2

THE TIME VALUE OF MONEY: ALL DOLLARS ARE NOT CREATED EQUAL

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CHAPTER FOCUS

Financial planning must consider the element of time in attempting to achieve goals. Very often, important goals will not be achieved until many years in the future. Clearly, the cost of such goals depends upon a variety of factors, not the least important of which are inflation rates and rates of return on invested assets. This chapter focuses on how the time value of money is used in making future plans. Before it can do that, however, it must first focus on how time-value-of-money calculations are performed. Thus, the elements of compounding and discounting are explained with respect to both single payments and annuities.

CHAPTER ORGANIZATION

I.   Compounding (Finding Future Values)

      A.   Future Value of a Single Payment

           1.   The Importance of Additional Yield

           2.   The Importance of Additional Time

           3.   Time Value of Money Tables

           4.   The Rule of 72

           5.   Time-Value-of-Money Calculations on the Internet

      B.   Future Value of an Annuity

II.   Discounting (Finding Present Values)

      A.   Present Value of a Single Payment

           1.   Using a Present Value Table

           2.   Present Value Graph

      B.   Present Value of an Annuity

      C   More Applications of Future and Present Values

           1.   Finding an Unknown Interest Rate

BOXES

Box 2.1	Saving Money	Time Value of Money—Big Bore or Big Help?
Box 2.2	Young Adults	Focus on the Asset, Not on the Daily Savings
Box 2.3	Finance News	Retirement: A Distant Goal or a Distant Dream?

SUPPLEMENTARY MATERIALS

Your Financial Review 2.1   Simple Savings Plan

Form for working through a simple savings plan to accompany the Marie Wilson goal planning exercise. This sheet gives you a simple approach to determine how much you must save each year to achieve all of your goals. To determine the annual savings amount, get the total cost of all goals and divide by the number of years. Example: $100,000/40 = $2,500 a year.

Your Financial Review 2.2   Determining a Savings Amount

Form for determining an annual savings amount. Excel sheet contains formulas for calculating future values and annual savings amount. Estimating an annual savings amount requires estimates for inflation and what can be earned on investments. Remember that the annual savings amount is not necessarily the actual amount you plan to save each year in the future.

Your Financial Review 2.3   Determining a Savings Plan

Now you can estimate an amount to save each year. Use your estimate from Sheet 2.1 as a starter, but then go through trial and error until you reach an annual savings amount that is in line with your annual income. Given an interest rate the Excel worksheet computes interest earned and ending balance.

Your Financial Review 2.4   Time Value of Money Calculator

This spreadsheet provides templates for time value of money calculations. You can review the needed Excel functions for calculating the present value and future value of lump-sum payments and annuities.

INSTRUCTIONAL OBJECTIVES

1. *To gain a general understanding that monies received or paid out at different points in time have different values and that these values can change dramatically depending upon interest rate levels and the points in time when they are received or paid*

    Students should recognize the time and interest elements in receiving or paying cash. If time didn't matter, who of us would care if we received money today, or 20 years from now? Moreover, if rates of interest weren't important, who of us would care if we earned high or low rates on our investments?

2. *To understand the process of compounding and to be able to compute a future value of a single payment or of an annuity*

    Students should be able to make future value calculations using appropriate tables. Some instructors may introduce financial calculators, or illustrate calculations at various Internet sites.

3. *To understand the process of discounting and to be able to compute the present value of a single payment or of an annuity*

    Students should see that discounting is simply the reverse of compounding. Finding discounted values simply involves using different time-value-of-money tables—discount tables rather than compound tables.

4. *To be able to find approximate values for unknown interest rates or unknown holding periods when the values for other financial variables are known*

    Time-value-of-money problems in the real world present themselves in a variety of ways. Students need the skill to recognize known and unknown values and to solve for the latter.

5. *To recognize the importance of planning in the effort to achieve future goals*

    Planning is the process that binds future goals to the saving and investing efforts that are required to achieve them. In the absence of planning, one has no idea how much to save or how much must be earned on the savings as they accumulate through time.

6. *To be able to compute a required annual savings amount to meet future goals*

    This calculation is vital to the entire planning process because the annual savings amount is a key input to creating an annual budget. Planners must be able to make a savings calculation.

7. *To be able to construct a savings plan designed to show how savings will grow over time and be used to meet future goals*

    A savings plan is in a sense a check on the planner's calculation of the required annual savings amount. It shows in concrete fashion how funds grow and are then used to achieve the planner's goals.

PERSONAL FINANCE ON THE NET

There are numerous Internet sites with financial calculators. Many of these are dedicated to performing only one or several functions, such as a savings calculator that tells you how much to save each year to accumulate college education funds. The instructor might wish to show how these various calculators can be employed in developing the savings plan discussed immediately above. Instructors might wish also to point out the importance of understanding the processes underlying the calculations.

TEACHING SUGGESTIONS

1.   Start a discussion with a problem: You have won a lottery and can take $1,000,000 today versus receiving equal annual payments of $100,000 for the next 15 years, the first payment beginning immediately. Do this as a written assignment so that each student has his/her own answer without being influenced by class discussion. Then, have a discussion with students explaining how they arrived at their answer.

2.   Ask students to prepare a list of goals they hope to achieve over the next ten years. Ask them to set a priority to each goal and to determine the amount of money needed to achieve each goal.

3.   Students (see suggestion 2) often do not consider the impact of inflation. Ask them to determine the future cost of a goal if today's cost is $10,000 and you expect inflation to be 5% a year. Assume the goal is reached in ten years.

4.   Ask students to prepare a savings plan that has only two goals: buying a $20,000 car in five years and making a $20,000 down payment on a home in ten years. Ask them how much they will save each year to achieve the two goals. Tell them to ignore the time value of money.

5.   Referring to suggestion 4 above, ask them how their annual savings amount would change if they could earn 10% on all amounts saved. Alternatively, ask them how their annual savings amount would change if each item experienced 10% inflation each year.

FOLLOW-UP ON THE STEELES

1.   Your Financial Review Form 2.2 Determining a Savings Amount may be used to complete this assignment.

Present Value	FVIF		Future Value	Future Value/ FVIFA (see text)
$12,000	1.8509	8%	$ 22,211	$ 2,088
12,000	1.9900	8%	23,880	1,912
24,000	2.1589	8%	51,814	3,577
24,000	2.3316	8%	55,958	3,362
12,000	2.5182	8%	30,218	1,592
12,000	2.7196	8%	32,635	1,518
40,000	2.4066	5%	96,264	2,571
10,000	2.6533	5%	26,533	580
100,000	3.9201	5%	392,010	4,112
		Required annual savings		$21,312

The required annual savings increases by $5,110 ($21,312 minus $16,202). This is a substantial increase, which could place a heavy stress on the Steeles' future budgets.

2.

Future Value	FVIFA	Future Value/ FVIFA
$19,126	see text	$ 1,798
20,274	see text	1,624
42,979	see text	2,967
45,559	see text	2,737
24,146	see text	1,272
25,595	see text	1,191
68,096	45.599	1,493
18,061	57.275	315
228,790	190.690	1,199

Required annual savings $14,596

The required annual savings decreases by $1,606 ($16,202 minus $14,596).

3.   The point made is a reasonable one. There is greater risk in deferring needed savings to later years. Conversely, though, the larger savings amounts in the earlier years place a heavy strain on the Steeles current consumption relative to their current incomes. They must weigh the advantages and disadvantages of each savings plan. Their choice to increase savings over time as their incomes increase is likely the approach most families would take.

ANSWERS TO PROBLEMS AND REVIEW QUESTIONS

1.   Compounding is the process of accumulating value over time.

2.   Compound interest refers to the total interest earned on an investment, which includes interest received by virtue of the investment's interest rate (simple interest) and interest earned by reinvesting the simple interest.

3.   Simple interest over the five years is $2,000 (8% x $5,000 x 5). The FVIF for 8% and 5 years is 1.4693; so, the total accumulation is $7,346.50. Return of principal is $5,000, leaving $2,346.50 in compound interest. The difference of $346.50 is interest earned on reinvested simple interest.

4.   A future value is a sum of money received or paid in the future.

5.   This answer is false, since the accumulation is much greater. For example, at 8% the future value of 1$ is $4.661; at 10%, it is $6.728. The latter accumulation is over 44% more [($6.728/$4.661) – 1.0].

6.	There is no constant proportionality. Rather, the difference in accumulations grows exponentially. As the text shows, invested at 10%, the accumulation at 40 years is 6.73 times the amount accumulated at 20 years ($45,259/$6,728).

7.	Use the rule of 72 and arrive at 12% (72/6).

8.	Use the rule of 72 and arrive at 24 years (72/3).

9.	An annuity is a series of equal payments over time. A future value of an annuity is the accumulation resulting from the annuity payments, assumed invested at a specified interest rate and for a specified period of time.

10.	An ordinary annuity assumes the payment takes place at the end of a period (usually a year) whereas an annuity due assumes the payment takes place at the beginning of the period. An annuity due provides a larger value since there is one extra compounding period, or one less discounting period.

11.	Compounding refers to moving forward in time from the current to a future point, whereas discounting refers to regressing back in time from a future point to the current time.

12.	(a)	$500 x 3.2071 (FVIF 6%, 20 years) = $1,603.55
	(b)	$800 x 15.937 (FVIFA 10%, 10 years) = $12,749.60
	(c)	$300 x 259.05 (FVIFA 8%, 40 years) = $77,715 = FVOA. FVAD = 1.08 x $77,715 = $83,932.20.

13.	(a)	$6,000 x 0.0573 (PVIF 10%, 30 years) = $343.80
	(b)	$4,000 x 7.3601 (PVIFA 6%, 10 years) = $29,440.40
	(c)	$2,000 x 2.5771 (PVIFA 8%, 3 years) = $5,154.20

14.	Since the expected future value of $10,000 earning 8% over 10 years is $21,589. This investment returns only $20,000; so, it is earning less than 8% and should be rejected.

15.	Yes, the present value of the $1,000 annuity at 6% for 10 years is $7,360.10. At a cost of $6,000 the contract is undervalued by $1,360.10.

16.	$25,000/$15,000 = 1.6667 = FVIF (??%, 4 years).
	14%, 4 years	1.6890
	? ?%, 4 years	1.6667
	12%, 4 years	1.5735

	?? = 12% + {[(1.6667 − 1.5735)] / [(1.6890 − 1.5735)] x 2%}
	  = 12% + {[0.0932 / 0.1135] x 2%}
	  = 12% + {0.8211 x 2%} = 12% + 1.6423% = 13.6423%

17.	$500,000/$40,000 = 12.5000 = FVIF (14%, ?? years).

Go down the 14% column of the future value table to the values closest to 12.5. They are 12.055 (19 years) and 13.743 (20 years). You can guess the answer at 19 years, three months.

18. $60,000/$3,000 = 20.000 = FVIFA (??%, 12 years)
        9%, 12 years   20.140
        ??%, 12 years   20.000
        8%, 12 years   18.997

        ?? = 8% + {[(20.000 − 18.997]/[(20.140 − 18.997)] x 1%}
           = 8% + {[1.003/1.143] x 1%}
           = 8% + {0.8775 x 1% = 8% + 0.8775% = 8.8775%

19.   Find the present value of the $20,000 annuity invested for 5 years at 6%.

            PV = (PVIFA) x $20,000 = 4.2124 X $20,000 = $84,248

        Since $84,248 is considerably greater than $60,000, take the extended pay out.

20.   A goal can be described as a dream with dollar amounts and due dates attached. In other words, goals can start as dreams, but at some point the dream must be made realistic. When this action takes place, it can become a goal.

21.   First, identify particular goals; second, set future years when goals are to be achieved; and third, establish dollar amounts for each goal.

22.   Inflation enters the goal planning process as an adjustment we must make to future amounts necessary to achieve future goals. The first step is to estimate the cost of a goal if attained today. The second step is to estimate the inflation rate affecting this goal over the period until its attainment. The third step is to calculate the future value; that is, its future cost.

23.   A required annual savings amount is the amount we must save each year to achieve our array of goals at the times we wish to achieve them. It takes into consideration estimated inflation rates and estimated rates of return on amounts saved and invested.

24.   A savings schedule is simply a worksheet that shows the flow of funds in terms of acquiring future funds amounts, which are then spent on future goals. It has an important added function of "proofing" the required savings amount. If the flow of funds shows significant negative balances at future points in time, we must go back and adjust the annual savings amounts.

25.   Clearly, the most important reason to monitor a savings plan is that adjustments will be necessary whenever estimated inflation rates and/or earnings rates change significantly. Savings plans are not static documents; they frequently require updating.

Case 2.1   Judy Shipley Plans for the Future

1.   Your Financial Review Form 2.2 Determining a Savings Amount may be used to complete this assignment. Determine required annual savings amounts:

Goals	Year	Amount Needed	Inflation Rate	Fv of $1	Adjusted Amount	FV of $1 Annuity	Required Savings
1. Trip to Asia	2	$ 20,000	4%	1.0816	$ 21,632	2.0600	$ 10,501
2. Purchase a Lexus	4	60,000	4%	1.1699	70,194	4.3746	16,046
3. Buy condo	6	80,000	8%	1.5869	126,952	6.9753	18,200
4. Furnish condo	8	100,000	4%	1.3686	136,860	9.8975	13,828
					Total		$58,575

2.   Prepare a savings plan.
        Here is one possible savings plan. Form 2.3 Determining a Savings Plan can be used to analyze various other savings plans.

Interest rate = 6%

Year		Deposit		Interest Earned		Withdrawals		Ending Balance
1	$	30,000.00	$	-	$	-	$	30,000.00
2	$	35,000.00		1,800.00		21,632.00		45,168.00
3	$	35,000.00		2,710.08		-		82,878.08
4	$	40,000.00		4,972.68		70,194.00		57,656.76
5	$	40,000.00		3,459.41		-		101,116.17
6	$	50,000.00		6,066.97		126,952.00		30,231.14
7	$	50,000.00		1,813.87		-		82,045.01
8	$	50,000.00		4,922.70		136,860.00		107.71

3. Judy's starting salary of $110,000 is very generous. Even with taxes and 401(K) contributions of 40%, she has $66,000 remaining. However, her first-year savings requirement is $30,000, which takes a good portion of her residual income, leaving very little for all other living expenses. Let's simplify and say that she must average about $40,000 a year in savings to achieve her goals. That amount is about 36% of her income, and it is very unlikely she can come close to saving that large a portion of income. Judy needs to prepare a more realistic budget as the next step in the planning process.

Case 2.2   The Rohrbachs Plan for Retirement

1. Social Security in 30 years will be $18,000 x 2.4273 = $43,691. (FVIF 3%, 30 years = 2.4273.)

   Income need in 30 years will be $60,000 x 2.4273 = $145,638.

   Shortfall = $145,638 - $43,691 = $101,947.

2. The amount in the bond fund = $50,000 x 4.3219 = $216,095 (FVIF 5%, 30 years = 4.3219.)

3. Value of the business = $100,000 x 17.449 = $1,744,900. (FVIF 10%, 30 years = 17.449.)

4. After selling the business, the Rohrbachs will have $1,960,995 in the bond fund. Earning 5% provides an annual income of $98,050 (0.05 x $1,960,995). This is close, but misses the shortfall amount by $3,897 ($101,947 - $98,050). In effect, they need $77,945 more in the account ($3,897/0.05). We can determine the added investment requirement each year.

   FV = (FVIFA 12%, 30 years) x Annual payment

Annual payment = FV/(FVIFA 12%, 30 years)

Annual payment = $77,945/ 241.33 = $322.98

So, the Rohrbachs must save about $323 a year, which they will invest to earn 12% annually.

Marie Wilson-Goal Planning Exercise

Specifying amounts for particular goals along with the number of years to achieve them allows us to undertake a quantitative analysis of her goal planning activities. Forms 2.1, 2.2 and 2.3 are available on the author's Web site.

Form 2.1
Simple Savings Plan

Goals in Action	Year	Amount Required for Goals	Planned Savings Amount	Savings Account Balance
	1		2,500.00	2,500.00
	2		2,500.00	5,000.00
	3		2,500.00	7,500.00
	4		2,500.00	10,000.00
Jake's golf clinic	5	4,000.00	2,500.00	8,500.00
	6		2,500.00	11,000.00
	7		2,500.00	13,500.00
	8		2,500.00	16,000.00
	9		2,500.00	18,500.00
Condo down payment	10	14,000.00	2,500.00	7,000.00
	11		2,500.00	9,500.00
	12		2,500.00	12,000.00
	13		2,500.00	14,500.00
Jake's college costs	14	8,000.00	2,500.00	9,000.00
Jake's college costs	15	8,000.00	2,500.00	3,500.00
Jake's college costs	16	8,000.00	2,500.00	(2,000.00)
Jake's college costs	17	8,000.00	2,500.00	(7,500.00)
	18		2,500.00	(5,000.00)
	19		2,500.00	(2,500.00)
	20		2,500.00	-
	21		2,500.00	2,500.00
	22		2,500.00	5,000.00
	23		2,500.00	7,500.00
	24		2,500.00	10,000.00
	25		2,500.00	12,500.00
	26		2,500.00	15,000.00
	27		2,500.00	17,500.00
	28		2,500.00	20,000.00

	29	2,500.00	22,500.00
	30	2,500.00	25,000.00
	31	2,500.00	27,500.00
	32	2,500.00	30,000.00
	33	2,500.00	32,500.00
	34	2,500.00	35,000.00
	35	2,500.00	37,500.00
	36	2,500.00	40,000.00
	37	2,500.00	42,500.00
	38	2,500.00	45,000.00
	39	2,500.00	47,500.00
Retirement fund	40	2,500.00	50,000.00

Form 2.1 shows a simple savings plan, which serves as an overall guide to future money flows. Actually, if we assume that the earning rate would be the same as the inflation rate (a pretty crude assumption), this simple plan may be all one needs. But, in the interest of greater accuracy, we move on to consider annual required savings amounts, given a 3% inflation rate and a 6% earning rate.

Form 2.2 Determining a Savings Amount

(1) Goal	(2) Years	(3) Amount Required	(4) FV of $1 3%Rate	(5) Inflation Adjusted	(6) FV of $1 Annuity 6% Rate	(7) Required Amount
1. Golf clinic	5	$ 4,000	1.1593	$ 4,637	5.6371	$ 823
2. Condo down payment	10	14,000	1.3439	18,815	13.1800	1,428
3. 1st year college	14	8,000	1.5126	12,101	21.0150	576
4. 2nd year college	15	8,000	1.5580	12,464	23.2760	535
5. 3rd year college	16	8,000	1.6047	12,838	25.6720	500
6. 4th year college	17	8,000	1.6528	13,222	28.2120	469
7. Retirement fund	40	50,000	3.2620	163,100	154.7600	1,054
					Total	$ 5,384

As Form 2.2 shows, the required annual savings amount is $5,384. Of course, as discussed in the text, Marie would not begin with such a large amount, since she anticipates her income to rise in the future. So, she tries $2,000 as a starting figure, which is increased $500 each five-year segment of time. However, as Form 2.3 indicates, this approach will leave some gaps (negative numbers). By the end of year 17, Marie will have borrowed $22,781 to meet her goals; and, she'll be $46,193 short at retirement. Unfortunately, Marie must try again. Instructors can suggest that students access the Winger Web site and use the forms supplied in the Your Financial Plan section.

Form 2.3 Determining a Savings Plan

Year	Goal	Deposit	Interest Earned	Withdrawals	Ending Balance
1		$ 2,000	$ -	$ -	$ 2,000
2		2,000	120	-	4,120
3		2,000	247	-	6,367
4		2,000	382	-	8,749
5	Camp	2,000	525	4,637	6,637
6		2,500	398	-	9,535
7		2,500	572	-	12,608
8		2,500	756	-	15,864
9		2,500	952	-	19,316
10	Condo	2,500	1,159	18,815	4,160
11		3,000	250	-	7,409
12		3,000	445	-	10,854
13		3,000	651	-	14,505
14	1st year	3,000	870	12,101	6,274
15	2nd year	3,000	376	12,464	(2,813)
16	3rd year	3,500	(169)	12,838	(12,320)
17	4th year	3,500	(739)	13,222	(22,781)
18		3,500	(1,367)	-	(20,648)
19		3,500	(1,239)	-	(18,387)
20		3,500	(1,103)	-	(15,990)
21		4,000	(959)	-	(12,949)
22		4,000	(777)	-	(9,726)
23		4,000	(584)	-	(6,310)
24		4,000	(379)	-	(2,688)
25		4,000	(161)	-	1,150
26		4,500	69	-	5,719
27		4,500	343	-	10,562
28		4,500	634	-	15,696
29		4,500	942	-	21,138
30		4,500	1,268	-	26,906
31		5,000	1,614	-	33,521
32		5,000	2,011	-	40,532

33	5,000	2,432	-	47,964
34	5,000	2,878	-	55,841
35	5,000	3,350	-	64,192
36	5,500	3,852	-	73,543
37	5,500	4,413	-	83,456
38	5,500	5,007	-	93,963
39	5,500	5,638	-	105,101
40 Retire	5,500	6,306	163,100	(46,193)