$$\phi(t,t_0) = \phi_{bc}(t,t_0) + \phi_{dc}(t,t_0)$$
$$= 1.347 + 2.432 = 3.779$$
$$J(t,t_0) = \frac{1}{E_{ci}(t_0)} + \frac{\phi(t,t_0)}{E_{\text{ci}}}$$
$$= \frac{1}{28,502.6} + \frac{3.779}{32,297.7} = 152.1\text{x}10^{-6}\,\text{MPa}^{-1}$$

For t = 2,190 days

$$\beta_{bc}(t,t_0) = \ln\left[\left(\frac{30}{t_{0,adj}} + 0.035\right)^2 \cdot (t - t_0) + 1\right]$$
$$= \ln\left[\left(\frac{30}{7} + 0.035\right)^2 \cdot (2,190 - 7) + 1\right] = 10.615$$
$$\phi_{bc}(t,t_0) = \beta_{bc}(f_{cm}) \cdot \beta_{bc}(t,t_0) = (0.153)(10.615) = 1.624$$
$$\beta_{dc}(t,t_0) = \left[\frac{(t-t_0)}{\beta_h + (t - t_0)}\right]^{\gamma(t_0)} = \left[\frac{(2,190-7)}{306.5 + (2,190 - 7)}\right]^{0.276} = 0.964$$
$$\phi_{dc}(t,t_0) = \beta_{dc}(f_{cm}) \cdot \beta(RH) \cdot \beta_{dc}(t_0) \cdot \beta_{dc}(t,t_0)$$
$$= (2.969)(1.53)(0.635)(0.964) = 2.782$$
$$\phi(t,t_0) = \phi_{bc}(t,t_0) + \phi_{dc}(t,t_0)$$
$$= 1.624 + 2.782 = 4.406$$
$$J(t,t_0) = \frac{1}{E_{ci}(t_0)} + \frac{\phi(t,t_0)}{E_{\text{ci}}}$$
$$= \frac{1}{28,502.6} + \frac{4.406}{32,297.7} = 171.5\text{x}10^{-6}\,\text{MPa}^{-1}$$

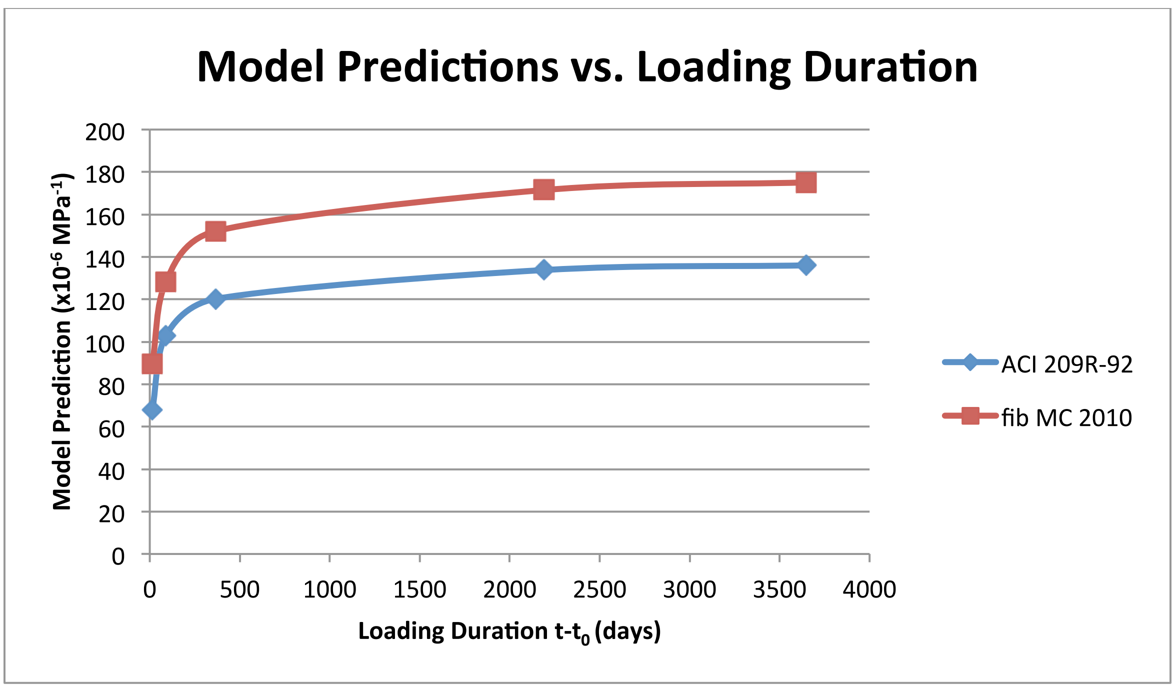
For t = 3,650 days

$$\beta_{bc}(t,t_0) = \ln\left[\left(\frac{30}{t_{0,adj}} + 0.035\right)^2 \cdot (t - t_0) + 1\right]$$
$$= \ln\left[\left(\frac{30}{7} + 0.035\right)^2 \cdot (3,650 - 7) + 1\right] = 11.127$$
$$\phi_{bc}(t,t_0) = \beta_{bc}(f_{cm}) \cdot \beta_{bc}(t,t_0) = (0.153)(11.127) = 1.702$$
$$\beta_{dc}(t,t_0) = \left[\frac{(t-t_0)}{\beta_h + (t - t_0)}\right]^{\gamma(t_0)} = \left[\frac{(3,650-7)}{306.5 + (3,650 - 7)}\right]^{0.276} = 0.978$$

$$\phi_{dc}(t,t_0) = \beta_{dc}(f_{cm}) \cdot \beta(RH) \cdot \beta_{dc}(t_0) \cdot \beta_{dc}(t,t_0)$$
$$= (2.969)(1.53)(0.635)(0.978) = 2.821$$
$$\phi(t,t_0) = \phi_{bc}(t,t_0) + \phi_{dc}(t,t_0)$$
$$= 1.702 + 2.821 = 4.523$$
$$J(t,t_0) = \frac{1}{E_{ci}(t_0)} + \frac{\phi(t,t_0)}{E_{\text{ci}}}$$
$$= \frac{1}{28,502.6} + \frac{4.523}{32,297.7} = 175.1\text{x}10^{-6}\,\text{MPa}^{-1}$$

Part b

$t\text{-}t_0$ (days)	ACI 209R-92 Predictions ($\text{x}10^{-6}$ MPa^{-1})	fib MC 2010 Predictions ($\text{x}10^{-6}$ MPa^{-1})
14	68.0	89.5
90	102.9	128.2
365	120.2	152.1
2,190	133.8	171.5
3,650	136	175.1



Part C

The ACI 209R-92 and fib MC 2010 models both predict rapid change in creep compliance for the first few days of loading. At later loading ages the models seem to be reaching a plateau. The difference between the models is that the fib MC 2010 model predicts higher levels of creep compliance.

2.16 A concrete specimen has the following properties: Humidity = 50%; $h_e = 2V/S = 51$ mm; $f_{cm28} = 16.5$ MPa; cement content (c) = 320 kg/m^3; w/c = 0.59; a/c = 5.669; $t_c = 28$ days; $\gamma = 2296.74$ kg/m^3; the specimen is Type I cement; and it was moist-cured. Use the B3 model and the GL 2000 model to answer the following:

a. Predict the amount of shrinkage the concrete specimen will undergo for ages: 41; 118; 2,010; 8,988; and 10,028 days.

b. Create a graph showing the predictions versus drying duration and discuss the results

Solution

Part a
B3:

$\varepsilon_s(t) = (\varepsilon_{\text{shu}})(K_h)S(t)$

Determination of εshu:

$\alpha_1 = 1.0$ (Table 2.8)

$\alpha_2 = 1.0$ (Table 2.9)

$$\varepsilon_{\text{shu}} = -\varepsilon_{\text{su}} \frac{E_{\text{cm607}}}{E_{\text{cm}(t_c+\tau_{\text{sh}})}}$$

$$w = \frac{w}{c} \cdot c = 0.59 \cdot 320 \text{ kg/m}^3 = 188.8 \text{ kg/m}^3$$

$$\varepsilon_{\text{su}} = -\alpha_1\alpha_2[0.019(w)^{2.1}(f_{\text{cm28}})^{-0.28} + 270]\times 10^{-6}$$

$$= -(1.0)(1.0)[0.019(188.8)^{2.1}(16.5)^{-0.28}+270]\times 10^{-6} = -791.7\times 10^{-6} \text{ mm/mm}$$

$$E_{\text{cm28}} = 4735\sqrt{f_{\text{cm28}}} = 4735\sqrt{16.5} = 19,233.7 \text{ MPa}$$

$k_s = 1.0$ (Since the type of member is not defined)

$$T_{\text{sh}} = 0.085(t_c)^{-0.08}(f_{\text{cm28}})^{-0.25}\left[2k_s\left(\frac{V}{S}\right)\right]^2$$

$$T_{\text{sh}} = 0.085(28)^{-0.08}(16.5)^{-0.25}\left[2(1)(25.5)\right]^2$$

$$= 84.0 \text{ days}$$

$$E_{\text{cm607}} = (1.167)^{1/2} E_{\text{cm28}}$$

$$= (1.167)^{1/2}(19{,}233.7) = 20{,}777.7 \text{ MPa}$$

$$E_{\text{cm}(t_c+\tau_{\text{sh}})} = \left(\frac{t_c+\tau_{\text{sh}}}{4+0.85(t_c+\tau_{\text{sh}})}\right)^{1/2} E_{\text{cm28}} = \left(\frac{28+84.0}{4+0.85(28+84.0)}\right)^{1/2}(19{,}233.7)$$

$$= 20{,}436.9 \text{ MPa}$$

$$\varepsilon_{\text{shu}} = -\varepsilon_{\text{su}}\frac{E_{\text{cm607}}}{E_{\text{cm}(t_c+\tau_{\text{sh}})}} = -(-791.7\times10^{-6})\frac{20{,}777.7}{20{,}436.9} = 804.9\times10^{-6} \text{ mm/mm}$$

Determination of Kh:
According to the Table 2.10, for H = 50%

$$K_h = 1-\left(\frac{H}{100}\right)^3 = 1-\left(\frac{50}{100}\right)^3 = 0.875$$

For t = 41 days:

$$S(t) = \tanh\sqrt{\frac{t-t_c}{T_{\text{sh}}}} = \tanh\sqrt{\frac{41-28}{84.0}} = 0.374$$

$$\varepsilon_s(t) = (\varepsilon_{\text{shu}})(K_h)S(t) = (804.9\times10^{-6})(0.875)(0.374) = 263.4\times10^{-6} \text{ mm/mm}$$

For t = 118 days:

$$S(t) = \tanh\sqrt{\frac{t-t_c}{T_{\text{sh}}}} = \tanh\sqrt{\frac{118-28}{84.0}} = 0.776$$

$$\varepsilon_s(t) = (\varepsilon_{\text{shu}})(K_h)S(t) = (804.9\times10^{-6})(0.875)(0.776) = 546.5\times10^{-6} \text{ mm/mm}$$

For t = 2,010 days:

$$S(t) = \tanh\sqrt{\frac{t-t_c}{T_{\text{sh}}}} = \tanh\sqrt{\frac{2{,}010-28}{84.0}} = 0.999$$

$$\varepsilon_s(t) = (\varepsilon_{\text{shu}})(K_h)S(t) = (804.9\times10^{-6})(0.875)(0.999) = 704.2\times10^{-6} \text{ mm/mm}$$

For t = 8,988 days:

$$S(t)=\tanh\sqrt{\frac{t-t_c}{T_{\text{sh}}}}=\tanh\sqrt{\frac{8{,}988-28}{84.0}}=1$$

.

$$\varepsilon_s(t)=(\varepsilon_{\text{shu}})(K_h)S(t)=(804.9\times10^{-6})(0.875)(1)=704.3\times10^{-6}\,\text{mm/mm}$$

For t = 10,028 days:

$$S(t)=\tanh\sqrt{\frac{t-t_c}{T_{\text{sh}}}}=\tanh\sqrt{\frac{10{,}028-28}{84.0}}=1$$

.

$$\varepsilon_s(t)=(\varepsilon_{\text{shu}})(K_h)S(t)=(804.9\times10^{-6})(0.875)(1)=704.3\times10^{-6}\,\text{mm/mm}$$

GL 2000:

$$\varepsilon_s(t)=\varepsilon_{\text{shu}}\beta(h)\beta(t-t_c)$$

$K=1.00$ (Table 2.12)

$$\varepsilon_{\text{shu}}=(900)K\left(\frac{30}{f_{\text{cm}28}}\right)^{1/2}\times10^{-6}=(900)(1.00)\left(\frac{30}{16.5}\right)^{1/2}\times10^{-6}=1{,}213.6\times10^{-6}\,\text{mm/mm}$$

$$\beta(h)=1-1.18\left(\frac{H}{100}\right)^4=1-1.18\left(\frac{50}{100}\right)^4=0.926$$

For t = 41 days:

$$\beta(t-t_c)=\left(\frac{t-t_c}{t-t_c+0.12(V/S)^2}\right)^{1/2}=\left(\frac{41-28}{41-28+0.12(25.5)^2}\right)^{1/2}=0.3779$$

$$\varepsilon_s(t)=\varepsilon_{\text{shu}}\beta(h)\beta(t-t_c)=(1{,}213.6\times10^{-6})(0.926)(0.3779)=424.7\times10^{-6}\,\text{mm/mm}$$

For t = 118 days:

$$\beta(t-t_c)=\left(\frac{t-t_c}{t-t_c+0.12(V/S)^2}\right)^{1/2}=\left(\frac{118-28}{118-28+0.12(25.5)^2}\right)^{1/2}=0.732$$

$$\varepsilon_s(t)=\varepsilon_{\text{shu}}\beta(h)\beta(t-t_c)=(1{,}213.6\times10^{-6})(0.926)(0.732)=822.6\times10^{-6}\,\text{mm/mm}$$

For t = 2,010 days:

$$\beta(t-t_c)=\left(\frac{t-t_c}{t-t_c+0.12(V/S)^2}\right)^{1/2}=\left(\frac{2{,}010-28}{2{,}010-28+0.12(25.5)^2}\right)^{1/2}=0.981$$

$$\varepsilon_s(t)=\varepsilon_{\text{shu}}\beta(h)\beta(t-t_c)=(1{,}213.6\times10^{-6})(0.926)(0.981)=1{,}102.4\times10^{-6}\,\text{mm/mm}$$

For t = 8,988 days:

$$\beta(t-t_c)=\left(\frac{t-t_c}{t-t_c+0.12(V/S)^2}\right)^{1/2}=\left(\frac{8,988\ -28}{8,988\ -28+0.12(25.5)^2}\right)^{1/2}=0.996$$

$$\varepsilon_s(t)=\varepsilon_{\text{shu}}\beta(h)\beta(t-t_c)=(1,213.6\times10^{-6})(0.926)(0.996)=1,119.3\times10^{-6}\,\text{mm/mm}$$

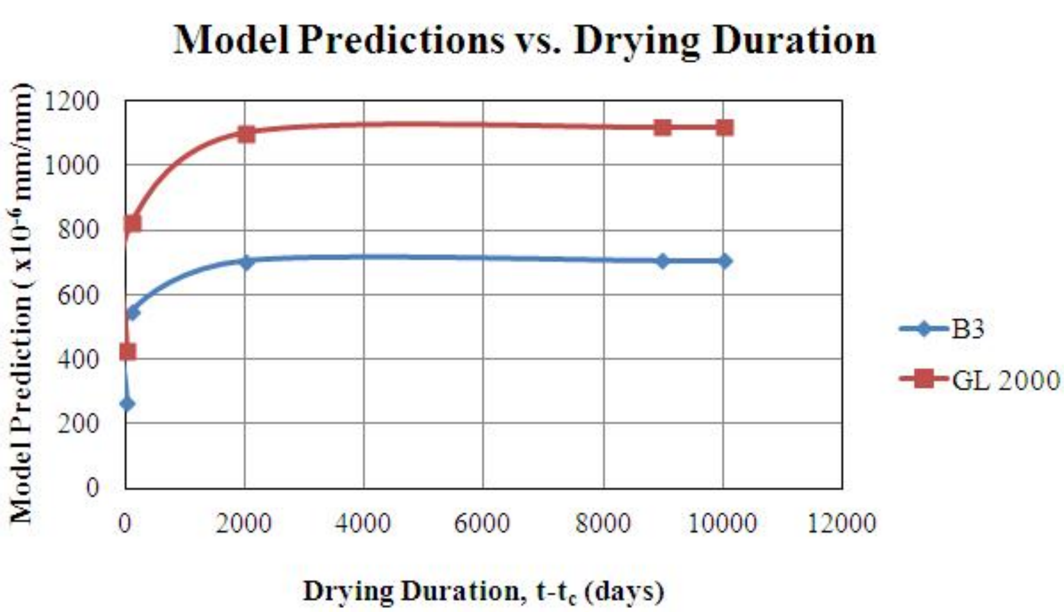
For t = 10,028 days:

$$\beta(t-t_c)=\left(\frac{t-t_c}{t-t_c+0.12(V/S)^2}\right)^{1/2}=\left(\frac{10,028-28}{10,028-28+0.12(25.5)^2}\right)^{1/2}=0.996$$

$$\varepsilon_s(t)=\varepsilon_{\text{shu}}\beta(h)\beta(t-t_c)=(1,213.6\times10^{-6})(0.926)(0.996)=1,119.3\times10^{-6}\,\text{mm/mm}$$

Part b

t-t_c (days)	B3 Prediction ($x10^{-6}$ mm/mm)	GL 2000 Predictions ($x10^{-6}$ mm/mm)
13	263.4	424.7
90	546.5	822.6
1982	704.2	1102.4
8960	704.3	1119.3
10000	704.3	1119.3



The GL 2000 model predicts higher levels of shrinkage for the concrete specimen compared to the B3 model. In addition, the B3 and GL 2000 models both reach a plateau at later drying durations.