1. Find the average rate of change of y with respect to x for $y = f(x) = \frac{3}{x^4}$ over the interval [1, 7].
   A) 0.375 B) –0.500 C) –0.500 D) –17.993 E) 2.999
   Ans: B
   Difficulty: Easy
   Section: 2.1

2. Find the average rate of change of y with respect to x over the interval [1, 5]. $y = f(x) = 3x^3$
   A) 62 B) 95 C) 93 D) 74 E) 372
   Ans: C
   Difficulty: Easy
   Section: 2.1

3. Find the instantaneous rate of change of $y = 4x^2$ with respect to x at $x_0 = 7$.
   A) 8 B) 56 C) 14 D) 28 E) 22
   Ans: B
   Difficulty: Easy
   Section: 2.1

4. Find the instantaneous rate of change of $y = \frac{9}{x}$ with respect to x at $x_0 = 5$.
   A) –225 B) –8.9600 C) 0.3600 D) –0.3600 E) –0.0617
   Ans: D
   Difficulty: Medium
   Section: 2.1

5. Find the instantaneous rate of change of $y = –4x^7$ with respect to x at a general point x_0.
   A) $–28x_0^7$ B) $–4x_0$ C) $–4x_0^7$ D) $–4x_0^6$ E) $–28x_0^6$
   Ans: E
   Difficulty: Easy
   Section: 2.1

6. Find the instantaneous rate of change of $y = \frac{2}{x^3}$ with respect to x at a general point x_0.
   A) $\frac{–6}{x_0^3}$ B) $\frac{2}{x_0^4}$ C) $\frac{–6}{x_0^4}$ D) $\frac{6}{x_0^4}$ E) $\frac{6}{x_0^3}$
   Ans: C
   Difficulty: Medium
   Section: 2.1

7. Find the slope of the tangent line to the graph of $f(x) = 7x^4 - 9$ at a general point x_0.
   A) $28x_0^3 - 9$ B) $7x_0^3$ C) $28x_0^3$ D) $7x_0^3 - 1$ E) $7x_0^3 - 9$
   Ans: C
   Difficulty: Easy
   Section: 2.1

8. Answer true or false. The slope of the tangent line to the graph of $f(x) = -2x^2 - 1$ at $x_0 = 3$ is –13.
   Ans: False
   Difficulty: Easy
   Section: 2.1

9. Answer true or false. Use a graphing utility to graph $y = 3t^2$ on [0, 4]. If this graph represents a position versus time curve for a particle, the instantaneous velocity of the particle is increasing over the graphed domain.
   Ans: True
   Difficulty: Easy
   Section: 2.1

10. Use a graphing utility to graph $y = t^2 - 7t + 10$ on [0, 10]. If this graph represents a position versus time curve for a particle, the instantaneous velocity of the particle is zero at what time? Assume time is in seconds.
    A) 6s B) 3s C) 3.5s D) 1.5s E) 7s
    Ans: C
    Difficulty: Medium
    Section: 2.1

11. A rock is dropped from a height of 2,704 feet and falls toward earth in a straight line. In t seconds the rock drops a distance of $16t^2$ feet. What is the instantaneous velocity downward when it hits the ground?
    A) 116,985,856 feet/s
    B) 416 feet/s
    C) 208 feet/s
    D) 32 feet/s
    E) 26 feet/s
    Ans: B
    Difficulty: Easy
    Section: 2.1

12. Answer true or false. The magnitude of the instantaneous velocity is always less than the magnitude of the average velocity.
    Ans: False
    Difficulty: Easy
    Section: 2.1

13. Answer true or false. If a rock is thrown straight upward to a height of 26 feet from the ground, when it returns to earth its average velocity will be its initial velocity.
Ans: False
Difficulty: Easy
Section: 2.1

14. Answer true or false. If an object is thrown straight upward with an instantaneous velocity of 35 m/s, its instantaneous velocity at the point where it stops rising is 0.
Ans: True
Difficulty: Easy
Section: 2.1

15. An object moves in a straight line so that after t s its distance in mm from its original position is given by $s = 7t^3 + 4t$. Its instantaneous velocity at $t = 4$s is
A) 336 mm B) 1,348 mm C) 5,380 mm D) 340 mm E) 116 mm
Ans: D
Difficulty: Medium
Section: 2.1

16. Find the instantaneous rate of change of y with respect to x at $x_0 = 4$. $y = 6x^2 - 2$
A) 48 B) 46 C) 24 D) 50 E) 96
Ans: A
Difficulty: Easy
Section: 2.1

17. Find the instantaneous rate of change of y with respect to x at $x_0 = 81$. $y = \sqrt{x} - 2$
A) $\frac{1}{18}$ B) $\frac{1}{9}$ C) $\frac{11}{9}$ D) $\frac{18}{17}$ E) $\frac{1}{81}$
Ans: A
Difficulty: Hard
Section: 2.1

18. Let $f(x) = \frac{1}{x^2}$. Find the average rate of change of y with respect to x over the interval [5, 6].
Ans: $-\frac{11}{900}$
Difficulty: Easy
Section: 2.1

19. Let $f(x) = \frac{1}{x^2}$. Find the instantaneous rate of change of y with respect to x at the point $x = 2$.

    Ans: $-\frac{1}{4}$
    Difficulty: Easy
    Section: 2.1

20. Let $y = x^2 + 2$. Find the average rate of change of y with respect to x over the interval $[-5, -1]$.
    Ans: -6
    Difficulty: Easy
    Section: 2.1

21. Let $y = x^2 + 6$. Find the instantaneous rate of change of y with respect to x at the point $x = -5$.
    Ans: -10
    Difficulty: Easy
    Section: 2.1

22. Let $y = \frac{1}{x-1}$. Find the average rate of change of y with respect to x over the interval $[2,4]$.

    Ans: $-\frac{1}{3}$
    Difficulty: Medium
    Section: 2.1

23. Let $y = \frac{1}{x-3}$. Find the instantaneous rate of change of y with respect to x at the point $x = 5$.

    Ans: $-\frac{1}{4}$
    Difficulty: Medium
    Section: 2.1

24. Let $y = \frac{2}{x+2}$. Find the average rate of change of y with respect to x over the given interval $[3,6]$.

    Ans: $-\frac{1}{20}$
    Difficulty: Medium
    Section: 2.1

25. Let $y = \dfrac{1}{x+4}$. Find the instantaneous rate of change of y with respect to x at the point $x = 1$.

Ans: $-\dfrac{1}{25}$

Difficulty: Medium
Section: 2.1

26. Let $f(x) = \dfrac{1}{5-x}$. Find the slope of the tangent to the graph of f at a general point x_0 using limits and find the slope of the tangent line at $x_0 = 4$

Ans: $\lim\limits_{x_1 \to x_0} \dfrac{1}{(5-x_1)(5-x_0)} = \dfrac{1}{(5-x_0)^2}$

The slope of the tangent line at $x_0 = 4$ is $\dfrac{1}{1}$.

Difficulty: Medium
Section: 2.1

27. Let $f(x) = \dfrac{1}{x-4}$. Find the slope of the tangent to the graph of f at a general point x_0 using limits and find the slope of the tangent at $x_0 = 5$.

Ans: $\lim\limits_{x_1 \to x_0} \dfrac{-1}{(x_1-4)(x_0-4)} = \dfrac{-1}{(x_0-4)^2}$

The slope of the tangent line at $x_0 = 5$ is $-\dfrac{1}{1}$.

Difficulty: Medium
Section: 2.1

28. Let $f(x) = \dfrac{4}{x^4}$. Find the slope of the tangent to the graph of f at a general point x_0 using limits and find the slope of the tangent at $x_0 = -5$.

Ans: $\lim\limits_{x_1 \to -5} \dfrac{\dfrac{4}{x_1^4} - \dfrac{4}{-5^4}}{x_1 + 5} = \lim\limits_{x_1 \to -5} \dfrac{4\left(-5^4 - x_1^4\right)}{625 x_1^4 (x_1+5)} = -\dfrac{16}{x_0^5}$

The slope of the tangent line at $x_0 = -5$ is $\dfrac{16}{3{,}125}$.

Difficulty: Medium
Section: 2.1

29. Let $f(x) = 4x^3$. Find the slope of the tangent to the graph of f at a general point x_0 using limits and find the slope of the tangent at $x_0 = 2$.

Ans: $\lim_{x_1 \to x_0} 4\left(x_1^2 + x_0^2\right) = 12x_0^2$

Slope of tangent at $x_0 = 2$ is 48

Difficulty: Easy
Section: 2.1

30. A rock is dropped from a height of 144 feet and falls toward the earth in a straight line. In t seconds, the rock drops a distance of $s = 16t^2$ feet. What is the average velocity of the rock while it is falling? Use limits to find the instantaneous velocity of the rock when it hits the ground.

Ans: Average velocity: 48 feet per second

Instantaneous velocity at ground = 96 feet per second

Difficulty: Medium
Section: 2.1

31. A particle moves in a straight line from its initial position so that after t seconds, its distance is given by $s = t^2 + t$ feet from its initial position. Find the average velocity of the particle over the interval [3,6] seconds. Use limits to find the instantaneous velocity of the particle at $t = 1$ second.

Ans: Average velocity = 10 feet per second

The instantaneous velocity at $t = 1$ second is 3 feet per second.

Difficulty: Medium
Section: 2.1

32. A particle moves in a straight line from its initial position so that after t seconds, its distance is given by $s = \dfrac{t}{t+1}$ feet from its initial position. Find the average velocity of the particle over the interval [4,8] seconds. Use limits to find the instantaneous velocity of the particle at $t = 4$ seconds.

Ans: Average velocity = $\dfrac{1}{45}$ feet per second.

The instantaneous velocity at $t = 4$ seconds is $\dfrac{1}{25}$ feet per second.

Difficulty: Medium
Section: 2.1

33. Let $f(x) = ax^2 + b$, where a and b are constant. Use the method of Section 3.1 to show that the slope of the tangent to the graph of f at $x = x_0$ is $2ax_0$.

Ans: $m_{\tan} = \lim_{x_1 \to x_0} \dfrac{\left(ax_1^2 + b\right) - \left(ax_0^2 + b\right)}{x_1 - x_0} = \lim_{x_1 \to x_0} \dfrac{a\left(x_1^2 - x_0^2\right)}{x_1 - x_0} = \lim_{x_1 \to x_0} a\left(x_1 + x_0\right) = 2ax_0$

Difficulty: Hard
Section: 2.1

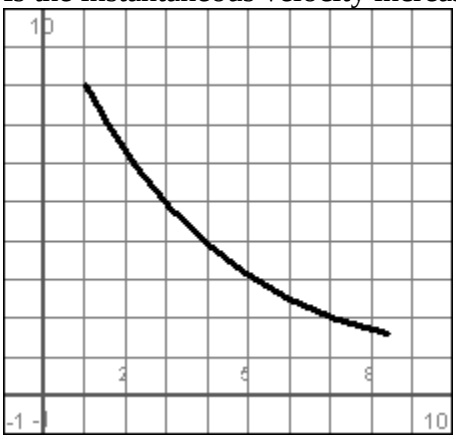
34. Let $f(x) = ax^3 + b$, where a and b are constants. Use the method of Section 3.1 to show that the slope of the tangent to the graph of f at $x = x_0$ is $3ax_0^2$.

Ans:

$$m_{\tan} = \lim_{x_1 \to x_0} \frac{\left(ax_1^3 + b\right) - \left(ax_0^3 + b\right)}{x_1 - x_0} = \lim_{x_1 \to x_0} \frac{ax_1^3 - ax_0^3}{x_1 - x_0} = \lim_{x_1 \to x_0} a\left(x_1^2 + x_1x_0 + x_0^2\right) = 3ax_0^2$$

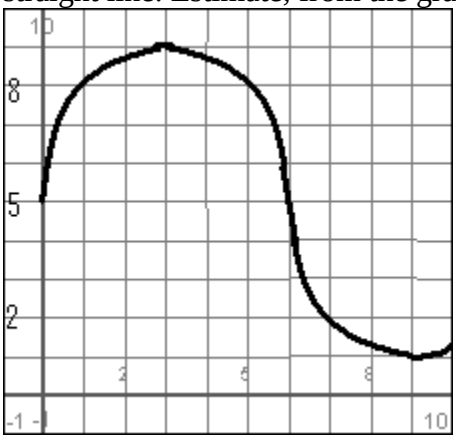
Difficulty: Medium
Section: 2.1

35. The graph shows the position versus time curve for a particle moving on a straight line. Is the instantaneous velocity increasing or decreasing with time?



Ans: decreasing
Difficulty: Easy
Section: 2.1

36. The figure shows the position versus time curve for a certain particle moving along a straight line. Estimate, from the graph, the average velocity over the interval 3 to 9.



Ans: -4/3
Difficulty: Easy
Section: 2.1

37. Given $f(x) = x^3 - 1$, find the slope of the graph of f at the x-value $x_0 = 4$.
Ans: 48
Difficulty: Medium
Section: 2.1

38. Given $f(x) = 13\sqrt{x}$, find the slope of the graph of f at $x_0 = 1$.
Ans: $\frac{13}{2}$
Difficulty: Medium
Section: 2.1

39. Find the instantaneous rate of change of $f(x) = \frac{2}{x^3}$ at $x_0 = 5$.
Ans: $-\frac{6}{625}$
Difficulty: Medium
Section: 2.1

40. Find the instantaneous rate of change of $f(x) = 5x^2 - 12$ at $x_0 = 5$.
Ans: 50
Difficulty: Medium
Section: 2.1

41. Find the instantaneous rate of change of $f(x) = 5x^2 - 6x + 9$ at $x_0 = 3$.
    Ans: 24
    Difficulty: Medium
    Section: 2.1