Solutions Manual

to accompany

***Principles of Highway Engineering and Traffic Analysis*, 4e**

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Chapter 2
Road Vehicle Performance

Metric Units

Preface

The solutions to the fourth edition of *Principles of Highway Engineering and Traffic Analysis* were prepared with the Mathcad[1] software program. You will notice several notation conventions that you may not be familiar with if you are not a Mathcad user. Most of these notation conventions are self-explanatory or easily understood. The most common Mathcad specific notations in these solutions relate to the equals sign. You will notice the equals sign being used in three different contexts, and Mathcad uses three different notations to distinguish between each of these contexts. The differences between these equals sign notations are explained as follows.

- The ':=' (colon-equals) is an assignment operator, that is, the value of the variable or expression on the left side of ':='is set equal to the value of the expression on the right side. For example, in the statement, L := 1234, the variable 'L' is assigned (i.e., set equal to) the value of 1234. Another example is x := y + z. In this case, x is assigned the value of y + z.

- The '**=**' (bold equals) is used when the Mathcad function solver was used to find the value of a variable in the equation. For example, in the equation $5.2 \cdot t - 0.005 \cdot t^2 \mathbf{=} 18.568 + 10 \cdot (t - 12.792)$, the **=** is used to tell Mathcad that the value of the expression on the left side needs to equal the value of the expression on the right side. Thus, the Mathcad solver can be employed to find a value for the variable 't' that satisfies this relationship. This particular example is from a problem where the function for arrivals at some time 't' is set equal to the function for departures at some time 't' to find the time to queue clearance.

- The '=' (standard equals) is used for a simple numeric evaluation. For example, referring to the x := y + z assignment used previously, if the value of y was 10 [either by assignment (with :=), or the result of an equation solution (through the use of **=**) and the value of z was 15, then the expression 'x =' would yield 25. Another example would be as follows: s := 1800/3600, with s = 0.5. That is, 's' was assigned the value of 1800 divided by 3600 (using :=), which equals 0.5 (as given by using =).

Another symbol you will see frequently is '→'. In these solutions, it is used to perform an evaluation of an assignment expression in a single statement. For example, in the following statement, $Q(t) := \text{Arrivals}(t) - \text{Departures}(t) \rightarrow 2.200 \cdot t - .1000 \cdot t^2$, Q(t) is assigned the value of Arrivals(t) – Departures(t), and this evaluates to $2.2t - 0.10t^2$.

Finally, to assist in quickly identifying the final answer, or answers, for what is being asked in the problem statement, yellow highlighting has been used (which will print as light gray).

[1] www.mathcad.com

Problem 2.1

Determine the power required to overcome aerodynamic drag.

$\rho := 1.2256 \quad C_D := 0.29 \quad A_f := 1.9$ (given)

$V := 160 \cdot \frac{1000}{3600}$ m/s $\quad V = 44.4$

solve for power

$$P_{Ra} := \frac{\rho}{2} \cdot C_D \cdot A_f V^3$$ (Eq. 2.4)

$P_{Ra} = 29643$ kW

Problem 2.2

Determine the final weight of the car.

$\rho := 1.2256 \quad C_D := 0.30 \quad A_f := 2 \quad W_o := 9300$ (given)

$V_{max} := 160 \cdot 0.2778$

At V_{max}, Power = $R_a V_{max} + R_{rl} V_{max}$

Add one kW per 9 N. additional vehicle weight

additional power = W_a x 1000/9

Solve for additional weight added to the vehicle, set resistance forces equal to additional kW

$$\frac{1000\ W_a}{9} = \frac{\rho}{2} \cdot C_D \cdot A_f \cdot (V_{max})^3 + 0.01 \cdot \left(1 + \frac{V_{max}}{44.73}\right) \cdot (W_o + W_a) \cdot V_{max}$$

$W_a = 367.69$

$Total := W_o + W_a \qquad Total = 9667.69$ N

Problem 2.3

Determine the distance from the vehicle's center of gravity to the front axle.

$W := 11000 \quad \mu := 0.6 \quad L := 3.05 \quad h := 0.55 \quad f_{rl} := 0.01$ (given)

FWD F_{max} = RWD F_{max}

$$\frac{\dfrac{\mu W(l_f - f_{rl} \cdot h)}{L}}{1 - \dfrac{\mu \cdot h}{L}} = \frac{\dfrac{\mu W(l_r + f_{rl} \cdot h)}{L}}{1 + \dfrac{\mu \cdot h}{L}}$$ (Eq. 2.14)

solve for l_r in terms of L and l_f, left with one unknown (l_f)

$l_r := L - l_f$

$$\frac{\dfrac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \dfrac{\mu \cdot h}{L}} = \frac{\dfrac{\mu \cdot W \cdot \left[(L - l_f) + f_{rl} \cdot h\right]}{L}}{1 + \dfrac{\mu \cdot h}{L}}$$

$l_f = 1.365$ m

Problem 2.4

Determine the minimum coefficient of road adhesion.

$W := 13300 \quad g := 9.807 \quad L := 5.10 \quad a := 4.6$ (given)

$h := .50 \quad f_{rl} := 0.01 \quad l_f := 3.55$

$$F = ma + R_{rl} = \text{rear } F_{max} = \frac{\mu W(l_f - f_{rl} h)/L}{1 - \mu h/L}$$ (Eq. 2.14)

substitute for m, R_{rl}

$m := \dfrac{W}{g} \qquad R_{rl} := f_{rl} \cdot W$ (Eq. 2.6)

$$\frac{W}{g} \cdot a + f_{rl} \cdot W = \frac{\dfrac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \dfrac{\mu \cdot h}{L}}$$

$\mu = 0.646$

Problem 2.5

Determine the distance from the vehicle's center of gravity to the rear axle.

$W := 3013000 \quad g := 9.807 \quad L := 5.10 \quad a := 11.77$ (given)

$h := 1.78 \quad f_{rl} := 0.01 \quad \mu := 0.95$

$$F = ma + R_{rl} = \text{rear } F_{max} = \frac{\mu W(l_f - f_{rl}h)/L}{1-\mu h/L}$$ (Eq. 2.14)

substitute for m, R_{rl}

$$m := \frac{W}{g} \qquad R_{rl} := f_{rl} \cdot W$$ (Eq. 2.6)

$$\frac{W}{g} \cdot a + f_{rl} \cdot W = \frac{\frac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}}$$

$l_f = 4.36$ m

Problem 2.6

Determine the lowest gear reduction ratio.

$W := 12000$ $r := 0.355$ $L := 2.5$ (given)

$\mu := 1.0$ $h := 0.450$ $f_{rl} := 0.01$ $l_f := 1.0$

$$F = \text{rear } F_{max} = \frac{\mu W(l_f - f_{rl} h)/L}{1 - \mu h/L} \quad \text{(Eq. 2.14)}$$

"highest possible acceleration" means F_e is equal to F_{max}

$$F_{max} := \frac{\frac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}}$$

$F_{max} = 5827.317$

$M_e := 730$ $\eta_d := 0.95$ (given)

solve for ε_0

$$F_{max} = \frac{M_e \cdot \varepsilon_0 \cdot \eta_d}{r} \qquad \varepsilon_0 := \frac{F_{max} \cdot r}{M_e \cdot \eta_d} \quad \text{(Eq. 2.17)}$$

$\varepsilon_0 = 3$

Determine the maximum acceleration from rest.

Problem 2.7

$\varepsilon_0 := 9$ $r := 0.355$ $g := 9.807$ $\mu := 1.0$ $f_{rl} := 0.01$

(given)

$h := 0.45$ $l_f := 1.3$ $L := 2.8$ $W := 10900$

$R_{rl} := W \cdot f_{rl}$ $R_{rl} = 109$ (Eq. 2.6)

$M_{ebase} := 250$ $M_{emod} := 290$ $\eta_d := 0.90$

solve for mass factor

$\gamma_m := 1.04 + 0.0025 \cdot \varepsilon_0^2$ $\gamma_m = 1.24$ (Eq. 2.20)

$F_{ebase} := \dfrac{M_{ebase} \cdot \varepsilon_0 \cdot \eta_d}{r}$ $F_{ebase} = 5704.23$ (Eq. 2.17)

$F_{max} := \dfrac{\dfrac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \dfrac{\mu \cdot h}{L}}$ $F_{max} = 6008.91$ (Eq. 2.14)

since $F_{ebase} < F_{max}$, use F_{ebase} for calculating acceleration with original engine

$a_{base} := \dfrac{F_{ebase}}{\gamma_m \cdot \left(\dfrac{W}{g}\right)}$ $a_{base} = 4.13$ $\dfrac{m}{s^2}$ (Eq. 2.19)

$F_{emod} := \dfrac{M_{emod} \cdot \varepsilon_0 \cdot \eta_d}{r}$ $F_{emod} = 6616.9$ (Eq. 2.17)

since $F_{emod} > F_{max}$, use F_{max} for calculating acceleration with modified engine

$a_{mod} := \dfrac{F_{max}}{\gamma_m \cdot \left(\dfrac{W}{g}\right)}$ $a_{mod} = 4.35$ $\dfrac{m}{s^2}$ (Eq. 2.19)

Determine the maximum acceleration rate. **Problem 2.8**

3000 rev/min = 50 rev/sec

$i := 0.035 \quad n_e := 50 \quad \varepsilon_o := 3.5 \quad r := 0.38 \quad g := 9.807$ (given)

solve for velocity

$$V := \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{\varepsilon_o} \qquad V = 32.9$$ (Eq. 2.18)

calculate aerodynamic resistance

$\rho := 1.2256 \quad C_D := 0.35 \quad A_f := 2$

$$R_a := \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2 \qquad R_a = 464.73$$ (Eq. 2.3)

calculate rolling resistance

$$f_{rl} := 0.01\left(1 + \frac{V}{44.73}\right) \qquad W := 13300$$ (Eq. 2.5)

$$R_{rl} := f_{rl} \cdot W \qquad R_{rl} = 230.87$$

calculate mass factor

$$\gamma_m := 1.04 + 0.0025 \cdot \varepsilon_o^{\;2} \qquad \gamma_m = 1.07$$ (Eq. 2.20)

$M_e := 340 \quad n_d := 0.90$

calculate engine-generated tractive effort

$$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \qquad F_e = 2818.42$$ (Eq. 2.17)

$$F_{net} = F - \sum_n R = \gamma_m \cdot m \cdot a$$

so $$a := \frac{F_e - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g}\right)} \qquad a = 1.46 \ \frac{m}{sec^2}$$ (Eq. 2.19)

Determine the drag coefficient.

Problem 2.9

$M_e := 200 \quad \varepsilon_0 := 3.0 \quad \eta_d := 0.90 \quad r := 0.38$ (given)

$i := 0.02 \quad W := 9500 \quad n_e := \frac{4500}{60}$

$$V := \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{\varepsilon_0} \qquad V = 58.496 \qquad \text{(Eq. 2.18)}$$

$$F_e := \frac{M_e \cdot \varepsilon_0 \cdot \eta_d}{r} \qquad F_e = 1.421 \times 10^3 \qquad \text{(Eq. 2.17)}$$

$$f_{rl} := 0.01 \cdot \left(1 + \frac{V}{44.73}\right) \qquad \text{(Eq. 2.5)}$$

$$R_{rl} := f_{rl} \cdot W \qquad R_{rl} = 219.238 \qquad \text{(Eq. 2.6)}$$

$A_f := 1.8 \quad \rho := 1.2256$

$$F_e = R_{rl} + \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2 \qquad C_D := \frac{2(F_e - R_{rl})}{A_f \cdot V^2 \cdot \rho}$$

$C_D = 0.318$

Problem 2.10

Determine the drag coefficient.

$M_e := 270$ $\varepsilon_o := 3.0$ $n_d := 0.90$ $r := 0.355$ (given)

$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r}$ $F_e = 2053.5$ (Eq. 2.17)

$V := 240 \cdot \frac{1000}{3600}$ $W := 11000$ $f_{rl} := 0.01 \cdot \left(1 + \frac{V}{44.7}\right)$ (Eq. 2.5)

$R_{rl} := f_{rl} \cdot W$ $R_{rl} = 274.057$ (Eq. 2.6)

$\rho := 1.2256$ $A_f := 2.3$

set F_e equal to the sum of the resistance forces and solve for C_D

$F_e = R_{rl} + \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2$ $C_D := \frac{2(F_e - R_{rl})}{A_f \cdot V^2 \cdot \rho}$

$C_D = 0.284$

Determine the maximum grade.

Problem 2.11

$i := 0.035 \quad n_e := \frac{3500}{60} \quad \varepsilon_o := 3.2 \quad r := 0.355 \quad g := 9.807$ (given)

calculate velocity

$$V := \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{\varepsilon_o} \qquad V = 39.2 \quad \text{m/s}$$ (Eq. 2.18)

calculate aerodynamic resistance

$\rho := 1.2256 \quad C_D := 0.35 \quad A_f := 2.3$

$$R_a := \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2 \qquad R_a = 759.49 \quad \text{N}$$ (Eq. 2.3)

calculate rolling resistance

$$f_{rl} := 0.01\left(1 + \frac{V}{44.73}\right) \qquad W := 11000 \quad \text{N}$$ (Eq. 2.5)

$$R_{rl} := f_{rl} \cdot W \qquad R_{rl} = 206.49 \quad \text{N}$$

calculate engine-generated tractive effort

$M_e := 270 \quad n_d := 0.90$

$$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \qquad F_e = 2190.42 \quad \text{N}$$ (Eq. 2.17)

$$R_g := F_e - R_a - R_{rl}$$ (Eq. 2.2)

$R_g = 1224.44$

$$G := \frac{R_g}{W}$$ (Eq. 2.9)

$G = 0.1113$ therefore G = 11.1%

Alternative calculation for grade, using trig relationships

$$\theta_g := \operatorname{asin}\left(\frac{R_g}{W}\right)$$

$\theta_g = 0.1115$ radians

$deg\theta_g := \theta_g \cdot \frac{180}{\pi}$ convert from radians to degrees

$deg\theta_g = 6.391$

tan deg = opposite side/adjacent side

$G := \tan(\theta_g) \cdot 100$ $G = 11.2$ %

Thus, error is minimal when assuming G = sin θ_g for small to medium grades

Problem 2.12

Determine the torque the engine is producing and the engine speed.

$F_e - \Sigma R = \gamma_m \cdot m \cdot a$

at top speed, acceleration = 0; thus, $F_e - \Sigma R = 0$

$\rho := 1.06$ $\quad C_D := 0.28$ $\quad A_f := 1.8$ $\quad V := 200 \cdot \frac{1000}{3600}$ $\quad V = 55.556$ $\quad$ (given)

calculate aerodynamic resistance

$R_a := \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2$ $\quad R_a = 824.444$ $\quad$ (Eq. 2.3)

$f_{rl} := 0.01 \cdot \left(1 + \frac{V}{44.73}\right)$ $\quad f_{rl} = 0.022$ $\quad$ (Eq. 2.5)

$W := 12000$ $\quad i := 0.03$ $\quad \eta_d := 0.90$ $\quad \varepsilon_0 := 2.5$ $\quad r := 0.320$ $\quad$ (given)

calculate rolling resistance

$R_{rl} := f_{rl} \cdot W$ $\quad R_{rl} = 269.042$ $\quad$ (Eq. 2.6)

$R_g := 0$

sum of resistances is equal to engine-generated tractive effort, solve for M_e

$F_e := R_a + R_{rl} + R_g$ $\quad F_e = 1093.487$ $\quad$ (Eq. 2.2)

$F_e = \frac{M_e \cdot \varepsilon_0 \cdot \eta_d}{r}$ $\quad M_e := \frac{F_e \cdot r}{\varepsilon_0 \cdot \eta_d}$ $\quad$ (Eq. 2.17)

$M_e = 155.518$

Knowing velocity, solve for n_e

$V = \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{\varepsilon_0}$ $\quad n_e := \frac{V \cdot \varepsilon_0}{2 \cdot \pi \cdot r \cdot (1 - i)}$ $\quad$ (Eq. 2.18)

$n_e = 71.214 \ \frac{rev}{s}$ $\quad n_e \cdot 60 = 4273 \ \frac{rev}{min}$

Problem 2.13

Determine the maximum acceleration from rest.

$W := 11000 \quad L := 2.03 \quad h := 0.50 \quad l_f := 0.76 \quad \mu := 0.75$

$f_{rl} := 0.01 \qquad g := 9.807$ (given)

$$F_{max} := \frac{\frac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}} \qquad F_{max} = 3763.6$$ (Eq. 2.14)

For maximum torque, derivative of torque equation equals zero

$$\frac{dM_e}{dn_e} = 6 - 0.09n_e = 0 \qquad n_e := \frac{6}{0.09} \qquad n_e = 66.67$$

plug this value into torque equation to get value of maximum torque

$$M_e := 6 \cdot n_e - 0.045 \cdot n_e^2 \qquad M_e = 200$$

$\varepsilon_o := 11 \qquad n_d := 0.75 \qquad r := 0.355$

$\varepsilon_o := 11 \qquad n_d := 0.75 \qquad r := 0.355$

find engine-generated tractive effort from maximum torque

$$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \qquad F_e = 4647.89$$ (Eq. 2.17)

calculate rolling resistance and mass factor

$$R_{rl} := f_{rl} \cdot W \qquad R_{rl} = 110$$ (Eq. 2.6)

$$\gamma_m := 1.04 + 0.0025 \cdot \varepsilon_o^2 \qquad \gamma_m = 1.34$$ (Eq. 2.20)

$F_{max} < F_e$ so F_{max} is used

calculate acceleration from maximum available tractive effort

$$a := \frac{F_{max} - f_{rl} \cdot W}{\gamma_m \cdot \left(\frac{W}{g}\right)} \qquad a = 2.43 \ \frac{m}{s^2}$$ (Eq. 2.19)

Problem 2.14

Determine speed of car.

$$\text{Power} = (2\pi M_e \cdot n_e) = 6.28 \cdot \left(6n_e^{\,2} - 0.045n_e^{\,3}\right) = 37.68n_e^{\,2} - 0.2826n_e^{\,3} \qquad \text{(Eq. 2.16)}$$

$$P(n_e) := 37.68n_e^{\,2} - 0.2826n_e^{\,3}$$

To find maximum power take derivative of power equation

$$\frac{d}{dn_e}P(n_e) \rightarrow 75.36 \cdot n_e - .8478 \cdot n_e^{\,2} \quad = 0$$

$$n_e := \frac{75.36}{0.8478} \qquad n_e = 88.89$$

$$i := 0.035 \qquad \varepsilon_o := 2 \qquad r := 0.355$$

Calculate maximum velocity at maximum engine power

$$V := \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{\varepsilon_o} \qquad \text{(Eq. 2.18)}$$

$$V = 95.66 \quad \frac{m}{s} \qquad \frac{V}{0.2778} = 344.4 \quad \frac{km}{h}$$

Problem 2.15

Determine the acceleration for front- and rear-wheel-drive options.

$M_e := 130 \quad \varepsilon_o := 4.5 \quad \eta_d := 0.80 \quad r := 0.33$ (given)

R_a, R_{rl}, and γ_m are as before

calculate engine-generated tractive effort

$$F_e := \frac{M_e \cdot \varepsilon_o \cdot \eta_d}{r} \qquad F_e = 1418.182$$ (Eq. 2.17)

$\gamma_m := 1.091 \quad W := 13300 \quad R_a := 6.26$ (given)

$R_{rl} := 146.22 \quad g := 9.807$

calculate maximum acceleration

$$a_{max} := \frac{F_e - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g}\right)} \qquad a_{max} = 0.855$$ (Eq. 2.19)

Rear - wheel drive

$\mu := 0.2 \quad f_{rl} := 0.011 \quad h := 0.50 \quad L := 3.05 \quad l_f := \frac{3.05}{2}$

$$F_{max} := \frac{\frac{\mu \cdot W \cdot \left(l_f - f_{rl} \cdot h\right)}{L}}{1 - \frac{\mu \cdot h}{L}} \qquad F_{max} = 1370.125$$ (Eq. 2.14)

$$a_{max} := \frac{F_{max} - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g}\right)} \qquad a_{max} = 0.823 \ \frac{m}{s^2} \qquad 0.823 < 0.855$$ (Eq. 2.19)

Front - wheel drive

$$F_{max} := \frac{\frac{\mu \cdot W \cdot \left(l_f + f_{rl} \cdot h\right)}{L}}{1 + \frac{\mu \cdot h}{L}} \qquad F_{max} = 1292.422$$ (Eq. 2.15)

$$a_{max} := \frac{F_{max} - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g}\right)} \qquad a_{max} = 0.77 \ \frac{m}{s^2} \qquad 0.77 < 0.855$$ (Eq. 2.19)

Problem 2.16

Determine weight and torque.

$\mu := 0.8 \quad W_o := 8900 \quad \varepsilon_o := 10 \quad n_d := 0.8 \quad r := .355$ (given)

$l_f := 1.40 \quad f_{rl} := 0.01 \quad h := 0.55 \quad L := 2.55$

$$F_e = \frac{M_e \cdot \varepsilon_o \cdot n_d}{r}$$ (Eq. 2.17)

$$F_{max} = \frac{\dfrac{\mu \cdot W_a \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \dfrac{\mu \cdot h}{L}}$$ (Eq. 2.14)

$$l_{fnew} = l_f - \frac{2.5 \cdot (13 \cdot M_e)}{89} \qquad W_a = W_o + 13 \cdot M_e$$

setting $F_e = F_{max}$ and solving for M_e gives

$$\frac{M_e \cdot \varepsilon_o \cdot n_d}{r} = \frac{\dfrac{\mu \cdot (W_o + 13 \cdot M_e) \cdot \left[\left[l_f - \dfrac{2.5 \cdot (13 \cdot M_e)}{89}\right] - f_{rl} \cdot h\right]}{L}}{1 - \dfrac{\mu \cdot h}{L}}$$

$M_e = 3.751$ N-m

$W_a := W_o + 13 \cdot M_e$ N $\qquad W_a = 8948.8$ N

Problem 2.17

Determine the difference in minimum theoretical stopping distances with and without aerodynamic resistance considered.

$\rho := 1.2256 \qquad C_D := 0.45 \qquad A_f := 2.3 \qquad V := 145 \cdot 0.2778$ (given)

$g := 9.807 \qquad \gamma_b := 1.04 \qquad W := 11000 \qquad \eta_b := 1.0$

$f_{rl} := 0.0145 \qquad \mu := 0.7 \qquad \theta := 5.71$

$K_a := \frac{\rho}{2} \cdot C_D \cdot A_f \qquad K_a = 0.634$ (Eq. 2.37)

$$S := \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln\left(1 + \frac{K_a \cdot V^2}{\eta_b \cdot \mu \cdot W + f_{rl} \cdot W - W \cdot \sin(\theta \cdot deg)}\right) \qquad S = 130.22$$ (Eq. 2.42)

compared to S = 134.97 m and S = 139.87 m

$134.97 - 130.22 = 4.75$ m

$139.87 - 130.22 = 9.65$ m

Problem 2.18

Determine the initial speed with and without aerodynamic resistance.

$C_D := 0.40 \quad A_f := 2.4 \quad \rho := 1.2256 \quad W := 15600$

$\mu := 0.5 \quad f_{rl} := 0.015 \quad \eta_b := 0.78$ (given)

$S := 76 \quad g := 9.807 \quad \gamma_b := 1.04$

$K_a := \frac{\rho}{2} \cdot C_D \cdot A_f \qquad K_a = 0.588$ (Eq. 2.37)

with aerodynamic resistance considered

$$S = \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln\left(1 + \frac{K_a \cdot V^2}{\mu \cdot \eta_b \cdot W + f_{rl} \cdot W}\right)$$ (Eq. 2.43)

$V = 24.423 \qquad \frac{V}{0.2778} = 87.915 \ \frac{km}{h}$

with aerodynamic resistance ignored

$$S = \frac{\gamma_b \cdot V^2}{2 \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl}\right)} \qquad V := \sqrt{\frac{S \cdot 2 \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl}\right)}{\gamma_b}}$$ (Eq. 2.42)

$V = 24.094 \qquad \frac{V}{0.2778} = 86.73 \ \frac{km}{h}$

Problem 2.19

Determine the unloaded braking efficiency, ignoring aerodynamic resistance.

$\mu := 0.75 \quad f_{rl} := 0.018 \quad \gamma_b := 1.04$ (given)

$g := 9.807 \quad S := 61 \quad V := 100 \cdot 0.2778$

solve Eq. 2.43 for braking efficiency

$$S = \frac{\gamma_b \cdot V^2}{2 \cdot g \cdot (\eta_b \cdot \mu + f_{rl})} \qquad \eta_b := \frac{\gamma_b \cdot V^2}{S \cdot 2 \cdot g \cdot \mu} - f_{rl} \qquad \text{(Eq. 2.43)}$$

$\eta_b = 0.8704$

$\eta_b \cdot 100 = 87.04$ %

Problem 2.20

Determine the braking efficiency.

$\mu := 0.60 \quad \gamma_b := 1.04 \quad g := 9.807 \quad S := 180 \quad G := 0.03$ (given)

$V_1 := 180 \cdot \frac{1000}{3600} \qquad V_1 = 50$

$V_2 := 90 \cdot \frac{1000}{3600} \qquad V_2 = 25$

$$f_{rl} := 0.01 \cdot \left(1 + \frac{\frac{V_1 + V_2}{2}}{44.73}\right) \qquad f_{rl} = 0.018 \qquad \text{(Eq. 2.5)}$$

solve for braking efficiency using theoretical stopping distance equation

$$S = \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot (\eta_b \cdot \mu + f_{rl} - G)} \qquad \eta_b := \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot S \cdot g \cdot \mu} - f_{rl} + G \qquad \text{(Eq. 2.43)}$$

$\eta_b = 0.9399 \qquad \eta_b \cdot 100 = 93.99$ %

Problem 2.21

Determine the maximum amount of cargo that can be carried.

$V_1 := 120 \cdot 0.2778$ $V_1 = 33.336$ m/s $V_2 := 0$ (vehicle is assumed to stop)

$\mu := 0.95$ $G := -0.04$ $g := 9.807$ (given)

$\gamma_b := 1.04$ $\eta_b := 0.80$ $S := 90$

$$V_{avg} := \frac{V_1 + V_2}{2} \qquad V_{avg} = 16.668$$

$$f_{rl} := 0.01 \cdot \left(1 + \frac{V_{avg}}{44.73}\right) \qquad f_{rl} = 0.0137 \qquad \text{(Eq. 2.5)}$$

solve for additional vehicle weight using theoretical stopping distance equation, ignoring aerodynamic resistance

$$S = \frac{\gamma_b \cdot V_1^{\,2}}{2 \cdot g \cdot \left[\left(\eta_b - \frac{W}{100 \cdot 445}\right) \cdot \mu + f_{rl} + G\right]} \qquad \text{(Eq. 2.43)}$$

$W = 3701.1$ N

Problem 2.22

Determine the speed of the car when it strikes the object.

$C_D := 0.5 \quad A_f := 2.3 \quad W := 15600 \quad \rho := 1.2256$ (given)

$S := 45 \quad \mu := 0.85 \quad g := 9.807 \quad \gamma_b := 1.04$

$f_{rl} := 0.018 \quad \eta_b := 0.80$

$V_1 := 130 \cdot 0.2778 \qquad V_1 = 36.114$

$K_a := \frac{\rho}{2} \cdot C_D \cdot A_f \qquad K_a = 0.705$ (Eq. 2.37)

How fast will the car be travelling when it strikes the object on a level surface?

$$S = \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln\left[\frac{\eta_b \cdot \mu \cdot W + K_a \cdot V_1^2 + f_{rl} \cdot W}{\eta_b \cdot \mu \cdot W + K_a \cdot \left(V_2^2\right) + f_{rl} \cdot W}\right]$$ (Eq. 2.39)

$V_2 = 25.961 \qquad \frac{V_2}{0.2778} = 93.45 \quad \frac{km}{h}$

How fast will the car be travelling when it strikes the object on a 5% grade?

$G := 0.05$

$$S = \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln\left[\frac{\eta_b \cdot \mu \cdot W + K_a \cdot V_1^2 + f_{rl} \cdot W + G \cdot W}{\eta_b \cdot \mu \cdot W + K_a \cdot \left(V_2^2\right) + f_{rl} \cdot W + G \cdot W}\right]$$ (Eq. 2.39)

$V_2 = 25.147 \qquad \frac{V_2}{0.2778} = 90.52 \quad \frac{km}{h}$

Problem 2.23

Determine the speed of the car just before it impacted the object.

$V_1 := 120 \cdot \frac{1000}{3600}$ $V_1 = 33.333$ (given)

$\gamma_b := 1.04$ $g := 9.807$

$\eta_b := 0.90$ $f_{rl} := 0.015$

$\mu_m := 0.6$ $\mu_s := 0.3$ (Table 2.4)

$S_{al} := 60$ $S_{skid} := 30$ (given)

Find velocity of the car when it starts to skid

$$S_{al} = \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu_m + f_{rl} - 0.03\right)}$$ (Eq. 2.43)

$$V_2 := \sqrt{V_1^2 - \frac{2 \cdot S_{al} \cdot g \cdot \left(\eta_b \cdot \mu_m + f_{rl} - 0.03\right)}{\gamma_b}}$$

$V_2 = 22.738$ $V_2 \cdot \frac{3600}{1000} = 81.858$ $\frac{km}{h}$

Vehicle's velocity at start of skid is $V_1 := 22.738$

Find velocity when the vehicle strikes the object

$$S_{skid} = \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu_s + f_{rl} - 0.03\right)}$$ (Eq. 2.43)

$$V_2 := \sqrt{V_1^2 - \frac{2 \cdot S_{skid} \cdot g \cdot \left(\eta_b \cdot \mu_s + f_{rl} - 0.03\right)}{\gamma_b}}$$

$V_2 = 19.306$

$V_2 \cdot \frac{3600}{1000} = 69.5$ $\frac{km}{h}$

Determine if the driver should appeal the ticket.

Problem 2.24

$\mu := 0.6$ (for good, wet pavement, and slide value because of skidding)

$\gamma_b := 1.04$ $g := 9.807$

$V_2 := 65 \cdot \frac{1000}{3600}$ $V_2 = 18.056$ (given)

$\eta_b := 0.95$ $S := 61$ $f_{rl} := 0.015$

Solve for the initial velocity of the car using theoretical stopping distance

$$S = \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl} - 0.04\right)} \qquad V_1 := \sqrt{\frac{2 \cdot S \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl} - 0.04\right)}{\gamma_b} + V_2^2}$$ (Eq. 2.43)

$V_1 = 30.871$

$V_1 \cdot \frac{3600}{1000} = 111.13$ $\frac{km}{h}$

No, the driver should not appeal the ticket as the initial velocity was higher than the speed limit, in addition to the road being wet.

Problem 2.25

Determine the shortest distance from the stalled car that the driver could apply the brakes and stop before hitting it.

$\eta_b := 0.90 \quad \gamma_b := 1.04 \quad f_{rl} := 0.013$ (given)

$V := 110 \cdot \frac{1000}{3600} \quad V = 30.556$

$S := 45 \quad g := 9.807$

$\mu_{dry} := 1.0$

$\mu_{wet} := 0.9$ (Table 2.4)

Solve for velocity when entering wet section of pavement

$$S = \frac{\gamma_b \cdot \left(V_1^2 - 0\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu_{wet} + f_{rl} - 0.03\right)} \qquad V_1 := \sqrt{\frac{2 \cdot S \cdot g \cdot \left(\eta_b \cdot \mu_{wet} + f_{rl} - 0.03\right)}{\gamma_b}}$$ (Eq. 2.43)

$V_1 = 25.942 \qquad V_1 \cdot \frac{3600}{1000} = 93.392$

Using this as final velocity, solve for distance to slow to this velocity

$$S = \frac{\gamma_b \cdot \left(V^2 - V_1^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu_{dry} + f_{rl} - 0.03\right)}$$ (Eq. 2.43)

$S = 15.651$

Add this distance to the 45 m of wet pavement

$45 + S = 60.65$ m

Problem 2.26

Determine the braking efficiency of car 1.

$\gamma_b := 1.04 \quad g := 9.807 \quad V := 100 \cdot 0.2778 \quad V = 27.78$ (given)

$t_{r1} := 2.5 \quad t_{r2} := 2.0 \quad \eta_{b2} := 0.75 \quad \mu := 0.80$

$f_{rl} := 0.01\left(1 + \frac{V}{2 \cdot 44.73}\right) \quad f_{rl} = 0.013$ (Eq. 2.5)

Total stopping distance is perception/reaction distance plus braking distance

Set stopping distance of two cars equal to each other and solve for η_{b1}

$$V \cdot t_{r1} + \frac{\gamma_b \cdot V^2}{2 \cdot g} \cdot \left(\frac{1}{\eta_{b1} \cdot \mu + f_{rl}}\right) = V \cdot t_{r2} + \frac{\gamma_b \cdot V^2}{2 \cdot g} \cdot \left(\frac{1}{\eta_{b2} \cdot \mu + f_{rl}}\right)$$

$\eta_{b1} = 0.9514 \quad \eta_{b1} \cdot 100 = 95.14$ %

Problem 2.27

Determine the student's associated perception reaction time.

$V_1 := 90 \cdot 0.2778 \quad V_1 = 25.002$ m/s (given)

$V_2 := 55 \cdot .2778 \quad V_2 = 15.28$ m/s

$g := 9.807 \quad G := 0 \quad a := 3.4$ ft/s^2

Solve for distance to slow from 90 km/h to 55 km/h

$$d := \frac{(V_1)^2 - (V_2)^2}{2 \cdot a}$$ (Eq. 2.45)

$d = 57.6$ m

Subtract this distance from total distance to sign (185 m) to find perception/reaction time

$d_s := 185$ (given)

$d_r := d_s - d \quad d_r := 185 - d \quad d_r = 127.4$

$t_r := \frac{d_r}{V_1} \quad t_r = 5.1$ sec (Eq. 2.49)

Comment on the student's claim.

Problem 2.28

Method 1:

$V_1 := 110 \cdot 0.2778$ $V_1 = 30.558$ (given)

$g := 9.807$ $a := 3.4$

Find practical stopping distance

$$d := \frac{V_1^2}{2 \cdot a} \qquad d = 137.322 \qquad \text{(Eq. 2.46)}$$

Subtract this distance from the total sight distance and solve for perception/reaction time

$d_s := 180$

$d_r := d_s - d$ $d_r = 42.678$

$$t_r := \frac{d_r}{V_1} \qquad t_r = 1.397 \text{ sec} \qquad \text{(Eq. 2.49)}$$

This reaction time is well below the design value; therefore, the student's claim is unlikely.

Method 2:

$V_1 := 110 \cdot 0.2778$ $V_1 = 30.558$

$g := 9.807$ $a := 3.4$ $t_r := 2.5$ $G := 0$

Stopping sight distance = practical stopping distance plus perception/reaction distance

$$SSD := \frac{V_1^2}{2 \cdot g \cdot \left[\left(\frac{a}{g}\right) + G\right]} + V_1 \cdot t_r \qquad SSD = 213.717 \text{ m} \qquad \text{(Eq. 2.47)}$$

180 ft < 214 m (required from Table 3.1) therefore 180 m is not enough for 110 km/h design speed.

Problem 2.29

<u>Determine the grade of the road.</u>

$V_1 := 90 \cdot 0.2778$ $V_1 = 25.002$ (given)

$t_r := 2.5$ $d_s := 140$

Solve for distance travelled during braking (total distance minus perception/reaction)

$d_r := V_1 \cdot t_r$ $d_r = 62.505$ (Eq. 2.50)

$d := d_s - d_r$ $d = 77.495$ (Eq. 2.49)

$a := 3.4$ $g := 9.807$ (given)

$$d = \frac{(V_1)^2}{2 \cdot g \cdot \left(\frac{a}{g} + G\right)} \qquad G := \frac{V_1^{\,2}}{2 \cdot d \cdot g} - \frac{a}{g}$$ (Eq. 2.47)

$G = 0.0646$ $G \cdot 100 = 6.46$ %

Problem 2.30

<u>Determine the driver's perception/reaction time before and after drinking.</u>

$V_1 := 90 \cdot 0.2778$ $\quad V_1 = 25.002$ m/s (given)

$g := 9.807$ $\quad a := 3.4$

while sober, $d_s := 160$

solve for perception/reaction time using total stopping distance formula

$$d := \frac{V_1^2}{2 \cdot a}$$ (Eq. 2.46)

$d_s := d_r + d$ $\quad d_r := d_s - d$ (Eq. 2.50)

$d_r := V_1 \cdot t_r$ $\quad t_r := \frac{d_r}{V_1}$ (Eq. 2.49)

substituting Eqs. 2.46 and 2.50 into Eq. 2.49 gives

$$t_r := \frac{d_s}{V_1} - \frac{V_1}{2a}$$ $t_r = 2.72$ sec

after drinking, driver strikes the oject at $V_2 := 55 \cdot 0.2778$ m/s

solve for perception/reaction time using total stopping distance formula

$$t_r := \frac{d_s}{V_1} - \frac{V_1^2 - V_2^2}{2a \cdot V_1}$$ $t_r = 4.1$ sec (Eq. 2.50)

Solutions Manual
to accompany
***Principles of Highway Engineering and Traffic Analysis*, 4e**

By
Fred L. Mannering, Scott S. Washburn, and Walter P. Kilareski

Chapter 2
Road Vehicle Performance

U.S. Customary Units

Preface

The solutions to the fourth edition of *Principles of Highway Engineering and Traffic Analysis* were prepared with the Mathcad[1] software program. You will notice several notation conventions that you may not be familiar with if you are not a Mathcad user. Most of these notation conventions are self-explanatory or easily understood. The most common Mathcad specific notations in these solutions relate to the equals sign. You will notice the equals sign being used in three different contexts, and Mathcad uses three different notations to distinguish between each of these contexts. The differences between these equals sign notations are explained as follows.

- The ':=' (colon-equals) is an assignment operator, that is, the value of the variable or expression on the left side of ':='is set equal to the value of the expression on the right side. For example, in the statement, L := 1234, the variable 'L' is assigned (i.e., set equal to) the value of 1234. Another example is x := y + z. In this case, x is assigned the value of y + z.

- The '**=**' (bold equals) is used when the Mathcad function solver was used to find the value of a variable in the equation. For example, in the equation $5.2 \cdot t - 0.005 \cdot t^2 \mathbf{=} 18.568 + 10 \cdot (t - 12.792)$, the **=** is used to tell Mathcad that the value of the expression on the left side needs to equal the value of the expression on the right side. Thus, the Mathcad solver can be employed to find a value for the variable 't' that satisfies this relationship. This particular example is from a problem where the function for arrivals at some time 't' is set equal to the function for departures at some time 't' to find the time to queue clearance.

- The '=' (standard equals) is used for a simple numeric evaluation. For example, referring to the x := y + z assignment used previously, if the value of y was 10 [either by assignment (with :=), or the result of an equation solution (through the use of **=**) and the value of z was 15, then the expression 'x =' would yield 25. Another example would be as follows: s := 1800/3600, with s = 0.5. That is, 's' was assigned the value of 1800 divided by 3600 (using :=), which equals 0.5 (as given by using =).

Another symbol you will see frequently is '→'. In these solutions, it is used to perform an evaluation of an assignment expression in a single statement. For example, in the following statement, $Q(t) := Arrivals(t) - Departures(t) \rightarrow 2.200 \cdot t - .1000 \cdot t^2$, Q(t) is assigned the value of Arrivals(t) – Departures(t), and this evaluates to $2.2t - 0.10t^2$.

Finally, to assist in quickly identifying the final answer, or answers, for what is being asked in the problem statement, yellow highlighting has been used (which will print as light gray).

[1] www.mathcad.com

Problem 2.1

Determine the power required to overcome aerodynamic drag.

$\rho := 0.002378 \quad C_D := 0.29 \quad A_f := 20 \quad ft^2$ (given)

$V := 100 \cdot \frac{5280}{3600}$ ft/s $\quad V = 146.7$

solve for horsepower

$$hp := \frac{\rho \cdot C_D \cdot A_f \cdot V^3}{1100}$$ (Eq. 2.4)

$hp = 39.6$

Problem 2.2

Determine the final weight of the car.

$\rho := 0.002378 \quad C_D := 0.30 \quad A_f := 21 \quad W_o := 2100$ (given)

$V_{max} := 100 \cdot \frac{5280}{3600}$

At $V_{max,}$ Power = $R_a V_{max} + R_{rl} V_{max}$

Add one horsepower per 2 lbs. additional vehicle weight

additional power = W_a x 550/2

Solve for additional weight added to the vehicle, set resistance forces equal to additional hp

$$\frac{550\ W_a}{2} = \frac{\rho}{2} \cdot C_D \cdot A_f \cdot (V_{max})^3 + 0.01 \cdot \left(1 + \frac{V_{max}}{147}\right) \cdot (W_o + W_a) \cdot V_{max}$$

$W_a = 109.48$

$Total := W_o + W_a$

$Total = 2209.48$ lb

Problem 2.3

Determine the distance from the vehicle's center of gravity to the front axle.

$W := 2500 \quad \mu := 0.6 \quad L := 120 \quad h := 22 \quad f_{rl} := 0.01$ (given)

FWD F_{max} = RWD F_{max}

$$\frac{\dfrac{\mu W\left(l_f - f_{rl}\cdot h\right)}{L}}{1 - \dfrac{\mu\cdot h}{L}} = \frac{\dfrac{\mu W\left(l_r + f_{rl}\cdot h\right)}{L}}{1 + \dfrac{\mu\cdot h}{L}} \qquad \text{(Eq. 2.14)}$$

solve for l_r in terms of L and l_f, left with one unknown (l_f)

$l_r := L - l_f$

$$\frac{\dfrac{\mu\cdot W\cdot\left(l_f - f_{rl}\cdot h\right)}{L}}{1 - \dfrac{\mu\cdot h}{L}} = \frac{\dfrac{\mu\cdot W\cdot\left[\left(L - l_f\right) + f_{rl}\cdot h\right]}{L}}{1 + \dfrac{\mu\cdot h}{L}}$$

$l_f = 53.62$ inches

Problem 2.4

Determine the minimum coefficient of road adhesion.

$W := 3000 \quad g := 32.2 \quad L := 200$ (given)

$a := 15 \quad h := 20 \quad f_{rl} := 0.01 \quad l_f := 140$

$$F = ma + R_{rl} = \text{rear } F_{max} = \frac{\mu W(l_f - f_{rl}h)/L}{1 - \mu h/L}$$ (Eq. 2.14)

substitute for m, R_{rl}

$$m := \frac{W}{g} \qquad R_{rl} := f_{rl} \cdot W$$ (Eq. 2.6)

$$\frac{W}{g} \cdot a + f_{rl} \cdot W = \frac{\frac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}}$$

$\mu = 0.637$

Problem 2.5

Determine the distance from the vehicle's center of gravity to the rear axle.

$W := 3000 \quad g := 32.2 \quad L := 200$

(given)

$a := 32.2 \quad h := 36 \quad f_{rl} := 0.01 \quad \mu := 1.0$

$$F = ma + R_{rl} = \text{rear } F_{max} = \frac{\mu W(l_f - f_{rl} h)/L}{1 - \mu h/L}$$ (Eq. 2.14)

substitute for m, R_{rl}

$$m := \frac{W}{g} \qquad R_{rl} := f_{rl} \cdot W$$ (Eq. 2.6)

$$\frac{W}{g} \cdot a + f_{rl} \cdot W = \frac{\frac{\mu \cdot W \cdot \left(l_f - f_{rl} \cdot h\right)}{L}}{1 - \frac{\mu \cdot h}{L}}$$

$l_f = 166$ inches

Problem 2.6

Determine the lowest gear reduction ratio.

$W := 2700 \quad r := \frac{14}{12} \quad L := 8.2 \cdot 12$

(given)

$\mu := 1.0 \quad h := 18 \quad f_{rl} := 0.01 \quad l_f := 3.3 \cdot 12$

$F = \text{ rear } F_{max} = \frac{\mu W(l_f - f_{rl} h)/L}{1 - \mu h/L}$ (Eq. 2.14)

"highest possible acceleration" means F_e is equal to F_{max}

$$F_{max} := \frac{\frac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}}$$

$F_{max} = 1323.806$

$M_e := 540 \quad \eta_d := 0.95$ (given)

solve for ε_0

$F_{max} = \frac{M_e \cdot \varepsilon_o \cdot \eta_d}{r} \qquad \varepsilon_0 := \frac{F_{max} \cdot r}{M_e \cdot \eta_d}$ (Eq. 2.17)

$\varepsilon_o = 3$

Problem 2.7

Determine the maximum acceleration from rest.

$\varepsilon_0 := 9 \qquad r := \frac{14}{12} \qquad g := 32.2 \qquad \mu := 1.0 \qquad f_{rl} := 0.01$

(given)

$h := \frac{18}{12} \qquad l_f := 4.3 \quad L := 9.2 \qquad W := 2450$

$R_{rl} := W \cdot f_{rl} \qquad R_{rl} = 24.5$ (Eq. 2.6)

$M_{ebase} := 185 \qquad M_{emod} := 215 \qquad \eta_d := 0.90$

solve for mass factor

$\gamma_m := 1.04 + 0.0025 \cdot \varepsilon_0^{\,2} \qquad \gamma_m = 1.24$ (Eq. 2.20)

$F_{ebase} := \frac{M_{ebase} \cdot \varepsilon_0 \cdot \eta_d}{r} \qquad F_{ebase} = 1284.43$ (Eq. 2.17)

$F_{max} := \frac{\frac{\mu \cdot W \cdot \left(l_f - f_{rl} \cdot h\right)}{L}}{1 - \frac{\mu \cdot h}{L}} \qquad F_{max} = 1363.41$ (Eq. 2.14)

since $F_{ebase} < F_{max}$, use F_{ebase} for calculating acceleration with original engine

$a_{base} := \frac{F_{ebase} - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g}\right)} \qquad a_{base} = 13.33 \quad \frac{ft}{s^2}$ (Eq. 2.19)

$F_{emod} := \frac{M_{emod} \cdot \varepsilon_0 \cdot \eta_d}{r} \qquad F_{emod} = 1492.71$ (Eq. 2.17)

since $F_{emod} > F_{max}$, use F_{max} for calculating acceleration with modified engine

$a_{mod} := \frac{F_{max} - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g}\right)} \qquad a_{mod} = 14.16 \quad \frac{ft}{s^2}$ (Eq. 2.19)

Problem 2.8

Determine the maximum acceleration rate.

3000 rev/min = 50 rev/sec

$i := 0.035 \quad n_e := 50 \quad \varepsilon_o := 3.5 \quad r := \frac{15}{12} \quad g := 32.2$ (given)

solve for velocity

$$V := \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{\varepsilon_o} \qquad V = 108.3$$ (Eq. 2.18)

$\rho := 0.002378 \quad C_D := 0.35 \quad A_f := 21$

calculate aerodynamic resistance

$$R_a := \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2 \qquad R_a = 102.45$$ (Eq. 2.3)

calculate rolling resistance

$$f_{rl} := 0.01\left(1 + \frac{V}{147}\right) \qquad W := 3000$$ (Eq. 2.5)

$$R_{rl} := f_{rl} \cdot W \qquad R_{rl} = 52.1$$

calculate mass factor

$$\gamma_m := 1.04 + 0.0025 \cdot \varepsilon_o^{\,2} \qquad \gamma_m = 1.07$$ (Eq. 2.20)

$M_e := 250 \quad n_d := 0.90$

calculate engine-generated tractive effort

$$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \qquad F_e = 630$$ (Eq. 2.17)

$$F_{net} = F - \sum_n R = \gamma_m \cdot m \cdot a$$

so

$$a := \frac{F_e - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g}\right)} \qquad a = 4.77 \ \frac{ft}{sec^2}$$ (Eq. 2.19)

Problem 2.9

Determine the drag coefficient.

$M_e := 150 \quad \varepsilon_0 := 3.0 \quad \eta_d := 0.90 \quad r := \frac{15}{12}$ (given)

$i := 0.02 \quad W := 2150 \quad n_e := \frac{4500}{60}$

$V := \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{\varepsilon_0} \qquad V = 192.4$ (Eq. 2.18)

$F_e := \frac{M_e \cdot \varepsilon_0 \cdot \eta_d}{r} \qquad F_e = 324$ (Eq. 2.17)

$f_{rl} := 0.01 \cdot \left(1 + \frac{V}{147}\right)$ (Eq. 2.5)

$R_{rl} := f_{rl} \cdot W \qquad R_{rl} = 49.643$ (Eq. 2.6)

$\rho := 0.002378 \qquad A_f := 19.4$

$F_e = R_{rl} + \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2 \qquad C_D := \frac{2(F_e - R_{rl})}{A_f \cdot V^2 \cdot \rho}$

$C_D = 0.321$

Problem 2.10

__Determine the drag coefficient.__

$M_e := 200 \quad \varepsilon_o := 3.0 \quad n_d := 0.90 \quad r := \frac{14}{12}$ (given)

$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \qquad F_e = 462.9$ (Eq. 2.17)

$V := 150 \cdot 1.467 \qquad W := 2500 \qquad f_{rl} := 0.01 \cdot \left(1 + \frac{V}{147}\right)$ (Eq. 2.5)

$R_{rl} := f_{rl} \cdot W \qquad R_{rl} = 62.423$ (Eq. 2.6)

$\rho := 0.002378 \qquad A_f := 25$

set F_e equal to the sum of the resistance forces and solve for C_D

$F_e = R_{rl} + \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2 \qquad C_D := \frac{2(F_e - R_{rl})}{A_f \cdot V^2 \cdot \rho}$

$C_D = 0.278$

Determine the maximum grade.

Problem 2.11

$i := 0.035 \quad n_e := \frac{3500}{60} \quad \varepsilon_o := 3.2 \quad r := \frac{14}{12} \quad W := 2500 \;\; \text{lb}$ (given)

assume $F=F_e$

calculate velocity

$$V := \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{\varepsilon_o} \qquad V = 128.9 \quad \text{ft/s}$$ (Eq. 2.18)

calculate aerodynamic resistance

$\rho := 0.002378 \quad C_D := 0.35 \quad A_f := 25$

$$R_a := \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2 \qquad R_a = 172.99 \quad \text{lb}$$ (Eq. 2.3)

calculate rolling resistance

$$f_{rl} := 0.01\left(1 + \frac{V}{147}\right)$$ (Eq. 2.5)

$$R_{rl} := f_{rl} \cdot W \qquad R_{rl} = 46.93 \quad \text{lb}$$

calculate engine-generated tractive effort

$M_e := 200 \quad n_d := 0.90$

$$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \qquad F_e = 493.71 \quad \text{lb}$$ (Eq. 2.17)

calculate grade resistance

$$R_g := F_e - R_a - R_{rl}$$ (Eq. 2.2)

$$R_g = 273.79$$

solve for G

$$G := \frac{R_g}{W}$$ (Eq. 2.9)

$G = 0.1095$ therefore G = 11.0%

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Alternative calculation for grade, using trig relationships

$$\theta_g := \text{asin}\left(\frac{R_g}{W}\right)$$

$\theta_g = 0.1097$ radians

$$\text{deg}\theta_g := \theta_g \cdot \frac{180}{\pi}$$ convert from radians to degrees

$\text{deg}\theta_g = 6.287$

tan deg = opposite side/adjacent side

$G := \tan(\theta_g) \cdot 100$ $G = 11.02$ %

Thus, error is minimal when assuming G = sin θ_g for small to medium grades

Problem 2.12

Determine the torque the engine is producing and the engine speed.

$F_e - \Sigma R = \gamma_m \cdot m \cdot a$

at top speed, acceleration = 0; thus, $F_e - \Sigma R = 0$

$V := 124 \cdot \frac{5280}{3600}$ $V = 181.867$ ft/s (given)

calculate aerodynamic resistance

$\rho := 0.00206$ $C_D := 0.28$ $A_f := 19.4$

$R_a := \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2$ $R_a = 185.056$ (Eq. 2.3)

calculate rolling resistance

$f_{rl} := 0.01 \cdot \left(1 + \frac{V}{147}\right)$ $f_{rl} = 0.022$ (Eq. 2.5)

$W := 2700$ (given)

$R_{rl} := f_{rl} \cdot W$ $R_{rl} = 60.404$ (Eq. 2.6)

$R_g := 0$

sum of resistances is equal to engine-generated tractive effort, solve for M_e

$F_e := R_a + R_{rl} + R_g$ $F_e = 245.46$ (Eq. 2.2)

$i := 0.03$ $\eta_d := 0.90$ $\varepsilon_0 := 2.5$ $r := \frac{12.6}{12}$

$F_e = \frac{M_e \cdot \varepsilon_0 \cdot \eta_d}{r}$ $M_e := \frac{F_e \cdot r}{\varepsilon_0 \cdot \eta_d}$ (Eq. 2.17)

$M_e = 114.548$ ft-lb

Knowing velocity, solve for n_e

$V = \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{\varepsilon_0}$ $n_e := \frac{V \cdot \varepsilon_0}{2 \cdot \pi \cdot r \cdot (1 - i)}$ (Eq. 2.18)

$n_e = 71.048$ $\frac{rev}{s}$ $n_e \cdot 60 = 4263$ $\frac{rev}{min}$

Problem 2.13

Determine the maximum acceleration from rest.

$W := 2500 \quad L := 80 \quad h := 20 \quad l_f := 30$ (given)

$\mu := 0.75 \quad f_{rl} := 0.01 \quad g := 32.2$

$$F_{max} := \frac{\dfrac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \dfrac{\mu \cdot h}{L}} \qquad F_{max} = 859.62$$ (Eq. 2.14)

For maximum torque, derivative of torque equation equals zero

$$\frac{dM_e}{dn_e} = 6 - 0.09 n_e = 0 \qquad n_e := \frac{6}{0.09} \qquad n_e = 66.67$$

plug this value into torque equation to get value of maximum torque

$$M_e := 6 \cdot n_e - 0.045 \cdot n_e^2 \qquad M_e = 200$$

$$\varepsilon_o := 11 \qquad n_d := 0.75 \qquad r := \frac{14}{12}$$

find engine-generated tractive effort from maximum torque

$$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \qquad F_e = 1414.29$$ (Eq. 2.17)

$$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \qquad F_e = 1414.29$$ (Eq. 2.17)

calculate rolling resistance and mass factor

$$R_{rl} := f_{rl} \cdot W \qquad R_{rl} = 25$$ (Eq. 2.6)

$$\gamma_m := 1.04 + 0.0025 \cdot \varepsilon_o^2 \qquad \gamma_m = 1.34$$ (Eq. 2.20)

$F_{max} < F_e$ so F_{max} is used

calculate acceleration from maximum available tractive effort

$$a := \frac{F_{max} - f_{rl} \cdot W}{\gamma_m \cdot \left(\dfrac{W}{g}\right)} \qquad a = 8.01 \frac{ft}{sec^2}$$ (Eq. 2.19)

Problem 2.14

Determine speed of car.

Power = $(2\pi M_e \cdot n_e) = 6.28 \cdot (6n_e^{\,2} - 0.045n_e^{\,3}) = 37.68n_e^{\,2} - 0.2826n_e^{\,3}$ (Eq. 2.16)

$$P(n_e) := 37.68n_e^{\,2} - 0.2826n_e^{\,3}$$

To find maximum power take derivative of power equation

$$\frac{d}{dn_e}P(n_e) \rightarrow 75.36 \cdot n_e - .8478 \cdot n_e^{\,2} \quad = 0$$

$$n_e := \frac{75.36}{0.8478} \qquad n_e = 88.89$$

$$i := 0.035 \qquad \varepsilon_o := 2 \qquad r := \frac{14}{12}$$

Calculate maximum velocity at maximum engine power

$$V := \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{\varepsilon_o}$$ (Eq. 2.18)

$$V = 314.39 \ \frac{\text{ft}}{\text{s}} \qquad \frac{V}{1.467} = 214.3 \ \frac{\text{mi}}{\text{h}}$$

Problem 2.15

Determine the acceleration for front- and rear-wheel-drive options.

$M_e := 95 \quad \varepsilon_o := 4.5 \quad n_d := 0.80 \quad r := \frac{13}{12}$ (given)

R_a, R_{rl}, and g_m are as before

calculate engine-generated tractive effort

$$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \qquad F_e = 315.692$$ (Eq. 2.17)

$\gamma_m := 1.091 \quad W := 3000 \quad R_a := 1.32$

$R_{rl} := 32.99 \quad g := 32.2$ (given)

calculate maximum acceleration

$$a_{max} := \frac{F_e - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g}\right)} \qquad a_{max} = 2.768$$ (Eq. 2.19)

Rear - wheel drive

$\mu := 0.2 \quad f_{rl} := 0.011 \quad h := 20 \quad L := 120 \quad l_f := 60$

$$F_{max} := \frac{\frac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}} \qquad F_{max} = 309.207$$ (Eq. 2.14)

$$a_{max} := \frac{F_{max} - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g}\right)} \qquad a_{max} = 2.704 \ \frac{ft}{sec^2} \qquad 2.704 < 2.768$$ (Eq. 2.19)

Front - wheel drive

$$F_{max} := \frac{\frac{\mu \cdot W \cdot (l_f + f_{rl} \cdot h)}{L}}{1 + \frac{\mu \cdot h}{L}} \qquad F_{max} = 291.387$$ (Eq. 2.15)

$$a_{max} := \frac{F_{max} - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g}\right)}$$ (Eq. 2.19)

$a_{max} = 2.529 \ \frac{ft}{sec^2} \qquad 2.529 < 2.768$

Problem 2.16

Determine weight and torque.

$\mu := 0.8$ $W_o := 2000$ $\varepsilon_o := 10$ $n_d := 0.8$

$l_f := 55$ $r := \frac{14}{12}$ $f_{rl} := 0.01$ $h := 22$ $L := 100$ (given)

$$F_e = \frac{M_e \cdot \varepsilon_o \cdot n_d}{r}$$ (Eq. 2.17)

$$F_{max} = \frac{\frac{\mu \cdot W_a \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}}$$ (Eq. 2.14)

$l_{fnew} = l_f - \frac{3 \cdot 1}{20} \cdot M_e$ $W_a = W_o + 3 \cdot M_e$

setting $F_e = F_{max}$ and solving for M_e gives

$$\frac{M_e \cdot \varepsilon_o \cdot n_d}{r} = \frac{\frac{\mu \cdot (W_o + 3 \cdot M_e) \cdot \left[\left(l_f - \frac{3 \cdot 1}{20} \cdot M_e\right) - f_{rl} \cdot h\right]}{L}}{1 - \frac{\mu \cdot h}{L}}$$

$M_e = 122.152$ ft-lb

$W_a := W_o + 3 \cdot M_e$ $W_a = 2366.5$ lb

Problem 2.17

Determine the difference in minimum theoretical stopping distances with and without aerodynamic resistance considered.

$\rho := 0.0024 \quad C_D := 0.45 \quad A_f := 25 \quad V := 90 \cdot 1.467$

$g := 32.2 \quad \gamma_b := 1.04 \quad W := 2500 \quad \eta_b := 1.0$ (given)

$f_{rl} := 0.019 \quad \mu := 0.7 \quad \theta := 5.71$

$K_a := \frac{\rho}{2} \cdot C_D \cdot A_f \quad K_a = 0.014$ (Eq. 2.37)

$$S := \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln\left(1 + \frac{K_a \cdot V^2}{\eta_b \cdot \mu \cdot W + f_{rl} \cdot W - W \cdot \sin(\theta \cdot deg)}\right) \quad S = 423.027$$ (Eq. 2.42)

compared to S = 444.07 and S = 457.53

$444.07 - 424.64 = 19.43$ ft

$457.53 - 424.64 = 32.89$ ft

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Problem 2.18

Determine the initial speed with and without aerodynamic resistance.

$C_D := 0.40 \quad A_f := 28 \quad \rho := 0.002378 \quad W := 3500$

$\mu := 0.5 \quad f_{rl} := 0.015 \quad \eta_b := 0.78$ (given)

$S := 250 \quad g := 32.2 \quad \gamma_b := 1.04$

$K_a := \frac{\rho}{2} \cdot C_D \cdot A_f \qquad K_a = 0.013$ (Eq. 2.37)

with aerodynamic resistance considered

$$S = \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln\left(1 + \frac{K_a \cdot V^2}{\mu \cdot \eta_b \cdot W + f_{rl} \cdot W}\right)$$ (Eq. 2.43)

$V = 80.362 \qquad \frac{V}{1.467} = 54.78$

with aerodynamic resistance ignored

$$S = \frac{\gamma_b \cdot V^2}{2 \cdot g \cdot (\eta_b \cdot \mu + f_{rl})} \qquad V := \sqrt{\frac{S \cdot 2 \cdot g \cdot (\eta_b \cdot \mu + f_{rl})}{\gamma_b}}$$ (Eq. 2.42)

$V = 79.182 \qquad \frac{V}{1.467} = 53.98 \ \frac{mi}{h}$

Problem 2.19

Determine the unloaded braking efficiency, ignoring aerodynamic resistance.

$\mu := 0.75 \quad f_{rl} := 0.018 \quad \gamma_b := 1.04$ (given)

$g := 32.2 \quad S := 200 \quad V := 60 \cdot 1.467$

solve Eq. 2.43 for braking efficiency

$$S = \frac{\gamma_b \cdot V^2}{2 \cdot g \cdot (\eta_b \cdot \mu + f_{rl})} \qquad \eta_b := \frac{\gamma_b \cdot V^2}{S \cdot 2 \cdot g \cdot \mu} - f_{rl} \qquad \text{(Eq. 2.43)}$$

$\eta_b = 0.8101$

$\eta_b \cdot 100 = 81.01$ %

Problem 2.20

Determine the braking efficiency.

$\mu := 0.60 \quad \gamma_b := 1.04 \quad g := 32.2 \quad S := 590 \quad G := 0.03$ (given)

$V_1 := 110 \cdot \frac{5280}{3600} \qquad V_1 = 161.333$

$V_2 := 55 \cdot \frac{5280}{3600} \qquad V_2 = 80.667$

$$f_{rl} := 0.01 \cdot \left(1 + \frac{\frac{V_1 + V_2}{2}}{147}\right) \qquad f_{rl} = 0.018 \qquad \text{(Eq. 2.5)}$$

solve for braking efficiency using theoretical stopping distance equation

$$S = \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot (\eta_b \cdot \mu + f_{rl} - G)} \qquad \eta_b := \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot S \cdot g \cdot \mu} - f_{rl} + G \qquad \text{(Eq. 2.43)}$$

$\eta_b = 0.9102 \qquad \eta_b \cdot 100 = 91.02$ %

Problem 2.21

Determine the maximum amount of cargo that can be carried.

$V_1 := 75 \cdot 1.467$ $\quad V_1 = 110.025$ ft/s $\quad V_2 := 0$ (vehicle is assumed to stop)

$\mu := 0.95$ $\quad G := -0.04$ $\quad g := 32.2$ (given)

$\gamma_b := 1.04$ $\quad \eta_b := 0.80$ $\quad S := 300$

$$V_{avg} := \frac{V_1 + V_2}{2} \qquad V_{avg} = 55.013$$

$$f_{rl} := 0.01 \cdot \left(1 + \frac{V_{avg}}{147}\right) \qquad f_{rl} = 0.0137 \qquad \text{(Eq. 2.5)}$$

solve for additional vehicle weight using theoretical stopping distance equation, ignoring aerodynamic resistance

$$S = \frac{\gamma_b \cdot V_1^{\,2}}{2 \cdot g \cdot \left[\left(\eta_b - \frac{W}{100 \cdot 100}\right) \cdot \mu + f_{rl} + G\right]} \qquad \text{(Eq. 2.43)}$$

$W = 864.2$ lb

Problem 2.22

Determine the speed of the car when it strikes the object.

$C_D := 0.5 \quad A_f := 25 \quad W := 3500 \quad \rho := 0.002378$

$S := 150 \quad \mu := 0.85 \quad g := 32.2 \quad \gamma_b := 1.04$ (given)

$f_{rl} := 0.018 \quad \eta_b := 0.80$

$V_1 := 80 \cdot 1.467 \quad V_1 = 117.36$

$K_a := \frac{\rho}{2} \cdot C_D \cdot A_f \quad K_a = 0.015$ (Eq. 2.37)

How fast will the car be travelling when it strikes the object on a level surface?

$$S = \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln\left[\frac{\eta_b \cdot \mu \cdot W + K_a \cdot V_1^{\,2} + f_{rl} \cdot W}{\eta_b \cdot \mu \cdot W + K_a \cdot \left(V_2^{\,2}\right) + f_{rl} \cdot W}\right] \qquad \text{(Eq. 2.39)}$$

$V_2 = 82.967 \qquad \frac{V_2}{1.467} = 56.56 \quad \frac{mi}{h}$

How fast will the car be travelling when it strikes the object on a 5% grade?

$G := 0.05$

$$S = \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln\left[\frac{\eta_b \cdot \mu \cdot W + K_a \cdot V_1^{\,2} + f_{rl} \cdot W + G \cdot W}{\eta_b \cdot \mu \cdot W + K_a \cdot \left(V_2^{\,2}\right) + f_{rl} \cdot W + G \cdot W}\right] \qquad \text{(Eq. 2.39)}$$

$V_2 = 80.176 \qquad \frac{V_2}{1.467} = 54.65 \quad \frac{mi}{h}$

Problem 2.23

<u>Determine the speed of the car just before it impacted the object.</u>

$V_1 := 75 \cdot \frac{5280}{3600}$ $V_1 = 110$ $\gamma_b := 1.04$ $g := 32.2$ (given)

$\eta_b := 0.90$ $f_{rl} := 0.015$

$\mu_m := 0.6$ $\mu_s := 0.3$ (Table 2.4)

$S_{al} := 200$ $S_{skid} := 100$ (given)

Find velocity of the car when it starts to skid

$$S_{al} = \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu_m + f_{rl} - 0.03\right)}$$ (Eq. 2.43)

$$V_2 := \sqrt{V_1^2 - \frac{2 \cdot S_{al} \cdot g \cdot \left(\eta_b \cdot \mu_m + f_{rl} - 0.03\right)}{\gamma_b}}$$

$V_2 = 74.82$ $V_2 \cdot \frac{3600}{5280} = 51.014$ $\frac{mi}{h}$

Vehicle's velocity at start of skid is $V_1 := 74.82$

Find velocity when the vehicle strikes the object

$$S_{skid} = \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu_s + f_{rl} - 0.03\right)}$$ (Eq. 2.43)

$$V_2 := \sqrt{V_1^2 - \frac{2 \cdot S_{skid} \cdot g \cdot \left(\eta_b \cdot \mu_s + f_{rl} - 0.03\right)}{\gamma_b}}$$

$V_2 = 63.396$

$V_2 \cdot \frac{3600}{5280} = 43.22$ $\frac{mi}{h}$

Problem 2.24

Determine if the driver should appeal the ticket.

$\mu := 0.6$ (for good, wet pavement, and slide value because of skidding)

$\gamma_b := 1.04$ $g := 32.2$

$V_2 := 40 \cdot \frac{5280}{3600}$ $V_2 = 58.667$ (given)

$\eta_b := 0.95$ $S := 200$ $f_{rl} := 0.015$

Solve for the initial velocity of the car using theoretical stopping distance

$$S = \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl} - 0.04\right)} \qquad V_1 := \sqrt{\frac{2 \cdot S \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl} - 0.04\right)}{\gamma_b} + V_2^2}$$ (Eq. 2.43)

$V_1 = 100.952$

$V_1 \cdot \frac{3600}{5280} = 68.83$ $\frac{mi}{h}$

No, the driver should not appeal the ticket as the initial velocity was higher than the speed limit, in addition to the road being wet.

Problem 2.25

Determine the shortest distance from the stalled car that the driver could apply the brakes and stop before hitting it.

$\eta_b := 0.90 \quad \gamma_b := 1.04 \quad f_{rl} := 0.013$ (given)

$V := 70 \cdot \frac{5280}{3600} \qquad V = 102.667$ ft/s

$S := 150 \qquad g := 32.2$

$\mu_{dry} := 1.0 \qquad \mu_{wet} := 0.9$ (Table 2.4)

$$S = \frac{\gamma_b \cdot \left(V_1^2 - 0\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu_{wet} + f_{rl} - 0.03\right)} \qquad V_1 := \sqrt{\frac{2 \cdot S \cdot g \cdot \left(\eta_b \cdot \mu_{wet} + f_{rl} - 0.03\right)}{\gamma_b}}$$ (Eq. 2.43)

$V_1 = 85.824 \qquad V_1 \cdot \frac{3600}{5280} = 58.516$ ft/s

Using this as final velocity, solve for distance to slow to this velocity

$$S = \frac{\gamma_b \cdot \left(V^2 - V_1^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu_{dry} + f_{rl} - 0.03\right)}$$ (Eq. 2.43)

$S = 58.062$

Add this distance to the 150 ft of wet pavement, $150 + S = 208.06$ ft

Problem 2.26

Determine the braking efficiency of car 1.

$\gamma_b := 1.04$ $g := 32.2$ $V := 60 \cdot 1.467$ $V = 88.02$ ft/s (given)

$t_{r1} := 2.5$ $t_{r2} := 2.0$ $\eta_{b2} := 0.75$ $\mu := 0.80$

$f_{rl} := 0.01\left(1 + \frac{V}{2 \cdot 147}\right)$ $f_{rl} = 0.013$ (Eq. 2.5)

Total stopping distance is perception/reaction distance plus braking distance

Set stopping distance of two cars equal to each other and solve for η_{b1}

$$V \cdot t_{r1} + \frac{\gamma_b \cdot V^2}{2 \cdot g} \cdot \left(\frac{1}{\eta_{b1} \cdot \mu + f_{rl}}\right) = V \cdot t_{r2} + \frac{\gamma_b \cdot V^2}{2 \cdot g} \cdot \left(\frac{1}{\eta_{b2} \cdot \mu + f_{rl}}\right)$$

$\eta_{b1} := \text{Find}(\eta_{b1})$ $\eta_{b1} \cdot 100 = 96.06$ %

Problem 2.27

Determine the student's associated perception reaction time.

$V_1 := 55 \cdot 1.467$ $V_1 = 80.685$ ft/s

$V_2 := 35 \cdot 1.467$ $V_2 = 51.345$ ft/s (given)

$g := 32.2$ $G := 0$ $a := 11.2$ ft/s^2

Solve for distance to slow from 55 mi/h to 35 mi/h

$$d := \frac{(V_1)^2 - (V_2)^2}{2 \cdot a}$$ (Eq. 2.45)

$d = 172.94$ ft

Subtract this distance from total distance to sign (600 ft) to find perception/reaction time

$d_s := 600$ (given)

$d_r := d_s - d$ $d_r := 600 - d$ $d_r = 427.06$

$t_r := \frac{d_r}{V_1}$ $t_r = 5.29$ sec (Eq. 2.49)

Problem 2.28

Comment on the student's claim.

Method 1:

$V_1 := 70 \cdot 1.467$ $\quad V_1 = 102.69$ ft/s (given)

$g := 32.2$ $\quad a := 11.2$

Find practical stopping distance

$$d := \frac{V_1^2}{2 \cdot a} \qquad d = 470.77$$ (Eq. 2.46)

Subtract this distance from the total sight distance and solve for perception/reaction time

$d_s := 590$

$d_r := d_s - d$ $\quad d_r = 119.23$

$$t_r := \frac{d_r}{V_1} \qquad t_r = 1.16 \text{ sec}$$ (Eq. 2.49)

This reaction time is well below the design value; therefore, the student's claim is unlikely.

Method 2:

$V_1 := 70 \cdot 1.467$

$g := 32.2$ $\quad a := 11.2$ $\quad G := 0$ $\quad t_r := 2.5$

Stopping sight distance = practical stopping distance plus perception/reaction distance

$$SSD := \frac{V_1^2}{2 \cdot g \cdot \left[\left(\frac{a}{g}\right) + G\right]} + V_1 \cdot t_r \qquad SSD = 727.49 \text{ ft}$$ (Eq. 2.47)

590 ft < 730 ft (required from Table 3.1) therefore 590 ft is not enough for 70 mi/h design speed.

Problem 2.29

Determine the grade of the road.

$V_1 := 55 \cdot 1.467$ $V_1 = 80.69$ ft/s (given)

$t_r := 2.5$ $d_s := 450$

Solve for distance travelled during braking (total distance minus perception/reaction)

$d_r := V_1 \cdot t_r$ $d_r = 201.71$ (Eq. 2.49)

$d := d_s - d_r$ $d = 248.29$ (Eq. 2.50)

$a := 11.2$ $g := 32.2$ (given)

Using practical stopping distance formula and solve for grade

$$d = \frac{(V_1)^2}{2 \cdot g \cdot \left(\frac{a}{g} + G\right)} \qquad G := \frac{V_1^2}{2 \cdot d \cdot g} - \frac{a}{g}$$ (Eq. 2.47)

$G = 0.059$ $G \cdot 100 = 5.93$ %

<u>Determine the driver's perception/reaction time before and after drinking.</u>

Problem 2.30

$V_1 := 55 \cdot 1.467$ ft/s $\quad g := 32.2 \quad a := 11.2$ (given)

while sober, $d_s := 520$

solve for perception/reaction time using total stopping distance formula

$$d := \frac{V_1^2}{2 \cdot a} \qquad \text{(Eq. 2.46)}$$

$$d_s := d_r + d \qquad d_r := d_s - d \qquad \text{(Eq. 2.50)}$$

$$d_r := V_1 \cdot t_r \qquad t_r := \frac{d_r}{V_1} \qquad \text{(Eq. 2.49)}$$

substituting Eqs. 2.46 and 2.50 into Eq. 2.49 gives

$$t_r := \frac{d_s}{V_1} - \frac{V_1}{2a} \qquad t_r = 2.84 \text{ sec} \qquad \text{(Eq. 2.50)}$$

after drinking, driver strikes the object at $V_2 := 35 \cdot 1.467$ ft/s

solve for perception/reaction time using total stopping distance formula

$$t_r := \frac{d_s}{V_1} - \frac{V_1^2 - V_2^2}{2a \cdot V_1} \qquad t_r = 4.3 \text{ sec} \qquad \text{(Eq. 2.50)}$$

Multiple Choice Problems

Determine the minimum tractive effort.

Problem 2.31

$C_D := 0.35$ $\quad A_f := 20 \ \ ft^2$ $\quad \rho := 0.002045 \ \frac{slugs}{ft^3}$ (given)

$V := 70 \cdot \left(\frac{5280}{3600}\right) \ \frac{ft}{s}$ $\quad W := 2000 \ \ lb$

$G := 0.05$

grade resistance

$R_g := 2000 \cdot G$ $\quad R_g = 100 \ \ lb$ (Eq. 2.9)

aerodynamic resistance

$R_a := \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2$ $\quad R_a = 75.44 \ \ lb$ (Eq. 2.3)

rolling resistance

$f_{rl} := 0.01 \cdot \left(1 + \frac{V}{147}\right)$ $\quad f_{rl} = 0.02$ (Eq. 2.5)

$R_{rl} := f_{rl} \cdot W$ $\quad R_{rl} = 33.97 \ \ lb$ (Eq. 2.6)

summation of resistances

$F := R_a + R_{rl} + R_g$ $\quad F = 209.41 \ lb$ (Eq. 2.2)

Alternative Answers:

1) Using mi/h instead of ft/s for velocity

$V := 70 \ \frac{mi}{h}$

$f_{rl} := 0.01 \cdot \left(1 + \frac{V}{147}\right)$ $\quad f_{rl} = 0.01$

$R_{rl} := f_{rl} \cdot W$ $\quad R_{rl} = 29.52 \ \ lb$

$R_a := \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2$ $\quad R_a = 35.07 \ \ lb$

$F := R_a + R_{rl} + R_g$ $\quad F = 164.6 \ \ lb$

2) not including aerodynamic resistance

$V := 70 \cdot \frac{5280}{3600}$

$F := R_{rl} + R_g$ $\quad F = 129.52 \ \ lb$

3) not including rolling resistance

$F := R_a + R_g$ $\quad F = 135.07 \ \ lb$

Determine the acceleration.

Problem 2.32

$V := 20 \cdot \frac{5280}{3600}$

$V = 29.33 \quad \frac{ft}{s}$

$\mu := 1$

$C_d := 0.3$

$A_f := 20 \quad ft^2$

$\rho := 0.002045 \quad \frac{slugs}{ft^3}$

$M_e := 95$ ft-lb

$r := \frac{14}{12} \quad ft$

$h := 20 \quad in$

$W := 2500 \quad lb$

$L := 110 \quad in$

$l_f := 50 \quad in$

$\varepsilon_o := 4.5$

$\eta_d := 0.90$

(given)

aerodynamic resistnace

$R_a := \frac{\rho}{2} \cdot C_d \cdot A_f \cdot V^2 \qquad R_a = 5.28 \quad lb$ (Eq. 2.3)

rolling resistance

$f_{rl} := 0.01 \left(1 + \frac{V}{147}\right)$ (Eq. 2.5)

$R_{rl} := 0.01 \cdot \left(1 + \frac{V}{147}\right) \cdot 2500 \qquad R_{rl} = 29.99 \quad lb$ (Eq. 2.6)

engine-generated tractive effort

$F_e := \frac{M_e \cdot \varepsilon_o \cdot \eta_d}{r} \qquad F_e = 329.79 \quad lb$ (Eq. 2.17)

mass factor

$\gamma_m := 1.04 + 0.0025 \varepsilon_o^2 \qquad \gamma_m = 1.09$ (Eq. 2.20)

$l_r := 120 - l_f$

acceleration

$F_{max} := \frac{\frac{\mu \cdot W \cdot (l_r + f_{rl} \cdot h)}{L}}{1 + \frac{\mu \cdot h}{L}} \qquad F_{max} = 1350.77 \quad lb$ (Eq. 2.15)

$a := \frac{F_e - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{2500}{32.2}\right)} \qquad a = 3.48 \quad \frac{ft}{s^2}$ (Eq. 2.19)

Alternative Answers:

1) Use a mass factor of 1.04 $\gamma_m := 1.04$ $a := \frac{F_e - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{2500}{32.2}\right)}$ $a = 3.65 \frac{ft}{s^2}$

2) Use F_{max} instead of F_e $\gamma_m := 1.091$ $a := \frac{F_{max} - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{2500}{32.2}\right)}$ $a = 15.53 \frac{ft}{s^2}$

3) Rear wheel instead of front wheel drive

$$F_{max} := \frac{\frac{\mu \cdot W \cdot \left(l_f - f_{rl} \cdot h\right)}{L}}{1 - \frac{\mu \cdot h}{L}}$$ $F_{max} = 1382.22$ $a := \frac{F_{max} - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{2500}{32.2}\right)}$ $a = 15.9 \frac{ft}{s^2}$

Determine the percentage of braking force.

Problem 2.33

$V := 65 \cdot \frac{5280}{3600} \frac{ft}{s}$ $\mu := 0.90$

$L := 120$ in $l_f := 50$ in (given)

$h := 20$ in $l_r := L - l_f$ in

determine the coefficient of rolling resistance

$f_{rl} := 0.01 \cdot \left(1 + \frac{V}{147}\right)$ $f_{rl} = 0.02$ (Eq. 2.5)

determine the brake force ratio

$BFR_{frmax} := \frac{l_r + h \cdot (\mu + f_{rl})}{l_f - h \cdot (\mu + f_{rl})}$ $BFR_{frmax} = 2.79$ (Eq. 2.30)

calculate percentage of braking force allocated to rear axle

$PBF_r := \frac{100}{1 + BFR_{frmax}}$ $PBF_r = 26.39$ % (Eq. 2.32)

Alternative Answers:

1) Use front axle equation

$PBF_f := 100 - \frac{100}{1 + BFR_{frmax}}$ $PBF_f = 73.61$ % (Eq 2.31)

2) Use incorrect brake force ratio equation

$BFR_{frmax} := \frac{l_r - h \cdot (\mu + f_{rl})}{l_f + h \cdot (\mu + f_{rl})}$ $BFR_{frmax} = 0.76$

$PBF_r := \frac{100}{1 + BFR_{frmax}}$ $PBF_r = 56.94$ %

3) Switch l_f and l_r in brake force ratio equation

$BFR_{frmax} := \frac{l_f + h \cdot (\mu + f_{rl})}{l_r - h \cdot (\mu + f_{rl})}$ $BFR_{frmax} = 1.32$

$PBF_r := \frac{100}{1 + BFR_{frmax}}$ $PBF_r = 43.06$ %

Determine the theoretical stopping distance on level grade.

Problem 2.34

$C_D := 0.59$ $\quad V := 80 \cdot \frac{5280}{3600} \frac{ft}{s}$ (given)

$A_f := 26\ ft^2$ $\quad \mu := 0.7$

$\gamma_b := 1.04$ $\quad \eta_b := 0.75$ (assumed values)

Coefficient of Rolling Resistance

$$f_{rl} := 0.01 \cdot \left(1 + \frac{\frac{V}{2}}{147}\right) \qquad f_{rl} = 0.014$$ (Eq. 2.5)

Theoretical Stopping Distance

$$S := \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl}\right)} \qquad S = 412.8 \frac{s^2}{ft}$$ (Eq. 2.43)

Alternative Answers:

1) Not dividing the velocity by 2 for the coeffeicent of rolling resistance

$$f_{rl} := 0.01 \cdot \left(1 + \frac{V}{147}\right)$$

$$S := \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl}\right)} \qquad S = 409.8 \frac{s^2}{ft}$$

2) Using mi/h instead of ft/s for the velocity

$V_1 := 80 \quad V_2 := 0 \quad V := 80$

$$f_{rl} := 0.01 \cdot \left(1 + \frac{\frac{V}{2}}{147}\right)$$

$$S := \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl}\right)} \qquad S = 192.4 \frac{s^2}{ft}$$

3) Using γ = 1.0 value

$\gamma_b := 1.0$

$$S := \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl}\right)} \qquad S = 397.9 \frac{s^2}{ft}$$

Determine the stopping sight distance.

Problem 2.35

$V := 45 \cdot \frac{5280}{3600}$ ft/s (given)

$a := 11.2 \ \frac{ft}{s^2}$ $t_r := 2.5 \ s$ $g := 32.2 \ \frac{ft}{s^2}$ (assumed)

Braking Distance

$$d := \frac{V^2}{2 \cdot g \cdot \left(\frac{a}{g}\right)} \qquad d = 194.46 \ ft \qquad \text{(Eq. 2.47)}$$

Perception/Reaction Distance

$d_r := V \cdot t_r$ $d_r = 165.00$ ft (Eq. 2.49)

Total Stopping Distance

$d_s := d + d_r$ $d_s = 359.46$ ft (Eq. 2.50)

Alternative Answers:

1) just the braking distance value

$d = 194.46$ ft

2) just the perception/reaction distance value

$d_r = 165.00$ ft

3) use the yellow signal interval deceleration rate

$a := 10.0$

$d := \frac{V^2}{2 \cdot g \cdot \left(\frac{a}{g}\right)}$ $d = 217.80$ ft

$d_s := d + d_r$ $d_s = 382.80$ ft

Determine the vehicle speed.

Problem 2.36

$C_D := 0.35$ $G := 0.04$ $\gamma_b := 1.04$

$A_f := 16$ ft^2 $S := 150$ ft $\eta_b := 1$ (given)

$W := 2500$ lb $\rho := 0.002378 \frac{slugs}{ft^3}$ $\mu := 0.8$

$V_1 := 88 \cdot \frac{5280}{3600} \frac{ft}{s}$ $g := 32.2 \frac{ft}{s^2}$ $f_{rl} := 0.017$

$K_a := \frac{\rho}{2} \cdot C_D \cdot A_f$ $K_a = 0.007$

Given

$V_2 := 0$

$$S = \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln\left[\frac{\eta_b \cdot \mu \cdot W + K_a \cdot V_1^2 + f_{rl} \cdot W + W \cdot G}{\eta_b \cdot \mu \cdot W + K_a \cdot \left(V_2^2\right) + f_{rl} \cdot W + W \cdot G}\right]$$ (Eq. 2.39)

$V_2 := \mathrm{Find}(V_2)$

$V_2 = 91.6$ $\frac{V_2}{1.467} = 62.43$ $\frac{mi}{h}$

Alternative Answers:

1) Use 0% grade

$G := 0.0$

Given

$V_2 := 0$

$$S = \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln\left[\frac{\eta_b \cdot \mu \cdot W + K_a \cdot V_1^2 + f_{rl} \cdot W + W \cdot G}{\eta_b \cdot \mu \cdot W + K_a \cdot \left(V_2^2\right) + f_{rl} \cdot W + W \cdot G}\right]$$

$V_2 := \text{Find}(V_2)$

$V_2 = 93.6$ $\qquad \frac{V_2}{1.467} = 63.78 \qquad \frac{mi}{h}$

2) Ignoring aerodynamic resistance

$G := 0.04$

$$V_2 := \sqrt{V_1^2 - \frac{S \cdot 2 \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl} + G\right)}{\gamma_b}} \qquad V_2 = 93.3$$

(Eq 2.43) rearranged to solve for V_2

$\frac{V_2}{1.467} = 63.6 \qquad \frac{mi}{h}$

3) Ignoring aerodynamic resistance and using G = 0

$G := 0$

$$V_2 := \sqrt{V_1^2 - \frac{S \cdot 2 \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl} + G\right)}{\gamma_b}} \qquad V_2 = 95.2$$

$\frac{V_2}{1.467} = 64.9 \qquad \frac{mi}{h}$