Chapter 2

2.1

a) Overall mass balance:

$$\frac{d(\rho V)}{dt} = w_1 + w_2 - w_3 \tag{1}$$

Energy balance:

$$\begin{aligned} C\frac{d\left[\rho V(T_3 - T_{ref})\right]}{dt} &= w_1 C(T_1 - T_{ref}) + w_2 C(T_2 - T_{ref}) \\ &\quad - w_3 C\left(T_3 - T_{ref}\right) \end{aligned} \tag{2}$$

Because ρ = constant and $V = \overline{V}$ = constant, Eq. 1 becomes:

$$w_3 = w_1 + w_2 \tag{3}$$

b) From Eq. 2, substituting Eq. 3

$$\begin{aligned} \rho C\overline{V}\frac{d(T_3 - T_{ref})}{dt} = \rho C\overline{V}\frac{dT_3}{dt} &= w_1 C(T_1 - T_{ref}) + w_2 C(T_2 - T_{ref}) \\ &\quad -(w_1 + w_2)C\left(T_3 - T_{ref}\right) \end{aligned} \tag{4}$$

Constants C and T_{ref} can be cancelled:

$$\rho\overline{V}\frac{dT_3}{dt} = w_1 T_1 + w_2 T_2 - (w_1 + w_2)T_3 \tag{5}$$

The simplified model now consists only of Eq. 5.

Degrees of freedom for the simplified model:

Parameters : ρ, $\overline{V}$

Variables : w_1, w_2, T_1, T_2, T_3
$N_E = 1$
$N_V = 5$

Thus, $N_F = 5 - 1 = 4$

Because w_1, w_2, T_1 and T_2 are determined by upstream units, we assume they are known functions of time:

$w_1 = w_1(t)$
$w_2 = w_2(t)$
$T_1 = T_1(t)$
$T_2 = T_2(t)$

Thus, N_F is reduced to 0.

2.2

Energy balance:

$$C_p \frac{d\left[\rho V(T-T_{ref})\right]}{dt} = wC_p(T_i - T_{ref}) - wC_p(T - T_{ref}) - UA_s(T - T_a) + Q$$

Simplifying

$$\rho V C_p \frac{dT}{dt} = wC_pT_i - wC_pT - UA_s(T - T_a) + Q$$

$$\rho V C_p \frac{dT}{dt} = wC_p(T_i - T) - UA_s(T - T_a) + Q$$

b) T increases if T_i increases and vice versa.

T decreases if w increases and vice versa if $(T_i - T) < 0$. In other words, if $Q > UA_s(T-T_a)$, the contents are heated, and $T > T_i$.

2.3

a) Mass Balances:

$$\rho A_1 \frac{dh_1}{dt} = w_1 - w_2 - w_3 \tag{1}$$

$$\rho A_2 \frac{dh_2}{dt} = w_2 \tag{2}$$

Flow relations:

Let P_1 be the pressure at the bottom of tank 1.

Let P_2 be the pressure at the bottom of tank 2.

Let P_a be the ambient pressure.

Then $$w_2 = \frac{P_1 - P_2}{R_2} = \frac{\rho g}{g_c R_2}(h_1 - h_2) \tag{3}$$

$$w_3 = \frac{P_1 - P_a}{R_3} = \frac{\rho g}{g_c R_3} h_1 \tag{4}$$

b) Seven parameters: ρ, A_1, A_2, g, g_c, R_2, R_3

Five variables : h_1, h_2, w_1, w_2, w_3

Four equations

Thus $N_F = 5 - 4 = 1$

1 input = w_1 (specified function of time)
4 outputs = h_1, h_2, w_2, w_3

2.4

Assume constant liquid density, ρ . The mass balance for the tank is

$$\frac{d(\rho Ah + m_g)}{dt} = \rho(q_i - q)$$

Because ρ, A, and m_g are constant, this equation becomes

$$A\frac{dh}{dt} = q_i - q \qquad (1)$$

The square-root relationship for flow through the control valve is

$$q = C_v\left(P_g + \frac{\rho gh}{g_c} - P_a\right)^{1/2} \qquad (2)$$

From the ideal gas law,

$$P_g = \frac{(m_g / M)RT}{A(H-h)} \qquad (3)$$

where T is the absolute temperature of the gas.

Equation 1 gives the unsteady-state model upon substitution of q from Eq. 2 and of P_g from Eq. 3:

$$A\frac{dh}{dt} = q_i - C_v\left[\frac{(m_g / M)RT}{A(H-h)} + \frac{\rho gh}{g_c} - P_a\right]^{1/2} \qquad (4)$$

Because the model contains P_a, operation of the system is not independent of P_a. For an open system $P_g = P_a$ and Eq. 2 shows that the system is independent of P_a.

2.5

a) For linear valve flow characteristics,

$$w_a = \frac{P_d - P_1}{R_a}, \quad w_b = \frac{P_1 - P_2}{R_b}, \quad w_c = \frac{P_2 - P_f}{R_c} \tag{1}$$

Mass balances for the surge tanks

$$\frac{dm_1}{dt} = w_a - w_b, \qquad \frac{dm_2}{dt} = w_b - w_c \tag{2}$$

where m_1 and m_2 are the masses of gas in surge tanks 1 and 2, respectively.

If the ideal gas law holds, then

$$P_1V_1 = \frac{m_1}{M}RT_1, \qquad P_2V_2 = \frac{m_2}{M}RT_2 \tag{3}$$

where M is the molecular weight of the gas
T_1 and T_2 are the temperatures in the surge tanks.

Substituting for m_1 and m_2 from Eq. 3 into Eq. 2, and noticing that V_1, T_1, V_2, and T_2 are constant,

$$\frac{V_1M}{RT_1}\frac{dP_1}{dt} = w_a - w_b \quad \text{and} \quad \frac{V_2M}{RT_2}\frac{dP_2}{dt} = w_b - w_c \tag{4}$$

The dynamic model consists of Eqs. 1 and 4.

b) For adiabatic operation, Eq. 3 is replaced by

$$P_1\left(\frac{V_1}{m_1}\right)^{\gamma} = P_2\left(\frac{V_2}{m_2}\right)^{\gamma} = C, \text{ a constant} \tag{5}$$

or

$$m_1 = \left(\frac{P_1V_1^{\gamma}}{C}\right)^{1/\gamma} \quad \text{and} \quad m_2 = \left(\frac{P_2V_2^{\gamma}}{C}\right)^{1/\gamma} \tag{6}$$

Substituting Eq. 6 into Eq. 2 gives,

$$\frac{1}{\gamma}\left(\frac{V_1^{\gamma}}{C}\right)^{1/\gamma} P_1^{(1-\gamma)/\gamma}\frac{dP_1}{dt} = w_a - w_b$$

$$\frac{1}{\gamma}\left(\frac{V_2{}^{\gamma}}{C}\right)^{1/\gamma} P_2{}^{(1-\gamma)/\gamma}\,\frac{dP_2}{dt} = w_b - w_c$$

as the new dynamic model. If the ideal gas law were not valid, one would use an appropriate equation of state instead of Eq. 3.

2.6

a) Assumptions:

1. Each compartment is perfectly mixed.
2. ρ and *C* are constant.
3. No heat losses to ambient.

<u>Compartment 1</u>:
Overall balance (No accumulation of mass):

$$0 = \rho q - \rho q_1 \qquad \text{thus} \qquad q_1 = q \tag{1}$$

Energy balance (No change in volume):

$$V_1\rho C\frac{dT_1}{dt} = \rho qC(T_i - T_1) - UA(T_1 - T_2) \tag{2}$$

<u>Compartment 2</u>:

Overall balance:

$$0 = \rho q_1 - \rho q_2 \qquad \text{thus} \qquad q_2 = q_1 = q \tag{3}$$

Energy balance:

$$V_2\rho C\frac{dT_2}{dt} = \rho qC(T_1 - T_2) + UA(T_1 - T_2) - U_cA_c(T_2 - T_c) \tag{4}$$

b) Eight parameters: ρ, V_1, V_2, C, U, A, U_c, A_c
Five variables: T_i, T_1, T_2, q, T_c
Two equations: (2) and (4)

Thus $N_F = 5 - 2 = 3$

2 outputs = T_1, T_2
3 inputs = T_i, T_c, q (specify as functions of t)

c) Three new variables: c_i, c_1, c_2 (concentration of species A).
Two new equations: Component material balances on each compartment.
c_1 and c_2 are new outputs. c_i must be a known function of time.

2.7

As in Section 2.4.2, there are two equations for this system:

$$\frac{dV}{dt} = \frac{1}{\rho}(w_i - w)$$

$$\frac{dT}{dt} = \frac{w_i}{V\rho}\left(T_i - T\right) + \frac{Q}{\rho VC}$$

Results:

(a) Since w is determined by hydrostatic forces, we can substitute for this variable in terms of the tank volume as in Section 2.4.5 case 3.

$$\frac{dV}{dt} = \frac{1}{\rho}\left(w_i - C_v\sqrt{\frac{V}{A}}\right)$$

$$\frac{dT}{dt} = \frac{w_i}{\rho V}\left(T_i - T\right) + \frac{Q}{\rho VC}$$

This leaves us with the following:

5 variables: V, T, w_i, T_i, Q
4 parameters: C, ρ, C_v, A
2 equations

The degrees of freedom are $5 - 2 = 3$. To make sure the system is specified, we have:

2 output variables: T, V

2 manipulated variables: Q, w_i

1 disturbance variable: T_i

(b) In this part, two controllers have been added to the system. Each controller provides an additional equation. Also, the flow out of the tank is now a manipulated variable being adjusted by the controller. So, we have

4 parameters: C, ρ, T_{sp}, V_{sp}

6 variables: V, T, w_i, T_i, Q, w

4 equations

The degrees of freedom are $6-4=2$. To specify the two degrees of freedom, we set the variables as follows:

2 output variables: T, V

2 manipulated variables (determined by controller equations): Q, w

2 disturbance variables: T_i, w_i

2.8

Additional assumptions:

(i) Density of the liquid, ρ, and density of the coolant, ρ_J, are constant.
(ii) Specific heat of the liquid, C, and of the coolant, C_J, are constant.

Because V is constant, the mass balance for the tank is:

$$\rho \frac{dV}{dt} = q_F - q = 0\text{; thus } q = q_F$$

Energy balance for tank:

$$\rho V C \frac{dT}{dt} = q_F \rho C (T_F - T) - K q_J^{\,0.8} A (T - T_J) \tag{1}$$

Energy balance for the jacket:

$$\rho_J V_J C_J \frac{dT_J}{dt} = q_J \rho_J C_J (T_i - T_J) + K q_J^{\,0.8} A (T - T_J) \tag{2}$$

where A is the heat transfer area (in ft^2) between the process liquid and the coolant.

Eqs.1 and 2 comprise the dynamic model for the system.

2.9

Assume that the feed contains only A and B, and no C. Component balances for A, B, C over the reactor give.

$$V\frac{dc_A}{dt} = q_i c_{Ai} - qc_A - Vk_1 e^{-E_1/RT} c_A \tag{1}$$

$$V\frac{dc_B}{dt} = q_i c_{Bi} - qc_B + V(k_1 e^{-E_1/RT} c_A - k_2 e^{-E_2/RT} c_B) \tag{2}$$

$$V\frac{dc_C}{dt} = -qc_C + Vk_2 e^{-E_2/RT} c_B \tag{3}$$

An overall mass balance over the jacket indicates that $q_c = q_{ci}$ because the volume of coolant in jacket and the density of coolant are constant.

Energy balance for the reactor:

$$\begin{aligned}\frac{d\left[\left(Vc_A M_A S_A + Vc_B M_B S_B + Vc_C M_C S_C\right)T\right]}{dt} &= \left(q_i c_{Ai} M_A S_A + q_i c_{Bi} M_B S_B\right)(T_i - T) \\ &\quad - UA(T - T_c) + (-\Delta H_1)Vk_1 e^{-E_1/RT} c_A + (-\Delta H_2)Vk_2 e^{-E_2/RT} c_B \end{aligned} \tag{4}$$

where M_A, M_B, M_C are molecular weights of A, B, and C, respectively
S_A, S_B, S_C are specific heats of A, B, and C.
U is the overall heat transfer coefficient
A is the surface area of heat transfer

Energy balance for the jacket:

$$\rho_j S_j V_j \frac{dT_c}{dt} = \rho_j S_j q_{ci}(T_{ci} - T_c) + UA(T - T_c) \tag{5}$$

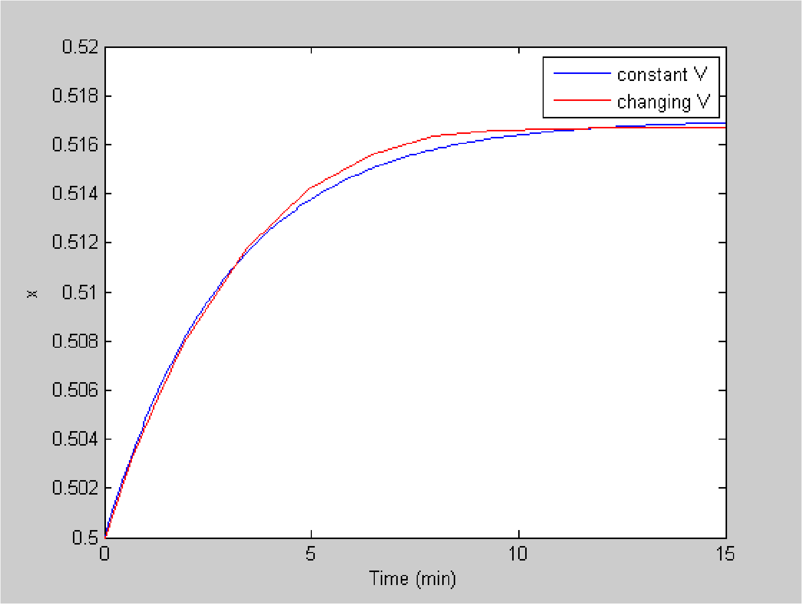
where:

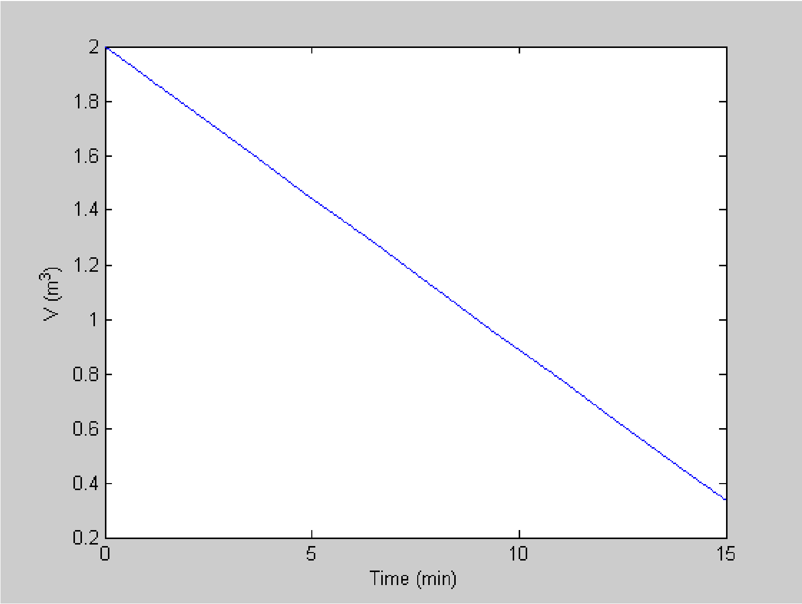
ρ_j, S_j	are density and specific heat of the coolant.
V_j	is the volume of coolant in the jacket.

Eqs. 1 - 5 represent the dynamic model for the system.

2.10

The plots should look as shown below:

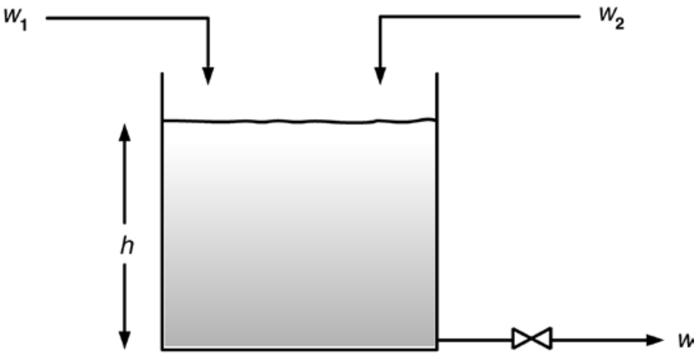




Notice that the functions are only good for *t* = 0 to *t* = 18, at which point the tank is completely drained. The concentration function blows up because the volume function is negative.

2.11

a)



Note that the only conservation equation required to find *h* is an overall mass balance:

$$\frac{dm}{dt} = \frac{d(\rho Ah)}{dt} = \rho A \frac{dh}{dt} = w_1 + w_2 - w \tag{1}$$

Valve equation: $w = C_v' \sqrt{\frac{\rho g}{g_c} h} = C_v \sqrt{h}$ (2)

$$\text{where } C_v = C_v' \sqrt{\frac{\rho g}{g_c}} \tag{3}$$

Substituting the valve equation into the mass balance,

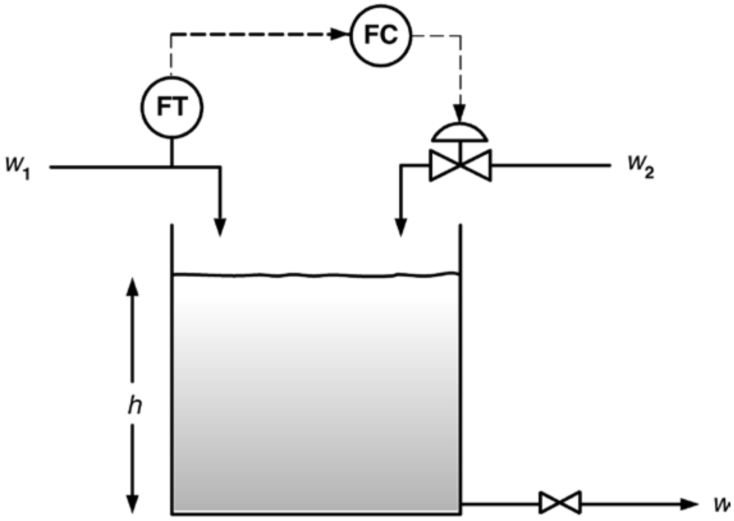
$$\frac{dh}{dt} = \frac{1}{\rho A}(w_1 + w_2 - C_v \sqrt{h}) \tag{4}$$

Steady-state model:

$$0 = \overline{w_1} + \overline{w_2} - C_v \sqrt{\overline{h}} \tag{5}$$

b) $$C_v = \frac{\overline{w_1} + \overline{w_2}}{\sqrt{\overline{h}}} = \frac{2.0 + 1.2}{\sqrt{2.25}} = \frac{3.2}{1.5} = 2.13 \frac{\text{kg/s}}{\text{m}^{1/2}}$$

c) Feedforward control



Rearrange Eq. 5 to get the feedforward (FF) controller relation,

$$w_2 = C_v\sqrt{\overline{h_R}} - w_1 \qquad \text{where } \overline{h_R} = 2.25 \text{ m}$$

$$w_2 = (2.13)(1.5) - w_1 = 3.2 - w_1 \tag{6}$$

Note that Eq. 6, for a value of $w_1 = 2.0$, gives

$w_2 = 3.2 - 1.2 = 2.0$ kg/s which is the desired value.

If the actual FF controller follows the relation, $w_2 = 3.2 - 1.1w_1$ (flow transmitter 10% higher), w_2 will change as soon as the FF controller is turned on,

$w_2 = 3.2 - 1.1\ (2.0) = 3.2 - 2.2 = 1.0$ kg/s

(instead of the correct value, 1.2 kg/s)

Then $C_v\sqrt{\bar{h}} = 2.13\sqrt{\bar{h}} = 2.0 + 1.0$

or $\sqrt{\bar{h}} = \dfrac{3}{2.13} = 1.408$ and $\bar{h} = 1.983$ m (instead of 2.25 m)

Error in desired level = $\dfrac{2.25 - 1.983}{2.25} \times 100\% = 11.9\%$

The sensitivity does not look too bad in the sense that a 10% error in flow measurement gives ~12% error in desired level. Before making this conclusion, however, one should check how well the operating FF controller works for a change in w_1 (e.g., $\Delta w_1 = 0.4$ kg/s).

2.12

a) Model of tank (normal operation):

$$\rho A \frac{dh}{dt} = w_1 + w_2 - w_3 \qquad \text{(Below the leak point)}$$

$$A = \frac{\pi(2)^2}{4} = \pi = 3.14 \text{ m}^2$$

$$(800)(3.14)\frac{dh}{dt} = 120 + 100 - 200 = 20$$

$$\frac{dh}{dt} = \frac{20}{(800)(3.14)} = 0.007962 \ \text{ m/min}$$

Time to reach leak point ($h = 1$ m) = 125.6 min.

b) Model of tank with leak and w_1, w_2, w_3 constant:

$$\rho A \frac{dh}{dt} = 20 - \delta q_4 = 20 - \rho(0.025)\sqrt{h-1} = 20 - 20\sqrt{h-1} \quad , \quad h \geq 1$$

To check for overflow, one can simply find the level h_m at which dh/dt = 0. That is the maximum value of level when no overflow occurs.

$$0 = 20 - 20\sqrt{h_m - 1} \quad \text{or} \quad h_m = 2 \text{ m}$$

Thus, overflow does not occur for a leak occurring because $h_m < 2.25$ m.

2.13

Model of process

Overall material balance:

$$\rho A_T \frac{dh}{dt} = w_1 + w_2 - w_3 = w_1 + w_2 - C_v\sqrt{h} \tag{1}$$

Component:

$$\rho A_T \frac{d(hx_3)}{dt} = w_1x_1 + w_2x_2 - w_3x_3$$

$$\rho A_T h\frac{dx_3}{dt} + \rho A_T x_3 \frac{dh}{dt} = w_1x_1 + w_2x_2 - w_3x_3$$

Substituting for *dh/dt* (Eq. 1)

$$\rho A_T h\frac{dx_3}{dt} + x_3(w_1 + w_2 - w_3) = w_1x_1 + w_2x_2 - w_3x_3$$

$$\rho A_T h\frac{dx_3}{dt} = w_1(x_1 - x_3) + w_2(x_2 - x_3) \tag{2}$$

or $$\frac{dx_3}{dt} = \frac{1}{\rho A_T h}\left[w_1(x_1 - x_3) + w_2(x_2 - x_3)\right] \tag{3}$$

a) At initial steady state ,

$$\overline{w_3} = \overline{w_1} + \overline{w_2} = 120 + 100 = 220 \text{ Kg/min}$$

$$C_v = \frac{220}{\sqrt{1.75}} = 166.3$$

b) If x_1 is suddenly changed from 0.5 to 0.6 without changing flowrates, then level remains constant and Eq.3 can be solved analytically or numerically to find the time to achieve 99% of the x_3 response. From the material balance, the final value of $x_3 = 0.555$. Then,

$$\frac{dx_3}{dt} = \frac{1}{(800)(1.75)\pi}\left[120(0.6 - x_3) + 100(0.5 - x_3)\right]$$

$$= \frac{1}{(800)(1.75)\pi}\left[(72+50)-220x_3)\right]$$

$$= 0.027738 - 0.050020x_3$$

Integrating,

$$\int_{x_{3o}}^{x_{3f}} \frac{dx_3}{0.027738 - 0.050020x_3} = \int_0^t dt$$

where x_{3o}=0.5 and x_{3f} =0.555 – (0.555)(0.01) = 0.549

Solving,

t = 47.42 min

c) If w_1 is changed to 100 kg/min without changing any other input variables, then x_3 will not change and Eq. 1 can be solved to find the time to achieve 99% of the h response. From the material balance, the final value of the tank level is h =1.446 m.

$$800\pi \frac{dh}{dt} = 100 + 100 - C_v\sqrt{h}$$

$$\frac{dh}{dt} = \frac{1}{800\pi}\left[200 - 166.3\sqrt{h}\quad\right]$$

$$= 0.079577 - 0.066169\sqrt{h}$$

where h_o=1.75 and h_f =1.446 + (1.446)(0.01) = 1.460

By using the MATLAB command ode45 ,

t = 122.79 min

Numerical solution of the ode is shown in Fig. S2.13

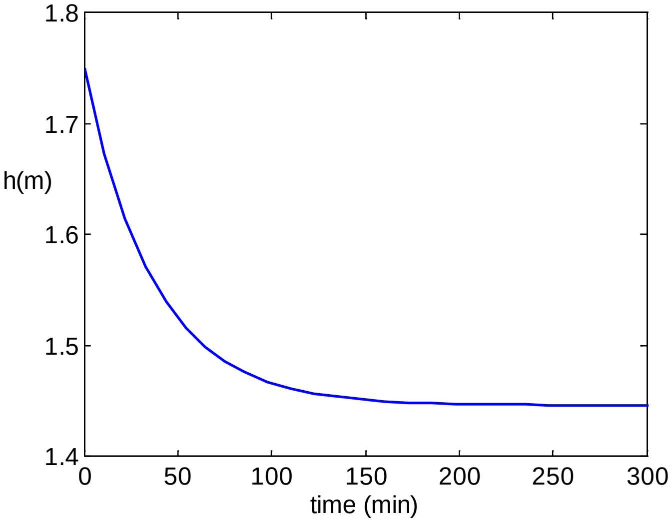


Figure S2.13. *Numerical solution of the ode for part c)*

d) In this case, both h and x_3 will be changing functions of time. Therefore, both Eqs. 1 and 3 will have to be solved simultaneously. Since concentration does not appear in Eq. 1, we would anticipate no effect on the h response.

2.14

a) The dynamic model for the chemostat is given by:

Cells: $$V\frac{dX}{dt} = Vr_g - FX \quad \text{or} \quad \frac{dX}{dt} = r_g - \left(\frac{F}{V}\right)X \qquad (1)$$

Product: $$V\frac{dP}{dt} = Vr_p - FP \quad \text{or} \quad \frac{dP}{dt} = r_p - \left(\frac{F}{V}\right)P \qquad (2)$$

Substrate: $$V\frac{dS}{dt} = F(S_f - S) - \frac{1}{Y_{X/S}}Vr_g$$

or

$$\frac{dS}{dt} = \left(\frac{F}{V}\right)(S_f - S) - \frac{1}{Y_{X/S}}r_g - \frac{1}{Y_{P/S}}r_P \qquad (3)$$

b) At steady state,

$$\frac{dX}{dt} = 0 \qquad \therefore \qquad r_g = DX$$

then,

$$\mu X = DX \qquad \therefore \qquad D = \mu \tag{4}$$

A simple feedback strategy can be implemented where the growth rate is controlled by manipulating the mass flow rate, *F*, so that *F/V* stays constant.

c) Washout occurs if dX/dt is negative for an extended period of time; that is,

$$r_g - DX < 0 \qquad \text{or} \qquad D > \mu$$

Thus, if $D > \mu$ the cells will be washed out.

d) At steady state, the dynamic model given by Eqs. 1, 2 and 3 becomes:

$$0 = r_g - DX \qquad\qquad DX = r_g \tag{5}$$

$$0 = r_p - DP \qquad\qquad DP = r_p \tag{6}$$

$$0 = D(S_f - S) - \frac{1}{Y_{X/S}}\, r_g \tag{7}$$

From Eq. 5,

$$DX = r_g \tag{8}$$

From Eq. 7

$$r_g = Y_{X/S}(S_f - S)D \tag{9}$$

Substituting Eq. 9 into Eq. 8,

$$DX = Y_{X/S}(S_f - S)D \tag{10}$$

From Eq. 4

$$S = \frac{DK_S}{\mu_{\max} - D}$$

Substituting these two equations into Eq. 10,

$$DX = Y_{X/S}\left(S_f - \frac{DK_S}{\mu_{\max} - D}\right)D \qquad (11)$$

For $Y_{x/s} = 0.5$, $S_f = 10$, $K_s = 1$, $X = 2.75$, $\mu_{\max} = 0.2$, the following plot can be generated based on Eq. 11.

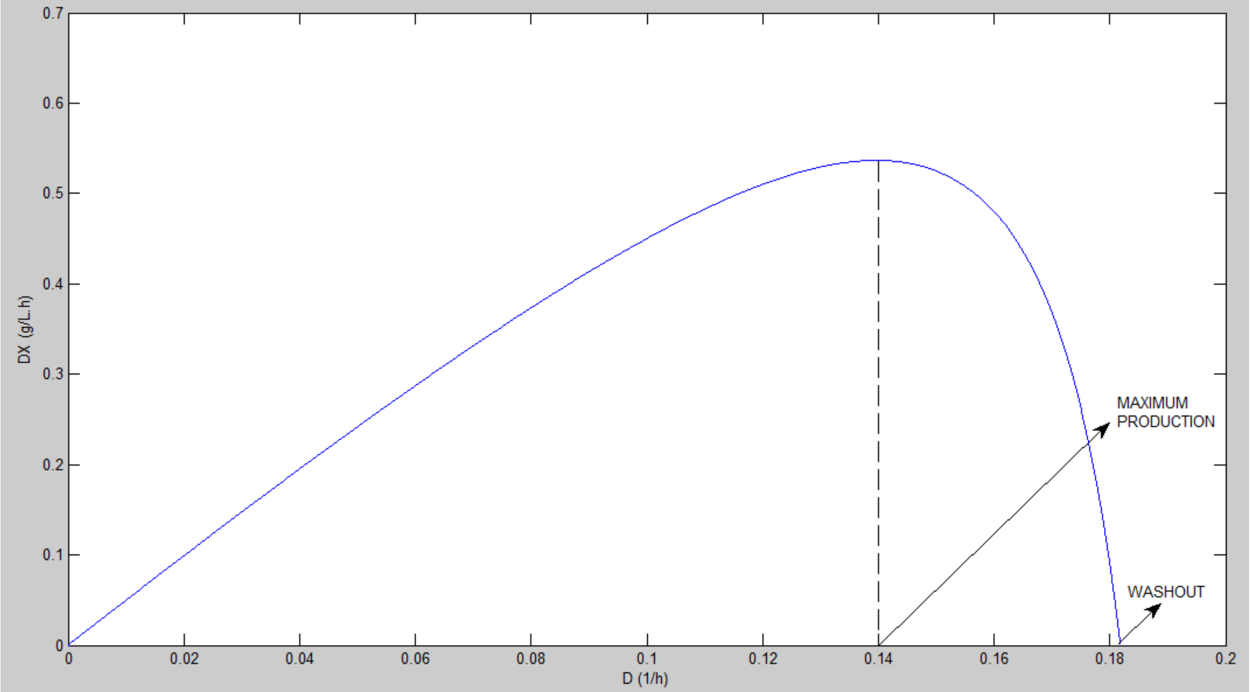


Figure S2.14. Steady-state cell production rate DX as a function of dilution rate D.

From Figure S2.14, washout occurs at $D = 0.18\ \text{h}^{-1}$ while the maximum production occurs at $D = 0.14\ \text{h}^{-1}$. Notice that maximum and washout points are dangerously close to each other, so special care must be taken when increasing cell productivity by increasing the dilution rate.

2.15

a) We can assume that ρ and h are approximately constant. The dynamic model is given by:

$$r_d = -\frac{dM}{dt} = kAc_s \qquad (1)$$

Notice that:

$$M = \rho V \qquad \therefore \qquad \frac{dM}{dt} = \rho\frac{dV}{dt} \qquad (2)$$

$$V = \pi r^2 h \qquad \therefore \qquad \frac{dV}{dt} = (2\pi rh)\frac{dr}{dt} = A\frac{dr}{dt} \qquad (3)$$

Substituting (3) into (2) and then into (1),

$$-\rho A\frac{dr}{dt} = kAc_s \qquad \therefore \qquad -\rho\frac{dr}{dt} = kc_s$$

Integrating,

$$\int_{r_o}^{r} dr = -\frac{kc_s}{\rho}\int_0^t dt \qquad \therefore \qquad r(t) = r_o - \frac{kc_s}{\rho}t \qquad (4)$$

Finally,

$$M = \rho V = \rho\pi h r^2$$

then

$$M(t) = \rho\pi h\left(r_o - \frac{kc_s}{\rho}t\right)^2$$

b) The time required for the pill radius r to be reduced by 90% is given by Eq. 4:

$$0.1r_o = r_o - \frac{kc_s}{\rho}t \qquad \therefore \qquad t = \frac{0.9r_o\rho}{kc_s} = \frac{(0.9)(0.4)(1.2)}{(0.016)(0.5)} = 54\,\text{min}$$

Therefore, $t = 54\,\text{min}$.

2.16

For V = constant and $F = 0$, the simplified dynamic model is:

$$\frac{dX}{dt} = r_g = \mu_{\max} \frac{S}{K_s + S} X$$

$$\frac{dP}{dt} = r_p = Y_{P/X} \mu_{\max} \frac{S}{K_s + S} X$$

$$\frac{dS}{dt} = -\frac{1}{Y_{X/S}} r_g - \frac{1}{Y_{P/X}} r_P$$

Substituting numerical values:

$$\frac{dX}{dt} = 0.2 \frac{SX}{1+S}$$

$$\frac{dP}{dt} = (0.2)(0.2) \frac{SX}{1+S}$$

$$\frac{dS}{dt} = 0.2 \frac{SX}{1+S} \left[-\frac{1}{0.5} - \frac{0.2}{0.1} \right]$$

By using MATLAB, this system of differential equations can be solved. The time to achieve a 90% conversion of S is $t = 22.15$ h.

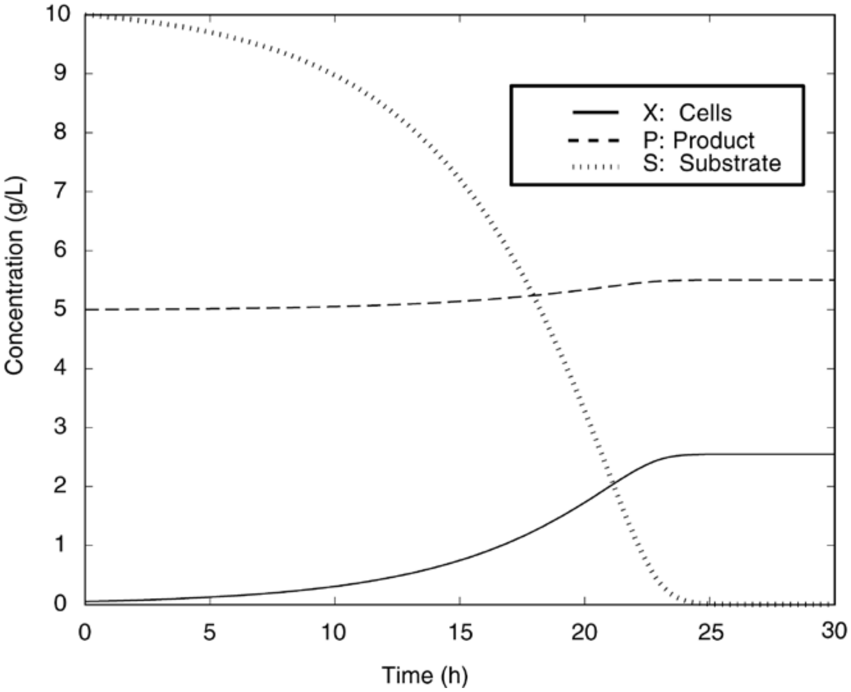


Figure S2.16. Fed-batch bioreactor dynamic behavior.

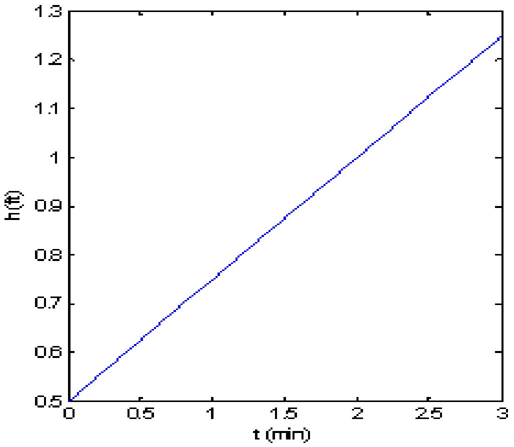
2.17

(a) Using a simple volume balance, for the system when the drain is closed ($q = 0$)

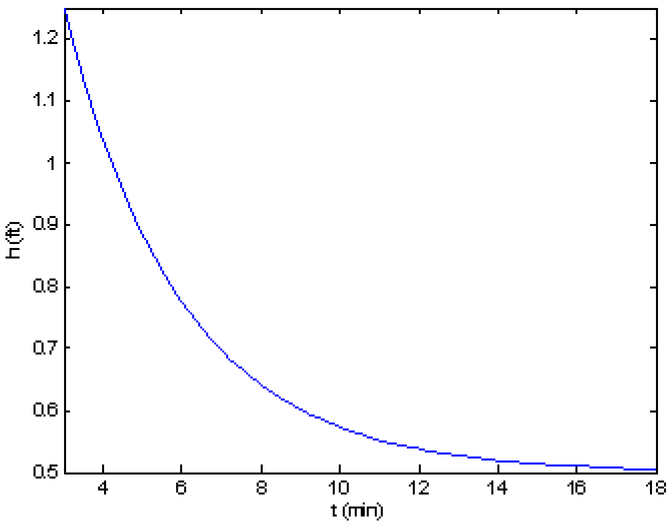
$$A\frac{dh}{dt} = q_1 \tag{1}$$

Solving this ODE with the given initial condition gives a height that is increasing at a rate of 0.25 ft/min.

So the height in this time range will look like:

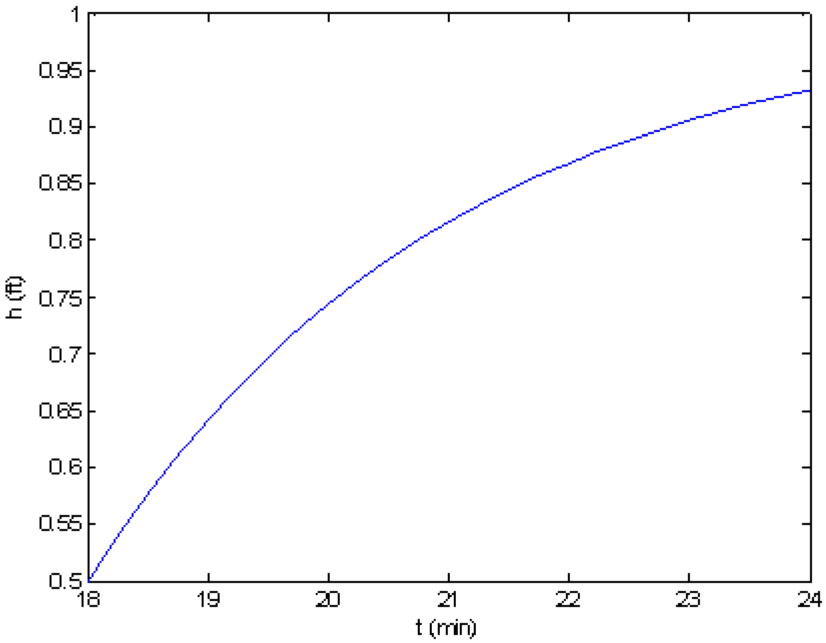


(b) the drain is opened for 15 mins; assume a time constant in a linear transfer function of 3 mins, so a steady state is essentially reached. ($3 \leq t \leq 18$). Assume that the process will return to its previous steady state in an exponential manner, reaching 63.2% of the response in three minutes.



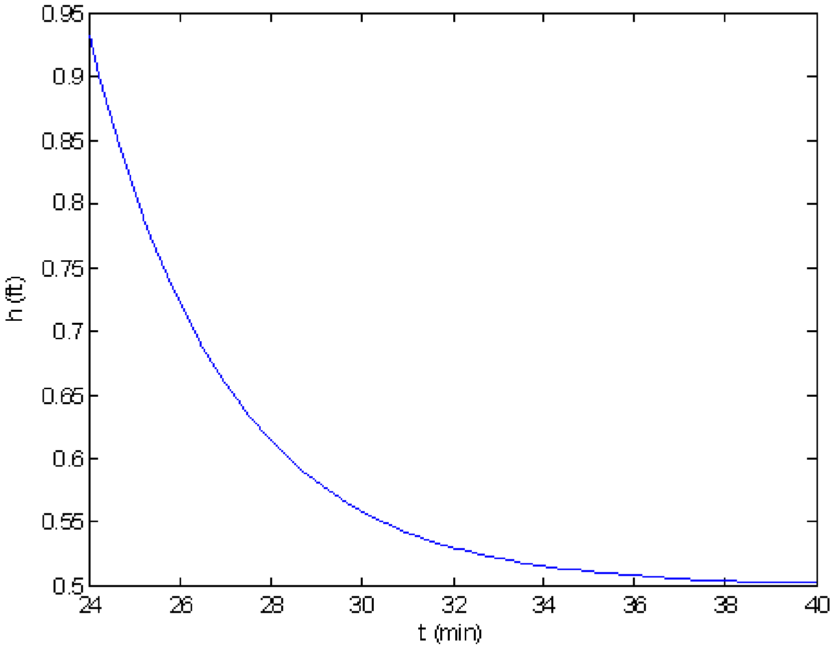
(c) the inflow rate is doubled for 6 minutes ($18 \leq t \leq 24$)

The height should rise exponentially towards a new steady state value double that of the steady state value in part b), but it should be apparent that the height does not reach this new steady state value at t = 24 min.. The new steady state would be 1 ft.

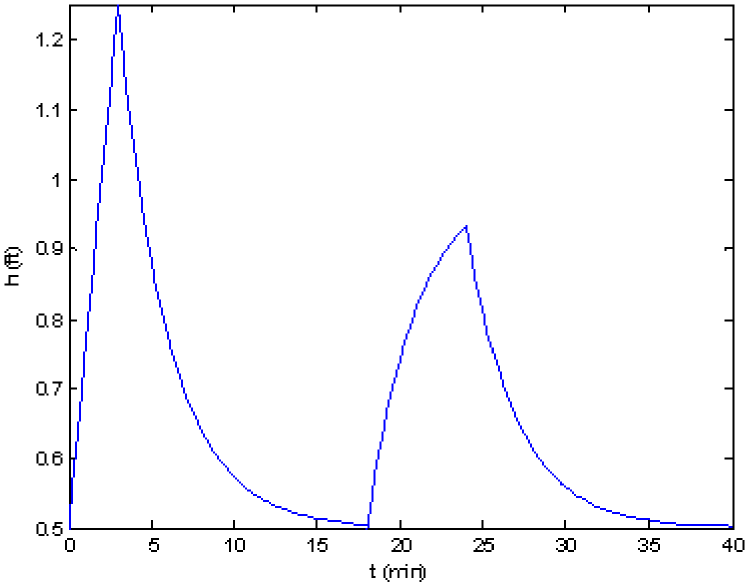


(d) the inflow rate is returned to its original value for 16 minutes ($24 \leq t \leq 40$)

The graph should show an exponential decrease to the previous steady state of 0.5 ft. The initial value should coincide with the final value from part (c).



Putting all the graphs together would look like this:



2.18

Parameters (fixed by design process): m, C, m_e, C_e, h_e, A_e.

CVs: T and T_e.

Input variables (disturbance): w, T_i. Input variables (manipulated): Q.

Degrees of freedom = (11-6) (number of variables) – 2 (number of equations) = 3

The three input variables (w, T_i, Q) are assigned and the resulting system has zero degrees of freedom.

2.19

(a) First we simulate a step change in the vapor flow rate from 0.033 to 0.045 m^3/s.
The resulting plots of xD and xB are shown below.

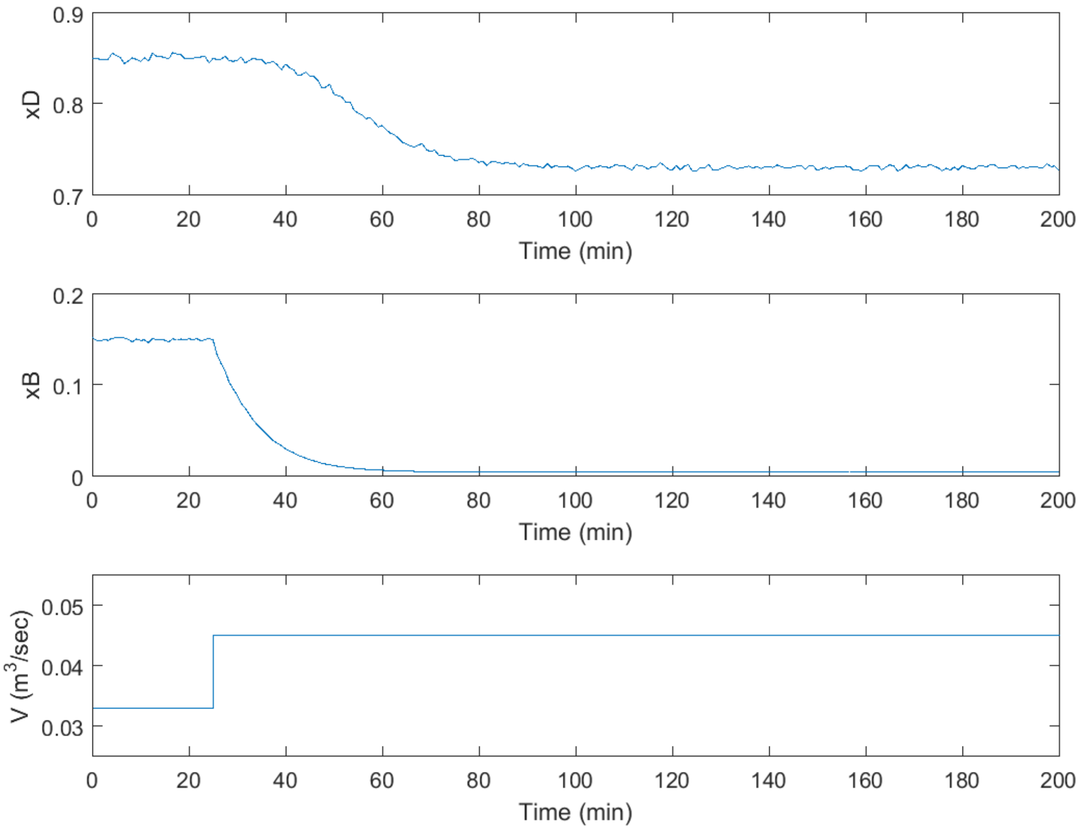


Figure: Plot of xD, xB, and V versus time for a step change in V from 0.033 to 0.045 m^3/s.

By examining the resulting data, we can find the steady-state values of xD and xB before and after the step change in V.

	Start	End	Change
xD	0.85	0.73	-0.12
xB	0.15	0.0050	-0.145

(b) Next we simulate a step change in the feed composition (zF) from 0.5 to 0.55. Note that the vapor flow rate, V, is still set at 0.045 m^3/s.

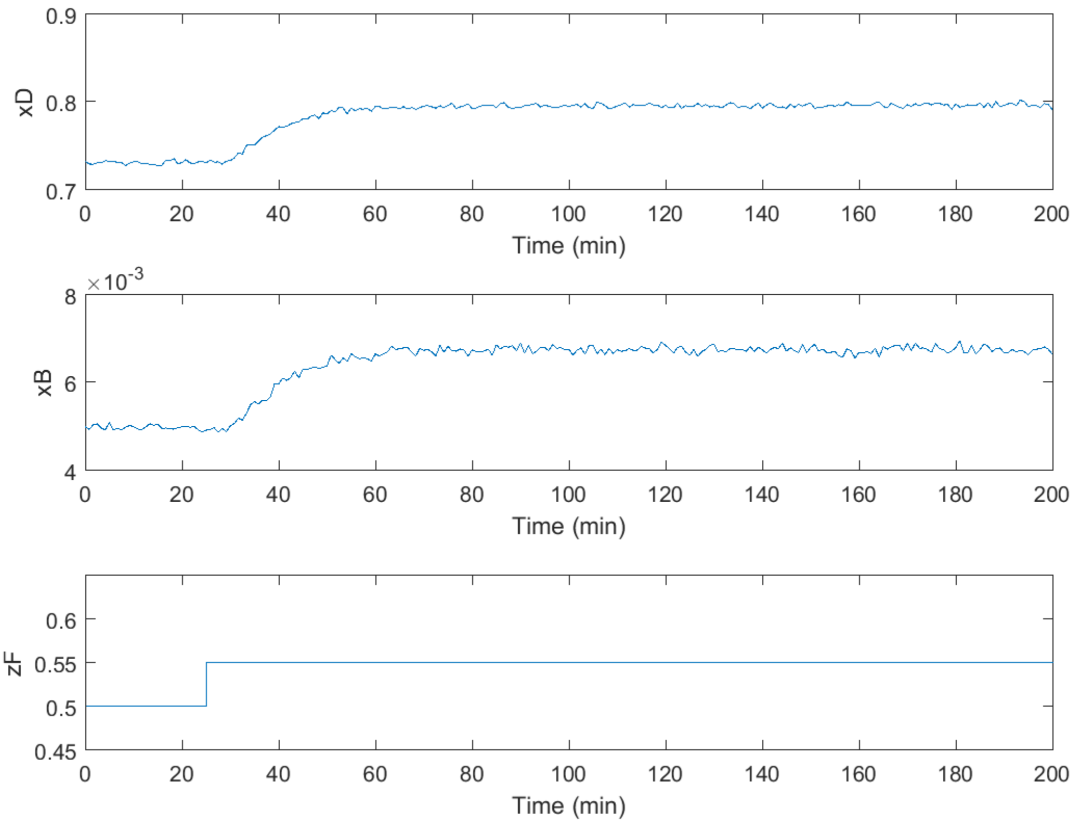


Figure: Plot of xD, xB, and zF versus time for a step change in zF from 0.5 to 0.55

By examining the resulting data, we can find the steady-state values of xD and xB before and after the step change in zF.

	Start	End	Change
xD	0.73	0.80	+0.066
xB	0.0050	0.0068	+0.0018

(c) Increasing V causes xD and xB to decrease, while increasing zF causes both xD and xB to increase. The magnitude of the effect is greater for changing V than for changing zF. When changing V, xB changes more quickly than xD.

2.20

(a) First we simulate a step change in the Fuel Gas Purity (FG_pur) from 1 to 0.95.
The resulting plots of Oxygen Exit Concentration (C_O2) and Hydrocarbon Outlet Temperature (T_HC) are shown below.

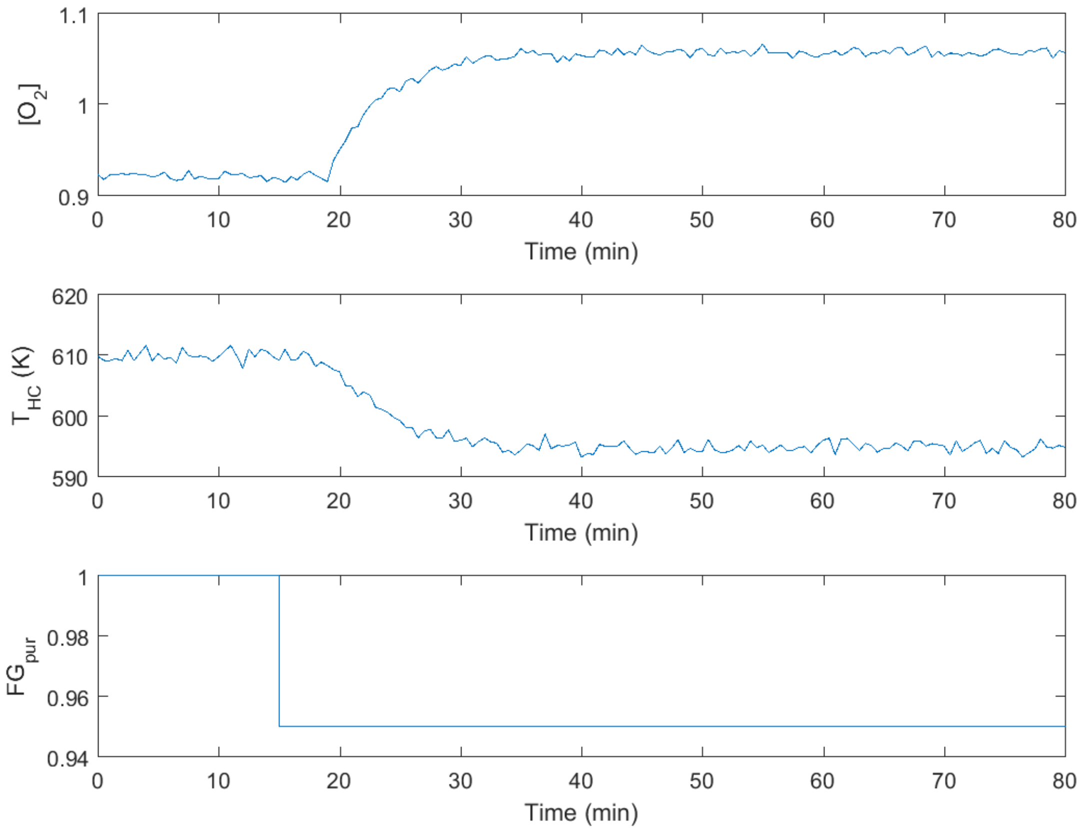


Figure: Plot of C_O2, T_HC, and FG_pur versus time for a step change in FG_pur from 1 to 0.95.

By examining the resulting data, we can find the steady-state values of C_O2 and T_HC before and after the step change in FG_pur.

	Start	End	Change
C_O2	0.92	1.06	0.14
T_HC	609	595	-14

(b) Next we simulate a step change in the Hydrocarbon Flow Rate (F_HC_sp) from 0.035 to 0.0385. Note that the Fuel Gas Purity, FG_pur, is still set at 0.95.

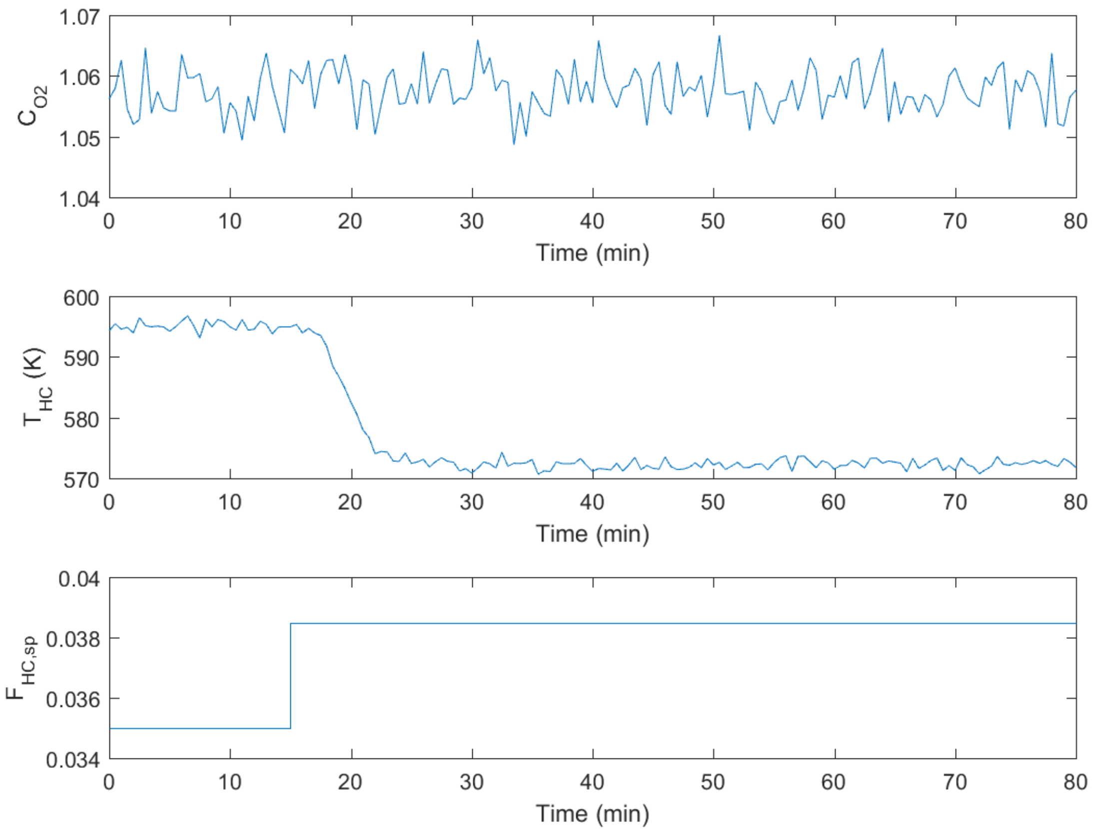


Figure: Plot of C_O2, T_HC, and F_HC_sp versus time for a step change in F_HC_sp from 0.035 to 0.0385.

By examining the resulting data, we can find the steady-state values of C_O2 and T_HC before and after the step change in F_HC_sp.

	Start	End	Change
C_O2	1.06	1.06	0
T_HC	595	572	-23

(c) Decreasing FG_pur causes C_O2 to increase, while T_HC decreases. Increasing F_HC_sp causes T_HC to decrease while C_O2 stays the same. The change in T_HC occurs more quickly when changing F_HC_sp versus changing FG_pur.

2.21

The key to this problem is solving the mass balance of the tank in each part.

Mass balance:

$$\frac{d}{dt}(\rho A h) = \rho q_i - \rho q_o$$

- ρ (density) and A (tank cross-sectional area) are constants, therefore:

$$A\frac{dh}{dt} = q_i - q_o$$

- The problem specifies q_o is linearly related to the tank height

$$q_o = \frac{1}{R}h$$

$$A\frac{dh}{dt} = q_i - \frac{1}{R}h$$

- Next, we can obtain R (valve constant) from the steady state information in the problem

$$\frac{dh}{dt} = 0 \quad \text{at steady state}$$

$$0 = \overline{q}_i - \frac{1}{R}\overline{h}$$

$$0 = 2 - \frac{1}{R}(1)$$

$$\therefore \ \frac{1}{R} = 2 \quad R = 0.5 \quad \frac{\text{ft}^2}{\text{min}}$$

- In addition, we can find that

$$\tau = AR = (4)\left(\frac{1}{2}\right) = 2 \text{ min}$$

Part a

$$A\ \frac{dh}{dt} = q_i - q_o \qquad \text{(Mass Balance)}$$

$$4\ \frac{dh}{dt} = 2 \qquad \text{(Separable ODE)}$$

$$\int dh = \int \frac{1}{2}\ dt$$

$$h(t) = \frac{1}{2}t + C \qquad h(0) = 1$$

$$\boxed{h(t) = \frac{1}{2}t + 1 \qquad 0 \le t < 3}$$

Part b

$$A\frac{dh}{dt} = q_i - \frac{1}{R}h \qquad \text{(Mass Balance)}$$

$$4\frac{dh}{dt} = 2 - 2h$$

$$\frac{dh}{dt} + \frac{1}{2}h = \frac{1}{2} \qquad \text{(Solution by integrating factor = } e^{t/2}\text{)}$$

$$\int d(e^{t/2}h) = \int \frac{1}{2}\, e^{t/2}\ dt$$

$$he^{t/2} = 1e^{t/2} + c \qquad h(3) = 2.5$$

$$h = 1 + ce^{-t/2}$$

$$2.5 = 1 + ce^{-3/2}$$

$$c = 1.5e^{3/2}$$

$$\boxed{h(t) = 1 + (1.5)e^{-(t-3)/2} \qquad 3 \le t < 18}$$

Part c

$$4\frac{dh}{dt} = 4 - 2h \qquad \text{(Mass balance)}$$

$$\frac{dh}{dt} + \frac{1}{2}h = 1 \qquad \text{(Solution by integrating factor)}$$

$$\int d(e^{t/2}h) \ = \ \int 1e^{t/2}dt \qquad h(18) = 1$$

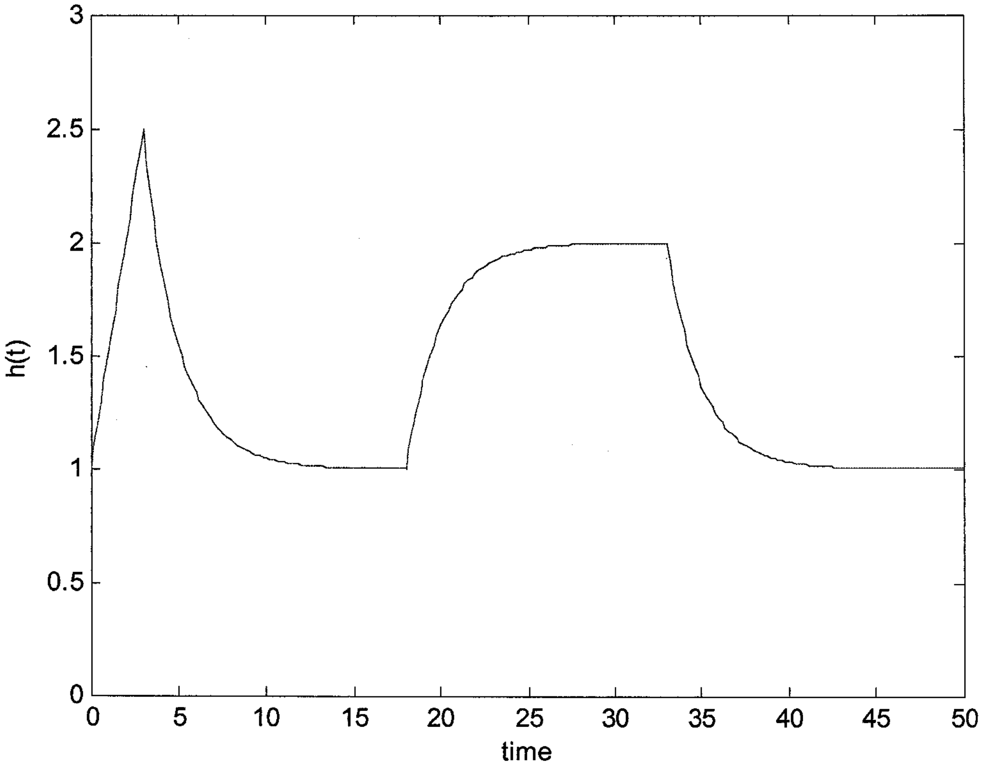
- Method is same as part b.

$$\boxed{h(t) = 2 - e^{-(t-18)/2} \qquad 18 \le t < 33}$$

Part d

Same as part b with *h* (33) = 2

$$\boxed{h(t) = 1 + e^{-(t-33)/2} \qquad 33 \le t \le 50}$$



2.22

To solve the problem, we start by writing the mass balance for each tank 1-4.
To write the mass balance for each tank, we start with the most general form, where the change in mass in the tank over time is equal to the mass flowing into the tank minus the mass flowing out of the tank. The general form of the equations are shown below, where i represents the tank number (1, 2, 3, 4). The mass can be written as the density multiplied by the tank volume, and the mass flow rates can be written as the density multiplied by the volumetric flow rate.

$$\frac{d(\rho V_i)}{dt} = \rho q_{in,i} - \rho q_{out,i}$$

With density assumed constant over time, it can be pulled out of the derivative. Also, we write the volume of the tank as the height of liquid in the tank, h_i, multiplied by the cross-sectional tank area, A_i.

$$\frac{\rho A_i d(h_i)}{dt} = \rho q_{in,i} - \rho q_{out,i}$$

$$\frac{A_i d(h_i)}{dt} = q_{in,i} - q_{out,i}$$

The flow exiting each tank through the bottom can be written as:

$$q_{exit,i} = C_i \sqrt{h_i}$$

Where C_i is the proportionality constant for each tank.

Results:

a) The final equations for the height of liquid in each tank are shown below.

$$\frac{dh_1}{dt}=-\frac{C_1}{A_1}\sqrt{h_1}+\frac{C_3}{A_1}\sqrt{h_3}+\frac{\gamma_1}{A_1}F_1 \qquad (1)$$

$$\frac{dh_2}{dt}=-\frac{C_2}{A_2}\sqrt{h_2}+\frac{C_4}{A_2}\sqrt{h_4}+\frac{\gamma_2}{A_2}F_2 \qquad (2)$$

$$\frac{dh_3}{dt}=-\frac{C_3}{A_3}\sqrt{h_3}+\frac{(1-\gamma_2)}{A_3}F_2 \qquad (3)$$

$$\frac{dh_4}{dt}=-\frac{C_4}{A_4}\sqrt{h_4}+\frac{(1-\gamma_1)}{A_4}F_1 \qquad (4)$$

b) Now we can substitute $\gamma_1=\gamma_2=0.5$

$$\frac{dh_1}{dt}=-\frac{C_1}{A_1}\sqrt{h_1}+\frac{C_3}{A_1}\sqrt{h_3}+\frac{0.5}{A_1}F_1$$

$$\frac{dh_2}{dt}=-\frac{C_2}{A_2}\sqrt{h_2}+\frac{C_4}{A_2}\sqrt{h_4}+\frac{0.5}{A_2}F_2$$

$$\frac{dh_3}{dt}=-\frac{C_3}{A_3}\sqrt{h_3}+\frac{0.5}{A_3}F_2$$

$$\frac{dh_4}{dt}=-\frac{C_4}{A_4}\sqrt{h_4}+\frac{0.5}{A_4}F_1$$

The differential equations for the tank heights are coupled, so the heights cannot be solved for or controlled independently. F_1 and F_2 can be used to control h_3 and h_4 independently, but h_1 and h_2 will be affected in an uncontrolled manner.

c) In the extreme case where $\gamma_1=\gamma_2=0$, we get:

$$\frac{dh_1}{dt}=-\frac{C_1}{A_1}\sqrt{h_1}+\frac{C_3}{A_1}\sqrt{h_3}$$

$$\frac{dh_2}{dt}=-\frac{C_2}{A_2}\sqrt{h_2}+\frac{C_4}{A_2}\sqrt{h_4}$$

$$\frac{dh_3}{dt}=-\frac{C_3}{A_3}\sqrt{h_3}+\frac{F_2}{A_3}$$

$$\frac{dh_4}{dt}=-\frac{C_4}{A_4}\sqrt{h_4}+\frac{F_1}{A_4}$$

These equations make sense with the process diagram because now F_1 and F_2 only affect tanks h_3 and h_4 directly (they no longer flow into tanks 1 and 2 at all). However, F_1 and F_2 indirectly affect tanks 1 and 2 through h_3 and h_4.