Chapter 2 Answer Key

1. Given the analysis presented in this chapter, where would you put the capital of Logistica and why? What factors went into your decision?

This is a bit of an open-ended question and could be good for class discussion. But it is not that difficult—the students should use the statistics from the Logistica spreadsheet (see http://networkdesignbook.com/academic-use/chapter-2-material/). Better answers will pick a city that has a low average distance and a high percentage of citizens within 100, 200, and 300 miles of the capital. Students will have different ways of weighting these factors, so you may see different answers and subtle trade-offs:

- City 11 minimizes Weighted Average Distance at 371.
- Cities 4 and 10 maximize the percent within 100 miles at 24%. City 7 is not far behind at 23%.
- City 10 maximizes the percent within 200 miles at 49%. City 4 is not far behind at 48%.
- Cities 7, 10, and 11 maximize the percent within 300 miles at 58%. Cities 3 and 4 are not far behind at 56%, while City 1 is at 54%.

A good answer or a good class discussion should talk about these quantitative factors as well as touch on the quality of the infrastructure of the cities, maybe future growth rates, and other factors (see the question below).

2. Besides the weighted-average distance and the percentage of customers within a certain distance of the capital, what other factors might the citizens of Logistica want to consider? Of these factors, which are quantifiable and which are qualitative?

This question relates to the preceding question. You would certainly want to consider the infrastructure, future growth rates, maybe the weather (if certain cities are cut off from others because of storms or snow), cost to build in different locations, maybe the type of workforce available, security of the site, and other factors.

3. If the planners of Logistica had been lucky enough for their first calculation to have picked an existing city, why should they still analyze other cities?

You may need to clarify this question. The first calculation is the physics center of gravity that ended up putting their capital in the shark-infested waters off a deserted coast.

But, what if the populations were such that the capital ended up in an existing city? They should still look at other sites for the following reasons:

- The physics center of gravity minimized the sum of the demand multiplied by the distance squared—not exactly what they wanted.
- The more detailed analysis showed subtle trade-offs—is average distance more important than a lot of citizens very close to the capital?

4. Name a reason why minimizing weighted-average distance is more important than maximizing the percentage of customers within a certain distance. Now, name a reason why maximizing the percentage of customers within a certain distance is more important than why minimizing weighted-average distance.

This question could also be a nice class discussion. This gets students thinking about objectives and how to compare one solution to another (you can refer back to the optimization discussion in Chapter 1).

You may get some creative answers here.

Some reasons for minimizing weighted average distance:

- It can be viewed as a more fair measure since all citizens are factored in.
- If all citizens have to drive to the capital, this would minimize the average drive time and cost.
- This measure helps the citizens who are far away.

Some reasons for maximizing the percent within a given radius:

- Sometimes, once you pass a certain distance, everything that is far away has a similar cost—maybe in this case, after 300 miles, the citizens will fly, and so the distance is no longer that important.
- It can be beneficial to have a large concentration of citizens very close—maybe there is more feedback, participation, and demand relative to the distance. This is the opposite of the previous point—if a citizen is close to the capital, he gets even more benefit. In a business setting, if the facility is close to customers, maybe there is extra demand.

5. If instead of weighting the problem by the population of each city, assume that the analysis was done with each city having equal weight. That is, what matters is how close a city is to the capital, not how many people live there.

a) Which capital location is now the best from an average-distance point of view?
b) Which capital is now the best in terms of the number of cities within 100, 200, and 300 miles?
c) Does this make the analysis easier or harder? Why?

To answer this question, you want to use the Logistica spreadsheet and change the population to 1 for all cities. Now, each city has an equal weight (you could use any number as long as it was the same).

a). City 15 is the best with an average of 401. It is interesting that City 16 is only 5 miles different at 406 despite being far away from 15. It is also interesting that City 19 is at 414 despite being close to 16. You

could discuss these findings. 15 is good because it balances the far western cities. 16 is also good because it is close to all the cities in the North East. The average for 19 seems to jump up because it is quickly moving away from the North East cities and is far from the Western cities.

b). Cities 10 and 4 have the most cities within 100 miles at 20%. Cities 4, 10, and 11 are the best at 200 miles with 36%. And, Cities 7, 10, and 11 are the best at 48%.

c). Sometimes students will think that this makes the problem easier. However, there are still trade-offs to make. The cities that showed up in Part (a) are very different from those in Part (b).

Here is what the new input for the spreadsheet looks like:

City #	Latitude	Longitude	Population
1	-16.6	154.8	1
2	-16.7	156.8	1
3	-16.8	153.2	1
4	-17	154	1
5	-17	152	1
6	-17.2	144.9	1
7	-17.5	155.7	1
8	-17.4	147.1	1
9	-17.5	141.1	1
10	-17.8	155.1	1
11	-17.9	153.8	1
12	-18	144.6	1
13	-18.4	142.4	1
14	-18.9	156.8	1
15	-19.3	148.3	1
16	-19.4	152.9	1
17	-19.4	142.8	1
18	-19.9	143.7	1
19	-20.3	152.5	1
20	-21.2	143.7	1
21	-21.6	155.6	1
22	-22.6	140.1	1
23	-23.4	155.8	1
24	-24	144.4	1
25	-24.9	146.4	1

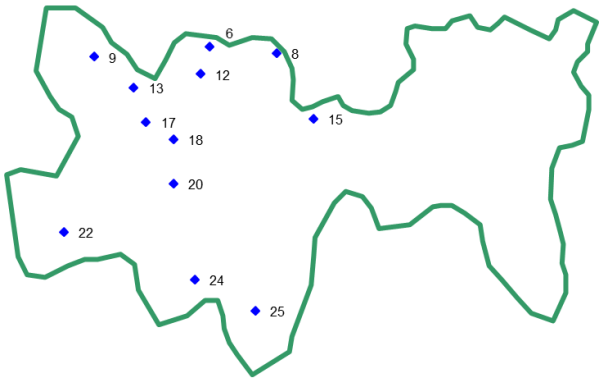
And, here is the output (the table at the bottom is the percent within 100, 200, and 300 miles):

Approx Distance to Capital for Each City (miles)																								
COG 1	COG 2	COG 3	COG 4	COG 5	COG 6	COG 7	COG 8	COG 9	COG 10	COG 11	COG 12	COG 13	COG 14	COG 15	COG 16	COG 17	COG 18	COG 19	COG 20	COG 21	COG 22	COG 23	COG 24	COG 25
0	138	111	62	195	684	88	534	947	85	113	710	865	210	486	233	850	799	301	829	349	1096	474	881	815
138	0	248	194	332	822	94	671	1085	140	223	847	1001	152	613	327	984	930	387	956	348	1222	467	993	914
111	248	0	57	84	573	179	423	836	148	86	599	753	288	380	181	740	690	246	722	370	989	489	785	730
62	194	57	0	138	628	122	477	891	94	64	652	806	233	424	182	790	738	250	768	336	1034	459	820	756
195	332	84	138	0	490	258	339	753	221	139	515	669	356	301	177	656	607	230	642	403	907	514	713	668
684	822	573	628	490	0	745	152	263	705	616	59	191	829	276	572	210	204	566	288	798	499	865	470	541
88	94	179	122	258	745	0	593	1007	46	134	767	920	123	525	233	900	844	293	866	283	1132	407	899	820
534	671	423	477	339	152	593	0	414	553	464	177	332	677	155	423	327	291	423	352	654	602	729	492	520
947	1085	836	891	753	263	1007	414	0	966	877	244	109	1088	512	825	176	244	810	312	1040	359	1093	503	628
85	140	148	94	221	705	46	553	966	0	90	725	877	140	480	188	856	800	249	821	264	1087	389	853	775
113	223	86	64	139	616	134	464	877	90	0	635	787	218	392	121	766	710	188	733	284	999	404	773	703
710	847	599	652	515	59	767	177	244	725	635	0	154	844	271	581	157	145	568	229	799	444	858	414	492
865	1001	753	806	669	191	920	332	109	877	787	154	0	994	412	728	74	137	709	213	937	330	987	410	527
210	152	288	233	356	829	123	677	1088	140	218	844	994	0	587	271	967	907	312	918	204	1180	318	925	828
486	613	380	424	301	276	525	155	512	480	392	271	412	587	0	317	380	320	298	343	528	610	590	421	408
233	327	181	182	177	572	233	423	825	188	121	581	728	271	317	0	697	636	68	647	240	910	341	667	588
850	984	740	790	656	210	900	327	176	856	766	157	74	967	380	697	0	71	672	139	896	289	939	336	454
799	930	690	738	607	204	844	291	244	800	710	145	137	907	320	636	71	0	608	90	829	311	869	287	392
301	387	246	250	230	566	293	423	810	249	188	568	709	312	298	68	672	608	0	610	232	870	312	614	527
829	956	722	768	642	288	866	352	312	821	733	229	213	918	343	647	139	90	610	0	822	267	849	199	316
349	348	370	336	403	798	283	654	1040	264	284	799	937	204	528	240	896	829	232	822	0	1072	125	790	674
1096	1222	989	1034	907	499	1132	602	359	1087	999	444	330	1180	610	910	289	311	870	267	1072	0	1085	312	463
474	467	489	459	514	865	407	729	1093	389	404	858	987	318	590	341	939	869	312	849	125	1085	0	788	657
881	993	785	820	713	470	899	492	503	853	773	414	410	925	421	667	336	287	614	199	790	312	788	0	151
815	914	730	756	668	541	820	520	628	775	703	492	527	828	408	588	454	392	527	316	674	463	657	151	0
474	564	428	439	412	482	491	431	639	462	421	475	557	543	401	406	533	498	414	517	531	723	600	580	574
16%	8%	16%	20%	8%	8%	16%	4%	4%	20%	16%	8%	8%	4%	4%	8%	12%	12%	8%	8%	4%	4%	4%	4%	4%
32%	24%	32%	36%	24%	16%	32%	16%	12%	36%	36%	24%	24%	16%	8%	28%	24%	20%	12%	16%	8%	4%	8%	12%	8%
40%	32%	44%	44%	36%	36%	48%	20%	24%	48%	48%	36%	28%	40%	20%	44%	32%	36%	40%	32%	32%	12%	8%	16%	8%

6. What if the western part of Logistica decided it needed a capital city as well? Assume that City 15 was the easternmost city in this region. What is the best location for the capital of the western half of Logistica? Why?

This question allows the students to practice the analysis without knowing the answer. Of course, this is made easier because they have the spreadsheet. (You can make up a similar problem by creating your own data sets with more or fewer points and have the students re-create the analysis—you may find better ways to lay out the spreadsheet and map the points.)

The key here is to turn off the cities in the eastern part of the country. We did this by moving the cities to (0,0) and setting the population to zero. The cities you need to get rid of are 1-5, 7, 10-11, 14, 16, 19, 21, and 23. The new map looks like this:



Then, you can use the spreadsheet showing all possibilities to get the statistics:

City #	Latitude	Longitude	Population	COG 6	COG 8	COG 9	COG 12	COG 13	COG 15	COG 17	COG 18	COG 20	COG 22	COG 24	COG 25
1	0	0	-	10068	10221	9810	10054	9907	10319	9944	10010	10023	9792	10100	10247
2	0	0	-	10068	10221	9810	10054	9907	10319	9944	10010	10023	9792	10100	10247
3	0	0	-	10068	10221	9810	10054	9907	10319	9944	10010	10023	9792	10100	10247
4	0	0	-	10068	10221	9810	10054	9907	10319	9944	10010	10023	9792	10100	10247
5	0	0	-	10068	10221	9810	10054	9907	10319	9944	10010	10023	9792	10100	10247
6	-17.2	144.9	665,000	0	152	263	59	191	276	210	204	288	499	470	541
7	0	0	-	10068	10221	9810	10054	9907	10319	9944	10010	10023	9792	10100	10247
8	-17.4	147.1	885,000	152	0	414	177	332	155	327	291	352	602	492	520
9	-17.5	141.1	1,116,000	263	414	0	244	109	512	176	244	312	359	503	628
10	0	0	-	10068	10221	9810	10054	9907	10319	9944	10010	10023	9792	10100	10247
11	0	0	-	10068	10221	9810	10054	9907	10319	9944	10010	10023	9792	10100	10247
12	-18	144.6	148,000	59	177	244	0	154	271	157	145	229	444	414	492
13	-18.4	142.4	854,000	191	332	109	154	0	412	74	137	213	330	410	527
14	0	0	-	10068	10221	9810	10054	9907	10319	9944	10010	10023	9792	10100	10247
15	-19.3	148.3	615,000	276	155	512	271	412	0	380	320	343	610	421	408
16	0	0	-	10068	10221	9810	10054	9907	10319	9944	10010	10023	9792	10100	10247
17	-19.4	142.8	627,000	210	327	176	157	74	380	0	71	139	289	336	454
18	-19.9	143.7	542,000	204	291	244	145	137	320	71	0	90	311	287	392
19	0	0	-	10068	10221	9810	10054	9907	10319	9944	10010	10023	9792	10100	10247
20	-21.2	143.7	964,000	288	352	312	229	213	343	139	90	0	267	199	316
21	0	0	-	10068	10221	9810	10054	9907	10319	9944	10010	10023	9792	10100	10247
22	-22.6	140.1	706,000	499	602	359	444	330	610	289	311	267	0	312	463
23	0	0	-	10068	10221	9810	10054	9907	10319	9944	10010	10023	9792	10100	10247
24	-24	144.4	669,000	470	492	503	414	410	421	336	287	199	312	0	151
25	-24.9	146.4	931,000	541	520	628	492	527	408	454	392	316	463	151	0
				282	333	310	254	246	358	226	219	233	371	332	404
			% within 100 miles	9%	10%	13%	9%	17%	7%	23%	24%	17%	8%	8%	11%
			% within 200 miles	29%	27%	30%	43%	45%	17%	49%	36%	32%	8%	29%	18%
			% within 300 miles	74%	33%	45%	74%	56%	27%	64%	74%	59%	26%	36%	18%

(We've hidden the columns representing the eastern cities.)

City 18 does best on the average distance, but others do better within 100 and 200 miles.

7. Often, when logistics practitioners think about having some automatic way of picking a latitude and longitude, they think about a problem such as where the four cities form a square on a map. In this case, let's assume that each city has a population of one million, and the latitude and longitude of the four points are (15,155), (15,158), (18,155), and (18,158).

a. What is the latitude and longitude of the physics center of gravity?

b. What is the average distance to each of the cities from this point?

c. Is there a better point that would minimize the weighted-average distance?

d. If the population of City 1 were five million, what would be the physics center of gravity latitude and longitude?

Part (a) and (b). The physics center of gravity point is 16.5, 156.5. See the following chart:

City #	Latitude	Longitude	Population	Approx Distance (miles)
1	15	155	1,000,000	146
2	15	158	1,000,000	146
3	18	155	1,000,000	146
4	18	158	1,000,000	146
Test	**16.5**	**156.5**		146

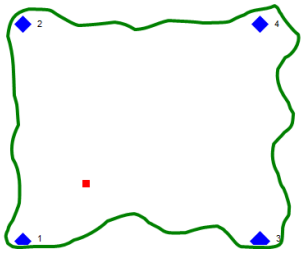
Part (c). In this case, with a perfect square and an equal population, the physics center of gravity is also the point that minimizes the average distance. For example, if you place the site at any of the four points, the average goes up to 177. (It is a fun question to ask how frequently this situation happens—almost never!)

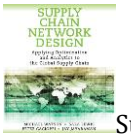
City #	Latitude	Longitude	Population	Approx Distance (miles)
1	15	155	1,000,000	0
2	15	158	1,000,000	207
3	18	155	1,000,000	207
4	18	158	1,000,000	293
COG	**15**	**155**		177

Part (d). If the population of City 1 was 5 million, then the physics center gravity moves much closer to City 1, but not all the way there. It goes to 15.8, 155.8. You can see below that locating the site at City 1 significantly lowers the average distance.

City #	Latitude	Longitude	Population	Approx Distance (miles)
1	15	155	5,000,000	78
2	15	158	1,000,000	162
3	18	155	1,000,000	162
4	18	158	1,000,000	215
Test	**15.8**	**155.8**		116

City #	Latitude	Longitude	Population	Approx Distance (miles)
1	15	155	5,000,000	0
2	15	158	1,000,000	207
3	18	155	1,000,000	207
4	18	158	1,000,000	293
COG	**15**	**155**		88





The square represents the physics of gravity when the population of City 1 is 5 million.