Exercises 2-1

1.
$$\frac{f(b)-f(a)}{b-a}=\frac{f(5)-f(3)}{5-3}=\frac{5-1}{2}=\frac{4}{2}=2$$

2.
$$\frac{v(b)-v(a)}{b-a}=\frac{v(1)-v(-1)}{1-(-1)}=\frac{-1-(1)}{2}=\frac{-2}{2}=-1$$

3.
$$\frac{v(b)-v(a)}{b-a}=\frac{v(4)-v(-3)}{4-(-3)}=\frac{12-12}{7}=\frac{0}{7}=0$$

4.
$$\frac{z(b)-z(a)}{b-a}=\frac{z(5)-z(1)}{5-1}=\frac{\frac{\ln(5)}{5}-\frac{\ln(1)}{1}}{4}=\frac{\frac{\ln(5)}{5}-0}{4}=\frac{\ln(5)}{20}\approx 0.0805$$

5.
$$\frac{q(b)-q(a)}{b-a}=\frac{q(6)-q(0)}{6-0}=\frac{\sqrt{8}-\sqrt{2}}{6}=\frac{2\sqrt{2}-\sqrt{2}}{6}=\frac{\sqrt{2}}{6}\approx 0.2357$$

6. average rate of change
$$=\frac{\text{change in number of warehouses}}{\text{change in years}}=\frac{(608-512)\text{ warehouses}}{(2012-2008)\text{ years}}=24\text{ warehouses per year}$$

7. average rate of change
$$=\frac{\text{change in number of Gold Star members}}{\text{change in years}}=\frac{(24{,}846-21{,}445)\text{ thousands}}{2011-2009}=1700.5\text{ thousand members per year}$$

8. average rate of change
$$=\frac{\text{change in number of Gold Star members/warehouse}}{\text{change in number of warehouses}}=\frac{(26{,}736-20{,}181)\text{ thousands}}{608-512}\approx 68.28\text{ thousand Gold Star members per warehouse}$$

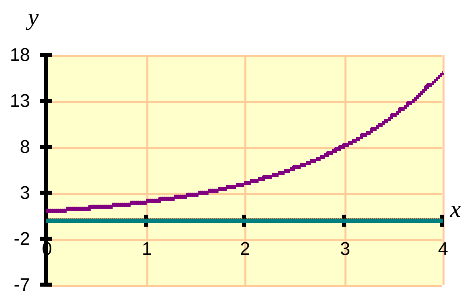
9. average rate of change

$$= \frac{\text{change in Starbucks net revenue}}{\text{change in years}}$$
$$= \frac{(11.7 - 9.4)\ \text{billion dollars}}{2011 - 2007}$$
$$= 0.575 \text{ billion dollars per year}$$

10. average rate of change

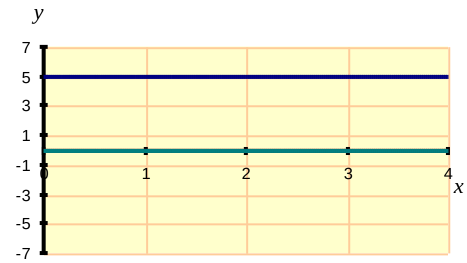
$$= \frac{\text{number of Starbucks stores}}{\text{change in years}}$$
$$= \frac{(16{,}858 - 16{,}680)\ \text{stores}}{2010 - 2008}$$
$$= 89 \text{ stores per year}$$

11. $f(x) = 2^x$



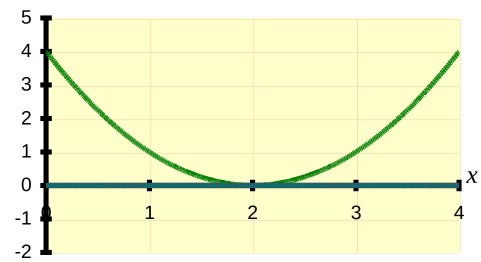
$$\frac{f(3) - f(1)}{3 - 1} = \frac{8 - 2}{2}$$
$$= \frac{6}{2}$$
$$= 3$$

12. $f(x) = 5$



$$\frac{f(3) - f(1)}{3 - 1} = \frac{5 - 5}{2}$$
$$= \frac{0}{2}$$
$$= 0$$

13. $f(x) = (x - 2)^2$



$$\frac{f(3) - f(1)}{3 - 1} = \frac{1 - 1}{2}$$
$$= \frac{0}{2}$$
$$= 0$$

14.

$$\frac{f(3) - f(1)}{3 - 1} = \frac{4 - 0}{2}$$
$$= \frac{4}{2}$$
$$= 2$$

15.

$$\frac{f(3)-f(1)}{3-1}=\frac{1-3}{2}$$
$$=\frac{-2}{2}$$
$$=-1$$

16.

$$\frac{m(700)-m(600)}{700-600}\ \frac{\text{GS members}}{\text{warehouses}}$$
$$\frac{31719-26116}{100}\ \frac{\text{GS members}}{\text{warehouses}}$$
$$\approx 56.03 \text{ thousand Gold Star members per warehouse}$$

Increasing the number of warehouses from 600 to 700 is predicted to increase the number of Gold Star members at an average rate of 56.03 thousand members per warehouse.

17.

$$\frac{p(5)-p(3)}{5-3}\ \frac{\text{dollars per pound}}{\text{years}}$$
$$\frac{2.75-2.0028}{2}\ \frac{\text{dollars per pound}}{\text{years}}$$
$$\approx 0.374 \text{ dollars per pound per year}$$

Between 2010 and 2012, the price of peanut butter increased at an average rate of $0.374 per pound per year.

18.

$$\frac{p(5)-p(1)}{5-1}\ \frac{\text{dollars per pound}}{\text{years}}$$
$$\frac{5.0025-4.0481}{4}\ \frac{\text{dollars per pound}}{\text{years}}$$
$$\approx 0.24 \text{ dollars per pound per year}$$

Between 2008 and 2012, the price of potato chips increased at an average rate of $0.24 per pound per year.

19.

$$\frac{j(20)-j(10)}{20-10}\ \frac{\text{thousand jobs}}{\text{years}}$$
$$\frac{347.8-334.7}{10}\ \frac{\text{thousand jobs}}{\text{years}}$$
$$=1.31 \text{ thousand jobs per year}$$

Between 2008 and 2018, the number of jobs in the dry cleaning and laundry industry is predicted to increase at an average rate of 1.31 jobs per year.

20.

$$\frac{j(20)-j(15)}{20-15}\ \frac{\text{thousand jobs}}{\text{years}}$$
$$\frac{140-150.925}{5}\ \frac{\text{thousand jobs}}{\text{years}}$$
$$=-2.185 \text{ thousand jobs per year}$$

Between 2008 and 2018, the number of jobs in the ship- and boat-building industry is predicted to decrease at an average rate of 2.185 thousand jobs per year.

Exercises 2-2

1.

$h = 0.1$

$$\Rightarrow \frac{f(2+0.1)-f(2)}{0.1} = \frac{4.41-4}{0.1} = \frac{0.41}{0.1} = 4.1$$

$h = 0.01$

$$\Rightarrow \frac{f(2+0.01)-f(2)}{0.01} = \frac{4.0401-4}{0.01} = \frac{0.0401}{0.01} = 4.01$$

$h = 0.001$

$$\Rightarrow \frac{f(2+0.001)-f(2)}{0.001} = \frac{4.004001-4}{0.001} = \frac{0.004001}{0.001} = 4.001$$

$\therefore$ inst. rate of change ≈ 4

2.

$h = 0.1$

$$\Rightarrow \frac{s(2+0.1)-s(2)}{0.1} = \frac{-6.56-0}{0.1} = -65.6$$

$h = 0.01$

$$\Rightarrow \frac{s(2+0.01)-s(2)}{0.01} = \frac{-0.6416-0}{0.01} = -64.16$$

$h = 0.001$

$$\Rightarrow \frac{s(2+0.001)-s(2)}{0.001} = \frac{-0.064016-0}{0.001} = -64.016$$

$\therefore$ inst. rate of change ≈ -64

3.

$$\begin{aligned}
&h = 0.1\\
&\Rightarrow \frac{w(5+0.1)-w(5)}{0.1}\\
&= \frac{22.4-22}{0.1}\\
&= \frac{0.4}{0.1}\\
&= 4\\
&h = 0.01\\
&\Rightarrow \frac{w(5+0.01)-w(5)}{0.01}\\
&= \frac{22.04-22}{0.01}\\
&= \frac{0.04}{0.01}\\
&= 4\\
&h = 0.001\\
&\Rightarrow \frac{w(5+0.001)-w(5)}{0.001}\\
&= \frac{22.004-22}{0.001}\\
&= \frac{0.004}{0.001}\\
&= 4
\end{aligned}$$

$\therefore$ inst. rate of change ≈ 4

4.

$$\begin{aligned}
&h = 0.1\\
&\Rightarrow \frac{P(5+0.1)-P(5)}{0.1}\\
&= \frac{5-5}{0.1}\\
&= \frac{0}{0.1}\\
&= 0\\
&h = 0.01\\
&\Rightarrow \frac{P(5+0.01)-P(5)}{0.01}\\
&= \frac{5-5}{0.01}\\
&= \frac{0}{0.01}\\
&= 0\\
&h = 0.001\\
&\Rightarrow \frac{P(5+0.001)-P(5)}{0.001}\\
&= \frac{5-5}{0.001}\\
&= \frac{0}{0.001}\\
&= 0
\end{aligned}$$

$\therefore$ inst. rate of change ≈ 0

5.

$h = 0.1$

$$\Rightarrow \frac{P(0.07+0.1)-P(0.07)}{0.1}$$
$$= \frac{684.45-572.45}{0.1}$$
$$= \frac{112}{0.1}$$
$$= 1120$$

$h = 0.01$

$$\Rightarrow \frac{P(0.07+0.01)-P(0.07)}{0.01}$$
$$= \frac{583.2-572.45}{0.01}$$
$$= \frac{10.75}{0.01}$$
$$= 1075$$

$h = 0.001$

$$\Rightarrow \frac{P(0.07+0.001)-P(0.07)}{0.001}$$
$$= \frac{573.5205-572.45}{0.001}$$
$$= \frac{1.0705}{0.001}$$
$$= 1070.5$$

$\therefore$ inst. rate of change ≈ 1070

6.

$$f'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$
$$= \lim_{h\to 0}\frac{(2+h)^2-(2)^2}{h}$$
$$= \lim_{h\to 0}\frac{4+4h+h^2-4}{h}$$
$$= \lim_{h\to 0}\frac{4h+h^2}{h}$$
$$= \lim_{h\to 0}\frac{h(4+h)}{h}$$
$$= \lim_{h\to 0}(4+h)$$
$$= 4$$

7.

$$s'(2) = \lim_{h\to 0}\frac{s(2+h)-s(2)}{h}$$
$$= \lim_{h\to 0}\frac{\left[-16(2+h)^2+64\right]-\left[-16(2)^2+64\right]}{h}$$
$$= \lim_{h\to 0}\frac{\left[-16(4+4h+h^2)+64\right]-\left[-16(4)+64\right]}{h}$$
$$= \lim_{h\to 0}\frac{\left[-64-64h-16h^2+64\right]-\left[-64+64\right]}{h}$$
$$= \lim_{h\to 0}\frac{\left[-64h-16h^2\right]-\left[0\right]}{h}$$
$$= \lim_{h\to 0}\frac{-16h\left(4+h\right)}{h}$$
$$= \lim_{h\to 0}\left(-16(4+h)\right)$$
$$= -64$$

8.

$$\begin{aligned} w'(5) &= \lim_{h\to 0}\frac{w(5+h)-w(5)}{h} \\ &= \lim_{h\to 0}\frac{\left[4(5+h)+2\right]-\left[4(5)+2\right]}{h} \\ &= \lim_{h\to 0}\frac{\left[20+4h+2\right]-\left[22\right]}{h} \\ &= \lim_{h\to 0}\frac{4h}{h} \\ &= \lim_{h\to 0}(4) \\ &= 4 \end{aligned}$$

9.

$$\begin{aligned} P'(25) &= \lim_{h\to 0}\frac{P(25+h)-P(25)}{h} \\ &= \lim_{h\to 0}\frac{5-5}{h} \\ &= \lim_{h\to 0}\frac{0}{h} \\ &= 0 \end{aligned}$$

10.

$$\begin{aligned} P'(0.07) &= \lim_{h\to 0}\frac{P(0.07+h)-P(0.07)}{h} \\ &= \lim_{h\to 0}\frac{\left[500(1+0.07+h)^2\right]-\left[500(1+0.07)^2\right]}{h} \\ &= \lim_{h\to 0}\frac{\left[500(1.07+h)^2\right]-\left[500(1.07)^2\right]}{h} \\ &= \lim_{h\to 0}\frac{\left[500(1.1449+2.14h+h^2)\right]-\left[500(1.1449\right]}{h} \\ &= \lim_{h\to 0}\frac{500\left[(1.1449+2.14h+h^2)-1.1449\right]}{h} \\ &= \lim_{h\to 0}\frac{500\left[2.14h+h^2\right]}{h} \\ &= \lim_{h\to 0}\frac{500h\left[2.14+h\right]}{h} \\ &= \lim_{h\to 0}500(2.14+h) \\ &= 1070 \end{aligned}$$

11. $f\left(x\right)=2x^{-3}; x=3$

h	**diff quotient**
0.1	–0.06940
0.01	–0.07358
0.001	–0.07402
Inst. rate of change ≈ –0.74	

12. $P(t)=230(0.9)^t; t=25$

h	**diff quotient**
0.1	–1.7305
0.01	–1.7388
0.001	–1.7396
Inst. rate of change ≈ -1.740	

13. $P(r)=5r^2; r=1.2$

h	**diff quotient**
0.1	12.5
0.01	12.05
0.001	12.005
Inst. rate of change ≈ 12	

14. $y=\ln(x); x=2$

h	**diff quotient**
0.1	0.4879
0.01	0.4988
0.001	0.4999
Inst. rate of change ≈ 5.0	

15. $g(x)=e^{3x}; x=1$

h	**diff quotient**
0.1	70.271
0.01	61.170
0.001	60.347
Inst. rate of change ≈ 60.0	

16. Using the *Calculating the Difference Quotient for Different Values of h* Tech Card with $h = 0.0001$, we calculate the instantaneous rate of change.

    $y'(10) = 361.94$

    In 2007, yogurt production is expected to be increasing by 361.94 million pounds per year.

17. Using the *Calculating the Difference Quotient for Different Values of h* Tech Card with $h = 0.0001$, we calculate the instantaneous rate of change.

    $R'(55) = -7.597$

    In 2035, the death rate due to heart disease is expected to be decreasing by 7.597 deaths per 100,000 people per year.

18. Using the *Calculating the Difference Quotient for Different Values of h* Tech Card with $h = 0.0001$, we calculate the instantaneous rate of change.

    $D'(100) \approx -282.1$

    When the price of a DVD player is \$100, the number of DVD players sold is expected to be decreasing by 282.1 thousand DVD players per dollar of price. That is, increasing the price by \$1 is predicted to reduce DVD player sales by 282,100 units.

19. Using the *Calculating the Difference Quotient for Different Values of h* Tech Card with $h = 0.0001$, we calculate the instantaneous rate of change.

    $T'(4) \approx 15.48$

    Increasing the number of days that the ticket authorizes entrance into the park from four days to five days is predicted to increase the ticket price by about \$15.48.

20. Using the *Calculating the Difference Quotient for Different Values of h* Tech Card with $h = 0.0001$, we calculate the instantaneous rate of change.

    $U'(10) = 2922.36$

    In 2015, the population of the United States is predicted to be increasing by 2922.36 thousand people per year.

21. $h_0 = 40; v_0 = 0 \ {}^{ft}\!/_{s}$. Thus

    $s(t) = -16t^2 + (0)t + 40.$

    Find time when the can hits the pool bottom:

    $$s(t) = 0$$
    $$-16t^2 + 40 = 0$$
    $$-16t^2 = -40$$
    $$t^2 = \frac{-40}{-16} = 2.5$$
    $$t = \sqrt{2.5} \approx 1.58, \text{ and then}$$
    $$s'(1.58) \approx -50.6 \ {}^{\text{ft}}\!/_{s}$$

    (using the tech. tip)

    $$\text{speed } \approx 50.6 \ {}^{\text{ft}}\!/_{\text{s}}$$

22. $h_0 = 4\ ft; v_0 = 20 \ {}^{ft}\!/_{s}$; Thus

    $s(t) = -16t^2 + 20t + 4$

    $s'(t) = -32t + 20$

    $s'(1) = -12 \ {}^{ft}\!/_{s}$

    At 1 second, the velocity of the ball is – 12 feet per second.

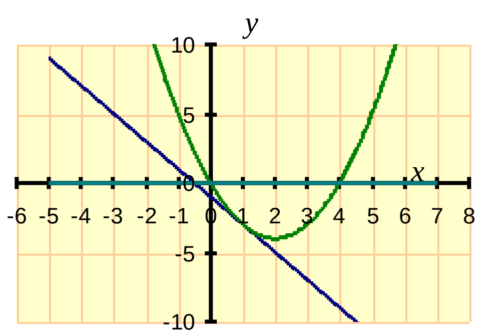
Exercises 2-3

1. $f(x) = x^2 - 4x; \quad (1, -3)$

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$
$$= \lim_{h \to 0} \frac{\left[(1+h)^2 - 4(1+h)\right] - (-3)}{h}$$
$$= \lim_{h \to 0} \frac{\left[1 + 2h + h^2 - 4 - 4h\right] - (-3)}{h}$$
$$= \lim_{h \to 0} \frac{\left[-3 - 2h + h^2\right] + 3}{h}$$
$$= \lim_{h \to 0} \frac{-2h + h^2}{h}$$
$$= \lim_{h \to 0} \frac{h(-2+h)}{h}$$
$$= \lim_{h \to 0} (-2+h)$$
$$= -2$$

Equation of line

$$y = mx + b$$
$$-3 = (-2)(1) + b$$
$$-3 = -2 + b$$
$$-1 = b$$
$$\therefore y = -2x - 1$$



2. $f(x) = -x^2 + 6; (2,2)$

$$\begin{aligned}
f'(2) &= \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} \\
&= \lim_{h\to 0} \frac{\left[-(2+h)^2 + 6\right] - (2)}{h} \\
&= \lim_{h\to 0} \frac{\left[-(4+4h+h^2) + 6\right] - 2}{h} \\
&= \lim_{h\to 0} \frac{\left[-4-4h-h^2+6\right] - 2}{h} \\
&= \lim_{h\to 0} \frac{\left[2-4h-h^2\right] - 2}{h} \\
&= \lim_{h\to 0} \frac{-4h-h^2}{h} \\
&= \lim_{h\to 0} \frac{h(-4-h)}{h} \\
&= \lim_{h\to 0} (-4-h) = -4
\end{aligned}$$

Equation of line

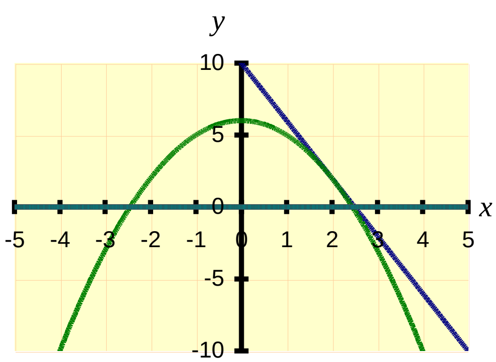
$y = mx + b$

$2 = (-4)(2) + b$

$2 = -8 + b$

$10 = b$

$\therefore y = -4x + 10$



3. $g(x) = x^2 + 2x + 1; (0,1)$

$$\begin{aligned}
g'(0) &= \lim_{h\to 0} \frac{g(0+h) - g(0)}{h} \\
&= \lim_{h\to 0} \frac{\left[h^2 + 2h + 1\right] - (1)}{h} \\
&= \lim_{h\to 0} \frac{h^2 + 2h}{h} \\
&= \lim_{h\to 0} \frac{h(h+2)}{h} \\
&= \lim_{h\to 0} (h+2) \\
&= 2
\end{aligned}$$

Equation of line

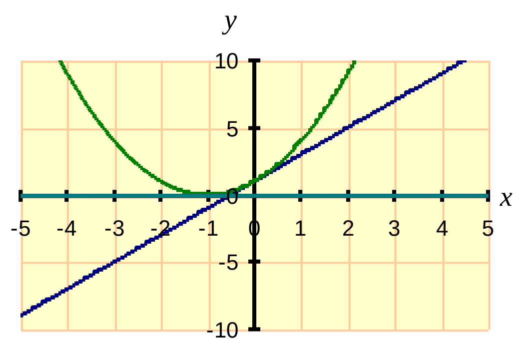
$y = mx + b$

$1 = (2)(0) + b$

$1 = 0 + b$

$1 = b$

$\therefore y = 2x + 1$

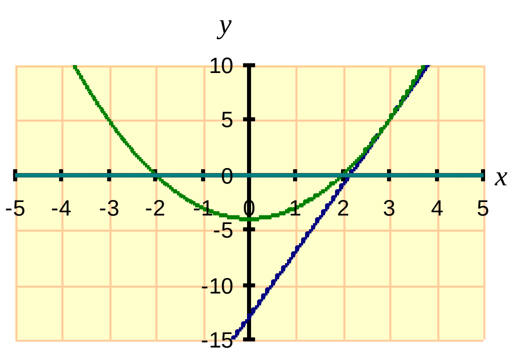


4. $g(x) = x^2 - 4; (3, 5)$

$$
\begin{aligned}
g'(2) &= \lim_{h \to 0} \frac{g(3+h) - g(3)}{h} \\
&= \lim_{h \to 0} \frac{\left[(3+h)^2 - 4\right] - (5)}{h} \\
&= \lim_{h \to 0} \frac{\left[(9 + 6h + h^2) - 4\right] - 5}{h} \\
&= \lim_{h \to 0} \frac{\left[5 + 6h + h^2\right] - 5}{h} \\
&= \lim_{h \to 0} \frac{6h + h^2}{h} \\
&= \lim_{h \to 0} \frac{h(6+h)}{h} \\
&= \lim_{h \to 0} (6+h) \\
&= 6
\end{aligned}
$$

Equation of line

$$
\begin{aligned}
y &= mx + b \\
5 &= (6)(3) + b \\
5 &= 18 + b \\
-13 &= b \\
\therefore y &= 6x - 13
\end{aligned}
$$

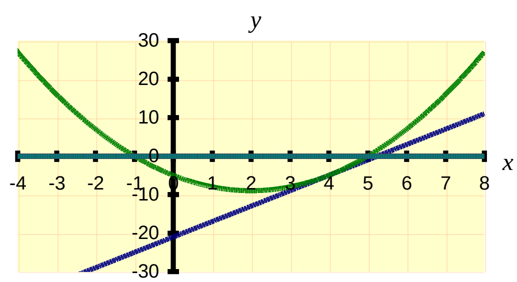


5. $g(x) = x^2 - 4x - 5; (4, -5)$

$$
\begin{aligned}
g'(4) &= \lim_{h \to 0} \frac{g(4+h) - g(4)}{h} \\
&= \lim_{h \to 0} \frac{\left[(4+h)^2 - 4(4+h) - 5\right] - (-5)}{h} \\
&= \lim_{h \to 0} \frac{\left[16 + 8h + h^2 - 16 - 4h - 5\right] + 5}{h} \\
&= \lim_{h \to 0} \frac{\left[4h + h^2 - 5\right] + 5}{h} \\
&= \lim_{h \to 0} \frac{4h + h^2}{h} \\
&= \lim_{h \to 0} \frac{h(4+h)}{h} \\
&= \lim_{h \to 0} (4+h) \\
&= 4
\end{aligned}
$$

Equation of line

$$
\begin{aligned}
y &= mx + b \\
-5 &= (4)(4) + b \\
-5 &= 16 + b \\
-21 &= b \\
\therefore y &= 4x - 21
\end{aligned}
$$



6. $f(x) = x^3; \quad (2,8)$

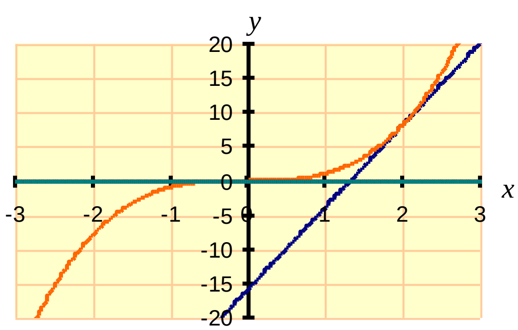
$$\begin{aligned} note: (2+h)^3 &= (2+h)(2+h)^2 \\ &= (2+h)(4+4h+h^2) \\ &= 8+8h+2h^2 \\ &\quad +4h+4h^2+h^3 \\ &= 8+12h+6h^2+h^3 \end{aligned}$$

so now

$$\begin{aligned} f'(2) &= \lim_{h\to 0}\frac{f(2+h)-f(2)}{h} \\ &= \lim_{h\to 0}\frac{(2+h)^3-(8)}{h} \\ &= \lim_{h\to 0}\frac{(8+12h+6h^2+h^3)-(8)}{h} \\ &= \lim_{h\to 0}\frac{12h+6h^2+h^3}{h} \\ &= \lim_{h\to 0}\frac{h(12+6h+h^2)}{h} \\ &= \lim_{h\to 0}(12+6h+h^2) \\ &= 12 \end{aligned}$$

Equation of line

$$\begin{aligned} y &= mx+b \\ 8 &= (12)(2)+b \\ 8 &= 24+b \\ -16 &= b \\ \therefore y &= 12x-16 \end{aligned}$$

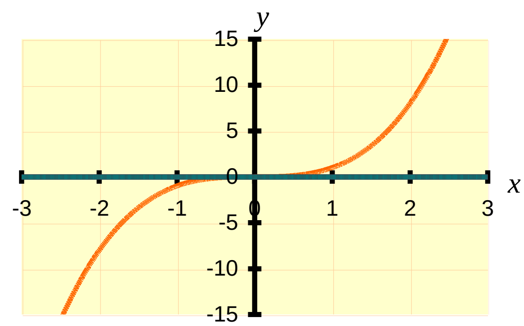


7. $f(x) = x^3; \ (0,0)$

$$\begin{aligned} f'(0) &= \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} \\ &= \lim_{h\to 0}\frac{(0+h)^3-(0)}{h} \\ &= \lim_{h\to 0}\frac{h^3}{h} \\ &= \lim_{h\to 0}(h^2) \\ &= 0 \end{aligned}$$

Equation of line (note horizontal line)

$$\begin{aligned} y &= mx+b \\ 0 &= (0)(0)+b \\ 0 &= b \\ \therefore y &= 0 \end{aligned}$$



8. $f(x) = x^3;\ \ (1,1)$

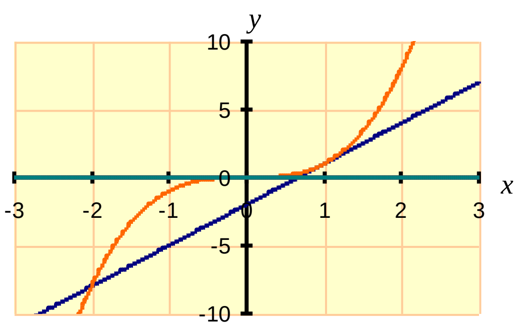
$$
\begin{aligned}
&note: (1+h)^3 = (1+h)(1+h)^2 \\
&= (1+h)(1+2h+h^2) \\
&= 1+2h+h^2 \\
&\quad + h + 2h^2 + h^3 \\
&= 1+3h+3h^2+h^3
\end{aligned}
$$

and now

$$
\begin{aligned}
f'(1) &= \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} \\
&= \lim_{h\to 0}\frac{(1+h)^3-(1)}{h} \\
&= \lim_{h\to 0}\frac{(1+3h+3h^2+h^3)-(1)}{h} \\
&= \lim_{h\to 0}\frac{3h+3h^2+h^3}{h} \\
&= \lim_{h\to 0}\frac{h(3+3h+h^2)}{h} \\
&= \lim_{h\to 0}(3+3h+h^2) \\
&= 3
\end{aligned}
$$

Equation of line

$$
\begin{aligned}
y &= mx+b \\
1 &= (3)(1)+b \\
1 &= 3+b \\
-2 &= b \\
\therefore\ y &= 3x-2
\end{aligned}
$$

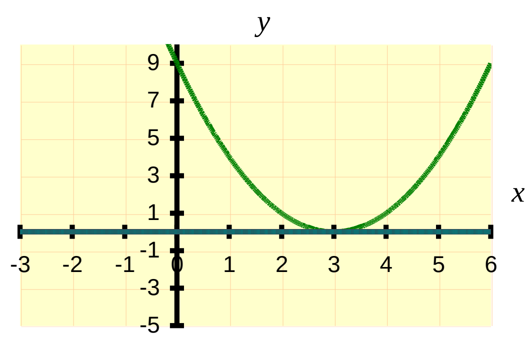


9. $f(x) = (x-3)^2\ \ ;(3,0)$

$$
\begin{aligned}
f'(3) &= \lim_{h\to 0}\frac{f(3+h)-f(3)}{h} \\
&= \lim_{h\to 0}\frac{\left[(3+h-3)^2\right]-(0)}{h} \\
&= \lim_{h\to 0}\frac{\left[h^2\right]-(0)}{h} \\
&= \lim_{h\to 0}\frac{h^2}{h} \\
&= \lim_{h\to 0}(h) \\
&= 0
\end{aligned}
$$

Equation of line (note horizontal line)

$$
\begin{aligned}
y &= mx+b \\
0 &= (0)(3)+b \\
0 &= 0+b \\
0 &= b \\
\therefore\ y &= 0
\end{aligned}
$$

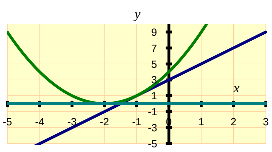


10. $f(x)=(x+2)^2; (-1,1)$

$$\begin{aligned} f'(-1) &= \lim_{h\to 0}\frac{f(-1+h)-f(-1)}{h} \\ &= \lim_{h\to 0}\frac{\left[(-1+h+2)^2\right]-(1)}{h} \\ &= \lim_{h\to 0}\frac{\left[(h+1)^2\right]-1}{h} \\ &= \lim_{h\to 0}\frac{\left[h^2+2h+1\right]-1}{h} \\ &= \lim_{h\to 0}\frac{h^2+2h}{h} \\ &= \lim_{h\to 0}\frac{h(h+2)}{h} \\ &= \lim_{h\to 0}(h+2) \\ &= 2 \end{aligned}$$

Equation of line

$$\begin{aligned} y &= mx+b \\ 1 &= (2)(-1)+b \\ 1 &= -2+b \\ 3 &= b \\ \therefore y &= 2x+3 \end{aligned}$$



In Exercises 11 – 20, the slope of the tangent line was computed using the procedures on the Chapter 2 Tech Card.

11. $W'(6)=-55.4$ thousand dollars per year. In 2009, the median sales price of a new home was decreasing at a rate of $55,400 dollars per year.

Since $W(6)=251.8$, the point $(6,251.8)$ lies on the tangent line.

$$\begin{aligned} y &= -55.4x+b \\ 251.8 &= -55.4(6)+b \\ b &= 584.2 \\ y &= -55.4x+584.2 \end{aligned}$$

We evaluate the tangent line at $x=7$.

$$\begin{aligned} y &= -55.4(7)+584.2 \\ &= 196.4 \end{aligned}$$

According to the model, the estimated median sales price of a new home in 2010 was $196.4 thousand.

12. *In the function $H(t)$, a decimal point was mistakenly dropped. The correct function is* $H(t)=-0.0023t^2+34.47t+2265$. *The solution is based on this corrected function.*

$H'(8)=34.43$ square feet per year. In 2008, the average number of square feet of a new home was increasing at a rate of 34.43 square feet per year. Since $H(8)\approx 2541$, the point $(8,2541)$ lies on the tangent line.

$$\begin{aligned} y &= 34.43x+b \\ 2541 &= 34.43(8)+b \\ b &\approx 2266 \\ y &= 34.43x+2266 \end{aligned}$$

We evaluate the tangent line at $x=9$.

$$\begin{aligned} y &= 34.43(9)+2266 \\ &\approx 2576 \end{aligned}$$

According to the model, the estimated size of a new home in 2009 was 2576 square feet.

13. $R'(13) \approx -0.1179$ students per year.
In 2008, the student-to-teacher ratio was dropping at a rate of 0.1179 students per year. Since $R(13) \approx 13.09$, the point $(13, 13.09)$ lies on the tangent line.

$$y = -0.1179x + b$$
$$13.09 = -0.1179(13) + b$$
$$b \approx 14.62$$
$$y = -0.1179x + 14.62$$

We evaluate the tangent line at $x = 14$.

$$y = -0.1179(14) + 14.62$$
$$\approx 12.97$$

According to the model, the student-to-teacher ratio in 2009 was approximately 12.97.

14. $V'(250) \approx 7.775 \ \dfrac{\text{million dollars}}{\text{million cassettes}}$
When the number of cassette tapes shipped is 250 million, the value of the tapes is increasing at a rate of 7.775 million dollars per million cassette tapes shipped.
$V(250) = 2126.4$ million dollars

Tangent at (250,2126.4)

$$T(s) = ms + b$$
$$2126.4 \approx 7.775(250) + b$$
$$182.65 \approx b$$
$$T(s) \approx 7.775t + 182.65$$
$$T(251) \approx 7.775(251) + 182.65$$
$$\approx 2134$$

The estimated shipment value when 251 million cassette tapes are shipped is about 2134 million dollars.

15. Since this is a linear function the slope of the tangent line (instantaneous rate of change) will be the same as the slope of the line. That is,

$$P'(x) = 0.340$$

Since the derivative does not depend on the value of *x*, it will be the same for all values of *x*. So the rate of change in *private* college enrollment when 12,752 thousand *public* students are enrolled is 0.340 thousand *private* college enrollments per thousand *public* college enrollments. (In other words, 340 *private* enrollments per 1000 *public* enrollments.)

Since the original function is linear, it is its own tangent line. So to estimate the number of private college enrollments when the public college enrollment is 12,753 thousand, we simply evaluate the original function at 12,753.

$$y = 0.340(12,753) - 457$$
$$\approx 3879$$

We estimate that when 12,753 *public* college students are enrolled, 3879 *private* college students are enrolled.

16. We estimate $W'(6)$ with an average rate of change.

$$W'(6) \approx \frac{67,055 - 62,051}{7 - 5}$$
$$\approx 2502$$

In 2006, the average annual salary in the motion picture and sound recording industry was increasing by approximately \$2502 per year.

17. We estimate $I'(3)$ with an average rate of change.

$$I'(3) \approx \frac{4.28-3.90}{4-2}$$
$$\approx 0.19$$

In 2007, the retail price of a ½ gallon of ice cream was increasing by approximately \$0.19 per year.

18. We estimate $f'(4)$ and $f'(5)$ with an average rate of change.

$$f'(4) \approx \frac{13,344-12,106}{5-3} = 619$$
$$f'(5) \approx \frac{13,344-12,832}{5-4} = 512$$

In 2008-2009, full-time resident tuition and fees were increasing at a rate of approximately \$619 per year. In 2009-2010, the rate of increase was approximately \$512 per year.

19. We estimate $S'(3)$ with an average rate of change.

$$S'(3) \approx \frac{6.07-5.79}{4-2}$$
$$\approx 0.14$$

In 2007, the price per pound of boneless sirloin steak was increasing at a rate of \$0.14 per year.

20. We estimate $B'(2)$ with an average rate of change.

$$B'(2) \approx \frac{1.81-1.29}{3-1}$$
$$\approx 0.26$$

In 2006, the price per loaf of whole wheat bread was increasing at a rate of approximately \$0.26 per year.

Exercises 2-4

1.

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f\left(x+h\right)-f\left(x\right)}{h} \\ &= \lim_{h\to 0}\frac{\left[(x+h)^2-4\left(x+h\right)\right]-\left[x^2-4x\right]}{h} \\ &= \lim_{h\to 0}\frac{\left[x^2+2xh+h^2-4x-4h\right]-x^2+4x}{h} \\ &= \lim_{h\to 0}\frac{2xh+h^2-4h}{h} \\ &= \lim_{h\to 0}\frac{h(2x+h-4)}{h} \\ &= \lim_{h\to 0}(2x+h-4) \\ &= 2x-4 \end{aligned}$$

2.

$$\begin{aligned} g'(x) &= \lim_{h\to 0}\frac{g\left(x+h\right)-g\left(x\right)}{h} \\ &= \lim_{h\to 0}\frac{\left[(x+h)^2+2\left(x+h\right)+1\right]-\left[x^2+2x+1\right]}{h} \\ &= \lim_{h\to 0}\frac{\left[x^2+2xh+h^2+2x+2h+1\right]-x^2-2x-1}{h} \\ &= \lim_{h\to 0}\frac{2xh+h^2+2h}{h} \\ &= \lim_{h\to 0}\frac{h(2x+h+2)}{h} \\ &= \lim_{h\to 0}(2x+h+2) \\ &= 2x+2 \end{aligned}$$

3.

$$\begin{aligned}
g'(x) &= \lim_{h\to 0}\frac{g(x+h)-g(x)}{h} \\
&= \lim_{h\to 0}\frac{\left[(x+h)^2-4(x+h)-5\right]-\left[x^2-4x-5\right]}{h} \\
&= \lim_{h\to 0}\frac{\left[x^2+2xh+h^2-4x-4h-5\right]-x^2+4x+5}{h} \\
&= \lim_{h\to 0}\frac{2xh+h^2-4h}{h} \\
&= \lim_{h\to 0}\frac{h(2x+h-4)}{h} \\
&= \lim_{h\to 0}(2x+h-4) \\
&= 2x-4
\end{aligned}$$

4.

$$\begin{aligned}
j'(x) &= \lim_{h\to 0}\frac{j(x+h)-j(x)}{h} \\
&= \lim_{h\to 0}\frac{\left[(x+h)^3+2\right]-\left[x^3+2\right]}{h} \\
&= \lim_{h\to 0}\frac{\left[x^3+3x^2h+3xh^2+h^3+2\right]-x^3-2}{h} \\
&= \lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h} \\
&= \lim_{h\to 0}\frac{h(3x^2+3xh+h^2)}{h} \\
&= \lim_{h\to 0}(3x^2+3xh+h^2) \\
&= 3x^2
\end{aligned}$$

5.

$$
\begin{aligned}
f(t) &= t^2 - 6t + 9 \\
f'(t) &= \lim_{h\to 0}\frac{f(t+h) - f(t)}{h} \\
&= \lim_{h\to 0}\frac{\left[(t+h)^2 - 6(t+h) + 9\right] - \left[t^2 - 6t + 9\right]}{h} \\
&= \lim_{h\to 0}\frac{\left[t^2 + 2th + h^2 - 6t - 6h + 9\right] - t^2 + 6t - 9}{h} \\
&= \lim_{h\to 0}\frac{2th + h^2 - 6h}{h} \\
&= \lim_{h\to 0}\frac{h(2t + h - 6)}{h} \\
&= \lim_{h\to 0}(2t + h - 6) \\
&= 2t - 6
\end{aligned}
$$

6.

$$
\begin{aligned}
g'(x) &= \lim_{h\to 0}\frac{g(x+h) - g(x)}{h} \\
&= \lim_{h\to 0}\frac{\left[2(x+h)^2 + (x+h) - 1\right] - \left[2x^2 + x - 1\right]}{h} \\
&= \lim_{h\to 0}\frac{\left[2(x^2 + 2xh + h^2) + x + h - 1\right] - 2x^2 - x + 1}{h} \\
&= \lim_{h\to 0}\frac{\left[2x^2 + 4xh + 2h^2 + x + h - 1\right] - 2x^2 - x + 1}{h} \\
&= \lim_{h\to 0}\frac{4xh + 2h^2 + h}{h} \\
&= \lim_{h\to 0}\frac{h(4x + 2h + 1)}{h} \\
&= \lim_{h\to 0}(4x + 2h + 1) \\
&= 4x + 1
\end{aligned}
$$

$$
\begin{aligned}
g'(1) &= 4(1) + 1 = 5 \\
g'(3) &= 4(3) + 1 = 13 \\
g'(5) &= 4(5) + 1 = 21
\end{aligned}
$$

7.

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{\left[(x+h)^2-2(x+h)\right]-\left[x^2-2x\right]}{h}\\
&= \lim_{h\to 0}\frac{\left[x^2+2xh+h^2-2x-2h\right]-x^2+2x}{h}\\
&= \lim_{h\to 0}\frac{2xh+h^2-2h}{h}\\
&= \lim_{h\to 0}\frac{h(2x+h-2)}{h}\\
&= \lim_{h\to 0}(2x+h-2)\\
&= 2x-2\\
f'(1) &= 2(1)-2\\
&= 0\\
f'(3) &= 2(3)-2\\
&= 4\\
f'(5) &= 2(5)-2\\
&= 8
\end{aligned}
$$

8. Since *j(x)* is linear with a slope of 0, $j'(x)=0$ for all *x*-values.

9. Since *W(x)* is linear with a slope of $-4, W'(x)=-4$ for all *x*-values.

10.

$$
\begin{aligned}
S'(x) &= \lim_{h\to 0}\frac{S(x+h)-S(x)}{h} \\
&= \lim_{h\to 0}\frac{\left[3(x+h)^2-2(x+h)+1\right]-\left[3x^2-2x+1\right]}{h} \\
&= \lim_{h\to 0}\frac{\left[3(x^2+2xh+h^2)-2x-2h+1\right]-3x^2+2x-1}{h} \\
&= \lim_{h\to 0}\frac{\left[3x^2+6xh+3h^2-2x-2h+1\right]-3x^2+2x-1}{h} \\
&= \lim_{h\to 0}\frac{6xh+3h^2-2h}{h} \\
&= \lim_{h\to 0}\frac{h(6x+3h-2)}{h} \\
&= \lim_{h\to 0}(6x+3h-2) \\
&= 6x-2
\end{aligned}
$$

$$S'(1)=6(1)-2=4$$

$$S'(3)=6(3)-2=18-2=16$$

$$S'(5)=6(5)-2=30-2=28$$

11.

$$
\begin{aligned}
P'(x) &= \lim_{h\to 0}\frac{P(x+h)-P(x)}{h} \\
&= \lim_{h\to 0}\frac{3^{x+h}-3^x}{h} \\
&= \lim_{h\to 0}\frac{(3^x)(3^h)-3^x}{h} \\
&= \lim_{h\to 0}\frac{3^x[3^h-1]}{h} \\
&= (3^x)\lim_{h\to 0}\frac{(3^h-1)}{h} \\
&\approx (3^x)\frac{(3^{0.001}-1)}{0.001} \\
&\approx 1.099(3^x)
\end{aligned}
$$

12.

$$\begin{aligned}
C'(x) &= \lim_{h\to 0}\frac{C(x+h)-C(x)}{h}\\
&= \lim_{h\to 0}\frac{-3(4^{x+h})-[-3(4^{x})]}{h}\\
&= \lim_{h\to 0}\frac{-3(4^{x})(4^{h})+3(4^{x})}{h}\\
&= \lim_{h\to 0}\frac{-3(4^{x})[4^{h}-1]}{h}\\
&= -3(4^{x})\lim_{h\to 0}\frac{(4^{h}-1)}{h}\\
&\approx -3(4^{x})\frac{(4^{0.001}-1)}{0.001}\\
&\approx -3(1.387)(4^{x})\\
&\approx -4.161(4^{x})
\end{aligned}$$

13. $R(x)=5.042\cdot(0.98)^{x}$

$$\begin{aligned}
R'(x) &= \lim_{h\to 0}\frac{R(x+h)-R(x)}{h}\\
&= \lim_{h\to 0}\frac{5.042(0.98^{x+h})-[5.042(0.98^{x})]}{h}\\
&= \lim_{h\to 0}\frac{5.042(0.98^{x})(0.98^{h})-5.042(0.98^{x})}{h}\\
&= \lim_{h\to 0}\frac{5.042(0.98^{x})[0.98^{h}-1]}{h}\\
&= 5.042(0.98^{x})\lim_{h\to 0}\frac{5.042(0.98^{x})(0.98^{h}-1)}{h}\\
&\approx 5.042(0.98^{x})\frac{(0.98^{0.001}-1)}{0.001}\\
&\approx 5.042\left(-0.020\right)(0.98^{x})\\
&\approx 0.1008(0.98^{x})
\end{aligned}$$

14. We find $W'(t)$.

$$\begin{aligned}
W(t+h) &= -9.2(t+h)^2 + 55(t+h) + 253 \\
&= -9.2\left(t^2 + 2ht + h^2\right) + 55t + 55h + 253 \\
&= -9.2t^2 - 18.4ht - 9.2h^2 + 55t + 55h + 253
\end{aligned}$$

$$\begin{aligned}
W'(t) &= \lim_{h\to 0} \frac{W(t+h) - W(t)}{h} \\
&= \lim_{h\to 0} \frac{\left(-9.2t^2 - 18.4ht - 9.2h^2 + 55t + 55h + 253\right) - \left(-9.2t^2 + 55t + 253\right)}{h} \\
&= \lim_{h\to 0} \frac{-18.4ht - 9.2h^2 + 55h}{h} \\
&= \lim_{h\to 0} \left(-18.4t - 9.2h + 55\right) \\
&= -18.4t + 55
\end{aligned}$$

$$\begin{aligned}
W'(3) &= -18.4(3) + 55 \\
&= -0.2 \text{ thousand dollars per year} \\
W'(5) &= -18.4(5) + 55 \\
&= -37 \text{ thousand dollars per year}
\end{aligned}$$

The median sales price was changing more quickly at the end of 2008.

15. We find $U'(t)$.

$$\begin{aligned} U(t+h) &= -5.1(t+h)^2 + 33(t+h) + 194 \\ &= -5.1\left(t^2 + 2ht + h^2\right) + 33t + 33h + 194 \\ &= -5.1t^2 - 10.2ht - 5.1h^2 + 33t + 33h + 194 \end{aligned}$$

$$\begin{aligned} U'(t) &= \lim_{h\to 0} \frac{U(t+h) - U(t)}{h} \\ &= \lim_{h\to 0} \frac{\left(-5.1t^2 - 10.2ht - 5.1h^2 + 33t + 33h + 194\right) - \left(-5.1t^2 + 33t + 194\right)}{h} \\ &= \lim_{h\to 0} \frac{-10.2ht - 5.1h^2 + 33h}{h} \\ &= \lim_{h\to 0} (-10.2t - 5.1h + 33) \\ &= -10.2t - 5.1(0) + 33 \\ &= -10.2t + 33 \end{aligned}$$

$$\begin{aligned} U'(4) &= -10.2(4) + 33 \\ &= -7.8 \text{ thousand dollars per year} \\ U'(6) &= -10.2(6) + 33 \\ &= -28.2 \text{ thousand dollars per year} \end{aligned}$$

The median sales price was changing more quickly at the end of 2009.

16. We find $s'(t)$.

$$\begin{aligned}s(t+h) &= 0.8636(t+h)^2 + 14.39(t+h) + 84.72\\ &= 0.8636(t^2 + 2ht + h^2) + 14.39t + 14.39h + 84.72\\ &= 0.8636t^2 + 1.7272ht + 0.8636h^2 + 14.39t + 14.39h + 84.72\end{aligned}$$

$$\begin{aligned}s'(t) &= \lim_{h\to 0}\frac{s(t+h)-s(t)}{h}\\ &= \lim_{h\to 0}\frac{(0.8636t^2 + 1.7272ht + 0.8636h^2 + 14.39t + 14.39h + 84.72) - (0.8636t^2 + 1.7272t + 84.72)}{h}\\ &= \lim_{h\to 0}\frac{1.7272ht + 0.8636h^2 + 14.39h}{h}\\ &= \lim_{h\to 0}(1.7272t + 0.8636h + 14.39)\\ &= 1.7272t + 0.8636(0) + 14.39\\ &= 1.7272t + 14.39\end{aligned}$$

$$\begin{aligned}s'(10) &= 1.7272(10) + 14.39\\ &\approx 31.66 \text{ billion dollars per year}\\ s'(9) &= 1.7272(9) + 14.39\\ &\approx 29.93 \text{ billion dollars per year}\end{aligned}$$

$$31.66 - 29.93 = 1.73$$

According to the model, Wal-Mart net sales were increasing 1.73 billion dollars per year more rapidly in 2006 than in 2005.

17. We have

$$\begin{aligned}P(t+h) &= 2.889(t+h)^2 - 2.613(t+h) + 158.7\\ &= 2.889(t^2+2ht+h^2) - 2.613t - 2.613h + 158.7\\ &= 2.889t^2 + 5.778ht + 2.889h^2 - 2.613t - 2.613h + 158.7\\ &= 5.778ht + 2.889h^2 - 2.613h + \left(2.889t^2 - 2.613t + 158.7\right)\\ &= 5.778ht + 2.889h^2 - 2.613h + P(t)\end{aligned}$$

$$\begin{aligned}P'(t) &= \lim_{h\to 0}\frac{P(t+h)-P(t)}{h}\\ &= \lim_{h\to 0}\frac{\left(5.778ht + 2.889h^2 - 2.613h + P(t)\right) - P(t)}{h}\\ &= \lim_{h\to 0}\frac{5.778ht + 2.889h^2 - 2.613h}{h}\\ &= \lim_{h\to 0}\frac{h\left(5.778t + 2.889h - 2.613\right)}{h}\\ &= \lim_{h\to 0}\left(5.778t + 2.889h - 2.613\right)\\ &= 5.778t - 2.613\end{aligned}$$

$$\begin{aligned}P'(10) &= 5.778(10) - 2.613\\ &= 55.167\end{aligned}$$

According to the model, per capita prescription drug spending was increasing at a rate of about \$55.17 per year in 2000. We want to find out when it was increasing at twice this rate.

$$\begin{aligned}2(55.167) &= 5.778t - 2.613\\ 110.334 &= 5.778t - 2.613\\ 112.947 &= 5.778t\\ t &\approx 19.5\end{aligned}$$

Since $t = 19$ is the end of 2009, we estimate that in mid–2010 per capita prescription drug spending will be increasing at a rate twice that of the 2000 rate.

18. We find $b'(t)$.

$$\begin{aligned} b(t+h) &= 4.29(t+h)^2 - 278(t+h) + 2250 \\ &= 4.29(t^2 + 2ht + h^2) - 278t - 278h + 2250 \\ &= 4.29t^2 + 8.58ht + 4.29h^2 - 278t - 278h + 2250 \end{aligned}$$

$$\begin{aligned} b'(t) &= \lim_{h\to 0} \frac{b(t+h) - b(t)}{h} \\ &= \lim_{h\to 0} \frac{(4.29t^2 + 8.58ht + 4.29h^2 - 278t - 278h + 2250) - (4.29t^2 - 278t + 2250)}{h} \\ &= \lim_{h\to 0} \frac{8.58ht + 4.29h^2 - 278h}{h} \\ &= \lim_{h\to 0} (8.58t + 4.29h - 278) \\ &= 8.58t + 4.29(0) - 278 \\ &= 8.58t - 278 \end{aligned}$$

$$\begin{aligned} b'(15) &= 8.58(15) - 278 \\ &= -149.3 \text{ million barrels per year} \\ b'(20) &= 8.58(20) - 278 \\ &= -106.4 \text{ million barrels per year} \end{aligned}$$

According to the model, the difference in U.S. oil field production and net oil imports was changing at a rate of –149.3 million barrels per year in 2000 at a rate of –106.4 million barrels per year in 2005.

19.

$$\begin{aligned} g'(x) &= 2x + f'(x) \\ &= 2x + 3x \\ &= 5x \end{aligned}$$

20.

$$\begin{aligned} &f'(x) = 3x^2 - 3, \text{ so} \\ &f'(x) = 0 \Rightarrow \\ &3x^2 - 3 = 0 \\ &3x^2 = 3 \\ &x^2 = 1 \\ &x = \pm 1 \end{aligned}$$

Exercises 2-5

1. $W(10) = 76.64$ => The weight of a 10-year old girl is about 76.64 pounds.
$W'(10) = 8.24$ => A 10-year-old girl is gaining weight at about 8.24 pounds per year.
$$W(11) \approx W(10) + W'(10)$$
$$\approx (76.64 + 8.24) \text{ pounds}.$$
$$\approx 84.88 \text{ pounds},$$
the weight of an 11-year old girl.

2. The boys weight gain between their tenth and eleventh year is about 7.29 pounds, while the girls is about 8.24 pounds per year, so the girls can be expected to gain about 0.95 more pounds.

3. In 2005, carbon monoxide pollution in the United States was 2.27 parts per million and was decreasing at a rate of 0.248 parts per million per year.

4. The population of Kazakhstan was predicted to be 13,819 thousand people in 2015 and expected to be decreasing at a rate of 97.08 thousand people per year.

5. The population of India was predicted to be 1,188,644 thousand people in 2010 and expected to be increasing at a rate of 17,697 thousand people per year.

6. In 2005, the 44.69% of highway accidents in the United States resulted in injuries. In 2005, the percentage of accidents with injuries was decreasing by 0.295 percentage points per year.

7. When there are 14,000 thousand public college students, there are 4303 thousand private college students and the number private college students is increasing by 0.340 thousand private college students per thousand public college students. In other words, a 1000 student increase in the number of public college students corresponds with a 340 student increase in private college students.

8. When 20% of people smoke, the heart disease death rate is 228 deaths per 100,000 people and is increasing by 14.08 deaths per 100,000 people per percentage point. In other words, a 1 percentage point increase in the percentage of people who smoke corresponds with a 14.08 deaths per 100,000 people increase in the heart disease death rate.

9. When 500 billion dollars are spent on farm foods at home, 356.9 billion dollars are spent on farm foods away from home. When 500 billion dollars are spend on farm foods at home, the amount of money spent on farm foods away from home is increasing at a rate of 0.7905 billion dollars per billion dollars spent on farm foods at home. In other words, a 1 billion dollar increase in spending on farm foods at home corresponds with a 0.7905 billion dollar increase in spending on farm foods away from home.

10. When 30 million cassette tapes are shipped the value of the shipped cassettes is 226 million dollars and is increasing at a rate of 9.5 million dollars per million cassettes. In other words, when 30 million cassettes are shipped, a 1 million cassette increase in cassettes shipped corresponds with a 9.5 million dollar increase in the value of the shipped cassettes.

    One comparatively easy way to do a tangent line approximation is to add the value of the derivative to the value of the function.*

    $$\begin{aligned} V(31) &\approx V(30) + V'(30) \\ &= 226 + 9.5 \\ &= 235.5 \end{aligned}$$

    When 31 million cassette tapes are shipped, the value of the shipment is predicted to be 235.5 million dollars

    *The strategy of adding the value of the derivative to the function value is based on the point-slope form of the line as demonstrated below.

    $$\begin{aligned} y - 266 &= 9.5(x - 25) \\ y &= 9.5(x - 25) + 266 \\ y &= 9.5(26 - 25) + 266 \quad \bullet \text{ Let } x = 26 \\ y &= 9.5 + 266 \\ y &= 235.5 \end{aligned}$$

11. In 2005 there were 42.0 million Medicare enrollees and the number of enrollees was increasing at a rate of 0.427 million enrollees per year.

    One comparatively easy way to do a tangent line approximation is to add the value of the derivative to the value of the function.

    $$\begin{aligned} M(26) &\approx M(25) + M'(25) \\ &= 42.0 + 0.427 \\ &\approx 42.4 \end{aligned}$$

12. When there are 42 million Medicare enrollees there are 3331 million prescriptions and the number of prescriptions is increasing at a rate of 79.84 million prescriptions per million Medicare enrollees.

    $$\begin{aligned} p(43) &\approx p(42) + p'(42) \\ &= 3331 + 79.84 \\ &\approx 3411 \text{ million prescriptions} \end{aligned}$$

    When there are 43 million Medicare enrollees the number of prescriptions is predicted to be 3411 million.

13. In 2007, the population of the United States was 302,366 thousand people and was increasing at a rate of 2742 thousand people per year.

    $$\begin{aligned} P(18) &\approx P(17) + P'(17) \\ &\approx 302,366 + 2742 \\ &\approx 305,108 \end{aligned}$$

    In 2008, the population of the United States was predicted to be 305,108 thousand people.

14. In 2003, Wal-Mart net sales were 227.8 billion dollars and were increasing at a rate of 26.48 billion dollars per year.

    $$\begin{aligned} s(8) &\approx s(7) + s'(7) \\ &\approx 227.8 + 26.48 \\ &\approx 254.3 \end{aligned}$$

    In 2004, Wal-Mart net sales were approximately 254.3 billion dollars, according to the model.

15. When the amount of private money spent on health care is 1500 billion dollars the amount of public money spent on health care is 1448 billion dollars and is increasing by 1.228 billion dollars of public money per billion dollars of private money.
$$G(1501) \approx G(1500) + G'(1500)$$
$$\approx 1448 + 1.228$$
$$\approx 1449$$
When 1501 billion dollars of private money are spent on health care, 1449 billion dollars of public money is spent on health care.

16. In 2010, there were 4896 million pounds of yogurt produced in the United States and yogurt production was increasing at a rate of 451.9 million pounds per year.
$$y(14) \approx y(13) + y'(13)$$
$$\approx 4896 + 451.9$$
$$\approx 5348$$
In 2011, yogurt production was predicted to be approximately 5348 million pounds.

17. When a 3-day ticket is purchased the price is \$157.10 and is increasing at a rate of 26.90 per additional day of park admittance.
$$T(4) \approx T(3) + T'(3)$$
$$\approx 157.10 + 26.90$$
$$\approx 184$$
The price of a 4-day ticket is predicted to be approximately \$184.

18. When the life expectancy of an American male is 75 years, the life expectancy of an American female is 80.2 years and is increasing at a rate of 1.22 years of female life per year of male life.
$$f(76) \approx f(75) + f'(75)$$
$$\approx 80.2 + 1.22$$
$$\approx 81.42$$
When the life expectancy of an American male is 76 years, the life expectancy of an American female is predicted to be 81.42 years.

19. We are looking for where the graph is increasing most rapidly and where the graph is decreasing most rapidly. The increasing portion of the graph appears to be steepest at approximately $t = 8$ and the decreasing portion of the graph appears to be steepest at $t = 14$. In 1989, the number of AIDS deaths was increasing most rapidly. In 1995, the number of AIDS deaths were decreasing most rapidly.

20. Since $f(a) \geq f(x)$ for all x, $f(a)$ is a maximum value for *f*. In addition f is a continuous, smooth function on the interval $(-\infty, \infty)$, so the tangent line at $(x, f(a))$ must be horizontal; hence $f'(a) = 0$.