned}$$

$$m = f'(6) = \frac{1}{2\sqrt{6 + 3}} = \frac{1}{6}$$

$$y - 3 = \frac{1}{6}(x - 6)$$

$$y - 3 = \frac{1}{6}x - 1$$

$$y = \frac{1}{6}x + 2$$



47. $f(x) = \frac{1}{5x}$ at $\left(-\frac{1}{5}, -1\right)$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{5(x + \Delta x)} - \frac{1}{5x}}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{x - (x + \Delta x)}{5x(x + \Delta x)}}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{x - (x + \Delta x)}{5x(x + \Delta x)} \cdot \frac{1}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-\Delta x}{5x \cdot \Delta x \cdot (x + \Delta x)}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-1}{5x(x + \Delta x)}$$

$$= -\frac{1}{5x^2}$$

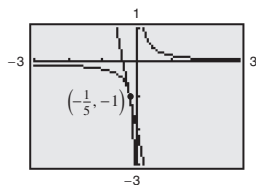
$$m = f'\left(-\frac{1}{5}\right) = -\frac{1}{5\left(-\frac{1}{5}\right)^2} = -\frac{1}{5\left(\frac{1}{25}\right)} = -5$$

$$y - (-1) = -5\left(x - \left(-\frac{1}{5}\right)\right)$$

$$y + 1 = -5\left(x + \frac{1}{5}\right)$$

$$y + 1 = -5x - 1$$

$$y = -5x - 2$$



48. $f(x) = \frac{1}{x-3}$ at $(2, -1)$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{x + \Delta x - 3} - \frac{1}{x - 3}}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{x + \Delta x - 3} \cdot \frac{x - 3}{x - 3} - \frac{1}{x - 3} \cdot \frac{x + \Delta x - 3}{x + \Delta x - 3}}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{x - 3 - (x + \Delta x - 3)}{(x + \Delta x - 3)(x - 3)}}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-\Delta x}{(x + \Delta x - 3)(x - 3)\Delta x}$$

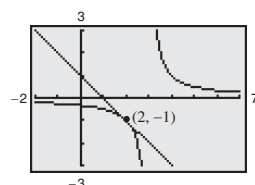
$$= \lim_{\Delta x \rightarrow 0} \frac{-1}{(x + \Delta x - 3)(x - 3)} = -\frac{1}{(x - 3)^2}$$

$$m = f'(2) = -\frac{1}{(2 - 3)^2} = -1$$

$$y - (-1) = -1(x - 2)$$

$$y + 1 = -x + 2$$

$$y = -x + 1$$



49. $f(x) = -\frac{1}{4}x^2$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{4}(x + \Delta x)^2 - (-\frac{1}{4}x^2)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{4}x^2 - \frac{1}{2}x\Delta x - \frac{1}{4}(\Delta x)^2 + \frac{1}{4}x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{2}x\Delta x - \frac{1}{4}(\Delta x)^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(-\frac{1}{2}x - \frac{1}{4}\Delta x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \left(-\frac{1}{2}x - \frac{1}{4}\Delta x\right) \\ &= -\frac{1}{2}x \end{aligned}$$

Since the slope of the given line is -1 ,

$$-\frac{1}{2}x = -1$$

$$x = 2 \text{ and } f(2) = -1.$$

At the point $(2, -1)$, the tangent line parallel to

$$x + y = 0 \text{ is } y - (-1) = -1(x - 2)$$

$$y = -x + 1.$$

51. $f(x) = -\frac{1}{3}x^3$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{3}(x + \Delta x)^3 - (-\frac{1}{3}x^3)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{3}(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3) + \frac{1}{3}x^3}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{3}x^3 - x^2\Delta x - x(\Delta x)^2 - \frac{1}{3}(\Delta x)^3 + \frac{1}{3}x^3}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(-x^2 - x\Delta x - \frac{1}{3}(\Delta x)^2)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} (-x^2 - x\Delta x - \frac{1}{3}(\Delta x)^2) = -x^2 \end{aligned}$$

Since the slope of the given line is -9 ,

$$-x^2 = -9$$

$$x^2 = 9$$

$$x = \pm 3 \text{ and } f(3) = -9 \text{ and } f(-3) = 9.$$

At the point $(3, -9)$, the tangent line parallel to $9x + y - 6 = 0$ is

$$y - (-9) = -9(x - 3)$$

$$y = -9x + 18.$$

At the point $(-3, 9)$, the tangent line parallel to $9x + y - 6 = 0$ is

$$y - 9 = -9(x - (-3))$$

$$y = -9x - 18.$$

50. $f(x) = x^2 - 7$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{[(x + \Delta x)^2 - 7] - (x^2 - 7)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 7 - x^2 + 7}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} (2x + \Delta x) = 2x \end{aligned}$$

Since the slope of the given line is -2 ,

$$2x = -2$$

$$x = -1 \text{ and } f(-1) = -6.$$

At the point $(-1, -6)$, the tangent line parallel to

$$2x + y = 0 \text{ is}$$

$$y - (-6) = -2(x - (-1))$$

$$y = -2x - 8.$$

52. $f(x) = x^3 + 2$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^3 + 2 - (x^3 + 2)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 + 2 - x^3 - 2}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{3x^2\Delta x + 3(\Delta x)^2 + (\Delta x)^3}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(3x^2 + 3x\Delta x + (\Delta x)^2)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} (3x^2 + 3x\Delta x + (\Delta x)^2) \\
 &= 3x^2
 \end{aligned}$$

The slope of the given line is

$$3x - y - 4 = 0$$

$$y = 3x - 4$$

$$m = 3.$$

$$3x^2 = 3$$

$$x^2 = 1$$

$$x = \pm 1$$

$$x = 1 \text{ and } f(1) = 3$$

$$x = -1 \text{ and } f(-1) = 1$$

At the point (1, 3), the tangent line parallel to $3x - y - 4 = 0$ is

$$y - 3 = 3(x - 1)$$

$$y - 3 = 3x - 3$$

$$y = 3x.$$

At the point (-1, 1), the tangent line parallel to $3x - y - 4 = 0$ is

$$y - 1 = 3(x - (-1))$$

$$y - 1 = 3(x + 1)$$

$$y - 1 = 3x + 3$$

$$y = 3x + 4.$$

53. y is differentiable for all $x \neq -3$.

At $(-3, 0)$, the graph has a node.

54. y is differentiable for all $x \neq \pm 3$.

At $(\pm 3, 0)$, the graph has a cusp.

55. y is differentiable for all $x \neq -\frac{1}{2}$.

At $(-\frac{1}{2}, 0)$, the graph has a vertical tangent line.

56. y is differentiable for all $x > 1$.

The derivative does not exist at endpoints.

57. y is differentiable for all $x \neq \pm 2$.

The function is not defined at $x = \pm 2$.

58. y is differentiable for all $x \neq 0$.

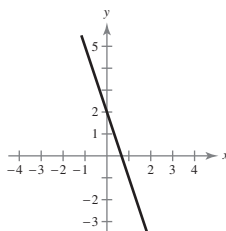
The function is discontinuous at $x = 0$.

59. Since $f'(x) = -3$ for all x , f is a line of the form

$$f(x) = -3x + b.$$

$$f(0) = 2, \text{ so } 2 = (-3)(0) + b, \text{ or } b = 2.$$

$$\text{Thus, } f(x) = -3x + 2.$$



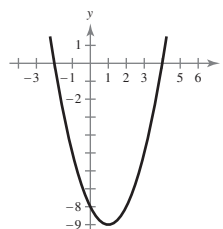
60. Sample answer: Since $f(-2) = f(4) = 0$, $(x + 2)(x - 4) = 0$.

A function with these zeros is $f(x) = x^2 - 2x - 8$.

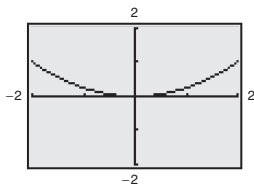
$$\begin{aligned} \text{Then } f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{[(x + \Delta x)^2 - 2(x + \Delta x) - 8] - (x^2 - 2x - 8)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 2x - 2\Delta x - 8 - x^2 + 2x + 8}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2 - 2\Delta x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} 2x + \Delta x - 2 \\ &= 2x - 2. \end{aligned}$$

So $f'(1) = 2(1) - 2 = 0$. Sketching $f(x)$ shows that

$$f'(x) < 0 \text{ for } x < 1 \text{ and } f'(0) > 0 \text{ for } x > 1.$$



- 61.

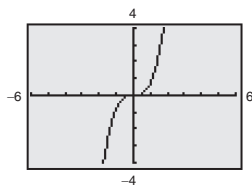


x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	1	0.5625	0.25	0.0625	0	0.0625	0.25	0.5625	1
$f'(x)$	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1

Analytically, the slope of $f(x) = \frac{1}{4}x^2$ is

$$\begin{aligned} m &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{4}(x + \Delta x)^2 - \frac{1}{4}x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{4}[x^2 + 2x(\Delta x) + (\Delta x)^2] - \frac{1}{4}x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{4}x^2 + \frac{1}{2}x\Delta x + \frac{1}{4}(\Delta x)^2 - \frac{1}{4}x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{2}x\Delta x + \frac{1}{4}(\Delta x)^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(\frac{1}{2}x + \frac{1}{4}\Delta x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} (\frac{1}{2}x + \frac{1}{4}\Delta x) \\ &= \frac{1}{2}x. \end{aligned}$$

62.

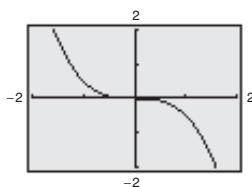


x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	-6	-2.53	-0.75	-0.1	0	0.1	0.75	2.53	6
$f'(x)$	9	5.0625	2.25	0.5625	0	0.5625	2.25	5.0625	9

Analytically, the slope of $f(x) = \frac{3}{4}x^3$ is

$$\begin{aligned}
 m &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\frac{3}{4}(x + \Delta x)^3 - \frac{3}{4}x^3}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\frac{3}{4}(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3) - \frac{3}{4}x^3}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\frac{3}{4}x^3 + \frac{9}{4}x^2\Delta x + \frac{9}{4}x(\Delta x)^2 + \frac{3}{4}(\Delta x)^3 - \frac{3}{4}x^3}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\frac{9}{4}x^2\Delta x + \frac{9}{4}x(\Delta x)^2 + \frac{3}{4}(\Delta x)^3}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x \left(\frac{9}{4}x^2 + \frac{9}{4}x\Delta x + \frac{3}{4}(\Delta x)^2 \right)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \left(\frac{9}{4}x^2 + \frac{9}{4}x\Delta x + \frac{3}{4}(\Delta x)^2 \right) \\
 &= \frac{9}{4}x^2.
 \end{aligned}$$

63.

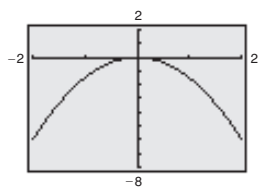


x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	4	1.6875	0.5	0.0625	0	-0.0625	-0.5	-1.6875	-4
$f'(x)$	-6	-3.375	-1.5	-0.375	0	-0.375	-1.5	-3.375	-6

Analytically, the slope of $f(x) = -\frac{1}{2}x^3$ is

$$\begin{aligned}
 m &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{2}(x + \Delta x)^3 + \frac{1}{2}x^3}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{2}[x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3] + \frac{1}{2}x^3}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{2}[3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3]}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} -\frac{1}{2}[3x^2 + 3x(\Delta x) + (\Delta x)^2] \\
 &= -\frac{3}{2}x^2.
 \end{aligned}$$

64.

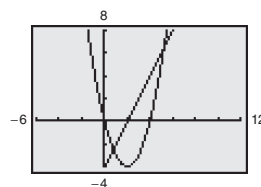


x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	-6	-3.375	-1.5	-0.375	0	-0.375	-1.5	-3.375	-6
$f'(x)$	6	4.5	3	1.5	0	-1.5	-3	-4.5	-6

Analytically, the slope of $f(x) = -\frac{3}{2}x^2$ is

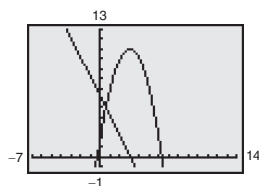
$$\begin{aligned}
 m &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{3}{2}(x + \Delta x)^2 - (-\frac{3}{2}x^2)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{3}{2}[x^2 + 2x\Delta x + (\Delta x)^2] + \frac{3}{2}x^2}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{3}{2}[2x\Delta x + (\Delta x)^2]}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} -\frac{3}{2}(2x + \Delta x) \\
 &= -3x.
 \end{aligned}$$

$$\begin{aligned}
 65. \quad f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - 4(x + \Delta x) - (x^2 - 4x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2 - 4\Delta x}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} (2x + \Delta x - 4) \\
 &= 2x - 4
 \end{aligned}$$



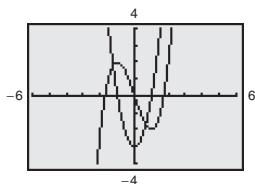
The x -intercept of the derivative indicates a point of horizontal tangency for f .

$$\begin{aligned}
 66. \quad f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{2 + 6(x + \Delta x) - (x + \Delta x)^2 - (2 + 6x - x^2)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{6\Delta x - 2x\Delta x - (\Delta x)^2}{\Delta x} = \lim_{\Delta x \rightarrow 0} (6 - 2x - \Delta x) = 6 - 2x
 \end{aligned}$$



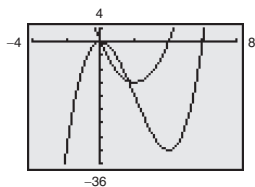
The x -intercept of the derivative indicates a point of horizontal tangency for f .

$$\begin{aligned}
 67. \quad f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^3 - 3(x + \Delta x) - (x^3 - 3x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 3x - 3\Delta x - x^3 + 3x}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 3\Delta x}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} (3x^2 + 3x\Delta x + (\Delta x)^2 - 3) \\
 &= 3x^2 - 3
 \end{aligned}$$



The x -intercepts of the derivative indicate points of horizontal tangency for f .

$$\begin{aligned}
 68. \quad f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^3 - 6(x + \Delta x)^2 - (x^3 - 6x^2)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{x^3 - 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 6(x^2 + 2x\Delta x + (\Delta x)^2) - x^3 + 6x^2}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} (3x^2 + 3x\Delta x + (\Delta x)^2 - 12x - 6\Delta x) \\
 &= 3x^2 - 12x
 \end{aligned}$$



The x -intercepts of the derivative indicate points of horizontal tangency for f .

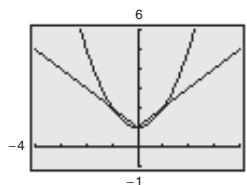
69. True. The slope of the graph is given by $f'(x) = 2x$, which is different for each different x value.

70. False. $f(x) = |x|$ is continuous, but not differentiable at $x = 0$.

71. True. See page 122.

72. True. See page 115.

73. The graph of $f(x) = x^2 + 1$ is smooth at $(0, 1)$, but the graph of $g(x) = |x| + 1$ has a node at $(0, 1)$. The function g is not differentiable at $(0, 1)$.



Section 2.2 Some Rules for Differentiation

Skills Warm Up

1. (a) $2x^2, x = 2$

$$2(2^2) = 2(4) = 8$$

(b) $(5x)^2, x = 2$

$$[5(2)]^2 = 10^2 = 100$$

(c) $6x^{-2}, x = 2$

$$6(2)^{-2} = 6\left(\frac{1}{4}\right) = \frac{3}{2}$$

2. (a) $\frac{1}{(3x)^2}, x = 2$

$$\frac{1}{[3(2)]^2} = \frac{1}{6^2} = \frac{1}{36}$$

(b) $\frac{1}{4x^3}, x = 2$

$$\frac{1}{4(2^3)} = \frac{1}{4(8)} = \frac{1}{32}$$

(c) $\frac{(2x)^{-3}}{4x^{-2}}, x = 2$

$$\frac{[2(2)]^{-3}}{4(2)^{-2}} = \frac{4^{-3}}{4(2)^{-2}} = \frac{2^2}{4(4^3)} = \frac{1}{64}$$

3. $4(3)x^3 + 2(2)x = 12x^3 + 4x = 4x(3x^2 + 1)$

4. $\frac{1}{2}(3)x^2 - \frac{3}{2}x^{1/2} = \frac{3}{2}x^2 - \frac{3}{2}\sqrt{x} = \frac{3}{2}x^{1/2}(x^{3/2} - 1)$

5. $\left(\frac{1}{4}\right)x^{-3/4} = \frac{1}{4x^{3/4}}$

6.
$$\begin{aligned} \frac{1}{3}(3)x^2 - 2\left(\frac{1}{2}\right)x^{-1/2} + \frac{1}{3}x^{-2/3} &= x^2 - x^{-1/2} + \frac{1}{3}x^{-2/3} \\ &= x^2 - \frac{1}{\sqrt{x}} + \frac{1}{3\sqrt[3]{x^2}} \end{aligned}$$

7. $3x^2 + 2x = 0$

$$x(3x + 2) = 0$$

$$x = 0$$

$$3x + 2 = 0 \rightarrow x = -\frac{2}{3}$$

8. $x^3 - x = 0$

$$x(x^2 - 1) = 0$$

$$x(x + 1)(x - 1) = 0$$

$$x = 0$$

$$x + 1 = 0 \rightarrow x = -1$$

$$x - 1 = 0 \rightarrow x = 1$$

9. $x^2 + 8x - 20 = 0$

$$(x + 10)(x - 2) = 0$$

$$x + 10 = 0 \rightarrow x = -10$$

$$x - 2 = 0 \rightarrow x = 2$$

10. $3x^2 - 10x + 8 = 0$

$$(3x - 4)(x - 2) = 0$$

$$3x - 4 = 0 \rightarrow x = \frac{4}{3}$$

$$x - 2 = 0 \rightarrow x = 2$$

1. $y = 3$

$$y' = 0$$

2. $f(x) = -8$

$$f'(x) = 0$$

3. $y = x^5$

$$y' = 5x^4$$

4. $f(x) = \frac{1}{x^6} = x^{-6}$

$$f'(x) = -6x^{-7} = -\frac{6}{x^7}$$

5. $h(x) = 3x^3$

$$h'(x) = 9x^2$$

6. $h(x) = 6x^5$

$$h'(x) = 30x^4$$

7. $y = \frac{5x^4}{3}$

$$y' = \frac{20}{6}x^3 = \frac{10}{3}x^3$$

8. $g(t) = \frac{3t^2}{4}$

$$g'(t) = \frac{3}{2}t$$

9. $f(x) = 4x$

$$f'(x) = 4$$

$$10. \quad g(x) = \frac{x}{3} = \frac{1}{3}x$$

$$g'(x) = \frac{1}{3}$$

$$11. \quad y = 8 - x^3$$

$$y' = -3x^2$$

$$12. \quad y = t^2 - 6$$

$$y' = 2t$$

$$13. \quad f(x) = 4x^2 - 3x$$

$$f'(x) = 8x - 3$$

$$14. \quad g(x) = 3x^2 + 5x^3$$

$$g'(x) = 6x + 15x^2 = 15x^2 + 6x$$

$$15. \quad f(t) = -3t^2 + 2t - 4$$

$$f'(t) = -6t + 2$$

$$16. \quad y = 7x^3 - 9x^2 + 8$$

$$y' = 21x^2 - 18x$$

$$17. \quad s(t) = 4t^4 - 2t + t + 3$$

$$s'(t) = 16t^3 - 4t + 1$$

$$18. \quad y = 2x^3 - x^2 + 3x - 1$$

$$y' = 6x^2 - 2x + 3$$

$$19. \quad g(x) = x^{2/3}$$

$$g'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$$

$$20. \quad h(x) = x^{5/2}$$

$$h'(x) = \frac{5}{2}x^{3/2}$$

$$21. \quad y = 4t^{4/3}$$

$$y' = 4\left(\frac{4}{3}\right)t^{1/3} = \frac{16}{3}t^{1/3}$$

$$22. \quad f(x) = 10x^{1/6}$$

$$f'(x) = \frac{5}{3}x^{-5/6} = \frac{5}{3x^{5/6}} = \frac{5}{3\sqrt[6]{x^5}}$$

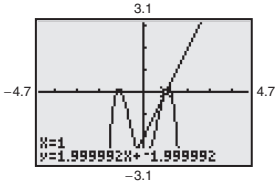
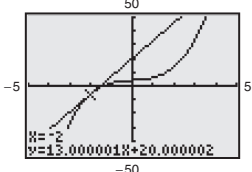
$$23. \quad y = 4x^{-2} + 2x^2$$

$$y' = -8x^{-3} + 4x^1 = -\frac{8}{x^3} + 4x$$

$$24. \quad s(t) = 8t^{-4} + t$$

$$s'(t) = 8(-4t^{-5}) + 1 = -\frac{32}{t^5} + 1$$

Function	Rewrite	Differentiate	Simplify
25. $y = \frac{2}{7x^4}$	$y = \frac{2}{7}x^{-4}$	$y' = \frac{-8}{7}x^{-5}$	$y' = -\frac{8}{7x^5}$
26. $y = \frac{2}{3x^2}$	$y = \frac{2}{3}x^{-2}$	$y' = -\frac{4}{3}x^{-3}$	$y' = -\frac{4}{3x^3}$
27. $y = \frac{1}{(4x)^3}$	$y = \frac{1}{64}x^{-3}$	$y' = -\frac{3}{64}x^{-4}$	$y' = -\frac{3}{64x^4}$
28. $y = \frac{\pi}{(2x)^6}$	$y = \frac{\pi}{64}x^{-6}$	$y' = -\frac{6\pi}{64}x^{-7}$	$y' = -\frac{3\pi}{32x^7}$
29. $y = \frac{4}{(2x)^{-5}}$	$y = 128x^5$	$y' = 128(5)x^4$	$y' = 640x^4$
30. $y = \frac{4x}{x^{-3}}$	$y = 4x^4$	$y' = 4(4)x^3$	$y' = 16x^3$
31. $y = 6\sqrt{x}$	$y = 6x^{1/2}$	$y' = 6\left(\frac{1}{2}\right)x^{-1/2}$	$y' = \frac{3}{\sqrt{x}}$
32. $y = \frac{3\sqrt{x}}{4}$	$y = \frac{3}{4}x^{1/2}$	$y' = \frac{3}{4}\left(\frac{1}{2}\right)x^{-1/2}$	$y' = \frac{3}{8\sqrt{x}}$

- | Function | Rewrite | Differentiate | Simplify |
|------------------------------------|-----------------------------------|---|--|
| 33. $y = \frac{1}{7\sqrt[6]{x}}$ | $y = \frac{1}{7}x^{-1/6}$ | $y' = \frac{1}{7}\left(-\frac{1}{6}\right)x^{-7/6}$ | $y' = -\frac{1}{42\sqrt[6]{x^7}}$ |
| 34. $y = \frac{3}{2\sqrt[4]{x^3}}$ | $y = \frac{3}{2}x^{-3/4}$ | $y' = \frac{3}{2}\left(-\frac{3}{4}\right)x^{-7/4}$ | $y' = -\frac{9}{8\sqrt[4]{x^7}}$ |
| 35. $y = \sqrt[5]{8x}$ | $y = (8x)^{1/5} = 8^{1/5}x^{1/5}$ | $y' = 8^{1/5}\left(\frac{1}{5}x^{-4/5}\right)$ | $y' = 8^{1/5}\frac{1}{5x^{4/5}} = \frac{\sqrt[5]{8}}{5\sqrt[5]{x^4}} = \frac{\sqrt[5]{8}}{5x}$ |
| 36. $y = \sqrt[3]{6x^2}$ | $y = \sqrt[3]{6}(x)^{2/3}$ | $y' = \sqrt[3]{6}\left(\frac{2}{3}\right)x^{-1/3}$ | $y' = \frac{2\sqrt[3]{6}}{3\sqrt[3]{x}}$ |
37. $y = x^{3/2}$
 $y' = \frac{3}{2}x^{1/2}$
 At the point $(1, 1)$, $y' = \frac{3}{2}(1)^{1/2} = \frac{3}{2} = m$.
38. $y = x^{-1}$
 $y' = x^{-2} = -\frac{1}{x^2}$
 At the point $\left(\frac{3}{4}, \frac{4}{3}\right)$, $y' = -\frac{1}{\left(\frac{3}{4}\right)^2} = -\frac{16}{9} = m$.
39. $f(t) = t^{-4}$
 $f'(t) = -4t^{-5} = -\frac{4}{t^5}$
 At the point $\left(\frac{1}{2}, 16\right)$, $f'\left(\frac{1}{2}\right) = -\frac{4}{\left(\frac{1}{2}\right)^5} = -\frac{4}{\frac{1}{32}} = -128 = m$.
40. $f(x) = x^{-1/3}$
 $f'(x) = -\frac{1}{3}x^{-4/3} = -\frac{1}{3x^{4/3}}$
 At the point $\left(8, \frac{1}{2}\right)$, $f'(8) = -\frac{1}{3(8)^{4/3}} = -\frac{1}{48} = m$.
41. $f(x) = 2x^3 + 8x^2 - x - 4$
 $f'(x) = 6x^2 + 16x - 1$
 At the point $(-1, 3)$, $f'(-1) = 6(-1)^2 + 16(-1) - 1 = -11 = m$.
42. $f(x) = x^4 - 2x^3 + 5x^2 - 7x$
 $f'(x) = 4x^3 - 6x^2 + 10x - 7$
 At the point $(-1, 15)$, $f'(-1) = 4(-1)^3 - 6(-1)^2 + 10(-1) - 7 = -4 - 6 - 10 - 7 = -27 = m$.
43. $f(x) = -\frac{1}{2}x(1 + x^2) = -\frac{1}{2}x - \frac{1}{2}x^3$
 $f'(x) = -\frac{1}{2} - \frac{3}{2}x^2$
 $f'(1) = -\frac{1}{2} - \frac{3}{2} = -2$
44. $f(x) = 3(5 - x)^2 = 75 - 30x + 3x^2$
 $f'(x) = -30 + 6x$
 $f'(5) = -30 + (6)(5) = 0$
45. (a) $y = -2x^4 + 5x^2 - 3$
 $y' = -8x^3 + 10x$
 $m = y'(1) = -8 + 10 = 2$
 The equation of the tangent line is
 $y - 0 = 2(x - 1)$
 $y = 2x - 2$.
- (b) and (c)
- 
46. (a) $y = x^3 + x + 4$
 $y' = 3x^2 + 1$
 $m = y'(-2) = 3(-2)^2 + 1 = 13$
 The equation of the tangent line is
 $y - (-6) = 13[x - (-2)]$
 $y + 6 = 13x + 26$
 $y = 13x + 20$.
- (b) and (c)
- 

$$47. (a) f(x) = \sqrt[3]{x} + \sqrt[5]{x} = x^{1/3} + x^{1/5}$$

$$f'(x) = \frac{1}{3}x^{-2/3} + \frac{1}{5}x^{-4/5} = \frac{1}{3x^{2/3}} + \frac{1}{5x^{4/5}}$$

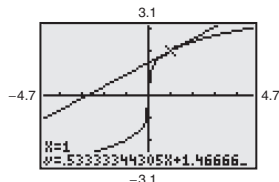
$$m = f'(1) = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$$

The equation of the tangent line is

$$y - 2 = \frac{8}{15}(x - 1)$$

$$y = \frac{8}{15}x + \frac{22}{15}$$

(b) and (c)



$$48. (a) f(x) = \frac{1}{\sqrt[3]{x^2}} - x = x^{-2/3} - x$$

$$f'(x) = -\frac{2}{3}x^{-5/3} - 1$$

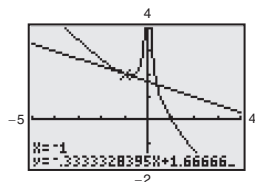
$$m = f'(-1) = \frac{2}{3} - 1 = -\frac{1}{3}$$

The equation of the tangent line is

$$y - 2 = -\frac{1}{3}(x + 1)$$

$$y = -\frac{1}{3}x + \frac{5}{3}$$

(b) and (c)



$$49. (a) y = 3x\left(x^2 - \frac{2}{x}\right)$$

$$y = 3x^3 - 6$$

$$y' = 9x^2$$

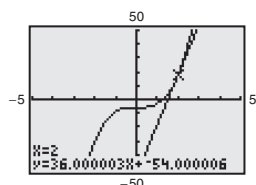
$$m = y' = 9(2)^2 = 36$$

The equation of the tangent line is

$$y - 18 = 36(x - 2)$$

$$y = 36x - 54$$

(b) and (c)



$$50. (a) y = (2x + 1)^2$$

$$y = 4x^2 + 4x + 1$$

$$y' = 8x + 4$$

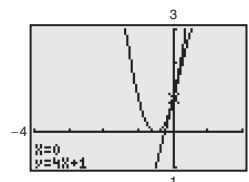
$$m = y' = 8(0) + 4 = 4$$

The equation of the tangent line is

$$y - 1 = 4(x - 0)$$

$$y = 4x + 1$$

(b) and (c)



$$51. f(x) = x^2 - 4x^{-1} - 3x^{-2}$$

$$f'(x) = 2x + 4x^{-2} + 6x^{-3} = 2x + \frac{4}{x^2} + \frac{6}{x^3}$$

$$52. f(x) = 6x^2 - 5x^{-2} + 7x^{-3}$$

$$f'(x) = 12x + 10x^{-3} - 21x^{-4} = 12x + \frac{10}{x^3} - \frac{21}{x^4}$$

$$53. f(x) = x^2 - 2x - \frac{2}{x^4} = x^2 - 2x - 2x^{-4}$$

$$f'(x) = 2x - 2 + 8x^{-5} = 2x - 2 + \frac{8}{x^5}$$

$$54. f(x) = x^2 + 4x + \frac{1}{x} = x^2 + 4x + x^{-1}$$

$$f'(x) = 2x + 4 - x^{-2} = 2x + 4 - \frac{1}{x^2}$$

$$55. f(x) = x^{4/5} + x$$

$$f'(x) = \frac{4}{5}x^{-1/5} + 1 = \frac{4}{5x^{1/5}} + 1$$

$$56. f(x) = x^{1/3} - 1$$

$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$$

$$57. f(x) = x(x^2 + 1) = x^3 + x$$

$$f'(x) = 3x^2 + 1$$

$$58. f(x) = (x^2 + 2x)(x + 1) = x^3 + 3x^2 + 2x$$

$$f'(x) = 3x^2 + 6x + 2$$

$$59. f(x) = \frac{2x^3 - 4x^2 + 3}{x^2} = 2x - 4 + 3x^{-2}$$

$$f'(x) = 2 - 6x^{-3} = 2 - \frac{6}{x^3} = \frac{2x^3 - 6}{x^3} = \frac{2(x^3 - 3)}{x^3}$$

$$60. f(x) = \frac{2x^2 - 3x + 1}{x} = 2x - 3 + x^{-1}$$

$$f'(x) = 2 - x^{-2} = 2 - \frac{1}{x^2} = \frac{2x^2 - 1}{x^2}$$

$$61. f(x) = \frac{4x^3 - 3x^2 + 2x + 5}{x^2} = 4x - 3 + 2x^{-1} + 5x^{-2}$$

$$f'(x) = 4 - 2x^{-2} - 10x^{-3} = 4 - \frac{2}{x^2} - \frac{10}{x^3} = \frac{4x^3 - 2x - 10}{x^3}$$

$$62. f(x) = \frac{-6x^3 + 3x^2 - 2x + 1}{x} = -6x^2 + 3x - 2 + x^{-1}$$

$$f'(x) = -12x + 3 - x^{-2} = -12x + 3 - \frac{1}{x^2}$$

$$63. y = x^4 - 2x + 3$$

$$y' = 4x^3 - 4x = 4x(x^2 - 1) = 0 \text{ when } x = 0, \pm 1$$

$$\text{If } x = \pm 1, \text{ then } y = (\pm 1)^4 - 2(\pm 1) + 3 = 2.$$

The function has horizontal tangent lines at the points $(0, 3)$, $(1, 2)$, and $(-1, 2)$.

$$64. y = x^3 + 3x^2$$

$$y' = 3x^2 + 6x = 3x(x + 2) = 0 \text{ when } x = 0, -2.$$

The function has horizontal tangent lines at the points $(0, 0)$ and $(-2, 4)$.

$$65. y = \frac{1}{2}x^2 + 5x$$

$$y' = x + 5 = 0 \text{ when } x = -5.$$

The function has a horizontal tangent line at the point $(-5, -\frac{25}{2})$.

$$66. y = x^2 + 2x$$

$$y' = 2x + 2 = 0 \text{ when } x = -1.$$

The function has a horizontal tangent line at the point $(-1, -1)$.

$$67. y = x^2 + 3$$

$$y' = 2x$$

$$\text{Set } y' = 4.$$

$$2x = 4$$

$$x = 2$$

$$\text{If } x = 2, y = (2)^2 + 3 = 7 \rightarrow (2, 7).$$

The graph of $y = x^2 + 3$ has a tangent line with slope $m = 4$ at the point $(2, 7)$.

$$68. y = x^2 + 2x$$

$$y' = 2x + 2$$

$$\text{Set } y' = 10.$$

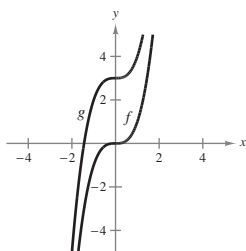
$$2x + 2 = 10$$

$$x = 4$$

$$\text{If } x = 4, y = (4)^2 + 2(4) = 24 \rightarrow (4, 24).$$

The graph of $y = x^2 + 2x$ has a tangent line with slope $m = 10$ at the point $(4, 24)$.

69. (a)



$$(b) \quad f'(x) = g'(x) = 3x^2$$

$$f'(1) = g'(1) = 3$$

(c) Tangent line to f at $x = 1$:

$$f(1) = 1$$

$$y - 1 = 3(x - 1)$$

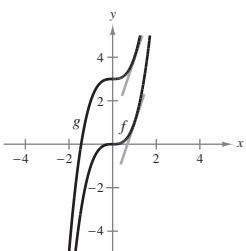
$$y = 3x - 2$$

Tangent line to g at $x = 1$:

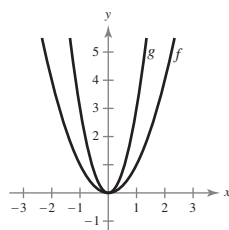
$$g(1) = 4$$

$$y - 4 = 3(x - 1)$$

$$y = 3x + 1$$

(d) f' and g' are the same.

70. (a)



$$(b) \quad f'(x) = 2x$$

$$f'(1) = 2$$

$$g'(x) = 6x$$

$$g'(1) = 6$$

(c) Tangent line to f at $x = 1$:

$$f(1) = 1$$

$$y - 1 = 2(x - 1)$$

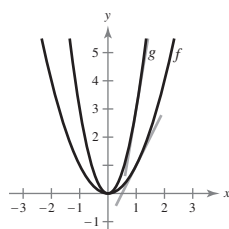
$$y = 2x - 1$$

Tangent line to g at $x = 1$:

$$g(1) = 3$$

$$y - 3 = 6(x - 1)$$

$$y = 6x - 3$$

(d) g' is 3 times f' .71. If $g(x) = f(x) + 6$, then $g'(x) = f'(x)$ because the derivative of a constant is 0, $g'(x) = f'(x)$.72. If $g(x) = 2f(x)$, then $g'(x) = 2f'(x)$ because of the Constant Multiple Rule.73. If $g(x) = -5f(x)$, then $g'(x) = -5f'(x)$ because of the Constant Multiple Rule.74. If $g(x) = 3f(x) - 1$, then $g'(x) = 3f'(x)$ because of the Constant Multiple Rule and the derivative of a constant is 0.

75. (a) $R = -4.1685t^3 + 175.037t^2 - 1950.88t + 7265.3$

$$R' = -12.5055t^2 + 350.074t - 1950.88$$

2009: $R'(9) = -12.5055(9)^2 + 350.074(9) - 1950.88 \approx \186.8 million per year

2011: $R'(11) = -12.5055(11)^2 + 350.074(11) - 1950.88 \approx \386.8 million per year

(b) These results are close to the estimates in Exercise 13 in Section 2.1.

(c) The slope of the graph at time t is the rate at which sales are increasing in millions of dollars per year.

76. (a) $R = -2.67538t^4 + 94.0568t^3 - 1155.203t^2 + 6002.42t - 9794.2$

$$R' = -10.70152t^3 + 282.1704t^2 - 2310.406t + 6002.42$$

$$2010: R'(10) = -10.70152(10)^3 + 282.1704(10)^2 - 2310.406(10) + 6002 \approx \$413.88 \text{ million per year}$$

$$2012: R'(12) = -10.70152(12)^3 + 282.1704(12)^2 - 2310.406(12) + 6002 \approx \$417.86 \text{ million per year}$$

(b) These results are close to the estimates in Exercise 14 in Section 2.1.

(c) The slope of the graph at time t is the rate at which sales are increasing in millions of dollars per year.

77. (a) More men and women seem to suffer from migraines between 30 and 40 years old. More females than males suffer from migraines. Fewer people whose income is greater than or equal to \$30,000 suffer from migraines than people whose income is less than \$10,000.

(b) The derivatives are positive up to approximately 37 years old and negative after about 37 years of age. The percent of adults suffering from migraines increases up to about 37 years old, then decreases. The units of the derivative are percent of adults suffering from migraines per year.

78. (a) The attendance rate for football games, $g'(t)$, is greater at game 1.

(b) The attendance rate for basketball games, $f'(t)$, is greater than the rate for football games, $g'(t)$, at game 3.

(c) The attendance rate for basketball games, $f'(t)$, is greater than the rate for football games, $g'(t)$, at game 4. In addition, the attendance rate for football games is decreasing at game 4.

(d) At game 5, the attendance rate for football continues to increase, while the attendance rate for basketball continues to decrease.

79. $C = 7.75x + 500$

$$C' = 7.75, \text{ which equals the variable cost.}$$

80. $C = 150x + 7000$

$$P = R - C$$

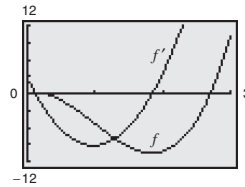
$$P = 500x - (150x + 7000)$$

$$P = 350x - 7000$$

$$P' = 350, \text{ which equals the profit on each dinner sold.}$$

81. $f(x) = 4.1x^3 - 12x^2 + 2.5x$

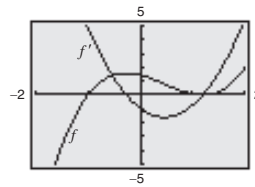
$$f'(x) = 12.3x^2 - 24x + 2.5$$



f has horizontal tangents at $(0.110, 0.135)$ and $(1.841, -10.486)$.

82. $f(x) = x^3 - 1.4x^2 - 0.96x + 1.44$

$$f'(x) = 3x^2 - 2.8x - 0.96$$



f has horizontal tangents at $(1.2, 0)$ and $(-0.267, 1.577)$.

83. False. Let $f(x) = x$ and $g(x) = x + 1$.

Then $f'(x) = g'(x) = 1$, but $f(x) \neq g(x)$.

84. True. c is a constant.

Section 2.3 Rates of Change: Velocity and Marginals

Skills Warm Up

$$1. \frac{-63 - (-105)}{21 - 7} = \frac{42}{14} = 3$$

$$2. \frac{-43 - 35}{6 - (-7)} = \frac{-78}{13} = -6$$

$$3. \frac{24 - 33}{9 - 6} = \frac{-9}{3} = -3$$

$$4. \frac{40 - 16}{18 - 8} = \frac{24}{10} = \frac{12}{5}$$

$$5. y = 4x^2 - 2x + 7 \\ y' = 8x - 2$$

$$6. s = -2t^3 + 8t^2 - 7t \\ s' = -6t^2 + 16t - 7$$

$$7. s = -16t^2 + 24t + 30 \\ s' = -32t + 24$$

$$8. y = -16x^2 + 54x + 70 \\ y' = -32x + 54$$

$$9. A = \frac{1}{10}(-2r^3 + 3r^2 + 5r) \\ A' = \frac{1}{10}(-6r^2 + 6r + 5) \\ A' = -\frac{3}{5}r^2 + \frac{3}{5}r + \frac{1}{2}$$

$$10. y = \frac{1}{9}(6x^3 - 18x^2 + 63x - 15) \\ y' = \frac{1}{9}(18x^2 - 36x + 63) \\ y' = 2x^2 - 4x + 7$$

$$11. y = 12x - \frac{x^2}{5000} \\ y' = 12 - \frac{2x}{5000} \\ y' = 12 - \frac{x}{2500}$$

$$12. y = 138 + 74x - \frac{x^3}{10,000} \\ y' = 74 - \frac{3x^2}{10,000}$$

$$1. (a) 1980-1986: \frac{120 - 63}{6 - 0} = \$9.5 \text{ billion/yr}$$

$$(b) 1986-1992: \frac{165 - 120}{12 - 6} = \$7.5 \text{ billion/yr}$$

$$(c) 1992-1998: \frac{226 - 165}{18 - 12} \approx \$10.2 \text{ billion/yr}$$

$$(d) 1998-2004: \frac{305 - 226}{24 - 18} \approx \$13.2 \text{ billion/yr}$$

$$(e) 2004-2010: \frac{408 - 305}{30 - 24} \approx \$17.2 \text{ billion/yr}$$

$$(f) 1980-2012: \frac{453 - 63}{32 - 0} \approx \$12.2 \text{ billion/yr}$$

$$(g) 1990-2012: \frac{453 - 152}{32 - 10} \approx \$13.7 \text{ billion/yr}$$

$$(h) 2000-2012: \frac{453 - 269}{32 - 20} \approx \$15.3 \text{ billion/yr}$$

2. (a) Imports:

$$1980-1990: \frac{495 - 245}{10 - 0} = \$25 \text{ billion/yr}$$

(b) Exports:

$$1980-1990: \frac{394 - 226}{10 - 0} = \$16.8 \text{ billion/yr}$$

(c) Imports:

$$1990-2000: \frac{1218 - 495}{20 - 10} \approx \$72.3 \text{ billion/yr}$$

(d) Exports:

$$1990-2000: \frac{782 - 394}{20 - 10} = \$38.8 \text{ billion/yr}$$

(e) Imports:

$$2000-2010: \frac{1560 - 1218}{29 - 20} = \$38.0 \text{ billion/yr}$$

(f) Exports:

$$2000-2010: \frac{1056 - 782}{29 - 20} = \$30.4 \text{ billion/yr}$$

(g) Imports:

$$1980-2013: \frac{2268 - 245}{33 - 0} \approx \$61.3 \text{ billion/yr}$$

(h) Exports:

$$1980-2013: \frac{1580 - 226}{33 - 0} \approx \$41.0 \text{ billion/yr}$$

3. $f(t) = 3t + 5; [1, 2]$

Average rate of change:

$$\frac{\Delta y}{\Delta t} = \frac{f(2) - f(1)}{2 - 1} = \frac{11 - 8}{1} = 3$$

$$f'(t) = 3$$

Instantaneous rates of change: $f'(1) = 3, f'(2) = 3$

4. $h(x) = 7 - 2x; [1, 3]$

Average rate of change:

$$\frac{\Delta h}{\Delta t} = \frac{h(3) - h(1)}{3 - 1} = \frac{1 - 5}{2} = -2$$

$$h'(t) = -2$$

Instantaneous rates of change: $h(1) = -2, h(3) = -2$

5. $h(x) = x^2 - 4x + 2; [-2, 2]$

Average rate of change:

$$\frac{\Delta h}{\Delta x} = \frac{h(2) - h(-2)}{2 - (-2)} = \frac{-2 - 14}{4} = -4$$

$$h'(x) = 2x - 4$$

Instantaneous rates of change: $h'(-2) = -8, h'(2) = 0$

6. $f(x) = -x^2 - 6x - 5; [-3, 1]$

Average rate of change:

$$\frac{\Delta f}{\Delta x} = \frac{f(1) - f(-3)}{1 - (-3)} = \frac{-12 - 4}{4} = -4$$

$$f'(x) = -2x - 6$$

Instantaneous rates of change: $f'(-3) = 0, f'(1) = -8$

7. $f(x) = 3x^{4/3}; [1, 8]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(8) - f(1)}{8 - 1} = \frac{48 - 3}{7} = \frac{45}{7}$$

$$f'(x) = 4x^{1/3}$$

Instantaneous rates of change: $f'(1) = 4, f'(8) = 8$

8. $f(x) = x^{3/2}; [1, 4]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(4) - f(1)}{4 - 1} = \frac{8 - 1}{3} = \frac{7}{3}$$

$$f'(x) = \frac{3}{2}x^{1/2}$$

Instantaneous rates of change: $f'(1) = \frac{3}{2}, f'(4) = 3$

9. $f(x) = \frac{1}{x}; [1, 5]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(5) - f(1)}{5 - 1} = \frac{\frac{1}{5} - 1}{4} = \frac{-\frac{4}{5}}{4} = -\frac{1}{5}$$

$$f'(x) = -\frac{1}{x^2}$$

Instantaneous rates of change:

$$f'(1) = -1, f'(5) = -\frac{1}{25}$$

10. $f(x) = \frac{1}{\sqrt{x}}; [1, 9]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(9) - f(1)}{9 - 1} = \frac{\frac{1}{3} - 1}{8} = \frac{-\frac{2}{3}}{8} = -\frac{1}{6}$$

$$f'(x) = \frac{1}{2x^{3/2}}$$

Instantaneous rates of change:

$$f'(1) = -\frac{1}{2}, f'(9) = \frac{1}{54}$$

11. $f(t) = t^4 - 2t^2; [-2, -1]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(-1) - f(-2)}{-1 - (-2)} = \frac{-1 - 8}{1} = -9$$

$$f'(t) = 4t^3 - 4t$$

Instantaneous rates of change:

$$f'(-2) = -24, f'(-1) = 0$$

12. $g(x) = x^3 - 1; [-1, 1]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{g(1) - g(-1)}{1 - (-1)} = \frac{0 - (-2)}{2} = 1$$

$$g'(x) = 3x^2$$

Instantaneous rates of change:

$$g'(-1) = 3, g'(1) = 3$$

13. (a) $\approx \frac{0 - 1400}{3} \approx -467$

The number of visitors to the park is decreasing at an average rate of 467 people per month from September to December.

(b) Answers will vary. Sample answer: $[4, 11]$

Both the instantaneous rate of change at $t = 8$ and the average rate of change on $[4, 11]$ are about zero.

14. (a) $\frac{\Delta M}{\Delta t} = \frac{800 - 200}{3 - 1} = \frac{600}{2} = 300$ mg/hr

(b) Answers will vary. Sample answer: $[2, 5]$

Both the instantaneous rate of change at $t = 4$ and the average rate of change on $[2, 5]$ is about zero.

15. $s = -4.9t^2 + 9t + 76$

Instantaneous: $v(t) = s'(t) = -9.8t + 9$ (a) Average: $[0, 1]$:

$$\frac{s(1) - s(0)}{1 - 0} = \frac{80.1 - 76}{1} = 4.1 \text{ m/sec}$$

$$v(0) = s'(0) = 9 \text{ m/sec}$$

$$v(1) = s'(1) = -0.8 \text{ m/sec}$$

(b) Average: $[1, 2]$:

$$\frac{s(2) - s(1)}{2 - 1} = \frac{74.4 - 80.1}{1} = -5.7 \text{ m/sec}$$

$$v(1) = s'(1) = -0.8 \text{ m/sec}$$

$$v(2) = s'(2) = -10.6 \text{ m/sec}$$

(c) Average: $[2, 3]$:

$$\frac{s(3) - s(2)}{3 - 2} = \frac{58.9 - 74.4}{1} = -15.5 \text{ m/sec}$$

$$v(2) = s'(2) = -10.6 \text{ m/sec}$$

$$v(3) = s'(3) = -20.4 \text{ m/sec}$$

(d) Average: $[3, 4]$:

$$\frac{s(4) - s(3)}{4 - 3} = \frac{33.6 - 58.9}{1} = -25.3 \text{ m/sec}$$

$$v(3) = s'(3) = -20.4 \text{ m/sec}$$

$$v(4) = s'(4) = -30.2 \text{ m/sec}$$

16. (a) $H'(v) = 33 \left[10 \left(\frac{1}{2} v^{-1/2} \right) - 1 \right] = 33 \left[\frac{5}{\sqrt{v}} - 1 \right]$

Rate of change of heat loss with respect to wind speed.

(b) $H'(2) = 33 \left[\frac{5}{\sqrt{2}} - 1 \right]$

$$\approx 83.673 \frac{\text{kcal/m}^2/\text{hr}}{\text{m/sec}}$$

$$= 83.673 \frac{\text{kcal}}{\text{m}^3} \cdot \frac{\text{sec}}{\text{hr}}$$

$$= 83.673 \frac{\text{kcal}}{\text{m}^3} \cdot \frac{1}{3600}$$

$$= 0.023 \text{ kcal/m}^3$$

$$H'(5) = 33 \left[\frac{5}{\sqrt{5}} - 1 \right]$$

$$\approx 40.790 \frac{\text{kcal/m}^2/\text{hr}}{\text{m/sec}}$$

$$= 40.790 \frac{\text{kcal}}{\text{m}^3} \cdot \frac{\text{sec}}{\text{hr}}$$

$$= 40.790 \frac{\text{kcal}}{\text{m}^3} \cdot \frac{1}{3600}$$

$$= 0.11 \text{ kcal/m}^3$$

17. $s = -4.9t^2 + 170$

$$\begin{aligned} \text{(a) Average velocity} &= \frac{s(3) - s(2)}{3 - 2} \\ &= \frac{125.9 - 150.4}{1} \\ &= -24.5 \text{ m/sec} \end{aligned}$$

$$\begin{aligned} \text{(b) } v &= s'(t) = -9.8t, \quad v(2) = -19.6 \text{ m/sec}, \\ v(3) &= -29.4 \text{ m/sec} \end{aligned}$$

$$\begin{aligned} \text{(c) } s &= -4.9t^2 + 170 = 0 \\ 4.9t^2 &= 170 \end{aligned}$$

$$t^2 = \frac{170}{4.9}$$

$$t \approx 5.89 \text{ seconds}$$

$$\text{(d) } v(5.89) \approx -57.7 \text{ m/sec}$$

18. (a) $s(t) = -4.9t^2 - 5.5t + 64$

$$v(t) = s'(t) = -9.8t - 5.5$$

(b) $[1, 2]: \frac{s(2) - s(1)}{2 - 1} = \frac{33.4 - 53.6}{1} = -20.2 \text{ m/sec}$

(c) $v(1) = -15.3 \text{ m/sec}$

$$v(2) = -25.1 \text{ m/sec}$$

(d) Set $s(t) = 0$.

$$-4.9t^2 - 5.5t + 64 = 0$$

$$-49t^2 - 55t + 640 = 0$$

$$t = \frac{-(-55) \pm \sqrt{(-55)^2 - 4(-49)(640)}}{2(-49)} = \frac{55 \pm \sqrt{128,465}}{-98} \approx 3.10 \text{ sec}$$

(e) $v(3.10) = -35.88 \text{ m/sec}$

19. $C = 205,000 + 9800x$

$$\frac{dC}{dx} = 9800$$

20. $C = 150,000 + 7x^3$

$$\frac{dC}{dx} = 21x^2$$

21. $C = 55,000 + 470x - 0.25x^2, 0 \leq x \leq 940$

$$\frac{dC}{dx} = 470 - 0.5x$$

22. $C = 100(9 + 3\sqrt{x})$

$$\frac{dC}{dx} = 100 \left[0 + 3 \left(\frac{1}{2} x^{-1/2} \right) \right] = \frac{150}{\sqrt{x}}$$

23. $R = 50x - 0.5x^2$

$$\frac{dR}{dx} = 50 - x$$

24. $R = 30x - x^2$

$$\frac{dR}{dx} = 30 - 2x$$

25. $R = -6x^3 + 8x^2 + 200x$

$$\frac{dR}{dx} = -18x^2 + 16x + 200$$

32. $R = 2x(900 + 32x - x^2)$

(a) $R = 1800x + 64x^2 - 2x^3$

$$R'(x) = 1800 + 128x - 6x^2$$

$$R'(14) = \$2416$$

(b) $R(15) - R(14) = 2(15)[900 + 32(15) - 15^2] - 2(14)[900 + 32(14) - 14^2]$

$$= 34,650 - 32,256 = \$2394$$

(c) The answers are close.

26. $R = 50(20x - x^{3/2})$

$$\frac{dR}{dx} = 50 \left[20 - \frac{3}{2} x^{1/2} \right] = 1000 - 75\sqrt{x}$$

27. $P = -2x^2 + 72x - 145$

$$\frac{dP}{dx} = -4x + 72$$

28. $P = -0.25x^2 + 2000x - 1,250,000$

$$\frac{dP}{dx} = -0.5x + 2000$$

29. $P = 0.0013x^3 + 12x$

$$\frac{dP}{dx} = 0.0039x^2 + 12$$

30. $P = -0.5x^3 + 30x^2 - 164.25x - 1000$

$$\frac{dP}{dx} = -1.5x^2 + 60x - 164.25$$

31. $C = 3.6\sqrt{x} + 500$

(a) $C'(x) = 1.8/\sqrt{x}$

$$C'(9) = \$0.60 \text{ per unit.}$$

(b) $C(10) - C(9) \approx \$0.584$

(c) The answers are close.

33. $P = -0.04x^2 + 25x - 1500$

(a) $\frac{dP}{dx} = -0.08x + 25 = P'(x)$
 $P'(150) = \$13$

(b) $\frac{\Delta P}{\Delta x} = \frac{P(151) - P(150)}{151 - 150} = \frac{1362.96 - 1350}{1} = \12.96

(c) The results are close.

34. $P = 36,000 + 2048\sqrt{x} - \frac{1}{8x^2}, 150 \leq x \leq 275$

$$\begin{aligned}\frac{dP}{dx} &= 2048\left(\frac{1}{2}x^{-1/2}\right) - \frac{1}{8}(-2x^{-3}) \\ &= \frac{1024}{\sqrt{x}} + \frac{1}{4x^3}\end{aligned}$$

(a) When $x = 150$, $\frac{dP}{dx} \approx \$83.61$. (b) When $x = 175$, $\frac{dP}{dx} \approx \$77.41$. (c) When $x = 200$, $\frac{dP}{dx} \approx \$72.41$.
 (d) When $x = 225$, $\frac{dP}{dx} \approx \$68.27$. (e) When $x = 250$, $\frac{dP}{dx} \approx \$64.76$. (f) When $x = 275$, $\frac{dP}{dx} \approx \$61.75$.

35. $P = 1.73t^2 + 190.6t + 16,994$

(a) $P(0) = 16,994$ thousand people

$P(3) = 17,581.37$ thousand people

$P(6) = 18,199.88$ thousand people

$P(9) = 18,849.53$ thousand people

$P(12) = 19,530.32$ thousand people

$P(15) = 20,242.25$ thousand people

$P(18) = 20,985.32$ thousand people

$P(21) = 21,759.53$ thousand people

The population is increasing from 1990 to 2011.

(b) $\frac{dP}{dt} = P'(t) = 3.46t + 190.6$

$\frac{dP}{dt}$ represents the population growth rate.

(c) $P'(0) = 190.6$ thousand people per year

$P'(3) = 200.98$ thousand people per year

$P'(6) = 211.36$ thousand people per year

$P'(9) = 221.74$ thousand people per year

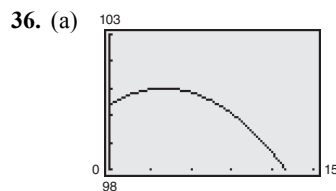
$P'(12) = 232.12$ thousand people per year

$P'(15) = 242.5$ thousand people per year

$P'(18) = 252.88$ thousand people per year

$P'(21) = 263.26$ thousand people per year

The rate of growth is increasing.



(b) For $t < 4$, the slopes are positive, and the fever is increasing. For $t > 4$, the slopes are negative, and the fever is decreasing.

(c) $T(0) = 100.4^\circ\text{F}$

$T'(4) = 101^\circ\text{F}$

$T(8) = 100.4^\circ\text{F}$

$T(12) = 98.6^\circ\text{F}$

(d) $\frac{dT}{dt} = -0.075t + 0.3$; the rate of change of temperature with respect to time

(e) $T'(0) = 0.3^\circ\text{F per hour}$

$T'(4) = 0^\circ\text{F per hour}$

$T'(8) = -0.3^\circ\text{F per hour}$

$T(12) = -0.6^\circ\text{F per hour}$

For $0 \leq t < 4$, the rate of change of the temperature is positive; therefore, the temperature is increasing. For $4 < t \leq 12$, the rate of change of the temperature is decreasing; therefore, the temperature is decreasing back to a normal temperature of 98.6°F .

37. (a) $TR = -10Q^2 + 160Q$

(b) $(TR)' = MR = -20Q + 160$

Q	0	2	4	6	8	10
Model	160	120	80	40	0	-40
Table	-	130	90	50	10	-30

38. (a) $R = xp = x(5 - 0.001x) = 5x - 0.001x^2$

(b) $P = R - C = (5x - 0.001x^2) - (35 + 1.5x)$
 $= -0.001x^2 + 3.5x - 35$

(c) $\frac{dR}{dx} = 5 - 0.002x$
 $\frac{dP}{dx} = 3.5 - 0.002x$

x	600	1200	1800	2400	3000
dR/dx	3.8	2.6	1.4	0.2	-1.0
dP/dx	2.3	1.1	-0.1	-1.3	-2.5
P	1705	2725	3025	2605	1465

39. (a) $(400, 1.75), (500, 1.50)$

$$\text{Slope} = \frac{1.50 - 1.75}{500 - 400} = -0.0025$$

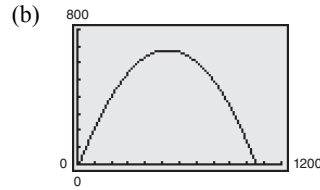
$$p - 1.75 = -0.0025(x - 400)$$

$$p = -0.0025x + 2.75$$

$$P = R - C = xp - c$$

$$= x(-0.0025x + 2.75) - (0.1x + 25)$$

$$= -0.0025x^2 + 2.65x - 25$$



At $x = 300$, P has a positive slope.

At $x = 530$, P has a 0 slope.

At $x = 700$, P has a negative slope.

(c) $P'(x) = -0.005x + 2.65$

$$P'(300) = \$1.15 \text{ per unit}$$

$$P'(530) = \$0 \text{ per unit}$$

$$P'(700) = -\$0.85 \text{ per unit}$$

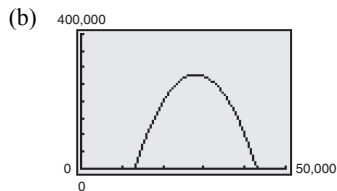
40. $(36,000, 30), (32,000, 35)$

$$\text{Slope} = \frac{35 - 30}{32,000 - 36,000} = -\frac{5}{4000} = -\frac{1}{800}$$

$$p - 30 = -\frac{1}{800}(x - 36,000)$$

$$p = -\frac{1}{800}x + 75 \text{ (demand function)}$$

(a) $P = R - C = xp - C = x\left(-\frac{1}{800}x + 75\right) - (5x + 700,000) = -\frac{1}{800}x^2 + 70x - 700,000$



At $x = 18,000$, P has a positive slope.

At $x = 28,000$, P has a 0 slope.

At $x = 36,000$, P has a negative slope.

(c) $P'(x) = -\frac{1}{400}x + 70$

$$P'(18,000) = \$25 \text{ per ticket}$$

$$P'(28,000) = \$0 \text{ per ticket}$$

$$P'(36,000) = -\$20 \text{ per ticket}$$

$$41. (a) C(x) = \left(\frac{25,000 \text{ km}}{\text{yr}} \right) \left(\frac{1 \text{ L}}{x \text{ km}} \right) \left(\frac{0.70 \text{ dollar}}{1 \text{ L}} \right)$$

$$C(x) = \frac{17,500}{x} \frac{\text{dollars}}{\text{yr}}$$

$$(b) \frac{dC}{dx} = -\frac{17,500}{x^2} \frac{\frac{\text{dollars}}{\text{yr}}}{\frac{\text{km}}{\text{L}}}$$

The marginal cost is the change of savings for a 1-kilometer per liter increase in fuel efficiency.

(c)

x	10	15	20	25	30	35	40
C	1750	1166.67	875	700	583.33	500	437.5
dC/dx	-175	-77.78	-43.75	-28	-19.44	-14.29	-10.94

- (d) The driver who gets 15 kilometers per liter would benefit more than the driver who gets 35 kilometers per liter. The value of dC/dx is a greater savings for $x = 15$ than for $x = 35$.

42. (a) $f'(0.789)$ is the rate of change of the number of liters of gasoline sold when the price is \$0.789/liter.

- (b) In general, it should be negative. Demand tends to decrease as price increases. Answers will vary.

43. (a) Average rate of change from 2000 to 2013: $\frac{\Delta p}{\Delta t} = \frac{16,576.66 - 10,786.85}{13 - 0} \approx \$445.37/\text{yr}$

(b) Average rate of change from 2003 to 2007: $\frac{\Delta p}{\Delta t} = \frac{13,264.82 - 10,453.92}{7 - 3} \approx \$702.73/\text{yr}$

So, the instantaneous rate of change for 2005 is $p'(5) \approx \$702.73/\text{yr}$.

(c) Average rate of change from 2004 to 2006: $\frac{\Delta p}{\Delta t} = \frac{12,463.15 - 10,783.01}{6 - 4} \approx \$840.07/\text{yr}$

So, the instantaneous rate of change for 2005 is $p'(5) \approx \$840.07/\text{yr}$.

- (d) The average rate of change from 2004 to 2006 is a better estimate because the data is closer to the years in question.

44. Answers will vary. *Sample answer:*

The rate of growth in the lag phase is relatively slow when compared with the rapid growth in the acceleration phase.

The population grows slower in the deceleration phase, and there is no growth at equilibrium. These changes could be explained by food supply or seasonal growth.

Section 2.4 The Product and Quotient Rules

Skills Warm Up

$$\begin{aligned} 1. (x^2 + 1)(2) + (2x + 7)(2x) &= 2x^2 + 2 + 4x^2 + 14x \\ &= 6x^2 + 14x + 2 \\ &= 2(3x^2 + 7x + 1) \end{aligned}$$

$$\begin{aligned} 2. (2x - x^3)(8x) + (4x^2)(2 - 3x^2) &= 16x^2 - 8x^4 + 8x^2 - 12x^4 \\ &= 24x^2 - 20x^4 \\ &= 4x^2(6 - 5x^2) \end{aligned}$$

$$3. x(4)(x^2 + 2)^3(2x) + (x^2 + 4)(1) = 8x^2(x^2 + 2)^3(x^2 + 4)$$

Skills Warm Up —continued—

$$4. \quad x^2(2)(2x+1)(2) + (2x+1)^4(2x) = 4x^2(2x+1) + 2x(2x+1)^4 \\ = 2x(2x+1)\left[2x + (2x+1)^3\right]$$

$$5. \quad \frac{(2x+7)(5) - (5x+6)(2)}{(2x+7)^2} = \frac{10x+35-10x-12}{(2x+7)^2} \\ = \frac{23}{(2x+7)^2}$$

$$6. \quad \frac{(x^2-4)(2x+1) - (x^2+x)(2x)}{(x^2-4)^2} = \frac{2x^3+x^2-8x-4-2x^3-2x^2}{(x^2-4)^2} \\ = \frac{-x^2-8x-4}{(x^2-4)^2}$$

$$7. \quad \frac{(x^2+1)(2) - (2x+1)(2x)}{(x^2+1)^2} = \frac{2x^2+2-4x^2-2x}{(x^2+1)^2} \\ = \frac{-2x^2-2x+2}{(x^2+1)^2} \\ = \frac{-2(x^2+x-1)}{(x^2+1)^2}$$

$$8. \quad \frac{(1-x^4)(4) - (4x-1)(-4x^3)}{(1-x^4)^2} = \frac{4-4x^4+16x^4-4x^3}{(1-x^4)^2} \\ = \frac{12x^4-4x^3+4}{(1-x^4)^2} \\ = \frac{4(3x^4-x^3+1)}{(1-x^4)^2}$$

$$9. \quad (x^{-1}+x)(2) + (2x-3)(-x^{-2}+1) = 2x^{-1}+2x + (-2x^{-1}+2x+3x^{-2}-3) \\ = 4x+3x^{-2}-3 \\ = 4x+\frac{3}{x^2}-3 \\ = \frac{4x^3-3x^2+3}{x^2}$$

$$10. \quad \frac{(1-x^{-1})(1) - (x-4)(x^{-2})}{(1-x^{-1})^2} = \left(\frac{1-x^{-1}-x^{-1}+4x^{-2}}{1-2x^{-1}+x^{-2}}\right)\left(\frac{x^2}{x^2}\right) \\ = \frac{x^2-2x+4}{x^2-2x+1} \\ = \frac{x^2-2x+4}{(x-1)^2}$$

Skills Warm Up —continued—

11. $f(x) = 3x^2 - x + 4$

$f'(x) = 6x - 1$

$f'(2) = 6(2) - 1$

$= 12 - 1$

$= 11$

12. $f(x) = -x^3 + x^2 + 8x$

$f'(x) = -3x^2 + 2x + 8$

$f'(2) = -3(2^2) + 2(2) + 8$

$= -3(4) + 4 + 8$

$= 0$

13. $f(x) = \frac{2}{7x} = \frac{2}{7}x^{-1}$

$f'(x) = -\frac{2}{7}x^{-2} = -\frac{2}{7x^2}$

$f'(2) = -\frac{2}{7(2)^2}$

$= -\frac{1}{14}$

14. $f(x) = x^2 - \frac{1}{x^2}$

$f'(x) = 2x + \frac{2}{x^3}$

$f'(2) = 2(2) + \frac{2}{2^3}$

$= 4 + \frac{2}{8}$

$= 4 + \frac{1}{4}$

$= \frac{17}{4}$

1. $f(x) = (2x - 3)(1 - 5x)$

$f'(x) = (2x - 3)(-5) + (1 - 5x)(2)$

$= -10x + 15 + 2 - 10x$

$= -20x + 17$

2. $g(x) = (4x - 7)(3x + 1)$

$g'(x) = (4x - 7)(3) + (3x + 1)(4)$

$= 12x - 21 + 12x + 4$

$= 24x - 17$

3. $f(x) = (6x - x^2)(4 + 3x)$

$f'(x) = (6x - x^2)(3) + (4 + 3x)(6 - 2x)$

$= 18x - 3x^2 + 24 - 8x + 18x - 6x^2$

$= -9x^2 + 28x + 24$

4. $f(x) = (5x - x^3)(2x + 9)$

$f'(x) = (5x - x^3)(2) + (2x + 9)(5 - 3x^2)$

$= 10x - 2x^3 + 10 - 6x^3 + 45 - 27x^2$

$= -8x^3 - 27x^2 + 20x + 45$

5. $f(x) = x(x^2 + 3)$

$f'(x) = x(2x) + (x^2 + 3)(1)$

$= 2x^2 + x^2 + 3$

$= 3x^2 + 3$

6. $f(x) = x^2(3x^3 - 1)$

$f'(x) = x^2(9x^2) + (3x^3 - 1)(2x)$

$= 9x^4 + 6x^4 - 2x$

$= 15x^4 - 2x$

7. $h(x) = \left(\frac{2}{x} - 3\right)(x^2 + 7) = (2x^{-1} - 3)(x^2 + 7)$

$h'(x) = (2x^{-1} - 3)(2x) + (x^2 + 7)(-2x^{-2})$

$= 4 - 6x - 2 - 14x^{-2}$

$= -6x + 2 - \frac{14}{x^2}$

8. $f(x) = (3 - x)\left(\frac{4}{x^2} - 5\right) = (3 - x)(4x^{-2} - 5)$

$f'(x) = (3 - x)(-8x^{-3}) + (4x^{-2} - 5)(-1)$

$= -24x^{-3} + 8x^{-2} - 4x^{-2} + 5$

$= -\frac{24}{x^3} + \frac{4}{x^2} + 5$

$$\begin{aligned}
 9. \quad g(x) &= (x^2 - 4x + 3)(x - 2) \\
 g'(x) &= (x^2 - 4x + 3)(1) + (x - 2)(2x - 4) \\
 &= x^2 - 4x + 3 + 2x^2 - 4x - 4x + 8 \\
 &= 3x^2 - 12x + 11
 \end{aligned}$$

$$\begin{aligned}
 10. \quad g(x) &= (x^2 - 2x + 1)(x^3 - 1) \\
 g'(x) &= (x^2 - 2x + 1)(3x^2) + (x^3 - 1)(2x - 2) \\
 &= 3x^4 - 6x^3 + 3x^2 + 2x^4 - 2x^3 - 2x + 2 \\
 &= 5x^4 - 8x^3 + 3x^2 - 2x + 2
 \end{aligned}$$

$$\begin{aligned}
 11. \quad h(x) &= \frac{x}{x - 5} \\
 h'(x) &= \frac{(x - 5)(1) - x(1)}{(x - 5)^2} = \frac{x - 5 - x}{(x - 5)^2} = -\frac{5}{(x - 5)^2}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad h(x) &= \frac{x^2}{x + 3} \\
 h'(x) &= \frac{(x + 3)(2x) - x^2(1)}{(x + 3)^2} \\
 &= \frac{2x^2 + 6x - x^2}{(x + 3)^2} \\
 &= \frac{x^2 + 6x}{(x + 3)^2}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad f(t) &= \frac{2t^2 - 3}{3t + 1} \\
 f'(t) &= \frac{(3t + 1)(4t) - (2t^2 - 3)(3)}{(3t + 1)^2} \\
 &= \frac{12t^2 + 4t - 6t^2 + 9}{(3t + 1)^2} \\
 &= \frac{6t^2 + 4t + 9}{(3t + 1)^2}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad f(x) &= \frac{7x + 3}{4x - 9} \\
 f'(x) &= \frac{(4x - 9)(7) - (7x + 3)(4)}{(4x - 9)^2} \\
 &= \frac{28x - 63 - 28x - 12}{(4x - 9)^2} \\
 &= -\frac{75}{(4x - 9)^2}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad f(t) &= \frac{t + 6}{t^2 - 8} \\
 f'(t) &= \frac{(t^2 - 8)(1) - (t + 6)(2t)}{(t^2 - 8)^2} \\
 &= \frac{t^2 - 8t - 2t^2 - 12t}{(t^2 - 8)^2} \\
 &= \frac{-t^2 - 12t - 8}{(t^2 - 8)^2}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad g(x) &= \frac{4x - 5}{x^2 - 1} \\
 g'(x) &= \frac{(x^2 - 1)(4) - (4x - 5)(2x)}{(x^2 - 1)^2} \\
 &= \frac{4x^2 - 4 - 8x^2 + 10x}{(x^2 - 1)^2} \\
 &= \frac{-4x^2 + 10x - 4}{(x^2 - 1)^2}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad f(x) &= \frac{x^2 + 6x + 5}{2x - 1} \\
 f'(x) &= \frac{(2x - 1)(2x + 6) - (x^2 + 6x + 5)(2)}{(2x - 1)^2} \\
 &= \frac{4x^2 + 12x - 2x - 6 - 2x^2 - 12x - 10}{(2x - 1)^2} \\
 &= \frac{2x^2 - 2x - 16}{(2x - 1)^2}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad f(x) &= \frac{4x^2 - x + 2}{3 - 4x} \\
 f'(x) &= \frac{(3 - 4x)(8x - 1) - (4x^2 - x + 2)(-4)}{(3 - 4x)^2} \\
 &= \frac{24x - 3 - 32x^2 + 4x + 16x^2 - 4x + 8}{(3 - 4x)^2} \\
 &= \frac{-16x^2 + 24x + 5}{(3 - 4x)^2}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad f(x) &= \frac{6 + 2x^{-1}}{3x - 1} \\
 f'(x) &= \frac{(3x - 1)(-2x^{-2}) - (6 + 2x^{-1})(3)}{(3x - 1)^2} \\
 &= \frac{-6x^{-1} + 2x^{-2} - 18 - 6x^{-1}}{(3x - 1)^2} \\
 &= \frac{2x^{-2} - 12x^{-1} - 18}{(3x - 1)^2} \\
 &= \frac{\frac{2}{x^2} - \frac{12}{x} - 18}{(3x - 1)^2} = \frac{2 - 12x - 18x^2}{x^2(3x - 1)^2}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad f(x) &= \frac{5 - x^{-2}}{x + 2} \\
 f'(x) &= \frac{(x + 2)(2x^{-3}) - (5 - x^{-2})(1)}{x + 2} \\
 &= \frac{2x^{-2} + 4x^{-3} - 5 + x^{-2}}{(x + 2)^2} \\
 &= \frac{4x^{-3} + 3x^{-2} - 5}{(x + 2)^2} \\
 &= \frac{\frac{4}{x^3} + \frac{3}{x^2} - 5}{(x + 2)^2} \\
 &= \frac{4 + 2x - 5x^3}{x^3(x + 2)^2}
 \end{aligned}$$

Function	Rewrite	Differentiate	Simplify
21. $f(x) = \frac{x^3 + 6x}{3}$	$f(x) = \frac{1}{3}x^3 + 2x$	$f'(x) = x^2 + 2$	$f'(x) = x^2 + 2$
22. $f(x) = \frac{x^3 + 2x^2}{10}$	$f(x) = \frac{1}{10}x^3 + \frac{1}{5}x^2$	$f'(x) = \frac{3}{10}x^2 + \frac{2}{5}x$	$f'(x) = \frac{3}{10}x^2 + \frac{2}{5}x$
23. $y = \frac{7x^2}{5}$	$y = \frac{7}{5}x^2$	$y' = \frac{7}{5} \cdot 2x$	$y' = \frac{14}{5}x$
24. $y = \frac{2x^4}{9}$	$y = \frac{2}{9}x^4$	$y' = \frac{2}{9} \cdot 4x^3$	$y' = \frac{8}{9}x^3$
25. $y = \frac{7}{3x^3}$	$y = \frac{7}{3}x^{-3}$	$y' = -7x^{-4}$	$y' = -\frac{7}{x^4}$
26. $y = \frac{4}{5x^2}$	$y = \frac{4}{5}x^{-2}$	$y' = -\frac{8}{5}x^{-3}$	$y' = -\frac{8}{5x^3}$
27. $y = \frac{4x^2 - 3x}{8\sqrt{x}}$	$y = \frac{1}{2}x^{3/2} - \frac{3}{8}x^{1/2}, x \neq 0$	$y' = \frac{3}{4}x^{1/2} - \frac{3}{16}x^{-1/2}$	$y' = \frac{3}{4}\sqrt{x} - \frac{3}{16\sqrt{x}}$
28. $y = \frac{5(3x^2 + 2x)}{6\sqrt[3]{x}}$	$y = \frac{5}{2}x^{5/3} + \frac{5}{3}x^{2/3}, x \neq 0$	$y' = \frac{25}{6}x^{2/3} + \frac{10}{9}x^{-1/3}, x \neq 0$	$y' = \frac{25}{6}\sqrt[3]{x^2} + \frac{10}{9\sqrt[3]{x}}$
29. $y = \frac{x^2 - 4x + 3}{2(x - 1)}$	$y = \frac{1}{2}(x - 3), x \neq 1$	$y' = \frac{1}{2}(1), x \neq 1$	$y' = \frac{1}{2}, x \neq 1$
30. $y = \frac{x^2 - 4}{4(x + 2)}$	$y = \frac{1}{4}(x - 2), x \neq -2$	$y' = \frac{1}{4}(1), x \neq -2$	$y' = \frac{1}{4}, x \neq -2$
31. $f'(x) = (x^3 - 3x)(4x + 3) + (3x^2 - 3)(2x^2 + 3x + 5)$ $= 4x^4 + 3x^3 - 12x^2 - 9x + 6x^4 + 9x^3 + 9x^2 - 9x - 15$ $= 10x^4 + 12x^3 - 3x^2 - 18x - 15$			

Product Rule and Simple Power Rule

$$\begin{aligned}
 32. \quad h'(t) &= (t^5 - 1)(8t - 7) + (5t^4)(4t^2 - 7t - 3) \\
 &= 8t^6 - 7t^5 - 8t + 7 + 20t^6 - 35t^5 - 15t^4 \\
 &= 28t^6 - 42t^5 - 15t^4 - 8t + 7
 \end{aligned}$$

Product Rule and Simple Power Rule

$$\begin{aligned}
 33. \quad h(t) &= \frac{1}{3}(6t - 4) \\
 h'(t) &= \frac{1}{3}(6) = 2
 \end{aligned}$$

Constant Multiple and Simple Power Rules

$$\begin{aligned}
 34. \quad f(x) &= \frac{1}{2}(3x - 8) \\
 f'(x) &= \frac{1}{2}(3) = \frac{3}{2}
 \end{aligned}$$

Constant Multiple and Simple Power Rules

$$\begin{aligned}
 35. \quad f'(x) &= \frac{(x^2 - 1)(3x^2 + 3) - (x^3 + 3x + 2)(2x)}{(x^2 - 1)^2} \\
 &= \frac{3x^4 - 3 - 2x^4 - 6x^2 - 4x}{(x^2 - 1)^2} \\
 &= \frac{x^4 - 6x^2 - 4x - 3}{(x^2 - 1)^2}
 \end{aligned}$$

Quotient Rule and Simple Power Rule

$$\begin{aligned}
 36. \quad f(x) &= \frac{2x^3 - 4x^2 - 9}{x^3 - 5} \\
 f'(x) &= \frac{(x^3 - 5)(3x^2 - 8x) - (2x^3 - 4x^2 - 9)(3x^2)}{(x^3 - 5)^2} \\
 &= \frac{3x^5 - 8x^4 - 15x^2 + 40x - 6x^5 + 12x^4 - 27x^2}{(x^3 - 5)^2} \\
 &= \frac{-3x^5 + 4x^4 - 42x^2 + 40x}{(x^3 - 5)^2}
 \end{aligned}$$

Quotient Rule and Simple Power Rule

$$37. \quad f(x) = \frac{x^2 - x - 20}{x + 4} = \frac{(x - 5)(x + 4)}{(x + 4)} = x - 5, x \neq -4$$

$$f'(x) = 1$$

Simple Power Rule

$$38. \quad h(t) = \frac{3t^2 + 22t + 7}{t + 7} = \frac{(3t + 1)(t + 7)}{t + 7} = 3t + 1, t \neq -7$$

$$h'(t) = 3, t \neq -7$$

Simple Power Rule

$$\begin{aligned}
 39. \quad g(t) &= (2t^3 - 1)^2 = (2t^3 - 1)(2t^3 - 1) \\
 g'(t) &= (2t^3 - 1)(6t^2) + (2t^3 - 1)(6t^2) \\
 &= 12t^2(2t^3 - 1)
 \end{aligned}$$

Product Rule and Simple Power Rule

$$\begin{aligned}
 40. \quad f(x) &= (4x^3 - 2x - 3)^2 = (4x^3 - 2x - 3)(4x^3 - 2x - 3) \\
 f'(x) &= (4x^3 - 2x - 3)(12x^2 - 2) + (4x^3 - 2x - 3)(12x^2 - 2) \\
 &= 48x^5 - 24x^3 - 36x^2 - 8x^3 + 4x + 6 + 48x^5 - 24x^3 - 36x^2 - 8x^3 + 4x + 6 \\
 &= 96x^5 - 48x^3 - 72x^2 - 16x^3 + 8x + 12
 \end{aligned}$$

Product Rule and Simple Power Rule

$$\begin{aligned}
 41. \quad g(s) &= \frac{s^2 - 2s + 5}{\sqrt{s}} = \frac{s^2 - 2s + 5}{s^{1/2}} \\
 g'(s) &= \frac{s^{1/2}(2s - 2) - (s^2 - 2s + 5)(\frac{1}{2}s^{-1/2})}{s} \\
 &= \frac{2s^{3/2} - 2s^{1/2} - \frac{1}{2}s^{3/2} + s^{1/2} - \frac{5}{2}s^{-1/2}}{s} \\
 &= \frac{3}{2}s^{1/2} - s^{-1/2} - \frac{5}{2}s^{-3/2} = \frac{3s^2 - 2s - 5}{2s^{3/2}}
 \end{aligned}$$

Quotient Rule and Simple Power Rule

$$\begin{aligned}
 42. \quad f(x) &= \frac{x^3 - 5x^2 - 6x}{\sqrt{x}} = \frac{x^3 - 5x^2 - 6x}{x^{1/2}} \\
 &= x^{5/2} - 5x^{3/2} - 6x^{1/2} \\
 f'(x) &= \frac{5}{2}x^{3/2} - \frac{15}{2}x^{1/2} - 3x^{-1/2} \\
 &= \frac{5}{2}x^{3/2} - \frac{15}{2}x^{1/2} - \frac{3}{x^{1/2}} \\
 &= \frac{5x^2 - 15x - 6}{2x^{1/2}}
 \end{aligned}$$

Constant Multiple and Simple Power Rules

$$\begin{aligned}
 43. \quad f(x) &= \frac{(x-2)(3x+1)}{4x+2} = \frac{3x^2 - 5x - 2}{4x+2} \\
 f'(x) &= \frac{(4x+2)(6x-5) - (3x^2 - 5x - 2)(4)}{(4x+2)^2} \\
 &= \frac{24x^2 - 8x - 10 - 12x^2 + 20x + 8}{(4x+2)^2} \\
 &= \frac{12x^2 + 12x - 2}{4(2x+1)^2} \\
 &= \frac{2(6x^2 + 6x - 1)}{2(2x+1)^2} \\
 &= \frac{6x^2 + 6x - 1}{2(2x+1)^2}
 \end{aligned}$$

Quotient Rule and Simple Power Rule

$$\begin{aligned}
 44. \quad f(x) &= \frac{(x+1)(2x-7)}{2x+1} = \frac{2x^2 - 5x - 7}{2x+1} \\
 f'(x) &= \frac{(2x+1)(4x-5) - (2x^2 - 5x - 7)(2)}{(2x+1)^2} \\
 &= \frac{8x^2 - 6x - 5 - 4x^2 + 10x + 14}{(2x+1)^2} \\
 &= \frac{4x^2 + 4x + 9}{(2x+1)^2}
 \end{aligned}$$

Quotient Rule and Simple Power Rule

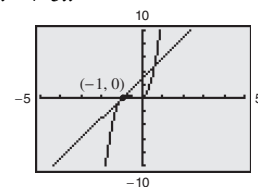
$$\begin{aligned}
 45. \quad f(x) &= (x+4)(2x+9)(x-3) \\
 &= (2x^2 + 17x + 36)(x-3) \\
 f'(x) &= (2x^2 + 17x + 36)(1) + (x-3)(4x+17) \\
 &= (2x^2 + 17x + 36) + (4x^2 + 5x - 51) \\
 &= 6x^2 + 22x - 15
 \end{aligned}$$

Product Rule and Simple Power Rule

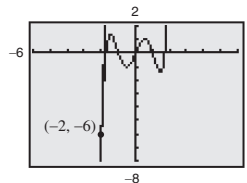
$$\begin{aligned}
 46. \quad f(x) &= (3x^3 + 4x)(x-5)(x+1) \\
 &= (3x^3 + 4x)(x^2 - 4x - 5) \\
 f'(x) &= (3x^3 + 4x)(2x-4) + (x^2 - 4x - 5)(9x^2 + 4) \\
 &= (6x^4 - 12x^3 + 8x^2 - 16x) \\
 &\quad + (9x^4 - 36x^3 - 41x^2 - 16x - 20) \\
 &= 15x^4 - 48x^3 - 33x^2 - 32x - 20
 \end{aligned}$$

Product Rule and Simple Power Rule

$$\begin{aligned}
 47. \quad f(x) &= (5x+2)(x^2+x) \\
 f'(x) &= (5x+2)(2x+1) + (x^2+x)(5) \\
 &= 10x^2 + 9x + 2 + 5x^2 + 5x \\
 &= 15x^2 + 14x + 2 \\
 m &= f'(-1) = 3 \\
 y - 0 &= 3(x - (-1)) \\
 y &= 3x + 3
 \end{aligned}$$



$$\begin{aligned}
 48. \quad f(x) &= (x^2 - 1)(x^3 - 3x) \\
 f'(x) &= (x^2 - 1)(3x^2 - 3) + (x^2 - 3x)(2x) \\
 &= 3x^4 - 6x^2 + 3 + 2x^4 - 6x^2 \\
 &= 5x^4 - 12x^2 + 3 \\
 m &= f'(-2) = 5(-2)^4 - 12(-2)^2 + 3 = 35 \\
 y - (-6) &= 35(x - (-2)) \\
 y + 6 &= 35x + 70 \\
 y &= 35x + 64
 \end{aligned}$$



49. $f(x) = x^3(x^2 - 4)$

$$f'(x) = x^2(2x) + (x^2 - 4)(3x^2)$$

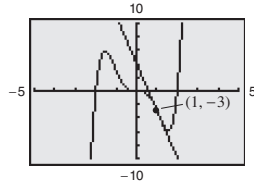
$$= 2x^4 + 3x^4 - 12x^2$$

$$= 5x^4 - 12x^2$$

$$m = f'(1) = -7$$

$$y - (-3) = -7(x - 1)$$

$$y = -7x + 4$$



50. $f(x) = \sqrt{x}(x - 3) = x^{1/2}(x - 3)$

$$f'(x) = x^{1/2}(1) + (x - 3)\left(\frac{1}{2}x^{-1/2}\right)$$

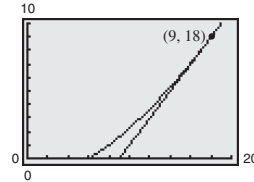
$$= x^{1/2} + \frac{1}{2}x^{1/2} - \frac{3}{2}x^{-1/2}$$

$$= \frac{3}{2}x^{1/2} - \frac{3}{2}x^{-1/2}$$

$$m = f'(9) = \frac{3}{2}(9)^{1/2} - \frac{3}{2}(9)^{-1/2} = \frac{9}{2} - \frac{1}{2} = 4$$

$$y - 18 = 4(x - 9)$$

$$y = 4x - 18$$



51. $f(x) = \frac{3x - 2}{x + 1}$

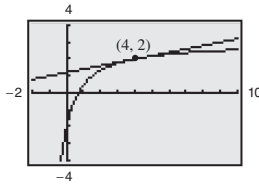
$$f'(x) = \frac{(x + 1)(3) - (3x - 2)(1)}{(x + 1)^2} = \frac{3x + 3 - 3x + 2}{(x + 1)^2} = \frac{5}{(x + 1)^2}$$

$$f'(4) = \frac{1}{5}$$

$$y - 2 = \frac{1}{5}(x - 4)$$

$$y - 2 = \frac{1}{5}x - \frac{4}{5}$$

$$y = \frac{1}{5}x + \frac{6}{5}$$

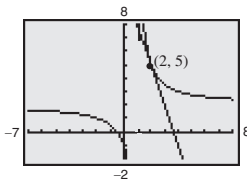


52. $f'(x) = \frac{(x - 1)2 - (2x + 1)}{(x - 1)^2} = \frac{-3}{(x - 1)^2}$

$$f'(2) = -3$$

$$y - 5 = -3(x - 2)$$

$$y = -3x + 11$$



53. $f(x) = \frac{(3x - 2)(6x + 5)}{2x - 3} = \frac{18x^2 + 3x - 10}{2x - 3}$

$$f'(x) = \frac{(2x - 3)(36x + 3) - (18x^2 + 3x - 10)(2)}{(2x - 3)^2}$$

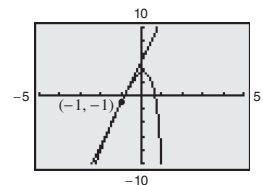
$$= \frac{72x^2 - 102x - 9 - 36x^2 - 6x - 20}{(2x - 3)^2}$$

$$= \frac{36x^2 - 108x + 11}{(2x - 3)^2}$$

$$m = f'(-1) = \frac{36(-1)^2 - 108(-1) + 11}{(2(-1) - 3)^2} = \frac{31}{5}$$

$$y - (-1) = \frac{31}{5}(x - (-1))$$

$$y = \frac{31}{5}x + \frac{26}{5}$$

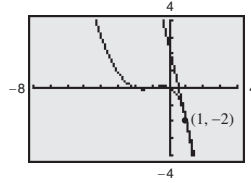


$$54. f(x) = \frac{(x+2)(x^2+x)}{x-4} = \frac{x^3+3x^2+2x}{x-4}$$

$$\begin{aligned} f'(x) &= \frac{(x-4)(3x^2+6x+2) - (x^3+3x^2+2x)(1)}{(x-4)^2} \\ &= \frac{3x^3-12x^2+6x^2-24x+2x-8-x^3-3x^2-2x}{(x-4)^2} \\ &= \frac{2x^3-9x^2-24x-8}{(x-4)^2} \end{aligned}$$

$$m = f'(1) = \frac{2(1)^3-9(1)^2-24(1)-8}{(1-4)^2} = -\frac{13}{3}$$

$$\begin{aligned} y - (-2) &= -\frac{13}{3}(x-1) \\ y &= \frac{13}{3}x + \frac{7}{3} \end{aligned}$$



$$55. f'(x) = \frac{(x-1)(2x) - x^2(1)}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2}$$

$f'(x) = 0$ when $x^2 - 2x = x(x-2) = 0$, which implies that $x = 0$ or $x = 2$. Thus, the horizontal tangent lines occur at $(0, 0)$ and $(2, 4)$.

$$56. f'(x) = \frac{(x^2+1)(2x) - (x^2)(2x)}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2}$$

$f'(x) = 0$ when $2x = 0$, which implies that $x = 0$. Thus, the horizontal tangent line occurs at $(0, 0)$.

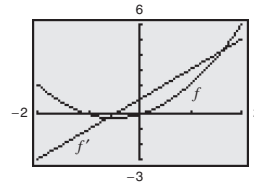
$$57. f'(x) = \frac{(x^3+1)(4x^3) - x^4(3x^2)}{(x^3+1)^2} = \frac{x^6+4x^3}{(x^3+1)^2}$$

$f'(x) = 0$ when $x^6 + 4x^3 = x^3(x^3 + 4) = 0$, which implies that $x = 0$ or $x = \sqrt[3]{-4}$. Thus, the horizontal tangent lines occur at $(0, 0)$ and $(\sqrt[3]{-4}, -2.117)$.

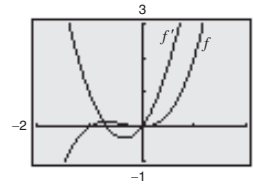
$$\begin{aligned} 58. f'(x) &= \frac{(x^2+1)(4x^3) - (x^4+3)(2x)}{(x^2+1)^2} \\ &= \frac{2x(x^2+3)(x^2-1)}{(x^2+1)^2} \end{aligned}$$

$f'(x) = 0$ when $2x(x^2+3)(x^2-1) = 0$, which implies that $x = 0$ or $x = \pm 1$. Thus, the horizontal tangent lines occur at $(0, 3)$, $(1, 2)$, and $(-1, 2)$.

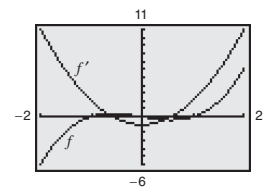
$$\begin{aligned} 59. f(x) &= x(x+1) = x^2+x \\ f'(x) &= 2x+1 \end{aligned}$$



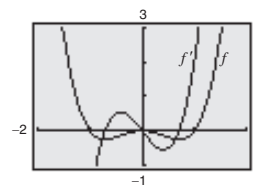
$$\begin{aligned} 60. f(x) &= x^2(x+1) = x^3+x^2 \\ f'(x) &= 3x^2+2x = x(3x+2) \end{aligned}$$



$$\begin{aligned} 61. f(x) &= x(x+1)(x-1) \\ &= x^3-x \\ f'(x) &= 3x^2-1 \end{aligned}$$



$$\begin{aligned} 62. f(x) &= x^2(x+1)(x-1) = x^4-x^2 \\ f'(x) &= 4x^3-2x = 2x(2x^2-1) \end{aligned}$$



$$63. \quad x = 275 \left(1 - \frac{3p}{5p+1} \right)$$

$$\frac{dx}{dp} = -275 \left[\frac{(5p+1)3 - (3p)(5)}{(5p+1)^2} \right] = -275 \left[\frac{3}{(5p+1)^2} \right]$$

When $p = 4$, $\frac{dx}{dp} = -275 \left[\frac{3}{(21)^2} \right] \approx -1.87$ units
per dollar.

$$64. \quad \frac{dx}{dp} = 0 - 1 - \frac{(p+1)(2) - (2p)(1)}{(p+1)^2}$$

$$= -1 - \frac{2}{(p+1)^2}$$

$$= \frac{-(p+1)^2 - 2}{(p+1)^2}$$

$$= \frac{-p^2 - 2p - 3}{(p+1)^2}$$

When $p = 3$, $\frac{dx}{dp} = \frac{-9 - 6 - 3}{16} \approx -1.13$ units
per dollar.

$$65. \quad P' = 500 \left[\frac{(50+t^2)(4) - (4t)(2t)}{(50+t^2)^2} \right] = 500 \left[\frac{200 - 4t^2}{(50+t^2)^2} \right]$$

When $t = 2$, $P' = 500 \left[\frac{184}{(54)^2} \right] \approx 31.55$ bacteria/hour.

$$68. \quad T = 10 \left(\frac{4t^2 + 16t + 75}{t^2 + 4t + 10} \right)$$

Initial temperature: $T(0) = 75^\circ\text{F}$

$$T'(t) = 10 \left(\frac{(t^2 + 4t + 10)(8t + 16) - (4t^2 + 16t + 75)(2t + 4)}{(t^2 + 4t + 10)^2} \right) = \frac{-700(t+2)}{(t^2 + 4t + 10)^2}$$

$$(a) \quad T'(1) \approx -9.33^\circ\text{F/hr}$$

$$(b) \quad T'(3) \approx -3.64^\circ\text{F/hr}$$

$$(c) \quad T'(5) \approx -1.62^\circ\text{F/hr}$$

$$(d) \quad T'(10) \approx -0.37^\circ\text{F/hr}$$

Each rate in parts (a), (b), (c), and (d) is the rate at which the temperature of the food in the refrigerator is changing at that particular time.

$$66. \quad \frac{dP}{dt} = \frac{50(t+2)(1) - (t+1750)(50)}{[50(t+2)]^2}$$

$$= \frac{50[(t+2) - (t+1750)]}{2500(t+2)^2}$$

$$= \frac{-1748}{50(t+2)^2}$$

$$= \frac{-874}{25(t+2)^2}$$

$$(a) \quad \text{When } t = 1, \frac{dP}{dt} = \frac{-874}{225} \approx -3.88 \text{ percent/day.}$$

$$(b) \quad \text{When } t = 10, \frac{dP}{dt} = \frac{-874}{3600}$$

$$= \frac{437}{1800}$$

$$\approx -0.24 \text{ percent/day.}$$

$$67. \quad P = \frac{t^2 - t + 1}{t^2 + 1}$$

$$P' = \frac{(t^2 + 1)(2t - 1) - (t^2 - t + 1)(2t)}{(t^2 + 1)^2} = \frac{t^2 - 1}{(t^2 + 1)^2}$$

$$(a) \quad P'(0.5) = -0.480/\text{week}$$

$$(b) \quad P'(2) = 0.120/\text{week}$$

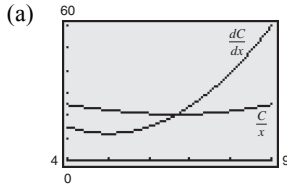
$$(c) \quad P'(8) = 0.015/\text{week}$$

Each rate in parts (a), (b), and (c) is the rate at which the level of oxygen in the pond is changing at that particular time.

69. $C = x^3 - 15x^2 + 87x - 73$, $4 \leq x \leq 9$

Marginal cost: $\frac{dC}{dx} = 3x^2 - 30x + 87$

Average cost: $\frac{C}{x} = x^2 - 15x + 87 - \frac{73}{x}$



(b) Point of intersection:

$$3x^2 - 30x + 87 = x^2 - 15x + 87 - \frac{73}{x}$$

$$2x^2 - 15x + \frac{73}{x} = 0$$

$$2x^3 - 15x^2 + 73 = 0$$

$$x \approx 6.683$$

When $x = 6.683$, $\frac{C}{x} = \frac{dC}{dx} \approx 20.50$.

Thus, the point of intersection is (6.683, 20.50).

At this point average cost is at a minimum.

70. (a) As time passes, the percent of people aware of the product approaches approximately 95%.

(b) As time passes, the rate of change of the percent of people aware of the product approaches zero.

71. $C = 100\left(\frac{200}{x^2} + \frac{x}{x+30}\right)$, $x \geq 1$

$$C' = 100\left[-2(200x^{-3}) + \frac{(x+30) - x}{(x+30)^2}\right]$$

$$= 100\left[-\frac{400}{x^3} + \frac{30}{(x+30)^2}\right]$$

(a) $C'(10) = 100\left(-\frac{400}{10^3} + \frac{30}{40^2}\right) = -38.125$

(b) $C'(15) \approx -10.37$

(c) $C'(20) \approx -3.8$

Increasing the order size reduces the cost per item.

An order size of 2000 should be chosen since the cost per item is the smallest at $x = 20$.

72. (a) $P = ax^2 + bx + c$

When $x = 10$, $P = 50$: $50 = 100a + 10b + c$.

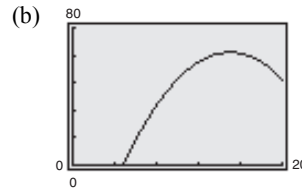
When $x = 12$, $P = 60$: $60 = 144a + 12b + c$.

When $x = 14$, $P = 65$: $65 = 196a + 14b + c$.

Solving this system, we have

$$a = -\frac{5}{8}, b = \frac{75}{4}, \text{ and } c = -75.$$

Thus, $P = -\frac{5}{8}x^2 + \frac{75}{4}x - 75$.



(c) Marginal profit: $P' = -\frac{5}{4}x + \frac{75}{4} = 0 \Rightarrow x = 15$

This is the maximum point on the graph of P , so selling 15 units will maximize the profit.

73. $f(x) = 2g(x) + h(x)$

$$f'(x) = 2g'(x) + h'(x)$$

$$f'(2) = 2(-2) + 4 = 0$$

74. $f(x) = 3 - g(x)$

$$f'(x) = -g'(x)$$

$$f'(2) = -(-2) = 2$$

75. $f(x) = g(x)h(x)$

$$f'(x) = g(x)h'(x) + h(x)g'(x)$$

$$f'(2) = g(2)h'(2) + h(2)g'(2)$$

$$= (3)(4) + (-1)(-2)$$

$$= 14$$

76. $f(x) = \frac{g(x)}{h(x)}$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$$

$$f'(2) = \frac{(-1)(-2) - (3)(4)}{(-1)^2} = -10$$

77. Answers will vary.

Chapter 2 Quiz Yourself

1. $f(x) = 5x + 3$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{[5(x + \Delta x) + 3] - (5x + 3)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{5x + 5\Delta x + 3 - 5x - 3}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{5\Delta x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} 5 = 5 \end{aligned}$$

At $(-2, -7)$: $m = 5$

2. $f(x) = \sqrt{x + 3}$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x + 3} - \sqrt{x + 3}}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x + \Delta x + 3 - (x + 3)}{\Delta x(\sqrt{x + \Delta x + 3} + \sqrt{x + 3})} \\ &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x + 3} + \sqrt{x + 3}} \\ &= \frac{1}{2\sqrt{x + 3}} \end{aligned}$$

At $(1, 2)$: $m = \frac{1}{2\sqrt{1 + 3}} = \frac{1}{4}$

3. $f(x) = 3x - x^2$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{[3(x + \Delta x) - (x + \Delta x)^2] - (3x - x^2)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{3x + 3\Delta x - (x^2 + 2x(\Delta x) + (\Delta x)^2) - (3x - x^2)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{3x + 3\Delta x - x^2 - 2x(\Delta x) - (\Delta x)^2 - 3x + x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{3x - 2x(\Delta x) - (\Delta x)^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(3 - 2x - \Delta x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} (3 - 2x - \Delta x) = 3 - 2x \end{aligned}$$

At $(4, -4)$: $m = 3 - 2(4) = 3 - 8 = -5$

4. $f(x) = 12$

$f'(x) = 0$

5. $f(x) = 19x + 9$

$f'(x) = 19$

6. $f(x) = x^4 - 3x^3 - 5x^2 + 8$

$f'(x) = 4x^3 - 9x^2 - 10x$

7. $f(x) = 12x^{1/4}$

$f'(x) = 3x^{-3/4} = \frac{3}{x^{3/4}}$

8. $f(x) = 4x^{-2}$

$f'(x) = -8x^{-3} = -\frac{8}{x^3}$

9. $f(x) = 10x^{-1/5} + x^{-3}$

$f'(x) = -2x^{-6/5} - 3x^{-4} = -\frac{2}{x^{6/5}} - \frac{3}{x^4}$

$$10. f(x) = \frac{2x+3}{3x+2}$$

$$\begin{aligned} f'(x) &= \frac{(3x+2)(2) - (2x+3)(3)}{(3x+2)^2} \\ &= \frac{6x+4-6x-9}{(3x+2)^2} \\ &= -\frac{5}{(3x+2)^2} \end{aligned}$$

$$11. f(x) = (x^2 + 1)(-2x + 4)$$

$$\begin{aligned} f'(x) &= (x^2 + 1)(-2) + (-2x + 4)(2x) \\ &= -6x^2 + 8x - 2 \end{aligned}$$

$$12. f(x) = (x^2 + 3x + 4)(5x - 2)$$

$$\begin{aligned} f'(x) &= (x^2 + 3x + 4)(5) + (5x - 2)(2x + 3) \\ &= 5x^2 + 15x + 20 + 10x^2 + 11x - 6 \\ &= 15x^2 + 26x + 14 \end{aligned}$$

$$13. f(x) = \frac{4x}{x^2 + 3}$$

$$\begin{aligned} f'(x) &= \frac{(x^2 + 3)(4) - 4x(2x)}{(x^2 + 3)^2} \\ &= \frac{4x^2 + 12 - 8x^2}{(x^2 + 3)^2} \\ &= \frac{-4x^2 + 12}{(x^2 + 3)^2} \\ &= \frac{-4(x^2 - 3)}{(x^2 + 3)^2} \end{aligned}$$

$$14. f(x) = x^2 - 3x + 1; [0, 3]$$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(3) - f(0)}{3 - 0} = \frac{1 - 1}{3} = 0$$

$$f'(x) = 2x - 3$$

Instantaneous rates of change: $f'(0) = -3$, $f'(3) = 3$

$$15. f(x) = 2x^3 + x^3 - x + 4; [-1, 1]$$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{6 - 4}{2} = 1$$

$$f'(x) = 6x^2 + 2x - 1$$

Instantaneous rates of change: $f'(-1) = 3$, $f'(1) = 7$

$$16. f(x) = \frac{1}{3x}; [-5, -2]$$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(-2) - f(5)}{-2 - (-5)} = \frac{-\frac{1}{6} - \left(-\frac{1}{15}\right)}{3} = \frac{-\frac{3}{30}}{3} = -\frac{1}{30}$$

$$f'(x) = -\frac{1}{3x^2}$$

Instantaneous rates of change:

$$f'(-2) = -\frac{1}{12}, f'(-5) = -\frac{1}{75}$$

$$17. f(x) = \sqrt[3]{x}; [8, 27]$$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(27) - f(8)}{27 - 8} = \frac{3 - 2}{19} = \frac{1}{19}$$

$$f(x) = \sqrt[3]{x} = x^{1/3}$$

$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$$

Instantaneous rates of change: $f'(8) = \frac{1}{12}$,

$$f'(27) = \frac{1}{27}$$

$$18. P = -0.0125x^2 + 16x - 600$$

$$(a) \frac{dP}{dx} = -0.025x + 16$$

$$\text{When } x = 175, \frac{dP}{dx} = \$11.625.$$

$$(b) P(176) - P(175) = 1828.8 - 1817.1875 = \$11.6125$$

(c) The results are approximately equal.

$$19. f(x) = 5x^2 + 6x - 1$$

$$f'(x) = 10x + 6$$

At $(-1, -2)$, $m = -4$.

$$y + 2 = -4(x + 1)$$

$$y = -4x - 6$$



$$20. f(x) = \frac{8}{\sqrt{x^3}} = 8x^{-3/2}$$

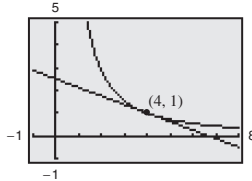
$$f'(x) = -12x^{-5/2} = -\frac{12}{x^{5/2}} = -\frac{12}{x^2\sqrt{x}}$$

$$m = f'(4) = -\frac{12}{(4)^2\sqrt{4}} = -\frac{3}{8}$$

$$y - 1 = -\frac{3}{8}(x - 4)$$

$$y - 1 = -\frac{3}{8}x + \frac{3}{2}$$

$$y = -\frac{3}{8}x + \frac{5}{2}$$



$$22. f(x) = \frac{5x + 4}{2 - 3x}$$

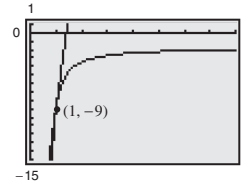
$$f'(x) = \frac{(2 - 3x)(5) - (5x + 4)(-3)}{(2 - 3x)^2} = \frac{10 - 15x + 15x + 12}{(2 - 3x)^2} = \frac{22}{(2 - 3x)^2}$$

$$m = f'(1) = \frac{22}{(2 - 3(1))^2} = 22$$

$$y - (-9) = 22(x - 1)$$

$$y + 9 = 22x - 22$$

$$y = 22x - 31$$



$$23. S = -0.01722t^3 + 0.7333t^2 - 7.657t + 45.47, 7 \leq t \leq 13$$

$$(a) \frac{dS}{dt} = S'(t) = -0.051666t^2 + 1.4666t - 7.657$$

$$(b) 2008: S'(8) \approx \$0.77/\text{yr}$$

$$2011: S'(11) \approx \$2.22/\text{yr}$$

$$2012: S'(12) \approx \$2.50/\text{yr}$$

Section 2.5 The Chain Rule

Skills Warm Up

$$1. \sqrt[5]{(1 - 5x)^2} = (1 - 5x)^{2/5}$$

$$2. \sqrt[4]{(2x - 1)^3} = (2x - 1)^{3/4}$$

$$3. \frac{1}{\sqrt{4x^2 + 1}} = (4x^2 + 1)^{-1/2}$$

$$4. \frac{1}{\sqrt[6]{2x^3 + 9}} = (2x^3 + 9)^{-1/6}$$

$$5. \frac{\sqrt{x}}{\sqrt[3]{1 - 2x}} = x^{1/2}(1 - 2x)^{-1/3}$$

$$6. \frac{\sqrt{(3 - 7x)^3}}{2x} = \frac{(3 - 7x)^{3/2}}{2x} = (2x)^{-1}(3 - 7x)^{3/2}$$

$$7. 3x^3 - 6x^2 + 5x - 10 = 3x^2(x - 2) + 5(x - 2) \\ = (3x^2 + 5)(x - 2)$$

Skills Warm Up —continued—

$$\begin{aligned} 8. \quad 5x\sqrt{x} - x - 5\sqrt{x} + 1 &= x(5\sqrt{x} - 1) - 1(5\sqrt{x} - 1) \\ &= (x - 1)(5\sqrt{x} - 1) \end{aligned}$$

$$\begin{aligned} 9. \quad 4(x^2 + 1)^2 - x(x^2 + 1)^3 &= (x^2 + 1)^2[4 - x(x^2 + 1)] \\ &= (x^2 + 1)^2(4 - x^3 - x) \end{aligned}$$

$$\begin{aligned} 10. \quad -x^5 + 6x^3 + 7x^2 - 42 &= -x^3(x^2 - 6) + 7(x^2 - 6) \\ &= (-x^3 + 7)(x^2 - 6) \\ &= -(x^3 - 7)(x^2 - 6) \end{aligned}$$

$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1. $y = (6x - 5)^4$	$u = 6x - 5$	$y = u^4$
2. $y = (x^2 - 2x + 3)^3$	$u = x^2 - 2x + 3$	$y = u^3$
3. $y = \sqrt{5x - 2}$	$u = 5x - 2$	$y = \sqrt{u}$
4. $y = \sqrt[3]{9 - x^2}$	$u = 9 - x^2$	$y = \sqrt[3]{u}$
5. $y = (3x + 1)^{-1}$	$u = 3x + 1$	$y = u^{-1}$
6. $y = (x^2 - 3)^{-1/2}$	$u = x^2 - 3$	$y = u^{-1/2}$
7. $y = (4x + 7)^2$ $y' = 2(4x + 7)^1(4)$ $y' = 8(4x + 7)$ $= 32x + 56$		
8. $y = (3x^2 - 2)^3$ $y' = 3(3x^2 - 2)^2(6x)$ $y' = 18x(3x^2 - 2)^2$		
9. $y = \sqrt{3 - x^2} = (3 - x^2)^{1/2}$ $y' = \frac{1}{2}(3 - x^2)^{-1/2}(-2x)$ $y' = -x(3 - x^2)^{-1/2} = -\frac{x}{(3 - x^2)^{1/2}}$ $y' = -\frac{x}{\sqrt{3 - x^2}}$		
10. $y = 4\sqrt[4]{6x + 5} = 4(6x + 5)^{1/4}$ $y' = 4\left(\frac{1}{4}\right)(6x + 5)^{-5/4}(6)$ $y' = 6(6x + 5)^{-5/4} = \frac{6}{(6x + 5)^{5/4}}$		

11. $y = (5x^4 - 2x)^{2/3}$ $y' = \left(\frac{2}{3}\right)(5x^4 - 2x)^{-1/3}(20x^3 - 2)$ $y' = \left(\frac{2}{3}\right)(5x^4 - 2x)^{-1/3}(2)(10x^3 - 1)$ $y' = \left(\frac{4}{3}\right)(5x^4 - 2x)^{-1/3}(10x^3 - 1)$ $y' = \frac{4(10x^3 - 1)}{3(5x^4 - 2x)^{1/3}} = \frac{40x^3 - 4}{3\sqrt[3]{5x^4 - 2x}}$	
12. $y = (x^3 + 2x^2)^{-1}$ $y' = (-1)(x^3 + 2x^2)^{-2}(3x^2 + 4x)$ $y' = -\frac{3x^2 + 4x}{(x^3 + 2x^2)^2}$	
13. $f(x) = \frac{2}{1 - x^3} = 2(1 - x^3)^{-1}$; (c) General Power Rule	
14. $f(x) = \frac{7}{(1 - x)^3} = 7(1 - x)^{-3}$; (c) General Power Rule	
15. $f(x) = \sqrt[3]{8^2}$; (b) Constant Rule	
16. $f(x) = \sqrt[3]{x^2} = x^{2/3}$; (a) Simple Power Rule	
17. $f(x) = \frac{x^2 + 9}{x^3 + 4x^2 - 6}$; (d) Quotient Rule	
18. $f(x) = \frac{x^{1/2}}{x^3 + 2x - 5}$; (d) Quotient Rule	
19. $y' = 3(2x - 7)^2(2) = 6(2x - 7)^2$	
20. $y = (3 - 5x)^4$ $y' = 4(3 - 5x)^3(-5) = -20(3 - 5x)^3$	
21. $h'(x) = 2(6x - x^3)(6 - 3x^2) = 6x(6 - x^2)(2 - x^2)$	

$$22. f(x) = (2x^3 - 6x)^{4/3}$$

$$f'(x) = \left(\frac{4}{3}\right)(2x^3 - 6x)^{1/3}(6x^2 - 6)$$

$$f'(x) = \left(\frac{4}{3}\right)(2x^3 - 6x)^{1/3}(6)(x^2 - 1)$$

$$f'(x) = 8(2x^3 - 6x)^{1/3}(x^2 - 1)$$

$$23. f(t) = \sqrt{t+1} = (t+1)^{1/2}$$

$$f'(t) = \frac{1}{2}(t+1)^{-1/2}(1) = \frac{1}{2\sqrt{t+1}}$$

$$24. g(x) = \sqrt{5-3x} = (5-3x)^{1/2}$$

$$g'(x) = \frac{1}{2}(5-3x)^{-1/2}(-3) = -\frac{3}{2\sqrt{5-3x}}$$

$$25. s(t) = \sqrt{2t^2 + 5t + 2} = (2t^2 + 5t + 2)^{1/2}$$

$$s'(t) = \frac{1}{2}(2t^2 + 5t + 2)^{-1/2}(4t + 5) = \frac{4t + 5}{2\sqrt{2t^2 + 5t + 2}}$$

$$26. y = 9\sqrt[3]{4x^2 + 3} = 9(4x^2 + 3)^{1/3}$$

$$y' = 9\left(\frac{1}{3}\right)(4x^2 + 3)^{-2/3}(8x)$$

$$y' = 24x(4x^2 + 3)^{-2/3}$$

$$y' = \frac{24x}{(4x^2 + 3)^{2/3}}$$

$$27. f(x) = 2(2 - 9x)^{-3}$$

$$f'(x) = 2(-3)(2 - 9x)^{-4}(-9) = \frac{54}{(2 - 9x)^4}$$

$$28. g(x) = \frac{3}{(7x^2 + 6x)^5} = 3(7x^2 + 6x)^{-5}$$

$$g'(x) = 3(-5)(7x^2 + 6x)^{-6}(14x + 6)$$

$$g'(x) = -15(7x^2 + 6x)^{-6}(14x + 6)$$

$$g'(x) = -\frac{15(14x + 6)}{(7x^2 + 6x)^6}$$

$$29. f(x) = \frac{1}{\sqrt{(x^2 + 11)^7}} = (x^2 + 11)^{-7/2}$$

$$f'(x) = \left(-\frac{7}{2}\right)(x^2 + 11)^{-9/2}(2x)$$

$$f'(x) = -7x(x^2 + 11)^{-9/2}$$

$$f'(x) = -\frac{7x}{(x^2 + 11)^{9/2}} = -\frac{7x}{\sqrt{(x^2 + 11)^9}}$$

$$30. y = (4 - x^3)^{-4/3}$$

$$y' = \left(-\frac{4}{3}\right)(4 - x^3)^{-7/3}(-3x^2) = \frac{4x^2}{3(4 - x^2)^{7/3}}$$

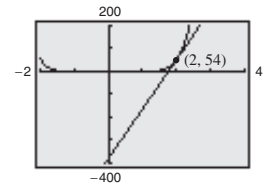
$$31. f'(x) = 2(3)(x^2 - 1)^2(2x) = 12x(x^2 - 1)^2$$

$$f'(2) = 24(3^2) = 216$$

$$f(2) = 54$$

$$y - 54 = 216(x - 2)$$

$$y = 216x - 378$$



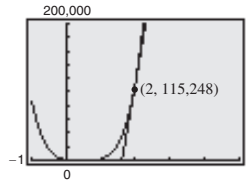
$$32. f'(x) = 12(9x - 4)^3(9) = 108(9x - 4)^3$$

$$f'(2) = 12(14)^3(9) = 296,352$$

$$f(2) = 3(14)^4 = 115,248$$

$$y - 115,248 = 296,352(x - 2)$$

$$y = 296,352x - 477,456$$



$$33. f(x) = \sqrt{4x^2 - 7} = (4x^2 - 7)^{1/2}$$

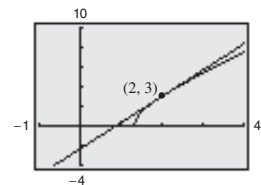
$$f'(x) = \frac{1}{2}(4x^2 - 7)^{-1/2}(8x) = \frac{4x}{\sqrt{4x^2 - 7}}$$

$$f'(2) = \frac{8}{3}$$

$$f(2) = 3$$

$$y - 3 = \frac{8}{3}(x - 2)$$

$$y = \frac{8}{3}x - \frac{7}{3}$$



$$34. f(x) = x\sqrt{x^2 + 5} = x(x^2 + 5)^{1/2}$$

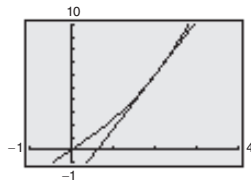
$$\begin{aligned} f'(x) &= x \left[\frac{1}{2}(x^2 + 5)^{-1/2}(2x) \right] + (x^2 + 5)^{1/2}(1) \\ &= x^2(x^2 + 5)^{-1/2} + (x^2 + 5)^{1/2} \\ &= (x^2 + 5)^{-1/2} [x^2 + (x^2 + 5)] \\ &= \frac{2x^2 + 5}{\sqrt{x^2 + 5}} \end{aligned}$$

$$f'(2) = \frac{13}{3}$$

$$f(2) = 6$$

$$y - 6 = \frac{13}{3}(x - 2)$$

$$y = \frac{13}{3}x - \frac{8}{3}$$



$$35. f(x) = \sqrt{x^2 - 2x + 1} = (x^2 - 2x + 1)^{1/2}$$

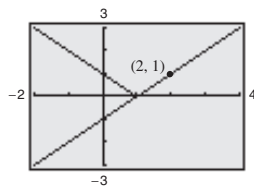
$$\begin{aligned} f'(x) &= \frac{1}{2}(x^2 - 2x + 1)^{-1/2}(2x - 2) \\ &= \frac{x - 1}{\sqrt{x^2 - 2x + 1}} \\ &= \frac{x - 1}{|x - 1|} \end{aligned}$$

$$f'(2) = 1$$

$$f(2) = 1$$

$$y - 1 = 1(x - 2)$$

$$y = x - 1$$



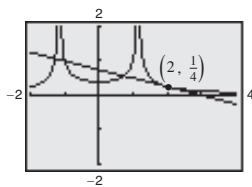
$$36. f'(x) = -\frac{2}{3}(4 - 3x^2)^{-5/3}(-6x) = \frac{4x}{(4 - 3x^2)^{5/3}}$$

$$f'(2) = \frac{4(2)}{(-8)^{5/3}} = \frac{8}{-32} = -\frac{1}{4}$$

$$f(2) = (-8)^{-2/3} = \frac{1}{4}$$

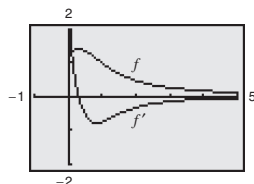
$$y - \frac{1}{4} = -\frac{1}{4}(x - 2)$$

$$y = -\frac{1}{4}x + \frac{3}{4}$$



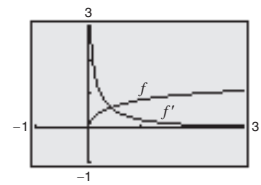
$$37. f'(x) = \frac{1 - 3x^2 - 4x^{3/2}}{2\sqrt{x}(x^2 + 1)^2}$$

f has a horizontal tangent when $f' = 0$.



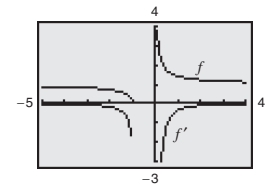
$$38. f'(x) = \frac{\sqrt{2}}{2\sqrt{x}(x+1)^{3/2}}$$

f' is never 0.



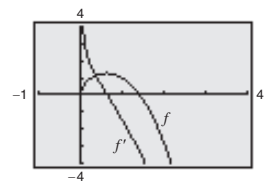
$$39. f'(x) = -\frac{\sqrt{(x+1)/x}}{2x(x+1)}$$

f' is never 0.



$$40. f'(x) = \frac{2 - 5x^2}{2\sqrt{x}}$$

f has a horizontal tangent when $f' = 0$.



$$41. y = (4 - x^2)^{-1}$$

$$\begin{aligned} y' &= (-1)(4 - x^2)^{-2}(-2x) \\ &= \frac{2x}{(4 - x^2)^2} \end{aligned}$$

General Power Rule

$$42. s(t) = \frac{1}{t^2 + 3t - 1} = (t^2 + 3t - 1)^{-1}$$

$$\begin{aligned} s'(t) &= -1(t^2 + 3t - 1)^{-2}(2t + 3) \\ &= -\frac{2t + 3}{(t^2 + 3t - 1)^2} \end{aligned}$$

General Power Rule

$$43. y = -\frac{5t}{(t+8)^2}$$

$$y' = \frac{(t+8)^2(5) - (5t)(2)(t+8)(1)}{(t+8)^4}$$

$$y' = \frac{5(t+8)[(t+8) - 2t]}{(t+8)^4}$$

$$y' = \frac{5(t+8)(-t+8)}{(t+8)^4} = \frac{5(t-8)}{(t+8)^3}$$

Quotient Rule and Chain Rule

$$44. f(x) = 3x(x^3 - 4)^{-2}$$

$$\begin{aligned} f'(x) &= 3x \left[(-2)(x^3 - 4)^{-3}(3x^2) \right] + (x^3 - 4)^{-2}(3) \\ &= -18x^3(x^3 - 4)^{-3} + 3(x^3 - 4)^{-2} \\ &= -3(x^3 - 4)^{-3} [6x^3 - (x^3 - 4)] \\ &= \frac{-3(5x^3 + 4)}{(x^3 - 4)^3} \end{aligned}$$

Product Rule and Chain Rule

$$45. f(x) = (2x - 1)(9 - 3x^2)$$

$$\begin{aligned} f'(x) &= (2x - 1)(-6x) + (9 - 3x^2)(2) \\ &= -12x^2 + 6x + 18 - 6x^2 \\ &= 18 + 6x - 12x^2 \\ &= -6(3x^2 - 2x - 3) \end{aligned}$$

Product Rule and Simple Power Rule

$$46. y = (7x + 4)(x^3 - 2x^2)$$

$$\begin{aligned} y' &= (7x + 4)(3x^2 - 4x) + (x^3 - 2x^2)(7) \\ &= 21x^3 - 16x^2 - 16x + 7x^3 - 14x^2 \\ &= 28x^3 - 30x^2 - 16x \end{aligned}$$

Product Rule and Simple Power Rule

$$47. y = \frac{1}{\sqrt{x+2}} = (x+2)^{-1/2}$$

$$y' = -\frac{1}{2}(x+2)^{-3/2} = -\frac{1}{2(x+2)^{3/2}}$$

General Power Rule

$$48. g(x) = \frac{3}{\sqrt[3]{x^3 - 1}} = 3(x^3 - 1)^{-1/3}$$

$$g'(x) = 3 \left(-\frac{1}{3} \right) (x^3 - 1)^{-4/3} (3x^2) = -\frac{3x^2}{(x^3 - 1)^{4/3}}$$

General Power Rule

$$49. f(x) = x(3x - 9)^3$$

$$\begin{aligned} f'(x) &= x(3)(3x - 9)^2(3) + (3x - 9)^3(1) \\ &= (3x - 9)^2 [9x + (3x - 9)] \\ &= 9(x - 3)^2 (12x - 9) \\ &= 27(x - 3)^2 (4x - 3) \end{aligned}$$

Product and General Power Rule

$$50. f(x) = x^3(x - 4)^2$$

$$\begin{aligned} &= x^3(x^2 - 8x + 16) \\ &= x^5 - 8x^4 + 16x^3 \\ f'(x) &= 5x^4 - 32x^3 + 48x^2 \\ &= x^2(5x^2 - 32x + 48) \\ &= x^2(5x - 12)(x - 4) \end{aligned}$$

Simple Power Rule

$$51. y = x\sqrt{2x+3} = x(2x+3)^{1/2}$$

$$\begin{aligned} y' &= x \left[\frac{1}{2}(2x+3)^{-1/2}(2) \right] + (2x+3)^{1/2} \\ &= (2x+3)^{-1/2} [x + (2x+3)] \\ &= \frac{3(x+1)}{\sqrt{2x+3}} \end{aligned}$$

Product and General Power Rule

$$52. y = 2t\sqrt{t+6} = 2t(t+6)^{1/2}$$

$$\begin{aligned} y' &= 2t \left[\frac{1}{2}(t+6)^{-1/2}(1) \right] + (t+6)^{1/2}(2) \\ &= t(t+6)^{-1/2} + 2(t+6)^{1/2} \\ &= (t+6)^{-1/2} [t + 2(t+6)] \\ &= (t+6)^{-1/2} (3t+12) \\ &= \frac{3t+12}{\sqrt{t+6}} = \frac{3(t+4)}{\sqrt{t+6}} \end{aligned}$$

Product and General Power Rule

$$53. y = t^2\sqrt{t-2} = t^2(t-2)^{1/2}$$

$$\begin{aligned} y' &= t^2 \left[\frac{1}{2}(t-2)^{-1/2}(1) \right] + 2t(t-2)^{1/2} \\ &= \frac{1}{2}(t-2)^{-1/2} [t^2 + 4t(t-2)] \\ &= \frac{t^2 + 4t(t-2)}{2\sqrt{t-2}} \\ &= \frac{t(5t-8)}{2\sqrt{t-2}} \end{aligned}$$

Product and General Power Rule

$$\begin{aligned}
 54. \quad y &= \sqrt{x}(x-2)^2 = x^{1/2}(x-2)^2 \\
 y' &= x^{1/2} \left[2(x-2)^1(1) \right] + (x-2)^2 \left(\frac{1}{2}x^{-1/2} \right) \\
 &= 2\sqrt{x}(x-2) + \frac{(x-2)^2}{2\sqrt{x}} \\
 &= \frac{4x(x-2) + (x-2)^2}{2\sqrt{x}} \\
 &= \frac{(x-2)[4x + (x-2)]}{2\sqrt{x}} \\
 &= \frac{(x-2)(5x-2)}{2\sqrt{x}}
 \end{aligned}$$

Product and General Power Rule

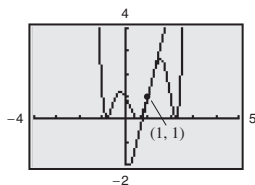
$$\begin{aligned}
 55. \quad y &= \left(\frac{6-5x}{x^2-1} \right)^2 \\
 y' &= 2 \left(\frac{6-5x}{x^2-1} \right) \left[\frac{(x^2-1)(-5) - (6-5x)(2x)}{(x^2-1)^2} \right] \\
 &= \frac{2(6-5x)(5x^2-12x+5)}{(x^2-1)^3}
 \end{aligned}$$

Quotient and General Power Rule

$$\begin{aligned}
 56. \quad y &= \left(\frac{4x^2-5}{2-x} \right)^3 \\
 y' &= 3 \left(\frac{4x^2-5}{2-x} \right)^2 \left[\frac{(2-x)(8x) - (4x^2-5)(-1)}{(2-x)^2} \right] \\
 &= 3 \left(\frac{4x^2-5}{2-x} \right)^2 \left[\frac{16x-8x^2+4x^2-5}{(2-x)^2} \right] \\
 &= 3 \left(\frac{4x^2-5}{2-x} \right)^2 \left[\frac{-4x^2+16x-5}{(2-x)^2} \right] \\
 &= \frac{3(4x^2-5)^2(-4x^2+16x-5)}{(2-x)^3}
 \end{aligned}$$

Quotient and General Power Rule

$$\begin{aligned}
 57. \quad y &= (x^3 - 2x^2 - x + 1)^2 \\
 y' &= 2(x^3 - 2x^2 - x + 1)(3x^2 - 4x - 1) \\
 m &= y'(1) = 2(1^3 - 2(1)^2 - (1) + 1)(3(1)^2 - 4(1) - 1) \\
 &= 2(-1)(-2) = 4 \\
 y - 1 &= 4(x - 1) \\
 y &= 4x - 3
 \end{aligned}$$

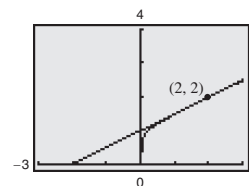


$$\begin{aligned}
 58. \quad f(x) &= (3x^3 + 4x)^{1/5} \\
 f'(x) &= \frac{1}{5}(3x^3 + 4x)^{-4/5}(9x^2 + 4) \\
 &= \frac{9x^2 + 4}{5(3x^3 + 4x)^{4/5}}
 \end{aligned}$$

$$m = f'(2) = \frac{1}{2}$$

$$y - 2 = \frac{1}{2}(x - 2)$$

$$y = \frac{1}{2}x + 1$$



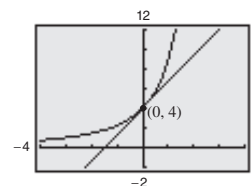
$$59. \quad f(t) = \frac{36}{(3-t)^2} = 36(3-t)^{-2}$$

$$f'(t) = -72(3-t)^{-3}(-1) = \frac{72}{(3-t)^3}$$

$$f'(0) = \frac{72}{27} = \frac{8}{3}$$

$$y - 4 = \frac{8}{3}(t - 0)$$

$$y = \frac{8}{3}t + 4$$



$$60. \quad s(x) = \frac{1}{\sqrt{x^2 - 3x + 4}} = (x^2 - 3x + 4)^{-1/2}$$

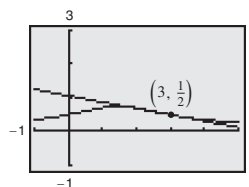
$$s'(x) = -\frac{1}{2}(x^2 - 3x + 4)^{-3/2}(2x - 3)$$

$$= \frac{3 - 2x}{2(x^2 - 3x + 4)^{3/2}}$$

$$s'(3) = \frac{3-6}{2(4)^{3/2}} = -\frac{3}{16}$$

$$y - \frac{1}{2} = -\frac{3}{16}(x - 3)$$

$$y = -\frac{3}{16}x + \frac{17}{16}$$



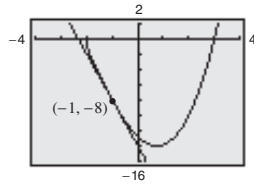
$$61. f(t) = (t^2 - 9)\sqrt{t+2} = (t^2 - 9)(t+2)^{1/2}$$

$$\begin{aligned} f'(t) &= (t^2 - 9)\left[\frac{1}{2}(t+2)^{-1/2}\right] + (t+2)^{1/2}(2t) \\ &= \frac{1}{2}(t^2 - 9)(t+2)^{-1/2} + 2t(t+2)^{1/2} \\ &= (t+2)^{-1/2}\left[\frac{1}{2}(t^2 - 9) + 2t(t+2)\right] \\ &= (t+2)^{-1/2}\left[\frac{1}{2}t^2 - \frac{9}{2} + 2t^2 + 4t\right] \\ &= (t+2)^{-1/2}\left(\frac{5}{2}t^2 + 4t - \frac{9}{2}\right) \\ &= \frac{\frac{5}{2}t^2 + 4t - \frac{9}{2}}{\sqrt{t+2}} \end{aligned}$$

$$f'(-1) = -6$$

$$y - (-8) = -6[t - (-1)]$$

$$y = -6t - 14$$



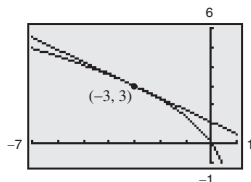
$$\begin{aligned} 62. y &= -\frac{2x}{\sqrt{1-x}} = -\frac{2x}{(1-x)^{1/2}} \\ y' &= -\left[\frac{(1-x)^{1/2}(2) - \left(\frac{1}{2}\right)(1-x)^{-1/2}(-1)(2x)}{\left((1-x)^{1/2}\right)^2}\right] \\ &= -\left[\frac{2(1-x)^{1/2} + x(1-x)^{-1/2}}{1-x}\right] \\ &= -\left[\frac{(1-x)^{-1/2}(2(1-x) + x)}{1-x}\right] \\ &= -\left[\frac{(1-x)^{-1/2}(2-2x+x)}{1-x}\right] \\ &= -\left[\frac{(1-x)^{-1/2}(2-x)}{1-x}\right] \\ &= -\left[\frac{(2-x)}{(1-x)^{3/2}}\right] \\ &= \frac{x-2}{(1-x)^{3/2}} \end{aligned}$$

$$y'(-3) = \frac{(-3) - 2}{(1 - (-3))^{3/2}} = \frac{-5}{4^{3/2}} = -\frac{5}{8}$$

$$y - 3 = -\frac{5}{8}(x - (-3))$$

$$y - 3 = -\frac{5}{8}x - \frac{15}{8}$$

$$y = -\frac{5}{8}x + \frac{9}{8}$$



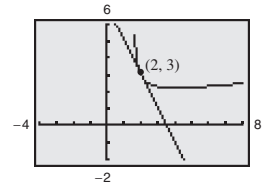
$$\begin{aligned} 63. f(x) &= \frac{x+1}{\sqrt{2x-3}} = \frac{x+1}{(2x-3)^{1/2}} \\ f'(x) &= \frac{(2x-3)^{1/2}(1) - (x+1)\left(\frac{1}{2}\right)(2x-3)^{-1/2}(2)}{(2x-3)} \end{aligned}$$

$$\begin{aligned} &= \frac{(2x-3) - (x+1)}{(2x-3)^{3/2}} \\ &= \frac{x-4}{(2x-3)^{3/2}} \end{aligned}$$

$$f'(2) = \frac{1-3}{1} = -2$$

$$y - 3 = -2(x - 2)$$

$$y = -2x + 7$$

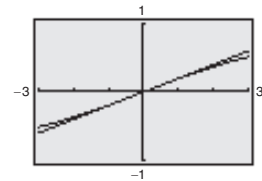


$$\begin{aligned} 64. y &= \frac{x}{\sqrt{25+x^2}} = x(25+x^2)^{-1/2} \\ y' &= x\left[-\frac{1}{2}(25+x^2)^{-3/2}(2x)\right] + (25+x^2)^{-1/2}(1) \\ &= -x^2(25+x^2)^{-3/2} + (25+x^2)^{-1/2} \\ &= (25+x^2)^{-3/2}[-x^2 + (25+x^2)] \\ &= \frac{25}{(25+x^2)^{3/2}} \end{aligned}$$

$$y'(0) = \frac{1}{5}$$

$$y - 0 = \frac{1}{5}(x - 0)$$

$$y = \frac{1}{5}x$$



$$65. f(x) = \sqrt[3]{x^2 + 4} = (x^2 + 4)^{1/3}$$

$$f'(x) = \frac{1}{3}(x^2 + 4)^{-2/3}(2x)$$

$$f'(x) = \frac{2x}{3(x^2 + 4)^{2/3}}$$

$$\text{Set } f'(x) = \frac{2x}{3(x^2 + 4)^{2/3}} = 0.$$

$$2x = 0$$

$$x = 0 \rightarrow y = f(0) = \sqrt[3]{4}$$

Horizontal tangent at: $(0, \sqrt[3]{4})$

$$66. f(x) = \sqrt{5x^2 + x - 3} = (5x^2 + x - 3)^{1/2}$$

$$f'(x) = \frac{1}{2}(5x^2 + x - 3)^{-1/2}(10x + 1)$$

$$f'(x) = \frac{10x + 1}{2(5x^2 + x - 3)^{1/2}}$$

$$\text{Set } f'(x) = \frac{10x + 1}{2(5x^2 + x - 3)^{1/2}} = 0.$$

$$10x + 1 = 0$$

$$x = -\frac{1}{10} \rightarrow y = f\left(-\frac{1}{10}\right) = \sqrt{-\frac{61}{20}}$$

Because $\sqrt{-\frac{61}{20}}$ is not a real number, there is no point of horizontal tangency.

$$67. f(x) = \frac{x}{\sqrt{2x-1}} = \frac{x}{(2x-1)^{1/2}}$$

$$f'(x) = \frac{(2x-1)^{1/2}(1) - x\left(\frac{1}{2}(2x-1)^{-1/2}(2)\right)}{\left[(2x-1)^{1/2}\right]^2}$$

$$f'(x) = \frac{(2x-1)^{1/2} - x(2x-1)^{-1/2}}{(2x-1)}$$

$$f'(x) = \frac{(2x-1)^{-1/2}[(2x-1) - x]}{(2x-1)}$$

$$f'(x) = \frac{x-1}{(2x-1)^{3/2}}$$

$$\text{Set } f'(x) = \frac{x-1}{(2x-1)^{3/2}} = 0.$$

$$x - 1 = 0$$

$$x = 1 \rightarrow y = f(1) = \frac{1}{\sqrt{1}} = 1$$

Horizontal tangent at: (1, 1)

$$68. f(x) = \frac{5x}{\sqrt{3x-2}} = \frac{5x}{(3x-2)^{1/2}}$$

$$f'(x) = \frac{(3x-2)^{1/2}(5) - 5x\left(\frac{1}{2}(3x-2)^{-1/2}(3)\right)}{\left[(3x-2)^{1/2}\right]^2}$$

$$f'(x) = \frac{5(3x-2)^{1/2} - \frac{15}{2}x(3x-2)^{-1/2}}{(2x+1)}$$

$$f'(x) = \frac{\frac{5}{2}(3x-2)^{-1/2}[2(3-x) - 3x]}{(2x-1)}$$

$$f'(x) = \frac{5(3x-4)}{2(3x-2)^{3/2}}$$

$$\text{Set } f'(x) = \frac{5(3x-4)}{2(3x-2)^{3/2}} = 0.$$

$$5(3x-4) = 0$$

$$3x - 4 = 0$$

$$x = \frac{4}{3} \rightarrow y = f\left(\frac{4}{3}\right) = \frac{20}{3\sqrt{2}}$$

Horizontal tangent at: $\left(\frac{4}{3}, \frac{10}{3\sqrt{2}}\right)$

$$69. A' = 1000(60)\left(1 + \frac{r}{12}\right)^{59}\left(\frac{1}{12}\right) = 5000\left(1 + \frac{r}{12}\right)^{59}$$

$$(a) A'(0.08) = 50\left(1 + \frac{0.08}{12}\right)^{59} \\ \approx \$74.00 \text{ per percentage point}$$

$$(b) A'(0.10) = 50\left(1 + \frac{0.10}{12}\right)^{59} \\ \approx \$81.59 \text{ per percentage point}$$

$$(c) A'(0.12) = 50\left(1 + \frac{0.12}{12}\right)^{59} \\ \approx \$89.94 \text{ per percentage point}$$

$$70. N = 400[1 - 3(t^2 + 2)^{-2}]$$

$$\begin{aligned}\frac{dN}{dt} &= N'(t) = 400[(-3)(-2)(t^2 + 2)^{-3}(2t)] \\ &= \frac{4800t}{(t^2 + 2)^3}\end{aligned}$$

- (a) $N'(0) = 0$ bacteria/day
 (b) $N'(1) \approx 177.8$ bacteria/day
 (c) $N'(2) \approx 44.4$ bacteria/day
 (d) $N'(3) \approx 10.8$ bacteria/day
 (e) $N'(4) \approx 3.3$ bacteria/day
 (f) The rate of change of the population is decreasing as time passes.

$$71. V = \frac{k}{\sqrt{t+1}}$$

When $t = 0$, $V = 10,000$.

$$10,000 = \frac{k}{\sqrt{0+1}} \Rightarrow k = 10,000$$

$$V = \frac{10,000}{\sqrt{t+1}}$$

$$V = 10,000(t+1)^{-1/2}$$

$$\frac{dV}{dt} = -5000(t+1)^{-3/2}(1) = -\frac{5000}{(t+1)^{3/2}}$$

When $t = 1$,

$$\frac{dV}{dt} = -\frac{5000}{(2)^{3/2}} = -\frac{2500}{\sqrt{2}} \approx -\$1767.77 \text{ per year.}$$

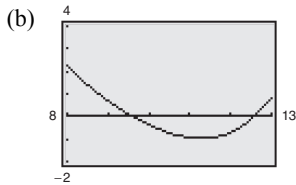
$$\text{When } t = 3, \frac{dV}{dt} = -\frac{5000}{(4)^{3/2}} = -\$625.00 \text{ per year.}$$

72. (a) From the graph, the tangent line at $t = 4$ is steeper than the tangent line at $t = 1$. So, the rate of change after 4 hours is greater.
 (b) The cost function is a composite function of x units, which is a function of the number of hours, which is not a linear function.

$$73. (a) r = (0.3017t^4 - 9.657t^3 + 97.35t^2 - 266.8t - 242)^{1/2}$$

$$\begin{aligned}\frac{dr}{dt} &= r'(t) = \frac{1}{2}(0.3017t^4 - 9.657t^3 + 97.35t^2 - 266.8t - 242)^{-1/2} \cdot (1.2068t^3 - 28.971t^2 + 194.7t - 266.8) \\ &= \frac{1.2068t^3 - 28.971t^2 + 194.7t - 266.8}{2\sqrt{0.3017t^4 - 9.657t^3 + 97.35t^2 - 266.8t - 242}}\end{aligned}$$

Chain Rule



- (c) The rate of change appears to be the greatest when $t = 8$ or 2008.
 The rate of change appears to be the least when $t \approx 9.60$, or 2009, and when $t \approx 12.57$, or 2012.

Section 2.6 Higher-Order Derivatives

Skills Warm Up

$$1. -16t^2 + 292 = 0$$

$$-16t^2 = -292$$

$$t^2 = \frac{73}{4}$$

$$t = \pm \frac{\sqrt{73}}{2}$$

$$2. -16t^2 + 88t = 0$$

$$-8t(2t - 11) = 0$$

$$-8t = 0 \rightarrow t = 0$$

$$2t - 11 = 0 \rightarrow t = \frac{11}{2}$$

Skills Warm Up —continued—

3. $-16t^2 + 128t + 320 = 0$

$$-16(t^2 - 8t - 20) = 0$$

$$-16(t - 10)(t + 2) = 0$$

$$t - 10 = 0 \rightarrow t = 10$$

$$t + 2 = 0 \rightarrow t = -2$$

4. $-16t^2 + 9t + 1440 = 0$

$$t = \frac{-9 \pm \sqrt{9^2 - 4(-16)(1440)}}{2(-16)}$$

$$= \frac{-9 \pm \sqrt{92241}}{-32}$$

$$= \frac{9 \pm 3\sqrt{10249}}{32}$$

$$t \approx -9.21 \text{ and } t \approx 9.77$$

5. $y = x^2(2x + 7)$

$$\frac{dy}{dx} = x^2(2) + 2x(2x + 7)$$

$$= 2x^2 + 4x^2 + 14x$$

$$= 6x^2 + 14x$$

6. $y = (x^2 + 3x)(2x^2 - 5)$

$$\frac{dy}{dx} = (x^2 + 3x)(4x) + (2x + 3)(2x^2 - 5)$$

$$= 4x^3 + 12x^2 + 4x^3 - 10x + 6x^2 - 15$$

$$= 8x^3 + 18x^2 - 10x - 15$$

7. $y = \frac{x^2}{2x + 7}$

$$\frac{dy}{dx} = \frac{(2x + 7)(2x) - (x^2)(2)}{(2x + 7)^2}$$

$$= \frac{4x^2 + 14x - 2x^2}{(2x + 7)^2}$$

$$= \frac{2x^2 + 14x}{(2x + 7)^2}$$

$$= \frac{2x(x + 7)}{(2x + 7)^2}$$

8. $y = \frac{x^2 + 3x}{2x^2 - 5}$

$$\frac{dy}{dx} = \frac{(2x^2 - 5)(2x + 3) - (x^2 + 3x)(4x)}{(2x^2 - 5)^2}$$

$$= \frac{4x^3 + 6x^2 - 10x - 15 - 4x^3 - 12x^2}{(2x^2 - 5)^2}$$

$$= \frac{-6x^2 - 10x - 15}{(2x^2 - 5)^2}$$

9. $f(x) = x^2 - 4$

$$\text{Domain: } (-\infty, \infty)$$

$$\text{Range: } [-4, \infty)$$

10. $f(x) = \sqrt{x - 7}$

$$\text{Domain: } [7, \infty)$$

$$\text{Range: } [0, \infty)$$

1. $f(x) = 9 - 2x$

$$f'(x) = -2$$

$$f''(x) = 0$$

2. $f(x) = 4x + 15$

$$f'(x) = 4$$

$$f''(x) = 0$$

3. $f(x) = x^2 + 7x - 4$

$$f'(x) = 2x + 7$$

$$f''(x) = 2$$

4. $f(x) = 3x^2 + 4x$

$$f'(x) = 6x + 4$$

$$f''(x) = 6$$

5. $g(t) = \frac{1}{3}t^3 - 4t^2 + 2t$

$$g'(t) = t^2 - 8t + 2$$

$$g''(t) = 2t - 8$$

6. $f(x) = -\frac{5}{4}x^4 + 3x^2 - 6x$

$$f'(x) = -5x^3 + 6x - 6$$

$$f''(x) = -15x^2 + 6$$

$$7. f(t) = \frac{2}{t^3} = 2t^{-3}$$

$$f'(t) = -6t^{-4}$$

$$f''(t) = 24t^{-5} = \frac{24}{t^5}$$

$$8. g(t) = \frac{5}{6t^4} = \frac{5}{6}t^{-4}$$

$$g'(t) = -\frac{10}{3}t^{-5}$$

$$g''(t) = \frac{50}{3}t^{-6} = \frac{50}{3t^6}$$

$$9. f(x) = 3(2 - x^2)^3$$

$$f'(x) = 9(2 - x^2)^2(-2x) = -18x(2 - x^2)^2$$

$$\begin{aligned} f''(x) &= (-18x)2(2 - x^2)(-2x) + (2 - x^2)^2(-18) \\ &= 18(2 - x^2)[4x^2 - (2 - x^2)] \\ &= 18(2 - x^2)(5x^2 - 2) \end{aligned}$$

$$10. y = 4(x^2 + 5x)^3$$

$$y' = 4(3)(x^2 + 5x)^2(2x + 5)$$

$$= (24x + 60)(x^4 + 10x^3 + 25x^2)$$

$$= 24x^5 + 300x^4 + 1200x^3 + 1500x^2$$

$$y'' = 120x^4 + 1200x^3 + 3600x^2 + 3000x$$

$$11. f(x) = \frac{x+1}{x-1}$$

$$f'(x) = \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2}$$

$$= -\frac{2}{(x-1)^2} = -2(x-1)^{-2}$$

$$f''(x) = 4(x-1)^{-3}(1) = \frac{4}{(x-1)^3}$$

$$12. g(x) = \frac{1-4x}{x-3}$$

$$g'(x) = \frac{(x-3)(-4) - (1-4x)(1)}{(x-3)^2}$$

$$= \frac{-4x + 12 - 1 + 4x}{(x-3)^2}$$

$$= \frac{11}{(x-3)^2} = 11(x-3)^{-2}$$

$$g''(x) = -22(x-3)^{-3}(1) = -\frac{22}{(x-3)^3}$$

$$13. f(x) = x^5 - 3x^4$$

$$f'(x) = 5x^4 - 12x^3$$

$$f''(x) = 20x^3 - 36x^2$$

$$f'''(x) = 60x^2 - 72x$$

$$14. f(x) = x^4 - 2x^3$$

$$f'(x) = 4x^3 - 6x^2$$

$$f''(x) = 12x^2 - 12x$$

$$f'''(x) = 24x - 12 = 12(2x - 1)$$

$$15. f(x) = 5x(x+4)^3$$

$$= 5x(x^3 + 12x^2 + 48x + 64)$$

$$= 5x^4 + 60x^3 + 240x^2 + 320x$$

$$f'(x) = 20x^3 + 180x^2 + 480x + 320$$

$$f''(x) = 60x^2 + 360x + 480$$

$$f'''(x) = 120x + 360$$

$$16. f(x) = (x^3 - 6)^4$$

$$f'(x) = 4(x^3 - 6)^3(3x^2)$$

$$= 12x^{11} - 216x^8 + 1296x^5 - 2592x^2$$

$$f''(x) = 132x^{10} - 1728x^7 + 6480x^4 - 5184x$$

$$f'''(x) = 1320x^9 - 12,096x^6 + 25,920x^3 - 5184$$

$$17. f(x) = \frac{3}{8x^4} = \frac{3}{8}x^{-4}$$

$$f'(x) = -\frac{3}{2}x^{-5}$$

$$f''(x) = \frac{15}{2}x^{-6}$$

$$f'''(x) = -45x^{-7} = -\frac{45}{x^7}$$

$$18. f(x) = -\frac{2}{25x^5}$$

$$f'(x) = -\frac{2}{25}x^{-5}$$

$$f''(x) = \frac{2}{5}x^{-6}$$

$$f'''(x) = -\frac{12}{5}x^{-7}$$

$$f''''(x) = \frac{84}{5}x^{-8} = \frac{84}{5x^8}$$

19. $g(t) = 5t^4 + 10t^2 + 3$

$g'(t) = 20t^3 + 20t$

$g''(t) = 60t^2 + 20$

$g''(2) = 60(4) + 20 = 260$

20. $f(x) = 9 - x^2$

$f'(x) = -2x$

$f''(x) = -2$

$f''(-\sqrt{5}) = -2$

21. $f(x) = \sqrt{4-x} = (4-x)^{1/2}$

$f'(x) = -\frac{1}{2}(4-x)^{-1/2}$

$f''(x) = -\frac{1}{4}(4-x)^{-3/2}$

$f'''(x) = -\frac{3}{8}(4-x)^{-5/2} = \frac{-3}{8(4-x)^{5/2}}$

$f'''(-5) = \frac{-3}{8(9)^{5/2}} = -\frac{1}{648}$

22. $f(t) = \sqrt{2t+3} = (2t+3)^{1/2}$

$f'(t) = \frac{1}{2}(2t+3)^{-1/2}(2) = (2t+3)^{-1/2}$

$f''(t) = -\frac{1}{2}(2t+3)^{-3/2}(2) = -(2t+3)^{-3/2}$

$f'''(t) = \frac{3}{2}(2t+3)^{-5/2}(2) = \frac{3}{(2t+3)^{5/2}}$

$f'''(\frac{1}{2}) = \frac{3}{32}$

23. $f(x) = (x^3 - 2x)^3 = x^9 - 6x^7 + 12x^5 - 8x^3$

$f'(x) = 9x^8 - 42x^6 + 60x^4 - 24x^2$

$f''(x) = 72x^7 - 252x^5 + 240x^3 - 48x$

$f''(1) = 12$

24. $g(x) = (x^2 + 3x)^4 = x^8 + 12x^7 + 54x^6 + 108x^5 + 81x^4$

$g'(x) = 8x^7 + 84x^6 + 324x^5 + 540x^4 + 324x^3$

$g''(x) = 56x^6 + 504x^5 + 1620x^4 + 2160x^3 + 972x^2$

$g''(-1) = -16$

25. $f'(x) = 2x^2$

$f''(x) = 4x$

26. $f'''(x) = 20x^3 - 36x^2$

$f'''(x) = 60x^2 - 72x = 12x(5x - 6)$

27. $f'''(x) = 4x^{-4}$

$f^{(4)}(x) = -16x^{-5}$

$f^{(5)}(x) = 80x^{-6} = \frac{80}{x^6}$

28. $f''(x) = 4\sqrt{x-2} = 4(x-2)^{1/2}$

$f'''(x) = 4\left(\frac{1}{2}\right)(x-2)^{-1/2}(1) = 2(x-2)^{-1/2}$

$f^{(4)}(x) = 2\left(-\frac{1}{2}\right)(x-2)^{-3/2}(1) = -(x-2)^{-3/2}$

$f^{(5)}(x) = \frac{3}{2}(x-2)^{-5/2}(1) = \frac{3}{2(x-2)^{5/2}}$

29. $f^{(5)}(x) = 2(x^2 + 1)(2x)$

$= 4x^3 + 4x$

$f^{(6)}(x) = 12x^2 + 4$

30. $f'''(x) = 4x + 7$

$f^{(4)}(x) = 4$

$f^{(5)}(x) = 0$

31. $f'(x) = 3x^2 - 18x + 27$

$f''(x) = 6x - 18$

$f'''(x) = 0 \Rightarrow 6x = 18$

$x = 3$

32. $f(x) = (x+2)(x-2)(x+3)(x-3)$

$= (x^2 - 4)(x^2 - 9)$

$= x^4 - 13x^2 + 36$

$f'(x) = 4x^3 - 26x$

$f''(x) = 12x^2 - 26$

$f'''(x) = 0 \Rightarrow 12x^2 = 26$

$x = \pm\sqrt{\frac{13}{6}} = \pm\frac{\sqrt{78}}{6}$

33. $f(x) = x\sqrt{x^2 - 1} = x(x^2 - 1)^{1/2}$

$$f'(x) = x \frac{1}{2}(x^2 - 1)^{-1/2}(2x) + (x^2 - 1)^{1/2} = \frac{x^2}{(x^2 - 1)^{1/2}} + (x^2 - 1)^{1/2}$$

$$\begin{aligned} f''(x) &= \frac{(x^2 - 1)^{1/2}(2x) - x^2 \left(\frac{1}{2}\right)(x^2 - 1)^{-1/2}(2x)}{x^2 - 1} + \frac{1}{2}(x^2 - 1)^{-1/2}(2x) \\ &= \frac{(x^2 - 1)(2x) - x^3}{(x^2 - 1)^{3/2}} + \frac{x}{(x^2 - 1)^{1/2}} \cdot \frac{x^2 - 1}{x^2 - 1} \\ &= \frac{2x^3 - 3x}{(x^2 - 1)^{3/2}} \end{aligned}$$

$$f''(x) = 0 \Rightarrow 2x^3 - 3x = x(2x^2 - 3) = 0$$

$$x = \pm \sqrt{\frac{3}{2}} = \pm \frac{\sqrt{6}}{2}$$

$x = 0$ is not in the domain of f .

34. $f'(x) = \frac{(x^2 + 3)(1) - (x)(2x)}{(x^2 + 3)^2} = \frac{3 - x^2}{(x^2 + 3)^2} = (3 - x^2)(x^2 + 3)^{-2}$

$$\begin{aligned} f''(x) &= (3 - x^2) \left[-2(x^2 + 3)^{-3}(2x) \right] + (x^2 + 3)^{-2}(-2x) \\ &= -2x(x^2 + 3)^{-3} \left[2(3 - x^2) + (x^2 + 3) \right] \\ &= \frac{-2x(9 - x^2)}{(x^2 + 3)^3} \\ &= \frac{2x(x^2 - 9)}{(x^2 + 3)^3} \end{aligned}$$

$$f'''(x) = 0 \Rightarrow 2x(x^2 - 9) = 0$$

$$x = 0, \pm 3$$

35. (a) $s(t) = -4.9t^2 + 44.1t$

$$v(t) = s'(t) = -9.8t + 44.1$$

$$a(t) = v'(t) = s''(t) = -9.8$$

(b) $s(3) = 88.2$ m

$$v(3) = 14.7$$
 m/sec

$$a(3) = -9.8$$
 m/sec²

(c) $v(t) = 0$

$$-9.8t + 44.1 = 0$$

$$-9.8t = -44.1$$

$$t = 4.5$$
 sec

$$s(4.5) = 99.225$$
 m

(d) $s(t) = 0$

$$-4.9t^2 + 44.1t = 0$$

$$-4.9t(t - 9) = 0$$

$$t = 0 \text{ sec} \quad t = 9 \text{ sec}$$

$$v(9) = -9.8(9) + 44.1 = -44.1$$
 m/sec

This is the same speed as the initial velocity.

36. (a) $s(t) = -4.9t^2 + 381$

$$v(t) = s'(t) = -9.8t$$

$$a(t) = v'(t) = -9.8$$

(b) $s(t) = 0$ when $4.9t^2 = 381$, or

$$t = \sqrt{\frac{381}{4.9}} \approx 8.8 \text{ sec.}$$

(c) $v(8.8) = -86.42$ m/sec

$$37. \frac{dv}{dt} = \frac{(t+10)(27.5) - (27.5t)(1)}{(t+10)^2} = \frac{275}{(t+10)^2}$$

t	0	10	20	30	40	50	60
v	0	13.75	18.33	20.63	22	22.92	23.57
$\frac{dv}{dt}$	2.75	0.69	0.31	0.17	0.11	0.08	0.06

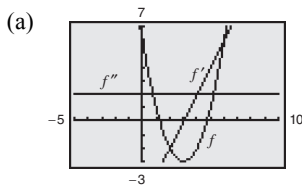
As time increases, the acceleration decreases. After 1 minute, the automobile is traveling at about 23.57 meters per second.

$$38. \begin{aligned} s(t) &= -2.5t^2 + 20t \\ v(t) &= s'(t) = -5t + 20 \\ a(t) &= s''(t) = -5 \end{aligned}$$

t	0	1	2	3	4
$s(t)$	0	17.5	30	37.5	40
$v(t)$	20	15	10	5	0
$a(t)$	-5	-5	-5	-5	-5

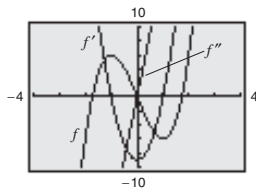
It takes 4 seconds for the car to stop, at which time it has traveled 40 meters.

$$39. \begin{aligned} f(x) &= x^2 - 6x + 6 \\ f'(x) &= 2x - 6 \\ f''(x) &= 2 \end{aligned}$$



(b) The degree decreased by 1 for each successive derivative.

$$\begin{aligned} (c) \quad f(x) &= 3x^2 - 9x \\ f'(x) &= 6x - 9 \\ f''(x) &= 6 \end{aligned}$$



(d) The degree decreases by 1 for each successive derivative.

40. Graph A is the position function. Graph B is the velocity function. Graph C is the acceleration function. Explanations will vary. Sample explanation: The position function appears to be a third-degree function, while the velocity is a second-degree function, and the acceleration is a linear function.

$$41. (a) \quad y(t) = -21.944t^3 + 701.75t^2 - 6969.4t + 27,164$$

$$(b) \quad \begin{aligned} y'(t) &= -65.832t^2 + 1403.5t - 6969.4 \\ y''(t) &= -131.664t + 1403.5 \end{aligned}$$

(c) Over the interval $8 \leq t \leq 13$, $y'(t) > 0$; therefore, y is increasing over $8 \leq t \leq 13$, or from 2008 to 20013.

$$\begin{aligned} (d) \quad y''(t) &= 0 \\ -131.664t + 1403.5 &= 0 \\ -131.664t &= -1403.5 \\ t &\approx 10.66 \text{ or } 2010 \end{aligned}$$

$$42. \text{ Let } y = xf(x).$$

$$\begin{aligned} \text{Then, } y' &= xf'(x) + f(x) \\ y'' &= xf''(x) + f'(x) + f'(x) \\ &= xf''(x) + 2f'(x) \\ y''' &= xf'''(x) + f''(x) + 2f''(x) \\ &= xf'''(x) + 3f''(x). \end{aligned}$$

$$\text{In general } y^{(n)} = [xf(x)]^{(n)} = xf^{(n)}(x) + nf^{(n-1)}(x).$$

43. True. If $y = (x+1)(x+2)(x+3)(x+4)$, then y is a fourth-degree polynomial function and its fifth derivative $\frac{d^5y}{dx^5}$ equals 0.

44. True. The second derivative represents the rate of change of the first derivative, the same way that the first derivative represents the rate of change of the function.

45. Answers will vary.

Section 2.7 Implicit Differentiation

Skills Warm Up

1. $x - \frac{y}{x} = 2$

$$x^2 - y = 2x$$

$$-y = 2x - x^2$$

$$y = x^2 - 2x$$

2. $\frac{4}{x-3} = \frac{1}{y}$

$$4y = x - 3$$

$$y = \frac{x-3}{4}$$

3. $xy - x + 6y = 6$

$$xy + 6y = 6 + x$$

$$y(x+6) = 6+x$$

$$y = \frac{6+x}{x+6}$$

$$y = 1, x \neq -6$$

4. $7 + 4y = 3x^2 + x^2y$

$$4y - x^2y = 3x^2 - 7$$

$$y(4 - x^2) = 3x^2 - 7$$

$$y = \frac{3x^2 - 7}{4 - x^2}, x \neq \pm 2$$

5. $x^2 + y^2 = 5$

$$y^2 = 5 - x^2$$

$$y = \pm\sqrt{5 - x^2}$$

6. $x = \pm\sqrt{6 - y^2}$

$$x^2 = 6 - y^2$$

$$x^2 - 6 = -y^2$$

$$6 - x^2 = y^2$$

$$\pm\sqrt{6 - x^2} = y$$

7. $\frac{3x^2 - 4}{3y^2}, (2, 1)$

$$\frac{3(2^2) - 4}{3(1^2)} = \frac{3(4) - 4}{3} = \frac{8}{3}$$

8. $\frac{x^2 - 2}{1 - y}, (0, -3)$

$$\frac{0^2 - 2}{1 - (-3)} = \frac{-2}{4} = -\frac{1}{2}$$

9. $\frac{7x}{4y^2 + 13y + 3}, \left(-\frac{1}{7}, -2\right)$

$$\frac{7\left(-\frac{1}{7}\right)}{4(-2)^2 + 13(-2) + 3} = \frac{-1}{16 - 26 + 3} = \frac{-1}{-7} = \frac{1}{7}$$

1. $x^3y = 6$

$$x^3 \frac{dy}{dx} + 3x^2y = 0$$

$$x^3 \frac{dy}{dx} = -3x^2y$$

$$\frac{dy}{dx} = -\frac{3x^2}{x^3}y = -\frac{3}{x}y$$

2. $3x^2 - y = 8x$

$$6x - \frac{dy}{dx} = 8$$

$$-\frac{dy}{dx} = 8 - 6x$$

$$\frac{dy}{dx} = 6x - 8$$

3. $y^2 = 1 - x^2$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$4. \quad y^3 = 5x^3 + 8x$$

$$3y^2 \frac{dy}{dx} = 15x^2 + 8$$

$$\frac{dy}{dx} = \frac{15x^2 + 8}{3y^2}$$

$$5. \quad y^4 - y^2 + 7y - 6x = 9$$

$$4y^3 \frac{dy}{dx} - 2y \frac{dy}{dx} + 7 \frac{dy}{dx} - 6 = 0$$

$$(4y^3 - 2y + 7) \frac{dy}{dx} = 6$$

$$\frac{dy}{dx} = \frac{6}{4y^3 - 2y + 7}$$

$$6. \quad 4y^3 + 5y^2 - y - 3x^3 = 8x$$

$$12y^2 \frac{dy}{dx} + 10y \frac{dy}{dx} - \frac{dy}{dx} - 9x^2 = 8$$

$$(12y^2 + 10y - 1) \frac{dy}{dx} = 8 + 9x^2$$

$$\frac{dy}{dx} = \frac{8 + 9x^2}{12y^2 + 10y - 1}$$

$$7. \quad xy^2 + 4xy = 10$$

$$y^2 + 2xy \frac{dy}{dx} + 4y + 4x \frac{dy}{dx} = 0$$

$$(2xy + 4x) \frac{dy}{dx} = -y^2 - 4y$$

$$\frac{dy}{dx} = -\frac{y^2 + 4y}{2xy + 4x}$$

$$8. \quad 2xy^3 - x^2y = 2$$

$$2y^3 + 6xy^2 \frac{dy}{dx} - 2xy - x^2 \frac{dy}{dx} = 0$$

$$(6xy^2 - x^2) \frac{dy}{dx} = 2xy - 2y^3$$

$$\frac{dy}{dx} = \frac{2xy - 2y^3}{6xy^2 - x^2}$$

$$9. \quad \frac{2x + y}{x - 5y} = 1$$

$$2x + y = x - 5y$$

$$6y = -x$$

$$y = -\frac{1}{6}x$$

$$\frac{dy}{dx} = -\frac{1}{6}$$

$$10. \quad \frac{xy - y^2}{y - x} = 1$$

$$xy - y^2 = y - x$$

$$y(x - y) = -(x - y)$$

$$y = -1$$

$$\frac{dy}{dx} = 0$$

$$11. \quad \frac{2y}{y^2 + 3} = 4x$$

$$2y = 4x(y^2 + 3)$$

$$2y = 4xy^2 + 12x$$

$$2 \frac{dy}{dx} = 8xy \frac{dy}{dx} + 4y^2 + 12$$

$$2 \frac{dy}{dx} - 8xy \frac{dy}{dx} = 4y^2 + 12$$

$$\frac{dy}{dx} = \frac{4y^2 + 12}{2 - 8xy}$$

$$12. \quad \frac{4y^2}{y^2 - 9} = x^2$$

$$\frac{(y^2 - 9) \left(8y \frac{dy}{dx} \right) - 4y^2 \left(2y \frac{dy}{dx} \right)}{(y^2 - 9)^2} = 2x$$

$$\frac{8y \frac{dy}{dx} (y^2 - 9 - y^2)}{(y^2 - 9)^2} = 2x$$

$$\frac{-72y \frac{dy}{dx}}{(y^2 - 9)^2} = 2x$$

$$\frac{dy}{dx} = \frac{2x(y^2 - 9)^2}{-72y}$$

$$\frac{dy}{dx} = -\frac{x(y^2 - 9)^2}{36y}$$

$$13. \quad x^2 + y^2 = 16$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\text{At } (0, 4), \frac{dy}{dx} = -\frac{0}{4} = 0.$$

14. $x^2 - y^2 = 25$

$$2x - 2y \frac{dy}{dx} = 0$$

$$-2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{x}{y}$$

At (5, 0), $\frac{dy}{dx}$ is undefined.

15. $y + xy = 4$

$$\frac{dy}{dx} + x \frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx}(1 + x) = -y$$

$$\frac{dy}{dx} = -\frac{y}{x+1}$$

At (-5, -1), $\frac{dy}{dx} = -\frac{1}{4}$.

16. $xy - 3y^2 = 2$

$$x \frac{dy}{dx} + y - 6y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x - 6y) = -y$$

$$\frac{dy}{dx} = -\frac{y}{x - 6y}$$

At (7, 2), $\frac{dy}{dx} = -\frac{2}{7 - 6(2)} = \frac{2}{5}$.

17. $x^2 - xy + y^2 = 4$

$$2x - \left(x \frac{dy}{dx} + y\right) + 2y \frac{dy}{dx} = 0$$

$$2x - x \frac{dy}{dx} - y + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2y - x) = y - 2x$$

$$\frac{dy}{dx} = \frac{y - 3x^2}{2y - x}$$

At (-2, -1), $\frac{dy}{dx} = \frac{(-1) - 3(-2)^2}{2(-1) - (-2)} = \frac{-7}{0}$, $\frac{dy}{dx}$ is

undefined.

18. $x^2y + y^3x = -6$

$$x^2 \frac{dy}{dx} + 2xy + y^3 + 3y^2 \frac{dy}{dx}x = 0$$

$$\frac{dy}{dx}(x^2 + 2xy) = -2xy - y^3$$

$$\frac{dy}{dx} = \frac{-(2xy + y^3)}{x^2 + 3xy^2}$$

$$\frac{dy}{dx} = \frac{y(2x + y^2)}{x(x + 3y^2)}$$

At (2, -1), $\frac{dy}{dx} = \frac{(-1)(2(2) + (-1)^2)}{(2)((2) + 3(-1)^2)} = \frac{-5}{10} = -\frac{1}{2}$.

19. $xy - x = y$

$$x \frac{dy}{dx} + y - 1 = \frac{dy}{dx}$$

$$x \frac{dy}{dx} - \frac{dy}{dx} = 1 - y$$

$$\frac{dy}{dx}(x - 1) = 1 - y$$

$$\frac{dy}{dx} = \frac{1 - y}{x - 1} = -\frac{y - 1}{x - 1}$$

At $\left(\frac{3}{2}, 3\right)$, $\frac{dy}{dx} = -\frac{3 - 1}{\frac{3}{2} - 1} = -\frac{2}{\frac{1}{2}} = -4$.

20. $x^3 + y^3 = 6xy$

$$3x^2 + 3y^2 \frac{dy}{dx} = 6\left(x \frac{dy}{dx} + y\right)$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 6x \frac{dy}{dx} + 6y$$

$$(3y^2 - 6x) \frac{dy}{dx} = 6y - 3x^2$$

$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$

$$\frac{dy}{dx} = \frac{3(2y - x^2)}{3(y^2 - 2x)}$$

$$\frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}$$

At $\left(\frac{4}{3}, \frac{8}{3}\right)$, $\frac{dy}{dx} = \frac{4}{5}$.

21. $x^{1/2} + y^{1/2} = 9$

$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2} \frac{dy}{dx} = 0$$

$$x^{-1/2} + y^{-1/2} \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{-1/2}}{y^{-1/2}} = -\sqrt{\frac{y}{x}}$$

At (16, 25), $\frac{dy}{dx} = -\frac{5}{4}$.

22. $x^{2/3} + y^{2/3} = 5$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{-1/3}}{y^{-1/3}} = -\frac{y^{1/3}}{x^{1/3}} = -\sqrt[3]{\frac{y}{x}}$$

At (8, 1), $\frac{dy}{dx} = -\frac{1}{2}$.

23.

$$\sqrt{xy} = x - 2y$$

$$\sqrt{x}\sqrt{y} = x - 2y$$

$$\sqrt{x}\left(\frac{1}{2}y^{-1/2}\frac{dy}{dx}\right) + \sqrt{y}\left(\frac{1}{2}x^{-1/2}\right) = 1 - 2\frac{dy}{dx}$$

$$\frac{\sqrt{x}}{2\sqrt{y}}\frac{dy}{dx} + 2\frac{dy}{dx} = 1 - \frac{\sqrt{y}}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{1 - \frac{\sqrt{y}}{2\sqrt{x}}}{\frac{\sqrt{x}}{2\sqrt{y}} + 2} \cdot \frac{2\sqrt{x}\sqrt{y}}{2\sqrt{x}\sqrt{y}}$$

$$= \frac{2\sqrt{xy} - y}{x + 4\sqrt{xy}}$$

$$= \frac{2(x - 2y) - y}{x + 4(x - 2y)}$$

$$= \frac{2x - 5y}{5x - 8y}$$

At (4, 1), $\frac{dy}{dx} = \frac{1}{4}$.

24.

$$(x + y)^3 = x^3 + y^3$$

$$3(x + y)^2\left(1 + \frac{dy}{dx}\right) = 3x^2 + 3y^2\frac{dy}{dx}$$

$$3(x + y)^2 + 3(x + y)^2\frac{dy}{dx} = 3x^2 + 3y^2\frac{dy}{dx}$$

$$(x + y)^2\frac{dy}{dx} - y^2\frac{dy}{dx} = x^2 - (x + y)^2$$

$$\frac{dy}{dx}\left[(x + y)^2 - y^2\right] = x^2 - (x^2 + 2xy + y^2)$$

$$\frac{dy}{dx} = \frac{-(2xy + y^2)}{x^2 + 2xy} = -\frac{y(2x + y)}{x(x + 2y)}$$

At (-1, 1), $\frac{dy}{dx} = -1$.

25. $y^2(x^2 + y^2) = 2x^2$

$$y^2\left(2x + 2y\frac{dy}{dx}\right) + (x^2 + y^2)\left(2y\frac{dy}{dx}\right) = 4x$$

$$2xy^2 + 2y^3\frac{dy}{dx} + 2x^2y\frac{dy}{dx} + 2y^3\frac{dy}{dx} = 4x$$

$$\frac{dy}{dx}(4y^3 + 2x^2y) = 4x - 2xy^2$$

$$\frac{dy}{dx} = \frac{2x(2 - y^2)}{2y(2y^2 + x^2)}$$

$$\frac{dy}{dx} = \frac{x(2 - y^2)}{y(2y^2 + x^2)}$$

At (1, 1), $\frac{dy}{dx} = \frac{1}{3}$.

26.

$$(x^2 + y^2)^2 = 8x^2y$$

$$2(x^2 + y^2)\left(2x + 2y\frac{dy}{dx}\right) = 8x^2\frac{dy}{dx} + y(16x)$$

$$4x^3 + 4x^2y\frac{dy}{dx} + 4xy^2 + 4y^3\frac{dy}{dx} = 8x^2\frac{dy}{dx} + 16xy$$

$$\frac{dy}{dx}(4x^2y + 4y^3 - 8x^2) = 16xy - 4x^3 - 4xy^2$$

$$\frac{dy}{dx} = \frac{4(4xy - x^3 - xy^2)}{4(x^2y + y^3 - 2x^2)}$$

$$\frac{dy}{dx} = \frac{x(4y - x^2 - y^2)}{x^2y + y^3 - 2x^2}$$

At (2, 2), $\frac{dy}{dx} = 0$.

27. $3x^2 - 2y + 5 = 0$

$$6x - 2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 3x$$

At (1, 4), $\frac{dy}{dx} = 3$.

28. $4x^2 + 2y - 1 = 0$

$$8x + 2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{8x}{2} = -4x$$

$$\frac{dy}{dx}(-1) = -4(-1) = 4$$

$$29. \quad x^2 + y^2 = 4$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\text{At } (\sqrt{3}, 1), \frac{dy}{dx} = -\frac{\sqrt{3}}{1} = -\sqrt{3}.$$

$$30. \quad 4x^2 + 9y^2 = 36$$

$$8x + 18y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{4x}{9y}$$

$$\text{At } \left(\sqrt{5}, \frac{4}{3}\right), \frac{dy}{dx} = -\frac{4\sqrt{5}}{9(4/3)} = -\frac{\sqrt{5}}{3}.$$

$$31. \quad x^2 - y^3 = 0$$

$$2x - 3y^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{2x}{3y^2}$$

$$\text{At } (-1, 1), \frac{dy}{dx} = -\frac{2}{3}.$$

$$32. \quad (4 - x)y^2 = x^3$$

$$y^2 = \frac{x^3}{4 - x}$$

$$2y \frac{dy}{dx} = \frac{(4 - x)(3x^2) - x^3(-1)}{(4 - x)^2}$$

$$2y \frac{dy}{dx} = \frac{12x^2 - 3x^3 + x^3}{(4 - x)^2}$$

$$2y \frac{dy}{dx} = \frac{12x^2 - 2x^3}{(4 - x)^2}$$

$$\frac{dy}{dx} = -\frac{2x^2(x - 6)}{2y(4 - x)^2}$$

$$\frac{dy}{dx} = -\frac{x^2(x - 6)}{y(4 - x)^2}$$

$$\text{At } (2, 7), \frac{dy}{dx} = 2.$$

$$33. \text{ Implicitly: } 1 - 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{1}{2y}$$

$$\text{Explicitly: } y = \pm\sqrt{x - 1}$$

$$= \pm(x - 1)^{1/2}$$

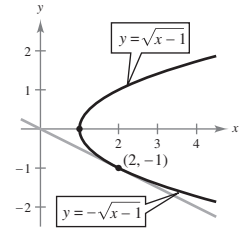
$$\frac{dy}{dx} = \pm \frac{1}{2}(x - 1)^{-1/2}(1)$$

$$= \pm \frac{1}{2\sqrt{x - 1}}$$

$$= \frac{1}{2(\pm\sqrt{x - 1})}$$

$$= \frac{1}{2y}$$

$$\text{At } (2, -1), \frac{dy}{dx} = -\frac{1}{2}.$$



$$34. \text{ Implicitly: } 8y \frac{dy}{dx} - 2x = 0$$

$$\frac{dy}{dx} = \frac{x}{4y}$$

$$\text{Explicitly: } y = \pm \frac{1}{2}\sqrt{x^2 + 7}$$

$$= \pm \frac{1}{2}(x^2 + 7)^{1/2}$$

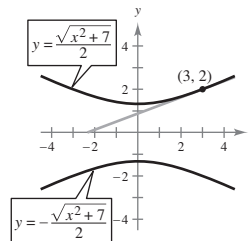
$$\frac{dy}{dx} = \pm \frac{1}{4}(x^2 + 7)^{-1/2}(2x)$$

$$= \pm \frac{x}{2\sqrt{x^2 + 7}}$$

$$= \frac{x}{4\left(\pm \frac{1}{2}\sqrt{x^2 + 7}\right)}$$

$$= \frac{x}{4y}$$

$$\text{At } (3, 2), \frac{dy}{dx} = \frac{3}{8}.$$



35. $x^2 + y^2 = 100$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

At (8, 6):

$$m = -\frac{4}{3}$$

$$y - 6 = -\frac{4}{3}(x - 8)$$

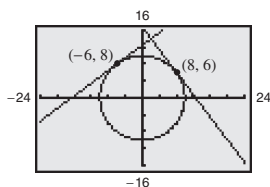
$$y = -\frac{4}{3}x + \frac{50}{3}$$

At (-6, 8):

$$m = \frac{3}{4}$$

$$y - 8 = \frac{3}{4}(x + 6)$$

$$y = \frac{3}{4}x + \frac{25}{2}$$



36. $x^2 + y^2 = 9$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

At (0, 3):

$$m = 0$$

$$y - 3 = 0(x - 0)$$

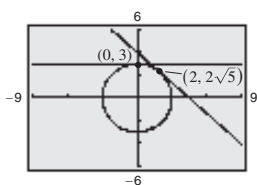
$$y = 3$$

At $(2, \sqrt{5})$:

$$m = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$y - \sqrt{5} = -\frac{2\sqrt{5}}{5}(x - 2)$$

$$y = -\frac{2\sqrt{5}}{5}x + \frac{9\sqrt{5}}{5}$$



37. $y^2 = 5x^3$

$$2y \frac{dy}{dx} = 15x^2$$

$$\frac{dy}{dx} = \frac{15x^2}{2y}$$

At $(1, \sqrt{5})$:

$$m = \frac{15}{2\sqrt{5}} = \frac{3\sqrt{5}}{2}$$

$$y - \sqrt{5} = \frac{3\sqrt{5}}{2}(x - 1)$$

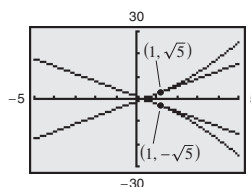
$$y = \frac{3\sqrt{5}}{2}x - \frac{\sqrt{5}}{2}$$

At $(1, -\sqrt{5})$:

$$m = \frac{-15}{2\sqrt{5}} = -\frac{3\sqrt{5}}{2}$$

$$y + \sqrt{5} = -\frac{3\sqrt{5}}{2}(x - 1)$$

$$y = -\frac{3\sqrt{5}}{2}x + \frac{\sqrt{5}}{2}$$



38. $4xy + x^2 = 5$

$$4x \frac{dy}{dx} + 4y + 2x = 0$$

$$\frac{dy}{dx} = -\frac{4y + 2x}{4x} = -\frac{2y + x}{2x}$$

At (1, 1):

$$m = -\frac{3}{2}$$

$$y - 1 = -\frac{3}{2}(x - 1)$$

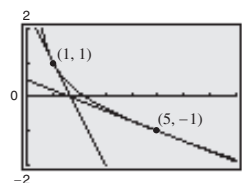
$$y = -\frac{3}{2}x + \frac{5}{2}$$

At (5, -1):

$$m = -\frac{3}{10}$$

$$y + 1 = -\frac{3}{10}(x - 5)$$

$$y = -\frac{3}{10}x + \frac{1}{2}$$



39. $x^3 + y^3 = 8$

$$3x^2 + 3y^2 \frac{dy}{dx} = 0$$

$$3y^2 \frac{dy}{dx} = -3x^2$$

$$\frac{dy}{dx} = -\frac{x^2}{y^2}$$

At (0, 2):

$$m = \frac{dy}{dx} = 0$$

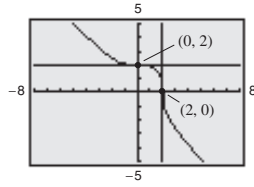
$$y - 2 = 0(x - 0)$$

$$y = 2$$

At (2, 0):

$$m = \frac{dy}{dx} \text{ is undefined.}$$

The tangent line is $x = 2$.



40. $x^2y - 8 = -4y$

$$x^2y + 4y = 8$$

$$y(x^2 + 4) = 8$$

$$y = \frac{8}{x^2 + 4} = 8(x^2 + 4)^{-1}$$

$$\frac{dy}{dx} = 8(-1)(x^2 + 4)^{-2}(2x)$$

$$\frac{dy}{dx} = -\frac{16x}{(x^2 + 4)^2}$$

At (-2, 1):

$$m = \frac{dy}{dx} = -\frac{16(-2)}{((-2)^2 + 4)^2} = \frac{32}{64} = \frac{1}{2}$$

$$y - 1 = \frac{1}{2}(x - (-2))$$

$$y = \frac{1}{2}x + 2$$

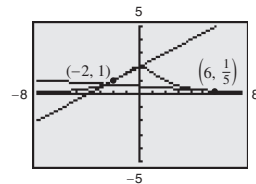
At $(6, \frac{1}{5})$:

$$m = \frac{dy}{dx} = -\frac{16(6)}{[(6)^2 + 4]^2} = -\frac{96}{1600} = -\frac{3}{50}$$

$$y - \frac{1}{5} = -\frac{3}{50}(x - 6)$$

$$y - \frac{1}{5} = -\frac{3}{50}x + \frac{9}{25}$$

$$y = -\frac{3}{50}x + \frac{14}{25}$$

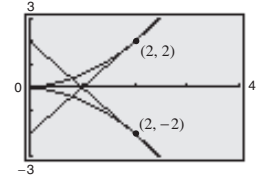


41. $y^2 = \frac{x^3}{4 - x}$

$$2y \frac{dy}{dx} = \frac{(4 - x)(3x^2) - (x^3)(-1)}{(4 - x)^2}$$

$$2y \frac{dy}{dx} = \frac{2x^2(6 - x)}{(4 - x)^2}$$

$$\frac{dy}{dx} = \frac{x^2(6 - x)}{y(4 - x)^2}$$



At (2, 2):

$$m = 2$$

$$y - 2 = 2(x - 2)$$

$$y = 2x - 2$$

At (2, -2):

$$m = -2$$

$$y + 2 = -2(x - 2)$$

$$y = -2x + 2$$

42. $x + y^3 = 6xy^3 - 1$

$$y^3 - 6xy^3 = -1 - x$$

$$y^3(1 - 6x) = -(1 + x)$$

$$y^3 = \frac{x + 1}{6x - 1}$$

$$3y^2 \frac{dy}{dx} = \frac{(6x - 1)(1) - (x + 1)(6)}{(6x - 1)^2}$$

$$3y^2 \frac{dy}{dx} = \frac{6x - 1 - 6x - 6}{(6x - 1)^2}$$

$$\frac{dy}{dx} = -\frac{7}{3y^2(6x - 1)^2}$$

At (-1, 0):

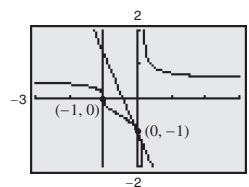
$$m = \frac{dy}{dx} \text{ is undefined. The tangent line is } x = -1.$$

At (0, -1):

$$m = \frac{dy}{dx} = -\frac{7}{3}$$

$$y - (-1) = -\frac{7}{3}(x - 0)$$

$$y = -\frac{7}{3}x - 1$$



$$43. \quad p = \frac{2}{0.00001x^3 + 0.1x}, \quad x \geq 0$$

$$\begin{aligned} 0.00001x^3 + 0.1x &= \frac{2}{p} \\ 0.00003x^2 \frac{dx}{dp} + 0.1 \frac{dx}{dp} &= -\frac{2}{p^2} \\ (0.00003x^2 + 0.1) \frac{dx}{dp} &= -\frac{2}{p^2} \\ \frac{dx}{dp} &= -\frac{2}{p^2(0.00003x^2 + 0.1)} \end{aligned}$$

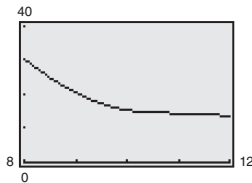
$$44. \quad p = \frac{4}{0.000001x^2 + 0.05x + 1}, \quad x \geq 0$$

$$\begin{aligned} 0.000001x^2 + 0.05x + 1 &= \frac{4}{p} \\ 0.000002x \frac{dx}{dp} + 0.05 \frac{dx}{dp} &= -\frac{4}{p^2} \\ (0.000002x + 0.05) \frac{dx}{dp} &= -\frac{4}{p^2} \\ \frac{dx}{dp} &= -\frac{4}{p^2(0.000002x + 0.05)} \end{aligned}$$

$$45. \quad p = \sqrt{\frac{200 - x}{2x}}, \quad 0 < x \leq 200$$

$$\begin{aligned} 2xp^2 &= 200 - x \\ 2x(2p) + p^2 \left(2 \frac{dx}{dp} \right) &= -\frac{dx}{dp} \\ (2p^2 + 1) \frac{dx}{dp} &= -4xp \\ \frac{dx}{dp} &= -\frac{4xp}{2p^2 + 1} \end{aligned}$$

$$\begin{aligned} 49. \quad (a) \quad y^2 - 35,892.5 &= -27.0021t^3 + 888.789t^2 - 9753.25t \\ y^2 &= -27.0021t^3 + 888.789t^2 - 9753.25t + 35,892.5 \\ y &= \pm \sqrt{-27.0021t^3 + 888.789t^2 - 9753.25t + 35,892.5} \end{aligned}$$



The numbers of cases of Chickenpox decreases from 2008 to 2012.

- (b) It appears that the number of reported cases was decreasing at the greatest rate during 2008, $t = 8$.

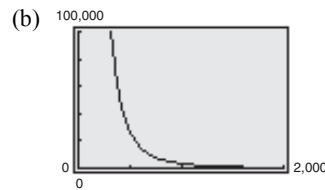
$$46. \quad p = \sqrt{\frac{500 - x}{2x}}, \quad 0 < x \leq 500$$

$$\begin{aligned} 2xp^2 &= 500 - x \\ 2x(2p) + p^2 \left(2 \frac{dx}{dp} \right) &= -\frac{dx}{dp} \\ \frac{dx}{dp} (2p^2 + 1) &= -4xp \\ \frac{dx}{dp} &= -\frac{4xp}{2p^2 + 1} \end{aligned}$$

$$47. \quad (a) \quad 100x^{0.75}y^{0.25} = 135,540$$

$$\begin{aligned} 100x^{0.75} \left(0.25y^{-0.75} \frac{dy}{dx} \right) + y^{0.25} (75x^{-0.25}) &= 0 \\ \frac{25x^{0.75}}{y^{0.75}} \cdot \frac{dy}{dx} &= -\frac{75y^{0.25}}{x^{0.25}} \\ \frac{dy}{dx} &= -\frac{3y}{x} \end{aligned}$$

When $x = 1500$ and $y = 1000$, $\frac{dy}{dx} = -2$.



If more labor is used, then less capital is available.

If more capital is used, then less labor is available.

48. (a) As price increases, the demand decreases.
 (b) For $x > 0$, the rate of change of demand, x , with respect to the price, p , is always decreasing; that is, for $x > 0$, $\frac{dx}{dp}$ is never increasing.

$$(c) \quad y^2 - 35,892.5 = -27.0021t^3 + 888.789t^2 - 9753.25t$$

$$2y \frac{dy}{dt} = -81.0063t^2 + 1777.578t - 9753.25$$

$$y' = \frac{dy}{dt} = \frac{-81.0063t^2 + 1777.578t - 9753.25}{2y}$$

t	8	9	10	11	12
y	30.40	20.51	15.39	14.51	13.40
y'	-11.79	-7.71	-2.54	-0.06	-3.26

The table of values for y' agrees with the answer in part (b) when the greatest value of y' is -11.79 thousand cases per year.

Section 2.8 Related Rates

Skills Warm Up

1. $A = \pi r^2$

2. $V = \frac{4}{3}\pi r^3$

3. $SA = 6s^2$

4. $V = s^3$

5. $V = \frac{1}{3}\pi r^2 h$

6. $A = \frac{1}{2}bh$

7. $x^2 + y^2 = 9$

$$\frac{d}{dx}[x^2 + y^2] = \frac{d}{dx}[9]$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$= \frac{-x}{y}$$

8. $3xy - x^2 = 6$

$$\frac{d}{dx}[3xy - x^2] = \frac{d}{dx}[6]$$

$$3y + 3x \frac{dy}{dx} - 2x = 0$$

$$3x \frac{dy}{dx} = 2x - 3y$$

$$\frac{dy}{dx} = \frac{2x - 3y}{3x}$$

9. $x^2 + 2y + xy = 12$

$$\frac{d}{dx}[x^2 + 2y + xy] = \frac{d}{dx}(12)$$

$$2x + 2 \frac{dy}{dx} + y + x \frac{dy}{dx} = 0$$

$$2 \frac{dy}{dx} + x \frac{dy}{dx} = -y - 2x$$

$$\frac{dy}{dx}(2 + x) = -y - 2x$$

$$\frac{dy}{dx} = \frac{-y - 2x}{2 + x}$$

10. $x + xy^2 - y^2 = xy$

$$\frac{d}{dx}[x + xy^2 - y^2] = \frac{d}{dx}[xy]$$

$$1 + y^2 + 2xy \frac{dy}{dx} - 2y \frac{dy}{dx} = y + x \frac{dy}{dx}$$

$$2xy \frac{dy}{dx} - 2y \frac{dy}{dx} - x \frac{dy}{dx} = y - y^2 - 1$$

$$\frac{dy}{dx}(2xy - 2y - x) = y - y^2 - 1$$

$$\frac{dy}{dx} = \frac{y - y^2 - 1}{2xy - 2y - x}$$

$$1. y = \sqrt{x}, \frac{dy}{dt} = \frac{1}{2}x^{-1/2} \frac{dx}{dt} = \frac{1}{2\sqrt{x}} \frac{dx}{dt}, \frac{dx}{dt} = 2\sqrt{x} \frac{dy}{dt}$$

$$(a) \text{ When } x = 4 \text{ and } \frac{dx}{dt} = 3, \frac{dy}{dt} = \left(\frac{1}{2\sqrt{4}}\right)(3) = \frac{3}{4}.$$

$$(b) \text{ When } x = 25 \text{ and } \frac{dy}{dt} = 2, \frac{dx}{dt} = 2\sqrt{25}(2) = 20.$$

$$2. y = 3x^2 - 5x, \frac{dy}{dt} = 6x \frac{dx}{dt} - 5 \frac{dx}{dt}, \frac{dy}{dt} = (6x - 5) \frac{dx}{dt}, \frac{dy}{6x - 5} = \frac{dx}{dt}$$

$$(a) \text{ When } x = 3 \text{ and } \frac{dx}{dt} = 2, \frac{dy}{dt} = (6(3) - 5(2)) = 26.$$

$$(b) \text{ When } x = 2 \text{ and } \frac{dy}{dt} = 4, \frac{4}{6(2) - 5} = \frac{4}{7} = \frac{dx}{dt}.$$

$$3. xy = 4, x \frac{dy}{dt} + y \frac{dx}{dt} = 0, \frac{dy}{dt} = \left(-\frac{y}{x}\right) \frac{dx}{dt}, \frac{dx}{dt} = \left(-\frac{x}{y}\right) \frac{dy}{dt}$$

$$(a) \text{ When } x = 8, y = \frac{1}{2}, \text{ and } \frac{dx}{dt} = 10, \frac{dy}{dt} = -\frac{1/2}{8}(10) = -\frac{5}{8}.$$

$$(b) \text{ When } x = 1, y = 4, \text{ and } \frac{dy}{dt} = -6, \frac{dx}{dt} = -\frac{1}{4}(-6) = \frac{3}{2}.$$

$$4. x^2 + y^2 = 25, 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0, \frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}, \frac{dx}{dt} = -\frac{y}{x} \frac{dy}{dt}$$

$$(a) \text{ When } x = 3, y = 4, \text{ and } \frac{dx}{dt} = 8, \frac{dy}{dt} = -\frac{3}{4}(8) = -6.$$

$$(b) \text{ When } x = 4, y = 3, \text{ and } \frac{dy}{dt} = -2, \frac{dx}{dt} = -\frac{3}{4}(-2) = \frac{3}{2}.$$

$$5. A = \pi r^2, \frac{dA}{dt} = 3, \frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 6\pi r$$

$$(a) \text{ When } r = 6, \frac{dA}{dt} = 2\pi(6)(3) = 36\pi \text{ cm}^2/\text{min}.$$

$$(b) \text{ When } r = 24, \frac{dA}{dt} = 2\pi(24)(3) = 144\pi \text{ cm}^2/\text{min}.$$

$$6. V = \frac{4}{3}\pi r^3, \frac{dV}{dt} = 3, \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 12\pi r^2$$

$$(a) \text{ When } r = 9, \frac{dV}{dt} = 12\pi(9)^2 = 972\pi \text{ cm}^3/\text{min}.$$

$$(b) \text{ When } r = 16, \frac{dV}{dt} = 12\pi(16)^2 = 3072\pi \text{ cm}^3/\text{min}.$$

$$7. A = \pi r^2, \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

If $\frac{dr}{dt}$ is constant, then $\frac{dA}{dt}$ is not constant; $\frac{dA}{dt}$ is proportional to r .

$$8. V = \frac{4}{3}\pi r^3, \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

If $\frac{dr}{dt}$ is constant, $\frac{dV}{dt}$ is *not* constant since it is proportional to the square of r .

$$9. V = \frac{4}{3}\pi r^3, \frac{dV}{dt} = 10, \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt},$$

$$\frac{dr}{dt} = \left(\frac{1}{4\pi r^2}\right) \frac{dV}{dt}$$

$$(a) \text{ When } r = 1, \frac{dr}{dt} = \frac{1}{4\pi(1)^2}(10) = \frac{5}{2\pi} \text{ m/min}.$$

$$(b) \text{ When } r = 2, \frac{dr}{dt} = \frac{1}{4\pi(2)^2}(10) = \frac{5}{8\pi} \text{ m/min}.$$

$$10. V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(3r) = \pi r^3$$

$$\frac{dV}{dt} = 3\pi r^2 \frac{dr}{dt} = 6\pi r^2$$

$$(a) \text{ When } r = 6, \frac{dV}{dt} = 6\pi(6)^2 = 216\pi \text{ cm}^3/\text{min}.$$

$$(b) \text{ When } r = 24, \frac{dV}{dt} = 6\pi(24)^2 = 3456\pi \text{ cm}^3/\text{min}.$$

$$11. (a) \frac{dC}{dt} = 0.75 \frac{dx}{dt} = 0.75(150) \\ = 112.5 \text{ dollars per week}$$

$$(b) \frac{dR}{dt} = 250 \frac{dx}{dt} - \frac{1}{5}x \frac{dx}{dt} \\ = 250(150) - \frac{1}{5}(1000)(150) \\ = 7500 \text{ dollars per week}$$

$$(c) P = R - C \\ \frac{dP}{dt} = \frac{dR}{dt} - \frac{dC}{dt} = 7500 - 112.5 \\ = 7387.5 \text{ dollars per week}$$

$$12. (a) \frac{dC}{dt} = 1.05 \frac{dx}{dt} = 1.05(250) = 262.5 \text{ dollars/week}$$

$$(b) \frac{dR}{dt} = \left(500 - \frac{2x}{25}\right) \frac{dx}{dt} = \left(500 - \frac{2(5000)}{25}\right)(250) \\ = 25,000 \text{ dollars/week}$$

$$(c) P = R - C \\ \frac{dP}{dt} = \frac{dR}{dt} - \frac{dC}{dt} = 25,000 - 262.5 \\ = 24,737.5 \text{ dollars/week}$$

$$13. R = 1200x - x^2, \frac{dR}{dt} = 1200 \frac{dx}{dt} - 2x \frac{dx}{dt}, \\ \frac{dR}{dt} = (1200 - 2x) \frac{dx}{dt}$$

$$(a) \text{ When } \frac{dx}{dt} = 23 \text{ units/day and } x = 300 \text{ units,} \\ \frac{dR}{dt} = [1200 - 2(300)](23) = \$13,800 \text{ per day.}$$

$$(b) \text{ When } \frac{dx}{dt} = 23 \text{ units/day and } x = 450 \text{ units,} \\ \frac{dR}{dt} = [1200 - 2(450)](23) = \$6900 \text{ per day.}$$

$$14. R = 510x - 0.3x^2, \frac{dR}{dt} = 510 \frac{dx}{dt} - 0.6x \frac{dx}{dt}, \\ \frac{dR}{dt} = (510 - 0.6x) \frac{dx}{dt}$$

$$(a) \text{ When } \frac{dx}{dt} = 9 \text{ units/day and } x = 400 \text{ units,} \\ \frac{dR}{dt} = [510 - 0.6(400)](9) = \$2430 \text{ per day.}$$

$$(b) \text{ When } \frac{dx}{dt} = 9 \text{ units/day and } x = 600 \text{ units,} \\ \frac{dR}{dt} = [510 - 0.6(600)](9) = \$1350 \text{ per day.}$$

$$15. V = x^3, \frac{dV}{dt} = 6x^2 \frac{dx}{dt}$$

$$(a) \text{ When } x = 2, \frac{dV}{dt} = 3(2)^2(6) = 72 \text{ cm}^3/\text{sec}.$$

$$(b) \text{ When } x = 10, \frac{dV}{dt} = 3(10)^2(6) = 1800 \text{ cm}^3/\text{sec}.$$

$$16. A = 6x^2, \frac{dA}{dt} = 12x \frac{dx}{dt}$$

$$(a) \text{ When } x = 2, \frac{dA}{dt} = 12(2)(6) = 144 \text{ cm}^2/\text{sec}.$$

$$(b) \text{ When } x = 10, \frac{dA}{dt} = 12(10)(6) = 720 \text{ cm}^2/\text{sec}.$$

17. Let x be the distance from the boat to the dock and y be the length of the rope.

$$3^2 + x^2 = y^2$$

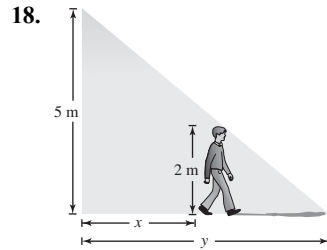
$$\frac{dy}{dt} = -1$$

$$2x \frac{dx}{dt} = 2y \frac{dy}{dt}$$

$$\frac{dx}{dt} = \frac{y}{x} \frac{dy}{dt}$$

$$\text{When } y = 5, x = 4 \text{ and } \frac{dx}{dt} = \frac{5}{4}(-1) = -1.25 \text{ m/sec.}$$

As $x \rightarrow 0$, $\frac{dx}{dt}$ increases.



$$(a) \quad \frac{5}{2} = \frac{y}{y-x} \Rightarrow 5y - 5x = 2y$$

$$3y = 5x$$

$$y = \frac{5}{3}x$$

Find $\frac{dy}{dt}$ when $\frac{dx}{dt} = 1.5$ m/sec and $x = 3$ m.

$$\frac{dy}{dt} = \frac{5}{3} \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{5}{3}(1.5) = 2.5 \text{ m/sec}$$

(b) Find $\frac{d}{dt}(y-x)$ when $\frac{dx}{dt} = 1.5$ m/sec and

$$\frac{dy}{dt} = 2.5 \text{ m/sec when } x = 3 \text{ m.}$$

$$\frac{d}{dt}(y-x) = \frac{dy}{dt} - \frac{dx}{dt}$$

$$= 2.5 - 1.5 = 1 \text{ m/sec}$$

19. $x^2 + 10^2 = s^2$

$$2x \frac{dx}{dt} = 2s \frac{ds}{dt}$$

$$\frac{dx}{dt} = \frac{s}{x} \frac{ds}{dt}$$

When $s = 26$, $x = 24$ and $\frac{ds}{dt} = 384$:

$$\frac{dx}{dt} = \frac{26}{24}(384) = 416 \text{ km/h.}$$

20. $s^2 = 90^2 + x^2$, $x = 26$, $\frac{dx}{dt} = -30$

$$2s \frac{ds}{dt} = 2x \frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{x}{s} \frac{dx}{dt}$$

When $x = 26$,

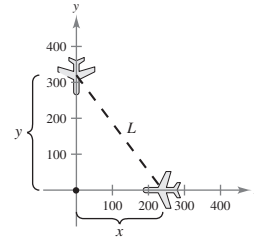
$$\frac{ds}{dt} = \frac{26}{\sqrt{90^2 + 26^2}}(-30) \approx -8.33 \text{ ft/sec.}$$

21. (a) $L^2 = x^2 + y^2$, $\frac{dx}{dt} = -720$, $\frac{dy}{dt} = -960$, and

$$\frac{dL}{dt} = \frac{x(dx/dt) + y(dy/dt)}{L}$$

When $x = 240$ and $y = 320$, $L = 400$ and

$$\frac{dL}{dt} = \frac{240(-720) + 320(-960)}{400} = 1200 \text{ km/h.}$$



(b) $t = \frac{400}{1200} = \frac{1}{3} \text{ hr} = 20 \text{ min}$

22. $S = 2250 + 50x + 0.35x^2$

$$\frac{dS}{dt} = 50 \frac{dx}{dt} + 0.70x \frac{dx}{dt}$$

$$\frac{dS}{dt} = 50(125) + 0.70(1500)(125)$$

$$= \$137,500 \text{ per week}$$

23. $V = \pi r^2 h$, $h = 0.025$, $V = 0.025\pi r^2$,

$$\frac{dV}{dt} = 0.05\pi r \frac{dr}{dt}$$

When $r = 50$ and $\frac{dr}{dt} = \frac{1}{4}$,

$$\frac{dV}{dt} = 0.05\pi(50)\left(\frac{1}{4}\right) = 0.625\pi \approx 1.96 \text{ m}^3/\text{min.}$$

24. $P = R - C$

$$= xp - C$$

$$= x(50 - 0.01x) - (4000 + 40x - 0.02x^2)$$

$$= 50x - 0.01x^2 - 4000 - 40x + 0.02x^2$$

$$= 0.01x^2 + 10x - 4000$$

$$\frac{dP}{dt} = 0.02x \frac{dx}{dt} + 10 \frac{dx}{dt}$$

When $x = 800$ and $\frac{dx}{dt} = 25$,

$$\frac{dP}{dt} = 0.02(800)(25) + (10)(25) = \$650/\text{week.}$$

$$25. P = R - C = xp - C = x(6000 - 25x) - (2400x + 5200) \\ = -25x^2 + 3600x - 5200$$

$$\frac{dP}{dt} = -50x \frac{dx}{dt} + 3600 \frac{dx}{dt} \\ \frac{dx}{dt} = \frac{1}{3600 - 50x} \frac{dP}{dt}$$

$$\text{When } x = 44 \text{ and } \frac{dP}{dt} = 5600, \frac{dx}{dt} = \frac{1}{3600 - 50(44)}(5600) = 4 \text{ units per week.}$$

26. (a) For supply, if $\frac{dx}{dt}$ is negative, then $\frac{dp}{dt}$ is negative. For demand, if $\frac{dx}{dt}$ is negative, then $\frac{dp}{dt}$ is positive.

(b) For supply, if $\frac{dp}{dt}$ is positive, then $\frac{dx}{dt}$ is positive. For demand, if $\frac{dp}{dt}$ is positive, then $\frac{dx}{dt}$ is negative.

Review Exercises for Chapter 2

1. Slope $\approx \frac{-4}{2} = -2$

2. Slope $\approx \frac{4}{2} = 2$

3. Slope ≈ 0

4. Slope $\approx \frac{-2}{4} = -\frac{1}{2}$

5. Answers will vary. Sample answer:

$t = 8$; slope $\approx$ \$225 million/yr; Revenue was increasing by about \$225 million per year in 2008.

$t = 10$; slope $\approx$ \$350 million/yr; Revenue was increasing by about \$350 million per year in 2010.

6. Answers will vary. Sample answer:

$t = 10$; slope ≈ -20 thousand/year; The number of farms was decreasing by about 20 thousand per year in 2010.

$t = 12$; slope ≈ -10 thousand/year; The number of farms was decreasing by about 10 thousand per year in 2012.

7. Answers will vary. Sample answer:

$t = 1$: $m \approx 65$ hundred thousand visitors/month; The number of visitors to the national park is increasing at about 65,000,000/per month in January.

$t = 8$: $m \approx 0$ visitors/month; The number of visitors to the national park is neither increasing nor decreasing in August.

$t = 12$: $m \approx -1000$ hundred thousand/month; The number of visitors to the national park is decreasing at about 1,000,000,000 visitors per month in December.

8. (a) At t_1 , the slope of $g(t)$ is greater than the slope of $f(t)$, so the rafter whose progress is given by $g(t)$ is traveling faster.

(b) At t_2 , the slope of $f(t)$ is greater than the slope of $g(t)$, so the rafter whose progress is given by $f(t)$ is traveling faster.

(c) At t_3 , the slope of $f(t)$ is greater than the slope of $g(t)$, so the rafter whose progress is given by $f(t)$ is traveling faster.

(d) The rafter whose progress is given by $f(t)$ finishes first. The value of t where $f(t) = 9$ is smaller than the value of t where $g(t) = 9$.

9. $f(x) = -3x - 5; (-2, 1)$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ = \lim_{\Delta x \rightarrow 0} \frac{-3(x + \Delta x) - 5 - (-3x - 5)}{\Delta x} \\ = \lim_{\Delta x \rightarrow 0} \frac{-3\Delta x}{\Delta x} = -3$$

$$f'(-2) = -3$$

10. $f(x) = 7x + 3; (-1, -4)$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ = \lim_{\Delta x \rightarrow 0} \frac{7(x + \Delta x) + 3 - (7x + 3)}{\Delta x} \\ = \lim_{\Delta x \rightarrow 0} \frac{7\Delta x}{\Delta x} = 7$$

$$f'(-1) = 7$$

11. $f(x) = x^2 + 9; (3, 18)$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 + 9 - (x^2 + 9)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 9 - x^2 - 9}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} (2x + \Delta x) = 2x \\
 f'(3) &= 2(3) = 6
 \end{aligned}$$

12. $f(x) = x^2 - 7x; (1, -6)$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - 7(x + \Delta x) - (x^2 - 7x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 7x - 7\Delta x - x^2 + 7x}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2 - 7\Delta x}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \Delta x - 7)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} (2x + \Delta x - 7) = 2x - 7 \\
 f'(1) &= 2(1) - 7 = -5
 \end{aligned}$$

13. $f(x) = \sqrt{x + 9}; (-5, 2)$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x + 9} - \sqrt{x + 9}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x + 9} + \sqrt{x + 9}}{\sqrt{x + \Delta x + 9} + \sqrt{x + 9}} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x + 9) - (x + 9)}{\Delta x [\sqrt{x + \Delta x + 9} + \sqrt{x + 9}]} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x + 9} + \sqrt{x + 9}} = \frac{1}{2\sqrt{x + 9}} \\
 f'(-5) &= \frac{1}{4}
 \end{aligned}$$

14. $f(x) = \sqrt{x - 1}; (10, 3)$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x - 1} - \sqrt{x - 1}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x - 1} + \sqrt{x - 1}}{\sqrt{x + \Delta x - 1} + \sqrt{x - 1}} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x - 1) - (x - 1)}{\Delta x [\sqrt{x + \Delta x - 1} + \sqrt{x - 1}]} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x - 1} + \sqrt{x - 1}} = \frac{1}{2\sqrt{x - 1}} \\
 f'(10) &= \frac{1}{6}
 \end{aligned}$$

$$15. f(x) = \frac{1}{x-5}; (6, 1)$$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{x + \Delta x - 5} - \frac{1}{x - 5}}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{(x - 5) - (x + \Delta x - 5)}{\Delta x(x + \Delta x - 5)(x - 5)} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-1}{(x + \Delta x - 5)(x - 5)} = -\frac{1}{(x - 5)^2} \end{aligned}$$

$$f'(6) = -1$$

$$16. f(x) = \frac{1}{x + 6}$$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{x + \Delta x + 6} - \frac{1}{x + 6}}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{(x + 6) - (x + \Delta x + 6)}{\Delta x[(x + \Delta x + 6)(x + 6)]} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-1}{(x + \Delta x + 6)(x + 6)} \\ &= -\frac{1}{(x + 6)^2} \end{aligned}$$

$$f'(-3) = -\frac{1}{(-2 + 6)^2} = -\frac{1}{16}$$

$$19. f(x) = -\frac{1}{2}x^2 + 2x$$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\left[-\frac{1}{2}(x + \Delta x)^2 + 2(x + \Delta x)\right] - \left(-\frac{1}{2}x^2 + 2x\right)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-\frac{1}{2}x^2 - x(\Delta x) - \frac{1}{2}(\Delta x)^2 + 2x + 2\Delta x + \frac{1}{2}x^2 - 2x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(-x - \frac{1}{2}\Delta x + 2)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \left(-x - \frac{1}{2}\Delta x + 2\right) = -x + 2 \end{aligned}$$

$$17. f(x) = 9x + 1$$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{[9(x + \Delta x) + 1] - (9x + 1)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{9x + 9\Delta x + 1 - 9x - 1}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{9\Delta x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} 9 = 9 \end{aligned}$$

$$18. f(x) = 1 - 4x$$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{[1 - 4(x + \Delta x)] - (1 - 4x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{1 - 4x - 4\Delta x - 1 + 4x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-4\Delta x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} -4 = -4 \end{aligned}$$

20. $f(x) = 3x^2 - \frac{1}{4}x$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\left[3(x + \Delta x)^2 - \frac{1}{4}(x + \Delta x) \right] - \left(3x^2 - \frac{1}{4}x \right)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{3x^2 + 6x(\Delta x) + 3(\Delta x)^2 - \frac{1}{4}x - \frac{1}{4}\Delta x - 3x^2 + \frac{1}{4}x}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{6x(\Delta x) + 3(\Delta x)^2 - \frac{1}{4}\Delta x}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x \left(6x + 3(\Delta x) - \frac{1}{4} \right)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \left(6x + 3(\Delta x) - \frac{1}{4} \right) = 6x - \frac{1}{4}
 \end{aligned}$$

21. $f(x) = \sqrt{x - 5}$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x - 5} - \sqrt{x - 5}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x - 5} + \sqrt{x - 5}}{\sqrt{x + \Delta x - 5} + \sqrt{x - 5}} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x - 5) - (x - 5)}{\Delta x (\sqrt{x + \Delta x - 5} + \sqrt{x - 5})} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x - 5} + \sqrt{x - 5}} = \frac{1}{2\sqrt{x - 5}}
 \end{aligned}$$

22. $f(x) = \sqrt{x} + 3$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{[\sqrt{x + \Delta x} + 3] - (\sqrt{x} + 3)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x) - x}{\Delta x (\sqrt{x + \Delta x} + \sqrt{x})} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}
 \end{aligned}$$

23. $f(x) = \frac{5}{x}$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\frac{5}{x + \Delta x} - \frac{5}{x}}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{5x - 5(x + \Delta x)}{\Delta x [x(x + \Delta x)]} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{-5\Delta x}{\Delta x [x(x + \Delta x)]} \\
 &= \lim_{\Delta x \rightarrow 0} -\frac{5}{x(x + \Delta x)} = -\frac{5}{x^2}
 \end{aligned}$$

$$24. f(x) = \frac{1}{x+4}$$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\frac{1}{x + \Delta x + 4} - \frac{1}{x + 4}}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\frac{(x + 4) - (x + \Delta x + 4)}{(x + 4)(x + \Delta x + 4)}}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{-\Delta x}{\Delta x[(x + 4)(x + \Delta x + 4)]} \\ &= \lim_{\Delta x \rightarrow 0} -\frac{1}{(x + 4)(x + \Delta x + 4)} = -\frac{1}{(x + 4)^2} \end{aligned}$$

25. y is not differentiable at $x = -1$. At $(-1, 0)$, the graph has a vertical tangent line.

26. y is not differentiable at $x = 0$. At $(0, 3)$, the graph has a node.

27. y is not differentiable at $x = 0$. The function is discontinuous at $x = 0$.

28. y is not differentiable at $x = -1$. At $(-1, 0)$, the graph has a cusp.

$$29. y = -6 \\ y' = 0$$

$$30. f(x) = 5 \\ f'(x) = 0$$

$$31. f(x) = x^7 \\ f'(x) = 7x^6$$

$$32. h(x) = \frac{1}{x^{-4}} \\ h(x) = x^{-4} \\ h'(x) = -4x^{-5} \\ h'(x) = \frac{-4}{x^5}$$

$$33. f(x) = 4x^2 \\ f'(x) = 8x$$

$$34. g(t) = 8t^6 \\ g'(t) = 48t^5$$

$$35. f(x) = \frac{5x^3}{4} \\ f'(x) = \frac{15x^2}{4}$$

$$36. y = 3x^{2/3} \\ y' = 2x^{-1/3} \\ y' = \frac{2}{x^{1/3}}$$

$$37. g(x) = 2x^4 + 3x^2 \\ g'(x) = 8x^3 + 6x$$

$$38. f(x) = 6x^2 - 4x \\ f'(x) = 12x - 4$$

$$39. y = x^2 + 6x - 7 \\ y' = 2x + 6$$

$$40. y = 2x^4 - 3x^3 + x \\ y' = 8x^3 - 9x^2 + 1$$

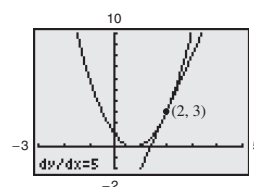
$$41. f(x) = 2x^{-1/2}; (4, 1) \\ f'(x) = -x^{-3/2} \\ f'(4) = -(4)^{-3/2} = -0.125$$

$$42. y = \frac{3}{2x} + 3; \left(\frac{1}{2}, 6\right) \\ y = \frac{3}{2}x^{-1} + 3 \\ y' = -\frac{3}{2}x^{-2} = -\frac{3}{2x^2} \\ y'\left(\frac{1}{2}\right) = \frac{3}{2\left(\frac{1}{2}\right)^2} = 6$$

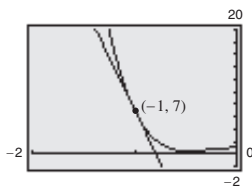
$$43. g(x) = x^3 - 4x^2 - 6x + 8; (-1, 9) \\ g'(x) = 3x^2 - 8x - 6 \\ g'(-1) = 3(-1)^2 - 8(-1) - 6 = 5$$

$$44. y = 2x^4 - 5x^3 + 6x^2 - x; (1, 2) \\ y' = 8x^3 - 15x^2 + 12x - 1 \\ y'(1) = 8(1)^3 - 15(1)^2 + 12(1) - 1 = 4$$

$$45. f'(x) = 4x - 3 \\ f'(2) = 5 \\ y - 3 = 5(x - 2) \\ y = 5x - 7$$



$$\begin{aligned}
 46. \quad y' &= 44x^3 - 10x \\
 y'(-1) &= -34 \\
 y - 7 &= -34(x + 1) \\
 y &= -34x - 27
 \end{aligned}$$



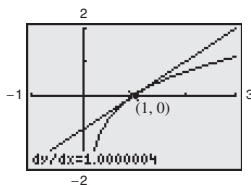
$$47. \quad f(x) = \sqrt{x} - \frac{1}{\sqrt{x}} = x^{1/2} - x^{-1/2}$$

$$\begin{aligned}
 f'(x) &= \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2} \\
 &= \frac{1}{2\sqrt{x}} + \frac{1}{2x^{3/2}}
 \end{aligned}$$

$$f'(1) = 1$$

$$y - 0 = 1(x - 1)$$

$$y = x - 1$$



$$48. \quad f(x) = \sqrt[3]{x} - x = x^{1/3} - x$$

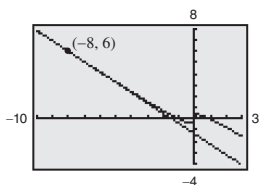
$$f'(x) = \frac{1}{3}x^{-2/3} - 1 = \frac{1}{3\sqrt[3]{x^2}} - 1$$

$$f'(-8) = \frac{1}{3\sqrt[3]{(-8)^2}} - 1 = \frac{1}{12} - 1 = -\frac{11}{12}$$

$$y - 6 = -\frac{11}{12}(x + 8)$$

$$y - 6 = -\frac{11}{12}x - \frac{22}{3}$$

$$y = -\frac{11}{12}x - \frac{4}{3}$$



$$49. \quad R = -0.5972t^3 + 51.187t^2 - 485.54t + 2199.0$$

$$\frac{dR}{dt} = R'(t) = -1.7916t^2 + 102.374t - 485.54$$

$$(a) \quad 2008: R'(8) = m \approx 218.8$$

$$2010: R'(10) = m \approx 359.0$$

(b) Results should be similar.

(c) The slope shows the rate at which sales were increasing or decreasing in that particular year, or value of t .

In 2008, the revenue was increasing about \$218.8 million per year, and in 2010, revenue was increasing about \$359.0 million per year.

$$50. \quad N = 0.2083t^4 - 7.954t^3 + 11.96t^2 - 706.5t + 3891$$

$$\frac{dN}{dt} = N'(t) = 0.8332t^3 - 23.862t^2 + 23.92t - 706.5$$

$$(a) \quad 2010: N'(10) = m \approx -2020.33$$

$$2012: N'(12) = m \approx -2415.85$$

(b) Results should be similar.

(c) The slope shows the rate at which the number of farms was increasing or decreasing in that particular year, or value of t .

In 2010, the number of farms was decreasing about 2020.33 thousand per year, and in 2012, the number of farms was decreasing about 2415.85 thousand per year.

$$51. \quad f(t) = 4t + 3; [-3, 1]$$

Average rate of change:

$$\frac{f(1) - f(-3)}{1 - (-3)} = \frac{7 - (-9)}{4} = 4$$

Instantaneous rate of change:

$$f'(t) = 4$$

$$f'(1) = 4$$

$$f'(-3) = 4$$

$$52. \quad f(x) = x^2 + 3x - 4; [0, 1]$$

$$\text{Average rate of change: } \frac{f(1) - f(0)}{1 - 0} = \frac{0 - (-4)}{1} = 4$$

Instantaneous rate of change:

$$f'(x) = 2x + 3$$

$$f'(1) = 5$$

$$f'(0) = 3$$

$$53. \quad f(x) = x^{2/3}; [1, 8]$$

$$\text{Average rate of change: } \frac{f(8) - f(1)}{8 - 1} = \frac{4 - 1}{7} = \frac{3}{7}$$

Instantaneous rate of change:

$$f'(x) = \frac{2}{3x^{1/3}}$$

$$f'(8) = \frac{1}{3}$$

$$f'(1) = \frac{2}{3}$$

54. $f(x) = x^3 - x^2 + 3; [-2, 2]$

Average rate of change: $\frac{f(2) - f(-2)}{2 - (-2)} = \frac{7 - (-9)}{4} = 4$

Instantaneous rate of change: $f'(x) = 3x^2 - 2x$

$$f'(2) = 8$$

$$f'(-2) = 16$$

55. $s(t) = -4.9t^2 - 10t + 200$

(a) Average velocity = $\frac{s(3) - s(1)}{3 - 1} = \frac{125.9 - 185.1}{2} = -29.6$ m/sec

(b) $v(t) = s'(t) = -9.8t - 10$

$$v(1) = -19.8 \text{ m/sec}$$

$$v(3) = -39.4 \text{ m/sec}$$

(c) $s(t) = 0$

$$-4.9t^2 - 10t + 200 = 0$$

$$4.9t^2 + 10t - 200 = 0$$

$$t = \frac{-10 \pm \sqrt{10^2 - 4(4.9)(-200)}}{2(4.9)} = \frac{-10 \pm \sqrt{4020}}{9.8}$$

$$t \approx 5.45 \text{ sec}$$

(d) $v(t) = s'(5.45) = -9.8(5.45) - 10 \approx -63.4$ m/sec

56. (a) $s(t) = -4.9t^2 + 84$

$$v(t) = s'(t) = -9.8t$$

(b) Average velocity = $\frac{s(2) - s(0)}{2 - 0}$

$$= \frac{64.4 - 84}{2}$$

$$= -9.8 \text{ m/sec}$$

(c) $v(t) = -9.8t$

$$v(2) = -19.6 \text{ m/sec}$$

$$v(3) = -29.4 \text{ m/sec}$$

(d) $s(t) = 0$

$$-4.9t^2 + 84 = 0$$

$$4.9t^2 = 84$$

$$t^2 = \frac{84}{4.9}$$

$$t \approx 4.14 \text{ sec}$$

(e) $v(4.14) = -40.6$ m/sec

57. $C = 2500 + 320x$

$$\frac{dC}{dx} = 320$$

58. $C = 24,000 + 450x - x^2, 0 \leq x \leq 225$

$$\frac{dC}{dx} = 450 - 2x$$

59. $C = 370 + 2.55\sqrt{x} = 370 + 2.25x^{1/2}$

$$\frac{dC}{dx} = \frac{1}{2}(2.55)(x^{-1/2}) = \frac{1.275}{\sqrt{x}}$$

60. $C = 475 + 5.25x^{2/3}$

$$\frac{dC}{dx} = 5.25\left(\frac{2}{3}x^{-1/3}\right) = \frac{3.5}{\sqrt[3]{x}}$$

61. $R = 150x - 0.6x^2$

$$\frac{dR}{dx} = 150 - 1.2x$$

62. $R = 150x - \frac{3}{4}x^2$

$$\frac{dR}{dx} = 150 - \frac{3}{2}x$$

63. $R = -4x^3 + 2x^2 + 100x$

$$\frac{dR}{dx} = -12x^2 + 4x + 100$$

64. $R = 4x + 10x^{1/2}$

$$\frac{dR}{dx} = 4 + \frac{5}{x^{1/2}}$$

65. $P = -0.0002x^3 + 6x^2 - x - 2000$

$$\frac{dP}{dx} = -0.0006x^2 + 12x - 1$$

66. $P = -\frac{1}{15}x^3 + 4000x^2 - 120x - 144,000$

$$\frac{dP}{dx} = -\frac{1}{5}x^2 + 8000x - 120$$

67. $P = -0.05x^2 + 20x - 1000$

(a) Find $\frac{dP}{dx}$ when $x = 100$.

$$\frac{dP}{dx} = -0.1x + 20 = P'(x)$$

When $x = 100$, $\frac{dP}{dx} = P'(100) = \10 .

(b) Find $\frac{\Delta P}{\Delta x}$ for $100 \leq x \leq 101$.

$$\frac{P(101) - P(100)}{101 - 100} = 509.95 - 500 = \$9.95$$

(c) Parts (a) and (b) differ by only \$0.05.

68. $P = -0.021t^2 + 2.77t + 148.9$

(a) $P(0) = 148.9$

$$P(4) = 159.644$$

$$P(8) = 169.716$$

$$P(12) = 179.116$$

$$P(16) = 187.844$$

$$P(20) = 195.9$$

$$P(23) = 201.501$$

These values are the populations in millions for Brazil from 1990 to 2013.

(b) $\frac{dP}{dt} = -0.042t + 2.77 = P'(t)$

(c) $P'(0) = 2.77$

$$P'(4) = 2.602$$

$$P'(8) = 2.434$$

$$P'(12) = 2.266$$

$$P'(16) = 2.098$$

$$P'(20) = 1.93$$

$$P'(23) = 1.804$$

These are the rates at which the population of Brazil is changing in millions per year from 1990 to 2013.

69. $f(x) = x^3(5 - 3x^2) = 5x^3 - 3x^5$

$$f'(x) = 15x^2 - 15x^4 = 15x^2(1 - x^2)$$

Simple Power Rule

70. $f(x) = 4x^2(2x^2 - 5) = 8x^4 - 20x^2$

$$f'(x) = 32x^3 - 40x = 8x(4x^2 - 5)$$

Simple Power Rule

71. $y = (4x - 3)(x^3 - 2x^2)$

$$y' = (4x - 3)(3x^2 - 4x) + 4(x^3 - 2x^2)$$

$$= 12x^3 - 25x^2 + 12x + 4x^3 - 8x^2$$

$$= 16x^3 - 33x^2 + 12x$$

Product Rule and Simple Power Rule

72. $s = \left(4 - \frac{1}{t^2}\right)(t^2 - 3t) = (4 - t^{-2})(t^2 - 3t)$

$$s' = (4 - t^{-2})(2t - 3) + (t^2 - 3t)(2t^{-3})$$

$$= 8t - 12 - 2t^{-1} + 3t^{-2} + 2t^{-1} - 6t^{-2}$$

$$= 8t - 12 - 3t^{-2}$$

Product Rule and Simple Power Rule

73. $g(x) = \frac{x}{x+3}$

$$g'(x) = \frac{(x+3)(1) - x(1)}{(x+3)^2}$$

$$g'(x) = \frac{3}{(x+3)^2}$$

Quotient Rule and Simple Power Rule

74. $f(x) = \frac{2-5x}{3x+1}$

$$f'(x) = \frac{(3x+1)(-5) - (2-5x)(3)}{(3x+1)^2}$$

$$f'(x) = \frac{-15x - 5 - 6 + 15x}{(3x+1)^2}$$

$$f'(x) = -\frac{11}{(3x+1)^2}$$

Quotient Rule and Simple Power Rule

$$\begin{aligned}
 75. \quad f(x) &= \frac{6x - 5}{x^2 + 1} \\
 f'(x) &= \frac{(x^2 + 1)(6) - (6x - 5)(2x)}{(x^2 + 1)^2} \\
 &= \frac{6 + 10x - 6x^2}{(x^2 + 1)^2} \\
 &= \frac{2(3 + 5x - 3x^2)}{(x^2 + 1)^2}
 \end{aligned}$$

Quotient Rule and Simple Power Rule

$$\begin{aligned}
 76. \quad f(x) &= \frac{x^2 + x - 1}{x^2 - 1} \\
 f'(x) &= \frac{(x^2 - 1)(2x + 1) - (x^2 + x - 1)(2x)}{(x^2 - 1)^2} \\
 &= \frac{2x^3 + x^2 - 2x - 1 - 2x^3 - 2x^2 + 2x}{(x^2 - 1)^2} \\
 &= \frac{-x^2 - 1}{(x^2 - 1)^2} = -\frac{x^2 + 1}{(x^2 - 1)^2}
 \end{aligned}$$

Quotient Rule and Simple Power Rule

$$\begin{aligned}
 77. \quad f(x) &= (5x^2 + 2)^3 \\
 f'(x) &= 3(5x^2 + 2)^2(10x) \\
 &= 30x(5x^2 + 2)^2
 \end{aligned}$$

General Power Rule

$$\begin{aligned}
 78. \quad f(x) &= \sqrt[3]{x^2 - 1} = (x^2 - 1)^{1/3} \\
 f'(x) &= \frac{1}{3}(x^2 - 1)^{-2/3}(2x) = \frac{2x}{3(x^2 - 1)^{2/3}}
 \end{aligned}$$

General Power Rule

$$\begin{aligned}
 79. \quad h(x) &= \frac{2}{\sqrt{x+1}} = 2(x+1)^{-1/2} \\
 h'(x) &= 2\left(-\frac{1}{2}\right)(x+1)^{-3/2} \\
 &= -\frac{1}{(x+1)^{3/2}}
 \end{aligned}$$

General Power Rule

$$\begin{aligned}
 80. \quad g(x) &= \frac{6}{(3x^2 - 5x)^4} = 6(3x^2 - 5x)^{-4} \\
 g'(x) &= 6(-4)(3x^2 - 5x)^{-5}(6x - 5) \\
 &= -\frac{24(6x - 5)}{(3x^2 - 5x)^5}
 \end{aligned}$$

General Power Rule

$$\begin{aligned}
 81. \quad g(x) &= x\sqrt{x^2 + 1} = x(x^2 + 1)^{1/2} \\
 g'(x) &= x\left[\frac{1}{2}(x^2 + 1)^{-1/2}(2x)\right] + (1)(x^2 + 1)^{1/2} \\
 &= (x^2 + 1)^{-1/2}[x^2 + (x^2 + 1)] \\
 &= \frac{2x^2 + 1}{\sqrt{x^2 + 1}}
 \end{aligned}$$

Product and General Power Rule

$$\begin{aligned}
 82. \quad g(t) &= \frac{t}{(1-t)^3} \\
 g'(t) &= \frac{(1-t)^3(1) - t(3)(1-t)^2(-1)}{(1-t)^6} \\
 &= \frac{(1-t)^3 + 3t(1-t)^2}{(1-t)^6} \\
 &= \frac{(1-t) + 3t}{(1-t)^4} = \frac{2t+1}{(1-t)^4}
 \end{aligned}$$

Quotient Rule and General Power Rule

$$\begin{aligned}
 83. \quad f(x) &= x(1 - 4x^2)^2 \\
 f'(x) &= x(2)(1 - 4x^2)(-8x) + (1 - 4x^2)^2 \\
 &= -16x^2(1 - 4x^2) + (1 - 4x^2)^2 \\
 &= (1 - 4x^2)[-16x^2 + (1 - 4x^2)] \\
 &= (1 - 4x^2)(1 - 20x^2)
 \end{aligned}$$

Product and General Power Rule

$$\begin{aligned}
 84. \quad f(x) &= \left(x^2 + \frac{1}{x}\right)^5 = (x^2 + x^{-1})^5 \\
 f'(x) &= 5(x^2 + x^{-1})^4(2x - x^{-2}) \\
 &= 5\left(x^2 + \frac{1}{x}\right)^4\left(2x - \frac{1}{x^2}\right)
 \end{aligned}$$

General Power Rule

$$\begin{aligned}
 85. \quad h(x) &= [x^2(2x+3)]^3 = x^6(2x+3)^3 \\
 h'(x) &= x^6[3(2x+3)^2(2)] + 6x^5(2x+3)^3 \\
 &= 6x^5(2x+3)^2[x + (2x+3)] \\
 &= 18x^5(2x+3)^2(x+1)
 \end{aligned}$$

Product and General Power Rule

$$\begin{aligned}
 86. \quad f(x) &= [(x-2)(x+4)]^2 \\
 f'(x) &= 2[(x-2)(x+4)][(x-2) + (x+4)] \\
 &= 2(x-2)(x+4)(2x+2) \\
 &= 4(x-2)(x+4)(x+1)
 \end{aligned}$$

Product and General Power Rule

$$\begin{aligned}
 89. \quad h(t) &= \frac{\sqrt{3t+1}}{(1-3t)^2} = \frac{(3t+1)^{1/2}}{(1-3t)^2} \\
 h'(t) &= \frac{(1-3t)^2(1/2)(3t+1)^{-1/2}(3) - (3t+1)^{1/2}(2)(1-3t)(-3)}{(1-3t)^4} \\
 &= \frac{(3t+1)^{-1/2}[(1-3t)(3/2) + (3t+1)6]}{(1-3t)^3} \\
 &= \frac{3(9t+5)}{2\sqrt{3t+1}(1-3t)^3}
 \end{aligned}$$

Quotient and General Power Rule

$$\begin{aligned}
 90. \quad g(x) &= \left(\frac{3x+1}{x^2+1}\right)^2 = \frac{(3x+1)^2}{(x^2+1)^2} \\
 g'(x) &= \frac{(x^2+1)^2(2)(3x+1)(3) - (3x+1)^2 2(x^2+1)2x}{(x^2+1)^4} \\
 &= \frac{6(x^2+1)^2(3x+1) - 4x(3x+1)^2(x^2+1)}{(x^2+1)^4} \\
 &= \frac{2(3x+1)(x^2+1)[3(x^2+1) - 2x(3x+1)]}{(x^2+1)^4} \\
 &= \frac{2(3x+1)(x^2+1)(-3x^2-2x+3)}{(x^2+1)^4} \\
 &= \frac{2(3x+1)(-3x^2-2x+3)}{(x^2+1)^3}
 \end{aligned}$$

Quotient and General Power Rule.

$$\begin{aligned}
 87. \quad f(x) &= x^2(x-1)^5 \\
 f'(x) &= 5x^2(x-1)^4 + 2x(x-1)^5 \\
 &= x(x-1)^4[5x + 2(x-1)] \\
 &= x(x-1)^4(7x-2)
 \end{aligned}$$

Product and General Power Rule

$$\begin{aligned}
 88. \quad f(s) &= s^3(s^2-1)^{5/2} \\
 f'(s) &= s^3\left(\frac{5}{2}\right)(s^2-1)^{3/2}(2s) + 3s^2(s^2-1)^{5/2} \\
 &= s^2(s^2-1)^{3/2}[5s^2 + 3(s^2-1)] \\
 &= s^2(s^2-1)^{3/2}(8s^2-3)
 \end{aligned}$$

Product and General Power Rule

$$91. \quad T = \frac{1300}{t^2 + 2t + 25} = 1300(t^2 + 2t + 25)^{-1}$$

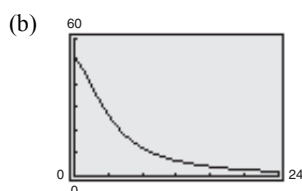
$$\begin{aligned} T'(t) &= -1300(t^2 + 2t + 25)^{-2}(2t + 2) \\ &= -\frac{2600(t + 1)}{(t^2 + 2t + 25)^2} \end{aligned}$$

$$(a) \quad T'(1) = -\frac{325}{49} \approx -6.63^\circ\text{F/hr}$$

$$T'(3) = -\frac{13}{2} \approx -6.5^\circ\text{F/hr}$$

$$T'(5) = -\frac{13}{3} \approx -4.33^\circ\text{F/hr}$$

$$T'(10) = -\frac{1144}{841} \approx -1.36^\circ\text{F/hr}$$



The rate of decrease is approaching zero.

92. When $L = 12$,

$$V = \frac{L}{16}(D - 4)^2 = \frac{12}{16}(D - 4)^2 = \frac{3}{4}(D - 4)^2$$

$$\frac{dV}{dD} = \frac{3}{2}(D - 4).$$

$$(a) \quad \text{When } D = 8, \quad \frac{dV}{dD} = \left(\frac{3}{2}\right)(8 - 4) = 6 \text{ board ft/in.}$$

$$(b) \quad \text{When } D = 16,$$

$$\frac{dV}{dD} = \left(\frac{3}{2}\right)(16 - 4) = 18 \text{ board ft/in.}$$

$$(c) \quad \text{When } D = 24,$$

$$\frac{dV}{dD} = \left(\frac{3}{2}\right)(24 - 4) = 30 \text{ board ft/in.}$$

$$(d) \quad \text{When } D = 36,$$

$$\frac{dV}{dD} = \left(\frac{3}{2}\right)(36 - 4) = 48 \text{ board ft/in.}$$

$$93. \quad f(x) = 3x^2 + 7x + 1$$

$$f'(x) = 6x + 7$$

$$f''(x) = 6$$

$$94. \quad f'(x) = 5x^4 - 6x^2 + 2x$$

$$f''(x) = 20x^3 - 12x + 2$$

$$f'''(x) = 60x^2 - 12 = 12(5x^2 - 1)$$

$$95. \quad f'''(x) = -\frac{3}{x^4} = -3x^{-4}$$

$$f^{(4)}(x) = 12x^{-5}$$

$$f^{(5)}(x) = -60x^{-6}$$

$$f^{(6)}(x) = 360x^{-7} = \frac{360}{x^7}$$

$$96. \quad f(x) = \sqrt{x} = x^{1/2}$$

$$f'(x) = \frac{1}{2}x^{-1/2}$$

$$f''(x) = -\frac{1}{4}x^{-3/2}$$

$$f'''(x) = \frac{3}{8}x^{-5/2}$$

$$f^{(4)}(x) = -\frac{15}{16}x^{-7/2} = -\frac{15}{16x^{7/2}}$$

$$97. \quad f'(x) = 8x^{5/2}$$

$$f''(x) = 20x^{3/2}$$

$$f'''(x) = 30x^{1/2}$$

$$f^{(4)}(x) = 15x^{-1/2} = \frac{15}{x^{1/2}}$$

$$98. \quad f''(x) = 9\sqrt[3]{x} = 9x^{1/3}$$

$$f'''(x) = 3x^{-2/3}$$

$$f^{(4)}(x) = -2x^{-5/3}$$

$$f^{(5)}(x) = \frac{10}{3}x^{-8/3} = \frac{10}{3x^{8/3}}$$

$$99. \quad f(x) = x^2 + \frac{3}{x} = x^2 + 3x^{-1}$$

$$f'(x) = 2x - 3x^{-2}$$

$$f''(x) = 2 + 6x^{-3} = 2 + \frac{6}{x^3}$$

$$100. \quad f'''(x) = 20x^4 - \frac{2}{x^3} = 20x^4 - 2x^{-3}$$

$$f^{(4)}(x) = 80x^3 + 6x^{-4}$$

$$f^{(5)}(x) = 240x^2 - 24x^{-5} = 240x^2 - \frac{24}{x^5}$$

101. (a) $s(t) = -4.9t^2 + 1.5t + 10$

$$v(t) = s'(t) = -9.8t + 1.5$$

$$a(t) = v'(t) = s''(t) = -9.8$$

(b) $s(t) = 0 = -4.9t^2 + 1.5t + 10$

Using the Quadratic Formula, $t \approx 1.59$ seconds.

(c) $v(t) = s'(t) = -9.8t + 1.5$

$$v(1.59) \approx -14.08 \text{ m/sec}$$

(d) $a(t) = v'(t) = -9.8 \text{ m/sec}^2$

102. $s(t) = \frac{1}{t^2 + 2t + 1} = (t + 1)^{-2}$

$$v(t) = s'(t) = -2(t + 1)^{-3} = -\frac{2}{(t + 1)^3}$$

$$a(t) = v'(t) = 6(t + 1)^{-4} = \frac{6}{(t + 1)^4}$$

103. $x^2 + 3xy + y^3 = 10$

$$2x + 3x \frac{dy}{dx} + 3y + 3y^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(3x + 3y^2) = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{3x + 3y^2} = -\frac{2x + 3y}{3(x + y^2)}$$

104. $x^2 + 9xy + y^2 = 0$

$$2x + 9y + 9x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$(9x + 2y) \frac{dy}{dx} = -2x - 9y$$

$$\frac{dy}{dx} = \frac{-2x - 9y}{9x + 2y} = -\frac{2x + 9y}{9x + 2y}$$

105. $y^2 - x^2 + 8x - 9y - 1 = 0$

$$2y \frac{dy}{dx} - 2x + 8 - 9 \frac{dy}{dx} = 0$$

$$(2y - 9) \frac{dy}{dx} = 2x - 8$$

$$\frac{dy}{dx} = \frac{2x - 8}{2y - 9}$$

106. $y^2 + x^2 - 6y - 2x - 5 = 0$

$$2y \frac{dy}{dx} + 2x - 6 \frac{dy}{dx} - 2 = 0$$

$$\frac{dy}{dx}(2y - 6) = 2 - 2x$$

$$\frac{dy}{dx} = \frac{2 - 2x}{2y - 6} = \frac{1 - x}{y - 3}$$

107. $y^2 = x - y$

$$2y \frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$2y \frac{dy}{dx} + \frac{dy}{dx} = 1$$

$$(2y + 1) \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2y + 1}$$

At (2, 1), $\frac{dy}{dx} = \frac{1}{3}$.

$$y - 1 = \frac{1}{3}(x - 2)$$

$$y = \frac{1}{3}x + \frac{1}{3}$$

108. $2x^{1/3} + 3y^{1/2} = 10$

$$\frac{2}{3}x^{-2/3} + \frac{3}{2}y^{-1/2} \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{4y^{1/2}}{9x^{2/3}}$$

At (8, 4), $\frac{dy}{dx} = -\frac{2}{9}$.

$$y - 4 = -\frac{2}{9}(x - 8)$$

$$y = -\frac{2}{9}x + \frac{52}{9}$$

109. $y^2 - 2x = xy$

$$2y \frac{dy}{dx} - 2 = x \frac{dy}{dx} + y$$

$$(2y - x) \frac{dy}{dx} = y + 2$$

$$\frac{dy}{dx} = \frac{y + 2}{2y - x}$$

At (1, 2), $\frac{dy}{dx} = \frac{4}{3}$.

$$y - 2 = \frac{4}{3}(x - 1)$$

$$y = \frac{4}{3}x + \frac{2}{3}$$

110. $y^3 - 2x^2y + 3xy^2 = -1$

$$3y^2 \frac{dy}{dx} - 2x^2 \frac{dy}{dx} - 4xy + 6xy \frac{dy}{dx} + 3y^2 = 0$$

$$\frac{dy}{dx}(3y^2 - 2x^2 + 6xy) = 4xy - 3y^2$$

$$\frac{dy}{dx} = \frac{4xy - 3y^2}{3y^2 - 2x^2 + 6xy}$$

At $(0, -1)$, $\frac{dy}{dx} = -1$.

$$y + 1 = -1(x - 0)$$

$$y = -x - 1$$

111. $A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

(a) Find $\frac{dA}{dt}$ when $r = 3$ cm and $\frac{dr}{dt} = 2$ cm/min.

$$\begin{aligned} \frac{dA}{dt} &= 2\pi(3)(2) = 12\pi \text{ cm}^2/\text{min} \\ &\approx 37.7 \text{ cm}^2/\text{min} \end{aligned}$$

(b) Find $\frac{dA}{dt}$ when $r = 10$ cm and $\frac{dr}{dt} = 2$ cm/min.

$$\begin{aligned} \frac{dA}{dt} &= 2\pi(10)(2) = 40\pi \text{ cm}^2/\text{min} \\ &\approx 125.7 \text{ cm}^2/\text{min} \end{aligned}$$

112. $P = 375x - 1.5x^2$

$$\frac{dP}{dt} = 375 \frac{dx}{dt} - 3.0x \frac{dx}{dt}$$

$$\frac{dP}{dt} = (375 - 3.0x) \frac{dx}{dt}$$

(a) $\frac{dP}{dt} = [375 - 3.0(50)](2) = \$450/\text{day}$

(b) $\frac{dP}{dt} = [375 - 3.0(100)](2) = \$150/\text{day}$

113. Let b be the horizontal distance of the water and h be the depth of the water at the deep end.

Then $b = 6h$ for $0 \leq h \leq 2$.

$$V = \frac{1}{2}bh(6) = 3bh = 3(6h)h = 18h^2$$

$$\frac{dV}{dt} = 36h \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{36h} \frac{dV}{dt} = \frac{1}{36h}(2) = \frac{1}{18h}$$

When $h = 1$, $\frac{dh}{dt} = \frac{1}{18(1)} = \frac{1}{18}$ m/min.

114. $P = R - C$

$$= xp - C$$

$$= x(211 - 0.002x) - (30x + 1,500,000)$$

$$= 181x - 0.002x^2 - 1,500,000$$

$$\frac{dP}{dt} = 181 \frac{dx}{dt} - 0.004x \frac{dx}{dt}$$

$$\frac{dP}{dt} = (181 - 0.004x) \frac{dx}{dt}$$

$$\frac{dP}{dt} = [181 - 0.004(1600)](15) = \$2619/\text{week}$$

Chapter 2 Test Yourself

1. $f(x) = x^2 + 3; (3, 12)$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 + 3 - (x^2 + 3)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 3 - (x^2 + 3)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x[2x + (\Delta x)]}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} 2x + \Delta x \\
 &= 2x
 \end{aligned}$$

At $(3, 12)$: $m = 2(3) = 6$

2. $f(x) = \sqrt{x} - 2; (4, 0)$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} - 2 - (\sqrt{x} - 2)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} \\
 &= \frac{1}{2\sqrt{x}}
 \end{aligned}$$

At $(4, 0)$: $m = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

3. $f(t) = t^3 + 2t$

$f'(t) = 3t^2 + 2$

4. $f(x) = 4x^2 - 8x + 1$

$f'(x) = 8x - 8$

5. $f(x) = x^{3/2} + 6x^{1/2}$

$$f'(x) = \frac{3}{2}x^{1/2} + 3x^{-1/2} = \frac{3\sqrt{x}}{2} + \frac{3}{\sqrt{x}}$$

6. $f(x) = 5x^2 - \frac{3}{x^3} = 5x^2 - 3x^{-3}$

$$f'(x) = 10 + 9x^{-4} = 10x + \frac{9}{x^4}$$

7. $f(x) = (x + 3)(x^2 + 2x)$

$f(x) = x^3 + 5x^2 + 6x$

$f'(x) = 3x^2 + 10x + 6$

(Or use the Product Rule.)

8. $f(x) = \sqrt{x}(5 + x) = 5x^{1/2} + x^{3/2}$

$$f'(x) = \frac{5}{2}x^{-1/2} + \frac{3}{2}x^{1/2} = \frac{5}{2\sqrt{x}} + \frac{3\sqrt{x}}{2}$$

9. $f(x) = (3x^2 + 4)^2$

$f'(x) = 2(3x^2 + 4)(6x)$

$= 36x^3 + 48x$

10. $f(x) = \sqrt{1 - 2x} = (1 - 2x)^{1/2}$

$$f'(x) = \frac{1}{2}(1 - 2x)^{-1/2}(-2)$$

$$= -\frac{1}{\sqrt{1 - 2x}}$$

11. $f(x) = \frac{(5x - 1)^3}{x}$

$$\begin{aligned}
 f'(x) &= \frac{x(3)(5x - 1)^2(5) - (5x - 1)^3}{x^2} \\
 &= \frac{(5x - 1)^2[15x - (5x - 1)]}{x^2} \\
 &= \frac{(5x - 1)^2(10x + 1)}{x^2}
 \end{aligned}$$

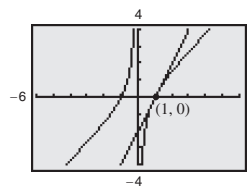
12. $f(x) = x - \frac{1}{x}$

$f'(x) = 1 + \frac{1}{x^2}$

$f'(1) = 1 + \frac{1}{1^2} = 2$

$y - 0 = 2(x - 1)$

$y = 2x - 2$



13. $S = -2.1083t^3 + 70.811t^2 - 777.05t + 2893.6$

(a) $\frac{\Delta S}{\Delta t}$ for $10 \leq t \leq 12$

$$\frac{S(12) - S(10)}{12 - 10} = \frac{122.6416 - 95.9}{2} = \$13.3708 \text{ billion/yr}$$

(b) $S'(t) = -6.3249t^2 + 141.622t - 777.05$

2010: $S'(10) = \$6.68 \text{ billion/yr}$

2012: $S'(12) = \$11.6284 \text{ billion/yr}$

(c) The annual sales of CVS Caremark from 2010 to 2012 increased by an average of about \$13.37 billion per year, and the instantaneous rates of change for 2010 and 2012 are \$6.68 billion per year and \$11.63 billion per year, respectively.

14. $P = 1700 - 0.016x$, $C = 715,000 + 240x$

Profit = Revenue - Cost

(a) Revenue: $R = xp$

$$R = x(1700 - 0.016x)$$

$$R = 1700x - 0.016x^2$$

$$P = R - C$$

$$P = (1700x - 0.016x^2) - (715,000 + 240x)$$

$$P = -0.016x^2 + 1460x - 715,000$$

(b) $\frac{dP}{dx} = -0.032x + 1460 = P'(x)$

$$P'(700) = \$1437.60$$

15. $f(x) = 2x^2 + 3x + 1$

$$f'(x) = 4x + 3$$

$$f''(x) = 4$$

$$f'''(x) = 0$$

16. $f(x) = \sqrt{3-x} = (3-x)^{1/2}$

$$f'(x) = \frac{1}{2}(3-x)^{-1/2}(-1) = -\frac{1}{2}(3-x)^{-1/2}$$

$$f''(x) = -\frac{1}{2}\left(-\frac{1}{2}\right)(3-x)^{-3/2}(-1) = -\frac{1}{4}(3-x)^{-3/2}$$

$$f'''(x) = -\frac{1}{4}\left(-\frac{3}{2}\right)(3-x)^{-5/2}(-1)$$

$$= -\frac{3}{8}(3-x)^{-5/2}$$

$$= -\frac{3}{8(3-x)^{5/2}}$$

17. $f(x) = \frac{2x+1}{2x-1}$

$$f'(x) = \frac{(2x-1)(2) - (2x+1)(2)}{(2x-1)^2}$$

$$= \frac{4}{(2x-1)^2}$$

$$= -4(2x-1)^{-2}$$

$$f''(x) = 8(2x-1)^{-3}(2) = 16(2x-1)^{-3}$$

$$f'''(x) = -48(2x-1)^{-4}(2) = -\frac{96}{(2x-1)^4}$$

18. $s(t) = -4.9t^2 + 10t + 25$

$$v(t) = s'(t) = -9.8t + 10$$

$$a(t) = v'(t) = s''(t) = -9.8$$

At $t = 2$: $s(2) = 25.4 \text{ m}$

$$v(2) = -9.6 \text{ m/sec}$$

$$a(2) = -9.8 \text{ m/sec}^2$$

19. $x + xy = 6$

$$1 + x\frac{dy}{dx} + y = 0$$

$$x\frac{dy}{dx} = -y - 1$$

$$\frac{dy}{dx} = -\frac{y+1}{x}$$

20. $y^2 + 2x - 2y + 1 = 0$

$$2y\frac{dy}{dx} + 2 - 2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2y - 2) = -2$$

$$\frac{dy}{dx} = -\frac{1}{y-1}$$

21. $4x^2 - 3y^2 + x^3y = 5$

$$8x - 6y\frac{dy}{dx} + x^3\frac{dy}{dx} + 3x^2y = 0$$

$$-6y\frac{dy}{dx} + x^3\frac{dy}{dx} = -8x - 3x^2y$$

$$(x^3 - 6y)\frac{dy}{dx} = -(8x + 3x^2y)$$

$$\frac{dy}{dx} = -\frac{8x + 3x^2y}{x^3 - 6y}$$

$$\frac{dy}{dx} = \frac{8x + 3x^2y}{6y - x^3} = \frac{x(8 + 3xy)}{6y - x^3}$$

22. $V = \pi r^2 h = 20\pi r^3$

$$\frac{dV}{dt} = 60\pi r^2 \frac{dr}{dt}$$

(a) $\frac{dV}{dt} = 60\pi(0.5)^2(0.25) = 3.75\pi \text{ cm}^3/\text{min}$

(b) $\frac{dV}{dt} = 60\pi(1)^2(0.25) = 15\pi \text{ cm}^3/\text{min}$