

COMPLETE SOLUTIONS MANUAL
for Stewart's
SINGLE VARIABLE CALCULUS
CONCEPTS AND CONTEXTS
FOURTH EDITION

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□ PREFACE

This Complete Solutions Manual contains solutions to all exercises in the texts *Single Variable Calculus: Concepts and Contexts*, Fourth Edition, and Chapters 1–8 of *Calculus: Concepts and Contexts*, Fourth Edition, by James Stewart. A student version of this manual is also available; it contains solutions to the odd-numbered exercises in each chapter section, the review sections, the True-False Quizzes, and the Focus on Problem Solving sections, as well as solutions to all the exercises in the Concept Checks. No solutions to the Projects appear in the student version. It is our hope that by browsing through the solutions, professors will save time in determining appropriate assignments for their particular classes.

Some nonstandard notation is used in order to save space. If you see a symbol that you don't recognize, refer to the Table of Abbreviations and Symbols on page v.

We appreciate feedback concerning errors, solution correctness or style, and manual style. Any comments may be sent directly to us at jeff.cole@anokaramsey.edu or tim@andrew.cmu.edu, or in care of the publisher: Cengage Learning Brooks/Cole, 20 Davis Drive, Belmont, CA 94002.

We would like to thank Jim Stewart, for his guidance; Brian Betsill, Kathi Townes, and Rebekah Million, of TECH-arts, for their production services; and Richard Stratton and Jeannine Lawless, of Cengage Learning Brooks/Cole, for entrusting us with this project as well as for their patience and support.

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□ ABBREVIATIONS AND SYMBOLS

CD	concave downward
CU	concave upward
D	the domain of f
FDT	First Derivative Test
HA	horizontal asymptote(s)
I	interval of convergence
I/D	Increasing/Decreasing Test
IP	inflection point(s)
R	radius of convergence
VA	vertical asymptote(s)
$\overset{\text{CAS}}{=}$	indicates the use of a computer algebra system.
$\overset{\text{H}}{=}$	indicates the use of l'Hospital's Rule.
$\overset{j}{=}$	indicates the use of Formula j in the Table of Integrals in the back endpapers.
$\overset{s}{=}$	indicates the use of the substitution $\{u = \sin x, du = \cos x dx\}$.
$\overset{c}{=}$	indicates the use of the substitution $\{u = \cos x, du = -\sin x dx\}$.

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COMPLETE SOLUTIONS MANUAL

for Stewart's

**MULTIVARIABLE CALCULUS:
CONCEPTS AND CONTEXTS**

FOURTH EDITION

DAN CLEGG
Palomar College



Australia · Brazil · Japan · Korea · Mexico · Singapore · Spain · United Kingdom · United States

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□ PREFACE

This *Complete Solutions Manual* contains detailed solutions to all exercises in the text *Multivariable Calculus: Concepts and Contexts*, Fourth Edition (Chapters 8–13 of *Calculus: Concepts and Contexts*, Fourth Edition) by James Stewart. A *Student Solutions Manual* is also available, which contains solutions to the odd-numbered exercises in each chapter section, review section, True-False Quiz, and Focus on Problem Solving section as well as all solutions to the Concept Check questions. (It does not, however, include solutions to any of the projects.)

While I have extended every effort to ensure the accuracy of the solutions presented, I would appreciate correspondence regarding any errors that may exist. Other suggestions or comments are also welcome, and can be sent to me at the email address or mailing address below.

I would like to thank James Stewart for entrusting me with the writing of this manual and offering suggestions, Kathi Townes, Stephanie Kuhns, and Rebekah Steele of TECH-arts for typesetting and producing this manual, and Brian Betsill of TECH-arts for creating the illustrations. Brian Karasek prepared solutions for comparison of accuracy and style in addition to proofreading manuscript; his assistance and suggestions were very helpful and much appreciated. Finally, I would like to thank Richard Stratton and Elizabeth Neustaetter of Brooks/Cole, Cengage Learning for their trust, assistance, and patience.

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8 INFINITE SEQUENCES AND SERIES

8.1 Sequences

1. (a) A sequence is an ordered list of numbers. It can also be defined as a function whose domain is the set of positive integers.
- (b) The terms a_n approach 8 as n becomes large. In fact, we can make a_n as close to 8 as we like by taking n sufficiently large.
- (c) The terms a_n become large as n becomes large. In fact, we can make a_n as large as we like by taking n sufficiently large.
2. (a) From Definition 1, a convergent sequence is a sequence for which $\lim_{n \rightarrow \infty} a_n$ exists. Examples: $\{1/n\}$, $\{1/2^n\}$
- (b) A divergent sequence is a sequence for which $\lim_{n \rightarrow \infty} a_n$ does not exist. Examples: $\{n\}$, $\{\sin n\}$
3. The first six terms of $a_n = \frac{n}{2n+1}$ are $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}$. It appears that the sequence is approaching $\frac{1}{2}$.
- $$\lim_{n \rightarrow \infty} \frac{n}{2n+1} = \lim_{n \rightarrow \infty} \frac{1}{2+1/n} = \frac{1}{2}$$
4. $\{\cos(n\pi/3)\}_{n=1}^9 = \{\frac{1}{2}, -\frac{1}{2}, -1, -\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}, -\frac{1}{2}, -1\}$. The sequence does not appear to have a limit. The values will cycle through the first six numbers in the sequence—never approaching a particular number.
5. $\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \dots\}$. The denominator of the n th term is the n th positive odd integer, so $a_n = \frac{1}{2n-1}$.
6. $\{1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \dots\}$. The denominator of the n th term is the $(n-1)$ st power of 3, so $a_n = \frac{1}{3^{n-1}}$.
7. $\{2, 7, 12, 17, \dots\}$. Each term is larger than the preceding one by 5, so $a_n = a_1 + d(n-1) = 2 + 5(n-1) = 5n - 3$.
8. $\{-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \dots\}$. The numerator of the n th term is n and its denominator is $(n+1)^2$. Including the alternating signs, we get $a_n = (-1)^n \frac{n}{(n+1)^2}$.
9. $\{1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \dots\}$. Each term is $-\frac{2}{3}$ times the preceding one, so $a_n = (-\frac{2}{3})^{n-1}$.
10. $\{5, 1, 5, 1, 5, 1, \dots\}$. The average of 5 and 1 is 3, so we can think of the sequence as alternately adding 2 and -2 to 3. Thus, $a_n = 3 + (-1)^{n+1} \cdot 2$.
11. $a_n = \frac{3+5n^2}{n+n^2} = \frac{(3+5n^2)/n^2}{(n+n^2)/n^2} = \frac{5+3/n^2}{1+1/n}$, so $a_n \rightarrow \frac{5+0}{1+0} = 5$ as $n \rightarrow \infty$. Converges
12. $a_n = \frac{n^3}{n^3+1} = \frac{n^3/n^3}{(n^3+1)/n^3} = \frac{1}{1+1/n^3}$, so $a_n \rightarrow \frac{1}{1+0} = 1$ as $n \rightarrow \infty$. Converges
13. $a_n = 1 - (0.2)^n$, so $\lim_{n \rightarrow \infty} a_n = 1 - 0 = 1$ by (7). Converges

14. $a_n = \frac{n^3}{n+1} = \frac{n^3/n}{(n+1)/n} = \frac{n^2}{1+1/n^2}$, so $a_n \rightarrow \infty$ as $n \rightarrow \infty$ since $\lim_{n \rightarrow \infty} n^2 = \infty$ and $\lim_{n \rightarrow \infty} (1+1/n^2) = 1$. Diverges

15. Because the natural exponential function is continuous at 0, Theorem 5 enables us to write

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} e^{1/n} = e^{\lim_{n \rightarrow \infty} (1/n)} = e^0 = 1. \quad \text{Converges}$$

16. $a_n = \frac{3^{n+2}}{5^n} = \frac{3^2 3^n}{5^n} = 9 \left(\frac{3}{5}\right)^n$, so $\lim_{n \rightarrow \infty} a_n = 9 \lim_{n \rightarrow \infty} \left(\frac{3}{5}\right)^n = 9 \cdot 0 = 0$ by (7) with $r = \frac{3}{5}$. Converges

17. If $b_n = \frac{2n\pi}{1+8n}$, then $\lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \frac{(2n\pi)/n}{(1+8n)/n} = \lim_{n \rightarrow \infty} \frac{2\pi}{1/n+8} = \frac{2\pi}{8} = \frac{\pi}{4}$. Since \tan is continuous at $\frac{\pi}{4}$, by

$$\text{Theorem 5, } \lim_{n \rightarrow \infty} \tan\left(\frac{2n\pi}{1+8n}\right) = \tan\left(\lim_{n \rightarrow \infty} \frac{2n\pi}{1+8n}\right) = \tan \frac{\pi}{4} = 1. \quad \text{Converges}$$

18. Using the last limit law for sequences and the continuity of the square root function,

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \sqrt{\frac{n+1}{9n+1}} = \sqrt{\lim_{n \rightarrow \infty} \frac{n+1}{9n+1}} = \sqrt{\lim_{n \rightarrow \infty} \frac{1+1/n}{9+1/n}} = \sqrt{\frac{1}{9}} = \frac{1}{3}. \quad \text{Converges}$$

19. $a_n = \frac{(-1)^{n-1} n}{n^2+1} = \frac{(-1)^{n-1}}{n+1/n}$, so $0 \leq |a_n| = \frac{1}{n+1/n} \leq \frac{1}{n} \rightarrow 0$ as $n \rightarrow \infty$, so $a_n \rightarrow 0$ by the Squeeze Theorem and Theorem 4. Converges

20. $a_n = \frac{(-1)^n n^3}{n^3+2n^2+1}$. Now $|a_n| = \frac{n^3}{n^3+2n^2+1} = \frac{1}{1+\frac{2}{n}+\frac{1}{n^3}} \rightarrow 1$ as $n \rightarrow \infty$, but the terms of the sequence $\{a_n\}$ alternate in sign, so the sequence a_1, a_3, a_5, \dots converges to -1 and the sequence a_2, a_4, a_6, \dots converges to $+1$. This shows that the given sequence diverges since its terms don't approach a single real number.

21. $a_n = \frac{e^n + e^{-n}}{e^{2n} - 1} \cdot \frac{e^{-n}}{e^{-n}} = \frac{1 + e^{-2n}}{e^n - e^{-n}} \rightarrow 0$ as $n \rightarrow \infty$ because $1 + e^{-2n} \rightarrow 1$ and $e^n - e^{-n} \rightarrow \infty$. Converges

22. $a_n = \cos(2/n)$. As $n \rightarrow \infty$, $2/n \rightarrow 0$, so $\cos(2/n) \rightarrow \cos 0 = 1$ because \cos is continuous. Converges

23. $a_n = n^2 e^{-n} = \frac{n^2}{e^n}$. Since $\lim_{x \rightarrow \infty} \frac{x^2}{e^x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{2x}{e^x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{2}{e^x} = 0$, it follows from Theorem 2 that $\lim_{n \rightarrow \infty} a_n = 0$. Converges

24. $2n \rightarrow \infty$ as $n \rightarrow \infty$, so since $\lim_{x \rightarrow \infty} \arctan x = \frac{\pi}{2}$, we have $\lim_{n \rightarrow \infty} \arctan 2n = \frac{\pi}{2}$. Converges

25. $0 \leq \frac{\cos^2 n}{2^n} \leq \frac{1}{2^n}$ [since $0 \leq \cos^2 n \leq 1$], so since $\lim_{n \rightarrow \infty} \frac{1}{2^n} = 0$, $\left\{\frac{\cos^2 n}{2^n}\right\}$ converges to 0 by the Squeeze Theorem.

26. $a_n = n \cos n\pi = n(-1)^n$. Since $|a_n| = n \rightarrow \infty$ as $n \rightarrow \infty$, the given sequence diverges.

27. $y = \left(1 + \frac{2}{x}\right)^x \Rightarrow \ln y = x \ln \left(1 + \frac{2}{x}\right)$, so

$$\lim_{x \rightarrow \infty} \ln y = \lim_{x \rightarrow \infty} \frac{\ln(1+2/x)}{1/x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{\left(\frac{1}{1+2/x}\right)\left(-\frac{2}{x^2}\right)}{-1/x^2} = \lim_{x \rightarrow \infty} \frac{2}{1+2/x} = 2 \Rightarrow$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x = \lim_{x \rightarrow \infty} e^{\ln y} = e^2, \text{ so by Theorem 2, } \lim_{n \rightarrow \infty} \left(1 + \frac{2}{n}\right)^n = e^2. \quad \text{Convergent}$$

28. $a_n = \sqrt[n]{2^{1+3n}} = (2^{1+3n})^{1/n} = (2^1 2^{3n})^{1/n} = 2^{1/n} 2^3 = 8 \cdot 2^{1/n}$, so

$$\lim_{n \rightarrow \infty} a_n = 8 \lim_{n \rightarrow \infty} 2^{1/n} = 8 \cdot 2^{\lim_{n \rightarrow \infty} (1/n)} = 8 \cdot 2^0 = 8 \text{ by Theorem 5, since the function } f(x) = 2^x \text{ is continuous at 0.}$$

Convergent

29. $a_n = \frac{(2n-1)!}{(2n+1)!} = \frac{(2n-1)!}{(2n+1)(2n)(2n-1)!} = \frac{1}{(2n+1)(2n)} \rightarrow 0$ as $n \rightarrow \infty$. Converges

30. $a_n = \frac{\sin 2n}{1 + \sqrt{n}}$. $|a_n| \leq \frac{1}{1 + \sqrt{n}}$ and $\lim_{n \rightarrow \infty} \frac{1}{1 + \sqrt{n}} = 0$, so $\frac{-1}{1 + \sqrt{n}} \leq a_n \leq \frac{1}{1 + \sqrt{n}} \Rightarrow \lim_{n \rightarrow \infty} a_n = 0$ by the Squeeze Theorem. Converges

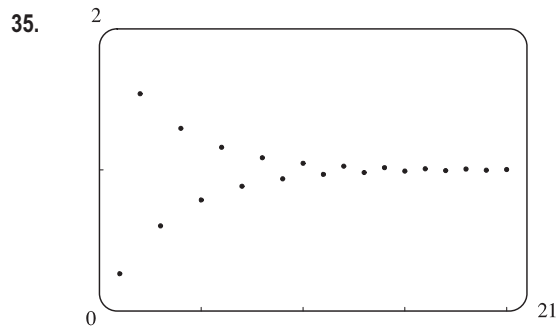
31. $\{0, 1, 0, 0, 1, 0, 0, 0, 1, \dots\}$ diverges since the sequence takes on only two values, 0 and 1, and never stays arbitrarily close to either one (or any other value) for n sufficiently large.

32. $\lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{2(\ln x)(1/x)}{1} = 2 \lim_{x \rightarrow \infty} \frac{\ln x}{x} \stackrel{H}{=} 2 \lim_{x \rightarrow \infty} \frac{1/x}{1} = 0$, so by Theorem 3, $\lim_{n \rightarrow \infty} \frac{(\ln n)^2}{n} = 0$. Convergent

33. $a_n = \ln(2n^2 + 1) - \ln(n^2 + 1) = \ln\left(\frac{2n^2 + 1}{n^2 + 1}\right) = \ln\left(\frac{2 + 1/n^2}{1 + 1/n^2}\right) \rightarrow \ln 2$ as $n \rightarrow \infty$. Convergent

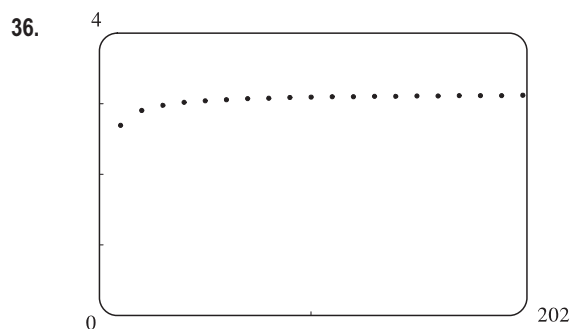
34. $0 < |a_n| = \frac{3^n}{n!} = \frac{3}{1} \cdot \frac{3}{2} \cdot \frac{3}{3} \cdots \frac{3}{(n-1)} \cdot \frac{3}{n} \leq \frac{3}{1} \cdot \frac{3}{2} \cdot \frac{3}{n}$ [for $n > 2$] $= \frac{27}{2n} \rightarrow 0$ as $n \rightarrow \infty$, so by the Squeeze

Theorem and Theorem 4, $\{(-3)^n/n!\}$ converges to 0.



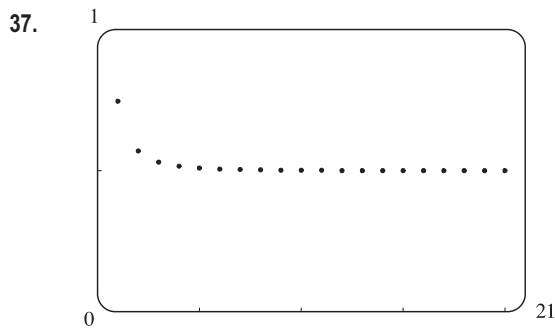
From the graph, it appears that the sequence converges to 1.

$\{(-2/e)^n\}$ converges to 0 by (7), and hence $\{1 + (-2/e)^n\}$ converges to $1 + 0 = 1$.



From the graph, it appears that the sequence converges to a number greater than 3.

$$\begin{aligned} \lim_{n \rightarrow \infty} a_n &= \lim_{n \rightarrow \infty} \sqrt{n} \sin\left(\frac{\pi}{\sqrt{n}}\right) = \lim_{n \rightarrow \infty} \frac{\sin\left(\frac{\pi}{\sqrt{n}}\right)}{\frac{\pi}{\sqrt{n}}} \cdot \pi \\ &= \lim_{x \rightarrow 0^+} \frac{\sin x}{x} \cdot \pi \quad \left[x = \frac{\pi}{\sqrt{n}}\right] = 1 \cdot \pi = \pi. \end{aligned}$$

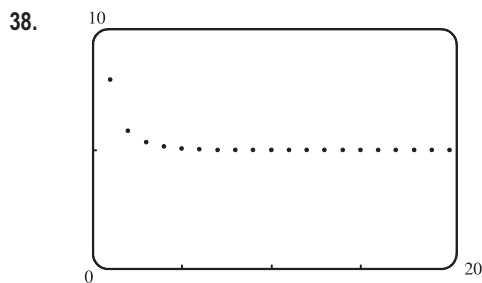


From the graph, it appears that the sequence converges to $\frac{1}{2}$.

As $n \rightarrow \infty$,

$$a_n = \sqrt{\frac{3 + 2n^2}{8n^2 + n}} = \sqrt{\frac{3/n^2 + 2}{8 + 1/n}} \Rightarrow \sqrt{\frac{0 + 2}{8 + 0}} = \sqrt{\frac{1}{4}} = \frac{1}{2},$$

so $\lim_{n \rightarrow \infty} a_n = \frac{1}{2}$.



From the graph, it appears that the sequence converges to 5.

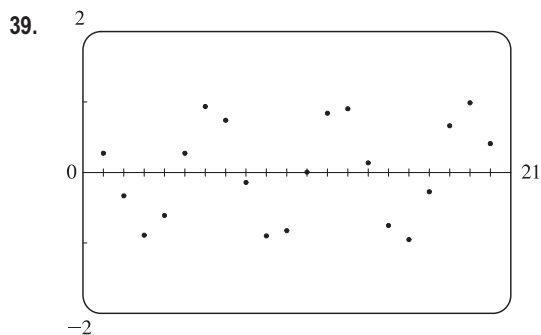
$$\begin{aligned} 5 &= \sqrt[n]{5^n} \leq \sqrt[n]{3^n + 5^n} \leq \sqrt[n]{5^n + 5^n} = \sqrt[n]{2} \sqrt[n]{5^n} \\ &= \sqrt[n]{2} \cdot 5 \rightarrow 5 \text{ as } n \rightarrow \infty \quad \left[\lim_{n \rightarrow \infty} 2^{1/n} = 2^0 = 1 \right] \end{aligned}$$

Hence, $a_n \rightarrow 5$ by the Squeeze Theorem.

Alternate solution: Let $y = (3^x + 5^x)^{1/x}$. Then

$$\lim_{x \rightarrow \infty} \ln y = \lim_{x \rightarrow \infty} \frac{\ln(3^x + 5^x)}{x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{3^x \ln 3 + 5^x \ln 5}{3^x + 5^x} = \lim_{x \rightarrow \infty} \frac{\left(\frac{3}{5}\right)^x \ln 3 + \ln 5}{\left(\frac{3}{5}\right)^x + 1} = \ln 5,$$

so $\lim_{x \rightarrow \infty} y = e^{\ln 5} = 5$, and so $\{\sqrt[n]{3^n + 5^n}\}$ converges to 5.



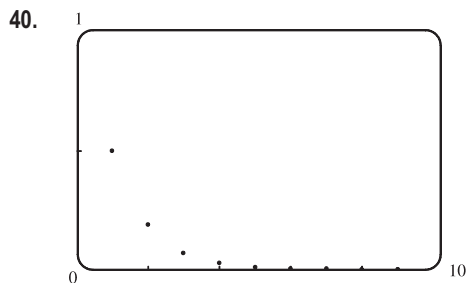
From the graph, it appears that the sequence $\{a_n\} = \left\{ \frac{n^2 \cos n}{1 + n^2} \right\}$ is

divergent, since it oscillates between 1 and -1 (approximately). To

prove this, suppose that $\{a_n\}$ converges to L . If $b_n = \frac{n^2}{1 + n^2}$, then

$\{b_n\}$ converges to 1, and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \frac{L}{1} = L$. But $\frac{a_n}{b_n} = \cos n$, so

$\lim_{n \rightarrow \infty} \frac{a_n}{b_n}$ does not exist. This contradiction shows that $\{a_n\}$ diverges.



From the graph, it appears that the sequence approaches 0.

$$\begin{aligned} 0 < a_n &= \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{(2n)^n} = \frac{1}{2n} \cdot \frac{3}{2n} \cdot \frac{5}{2n} \cdots \frac{2n-1}{2n} \\ &\leq \frac{1}{2n} \cdot (1) \cdot (1) \cdots (1) = \frac{1}{2n} \rightarrow 0 \text{ as } n \rightarrow \infty \end{aligned}$$

So by the Squeeze Theorem, $\left\{ \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{(2n)^n} \right\}$ converges to 0.

41. (a) $a_n = 1000(1.06)^n \Rightarrow a_1 = 1060, a_2 = 1123.60, a_3 = 1191.02, a_4 = 1262.48, \text{ and } a_5 = 1338.23.$

(b) $\lim_{n \rightarrow \infty} a_n = 1000 \lim_{n \rightarrow \infty} (1.06)^n$, so the sequence diverges by (7) with $r = 1.06 > 1$.

47. (a) Let a_n be the number of rabbit pairs in the n th month. Clearly $a_1 = 1 = a_2$. In the n th month, each pair that is 2 or more months old (that is, a_{n-2} pairs) will produce a new pair to add to the a_{n-1} pairs already present. Thus, $a_n = a_{n-1} + a_{n-2}$, so that $\{a_n\} = \{f_n\}$, the Fibonacci sequence.

(b) $a_n = \frac{f_{n+1}}{f_n} \Rightarrow a_{n-1} = \frac{f_n}{f_{n-1}} = \frac{f_{n-1} + f_{n-2}}{f_{n-1}} = 1 + \frac{f_{n-2}}{f_{n-1}} = 1 + \frac{1}{f_{n-1}/f_{n-2}} = 1 + \frac{1}{a_{n-2}}$. If $L = \lim_{n \rightarrow \infty} a_n$,

then $L = \lim_{n \rightarrow \infty} a_{n-1}$ and $L = \lim_{n \rightarrow \infty} a_{n-2}$, so L must satisfy $L = 1 + \frac{1}{L} \Rightarrow L^2 - L - 1 = 0 \Rightarrow L = \frac{1 + \sqrt{5}}{2}$

[since L must be positive].

48. For $\left\{ \sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots \right\}$, $a_1 = 2^{1/2}$, $a_2 = 2^{3/4}$, $a_3 = 2^{7/8}$, \dots , so $a_n = 2^{(2^n - 1)/2^n} = 2^{1 - (1/2^n)}$.

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} 2^{1 - (1/2^n)} = 2^1 = 2.$$

Alternate solution: Let $L = \lim_{n \rightarrow \infty} a_n$. (We could show the limit exists by showing that $\{a_n\}$ is bounded and increasing.)

Then L must satisfy $L = \sqrt{2 \cdot L} \Rightarrow L^2 = 2L \Rightarrow L(L - 2) = 0$. $L \neq 0$ since the sequence increases, so $L = 2$.

49. $a_n = \frac{1}{2n + 3}$ is decreasing since $a_{n+1} = \frac{1}{2(n+1) + 3} = \frac{1}{2n + 5} < \frac{1}{2n + 3} = a_n$ for each $n \geq 1$. The sequence is

bounded since $0 < a_n \leq \frac{1}{5}$ for all $n \geq 1$. Note that $a_1 = \frac{1}{5}$.

50. $a_n = \frac{2n - 3}{3n + 4}$ defines an increasing sequence since for $f(x) = \frac{2x - 3}{3x + 4}$,

$$f'(x) = \frac{(3x + 4)(2) - (2x - 3)(3)}{(3x + 4)^2} = \frac{17}{(3x + 4)^2} > 0. \text{ The sequence is bounded since } a_n \geq a_1 = -\frac{1}{7} \text{ for } n \geq 1,$$

and $a_n < \frac{2n - 3}{3n} < \frac{2n}{3n} = \frac{2}{3}$ for $n \geq 1$.

51. The terms of $a_n = n(-1)^n$ alternate in sign, so the sequence is not monotonic. The first five terms are $-1, 2, -3, 4,$ and -5 .

Since $\lim_{n \rightarrow \infty} |a_n| = \lim_{n \rightarrow \infty} n = \infty$, the sequence is not bounded.

52. $a_n = n + \frac{1}{n}$ defines an increasing sequence since the function $g(x) = x + \frac{1}{x}$ is increasing for $x > 1$. [$g'(x) = 1 - 1/x^2 > 0$

for $x > 1$.] The sequence is unbounded since $a_n \rightarrow \infty$ as $n \rightarrow \infty$. (It is, however, bounded below by $a_1 = 2$.)

53. Since $\{a_n\}$ is a decreasing sequence, $a_n > a_{n+1}$ for all $n \geq 1$. Because all of its terms lie between 5 and 8, $\{a_n\}$ is a bounded sequence. By the Monotonic Sequence Theorem, $\{a_n\}$ is convergent; that is, $\{a_n\}$ has a limit L . L must be less than 8 since $\{a_n\}$ is decreasing, so $5 \leq L < 8$.

54. (a) Let P_n be the statement that $a_{n+1} \geq a_n$ and $a_n \leq 3$. P_1 is obviously true. We will assume that P_n is true and

then show that as a consequence P_{n+1} must also be true. $a_{n+2} \geq a_{n+1} \Leftrightarrow \sqrt{2 + a_{n+1}} \geq \sqrt{2 + a_n} \Leftrightarrow$

$$2 + a_{n+1} \geq 2 + a_n \Leftrightarrow a_{n+1} \geq a_n, \text{ which is the induction hypothesis. } a_{n+1} \leq 3 \Leftrightarrow \sqrt{2 + a_n} \leq 3 \Leftrightarrow$$

$$2 + a_n \leq 9 \Leftrightarrow a_n \leq 7, \text{ which is certainly true because we are assuming that } a_n \leq 3. \text{ So } P_n \text{ is true for all } n, \text{ and so}$$

$a_1 \leq a_n \leq 3$ (showing that the sequence is bounded), and hence by the Monotonic Sequence Theorem, $\lim_{n \rightarrow \infty} a_n$ exists.

$$(b) \text{ If } L = \lim_{n \rightarrow \infty} a_n, \text{ then } \lim_{n \rightarrow \infty} a_{n+1} = L \text{ also, so } L = \sqrt{2 + L} \Rightarrow L^2 = 2 + L \Leftrightarrow L^2 - L - 2 = 0 \Leftrightarrow$$

$$(L + 1)(L - 2) = 0 \Leftrightarrow L = 2 \text{ [since } L \text{ can't be negative].}$$

55. $a_1 = 1, a_{n+1} = 3 - \frac{1}{a_n}$. We show by induction that $\{a_n\}$ is increasing and bounded above by 3. Let P_n be the proposition

$$\text{that } a_{n+1} > a_n \text{ and } 0 < a_n < 3. \text{ Clearly } P_1 \text{ is true. Assume that } P_n \text{ is true. Then } a_{n+1} > a_n \Rightarrow \frac{1}{a_{n+1}} < \frac{1}{a_n} \Rightarrow$$

$$-\frac{1}{a_{n+1}} > -\frac{1}{a_n}. \text{ Now } a_{n+2} = 3 - \frac{1}{a_{n+1}} > 3 - \frac{1}{a_n} = a_{n+1} \Leftrightarrow P_{n+1}. \text{ This proves that } \{a_n\} \text{ is increasing and bounded}$$

above by 3, so $1 = a_1 < a_n < 3$, that is, $\{a_n\}$ is bounded, and hence convergent by the Monotonic Sequence Theorem.

$$\text{If } L = \lim_{n \rightarrow \infty} a_n, \text{ then } \lim_{n \rightarrow \infty} a_{n+1} = L \text{ also, so } L \text{ must satisfy } L = 3 - 1/L \Rightarrow L^2 - 3L + 1 = 0 \Rightarrow L = \frac{3 \pm \sqrt{5}}{2}.$$

$$\text{But } L > 1, \text{ so } L = \frac{3 + \sqrt{5}}{2}.$$

56. $a_1 = 2, a_{n+1} = \frac{1}{3 - a_n}$. We use induction. Let P_n be the statement that $0 < a_{n+1} \leq a_n \leq 2$. Clearly P_1 is true, since

$$a_2 = 1/(3 - 2) = 1. \text{ Now assume that } P_n \text{ is true. Then } a_{n+1} \leq a_n \Rightarrow -a_{n+1} \geq -a_n \Rightarrow 3 - a_{n+1} \geq 3 - a_n \Rightarrow$$

$$a_{n+2} = \frac{1}{3 - a_{n+1}} \leq \frac{1}{3 - a_n} = a_{n+1}. \text{ Also } a_{n+2} > 0 \text{ [since } 3 - a_{n+1} \text{ is positive] and } a_{n+1} \leq 2 \text{ by the induction}$$

$$\text{hypothesis, so } P_{n+1} \text{ is true. To find the limit, we use the fact that } \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} a_{n+1} \Rightarrow L = \frac{1}{3 - L} \Rightarrow$$

$$L^2 - 3L + 1 = 0 \Rightarrow L = \frac{3 \pm \sqrt{5}}{2}. \text{ But } L \leq 2, \text{ so we must have } L = \frac{3 - \sqrt{5}}{2}.$$

$$57. (0.8)^n < 0.000001 \Rightarrow \ln(0.8)^n < \ln(0.000001) \Rightarrow n \ln(0.8) < \ln(0.000001) \Rightarrow n > \frac{\ln(0.000001)}{\ln(0.8)} \Rightarrow$$

$n > 61.9$, so n must be at least 62 to satisfy the given inequality.

58. (a) If f is continuous, then $f(L) = f\left(\lim_{n \rightarrow \infty} a_n\right) = \lim_{n \rightarrow \infty} f(a_n) = \lim_{n \rightarrow \infty} a_{n+1} = \lim_{n \rightarrow \infty} a_n = L$ by Exercise 46(a).

(b) By repeatedly pressing the cosine key on the calculator (that is, taking cosine of the previous answer) until the displayed value stabilizes, we see that $L \approx 0.73909$.

$$59. (a) \text{ Suppose } \{p_n\} \text{ converges to } p. \text{ Then } p_{n+1} = \frac{bp_n}{a + p_n} \Rightarrow \lim_{n \rightarrow \infty} p_{n+1} = \frac{b \lim_{n \rightarrow \infty} p_n}{a + \lim_{n \rightarrow \infty} p_n} \Rightarrow p = \frac{bp}{a + p} \Rightarrow$$

$$p^2 + ap = bp \Rightarrow p(p + a - b) = 0 \Rightarrow p = 0 \text{ or } p = b - a.$$

$$(b) p_{n+1} = \frac{bp_n}{a + p_n} = \frac{\left(\frac{b}{a}\right)p_n}{1 + \frac{p_n}{a}} < \left(\frac{b}{a}\right)p_n \text{ since } 1 + \frac{p_n}{a} > 1.$$

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