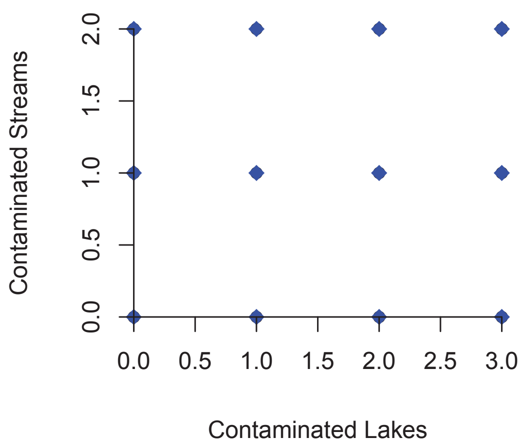
Chapter 3

PROBABILITY

3.1 (a) A sketch of the 12 points of the sample space is as follows:



(b) R={(0,0),(1,1),(2,2)}. T={(0,0),(1,0),(2,0),(3,0)}. U={(0,1),(0,2),(1,2)}.

3.2 R and T are not mutually exclusive. R and U are mutually exclusive. T and U are mutually exclusive.

3.3 (a) $R \cup U$={(0,0),(1,1),(2,2),(0,1),(0,2),(1,2)}. $R \cup U$ is the event that the number of contaminated streams is greater than or equal to the number of contaminated lakes.

(b) $R \cap T$={(0,0)}. $R \cap T$ is the event that none of the streams or lakes is contaminated.

(c) $\overline{T}$={(0,1),(1,1),(2,1),(3,1),(0,2),(1,2),(2,2),(3,2)}. $\overline{T}$ is the event that at least one stream is contaminated.

3.4 (a) F is the event that exactly one plant will be located in Texas.

(b) G is the event that there will be exactly 2 new plants in Texas and California.

(c) $F \cap G$={(1,1)}. $F \cap G$ is the event that exactly one plant will be in California and exactly one plant will be in Texas.

3.5 A={3,4}, B={2,3}, C={4,5}.

(a) $A \cup B$={2,3,4}. Work is easy, average or difficult on this model.

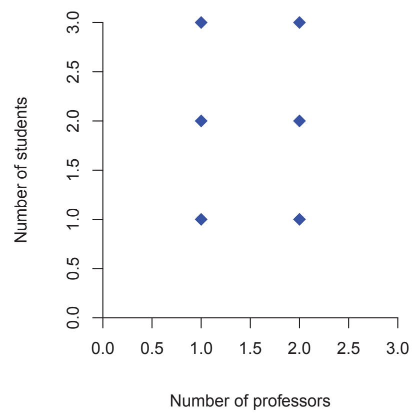
(b) $A \cap B$={3}. Work is average on this model.

(c) $\overline{B}$={1,4,5}. Thus $A \cup \overline{B}$={1,3,4,5}. Work is not easy on this model.

(d) $\overline{C}$={1,2,3}. Work is very easy, easy or average on this model.

3.6 A and B are not mutually exclusive. A and C are not mutually exclusive. B and C are mutually exclusive.

3.7 (a) A sketch of the 6 points of the sample space is



(b) B is the event that 3 graduate students are present. C is the event that the same number of professors and graduate students are present. D is the event that the total number of graduate students and professors is 3.

(c) $C \cup D$={(1,1),(1,2),(2,1),(2,2)}. $C \cup D$ is the event that at most 2 graduate students are present.

(d) B and D are mutually exclusive.

3.8 (a) Countably infinite. The counter, presumably, could register anywhere from 0 counts to an arbitrarily large number.

(b) Finite. A finite subset of a finite set is being chosen. This is finite.

(c) Continuous. The sample space is an interval.

(d) Finite. Possible outcomes range from 0 to 450.

(e) Continuous. The sample space is an interval.

(f) Countably infinite. If it is a tough watch it could, presumably, be dropped an arbitrarily large number of times without stopping.

3.9 Region 1 is the event that the ore contains both uranium and copper. Region 2 is the event that the ore contains copper but not uranium. Region 3 is the event that the ore contains uranium but not copper. Region 4 is the event that the ore contains neither uranium nor copper.

3.10 (a) Regions 1 and 3 together represent the event that the ore contains uranium.

(b) Regions 3 and 4 together represent the event that the ore does not contain copper.

(c) Regions 1, 2 and 3 together represent the event that the ore contains either copper or uranium or both.

3.11 (a) Region 5 represents the event that the windings are improper, but the shaft size is not too large and the electrical connections are satisfactory.

(b) Regions 4 and 6 together represent the event that the electrical connections are unsatisfactory, but the windings are proper.

(c) Regions 7 and 8 together represent the event that the windings are proper and the electrical connections are satisfactory.

(d) Regions 1, 2, 3, and 5 together represent the event that the windings are improper.

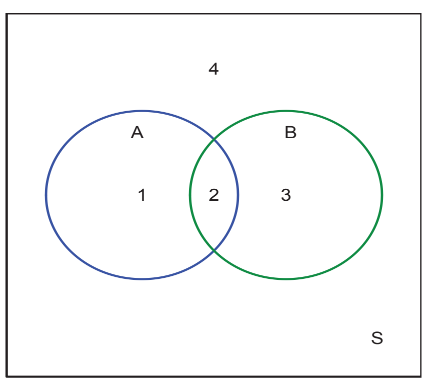
3.12 (a) Region 8.

(b) Regions 1 and 2 together.

(c) Regions 2, 5, and 7 together.

(d) Regions 1, 2, 3, 4, and 6 together.

3.13 The following Venn diagram will be used in parts (a), (b), (c) and (d).

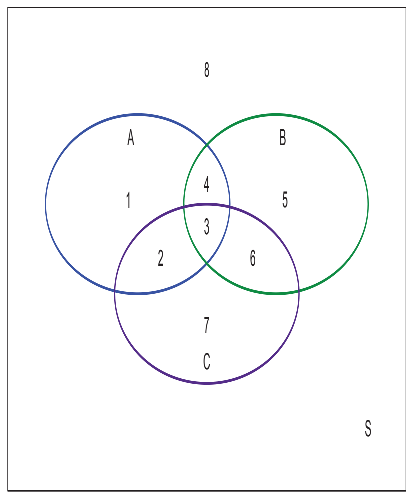


(a) $A \cap B$ is region 2 in the figure. $\overline{(A \cap B)}$ is the region composed of areas 1, 3, and 4. $\overline{A}$ is the region composed of areas 3 and 4. $\overline{B}$ is the region composed of areas 1 and 4. $\overline{A} \cup \overline{B}$ is the region composed of areas 1, 3, and 4. This corresponds to $\overline{(A \cap B)}$.

(b) $A \cap B$ is the region 2 in the figure. A is the region composed of areas 1 and 2. Since $A \cap B$ is entirely contained in A, $A \cup (A \cap B) = A$.

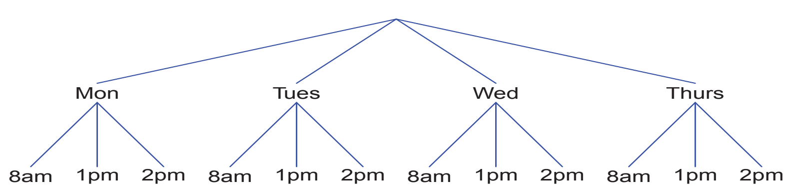
(c) $A \cap B$ is region 2. $A \cap \overline{B}$ is region 1. Thus, $(A \cap B) \cup (A \cap \overline{B})$ is the region composed of areas 1 and 2 which is A.

(d) From part (c), we have $(A \cap B) \cup (A \cap \overline{B}) = A$. Thus, we must show that $(A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B) = A \cup (\overline{A} \cap B) = A \cup B$. A is the region composed of areas 1 and 2 and $\overline{A} \cap B$ is region 3. Thus, $A \cup (\overline{A} \cap B)$ is the region composed of areas 1, 2, and 3.

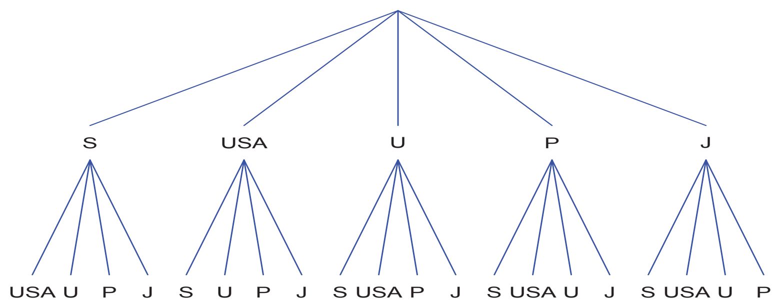


(e) In the figure above, $A \cup B$ is the region composed of areas 1, 2, 3, 4, 5, and 6. $A \cup C$ is the region composed of areas 1, 2, 3, 4, 6, and 7, so $(A \cup B) \cap (A \cup C)$ is the region composed of areas 1, 2, 3, 4, and 6. $B \cap C$ is the region composed of areas 3, and 6, and A is the region composed of areas 1, 2, 3, and 4. Thus, $A \cup (B \cap C)$ is the region composed of areas 1, 2, 3, 4, and 6. Thus $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

3.14 The tree diagram is



3.15 Using the labels S = Spain, U = Uruguay, P = Portugal and J = Japan, the tree diagram is



3.16 There are 30 choices for the first number, and since a number cannot be immediately repeated, there are $30 \times 29 \times 29 = 25,230$ ways.

3.17 (a) $3 + 4 = 7$ opportunities. (b) $3 \times 4 = 12$ opportunities.

3.18 There are $10 \times 10 \times 10 \times 10 = 10,000$ ways.

3.19 (a) The men and women can be chosen in

$$_6C_2 = \binom{6}{2} = \frac{6!}{4!\,2!} = 15 \qquad \text{and} \qquad _4C_2 = \binom{4}{2} = \frac{4!}{2!\,2!} = 6$$

ways so there are $15 \times 6 = 90$ different project teams consisting of 2 men and 2 women.

(b) We are restricted from the choice of having the two women in question both selected giving a total of $6 - 1 = 5$ choices for two women. The number of project teams is reduced to $15 \times 5 = 75$.

3.20 $_9P_3 = 9 \cdot 8 \cdot 7 = 504$.

3.21 $6! = 720$.

3.22 (a) There are $5!/2! = 60$ permutations.

(b) There are 6 commercials, 3 of which are alike. Thus, there are $6!/3! = 120$ ways to fill the time slots.

3.23 Since order does not matter, there are

$$_{15}C_2 = \binom{15}{2} = \frac{15!}{13!\,2!} = 105$$

ways.

3.24 There are

$$ {}_{12}C_4 = \binom{12}{4} = \frac{12!}{4!\,8!} = 495 \qquad \text{ways} $$

to select the 4 candidates to interview.

3.25 There are ${}_{12}C_3 = 220$ ways to draw the three rechargeable batteries.

There are ${}_{11}C_3 = 165$ ways to draw none are defective.

(a) The number of ways to get the one that is defective is $220 - 165 = 55$.

(b) There are 165 ways not to get the one that is defective.

3.26 (a) There are ${}_{10}C_3 = 120$ ways to get no defective batteries.

(b) There are $2 \cdot {}_{10}C_2 = 90$ ways to get 1 defective battery.

(c) There are ${}_{10}C_1 = 10$ ways to get both defective batteries.

3.27 There are ${}_8C_2$ ways to choose the electric motors and ${}_5C_2$ ways to choose the switches. Thus, there are

$$ {}_8C_2 \cdot {}_5C_2 = 28 \cdot 10 = 280 $$

ways to choose the motors and switches for the experiment.

3.28 (a) Using the long run relative frequency approximation to probability, we estimate the probability of a required warranty repair by

$$ P[\text{ Warranty repair required }] = \frac{72}{880} = 0.0818 $$

The number of trials 880 is quite large so the relative frequency should be close to the probability.

(b) Using the data from last year, the long run relative frequency approximation to probability gives the estimate

$$ P[\text{Receive season ticket }] = \frac{6000}{8400} = 0.714 $$

One factor is the expected quality of the team next year. If the team is expected to be much better next year more students will apply for tickets.

3.29 The outcome space is given in Figure 3.1.

(a) The 6 outcomes summing to 7 are marked by squares. Thus, the probability is $6/36 = 1/6$.

(b) There are 2 outcomes summing to 11, which are marked by diamonds. Thus, the probability is $2/36 = 1/18$.

(c) These events are mutually exclusive. Thus, the probability is $6/36 + 2/36 = 2/9$.

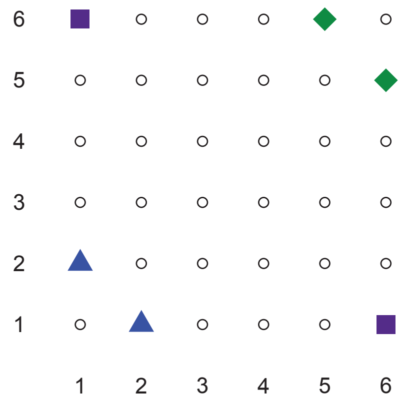


Figure 3.1: The outcome space for Exercise 3.29.

(d) The 2 outcomes are marked by triangles. Thus, the probability is $2/36 = 1/18$.

(e) There are two such outcomes, (1,1) and (6,6). Thus, the probability is $2/36 = 1/18$.

(f) There are four such outcomes, (1,1), (1,2), (2,1) and (6,6). Thus, the probability is $4/36 = 1/9$.

3.30 There are 250 numbers divisible by 200. Thus, the probability is $250/50{,}000 = 1/200$.

3.31 There are $19 + 12 = 31$ cars. Thus, there are ${}_{31}C_4$ ways to choose the cars for inspection. There are ${}_{19}C_2$ ways to get the compacts and ${}_{12}C_2$ ways to get the intermediates. Thus, the probability is:

$$\frac{\binom{19}{2}\binom{12}{2}}{\binom{31}{4}} = \frac{11,286}{31,465} = .359.$$

3.32 Using the long run relative frequency, we the estimate of the probability is $12/365 = .033$.

3.33 The number of students enrolled in the statistics course or the operations research course is $92 + 63 - 40 = 115$. Thus, $160 - 115 = 45$ are not enrolled in either course.

3.34 (a) $N(A) = 2 + 8 + 54 + 20 = 84$.

(b) $N(B) = 20 + 54 + 9 + 16 = 99$.

(c) $N(C) = 8 + 54 + 9 + 14 = 85$.

(d) $N(A \cap B) = 20 + 54 = 74$.

(e) $N(A \cap C) = 8 + 54 = 62$.

(f) $N(A \cap B \cap C) = 54$.

(g) $N(A \cup B) = 2 + 8 + 20 + 54 + 9 + 16 = 109$.

(h) $N(B \cup C) = 16 + 20 + 54 + 9 + 8 + 14 = 121$.

(i) $N(\overline{A} \cup \overline{B} \cup C) = 27 + 2 + 16 + 8 + 54 + 9 + 14 = 130$. Another way to do this is $N(\overline{A} \cup \overline{B} \cup C) = 150 - 20 = 130$.

(j) $N(B \cap (A \cup C)) = 20 + 54 + 9 = 83$.

3.35 (a) Yes. $P(A) + P(B) + P(C) + P(D) = 1$.

(b) No. $P(A) + P(B) + P(C) + P(D) = 1.01 > 1$.

(c) No. $P(C) = -.06 < 0$.

(d) No. $P(A) + P(B) + P(C) + P(D) = 15/16 < 1$.

(e) Yes. $P(A) + P(B) + P(C) + P(D) = 1$.

3.36 (a) The probabilities are permissible: they are all between 0 and 1, and they sum to 1.

(b) $P(R) = .060 + .014 + .080 = .154$.
$P(T) = .060 + .067 + .260 + .166 = .553$.
$P(U) = .012 + .006 + .092 + .166 = .110$.

(c) P(0 Contaminated Streams)$=.060 + .067 + .260 + .166 = P(T)$.
P(1 Contaminated Stream)$=.012 + .014 + .027 + .110 = .163$.
P(2 Contaminated Streams)$=.006 + .092 + .080 + .106 = .284$.

3.37 (a) There is 1 point where $i + j = 2$. There are 2 points where $i + j = 3$. There are 2 points where $i + j = 4$. There is 1 point where $i + j = 5$. Thus,

$$\begin{aligned} &\frac{15/28}{2} + 2 \cdot \frac{15/28}{3} + 2 \cdot \frac{15/28}{4} + \frac{15/28}{5} \\ &= \frac{15}{28} \cdot (1/2 + 2/3 + 1/2 + 1/5) = \frac{15}{28} \cdot \frac{28}{15} = 1. \end{aligned}$$

Since each probability is between 0 and 1, the assignment is permissible.

(b) $P(B) = 15/28 \cdot (1/4 + 1/5) = 5/28 \cdot 9/20 = 135/560 = 27/112$.
$P(C) = 15/28 \cdot (1/2 + 1/4) = 15/28 \cdot 3/4 = 45/112$.
$P(D) = 15/28 \cdot (1/3 + 1/3) = 15/28 \cdot 2/3 = 5/14$.

(c) The probability that 1 graduate student will be supervising the lab is:

$$\frac{15/28}{2} + \frac{15/28}{3} = \frac{25}{56} = .446.$$

The probability that 2 graduate students will be supervising the lab is:

$$\frac{15/28}{3} + \frac{15/28}{4} = \frac{35}{112} = \frac{5}{16} = .3125.$$

The probability that 3 graduate students will be supervising the lab is:

$$\frac{15/28}{4} + \frac{15/28}{5} = \frac{27}{112} = .241.$$

3.38 (a) .38 and .53 do not sum to 1.

(b) Probability can not be negative.

(c) The probability that the compressor or the fan motor is all right would be $.82 + .64 - .41 = 1.05$ which is greater than 1.

3.39 (a) $(A \cap B) \cup (A \cap \overline{B}) = A$, and $A \cap B$ and $A \cap \overline{B}$ are disjoint. Thus

$$P((A \cap B) \cup (A \cap \overline{B})) = P(A \cap B) + P(A \cap \overline{B}) = P(A).$$

Since $P(A \cap \overline{B}) \geq 0$, we have proved that $P(A \cup B) \leq P(A)$.

(b) Combining (d) and (c) of Exercise 3.13 gives us

$$A \cup B = (A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B) = A \cup (\overline{A} \cap B).$$

But A and $\overline{A} \cap B$ are disjoint. Thus,

$$P(A \cup B) = P(A) + P(\overline{A} \cap B).$$

Since $P(\overline{A} \cap B) \geq 0$, we have proved that $P(A \cup B) \geq P(A)$.

3.40 (a) Let A be the event that the student gets an A , and B be the event that the student gets a B. Then, $P(A \cup B) \geq P(A)$. But $.27 < .32$.

(b) The probability that both are completed on time is the intersection of the events that the larger is completed on time and the smaller is completed on time. Thus, the probability that both are completed on time must be no greater than the probability that the larger is completed on time. But $.42 > .35$.

3.41 (a) $P(\overline{A}) = 1 - P(A) = 1 - .45 = .55$.

(b) $P(A \cup B) = P(A) + P(B) = .45 + .30 = .75$, since A and B are mutually exclusive.

(c) $P(A \cap \overline{B}) = P(A) = .45$, since A and B are mutually exclusive.

(d) $P(\overline{A} \cap \overline{B}) = P(\overline{(A \cup B)}) = 1 - P(A \cup B) = 1 - .45 - .30 = .25$.

3.42 (a) $P(A) = N(A)/150 = 84/150 = .56$.

(b) $P(B) = N(B)/150 = 99/150 = .66$.

(c) $P(\overline{A} \cap \overline{C}) = P(\overline{(A \cup C)}) = (16 + 27)/150 = .287$.

(d) $P(B \cap C) = (54 + 9)/150 = .42$.

3.43 (a) This probability is given by $.22 + .21 = .43$.

(b) $.17 + .29 + .21 = .67$.

(c) $.03 + .21 = .24$.

(d) $.22 + .29 + .08 = .59$.

3.44 (a) $P(\text{at most 4 complaints}) = .01 + .03 + .07 + .15 + .19 = .45$.

(b) $P(\text{at least 6 complaints}) = .14 + .12 + .09 + .02 = .37$.

(c) $P(\text{from 5 to 8 complaints}) = .18 + .14 + .12 + .09 = .53$.

3.45 (a) 15/32 (b) 13/32 (c) 5/32 (d) 23/32 (e) 8/32 (f) 9/32.

3.46 (a) The probability that a chip will have either defective etching or a crack defect is $.06+.03-.02 = .07$.

(b) The probability is $1 - .07 = .93$.

3.47 (a) "At least one award" is the same as "design or efficiency award". Thus, the probability is $.16 + .24 - .11 = .29$.

(b) This the probability of "at least one award" minus the probability of both awards or $.29 - .11 = .18$.

3.48 (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .30 + .62 - .12 = .80$.

(b) $P(\overline{A} \cap B) = P(B) - P(A \cap B) = .62 - .12 = .50$.

(c) $P(A \cap \overline{B}) = P(A) - P(A \cap B) = .30 - .12 = .18$.

(d) $P(\overline{A} \cup \overline{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - .12 = .88$.

3.49

$$\begin{array}{ll}
P(A \cup B \cup C) = 1 - .11 = .89, & P(A) = .24 + .06 + .04 + .16 = .5, \\
P(B) = .19 + .06 + .04 + .11 = .4, & P(C) = .09 + .16 + .04 + .11 = .4, \\
P(A \cap B) = .06 + .04 = .1, & P(A \cap C) = .16 + .04 = .2, \\
P(B \cap C) = .04 + .11 = .15, & P(A \cap B \cap C) = .04.
\end{array}$$

The formula is verified since

$$.5 + .4 + .4 - .1 - .2 - .15 + .04 = .89.$$

3.50 These probabilities are shown in Figure 3.2.
The probability of at least one of these errors is:

$$.0006 + .0001 + .0001 + .0002 + .0002 + .0005 + .0007 = .0024.$$

We can also use the formula given in Exercise 3.49 to calculate the probability:

$$P(P \cup F \cup R) = .001 + .0009 + .0012 - .0002 - .0003 - .0003 + .0001 = .0024.$$

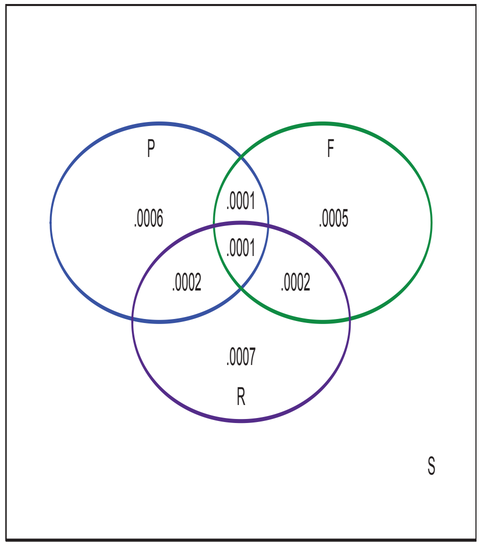


Figure 3.2: Diagram for Exercise 3.50: P = Processing, F = Filing and R = Retrieving.

3.51 (a) The odds for are $(4/7)/(3/7) = 4$ to 3.

(b) The odds against are $.95/.05 = 19$ to 1 against.

(c) The odds for are $.80/.20 = 4$ to 1.

3.52 If the probability of A is p and the odds for A are a to b, then $p/(1-p) = a/b$. Solving for p gives $p = a/(a+b)$.

3.53 (a) $p = 3/(3+2) = 3/5$.

(b) $30/(30+10) = 3/4 \leq p < 40/(10+40) = 4/5$.

3.54 (a) The supplier feels that:

$$P(\text{arrives late}) = 5/(5+7) = 5/12,$$
$$P(\text{doesn't arrive}) = 1/(1+11) = 1/12,$$
$$P(\text{arrives late or doesn't arrive at all}) = 50/(50+50) = 1/2.$$

These subjective probabilities are consistent, because

$$P(\text{arrives late}) + P(\text{doesn't arrive})$$
$$= P(\text{arrives late or doesn't arrive at all}).$$

(b) $P(\text{first wins}) = 1/(2+1) = 1/3$, $P(\text{second wins}) = 1/(3+1) = 1/4$,
$P(\text{either wins}) < 50/(50+50) = 1/2$.
It must be true that

$$P(A \text{ wins}) + P(B \text{ wins}) = P(\text{either wins}).$$

But $1/3 + 1/4 = 7/12 > 1/2$ which is a contradiction.

3.55 $P(I \cap D) = 10/500$, $P(D) = 15/500$, $P(I \cap \overline{D}) = 20/500$, $P(\overline{D}) = 485/500$.

$$P(I|D) = \frac{P(I \cap D)}{P(D)} = \frac{10}{15} = \frac{2}{3},$$

$$P(I|\overline{D}) = \frac{P(I \cap \overline{D})}{P(\overline{D})} = \frac{20}{485} = \frac{4}{97}.$$

3.56 (a) The probability of major repairs would increase. High mileage causes many major parts to wear out.

(b) The probability of knowing the second law of thermo-
dynamics would increase. The typical college senior would not know the law but most mechanical engineers would learn the laws of thermodynamics.

(c) Let A = [major repairs required] and B = [high mileage]. The question concerns the relation of the conditional probability $P(\,A\,|\,B\,)$ to the unconditional probability $P(\,A\,)$.

3.57 (a) The sample space is C. Thus the probability is given by:

$$\frac{N(A \cap C)}{N(C)} = \frac{8+15}{8+54+9+14} = \frac{62}{85} = .73.$$

(b) This is given by:

$$\frac{N(A \cap B)}{N(A)} = \frac{20+54}{20+54+8+2} = \frac{74}{84} = .881.$$

(c) This is given by:

$$\frac{N(\overline{C} \cap \overline{B})}{N(\overline{B})} = \frac{N(\overline{(C \cup B)})}{N(\overline{B})} = \frac{150-121}{105-99} = \frac{29}{51} = .569.$$

3.58 (a)

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.06+.04}{.06+.04+.19+.11} = .25.$$

(b)

$$P(B|\overline{C}) = \frac{P(B \cap \overline{C})}{P(\overline{C})} = \frac{.19+.06}{1-.4} = .417.$$

(c)

$$P(A \cap B|C) = \frac{P(A \cap B \cap C)}{P(C)} = \frac{.04}{.4} = .1.$$

(d)

$$P(B \cup C|\overline{A}) = \frac{P((B \cup C) \cap \overline{A})}{P(\overline{A})} = \frac{.09+.11.+.19}{1-.5} = .78.$$

(e)

$$P(A|B \cup C) = \frac{P(A \cap (B \cup C))}{P(B \cup C)} = \frac{.06+.04+.16}{.06+.04+.16+.19+.11+.09} = \frac{.26}{.65} = .4$$

(f)

$$P(A|B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{.04}{.04 + .11} = .267.$$

(g)

$$P(A \cap B \cap C|B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = .267.$$

(h)

$$P(A \cap B \cap C|B \cup C) = \frac{P(A \cap B \cap C)}{P(B \cup C)} = \frac{.04}{.65} = .062.$$

3.59 From the used car example, we know that $P(M_1) = 0.20$, $P(P_1) = 0.75$, $P(M_1 \cap P_1) = 0.16$, and $P(C_3) = 0.40$

(a)

$$P(M_1|P_1) = \frac{0.16}{0.75} = .213 \quad \text{which is different from} \quad P(M_1)$$

(b) We first obtain $P(C_3 \cap P_2) = 0.01 + 0.06 + 0.02 = 0.09$ and

$$P(P_2) = 0.02 + 0.01 + 0.01 + 0.02 + 0.07 + 0.06 + 0.01 + 0.03 + 0.02 = 0.25$$

Hence, $P(C_3|P_2) = 0.09/0.25 = .36$ which is different from $P(C_3)$.

(c) We first obtain $P(M_1 \cap P_1 \cap C_3) = 0.07$ and $P(P_1 \cap C_3) = .07 + .10 + .14 = .31$ so that

$$P(M_1|P_1 \cap C_3) = \frac{.07}{.31} = .2258 \quad \text{which is different from} \quad P(M_1)$$

3.60 (a) $P(\text{both awards}) = .11$, and $P(\text{efficiency award}) = .24$. Thus,

$$P(\text{design award given efficiency award}) = .11/.24 = .458.$$

(b) $P(\text{design award and no efficiency award}) = .16 - .11 = .05$, and $P(\text{no efficiency award}) = 1 - .24 = .76$. Thus,

$$P(\text{design award given no efficiency award}) = .05/.76 = .066.$$

3.61 $P(A|B) = P(A \cap B)/P(B)$ by definition. Thus, $P(A|B) = P(A)$ implies that $P(A \cap B)/P(B) = P(A)$, which implies $P(A \cap B)/P(A) = P(B)$, since both $P(A)$ and $P(B)$ are not zero. Thus $P(B|A) = P(B)$.

3.62 Let A be the event that a car is stopped for speeding and and B be the event that it is red. Then, $P(A|B) = .06$, $P(A|\overline{B}) = .02$ and $P(A) = .09$.

(a)

$$P(A) = P(A|B)P(B) + P(A|\overline{B})P(\overline{B}) = .06 \times .09 + .02 \times .91 = .0236$$

(b)

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\overline{B})P(\overline{B})} = \frac{.0054}{.0236} = .229$$

3.63 (a) $P(A|B) = P(A \cap B)/P(B) = .24/.40 = .6 = P(A)$.

(b) $P(A|\overline{B}) = P(A \cap \overline{B})/P(\overline{B}) = (P(A) - P(A \cap B))/(1 - P(B))$
$= (.60 - .24)/(1 - .40) = .6 = P(A)$.

(c) $P(B|A) = P(B \cap A)/P(A) = .24/.60 = .4 = P(B)$.

(d) $P(B|\overline{A}) = P(B \cap \overline{A})/P(\overline{A}) = (P(B) - P(B \cap A))/(1 - P(A))$
$= (.40 - .24)/(1 - .60) = .4 = P(B)$.

3.64 (a) The probability of getting an error on the first draw is 4/24. The probability of getting an error on the second draw given that there was an error on the first draw is 3/23. Thus, the probability that both will contain errors is:

$$\frac{4}{24} \cdot \frac{3}{23} = \frac{1}{46} = .0217.$$

(b) The probability that neither will contain errors is :

$$\frac{20}{24} \cdot \frac{19}{23} = \frac{95}{138} = .688.$$

3.65 (a) The probability of drawing a Seattle-bound part on the first draw is 45/60. The probability of drawing a Seattle-bound part on the second draw given that a Seattle-bound part was drawn on the first draw is 44/59. Thus, the probability that both parts should have gone to Seattle is:

$$\frac{45}{60} \cdot \frac{44}{59} = .559.$$

(b) Using an approach similar to (a), the probability that both parts should have gone to Vancouver is:

$$\frac{15}{60} \cdot \frac{14}{59} = .059.$$

(c) The probability that one should have gone to Seattle and one to Vancouver is 1 minus the sum of the probability in parts (a) and (b) or .381.

3.66 Let A be the event that the service call results in a complaint and B be the event that the call was made by the contractor. Then, $P(A|B) = .1$, $P(A|\overline{B}) = .05$ and $P(B) = .85$.

(a)

$$P(A) = P(A|B)P(B) + P(A|\overline{B})P(\overline{B}) = .10 \times .85 + .05 \times .15 = .0925$$

(b)

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) \, + \, P(A|\overline{B})P(\overline{B})} \; = \; \frac{.085}{.0925} \; = \; .919$$

3.67 A and B are independent if and only if $P(A)P(B) = P(A \cap B)$. Since (0.60)(0.45)= 0.27, they are independent.

3.68 By the definition of odds, $P(M) = 3/8$, $P(N) = 2/3$, and $P(M \cap N)$ =1/5. Since

$$P(M)P(N) = (3/8)(2/3) = 1/4 \neq 1/5 \quad \text{the events M and N are not independent.}$$

3.69 (a) Each head has probability 1/2, and each toss is independent. Thus, the probability of 8 heads is $(1/2)^8 = 1/256$.

(b) P(three 3's and then a 4 or 5) $= (1/6)^3(1/3) = 1/648$.

(c) P(five questions answered correctly) $= (1/3)^5 = 1/243$.

3.70 (a) P(first three are blanks) $= (3/6)(2/5)(1/4) = 1/20$.

(b) $P(R_2 R_3 S_4 R_5 | R_1) = (.8)(.8)(1{-}.8)(.6) = .0768$, where R_i means it rained on day i, and S_i means it was sunny on day i.

(c) P(not promptly next 3 months | promptly this month) $= (1{-}.90)(.50)(.50) = .025$.

(d) P(4 picked do not meet standards) $= (5/12)(4/11)(3/10)(2/9) = .0101$.

3.71 (a) $P(A) = (.4)(.3) + (.6)(.8) = .60$.

(b) $P(B|A) = P(B \cap A)/P(A) = (.4)(.3)/(.60) =.20$.

(c) $P(B|\overline{A}) = P(B \cap \overline{A})/P(\overline{A}) = (.4)(.7)/(.4) =.70$.

3.72 We have $P(A) \,=\, .25$ and $P(B \,|\, A\,) \,=\, .0002$.

(a) Since $P(\,B \mid \overline{A}\,) \,=\, 0$, $P(B) = P(B|A)\,P(A) \,=\, .0002 \times .25 \,=\, .00005$

(b) Here $P(B) = P(B|A)\,P(A) \,=\, .0004 \times .25 \,=\, .0001$.

3.73 (a) We first complete the table with numbers of policy holders. The total **12,299** and other given values are displayed in bold italics and the other italic numbers follow by addition and subtraction. Dividing each entry by the total 12,299 we obtain the long run relative frequencies that closely approximate the probabilities

	B	$\overline{B}$	
A	**1032**	1041	**2073**
$\overline{A}$	4097	6129	10,226
	5129	7170	**12,299**

	B	$\overline{B}$	
A	0.0839	0.0846	0.1686
$\overline{A}$	0.3331	0.4983	0.8314
	0.4170	0.5830	1.0000

(b)

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A|B)\,P(B)}{P(A)} = \frac{\frac{1032}{5129} \times \frac{5129}{12299}}{\frac{1032}{5129} \times \frac{5129}{12299} + \frac{1041}{7170} \times \frac{7170}{12299}} = .498$$

(c) From the table

$$P(\,B\,|\,A\,) = \frac{P(\,A \cap B\,)}{P(\,A\,)} = \frac{.0839}{.1686} = .498$$

3.74 Let A be the event that an identity theft complaint is filed and B be the event that the person is in the 20-29 age group. We are given that 43.2 million persons belong to this age group and they filed 56,689 complaints.

(a) We estimate $P(A|B) = 56,689/43.2 \times 10^6 = .001312$.

(b) Since $P(B) = .139$, letting N be the relevant population size, we have $.139 = 43.2 \times 10^6/N$ or $N = 43.2 \times 10^6/.139$. The number of persons not in age group 20-29 is then $(1 - .139\,)N = (.861)43.2 \times 10^6\,/\,.139 = 267591367$ and they file $280,000 - 56,689 = 223,311$ complaints. We estimate $P(A\,|\,\overline{B}) = \frac{223311}{267591367} = .000835$. The rate is over fifty percent higher for the 20-29 age group.

(c)

$$P(A) = P(A|B)P(B) + P(A|\overline{B})P(\overline{B}) = .001312 \times .139 + .000835 \times .861 = .000901$$

(d)

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A|B)\,P(B)}{P(A)} = \frac{.001312 \times .139}{.000901} = .202$$

3.75 We now have $P(B_1) = .45$, $P(B_2) = .40, P(B_3) = .15$.

(a)

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3)$$

$$= .45 \times .95 + .40 \times .80 + .15 \times .65 = .845$$

(b)

$$P(B_2|A) = \frac{P(A|B_2)\,P(B_2)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3)} = \frac{.32}{.845} = .3787$$

3.76 We now have

(a)

$$P(B_1) = \frac{1750}{5000} = .3500 \qquad P(B_2) = \frac{3250}{5000} = .6500$$

$$P(A\,|\,B_1) = \frac{1570}{1750} = .8971 \qquad P(A\,|\,B_2) = \frac{300}{3250} = .0923$$

so the posterior probability of being spam, given that identified as spam, is

$$P(B_1|A) = \frac{P(A|B_1)\,P(B_1)}{P(A|B_1)P(B_1)\,+\,P(A|B_2)P(B_2)} = \frac{.8971 \times .3500}{.8971 \times .3500\,+\,.0923 \times .6500} = .840$$

(b) This new list increases the posterior probability from .819 to .840 which is quite an improvement.

3.77 (a)

$$\begin{aligned} &P(\text{Tom }\,|\,\text{incomplete repair}) \\ &\quad = \frac{(.6)(1/10)}{(.2)(1/20) + (.6)(1/10) + (.15)(1/10) + (.05)(1/20)} \\ &\quad = \frac{.06}{.0875} = .686. \end{aligned}$$

(b)

$$P(\text{George }\,|\,\text{incomplete repair}) = \frac{(.15)(1/10)}{.0875} = .171.$$

(c)

$$P(\text{Peter }\,|\,\text{incomplete repair}) = \frac{(.05)(1/20)}{.0875} = .0286.$$

3.78 (a)

$$P(\text{ V gets job}) = (.8)(.3) + (.4)(.7) = .52.$$

(b)

$$\begin{aligned} &P(\text{ W did not bid }\,|\,\text{ V gets job}) \\ &\quad = P(\text{ W did not bid and V gets job})/P(\text{V gets job}) \\ &\quad = (.3)(.8)/(.52) = .4615. \end{aligned}$$

3.79 Let A be the event that the test indicates corrosion inside of the pipe and C be the event that corrosion is present. We are given $P(A|C) = .7$, $P(A|\overline{C}) = .2$, and $P(C) = .1$.

(a) By Bayes' theorem

$$P(C|A) = \frac{P(A|C)P(C)}{P(A|C)P(C) + PA|\overline{C})P(\overline{C})}$$

$$= \frac{.7 \times .1}{.7 \times .1 + .2 \times .9} = \frac{.07}{.07 + .18} = .28$$

(b)

$$P(C|\overline{A}) = \frac{P(\overline{A}|C)P(C)}{P(\overline{A}|C)P(C) + P(\overline{A}|\overline{C})P(\overline{C})}$$

$$= \frac{(1 - .7) \times .1}{(1 - .7) \times .1 + (1 - .2) \times .9} = \frac{.03}{.03 + .72} = .04$$

3.80 Let A be the event that a circuit board passes the automated test and D be the event that the board is defective. We approximate $P(A|D) = 25/30$ and $P(A) = 890/900$

(a) We then approximate $P($ Pass test | board has defects $) = P(A\,|\,D)$ using the relation

$$\frac{25}{30} = P(A\,|\,D) = 1 - P(\overline{A}\,|\,D)$$

or $P(\overline{A}\,|\,D) = 5/30$.

(b) The approximation $P(A\,|\,D) = 5/30$ may be too small because the boards were intentionally made to have noticeable defects. Likely, many defects are not very noticeable.

(c) To proceed, we assume that $P(\overline{A}\,|\,\overline{D}) = 0$ or $P(A\,|\,\overline{D}) = 1$. By the law of total probability

$$P(A) = P(A\,|\,D)P(D) + P(A\,|\,\overline{D}\,)\,P(\overline{D})$$

so

$$\frac{890}{900} = \frac{5}{30} \times P(D) + 1 \times (1 - P(D)$$

$$\text{or} \quad \frac{25}{30}P(D) = \frac{10}{900} \qquad \text{so} \quad P(D) = \frac{1}{75} = .013$$

(d)

$$P(D\,|\,A) = \frac{P(A\,|\,D)P(D)}{P(A\,|\,D)P(D) + P(A\,|\,\overline{D})P(\overline{D})} = \frac{\frac{5}{30} \times \frac{1}{75}}{\frac{5}{30} \times \frac{1}{75} + 1 \times \frac{74}{75}} = \frac{5}{2225} = .00225$$

3.81 (a) Using the long run relative frequency approximation to probability, we estimate the probability

$$P\,[\,\text{Checked out}\,] = \frac{27}{300} = 0.09$$

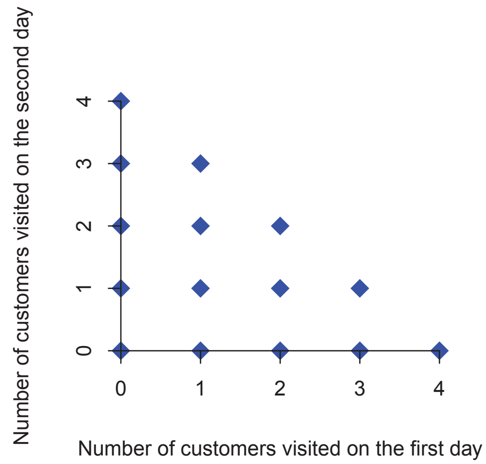
(b) Using the data from last year, the long run relative frequency approximation to probability gives the estimate

$$P\,[\,\text{Get internship}\,] = \frac{28}{380} = 0.074$$

One factor might be the quality of permanent jobs that interns received last year or even how enthusiastic they were about the internship. Both would likely increase the number of applicants.

Bad experiences may decrease the number of applicants.

3.82 (a) A graphic version of the sample space is



(b) A={(4,0),(3,1),(2,2),(1,3),(0,4)}.

B={(1,0),(2,0),(2,1),(3,0),(3,1),(4,0)}.

C={(0,3),(1,3),(0,4)}.

(c) A and B are not mutually exclusive. A and C are not mutually exclusive. But B and C are mutually exclusive.

3.83 (a) $\overline{A}$={(0,0),(0,1),(0,2),(0,3),(1,0),(1,1),(1,2),(2,0), (2,1),(3,0)}.

$\overline{A}$ is the event that the salesman will not visit all four of his customers.

(b) $A \cup B$={(4,0),(3,1),(2,2),(1,3),(0,4),(1,0),(2,0), (2,1),(3,0)}.

$A \cup B$ is the event that the salesman will visit all four customers or more on the first day than on the second day.

(c) $A \cap C$={(1,3),(0,4)}.

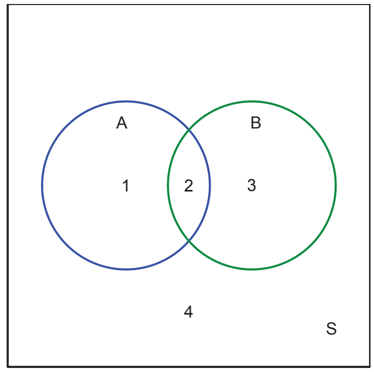
$A \cap C$ is the event that he will visit all four customers but at most one on the first day.

(d) $\overline{A} \cap B$={(1,0),(2,0),(2,1),(3,0)}.

$\overline{A} \cap B$ is the event that he will visit at most three of the customers and more on the first day than on the second day.

3.84 The Venn diagram is

(a) $A \cap B$ is the region 2. $\overline{A}$ is the region composed of areas 3 and 4. $\overline{B}$ is the region composed of areas 1 and 4. Since $\overline{A} \cup \overline{B}$ is the region composed of areas 1, 3 and 4, $\overline{(\overline{A} \cup \overline{B})}$ is the region 2. Hence $\overline{(\overline{A} \cup \overline{B})} = A \cap B$.



(b) A is the region composed of areas 1 and 2. $\overline{B}$ is the region composed of areas 1 and 4. Since $A \cap \overline{B}$ is the region 1, $\overline{(A \cap \overline{B})}$ is the region composed of areas 2, 3 and 4. On the other hand, $\overline{A}$ is the region composed of areas 3 and 4, and B is the region composed of areas 2 and 3. Hence $\overline{A} \cup B$ is the region composed of areas 2, 3,and 4, which is exactly the region $\overline{(A \cap \overline{B})}$.

3.85 The tree diagram is given in Figure 3.3

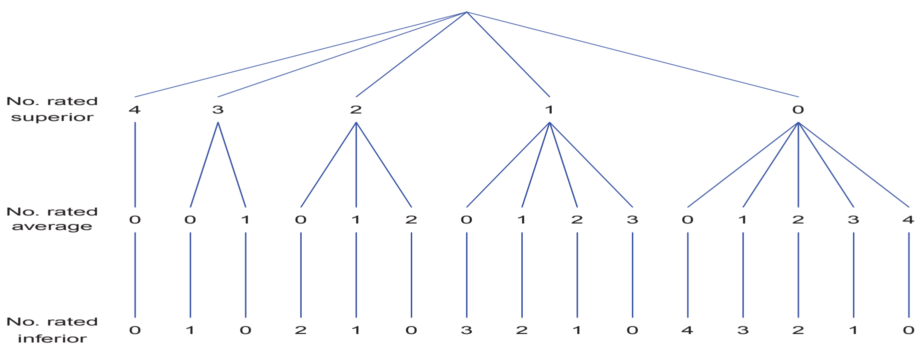


Figure 3.3: Tree diagram for Exercise 3.85.

3.86 There are 4!=24 ways to connect the alarm units.

3.87 There are ${}_7C_2 = 21$ ways to assign the chemical engineers.

3.88 (a) There are ${}_{20}C_4 = 4845$ possible selections of four motors and ${}_{16}C_4 = 1820$ ways to select all good motors.

$$P(\quad \text{all four good} \quad) = \frac{1820}{4845} = 0.3756$$

(b) The number of ways to choose are

$$_{16}C_3 = \binom{16}{3} = 560 \quad \text{for the 3 good and} \quad {}_4C_1 = \binom{4}{1} = 4 \;\; \text{for the one bad motor.}$$

Consequently

$$P(\quad \text{Three good and one bad} \quad) = \frac{560 \times 4}{4845} = 0.4623$$

3.89 (a) $P(A \cup B) = 0.30 + 0.40 - 0.20 = 0.50.$

(b) $P(\overline{A} \cap B) = P(B) - P(A \cap B) = 0.40 - 0.20 = 0.20$

(c) $P(A \cap \overline{B}) = P(A) - P(A \cap B) = 0.30 - 0.20 = 0.10$

(d) $P(\,\overline{A} \cup \overline{B}\,) = P(\overline{A \cap B}\,) = 1 - P(A \cap B) = 1 - .20 = 0.80.$

(e) A and B are not independent, since

$$P(A)P(B) = (.30)(.40) = .12 \neq P(A \cap B).$$

3.90 The estimated probability is 67/446= .15.

3.91 The total number of that said the product was reliable or easy to use is $165 + 117 - 88 = 195$. If 33 said neither, then there must be $195 + 33 = 228$ in the survey, not 200.

3.92 (a) The probability that it will not explode during lift-off is $1 - .0002 = .9998$.

(b) The probability is given by $.0002 + .0005 = .0007$, since these events are mutually exclusive.

(c) The probability is given by 1 minus the probability in (b) which is $1 - .0007 = .9993$.

3.93 (a) $P(A|B) = P(A \cap B)/P(B) = 0.09/0.45 = 0.2 = P(A).$

(b) Since $P(A \cap \overline{B}) = P(A) - P(A \cap B) = 0.2 - 0.09 = 0.11,$

$$P(A|\overline{B}) = P(A \cap \overline{B})/P(\overline{B}) = \frac{0.11}{1 - 0.45} = .2 = P(A).$$

(c) $P(B|A) = P(B \cap A)/P(A) = 0.09/0.20 = 0.45 = P(B).$

(d) Since $P(B \cap \overline{A}) = P(B) - P(B \cap A) = 0.45 - 0.09 = 0.36,$

$$P(B|\overline{A}) = P(B \cap \overline{A})/P(\overline{A}) = \frac{0.36}{1 - .2} = 0.45 = P(B).$$

3.94 (a) Since A and B are independent,

$$P(A \cap B) = P(A)P(B) = (.50)(.30) = .15.$$

(b) $P(A|B) = P(A) = .50$, since A and B are independent.

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .50 + .30 - .15 = .65$.

(d) Since A and B are independent, so are $\overline{A}$ and $\overline{B}$. Thus,

$$P(\overline{A} \cap \overline{B}) = P(\overline{A})P(\overline{B}) = (1 - P(A))(1 - P(B)) = (.50)(.70) = .35.$$

3.95 Let us denote the chemicals Arsenic, Barium, and Mercury by the letters A, B, and M respectively, and indicate the concentrations by the subscripts 'H' for high and 'L' for low. For instance, a high concentration of Mercury will be denoted by M_H.

(a) Of the 58 landfills, the number with M_H is 1 + 4 + 5 + 10 = 20.

Therefore, $P(M_H) = \frac{20}{58} = .344$

(b) The number of $M_L A_L B_H$ landfills is 8 so $P(M_L A_L B_H) = \frac{8}{58} = .138$

(c) There are three possibilities for landfills with two H's one L. The number of $A_H B_L M_H$ landfills is 5, the number of $A_L B_H M_H$ is 4, and the number of $A_H B_H M_L$ is 3, so the total is 12.

Therefore, $P(\text{two } H\text{'s and one } L\text{'s}) = \frac{12}{58} = .207$

(d) There are three possibilities for landfills with one H and two L's. The number of $A_H B_L M_L$ landfills is 9, the number of $A_L B_H M_L$ is 8, and the number of $A_L B_L M_H$ is 10, so the total is 27.

Therefore, $P(\text{one } H \text{ and two } L\text{'s}) = \frac{27}{58} = .466$

3.96 Let us denote the chemicals Arsenic, Barium, and Mercury by the letters A, B, and M respectively, and indicate the concentrations by the subscripts H for high and L for low. For instance, a high concentration of Mercury will be denoted by M_H. $P(B_H) = (1 + 3 + 4 + 8)/58 = 16/58$.

(a)

$$P(M_H|B_H) = \frac{P(M_H \cap B_H)}{P(B_H)} = \frac{5/58}{16/58} = \frac{5}{16}$$

(b)

$$P(A_H \cap M_H|B_H) = \frac{P((A_H \cap M_H) \cap B_H}{P(B_H)} = \frac{8/58}{16/58} = .5$$

(c)

$$P(A_H \cup M_H|B_H) = \frac{P((A_H \cup M_H) \cap B_H}{P(B_H)} = \frac{(1 + 4 + 3)/58}{16/58} = .5$$

3.97 Let events S.E. = static electricity, E = explosion, M = malfunction, O.F. = open flame, and P.E. = purposeful action. We need to find probabilities $P(\text{S.E.}|\text{E})$, $P(\text{M}|\text{E})$, $P(\text{O.F.}|\text{E})$, $P(\text{P.A.}|\text{E})$. Since

$$P(E) = (.30)(.25) + (.40)(.20) + (.15)(.40) + (.15)(.75) = .3275,$$

we have

$$P(S.E.|E) = (.30)(.25)/.3275 = .229, \qquad P(M|E) = (.40)(.20)/.3275 = .244,$$
$$P(O.F.|E) = (.15)(.40)/.3275 = .183, \qquad P(P.A.|E) = (.15)(.75)/.3275 = .344.$$

Thus, purposeful action is most likely.

3.98 Let A = [Laptop] and B = [Tablet]. We are given $P(A) = .9$, $P(B) = .3$ and $P(A \cap B) = .2$.

(a) First, $.9 = P(A) = P(A \cap B) + P(A \cap \overline{B})$ so that $P(A \cap \overline{B}) = .9 - .2 = .7$. Then, since $P(\overline{B}) = 1 - P(B) = .7$, we have

$$P(A \,|\, \overline{B}) = \frac{P(A \cap \overline{B})}{P(\overline{B})} = \frac{.7}{.7} = 1.0$$

(b) Since $.3 = P(B) = P(A \cap B) + P(\overline{A} \cap B)$, we have $P(\overline{A} \cap B) = .3 - .2 = .1$ and

$$P(A \cap \overline{B}) + P(\overline{A} \cap B) = .7 + .1 = .8$$

(c) Because $(A \cup B) \cap A = A$ and $P(A \cup B) = .9 + .3 - .2 = 1.0$

$$P(A \,|\, A \cup B) = \frac{P(A)}{P(A \cup B)} = \frac{.9}{1.0} = .9$$

3.99 We let A be the event route A is selected, B be the event route B is selected and C the event Amy arrives home at or before 6 p.m. We are given $P(A) = .4$ so $P(B) = 1 - .4 = .6$.

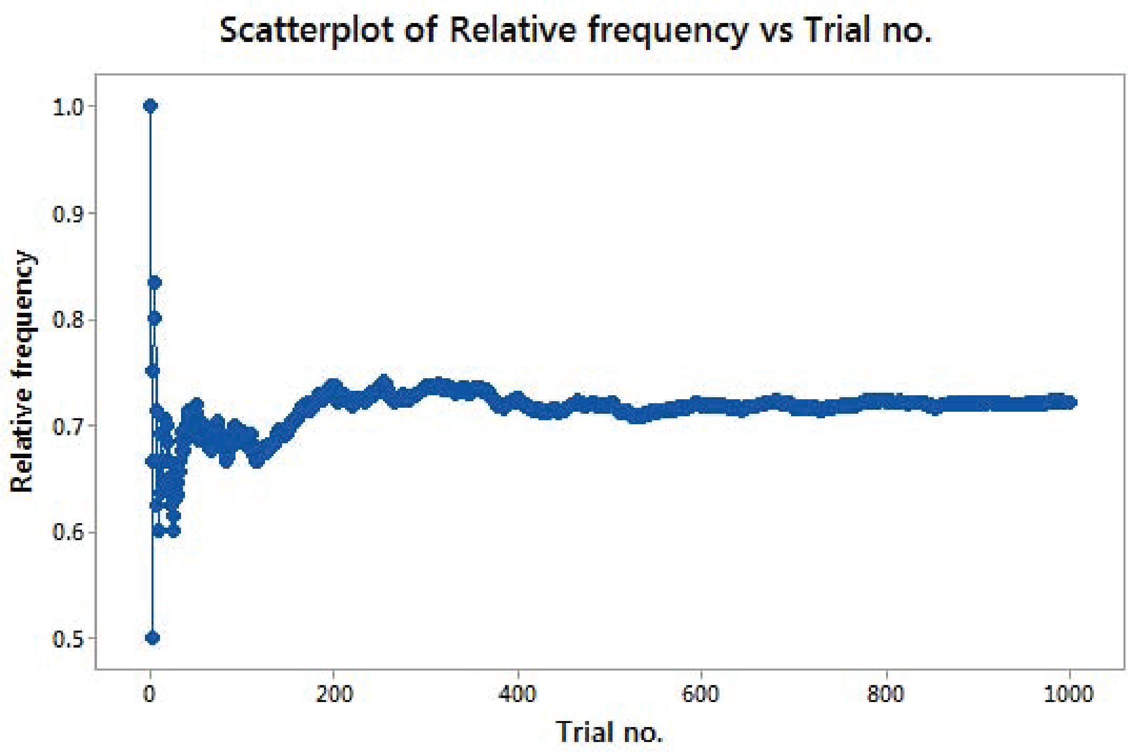
(a) By the law of total probability

$$P(C) = P(C|A)P(A) + P(C|B)P(B) = .8 \times .4 + .7 \times .6 = .74$$

(b) Using Bayes' rule

$$P(B|\overline{C}) = \frac{P(\overline{C}|B)P(B)}{P(\overline{C}|A)P(A) + P(\overline{C}|B)P(B)} = \frac{.3 \times .6}{.2 \times .4 + .3 \times .6} = .692$$

3.100 As the number of trials increases to a few hundred and beyond, the relative frequencies becomes much less variable from trial to trail. It approaches the probability of the event 0.7.


Scatterplot of Relative frequency vs Trial no.
Relative frequency
1.0
0.9
0.8
0.7
0.6
0.5
0
200
400
600
800
1000
Trial no.