Contents

Chapter 1

Functions

1.1 Review of Functions

1.1.1 A function is a rule which assigns each domain element to a unique range element. The independent variable is associated with the domain, while the dependent variable is associated with the range.

1.1.2 The independent variable belongs to the domain, while the dependent variable belongs to the range.

1.1.3 The vertical line test is used to determine whether a given graph represents a function. (Specifically, it tests whether the variable associated with the vertical axis is a function of the variable associated with the horizontal axis.) If every vertical line which intersects the graph does so in exactly one point, then the given graph represents a function. If any vertical line $x = a$ intersects the curve in more than one point, then there is more than one range value for the domain value $x = a$, so the given curve does not represent a function.

1.1.4 $f(2) = \frac{1}{2^3+1} = \frac{1}{9}$. $f(y^2) = \frac{1}{(y^2)^3+1} = \frac{1}{y^6+1}$.

1.1.5 Item (i) is true, since it is stipulated in the definition of function. However, item (ii) need not be true – for example, the function $f(x) = x^2$ has two different domain values associated with the one range value 4, since $f(2) = f(-2) = 4$.

1.1.6

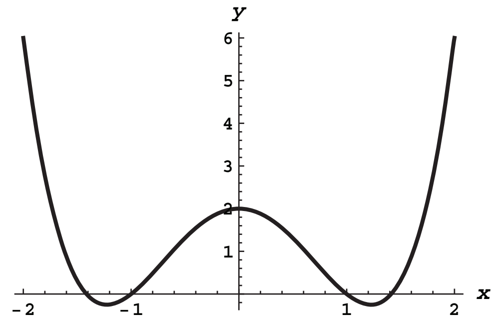
$$(f \circ g)(x) = f(g(x)) = f(x^3 - 2) = \sqrt{x^3 - 2}$$
$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = x^{3/2} - 2$$
$$(f \circ f)(x) = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$$
$$(g \circ g)(x) = g(g(x)) = g(x^3 - 2) = (x^3 - 2)^3 - 2 = x^9 - 6x^6 + 12x^3 - 10$$

1.1.7 $f(g(2)) = f(-2) = 2$ and $g(f(-2)) = g(2) = -2$.

1.1.8 The domain of $f \circ g$ is the subset of the domain of g whose range is in the domain of f. Thus, we need to look for elements x in the domain of g so that $g(x)$ is in the domain of f.

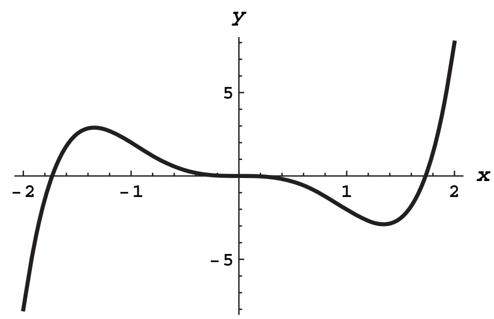
1.1.9

The defining property for an even function is that $f(-x) = f(x)$, which ensures that the graph of the function is symmetric about the y-axis.



1.1.10

The defining property for an odd function is that $f(-x) = -f(x)$, which ensures that the graph of the function is symmetric about the origin.

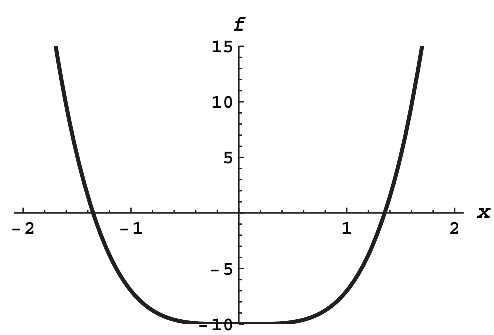


1.1.11 Graph A does not represent a function, while graph B does. Note that graph A fails the vertical line test, while graph B passes it.

1.1.12 Graph A does not represent a function, while graph B does. Note that graph A fails the vertical line test, while graph B passes it.

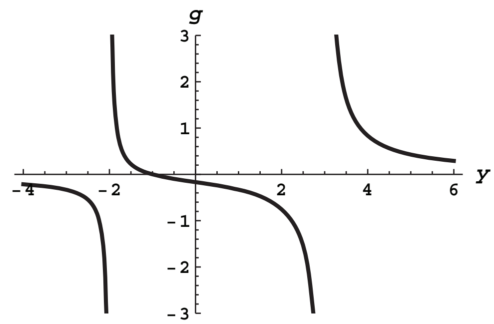
1.1.13

The natural domain of this function is the set of a real numbers. The range is $[-10, \infty)$.



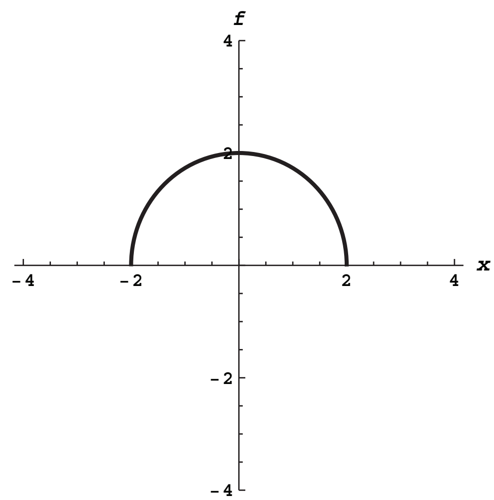
1.1.14

The natural domain of this function is $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$. The range is the set of all real numbers.



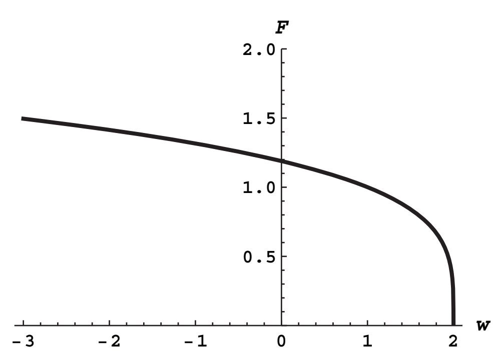
1.1.15

The natural domain of this function is $[-2, 2]$. The range is $[0, 2]$.



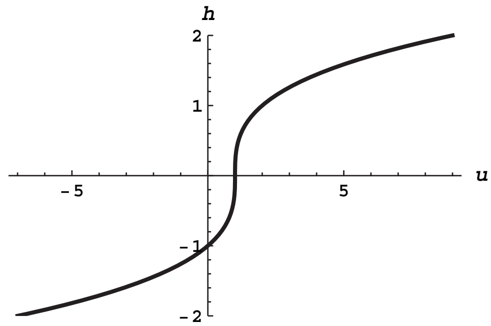
1.1.16

The natural domain of this function is $(-\infty, 2]$. The range is $[0, \infty)$.



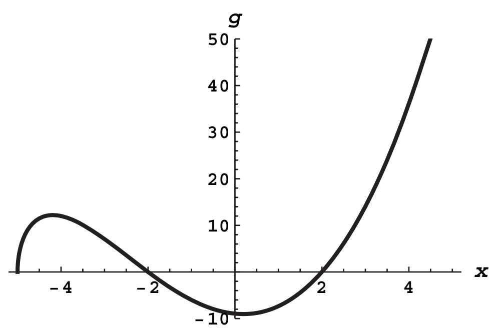
1.1.17

The natural domain and the range for this function are both the set of all real numbers.



1.1.18

The natural domain of this function is $[-5, \infty)$. The range is approximately $[-9.03, \infty)$.



1.1.19 The independent variable t is time and the dependent variable d is distance above the ground. The domain in context is $[0, 8]$

1.1.20 The independent variable n is the number of bicycles made and the dependent variable c is average cost. The domain in context is $\{n\colon n \geq 0\}$.

1.1.21 $f(10) = 96$

1.1.22 $f(p^2) = (p^2)^2 - 4 = p^4 - 4$

1.1.23 $g(1/z) = (1/z)^3 = \frac{1}{z^3}$

1.1.24 $F(y^4) = \frac{1}{y^4-3}$

1.1.25 $F(g(y)) = F(y^3) = \frac{1}{y^3-3}$

1.1.26 $f(g(w)) = f(w^3) = (w^3)^2 - 4 = w^6 - 4$

1.1.27 $g(f(u)) = g(u^2 - 4) = (u^2 - 4)^3$

1.1.28 $\dfrac{f(2+h) - f(2)}{h} = \dfrac{(2+h)^2 - 4 - 0}{h} = \dfrac{4 + 4h + h^2 - 4}{h} = \dfrac{4h + h^2}{h} = 4 + h$

1.1.29 $F(F(x)) = F\left(\dfrac{1}{x-3}\right) = \dfrac{1}{\frac{1}{x-3} - 3} = \dfrac{1}{\frac{1}{x-3} - \frac{3(x-3)}{x-3}} = \dfrac{1}{\frac{10-3x}{x-3}} = \dfrac{x-3}{10-3x}$

1.1.30 $g(F(f(x))) = g(F(x^2 - 4)) = g\left(\dfrac{1}{x^2 - 4 - 3}\right) = \left(\dfrac{1}{x^2 - 7}\right)^3$

1.1.31 $g(x) = x^3 - 5$ and $f(x) = x^{10}$. The domain of h is the set of all real numbers.

1.1.32 $g(x) = x^6 + x^2 + 1$ and $f(x) = \frac{2}{x^2}$. The domain of h is the set of all real numbers.

1.1.33 $g(x) = x^4 + 2$ and $f(x) = \sqrt{x}$. The domain of h is the set of all real numbers.

1.1.34 $g(x) = x^3 - 1$ and $f(x) = \frac{1}{\sqrt{x}}$. The domain of h is the set of all real numbers for which $x^3 - 1 > 0$, which corresponds to the set $(1, \infty)$.

1.1.35 $(f \circ g)(x) = f(g(x)) = f(x^2 - 4) = |x^2 - 4|$. The domain of this function is the set of all real numbers.

1.1.36 $(g \circ f)(x) = g(f(x)) = g(|x|) = |x|^2 - 4 = x^2 - 4$. The domain of this function is the set of all real numbers.

1.1.37 $(f \circ G)(x) = f(G(x)) = f\left(\frac{1}{x-2}\right) = \left|\frac{1}{x-2}\right|$. The domain of this function is the set of all real numbers except for the number 2.

1.1.38 $(f \circ g \circ G)(x) = f(g(G(x))) = f\left(g\left(\frac{1}{x-2}\right)\right) = f\left(\left(\frac{1}{x-2}\right)^2 - 4\right) = \left|\left(\left(\frac{1}{x-2}\right)^2 - 4\right)\right|$. The domain of this function is the set of all real numbers except for the number 2.

1.1.39 $(G \circ g \circ f)(x) = G(g(f(x))) = G(g(|x|)) = G(x^2 - 4) = \dfrac{1}{x^2 - 4 - 2} = \dfrac{1}{x^2 - 6}$. The domain of this function is the set of all real numbers except for the numbers $\pm\sqrt{6}$.

1.1.40 $(F \circ g \circ g)(x) = F(g(g(x))) = F(g(x^2 - 4)) = F((x^2 - 4)^2 - 4) = \sqrt{(x^2 - 4)^2 - 4} = \sqrt{x^4 - 8x^2 + 12}$. The domain of this function consists of the numbers x so that $x^4 - 8x^2 + 12 \geq 0$. Since $x^4 - 8x^2 + 12 = (x^2 - 6) \cdot (x^2 - 2)$, we see that this expression is zero for $x = \pm\sqrt{6}$ and $x = \pm\sqrt{2}$, By looking between these points, we see that the expression is greater than or equal to zero for the set $(-\infty, -\sqrt{6}] \cup [-\sqrt{2}, \sqrt{2}] \cup [\sqrt{2}, \infty)$.

1.1.41 Since $(x^2 + 3)^2 = x^4 + 6x^2 + 9$, it must be the case that $f(x) = x^2$.

1.1.42 Since $(x^2 + 3)^2 = x^4 + 6x^2 + 9$, and the given expression is 11 more than this, it must be the case that $f(x) = x^2 + 11$.

1.1.43 Since $(x^2)^2 + 3 = x^4 + 3$, this expression results from squaring x^2 and adding 3 to it. Thus we must have $f(x) = x^2$.

1.1.44 Since $x^{2/3} + 3 = (\sqrt[3]{x})^2 + 3$, we must have $f(x) = \sqrt[3]{x}$.

1.1.45

a. $f(g(2)) = f(2) = 4$

b. $g(f(2)) = g(4) = 1$

c. $f(g(4)) = f(1) = 3$

d. $g(f(5)) = g(6) = 3$

e. $f(g(7)) = f(4) = 7$

f. $f(f(8)) = f(8) = 8$

1.1.46

a. $h(g(0)) = h(0) = -1$

b. $g(f(4)) = g(-1) = -1$

c. $h(h(0)) = h(-1) = 0$

d. $g(h(f(4))) = g(h(-1)) = g(0) = 0$

e. $f(f(f(1))) = f(f(0)) = f(1) = 0$

f. $h(h(h(0))) = h(h(-1)) = h(0) = -1$

g. $f(h(g(2))) = f(h(3)) = f(0) = 1$

h. $g(f(h(4))) = g(f(4)) = g(-1) = -1$

i. $g(g(g(1))) = g(g(2)) = g(3) = 4$

j. $f(f(h(3))) = f(f(0)) = f(1) = 0$

1.1.47 This function is symmetric about the y-axis, since $f(-x) = (-x)^4 + 5(-x)^2 - 12 = x^4 + 5x^2 - 12 = f(x)$.

1.1.48 This function is symmetric about the origin, since $f(-x) = 3(-x)^5 + 2(-x)^3 - (-x) = -3x^5 - 2x^3 + x = -(3x^5 + 2x^3 - x) = f(x)$.

1.1.49 This function has none of the indicated symmetries. For example, note that $f(-2) = -26$, while $f(2) = 22$, so f is not symmetric about either the origin or about the y-axis, and is not symmetric about the x-axis because it is a function.

1.1.50 This function is symmetric about the y-axis. Note that $f(-x) = 2|-x| = 2|x| = f(x)$.

1.1.51 This curve (which is not a function) is symmetric about the x-axis, the y-axis, and the origin. Note that replacing either x by $-x$ or y by $-y$ (or both) yields the same equation. This is due to the fact that $(-x)^{2/3} = ((-x)^2)^{1/3} = (x^2)^{1/3} = x^{2/3}$, and a similar fact holds for the term involving y.

1.1.52 This function is symmetric about the origin. Writing the function as $y = f(x) = x^{3/5}$, we see that $f(-x) = (-x)^{3/5} = -(x)^{3/5} = -f(x)$.

1.1.53 Function A is symmetric about the y-axis, so is even. Function B is symmetric about the origin, so is odd. Function C is also symmetric about the y-axis, so is even.

1.1.54 Function A is symmetric about the y-axis, so is even. Function B is symmetric about the origin, so is odd. Function C is also symmetric about the origin, so is odd.

1.1.55

a. True. A real number z corresponds to the domain element $z/2+19$, since $f(z/2+19) = 2(z/2+19)-38 = z + 38 - 38 = z$.

b. False. The definition of function does not require that each range element comes from a unique domain element, rather that each domain element is paired with a unique range element.

c. True. $f(1/x) = \frac{1}{1/x} = x$, and $\frac{1}{f(x)} = \frac{1}{1/x} = x$.

d. False. For example, suppose that f is the straight line through the origin with slope 1, so that $f(x) = x$. Then $f(f(x)) = f(x) = x$, while $(f(x))^2 = x^2$.

e. False. For example, let $f(x) = x+2$ and $g(x) = 2x-1$. Then $f(g(x)) = f(2x-1) = 2x-1+2 = 2x+1$, while $g(f(x)) = g(x+2) = 2(x+2) - 1 = 2x + 3$.

f. True. In fact, this is the definition of $f \circ g$.

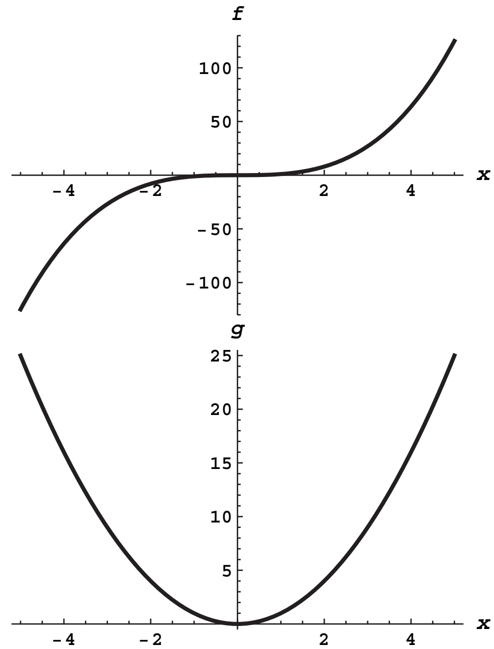
g. True. If f is even, then $f(-z) = f(z)$ for all z, so this is true in particular for $z = ax$. So if $g(x) = cf(ax)$, then $g(-x) = cf(-ax) = cf(ax) = g(x)$, so g is even.

h. False. For example, $f(x) = x$ is an odd function, but $h(x) = x + 1$ isn't, since $h(2) = 3$, while $h(-2) = -1$ which isn't $-h(2)$.

i. True. If $f(-x) = -f(x) = f(x)$, then in particular $-f(x) = f(x)$, so $0 = 2f(x)$, so $f(x) = 0$ for all x.

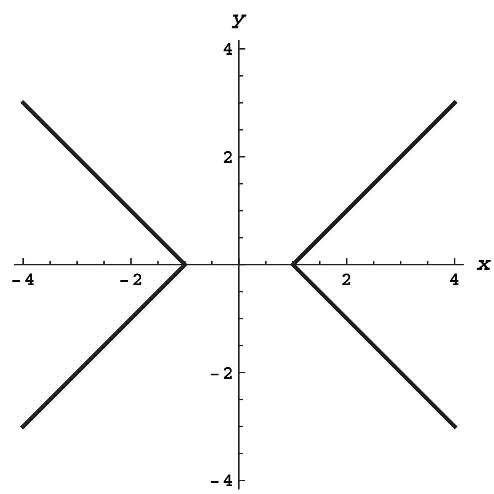
1.1.56

If n is odd, then $n = 2k+1$ for some integer k, and $(x)^n = (x)^{2k+1} = x(x)^{2k}$, which is less than 0 when $x < 0$ and greater than 0 when $x > 0$. For any number P (positive or negative) the number $\sqrt[n]{P}$ is a real number when n is odd, and $f(\sqrt[n]{P}) = P$. So the range of f in this case is the set of all real numbers.
If n is even, then $n = 2k$ for some integer k, and $x^n = (x^2)^k$. Thus $g(-x) = g(x) = (x^2)^k \geq 0$ for all x. Also, for any nonnegative number M, we have $g(\sqrt[n]{M}) = M$, so the range of g in this case is the set of all nonnegative numbers.



1.1.57

We will make heavy use of the fact that $|x|$ is x if $x > 0$, and is $-x$ if $x < 0$. In the first quadrant where x and y are both positive, this equation becomes $x - y = 1$ which is a straight line with slope 1 and y-intercept -1. In the second quadrant where x is negative and y is positive, this equation becomes $-x - y = 1$, which is a straight line with slope -1 and y-intercept -1. In the third quadrant where both x and y are negative, we obtain the equation $-x - (-y) = 1$, or $y = x + 1$, and in the fourth quadrant, we obtain $x + y = 1$. Graphing these lines and restricting them to the appropriate quadrants yields the following curve:



1.1.58

a. No. For example $f(x) = x^2 + 3$ is an even function, but $f(0)$ is not 0.

b. Yes. Since $f(-x) = -f(x)$, and since $-0 = 0$, we must have $f(-0) = f(0) = -f(0)$, so $f(0) = -f(0)$, and the only number which is its own additive inverse is 0, so $f(0) = 0$.

1.1.59 Since the composition of f with itself has first degree, we can assume that f has first degree as well, so let $f(x) = ax + b$. Then $(f \circ f)(x) = f(ax + b) = a(ax + b) + b = a^2x + (ab + b)$. Equating coefficients, we see that $a^2 = 9$ and $ab + b = -8$. If $a = 3$, we get that $b = -2$, while if $a = -3$ we have $b = 4$. So two possible answers are $f(x) = 3x - 2$ and $f(x) = -3x + 4$.

1.1.60 Since the square of a linear function is a quadratic, we let $f(x) = ax+b$. Then $f(x)^2 = a^2x^2+2abx+b^2$. Equating coefficients yields that $a = \pm 3$ and $b = \pm 2$. However, a quick check shows that the middle term is correct only when one of these is positive and one is negative. So the two possible such functions f are $f(x) = 3x - 2$ and $f(x) = -3x + 2$.

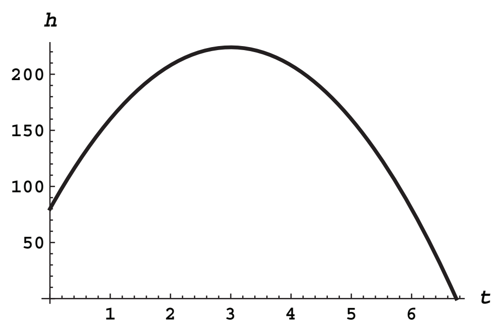
1.1.61 Let $f(x) = ax^2 + bx + c$. Then $(f \circ f)(x) = f(ax^2 + bx + c) = a(ax^2 + bx + c)^2 + b(ax^2 + bx + c) + c$. Expanding this expression yields $a^3x^4 + 2a^2bx^3 + 2a^2cx^2 + ab^2x^2 + 2abcx + ac^2 + abx^2 + b^2x + bc + c$, which simplifies to $a^3x^4 + 2a^2bx^3 + (2a^2c + ab^2 + ab)x^2 + (2abc + b^2)x + (ac^2 + bc + c)$. Equating coefficients yields $a^3 = 1$, so $a = 1$. Then $2a^2b = 0$, so $b = 0$. It then follows that $c = -6$, so the original function was $f(x) = x^2 - 6$.

1.1.62 Since the square of a quadratic is a quartic, we let $f(x) = ax^2 + bx + c$. Then the square of f is $c^2 + 2bcx + b^2x^2 + 2acx^2 + 2abx^3 + a^2x^4$. By equating coefficients, we see that $a^2 = 1$ and so $a = \pm 1$. Since the coefficient on x^3 must be 0, we have that $b = 0$. And the constant term reveals that $c = \pm 6$. A quick check shows that the only possible solutions are thus $f(x) = x^2 - 6$ and $f(x) = -x^2 + 6$.

1.1.63

a. The formula for the height of the rocket is valid from $t = 0$ until the rocket hits the ground, which is the positive solution to $-16t^2 + 96t + 80 = 0$, which the quadratic formula reveals is $t = 3 + \sqrt{14}$. Thus, the domain is $[0, 3\sqrt{14}]$.

b.



The maximum appears to occur at $t = 3$. The height at that time would be 224.

1.1.64

a. $d(0) = (10 - (2.2) \cdot 0)^2 = 100$.

b. An appropriate domain would be the interval from $t = 0$ to $t = t_e$ where t_e represents the time when the tank is first empty.

c. The tank is first empty when $d(t) = 0$, which is when $10 - (2.2)t = 0$, or $t = 50/11$.

1.1.65 This would not necessarily have either kind of symmetry. For example, $f(x) = x^2$ is an even function and $g(x) = x^3$ is odd, but the sum of these two is neither even nor odd.

1.1.66 This would be an odd function, so it would be symmetric about the origin. Suppose f is even and g is odd. Then $(f \cdot g)(-x) = f(-x)g(-x) = f(x) \cdot (-g(x)) = -(f \cdot g)(x)$.

1.1.67 This would be an odd function, so it would be symmetric about the origin. Suppose f is even and g is odd. Then $\frac{f}{g}(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\frac{f}{g}(x)$.

1.1.68 This would be an even function, so it would be symmetric about the y-axis. Suppose f is even and g is odd. Then $f(g(-x)) = f(-g(x)) = f(g(x))$.

1.1.69 This would be an even function, so it would be symmetric about the y-axis. Suppose f is even and g is even. Then $f(g(-x)) = f(g(x))$, since $g(-x) = g(x)$.

1.1.70 This would be an odd function, so it would be symmetric about the origin. Suppose f is odd and g is odd. Then $f(g(-x)) = f(-g(x)) = -f(g(x))$.

1.1.71 This would be an even function, so it would be symmetric about the y-axis. Suppose f is even and g is odd. Then $g(f(-x)) = g(f(x))$, since $f(-x) = f(x)$.

1.1.72

a. $\dfrac{f(x)-f(a)}{x-a} = \dfrac{3-2x-(3-2a)}{x-a} = \dfrac{-2x+2a}{x-a} = \dfrac{(-2)(x-a)}{x-a} = -2.$

b. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{3-2(x+h)-(3-2x)}{h} = \dfrac{3-2x-2h-3+2x}{h} = \dfrac{-2h}{h} = -2.$

1.1.73

a. $\dfrac{f(x)-f(a)}{x-a} = \dfrac{4x-3-(4a-3)}{x-a} = \dfrac{4x-4a}{x-a} = \dfrac{(4)(x-a)}{x-a} = 4.$

b. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{4(x+h)-3-(4x-3)}{h} = \dfrac{4x+4h-3-4x+3}{h} = \dfrac{4h}{h} = 4.$

1.1.74

a. $\dfrac{f(x)-f(a)}{x-a} = \dfrac{4x^2-1-(4a^2-1)}{x-a} = \dfrac{4x^2-4a^2}{x-a} = \dfrac{(4)(x-a)(x+a)}{x-a} = 4(x+a).$

b. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{4(x+h)^2-1-(4x^2-1)}{h} = \dfrac{4x^2+8xh+4h^2-1-4x^2+1}{h} = \dfrac{(8x+4h)(h)}{h} =$ $8x+4h.$

1.1.75

a. $\dfrac{f(x)-f(a)}{x-a} = \dfrac{\frac{1}{2x}-\frac{1}{2a}}{x-a} = \dfrac{\frac{a}{2ax}-\frac{x}{2ax}}{x-a} = \dfrac{a-x}{(2ax)(x-a)} = \dfrac{(-1)(x-a)}{(2ax)(x-a)} = \dfrac{-1}{2ax}.$

b. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\frac{1}{(2)(x+h)}-\frac{1}{2x}}{h} = \dfrac{\frac{x}{(2)(x+h)(x)}-\frac{x+h}{(2)(x+h)(x)}}{h} = \dfrac{-h}{(2)(x+h)(x)(h)} = \dfrac{-1}{(2)(x+h)(x)}.$

1.2 Representing Functions

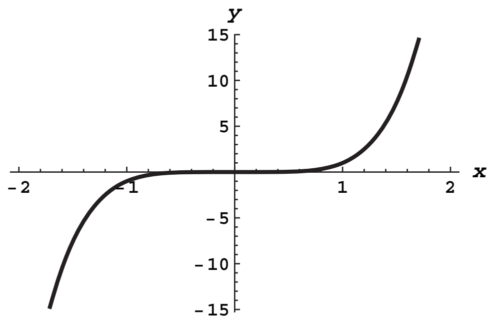
1.2.1 Functions can be defined and represented by a formula, through a graph, via a table, and by using words.

1.2.2 The domain of every polynomial is the set of all real numbers.

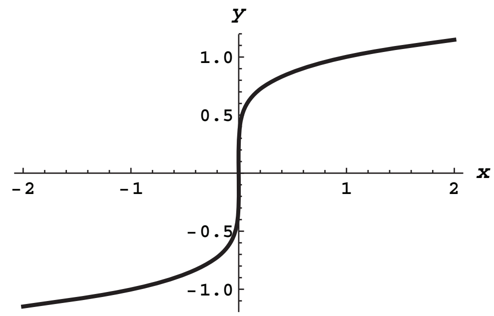
1.2.3 The domain of a rational function $\frac{p(x)}{q(x)}$ is the set of all real numbers for which $q(x) \neq 0$.

1.2.4 A piecewise linear function is one which is linear over intervals in the domain.

1.2.5



1.2.6



1.2.7 Compared to the graph of $f(x)$, the graph of $f(x+2)$ will be shifted 2 units to the left.

1.2.8 Compared to the graph of $f(x)$, the graph of $-3f(x)$ will be stretched vertically by a factor of 3 and flipped about the x axis.

1.2.9 Compared to the graph of $f(x)$, the graph of $f(3x)$ will be scaled horizontally by a factor of 3.

1.2.10 To produce the graph of $y = 4(x+3)^2 + 6$ from the graph of x^2, one must

1. shift the graph horizontally by 3 units to left
2. scale the graph vertically by a factor of 4
3. shift the graph vertically up 6 units.

1.2.11 The slope of the line shown is $m = \frac{-3-(-1)}{3-0} = -2/3$. The y-intercept is $b = -1$. Thus the function is given by $f(x) = (-2/3)x - 1$.

1.2.12 The slope of the line shown is $m = \frac{1-(5)}{5-0} = -4/5$. The y-intercept is $b = 5$. Thus the function is given by $f(x) = (-4/5)x + 5$.

1.2.13 Using price as the independent variable p and the average number of units sold per day as the dependent variable d, we have the ordered pairs $(250, 12)$ and $(200, 15)$. The slope of the line determined by these points is $m = \frac{15-12}{200-250} = \frac{3}{-50}$. Thus the demand function has the form $d(p) = (-3/50)p + b$ for some constant b. Using the point $(200, 15)$, we find that $15 = (-3/50) \cdot 200 + b$, so $b = 27$. Thus the demand function is $d = (-3/50)p + 27$. While the natural domain of this linear function is the set of all real numbers, the formula is only likely to be valid for some subset of the interval $(0, 450)$, since outside of that interval either $p \le 0$ or $d \le 0$.

1.2.14 The profit is given by $p = f(x) = 8x - 175$. The break-even point is when $p = 0$, which occurs when $x = 175/8 = 21.875$, so they need to sell at least 22 tickets to not have a negative profit.

1.2.15 For $x < 0$, the graph is a line with slope 1 and y- intercept 3, while for $x > 0$, it is a line with slope $-1/2$ and y-intercept 3. Note that both of these lines contain the point $(0, 3)$. The function shown can thus be written

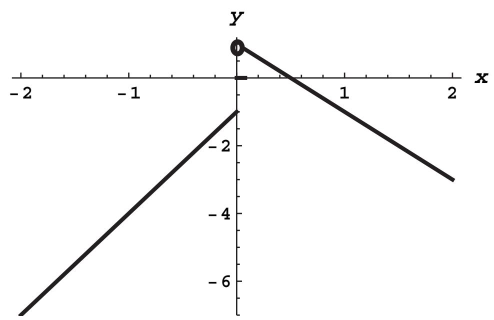
$$f(x) = \begin{cases} x+3 & \text{if } x \le 0; \\ (-1/2)x + 3 & \text{if } x > 0. \end{cases}$$

1.2.16 For $x < 3$, the graph is a line with slope 1 and y- intercept 1, while for $x > 3$, it is a line with slope $-1/3$. The portion to the right thus is represented by $y = (-1/3)x + b$, but since it contains the point $(6, 1)$, we must have $1 = (-1/3)(6) + b$ so $b = 3$. The function shown can thus be written

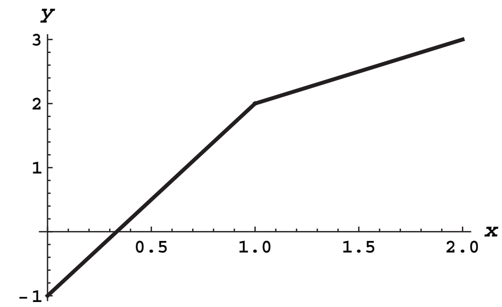
$$f(x) = \begin{cases} x+1 & \text{if } x < 3; \\ (-1/3)x + 3 & \text{if } x \ge 3. \end{cases}$$

Note that at $x = 3$ the value of the function is 2, as indicated by our formula.

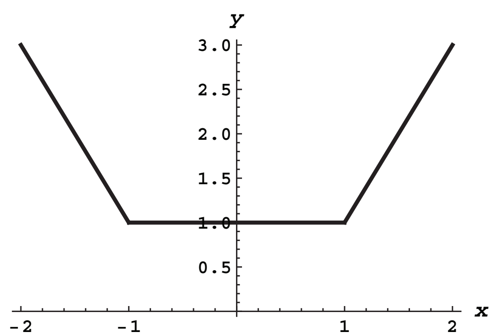
1.2.17



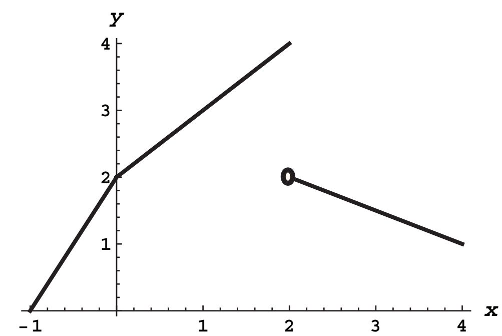
1.2.18



1.2.19

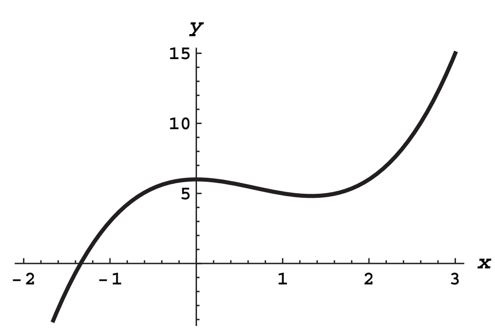


1.2.20



1.2.21

a.

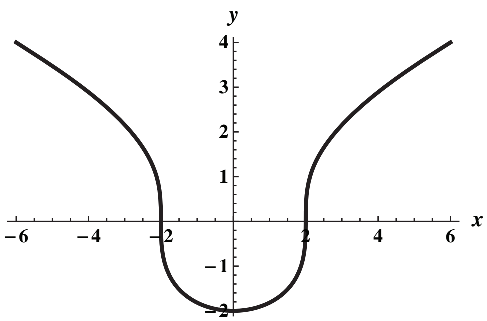


b. The function is a polynomial, so its domain is the set of all real numbers.

c. It has one peak near its y-intercept of $(0, 6)$ and one valley between $x = 1$ and $x = 2$. Its x-intercept is near $x = -4/3$.

1.2.22

a.

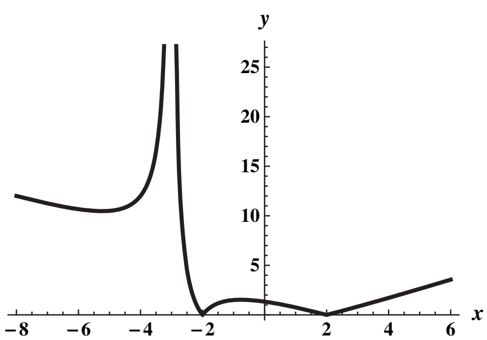


b. The function is an algebraic function. Its domain is the set of all real numbers.

c. It has a valley at the y-intercept of $(0, -2)$, and is very steep at $x = -2$ and $x = 2$ which are the x-intercepts. It is symmetric about the y-axis.

1.2.23

a.

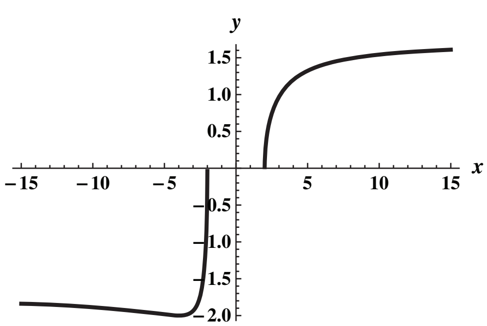


b. The domain of the function is the set of all real numbers except -3.

c. There is a valley near $x = -5.2$ and a peak near $x = -0.8$. The x-intercepts are at -2 and 2, where the curve does not appear to be smooth. There is a vertical asymptote at $x = -3$. The function is never below the x-axis. The y-intercept is $(0, 4/2)$.

1.2.24

a.



b. The domain of the function is $(-\infty, -2] \cup [2, \infty)$

c. x-intercepts are at -2 and 2. Since 0 isn't in the domain, there is no y-intercept. The function has a valley at $x = -4$..

1.2.25 The slope function is given by $s(x) = \begin{cases} 1 & \text{if } x < 0; \\ -1/2 & \text{if } x > 0. \end{cases}$

1.2.26 The slope function is given by $s(x) = \begin{cases} 1 & \text{if } x < 3; \\ -1/3 & \text{if } x > 3. \end{cases}$

1.2.27

a. Since the area under consideration is that of a rectangle with base 2 and height 6, $A(2) = 12$.

b. Since the area under consideration is that of a rectangle with base 6 and height 6, $A(6) = 36$.

c. Since the area under consideration is that of a rectangle with base x and height 6, $A(x) = 6x$.

1.2.28

a. Since the area under consideration is that of a triangle with base 2 and height 2, $A(2) = 2$.

b. Since the area under consideration can be divided into two triangles and a trapezoid, the area function at $x = 6$ is the sum of the areas from 0 to 2 (which is 2) plus the area from 2 to 4 (which is 4) plus the area 4 to 6 (which is 7), so $A(6) = 2 + 4 + 7 = 13$.

c. If x is between 0 and 2, we have a trapezoid whose base is x and whose average height is $\frac{2+(2-x)}{2} = 2 - (x/2)$, so the area in this case is $A(x) = x(2 - (x/2)) = 2x - (x^2/2)$. If x is between 2 and 4, the area in question is that of a triangle of area 2 plus that of a triangle over the interval $[2, x]$. The area of this second triangle is $\frac{1}{2}(x-2)(2x-4) = (x-2)^2 = x^2 - 4x + 4$. So the area from 0 to x for $2 < x < 4$ is $2 + x^2 - 4x + 4 = x^2 - 4x + 6$. If x is between 4 and 12, the area in question is the sum of a triangle of area 2, a triangle of area 4, and a trapezoid over the region $[4, x]$. The base of this trapezoid is $x - 4$ and the average height is $\frac{4+(-1/2)x+6}{2} = 5 - (x/4)$. Thus the area of this trapezoid is $(x-4)(5-(x/4)) = 5x - (x^2/4) - 20 + x = (-x^2/4) + 6x - 20$. So the area from 0 to x for $4 < x < 12$ is $2 + 4 + (-x^2/4) + 6x - 20 = (-x^2/4) + 6x - 14$. Thus we have

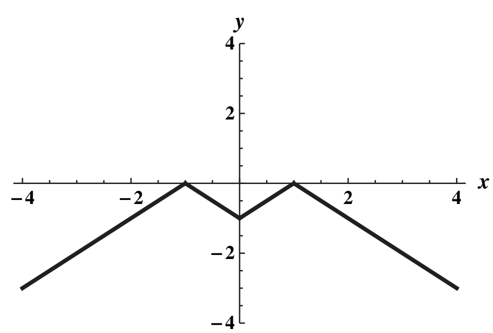
$$A(x) = \begin{cases} 2x - \frac{x^2}{2} & \text{if } 0 \le x \le 2; \\ x^2 - 4x + 6 & \text{if } 2 \le x \le 4; \\ \frac{-x^2}{4} + 6x - 14 & \text{if } 4 \le x \le 12. \end{cases}$$

1.2.29 $f(x) = |x - 2| + 3$, since the graph of f is obtained from that of $|x|$ by shifting 2 units to the right and 3 units up.

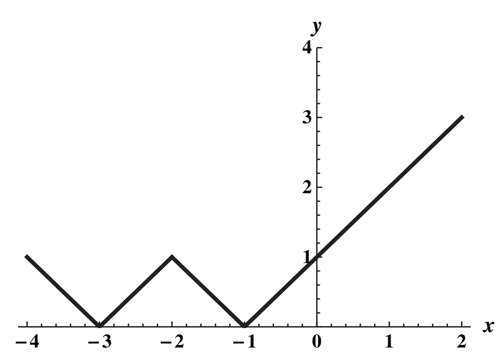
$g(x) = -|x + 2| - 1$, since the graph of g is obtained from the graph of $|x|$ by shifting 2 units to the left, then reflecting about the x-axis, and then shifting 1 unit down.

1.2.30

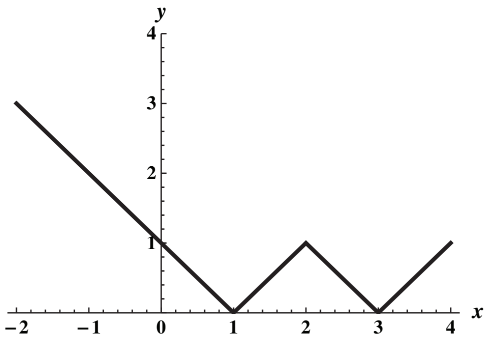
a.



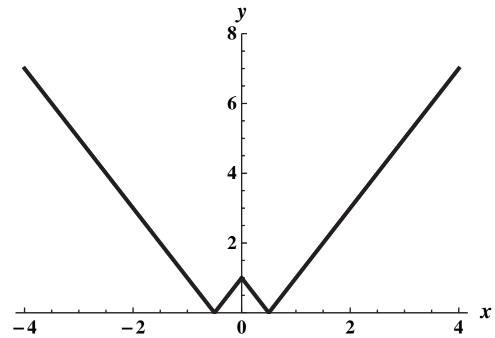
b.



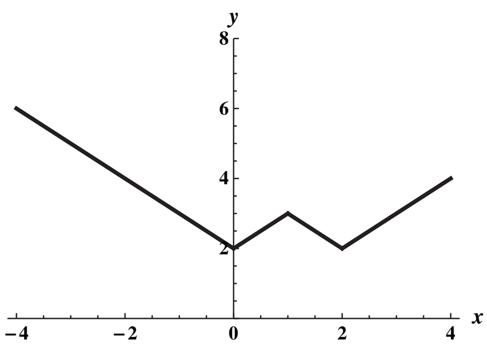
c.



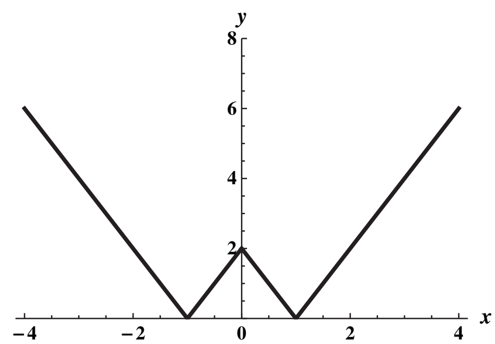
d.



e.

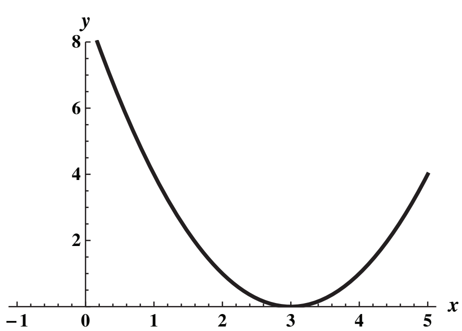


f.

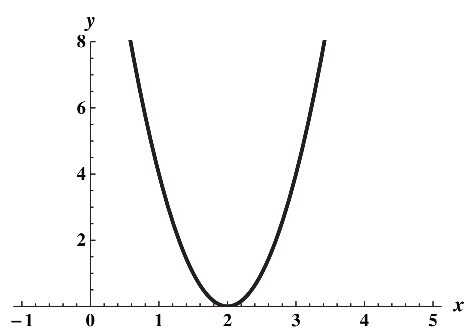


1.2.31

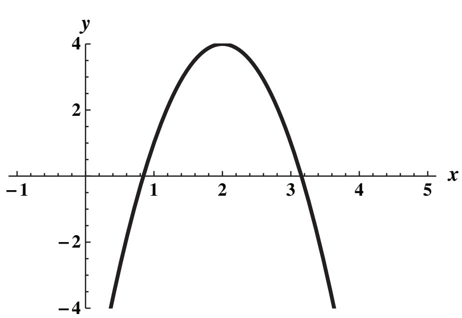
a.



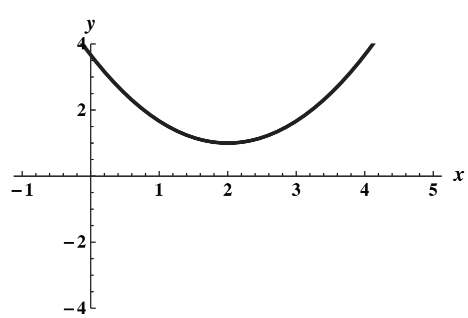
b.



c.

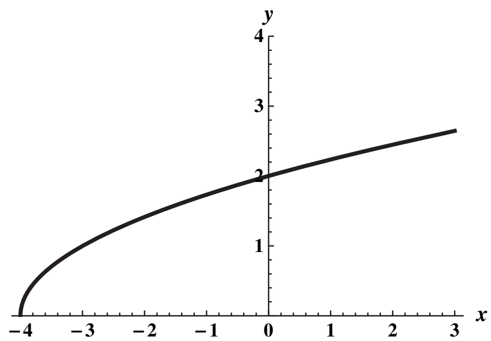


d.

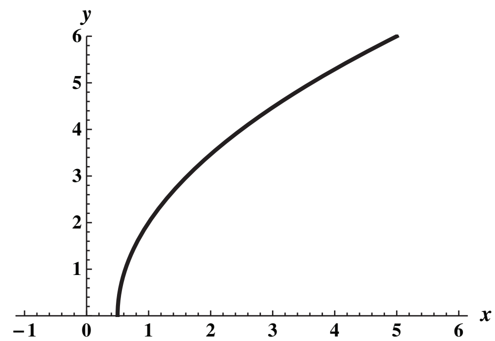


1.2.32

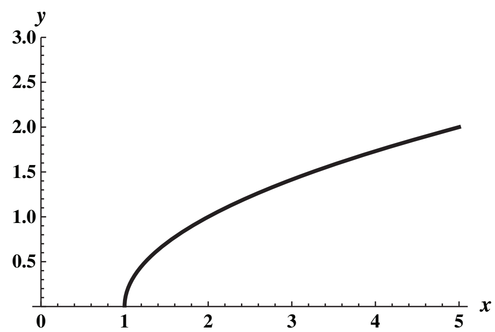
a.



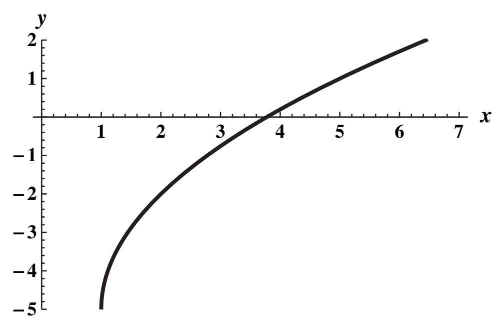
b.



c.

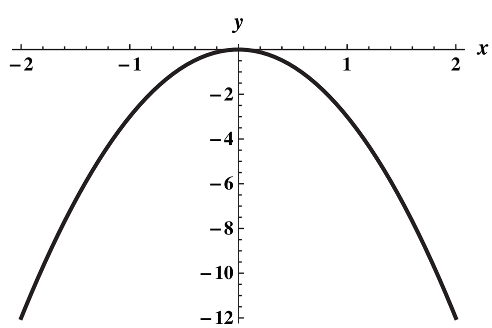


d.



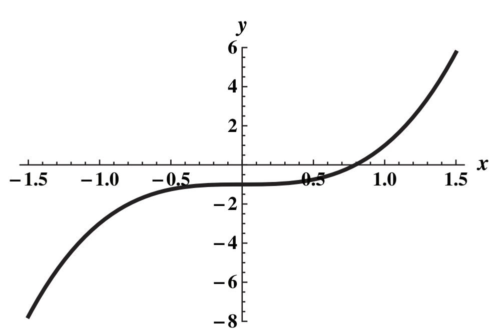
1.2.33

This function is $-3 \cdot f(x)$ where $f(x) = x^2$



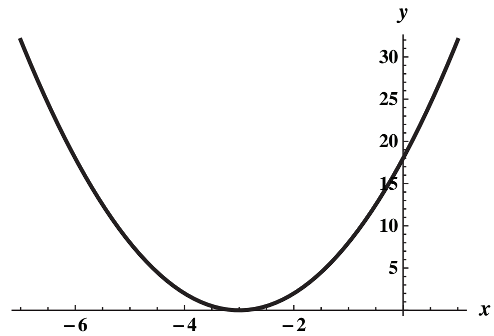
1.2.34

This function is $2 \cdot f(x) - 1$ where $f(x) = x^3$



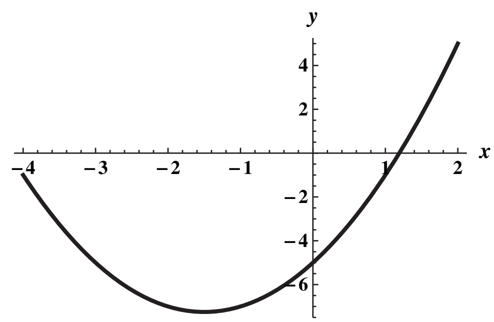
1.2.35

This function is $2 \cdot f(x+3)$ where $f(x) = x^2$



1.2.36

By completing the square, we have that $p(x) = (x^2+3x+(9/4))-(29/4) = (x+(3/2))^2-(29/4)$. So it is $f(x+(3/2))-(29/4)$ where $f(x) = x^2$.

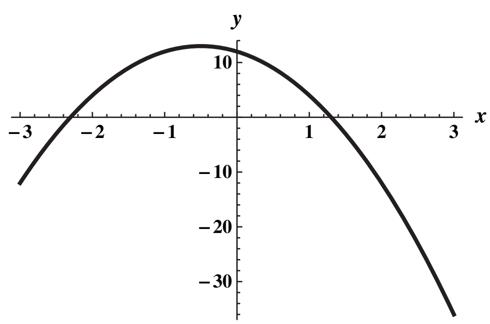


1.2.37

By completing the square, we have that

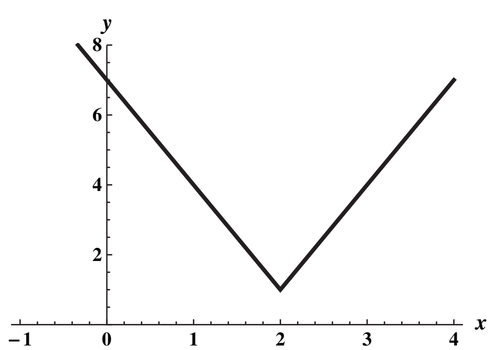
$$\begin{aligned} h(x) &= -4(x^2+x-3) \\ &= -4\left(x^2+x+\frac{1}{4}-\frac{1}{4}-3\right) \\ &= -4(x+(1/2))^2+13 \end{aligned}$$

So it is $-4f(x+(1/2))+13$ where $f(x) = x^2$.



1.2.38

Since $|3x-6|+1 = 3|x-2|+1$, this is $3f(x-2)+1$ where $f(x) = |x|$.



1.2.39

a. True. A polynomial $p(x)$ can be written as the ratio of polynomials $\frac{p(x)}{1}$, so it is a rational function. However, a rational function like $\frac{1}{x}$ is not a polynomial.

b. False. For example, if $f(x) = 2x$, then $(f \circ f)(x) = f(f(x)) = f(2x) = 4x$ is linear, not quadratic.

c. True. In fact, if f is degree m and g is degree n, then the degree of the composition of f and g is $m \cdot n$, regardless of the order they are composed.

d. False. The graph would be shifted two units to the left.

1.2.40 The points of intersection are found by solving $x^2 + 2 = x + 4$. This yields the quadratic equation $x^2 - x - 2 = 0$ or $(x-2)(x+1) = 0$. So the x-values of the points of intersection are 2 and -1. The actual points of intersection are $(2, 6)$ and $(-1, 3)$.

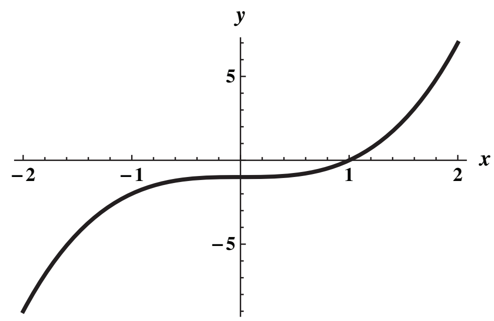
1.2.41 The points of intersection are found by solving $x^2 = -x^2 + 8x$. This yields the quadratic equation $2x^2 - 8x = 0$ or $(2x)(x-4) = 0$. So the x-values of the points of intersection are 0 and 4. The actual points of intersection are $(0, 0)$ and $(4, 16)$.

1.2.42 $y = x + 1$, since the y value is always 1 more than the x value.

1.2.43 $y = \sqrt{x} - 1$, since the y value is always 1 less than the square root of the x value.

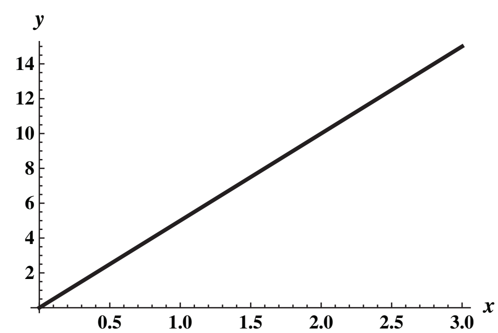
1.2.44

$y = x^3 - 1$. The domain is $(-\infty, \infty)$.



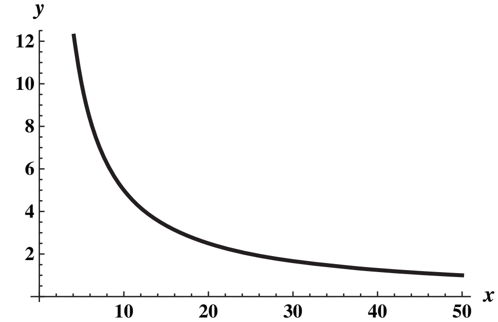
1.2.45

$y = 5x$. The natural domain for the situation is $[0, h]$ where h represents the maximum number of hours that you can run at that pace before keeling over.



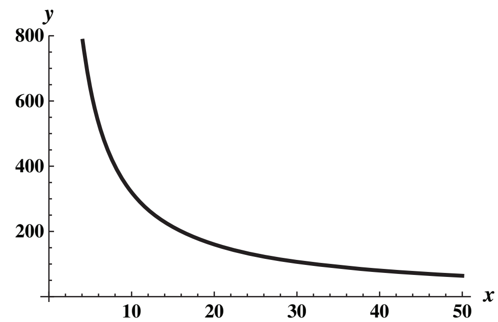
1.2.46

$y = \frac{50}{x}$. Theoretically the domain is $(0, \infty)$, but the world record for the "hour ride" is just short of 50 miles.

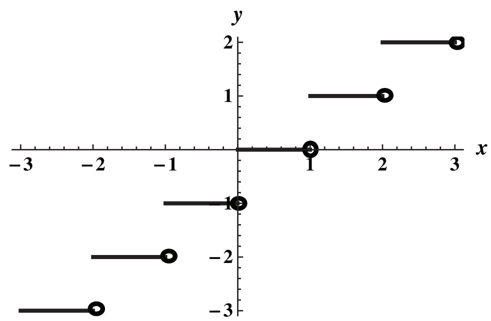


1.2.47

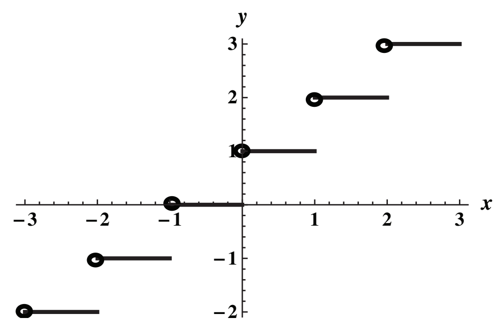
$y = \dfrac{3200}{x}$. Note that $\dfrac{x \text{ dollars per gallon}}{32 \text{ miles per gallon}} \cdot y$ miles would represents the numbers of dollars, so this must be 100. So we have $\frac{xy}{32} = 100$, or $y = \frac{3200}{x}$. We certainly have $x > 0$, but unfortunately, there appears to be no no upper bound for x, so the domain is $(0, \infty)$.



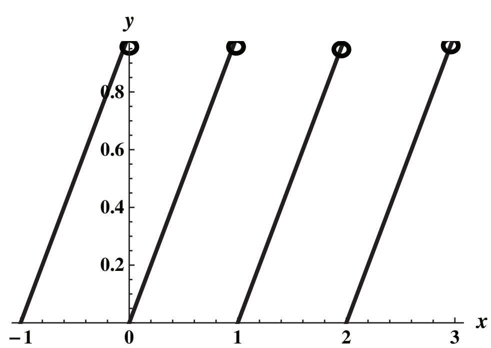
1.2.48



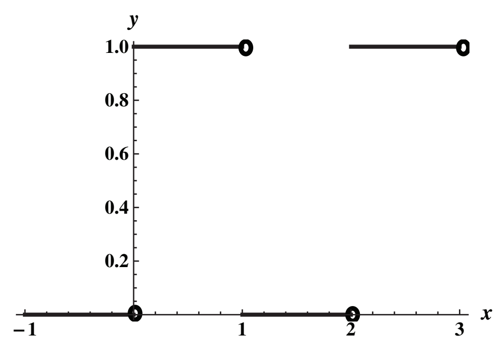
1.2.49



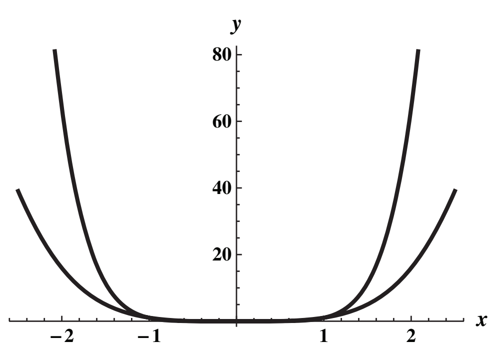
1.2.50



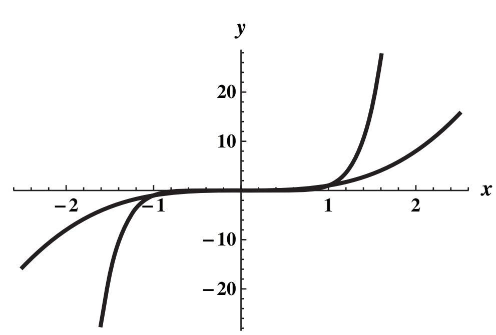
1.2.51



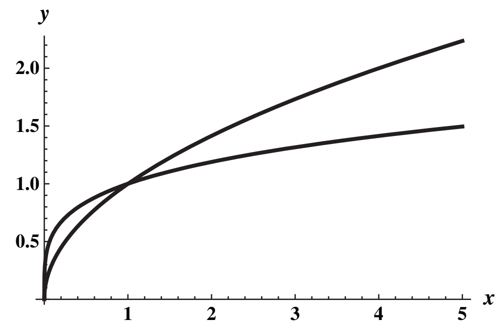
1.2.52



1.2.53



1.2.54



1.2.55

a. By comparing various pairs of points, it appears that the slope of the line is about 328.3. At $t = 0$, the value of p is 1875. Therefore a line which reasonably approximates the data is $p(t) = 328.3t + 1875$.

b. Using this line, we have that $p(9) = 4830$.

1.2.56

a. We know that the points $(32, 0)$ and $(212, 100)$ are on our line. The slope of our line is thus $\frac{100-0}{212-32} = \frac{100}{180} = \frac{5}{9}$. The function $f(F)$ thus has the form $C = (5/9)F + b$, and using the point $(32, 0)$ we see that $0 = (5/9)32 + b$, so $b = -(160/9)$. Thus $C = (5/9)F - (160/9)$

b. Solving the system of equations $C = (5/9)F - (160/9)$ and $C = F$, we have that $F = (5/9)F - (160/9)$, so $(4/9)F = -160/9$, so $F = -40$ when $C = -40$.

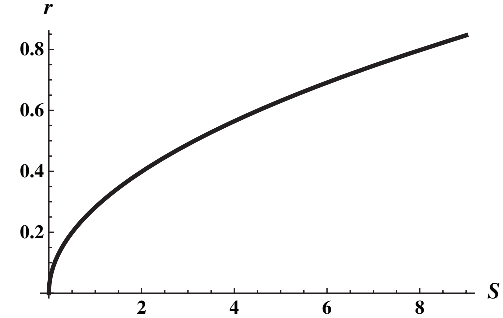
1.2.57

a. Since you are paying \$350 per month, the amount paid after m months is $y = 350m + 1200$.

b. After 4 years (48 months) you have paid $350 \cdot 48 + 1200 = 18000$ dollars. If you then buy the car for \$10,000, you will have paid a total of \$28,000 for the car instead of \$25,000. So you should buy the car instead of leasing it.

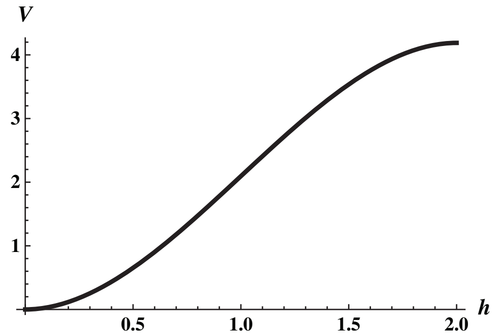
1.2.58

Since $S = 4\pi r^2$, we have that $r^2 = \frac{S}{4\pi}$, so $|r| = \frac{\sqrt{S}}{2\sqrt{\pi}}$, but since r is positive, we can write $r = \frac{\sqrt{S}}{2\sqrt{\pi}}$.



1.2.59

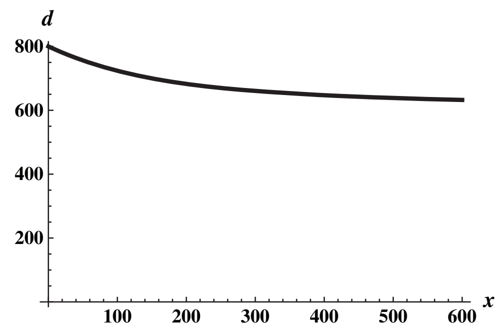
The function makes sense for $0 \leq h \leq 2$.



1.2.60

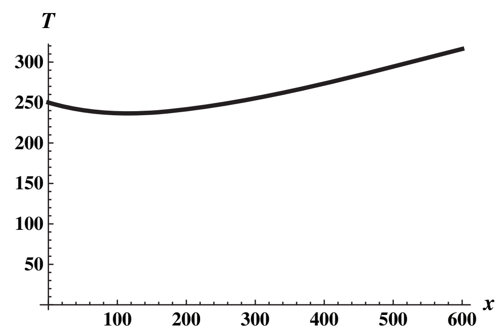
a. Note that the island, the point P on shore, and the point down shore x units from P form a right triangle. By the Pythagorean theorem, the length of the hypotenuse is $\sqrt{40000 + x^2}$. So Kelly must row this distance and then jog $600 - x$ meters to get home. So her total distance is

$$d(x) = \sqrt{40000 + x^2} + (600 - x).$$



b. Since distance is rate times time, we have that time is distance divided by rate. Thus

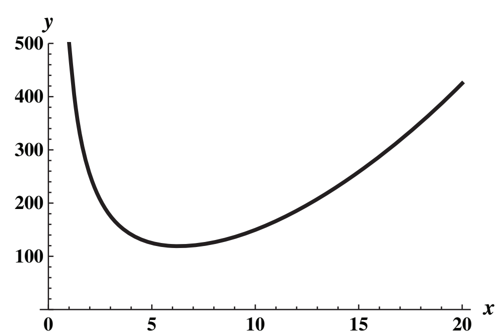
$$T(x) = \frac{\sqrt{40000 + x^2}}{2} + \frac{600 - x}{4}.$$



c. By inspection, it looks as though she should head to a point about 115 meters down shore from P. This would lead to a time of about 236.6 seconds.

1.2.61

a. The volume of the box is $x^2 h$, but since the box has volume 125 cubic feet, we have that $x^2 h = 125$, so $h = \frac{125}{x^2}$. The surface area of the box is given by x^2 (the area of the base) plus $4 \cdot hx$, since each side has area hx. Thus $S = x^2 + 4hx = x^2 + \frac{4 \cdot 125 \cdot x}{x^2} = x^2 + \frac{500}{x}$.



b. By inspection, it looks like the value of x which minimizes the surface area is about 6.3.

1.2.62 Let $f(x) = a_n x^n +$ some smaller degree terms and let $g(x) = b_m x^m +$ some smaller degree terms.

a. The largest degree term in $f \cdot f$ is $a_n x^n \cdot a_n x^n = a_n^2 x^{n+n}$, so the degree of this polynomial is $n+n = 2n$.

b. The largest degree term in $f \circ f$ involves $a_n \cdot (a_n x^n)^n$, so the degree is n^2.

c. The largest degree term in $f \cdot g$ is $a_n b_m x^{m+n}$, so the degree of the product is $m + n$.

d. The largest degree term in $f \circ g$ involves $a_n \cdot (b_m x^m)^n$, so the degree is mn.

1.2.63 Suppose that the parabola f crosses the x-axis at a and b, with $a < b$. Then a and b are roots of the polynomial, so $(x - a)$ and $(x - b)$ are factors. Thus the polynomial must be $f(x) = c(x - a)(x - b)$ for some non-zero real number c. So $f(x) = cx^2 - c(a + b)x + abc$. Since the vertex always occurs at the x value which is $\dfrac{-\text{coefficient of } x}{2 \cdot \text{coefficient on } x^2}$ we have that the vertex occurs at $\dfrac{c(a+b)}{2c} = \dfrac{a+b}{2}$, which is halfway between a and b.

1.2.64

a. We complete the square to rewrite the function f. Write $f(x) = ax^2 + bx + c$ as $f(x) = a(x^2 + \frac{b}{a}x + \frac{c}{a})$. Completing the square yields

$$a\left(\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a}\right) + \left(\frac{c}{a} - \frac{b^2}{4a}\right)\right) = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4}\right).$$

Thus the graph of f is obtained from the graph of x^2 by shifting $\frac{b}{2a}$ units to the left (and then doing some scaling and vertical shifting) – moving the vertex from 0 to $-\frac{b}{2a}$.

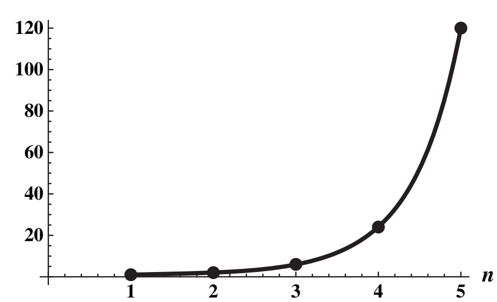
b. We know that the graph of f touches the x-axis if the equation $ax^2 + bx + c = 0$ has at least one real solution. By the quadratic formula, we know that this occurs exactly when the discriminant $b^2 - 4ac$ is non-negative. So the condition we seek is for $b^2 - 4ac \geq 0$, or $b^2 \geq 4ac$.

1.2.65

a.

n	1	2	3	4	5
$n!$	1	2	6	24	120

b.



c. Using trial and error and a calculator yields that 10! is more than a million, but 9! isn't.

1.2.66

a.

n	1	2	3	4	5	6	7	8	9	10
$S(n)$	1	3	6	10	15	21	28	36	45	55

b. The domain of this function consists of the positive integers. The range is a subset of the set of positive integers.

c. Using trial and error and a calculator yields that $S(n) > 1000$ for the first time for $n = 45$.

1.2.67

a.

n	1	2	3	4	5	6	7	8	9	10
$T(n)$	1	5	14	30	55	91	140	204	285	385

b. The domain of this function consists of the positive integers.

c. Using trial and error and a calculator yields that $T(n) > 1000$ for the first time for $n = 14$.

1.3 Trigonometric Functions and Their Inverses

1.3.1 Let O be the length of the side opposite the angle x, let A be length of the side adjacent to the angle x, and let H be the length of the hypotenuse. Then $\sin x = \frac{O}{H}$, $\cos x = \frac{A}{H}$, $\tan x = \frac{O}{A}$, $\csc x = \frac{H}{O}$, $\sec x = \frac{H}{A}$, and $\cot x = \frac{A}{O}$.

1.3.2 We consider the angle formed by the positive x axis and the ray from the origin through the point $P(x, y)$. A positive angle is one for which the rotation from the positive x axis to the other ray is counterclockwise. We then define the six trigonometric functions as follows: let $r = \sqrt{x^2 + y^2}$. Then $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$, $\csc\theta = \frac{r}{y}$, $\sec\theta = \frac{r}{x}$, and $\cot\theta = \frac{x}{y}$.

1.3.3 The radian measure of an angle θ is the length of the arc s on the unit circle associated with θ.

1.3.4 The period of a function is the smallest positive real number k so that $f(x + k) = f(x)$ for all x in the domain of the function. The sine, cosine, secant, and cosecant function all have period 2π. The tangent and cotangent functions have period π.

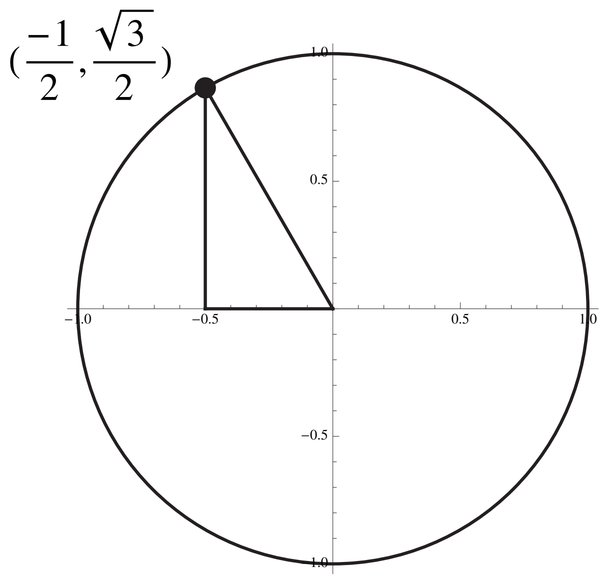
1.3.5 $\sin^2 x + \cos^2 x = 1$, $1 + \cot^2 x = \csc^2 x$, and $\tan^2 x + 1 = \sec^2 x$.

1.3.6 $\csc x = \frac{1}{\sin x}$, $\sec x = \frac{1}{\cos x}$, $\tan x = \frac{\sin x}{\cos x}$, and $\cot x = \frac{\cos x}{\sin x}$.

1.3.7 The tangent function is undefined where $\cos x = 0$, which is at all real numbers of the form $\frac{\pi}{2} + k\pi, k$ an integer.

1.3.8 $\sec x$ is defined wherever $\cos x \neq 0$, which is $\{x\colon x \neq \frac{\pi}{2} + k\pi, k \text{ an integer}\}$.

1.3.9 The point on the unit circle associated with $2\pi/3$ is $(-1/2, \sqrt{3}/2)$, so $\cos(2\pi/3) = -1/2$.



1.3.10 The point on the unit circle associated with $2\pi/3$ is $(-1/2, \sqrt{3}/2)$, so $\sin(2\pi/3) = \sqrt{3}/2$. See the picture from the previous problem.

1.3.11

The point on the unit circle associated with $-3\pi/4$ is $(-\sqrt{2}/2, -\sqrt{2}/2)$, so

$$\tan(-3\pi/4) = 1.$$