CHAPTER 2
How to Calculate Present Values

Answers to Problem Sets

1. If the discount factor is .507, then $.507*1.12^6 = \$1$

2. 125/139 = .899

3. $PV = 374/(1.09)^9 = 172.20$

4. $PV = 432/1.15 + 137/(1.15^2) + 797/(1.15^3) = 376 + 104 + 524 = \$1,003$

5. $FV = 100*1.15^8 = \$305.90$

6. NPV = -1,548 + 138/.09 = -14.67 (cost today plus the present value of the perpetuity)

7. PV = 4/(.14-.04) = $40

8. a. PV = 1/.10 = $10

   b. Since the perpetuity will be worth $10 in year 7, and since that is roughly double the present value, the approximate PV equals $5.
   $PV = (1 / .10)/(1.10)^7 = 10/2 = \$5$ (approximately)

   c. A perpetuity paying $1 starting now would be worth $10, whereas a perpetuity starting in year 8 would be worth roughly $5. The difference between these cash flows is therefore approximately $5. PV = 10 – 5= $5 (approximately)

   d. PV = C/(r-g) = 10,000/(.10-.05) = $200,000.

9. a. $PV = 10,000/(1.05^5) = \$7,835.26$ (assuming the cost of the car does not appreciate over those five years).

   b. You need to set aside (12,000 × 6-year annuity factor) = 12,000 × 4.623 =

$55,476.

c. At the end of 6 years you would have $1.08^6 \times (60{,}476 - 55{,}476) = \$7{,}934$.

10. a. $FV = 1{,}000e^{.12x5} = 1{,}000e^{.6} = \$1{,}822.12$.

b. $PV = 5e^{-.12 \text{ x } 8} = 5e^{-.96} = \$1.914$ million

c. $PV = C\,(1/r - 1/re^{rt}) = 2{,}000(1/.12 - 1/.12e^{.12 \text{ x} 15}) = \$13{,}912$

11.

a. $FV = 10{,}000{,}000x(1.06)^4 = 12{,}624{,}770$

b. $FV = 10{,}000{,}000x(1 + .06/12)^{(4x12)} = 12{,}704{,}892$

c. $FV = 10{,}000{,}000xe^{(4x.06)} = 12{,}712{,}492$

12.

a. $PV = \$100/1.01^{10} = \$90.53$

b. $PV = \$100/1.13^{10} = \$29.46$

c. $PV = \$100/1.25^{15} = \$ 3.52$

d. $PV = \$100/1.12 + \$100/1.12^2 + \$100/1.12^3 = \$240.18$

13. a. $DF_1 = \dfrac{1}{1+r_1} = 0.905 \Rightarrow r_1 = 0.1050 = 10.50\%$

b. $DF_2 = \dfrac{1}{(1+r_2)^2} = \dfrac{1}{(1.105)^2} = 0.819$

c. $AF_2 = DF_1 + DF_2 = 0.905 + 0.819 = 1.724$

d. PV of an annuity = $C \times$ [Annuity factor at r% for t years]

Here:

$\$24.65 = \$10 \times [AF_3]$

$AF_3 = 2.465$

e. $AF_3 = DF_1 + DF_2 + DF_3 = AF_2 + DF_3$

$2.465 = 1.724 + DF_3$

$DF_3 = 0.741$

14. The present value of the 10-year stream of cash inflows is:

$$PV = \$170{,}000 \times \left[\frac{1}{0.14} - \frac{1}{0.14 \times (1.14)^{10}}\right] = \$886{,}739.66$$

Thus:

$$NPV = -\$800{,}000 + \$886{,}739.66 = +\$86{,}739.66$$

At the end of five years, the factory's value will be the present value of the five remaining $170,000 cash flows:

$$PV = \$170{,}000 \times \left[\frac{1}{0.14} - \frac{1}{0.14 \times (1.14)^{5}}\right] = \$583{,}623.76$$

15.

$$NPV = \sum_{t=0}^{10} \frac{C_t}{(1.12)^t} = -\$380{,}000 + \frac{\$50{,}000}{1.12} + \frac{\$57{,}000}{1.12^2} + \frac{\$75{,}000}{1.12^3} + \frac{\$80{,}000}{1.12^4} + \frac{\$85{,}000}{1.12^5}$$

$$+ \frac{\$92{,}000}{1.12^6} + \frac{\$92{,}000}{1.12^7} + \frac{\$80{,}000}{1.12^8} + \frac{\$68{,}000}{1.12^9} + \frac{\$50{,}000}{1.12^{10}} = \$23{,}696.15$$

16. a. Let S_t = salary in year t

$$PV = \sum_{t=1}^{30} \frac{40{,}000\,(1.05)^{t-1}}{(1.08)^t}$$

$$= 40{,}000 \times \left[\frac{1}{(.08 - .05)} - \frac{(1.05)^{30}}{(.08 - .05) \times (1.08)^{30}}\right] = \$760{,}662.53$$

b. PV(salary) x 0.05 = $38,033.13

Future value = $38,018.96 x $(1.08)^{30}$ = $382,714.30

c.

$$PV = C \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^t} \right]$$

$$\$382{,}714.30 = C \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{20}} \right]$$

$$C = \$382{,}714.30 \bigg/ \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{20}} \right] = \$38{,}980.30$$

17.

Period		Present Value
0		–400,000.00
1	+100,000/1.12 =	+ 89,285.71
2	+200,000/1.12^2 =	+159,438.78
3	+300,000/1.12^3 =	+213,534.07
	Total = NPV =	$62,258.56

18. We can break this down into several different cash flows, such that the sum of these separate cash flows is the total cash flow. Then, the sum of the present values of the separate cash flows is the present value of the entire project. (All dollar figures are in millions.)

- Cost of the ship is $8 million
  PV = –$8 million
- Revenue is $5 million per year, operating expenses are $4 million. Thus, operating cash flow is $1 million per year for 15 years.

$$PV = \$1 \text{ million} \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{15}} \right] = \$8.559 \text{ million}$$

- Major refits cost $2 million each, and will occur at times t = 5 and t = 10.
  PV = (–$2 million)/1.08^5 + (–$2 million)/1.08^{10} = –$2.288 million
- Sale for scrap brings in revenue of $1.5 million at t = 15.
  PV = $1.5 million/1.08^{15} = $0.473 million

Adding these present values gives the present value of the entire project:

NPV = –$8 million + $8.559 million – $2.288 million + $0.473 million
NPV = –$1.256 million

19. a. PV = $100,000

b. PV = $180,000/1.12^5 = $102,136.83

c. PV = $11,400/0.12 = $95,000

d. $$PV = \$19{,}000 \times \left[\frac{1}{0.12} - \frac{1}{0.12 \times (1.12)^{10}}\right] = \$107{,}354.24$$

e. PV = $6,500/(0.12 – 0.05) = $92,857.14

Prize (d) is the most valuable because it has the highest present value.

20. Mr. Basset is buying a security worth $20,000 now. That is its present value. The unknown is the annual payment. Using the present value of an annuity formula, we have:

$$PV = C \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^t}\right]$$

$$\$20{,}000 = C \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{12}}\right]$$

$$C = \$20{,}000 \Big/ \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{12}}\right] = \$2{,}653.90$$

21. Assume the Zhangs will put aside the same amount each year. One approach to solving this problem is to find the present value of the cost of the boat and then equate that to the present value of the money saved. From this equation, we can solve for the amount to be put aside each year.

PV(boat) = $20,000/(1.10)^5 = $12,418

$$\text{PV(savings)} = \text{Annual savings} \times \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^5}\right]$$

Because PV(savings) must equal PV(boat):

$$\text{Annual savings} \times \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^5}\right] = \$12{,}418$$

$$\text{Annual savings } = \$12{,}418 \Bigg/ \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^5}\right] = \$3{,}276$$

Another approach is to use the future value of an annuity formula:

$$\text{Annual savings} \times \left[\frac{(1+.10)^5 - 1}{.10}\right] = \$20{,}000$$

Annual savings = $ 3,276

22. The fact that Kangaroo Autos is offering "free credit" tells us what the cash payments are; it does not change the fact that money has time value. A 10% annual rate of interest is equivalent to a monthly rate of 0.83%:

$$r_{monthly} = r_{annual} /12 = 0.10/12 = 0.0083 = 0.83\%$$

The present value of the payments to Kangaroo Autos is:

$$\$1{,}000 + \$300 \times \left[\frac{1}{0.0083} - \frac{1}{0.0083 \times (1.0083)^{30}}\right] = \$8{,}938$$

A car from Turtle Motors costs $9,000 cash. Therefore, Kangaroo Autos offers the better deal, i.e., the lower present value of cost.

23. The NPVs are:

$$\text{at } 5\% \quad \Rightarrow \text{NPV} = -\$170{,}000 - \frac{\$100{,}000}{1.05} + \frac{\$320{,}000}{(1.05)^2} = \$25{,}011$$

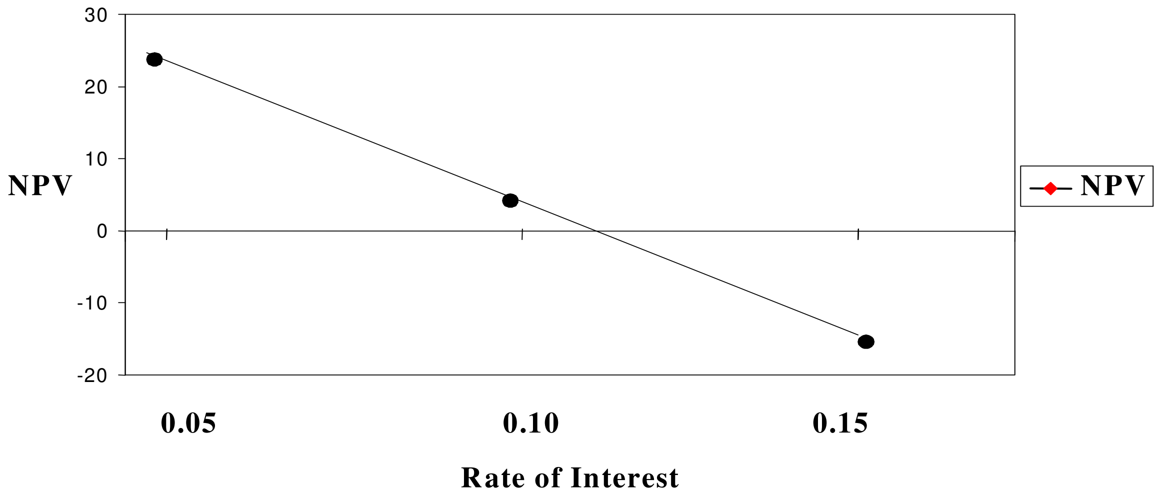
$$\text{at } 10\% \quad \Rightarrow \text{NPV} = -\$170{,}000 - \frac{\$100{,}000}{1.10} + \frac{320{,}000}{(1.10)^2} = \$3{,}554$$

$$\text{at } 15\% \quad \Rightarrow \text{NPV} = -\$170{,}000 - \frac{\$100{,}000}{1.15} + \frac{320{,}000}{(1.15)^2} = -\$14{,}991$$

The figure below shows that the project has zero NPV at about 11%.

As a check, NPV at 11% is:

$$NPV = -\$170{,}000 - \frac{\$100{,}000}{1.11} + \frac{320{,}000}{(1.11)^2} = -\$371$$



24. a. This is the usual perpetuity, and hence:

$$PV = \frac{C}{r} = \frac{\$100}{0.07} = \$1{,}428.57$$

b. This is worth the PV of stream (a) *plus* the immediate payment of $100:

PV = $100 + $1,428.57 = $1,528.57

c. The continuously compounded equivalent to a 7% annually compounded rate is approximately 6.77%, because:

$$e^{0.0677} = 1.0700$$

Thus:

$$PV = \frac{C}{r} = \frac{\$100}{0.0677} = \$1{,}477.10$$

Note that the pattern of payments in part (b) is more valuable than the pattern of payments in part (c). It is preferable to receive cash flows at the start of every year than to spread the receipt of cash evenly over the year; with the former pattern of payment, you receive the cash more quickly.

25. a. PV = \$1 billion/0.08 = \$12.5 billion

b. PV = \$1 billion/(0.08 – 0.04) = \$25.0 billion

c. $$PV = \$1 \text{ billion} \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{20}}\right] = \$9.818 \text{ billion}$$

d. The continuously compounded equivalent to an 8% annually compounded rate is approximately 7.7% , because:

$e^{0.0770} = 1.0800$

Thus:

$$PV = \$1 \text{ billion} \times \left[\frac{1}{0.077} - \frac{1}{0.077 \times e^{(0.077)(20)}}\right] = \$10.203 \text{ billion}$$

This result is greater than the answer in Part (c) because the endowment is now earning interest during the entire year.

26. With annual compounding: FV = \$100 $\times$ $(1.15)^{20}$ = \$1,636.65

With continuous compounding: FV = \$100 $\times$ $e^{(0.15\times20)}$ = \$2,008.55

27. One way to approach this problem is to solve for the present value of:

(1) \$100 per year for 10 years, and
(2) \$100 per year in perpetuity, with the first cash flow at year 11.

If this is a fair deal, these present values must be equal, and thus we can solve for the interest rate (r).

The present value of \$100 per year for 10 years is:

$$PV = \$100 \times \left[\frac{1}{r} - \frac{1}{(r) \times (1+r)^{10}}\right]$$

The present value, as of year 10, of \$100 per year forever, with the first payment in year 11, is: PV_{10} = \$100/r

At t = 0, the present value of PV_{10} is:

$$PV = \left[\frac{1}{(1+r)^{10}}\right] \times \left[\frac{\$100}{r}\right]$$

Equating these two expressions for present value, we have:

$$\$100 \times \left[\frac{1}{r} - \frac{1}{(r) \times (1+r)^{10}}\right] = \left[\frac{1}{(1+r)^{10}}\right] \times \left[\frac{\$100}{r}\right]$$

Using trial and error or algebraic solution, we find that r = 7.18%.

28. Assume the amount invested is one dollar.
Let A represent the investment at 12%, compounded annually.
Let B represent the investment at 11.7%, compounded semiannually.
Let C represent the investment at 11.5%, compounded continuously.

After one year:

$FV_A = \$1 \times (1 + 0.12)^1 = \$1.1200$

$FV_B = \$1 \times (1 + 0.0585)^2 = \$1.1204$

$FV_C = \$1 \times e^{(0.115 \times 1)} = \$1.1219$

After five years:

$FV_A = \$1 \times (1 + 0.12)^5 = \$1.7623$

$FV_B = \$1 \times (1 + 0.0585)^{10} = \$1.7657$

$FV_C = \$1 \times e^{(0.115 \times 5)} = \$1.7771$

After twenty years:

$FV_A = \$1 \times (1 + 0.12)^{20} = \$9.6463$

$FV_B = \$1 \times (1 + 0.0585)^{40} = \$9.7193$

$FV_C = \$1 \times e^{(0.115 \times 20)} = \$9.9742$

The preferred investment is C.

29. Because the cash flows occur every six months, we first need to calculate the equivalent semi-annual rate. Thus, $1.08 = (1 + r/2)^2$ => r = 7.85 semi-annually compounded APR. Therefore the rate for six months is 7.85/2 or 3.925%:

$$PV = \$100{,}000 + \$100{,}000 \times \left[\frac{1}{0.03925} - \frac{1}{0.03925 \times (1.03925)^9}\right] = \$846{,}081$$

30. a. Each installment is: \$9,420,713/19 = \$495,827

$$PV = \$495{,}827 \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{19}}\right] = \$4{,}761{,}724$$

b. If ERC is willing to pay \$4.2 million, then:

$$\$4{,}200{,}000 = \$495{,}827 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{19}}\right]$$

Using Excel or a financial calculator, we find that r = 9.81%.

31. a. $$PV = \$70{,}000 \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{8}}\right] = \$402{,}264.73$$

b.

Year	Beginning-of-Year Balance	Year-end Interest on Balance	Total Year-end Payment	Amortization of Loan	End-of-Year Balance
1	402,264.73	32,181.18	70,000.00	37,818.82	364,445.91
2	364,445.91	29,155.67	70,000.00	40,844.33	323,601.58
3	323,601.58	25,888.13	70,000.00	44,111.87	279,489.71
4	279,489.71	22,359.18	70,000.00	47,640.82	231,848.88
5	231,848.88	18,547.91	70,000.00	51,452.09	180,396.79
6	180,396.79	14,431.74	70,000.00	55,568.26	124,828.54
7	124,828.54	9,986.28	70,000.00	60,013.72	64,814.82
8	64,814.82	5,185.19	70,000.00	64,814.81	0.01

32. This is an annuity problem with the present value of the annuity equal to \$2 million (as of your retirement date), and the interest rate equal to 8% with 15 time periods. Thus, your annual level of expenditure (C) is determined as follows:

$$PV = C \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{t}}\right]$$

$$\$2{,}000{,}000 = C \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{15}}\right]$$

$$C = \$2{,}000{,}000 \Big/ \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{15}}\right] = \$233{,}659$$

With an inflation rate of 4% per year, we will still accumulate \$2 million as of our retirement date. However, because we want to spend a constant amount per year in real terms (R, constant for all t), the nominal amount (C_t) must increase each year. For each year t: $R = C_t/(1 + \text{inflation rate})^t$

Therefore:

$PV\ [\text{all } C_t] = PV\ [\text{all } R \times (1 + \text{inflation rate})^t] = \$2{,}000{,}000$

$$R\times\left[\frac{(1+0.04)^{1}}{(1+0.08)^{1}}+\frac{(1+0.04)^{2}}{(1+0.08)^{2}}+\ldots+\frac{(1+0.04)^{15}}{(1+0.08)^{15}}\right]=\$2{,}000{,}000$$

$R \times [0.9630 + 0.9273 + \ . \ . \ . \ + 0.5677] = \$2{,}000{,}000$

$R \times 11.2390 = \$2{,}000{,}000$

$R = \$177{,}952$

Alternatively, consider that the real rate is $\frac{(1+0.08)}{(1+0.04)}-1=.03846$. Then, redoing the steps above using the real rate gives a real cash flow equal to:

$$C=\$2{,}000{,}000\Big/\left[\frac{1}{0.03846}-\frac{1}{0.03846\times(1.03846)^{15}}\right]=\$177{,}952$$

Thus C_1 = (\$177,952 × 1.04) = \$185,070, C_2 = \$192,473, etc.

33. a. $$PV=\$50{,}000\times\left[\frac{1}{0.055}-\frac{1}{0.055\times(1.055)^{12}}\right]=\$430{,}925.89$$

b. The annually compounded rate is 5.5%, so the semiannual rate is:

$(1.055)^{(1/2)} - 1 = 0.0271 = 2.71\%$

Since the payments now arrive six months earlier than previously:

PV = \$430,925.89 × 1.0271 = \$442,603.98

34. In three years, the balance in the mutual fund will be:

$FV = \$1{,}000{,}000 \times (1.035)^{3} = \$1{,}108{,}718$

The monthly shortfall will be: \$15,000 – (\$7,500 + \$1,500) = \$6,000

Annual withdrawals from the mutual fund will be: \$6,000 × 12 = \$72,000

Assume the first annual withdrawal occurs three years from today, when the balance in the mutual fund will be \$1,108,718. Treating the withdrawals as an annuity due, we solve for t as follows:

$$PV=C\times\left[\frac{1}{r}-\frac{1}{r\times(1+r)^{t}}\right]\times(1+r)$$

$$\$1{,}108{,}718=\$72{,}000\times\left[\frac{1}{0.035}-\frac{1}{0.035\times(1.035)^{t}}\right]\times1.035$$

Using Excel or a financial calculator, we find that t = 22.5 years.

35. a. PV = 2/.12 = $16.667 million

b. $PV = \$2 \times \left[\frac{1}{0.12} - \frac{1}{0.12 \times (1.12)^{20}}\right] = \$14.939$ million

c. PV = 2/(.12-.03) = $22.222 million

d. $PV = \$2 \times \left[\frac{1}{(0.12-.03)} - \frac{1.03^{20}}{(0.12-.03) \times (1.12)^{20}}\right] = \$18.061$ million

36. a. Using the Rule of 72, the time for money to double at 12% is 72/12, or 6 years. More precisely, if x is the number of years for money to double, then:

$(1.12)^x = 2$

Using logarithms, we find:

x (ln 1.12) = ln 2

x = 6.12 years

b. With continuous compounding for interest rate r and time period x:

$e^{r\,x} = 2$

Taking the natural logarithm of each side:

r x = ln(2) = 0.693

Thus, if r is expressed as a percent, then x (the time for money to double) is: x = 69.3/(interest rate, in percent).

37. Spreadsheet exercise.

38. a. This calls for the growing perpetuity formula with a negative growth rate (g = –0.04):

$$PV = \frac{\$2 \text{ million}}{0.10 - (-0.04)} = \frac{\$2 \text{ million}}{0.14} = \$14.29 \text{ million}$$

b. The pipeline's value at year 20 (i.e., at t = 20), assuming its cash flows last forever, is:

$$PV_{20} = \frac{C_{21}}{r - g} = \frac{C_1 (1 + g)^{20}}{r - g}$$

With C_1 = \$2 million, g = –0.04, and r = 0.10:

$$PV_{20} = \frac{(\$2\ \text{million}) \times (1 - 0.04)^{20}}{0.14} = \frac{\$0.884\ \text{million}}{0.14} = \$6.314\ \text{million}$$

Next, we convert this amount to PV today, and subtract it from the answer to Part (a):

$$PV = \$14.29\,\text{million} - \frac{\$6.314\,\text{million}}{(1.10)^{20}} = \$13.35\,\text{million}$$

Principles of Corporate Finance, 2nd CONCISE Edition
Spreadsheet Templates - SOLUTION
MAIN MENU -- Chapter 3

Principles of Corporate Finance

Main Menu

2nd CONCISE Edition

Instructions

Navigating the Workbook

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Each chapter of the spreadsheets to accompany Principles of Corporate Finance contains links to help you navigate the workbook. These hyperlinks help you move around the workbook quickly. The Main Menu contains links to each problem from the chapter that contains the Excel icon. From the Main Menu, click on the question you wish to complete. You can always return to the main menu by clicking on the link located in the upper right corner of each worksheet.

You can move quickly around an Excel workbook by selecting the worksheet tab at the bottom of the screen. Each worksheet in an Excel workbook will have its own tab. In the spreadsheets to accompany Principles of Corporate Finance, you will see a separate tab for each problem, along with the Main Menu, Instructions and Help Topics worksheets.

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Entering your information

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For each question, you will see the following lists and boxes:

Student Name:	
Course Name:	
Student ID:	
Course Number:	

Enter your information in these cells before submitting your work.

Entering data

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To enter numbers or text for these questions, click the cell you want, type the data and press ENTER or TAB. Press ENTER to move down the column or TAB to move across the row.

For cells or columns where you want to enter text, select “Format,” and then “Cells” from Excel’s main menu at the top of your screen. Select the “Number” tab and then “Text” from the category list.

Printing

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Help

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There are two sources of help throughout these spreadsheet templates. First, you will find comments in specific cells (highlighted in red) providing tips to what formula or function is needed to complete the problem. Second, you will find links to Microsoft Office's online help page when an Excel Function is needed to complete the problem.

Enter the values in blue colored cells

Chapter 3

Question 3

Student Name:	SOLUTION
Course Name:	
Student ID:	
Course Number:	

Use Excel's PRICE function to find the value of the bond under the following assumptions:

Settlement Date	2/15/2009
Maturity Date	2/15/2026
Coupon Rate	0.06
YTM	0.035965
Price	130.37

For help with Excel's PRICE function

Enter the values in blue colored cells

Chapter 3
Question 4

Student Name:	SOLUTION
Course Name:	
Student ID:	
Course Number:	

Use Excel's YIELD function to find the YTM of the bond under each of the above assumptions:

Coupon Rate	2%	4%	8%
Price (%)	81.62	98.39	133.42
Settlement Date	8/15/2006	8/15/2006	8/15/2006
Maturity Date	8/15/2016	8/15/2016	8/15/2016
YTM	4.3%	4.2%	3.9%

For help with Excel's YIELD function

Enter the values in blue colored cells

Chapter 3
Question 7

Student Name:	SOLUTION
Course Name:	
Student ID:	
Course Number:	

	Year					
	1	**2**	**3**	**4**	**Bond Price**	**YTM**
Spot rate	4%	4%	4%	4%		
Discount factor	0.961538462	0.924556213	0.888996	0.8548042		
Bond A (8% coupon)						
Payment	80	1080				
Present Value	76.92307692	998.5207101			1075.443787	4.00%
Bond B (11% coupon)						
Payment	110	110	1110			
Present Value	105.7692308	101.7011834	986.786		1194.256372	4.00%
Bond C (6% coupon)						
Payment	60	60	60	1060		
Present Value	57.69230769	55.47337278	53.33978	906.09244	1072.597904	4.00%
Bond D						
Payment				1000		
Present Value				854.80419	854.804191	16.98%

Enter the values in blue colored cells

Chapter 3
Question 12

Student Name:	SOLUTION
Course Name:	
Student ID:	
Course Number:	

Use the model below to find the duration and volatiltiy for each security.

Security A

Period	Ct	PV(Ct)	Proportion of Total Value	Proportion of Total Value x Time	
1	40	37.04	0.359	0.359	
2	40	34.29	0.333	0.665	
3	40	31.75	0.308	0.924	
	V =	103.08	1.000	1.949	= Duration (years)
				1.804	= Volatility

Note:
Yield % 8%

Security B

Period	Ct	PV(Ct)	Proportion of Total Value	Proportion of Total Value x Time	
1	20	18.52	0.141	0.141	
2	20	17.15	0.131	0.262	
3	120	95.26	0.728	2.183	
	V =	130.93	1.000	2.586	= Duration (years)
				2.395	= Volatility

Note:
Yield % 8%

Security C

Period	Ct	PV(Ct)	Proportion of Total Value	Proportion of Total Value x Time	
1	10	9.26	0.088	0.088	
2	10	8.57	0.082	0.163	
3	110	87.32	0.830	2.491	
	V =	105.15	1.000	2.742	= Duration (years)
				2.539	= Volatility

Note:
Yield % 8%

Enter the values in blue colored cells

Chapter 3
Question 15

Student Name:	SOLUTION
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Course Number:	

Maturity in Years	10
Settlement date	1/1/2010
Maturity Date	1/1/2020
Face Value	100
Coupon	5%
Market rate	6%
Annual Payment	5

Bond's PV	92.63991295

For help with Excel's PRICE fu

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Enter the values in blue colored cells

Chapter 3
Question 16

Student Name:	SOLUTION
Course Name:	
Student ID:	
Course Number:	

Interest Payment	$275.00
Annuity Factor	15.44
PV of Interest Payments	$4,246.80
PV of Face Value	$5,984.84
Value of Bond	$10,231.64

For help with Excel's PV function

Interest Rate	Annuity Factor	PV of Interest Pmt	PV of Face Value	PV of Bond
1%	$18.99	$5,221.54	$9,050.63	$ 14,272.17
2%	$18.05	$4,962.53	$8,195.44	$ 13,157.97
3%	$17.17	$4,721.38	$7,424.70	$ 12,146.08
4%	$16.35	$4,496.64	$6,729.71	$ 11,226.36
5%	$15.59	$4,287.02	$6,102.71	$ 10,389.73
6%	$14.88	$4,091.31	$5,536.76	$ 9,628.06
7%	$14.21	$3,908.41	$5,025.66	$ 8,934.07
8%	$13.59	$3,737.34	$4,563.87	$ 8,301.21
9%	$13.01	$3,577.18	$4,146.43	$ 7,723.61
10%	$12.46	$3,427.11	$3,768.89	$ 7,196.00
11%	$11.95	$3,286.36	$3,427.29	$ 6,713.64
12%	$11.47	$3,154.23	$3,118.05	$ 6,272.28
13%	$11.02	$3,030.09	$2,837.97	$ 5,868.06
14%	$10.59	$2,913.35	$2,584.19	$ 5,497.54
15%	$10.19	$2,803.49	$2,354.13	$ 5,157.62

Enter the values in blue colored cells

Chapter 3

Question 17

Student Name:	SOLUTION
Course Name:	
Student ID:	
Course Number:	

a.

	Now	One Year Later
Interest Payment	50	50
Annuity Factor	5.417191444	4.579707187
PV of Interest Payments	270.8595722	228.9853594
PV of Face Value	837.4842567	862.6087844
Value of Bond	1,108.34	1,091.59
Rate of return	3.00%	

b.

	Now	One Year Later
Interest Payment	50	50
Annuity Factor	5.417191444	4.713459509
PV of Interest Payments	270.8595722	235.6729754
PV of Face Value	837.4842567	905.7308098
Value of Bond	1,108.34	1,141.40
Rate of return	7.49%	

Enter the values in blue colored cells

Chapter 3
Question 18

Student Name:	SOLUTION
Course Name:	
Student ID:	
Course Number:	

Use Excel's PRICE function to calculate the value of each bond.
Assume today's settlement date and a maturity date six years hence.

For help with the PRICE fu

Bond	YTM	Current Price	Settlement Date	Maturity Date
6 % Coupon	12%	75.33155606	8/1/2009	8/1/2015
10 % Coupon	8%	109.2457593	8/1/2009	8/1/2015

A purchase of 1.2 10% bonds results in the same cash flow as two 6% bonds.

What is the value of a portfolio that is long 2 6% bonds and short 1.2 10% bonds?	195.68
What is the cash flow in period 6 for this portfolio?	800
What is the six-year spot rate given the portfolio value and cash flow?	26.45%

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Enter the values in blue colored cells

Chapter 3
Question 20

Student Name:	SOLUTION
Course Name:	
Student ID:	
Course Number:	

a) What are the discount factors for each date (that is, the present value of $1 paid in year *t*)?

b) Calculate the PV of the following Treasury notes assuming annual coupons:

i. 5 percent, two-year bond.

	Year	
	1	2
Spot rate	5%	5%
Cash Flow	50.00	1,050.00
PV	47.62	116.67
Total PV	164.29	

ii. 5 percent, five-year bond.

	Year				
	1	2	3	4	5
Spot rate	5.00%	5.40%	5.70%	5.90%	6.00%
Cash Flow	50	50	50	50	1050
PV	47.62	45.01	42.34	39.75	784.62
Total PV	959.34				

For help with Excel's SUM function

iii. 10 percent, five-year bond.

	Year				
	1	2	3	4	5
Spot rate	5.00%	5.40%	5.70%	5.90%	6.00%
Cash Flow	100	100	100	100	1100
PV	95.24	90.02	84.68	79.51	821.98
Total PV	1171.43				

For help with Excel's SUM function

c) Explain intuitively why the yield to maturity on the 10 percent bond is less than that on the 5 percent bond.

Use the values you found in sections ii and iii of part c to find the yield for each bond.

	Year					
	0.00	1.00	2.00	3.00	4.00	5.00
5% five year	(959.34)	50.00	50.00	50.00	50.00	1050.00
10% five-year	(1171.43)	100.00	100.00	100.00	100.00	1100.00

5% five year	5.96%
10% five-year	5.94%

For help with Excel's IRR function

Enter the values in blue colored cells

Chapter 3

Question 20

Why is the 10 percent bond's yield less?

The yield depends upon both the coupon payment and the spot rate at the time of the coupon payment. The 10% bond has a slightly greater proportion of its total payments coming earlier, when interest rates are low, than does the 5% bond. Thus, the yield of the 10% bond is slightly lower.

d) What should be the yield to maturity on a five-year zero-coupon bond?

The yield to maturity on a five-year zero coupon bond is the five-year spot rate, here 6.00%.

e) Show that the correct yield to maturity on a five-year annuity is 5.75 percent.

Find the annuity factor for each year and sum these value to calculate the price of a five year annuity.

	Year				
	1	2	3	4	5
Spot rate	5.00%	5.40%	5.70%	5.90%	6.00%
Annuity Factors:	0.952380952	0.900158068	0.846788669	0.795089759	0.747258173

Total Value of Annuity: 4.241675621

Use this value to find the yield to maturity of this annuity.

	Year					
	0	1	2	3	4	5
Cash Flows	-4.24	1.00	1.00	1.00	1.00	1.00
	5.75%					

For help with Excel's IRR function

f) Explain intuitively why the yield on the five-year Treasury notes described in part (c) must lie between the yield on a five-year zero-coupon bond and a five-year annuity.

The yield on the five-year Treasury note lies between the yield on a five-year zero-coupon bond and the yield on a 5-year annuity because the cash flows of the Treasury note lie between the cash flows of these other two financial instruments. That is, the annuity has fixed, equal payments, the zero-coupon bond has one payment at the end, and the bond's payments are a combination of these.

Enter the values in blue colored cells

Chapter 3
Question 21

Student Name:	SOLUTION
Course Name:	
Student ID:	
Course Number:	

4% coupon bond

Settlement date	1-Feb-09
Maturity date	1-Feb-15
Maturity in yrs	6
Coupon on Bond	0.04
Frequency	2
Face Value	100
YTM	0.02

Period	Ct	PV(Ct)	Proportion of Total Value	Proportion of Total Value x Time
0.5	2	1.98	0.018	0.009
1	2	1.96	0.018	0.018
1.5	2	1.94	0.017	0.026
2	2	1.92	0.017	0.035
2.5	2	1.90	0.017	0.043
3	2	1.88	0.017	0.051
3.5	2	1.87	0.017	0.059
4	2	1.85	0.017	0.066
4.5	2	1.83	0.016	0.074
5	2	1.81	0.016	0.081
5.5	2	1.79	0.016	0.089
6	102	90.57	0.814	4.882
	PV =	111.31	1.000	5.43 = Duration
				5.325% = Modified Duration

Strip (Zero-Coupon Bond)

Settlement date	1-Feb-09
Maturity date	1-Feb-15
Maturity in yrs	6
Face Value	100
YTM	2%
Frequency	2

PV =	190.293

Duration =	6.000
Modified Duration =	5.88%

Confirm that modified duration predicts the impact of a 1% change in interest rates on the bond prices.

4% coupon bond

YTM	Price	Change %
2.0%	111.3143358	
2.5%	108.31	0.0270
1.5%	114.294	0.0261
Change in Price =		5.31%

Note: Percentage change in price is equal to Modified Duration calculated for 4% coupon bond above.

Enter the values in blue colored cells

Chapter 3

Question 22

Student Name:	SOLUTION
Course Name:	
Student ID:	
Course Number:	

Use Table 3.3 below to answer each question.

TABLE 3.3 Calculating duration of a bond

(a)

Coupon rate	8%
Yield	2%

Date	Year	Cash Payment	Discount Factor at 2%	PV	Fraction of Total Value	Year times Fraction of Value
Aug-09	0.5	4	0.990147543	3.96059	0.029593849	0.014796924
Feb-10	1	4	0.980392157	3.921569	0.029302277	0.029302277
Aug-10	1.5	4	0.970732885	3.882932	0.029013577	0.043520366
Feb-11	2	4	0.961168781	3.844675	0.028727722	0.057455445
Aug-11	2.5	4	0.951698907	3.806796	0.028444684	0.071111709
Feb-12	3	4	0.942322335	3.769289	0.028164434	0.084493301
Aug-12	3.5	4	0.933038144	3.732153	0.027886945	0.097604307
Feb-13	4	4	0.923845426	3.695382	0.02761219	0.110448759
Aug-13	4.5	4	0.914743279	3.658973	0.027340142	0.123030639
Feb-14	5	4	0.90573081	3.622923	0.027070774	0.135353872
Aug-14	5.5	4	0.896807136	3.587229	0.026804061	0.147422334
Feb-15	6	104.00	0.887971382	92.34902	0.690039346	4.140236076
TOTAL				133.83	1.00	5.054776008

(b)

Coupon rate	11.25%
Yield	6%

Date	Year	Cash Payment	Discount Factor at 2%	PV	Fraction of Total Value	Year times Fraction of Value
Aug-09	0.5	5.625	0.971285862	5.463483	0.043144001	0.021572001
Feb-10	1	5.625	0.943396226	5.306604	0.041905159	0.041905159
Aug-10	1.5	5.625	0.916307417	5.154229	0.040701888	0.061052832
Feb-11	2	5.625	0.88999644	5.00623	0.039533168	0.079066337
Aug-11	2.5	5.625	0.86444096	4.86248	0.038398008	0.095995019
Feb-12	3	5.625	0.839619283	4.722858	0.037295442	0.111886326
Aug-12	3.5	5.625	0.815510339	4.587246	0.036224535	0.126785874
Feb-13	4	5.625	0.792093663	4.455527	0.035184379	0.140737517
Aug-13	4.5	5.625	0.769349377	4.32759	0.03417409	0.153783405
Feb-14	5	5.625	0.747258173	4.203327	0.033192811	0.165964053
Aug-14	5.5	5.625	0.725801299	4.082632	0.032239708	0.177318392
Feb-15	6	105.625	0.70496054	74.46146	0.588006811	3.528040868
TOTAL				126.63	1.00	4.704107781

	Original	Change (a)	Change (b)
Duration	4.83	5.05	4.70
Volatility	4.73529412	5.053765255	4.701287009

Change (a) Coupon of 8%

Duration and volatility rise

Change (b) Bond Yield of 6%

Duration and volatility fall

Enter the values in blue colored cells

Chapter 3
Question 23

Student Name:	SOLUTION
Course Name:	
Student ID:	
Course Number:	

Zero-Coupon Bond

settlement date	1-Feb-10	
maturity date	1-Feb-25	
maturity in yrs	15	
Face Value	100	
YTM (initial, new)	5%	10%
frequency	1	

PV =	52.912
Duration of Zero-Coupon Bond =	15.000

Perpetual Bond

Face Value	100	
YTM (initial, new)	5%	10%
frequency	1	

Duration of Perpetual Bond =	21.000

As shown above, the duration of the Perpetual Bond is longer than a 15-year Zero Coupn Bond.

What if the yield is 10%?

Zero-Coupon Bond

PV =	383.027
Duration of Zero-Coupon Bond =	15.000

Perpetual Bond

Duration of Perpetual Bond =	11.000

As shown above, the situation reverses when the Yield changes to 10%.
The duration of the Zero-Coupon Bond is higher than the Perpetual Bond.

Enter the values in blue colored cells

Chapter 3
Question 24

Student Name:	SOLUTION
Course Name:	
Student ID:	
Course Number:	

Use Excel's PRICE function to calculate the new value for each of the ten bonds you select.

Bond	Coupon Rate	Maturity (Years)	Current Price	Price with 1% > YTM	Change in Price
1				FUNCTION	FORMULA
2					
3					
4					
5					
6					
7					
8					
9					
10					

For help with the PRICE function

Explain your answer

In general, yield changes have the greatest impact on long-maturity, low-coupon bonds.

Enter the values in blue colored cells

Chapter 3

Question 25

Student Name:	SOLUTION
Course Name:	
Student ID:	
Course Number:	

	Year				
	1	**2**	**3**	**4**	**Bond Price**
Spot Rate	4.60%	4.40%	4.20%	4.00%	
Discount factor	0.956022945	0.917485063	0.883887197	0.854804	
Bond A (8% coupon)					
Payment	80	1080			
Present Value	76.48183556	990.8838684			1067.365704
Bond B (11% coupon)					
Payment	110	110	1110		
Present Value	105.1625239	100.923357	981.1147884		1187.200669
Bond C (6% coupon)					
Payment	60	60	60	1060	
Present Value	57.36137667	55.0491038	53.0332318	906.0924	1071.536155
Bond D					
Payment				1000	
Present Value				854.8042	854.8042

YTM
4.41%
4.22%
4.03%
15.49%

Enter the values in blue colored cells

Chapter 3

Question 30

Student Name:	SOLUTION
Course Name:	
Student ID:	
Course Number:	

Use Excel's PRICE function to calculate the new value for each of the ten bonds you select.

Bond	Coupon Rate	Maturity (Years)	YTM	Price
1				FUNCTION
2				
3				
4				
5				
6				
7				
8				
9				
10				

For help with the PRIC

:E function

Enter the values in blue colored cells

Chapter 3
Question 31

Student Name:	SOLUTION
Course Name:	
Student ID:	
Course Number:	

Follow each of the steps below to determine if implied forward rates or spot rates differ.

Calculate the implied spot rates for years 2 and 3 using the zero coupon bonds.

Bond	YTM
A	10.00%
G	9.50%

Find the implied four year rate using a combination of bonds B and D

Bond	0	1	2	3	4
B	-842.30	50	50	50	1050
D	-980.57	100	100	100	1100
Net	-704.03	0	0	0	1000

Calculate the implied four-year spot rate. 9.17%

Use the above calculated rates to determine the one-year spot rate from Bond C.

Bond	0	1	2	3	4
Cash Flow	-1065.28	120	120	120	1120
PV	-1065.28		100.08	90.156	788.5136

Calculate the implied one-year spot rate. 38.68%

Use all four implied spot rates to value bonds B, D, E, and F.

Bond	PV	PV(1)	PV(2)	PV(3)	PV(4)
B	854.5508333	36.05433333	41.7	37.565	739.2315
D	1005.071667	72.10866667	83.4	75.13	774.433
E	1074.194133	100.9521333	116.76	856.482	
F	938.2999204	50.47606667	83.93285372	803.891	

What arbitrage opportunities exist?

Since the present value using the implied spot rates does not equal the market price, arbitrage opportunities exist.

Enter the values in blue colored cells

Chapter 3
Question 34

Student Name:	SOLUTION
Course Name:	
Student ID:	
Course Number:	

Follow each of the steps below to answer these questions.

Calculate the implied one-year spot rate. 7.00%

Find a position that provides a payoff in only year two.

Bond	0	1	2
	-94.92	4	104
	-93.46	100	0
Net	2279.54	0	2600

Calculate the implied two-year spot rate. 6.80%

Compute the forward rate for year 2 6.60%

Find a position that provides a payoff in only year three.

Bond	0	1	2	3
	-103.64	8	8	108
	-14.49	8	8	
Net	-89.15	0.00	0.00	108.00

Calculate the implied three-year spot rate. 6.60%

Compute the forward rate for year 3 6.21%

Use these rates to find the price of the 4 percent coupon bond.

Bond	0	1	2	3
Cash Flow	0	40	40	1040
PV	930.93	37.384	35.06984615	858.4740741

Is there a profit opportunity here? If so, how would you take advantage of it?

The actual price of the bond ($950) is significantly greater than the price deduced using the spot and forward rates embedded in the prices of the other bonds ($931). Hence, a profit opportunity exists. In order to take advantage of this opportunity, one should sell the 4 percent coupon bond short and purchase the 8 percent coupon bond.